Real Cash Flow Expectations and Asset Prices

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ABSTRACT

Using survey forecasts, we find that systematic errors in expectations of *long-term inflation* and *short-term nominal earnings growth* are the main driver of prices and return puzzles for bonds and stocks. We demonstrate this by deriving and testing a single necessary and sufficient condition based on accounting identities. Errors in expectations of short-term inflation and long-term nominal earnings growth do not play a role in either asset market. Because of these systematic errors, real cash flow expectations closely match aggregate bond and stock prices, leaving little room for time-varying discount rates. These expectations also accurately match key return puzzles for bonds and stocks: the rejection of the expectations hypothesis and stock return predictability. These results are consistent with a simple model in which agents believe the persistences of inflation and nominal earnings growth are magnified versions of the objective persistences.

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How do expectations of inflation and nominal earnings growth affect bond and stock markets? Prices are discounted expectations of real cash flows. Under standard rational expectations (RE), investors know the objective probability distribution where real cash flows empirically do not account for a large amount of price movements.[1] This implies that bond and stock markets are mainly driven by time-varying discount rates, which has motivated many models attempting to generate movements over time in investors' discount rates for safe and risky assets. However, under more general subjective expectations, real cash flow expectations may have a large impact on both markets due to systematic forecast errors.

In this paper, we measure subjective expectations from surveys and document the importance of expected *long-term inflation* and *short-term nominal earnings growth* in driving bond and stock markets.[2] Using accounting identities, we derive a single condition on forecast errors in subjective expectations which is (i) necessary and sufficient to explain price movements and return puzzles and (ii) easily testable. Testing this condition acts as a diagnostic tool, identifying where the errors relevant for asset pricing do or do not appear in subjective expectations. We document that expectations of long-term inflation and short-term nominal earnings growth satisfy this condition, while expectations of short-term inflation and long-term nominal earnings growth do not. Additionally, most movements in inflation expectations occur in long-term expectations while most movements in nominal earnings growth expectations occur in short-term expectations.

Because of these systematic forecast errors, time-varying discount rates play almost no role in aggregate stock prices and play only a secondary role in aggregate bond yields. Subjective expectations of real cash flows move one-for-one with the S&P 500 price-earnings ratio and price-dividend ratio and the ten-year Treasury yield, with R^2's of 0.81, 0.79, and 0.66, respectively. Additionally, these expectations replicate two key return puzzles: the predictability of returns for stocks and the rejection of the expectations hypothesis for bonds. These results, as well as our findings on the term structure of subjective expectations are consistent with a simple model in which agents believe the persistences of inflation and

[1] See Campbell and Shiller (1988); Ang and Piazzesi (2003); Buraschi and Jiltsov (2005); Cochrane and Piazzesi (2005); Cochrane (2011); Koijen and Van Nieuwerburgh (2011); Abrahams et al. (2016). Instead, these price movements predict future returns.

[2] For bonds, real cash flows solely depend on inflation. For stocks, real cash flows depend on inflation as well as nominal earnings growth.

nominal earnings growth are magnified versions of the objective persistences.

We first establish a single necessary and sufficient condition for subjective expectations of real cash flows to explain asset price movements, stock return predictability, and the rejection of the expectations hypothesis. The condition is that forecast errors for these expectations must be predictable using current prices. Because this condition is derived from accounting identities, it is general and holds for any deviation from RE, e.g. learning, rational inattention, bounded rationality, behavioral biases, etc. For stocks, predictable forecast errors for real earnings growth cause price ratios to predict future returns even if discount rates are constant. For bonds, predictable forecast errors for inflation cause holding returns to be excessively volatile relative to the expectations hypothesis, even if term premia are constant.

We then document the importance of long-term inflation expectations and short-term nominal earnings growth expectations using survey forecasts of CPI inflation and nominal earnings growth for the S&P 500. This is done in two ways. First, we find that long-term inflation expectations and short-term nominal earnings growth expectations are the only expectations that meet our single condition.[3] This highlights the role of these expectations in driving asset price movements and return puzzles. Second, we show that movements in inflation expectations are concentrated at long horizons while movements in nominal earnings growth expectations are concentrated at short horizons. This highlights the importance of long-term inflation expectations and short-term nominal earnings growth expectations even outside the context of asset pricing. Changes in short-term inflation expectations are associated with large changes in long-term expectations. In contrast, long-term nominal earnings growth expectations are relatively flat compared to short-term expectations.

To demonstrate that satisfying our single condition reduces the need for time-varying discount rates, we test how well subjective expectations of real cash flows match movements in bond and stock prices. Specifically, we calculate fundamental values for the S&P 500 price-earnings and price-dividend ratios and the ten-year zero-coupon Treasury yield based on the survey forecasts and constant discount rates. Over our 1976-2018 sample, these fundamental prices closely match the observed prices, with regression coefficients near 1 and high R^2's

[3]While short-term inflation expectations and long-term nominal earnings growth expectations may contain systematic forecast errors (see Coibion and Gorodnichenko (2015) and Bordalo et al. (2020)), these forecast errors do not satisfy the necessary and sufficient condition for explaining aggregate asset prices and return puzzles.

of 0.81, 0.79, 0.66, respectively. A direct corollary to this finding is that discount rates play only a secondary role in price movements. Even if we attribute all residual variation in the observed prices to movements in discount rates, this discount rate variation would only account for 19-21% of variation in aggregate stock price ratios and 34% of variation in aggregate bond yields.

Despite having constant discount rates, these fundamental prices also replicate two key return puzzles in the asset pricing literature: the predictability of stock returns and the rejection of the expectations hypothesis.[4] Empirically, movements in the price-earnings ratio primarily predict future returns, with regression coefficients of -0.77 and -0.59 for ten-year nominal and real returns, respectively. Movements in the fundamental price-earnings ratio also primarily predict future returns, with nearly identical coefficients of -0.77 and -0.54.[5] Intuitively, the fundamental price-earnings ratio predicts future returns because it reflects errors in expectations of real cash flows. For bonds, we focus on excess volatility in holding returns as documented in Shiller (1979); Singleton (1980).[6] Under RE, the expectations hypothesis places a bound on the variance of holding returns which is violated in the data by a factor of 5.60. Holding returns based on fundamental yields violate the bound by virtually the same factor, 5.22. Even though the expectations hypothesis holds for fundamental yields, variation in errors in expectations increase the volatility of holding returns.

Our empirical results have strong implications for models of expectation formation. Our diagnostic test reveals that some expectations, but not all expectations, contain significant errors relevant for asset prices and return puzzles and that the term structure of these errors differs between inflation expectations and nominal earnings growth expectations. In addition to discussing the implications for models of learning or behavioral biases, we also provide a concrete example of a simple model that successfully accounts for all of our main results. This model is based on the representativeness bias of Tversky and Kahneman (1974), which causes agents to exaggerate the objective persistence of forecasted variables, in line with the experimental findings of Afrouzi et al. (2021). For a process with a high (low) objective persistence, this causes errors to be concentrated in long-term (short-term) expectations.

[4] See Campbell and Shiller (1988); Shiller (1979) for early summaries of these puzzles and Cochrane (2011); Gürkaynak and Wright (2012) for more recent reviews of the literature.

[5] We also get almost identical coefficients of -0.88, -0.69 and -0.92, -0.69 for the observed price-dividend ratio and the fundamental price-dividend ratio.

[6] The holding return is the return from buying a long-term bond and selling it one period later.

This allows this one bias to match the stark differences in the term structure of inflation expectations and nominal earnings growth expectations. In the model, agents have constant discount rates and price stocks and bonds based on their expectations of real earnings growth and inflation, respectively. Due to predictable errors in their forecasts, stock price ratios predict future returns and holding returns on bonds are excessively volatile. We set the degree of representativeness bias to match subjective expectations of long-term inflation and short-term nominal earnings growth. The model quantitatively matches the concentration of movements and errors at long horizons for inflation expectations and short horizons for nominal earnings growth expectations, as well as the magnitude of both return predictability and excess volatility in holding returns.

A long-standing literature attempts to explain return puzzles through mispricing rather than time-varying discount rates.[7] Our empirical analysis guides the models in this literature, informing where errors should and should not arise in expectations. Because our single condition is derived from accounting identities, it applies to a broad set of models including but not limited to behavioral biases, learning, and rational inattention. By testing this condition on survey expectations, we show that the relevant errors for aggregate return puzzles occur in long-term inflation expectations and short-term nominal earnings growth expectations, regardless of the proposed mechanism. Beyond this, we document key differences between the term structure of inflation expectations and the term structure of nominal earnings growth expectations, which provides useful targets for disciplining a general model of expectation formation.

This paper also contributes to a growing empirical literature in finance which directly uses survey expectations, rather than expectations based on regressions, to understand aggregate asset price movements. This literature primarily focuses on expectations of returns and price growth (Bacchetta, Mertens, and Van Wincoop (2009); Greenwood and Shleifer (2014); Piazzesi, Salomao, and Schneider (2015); Giglio et al. (2021); Brunnermeier et al. (2021)). We emphasize the importance of real cash flow expectations in line with Nagel and Xu (2021). De la O and Myers (2021) (DM) and Bordalo et al. (2020) (BGLS) both study nominal earnings growth expectations for stocks through the Campbell-Shiller variance decomposition. DM show that nominal earnings growth expectations explain the majority of

[7]See Hirshleifer (2015) for a survey of the literature using behavioral biases and see Collin-Dufresne, Johannes, and Lochstoer (2017) for an example using Bayesian learning.

the variance of the price-earnings ratio using short-term expectations and an approximation for long-term expectations. BGLS directly measure long-term nominal expectations and confirm the findings of DM.[8] Additionally, BGLS document predictable errors in long-term nominal earnings growth expectations.

We make three key contributions to this literature. First, we study bond markets and term premia, as well as stocks, using inflation expectations to link the two markets. Second, we establish a single necessary and sufficient condition for forecast errors in expectations to explain asset prices and return puzzles. Testing the nominal earnings growth forecast errors, we find that only forecast errors in short-term expectations meet this condition. Errors in long-term nominal earnings growth expectations do not appear to play a role. Third, we document new systematic errors in long-term inflation expectations that explain bond price movements and substantially help in explaining stock price movements relative to just using nominal earnings growth expectations.

More broadly, a general literature has studied the accuracy of inflation expectations. Focusing on short-term expectations, Ang, Bekaert, and Wei (2007); Del Negro and Eusepi (2011); Chernov and Mueller (2012) do not find any significant errors and Cieslak (2018) finds that forecast errors in short-term inflation expectations do not contribute to return predictability in bonds. We study both short-term and long-term inflation expectations and document significant predictable errors in long-term expectations. These errors are predictable with yields for long-term bonds and stock prices, indicating that these errors influence both risk-free and risky asset prices and contribute to return puzzles. These errors have a similar effect on stock prices as the money illusion hypothesis of Modigliani and Cohn (1979), causing prices to be too low in periods of high expected inflation. However, this is not caused by investors failing to account for inflation when pricing stocks, but rather by investors accounting for inflation using biased expectations, similar to Katz, Lustig, and Nielsen (2017). Additionally, we find that the information or bias that drives long-term expectations must be highly correlated with current short-term expectations.

The sections are organized as follows. Section I establishes a single condition for forecast errors of real cash flows to explain asset price movements and return puzzles. Section II describes the data, establishes the importance of long-term inflation expectations and short-

[8]DM estimate that expectations explain 63% of price-earnings ratio variance. BGLS find that short-term and long-term expectations explain 62% of price-earnings ratio variance.

term nominal earnings growth expectations, and discusses the implications for models of expectation formation. Section III measures the ability of real cash flow expectations to match observed asset prices and return puzzles. Section IV proposes a model in which agents exaggerate objective persistences which matches our main findings. Section V concludes.

I. Prices and Returns under Subjective Expectations

Movements in bond and stock markets must be due to changes in investors' expectations of real cash flows or real returns. In this section, we establish a single necessary and sufficient condition for errors in expectations to explain asset prices and return puzzles. This single condition provides a useful diagnostic tool to determine if deviations from RE can account for price movements and return puzzles. Importantly, not all errors are relevant and we include an intuitive example of a systematic error that does not satisfy this condition.

We first show how subjective expectations of real cash flows can explain price movements, even if expectations of real cash flows under RE are uncorrelated with prices. We then derive two additional identities which show that stock return predictability and excess volatility of bond returns can be explained by forecast errors in real cash flow expectations rather time-varying discount rates. For all three identities, the key condition is that forecast errors must be predictable with current prices.[9]

A. *Price identities*

We start with price identities based on the definition of returns. For stocks, we use the approximate log-linearized return, which states the one-period real return in terms of real earnings growth $\Delta\tilde{e}_{t+1}$ and the price-earnings ratio pe_{t+1}, all in logs:

$$\tilde{r}_{t+1} \approx \kappa + \Delta\tilde{e}_{t+1} - pe_t + \rho pe_{t+1} \tag{1}$$

where κ is a constant, $\rho = e^{\bar{pd}} / \left(1 + e^{\bar{pd}}\right) < 1$ and $\bar{pd}$ is the mean value of the log price-dividend ratio. By imposing a no-bubble condition, $\lim_{T\to\infty} \rho^T pe_{t+T} = 0$, we can iterate this

[9] The Appendix gives the full derivation of all equations in this section.

equation and apply subjective expectations to get

$$pe_t \approx \frac{1}{1-\rho}\kappa + \sum_{j=1}^{\infty} \rho^{j-1} E_t^* \left[\Delta \tilde{e}_{t+j}\right] - \sum_{j=1}^{\infty} \rho^{j-1} E_t^* \left[\tilde{r}_{t+j}\right]. \tag{2}$$

Equation (2) states that an increase in the price-earnings ratio must reflect an increase in subjective real earnings growth expectations or a decrease in subjective real return expectations. These subjective expectations do not need to be rational. Because equation (2) is derived from the definition of real returns, it holds under any probability distribution.

For bonds, we use the return from holding an n-period zero-coupon bond for one period. Expressing all variables in logs, the holding return from t to $t+1$ is defined as

$$h_{t+1}^{(n)} = n y_t^{(n)} - (n-1)\, y_{t+1}^{(n-1)}. \tag{3}$$

Iterating and applying subjective expectations, the yield is

$$y_t^{(n)} = \frac{1}{n}\sum_{j=1}^{n} E_t^* \left[\pi_{t+j}\right] + \frac{1}{n}\sum_{j=1}^{n} E_t^* \left[\tilde{h}_{t+j}^{(n+1-j)}\right] \tag{4}$$

where $\tilde{h}_t^{(n)}$ is the real holding return. Intuitively, an increase in the bond yield must be due to higher subjective inflation expectations or higher subjective real holding return expectations. This is analogous to equation (2), where prices depend on expectations of real cash flows and real returns. For bonds, real cash flows only depend on inflation, as the nominal cash flow is known, whereas for stocks this will depend on inflation and nominal earnings growth Δe_{t+j}. For the zero-coupon bond, there is a single real cash flow at maturity, so inflation in different years all have the same impact on the yield. For stocks, there are real cash flows every period, so real earnings growth in earlier years will have a larger impact on prices than real earnings growth in later years, which is reflected in the ρ^{j-1} terms.

B. Price movements and return puzzles

Given these equations for the price-earnings ratio and bond yields, we establish three identities which connect discount rates and errors in expectations to asset prices and return puzzles. Our two return puzzles are the predictability of stock returns using price ratios, such as the price-earnings ratio or price-dividend ratio, and the rejection of the expectations hypothesis for bond yields. Other than the subjective expectations $E_t^*[\cdot]$, all operators use the objective probability distribution. For example, $Var(\cdot)$ and $Cov(\cdot,\cdot)$ denote the observable variance or covariance of variables. This allows us to use the subjective expectations observed in the survey data to explain empirical puzzles measured by an econometrician such as the comovement between current prices and future returns.

First, the comovement of prices with subjective expectations of real cash flows is

$$Cov\left(pe_t, E_t^*\left[\Delta\tilde{e}_{t+j}\right]\right) = Cov\left(pe_t, E_t\left[\Delta\tilde{e}_{t+j}\right]\right) - Cov\left(pe_t, f_t^{\Delta e_{t+j}} - f_t^{\pi_{t+j}}\right) \quad (5)$$

$$Cov\left(y_t^{(n)}, E_t^*\left[\pi_{t+j}\right]\right) = Cov\left(y_t^{(n)}, E_t\left[\pi_{t+j}\right]\right) - Cov\left(y_t^{(n)}, f_t^{\pi_{t+j}}\right) \quad (6)$$

where $E_t[\cdot]$ represents expectations under RE which use the objective probability distribution, $f_t^{\Delta e_{t+j}}$ is the forecast error for nominal earnings growth $\Delta e_{t+j} - E_t^*\left[\Delta e_{t+j}\right]$, and $f_t^{\pi_{t+j}}$ is the forecast error for inflation $\pi_{t+j} - E_t^*\left[\pi_{t+j}\right]$. Note that these identities hold regardless if subjective expectations $E_t^*[\cdot]$ are rational. If subjective expectations are not rational and the comovement of subjective expectations with prices differs from the comovement of rational expectations with prices, then there must be an observable comovement between prices and forecast errors.

Second, from equations (1)-(2), the ability of the price-earnings ratio to predict future real returns is

$$\begin{aligned} Cov\left(pe_t, \sum_{j=1}^{\infty}\rho^{j-1}\tilde{r}_{t+j}\right) &= Cov\left(pe_t, \sum_{j=1}^{\infty}\rho^{j-1}E_t^*\left[\tilde{r}_{t+j}\right]\right) \\ &+ Cov\left(pe_t, \sum_{j=1}^{\infty}\rho^{j-1}\left[f_t^{\Delta e_{t+j}} - f_t^{\pi_{t+j}}\right]\right). \end{aligned} \quad (7)$$

Note that identities (5) and (7) on comovement with prices and return predictability also hold if we replace the price-earnings ratio pe_t with the price-dividend ratio pd_t. Third, using equations (3)-(4), we generalize the Shiller (1979) variance bound for holding returns,

$$\begin{aligned} Var\left(h_{t+1}^{(n)}\right) &\leq \eta Var\left(y_t^{(1)}\right) + 2nCov\left(y_t^{(n)}, E_t^*\left[h_{t+1}^{(n)}\right] - y_t^{(1)}\right) \\ &- 2nCov\left(y_t^{(n)}, \sum_{j=2}^{n} f_t^{\pi_{t+j}} + f_t^{\tilde{h}_{t+j}^{(n+1-j)}}\right) \end{aligned} \quad (8)$$

where $\eta = n^2/(2n-1)$.

Under RE, equations (7)-(8) provide simple tests of time-varying discount rates and the expectations hypothesis. Forecast errors are unpredictable under RE so the final terms in both equations are zero. Therefore, the observed comovement between prices and future returns in (7) must be due to time-variation in investors' discount rates, $E_t^*[\tilde{r}_{t+j}]$. Similarly, under the expectations hypothesis, which posits that expected term premia $E_t^*\left[h_{t+1}^{(n)}\right] - y_t^{(1)}$ are constant, equation (8) reduces to the standard variance bound proposed in Shiller (1979). This bound restricts the variance of holding returns to be less than $\eta Var\left(y_t^{(1)}\right)$. Therefore, the observed excess volatility of holding returns beyond this bound rejects the expectations hypothesis and implies that expected term premia must be time-varying.

C. Single necessary and sufficient condition

If we allow for more general subjective expectations, then equations (5)-(8) show how expectations of real cash flows can explain asset prices, predictable stock returns, and excess volatility in bond holding returns. Importantly, all these phenomena share a single necessary and sufficient condition, namely that forecast errors for real cash flows must comove with current prices.[10] In other words, this single condition acts as a useful diagnostic tool to determine whether deviations from RE explain asset prices and return puzzles. Additionally, this allows us to measure the relative importance of subjective expectations at different horizons j by comparing the covariance of the forecast errors with current prices.

If prices predict forecast errors, then equations (5)-(6) show that subjective expectations of real cash flows $E_t^*[\cdot]$ will comove with prices even if prices are uncorrelated with expectations of real cash flows under RE $E_t[\cdot]$. Similarly, if the price-earnings ratio comoves with forecast errors, then the price-earnings ratio will predict future real returns even if discount rates $E_t^*[\tilde{r}_{t+j}]$ are constant over time.[11] Finally, forecast errors that comove with the yield relax the bound in equation (8), implying that excess volatility of holding returns beyond $\eta Var\left(y_t^{(1)}\right)$ does not violate the expectations hypothesis. Predictable forecast errors generate excess volatility even if the expectations hypothesis holds, i.e., expected term premia $E_t^*\left[h_{t+1}^{(n)}\right] - y_t^{(1)}$ are constant.

In contrast, errors that do not satisfy this condition will have no impact on equations (5)-(8). To give an intuitive example of a systematic error that does not satisfy the condition, suppose there is a variable x_t that predicts real earnings growth but investors do not know

[10] For stocks, this condition translates to pe_t negatively comoving with forecast errors for real earnings growth. For bonds, this condition translates to yields negatively comoving with forecast errors for inflation.

[11] By extension, these forecast errors also increase the comovement between prices and nominal returns $Cov(pe_t, \tilde{r}_{t+j}) + Cov(pe_t, \pi_{t+j})$.

this. For simplicity, assume that x_t is independent of the investors' information set I_t. In this example, an econometrician would find systematic errors in investors' expectations, as x_t predicts forecast errors. Further, the econometrician would find that x_t predicts real stock returns. However, these systematic errors do not drive price movements, as prices are only a function of I_t. As a result, these errors will not explain why prices or price ratios predict future returns.

II. Term Structure of Subjective Expectations

In this section, we emphasize the importance of long-term inflation expectations and short-term nominal earnings growth expectations. We show this empirically in two ways and then discuss the implications of these empirical results for models of learning and behavioral biases.

First, in Section II.A, we show that movements in inflation expectations are concentrated at long horizons while movements in nominal earnings growth expectations are concentrated at short horizons. This is informative even outside the context of asset pricing, as it demonstrates that individuals believe changes in inflation will be highly persistent and believe changes in nominal earnings growth will be fairly transitory. We also discuss the implications of these movements in inflation expectations and nominal earnings growth expectations for real earnings growth expectations.

Second, in Section II.B, we find that long-term inflation expectations and short-term nominal earnings growth expectations are the only expectations that satisfy the single necessary and sufficient condition from Section I for explaining asset price movements and return puzzles. This test acts as an important diagnostic tool, identifying where the errors relevant for asset pricing do and do not appear in subjective expectations. It is important to note that this test does not determine the primitive cause of the errors. The results could still be consistent with models of learning, extrapolation, or other mechanisms so long as these mechanisms impact asset prices through the relevant horizons identified by the single condition. In Section II.C, we discuss the implications of this test for models of expectation formation, specifically models of learning and behavioral biases. For example, for inflation our results favor models of learning or biased beliefs about the long-term components of inflation such as the long-run mean.

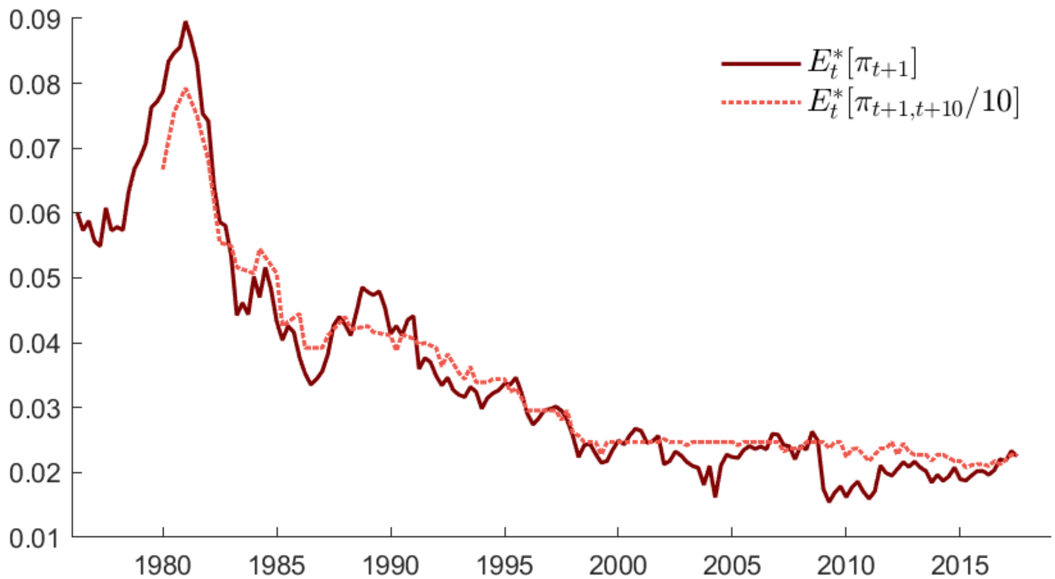

Figure 1. Subjective inflation expectations. The figure compares subjective expectations for one-year inflation (solid) and subjective expectations for annualized ten-year inflation (dotted). All variables are in logs.

A. Movements in subjective expectations

A.1. Inflation Expectations

We use quarterly median inflation forecasts from the Survey of Professional Forecasters. To understand the relation between short-term and long-term inflation expectations, we analyze one-year inflation expectations from 1976Q1-2018Q2 and expectations of average inflation over the next ten years from 1979Q4-2018Q2. Figure 1 shows that one-year expectations, denoted as $E_t^*[\pi_{t+1}]$, rise sharply in the late 1970's. These expectations reach a peak of 9% before falling steadily during the Volcker period in the 1980's and stabilize after 2000. Importantly, average ten-year expectations, denoted as $E_t^*[\pi_{t+1,t+10}/10]$, closely follow one-year expectation movements. When short-term expectations are high, average ten-year expectations are high and when short-term expectations are low, long-term expectations are low. This relationship suggests that analysts believe movements in expected one-year inflation will largely persist and be observed in long-term inflation.

To quantify the relationship between expectations of short-term and long-term inflation,

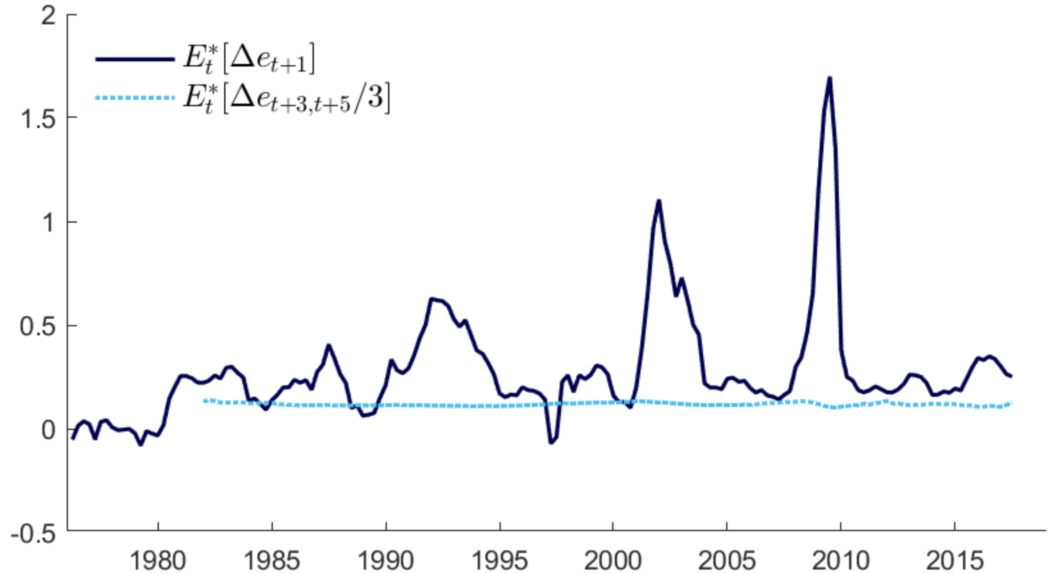

Figure 2. Subjective nominal earnings growth expectations. The figure compares subjective expectations of one-year nominal earnings growth (solid) and the subjective expectations for annualized three-to-five-year nominal earnings growth (dashed) for the S&P 500. All variables are in logs.

we measure the believed annual persistence ϕ_π from

$$E_t^*\left[\pi_{t+1+j}\right] \;=\; \alpha_{\pi,j} + \phi_\pi^j E_t^*\left[\pi_{t+1}\right] + \varepsilon_{t,j}^\pi. \tag{9}$$

An increase in one-year expectations is associated with a ϕ_π increase in $E_t^*\left[\pi_{t+2}\right]$, ϕ_π^2 increase in $E_t^*\left[\pi_{t+3}\right]$, and so on. Using one-year expectations and expectations of average inflation over the next ten years $E_t^*[\pi_{t+1,t+10}/10]$, we estimate ϕ_π as 0.96 (0.01). We choose this form because of its simplicity and the fact that it nests standard AR(1) processes. The constants $\alpha_{\pi,j}$ will not matter for our analysis as they will not affect comovements or variances.

A.2. Nominal Earnings Growth Expectations

We follow De la O and Myers (2021) to construct short–term (one-year) nominal earnings growth expectations for the S&P 500 from the Thomson Reuters I/B/E/S individual firm forecasts. We also estimate long-term (three-to-five year) nominal earnings growth expectations using the earnings-weighted average of long-term growth forecasts (LTG) for individual firms.[12] Figure 2 shows one-year nominal earnings growth expectations from 1976Q1 up to

[12]Full details on the construction can be found in the Appendix.

2018Q2, and average three-to-five-year nominal earnings growth expectations from 1986Q1 to 2018Q2. Expectations of one-year nominal earnings growth are volatile, with a standard deviation of 26.4%, rising substantially after the large decline in earnings during financial crisis. In comparison, expectations of three-to-five-year nominal earnings growth have much lower volatility, with a standard deviation of only 1.2%. This suggests that movements in nominal earnings growth expectations are primarily concentrated at short horizons. The fact that movements in short-term expectations are not matched by similar movements in long-term expectations indicates that analysts believe movements in expected one-nominal earnings growth will not persist through to long-term nominal earnings growth.

We quantify the relationship between expectations of short-term and long-term nominal earnings growth by measuring the believed annual persistence ϕ_e from

$$E_t^*\left[\Delta e_{t+j}\right] \quad = \quad \alpha_{e,j} + \phi_e^j E_t^*\left[\Delta e_{t+1}\right] + \varepsilon_{t,j}^e. \tag{10}$$

Using the one-year and three-to-five-year expectations, we estimate a small ϕ_e of 0.004 (0.075).[13] This sharply contrasts with the large, significant estimate of ϕ_π of 0.96.

A.3. Real earnings growth expectations

Throughout the paper, we split real earnings growth into nominal earnings growth and inflation to align with the available survey data. Looking at Figures 1 and 2, we see that short-term real earnings growth expectations are primarily driven by changes in short-term nominal earnings growth expectations, with short-term inflation expectations playing only a small part. However, inflation expectations do play an important role at long horizons. While long-term nominal earnings growth expectations have virtually no trend over time, long-term inflation expectations have dropped considerably. This implies that expectations of real earnings growth at long horizons have substantially increased over time.

One might expect that the lack of a decline in long-term nominal earnings growth expectations must represent a systematic error. However, over this period, realized nominal earnings growth also did not decline despite the large drop in realized inflation. Section II.B formally shows that forecast errors in expectations of long-term nominal earnings growth do not play a role in explaining asset prices. Instead, Section II.B finds that forecast errors in

[13] As with the estimation of the believed persistence of inflation, the constants $\alpha_{\pi,j}$ will not matter for our analysis as they will not affect comovements or variances.

long-term inflation expectations explain asset prices, indicating that the relevant errors in long-term real earnings growth expectations come from inflation expectations.

B. Testing single condition

Section I establishes a single necessary and sufficient condition for subjective expectations of real cash flows to explain asset prices, stock return predictability, and excess bond return volatility: current prices must predict forecast errors for real cash flows. For stocks, this means that the current price-earnings ratio positively predicts forecast errors for inflation and negatively predicts forecast errors for nominal earnings growth. For bonds, this means that the current yield negatively predicts forecast errors for inflation. In this section, we empirically test if subjective expectations satisfy this condition. We find that expectations of long-term inflation and short-term nominal earnings growth satisfy the condition while expectations of short-term inflation and long-term nominal earnings growth do not. Table I shows the results. For robustness, we also test the condition using the price-dividend ratio rather than the price-earnings ratio and find the same results.

We first show that forecast errors for short-term inflation expectations do not satisfy the condition for the price-earnings ratio. The first row of Panel A shows the comovement of the price-earnings ratio with expected and realized short-term inflation, as well as the comovement with the forecast error. We see that expected short-term inflation and realized short-term inflation both have significant negative comovements with the price-earnings ratio and that the magnitudes are quite similar. As a result, forecast errors do not significantly comove with the price-earnings ratio, meaning that forecast errors for short-term inflation do not explain movements in the price-earnings ratio or the stock return puzzle. This is consistent with a large literature documenting the accuracy of short-term inflation expectations.[14]

In contrast, we document new systematic errors in long-term inflation expectations that do satisfy the condition. Consistent with the findings of Section II.A, we find a large amount of action in long-term inflation expectations. The second row of Panel A shows that long-term inflation expectations have a significant negative comovement with the price-earnings ratio of -3.97 that is substantially larger than the comovement of short-term expectations with

[14] Ang, Bekaert, and Wei (2007); Del Negro and Eusepi (2011); Chernov and Mueller (2012); Cieslak (2018).

Table I

Testing single condition

This table tests the single necessary and sufficient condition for expectations to explain asset price movements and return puzzles. The condition is that prices must comove with forecast errors. This table shows the covariance of price ratios and bond yields with expected and realized inflation and nominal earnings growth and the forecast errors. In *Panel A*, the first row shows the covariance of the S&P 500 price-earnings ratio with short-term inflation expectations $E_t^*[\pi_{t+1}]$, realized short-term inflation π_{t+1}, and the forecast errors $\pi_{t+1} - E_t^*[\pi_{t+1}]$ from 1976Q1 to 2018Q2. The second row shows the covariance of the S&P 500 price-earnings ratio with long-term inflation expectations $E_t^*[\pi_{t+1,t+10}]$, realized long-term inflation $\pi_{t+1,t+10}$, and the forecast errors $\pi_{t+1,t+10} - E_t^*[\pi_{t+1,t+10}]$ using quarterly data from 1979Q4 to 2018Q2. The third and fourth rows show analogous results using the ten-year Treasury yield instead of the S&P 500 price-earnings ratio. In *Panel B* , the first row shows the covariance of the S&P 500 price-earnings ratio with short-term nominal earnings growth expectations $E_t^*[\Delta e_{t+1}]$, realized short-term nominal earnings growth Δe_{t+1}, and the forecast errors $\Delta e_{t+1} - E_t^*[\Delta e_{t+1}]$ from 1976Q1 to 2018Q2. The second row shows the covariances of the S&P 500 price ratios with long-term nominal earnings growth expectations $E_t^*[\Delta e_{t+3,t+5}]$, realized long-term nominal earnings growth $\Delta e_{t+3,t+5}$ and the forecast errors $\Delta e_{t+3,t+5} - E_t^*[\Delta e_{t+3,t+5}]$ from 1982Q1 to 2018Q2. Expectations expressed in percentages. We use small-sample adjusted Newey-West standard errors. Superscripts indicate significance at the 1% (***), 5% (**), and 10% (*) level.

	Panel A: Inflation			
	Horizon	Expected	Realized	Forecast Error
$Cov(pe_t, \cdot)$	Short-term (π_{t+1})	-0.66^{***}	-0.82^{***}	-0.16
	Long-term ($\pi_{t+1,t+10}$)	-3.97^{**}	-1.66^{*}	2.31^{***}
$Cov\left(y_t^{(10)}, \cdot\right)$	Short-term (π_{t+1})	0.04^{***}	0.04^{***}	0.00
	Long-term ($\pi_{t+1,t+10}$)	0.25^{***}	0.13^{***}	-0.13^{***}
	Panel B. Nominal Earnings Growth			
$Cov(pe_t, \cdot)$	Short-term (Δe_{t+1})	9.44^{***}	5.18	-4.26^{***}
	Long-term ($\Delta e_{t+3,t+5}$)	0.36	2.71	2.35

the price-earnings ratio of -0.66. This is also significantly larger than the covariance of the price-earnings ratio with future realized long-term inflation of -1.66, leading to significant forecast errors that are predictable with the current price-earnings ratio. This matches our high believed persistence ϕ_π of 0.96, as $E_t^*[\pi_{t+1}](1-\phi_\pi^{10})/(1-\phi_\pi)$ has a nearly identical covariance with the price-earnings ratio of -3.85 over the same sample.

We find the same results when we test the condition using bond yields rather than the price-earnings ratio, meaning that systematic errors in long-term inflation expectations also explain price movements and return puzzles for bonds. Expected and realized short-term inflation have virtually the same comovement with the ten-year bond yield at 0.04, leading to an insignificant comovement between the yield and forecast errors. Compared to short-term expectations, long-term expectations comove much more with the yield at 0.25, again emphasizing the importance of movements in longer horizon inflation expectations. This comovement is substantially larger than the comovement of future realized long-term inflation with the yield of 0.13, meaning that forecast errors significantly comove with the yield. Again, this matches our high persistence estimate ϕ_π of 0.96, as $E_t^*[\pi_{t+1}](1-\phi_\pi^{10})/(1-\phi_\pi)$ has a covariance of 0.23 with the yield.

Next, we test expectations of nominal earnings growth and show that forecast errors for short-term expectations satisfy the condition. The first row of Panel B shows that short-term nominal earnings growth expectations have a large and significant comovement with the price-earnings ratio, while realized short-term nominal earnings growth has a smaller and insignificant comovement. This means that forecast errors significantly comove with the price-earnings ratio, in line with the findings of De la O and Myers (2021).

Finally, we show that forecast errors for long-term nominal earnings growth do not satisfy the condition. In contrast with short-term expectations, long-term nominal earnings growth expectations have a small and insignificant comovement with the price-earnings ratio of 0.36. Consistent with our low estimate for the believed persistence ϕ_e of 0.004, this covariance is nearly two orders of magnitude smaller than the covariance of short-term nominal earnings growth expectations with the price-earnings ratio of 9.44. Importantly, realized long-term nominal earnings growth is also insignificantly related to the price-earnings ratio, meaning that forecast errors do not significantly comove with the price-earnings ratio. While expectations of long-term nominal earnings growth may fail other tests of rationality, as shown by Bordalo et al. (2020), the inability of the price-earnings ratio to predict the forecast errors

Table II

Testing single condition with price-dividend ratio

This table shows the covariance of the price-dividend ratio and bond yields with expected and realized inflation and nominal earnings growth and the forecast errors. In *Panel A*, the first row shows the covariance of the S&P 500 price-dividend ratio with short-term inflation expectations $E_t^*[\pi_{t+1}]$, realized short-term inflation π_{t+1}, and the forecast errors $\pi_{t+1} - E_t^*[\pi_{t+1}]$ from 1976Q1 to 2018Q2. The second row shows the covariance of the S&P 500 price-dividend ratio with long-term inflation expectations $E_t^*[\pi_{t+1,t+10}]$, realized long-term inflation $\pi_{t+1,t+10}$, and the forecast errors $\pi_{t+1,t+10} - E_t^*[\pi_{t+1,t+10}]$ using quarterly data from 1979Q4 to 2018Q2. In *Panel B* , the first row shows the covariance of the S&P 500 price-dividend ratio with short-term nominal earnings growth expectations $E_t^*[\Delta e_{t+1}]$, realized short-term nominal earnings growth Δe_{t+1}, and the forecast errors $\Delta e_{t+1} - E_t^*[\Delta e_{t+1}]$ from 1976Q1 to 2018Q2. The second row shows the covariances of the S&P 500 price-dividend ratio with long-term nominal earnings growth expectations $E_t^*[\Delta e_{t+3,t+5}]$, realized long-term nominal earnings growth $\Delta e_{t+3,t+5}$ and the forecast errors $\Delta e_{t+3,t+5} - E_t^*[\Delta e_{t+3,t+5}]$ from 1982Q1 to 2018Q2. Expectations expressed in percentages. We use small-sample adjusted Newey-West standard errors. Superscripts indicate significance at the 1% (***), 5% (**), and 10% (*) level.

	Panel A: Inflation			
	Horizon	Expected	Realized	Forecast Error
$Cov(pd_t, \cdot)$	Short-term (π_{t+1})	-0.70^{***}	-0.77^{***}	-0.08
	Long-term ($\pi_{t+1,t+10}$)	-5.13^{***}	-2.25^{***}	2.88^{***}
	Panel B. Nominal Earnings Growth			
$Cov(pd_t, \cdot)$	Short-term (Δe_{t+1})	2.52	-0.90	-3.43^{***}
	Long-term ($\Delta e_{t+3,t+5}$)	0.59	0.84	0.25

implies that these deviations from rationality will not help in matching prices or explaining return puzzles.[15] In fact, the lack of comovement between long-term nominal earnings growth expectations and the price-earnings ratio slightly worsens the ability to explain prices and return puzzles, as expected long-term nominal earnings growth comoves less with the price-earnings ratio than realized long-term nominal earnings growth.

For robustness, we repeat the test using the price-dividend ratio rather than the price-

[15]For robustness, in the Appendix we directly test the long-term nominal earnings growth expectations constructed in Bordalo et al. (2020). These expectations differ slightly from our main series as we use an earnings-weighted average across stocks to calculate aggregate S&P 500 long-term nominal earnings growth expectations and Bordalo et al. (2020) use a value-weighted average. We confirm that these differences do not alter the results in any noticeable way and the expectations do not satisfy the single condition. We thank Andrei Shleifer and Rafael La Porta for sharing these expectations.

earnings ratio to show that the choice of the normalizing variable does not change the results. In line with the results for the price-earnings ratio, Table II shows that forecast errors for long-term inflation and short-term nominal earnings growth significantly comove with the price-dividend ratio. Further, the magnitude of these comovements closely matches the results using the price-earnings ratio. For long-term inflation forecast errors, the comovement with the price-dividend ratio is 2.88 and the comovement with the price-earnings ratio is 2.31. Similarly, for short-term nominal earnings growth forecast errors, the comovement with the price-dividend ratio and price-earnings ratio is -3.43 and -4.26, respectively. Forecast errors for short-term inflation and long-term nominal earnings growth do not significantly comove with the price-dividend ratio, matching the results for the price-earnings ratio.

C. Implications for models of expectation formation

The results of Table I provide several insights for models of learning or behavioral biases that attempt to explain asset prices. For this section, we focus on the implications for models of inflation expectations but this procedure can be analogously used to discipline models of nominal earnings growth expectations.

First, the test acts as a diagnostic tool, showing that systematic errors should primarily arise in long-term inflation expectations. For a model of learning, this implies that agents must be learning about parameters that primarily impact long-term outcomes, such as the long-run mean level of inflation, rather than short-term outcomes, such as an unobserved mean-reverting state or regime. Importantly, this learning must produce large changes over time in long-term expectations to match the high comovement between long-term expectations and prices. This means that agents must receive a large amount of relevant information each period, have weak priors, or overweight new information.[16] These errors could also be directly attributed to behavioral errors, so long as the behavioral bias is parameterized so that errors are concentrated at long horizons. For example, for extrapolation models based on sentiment (e.g., Barberis et al. (2015)), concentrating errors at long horizons requires that agents believe sentiment is highly persistent.

Second, Table I shows that shocks solely to long-term inflation expectations do not play a large role. The fact that errors are concentrated in long-term expectations naturally

[16]For example, experiential (Nagel and Xu (2021)) or generational learning (Collin-Dufresne, Johannes, and Lochstoer (2017)) increase the weight placed on recent information.

raises the question whether there are important shocks that only impact long-term inflation expectations. As discussed in Section II.B, the covariance of prices with 10-year expectations in Table I almost exactly matches the covariance of prices with one-year expectation scaled by the believed persistence $E_t^*\left[\pi_{t+1}\right]\left(1-\phi_\pi^{10}\right)/\left(1-\phi_\pi\right)$. This means that shocks $\varepsilon_{t,j}^{\pi}$ from equation (9), which only affect long-term expectations, are not important in explaining the large comovement between long-term expectations and prices. For models of expectation formation, this means that the information or bias that drives long-term expectations must be highly correlated with current short-term expectations. Another way to see this is to note that short-term and long-term expectations are highly correlated (0.98), as shown in Figure 1. Because the correlation is nearly 1, long-term inflation expectations in these models should approximately be affine functions solely of short-term expectations.

Second, Table I shows that shocks solely to long-term inflation expectations do not play a large role. The fact that errors are concentrated in long-term expectations naturally raises the question what type of shocks impact long-term inflation expectations. For example, in equation (9), shocks $\varepsilon_{t,j}^{\pi}$ only affect long-term expectations and do not impact short-term expectations. Our findings suggests that this type of shock is not important in explaining the large comovement between long-term expectations and prices. This is because, as discussed in Section II.B, the covariance of prices with 10-year expectations in Table I almost exactly matches the covariance of prices with one-year expectation scaled by the believed persistence $E_t^*\left[\pi_{t+1}\right]\left(1-\phi_\pi^{10}\right)/\left(1-\phi_\pi\right)$. For models of expectation formation, this means that the information or bias that drives long-term expectations must be highly correlated with current short-term expectations. Another way to see this is to note that short-term and long-term expectations are highly correlated (0.98), as shown in Figure 1. Because the correlation is nearly 1, long-term inflation expectations in these models should approximately be affine functions solely of short-term expectations.

Third, the results apply to both stock and bond markets. Regardless if stock or bond prices are used to test the single condition, we consistently find that prices comove with forecast errors for long-term inflation expectations and do not comove with forecast errors for short-term inflation expectations. This points against models where agents make errors in one market but not the other. For example, money illusion models predict that inflation expectations lead to mispricing for stocks but not for bonds (Modigliani and Cohn (1979), and more recently Cohen, Polk, and Vuolteenaho (2005)).

III. Asset Prices and Puzzles

In this section, we show that subjective expectations of inflation and nominal earnings growth accurately match observed asset prices and return puzzles, leaving little room for time-varying discount rates. As discussed in Section II, this is due to errors in long-term inflation expectations and short-term nominal earnings growth expectations.

We construct fundamental stock and bond prices based on equations (2) and (4) using subjective expectations of real cash flows and constant discount rates. We focus on aggregate stock and bond prices, specifically the S&P 500 price-earnings ratio and price-dividend ratio and the ten-year zero-coupon Treasury yield. Fundamental prices closely match all three time-series. The quantitative results imply that time-varying discount rates play almost no role in aggregate stock prices and play only a secondary role in aggregate bond yields.

We then test whether these fundamental prices replicate two key puzzles in the asset pricing literature on stocks and bonds: the predictability of stock returns and the rejection of the expectations hypothesis for bonds. Specifically, we study the comovement of future stock returns with current price ratios and the excess volatility of bond holding returns. Under RE, the magnitude of these two puzzles is evidence that discount rates must vary over time and that this variation is substantial. In both cases, the fundamental prices closely match the observed puzzles despite having constant discount rates.

A. Price movements

Movements in observed asset prices are closely matched by subjective expectations of real cash flows. Setting the discount rate to a constant and using the believed persistence for inflation and nominal earnings growth measured in Section II, the fundamental price-earnings ratio is

$$pe_t^{fun} = c + \frac{1}{1-\rho\phi_e} E_t^* \left[\Delta e_{t+1}\right] - \frac{1}{1-\rho\phi_\pi} E_t^* \left[\pi_{t+1}\right] \tag{11}$$

where the discount rate simply falls into the constant c.[17] Note that the fundamental price-earnings ratio is simply a linear function of two time-series measured from survey forecasts, $E_t^* \left[\Delta e_{t+1}\right]$ and $E_t^* \left[\pi_{t+1}\right]$, and the parameters ϕ_e and ϕ_π are also estimated directly from the

[17] The term c conveniently condenses all of the constants. Given the constant discount rate $\tilde{r}$, the full expressions is $c = \frac{\kappa-\tilde{r}}{1-\rho} + \sum_{j=1}^{\infty} \rho^{j-1} \left(\alpha_{e,j} - \alpha_{\pi,j}\right)$.

survey data.[18] The mean price-earnings ratio is used to set c, but no other information about the observed price-earnings ratio is used when constructing the fundamental price-earnings ratio. By extension, the fundamental price-dividend ratio is

$$pd_t^{fun} = pe_t^{fun} + e_t - d_t. \tag{12}$$

Similarly, using a constant discount rate, the fundamental ten-year yield is

$$y_t^{(10),fun} = c^y + \frac{1}{10}\frac{1-\phi_\pi^{10}}{1-\phi_\pi} E_t^* \left[\pi_{t+1}\right] \tag{13}$$

where the second term represents the average expectation of inflation over the next ten years. The results are almost identical if we use the survey ten-year inflation expectations $E_t^* \left[\pi_{t+1,t+10}\right]$. This is because the correlation between one-year and ten-year inflation expectations is 0.98 and ϕ_π is estimated to match the ten-year expectations. The constant c^y is set to match the mean ten-year yield, but no other information about the observed yield is used in the fundamental yield.

Table III shows regressions of the observed stock price ratios and bond yield on the fundamental price ratios and bond yield. In short, subjective expectations of real cash flows explain virtually all movements in stock price ratios and 2/3 of movements in bond yields. For the price-earnings ratio and the price-dividend ratio, the observed values move almost 1-1 with the fundamental values with coefficients of 0.96 in both regressions and high R^2's of 0.81 and 0.79, respectively. Even for bond yields, which one may expect to be strongly driven by discount rates, we find a significant coefficient of 1.55 and an R^2 of 0.66. Additionally, we cannot reject that the observed prices and yields are equal to the fundamental prices and yields plus noise, i.e., $a = 0$ and $b = 1$. In this constrained case of $a = 0$ and $b = 1$, we continue to find high R^2's of roughly 0.8 for the stock price ratios and 0.58 for the bond yield.

The results of Table III leave little room for time-varying discount rates. The high R^2 values demonstrate that the majority of price movements for both stocks and bonds are attributed to movements in subjective expectations of real cash flows. Even if we ascribe all of the residual variation to movements in discount rates, the discount rate variation would play almost no role in stock prices and only a secondary role in bond yields. The inclusion

[18]In the Appendix, we also estimate a generalization of equations (9) and (10) where expectations of inflation can impact expectations of nominal earnings growth. When estimated on the survey expectations, we find no significant interaction and all of our results are robust to including this interaction.

Table III

Asset prices

This table shows a comparison between observed stock and bond prices and the fundamental prices constructed with the subjective expectations of inflation and nominal earnings growth. The table shows linear regressions of observed prices on fundamental prices and reports the intercept a, the slope b, and the R^2 of the regression. Additionally, we report the constrained R^2 obtained from the residuals of the constrained regression where the intercept is forced to be zero $a = 0$ and the slope is forced to be one $b = 1$. The first two rows show the results for the S&P 500 price-earnings ratio and price-dividend ratio. The third row shows the results for the 10-year Treasury yield. The fourth and fifth row show the results for the long-run component pe_t^{LR} and the short-run component pe_t^{SR} of the S&P 500 price-earnings ratio. The short-run and long-run component were obtained by applying a Hodrick-Prescott filter on both the observed and the fundamental price-earnings ratio with a smoothing parameter of 1600. All calculations use quarterly data. Small-sample adjusted Newey-West standard errors in parenthesis.

Regression	a	b	R^2	Constrained R^2 ($a = 0, b = 1$)
$pe_t = a + bpe_t^{fun} + \varepsilon_t$	0.13	0.96	0.81	0.81
	(0.21)	(0.09)		
$pd_t = a + bpd_t^{fun} + \varepsilon_t$	0.16	0.96	0.79	0.79
	(0.39)	(0.12)		
$y_t = a + by_t^{fun} + \varepsilon_t$	-0.03	1.55	0.66	0.58
	(0.02)	(0.30)		
$pe_t^{LR} = a + bpe_t^{fun,LR} + \varepsilon_t$	0.03	0.99	0.82	0.82
	(0.14)	(0.06)		
$pe_t^{SR} = a + bpe_t^{fun,SR} + \varepsilon_t$	0.00	0.91	0.81	0.80
	(0.01)	(0.05)		

of inflation expectations accounts for the entirety of the bond results and also substantially reduces the amount of residual variation in the stock prices that could be explained by discount rates. To measure this, we re-estimate the first row of Table III holding inflation expectations constant and only find an unconstrained R^2 of 0.54.[19]

To emphasize how closely subjective expectations of real cash flows match observed stock prices, we calculate two additional measures and test how well the fundamental prices explain long-run changes in the price-earnings ratio as well as short-run business cycle fluctuations. Using a standard Hodrick-Prescott filter, we calculate the long-run and short-run components

[19] Including long-term nominal earnings growth expectations as a separate independent variable in the regression only marginally improves the unconstrained R^2 to 0.57.

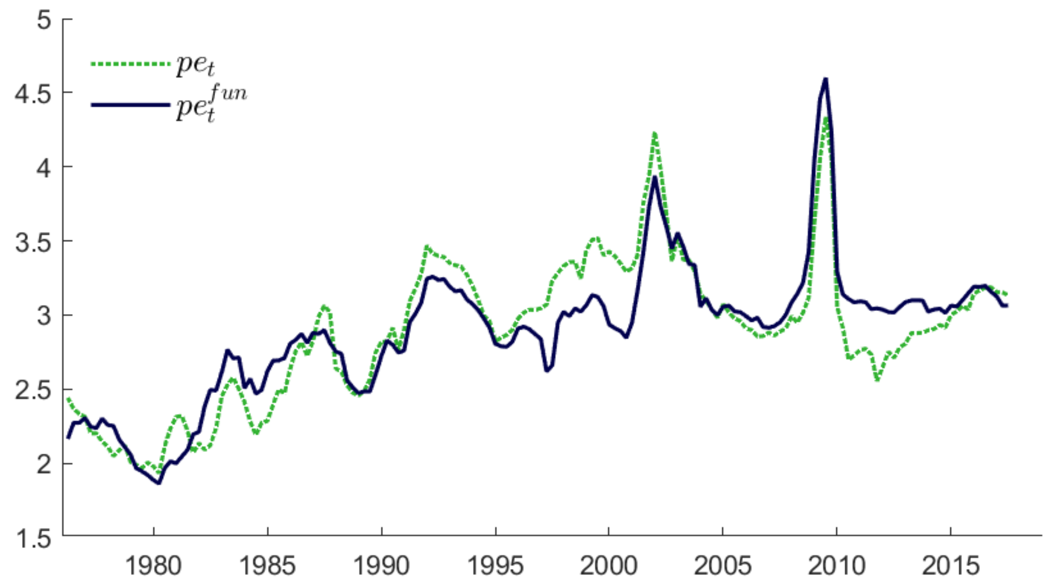

Figure 3. Fundamental and observed price-earnings ratio. The figure compares the observed price-earnings ratio for the S&P 500 (dotted green) with the fundamental price-earnings ratio (solid blue). The fundamental price-earnings ratio is the value of subjective expectations of real earnings growth plus a constant. All variables are in logs.

of the observed price-earnings ratio, pe_t^{LR} and pe_t^{SR}, and the fundamental price-earnings ratio, $pe_t^{fun,LR}$ and $pe_t^{fun,SR}$. The final two rows of Table III show that the fundamental values continue to move almost 1-1 with the observed values, with coefficients of 0.99 and 0.91 and high R^2's of 0.82 and 0.81 for the long-run and short-run components, respectively.

Figures 3 and 4 show the fundamental and observed price-earnings ratio, as well as the short-run and long-run components. Figure 3 shows that the fundamental price-earnings ratio pe_t^{fun} accurately tracks the observed price-earnings ratio pe_t. Additionally, Figure 4 shows that the fundamental price-earnings ratio captures almost all short-run business-cycle variation in the price-earnings ratio, as well as the long-run secular increase in the price-earnings ratio over the last 40 years.

B. Return predictability

A central argument for time-varying discount rates is the empirical fact that future stock returns comove significantly with current price ratios. As shown in equation (7), under RE

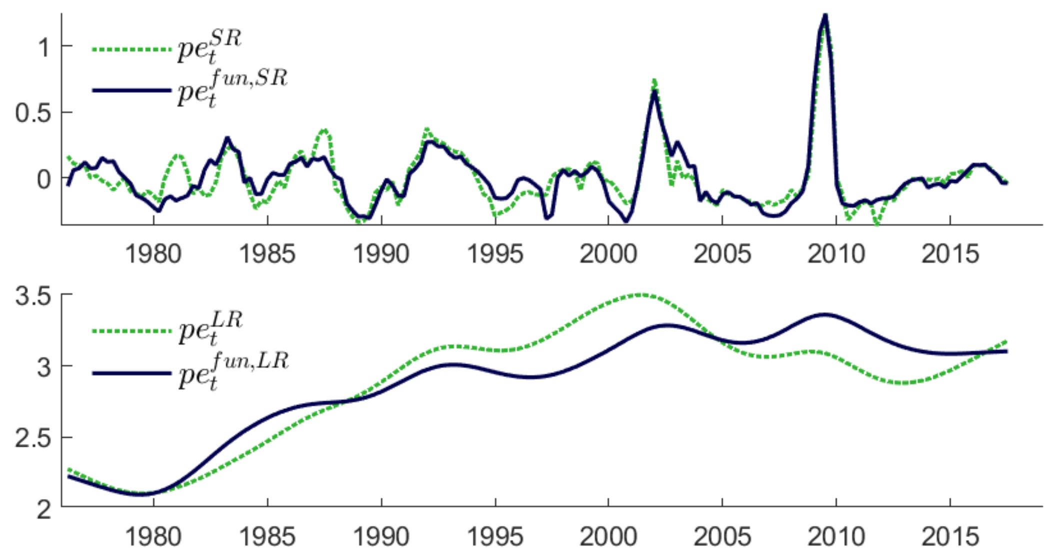

Figure 4. Short-run and long-run price movements. The top panel compares the short-run component of the price-earnings ratio for the S&P 500 (dotted green) with the short-run component of the fundamental price-earnings ratio (solid blue). The fundamental price-earnings ratio is the value of subjective expectations of real earnings growth plus a constant. The bottom panel compares the long-run component of the price-earnings ratio and the long-run component of the fundamental price-earnings ratio. All variables are in logs.

this comovement must be due to substantial variation in discount rates. In Table IV, we test whether this comovement is matched by fundamental price ratios. Intuitively, if predictable returns are driven by errors in subjective expectations of real cash flows, then fundamental prices with constant discount rates will also predict returns.

The first row of Table IV shows the results of univariate regressions of future nominal returns for the next ten years on current price ratios. Changes in the observed price ratios are almost completely reflected in future returns. All variables are in logs, so a 1% increase in the price ratios is associated with a 0.77% and 0.88% decrease in future ten-year returns, respectively. Importantly, increases in the fundamental price ratios are also reflected in future returns, with nearly identical coefficients of -0.77 and -0.92, respectively. The second row shows the analogous results using real returns for the next ten years. Again, the coefficients for the observed price ratios, -0.59 and -0.69, are nearly identical to the coefficients for the fundamental price ratios, -0.54 and -0.69, and all coefficients are significant. In all cases,

Table IV

Stock return predictability

This table shows the ability of real cash flow expectations to predict stock returns. The first and third columns show the coefficients from univariate linear regressions of ten-year nominal and real stock returns on the observed price-earnings ratio (pe_t) and price-dividend ratio (pd_t). The second and fourth columns show the coefficients from univariate linear regressions of ten-year nominal and real stock returns on the fundamental price-earnings ratio (pe_t^{fun}) and price-dividend ratio (pd_t^{fun}), which are constructed using real cash flow expectations and constant discount rates. For each regression, the table reports the slope coefficient β and the R^2. Small-sample adjusted Newey-West standard errors in parenthesis.

		Price-earnings ratio		Price-dividend ratio	
		Observed	Fundamental	Observed	Fundamental
Nominal returns $\sum_{j=1}^{10} r_{t+j}$	β	-0.77	-0.77	-0.88	-0.92
		(0.11)	(0.09)	(0.11)	(0.17)
	R^2	0.60	0.42	0.81	0.64
Real returns $\sum_{j=1}^{10} \tilde{r}_{t+j}$	β	-0.59	-0.54	-0.69	-0.69
		(0.14)	(0.13)	(0.15)	(0.19)
	R^2	0.45	0.26	0.65	0.46

the differences between the coefficients for the observed price ratios and the fundamental price ratios are small and statistically insignificant.

Further, looking at the R^2 values, we see that the majority of the R^2 for the observed price ratios is generated by the fundamental price ratios. For example, the observed price-dividend ratio explains 81% of variation in future returns and the fundamental price-dividend ratio explains 64%. Thus, even if we attribute the entire difference to time-variation in discount rates, subjective expectations of real cash flows would still be the primary factor driving return predictability.

Combined, the results of Table III and Table IV imply that statistical forecasts of future returns based on observed price ratios are nearly identical to statistical forecasts based on fundamental price ratios. Fundamental price ratios move nearly 1-1 with observed price ratios with high R^2's and have virtually identical coefficients for predicting future returns. In other words, an econometrician using observed price ratios and an econometrician using subjective expectations of real cash flows produce almost the same forecasts for future returns. This means that rather than reflecting time-variation in investors' discount rates, movements in statistical forecasts of returns primarily reflect predictable errors in subjective expectations of inflation and nominal earnings growth.

C. Rejection of expectations hypothesis

The expectations hypothesis posits that the n-period yield $y_t^{(n)}$ is the expectation of the average one-period yield $y_{t+j}^{(1)}$ over the next n periods, plus a constant term premium.[20] Under RE, the expectations hypothesis implies that movements in yields must ultimately be related to movements in $y_{t+j}^{(1)}$ and the volatility of holding returns is constrained by the volatility of movements in the one-period yield. Specifically, equation (8) shows that under RE and the expectations hypothesis, the variance of holdings returns $h_{t+1}^{(n)}$ is bounded by $Var\left(y_t^{(1)}\right)$ scaled by a constant η. The first column of Table V shows that this bound is decisively violated in the data by a factor of 5.6. Under RE, these large movements in $h_{t+1}^{(n)}$ must be due to large movements in term premia.

In this section, we show that subjective expectations of inflation can quantitatively match this violation without using time-varying term premia. The fundamental yield $y_t^{(n),fun}$ is simply equal to the expected average inflation over the next n-periods plus a constant, meaning that the expectations hypothesis holds by definition. The holding return using fundamental yields is then measured from equation (3).[21] The second row of Table V shows that fundamental yields violate this bound by a factor of 5.22, almost exactly matching the violation in the observed data. Using a one-sided F-test for equality of variance, we confidently reject that $Var\left(h_{t+1}^{(10)}\right)$ is below the bound for both the observed yields and the fundamental yields. We cannot reject that the two variance ratios are equal.

Equation (8) demonstrates how fundamental yields can violate this bound. Even when the expectations hypothesis holds and expected term premia are constant, $Var\left(h_{t+1}^{(10)}\right)$ can exceed $\eta Var\left(y_t^{(1)}\right)$ if yields predict forecast errors. Intuitively, if long-term inflation expectations are too high when yields are high, then these inflation expectations will have to be revised downwards as investors realize their mistake. This downward revision in long-term inflation expectations lowers yields, which produces a high holding return following equation (3). To an econometrician assuming RE, it appears as if the high yield and the high holding return are due investors demanding a high term premium. Time-variation in these

[20]Rejecting the expectations hypothesis is equivalent to stating that term premia $E_t^*\left[h_{t+1}^{(n)}\right] - y_t^{(1)}$ vary over time.

[21]Using the believed persistence ϕ_π, the holding return reduces to a constant plus $\frac{1-\phi_\pi^{10}}{1-\phi_\pi}E_t^*\left[\pi_{t+1}\right] - \frac{1-\phi_\pi^{9}}{1-\phi_\pi}E_{t+1}^*\left[\pi_{t+2}\right]$.

Table V

Rejection of expectations hypothesis

This table compares the excess volatility of holding returns for the observed yields and the fundamental yields. The variance ratio measures the variance of one-period holding returns on a ten-year bond divided by the variance of the one-period yield multiplied by a constant $\eta = 10^2/(20-1)$. Under RE and the expectations hypothesis, this ratio has an upper bound of 1. The first column shows this ratio in the observed detrended data. The second column shows the variance ratio based on fundamental yields constructed using detrended inflation expectations and constant discount rates. The second row shows the significance of the one-sided F-test for equality of variance adjusted for autocorrelation of samples as in Priestley (1981).

	Observed yields	Fundamental yields
$Var\left(h_{t+1}^{(10)}\right)/\eta Var\left(y_t^{(1)}\right)$	5.60	5.22
F-test	0.00001	0.00046

predictable inflation errors similarly makes it appear as if term premia are time-varying.

The results of Table V indicate that rejections of the expectations hypothesis may be due to errors in long-term inflation expectations, rather than time-variation in term premia. This is consistent with the results of Piazzesi, Salomao, and Schneider (2015), who find that survey expectations of the term premia $E_t^*\left[h_{t+1}^{(n)}\right] - y_t^{(1)}$ are relatively flat compared statistical measures of term premia. This is also consistent with Barr and Campbell (1997), who find that holding returns on inflation protected bonds are much less volatile.

IV. Model of Expectation Formation

The analysis of Sections I-III is applicable to a broad class of deviations from RE, such as behavioral biases or learning. Section II.C discusses general implications of these results for models of expectation formation. In this section, we provide a concrete example of a model that matches our empirical findings. Specifically, we illustrate that these findings can be reconciled by agents who exaggerate the objective persistence of inflation and nominal earnings growth. This mechanism not only matches our survey expectations but also is consistent with the experimental evidence of Afrouzi et al. (2021). We microfound these expectations using the representativeness bias of Tversky and Kahneman (1974), similar to the diagnostic models of Bordalo, Gennaioli, and Shleifer (2018) and Bordalo et al. (2019).

Given that we are only interested in variances and covariances, we demean all variables without loss of generality. Objectively, inflation and nominal earnings growth are AR(1) processes,

$$\begin{aligned} \pi_{t+1} &= \varphi_\pi \pi_t + \varepsilon_{\pi,t+1} \quad (14) \\ \Delta e_{t+1} &= \varphi_e \Delta e_t + \varepsilon_{e,t+1.} \quad (15) \end{aligned}$$

We use φ_π, φ_e to distinguish from the values of ϕ_π, ϕ_e used earlier in the empirical sections. Agents understand that inflation and nominal earnings growth are AR(1) but exaggerate the persistence of both processes. Specifically, agents believe the persistences are $\varphi_\pi^*, \varphi_e^*$ where $\frac{\varphi_\pi^*}{\varphi_\pi}, \frac{\varphi_e^*}{\varphi_e} \geq 1$. Agents have constant discount rates, so the price-earnings ratio and yield are simply equal to their expectation of real earnings growth and average inflation,

$$\begin{aligned} pe_t &= \frac{\varphi_e^*}{1-\rho\varphi_e^*}\Delta e_t - \frac{\varphi_\pi^*}{1-\rho\varphi_\pi^*}\pi_t \quad (16) \\ y_t^{(n)} &= \frac{1}{n}\varphi_\pi^* \frac{1-\varphi_\pi^{*n}}{1-\varphi_\pi^*}\pi_t. \quad (17) \end{aligned}$$

The condition $\frac{\varphi_\pi^*}{\varphi_\pi}, \frac{\varphi_e^*}{\varphi_e} \geq 1$ reflects representativeness bias, which states that when an outcome becomes objectively more likely, individuals overweight the likelihood of that outcome.[22] For example, given $\varphi_\pi > 0$, a high value for current inflation makes high future inflation more likely than it would be in the unconditional distribution. Agents understand the direction of this effect but exaggerate how much the current inflation increases the likelihood of high future inflation, believing the expected value for future inflation is $\varphi_\pi^*\pi_t$ rather than $\varphi_\pi\pi_t$.

A. *Term structure of subjective expectations*

We estimate the objective persistences φ_π, φ_e from the observed inflation and nominal earnings growth. Inflation is persistent with a coefficient of 0.76, while future nominal earnings growth tends to be negatively related to current nominal earnings growth giving a coefficient of -0.20. For φ_π^*, we use the value measured from the survey data in Section II of 0.96. We estimate φ_e^* from the comovement of one-year expectations with current realized nominal earnings growth to get -0.45. Consistent with the results of Section II, this implies that long-term nominal earnings growth expectations are relatively flat compared to short-term expectations while also matching the significant negative relationship between current nom-

[22]The Appendix gives an explicit microfoundation for the model expectations using representativeness bias.

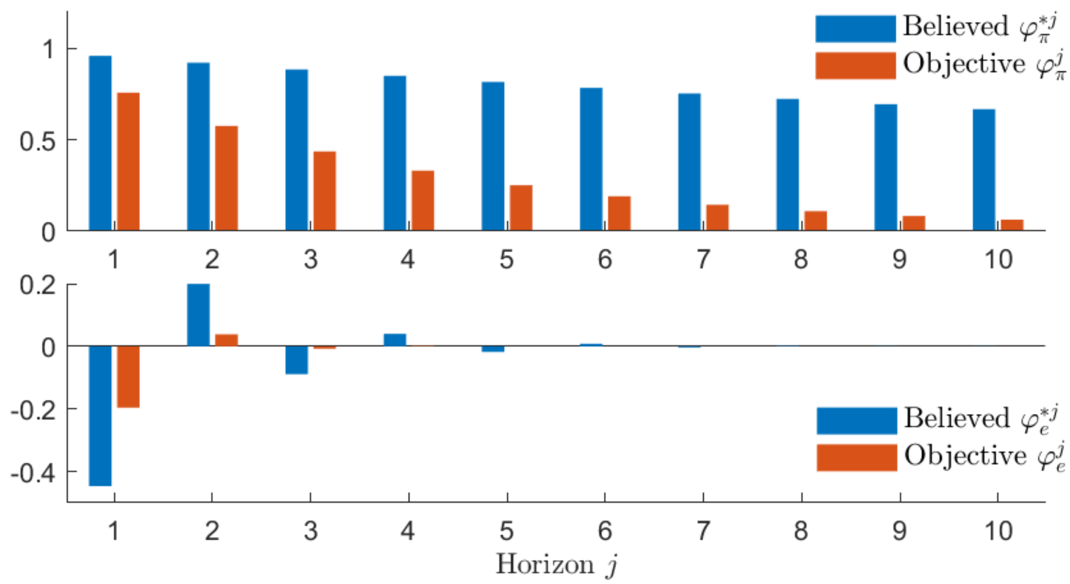

Figure 5. Term structure of expectations. The top panel compares the believed persistence of inflation φ_π^{*j} (blue) with the objective persistence φ_π^j (red) at multiple horizons. The bottom panel compares the believed persistence of nominal earnings growth φ_e^{*j} (blue) with the objective persistence φ_e^j (red).

inal earnings growth and short-term expectations. In line with the representativeness bias, $\varphi_\pi^*, \varphi_e^*$ exaggerate the true persistences.[23] The values for φ_e, φ_e^* are also consistent with the earnings growth reversal model of De la O and Myers (2021), where subjective expectations overstate how much negative current nominal earnings growth will be reversed by higher future nominal earnings growth.

Figure 5 shows that this simple model matches all four of our findings on the term structure of subjective expectations from Section II. Specifically, Figure 5 shows the believed persistence $\varphi_\pi^{*j}, \varphi_e^{*j}$ and the objective persistence $\varphi_\pi^j, \varphi_e^j$ at multiple horizons j. First, the values for φ_π^{*j} demonstrate that long-term inflation expectations move almost 1-1 with short-term expectations. Second, the values for φ_e^{*j} show that long-term nominal earnings growth expectations are flat relative to short-term expectations. Movements in one-year expectations are 20 times larger than movements in average three-to-five-year expectations, matching the results of Figure 2. Third, predictable errors in expectations of inflation, which are

[23]The Appendix confirms that forecast errors for inflation are negatively correlated with current inflation and forecast errors for nominal earnings growth are positively correlated with current nominal earnings growth.

represented by $\varphi_\pi^{*j} - \varphi_\pi^j$, are primarily concentrated at long horizons. Fourth, predictable errors in expectations of nominal earnings growth, $\varphi_e^{*j} - \varphi_e^j$, are primarily concentrated at short horizons.

Intuitively, the representativeness bias relates the term structure of subjective expectations to the objective persistence, which is why this one bias is able to match the starkly different results for inflation expectations and nominal earnings growth expectations. For an objectively persistent process like inflation, a bias that magnifies the persistence primarily impacts long-term expectations. As shown in Figure 5, the error in one-year expectations is relatively small compared to the size of the objective and subjective expectations. It is only at longer horizons that the difference between a high and a very high persistence becomes apparent, as the objective persistence φ_π^j steadily declines more rapidly than the believed persistence φ_π^{*j}. For an objectively transitory process like nominal earnings growth, a bias that magnifies the persistence primarily impacts short-term expectations. Even though φ_e^* has a greater magnitude than φ_e, both persistences are still low and they quickly converge towards zero. As a result, differences between the objective and subjective expectations are concentrated almost entirely at short horizons.

B. *Return predictability*

Let $\alpha_e \equiv \frac{\varphi_e}{1-\rho\varphi_e}$ and $\alpha_\pi \equiv \frac{\varphi_\pi}{1-\rho\varphi_\pi}$. Similarly, let $\alpha_e^* \equiv \frac{\varphi_e^*}{1-\rho\varphi_e^*}$ and $\alpha_\pi^* \equiv \frac{\varphi_\pi^*}{1-\rho\varphi_\pi^*}$ be the analogous values using the believed persistences. For this section, we impose that shocks are independent which provides a useful representation for return predictability. See the Appendix for all proofs.

Proposition 1. *Total future returns are* $\sum_{j=1}^{\infty} \rho^{j-1}\tilde{r}_{t+j} = \beta pe_t + \upsilon_{t,t+\infty}$ *where* $\upsilon_{t,t+\infty}$ *is uncorrelated with* pe_t *and* β *is the weighted average bias,*

$$\beta = w\left(\frac{\alpha_e}{\alpha_e^*} - 1\right) + (1-w)\left(\frac{\alpha_\pi}{\alpha_\pi^*} - 1\right). \tag{18}$$

The terms $\frac{\alpha_e}{\alpha_e^*} - 1$, $\frac{\alpha_\pi}{\alpha_\pi^*} - 1$ capture how much the subjective expectations deviate from the objective distribution. The representativeness bias $\frac{\varphi_\pi^*}{\varphi_\pi}, \frac{\varphi_e^*}{\varphi_e} \geq 1$ implies that $\frac{\alpha_e}{\alpha_e^*}, \frac{\alpha_\pi}{\alpha_\pi^*} \leq 1$, guaranteeing that the price-earnings ratio negatively predicts future returns. The weight $w \equiv \left(\alpha_e^{*2}\sigma_e^2\right) / \left(\alpha_e^{*2}\sigma_e^2 + \alpha_\pi^{*2}\sigma_\pi^2\right)$ represents the portion of the variation in pe_t that comes from

Table VI

Return puzzles in data and model

This table compares the return predictability and rejection of expectations hypothesis observed in the data with that generated by our constant discount rate model. The first row shows the coefficient from a univariate linear regression of ten-year real stock returns on the price-earnings ratio (pe_t) in the observed data and the model. The second row shows the variance of holding returns ($Var\left(h_{t+1}^{(10)}\right)$) relative to the upper bound imposed by RE and the expectations hypothesis ($\eta Var\left(y_t^{(1)}\right)$).

	Data	Model
$Cov\left(\sum_{j=1}^{10} \tilde{r}_{t+j}, pe_t\right)/Var\left(pe_t\right)$	-0.59	-0.79
$Var\left(h_{t+1}^{(10)}\right)/\eta Var\left(y_t^{(1)}\right)$	5.60	6.01

variation in subjective nominal earnings growth expectations where σ_e^2, σ_π^2 are the objective variances of nominal earnings growth and inflation.

This relationship between returns and the price-earnings ratio can easily be extended to average returns over a finite horizon.

Corollary 1. *Future returns over a finite horizon are* $\sum_{j=1}^{T} \tilde{r}_{t+j} = \beta pe_t + \upsilon_{t,t+T}$ *where* $\upsilon_{t,t+T}$ *is uncorrelated with* pe_t,

$$\beta = w\left(\frac{\alpha_e}{\alpha_e^*} - 1\right)\gamma_e + (1-w)\left(\frac{\alpha_\pi}{\alpha_\pi^*} - 1\right)\gamma_\pi, \tag{19}$$

and $\gamma_e \equiv (1-\rho\varphi_e)\frac{1-\varphi_e^T}{1-\varphi_e}$, $\gamma_\pi \equiv (1-\rho\varphi_\pi)\frac{1-\varphi_\pi^T}{1-\varphi_\pi}$.

The first row of Table VI shows that the model coefficient for predicting 10-year future returns matches the large, negative regression coefficient measured in the data. Even though the model price-earnings ratio uses constant discount rates, increases in the model price-earnings ratio predict future ten-year real returns with a coefficient of -0.79, in line with the value of -0.59 in the data. This occurs because increases in the price-earnings ratio largely reflect errors in subjective expectations of real earnings growth. Because these price increases are not matched by future increases in real earnings growth, the price increases instead lead to lower future returns.

C. Rejection of expectations hypothesis

By construction, the expectations hypothesis holds for the model yields. The model yields are simply expectations of average future inflation and therefore are also expectations of the average future short rate $y_t^{(1)}$. However, holding returns can still violate the variance bound of Shiller (1979); Singleton (1980), leading an econometrician that assumes RE to reject the expectations hypothesis.

Proposition 2. *The variance of holding returns relative to the variance of the short-rate is*

$$\frac{Var\left(h_{t+1}^{(n)}\right)}{Var\left(y_t^{(1)}\right)} = \left[1+\lambda\left(\varphi_\pi^* - \varphi_\pi\right)\right]^2 + \left(1-\varphi_\pi^2\right)\lambda^2 \tag{20}$$

where $\lambda \equiv \frac{1-(\varphi_\pi^*)^{n-1}}{1-\varphi_\pi^*}$.

Given a positive objective persistence φ_π, the representativeness bias $\varphi_\pi^* \geq \varphi_\pi$ raises the variance ratio in two ways: (i) by increasing $\varphi_\pi^* - \varphi_\pi$ and (ii) by increasing λ.

The second row of Table VI shows that the model closely matches the empirical violation of the variance bound. The variance of holding returns on ten-year bonds exceeds the bound $\eta Var\left(y_t^{(1)}\right)$ by a factor of 6.01 in the model, compared to 5.60 in the data. This excess volatility in bond holding returns is driven by errors in inflation expectations. Movements in long-term yields can be split into changes in objective expectations of future inflation, which also represent changes in objective expectations of the future short rate, and changes in the gap between subjective and objective expectations of inflation. Whenever the objective expectation of future inflation increases, the representativeness bias causes the gap to also increase, which magnifies the movement in long-term yields and increases the volatility of holding returns.

V. Conclusion

In this paper, we find that errors in expectations of real cash flows quantitatively explain asset price movements and return puzzles for both bonds and stocks. Using a single necessary and sufficient condition derived from accounting identities, we find that errors in long-term inflation expectations and short-term nominal earnings growth expectations drive these results, while errors in short-term inflation expectations and long-term nominal earnings growth expectations do not play a noticeable role. Rather than focusing on models that attempt to explain why investors' discount rates would fluctuate significantly over time, this evidence argues that research should focus on how investors form expectations of real cash flows. This paper provides diagnostic tools and results to guide models that depart from RE.

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Appendix

A. Derivation of formulas

In order to arrive to equation (1) we start with the one-year return identity

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{\left(\frac{P_{t+1}}{D_{t+1}} + 1\right)\frac{D_{t+1}}{D_t}}{\frac{P_t}{D_t}},$$

where P_t and D_t represent the current nominal price and nominal dividends of the S&P 500 index. Log-linearizing around a long-term average of P/D, we can state the price-dividend ratio pd_t in terms of future dividend growth, Δd_{t+1}, future returns, r_{t+1}, and the future price-dividend ratio, pd_{t+1}, all in logs:

$$r_{t+1} \quad = \quad \kappa^d + \Delta d_{t+1} - pd_t + \rho pd_{t+1}, \tag{A1}$$

where κ^d is a constant, $\rho = e^{\bar{pd}}/\left(1 + e^{\bar{pd}}\right) < 1$ and $\bar{pd}$ is the mean value of the log price-dividend ratio. Using the log payout ratio de_t, we can insert the identity $pe_t = pd_t + de_t$[24] into (A1) to obtain

$$\tilde{r}_{t+1} \quad \approx \quad \kappa + \Delta\tilde{e}_{t+1} - pe_t + \rho pe_{t+1} \tag{A2}$$

where we approximate $(1-\rho)\, de_{t+1}$ as 0 given that $1-\rho$ is close to 0.

To establish identities (5) and (6) we just need to use the fact that forecast errors are unpredictable with information at time t under RE, meaning that

$$Cov(pe_t, \Delta\tilde{e}_{t+j}) \quad = \quad Cov\left(pe_t, E_t\left[\Delta\tilde{e}_{t+j}\right]\right). \tag{A3}$$

Under subjective expectations, the predictability of forecast errors with the price-earnings ratio is:

$$Cov(pe_t, f^{\Delta\tilde{e}_{t+j}}) \quad = \quad Cov(pe_t, \Delta\tilde{e}_{t+j}) - Cov\left(pe_t, E_t^*\left[\Delta\tilde{e}_{t+j}\right]\right) \tag{A4}$$

Equation (A3) and (A4) lead to equation (5) which expresses the comovement of price-earnings ratio with subjective real earnings growth expectations in terms of its comovement with expectations under RE and the predictability of forecast errors for real earnings growth. In a similar fashion, the covariance of the 10-year bond yield with subjective inflation ex-

[24]Because we are using the aggregate S&P 500, we do not need to worry about very small or negative values for earnings.

pectations can be expressed in terms of its covariance with expectations under RE and the predictability of forecast errors for inflation as in equation (6).

To establish identity (7), we start from the fact that equation (2) is satisfied with and without applying expectations. Hence

$$\sum_{j=1}^{\infty} \rho^{j-1} \tilde{r}_{t+j} - \sum_{j=1}^{\infty} \rho^{j-1} E_t^* \left[\tilde{r}_{t+j}\right] = \sum_{j=1}^{\infty} \rho^{j-1} \Delta \tilde{e}_{t+j} - \sum_{j=1}^{\infty} \rho^{j-1} E_t^* \left[\Delta \tilde{e}_{t+j}\right].$$

By covarying both sides with the price-earnings ratio we arrive to equation (7). Finally, to establish the inequality in equation (8) we generalize the Shiller (1979) variance bound. From equation (3) we have that the variance of one-period holding returns can be expressed as

$$Var\left(h_{t+1}^{(n)}\right) = n^2 Var\left(y_t^{(n)}\right) + (n-1)^2 Var\left(y_{t+1}^{(n-1)}\right) - 2n(n-1) Cov\left(y_t^{(n)}, y_{t+1}^{(n-1)}\right).$$

The covariance term in the above equation can be expressed as

$$(n-1) Cov\left(y_{t+1}^{(n-1)}, y_t^{(n)}\right) = nVar\left(y_t^{(n)}\right) - Cov\left(y_t^{(1)}, y_t^{(n)}\right) - Cov\left(h_{t+1}^{(n)} - y_t^{(1)}, y_t^{(n)}\right)$$

and $Cov\left(h_{t+1}^{(n)} - y_t^{(1)}, y_t^{(n)}\right)$ can be expressed as

$$\begin{aligned} Cov\left(h_{t+1}^{(n)} - y_t^{(1)}, y_t^{(n)}\right) &= Cov\left(E_t^*\left[h_{t+1}^{(n)}\right] - y_t^{(1)}, y_t^{(n)}\right) + Cov\left(f_t^{h_{t+1}^{(n)}}, y_t^{(n)}\right) \\ &= Cov\left(E_t^*\left[h_{t+1}^{(n)}\right] - y_t^{(1)}, y_t^{(n)}\right) - Cov\left(\sum_{j=2}^{n} f_t^{h_{t+j}^{(n+1-j)}}, y_t^{(n)}\right). \end{aligned}$$

We plug these two values back into the variance of $h_{t+1}^{(n)}$ to get

$$\begin{aligned} Var\left(h_{t+1}^{(n)}\right) &= -n^2 Var\left(y_t^{(n)}\right) + (n-1)^2 Var\left(y_{t+1}^{(n-1)}\right) + 2nCov\left(y_t^{(1)}, y_t^{(n)}\right) \\ &\quad + 2nCov\left(E_t^*\left[h_{t+1}^{(n)}\right] - y_t^{(1)}, y_t^{(n)}\right) - 2nCov\left(\sum_{j=2}^{n} f_t^{h_{t+j}^{(n+1-j)}}, y_t^{(n)}\right). \end{aligned} \tag{A5}$$

Under RE and the expectations hypothesis, the fourth and fifth term of the equation would be zero. The hypothetical variance under these conditions, denoted as $\overline{Var}\left(h_{t+1}^{(n)}\right)$ is

$$\overline{Var}\left(h_{t+1}^{(n)}\right) = -n^2 Var\left(y_t^{(n)}\right) + (n-1)^2 Var\left(y_{t+1}^{(n-1)}\right) + 2nCov\left(y_t^{(1)}, y_t^{(n)}\right).$$

From Shiller (1979), we know that under these conditions the variance has an upper bound defined by

$$\overline{Var}\left(r_{t+1}^{(n)}\right) \;\leq\; \frac{n^2}{2n-1}Var\left(y_t^{(1)}\right).$$

This means that the generalized bound from (A5) is

$$Var\left(h_{t+1}^{(n)}\right) \;\leq\; \frac{n^2}{2n-1}Var\left(y_t^{(1)}\right) + 2nCov\left(E_t^*\left[h_{t+1}^{(n)}\right] - y_t^{(1)}, y_t^{(n)}\right) - 2nCov\left(\sum_{j=2}^{n} f_t^{h_{t+j}^{(n+1-j)}}, y_t^{(n)}\right)$$

as expressed in equation (8).

B. Data sources for Inflation and Nominal Earnings Growth Expectations

B.1. Inflation Expectations

Our main source for inflation expectations is the Survey of Professional Forecasters (SPF). The SPF contains quarterly median inflation forecasts of one-year inflation expectations from 1976Q1-2018Q2 and average ten-year inflation expectations from 1991Q1-2018Q2. As suggested by the technical documentation in the SPF, the average ten-year inflation expectation series is complemented backwards to 1979Q3 with the average ten-year inflation expectations from the Philadelphia Fed's Livingston Survey and from the Blue Chip Economic Indicators Survey.

For robustness, we also analyze an alternative measure of inflation expectations from the Survey of Consumer Finance which has both one-year inflation expectations and five-to-ten-year average inflation expectations. This alternative measure ranges from 1979Q3 to 2015Q4 and gives qualitatively very similar results.

B.2. Nominal Earnings Growth Expectations

We construct one-year cash flow expectations for the S&P 500 index following the aggregating procedure in De la O and Myers (2021). The Summary Statistics of the Thomson Reuters I/B/E/S database contains the median analyst forecasts for EPS (earnings per share) since 1976.[25] Using these individual forecasts, we measure aggregate earnings expectations using the constituents of the S&P 500 at each point in time. Expected earnings growth for the S&P 500 is then simply measured from the expectation of one-year aggregate earnings for

[25] Using the mean forecasts does not change the results in any noticeable way.

the S&P 500 and the current earnings of the S&P 500. The online appendix of De la O and Myers (2021) shows in detail the tests of using this methodology for the I/B/E/S database.

For longer horizons, analysts forecast growth rather than the future level of earnings. For any set of firms, the growth of total earnings for the entire set is equal to the earnings-weighted average of individual firm earnings growth. Thus, following Nagel and Xu (2021), we measure the expected long-term earnings growth of the S&P 500 using the earnings-weighted average of the individual firm forecasts, dropping firms with negative weights. To ensure few firms are dropped, we weight by 5-year average earnings as these are rarely negative. However, we do not find any noticeable change in our results if we weight by forecasted earnings for the end of the fiscal year as in Nagel and Xu (2021).

C. Robustness checks

C.1. Long-term nominal earnings growth expectations

In this section, we repeat our analysis using the value-weighted long-term nominal growth expectations of Bordalo et al. (2020) provided by the authors. Analysts in the IBES dataset provide long-term nominal earnings growth expectations for individual firms. For any set of firms such as the S&P 500, the growth of total earnings for the entire set is equal to the earnings-weighted average of individual firm earnings growth. This is why we use the earnings-weighted average long-term nominal earnings growth expectation for our main analysis. However, one can also construct a value-weighted average for long-term nominal earnings growth.

We find that using the value-weighted long-term nominal earnings growth expectations does not change our results in any noticeable way. Specifically, we continue to find that long-term nominal earnings growth expectations (i) have low volatility compared to short-term nominal earnings growth expectations, (ii) imply that the believed persistence of nominal earnings growth is quite low, and most importantly (iii) fail to satisfy the single necessary condition derived in Section I. Table AI shows the results. The standard deviations for the earnings-weighted and value-weighted expectations closely align at 1.2% and 1.4%, respectively, and are both substantially lower than the 26.4% standard deviation of short-term nominal earnings growth expectations. In other words, short-term expectations are still much more volatile than long-term expectations. A natural consequence of this is that the

Table AI

Value-weighted long-term nominal earnings growth expectations

This table compares the earnings-weighted long-term nominal earnings growth expectation to the value-weighted long-term nominal earnings growth expectation. The first row compares the standard deviation of the series. The second row reports the believed persistence estimated from equation (10) using the short-term and long-term nominal earnings growth expectations. The third and fourth rows show the covariance of the S&P 500 price-earnings ratio with the long-term nominal earnings growth expectations $E_t^*[\Delta e_{t+3,t+5}]$ and the forecast errors $\Delta e_{t+3,t+5} - E_t^*[\Delta e_{t+3,t+5}]$. All estimates are calculated using a sample of 1982Q1 to 2018Q2. Small-sample adjusted Newey-West standard errors reported in parenthesis.

	Earnings-weighted	Value-weighted
Standard deviation	1.2%	1.4%
Believed persistence ϕ_e	0.004	0.008
	(0.075)	(0.049)
Comovement with pe_t	0.36	0.55
	(0.40)	(0.48)
Comovement of forecast errors with pe_t	2.16	2.35
	(1.57)	(1.65)

believed persistence is nearly 0. The estimated value for ϕ_e from equation (10) using the earnings-weighted expectations and the value-weighted expectations are both small and insignificant at 0.004 (0.075) and 0.008 (0.049), respectively. Both of these estimates are far below the believed persistence of 0.96 (0.01) estimated for inflation expectations.

Finally, we find that value-weighted long-term nominal earnings growth expectations also fail to satisfy the single necessary and sufficient condition for explaining price movements and return puzzles. As detailed in Section I, errors in subjective expectations will only help to explain price movements and return puzzles if the price-earnings ratio negatively comoves with forecast errors, which is equivalent to the price-earnings ratio comoving more with expected earnings growth than with realized earnings growth. The third row of Table AI shows that the comovement of value-weighted expectations with the price-earnings ratio is small and insignificant at 0.55 (0.48), matching our findings using the earnings-weighted expectations of 0.36 (0.40). Crucially, forecast errors for both the value-weighted expectations and the earnings-weighted expectations have no significant comovement with the price-earnings ratio, as shown in the fourth row of Table AI. If anything, these forecast errors positively, rather than negatively, comove with the price-earnings ratio. In short, these expectations do

Table AII

Forecast error predictability with fundamentals

This table shows the covariance of current inflation and nominal earnings growth with expected and realized inflation and nominal earnings growth. In *Panel A*, the first row shows the covariance of current inflation with short-term inflation expectations $E_t^*[\pi_{t+1}]$, realized short-term inflation π_{t+1}, and the forecast errors $\pi_{t+1} - E_t^*[\pi_{t+1}]$ from 1976Q1 to 2018Q2. The second shows the covariance of current inflation with long-term inflation expectations $E_t^*[\pi_{t+1,t+10}]$, realized long-term inflation $\pi_{t+1,t+10}$, and the forecast errors $\pi_{t+1,t+10} - E_t^*[\pi_{t+1,t+10}]$using quarterly data from 1979Q4 to 2018Q2. In *Panel B* , the first row shows the covariance of current one-year nominal earnings growth with short-term nominal earnings growth expectations $E_t^*[\Delta e_{t+1}]$, realized short-term nominal earnings growth Δe_{t+1}, and the forecast errors $\Delta e_{t+1} - E_t^*[\Delta e_{t+1}]$ from 1976Q1 to 2018Q2. The second row shows the covariances of current one-year nominal earnings growth with long-term nominal earnings growth expectations $E_t^*[\Delta e_{t+3,t+5}]$, realized long-term nominal earnings growth $\Delta e_{t+3,t+5}$ and their forecast errors $\Delta e_{t+3,t+5} - E_t^*[\Delta e_{t+3,t+5}]$ from 1982Q1 to 2018Q2. Expectations expressed in percentages. Small-sample adjusted Newey-West standard errors in parenthesis. Superscripts indicate significance at the 1% (***), 5% (**), and 10% (*) level.

	Horizon	Expected	Realized	Forecast Error
Panel A: Inflation				
$Cov(\pi_t, \cdot)$	Short-term (π_{t+1})	0.04^{***}	0.05^{***}	0.01
	Long-term ($\pi_{t+1,t+10}$)	0.22^{***}	0.09^{***}	-0.13^{***}
Panel B. Nominal Earnings Growth				
$Cov(\Delta e_t, \cdot)$	Short-term (Δe_{t+1})	-6.47^{***}	-2.83	3.63^{***}
	Long-term ($\Delta e_{t+3,t+5}$)	-0.02	-4.77	-4.75

not comove enough with the price-earnings ratio to satisfy the single necessary and sufficient condition.

C.2. Forecast error predictability with fundamentals

Table AII shows the forecast error predictability of subjective expectations of inflation and nominal earnings growth with current realizations of inflation and nominal earnings growth. Similar to the results in Table I, we find large and significant predictable forecast errors for long-term inflation and short-term nominal earnings growth expectations and insignificant errors for short-term inflation and long-term nominal earnings growth expectations.

C.3. Generalized persistence structure

We calculate a more generalized version of equations (9) and (10) that allows inflation expectations to impact nominal earnings growth expectations,

$$\begin{pmatrix} E_t^*\left[\pi_{t+1+j}\right] \\ E_t^*\left[\Delta e_{t+1+j}\right] \end{pmatrix} = \begin{pmatrix} \alpha_{\pi,j} \\ \alpha_{e,j} \end{pmatrix} + \begin{pmatrix} \phi_\pi & 0 \\ \phi_{\pi,e} & \phi_e \end{pmatrix}^j \begin{pmatrix} E_t^*\left[\pi_{t+1}\right] \\ E_t^*\left[\Delta e_{t+1}\right] \end{pmatrix} + \begin{pmatrix} \varepsilon_{t,j}^\pi \\ \varepsilon_{t,j}^e \end{pmatrix}. \quad \text{(A1)}$$

This generalization has no impact on our estimate of ϕ_π of 0.96 (0.01). Using the short-term and long-term inflation expectations and nominal earnings growth expectations, we estimate ϕ_e as 0.001 (0.088) which is almost identical to our estimate from equation (10) of 0.004 (0.075). In other words, including this interaction does not impact our estimate of ϕ_e.

Importantly, when we estimate the interaction term $\phi_{\pi,e}$, we find a small and insignificant value of 0.22 (0.28). As discussed in Section II.A, the large decline in inflation expectations did not lead to a noticeable decline in nominal earnings growth expectations. If we include this interaction term, the fundamental price-earnings ratio would be

$$pe_t^{fun} = \frac{1}{1-\rho\phi_e} E_t^*\left[\Delta e_{t+1}\right] - \frac{1}{1-\rho\phi_\pi}\left(1 - \frac{\rho\phi_{\pi,e}}{1-\rho\phi_e}\right) E_t^*\left[\pi_{t+1}\right]. \quad \text{(A2)}$$

Note that this is identical to equation (11), except for the $\left(1 - \frac{\rho\phi_{\pi,e}}{1-\rho\phi_e}\right)$ scaling on inflation expectations. Given that $\phi_{\pi,e}$ and ϕ_e are both near 0 and insignificant, this scaling term is close to 1 at 0.79 and is not statistically significantly different from 1.

Because the scaling term is near 1, this interaction term has almost no impact on the asset pricing results and by extension the return predictability results. Regressing the observed price-earnings ratio on the fundamental price-earnings ratio from equation (A2) produces a coefficient of almost exactly 1 at 1.07 (0.11) with an R^2 of 0.81. This is virtually identical to the results of Table III where the regression coefficient is 0.96 (0.09) and the R^2 is 0.81.

Regressing future 10-year nominal returns on the fundamental price-earnings ratio from (A2) produces a coefficient of -0.85 (0.07) with an R^2 of 0.39. Similarly, regressing 10-year real returns on the fundamental price-earnings ratio from (A2) produces a coefficient of -0.59 (0.12) with an R^2 of 0.24. These are almost identical to the results in Table IV.

D. *Proofs for model*

D.1. Representativeness bias and believed persistence

In this section, we microfound our model of expectations using the representativeness bias. Given that the procedure is general for any AR(1) process, we just focus on the case of inflation. We demean all variables without loss of generality, as we are only interested in variances and covariances. Inflation is

$$\pi_{t+1} \quad = \quad \varphi\pi_t + \varepsilon_{t+1} \tag{A1}$$

where $\varepsilon_{t+1} \sim N\left(0, \sigma^2\right)$. Intuitively, high values for current inflation are indicative of high future inflation. In other words, high current inflation is “representative” of high future inflation relative to a baseline case of average inflation. Specifically, the relative frequency of high future inflation is larger when current inflation is high. When current inflation makes certain future outcomes more likely, representativeness bias causes agents to exaggerate the likelihood of those outcomes. This causes agents to exaggerate the likelihood of high future inflation when current inflation is high.

Let $h\left(\pi'|\pi\right)$ be the objective probability of next period inflation π' given current inflation π, which is simply the standard normal PDF of $\left(\pi' - \varphi\pi\right)/\sigma$. As in Bordalo et al. (2018), we formalize representativeness bias as distorting the objective probability density for next period inflation $h\left(\pi'|\pi = \pi_t\right)$ so that agents form beliefs using $h^\theta\left(\pi'|\pi = \pi_t\right)$. We assume that the background context is the case where current inflation is 0, i.e., the average given that all variables are demeaned. The distorted probability density is then (up to a normalization constant)

$$h^\theta\left(\pi'|\pi = \pi_t\right) \quad = \quad h\left(\pi'|\pi = \pi_t\right)\left[\frac{h\left(\pi'|\pi = \pi_t\right)}{h\left(\pi'|\pi = 0\right)}\right]^\theta. \tag{A2}$$

When current inflation makes π' more likely relative to the background context of $\pi = 0$, agents exaggerate the probability of this outcome.

Under this distorted probability density, expected future inflation is

$$E_t^*\left[\pi_{t+1}\right] \quad = \quad \left(1+\theta\right)\varphi\pi_t. \tag{A3}$$

In other words, agents believe that an increase in current inflation raises next year inflation on average by $\left(1+\theta\right)\varphi$ rather than the objective value of φ. This means that agents act as if inflation has persistence $\varphi^* = \left(1+\theta\right)\varphi$ which is a magnified version of the objective persistence φ. Given this distortion on one-period expectations, we assume that agent's expectations satisfy the law of iterated expectations, which implies that

$$E_t^*\left[\pi_{t+j}\right] \;=\; \varphi^{*j}\pi_t. \tag{A4}$$

D.2. Proof of Proposition 1

Let z_t be any linear combination of nominal earnings growth and negative inflation, $z_t = \chi_e \Delta e_t - \chi_\pi \pi_t$. Then, given equations (16) for the price-earnings ratio, we have

$$\begin{aligned}\frac{Cov\left(z_t, pe_t\right)}{Var\left(pe_t\right)} &= \frac{\chi_e \alpha_e^* \sigma_e^2 + \chi_\pi \alpha_\pi^* \sigma_\pi^2}{\alpha_e^{*2}\sigma_e^2 + \alpha_\pi^{*2}\sigma_\pi^2} \\ &= w\frac{\chi_e}{\alpha_e^*} + (1-w)\frac{\chi_\pi}{\alpha_\pi^*} \qquad \text{(A5)}\end{aligned}$$

where $\alpha_e^* = \frac{\varphi_e^*}{1-\rho\varphi_e^*}$, $\alpha_\pi^* = \frac{\varphi_\pi^*}{1-\rho\varphi_\pi^*}$ and $w = \frac{\alpha_e^{*2}\sigma_e^2}{\alpha_e^{*2}\sigma_e^2+\alpha_\pi^{*2}\sigma_\pi^2}$, as defined in Section IV.

From the Campbell-Shiller decomposition we have that the sum of total future returns is:

$$\begin{aligned}\sum_{j=1}^{\infty}\rho^{j-1}\tilde{r}_{t+j} &= \sum_{j=1}^{\infty}\rho^{j-1}\Delta\tilde{e}_{t+j} - pe_t \\ &= \alpha_e\Delta e_t - \alpha_\pi\pi_t - pe_t + \sum_{j=1}^{\infty}\rho^{j-1}\varepsilon_{t+j}^e - \sum_{j=1}^{\infty}\rho^{j-1}\varepsilon_{t+j}^\pi \\ &= \left(\alpha_e - \alpha_e^*\right)\Delta e_t - \left(\alpha_e - \alpha_e^*\right)\pi_t + \sum_{j=1}^{\infty}\rho^{j-1}\varepsilon_{t+j}^e - \sum_{j=1}^{\infty}\rho^{j-1}\varepsilon_{t+j}^\pi.\end{aligned}$$

Note that the white-noise shocks $\varepsilon_{t+j}^e, \varepsilon_{t+j}^\pi$ do not affect the covariance, so we can insert this into equation (A5) to conclude

$$\frac{Cov\left(\sum_{j=1}^{\infty}\rho^{j-1}\tilde{r}_{t+j}, pe_t\right)}{Var\left(pe_t\right)} = w\left(\frac{\alpha_e}{\alpha_e^*} - 1\right) + (1-w)\left(\frac{\alpha_\pi}{\alpha_\pi^*} - 1\right).$$

D.3. Proof of Corollary 1

First, we note that $Cov\left(\sum_{j=1}^{T}\tilde{r}_{t+j}, pe_t\right) = Cov\left(E_t\left[\sum_{j=1}^{T}\tilde{r}_{t+j}\right], pe_t\right)$ where $E_t\left[\cdot\right]$ is the objective expectation. Using equation (1), the objective expectation for the next period real return is

$$\begin{aligned}
E_t[\tilde{r}_{t+1}] &= E_t[\Delta\tilde{e}_{t+1}] - pe_t + \rho E_t[pe_{t+1}] \\
&= \varphi_e \Delta e_t - \varphi_\pi \pi_t - \left(\alpha_e^* \Delta e_t - \alpha_e^* \pi_t\right) + \rho\left(\alpha_e^* E_t[\Delta e_{t+1}] - \alpha_\pi^* E_t[\pi_{t+1}]\right) \\
&= \varphi_e \Delta e_t - \varphi_\pi \pi_t - \left(\alpha_e^* \Delta e_t - \alpha_e^* \pi_t\right) + \rho\left(\alpha_e^* \varphi_e \Delta e_t - \alpha_\pi^* \varphi_\pi \pi_t\right) \\
&= \frac{\varphi_e - \varphi_e^*}{1 - \rho\varphi_e^*} \Delta e_t - \frac{\varphi_\pi - \varphi_\pi^*}{1 - \rho\varphi_\pi^*} \pi_t
\end{aligned}$$

where the second line inserts the definition of price-earnings ratio from equation (16) and the third line substitutes in the objective expectation of inflation and nominal earnings growth.

We can then calculate the objective expectation of future real returns over finite horizon T as

$$\begin{aligned}
E_t\left[\sum_{j=1}^{T} \tilde{r}_{t+j}\right] &= E_t\left[\sum_{j=1}^{T} E_{t+j-1}\left[\tilde{r}_{t+j}\right]\right] \\
&= \frac{\varphi_e - \varphi_e^*}{1 - \rho\varphi_e^*} \frac{1 - \varphi_e^T}{1 - \varphi_e} \Delta e_t - \frac{\varphi_\pi - \varphi_\pi^*}{1 - \rho\varphi_\pi^*} \frac{1 - \varphi_\pi^T}{1 - \varphi_\pi} \pi_t.
\end{aligned}$$

Inserting this into equation (A5) gives us

$$\frac{Cov\left(\sum_{j=1}^{T} \tilde{r}_{t+j}, pe_t\right)}{Var\left(pe_t\right)} = \frac{Cov\left(E_t\left[\sum_{j=1}^{T} \tilde{r}_{t+j}\right], pe_t\right)}{Var\left(pe_t\right)} = w\left(\frac{\alpha_e}{\alpha_e^*} - 1\right)\gamma_e + (1-w)\left(\frac{\alpha_\pi}{\alpha_\pi^*} - 1\right)\gamma_\pi$$

where $\gamma_e = \left(1 - \rho\varphi_e\right) \frac{1-\varphi_e^T}{1-\varphi_e}$, $\gamma_\pi = \left(1 - \rho\varphi_\pi\right) \frac{1-\varphi_\pi^T}{1-\varphi_\pi}$.

D.4. Proof of Proposition 2

The one-year holding period returns are defined as

$$\begin{aligned}
h_{t+1}^{(n)} &= n y_t^{(n)} - (n-1)\, y_{t+1}^{(n-1)} \\
&= \varphi_\pi^* \pi_t \left[1 + \frac{1 - \varphi_\pi^{*(n-1)}}{1 - \varphi_\pi^*} \left(\varphi_\pi^* - \varphi_\pi\right)\right] - \varphi_\pi^* \frac{1 - \varphi_\pi^{*(n-1)}}{1 - \varphi_\pi^*} \varepsilon_{t+1}
\end{aligned}$$

Define $\lambda = \frac{1-\varphi_\pi^{*(n-1)}}{1-\varphi_\pi^*}$. Since $Var\left(y_t^{(1)}\right) = \varphi_\pi^* \pi_t$, we can obtain the ratio of variances as

$$\frac{Var\left(h_{t+1}^{(n)}\right)}{Var\left(y_t^{(1)}\right)} = \left[1 + \lambda\left(\varphi_\pi^* - \varphi_\pi\right)\right]^2 + \left(1 - \varphi_\pi^2\right)\lambda^2.$$