A p Theory of Government Debt, Taxes, and Inflation

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Conference in honor of Andrew Abel

Our Focus

- A Barro (1979) zero drift $\mu_b = 0$ prescription for government debt
- Forces that can reproduce that prescription and that can disrupt it

What We Don't Do

- Exploit analogy with Hall (1978) consumption smoothing model
  - For that see `https://python-advanced.quantecon.org/smoothing_tax.html`
- Study how Ramsey planner manipulates endogenous asset prices
  - For that see Lucas and Stokey (1983), Aiyagari et al. (2002), Farhi (2009), Bhandari et al. (2017, 2021), or various quantecon lectures
- Study dynamic Mirlees a la Golosov and others

What We Do

- Some back-of-the-envelope calculations
- Reverse engineer assertions of Barro (1979)
- Assume some indirect cost or utility or value functions and derive others
- Exploit analogy with a q theory of investment
- Explore some perturbations

What We Assume

In spirit of Barro (1979)

- exogenous asset prices
- hard-wired government motive to smooth tax rates across states
- a less than fully rationalized objective function

What We Study

- How dynamics of tax collections and debt/gdp are affected by these perturbations of Barro (1979)
  - Randomness
  - Default option or "balanced budget amendment"
  - Extra impatience

Possible Substantive Questions

- Cost of balanced budget amendment
- What is maximum sustainable debt-to-GDP ratio for a "small open economy"?
- How long should it take to reach that debt capacity?
- How does cost of servicing government debt depend on debt-to-GDP ratio?
- Should a government immediately issue more debt when $r < g$ (Blanchard, 2019)?

Two Barro Models

1974: Ricardian equivalence (indeterminacy of B and tax rate)

$$\int_0^\infty e^{-rt}(Y_t - \mathcal{T}_t)\,dt$$

$$B_0 \leq \underbrace{\int_0^\infty e^{-rt}(\mathcal{T}_t - \Gamma_t)\,dt}_{\text{PV of primary surpluses}}$$

1979: Zero drift debt-GDP, constant tax rate

$$\int_0^\infty e^{-rt}(Y_t - (\mathcal{T}_t + C_t))\,dt$$

Sources of Curvature

- Assumed tax distortion cost functions $c(\cdot), \hat{c}(\cdot)$
- Debt default option, a.k.a primary budget balance option

Models

Barro 1979: zero drift debt/GDP, constant tax rate

$$\int_0^\infty e^{-rt}(Y_t - (\mathcal{T}_t + C_t))\,dt$$

$$B_0 \leq \underbrace{\int_0^\infty e^{-rt}(\mathcal{T}_t - \Gamma_t)\,dt}_{\text{PV of primary surpluses}}$$

This paper: zero drift debt/GDP ? ... constant tax rate ?

$$\mathbb{E}\int_0^\infty e^{-\zeta t}\Lambda_t\left[(1-\mathbf{1}_t^{\mathcal{D}})[dU_t + (Y_t - (\mathcal{T}_t + C_t))\,dt] + \mathbf{1}_t^{\mathcal{D}}(\widehat{Y}_t - (\widehat{T}_t + \widehat{C}_t))dt\right]$$

$$B_0 \leq \underbrace{\mathbb{E}\int_0^{T^{\mathcal{D}}} \Lambda_t[(\mathcal{T}_t - \Gamma_t)\,dt - dU_t]}_{\text{PV of primary surpluses}}$$

Models

- Minimalist stochastic continuous-time models with

  A. Arrow securities and risk premia

  B. Barro (1979) Tax distortion costs

  C. Credit limits: default option

  D. Discounting for impatience

Models

- Minimalist stochastic continuous-time models with

A. Arrow securities and risk premia

- Black and Scholes (1973), Merton (1971, 1973), Harrison and Kreps (1979), Lucas (1978), Shiller (1994), Bohn (1995), and Jiang, Lustig, Van Nieuwerburgh, and Xiaolan (2022)

B. Barro Tax distortion costs

- Barro (1979)

C. Credit limits: default option

- Eaton and Gersovitz (1981), Thomas and Worrall (1988), Kehoe and Levine (1993), Kocherlakota (1996), Alvarez and Jermann (2000, 2001), Ai and Li (2015), Bolton, Wang, and Yang (2019), Rebelo, Wang, and Yang (2022)

D. Discounting for impatience

- Amador (2006), DeMarzo and Sannikov (2006), DeMarzo and Fishman (2007), Aguiar and Amador (2021)

A: GDP AND SDF

- No-default regime GDP Y_t dynamics:

$$dY_t/Y_t = g dt + \psi_h d\mathcal{Z}_t^h + \psi_m d\mathcal{Z}_t^m$$

- Risk management:
  - Idiosyncratic Brownian $d\mathcal{Z}_t^h$ and exposure: $-\Xi_t^h \cdot d\mathcal{Z}_t^h$
  - Systematic Brownian $d\mathcal{Z}_t^m$ and exposure: $-\Xi_t^m \cdot (\eta dt + d\mathcal{Z}_t^m)$
- A risk-free bond pays interest rate r
- **Arrow securities** are priced by SDF process Λ_t :

$$d\Lambda_t/\Lambda_t = -r dt - \eta d\mathcal{Z}_t^m\,, \quad \Lambda_0 = 1$$

Harrison and Kreps (1979), Black and Scholes (1973), Merton (1973), Lucas (1978), Duffie and Huang (1985)

Risk Management

- government insures systematic risk at price $\lambda = \psi_m \eta = \rho \sigma_Y \eta$
- government insures idiosyncratic risk at price 0

B: Gov't Spending, Taxes, & Debt

- Exogenous government spending: $\Gamma_t = \gamma Y_t$
- **Barro**'s tax distortion cost: $C_t = c(\tau_t) Y_t; \quad \tau_t = \mathcal{T}_t / Y_t$
- Budget constraint

$$B_0 \leq \underbrace{\mathbb{E} \int_0^{T^{\mathcal{D}}} \Lambda_t [(\mathcal{T}_t - \Gamma_t)\, dt - dU_t]}_{\text{PV of primary surpluses}} \qquad (1)$$

  dU_t is a transfer payment to household financed by **jump** in debt; B_0 is initial debt

- Debt dynamics

  $$dB_t = (rB_t + (\Gamma_t - \mathcal{T}_t))\, dt + dU_t - \Xi_t^h d\mathcal{Z}_t^h - \Xi_t^m (\eta dt + d\mathcal{Z}_t^m) \qquad (2)$$

- Upper bound on tax-GDP ratio (Keynes, 1923): $\tau_t \leq \overline{\tau} = .5$

C: Credit Limit

Default Consequences

- GDP hit: $\widehat{Y}_t = \alpha Y_t < Y_t$
- Bigger tax distortions:

$$\widehat{c}(\cdot) = \widehat{C}_t / \widehat{Y}_t = \kappa \, c(\cdot) > c(\cdot) = C_t / Y_t \qquad \kappa \geq 1$$

- Zero Primary Deficits: $\widehat{\mathcal{T}}_t = \Gamma_t = \gamma_t Y_t$
- Same upper bound on tax/GDP ratio (Keynes, 1923): $\widehat{\tau}_t \leq \overline{\tau}$

Default regime value function $\widehat{P}(\widehat{Y}_t)$ appears in **Credit Constraints**

D: Debt, Taxes, and Risk Management

- **Discounted-for-impatience SDF** of household: $e^{-\zeta t}\Lambda_t$
- Among admissible paths of debt jumps (dU_t), tax rates (τ_t and $\widehat{\tau}_t$), and idiosyncratic and systematic risk hedging demands (Π_t^h and Π_t^m), value function $P(B_0, Y_0)$ maximizes

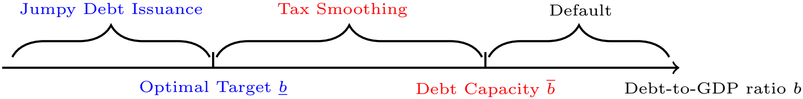
$$\mathbb{E}\int_0^\infty e^{-\zeta t}\Lambda_t \left[(1-\mathbf{1}_t^{\mathcal{D}})[dU_t + (Y_t - (\mathcal{T}_t + C_t))\,dt] + \mathbf{1}_t^{\mathcal{D}}(\widehat{Y}_t - (\widehat{\mathcal{T}}_t + \widehat{C}_t))dt\right]$$

subject to

- $$B_0 \le \underbrace{\mathbb{E}\int_0^{T^{\mathcal{D}}} \Lambda_t[(\mathcal{T}_t - \Gamma_t)\,dt - dU_t]}_{\text{PV of primary surpluses}}$$
- default-option-induced credit constraint for all $t \ge 0$: $P(B_t, Y_t) \ge \widehat{P}(\widehat{Y}_t)$, where $\widehat{P}(\widehat{Y}_t)$ is value function in autarky

3 Regions

- **Three regions** of debt-to-GDP ratio $b_t = B_t/Y_t$:



- In tax-smoothing (Barro) region
  - $\dot{b}_t \equiv \mu^b(b_t) = \underbrace{\gamma - \tau(b_t)}_{\text{primary deficit}} + \underbrace{r \times b_t}_{\text{interest payment}} - \underbrace{g \times b_t}_{\text{growth}} + \underbrace{\lambda \times b_t}_{\text{hedging cost}}$

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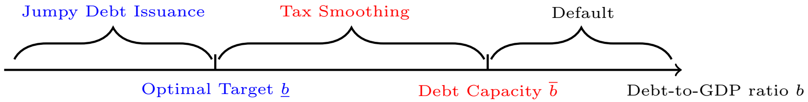

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  - scaled household value function $p(b_t) = P(B_t, Y_t)/Y_t$ solves

$$\Big[\underbrace{(r + \lambda + \zeta)}_{\text{discount rate}} - g\Big] p(b) = \underbrace{1 - \tau(b) - c(\tau(b))}_{\text{household net income}} + \underbrace{[(r + \lambda - g)b + \gamma - \tau(b)]}_{\text{drift of } b\text{: } \mu^b(b)} \cdot p'(b)$$

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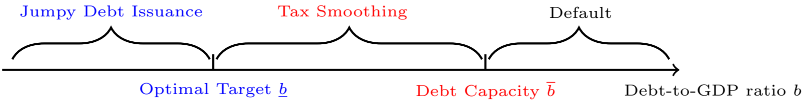

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  - first-order condition for taxes: $1 + c'(\tau(b)) = -p'(b)$

HJB Equations (interior)

- HJB for household value function $P(B,Y)$:

$$(\zeta + r)P(B,Y) = \max_{\mathcal{T},\,\Xi^h,\,\Xi^m} \left(Y - \mathcal{T} - C(\mathcal{T},Y)\right) + \left[rB + \Gamma - \mathcal{T}\,\right] P_B(B,Y) + \frac{(\Xi^h)^2 + (\Xi^m)^2}{2} P_{BB}(B,Y) + (g-\lambda) Y P_Y(B,Y) + \frac{\sigma_Y^2 Y^2}{2} P_{YY}(B,Y) - \left(\psi_h \Xi^h + \psi_m \Xi^m\right) Y P_{BY}(B,Y)$$

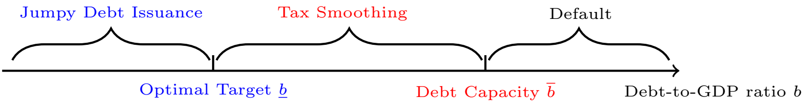
  - GDP growth volatility: $\sigma_Y^2 = \psi_m^2 + \psi_h^2$
  - GDP growth risk premium: $\lambda = \psi_m \eta = \rho \sigma_Y \eta$

- HJB for scaled value function $p(b) = \frac{P(B,Y)}{Y}$, where $b = \frac{B}{Y}$:

$$[\zeta + (r + \lambda - g)]\, p(b) = \max_{\tau} 1 - \tau - c(\tau) + [(r + \lambda - g)b + \gamma - \tau]\, p'(b)$$

2 + 2 + 2 Equations

- **Three regions** of debt-to-GDP ratio $b_t = B_t/Y_t$:



- 'Debt is cheap' region ($b < \underline{b}$)
  - $p'(\underline{b}) = -1$ and $p''(\underline{b}) = 0$ if $\underline{b} > 0$; otherwise $\underline{b} = 0$

- Off-equilibrium default region ($b > \overline{b}$):
  - zero drift: $\mu^b(\overline{b}) = 0 \iff \overline{b} = \frac{\tau(\overline{b}) - \gamma}{r + \lambda - g}$
  - $p(\overline{b}) = \alpha \widehat{p}$ or $\tau(\overline{b}) = \overline{\tau}$, whichever is tighter

Etymology of 'p theory'

q **theory of investment**	p **theory of debt and taxes**
capital stock	government debt
capital adjustment costs	tax deadweight costs
marginal q = MC of investing	$-$ marginal p = MC of taxing
MM holds	limited commitment
marginal $q \geq 1$	marginal $p \geq 1$

- q theory of investment (Lucas and Prescott, 1971; Hayashi, 1982; Abel and Eberly, 1994)

Quantitative Illustrations

- FRED: US annual debt-GDP ratio from 2000 to 2020
- Parameters to estimate: $\Omega = \{\zeta, \alpha, \varphi_\tau\}$
- Debt-to-GDP ratio with measurement errors ε:

$$b_{t_{i+1}} = b_{t_i} + \mu^b(b_{t_i}; \Omega)(t_{i+1} - t_i) + \varepsilon_{i+1}\,, \quad i = 1, 2, \cdots\,,$$

where $\mu^b(b_{t_i}; \Omega) = \dot{b}_t$ is model-implied deterministic debt-GDP drift

- Estimating Ω:

$$\widehat{\Omega} = \arg\max_{\Omega} \quad \sum_{i=1}^{20} \ln h\big(\widehat{b}_{t_{i+1}} - \widehat{b}_{t_i} + \mu^b(\widehat{b}_{t_i}; \Omega)\big)\,.$$

where $h(\,\cdot\,)$ is density function of measurement error ε

Parameters

Parameter	Symbol	Value
A. Borrowed parameters		
risk-free rate	r	1%
risk premium (Jiang et al., 2022)	λ	3%
average output growth rate	g	2%
government spending-GDP ratio	γ	20%
default deadweight loss	κ	1
B. Calibrated parameters		
impatience	ζ	0.1%
output recovery in the default regime	α	0.94
tax deadweight loss	φ_τ	2.8

- Quadratic deadweight loss function (Barro, 1979): $c(\tau) = \frac{\varphi_\tau}{2}\tau^2$
- Scaled total value: $s(b_t) = \frac{S(B_t, Y_t)}{Y_t} = p(b_t) + b_t$

Total value

Scaled total value:

$$s(b_t) = \frac{S(B_t, Y_t)}{Y_t} = p(b_t) + b_t$$

Ricardian Equivalence (A)

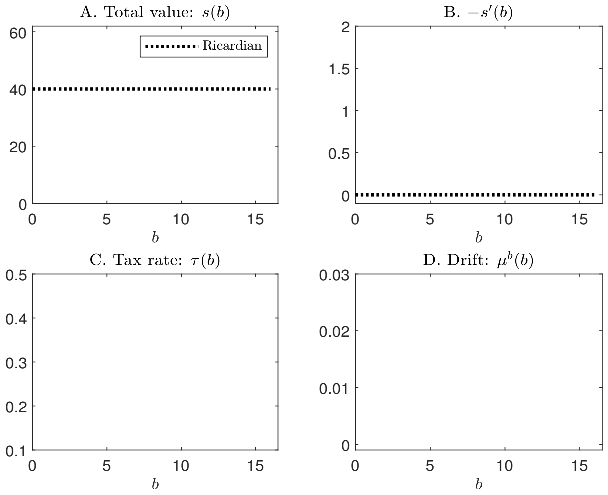


Figure: $r = 1\%$, $\lambda = 3\%$, $g = 2\%$, and $\gamma = 20\%$.

Stochastic Barro (A + B)

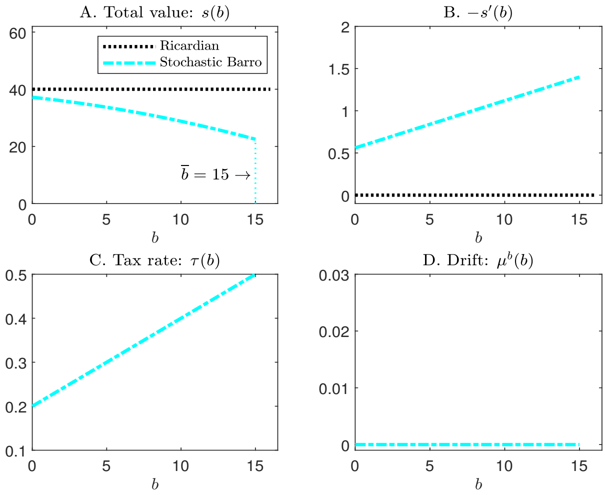


Figure: $r = 1\%$, $\lambda = 3\%$, $g = 2\%$, $\gamma = 20\%$, and $\varphi_\tau = 2.8$.

Credit Constraint (A + B + C)

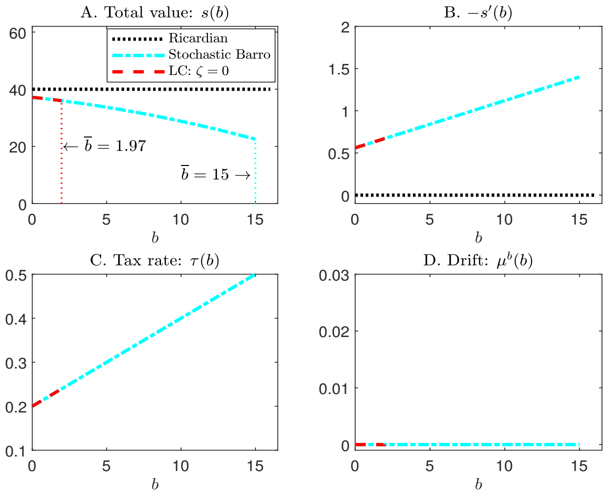


Figure: $r = 1\%$, $\lambda = 3\%$, $g = 2\%$, $\gamma = 20\%$, $\varphi = 2.8$, $\alpha = 0.94$ and $\kappa = 1$.

Full Model (A + B + C + D)

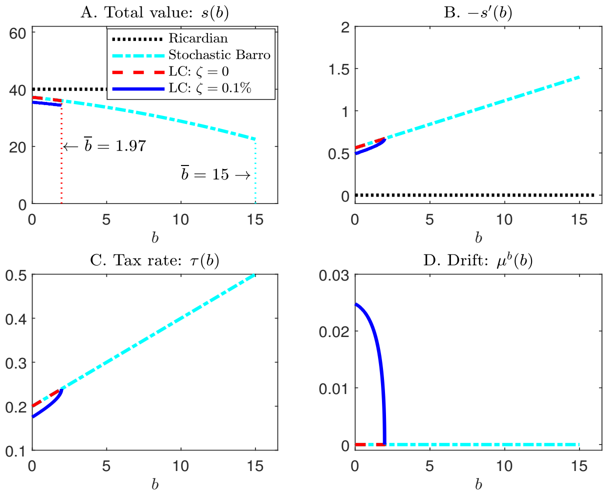


Figure: $r = 1\%$, $\lambda = 3\%$, $g = 2\%$, $\gamma = 20\%$, $\varphi_\tau = 2.8$, $\alpha = 0.94$, $\kappa = 1$, and $\zeta = 0.1\%$.

Models ABC and ABCD

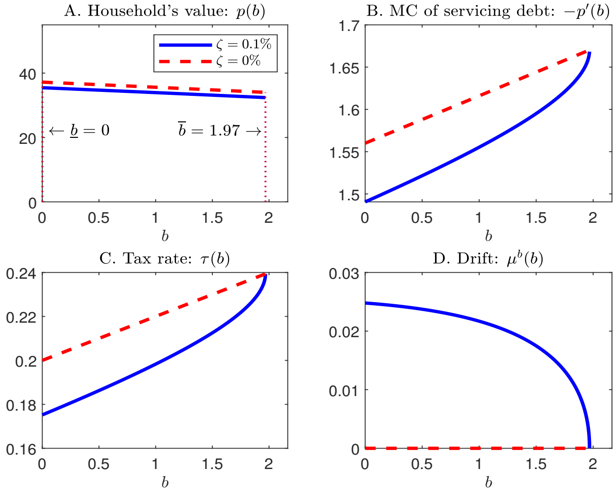
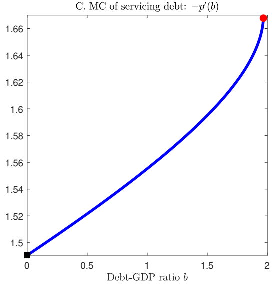
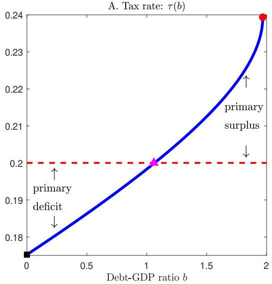


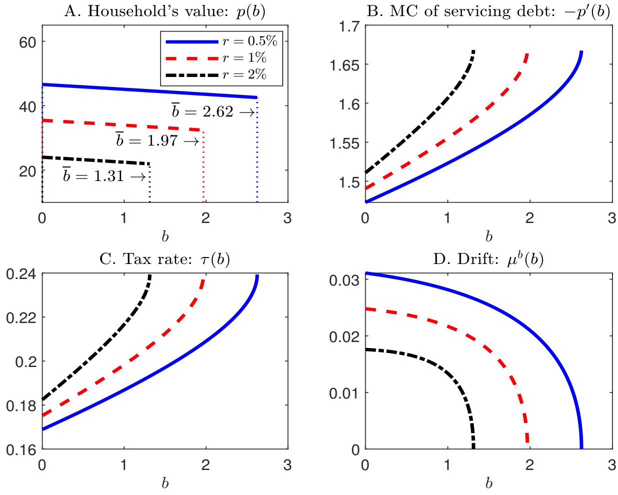
Figure: $r = 1\%$, $\lambda = 3\%$, $g = 2\%$, $\gamma = 20\%$, $\varphi_\tau = 2.8$, $\alpha = 0.94$, $\kappa = 1$, and $\zeta = 0.1\%$.

Marginal Cost $-p'(b)$ of Servicing Debt and Tax Rate $\tau(b)$

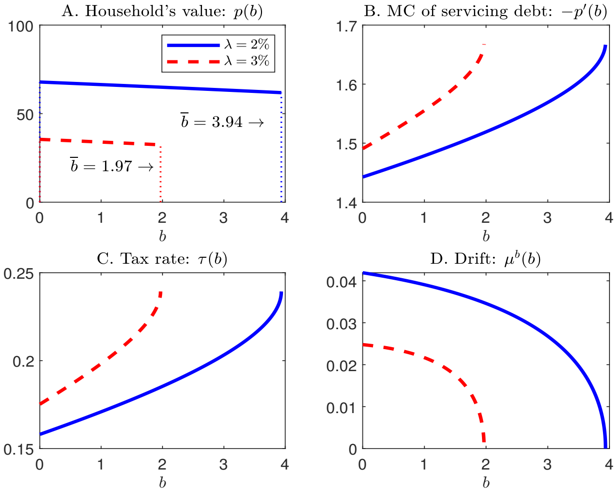




Alternative interest rates r



Alternative risk premia λ



Time to reach debt capacity alternative interest rates r

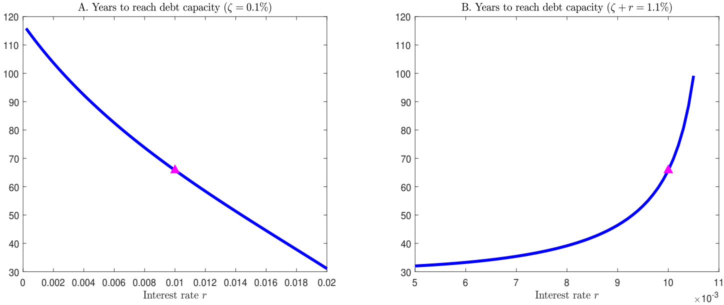


Figure: For both panels, $b_0 = 108.1\%$. In panel A, the impatience parameter is fixed at $\zeta = 0.1\%$. In panel B, household discount rate is fixed at $\zeta + r = 1.1\%$.

Dual: Govt Value Maximization

- Government value

$$F(W_t, Y_t) = \max_{\mathcal{T}, \Phi^m, \Phi^h, J, T^{\mathcal{D}}} \quad \mathbb{E}_t \int_t^{T^{\mathcal{D}}} \frac{\Lambda_s}{\Lambda_t} \underbrace{\left[(\mathcal{T}_s - \Gamma_s)\, ds - dJ_s \right]}_{\text{primary surplus}} .$$

- Continuation value W_t promised to household has dynamics

$$dW_t = \left[(\zeta + r) W_t - (Y_t - \mathcal{T}_t - C_t) - \eta \Phi_t^m \right] dt - dJ_t - \Phi_t^h d\mathcal{Z}_t^h - \Phi_t^m d\mathcal{Z}_t^m$$

- Scaled household continuation value $w_t = W_t / Y_t$ and scaled government value:

$$f(w_t) = F(W_t, Y_t) / Y_t$$

HJB and Valuation Equations

- HJB for government value function $F(W, Y)$:

$$rF(W,Y) = \max_{\mathcal{T},\Phi^h,\Phi^m} \quad (\mathcal{T} - \Gamma) + ((\zeta + r)W - (Y - \mathcal{T} - C(\mathcal{T}, Y)))\, F_W$$

$$+ (g - \rho\eta\sigma_Y)YF_Y + \frac{\sigma_Y^2 Y^2 F_{YY}}{2}$$

$$+ \frac{((\Phi^h)^2 + (\Phi^m)^2)F_{WW}}{2} - \left(\psi_h \Phi^h + \psi_m \Phi^m\right) YF_{WY}$$

- HJB equation for scaled government value function $f(w) = \frac{F(W,Y)}{Y}$, where tax rate is $\theta = \frac{\mathcal{T}}{Y}$:

$$(r + \lambda - g)f(w) = \max_{\theta} \theta - \gamma + [(\zeta + r + \lambda - g)w - (1 - \theta - c(\theta))]\, f'(w)$$

Prospects for b_t

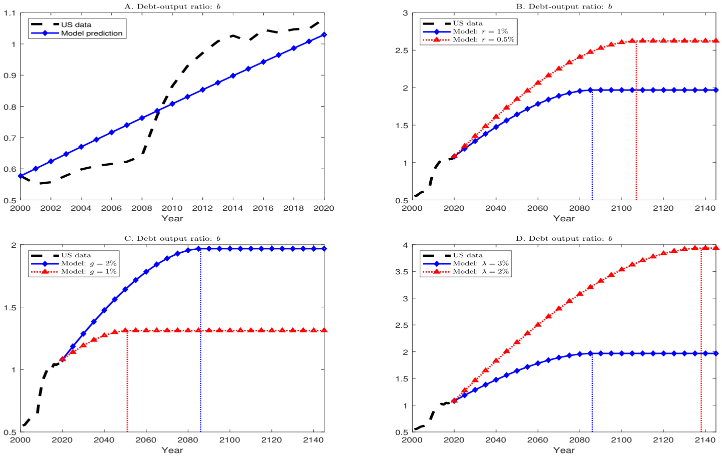


Figure: Steady-state debt capacity for our baseline calulation (blue lines in panels B, C, and D): $\bar{b} = 197\%$ within plausible range of $150 - 300\%$. US debt-GDP ratios in 2000 and 2020 were 57.5% and 108.1%, respectively.

Inflation Tax

Monetary Policy

- Quantity Theory of Money:

$$M_t v = \mathcal{P}_t Y_t$$

- Government chooses inflation process $\{\pi_t\}$:

$$\frac{d\mathcal{P}_t}{\mathcal{P}_t} = \pi_t dt$$

- Seigniorage flow:

$$\frac{dM_t}{\mathcal{P}_t} = \frac{Y_t}{v}\left((\pi_t + g)dt + \sigma_Y d\mathcal{Z}_t^Y\right)$$

- Deadweight loss:

$$C_t = C(\tau_t, \pi_t, Y_t) = c(\tau_t, \pi_t)Y_t$$

Autarky

- Output in autarky $\widehat{Y}_t = \alpha Y_t$
- Budget constraints in autarky:

$$\widehat{\Gamma}_t dt = \widehat{\mathcal{T}}_t dt + \frac{d\widehat{M_t}}{\widehat{\mathcal{P}}_t}$$

- Distortion costs of default: $\widehat{C}_t = \widehat{c}(\widehat{\tau}_t, \widehat{\pi}_t)\widehat{Y}_t$, where

$$\widehat{c}(\cdot\,,\cdot) = \kappa\, c(\cdot\,,\cdot)\,, \qquad \kappa \geq 1$$

- Same tax rate constraint (Keynes, 1923) as in no-default regime: $\widehat{\tau}_t \leq \overline{\tau}$

Optimal Government Plan

- Optimal tax and inflation

$$c_\tau(\tau,\pi) = -s'(b) = -p'(b) - 1\,, \tag{3}$$
$$c_\pi(\tau,\pi) = -s'(b)/v = (-p'(b) - 1)/v \tag{4}$$

- Debt-to-GDP dynamics
  - Recall without inflation:

$$\dot{b}_t \equiv \mu^b(b_t) = \underbrace{\gamma - \tau(b_t)}_{\text{primary deficit}} + \underbrace{r \times b_t}_{\text{interest payment}} - \underbrace{g \times b_t}_{\text{growth}} + \underbrace{\lambda \times b_t}_{\text{hedging cost}}$$

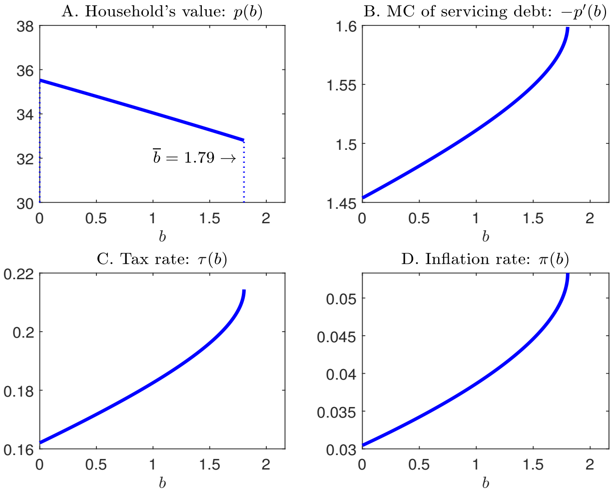
  - With inflation:

$$\dot{b}_t \equiv \mu^b(b_t) = \gamma - \tau(b) + r \times b_t - g \times b_t + \lambda \times b_t - \underbrace{\frac{\pi(b) + g - \lambda}{v}}_{\text{seigniorage}}$$

- Gordon growth formula at equilibrium debt capacity $\bar{b}$:

$$\bar{b} = \frac{1}{r + \lambda - g}\left(\tau(\bar{b}) - \gamma + \frac{\pi(\bar{b}) + g - \lambda}{v}\right)$$

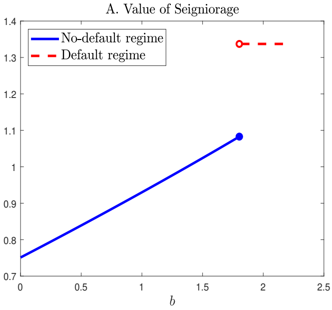
Monetary Economy

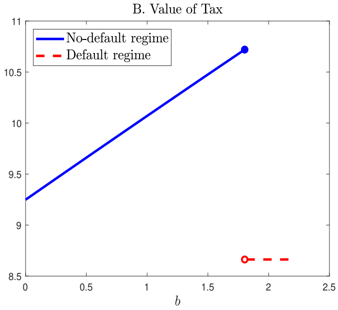


We set $c(\tau, \pi) = \frac{\varphi_\tau}{2}\tau^2 + \frac{\varphi_\pi}{\delta}\pi^\delta$ with $\varphi_\tau = 2.8$, $\varphi_\pi = 1.3, \delta = 1.5.$

Seigniorage versus Taxation

$$\text{PV of seigniorage} + \text{PV of taxation} = \text{PV of gov't spending} + \text{Debt value}$$

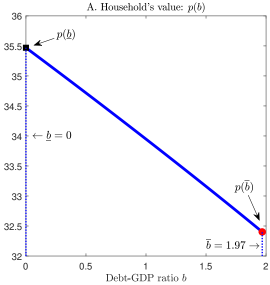




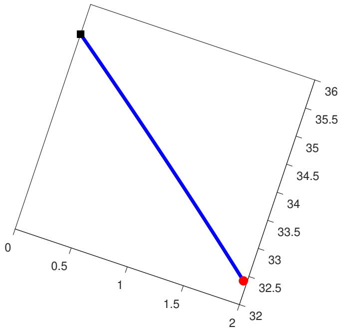
Execuses

- First pass at pedagogical p theory to organize thoughts about government debt and taxes: **SDF/intertemporal budget (with risk premium and impatience) approach with Barro's distortionary taxes and endogenous credit limit coming from limited commitment**
- Quantify consequences of alternative interest rate (r), risk premium (λ), growth (g) on transition dynamics, equilibrium debt capacity, taxes, and **MC of servicing debt** $-p'(b)$
- Interaction of fiscal and monetary policies (Sargent and Wallace, 1981): **inflation tax and nominal debt**
- On-going work
  - **Endogenous SDF** (Lucas and Stokey, 1983) and consequent incentives to manipulate asset prices

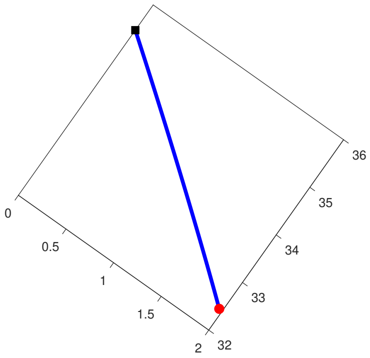
Duality: $w = p(b)\,, b = f(w)$



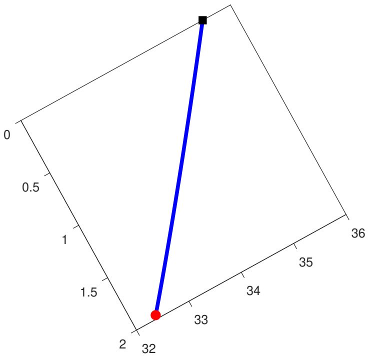
DUALITY: $w = p(b)\,, b = f(w)$



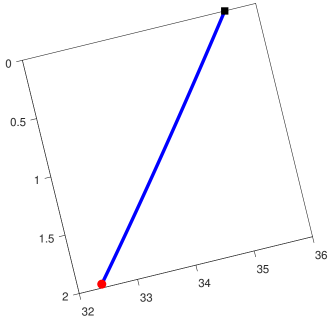
Duality: $w = p(b)\,, b = f(w)$



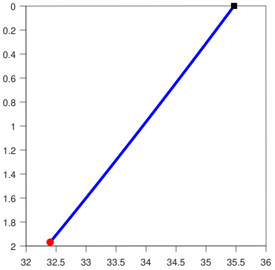
Duality: $w = p(b)\,, b = f(w)$



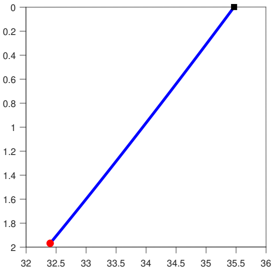
DUALITY: $w = p(b)\,, b = f(w)$



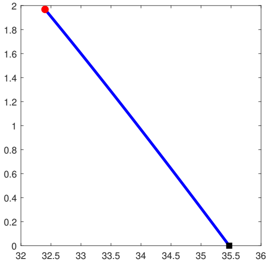
DUALITY: $w = p(b)\,, b = f(w)$



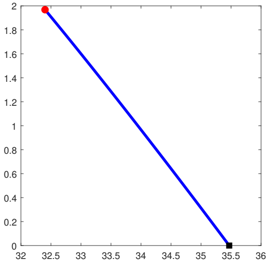
DUALITY: $w = p(b)\,, b = f(w)$



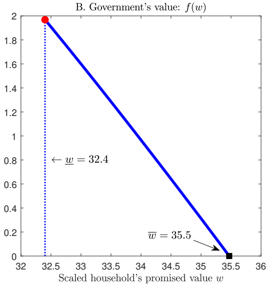
DUALITY: $w = p(b)\, , b = f(w)$



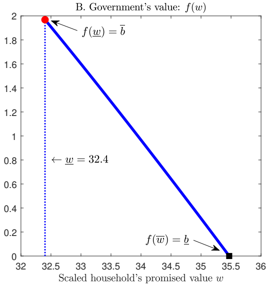
DUALITY: $w = p(b)\, , b = f(w)$



DUALITY: $w = p(b)\,, b = f(w)$



Duality: $w = p(b)\,, b = f(w)$



DUALITY: $p(b) = w$ AND $f(w) = b$