Funding Horizons, Interest Rates, and Growth

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Questions

To finance investment, entrepreneurs raise external funds against the value of the firm

The value of the firm (debt + equity) appears to have short horizons

Cash-flow-based borrowing: debt is limited mostly by 3 to 4.5 years worth of EBITDA, according to Lian-Ma (2021)

Stock analysts typically provide 5-year earning forecast

Why are the horizons of external funding short, even when the durations of businesses are long?

How do funding horizons interact with growth and business cycles?

Introductory Example

Engineer's investment technology:

$$\left.\begin{array}{c}\text{goods}\\ \text{a building}\end{array}\right\}\rightarrow\left\{\begin{array}{c}\text{a plant}\\ \text{maintenance capacity}\end{array}\right.$$

To finance investment, engineer sells the plant ownership to saver (equity finance)

With the engineer's continual maintenance, the plant yields returns $y_{t+1},\ y_{t+2},\ y_{t+3}, \dots$. Once missed the maintenance, the plant stops yielding returns forever

Engineer cannot commit to maintain the plant in future. Match between engineer and plant owner can change over time

The owner and engineer bilaterally match and bargain over "wage" every period. Liquidation needs engineer

$$\underset{w_t}{Max} \left(-w_t + \frac{1}{R}V_{t+1}\right)^{\theta} (w_t)^{1-\theta}$$

Engineer receives a fraction $1-\theta$ of continuation value as wage *every period*

$$\begin{aligned} w_t &= (1-\theta)\frac{1}{R}V_{t+1} \\ V_t &= y_t - w_t + \frac{1}{R}V_{t+1} \end{aligned}$$

The plant owner retains θ fraction of the continuation value, and derives the value largely from near-future revenue

$$V_t = y_t + \frac{\theta}{R}y_{t+1} + \left(\frac{\theta}{R}\right)^2 y_{t+2} + \left(\frac{\theta}{R}\right)^3 y_{t+3} + \left(\frac{\theta}{R}\right)^4 y_{t+4} \cdots$$

The engineer sells plant at price $\frac{1}{R}V_{t+1}$ to finance investment: her funding horizon is short

Model

Small open economy with an exogenous real interest rate R

Homogeneous perishable consumption/investment good at each date $t = 0, 1, 2, \ldots$(numeraire)

Continuum of agents, utility $U = E_0 \left[\sum_{t=0}^{\infty} \beta^t \ln c_t \right]$

Each agent sometimes has an investment opportunity (entrepreneur/engineer) and sometimes not (saver), Markov process

At each date, an engineer E can jointly produce plant and tools from goods and building: per unit of plant,

$$\left.\begin{array}{c} x \text{ goods} \\ 1 \text{ building} \end{array}\right\} \rightarrow \left\{\begin{array}{c} \text{plant of productivity } 1 \\ 1 \text{ E-tool} \end{array}\right.$$

Engineer raises funds by selling the plant to savers. Match between plant and engineer is not specific $\longrightarrow$ Plant owner hires engineers for maintenance in a competitive market at wage w. Engineer cannot precommit to work for less

At each date, the owner of plant of productivity z can hire any number h of tools (the engineer's expertise) to produce goods and maintain plant productivity: within the period, per unit of plant,

$$\left.\begin{array}{c}\text{productivity } z \text{ plant} \\ h \text{ tools}\end{array}\right\} \longrightarrow \left\{\begin{array}{c} y = az \quad \text{goods} \\ \lambda \text{ productivity } z' = z^{\theta} h^{\eta} \text{ plant} \\ \lambda h \text{ tools}\end{array}\right.$$

New buildings are supplied by foreigners

Alternative use of building by foreigners:

$$1 \text{ building} \longrightarrow \begin{cases} \;\; f \;\; \text{goods} \\ \lambda \text{ building} \end{cases}$$

Building supply is perfectly elastic at a constant rent f

$\longrightarrow$ Price of buildings

$$q = \frac{f}{R - \lambda}$$

The plant owner always has the option to stop and liquidate his plant into generic building. So his value of a unit of plant of productivity z at the beginning of the period is given by

$$V(z) = az + Max\left\{q,\ \max_{h}\left[-wh + \frac{\lambda}{R}V\left(z^{\theta}h^{\eta}\right)\right]\right\}$$

The plant owner must devise a long-term plan:

stop after a finite number of periods T, or

continue forever ($T = \infty$)?

An engineer raises fund by selling a new plant at price $b = \frac{1}{R}V(1)$

The budget constraint of an agent at date t who has h_t tools and d_t financial assets (maturing one-period discount bonds plus returns to plant ownership) is

$$c_t + (x + q - b)i_t + \frac{d_{t+1}}{R} = wh_t + d_t,$$

where h_t is positive iff the agent was engineer yesterday. Iff the agent is an engineer today, investment i_t is positive, and her tools tomorrow will be

$$h_{t+1} = \lambda h_t + i_t$$

The budget constraint can be written as

$$c_t + (x + q - b)h_{t+1} + \frac{d_{t+1}}{R} = [w + \lambda(x + q - b)]h_t + d_t \equiv n_t,$$

where n_t is net worth

When the rate of return on investment with maximal leverage, R^E, exceeds the interest rate

$$R^E = \frac{w + \lambda(x + q - b)}{x + q - b} > R,$$

the engineer's consumption and investment are

$$\begin{aligned} c_t &= (1 - \beta)n_t \\ (x + q - b)h_{t+1} &= \beta n_t \end{aligned}$$

A saver's are

$$c_t = (1 - \beta)n_t, \text{ and } \frac{d_{t+1}}{R} = \beta n_t$$

A steady state equilibrium of our small open economy is characterized by wage rate w, new-plant price b, and the quantity choices of savers/plant owners, engineers and foreigners, such that the markets for goods, tools, plant, and bonds all clear

Proposition. Pure Equilibrium with No Stopping: Low opportunity cost $f < f^{\text{critical}}$

(a) No plant owner stops

(b) Aggregate ratio of tools-to-plant stays one-to-one (because equal initial supply, equal depreciation, no stopping): $h_t = 1$

(c) Every plant is maintained at initial productivity: $z_t = 1$

(d) Every plant has output: $y_t = a$

(e) If $z' = z^{\theta} h^{1-\theta}$, the competitive equilibrium and the bilateral bargaining are equivalent

Optimal maintenance choice, $z_{t+1} = z_t^\theta h_t^\eta$ and $h_t = z_t = 1$

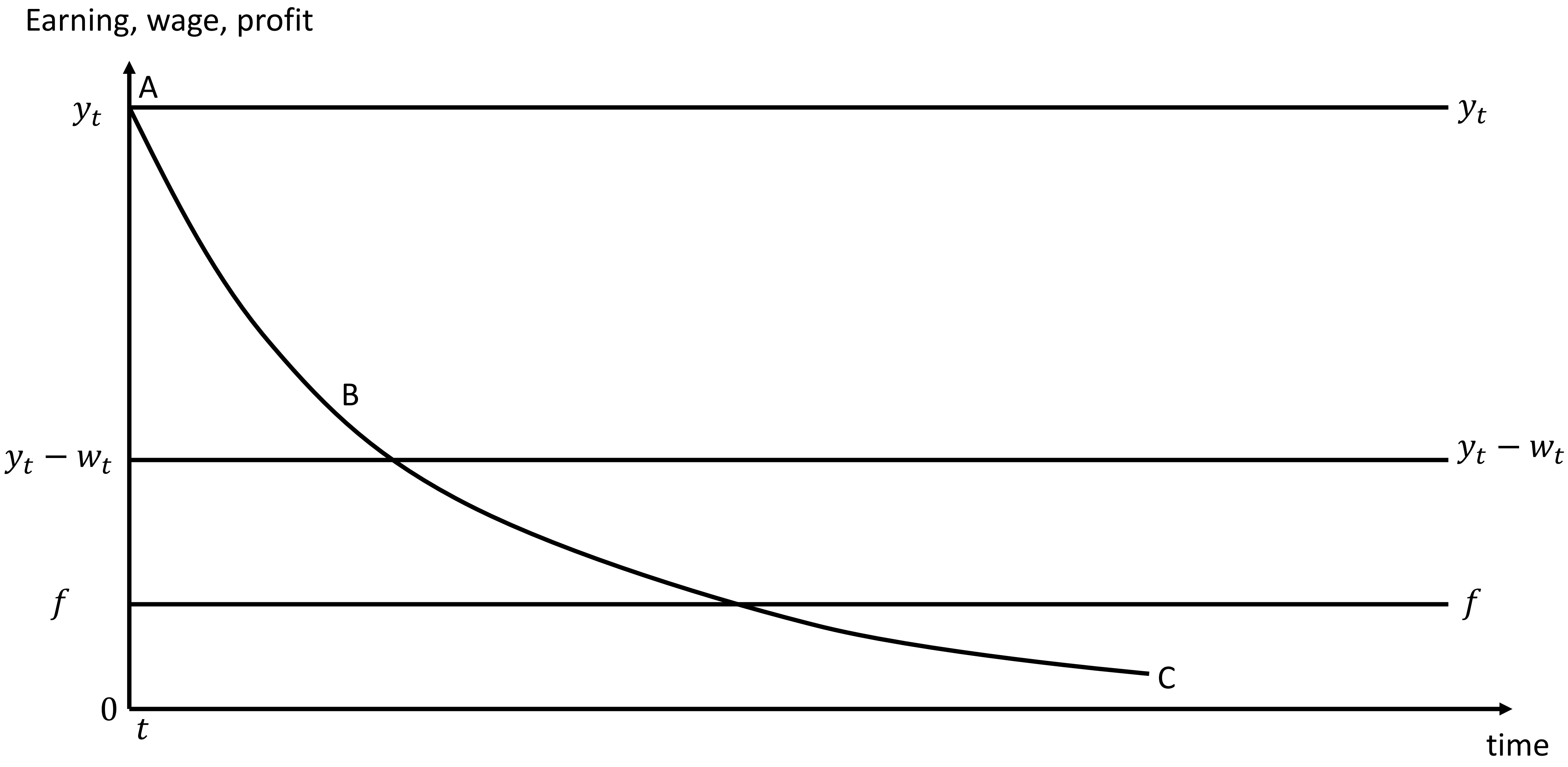
$$w = \frac{\lambda}{R}\eta a + \frac{\lambda^2}{R^2}\eta\theta a + \frac{\lambda^3}{R^3}\eta\theta^2 a + ...$$
$$= \text{ PV of marginal product of } h$$

$$b = \frac{1}{R}(a-w) \ + \ \frac{\lambda}{R^2}(a-w) + \frac{\lambda^2}{R^3}(a-w) + ..$$
$$= \frac{1}{R}a \ + \ \frac{\lambda}{R^2}a(1-\eta) + \frac{\lambda^2}{R^3}a(1-\eta-\eta\theta) + ..$$

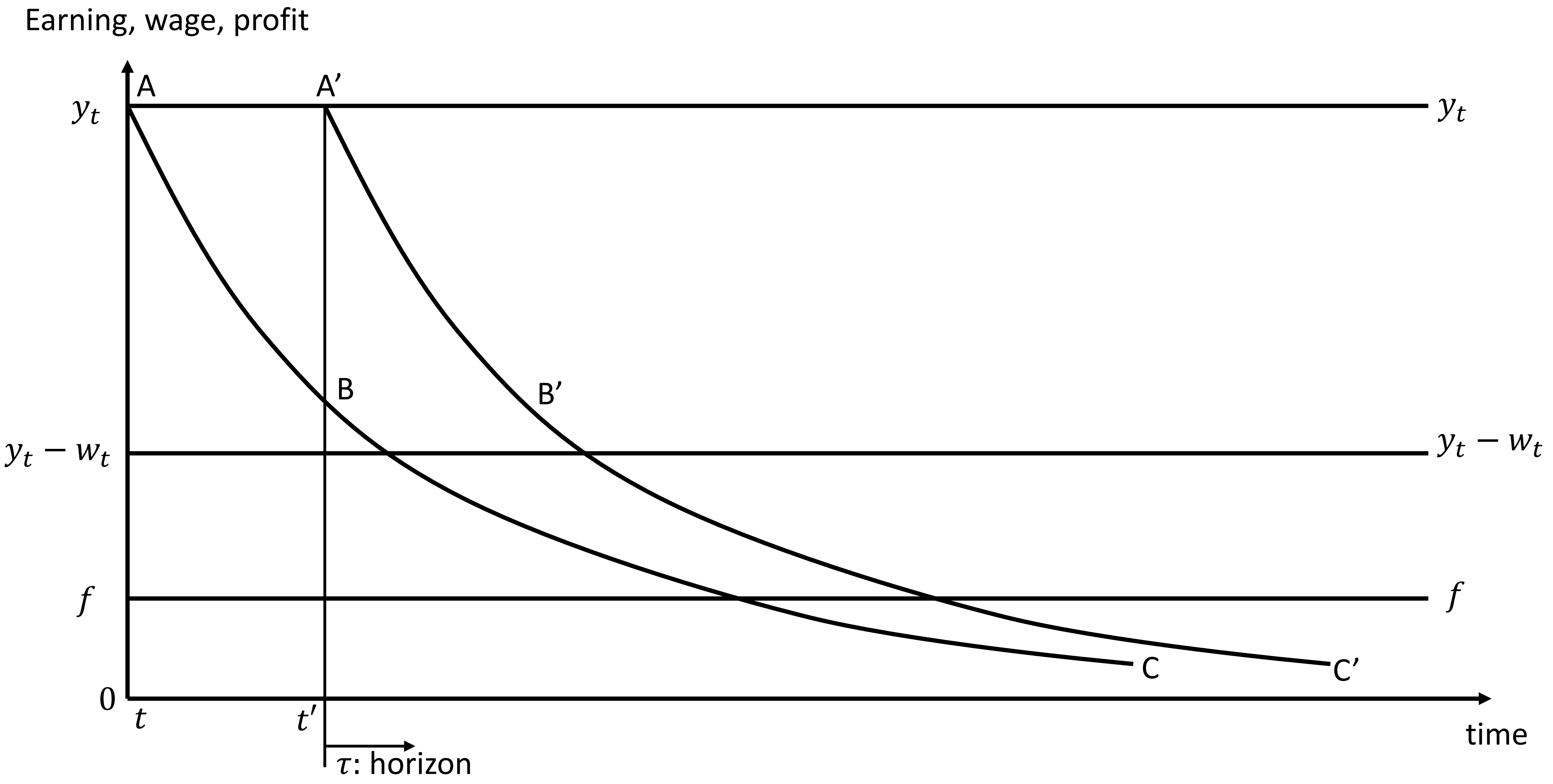
Engineers' share of output rises with horizon as 0, η, $\eta(1+\theta)$, $\eta(1+\theta+\theta^2), ...$

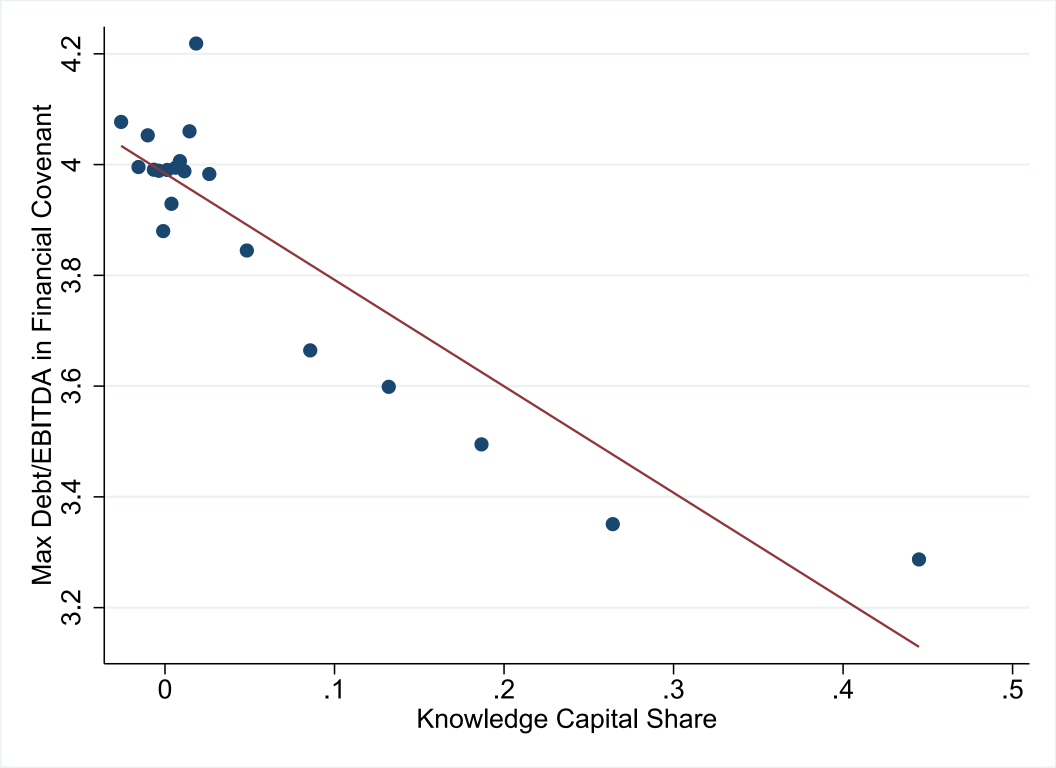
Plant owner's share from present plant declines with horizon as 1, $1-\eta$, $1-\eta-\eta\theta$, $1-\eta-\eta\theta-\eta\theta^2$, ...

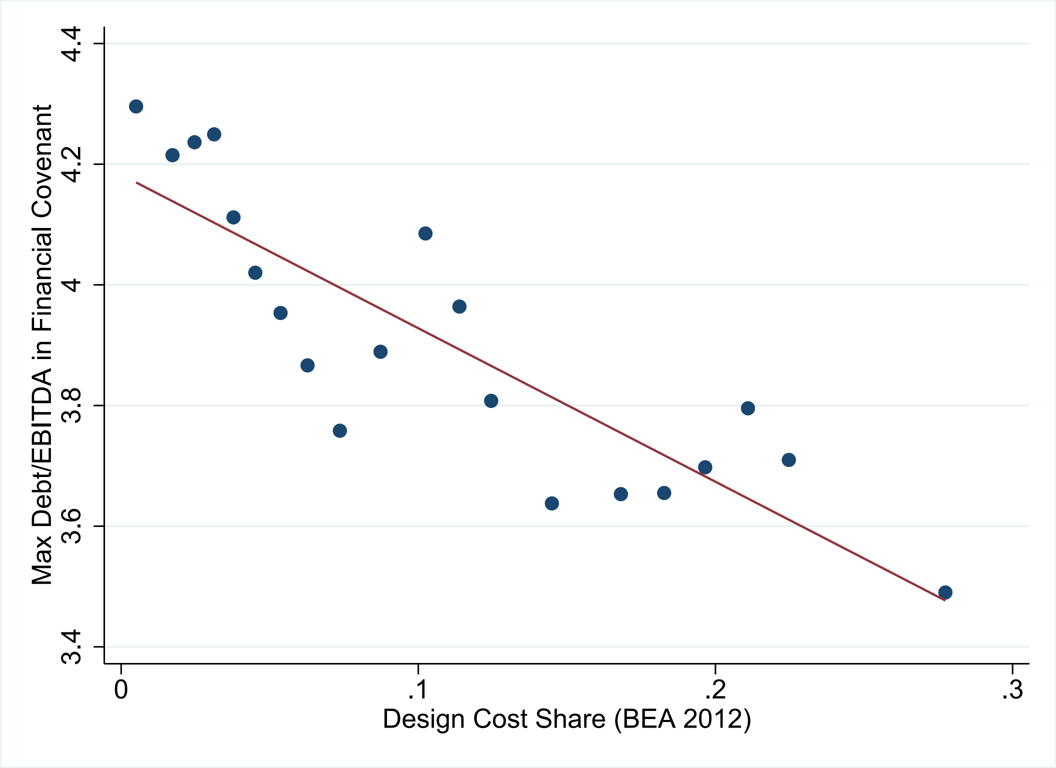
Shares of Owner and Engineer



Shares of Owner and Engineer

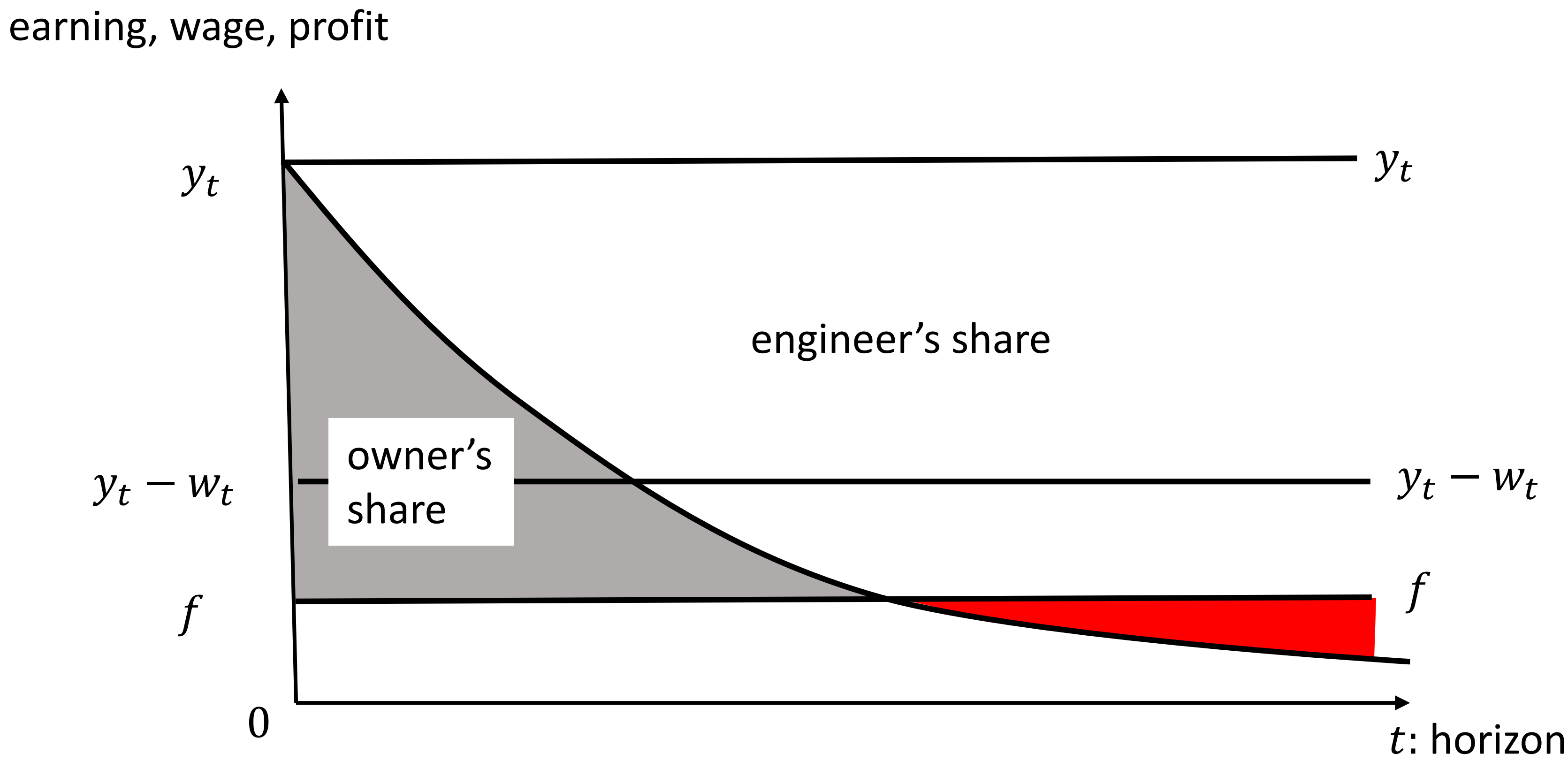



Max Debt/EBITDA in Financial Covenant
4.2
4
3.8
3.6
3.4
3.2
0
.1
.2
.3
.4
.5
Knowledge Capital Share



Max Debt/EBITDA in Financial Covenant
4.4
4.2
4
3.8
3.6
3.4
0
.1
.2
.3
Design Cost Share (BEA 2012)

Owner’s share net of building costs



Effects of a Permanent Fall in Interest Rate

The engineers' fund-raising capacity has a shorter duration than building

With a permanent fall in interest rate, the fund-raising capacity may fail to catch up with the investment cost. Can offset a rise in net worth – to stifle investment and growth:

$$\text{gross investment} \downarrow \; = \; \beta \times \frac{\text{net worth of engineers} \uparrow}{\text{investment cost } \; (x+q) \uparrow\uparrow - \; \text{fund-raising capacity } \; (b) \uparrow}$$

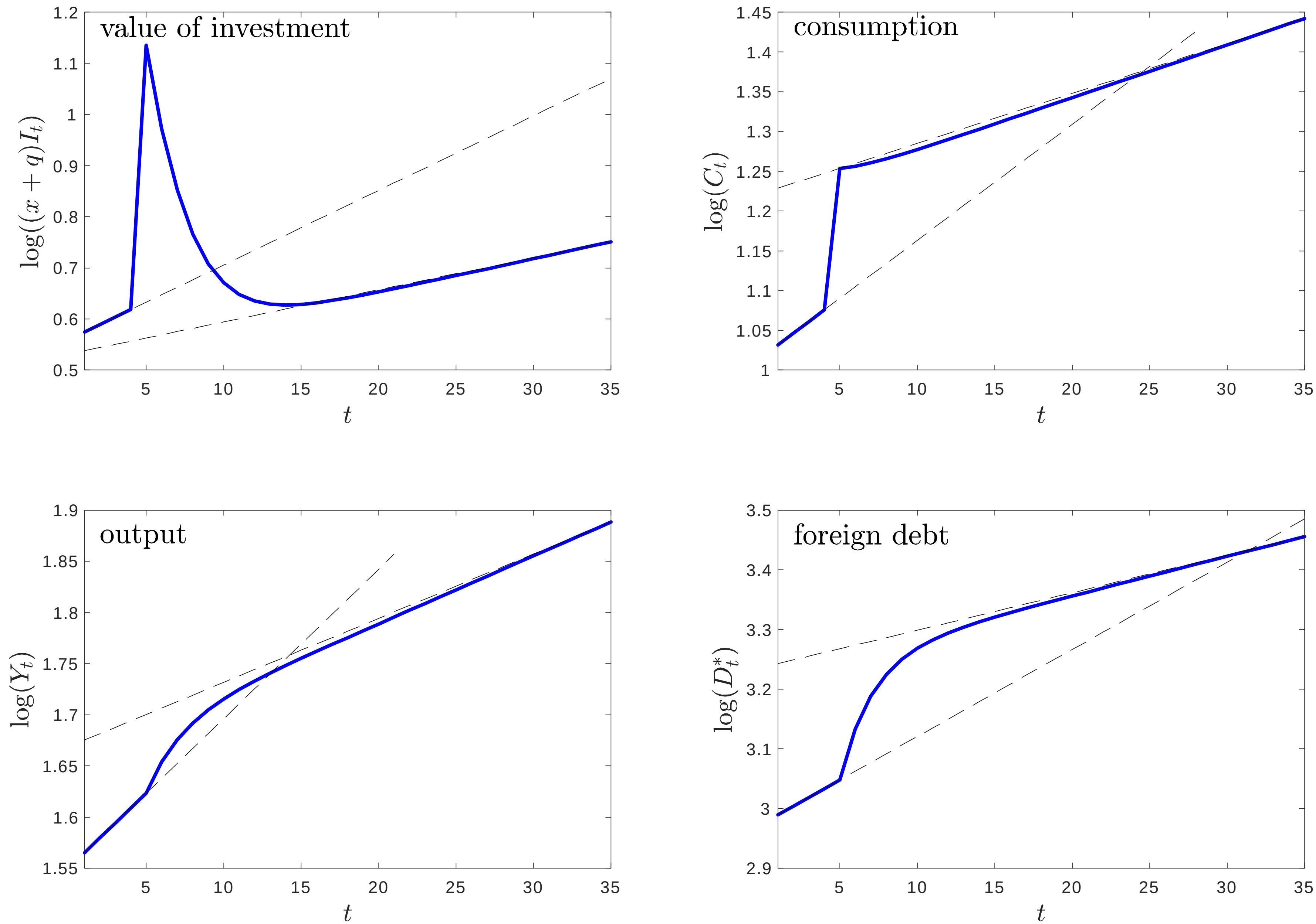

value of investment
consumption
output
foreign debt
$\log((x+q)I_t)$
$\log(C_t)$
$\log(Y_t)$
$\log(D^*_t)$
t

Figure 6: Interest rate fluctuation and business cycle

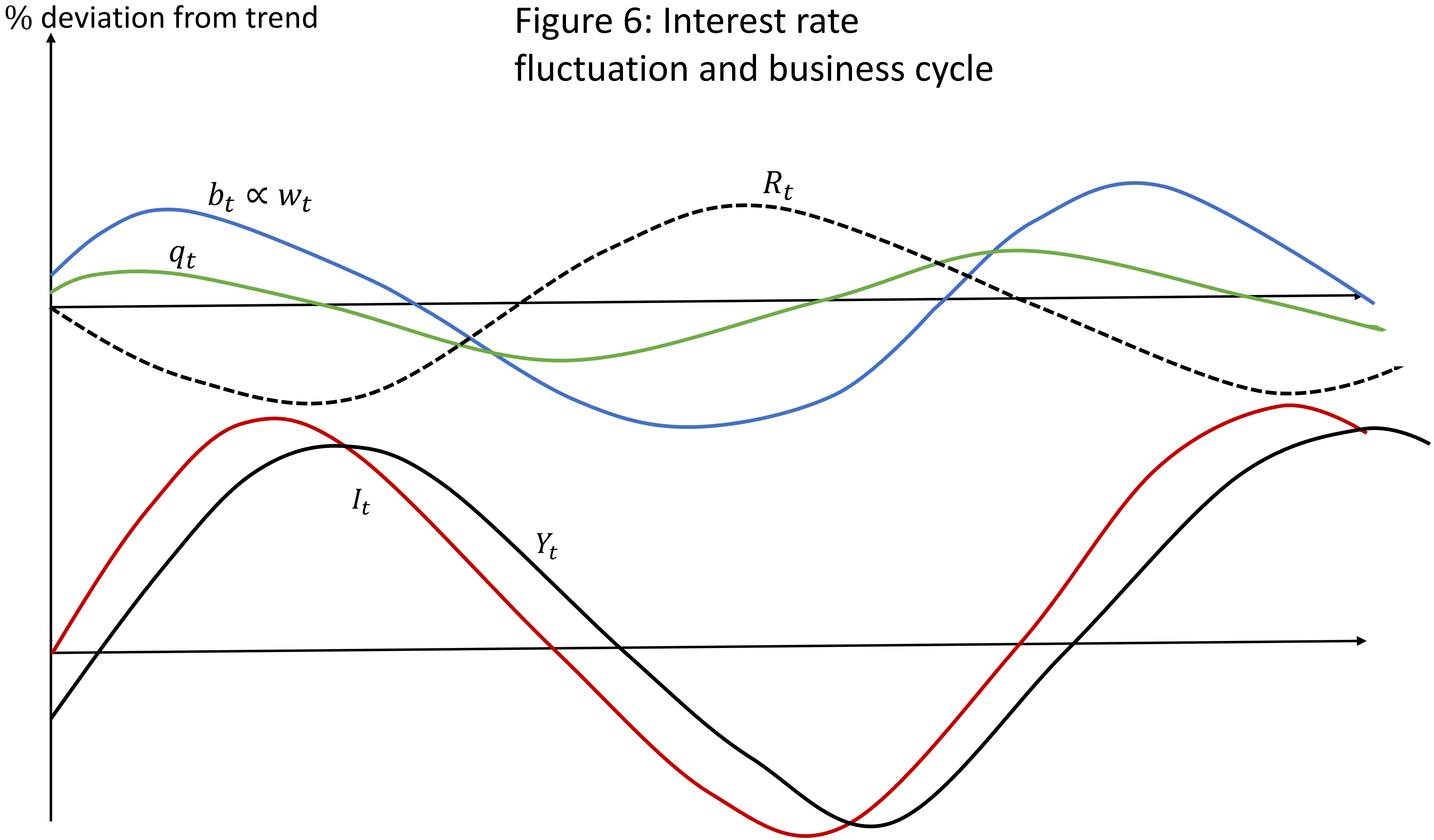
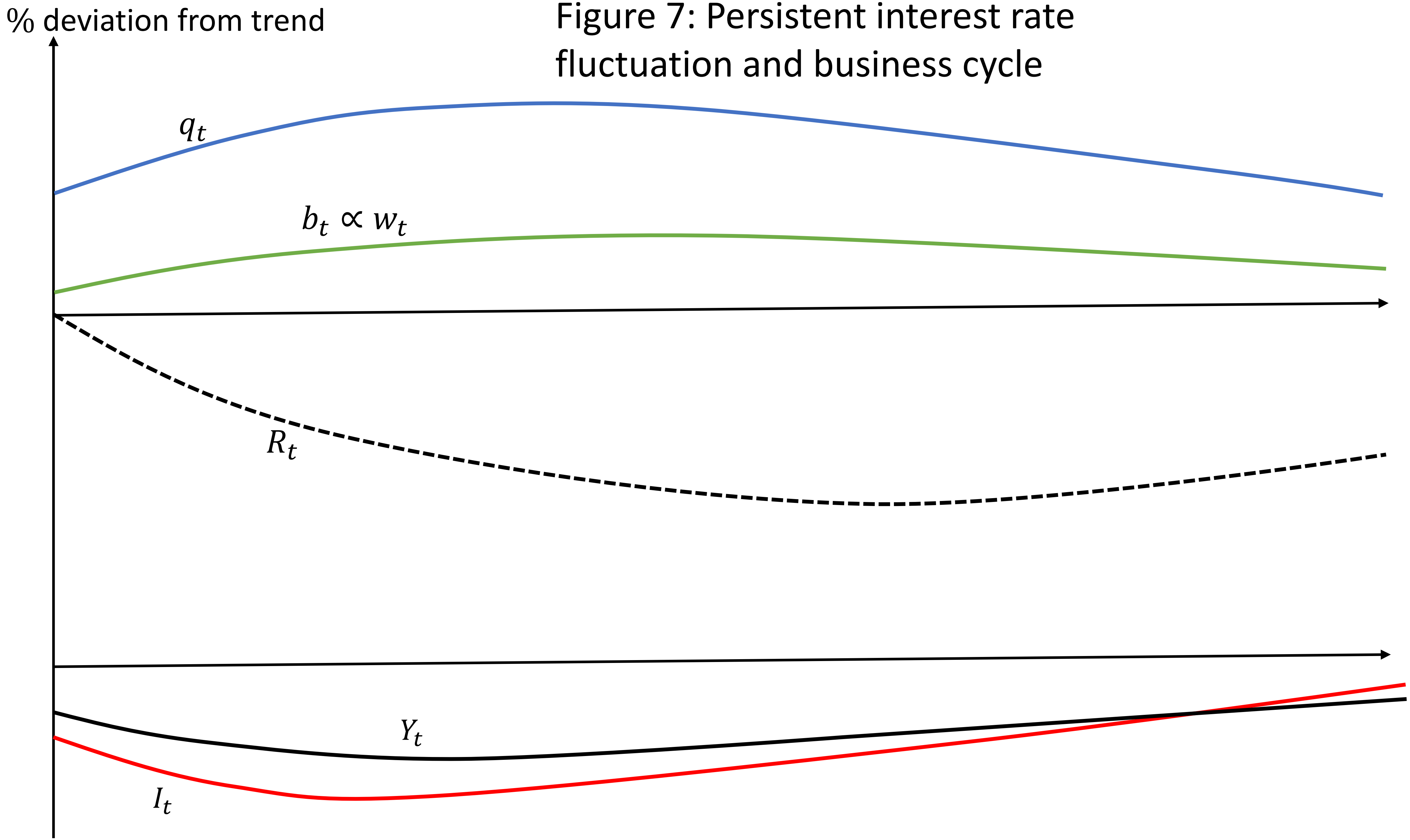


Figure 7: Persistent interest rate fluctuation and business cycle



Panel Data Evidences

UK firm Bureau van Dijk & Companies House panel data for 1997-2020

High-frequency interest rate surprise for short (3 month) and long (5 year) interest rates: Δr_t^{SR}, Δr_t^{LR}

For each of 10 firm size (employment) buckets, use $\frac{\#\text{ of directors}}{\#\text{ of employees}}$ to define funky (above median) and non-funky (below median) firms: $D_{f,i}$, $D_{nf,i}$

$$\frac{FA_{t+h,i} - FA_{t-1,i}}{\left(FA_{t+3,i} + FA_{t-1,i}\right)/2} = \beta_f^h \cdot D_{f,i} \cdot \Delta r_t^{SR} + \beta_{nf}^h \cdot D_{nf,i} \cdot \Delta r_t^{SR}$$

$$+\gamma_f^h \cdot D_{f,i} \cdot \Delta r_t^{LR} + \gamma_{nf}^h \cdot D_{nf,i} \cdot \Delta r_t^{LR} + \alpha_i^h + \varepsilon_{i,t}^h$$

Investment effects of interest rate shocks for 82,781 unique firms

Horizon h years	1	2	3
Funky♯SR	-0.27 (-0.07)	-3.87 (-0.84)	-10.3^{*} (-1.86)
Non-funky♯SR	4.23 (1.33)	2.88 (0.70)	-1.62 (-0.32)
Funky♯LR	-8.53 (-1.15)	-15.4 (-1.65)	-14.7 (-1.34)
Non-funky♯LR	-16.7^{***} (-2.72)	-22.9^{***} (-3.01)	-22.6^{**} (-2.53)
#Observations	$970,835$	$970,835$	$970,835$

Investment effects of long-term rate shocks, with short-term rate shock and firm fixed effects

Horizon=3Y	Funky$\sharp$LR	Non-funky$\sharp$LR
Size=1	-60.4^{*}	-47.1^{***}
Size=2	-37.3^{***}	-35.9^{***}
Size=3	-27.8^{***}	-25.5^{***}
Size=4	-16.8^{**}	-25.7^{***}
Size=5	-15.2	-18.7^{*}
Size=6	-12.7	-21.9^{**}
Size=7	-13.0	-23.2^{**}
Size=8	-8.4	-9.5
Size=9	-10.4	-5.1
Size=10	-13.4	-19.8

82,781 unique firms

Policy

Engineers do not maintain unless paid every period: Impossible to keep track of each engineer's trading history

If plant is easy to locate, then perhaps government could tax the plant owner's payroll at rate τ. Use the revenue to subsidize investment at rate s

$$\text{investment of engineers} = \frac{\text{engineers' net worth} \downarrow}{x + q - s - b \downdownarrows} \uparrow$$

$\longrightarrow$ Growth rate rises in pure equilibrium with no stopping

Population-weighted average expected discounted utility of engineers and savers raises with τ for small τ

Government acts as a social creditor to engineers

Proposition. Mixed Equilibrium: High rent $f > f^{critical}$

(a) Plant owners are initially indifferent between stopping in some finite time and continuing forever

(b) Aggregate ratio of tools-to-plant is larger than one-to-one for continuing plant: $h_t > 1$

(c) The productivity of continuing plant increases over time

(d) The productivity of stopping plant decreases over time

Lemma: There is no equilibrium in which all plant shut down in finite time

Debt effects of interest rate shocks for 24,200 unique firms

Horizon h years	1	2	3
Funky$\sharp$SR	-0.40 (-0.11)	-3.82 (-0.95)	-9.95^{*} (-1.79)
Non-funky$\sharp$SR	3.80 (0.83)	5.41 (0.98)	-1.35 (-0.19)
Funky$\sharp$LR	-3.57 (-0.65)	-14.3^{**} (-2.07)	-11.4 (-1.20)
Non-funky$\sharp$LR	-5.99 (-0.88)	-20.8^{**} (-2.32)	-20.0^{*} (-1.75)
#Observations	$141,260$	$141,260$	$141,260$

Investment effects of interest rate shocks with wage-based funkiness measure

Horizon h years	1	2	3
Funky♯SR	2.38 (0.79)	-1.61 (-0.43)	-8.35^{*} (-1.87)
Non-funky♯SR	3.49 (-1.07)	0.69 (0.17)	-5.08 (-1.01)
Funky♯LR	-14.2^{**} (-2.56)	-20.2^{***} (-2.97)	-17.3^{**} (-2.18)
Non-funky♯LR	-18.2^{***} (-3.16)	-22.1^{***} (-3.13)	-21.7^{***} (-2.71)
#Observations	1525269	1525269	1525269