Sustainable Spending and Portfolio Choice

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In the Spirit of Andy Abel

- I admire Andy's work for its transparent modeling.
    - An Abel model is never a "black box", you can understand how it works.
    - Yet the results are not obvious!
- Andy asks deceptively simple, yet fruitful questions.
    - For example, what is the meaning of a low riskless real interest rate in a risky economy?
    - Does it mean that society has overaccumulated capital? (Maybe not: Abel, Mankiw, Summers, and Zeckhauser 1989.)
    - Does it mean that the government can roll over debt forever without running primary surpluses to pay it off? (Maybe: Abel and Panageas 2022.)
- In this talk I will present two papers that also consider the implications of a low riskless real interest rate in a risky economy.
    - A paper with Roman Sigalov (*JFE* 2022)
    - A paper with Ian Martin (unpublished 2021).

Two Meanings of Sustainability

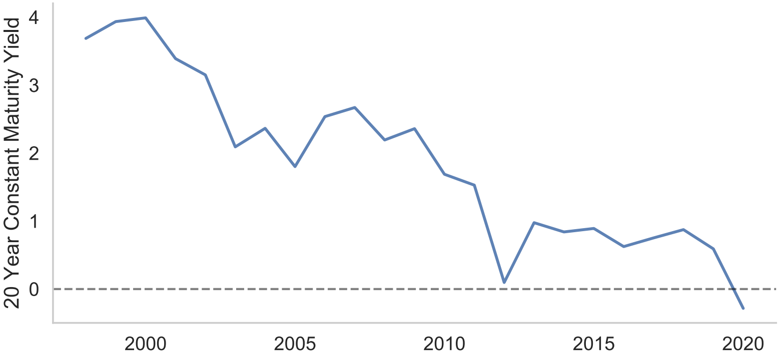
- Campbell and Sigalov (2022) study **sustainable spending**.
  - An institution has standard power utility with time preference, but is constrained only to spend the expected return (or expected log return) on its portfolio.
  - Zero drift in consumption (or log consumption).
  - Portfolio choice is distorted: the institution takes more risk when the riskless rate is low ("reaching for yield", RFY).
- Campbell and Martin (2021) study **sustainable utility**.
  - An ethical constraint that society should only spend an amount that allows future generations to have the same expected utility.
  - Zero drift in utility, upward drift in consumption to compensate future generations for risk.
  - Portfolio choice is undistorted.
  - In a benchmark case, spending is the average of the riskless rate and the expected return on the portfolio.

Reaching for Yield

- **Low for long:** Large and persistent decline in real interest rates during this century (partially reversed in 2022).
- **Reaching for yield (RFY):** The hypothesis that investors respond by taking more risk.
- Much discussed by central bankers, e.g. Stein (2013):

> *"A prolonged period of low interest rates, of the sort we are experiencing today, can create incentives for agents to take on greater duration or credit risks, or to employ additional financial leverage, in an effort to reach for yield."*

Low for Long



20-year constant maturity TIPS yield, 1997-2020.

The Theory Puzzle

- Standard finance theory does **not** predict RFY: risktaking depends on risk premium, risk, and risk aversion but **not** the riskfree interest rate.
- Recent literature has proposed a variety of institutional explanations for RFY:
  - Fixed nominal return target (Rajan 2013), possibly related to zero lower bound for retail deposit rates (Di Maggio and Kacperczyk 2017).
  - Low rates lower the opportunity cost of holding liquid assets (e.g. reserves) which are needed for leveraged risktaking (Drechsler, Savov, and Schnabl (2018).
  - Low rates worsen the underfunding of pension plans, which react by gambling for resurrection (Andonov, Bauer, and Cremers 2017).
  - Low rates lengthen the duration of insurance company liabilities, which react by lengthening the duration and hence the yield of their assets (Ozdagli and Wang 2019).

Our Theory

- Start with a standard model of an infinitely lived investor with power utility (Merton 1969, 1971).
- Add a sustainable spending constraint, realistic for endowments and sovereign wealth funds (SWFs):
  - The investor must consume the expected real return on wealth each period.
  - This implies that wealth is expected to remain constant: the investor cannot plan to run down or accumulate wealth.
  - Two variants, arithmetic expected return vs. geometric expected return, differ in detail but the main results are the same.
- This one change to the standard model implies
  - RFY
  - Stronger RFY when the real interest rate is already low
  - Risktaking responds perversely to the risk premium when the real interest rate is low
  - In a nominal variant of the model, stronger RFY when inflation is low.

The Sustainable Spending Constraint (1)

- Tobin (1974):

  "The trustees of an endowed institution are the guardians of the future against the claims of the present. Their task is to preserve equity among generations. The trustees of an endowed university like my own assume the institution to be immortal. They want to know, therefore, the rate of consumption from endowment which can be sustained indefinitely."

The Sustainable Spending Constraint (2)

- Harvard website:

  "The University's spending practice has to balance two competing goals: the need to fund the operating budget with a stable and predictable distribution, and the obligation to **maintain the long-term value of endowment assets after accounting for inflation***."*

- Norges Bank Investment Management (NBIM) website:

  "So that the fund benefits as many people as possible in the future too, politicians have agreed on a fiscal rule which ensures that **we do not spend more than the expected return on the fund.***"*

- These two formulations are equivalent.

Merton Model Setup

- Choose consumption and asset allocation to maximize expected utility.
- Utility function has constant relative risk aversion γ and time preference rate ρ.
- The consumer lives off financial wealth w_t invested in two different assets.
- There is a constant riskfree interest rate r_f.
- The risky asset has risk premium μ and volatility σ.
- We write the risky portfolio share as α and the consumption-wealth ratio c_t/w_t as θ. (Both are constant.)

Merton Model Solution

- The risky share is a constant α given by the famous formula

$$\alpha = \frac{\mu}{\gamma\sigma^2}.$$

- The risky share depends on the reward for taking risk and on risk aversion, but **not** on the riskfree interest rate.
- Because the consumption-wealth ratio is a constant θ, consumption and wealth grow at the same rate.
- The expected (desired) growth rate of consumption and wealth is increasing in the riskfree rate r_f and decreasing in the rate of time preference ρ.
- But what if this is not possible?

Definition of the Arithmetic Constraint

- **Consumption-wealth ratio equals the expected simple return** (aka arithmetic average return).

$$\theta = \frac{c_t}{w_t} = (r_f + \alpha\mu).$$

- From the budget constraint, then **the expected change in wealth is zero.**
  - Thus the two formulations of the constraint by Harvard and NBIM are equivalent.
  - Since c_t / w_t is constant, the expected change in consumption is also zero.
- The only choice variable in the problem is the risky portfolio share α.

Arithmetic Solution

- The risky share is

$$\alpha = \frac{-r_f + \sqrt{K}}{\mu(1+\gamma)},$$

where

$$K = r_f^2 + 2\rho \left(\frac{1+\gamma}{\gamma} \right) \left(\frac{\mu}{\sigma} \right)^2 .$$

- Standard properties:
  - Portfolio volatility $\alpha\sigma$ depends only on the Sharpe ratio μ/σ.
  - Risky share α is inversely related (although not inversely proportional) to σ^2 and γ.
- But there are nonstandard properties too!

Nonstandard Properties of the Arithmetic Model

Proposition

In the arithmetic model, the risky share α has the following properties.

1. *α is a decreasing and convex function of the riskfree rate r_f.*
2. *α is an increasing function of the rate of time preference ρ.*
3. *α is an increasing function of the risk premium μ when $r_f > 0$, and a decreasing function of μ when $r_f < 0$.*

Simple Intuition

- Lower riskfree rate or greater impatience lead the investor to want higher consumption (lower marginal utility) today relative to expected future consumption (marginal utility).
- In the standard model, this is achieved by dissaving.
- With a sustainable spending constraint, it is achieved by taking risk. This allows higher spending today, and the negative consequence (riskier consumption) is realized in the future.
- A lower risk premium has both a standard substitution effect (take less risk) and a nonstandard income effect similar to that of a lower riskfree rate.
- All the nonstandard effects get stronger as the riskfree rate declines.
    - Hence the interest in RFY today (this 2022 paper uses 1970s technology!)
    - The nonstandard effect of the risk premium dominates when $r_f < 0$.

Definition of the Geometric Constraint

- A problem with the arithmetic model is that although the average of future wealth is equal to current wealth, future wealth is more often than not lower than current wealth.
    - There are a few ultra-rich scenarios that counterbalance many impoverished scenarios.
- To fix this problem, we can alternatively impose a geometric constraint in which **the consumption-wealth ratio equals the expected log return** (aka geometric average return):

$$\theta = \frac{c_t}{w_t} = (r_f + \alpha\mu - \frac{1}{2}\alpha^2\sigma^2).$$

- Then **the expected change in log wealth is zero**.
- This implies that current wealth is the median of future wealth: 50% of the time future wealth will be higher, 50% of the time it will be lower than today.
- The solution to the geometric model is more complex than the solution to the arithmetic model, but it has similar properties.

Nonstandard Properties of the Geometric Model

Proposition

In the geometric average model with $\gamma > 1$, the risky share α has the following properties:

1. *α is a decreasing and convex function of the riskfree rate r_f.*
2. *α is an increasing function of the rate of time preference ρ.*
3. *Define $r_f^* = -\rho/(\gamma^2 - 1)$. When $r_f > r_f^*$, α is an increasing function of the risk premium μ and when $r_f < r_f^*$, α is a decreasing function of μ.*
4. *The growth-optimal risky share μ/σ^2 is an upper bound on α.*

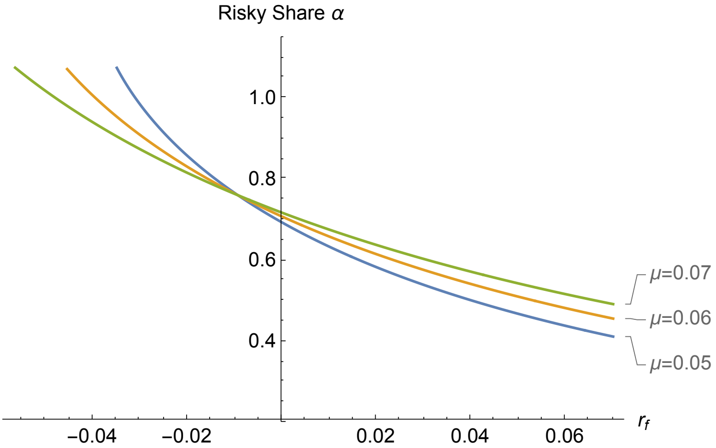
Interpretation

- The nonstandard effects of the riskfree rate and impatience have the same intuition as in the arithmetic model.
- The nonstandard effects get stronger as the riskfree rate declines.
- The nonstandard income effect of the risk premium dominates the standard substitution effect when the riskfree rate is sufficiently negative.
- The upper bound on risktaking is the growth-optimal portfolio with $\alpha = \mu/\sigma^2$, because this maximizes the expected log return and hence current consumption.

A Calibrated Example

- We illustrate the model in a calibrated example with $\rho = 7.5\%$, $\gamma = 3$, $\sigma = 18\%$.
- Base case has $\mu = 6\%$, market Sharpe ratio $= 33\%$, Merton $\alpha = 62\%$.
    - This is close to the classic 60% rule of thumb for endowments' risky share.
- In the base case, the sustainable spending constraint is nonbinding at $r_f \approx 2\%$.
    - At this level of r_f, the geometric expected portfolio return (consumption-wealth ratio) $= 5.1\%$.
    - This is slightly above the typical historical distribution rates for endowments reported by Dahiya and Yermack (2018).
- The interest rate at which μ does not affect α is $r_f^* = -0.94\%$, and the corresponding $\alpha^* = 76\%$.

RFY and the Risk Premium



$$\rho = 0.075,\ \gamma = 3,\ \sigma = 0.18.$$

Conclusion

- Classical finance theory separates risktaking from intertemporal choice.
- Our model breaks this separation using a sustainable spending constraint: investors take risk as a way to increase current consumption at the cost of more volatile future consumption.
    - The model predicts RFY, stronger when the riskfree rate is low, and more risktaking by impatient investors.
- Rampini and Viswanathan (2010, 2013, 2019) similarly break the separation using models of collateral constraints for firms or borrowing constraints for households.
    - Constrained firms will not put up collateral today, and constrained households will not pay insurance premia today, to manage their future risk exposures.
- The classical result is not as robust as finance theorists have supposed.

Is Impatience Ethical?

Three views:

- Preferences must be respected**:** impatience aka time preference is whatever it is.
- Any positive time preference is unethical, at least across generations (Ramsey 1928).
  - Influence on the low discount rates used in the Stern Report (2006).
- Ethics imposes a sustainability constraint on time preference.

What is Sustainability?

- World Commission on Environment and Development (1987): a sustainable consumption plan

  "meets the needs of the present without compromising the ability of future generations to meet their own needs."

- Solow (1993):

  "A sustainable national economy is one that allows every future generation the option of being as well off as its predecessors. The duty imposed by sustainability is to bequeath to posterity not any particular thing . . . but rather to endow them with whatever it takes to achieve a standard of living at least as good as our own and to look after their next generation similarly."

Ethics as a Constraint

- Expressing ethics as a constraint, rather than incorporating ethics in preferences, is unfamiliar to many economists who tend to be philosophical utilitarians.
- However, it is consistent with the positions of many moral philosophers including Rawls (*A Theory of Justice*, rev. ed. 1999):

> *"The principles of right, and so of justice, put limits on which satisfactions have value; they impose restrictions on what are reasonable conceptions of one's good. In drawing up plans and deciding on aspirations men are to take these constraints into account."*

Formalizing Sustainability in a Riskless World

- In a riskless world, sustainability is straightforward (Arrow et al. 2004).
- Assume a single form of capital with a constant riskless rate of return.
- Sustainability constraint: the social rate of time preference cannot exceed the riskless interest rate.
- When the constraint binds, time preference equals the riskless rate so society consumes the return on its capital and leaves wealth unchanged over time.
- Then consumption, utility of current consumption, marginal utility of current consumption, and social value (the discounted present value of utility) are all constant over time.

Sustainability Allows More Patience than Ramsey

- Sustainable consumption is only feasible when the riskless rate is positive.
- Then, positive time preference is sustainable so the sustainability constraint is not as tight as Ramsey's zero-time-preference constraint.
- Sustainability responds to the Koopmans (1960, 1967) critique of Ramsey:

> *"One cannot adopt ethical principles without regard to . . . the anticipated technological possibilities. Any proposed optimality criterion needs to be subjected to a mathematical screening, to determine whether it does indeed bear on the problem at hand, under the circumstances assumed. More specifically, too much weight given to generations far in the future turns out to be self-defeating. It does nobody any good. How much weight is too much has to be determined in each case."*

What About Risk?

- In a risky economy the return on capital is uncertain so it is infeasible to guarantee that social value remains constant over time.
- Instead, we impose a weaker constraint, suggested but not analyzed by Howarth (1995), that social value is not **expected** to decline over time.
- This approach restricts the drift in utility in the same way that Campbell and Sigalov restrict the drift in consumption.

Model Ingredients

- Two assets:
  - Safe log return r_f.
  - Risky asset has expected excess return μ, Brownian volatility σ, and jumps of size L that arrive at rate ω.
  - If no jumps occur, expected excess return is $\widehat{\mu} = \mu + \omega \mathrm{E} L$.
- Representative investor chooses consumption-wealth ratio $\theta = C_t / W_t$ and risky portfolio share α to maximize

$$U_0 = \mathrm{E} \int_0^\infty e^{-\rho t} \frac{C_t^{1-\gamma}}{1-\gamma} \, dt,$$

where

$$\frac{dC}{C} = \frac{dW}{W} = \left[r_f + \alpha \underbrace{(\mu + \omega \mathrm{E} L)}_{\widehat{\mu}} - \theta \right] dt + \alpha \sigma \, dZ - \alpha L \, dN.$$

Microfoundations

- We take the representative investor's objective function as given.
- One microfoundation could be as in Blanchard (1985):
  - Individual agents have a constant probability of death.
  - No population growth, so each deceased agent is replaced by a newborn agent.
  - Wealth of deceased agents is reallocated to newborns.
- Then all agents alive at a given time are identical, and the time preference rate ρ reflects
  - True time preference within each individual's lifetime.
  - The probability of death (which raises ρ).
  - Any degree of intergenerational altruism (which lowers ρ).
- The sustainability constraint protects the interests of future generations vs those currently alive.
- The framework can accommodate population growth, but we discuss this later.

Unconstrained Solution

$$U_0 = \frac{W_0^{1-\gamma}}{1-\gamma} \frac{\theta^{1-\gamma}}{\rho - (1-\gamma)\left(r_f + \alpha\widehat{\mu} - \frac{1}{2}\gamma\alpha^2\sigma^2 - \theta\right) - \omega \mathrm{E}\left[(1-\alpha L)^{1-\gamma} - 1\right]}.$$

- Expected utility is a function of current wealth W_0.
- Maximize with respect to the optimal portfolio decision, α, and consumption-savings decision, θ.
- The optimal consumption-wealth ratio is

$$\theta_{\text{unc}} = \frac{\rho + (\gamma - 1)\left(r_f + \alpha\widehat{\mu} - \frac{1}{2}\gamma\alpha^2\sigma^2\right) - \omega \mathrm{E}\left[(1-\alpha L)^{1-\gamma} - 1\right]}{\gamma}.$$

- Impatience (high ρ) implies a high consumption-wealth ratio.
- We will show that if $\rho > \theta_{\text{unc}}$, then expected utility is expected to decline without limit.

Unconstrained Solution

$$U_0 = \frac{W_0^{1-\gamma}}{1-\gamma} \frac{\theta^{1-\gamma}}{\rho - (1-\gamma)\left(r_f + \alpha\widehat{\mu} - \frac{1}{2}\gamma\alpha^2\sigma^2 - \theta\right) - \omega \mathrm{E}\left[(1-\alpha L)^{1-\gamma} - 1\right]}.$$

- The optimal risky portfolio share is defined implicitly by

$$\widehat{\mu} - \alpha\gamma\sigma^2 = \omega \mathrm{E}\left[L\,(1-\alpha L)^{-\gamma}\right].$$

  - This is the classic Merton formula, $\alpha = \mu/\gamma\sigma^2$, when there are no jumps.

Binding Sustainability Constraint

- Zero drift in expected utility requires zero drift in $X_t = W_t^{1-\gamma}$.
- The stochastic process for X_t is

$$\begin{aligned}\frac{dX}{X} &= (1-\gamma)\left(r_f + \alpha\widehat{\mu} - \theta - \frac{1}{2}\gamma\alpha^2\sigma^2\right)dt + (1-\gamma)\alpha\sigma dZ \\ &\quad + \left[(1-\alpha L)^{1-\gamma} - 1\right]dN\,.\end{aligned}$$

- Hence if the constraint binds, we have

$$\theta_{\text{con}} = r_f + \alpha\widehat{\mu} - \frac{1}{2}\gamma\alpha^2\sigma^2 + \omega\frac{\mathrm{E}\left[(1-\alpha L)^{1-\gamma} - 1\right]}{1-\gamma}.$$

- This allows us to eliminate θ from the objective function. Maximizing with respect to α gives the same solution as before.
  - The sustainability constraint does **not** distort portfolio choice.

Sustainable C/W Exceeds the Riskless Interest Rate

- We can use the solution for α to rewrite the constrained consumption-wealth ratio as

$$\theta_{\text{con}} = r_f + \frac{1}{2}\gamma\alpha^2\sigma^2 + \omega\frac{\mathrm{E}\left[(1-\alpha\gamma L)(1-\alpha L)^{-\gamma} - 1\right]}{1-\gamma}.$$

- When risk and the risky portfolio share are positive, both the second two terms are positive.
- Hence, the sustainable consumption-wealth ratio exceeds the riskless interest rate.
  - Risk is critical for this result!

Sustainability Constraint as an Inequality

- The constraint is one-sided. We allow expected utility to drift up, not down. Equivalently, we allow the consumption-wealth ratio to be lower than the constrained level:

$$\theta_{\text{sus}} = \min\left\{\theta_{\text{unc}}, \theta_{\text{con}}\right\}.$$

- Comparing the constrained and unconstrained solutions for θ, we have

$$\theta_{\text{unc}} = \frac{1}{\gamma}\rho + \left(1 - \frac{1}{\gamma}\right)\theta_{\text{con}}.$$

  - The constraint binds if and only if $\rho > \theta_{\text{con}}$, or equivalently $\rho > \theta_{\text{unc}}$.
  - The constraint has a smaller effect when γ is very large (because then the unconstrained consumption path is close to flat).
  - The constraint can be implemented by imposing an adjusted time preference rate $\widehat{\rho} = \min\left\{\rho, \theta_{\text{con}}\right\}$.

Sustainable Drift in Wealth

- Although utility (equivalently, $X = W^{1-\gamma}$) has zero drift when the sustainability constraint binds, wealth W has a positive drift.
- The positive drift in wealth implies that sustainable utility is a tighter constraint than the sustainable spending constraint imposed by Campbell and Sigalov (2021).
  - Sustainable spending distorts portfolio choice, sustainable utility does not.
- Intuitively, risk cumulates over time so later generations are exposed to more of it. To compensate them, they must have higher wealth in expectation.

Sustainable Drift in Marginal Utility

- Although utility (equivalently, $X = W^{1-\gamma}$) has zero drift when the sustainability constraint binds, marginal utility $M = W^{-\gamma}$ has a positive drift.
- This is another way to understand the result that $\theta_{\text{con}} > r_f$, since the first-order condition for optimal investment in the riskless asset implies that

$$\mathrm{E}\frac{dM}{M} = \theta_{\text{con}} - r_f.$$

- But note that

$$X = MW.$$

  - How is it possible that X has zero drift when both M and W have positive drifts?
  - The answer is that M and W covary negatively, and

$$\frac{dX}{X} = \frac{dM}{M} + \frac{dW}{W} + \frac{dM}{M}\frac{dW}{W}.$$

  - Once again risk is critical!

Explicit Solution for Brownian Motion

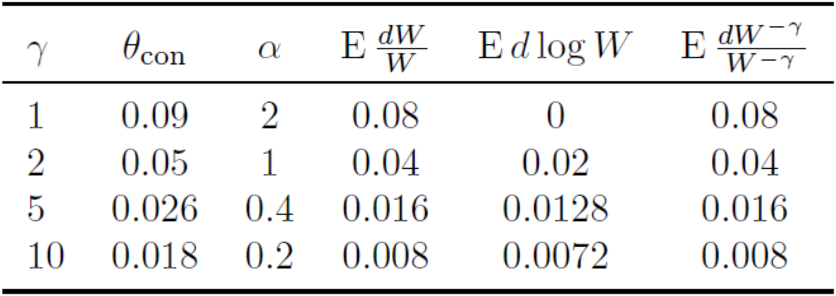
- If we turn off jumps, we get an explicit solution for the case of pure Brownian risk.
- Using the Merton portfolio rule for the Brownian case, $\alpha = \mu / \gamma \sigma^2$, we have

$$\theta_{\mathrm{con,BM}} = r_f + \frac{1}{2}\frac{\mu^2}{\gamma \sigma^2} = r_f + \frac{\alpha \mu}{2}.$$

- The sustainable consumption-wealth ratio is the average of the riskless interest rate and the return on the optimally invested portfolio.

Numerical Examples for Brownian Motion

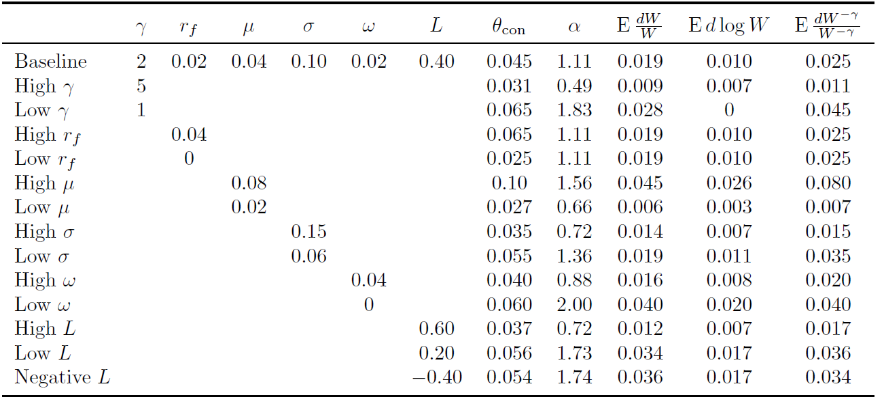
$r_f = 1\%$, $\mu = 8\%$, $\sigma = 20\%$, $\mu/\sigma = 0.4$.



γ	θ_{con}	α	$\mathrm{E}\,\frac{dW}{W}$	$\mathrm{E}\,d\log W$	$\mathrm{E}\,\frac{dW^{-\gamma}}{W^{-\gamma}}$
1	0.09	2	0.08	0	0.08
2	0.05	1	0.04	0.02	0.04
5	0.026	0.4	0.016	0.0128	0.016
10	0.018	0.2	0.008	0.0072	0.008

Numerical Examples with Jumps

Baseline: $r_f = 2\%$, $\mu = 4\%$, $\sigma = 10\%$, $L = 40\%$, $\omega = 4\%$.



	γ	r_f	μ	σ	ω	L	θ_{con}	α	$\mathrm{E}\,\frac{dW}{W}$	$\mathrm{E}\,d\log W$	$\mathrm{E}\,\frac{dW^{-\gamma}}{W^{-\gamma}}$
Baseline	2	0.02	0.04	0.10	0.02	0.40	0.045	1.11	0.019	0.010	0.025
High γ	5						0.031	0.49	0.009	0.007	0.011
Low γ	1						0.065	1.83	0.028	0	0.045
High r_f		0.04					0.065	1.11	0.019	0.010	0.025
Low r_f		0					0.025	1.11	0.019	0.010	0.025
High μ			0.08				0.10	1.56	0.045	0.026	0.080
Low μ			0.02				0.027	0.66	0.006	0.003	0.007
High σ				0.15			0.035	0.72	0.014	0.007	0.015
Low σ				0.06			0.055	1.36	0.019	0.011	0.035
High ω					0.04		0.040	0.88	0.016	0.008	0.020
Low ω					0		0.060	2.00	0.040	0.020	0.040
High L						0.60	0.037	0.72	0.012	0.007	0.017
Low L						0.20	0.056	1.73	0.034	0.017	0.036
Negative L						−0.40	0.054	1.74	0.036	0.017	0.034

Population Growth

- Population growth is notoriously challenging for intertemporal ethics, particularly when population is endogenous (Parfit 1984, Dasgupta 2001).
- But we can accommodate exogenous deterministic population growth, if we assume that the utility of a representative newborn individual must have zero drift.
- Since society's wealth must be shared among a growing number of people, we need to increase saving to compensate.
- The sustainable consumption-wealth ratio declines by the population growth rate g.
  - It can be lower than the riskfree rate.
  - But for realistic parameters, it will remain higher than the riskfree rate as our numerical examples show.

Conclusion

- In a risky world, there is no inconsistency between a substantial rate of time preference and the ethical criterion of sustainability.
  - Intuitively, high risky returns make it attractive for society to save for the future even if people are impatient and would dissave if only a safe asset with a low return were available.
- In a risky world, there is no unique discount rate for investments.
  - The low riskfree interest rate should be used to discount safe investments, and this is a lower discount rate than the sustainable rate of time preference.
- In the spirit of Andy Abel, we have made these points using a deliberately simple model.
  - We have ignored parameter uncertainty (Weitzman 2001).
  - We have ignored time-varying expected returns and the resulting term structure of discount rates (Gollier 2002, Bansal and Yaron 2004).