Credit Cycles with Market-Based Household Leverage*

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Abstract

We develop a general equilibrium model in which households' mortgage leverage is determined by supply and demand forces, where the price of credit impacts the quantity of leverage households choose. Mortgages are supplied by financial intermediaries, who offer households a menu of mortgage contracts whose pricing varies with intermediaries' equity capital. In the model, growth in the demand for safe assets that replicates the falling interest rates in the 2000s causes an empirically realistic boom in household borrowing, debt-financed consumption, and house prices. This boom results in a larger bust in asset prices and household borrowing in future financial crises.

JEL Codes: E44, G01, G21, G51 Keywords: Credit Constraints, Household Leverage, Financial Intermediaries, Housing Boom, Credit Crises, Safe Assets

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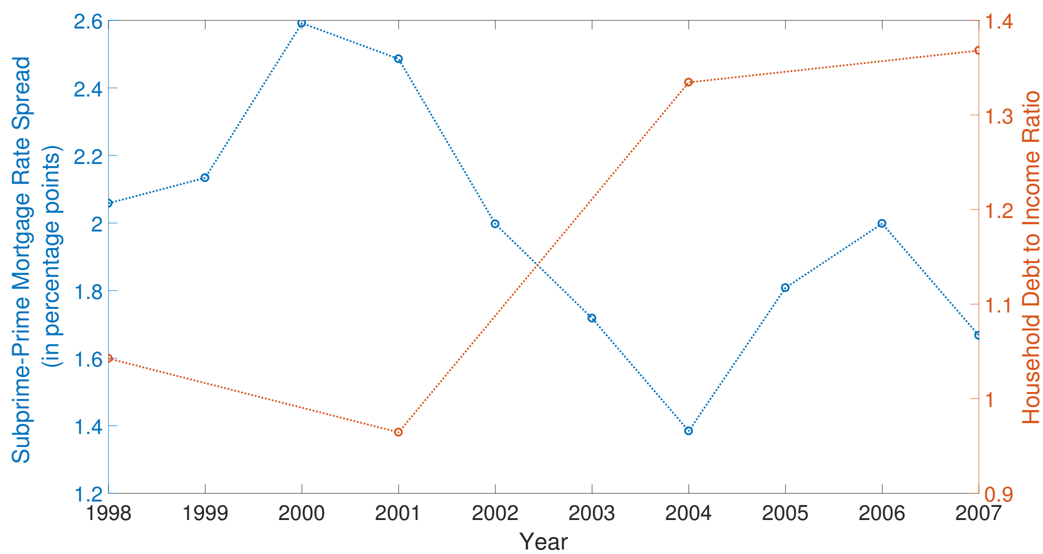
1 Introduction

In the 2000s, the U.S. economy experienced a credit boom followed by the financial crisis of 2007-2008. As shown in figure 1 below, the spread between prime and subprime mortgage rates dropped from a peak of 2.6% in 2000 to a low of 1.4% in 2004, while households' debt to income ratio surged from .94 in 2001 to 1.37 in 2007. This drop in the cost of risky borrowing and increase in the quantity borrowed by households was accompanied by rapid growth in the size of the financial sector. The simultaneous fall in price and rise in quantity household borrowing is consistent with an increase in credit supply, where the quantity of credit households demand increases as the cost of borrowing falls.

Figure 1: Prime-Subprime Rate Spread and Household Debt to Income Ratio During 2000s Housing Boom



The prime and subprime rates are for 30 year fixed-rate mortgages and are respectively from the FHFA's Monthly Interest Rate Survey and from the Moody's Non-Agency RMBS Database. The debt to income series is from the Survey of Consumer Finances.

This paper presents a quantitative general equilibrium model in which the pricing of credit offered by the financial sector impacts the quantity of leverage chosen by households. Most quantitative work on the 2000s credit cycle assumes that household leverage is exogenously constrained by a fraction of their home value or their income, making households' demand for leverage inelastic to its price. Booms and busts in such models are generated by exogenous loosening or tightening of these constraints. The modeling approach in this paper, where household leverage is determined by supply and demand forces like any other good, allows us to study how asset price fluctuations impact house-

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holds' leverage choices. As a result, our model features a key mechanism by which events on Wall Street that impact asset prices can have macroeconomic consequences for Main Street. We use the model to analyze how household leverage responds to a drop in interest rates caused by a growing demand for safe assets, which would have no response if household leverage were determined by exogenous constraints.

Our model has three key types of agents: young households who want to borrow, middle-aged households who want to save for retirement, and financial intermediaries. The model's financial sector follows recent work in the intermediary asset pricing literature, in which intermediaries issue riskless deposits, invest in risky assets such as mortgages, and face frictions in raising the equity capital needed to bear the risk in their asset portfolios. When intermediary capital is low, the price of risk in asset markets rises and high-leverage loans become expensive for borrowers. Young households expect rapid growth in their future income and therefore want to finance their consumption by borrowing from middle-aged households saving for retirement. However, households must pledge housing as collateral in order to borrow and can only borrow from intermediaries. The consumption of the young is therefore highly sensitive to the supply of credit from intermediaries, and they choose risky, highly levered mortgages.

Relative to existing quantitative models of credit cycles, our approach to modelling leverage is new and builds on recent theoretical work (Geanakoplos (2010), Simsek (2013), Diamond (2020)). Financial intermediaries offer households a menu of competitively priced mortgage contracts, where the severity of the intermediaries' financing constraints endogenously determines the risk premium they charge for default risk. This menu yields a "credit surface" which determines how the interest rate on a mortgage depends on the leverage of the mortgage and the creditworthiness of the borrower. Households choose their optimal mortgage from the credit surface, with no exogenous constraints placed on the amount of leverage they choose. When credit spreads on risky mortgages increase, households endogenously choose to reduce their leverage.

Although our model features rich heterogeneity and imperfect risk sharing between households, the total wealth of households in each generation is the only state variable required to describe their aggregate behavior. This aggregation result follows because households have homogeneous utility functions and choice sets that scale linearly in their wealth. Our innovation in this paper is to allow households to trade their endowment income with other households of the same age, but not with other agents.[1] This allows us to

[1]In a standard model of non-pledgeable endowment income, households become increasingly financially constrained as their liquid wealth decreases, so their choice set does not scale linearly in their wealth.

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model financial distortions due to the lifecycle dynamics of endowment income while maintaining enough tractability to solve the model.[2]

We use our model for three quantitative counterfactuals: comparing non-financial recessions driven by productivity with housing recessions, understanding the equilibrium effects of a drop in the equity capital of intermediaries, and analyzing the effects of a growing demand for safe assets on the financial system and real economy. Housing recessions in our model are caused by shocks to the cross-sectional dispersion of house values, which we refer to as "housing risk shocks". High house price dispersion pushes more borrowers underwater and causes more mortgage defaults. The resulting losses for intermediaries reduce their ability to bear risk going forward. This impaired risk-bearing capacity leads to a large increase in mortgage spreads and to a drop in the leverage of all households in a housing crisis, while the mortgage market is nearly unaffected by a productivity-driven recession. All households have similarly sized drops in consumption in the productivity driven recession, while in a housing crisis borrowing-constrained young households face a disproportionate drop in consumption.

Next, we analyze the effects of a 50% drop in the equity capital of financial intermediaries on the economy, without any other exogenous shocks. This equity loss impairs the ability of the intermediary to bear risk, leading to a 3% increase in mortgage rates for the young and 40 basis point increase for the middle-aged, despite a reduction in households' average loan to value ratio from 0.57 to 0.5. This implies that households' borrowing costs rise even though the riskiness of their mortgages decreases. The consumption of young, credit-constrained households remains depressed for several years as intermediaries gradually rebuild their equity capital and become more willing to provide risky mortgages. As intermediary capital converges back to its original level, other variables gradually revert as well.

As our main quantitative experiment, we consider the general equilibrium effects of a growing demand for safe assets, considering both the effects during the credit and housing boom of the 2000s and the following bust and crisis. As discussed by Bernanke (2005), Caballero and Farhi (2018), and others, an increase in the demand for safe assets was a key macroeconomic feature of the economy before the financial crisis. Relative to existing literature, our model allows us to study the indirect effects of this growing demand on intermediary leverage and risk taking, household leverage, consumption, and house prices. Along a range of dimensions, we find that this increase in the demand for safe as-

[2] As in Constantinides and Duffie (1996), households face only multiplicative shocks to their wealth, so they choose portfolios that scale linearly in their wealth even with incomplete financial markets.

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sets replicates features of the pre-crisis lending boom. In particular, we find that the size of the financial sector, the amount of mortage debt outstanding, the leverage of households, and the price of houses increase. Our baseline specification matches the roughly 25% increase in house price to rent ratios and about two thirds of the 30% increase in mortgage debt to income ratios observed from 1998 to 2008.

We then study how the economy with inflated house prices and mortgage debt, following the elevated demand for safe assets, responds to a housing risk shock, which causes more homeowners to default. Relative to an average economy drawn from the ergodic distribution, our high-safe-asset-demand economy is more vulnerable to this shock. The economy faces a sharp increase in mortgage defaults, which depletes the majority of the financial system's equity capital. This in turn induces banks to charge substantially higher spreads on mortgage debt for any given leverage, and as a result households cut back drastically on their mortgage leverage. Because our model connects household leverage choices with intermediary risk taking capacity, this counterfactual provides a rich illustration of how a growing demand for safe assets increases the size and riskiness of the financial sector and therefore the severity of financial crises.

We consider three variations on our baseline counterfactual, featuring financial deregulation, overoptimistic beliefs about the creditworthiness of borrowers, and adding a market in which the middle-aged can rent housing to the young. The first two variations increase the magnitude of the boom and bust in our model. The financial deregulation counterfactual roughly matches the observed size of the boom, while adding overoptimism overshoots the observed data. Both of these counterfactuals feature an empirically realistic reduction in the risk premium on subprime mortgages and a sharper reduction in young households' borrowing and consumption than in our baseline featuring only a growing demand for safe assets.

Our third variation that adds rental markets is motivated by the work of Kaplan, Mitman, and Violante (2020) showing in their model that a rental market for housing sharply reduces the impact of credit supply on house prices.[3] In our model with a rental market, we obtain a roughly 20% increase in the house price to rent ratio in our safe asset demand counterfactual. This is because our middle-aged landlords, even though they are not financially constrained like the young, demand a larger quantity of borrowing when mortgage rates are lower. As a result, the value of a home as collateral rises when interest rates fall, and shocks to bank credit supply impact house prices even when landlords are

[3]Greenwald and Guren (2020) show a similar disconnect between credit supply and house prices in a version of their model with a financially unconstrained landlord, though not in their benchmark model calibrated to empirical estimates of the effect of credit supply on house prices.

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wealthy and financially unconstrained.

Finally, we compare our mechanism to the literature with hard loan-to-value and debt-to-income constraints by incorporating such constraints in our model. We repeat the boom-bust experiment in the model with hard constraints. We conclude that the rise in asset demand causes a large implicit relaxation of debt-to-income, and a moderate relaxation of loan-to-value constraints in our model with endogenous leverage determination. In a model with a hard debt-to income constraint, a growing demand for safe assets has a minimal effect on the quantity of household debt, since households are not able to respond to a reduction in mortgage rates by increasing the amount they promise to repay.[4] Unlike a conventional model with binding exogenous constraints on household borrowing, our framework allows for household leverage and consumption decisions to respond to fluctuations in credit supply and asset prices. Because our model allows household leverage to be elastic to the price of credit, following standard supply-and-demand logic, it demonstrates how asset price fluctuations on Wall Street have consequences for borrowing and consumption decisions on Main Street.

Related Literature. A key feature of research in macroeconomics and finance since the 2008 financial crisis is a new understanding of how financial frictions impact the overall economy. A large and growing body of empirical research documents the macroeconomic roles of house prices, credit supply, and their impact on households' leverage and consumption.[5] Another recent body of empirical work documents how distressed financial institutions reduce the supply of credit to households and firms, contributing to a drop in output and employment.[6] Our goal is to develop a model that is consistent with and unifies findings in both empirical literatures as a framework for counterfactual analysis.

The quantitative macroeconomics literature after the housing boom has focused on models with exogenous housing collateral constraints following Iacoviello (2005), such as in Kiyotaki, Michaelides, and Nikolov (2011), Landvoigt, Piazzesi, and Schneider (2015), Favilukis, Ludvigson, and Van Nieuwerburgh (2017), and Guerrieri and Lorenzoni (2017). More recent work emphasized the importance of high household indebtedness and credit

[4]In a model like Greenwald (2018) where a household's mortgage payment is constrained to a fraction of its income, a reduction in mortgage rates can increase the amount the household can borrow today. We demonstrate however in subsection 4.3.1 that the introduction of such a constraint to our model sharply reduces the response of household borrowing to a drop in mortgage rates.

[5]For example Mian and Sufi (2011), Mian, Rao, and Sufi (2013), Favara and Imbs (2015), Di Maggio, Kermani, Keys, Piskorski, Ramcharan, Seru, and Yao (2017), Di Maggio and Kermani (2017), Adelino, Schoar, and Severino (2016), and Foote, Lowenstein, and Willen (2020)

[6]For example Chodorow-Reich (2014), Chodorow-Reich and Falato (2020), Benmelech, Meisenzahl, and Ramcharan (2014), and Ramcharan, Verani, and Van Den Heuvel (2016)

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frictions for the severity of the bust, for example Guren, Krishnamurthy, and McQuade (2018) and Hedlund and Garriga (2020), or the relevance of household-level credit frictions for the transmission of monetary policy and other aggregate shocks, e.g. Elenev (2018), Wong (2019) and Greenwald (2018). Papers in this literature tighten or loosen exogenous collateral, leverage, or payment to income constraints to simulate a boom or bust. As pointed out by Justiniano, Primiceri, and Tambalotti (forthcoming), among others, it is necessary in such a framework to shock both households' borrowing constraints and constraints on the supply of mortgages to explain movements in both the prices and quantity of high leverage mortgages during the 2000-2006 housing boom. Our framework, in which a reduction in intermediaries' funding cost both lowers the price and raises the leverage of household debt, provides a single explanation for these facts.

Corbae and Quintin (2015) is the only paper we know in this literature where households choose the leverage of their mortgage. They study a framework where households face a menu of mortgage contracts offered by a risk-neutral lender subject to exogenous constraints and select endogenously into high and low leverage mortgages. To our knowledge, ours is the first quantitative paper to model heterogeneous borrowers facing a menu of leverage choices priced by constrained intermediaries in general equilibrium. The fact that constrained intermediares price mortgages in our model is crucial for there to be fluctuations in mortgage risk premia driven by the supply of credit. In a model with risk-neutral lenders, mortgage risk premia would be quantitatively too small for households to meaningfully restrict their leverage. This is perhaps why the literature has followed the practice of exogenously constraining household leverage, so that a low price of mortgage risk does not lead to an unreasonably high quantity of mortgage leverage.

A separate research agenda in finance on intermediary asset pricing (He and Krishnamurthy (2013), Adrian, Etula, and Muir (2014), He, Kelly, and Manela (2017)) has shown empirically and quantitatively that the risk taking capacity of financial intermediaries is a key driver of asset prices. This approach to asset pricing successfully explains prices in a range of asset classes and is particularly important for pricing highly intermediated assets such as derivatives, bonds, and commodities (Haddad and Muir (2020)). Of particular relevance to our model is the empirical evidence (Gabaix, Krishnamurthy, and Vigeron (2007), Hanson (2014)) that the pricing of mortgage risk is sensitive to the risk taking capacity of specialized intermediaries. Relative to this literature, our contribution is to connect the pricing kernel of a constrained financial intermediary to the leverage choices of the agents that borrow from it.

Finally, our paper provides a potential resolution to the question whether the boom-

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bust episode was caused by loose credit constraints, or high expected future house prices (Landvoigt (2017), Kaplan, Mitman, and Violante (2020)). In our framework, a large positive shock to the demand for safe assets leads to a relaxation of credit constraints, and simultaneously puts the economy on a path of rising house prices. This integrates the three narratives about the origins of the financial crisis mentioned above, in a manner that depends crucially on the role of financial intermediaries in our model as both mortgage lenders and creators of safe assets. While existing work connects the demand for safe assets to financial fragility (Caballero and Krishnamurthy (2009)), the indirect effect of a safe asset shortage on househould leverage and consumption we find relies crucially on our supply-and-demand approach to modeling household borrowing.

2 Model

2.1 Income and housing endowment

There are two goods, non-durable consumption goods and housing, both of whose supply is assumed to be exogenous for tractability. The aggregate output of non-durable consumption goods Y_t is the product of a deterministic trend $\bar{Y}_t$ and a random cyclical component $\tilde{Y}_t$, so $Y_t = \tilde{Y}_t \bar{Y}_t$. The trend $\bar{Y}_t$ grows at the rate g, so $\bar{Y}_t = \bar{Y}_{t-1}\exp(g)$. The cyclical component $\tilde{Y}_t$ follows an AR(1) in logs

$$\log(\tilde{Y}_t) = (1-\rho_y)\mu_y + \rho_y \log(\tilde{Y}_{t-1}) + \varepsilon_t, \tag{1}$$

where ε_t is i.i.d. and normally distributed with mean zero, and (ρ_y, μ_y) are constants.

The economy is endowed with a constant stock of housing capital $\bar{H}$. Housing capital produces housing services s_t that can be consumed each period according to a linear technology whose productivity increases with the trend variable $\bar{Y}_t$, so h_t units of housing produces

$$s_t = n(h_t, \bar{Y}_t) = h_t \bar{Y}_t. \tag{2}$$

Owners of housing capital are required to spend non-durable consumption resources equal to δ_H of the market price of the housing they own on maintenance each period.

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2.2 Agents

The economy is populated by a continuum of households in three generations: old (O), middle-aged (M), and young (Y). To give households realistic life spans without having to track the precise age of every household, we assume that in each period households have a probability of aging into the next generation and otherwise do not age. The simplifying assumption that there are only 3 possible ages between which households move over many years drastically reduces the number of state variables we must track compared to keeping track of the number of years each household has lived.[7]

Young households have a probability π^Y of becoming middle-aged each period, drawn i.i.d across households. Middle-aged households (including those who age from young to middle-aged this period) have a probability π^M of becoming old, also drawn i.i.d across households. Old households live for one period. Each period, a measure one of new young households are born, and the same measure of old households die. The population of each generation is constant, with measure $\frac{1}{\pi^Y}$ young, $\frac{1}{\pi^M} - 1$ middle-aged, and 1 old households. In addition, there is a representative financial intermediary firm that makes loans to households, and issues riskless deposits and risky equity backed by its loan portfolio.

2.3 Markets

The model features competitive markets for housing, riskless bank deposits, mortgages, equity in the financial intermediary firm, and a special market in which households can trade claims to their endowment income with households of the same generation. We assume that certain markets are accessible only to certain agents. Housing and bank deposits can only be held by households. Intermediary equity can only be held by middle-aged households. Mortgages can only be held by the financial intermediary. These simplifying assumptions are consistent with the fact that equity ownership is highly concentrated among wealthy (older) investors, while mortgages are held primarily by financial institutions and not directly by households. In addition, for analytical tractability, we add a market in which households in each generation are able to rent housing but only from members of the same generation. We show in proposition 1 that this market is not used in equilibrium, so the equilibrium is equivalent to one with only owner-occupied housing.

At each time t, housing trades at a price p_t^h, riskless deposits pay an interest rate r_t,

[7] To our knowledge, lifecycle models which track each household's age seperately have only been feasible so far in partial equilibrium (Gourinchas and Parker (2002), Chen, Michaux, and Roussanov (2020)).

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intermediary equity trades at a price p_t^I, and members of generation $a \in \{Y, M\}$ can trade shares of their endowment at a generation-specific price p_t^a and rent housing at the generation-specific rate ρ_t^a. Young and middle-aged households can take out mortgages from the financial intermediary and choose the face values (m_t^Y, m_t^M) they owe as mortgage debt next period. The entire portfolios (α_t^Y, α_t^M) chosen by young and middle-aged households are given by

$$\alpha_t^Y = (d_t^Y, h_t^Y, b_t^{Y,Y}, m_t^Y) \tag{3}$$

$$\alpha_t^M = (d_t^M, h_t^M, b_t^{M,M}, b_t^{M,I}, m_t^M), \tag{4}$$

composed of deposits (d_t^Y, d_t^M), housing (h_t^Y, h_t^M), generation-specific endowment shares $(b_t^{Y,Y}, b_t^{M,M})$, mortgage face values (m_t^Y, m_t^M), and (only for the middle-aged) intermediary equity shares $b_t^{M,I}$. The ratio of a household's mortgage face value to the market value of its housing ($\frac{m_t^Y}{p_t^h h_t^Y}$ or $\frac{m_t^M}{p_t^h h_t^M}$) we will refer to as its leverage.

A household of generation a, that chooses a portfolio α_t^a is lent $q^a(\alpha_t^a, \mathcal{Z}_t) m_t^a$ by the intermediary, where $\mathcal{Z}_t$ indexes all aggregate state variables at time t. In addition, reflecting the tax-advantaged status of mortgage debt, the government provides a propotional subsidy τ^m, so the household receives

$$(1 + \tau^m) q^a(\alpha_t^a, \mathcal{Z}_t) m_t^a. \tag{5}$$

The function $q^a(\alpha_t^a, \mathcal{Z}_t)$ determines how the pricing of a mortgage varies first with a household's portfolio choice α_t^a, so that interest rates can be higher for riskier mortgages. Second, the function $q^a(\alpha_t^a, \mathcal{Z}_t)$ also determines how the pricing of mortgage risk varies with aggregate shocks $\mathcal{Z}_t$, so that mortgage credit can be more expensive when the financial intermediary is financially distressed. When an intermediary lends $q^a(\alpha_t^a, \mathcal{Z}_t) m_t^a$, it faces a proportional processing cost of $\zeta > 0$, so to fund the loan the intermediary actually has to pay

$$(1 + \zeta) q^a(\alpha_t^a, \mathcal{Z}_t) m_t^a. \tag{6}$$

One additional piece of notation we introduce is the vector of prices P_t^a and the associated quantities b_t^a, only be traded by generation $a \in \{Y, M\}$, given by

$$b_t^Y = b_t^{Y,Y}, \quad P_t^Y = p_t^Y, \quad b_t^M = (b_t^{M,M}, b_t^{M,I}), \quad P_t^M = (p_t^M, p_t^I). \tag{7}$$

This reflects the fact that only the young can trade their generation's endowment shares, while only the middle age can trade their generation's endowment shares and equity

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securities issued by the financial intermediary.

One crucial assumption made above is that all assets trade in competitive markets, even though some markets are not accessible to all agents. This implies that an individual household's choice set scales linearly in its wealth, so an agent with $k > 0$ as much wealth as another can buy k times as much of every asset. This linearity is crucial for proposition 1 below, which shows that all agents of each generation aggregate to a single representative agent and thereby makes our model tractable.

Housing Risk Shocks. After a household of generation a chooses the housing h_t^a it owns at time t, it is hit by an idiosyncratic shock ϵ_{t+1}^a at time $t+1$, leaving it with $\epsilon_{t+1}^a h_t^a$ at the beginning of period $t+1$. ϵ_{t+1}^a is a mean one lognormal random variable, drawn i.i.d. across households and across time. The standard deviations of ϵ_{t+1}^Y and ϵ_{t+1}^M are equal to each other and vary between a low value of σ_ϵ^0 and a high value of σ_ϵ^1. The transition between these two values of the standard deviation is determined by the realization of a binary Markov chain with transition matrix Γ. Realizations of this Markow chain are the second source of aggregate risk in the economy (the first source being output shocks ε_t). We refer to reaching the higher standard devation σ_ϵ^1 as a housing risk shock. Households choose whether to default on their mortgage after these shocks are realized. If they default, a fraction λ of their wealth is lost, and a fraction ξ of their house's value is lost when taken by their lender.

Because the shock ϵ_{t+1}^a to a household's housing is the only idiosyncratic shock it faces, all households' default decisions are driven by shocks to the value of their homes. This follows Elenev, Landvoigt, and Van Nieuwerburgh (2020) and provides a simple way of modelling mortgage default. Empirical work (e.g. Ganong and Noel (2021)) demonstrates that other shocks to households, such as job loss and medical care needs, play a large role in mortgage default. We abstract from these alternative causes of default for simplicity since our focus is on the impact of events within the financial system.[8]

Endowment income. The aggregate payoff of all households' endowment is Y_t^*, equal to total output Y_t plus some quantiatively small losses incurred during mortgage default and additional transfers specified in section 2.6, see equation (35). A fraction $0 < \nu < 1$ of the aggregate income Y_t^* is paid to the young households, with the remaining fraction $1 - \nu$ paid to the middle aged. Of the income paid to the middle-aged, a fraction δ_M is paid to the new middle-aged that were previously young with the remainder paid to

[8]See Campbell and Cochrane (1999a) for a richer model specifically of mortgage default.

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those who were also middle-aged last period. Shares of the middle-aged endowment that are owned by those who become old are sold by the old to other middle-aged households. The payoffs per share x_t^Y to the young, x_t^{Y+} to those aging from young to middle-aged this period, and x_t^M to those who remain middle-aged are

$$x_t^Y = \nu Y_t^*, \tag{8}$$

$$x_t^{Y+} = \frac{\delta^M}{\pi^Y}(1-\nu)Y_t^* \tag{9}$$

$$x_t^M = (1-\delta^M)(1-\nu)Y_t^*. \tag{10}$$

Parameters ν and δ^M allow us to parsimoniously specify the life-cycle income profile of households. If $\nu < 1/2$, young households receive a smaller fraction of the aggregate endowment each period in total, and thus face an upward-sloping lifetime income path. δ^M determines how financially wealthy middle-aged households are. If $\delta^M = 1$, the middle-aged have a large holding of wealth and no expected future endowment income, so their demand to buy financial assets (and bid up their price) is large. Conversely, if $\delta^M = 0$, the middle-aged expect large future endowment income but do not have a large quantity of financial wealth today that must be invested in other assets, so asset prices are not as high. We discuss the effect of δ^M further in the calibration section.

We make the simplifying assumption that households can trade their ownership stake of the "non-pledgeable" endowment with other members of the same generation. However, they cannot pledge their endowment to borrow from other generations. This assumption is crucial for our key aggregation result, since it implies that households can trade all assets they own. If each household received its own non-tradeable endowment stream, it would be necessary to track the entire distribution of wealth within each generation as in Krusell and Smith (1998). Although real households cannot trade their labor income, our assumption maintains tractability and can match the empirical fact that young households have growing future labor income that they are unable to borrow against (Gourinchas and Parker (2002)).

2.4 Individual household problem

Preferences and timing. All households maximize expected utility with discount factor β and constant relative risk aversion γ.

Old households only live for one period. They decide how to split their wealth between consumption c_t^O and bequests b_t^O that are paid into the endowment income of younger

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generations in order to maximize their utility function

$$u^O(c_t^O, b_t^O) = \frac{1}{1-\gamma}((c_t^O)^{1-\gamma} + \phi(b_t^O)^{1-\gamma}). \tag{11}$$

Middle-aged and young households obtain utility from consuming non-durables and housing as well as from their holdings of bank deposits. They can spend their wealth by consuming non-durables, renting housing, buying housing, buying shares of their generation's endowment, and (if they are middle-aged) buying shares of intermediary equity. In addition they can take out a mortgage from the bank collateralized by their house and can choose the loan-to-value ratio of the mortgage, taking as given the menu of contracts offered by the bank.

Denote by $a \in \{Y, M\}$ the generation of an individual young or middle-aged household, respectively. The utility function depends on nondurable consumption c_t^a, housing consumption s_t^a and real deposit holdings d_t^a as follows:

$$u^a(c_t^a, s_t^a, d_t^a) = \frac{1}{1-\gamma}((c_t^a)^{1-\theta}(s_t^a)^{\theta})^{1-\gamma} + \psi\frac{(d_t^a)^{1-\gamma}}{1-\gamma}. \tag{12}$$

Utility over non-durable and housing consumption is Cobb-Douglas as in Berger, Guerrieri, Lorenzoni, and Vavra (2008). This functional form choice sets households' elasticity of consumption between housing and non-durables equal to 1, similar to empirical estimates (e.g. Piazzesi, Schneider, and Tuzel (2007)) and provides tractable expressions for consumption decisions, see equations (65)-(66). We add to this a "bank-deposits-in-the-utility function" term $\psi\frac{(d_t^a)^{1-\gamma}}{1-\gamma}$, which generates a demand for holding bank deposits without explicitly modelling their role as special liquid assets (Stein (2012),Diamond (2020)).[9]

The precise timing of events within each period is:

1. Aggregate shocks, idiosyncratic housing shocks, and aging shocks are realized.

2. Households decide whether to default on their mortgage.[10]

3. All households make consumption and portfolio choices (including bequest choice for the old) given their post-default-decision wealth.

[9]This follows a long tradition of modeling money demand by putting money holdings in the utility function for tractability (Chetty (1969), Poterba and Rotermberg (1987)).

[10]Old households also have the option to default on their mortgages. Since the old only live for one period, they have the same beginning-of-period portfolio as the middle-aged, and their default decision is identical to that of middle-aged households.

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The post-default-decision wealth w_t^a is the only individual state variable of a household in generation a. Denote all other state variables exogenous to the household by $\mathcal{Z}_t$. Note in particular that after their post-default decision wealth is determined, all households (regardless of whether they defaulted in the past) face the same problem going forward. This abstracts from models where default can lead an agent to be excluded from financial markets (Kehone and Levine (1993), Alvarez and Jermann (2000)) and is crucial for tractability.

Old generation. The old begin the period with post-default-decision wealth w_t^O. Given their utility function in (11), they optimally choose to consume $c_t^O = \frac{1}{1+\phi^{\frac{1}{\gamma}}} w_t^O$ and bequeath $b_t^O = \frac{\phi^{\frac{1}{\gamma}}}{1+\phi^{\frac{1}{\gamma}}} w_t^O$ yielding a total amount of utility

$$V^O(w_t^O) = \frac{(w_t^O)^{1-\gamma}}{1-\gamma} \left[\left(\frac{1}{1+\phi^{\frac{1}{\gamma}}} \right)^{1-\gamma} + \phi \left(\frac{\phi^{\frac{1}{\gamma}}}{1+\phi^{\frac{1}{\gamma}}} \right)^{1-\gamma} \right]. \tag{13}$$

Recursive optimization problem for the young and middle-aged. Each household that is currently of generation a maximizes its expected utility over its lifetime and starts at time t with post-default-decision wealth w_t^a. It chooses this period its portfolio vector α_t^a, as given in expressions (3)-(4). At time t, the household faces the budget constraint

$$w_t^a = c_t^a + \rho_t^a s_t^a + h_t^a p_t^h - \rho_t^a h_t^a \bar{Y}_t + b_t^a \cdot P_t^a + \frac{d_t^a}{1+r_t} - (1+\tau^m) q^a(\alpha_t^a, \mathcal{Z}_t) m_t^a. \tag{14}$$

The household's budget constraint given in equation (14) shows how a household can allocate its wealth w_t^a in the post-default stage of the period. In addition to its wealth, the household receives $(1+\tau^m)q^a(\alpha_t^a, \mathcal{Z}_t)m_t^a$, given in equation (5), from taking out a new mortgage at time t as well as income from renting the $h_t^a \bar{Y}_t$ units of housing services, defined in equation (2), at the rental rate ρ_t^a. This income can be used to obtain non-durable consumption c_t^a, to buy housing at price p_t^h, rent housing services s_t at rental rate ρ_t^a, invest in generation-specific assets b_t^a at price vector P_t^a, given in equation (7), and invest in deposits paying d_t^a next period at an interest rate r_t.[11]

[11] Even though there are markets both for buying and renting housing, we show below that every household will choose $s_t = h_t^a \bar{Y}_t$, so this is equivalent to a setting with only owner-occupied housing.

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The full problem of a household of age a is

$$V^a(w_t^a, \mathcal{Z}_t) = \max_{c_t^a, s_t^a, \alpha_t^a} \frac{\left((c_t^a)^{1-\theta}(s_t^a)^{\theta}\right)^{1-\gamma}}{1-\gamma} + \psi \frac{(d_t^a)^{1-\gamma}}{1-\gamma}$$
$$+ \beta(1-\pi^a)\mathrm{E}_t\left[\max\left\{V^a(w_{t+1}^{a,nd}, \mathcal{Z}_{t+1}), V^a(w_{t+1}^{a,d}, \mathcal{Z}_{t+1})\right\}\right]$$
$$+ \beta\pi^a\mathrm{E}_t\left[\max\left\{V^{a+}(w_{t+1}^{a+,nd}, \mathcal{Z}_{t+1}), V^{a+}(w_{t+1}^{a+,d}, \mathcal{Z}_{t+1})\right\}\right], \tag{15}$$

subject to the budget constraint, equation (14), and the definition of next-period wealth for non-defaulters

$$w_{t+1}^{a,nd} = (1-\delta_H)p_{t+1}^h \epsilon_{t+1}^a h_t^a + d_t^a + b_t^a \cdot (P_{t+1}^a + x_{t+1}^a) - m_t^a, \tag{16}$$

and for defaulters

$$w_{t+1}^{a,d} = (1-\lambda)(d_t^a + b_t^a \cdot (P_{t+1}^a + x_{t+1}^a)), \tag{17}$$

where x_t^a is the vector of payoffs paid by the assets specifically available to generation a if the household *does not age*. For households that age, the wealth for non-defaulters is

$$w_{t+1}^{a+,nd} = (1-\delta_H)P_{t+1}\epsilon_{t+1}^a h_t^a + d_t^a + b_t^a \cdot (P_{t+1}^{a+} + x_{t+1}^{a+}) - m_t^a, \tag{18}$$

and for defaulters is

$$w_{t+1}^{a+,d} = (1-\lambda)(d_t^a + b_t^a \cdot (P_{t+1}^{a+} + x_{t+1}^{a+})), \tag{19}$$

where $x_t^{a^+}$ is the vector of payoffs paid by the assets specifically available to generation a if the household *ages*.

The household's next-period wealth depends both on whether the household ages and if it chooses to default on its mortgage. A household that defaults loses all of the housing it owns and keeps a fraction $(1-\lambda)$ of its deposits and generation-specific assets (its share of the endowment, plus bank equity holdings for the middle-aged). The price and dividend of these assets are (P_{t+1}^a, x_{t+1}^a) if the household does not age and $(P_{t+1}^{a+}, x_{t+1}^{a+})$ if it ages, yielding equations (17) and (19). If the household does not default, it enters owning housing $\epsilon_{t+1}^a h_t^a$ which has a market value of $p_t^h \epsilon_{t+1}^a h_t^a$. In addition, households keep their deposit wealth d_t^a and both the dividend and resale value of their generation-specific assets $b_t^a \cdot (P_{t+1}^a + x_{t+1}^a)$ if they do not age and $b_t^a \cdot (P_{t+1}^{a+} + x_{t+1}^{a+}))$ if they do, and have to pay back their mortgage m_t^a. Because households choose whether or not to default to maximize their value function, the value function next period is

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$\max\left\{V^a(w^{a,nd}_{t+1}, \mathcal{Z}_{t+1}), V^a(w^{a,d}_{t+1}, \mathcal{Z}_{t+1})\right\}$

To apply this general structure of the optimization problem to both generations, note that all middle-aged households who age become old, while a fraction π^M of young households who age immediately become old instead of middle-aged. We therefore have for $a = Y$ and $a = M$

$$V^{M+}(w_t, \mathcal{Z}_t) = V^O(w_t) \tag{20}$$

$$V^{Y+}(w_t, \mathcal{Z}_{t+1}) = \pi^M V^O(w_t) + (1-\pi^M)V^M(w_t, \mathcal{Z}_{t+1}). \tag{21}$$

where the old generation value function $V^O(w_t)$ is given by equation (13), and the middle-aged value function $V^M(w_t, \mathcal{Z}_{t+1})$ comes from solving the middle-aged generation's problem in equation (15) and then using this as an input for the young generation's problem.

2.4.1 Characterization of the Household Problem

The following proposition provides the key result for characterizing the optimization problem in (15).

Proposition 1. *1. The household value function has the form*

$$V^a(w^a_t, \mathcal{Z}_t) = v^a(\mathcal{Z}_t)\frac{(w^a_t)^{1-\gamma}}{1-\gamma}, \tag{22}$$

where $v^a(\mathcal{Z}_t)$ only depends on aggregate state variables.

2. *The choice vector $[c^a_t, s^a_t, \alpha^a_t]$ is linear in individual wealth w^a_t, conditional on the aggregate state. As a result, the total quantity of consumption, investment, and borrowing at time t by generation-a households is independent of the time-t wealth distribution within the generation.*

3. *Each household consumes the same amount of housing services as produced by the housing it owns, so the model is equivalent to one with only owner-occupied housing and no within-generation rental market.*

Proof. The proof follows from three properties. First, a choice vector $(c^a_t, s^a_t, \alpha^a_t)$ requires wealth w^a_t to satisfy the budget constraint in equation (14) if and only if $(kc^a_t, ks^a_t, k\alpha^a_t)$ requires wealth kw^a_t for any constant $k > 0$. Second, the household's utility function given in equation (12) is homogenous of degree $1-\gamma$ in the household's choice vector.

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Third, all four realizations of the household's next-period wealth (age/no age, default/no default) are linear functions of the household's choice vector.

Suppose that at time $t+1$ the household's value function is homogenous of degree $1-\gamma$ in its wealth. The first and third properties imply that the household's objective at time t is homogenous of degree $1-\gamma$ in the household's choice vector $(c^a_t, s^a_t, \alpha^a_t)$. The homogeneity of the household's objective function[12] and the first property that the household's budget set scales linearly in its wealth imply that if $(c^a_t, s^a_t, \alpha^a_t)$ is the optimal choice for a household with wealth w^a_t, $(kc^a_t, ks^a_t, k\alpha^a_t)$ is the optimal choice for a houshold with wealth kw^a_t for any $k > 0$. This proves part 2 of the proposition.

Because the household's choices scale linearly in its wealth and its objective function is homogenous of degree $1-\gamma$ in its choice vector, its value function must be homogenous of degree $1-\gamma$ in its wealth at time t, proving part 1 of the proposition.[13] Because a household's ownership and consumption of housing scale linearly in its wealth, the clearing of the housing rental market within generation a implies that each household must only consume the housing services it produces, proving part 3 of the proposition. □

Proposition 1 has three key implications. First, households within each generation behave like a representative agent in their consumption and portfolio choice ex-ante, even though they are not insured ex-post against idiosyncratic shocks. This aggregation result relies on two key features of the model. First, the idiosyncratic shock ϵ^a_{t+1} a household faces is multiplicative in the amount of housing it owns (resulting in property three used in the proof). As explained in Constantinides and Duffie (1996), this sort of multiplicative shock cannot be avoided by trading financial assets within the generation when agents have CRRA utility, so choices naturally aggregate as if each generation was a representative agent in autarky. In addition, property one used in the proof (that the agent's budget set scales linearly in its wealth) relies on the fact that all assets, including the agent's endowment income, can be traded in a competitive market.[14]

Second, the mortgages given to all households in the same generation are equally risky. Richer households borrow more and buy more housing than poorer ones, but all house-

[12] In particular, if $f(x)$ is a homogenous function of degree $1-\gamma$, $f(kx_1) - f(kx_2) = k^{1-\gamma}(f(x_1) - f(x_2))$, so $f(x_1) > f(x_2)$ is equivalent to $f(kx_1) > f(kx_2)$ for any constant $k > 0$.

[13] Strictly speaking, this argument has only proved that the household's Bellman operator preserves the property that the value function is homogenous of degree $1-\gamma$. A contraction mapping argument implies that the unique value function must have this property, see Alvarez and Stokey (1998) for details.

[14] If households had non-tradeable streams of income, as in Krusell and Smith (1998), it would be necessary to track the entire distribution of wealth within a generation, which is an infinite dimensional object. See Lenel (2020) for another model that is made tractable by allowing certain assets to trade in segmented markets.

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holds in a generation choose the same leverage- the ratio of their mortgage face value over the value of their housing. This result comes from the fact that the mortgage pricing function $q^a(\alpha_t^a, \mathcal{Z}_t)$ is homogeneous of degree zero in the portfolio vector α_t^a. That is, an agent with twice as much housing, twice as much mortgage face value, and twice the financial portfolio of another will be provided with twice as much of a loan by the intermediary. The property of the mortgage pricing function is derived from the optimal behavior of the financial intermediary below.

Third, only households that receive a sufficiently bad idiosyncratic housing shock ϵ_{t+1}^a default on their mortgage at time $t+1$. This is because households choose to default if and only if their wealth (inclusive of the costs of default) will be higher than if they did not, and all households choose mortgages with identical ex-ante risk at time t. Define the non-housing wealth of a household, conditional on the outcome of the age transition at $t+1$, as

$$w_{t+1}^{a_{t+1},nh} = d_t^a + b_t^a \cdot (P_{t+1}^{a_{t+1}} + x_{t+1}^{a_{t+1}}) \tag{23}$$

where

$$a_{t+1} = \begin{cases} a & \text{if the household does not age,} \\ a^+ & \text{if the household ages.} \end{cases} \tag{24}$$

Then households default for any realization of ϵ_{t+1}^a such that

$$(1-\lambda) w_{t+1}^{a_{t+1},nh} > \underbrace{\epsilon_{t+1}^a (1-\delta_H) P_{t+1} h_t^a - m_t^a}_{\text{home equity}} + w_{t+1}^{a_{t+1},nh}, \tag{25}$$

This default rule shows that negative home equity (if the value $\epsilon_{t+1}^a(1-\delta_H)P_{t+1}h_t^a$ of the household's housing is less than the mortgage debt m_t^a it owes) is necessary for a household to default, but only households with sufficiently large negative equity choose to default. In addition, the default rule defines a cutoff value $\bar{\epsilon}_{t+1}^{a_{t+1}}$, conditional on the age transition, such that the household defaults if and only if their realized ϵ_{t+1}^a is lower.

Corollary 1. *There exists default thresholds $\bar{\epsilon}_{t+1}^{a_{t+1}}$ such that generation-a households with $\epsilon_{t+1}^a < \bar{\epsilon}_{t+1}^{a_{t+1}}$ default. The aggregate default rate of generation a is $\pi^a F_{\epsilon,t+1}^a(\bar{\epsilon}_{t+1}^{a+}) + (1-\pi^a) F_{\epsilon,t+1}^a(\bar{\epsilon}_{t+1}^a)$.*

Based on corollary 1, the total payoff at time $t+1$ from mortgages issued to middle-aged households at time t divided by the face value m_t^M of mortgage debt owed by the

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middle-aged is

$$\mathcal{P}_{t+1}^{M} = 1 - F_{\epsilon,t+1}^{M}(\bar{\epsilon}_{t+1}^{M}) + F_{\epsilon,t+1}^{M}(\bar{\epsilon}_{t+1}^{M})(1-\zeta)(1-\delta_H)\frac{E_t(\epsilon^M|\epsilon^M < \bar{\epsilon}_{t+1}^{M})P_{t+1}h_t^M}{m_t^M}, \quad (26)$$

where a fraction ζ of the house value is lost when a house of a defaulting borrower is repossessed. Similarly, for the young generation, the payoff per dollar of face value is

$$\mathcal{P}_{t+1}^{Y} = (1-\pi^Y)(1-F_{\epsilon,t}^{Y}(\bar{\epsilon}_{t+1}^{Y})) + \pi^Y(1-F_{\epsilon,t+1}^{Y}(\bar{\epsilon}_{t+1}^{Y+})) + (1-\zeta)(1-\delta_H)\frac{P_{t+1}h_t^Y}{m_t^Y}\times$$
$$\left[(1-\pi^Y)E_t(\epsilon^Y|\epsilon^Y < \bar{\epsilon}_{t+1}^{Y})F_{\epsilon,t+1}^{Y}(\bar{\epsilon}_{t+1}^{Y}) + \pi^Y E_t(\epsilon^Y|\epsilon^Y < \bar{\epsilon}_{t+1}^{Y+})F_{\epsilon,t+1}^{Y}(\bar{\epsilon}_{t+1}^{Y+})\right]. \quad (27)$$

2.5 Financial Intermediary

The financial intermediary is a publicly traded firm in a competitive financial market which is owned in equilibrium by the middle-aged generation. It makes mortgages and issues deposits and equities backed by these mortgages with the goal of maximizing its value to its middle-aged owners. Because, as shown in proposition 1, all households of the same generation choose equally risky mortgages, the intermediary's balance sheet entering time $t+1$ can be charecterized by the face values of mortgages it made to the young and middle-aged, that we will denote by N_t^Y and N_t^M, and by the payment D_t it owes to depositors. Its balance sheet at time $t+1$ is charecterized by one state variable, that we call its inside equity[15]

$$e_{t+1} = N_t^Y\mathcal{P}_{t+1}^{Y} + N_t^M\mathcal{P}_{t+1}^{M} - D_t. \quad (28)$$

equal to the payments, given in equations (26)-(27), it receives from mortgages minus D_t it repays to depositors. The intermediary is required, for a constant $\eta > 0$, to pay a dividend

$$\eta e_t \quad (29)$$

to its owners at time t. The intermediary can also raise additional funding I_t from its owners at a cost of

$$C(I_t, \bar{Y}_t) = \frac{\chi}{\bar{Y}_t}I_t^2 \quad (30)$$

[15]By inside equity, we mean the market value of the intermediary's assets minus the market value of the intermediary's deposit liabilities, so that the value of all assets equals the value of all liabilities (debt plus equity).

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for some constant $\chi > 0$. That is, if the intermediary's owners pay I_t of additional funds, the intermedary only receives $I_t - C(I_t, \bar{Y}_t)$.[16]

The intermediary faces a "risk-weighted capital requirement" of the form

$$e_{t+1} \geq \bar{e}^Y N_t^Y \mathcal{P}^Y(z_{t+1}) + \bar{e}^M N_t^M \mathcal{P}^M(z_{t+1}) \ \forall z_{t+1} | \mathcal{Z}_t. \tag{31}$$

That is, for every realization of uncertainty z_{t+1} following the current state Z_t, the intermediary's inside equity e_{t+1} has to be bigger than the right hand side of inequality (31). This right hand side is motivated by how banks are regulated in practice. The bank at time $t+1$ has assets $N_a^M \mathcal{P}^a(z_{t+1})$ coming from its mortages to generation a, and these assets are multiplied by a "risk weight" $\bar{e}^a$ to determine how much equity the bank is required to hold. Riskier assets (in our calibration and in practice) have larger risk weights, so $\bar{e}^Y > \bar{e}^M$.

Finally, the intermediary faces the budget constraint

$$(1-\eta)e_t + I_t - C(I_t, \bar{Y}_t) + \frac{D_t}{1+r_t} = (1+\zeta)\left(N_t^Y q^Y(\alpha_t^Y, \mathcal{Z}_t) + N_t^M q^M(\alpha_t^M, \mathcal{Z}_t)\right). \tag{32}$$

On the left hand side, we have the inside equity $(1-\eta)e_t$ remaining after the mandatory dividend ηe_t is paid, the funds $I_t - C(I_t, \bar{Y}_t)$ obtained by raising additional funds from the intermediary's owners, and the funds $\frac{D_t}{1+r_t}$ raised in the deposit market by promising to repay D_t next period. This is used to fund loans of total face values N_t^Y, N_t^M to the young and middle-aged, which respectively require paying out $q^Y(\alpha_t^Y, \mathcal{Z}_t)$ per dollar of face value to the young and $q^M(\alpha_t^M, \mathcal{Z}_t)$ per dollar of face value to the middle-aged. At the time of origination, intermediaries pay a processing cost ζ that is proportional to the mortgage amount, reflecting intermediary operational expenses that we do not model explicitly. As a result, the funds $\left(N_t^Y q^Y(\alpha_t^Y, \mathcal{Z}_t) + N_t^M q^M(\alpha_t^M, \mathcal{Z}_t)\right)$ lent to households are multiplied by $1+\zeta$ on the right hand side, see equation (6).

Let $\mathcal{M}_{t,t+1}^M$ be the stochastic discount factor of the middle-aged, who own the intermediary's equity. The full optimization problem of the intermediary is

$$V^I(e_t, \mathcal{Z}_t) = \max_{I_t, D_t, N_t^M, N_t^Y} \eta e_t - I_t + \mathrm{E}_t\left[\mathcal{M}_{t,t+1}^M V^I(e_{t+1}, \mathcal{Z}_{t+1})\right], \tag{33}$$

subject to the budget constraint (32), definition of next period's inside equity (28), and risk-weighted capital requirement (31).

[16] This loss is rebated back into the economy's endowment so no resources are destroyed.

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Using the intermediary's value function, we can now provide an expression for the credit surface $q^a(\alpha_t^a, \mathcal{Z}_t)$ of loans available to borrowers of generation a. The intermediary is a price taker in a competitive market, so the equilibrium mortgage prices are such that the intermediary is indifferent to providing a marginal loan of $q^a(\alpha_t^a, \mathcal{Z}_t)$ to a borrower that chooses the vector α_t^a of consumption, investment and borrowing decisions. If the intermediary lends $q^a(\alpha_t^a, \mathcal{Z}_t)$ today per dollar of repaid face value m_t^a promised, it receives the (random) payoff $\mathcal{P}_{t+1}^a(\alpha_t^a, \mathcal{Z}_{t+1})$ in the next period. This results in the first-order condition

$$(1+\zeta)q^a(\alpha_t^a, \mathcal{Z}_t) = \mu_t^I(1-\bar{e}^a)\mathcal{P}^a(\alpha_t^a, \underline{z}_t) + \mathrm{E}_t\left[\mathcal{M}_{t,t+1}^M \frac{\frac{\partial V^I(e_{t+1}, \mathcal{Z}_{t+1})}{\partial e_{t+1}}}{\frac{\partial V^I(e_t, \mathcal{Z}_t)}{\partial e_t}} \mathcal{P}_{t+1}^a(\alpha_t^a, \mathcal{Z}_{t+1})\right], \quad (34)$$

where μ_t^I is the Lagrange multiplier on the risk-weighted capital requirement in the aggregate state where the intermediary's portfolio has the lowest payoff and $\underline{z}_t$ is the realization of $\mathcal{Z}_{t+1}$ in which that lowest payoff occurs given the state $\mathcal{Z}_t$.

The following proposition verifies that $q^a(\alpha_t^a, \mathcal{Z}_t)$ is homogenous of degree 0 in α_t^a as was assumed to prove proposition 1. It follows from the fact that the payoff per dollar of face value $\mathcal{P}_{t+1}^a(\alpha_t^a, \mathcal{Z}_{t+1})$ of a mortgage is also homogenous of degree 0 in α_t^a and that the intermediary has a well-defined stochastic discount factor for valuing mortgages to generation a.

Proposition 2. *The mortgage pricing functions $q^a(\alpha_t^a, \mathcal{Z}_t)$ for $a = Y, M$ are homogeneous of degree zero in the portfolio choices α_t^a of borrowing households.*

Proof. The household's default decision given in expression (25) has the property that if all of a household's assets and debts are multiplied by a constant $k > 0$, the states of the world where the household defaults remains unchanged. The expressions in equations (26)-(27) for the payoff $\mathcal{P}_{t+1}^a$ depend only on the default thresholds $\bar{\epsilon}_{t+1}^{a_{t+1}}$ and on the ratio $\frac{h_t^a}{m_t^a}$. These remain fixed if $k\alpha_t^a$ replaces α_t^a, so $\mathcal{P}_{t+1}^a(k\alpha_t^a, \mathcal{Z}_{t+1}) = \mathcal{P}_{t+1}^a(\alpha_t^a, \mathcal{Z}_{t+1})$. Since the right hand side of equation (34) stays fixed when $k\alpha_t^a$ replaces α_t^a, the left hand side must as well, so $q^a(k\alpha_t^a, \mathcal{Z}_t) = q^a(\alpha_t^a, \mathcal{Z}_t)$ as desired. □

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2.6 Aggregation and Equilibrium

Expression for Aggregate Endowment Income. This section provides an expression for the total payoff Y_t^* of the endowment of the young and middle-aged. Y_t^* is given by

$$Y_t^* = Y_t + B_t^O + \Lambda_t^\xi + \Lambda_t^\lambda + (\zeta - \tau^m)\left(N_t^Y q^Y(\alpha_t^Y, \mathcal{Z}_t) + N_t^M q^M(\alpha_t^M, \mathcal{Z}_t)\right), \tag{35}$$

where

$$\begin{aligned}\Lambda_t^\xi = \xi p_t^h \Big[& F_{\epsilon,t}^M(\bar{\epsilon}_t^M)\mathrm{E}_t[\epsilon^M|\epsilon^M < \bar{\epsilon}_t^M]H_{t-1}^M \\ & + \left((1-\pi^Y)F_\epsilon^M(\bar{\epsilon}_t^Y)\mathrm{E}_t[\epsilon^Y|\epsilon^Y < \bar{\epsilon}_t^Y] + \pi^Y F_\epsilon^Y(\bar{\epsilon}_t^{Y+})\mathrm{E}_t[\epsilon^Y|\epsilon^Y < \bar{\epsilon}_t^{Y+}]\right) H_{t-1}^Y\Big].\end{aligned} \tag{36}$$

is the total resources lost when a fraction ξ of the value of all houses owned by mortgage defaulters are lost, and

$$\begin{aligned}\Lambda_t^\lambda = {} & \lambda(1-\pi^Y)F_{\epsilon,t}^Y(\bar{\epsilon}_t^Y)(D_{t-1}^Y + \mathbf{1}\cdot(P_t^Y + x_t^Y)) + \lambda F_{\epsilon,t}^Y(\bar{\epsilon}_t^{Y+})(D_{t-1}^Y + \mathbf{1}\cdot(P_t^{Y+} + x_t^{Y+})) \\ & + \lambda F_{\epsilon,t}^M(\bar{\epsilon}_t^M)\left(D_{t-1}^M + \mathbf{1}\cdot(P_t^M + x_t^M)\right)\end{aligned} \tag{37}$$

is the among of wealth lost by defaulting households when a fraction λ of their non-housing wealth is lost in default. B_t^O is the aggregate bequest from the current old households. The last term equals the processing cost (ζ times the total funds $N_t^Y q^Y(\alpha_t^Y, \mathcal{Z}_t)+$ $N_t^M q^M(\alpha_t^M, \mathcal{Z}_t)$ lent in new mortgages) the intermediary pays to originate mortgages minus the subsidy (τ^m times the total funds lent) households receive when the mortgages are originated. This aggregate endowment income insures that the total consumption resources available in the economy equals the exogenous output Y_t, which allows us to maintain the simplicity of an endowment economy.

Equilibrium. This section states the market-clearing conditions necessary to define an equilibrium of our econony. Uppercase letters denote aggregate choice variables for young, middle-aged and old generations throughout. $(M_t^a, H_t^a, S_t^a, D_t^a, B_t^{a,a}, C_t^a)$ is the aggregate mortgage face value, housing ownership, housing consumption, endowment shares holdings, and non-durable consumption of generation a. $B_t^{M,I}$ are the aggregate holdings of intermediary equity by the middle-aged. Market clearing for mortgage debt requires that intermediaries purchase the full portfolio of mortgages of both borrowing

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generations:

$$N_t^Y = M_t^Y, \tag{38}$$

$$N_t^M = M_t^M. \tag{39}$$

Market clearing for housing capital requires that

$$H_t^Y + H_t^M = \bar{H}, \tag{40}$$

and the rental market needs to clear within each generation

$$S_t^Y = H_t^Y \bar{Y}_t, \tag{41}$$

$$S_t^M = H_t^M \bar{Y}_t. \tag{42}$$

Market clearing for intermediary liabilities requires

$$D_t = D_t^Y + D_t^M, \tag{43}$$

$$B_t^{M,I} = 1. \tag{44}$$

Shares to the endowment assets of the young and middle-aged are in unit supply such that

$$B_t^{Y,Y} = 1, \tag{45}$$

$$B_t^{M,M} = 1. \tag{46}$$

Finally, market clearing for non-durables requires that

$$Y_t = C_t^Y + C_t^M + C_t^O + C(I_t, \bar{Y}_t). \tag{47}$$

An equilibrium is a set of prices and allocations such that all 3 generations and the intermediary solve their optimization problems above, equations (15) and (33), and all markets clear.

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3 Calibration, Solution Method, and Model Fit

3.1 Parameterization

We calibrate the model to annual U.S. data. We choose 1998 as base year for the calibration, since in the boom-bust simulation below, this will be the starting point of the trend of declining real interest rates. Several parameters, listed in Table 1, are directly set to external estimates. The remaining 10 parameters, listed in Table 2, are chosen jointly so that simulated moments from a model-generated time series match a set of corresponding moments in the data.

For each of the 10 parameters calibrated in table 2, we report an associated moment that is intuitively related to the value of that parameter. While each calibrated parameter does impact all 10 moments, we show in a sensitivity analysis in Appendix C.2 that most parameters have a considerably larger impact on their associated moments than on other moments. We report the elasticity of each of the 10 moments to each of the 10 parameters. In some cases, we find that a parameter has a larger impact on 2 intuitively related moments than on the 8 others. Only the discount factor β has a large impact on many moments. The calibration is locally stable in the sense that deviations in any single parameter would worsen the overall fit.[17]

Growth Rate and Productivity shocks. We calibrate the trend growth rate and productivity shocks based on real disposable income per capita from 1929-2017. The annual growth rate is exactly 2%. The standard deviation and autocorrelation of the cyclical HP-filtered series are 2.7% and 45%, respectively. We convert the continuous AR(1) productivity process to a 3-state Markov chain using the Rouwenhorst (1995b) method. The aggregate endowment income per year is normalized to 1, as is the fixed housing stock $\bar{H}$.

Preferences and Life-Cycle. Risk aversion is set to a standard value of 1, implying log utility. A choice of $\gamma = 1$ or $\gamma = 2$ is chosen in all macro-housing and intermediary asset pricing models we know.[18] This risk aversion is considerably lower than in papers that

[17] In principle, one could choose these 10 parameters by minimizing a method of moments critereon function across all 10 parameters. Because a global estimation of parameters spanning the complete parameter space is computationally intractable, we chose parameter values that fit our target moments well without explicit optimization.

[18] For example, $\gamma = 1$ or $\gamma = 2$ is chosen in Greenwald (2018); Greenwald, Landvoigt, and Van Nieuwerburgh (2020); Corbae and Quintin (2015); Hedlund and Garriga (2020); He and Krishnamurthy (2013); Ia-

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Table 1: Pre-Set Parameters

Description	Par	Value	Source
Exogenous Shocks			
1. Growth rate	g	2%	Average growth rate income p.c.
2. Income shocks std.dev.	σ^Y	2.7%	Std. dev. HP-filtered income p.c.
3. Income shocks AC	ρ^Y	45%	Autocorrelation HP-filtered income p.c.
4. Trans. prob. $\sigma_\epsilon^0 \to \sigma_\epsilon^1$	$\Gamma_{1,2}$	5%	Jordà, Schularick, and Taylor (2016)
5. Trans. prob. $\sigma_\epsilon^1 \to \sigma_\epsilon^0$	$\Gamma_{2,1}$	20%	Jordà, Schularick, and Taylor (2016)
Population and Income			
6. Transition prob Y	π^Y	5%	Young lifespan 26-45
7. Transition prob M	π^M	5%	Middle-aged lifespan 46-65
8. Income share of young	ν	45%	Income share ages 26-45 (SCF)
Housing and Mortgages			
9. Forced maintenance	δ_H	2.5%	Housing depreciation (BLS)
10. Foreclosure loss to bank	ξ	35%	Campbell, Giglio, and Pathak (2011)
Preferences			
11. Risk aversion (1/IES)	γ	1	standard
12. Bequest parameter	ϕ	2	Bequest/GDP 15% (Alvaredo et al. (2017))
Intermediary			
13. Capital requirement M	$\bar{e}^M$	1%	Basel requirement Agency MBS
14. Capital requirement Y	$\bar{e}^Y$	8%	Basel requirement Mortgage Loans
15. Target payout ratio	τ	6.8%	Bank dividend ratio (Elenev et al. (2020))

match the equity premium (Campbell and Cochrane, 1999b; Bansal and Yaron, 2004), and such papers also deviate from CRRA utility to avoid other counterfactual implications of a large γ.

We choose the discount factor β to match the deposit rate in the model to the annualized real yield of 1-year treasury bills in 1998, which is 3.2%. Several preference and life-cycle related parameters are chosen to match moments from the Survey of Consumer Finances (SCF). We compute means for the target moments from the 1998 SCF wave, using SCF sampling weights. We categorize households by age of the household head, with the young being 26-44 years of age, the middle-aged between 46-65, and the old 66 and older. Accordingly, we set $\pi^Y = \pi^M = 5\%$ to achieve an average duration of 20 years spent in the young and middle-aged generation, respectively. We set the weight on housing in the Cobb-Douglas consumption aggregator to 0.135 to match the aggregate housing wealth-to-income ratio in the SCF. We pick the utility parameter ψ to match the liquidity premium estimated by Krishnamurthy and Vissing-Jorgensen (2012) of 73 bp for the middle-aged.[19]

coviello (2005); Kaplan, Moll, and Violante (2018); Kaplan, Mitman, and Violante (2020).

[19]We calculate the liquidity premium in the model as the difference of a counterfactual risk free rate that

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Table 2: Calibrated Parameters

Description	Par	Value	Target	Data	Model
Exogenous Shocks					
1. Housing shock low Y,M	σ_ϵ^0	19%	Charge-off rate real estate	0.47%	0.42%
2. Housing shock high Y,M	σ_ϵ^1	32%	Charge-off rate real estate, crisis	1.75%	1.66%
Preferences and Life-Cycle					
3. Patience	β	0.925	1998 real yield (1-year T-bill %)	3.21%	3.02%
4. Weight on housing	θ	0.135	Housing wealth/income (1998 SCF)	2.05	2.06
5. Liquidity pref.	ψ	0.015	Liquidity premium KVJ	0.70%	0.73%
6. Middle-aged income profile	δ^M	0.94	LTV of middle-aged (1998 SCF)	45.7%	46.02%
Housing and Mortgages					
7. Mortgage tax benefit	τ^m	1.2%	Effective tax rate for MID	30%	29.86%
8. Default penalty	λ	1%	LTV of young (1998 SCF)	64.2%	63.79%
Intermediary					
9. Origination cost	ζ	1.4%	Spread prime mortg. – treasury	1.5%	1.53%
10. Equity issuance cost	χ	850	Bank net payout rate (Elenev et al. (2020))	5.7%	5.73%

The share of the aggregate endowment received by the young, ν, is set to 45% to match the aggregate income share of the young in the 1998 SCF. The degree of front-loading of middle-aged income δ^M governs the need of middle-aged households to save for consumption smoothing over the rest of their life-cycle. A higher value of δ^M leads to lower leverage for the middle-aged. We set this parameter to 0.94 to match middle-aged leverage of 45% in the data. The bequest parameter ϕ directly determines aggregate bequests B^O. With $\phi = 2$ the model produces a bequest flow/GDP ratio of 15%, matching estimates for several European countries reported by Alvaredo, Garbinti, and Piketty (2017).[20]

Housing and Mortgages. We set the forced maintenance of housing δ_H to match depreciation of residential fixed assets based on the BEA fixed asset tables. Idiosyncratic house price dispersion follows a two-state Markov Chain with transition matrix Γ, with state 0 indicating normal times, and state 1 indicating elevated housing risk. The probability of staying in the normal state in the next year is 95% and the probability of staying in the crisis state in the next quarter is 80%. Under these parameters, the economy spends 80% of the time in the normal state and 20% in the high housing risk state, and the average duration of the high risk state is 4.5 years. These transition probabilities are independent of the aggregate endowment state. High housing risk by itself does not induce a housing

does not provide any liquidity services and the deposit rate. We calibrate to the reported benchmark of 73 bp in Krishnamurthy and Vissing-Jorgensen (2012) even though the sample mean of their AAA-Treasury spread we report in table 3 is slightly larger in the time sample we examine.

[20] As Alvaredo et al. (2017) explain, data limitations make a similar estimate for the U.S. hard to obtain.

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recession, which we define as a combination of high housing risk and low aggregate income, as explained in Section 4.1 below. Housing recessions occur with 5.2% probability unconditionally, consistent with evidence in Jordà, Schularick, and Taylor (2016). Dispersion during the low and high uncertainty state govern mortgage defaults rates during normal periods and housing recessions. Given foreclosure losses of $\zeta = 35\%$ of the house value (based on evidence in Campbell, Giglio, and Pathak (2011)), these parameters are chosen to match data moments on charge-off rate for residential real estate loans. Specifically, low-state dispersion of $\bar{\sigma}_{\epsilon,0} = 19\%$ matches an average charge-off rate on residential mortgages of 0.47% for the 1985-2017 period. The high dispersion value $\bar{\sigma}_{\omega,1} = 32\%$ leads to a model charge-off rate of 1.68% during housing recessions, matching the data for the 2008-2012 period of high foreclosure rates. We use the same values for idiosyncratic housing risk of young and middle-aged households.

Given the housing risk parameters, we choose the pecuniary default penalty λ to match mortgage leverage of young households in the SCF. Holding fixed other parameters, households choose higher leverage at a higher level of λ, as high costs of defaulting reduce the likelihood of default tomorrow for given leverage today. A value of $\lambda = 1\%$ delivers young leverage of 63.79% close to the data value of 64.2%.

We set the mortgage tax subsidy households receive to $\tau^m = 1.2\%$. This value implies an effective tax rate of 30% at which households can deduct mortgage interest payments from taxable income.[21]

Intermediary. We set the equity capital requirements for the intermediary sector based on Basel risk-weighted regulatory requirements for mortgage assets. Since mortgages of young households are far riskier than those of middle-aged borrowers in equilibrium, we assign 100% risk weight to these assets, which combined with a simple equity ratio requirement of 8% yields $\bar{e}^Y = 0.08$. We calibrate the capital requirement for middle-age mortgages, which are close to risk-free, to the risk weight of GSE-issued mortgage backed securities of 20%, yielding $\bar{e}^M = 0.01$.[22] We calibrate the remaining two parameters of the

[21] In the model, households of generation a receive $(1+\tau^m)q^a_t$ for each dollar of mortgage face value m^a_t. Tax deduction of interest at income tax rate $\tilde{\tau}^m$ would imply that agents receive $q^a_t + \tilde{\tau}^m r^a_t$ per dollar of face value, where $r^a_t = 1/q^a_t - 1$ is the mortgage interest rate. Then the effective tax rate at which households deduct interest payments implied by subsidy τ^m is

$$\tilde{\tau}^m = \frac{\tau^m (q^a_t)^2}{1 - q^a_t}.$$

[22] While we view our intermediary sector broadly as also including non-bank lenders that transform illiquid mortgages into safe and liquid assets, for example through securitization, we recognize that capital

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intermediary objective based on Elenev, Landvoigt, and Van Nieuwerburgh (2020), who construct time series of dividends, share repurchases, equity issuances, and book equity for all publicly-traded banks in the US. Over the period from 1974 to 2018, banks paid out an average 6.8% of their book equity per year as dividends and share repurchases, which is the value we set for target payout rate η. The *net* payout rate for the same banks (i.e., net of equity issuance) in the data is 5.75% of equity, which our model matches with an equity issuance parameter of $\chi = 850$. We pick the proportional origination cost to match the spread of middle-aged mortgage rates over the deposit rate in the model to the spread of 30 year prime fixed mortgage rates over the 10 year Treasury yield. For the period 1990-2007, the average spread in the data is 1.59%, which we round to 1.5%. The model matches this with a value of $\zeta = 1.4\%$.

3.2 Solution Method

We solve the model numerically using a global projection method. The two exogenous state variables of the economy are the cyclical component of the endowment, and the time-varying cross-sectional dispersion of idiosyncratic housing shocks. Both shocks are jointly approximated by a discrete-time Markov chain. The model features three endogenous aggregate state variables, which span the wealth distribution across the different optimizing agents. They are aggregate wealth of the young, the combined aggregate wealth of middle-aged and old, and intermediary equity. Since the wealth of all agents has to add up to aggregate tradable wealth, we only need to keep track of any two of these three endogenous state variables when computing the model.

The solution technique involves approximating the unknown functions that characterize the equilibrium of the economy over the domain of the state variables. The Appendix summarizes the set of equations and unknowns that fully characterize the equilibrium. For details on the solution method, see Elenev, Landvoigt, and Van Nieuwerburgh (2020).

3.3 Model Fit

We perform a long simulation of the calibrated model. We then compare model-generated outcomes to their counterparts in the data in Table 3. For the data variables, we re-

requirements reflect a key cost of mortgage lending for traditional banks. Further, Greenwald, Landvoigt, and Van Nieuwerburgh (2020) calculate from the Flow of Funds accounts that most securitized mortgage debt ends up on balance sheets of financial intermediaries; only 14.5% of aggregate mortgage debt is held outside of the levered financial sector on average over the 1991-2016 period.

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port the unconditional mean for the longest sample available (column "Mean" in the left panel). Since many of the data time series contain slow-moving secular trends that are not captured by our model, we Hamilton-filter these data series (Hamilton (2018)) at annual frequency, and report the standard deviation of the cyclical component (column "St.Dev."). We further compute the correlation of each variable with the cyclical component of Hamilton-filtered aggregate consumption (column "Corr.(·,C)"). Finally, we report the mean of the unfiltered series, conditional on years 2009-2012 (column "09-12"). For each data variable, Appendix C.1 describes source(s), how the variable was computed, and sample length.

For the model, we report means, standard deviations, and consumption correlations for the corresponding model simulated data series (columns "Mean", "St.Dev.", "Corr.(·,C)" in the right panel). As the model economy grows at the deterministic trend, the stationarized model variables do not need to be Hamilton-filtered. In the last column ("H.Rec."), we report means conditional on being in a housing recession, which is a period of simultaneous low endowment and high housing risk shock realizations.

We first inspect the model's quantitative performance with respect to several measures of mortgage quantities and risks. The model average of mortgage debt over consumption (row 1) is somewhat higher than in the data.[23] Similarly, average mortgage leverage (row 2) in the data is 45% for the 1945-2018 sample, yet significantly higher and closer to the model in 1998 at 51%. Importantly, both mortgage quantity and leverage have quantitatively very similar correlations with the cycle in model and data (correlation of around -0.4). The model does not deliver the large increase in leverage during the 2009-12 housing bust; this is due to the short-term nature of mortgage debt in the model. Next, we compare measures of mortgage default risk. High quality aggregate data on mortgage delinquencies is readily available (row 4); however, unlike in our model, only a fraction of delinquencies turn into foreclosures.Therefore, we compare the model's default rate both to data delinquencies and foreclosures (row 3). The model default rate is far below the delinquency rate and slightly above the foreclosure rate.[24] Importantly, the model matches the effective losses to banks from nonperforming mortgages: the model loss rate is close to the charge-off rate in the data (row 5); this is the case for mean, volatility, cyclical correlation and the spike during the housing bust. The model thus generates the correct quantity of aggregate mortgage credit risk. The middle-aged mortgage spread in

[23] The data series has a strong upward trend with a boom bust pattern. The model quantity reflects a calibration to 1998 values for housing wealth to income of *homeowners* only.

[24] Delinquencies that do not turn into foreclosures often still lead to charge-offs if they involve modifications or partial forgiveness. Since the model abstracts away from this distinction, it is reasonable that the model's default rate lies between real-world delinquencies and foreclosures.

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Table 3: Model Fit

Data					Model				
Series (Source)	**Mean**	**St.Dev.**	**Corr.(·,C)**	**09-12**	**Variable**	**Mean**	**St.Dev.**	**Corr.(·,C)**	**H. Rec.**
Mortgages									
1. Mort. debt/Cons. (FoF)	96.45	4.67	-0.37	132.90	Mort. debt/Cons.	113.79	8.96	-0.39	100.93
2. Mort. debt/housing wealth (FoF)	44.74	2.65	-0.33	67.02	Aggregate LTV	55.18	3.75	-0.41	49.99
3. Forclosure rate (Guren & McQuade)	0.75	-	-	1.50	Avg. Default Rate	0.88	1.19	-0.17	3.38
4. Delinquency rate (Fed)	3.93	1.43	-0.36	8.74					
5. Charge-off rate (Fed)	0.47	0.43	-0.38	1.74	Avg. Loss Rate	0.42	0.59	-0.16	1.66
6. Prime mort. spread	1.79	0.31	-0.81	1.87	Mort. Spread M	1.53	0.05	-0.37	1.57
7. Subprime mort. spread	3.86	0.48	-	-	Mort. Spread Y	2.94	0.37	0.12	3.47
Intermediation									
8. Deposits/Cons. (FoF)	89.94	6.61	-0.53	98.05	Deposits/Cons.	105.21	7.45	-0.43	95.23
9. Fin. sector equity (FoF)	6.94	1.27	0.68	7.53	Interm. equity ratio	7.29	1.17	-0.01	5.03
10. Fin. sector net payout (ELN)	5.75	5.71	0.39	-1.15	Bank equity net payout	5.73	0.44	0.02	4.78
11. Deposit rate (DSS)	2.70	1.52	0.49	0.70	Deposit rate	3.02	1.55	-0.95	4.04
12. Liquidity Premium (KVJ)	0.83	0.39	-0.26	1.05	Convenience Yield	0.73	0.05	0.39	0.80
Household Life-cycle									
13. Consumption M/Y (CEX)	110.20	3.27	0.00	113.10	Consumption M/Y	102.16	1.59	-0.69	103.88
14. Consumption Gr Y (CEX)	2.70	2.85	0.39	0.63	Consumption Gr Y	2.00	3.25	0.54	-2.52
15. Consumption Gr M (CEX)	1.96	3.30	0.52	0.94	Consumption Gr M	2.00	2.45	0.47	-1.59
16. Housing M/Y (SCF)	192.10	-	-	-	Housing M/Y	135.74	5.89	-0.24	149.27

The table compares moments from a long simulation of the calibrated benchmark model to corresponding data moments. All numbers are in percent. Columns: “Mean” – mean, “St.Dev.” – standard deviation, “Corr(·,C)” – correlation with aggregate consumption. Data column “09-12”: conditional mean 2009-12. Model column “H.Rec.” – mean conditional on housing recession (simultaneous low income and high housing risk shock). Some data series have different sample lengths and are Hamilton (2018)-filtered. See Appendix C.1 for details on data series construction.

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the model does well in matching the spread of prime mortgage rates over treasury bonds (row 6) in the data, although mortgage data mortgage spreads are much more volatile and cyclical. Since the model matches well the risk properties of mortgage payoffs, these data fluctuations are likely driven by other factors not captured in the model. The model spread on riskier young-generation mortgages come close to matching subprime mortgage spreads in the data (row 7).[25]

Next, we look at financial intermediation related outcomes. The model somewhat overstates the ratio of deposits to aggregate consumption (row 8), but this is not surprising given the broad notion of deposits in the model. Importantly, the model gets the cyclicality and volatility of deposits right. It also matches well the data properties of intermediary leverage (row 9), although the equity ratio of financial intermediaries in the data is more cyclical and volatile. Inspecting intermediary payout behavior (row 10), we can see that this difference is driven by more cyclical payouts in the data: in particular during the housing bust of 2009-12, banks were forced to issue new equity under TARP; neither this form of government bailout nor the associated mandated payout and issuance policies are present in our model. The model is close to matching the average level and volatility of deposit rates in the data, but misses the correlation with the business cycle: while data deposit rates are procyclical, the model's deposit rate is strongly countercyclical. This counterfactual interest rate pattern is a known feature of purely real models with CRRA preferences and mean-reverting productivity shocks such as ours.[26] Furthermore, while the model matches the level of liquidity premia (row 12) even for a longer sample than used for the calibration, it understates convenience yield volatility. The cyclicality of convenience yields in the model is mostly driven by liquidity supply – the quantity of deposits, which is countercyclical with respect to the regular business cycle, and therefore causes a procyclical convenience yield. In the data, the demand for liquidity increases in recessions due to precautionary motives; this effect is largely absent from the model. However, the model matches the rise in liquidity premia during the 2009-12 crisis with a rise in convenience yields during housing recessions. This is because during housing recessions, intermediary balance sheets contract and the supply of deposits shrinks.

Finally, we compare several lifecycle moments between model and data.[27] The ratio of middle-aged to young consumption (row 13) is roughly equal in model and data.

[25] Unfortunately, data for this market are only available for a limited sample, see Appendix C.1 for details.

[26] Boldrin, Christiano, and Fisher (2001) shows how preferences with habit formation can solve this issue without nominal shocks.

[27] Consumption data for young and middle-aged households are taken from the Consumer Expenditure Survey (CEX), which, as other have pointed out (e.g. Fernández-Villaverde and Krueger (2007)), is plagued by measurement issues especially at high frequencies.

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The data ratio is perfectly acyclical, whereas in the model the young consume a greater share of output during booms, when house prices rise and leverage becomes cheaper. Consumption growth of both generations (row 14 and 15) is equal to overall economic growth on average in the model, whereas in the data, young household consumption grows at a faster rate than that of the middle-aged. Consumption growth is procyclical for both generations in model and data. Interestingly, in the CEX data middle-aged consumption growth is more volatile than that of the young. This is contrary to the model, in which young households consumption growth fluctuates more with credit conditions. Finally, the middle-aged own significantly more housing than the young in the model, yet even more so in the SCF data (row 16). Young households in the data move more frequently and are smaller in size (more unmarried and childless households), two reasons that likely explain the discrepancy. However, the model matches well the leverage of middle-aged households (row 17).[28]

In sum, the model provides a reasonable fit for key untargeted data moments. The model's performance could be further improved by adding long-term mortgage debt, which would make leverage dynamics in housing crises more realistic, and by adding monetary policy or permanent productivity shocks that would yield the right cyclicality of interest rates. These features would increase the realism and complexity of the model. They should be added before applying the market based leverage mechanism to analyzing monetary policy pass-through to mortgage rates, for example. However, they are not crucial for understanding the contribution of market based leverage to the housing boom, the main application in this paper.

3.4 Credit Surfaces

Figure 2 plots the cost-adjusted average "credit surfaces" facing young and middle-aged borrowers. This credit surface defines the menu of mortgage contracts available to a borrower who then selects an optimal contract. Although the mortgage offered to a housing depends on its entire portfolio (α_t^Y or α_t^M), our plot presents a three-dimensional approximation to it. This approximate credit surface for a borrower reports the interest rate it would be charged on a loan if it had a given loan to value (LTV) ratio $\frac{P_{t+1}h_t^a}{m_t}$ and loan to wealth (LTW) $\frac{w_{t+1}^{a_{t+1},nh}}{m_t}$ ratio assuming that its wealth at time $t+1$ remains at the level of

[28]Since SCF data are only available every three years, we only report means from the 1998 sample that is also used for the calibration.

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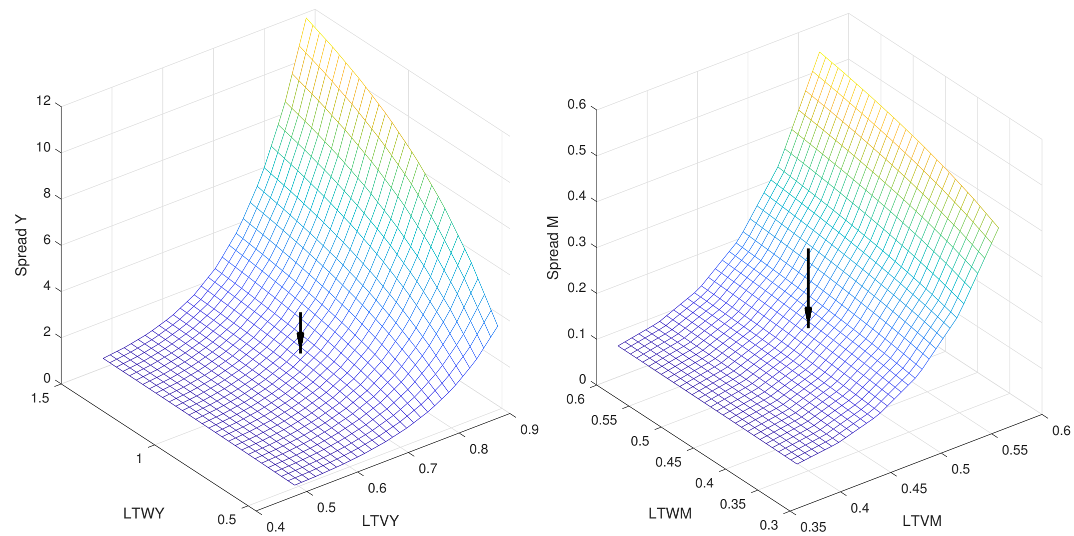
its wealth at time t.[29] The plots below are created by computing the (approximate) credit surface available to young and middle-aged borrowers at each point in the economy's state space, subtracting the mortgage origination cost ζ, and then reporting an average over the ergodic distribution.

Figure 2: Average Credit Surfaces



Left Panel: average credit surface for young. Right panel: middle-aged. X-axis: loan-to-value ratio (LTV). Y-axis: loan-to-wealth ratio (LTW). Z-axis: spread of mortgage interest rate over deposit rate, net of balance sheet cost ζ. Plots generated by computing the credit surface at each point in the economy's state space and reporting an average over the ergodic distribution.

As is intuitive, young borrowers face higher interest rates than middle-aged borrowers at the same LTV and LTW, since the non-pledgeability of their large future endowment income makes them more likely to default. In addition, the interest rate charged for a given young borrower is increasing and convex in LTW and LTV, consistent with empirical properties of credit surfaces estimated by Geanakoplos and Rappoport (2019). Middle-aged borrowers face a credit surface that is qualitatively similar to that of young households.

The black arrow on each credit surface reports the average LTV and LTW chosen by members of each generation. Young households choose considerably higher leverage than middle-aged households, and they are charged a much larger spread on their mortgage interest rate. Young households choose to increase their housing consumption by having a highly levered mortgage. This high leverage exposes the lender to default risk,

[29] As can be seen from expression 25, a household's default decision at time $t+1$ only depends on its LTV ratio and LTW ratio at time $t+1$. Our plot's approximation is equivalent to each household choosing to invest all their wealth in a riskless asset with interest rate 0. In the full model, households can choose to invest their wealth in assets with different degrees of risk that may impact their default decisions.

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who therefore requires a large credit spread. Middle-aged households do not need to use mortgages for financing consumption because they have liquid wealth that can be consumed. As a result, they choose a lower degree of leverage, and their mortgages are almost riskless. Why do middle-aged households lever up their house at all? The reason is that mortgage interest rates inherit part of the convenience yield of deposits through the competitive banking sector. Thus, mortgage rates are low relative to the convenience-yield-free discount rate of the middle-aged. In that sense, the middle-aged on aggregate "earn back" part of the convenience yield they pay on deposits. Our model provides a consistent explanation for the fact that even older and wealthier households in the data carry mortgage debt.[30]

We can compare model-generated credit surfaces to observed interest rate spread of mortgages in the data. To do so, we obtain the universe of all mortgage records originated since 1998 that were packaged into private-label mortgage-backed securities (MBS). We choose PLMBS mortgages for comparison, since the interest rate pricing of these mortgages is determined in a market and not subject to government-imposed constraints. For each mortgage, we know the interest rate at origination and several other borrower and mortgage characteristics. Importantly, we also know the borrower's credit score (FICO) and combined loan-to-value ratio (CLTV) at origination. We non-parametrically estimate the data credit surface by regressing origination interest rate on sets of indicator variables for bins of CLTV and FICO, and their interaction for the years 1998 to 2007; the regression also includes year dummies and various other loan-level controls.[31] We construct the credit surface by computing the average interest rate in the lowest CLTV-FICO bin (the omitted category in the regression), and then adding the respective regression coefficients for each other bin.

Figure 3 shows the resulting surface plot. As in the model, the credit spread is strongly increasing in LTV. Since FICO scores correlate with wealth and income, we proxy with these credit scores for the model's loan-to-wealth ratio. When comparing model and data surfaces, it is useful to be aware of two differences that help understand the quantitative discrepancies in spreads. First, in the data, we observe a collection of realized mortgage prices, whereas model surfaces in Figure 2 plot a menu of prices to choose from. It is natural that observed choices are less extreme. Second, mortgages in the model are one-period loan contracts, implying that LTV at origination and average LTV of all borrowers

[30] In both model and data, the mortgage tax advantage is another reason why middle-aged households choose to have mortgage debt. However, in our calibration this advantage is almost perfectly offset by the balance sheet cost of mortgages ζ.

[31] Appendix C.1.4 describes the credit surface regression in detail.

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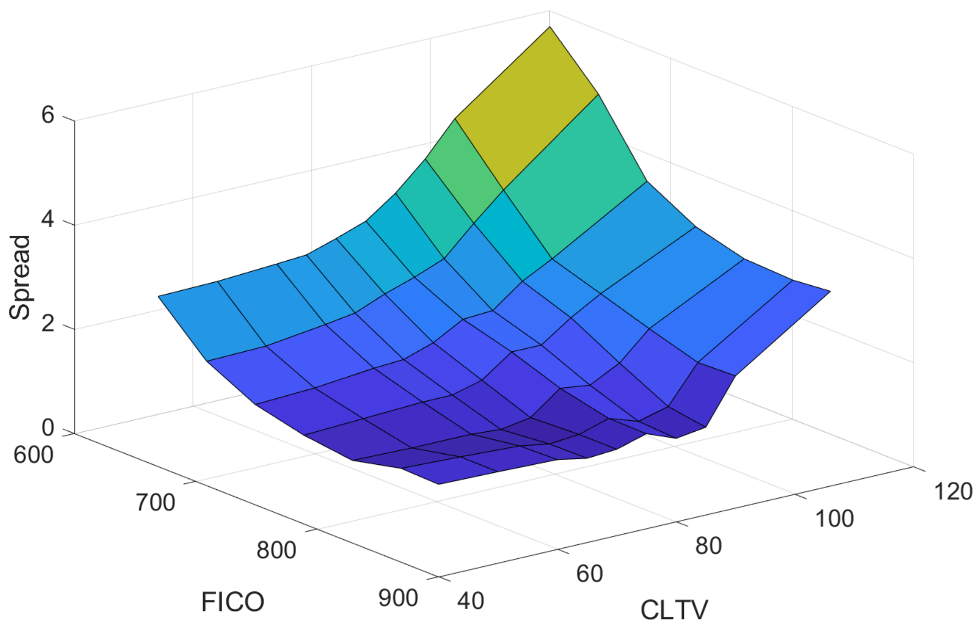
Figure 3: Data Credit Surface, 1998-2007



Average estimated interest spread at origination of mortgage sold into private-label MBS between 1998 – 2007. X-axis: combined loan-to-value ratio, Y-axis: borrower FICO score, both at origination. Z-axis: spread of origination interest rate over 10-year treasury yield for same period, and net of balance sheet cost ζ. See text for estimation details.

are the same. This is not the case for most mortgages in the data, which have fixed interest rates and amortization schedules. Hence, the model needs to match average data default rates with lower origination leverage, causing a higher sensitivity of credit spreads to LTV in the model. Third, while most mortgagees of private-label securitized loans were younger and less wealthy than the average mortgage borrower, there is not a one-to-one mapping from these data mortgages to young-generation mortgages in the model. Thus, when comparing model and data, we view the data credit surface as a weighted average of the young and middle-aged surfaces.

4 Results

We use the model for three quantitative exercises. First, we compare the behavior of the economy in regular productivity-driven recessions and in "housing recessions" that feature both low endowment realizations and high house price shock dispersion. Second,

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we show how the economy responds to an unanticipated 50% drop in intermediary capital. Finally, we consider the effects of an unanticipated shock: in the model calibrated to the 1998 base year, the relative supply of assets available for middle-aged savings declines. The decline is modeled as a a sequence of small shocks along the transition path. As result, the deposit rate gradually declines to 1.2%. We show how that this trend shock leads to a credit boom and increases the severity of future housing crisis.

4.1 Response to productivity and housing risk shocks

Next, we will examine the how the economy reacts to endowment (productivity) and housing risk shocks. Figure 4 shows impulse response functions to a pure negative endowment shock (blue) and the combination of a negative endowment shock and a housing risk shock (red). By construction, the blue and red lines coincide in the top left panel of Figure 4. However, as can be seen from the second panel in the top row, housing risk spikes during housing recessions, and reverts to normal levels over 15 years on average.[32]

Overall, we find that a housing risk shock causes a reduction in the size of the intermediary, an increase in mortgage spreads, and deleveraging by households. Deposits quantities fall by over 15%, bank equity falls by nearly 4% of the available endowment income (which is about 50% of the bank's equity), and mortgage spreads (averaged across both generations) rise by over 40 bps. In response to this, household leverage falls by over 6%, which forces credit-constrained young households to reduce their consumption. In addition, the reduced value of borrowing against a house contributes to a roughly 12% drop in house prices, compared to only a 7% drop in a regular recession. These results show that a wave of mortgage defaults depletes intermediaries' equity buffer, making mortgages more expensive and inducing households to choose less levered mortgages.

4.2 Bank equity and the price of risk

The results of the previous section suggest that the interaction between household leverage and constrained credit supply from intermediaries is a powerful amplification mechanism in housing recessions. However, the results are driven both by an increase in the riskiness of mortgages as well as a reduction in the amount of equity capital that allows intermediaries to bear risk.

[32] Recall that σ_ϵ is a two-state Markov chain with the average duration of a high-housing-risk episode being 4.5 years.

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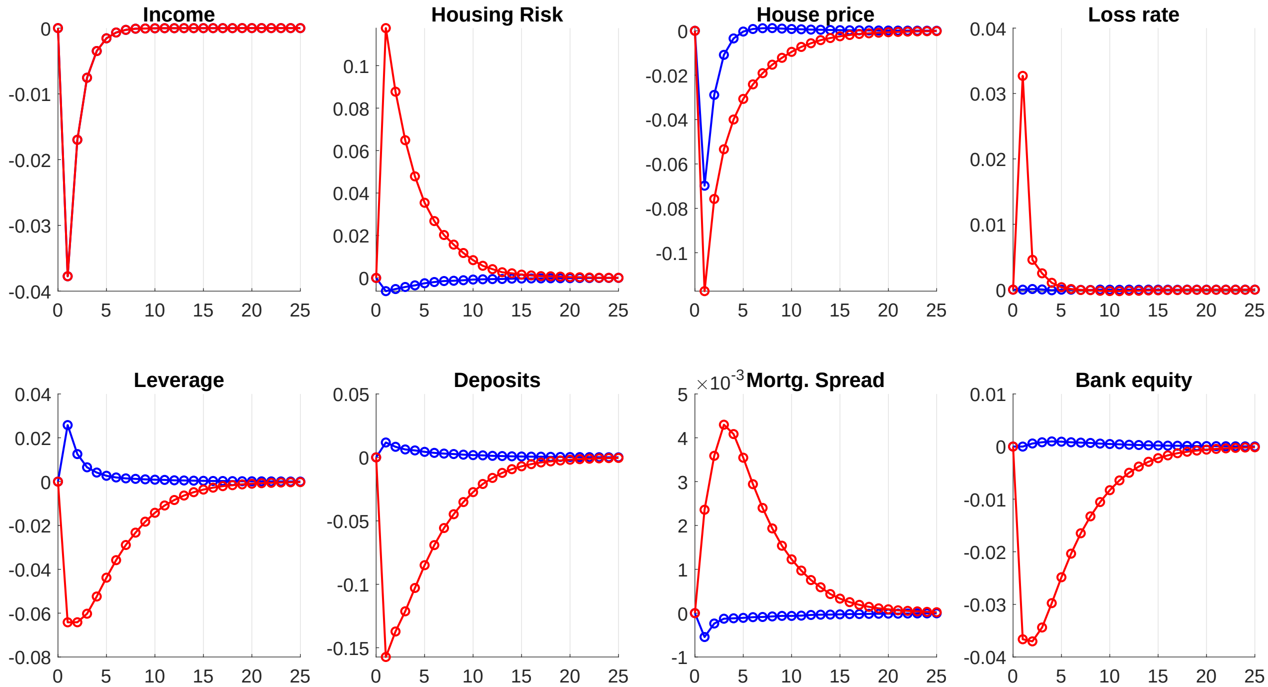
Figure 4: Regular vs. housing Recession (part 1)



Blue: regular recession, **Red:** housing recession. The generalized IRF plots are created by simulating the economy 10,000 times for 25 years, and plotting the average path of variables. The simulations are initialized at the ergodic distribution of the endogenous states, the mean income level, and in the low-housing-risk state $\bar{\sigma}_{\epsilon,0}$. The plots indicate deviations from the unconditional path in levels.

To understand a shock to the financial system that impacts the pricing of mortgages but not the creditworthiness of borrowers, we analyze the effect of an unanticipated reduction in internal bank equity, similar in magnitude to the losses banks suffer in a housing recession. Figure 5 illustrates the pure effect of a loss in bank equity. In the initial period, there is an unanticipated drop in bank equity by 50% (top left), roughly the same magnitude of drop that banks experience in housing recessions. With less equity, banks charge higher mortgage rates, with risk premia for the young rising by less than 1% to almost 4%, even though mortgage default risk as summarized by loss rates (second panel in top row) hardly changes. Households reduce their leverage from 0.57 to 0.5 in response to this rise in borrowing costs, forcing the young in particular to reduce their consumption from 0.47 to 0.4. Total mortgage debt declines by a comparable amount to a housing recession.

Overall, figure 5 illustrates how our supply-and-demand approach to modeling leverage results in a transmission mechanism from shocks to the financial system to household borrowing and consumption. This counterfactual is driven only by a shock to intermediary capital. This shock reduces risky asset prices and therefore causes an increase in the

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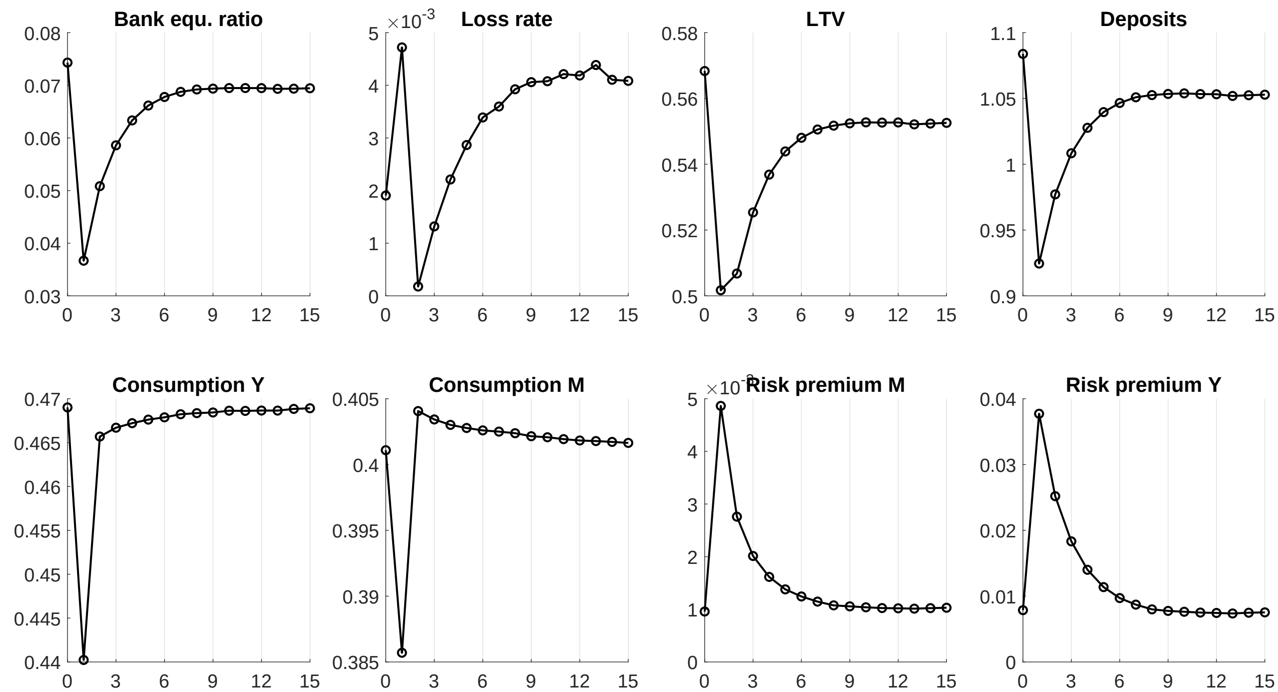
Figure 5: Effect of reduction in bank equity



The generalized IRF plots are created by simulating the economy 10,000 times for 15 years, and plotting the average path of variables. The simulations are initialized at the ergodic distribution of the exogenous and endogenous states. The plots report the evolution of variables in levels.

cost of borrowing for households. Households reduce their leverage in response to this rise in the price of borrowing, and those who are credit constrained must therefore reduce their consumption.

4.3 Housing boom-bust

To which extent can low interest rates, driven by a growing demand for safe assets, explain a credit cycle? To answer this question, we simulate a boom and bust episode motivated by the path of the U.S. economy from 1998 to the 2007-2008 financial crises. The simulation starts at the ergodic distribution, calibrated to 1998 data. The economy then experiences a sequence of two unanticipated shocks: (i) over the next 9 years, the fraction of middle-aged income y_t^M that goes to the newly middle-aged households linearly increases from 0.94 to 0.99. Over the same 9 years, the taste parameter ψ linearly increases from its current value of 0.015 to a new steady state value of 0.025. The parameter changes each year are unanticipated, so agents make decisions as if parameters will not change in

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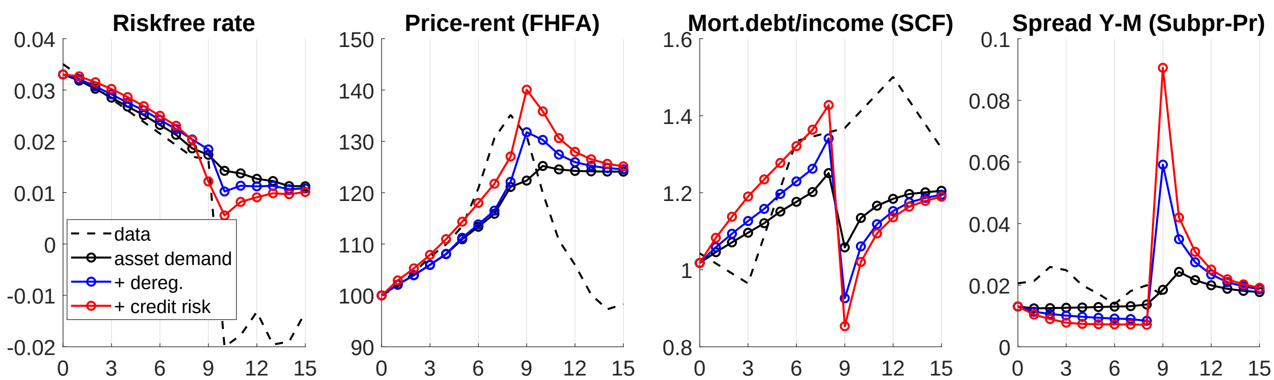
the future. Our first shock allocates the income of middle-aged households earlier in their life, effectively requiring them to hold a larger portfolio of financial assets in order to save for retirement. This greater imbalance between asset demand and supply exerts downward pressure on the risk-free interest rate. The shock can be interpreted broadly as one which increases the relative demand for savings compared to the supply of existing assets and therefore bids up asset prices (the "global savings glut"). Our second shock, increasing the preference for safe assets, makes it cheaper for intermediaries to issue riskless deposits and thus induces intermediaries to borrow more. The combination of both shocks causes a fall in the risk-free rate but also in the yields on risky assets, without leading to a counterfactually large rise in the convenience yield on safe assets.

Over the 9 years of these parameter adjustments, productivity shock realizations are at their median level in all periods, while the housing risk shock realizations are low. In year 9, a high housing risk shock is realized, triggering an event similar to the wave of mortgage defaults in 2007 (9 years after 1998) that contributed to the financial crisis. After that, the simulation progresses stochastically for 6 more years as the economy recovers with no additional parameter changes.

Figure 6: Housing boom and bust: comparison to data



The transition plots are created by simulating the economy 10,000 times for 15 years, and plotting the average path of variables. The simulations are initialized at the ergodic distribution of the exogenous and endogenous states in the base year (1998). All simulations have the same sequence of shocks for the first 9 years, which are 8 years of average endowment and low housing risk realizations, followed by a housing recession (low endowment, high housing risk) in year 9. The plots report the evolution of variables in levels.

Figure 6 shows the response of the economy to this sequence of shocks. The black line only considers the basic asset demand shocks described above, while the blue and the red lines consider additional credit supply shocks that amplify the basic rise in asset demand. The blue line adds a relaxation of bank capital requirements for young mortgages from 8% to 4%, for middle-aged mortgages from 1% to 0%, and a reduction in the equity

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issuance cost parameter from 850 to 400.[33] This combination of shocks, which we refer to as "deregulation", stands in for regulatory arbitrage through securitization that the mortgage industry engaged in during the boom. The red line adds misperception of credit risk: during the boom, agents believe that the realization of housing risk σ_t^ϵ that would occur in a housing recession declines linearly from its orignal value of 0.32 to 0.25.

The figure compares the model-generated paths for the risk-free deposit rate, house price-rent ratio, mortgage debt-income ratio, aggregated across all households, and the spread of young mortgages rates over middle-aged mortgage rates, to their counterparts in the data. For the first three panels, the model was calibrated to match the data in the first year, while the mortgage spread was not a targeted moment. The asset demand shock is calibrated to match the trend decline in the risk-free rate. The asset demand shock alone (black line) generates a 25% increase in the price-rent ratio, and a similar rise in the mortgage-debt-income ratio, accounting for roughly 70% of the rise of the ratios from 1998 to 2007. Adding bank deregulation (blue line) further increases these ratios, matching the size of the boom in the data. Finally, by adding underestimation of credit risk, the model yields a larger boom than observed in the data. In addition, our deregulation and overoptimism counterfactuals lead to empirically realistic reductions in the spread between prime (middle-aged) and subprime (young) mortgage rates.

Figure 7 shows the path of several other variables for the simulated boom-bust episode. Since both mortgage debt and house prices rise in lockstep, a large rise in mortgage debt only requires a moderate rise in leverage as a whole, consistent with the aggregate data for the boom period. Young households, whose consumption is highly sensitive to credit supply, increase their share of consumption from 0.47 to 0.49. Because of the exogenous aggregate endowment, market clearing requires that the middle-aged reduce their consumption.

After the realization of the high housing risk shock in period 9, a wave of mortgage defaults occur. This drives a wave of losses on banks' mortgage portfolios, depleting their equity capital, and also increases the riskiness of future mortgages, since the economy tends to stay in the high housing risk state for 4.5 years on average. Mortgage spreads spike, reflecting both elevated default risk and higher mortgage risk premiums. As a result, households sharply reduce leverage and debt/income, and house prices decline. Expected excess returns on mortgages, i.e. the risk premiums in mortgages, are flat for the pure asset demand shock despite a rise in household leverage and default risk. When

[33] Like in our baseline changes, these parameter changes happen linearly over the first 9 years of the simulation. Each subsequent parameter change is unanticipated.

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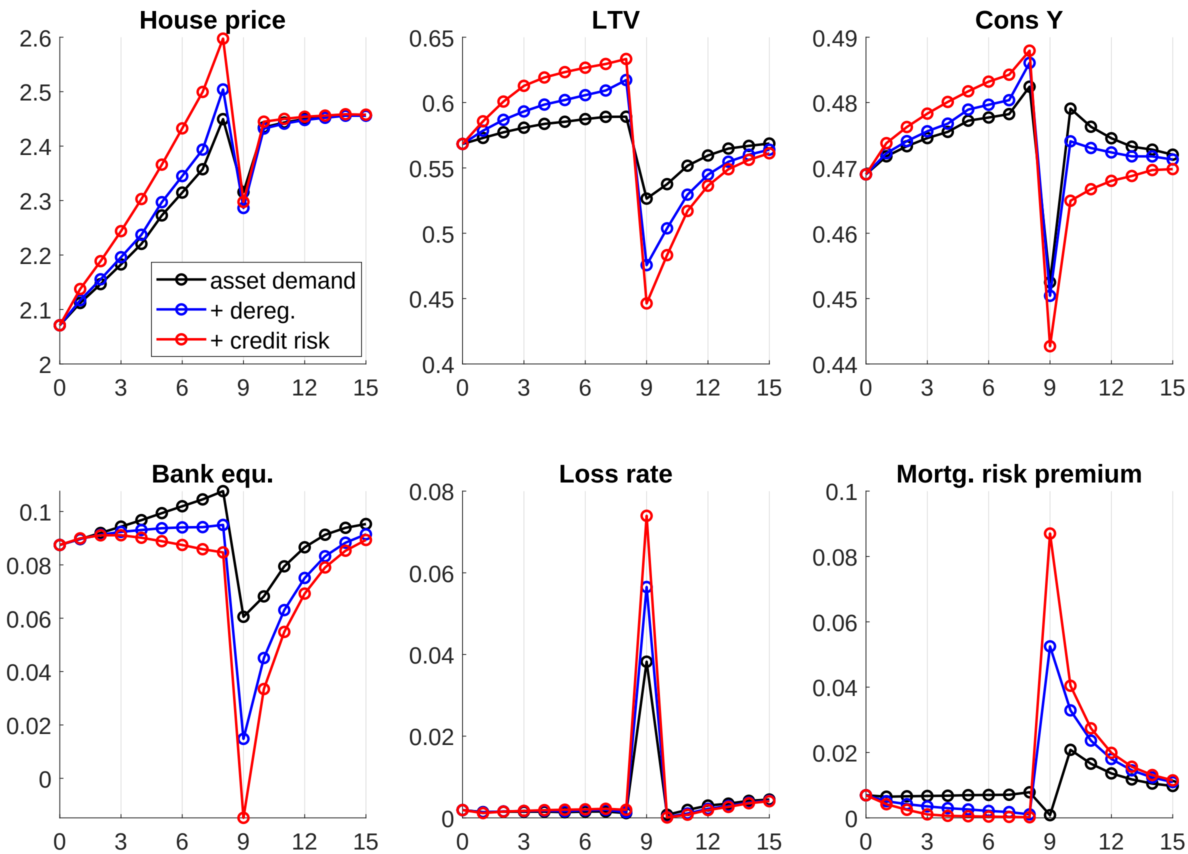
Figure 7: Housing boom and bust: leverage, consumption, and risk premia



The transition plots are created by simulating the economy 10,000 times for 15 years, and plotting the average path of variables. The simulations are initialized at the ergodic distribution of the exogenous and endogenous states in the base year (1998). All simulations have the same sequence of shocks for the first 9 years, which are 8 years of average endowment and low housing risk realizations, followed by a housing recession (low endowment, high housing risk) in year 9. The plots report the evolution of variables in levels.

combined with deregulation, risk premiums decline to almost zero during the boom in spite of an even greater increase in risk. Then risk premiums rise sharply in the bust (two panels on bottom right). The effects of a housing risk shock is stronger in each of our simulations than in a “standard" housing recession analyzed in section 4.1, and across specifications a larger boom ex ante results in a larger bust ex post. This demonstrates how a credit boom driven by a growing demand for safe assets makes our economy vulnerable to financial crises.

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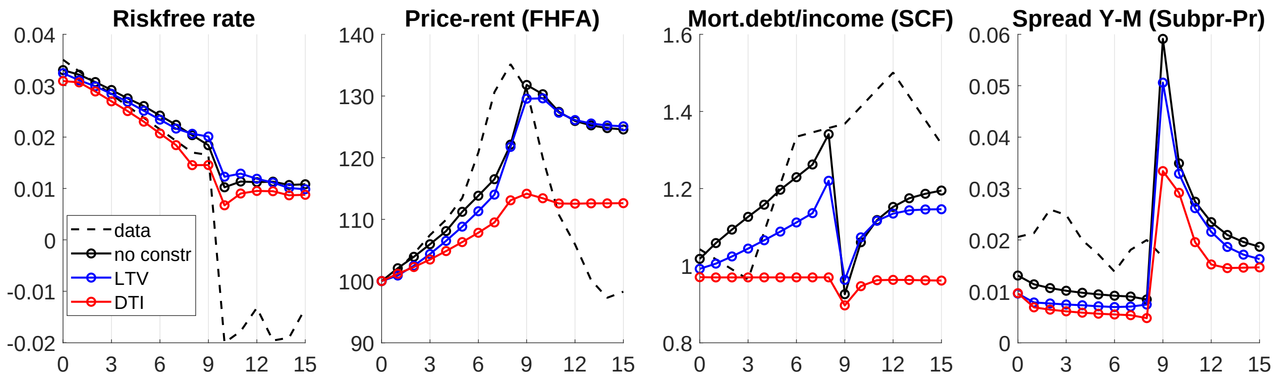
Figure 8: Housing boom and bust: hard constraints



The transition plots are created by simulating the economy 10,000 times for 15 years, and plotting the average path of variables. The simulations are initialized at the ergodic distribution of the exogenous and endogenous states in the base year (1998). All simulations have the same sequence of shocks for the first 9 years, which are 8 years of average endowment and low housing risk realizations, followed by a housing recession (low endowment, high housing risk) in year 9. The plots report the evolution of variables in levels.

4.3.1 Hard borrowing constraints

Next, we compare our simulation to one with loan-to-value (LTV) or debt-to-income (DTI) constraints common in the rest of the literature added to our model. Both constraints are set at at precisely the LTV and DTI levels observed in 1998, so that our simulations start at the same values as our benchmark but with future increases forbidden. For generation a, the LTV and DTI constraints respectively take the forms

$$\theta^a_{LTV} P_t h^a_t \geq m^a_t$$
$$\theta^a_{DTI} y^a_t b^{a,a}_t \geq m^a_t.$$

In each, the household's promised mortgage repayment $\hat{m}^a_t$ next period is constrained by a constant θ^a times either the value of its housing $P_t h^a_t$ or its income $y^a_t b^{a,a}_t$ today.

The black line in figure 8 is the “deregulation" counterfactual in our baseline model without hard constraints (shown in figure 6). The blue line imposes an LTV constraint limiting mortgage debt to be a fraction of the house value. The figure shows that a hard LTV constraint causes only a modest reduction in the rise in the aggregate mortgage debt/income and price-rent ratios. This is consistent with the fact that aggregate leverage only increased moderately during the boom, as household borrowing and house price levels increased together. In contrast, imposing a DTI constraint entirely stops the rise in debt/income and significantly reduces the rise in the price-rent ratio. This is true even

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though this counterfactual results in the greatest reduction in subprime-prime mortgage spreads, demonstrating how hard constraints effectively disconnect movements in the price and quantity of credit. This is particularly relevant since Greenwald (2018) shows in a model with hard constraints that relaxed debt-to-income constraints are quantitatively important for explaining the boom. Our results imply that as households rationally adjust their borrowing choices to changes in the price of credit, they behave as if a DTI constraint was relaxed.

4.3.2 Rental markets

Our final comparison is to a version of our model in which the young can rent housing from the middle-aged.[34] This is motivated by Kaplan, Mitman, and Violante (2020), who show in their model with rental markets that shocks to credit supply (which they model as relaxation of a hard constraint) do not impact house prices. Their result follows from the fact that housing is largely owned by financially unconstrained agents in their model, whose housing demand is not affected by credit constraints. In our model, while the middle-aged have sufficient pledgeable wealth to finance their desired consumption, they rationally choose to increase their mortgage borrowing as mortgage rates fall. This implies that the collateral value of housing capital increases as rates fall even for unconstrained middle-aged homeowners. As a result, we find a similar increase in debt to income ratios in our simulation with and without rental markets. In addition, house price to rent ratios increase by 20% even in the presence of a rental market.

This result demonstrates how our supply-and-demand approach to modeling leverage choices does not rely on a sharp distinction between "borrowing constrained" and "borrowing unconstrained" agents. Even agents with plentiful wealth make borrowing choices that respond to interest rates. This is consistent with the fact that wealthy homeowners often both own significant financial assets and have a mortgage, and they also actively refinance their mortgages in response to changing mortgage interest rates.

[34] This model is identical to our benchmark with two changes. First, we do not require that separate rental markets clear within each generation, but only that the total quantity of housing consumed either by renting or owning, equals the housing stock. Second, we add a term $\psi^{y,h} \frac{(h_t^y)^{1-\gamma}}{1-\gamma}$ to the utility function of the young, directly providing utility for owning housing. We calibrate this parameter so that the young own 54% of the housing they consume, which is the home ownership rate of young households in the 1998 SCF.

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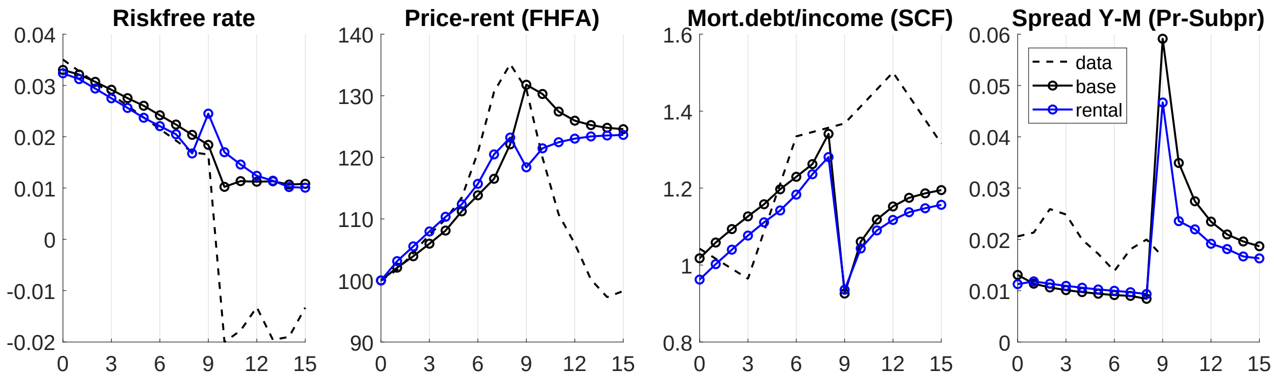
Figure 9: Housing boom and bust: rental markets



The transition plots are created by simulating the economy 10,000 times for 15 years, and plotting the average path of variables. The simulations are initialized at the ergodic distribution of the exogenous and endogenous states in the base year (1998). All simulations have the same sequence of shocks for the first 9 years, which are 8 years of average endowment and low housing risk realizations, followed by a housing recession (low endowment, high housing risk) in year 9. The plots report the evolution of variables in levels.

5 Conclusion

The main innovation of our model is that the supply of credit to borrowing-constrained households depends on the risk-taking capacity of financial intermediaries. As a result, when financial intermediaries are distressed, constrained households choose to reduce their mortgage leverage and must cut back on their consumption. This connection between the health of the financial sector and the real economy gives us a novel propagation mechanism for shocks to the financial system.

Our model mechanism generates an endogenous credit surface, which is the menu of leverage and interest rates that borrowers face. An exogenous reduction in the equity capital of intermediaries causes an upward shift and steepening of the credit surface, making mortgage leverage more expensive. This leads to an increase in mortgage spreads, a decrease in leverage, a drop in the consumption of the most constrained households, and a reduction in house prices. Since intermediary equity is endogenous in our model, it becomes a key state variable amplifying the effects of fluctuations in mortgage default risk.

We use our model to show that a growing demand for safe assets leads to many features of the housing and credit boom of the 2000s and increases the severity of future financial crises after a shock to mortgage default risk. In particular we find that an increase in the demand for safe assets causes intermediaries to expand their balance sheet

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and make riskier loans, which induces households to borrow more and boosts the consumption of constrained households. However, after an increase in mortgage default risk, this increased size and riskiness of the financial sector leads to a more severe drop in household leverage, consumption of constrained households, and house prices.

Broadly speaking, our model implies that shocks to intermediary capital emphasized by the intermediary asset pricing literature building on He and Krishnamurthy (2013) cause a negative credit supply shock that induces households to delever and consume less as emphasized by the literature following Mian and Sufi (2011). Our model therefore has a novel transmission mechanism of distress from Wall Street to Main Street, because leverage is endogenously determined. In future, we hope to enrich the general equilibrium effects of this transmission mechanism by making several features of our model endogenous. First, the fact that we have an endowment economy does not let us consider effects on output. Second, our current model of mortgage borrowing misses the fact that mortgages are long-term with costly refinancing. Third, our framework does not explore the interaction of our market-based leverage mechanism with specific features of the U.S. mortgage system, which includes the Government-Sponsored Enterprises and both bank and non-bank lenders.

Finally, while our model framework uses several stylized assumptions to achieve aggregation among households with idiosyncratic shocks, we believe that integrating our model of intermediation and market-based leverage with a full quantitative model of the wealth distribution, such as in Favilukis, Ludvigson, and Van Nieuwerburgh (2017), would be an important step forward for the literature. The tractability of our model comes from its relatively small number of state variables (since our households aggregate to a few representative agents), allowing us to use nonlinear global projection methods. Such nonlinear methods are crucial to accurately capture the dynamics of financial crises. A fully realistic model of household heterogeneity would result in a high-dimensional state space that is currently beyond the capabilities of nonlinear global solution methods. A more realistic model of the interaction between financial institutions and heterogeneous households requires advances on this technical problem.

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A Model

Transition Law for Wealth Shares. This section provides expressions for how the wealth of each generation evolves from period to period. In every period, there is a total measure of $1/\pi^Y$ young, a fraction $1-\pi^Y$ of whom are "incumbent" young and the remaining fraction π^Y are newly born households (with a fraction π^Y having moved on to middle-age). Denote the total beginning-of-period liquid wealth of the incumbent young that do not turn middle-aged as

$$\begin{aligned}\overrightarrow{W}_t^{Y,Y} =&(1-\pi^Y)\underbrace{\left(1-F^Y_{\epsilon,t}(\bar{\epsilon}^Y_t)\right)\left((1-\delta_H)\mathrm{E}_t[\epsilon^Y|\epsilon^Y>\bar{\epsilon}^Y_t]p^h_t H^Y_{t-1}-M^Y_{t-1}\right)}_{\text{home equity of non-defaulters}} \\ &+(1-\pi^Y)\underbrace{\left(1-\lambda F^Y_{\epsilon,t}(\bar{\epsilon}^Y_t)\right)\left(D^Y_{t-1}+\mathbf{1}\cdot(P^Y_t+x^Y_t)\right)}_{\text{other wealth}-\text{def. penalty}}. \end{aligned} \tag{48}$$

The home equity of the non-defaulters depends on the conditional expectation $\mathrm{E}_t[\epsilon^Y|\epsilon^Y > \bar{\epsilon}^Y_t]$, which is the average realization of the idiosyncratic house price shock conditional on not defaulting. Similarly, we define the aggregate wealth of the young that turn middle-aged as

$$\begin{aligned}\overrightarrow{W}_t^{Y,M} =&\pi^Y\underbrace{\left(1-F^Y_{\epsilon,t}(\bar{\epsilon}^{Y+}_t)\right)\left((1-\delta_H)\mathrm{E}_t[\epsilon^Y|\epsilon^Y>\bar{\epsilon}^{Y+}_t]p^h_t H^Y_{t-1}-M^Y_{t-1}\right)}_{\text{home equity of non-def.}} \\ &+\pi^Y\underbrace{\left(1-\lambda F^Y_{\epsilon,t}(\bar{\epsilon}^{Y+}_t)\right)\left(D^Y_{t-1}+\mathbf{1}\cdot(P^{Y+}_t+x^{Y+}_t)\right)}_{\text{other wealth}-\text{def. penalty}}. \end{aligned} \tag{49}$$

Since newly born households are endowed with one share of the young-generation-specific endowment asset, the aggregate wealth of the young generation is

$$W^Y_t = \pi^Y(y^Y_t + p^Y_t) + \overrightarrow{W}_t^{Y,Y}. \tag{50}$$

We further define the wealth of incumbent middle-aged as

$$\overrightarrow{W}_t^{M} = \left(1-F^M_{\epsilon,t}(\bar{\epsilon}^M_t)\right)\left((1-\delta_H)\mathrm{E}_t[\epsilon^M|\epsilon^M>\bar{\epsilon}^M_t]p^h_t H^M_{t-1}-M^M_{t-1}\right) \tag{51}$$

$$+\left(1-\lambda F^M_{\epsilon,t}(\bar{\epsilon}^M_t)\right)\left(D^M_{t-1}+\mathbf{1}\cdot(P^M_t+x^M_t)\right). \tag{52}$$

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Then the aggregate wealth of the middle generation is

$$W_t^M = (1 - \pi^M) \left(\overrightarrow{W}_t^{Y,M} + \overrightarrow{W}_t^M \right), \tag{53}$$

and the aggregate beginning-of-period wealth of the old is

$$W_t^O = \pi^M \left(\overrightarrow{W}_t^{Y,M} + \overrightarrow{W}_t^M \right). \tag{54}$$

A.1 Proofs

A.1.1 Proposition 1

In order to use variables that are stationary if the economy grows at a trend rate g, we renormalize the choices $(c_t^M, s_t^M, \alpha_t^M)$ as well as prices (p_t^h, p_t^I), and the bank dividend x_t^I. The rents ρ_t^a, interest rate r_t, and housing and asset holdings (h_t^a, b_t^a) are stationary variables along such a balanced growth path.

Thus, defining $G = exp(g)$, the detrended problem along the BGP of a household in generation $a \in \{Y, M\}$ is

$$\begin{aligned} V^a(w_t^a, \mathcal{Z}_t) = \max_{c_t^a, s_t^a, \alpha_t^a} & \frac{\left((c_t^a)^{1-\theta}(s_t^a)^\theta\right)^{1-\gamma}}{1-\gamma} + \psi \frac{(d_t^a)^{1-\gamma}}{1-\gamma} \\ & + \beta(1-\pi^a)\mathrm{E}_t\left[G^{1-\gamma}\max\left\{V^a(w_{t+1}^{a,d}, \mathcal{Z}_{t+1}), V^a(w_{t+1}^{a,nd}, \mathcal{Z}_{t+1})\right\}\right] \\ & + \beta\pi^a\mathrm{E}_t\left[G^{1-\gamma}\max\left\{V^{a^+}(w_{t+1}^{a+,d}, \mathcal{Z}_{t+1}), V^{a^+}(w_{t+1}^{a+,nd}, \mathcal{Z}_{t+1})\right\}\right], \end{aligned} \tag{55}$$

subject to

$$w_t^a = c_t^a + \rho_t^a s_t^a + (p_t^h - \rho_t^a)h_t^a - (1+\tau^m)q^a(\alpha_t^a, \mathcal{Z}_t)m_t^a + \frac{d_t^a}{1+r_t} + b_t^a \cdot P_t^a. \tag{56}$$

Next-period non-housing wealth, conditional on the age transition is $k \in \{a, a+\}$,

$$w_{t+1}^{k,nh} = \frac{d_t^a}{G} + b_t^a \cdot (P_{t+1}^k + x_{t+1}^k). \tag{57}$$

Thus next-period wealth conditional on defaulting is

$$w_{t+1}^{k,d} = (1-\lambda)w_{t+1}^{k,nh}, \tag{58}$$

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and next-period wealth conditional on not defaulting is

$$w_{t+1}^{k,nd} = \epsilon_{t+1}^{a}(1-\delta_H)P_{t+1}h_t^a - \frac{m_t^a}{G} + w_{t+1}^{k,nh}. \tag{59}$$

Denote the savings of an individual household by

$$\Sigma_t^a = h_t^a(p_t^h - \rho_t^a) + \frac{d_t^a}{1+r_t} - (1+\tau^m)q^a(\alpha_t^a, \mathcal{Z}_t)m_t^a + b_t^a \cdot P_t^a. \tag{60}$$

Further define the portfolio return conditional on defaulting and not defaulting, respectively, as

$$R_{t+1}^{k,j} = G\frac{w_{t+1}^{k,j}}{\Sigma_t^a}, \tag{61}$$

for $j \in \{d, nd\}$ and $k \in \{a, a+\}$, where

$$R_{t+1}^{k,nd} = G(1-\delta_H)P_{t+1}\epsilon_{t+1}^a \hat{h}_t^a - \hat{m}_t^a + \hat{d}_t^a + \hat{b}_t^a \cdot (P_{t+1}^k + x_{t+1}^k) \tag{62}$$

$$R_{t+1}^{k,d} = (1-\lambda)(\hat{d}_t^a + \hat{b}_t^a \cdot (P_{t+1}^k + x_{t+1}^k)) \tag{63}$$

$$\tag{64}$$

and we have defined quantity portfolio shares $\hat{h}_t^a = h_t^a/\Sigma_t^a$, $\hat{d}_t^a = d_t^a/\Sigma_t^a$, $\hat{m}_t^a = m_t^a/\Sigma_t^a$, and $\hat{b}_t^a = b_t^a/\Sigma_t^a$.

The usual results for Cobb-Douglas utility functions imply that the optimal expenditure on non-durable and housing services consumption are

$$c_t^a = (1-\theta)(w_t^a - \Sigma_t^a), \tag{65}$$

$$s_t^a = \frac{\theta}{\rho_t^a}(w_t^a - \Sigma_t^a). \tag{66}$$

We conjecture and then verify that the value function has the form

$$V^a(w_t^a, \mathcal{Z}_t) = v^a(\mathcal{Z}_t)\frac{(w_t^a)^{1-\gamma}}{1-\gamma}, \tag{67}$$

as in (15), where $v^a(\mathcal{Z}_t)$ only depends on aggregate states exogenous to the individual household.

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This allows us to rewrite the value function as

$$V^a(w_t^a, \mathcal{Z}_t) = \max_{\Sigma_t^a} \frac{\Theta^a(\mathcal{Z}_t)}{1-\gamma} (w_t^a - \Sigma_t^a)^{1-\gamma} + (\Sigma_t^a)^{1-\gamma} A^a(\mathcal{Z}_t), \tag{68}$$

where we defined the portfolio choice problem per dollar of savings

$$\begin{aligned} A^a(\mathcal{Z}_t) = \max_{\hat{\alpha}_t^a} \; & \psi \frac{(\hat{d}_t^a)^{1-\gamma}}{1-\gamma} \\ & + \beta(1-\pi^a)\mathrm{E}_t \left[\max \left\{ \frac{(R_{t+1}^{a,nd})^{1-\gamma}}{1-\gamma}, \frac{(R_{t+1}^{a,d})^{1-\gamma}}{1-\gamma} \right\} v^a(\mathcal{Z}_{t+1}) \right] \\ & + \beta\pi^a \mathrm{E}_t \left[\max \left\{ \frac{(R_{t+1}^{a+,nd})^{1-\gamma}}{1-\gamma}, \frac{(R_{t+1}^{a+,d})^{1-\gamma}}{1-\gamma} \right\} v^{a+}(\mathcal{Z}_{t+1}) \right] \end{aligned} \tag{69}$$

subject to the budget constraint

$$1 = \hat{h}_t^a(p_t^h - \rho_t^a) + \frac{\hat{d}_t^a}{1+r_t} + \hat{b}_t^a \cdot P_t^a - (1+\tau^m) q^a(\hat{\alpha}_t^a, \mathcal{Z}_t)\hat{m}_t^a \tag{70}$$

and where $\Theta^a(\mathcal{Z}_t) = \left((1-\theta)^{1-\theta} \left(\frac{\theta}{\rho_t^a} \right)^{\theta} \right)^{1-\gamma}$. The last term of the portfolio budget constraint (70) uses the property that the mortgage price $q^a(\alpha_t^a, \mathcal{Z}_t)$ is homogeneous of degree zero in household wealth and savings, conditional on the conjectured value function, such that $q^a(\hat{\alpha}_t^a, \mathcal{Z}_t) = q^a(\alpha_t^a, \mathcal{Z}_t)$, see also proposition 2.

Taking the first-order condition with respect to Σ_t^a and solving, we get

$$\Sigma_t^a = \underbrace{\frac{((1-\gamma)A^a(\mathcal{Z}_t))^{1/\gamma}}{\Theta^a(\mathcal{Z}_t)^{1/\gamma} + ((1-\gamma)A^a(\mathcal{Z}_t))^{1/\gamma}}}_{=B^a(\mathcal{Z}_t)} w_t^a. \tag{71}$$

Equation (71) implies that all households in generation a save the same fraction of their wealth, with this fraction given by $B^a(\mathcal{Z}_t)$.

Reinserting this solution for Σ_t^a into the value function gives

$$V^a(w_t^a, \mathcal{Z}_t) = \frac{(w_t^a)^{1-\gamma}}{1-\gamma} \left[\Theta^a(\mathcal{Z}_t) \left(1 - B^a(\mathcal{Z}_t)\right)^{1-\gamma} + (1-\gamma)A^a(\mathcal{Z}_t)B^a(\mathcal{Z}_t)^{1-\gamma} \right]. \tag{72}$$

This confirms the conjecture from (15) with

$$v^a(\mathcal{Z}_t) = \Theta^a(\mathcal{Z}_t) \left(1 - B^a(\mathcal{Z}_t)\right)^{1-\gamma} + (1-\gamma)A^a(\mathcal{Z}_t)B^a(\mathcal{Z}_t)^{1-\gamma}. \tag{73}$$

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Equation (73) is a recursion in $v^a(\mathcal{Z}_t)$, since $A^a(\mathcal{Z}_t)$ depends on the expectation of $v^a(\mathcal{Z}_{t+1})$ and $v^{a+}(\mathcal{Z}_t)$. In order for the proposition to hold, $V^{a+}(\mathcal{Z}_t)$ must also be homogeneous in wealth of degree $1-\gamma$. This is the case for both generations: for middle-aged households $V^{M+} = V^O$, which satisfies this property from (13). This implies that V^M is homogeneous of degree $1-\gamma$. Since $V^{Y+} = \pi^M V^O + (1-\pi^M)V^M$, it follows that V^Y inherits the same homogeneity.

Since the optimization problem in (69) is independent of individual wealth, all households in the same generation choose the same portfolio and savings shares, irrespective of their level of wealth.

A.1.2 Proposition 2

Proposition 2 was proven in the main text taking as given the stochastic discount factor of the intermediary. Here we derive the intermediary's stochastic discount factor from its optimization problem. As above, we normalize all variables to grow at a rate $G = exp(g)$ each period so that the first order conditions we derive are consistent with a balanced growth path. The proof does not assume this balanced growth path exists, but this provides expressions useful for the numerical solution of the model. The detrended value function is

$$V^I(e_t, \mathcal{Z}_t) = \max_{I_t, D_t, N_t^Y, N_t^M} \eta e_t - I_t - \frac{\chi}{2} I_t^2 + \mathrm{E}_t \left[G\mathcal{M}_{t,t+1}^M V^I(e_{t+1}, \mathcal{Z}_{t+1}) \right], \tag{74}$$

subject to the budget constraint

$$(1-\eta)e_t + I_t - C(I_t, \bar{Y}_t) + \frac{D_t}{1+r_t} = (1+\zeta)[N_t^Y q^Y(\alpha_t^Y) + N_t^M q^M(\alpha_t^M)], \tag{75}$$

the transition law for equity

$$e_{t+1} = \left(N_t^Y \mathcal{P}_{t+1}^Y + N_t^M \mathcal{P}_{t+1}^M - D_t \right) / G, \tag{76}$$

and the regulatory capital constraint for the worst-payoff state next period

$$D_t \leq (1-\bar{e}^Y) N_t^Y \mathcal{P}^Y(\underline{z}_t) + (1-\bar{e}^M) N_t^M \mathcal{P}^M(\underline{z}_t). \tag{77}$$

The regulatory capital constraint is effectively an endogenous leverage constraint.

The Lagrangian form of the problem, with Lagrange multiplier μ_t^{I*} on the (occasion-

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ally binding) regulatory capital constraint and multiplier κ_t^I on the intratemporal budget constraint, is

$$\max_{I_t, D_t, N_t^Y, N_t^M} \eta e_t - I_t + \mathrm{E}_t \left[G \mathcal{M}_{t,t+1}^M V^I \left(\left(N_t^Y \mathcal{P}_{t+1}^Y + N_t^M \mathcal{P}_{t+1}^M - D_t \right) / G, \mathcal{Z}_{t+1} \right) \right]$$
$$+ \mu_t^{I^*} \left[D_t - (1 - \bar{e}^Y) N_t^Y \mathcal{P}^Y(\underline{z}_t) - (1 - \bar{e}^M) N_t^M \mathcal{P}^M(\underline{z}_t) \right]$$
$$+ \kappa_t^I \left[(1-\eta) e_t + I_t - \frac{\chi}{2} I_t^2 + \frac{D_t}{1+r_t} - (1+\zeta) \left(N_t^Y q^Y(\alpha_t^Y) + N_t^M q^M(\alpha_t^M) \right) \right]. \tag{78}$$

Assets held by the intermediary have value for two reasons. First, their payoff in the worst aggregate state loosens the regulatory capital constraint if it is binding. Second, assets provide wealth in the future, which is valued by a stochastic discount factor determined by the intermediary's shadow value of equity. Taking the FOC for issuance I_t, the shadow value of internal funds is

$$\kappa_t^I = \frac{1}{1 - \chi I_t}. \tag{79}$$

Hence, the marginal value of equity is

$$\frac{\partial V^I(e_t, \mathcal{Z}_t)}{\partial e_t} = \eta + (1-\eta)\kappa_t^I = \eta + \frac{1-\eta}{1-\chi I_t}. \tag{80}$$

We define the intermediary's shadow value SDF (which captures only this second source of value) as

$$\mathcal{M}_{t,t+1}^I = \mathcal{M}_{t,t+1}^M (1 - \chi I_t) \left(\eta + \frac{1-\eta}{1 - \chi I_{t+1}} \right). \tag{81}$$

Letting $\mu_t^I = \mu_t^{I^*}(1 - \chi I_t)$ be a renormalization of the Lagrange multiplier on the constraint (77), the FOCs for deposits and loans are

$$\frac{1}{1+r_t} = \mu_t^I + \mathrm{E}_t \left[\mathcal{M}_{t,t+1}^I \right] \tag{82}$$
$$(1+\zeta) q^M(\alpha_t^M) = \mu_t^I (1 - \bar{e}^M) \mathcal{P}^M(\underline{z}_t) + \mathrm{E}_t \left[\mathcal{M}_{t,t+1}^I \mathcal{P}_{t+1}^M \right], \tag{83}$$
$$(1+\zeta) q^Y(\alpha_t^Y) = \mu_t^I (1 - \bar{e}^Y) \mathcal{P}^Y(\underline{z}_t) + \mathrm{E}_t \left[\mathcal{M}_{t,t+1}^I \mathcal{P}_{t+1}^Y \right]. \tag{84}$$

The first-order conditions (83) and (84) define the mortgage pricing functions faced by borrowers, $q^j(\alpha_t^j)$, which depend on mortgage payoffs $\mathcal{P}_{t+1}^j$. From the definitions of these payoffs in (26) and (27), it is clear that they depend on borrower choices through

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the inverse mortgage leverage ratio $P_{t+1}h_t^j/m_t^j$, and default thresholds, which depend on choices through ratios of non-default to default wealth $w_{t+1}^{a,nd}/w_{t+1}^{a,d}$. Then by proposition 1, these payoffs are homogeneous of degree zero in borrower wealth. Individual borrowers choose identical portfolio *shares* of wealth, thus keeping these ratios independent of wealth levels.

A.2 Characterization of Portfolio Problems

SDF. Since the solution to the optimization problem of households scales in individual wealth (Proposition 1), we can construct the stochastic discount factor of a representative household for generation a. To do so, first note that the growth of wealth of any generation a household, conditional on the default decision and age transition, is given by

$$\frac{w_{t+1}^{k,j}}{w_t^a} = \frac{\Sigma_t^a R_{t+1}^{k,j}}{w_t^a} = \frac{B^a(\mathcal{Z}_t)R_{t+1}^{k,j}w_t^a}{w_t^a} = B^a(\mathcal{Z}_t)R_{t+1}^{k,j}, \tag{85}$$

for $j \in \{nd, d\}$ and $k \in \{a, a+\}$.

Thus we can construct the SDF of generation a as

$$\mathcal{M}_{t+1}^{k,j} = \beta\frac{(B^a(\mathcal{Z}_t)R_{t+1}^{k,j})^{-\gamma}v^k(\mathcal{Z}_{t+1})}{v^a(\mathcal{Z}_t)}. \tag{86}$$

We can assemble the SDF for assets that only pay off in the non-default state, namely housing and mortgages, as

$$M_{t+1}^{a,nd} = \pi^a\int_{\bar{\epsilon}_{t+1}^{a+}}^{\infty}\mathcal{M}_{t+1}^{a+,nd}(\epsilon)dF_{\epsilon,t+1}^a(\epsilon) + (1-\pi^a)\int_{\bar{\epsilon}_{t+1}^{a}}^{\infty}\mathcal{M}_{t+1}^{a,nd}(\epsilon)F_{\epsilon,t+1}^a(\epsilon), \tag{87}$$

and the SDF of defaulters is

$$M_{t+1}^{a,d} = \pi^a(1-\lambda)F_{\epsilon,t+1}^a(\bar{\epsilon}_{t+1}^{a+})\mathcal{M}_{t+1}^{a+,d} + (1-\pi^a)(1-\lambda)F_{\epsilon,t+1}^a(\bar{\epsilon}_{t+1}^{a})\mathcal{M}_{t+1}^{a,d}. \tag{88}$$

The SDF for discounting payoffs that do not depend on the default decision or the age transition (deposit) is

$$\mathcal{M}_{t+1}^a = \mathcal{M}_{t+1}^{a,nd} + \mathcal{M}_{t+1}^{a,d}. \tag{89}$$

We can also construct SDFs for discounting the age-specific assets that condition on the

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age transition status, but not on the default decision

$$\mathcal{M}^{ak}_{t+1} = (1-\pi^a)^{\mathbb{1}[k=a]}(\pi^a)^{\mathbb{1}[k=a+]}\left[\int_{\bar{\epsilon}^k_{t+1}}^{\infty} \mathcal{M}^{k,nd}_{t+1}(\epsilon)dF^a_{\epsilon,t+1}(\epsilon) + (1-\lambda)F^a_{\epsilon,t+1}(\bar{\epsilon}^k_{t+1})\mathcal{M}^{k,d}_{t+1}\right], \tag{90}$$

for $k \in \{a, a+\}$.

First-order conditions. The portfolio problem of the young is analogous to that of the middle-aged. Using the SDF definitions in (87), (89), and (90), the first-order conditions are

$$\frac{1}{1+r_t} - (1+\tau^m)\hat{m}^a_t q^a_d(\hat{\alpha}^a_t) = \frac{\psi}{v^a(\mathcal{Z}_t)}(\hat{d}^a_t B^a(\mathcal{Z}_t))^{-\gamma} + \beta \mathrm{E}_t\left[\mathcal{M}^a_{t+1}\right] \tag{91}$$

$$(1+\tau^m)q^a(\hat{\alpha}^a_t) + (1+\tau^m)\hat{m}^a_t q^a_m(\hat{\alpha}^a_t) = \beta \mathrm{E}_t\left[\mathcal{M}^{a,nd}_{t+1}\right] \tag{92}$$

$$p^h_t - \rho^a_t - (1+\tau^m)\hat{m}^a_t q^a_h(\hat{\alpha}^a_t) = \beta \mathrm{E}_t\left[\mathcal{M}^{a,nd}_{t+1}(1-\delta_H)p^h_{t+1}\right] \tag{93}$$

$$P^a_t - (1+\tau^m)\hat{m}^a_t q^a_b(\hat{\alpha}^a_t) = \beta \mathrm{E}_t\left[\mathcal{M}^{aa}_{t+1}(P^a_{t+1} + x^a_{t+1}) + M^{aa+}_{t+1}(P^{a+}_{t+1} + x^{a+}_{t+1})\right], \tag{94}$$

where we use the shorthand notation

$$q^a_\ell(\hat{\alpha}^a_t) = \frac{\partial q^a(\hat{\alpha}^a_t, \mathcal{Z}_t)}{\partial \ell}. \tag{95}$$

The derivatives of the mortgage pricing function are provided by equation (112) below. Note that equations (91)-(94) only characterize the relative portfolio shares of assets that households invest in. To fully characterize the complete savings and portfolio choice problem of the middle generation, we can reduce these equation to three excess return equations by first defining the effective returns to mortgage borrowing and housing as

$$R^a_{t+1,m} = \frac{1}{(1+\tau^m)q^a(\hat{\alpha}^a_t) + (1+\tau^m)\hat{m}^a_t q^a_m(\hat{\alpha}^a_t)}, \tag{96}$$

and

$$R^a_{t+1,h} = G\frac{(1-\delta_H)p^h_{t+1}}{p^h_t - \rho^a_t - (1+\tau^m)\hat{m}^a_t q^a_h(\hat{\alpha}^a_t)}. \tag{97}$$

Further, the effective return to deposits is

$$R^a_{t+1,d} = \frac{1+r_t}{1-(1+r_t)(1+\tau^m)\hat{m}^a_t q^a_d(\hat{\alpha}^a_t)}, \tag{98}$$

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and to the generation-specific assets for $k = \in \{a, a+\}$

$$R^k_{t+1,b} = \frac{[P^k_{t+1} + x^k_{t+1}]}{[P^a_t - (1+\tau^m)\hat{m}^a_t q^a_b(\hat{\alpha}^a_t)]}. \tag{99}$$

The resulting excess return restrictions are

$$0 = \frac{\psi}{v^a(\mathcal{Z}_t)}(\hat{d}^a_t B^a(\mathcal{Z}_t))^{-\gamma} R^a_{t+1,d} + \beta \mathrm{E}_t \left[\mathcal{M}^a_{t+1} R^a_{t+1,d} - \mathcal{M}^{a,nd}_{t+1} R^a_{t+1,m}\right] \tag{100}$$

$$0 = \mathrm{E}_t \left[\mathcal{M}^{a,nd}_{t+1}(R^a_{t+1,m} - R^a_{t+1,h})\right], \tag{101}$$

$$0 = \mathrm{E}_t \left[\mathcal{M}^{a,nd}_{t+1} R^a_{t+1,h} - \left(\mathcal{M}^{aa}_{t+1} R^a_{t+1,b} + \mathcal{M}^{aa+}_{t+1} R^{a+}_{t+1,b}\right)\right]. \tag{102}$$

Jointly with the optimal savings choice (71) and the recursive definition of the value function (73), these equations fully characterize the dynamic problem of the middle-generation.

Mortgage pricing function derivatives. To compute the effective returns on all assets, we need to calculate the derivative of the mortgage pricing function $q^Y(\hat{\alpha}^Y_t, \mathcal{Z}_t)$ with respect to the elements of $\hat{\alpha}^Y_t$. The first step is to differentiate the payoff functions (26) and (27) with respect to these portfolio choices. We first define the home equity per dollar of mortgage debt of the marginal defaulter after bankruptcy losses, conditional on $k \in \{a, a+\}$,

$$\hat{\epsilon}^k_{t+1} = \frac{(1-\xi)(1-\delta_H)P_{t+1}h^a_t \bar{\epsilon}^k_{t+1}}{m^a_t} - 1. \tag{103}$$

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Then we get

$$\frac{\partial \mathcal{P}^a_{t+1}}{\partial m^a_t} = \frac{\hat{f}^a_{\epsilon,t+1}}{(1-\delta_H)P_{t+1}h^a_t} - \hat{F}^a_{\epsilon,t+1}(1-\xi)(1-\delta_H)\frac{P_{t+1}h^a_t}{(m^a_t)^2}, \tag{104}$$

$$\frac{\partial \mathcal{P}^a_{t+1}}{\partial h^a_t} = (1-\delta_H)P_{t+1}\left[\hat{F}^a_{\epsilon,t+1}\frac{1-\xi}{m^a_t} - \frac{\hat{\hat{f}}^a_{\epsilon,t+1}}{(1-\delta_H)P_{t+1}h^a_t}\right], \tag{105}$$

$$\frac{\partial \mathcal{P}^a_{t+1}}{\partial d^a_t} = -\hat{f}^a_{\epsilon,t+1}\frac{\lambda}{(1-\delta_H)P_{t+1}h^a_t}, \tag{106}$$

$$\frac{\partial \mathcal{P}^a_{t+1}}{\partial b^a_t} = -\hat{\hat{\hat{f}}}^a_{\epsilon,t+1}\frac{\lambda}{(1-\delta_H)P_{t+1}h^a_t}, \tag{107}$$

where we use the auxiliary functions

$$\hat{f}^a_{\epsilon,t+1} = \pi^a f^a_{\epsilon,t+1}(\bar{\epsilon}^{a+}_{t+1})\hat{\epsilon}^{a+}_{t+1} + (1-\pi^a) f^a_{\epsilon,t+1}(\bar{\epsilon}^{a}_{t+1})\hat{\epsilon}^{a}_{t+1}, \tag{108}$$

$$\hat{F}^a_{\epsilon,t+1} = \pi^a \int_0^{\bar{\epsilon}^{a+}_{t+1}} \epsilon F^a_{\epsilon,t+1}(\epsilon) + (1-\pi^a)\int_0^{\bar{\epsilon}^{a}_{t+1}} \epsilon F^a_{\epsilon,t+1}(\epsilon), \tag{109}$$

$$\hat{\hat{f}}^a_{\epsilon,t+1} = \pi^a f^a_{\epsilon,t+1}(\bar{\epsilon}^{a+}_{t+1})\hat{\epsilon}^{a+}_{t+1}(\bar{\epsilon}^{a+}_{t+1})^2 + (1-\pi^a) f^a_{\epsilon,t+1}(\bar{\epsilon}^{a}_{t+1})\hat{\epsilon}^{a}_{t+1}(\bar{\epsilon}^{a}_{t+1})^2, \tag{110}$$

$$\hat{\hat{\hat{f}}}^a_{\epsilon,t+1} = \pi^a f^a_{\epsilon,t+1}(\bar{\epsilon}^{a+}_{t+1})\hat{\epsilon}^{a+}_{t+1}(P^{a+}_{t+1} + x^{a+}_{t+1}) + (1-\pi^a) f^a_{\epsilon,t+1}(\bar{\epsilon}^{a}_{t+1})\hat{\epsilon}^{a}_{t+1}(P^{a}_{t+1} + x^{a}_{t+1}). \tag{111}$$

For any argument ℓ of the mortgage pricing function q^a we have that

$$(1+\zeta)\frac{\partial q^a(\alpha^a_t, \mathcal{Z}_t)}{\partial \ell} = \mu^I_t \frac{\partial \mathcal{P}^a(\underline{z}_t)}{\partial \ell} + \mathrm{E}_t\left[\mathcal{M}^I_{t,t+1}\frac{\partial \mathcal{P}^a_{t+1}}{\partial \ell}\right]. \tag{112}$$

With the first order conditions above, this characterizes the portfolio choice problem.

B Model Solution (Internet Appendix)

This section describes the computational solution procedure.

B.1 Model Equations and State Variables

The model's equilibrium can be characterized using two types of functions: transition functions map today's state into probability distributions of tomorrow's state, and policy functions determine agents' decisions and prices given the current state. Brumm, Kryczka, and Kubler (2018) analyze theoretical existence properties in this class of mod-

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els and discuss the literature.

The endogenous state variables need to determine the wealth distribution among the types of optimizing agents: aggregate wealth of the young generation (W_t^Y), the combined aggregate wealth of middle-aged and old (W_t^{MO}), and the intermediary's equity (e_t^I).

The minimal set of aggregate state variables is thus $[W_t^Y, W_t^{MO}, e_t^I]$, and the complete vector of aggregate state variables is $\mathcal{S}_t = [Y_t, \sigma_{\epsilon,t}, W_t^Y, W_t^{MO}, e_t^I]$. Since (i) the wealth of all agents has to add up to aggregate tradable wealth, we only need to keep track of any two of the three endogenous state variables when computing the model, and (ii) W_t^{MO} is split among the middle-age and old generations on a fixed fraction (the age transition probability), using only $[W_t^M, e_t^I]$, we can compute beginning-of-period wealth of the young and old generations.

Functions. We can characterize the equilibrium as a system of 20 functional equations that we list below. The equilibrium objects to be computed, such as the agent's optimal choices, market-clearing prices, and multipliers for occasionally binding constraints, are functions of the model's aggregate state variables and represent a solution to the functional equations. The equations in this appendix are the ones including the LTV and DTI constraints explored in subsection 4.3.1 in the main text; for the baseline model, we set parameters on these constraints high enough such that they are never binding. The functions to be computed are as follows:

For the young generation:

E1. Consumption: $c^Y(\mathcal{S}_t)$

E2. Deposits: $d^Y(\mathcal{S}_t)$

E3. Housing: $h^Y(\mathcal{S}_t)$

E4. Mortgage Balance: $m^Y(\mathcal{S}_t)$

E5. Household's value function[35]: $v^Y(\mathcal{S}_t)$

For the middle-age generation:

E6. Consumption: $c^M(\mathcal{S}_t)$

E7. Deposits: $d^M(\mathcal{S}_t)$

[35]In Proposition 1, we defined $v^a(\mathcal{S}_t)$ as the value of a dollar of wealth.

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E8. Housing: $h^M(\mathcal{S}_t)$

E9. Mortgage Balance: $m^M(\mathcal{S}_t)$

E10. Household's value function: $v^M(\mathcal{S}_t)$

For the Financial Intermediary:

E11. Multiplier of the regulatory capital constraint: $\mu^I(\mathcal{S}_t)$

Market prices:

E12. Risk free interest rate: $R(\mathcal{S}_t)$

E13. Housing price: $P(\mathcal{S}_t)$

E14. Young generation endowment asset price: $p^Y(\mathcal{S}_t)$

E15. Middle-age generation endowment asset price: $p^M(\mathcal{S}_t)$

E16. Equity price: $q(\mathcal{S}_t)$.

Multipliers for the LTV and DTI constraints for the young and middle-age generations:

E17. Multiplier for the LTV constraint (young generation): $\lambda^Y_{LTV}(\mathcal{S}_t)$

E18. Multiplier for the LTV constraint (middle-age generation): $\lambda^M_{LTV}(\mathcal{S}_t)$

E19. Multiplier for the DTI constraint (young generation): $\lambda^Y_{DTI}(\mathcal{S}_t)$

E20. Multiplier for the DTI constraint (middle-age generation): $\lambda^M_{DTI}(\mathcal{S}_t)$

All other choice variables and model outcomes have explicit closed-form solutions given the state variables and these 20 functions.

Equations. In equilibrium, functions E1 to E20 must jointly satisfy equations (E1) to (E20) below at each point in the aggregate state space. The equations are intertemporal FOCs to the agents' optimization problems, complementary slackness conditions for constraints, and market-clearing conditions.

For the middle-aged and young generations, we have the FOCs for deposits, housing, mortgage balance, and generation specific assets. These equilibrium conditions are

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derived in Appendix A and correspond to equations (91), (92), (93), and (94). Furthermore, we also include the optimal savings policy for each generation (71), and per-dollar value functions (73). Finally, as stated above we include the equations for LTV and DTI constraints.

For the intermediary sector, we have banks' FOCs for deposits, and mortgage balances for both the middle-age generation and the young generation, these correspond to equations (82), (83), (84) derived in Appendix A. We also add the banks' regulatory constraint (77) and budget constraint (32) in subsection 2.5 of the main text.

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Middle-aged generation equations

For the middle-aged, we have the generation-specific returns on bank equity

$$R^M_{t+1,b^{M,I}} = \frac{p^I_{t+1} + \eta e_{t+1} - I_{t+1}}{p^I_t - (1+\tau^m)\hat{m}^M_t q^M_{M,I}(\hat{\alpha}^M_t)},$$

and middle-aged endowment shares

$$R^M_{t+1,b^{M,M}} = \frac{p^M_{t+1} + (1-\delta^M) y^M_{t+1}}{p^M_t - (1+\tau^m)\hat{m}^M_t q^M_{M,M}(\hat{\alpha}^M_t)}.$$

With these return definitions, the block of middle-aged equations is

$$\begin{aligned}
0 &= \beta \mathrm{E}_t \left[\mathcal{M}^M_{t+1} R^M_{t+1,b^{M,I}} - \mathcal{M}^{M,nd}_{t+1} R^M_{t+1,h} \right] \\
&\quad - \lambda^M_{t,LTV} \cdot \theta^M_{LTV} \cdot \frac{B^M(\mathcal{Z}_t)^{-\gamma}}{v^M(\mathcal{Z}_t)} \left(\frac{P_t}{P_t - \rho^M_t - (1+\tau^m)\hat{m}^M_t \cdot q^M_h(\alpha^M_t)} \right) && \text{(E1)} \\
0 &= \frac{\psi}{v^M(\mathcal{Z}_t)} (\hat{d}^M_t B^M(\mathcal{Z}_t))^{-\gamma} R^M_{t+1,d} + \beta \mathrm{E}_t \left[\mathcal{M}^M_{t+1} R^M_{t+1,d} - \mathcal{M}^M_{t+1} R^M_{t+1,b^{M,I}} \right] && \text{(E2)} \\
0 &= \beta \mathrm{E}_t \left[\mathcal{M}^{M,nd}_{t+1} R^M_{t+1,h} - \mathcal{M}^{M,nd}_{t+1} R^M_{t+1,m} \right] \\
&\quad + \lambda^M_{t,LTV} \cdot \frac{B^M(\mathcal{Z}_t)^{-\gamma}}{v^M(\mathcal{Z}_t)} \left[\theta^M_{LTV} \left(\frac{P_t}{P_t - \rho^M_t - (1+\tau^m)\hat{m}^M_t \cdot q^M_h(\alpha^M_t)} \right) - R^M_{t+1,m} \right] \\
&\quad - \lambda^M_{t,PTI} \cdot \frac{B^M(\mathcal{Z}_t)^{-\gamma}}{v^M(\mathcal{Z}_t)} \cdot R^M_{t+1,m} && \text{(E3)} \\
0 &= \beta \mathrm{E}_t \left[\mathcal{M}^M_{t+1} R^M_{t+1,b^{M,I}} - \mathcal{M}^M_{t+1} R^M_{t+1,b^{M,M}} \right] \\
&\quad - \lambda^M_{t,PTI} \cdot y^M_t \cdot \frac{B^M(\mathcal{Z}_t)^{-\gamma}}{v^M(\mathcal{Z}_t)} \left(\frac{1}{p^M_t - (1+\tau^m)\hat{m}^M_t \cdot q^M_{b^M}(\alpha^M_t)} \right) && \text{(E4)} \\
\Sigma^M_t &= B^M(\mathcal{Z}_t) \cdot W^M_t && \text{(E5)} \\
v^M(\mathcal{Z}_t) &= \Theta^M(\mathcal{Z}_t) \left(1 - B^M(\mathcal{Z}_t) \right)^{1-\gamma} + (1-\gamma) A^M(\mathcal{Z}_t) B^M(\mathcal{Z}_t)^{1-\gamma} && \text{(E6)}
\end{aligned}$$

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Young generation equations

For the young, we define the age-dependent return on endowment shares

$$R^M_{t+1,b^{Y,Y}} = \frac{p^Y_{t+1} + (1-\delta^M)y^M_{t+1}}{p^Y_t - (1+\tau^m)\hat{m}^Y_t q^Y_{Y,Y}(\hat{\alpha}^Y_t)},$$

$$R^M_{t+1,b^{Y,M}} = \frac{\frac{\delta^M}{\pi^Y} y^M_{t+1}}{p^Y_t - (1+\tau^m)\hat{m}^Y_t q^Y_{Y,Y}(\hat{\alpha}^Y_t)}.$$

Then the block of equations for the young is

$$0 = \frac{\psi}{v^Y(\mathcal{Z}_t)}(\hat{d}^Y_t B^Y(\mathcal{Z}_t))^{-\gamma} R^Y_{t+1,d} + \beta \mathrm{E}_t \left[\mathcal{M}^Y_{t+1} R^Y_{t+1,d} - \mathcal{M}^{Y,nd}_{t+1} R^Y_{t+1,m}\right]$$

$$- \left(\lambda^Y_{t,LTV} + \lambda^Y_{t,PTI}\right) \cdot \frac{B^Y(\mathcal{Z}_t)^{-\gamma}}{v^Y(\mathcal{Z}_t)} \cdot R^Y_{t+1,m} \tag{E7}$$

$$0 = \beta \mathrm{E}_t \left[\mathcal{M}^{Y,nd}_{t+1} R^Y_{t+1,h} - \mathcal{M}^{Y,nd}_{t+1} R^Y_{t+1,m}\right]$$

$$+ \lambda^Y_{t,LTV} \cdot \frac{B^Y(\mathcal{Z}_t)^{-\gamma}}{v^Y(\mathcal{Z}_t)} \left[\theta^Y_{LTV}\left(\frac{P_t}{P_t - \rho^Y_t - (1+\tau^m)\hat{m}^Y_t \cdot q^Y_h(\alpha^Y_t)}\right) - R^Y_{t+1,m}\right]$$

$$- \lambda^Y_{t,PTI} \cdot \frac{B^Y(\mathcal{Z}_t)^{-\gamma}}{v^Y(\mathcal{Z}_t)} \cdot R^Y_{t+1,m} \tag{E8}$$

$$0 = \beta \mathrm{E}_t \left[\mathcal{M}^{Y,nd}_{t+1} R^Y_{t+1,h} - \mathcal{M}^{YY}_{t+1} R^Y_{t+1,b^{Y,Y}} - \mathcal{M}^{YM+}_{t+1} R^{Y+}_{t+1,b^{Y,Y}}\right]$$

$$+ \lambda^Y_{t,LTV} \cdot \theta^Y_{LTV} \cdot \frac{B^Y(\mathcal{Z}_t)^{-\gamma}}{v^Y(\mathcal{Z}_t)} \left(\frac{P_t}{P_t - \rho^Y_t - (1+\tau^m)\hat{m}^Y_t \cdot q^Y_h(\alpha^Y_t)}\right)$$

$$- \lambda^Y_{t,PTI} \cdot y^Y_t \cdot \frac{B^Y(\mathcal{Z}_t)^{-\gamma}}{v^Y(\mathcal{Z}_t)} \left(\frac{1}{p^Y_t - (1+\tau^m)\hat{m}^Y_t \cdot q^Y_{b^M}(\alpha^Y_t)}\right) \tag{E9}$$

$$\Sigma^Y_t = B^Y(\mathcal{Z}_t) \cdot W^Y_t \tag{E10}$$

$$v^Y(\mathcal{Z}_t) = \Theta^Y(\mathcal{Z}_t)\left(1 - B^Y(\mathcal{Z}_t)\right)^{1-\gamma} + (1-\gamma)A^Y(\mathcal{Z}_t)B^Y(\mathcal{Z}_t)^{1-\gamma} \tag{E11}$$

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Financial Intermediaries

$$R_t = \mu_t^I + \mathrm{E}_t\left[\mathcal{M}_{t,t+1}^I\right], \tag{E12}$$

$$(1+\zeta)q^Y(\alpha_t^Y) = \mu_t^I \mathcal{P}^Y(\underline{z}_t) + \mathrm{E}_t\left[\mathcal{M}_{t,t+1}^I \mathcal{P}_{t+1}^Y\right], \tag{E13}$$

$$(1+\zeta)q^M(\alpha_t^M) = \mu_t^I \mathcal{P}^M(\underline{z}_t) + \mathrm{E}_t\left[\mathcal{M}_{t,t+1}^I \mathcal{P}_{t+1}^M\right], \tag{E14}$$

$$0 = \mu_t^I[(1-\bar{e}^Y)N_t^Y \mathcal{P}^Y(\underline{z}_t) + (1-\bar{e}^M)N_t^M \mathcal{P}^M(\underline{z}_t) - D_t], \tag{E15}$$

$$(1-\eta)e_t + I_t - C(I_t, \bar{Y}_t) + \frac{D_t}{1+r_t} = (1+\zeta)[N_t^Y q^Y(\alpha_t^Y) + N_t^M q^M(\alpha_t^M)]. \tag{E16}$$

Complementary Slackness for LTV and DTI

$$0 = \lambda_{t,LTV}^Y(\theta_{LTV}^Y P_t \hat{h}_t^Y - \hat{m}_t^Y) \tag{E17}$$

$$0 = \lambda_{t,DTI}^Y(\theta_{DTI}^Y y_t^Y \hat{b}_t^{Y,Y} - \hat{m}_t^Y) \tag{E18}$$

$$0 = \lambda_{t,LTV}^M(\theta_{LTV}^M P_t \hat{h}_t^M - \hat{m}_t^M) \tag{E19}$$

$$0 = \lambda_{t,DTI}^M(\theta_{DTI}^M y_t^M \hat{b}_t^{M,M} - \hat{m}_t^M) \tag{E20}$$

These equations represent the minimal set of conditions that define the economy's equilibrium. In other words, functions F1 to F20 are only implicitly defined by the system (E1) to (E20), and we need to solve for equilibrium by numerically finding the root of the system of equations.

Transitions. The rational expectations equilibrium requires that agents correctly forecast the law of motion of the aggregate state variables. The exogenous state variables $[Y_t, \sigma_{\epsilon,t}]$ follow a discrete Markov chain, with states and transition probabilities known to agents. The endogenous state variables evolve according to equations (T1) to (T3) listed below. (T1) and (T2) describe the evolution of aggregate wealth for the young and middle-age generations, these equations were derived in subsection 2.6 in the main text. (T3) refers to the intermediary's inside equity evolution found in subsection 2.5 in the main text.

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Aggregate Wealth for the young generation:

$$W^{Y}_{t+1} = \pi^{Y}(y^{Y}_{t+1} + p^{Y}_{t+1}) + \overrightarrow{W}^{Y,Y}_{t+1} \tag{T1}$$

where:

$$\begin{aligned}\overrightarrow{W}^{Y,Y}_{t+1} =& (1-\pi^{Y})\left(1 - F^{Y}_{\epsilon,t+1}(\bar{\epsilon}^{Y}_{t+1})\right)((1-\delta_H)\mathrm{E}_{t+1}[\epsilon^{Y}|\epsilon^{Y} > \bar{\epsilon}^{Y}_{t+1}]P_{t+1}H^{Y}_{t} - M^{Y}_{t}) \\ &+ (1-\pi^{Y})\left(1 - \lambda^{Y}F^{Y}_{\epsilon,t+1}(\bar{\epsilon}^{Y}_{t+1})\right)(D^{Y}_{t} + \mathbf{1}\cdot(P^{Y}_{t+1} + x^{Y}_{t+1})).\end{aligned}$$

Aggregate Wealth for the middle-age generation:

$$W^{MO}_{t+1} = \overrightarrow{W}^{Y,M}_{t+1} + \overrightarrow{W}^{M}_{t+1} \tag{T2}$$

where:

$$\begin{aligned}\overrightarrow{W}^{Y,M}_{t+1} =& \pi^{Y}\left(1 - F^{Y}_{\epsilon,t+1}(\bar{\epsilon}^{Y+}_{t+1})\right)((1-\delta_H)\mathrm{E}_{t+1}[\epsilon^{Y}|\epsilon^{Y} > \bar{\epsilon}^{Y+}_{t+1}]P_{t+1}H^{Y}_{t} - M^{Y}_{t}) \\ &+ \pi^{Y}\left(1 - \lambda^{Y}F^{Y}_{\epsilon,t+1}(\bar{\epsilon}^{Y+}_{t+1})\right)\left(D^{Y}_{t} + \mathbf{1}\cdot(P^{Y+}_{t+1} + x^{Y+}_{t+1})\right).\end{aligned}$$

$$\begin{aligned}\overrightarrow{W}^{M}_{t+1} =& \left(1 - F^{M}_{\epsilon,t+1}(\bar{\epsilon}^{M}_{t+1})\right)((1-\delta_H)\mathrm{E}_{t+1}[\epsilon^{M}|\epsilon^{M} > \bar{\epsilon}^{M}_{t+1}]P_{t+1}H^{M}_{t} - M^{M}_{t}) \\ &+ \left(1 - \lambda^{M}F^{M}_{\epsilon,t+1}(\bar{\epsilon}^{M}_{t+1})\right)\left(D^{M}_{t} + \mathbf{1}\cdot(P^{M}_{t+1} + x^{M}_{t+1})\right).\end{aligned}$$

Intermediary's equity:

$$e_{t+1} = N^{Y}_{t}\mathcal{P}^{Y}_{t+1} + N^{M}_{t}\mathcal{P}^{M}_{t+1} - D_t \tag{T3}$$

where $\mathcal{P}^{M}_{t+1}$ and $\mathcal{P}^{Y}_{t+1}$ can be found in equations (26) and (27), respectively.

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Table 4: Table of Notation

Variable	Definition	Section Introduced
Y_t	Aggregate output of non-durable consumption goods	Income and housing endowment
$\bar{H}, s_t$	Stock of housing capital, housing services produced	Income and housing endowment
δ_H	Fraction of housing value paid as maintenance each period	Income and housing endowment
π^Y, π^M	Probabilities that young and middle aged age each period	Agents
p_t^h, p_t^I, p_t^a	Prices of housing, intermediary equity, endowment shares	Markets
r_t, ρ_t^a	Risk-free rate, housing rental rate for generation a	Markets
$(d_t^Y, h_t^Y, b_t^{Y,Y}), (d_t^M, h_t^M, b_t^{M,M})$	Deposits, housing, endowment shares young/middle-aged	Markets
(m_t^Y, m_t^M)	Mortgage face values of young/middle-aged	Markets
$b_t^{M,I}$	Intermediary equity holdings of middle-aged	Markets
α_t^Y, α_t^M	Entire portfolios of young/middle-aged	Markets
$\mathcal{Z}_t$	All aggregate state variables	Markets
$q^a(\alpha_t^a, \mathcal{Z}_t)$	Amount lent to generation a per $ of mortgage face value	Markets
τ^m, ζ	Mortgage subsidy to household/processing cost to intermediary	Markets
P_t^a, b_t^a	Prices and quantities of assets available only to generation a	Markets
ϵ_t^a	Shock to generation a's housing at time t	Housing Risk Shocks
$\sigma_\epsilon^0, \sigma_\epsilon^1$	Low and high realizations of standard deviation of ϵ_t^a	Housing Risk Shocks
Γ	Markov transition matrix for standard deviation of ϵ_t^a	Housing Risk Shocks
λ, ξ	Fraction of household wealth and house value lost in default	Housing Risk Shocks
Y_t^*, ν	Total endowment payoff (defined below), fraction paid to young	Endowment income
δ_M	Fraction of middle-aged endowment payoff to new middle-aged	Endowment income
x_t^Y, x_t^{Y+}, x_t^M	Endowment payoff to young, young who age, middle-aged	Endowment income
$\beta, \gamma, \psi, \theta$	Discount factor, risk aversion, deposit preference, housing preference	Preferences and timing
b_t^O	Bequests of old generation	Preferences and timing
c_t^a, s_t^a	Consumption of non-durables/housing by generation a	Preferences and timing
w_t^a	Wealth of generation a after mortgage default decision	Preferences and timing
$V^a(w_t^a, \mathcal{Z}_t), V^O(w_t^O)$	Value function of generation $a \in \{Y, M\}$, value function of old	Preferences and timing
$w_t^{a,nd}, w_t^{a,d}, w_t^{a+,nd}, w_t^{a+,d}$	Wealth after no default (nd) or default (d), with + if aging	Recursive optimization problem
N_t^Y, N_t^M	Young/middle aged mortgage face values	Financial Intermediary
D_t	Deposits from intermediary	Financial Intermediary
e_t, η, I_t	Intermediary equity, fraction of equity paid as dividend	Financial Intermediary
I_t	New equity issued	Financial Intermediary
$C(I_t, \bar{Y}_t), \chi$	Equity issuance cost function, parameter determining cost	Financial Intermediary
$\mathcal{P}_t^Y, \mathcal{P}_t^M$	Payoff per unit of young/middle-aged mortgage portfolios	Financial Intermediary
$\mathcal{M}_{t,t+1}^M$	Stochastic discount factor of owners of intermediary equity	Financial Intermediary
$V^I(e_t, \mathcal{Z}_t)$	Intermediary's value function	Financial Intermediary
μ_t^I	Lagrange multiplier on risk-weighted capital requirement	Financial Intermediary
$\underline{z}_t$	Given $\mathcal{Z}_t$, realized $\mathcal{Z}_{t+1}$ with lowest value of intermediary assets	Financial Intermediary
Y_t^*, B_t^O	Total endowment income (defined here), total bequest payment	Aggregation and Equilibrium
$\Lambda_t^\xi, \Lambda_t^\lambda$	Total housing, weath loss from mortgage default	Aggregation and Equilibrium
$(M_t^a, H_t^a, S_t^a, D_t^a, B_t^{a,a}, C_t^a)$	Generation-specific aggregates of lower case variables	Aggregation and Equilibrium

This table reports a concise definition for each variable and the section in which it was introduced in more detail.

B.2 Numerical Solution Method

We solve the model numerically using policy function iteration Judd (1998). Our global nonlinear method, described in detail in Elenev, Landvoigt, and Van Nieuwerburgh (2020), allows us to compute a numerical solution to the economy's equilibrium with high accuracy. While a local method that approximates the equilibrium around the deterministic "steady state" would be simpler, it would not provide a reliable approximation to our model economy. First, portfolio restrictions such as banks' leverage constraints are only occasionally binding in the true stochastic equilibrium. Generally, a local approximation around the steady state (with a binding or slack constraint) will inaccurately capture nonlinear dynamics when constraints go from slack to binding. Further, local methods have difficulties in dealing with highly nonlinear functions within the model such as probability distributions or option-like payoffs, as is the case for the quantitative model in this paper. Finally, in models with rarely occurring bad shocks (such as the financial recessions in our model), the steady state used by local methods may not properly capture the ergodic distribution of the true dynamic equilibrium due to precautionary motives and risk premia.

Global projection methods avoid these problems by not relying on the deterministic steady state. Rather, they directly approximate the transition and policy functions in the relevant area of the state space.

B.3 Solution Procedure

The projection-based solution approach used in this paper has three main steps.

Step 1. Define approximating basis for the policy and transition functions. To approximate these unknown functions, we discretize the state space and use multivariate linear interpolation. Our solution method requires approximation of three sets of functions defined on the domain of the state variables. The first set, the "policy" functions, determine the values of endogenous objects specified in the equilibrium definition at every point in the state space. These are the functions F1 to F20 listed in Section B.1. The second set, the "transition" functions, determine the next-period endogenous state variable realizations as a function of the state in the current period and the next-period realization of exogenous shocks, corresponding to transition laws (T1) to (T3) in Section B.1. The third set are "forecasting functions", which map the state into variables sufficient to compute expectations terms in the nonlinear functional equations that characterize equilibrium. They partially coincide with

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the policy functions, but contain some additional information, for example, the recursive utility of each agent.

Step 2. **Iteratively solve for the unknown functions.** Given an initial guess for policy and transition functions, at each point in the discretized state space compute the current-period optimal policies. Using the solutions, compute the next iterate of the transition functions. Repeat until convergence. The system of nonlinear equations at each point in the state space is solved using a standard nonlinear equation solver. Kuhn-Tucker conditions can be rewritten as equality constraints for this purpose. This step is completely parallelized across points in the state space within each iterate. The sub-steps are as follows:

A. **Initialize** the algorithm by specifying a guess for the policy and transition functions.

B. **Compute forecasting values.** For each point in the discretized state space, perform the following steps:

i. Evaluate the transition functions at each possible realization of the aggregate state combined with each possible realization of the exogenous shocks.

ii. Evaluate the forecasting functions at these future state variable realizations.

The end result is a matrix, with each entry being a vector of the next-period realization of the forecasting functions for each possible combination of the current state and the next-period exogenous state.

C. **Solve system of nonlinear equations.** At each point in the discretized state space, solve the system of nonlinear equations that characterize equilibrium in the equally many "policy" variables, given the forecasting matrix from step B. This amounts to solving a nonlinear system of 20 equations in 20 unknowns at each point in the state space, with the unknowns being the function values for F1 to F20 and the equations given by (E1) to (E20).

Expectations are computed as weighted sums, with the weights being the conditional transition probabilities of the exogenous states. The expressions in expectations generally depend on the forecasting matrix, which we pre-computed in step B.

To solve the system in practice, we use a nonlinear equation solver that relies on a variant of Newton's method, using policy functions from the last iteration

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as the initial guess. See section B.4 below for further details.

The final output of this step is a matrix, where each row is the solution vector that solves the system (E1) to (E20) at a specific point in the discretized state space. This is the numerical representations of functions F1 to F20.

D. Update forecasting, transition, and policy functions. Given the policy matrix from step C, update the policy and forecasting functions.

Finally, updating transition functions for the endogenous state variables according to (T1) to (T3) gives the complete set of functions for the next iteration.

E. Check convergence. Compute a distance measure on the forecasting, policy and/or transition function between current and previous iterate. If the distance is below the convergence threshold, stop and use the current functions as approximate solution. Otherwise reset all functions to the current iterate and go to step B.

Step 3. Simulate the model for many periods using approximated functions. Verify that the simulated time path stays within the bounds of the state space for which policy and transition functions were computed. Calculate relative Euler equation errors to assess the computational accuracy of the solution. If the simulated time path leaves the state space boundaries or errors in the transition functions (T1)-(T3) are too large, the solution procedure may have to be repeated with optimized grid bounds or positioning of grid points.

B.4 Implementation

Solving the system of equations. We solve the system of nonlinear equations at each point in the state space using a standard nonlinear equation solver (MATLAB's fsolve). This nonlinear equation solver uses a variant of Newton's method to find a "zero" of the system. We employ several simple modifications of the system (E1) to (E20) to avoid common pitfalls at this step of the solution procedure. Nonlinear equation solvers are notoriously bad at addressing complementary slackness conditions associated with a constraint. Judd, Kubler, and Schmedders (2002) discuss the reasons for this and also show how Kuhn-Tucker conditions can be rewritten as additive equations for this purpose.

Similarly, certain solution variables are restricted to positive values due to the economic structure of the problem. For example, given the utility function, optimal consumption is always strictly positive. To avoid the solver trying out negative consumption values (and thus output becoming ill-defined), we use $\log(c_t^a)$ as the solution variable for

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the solver. This means that the solver can make consumption arbitrarily small but not negative.

Grid Configuration. The three endogenous state variables of the model need to reflect the distribution of wealth across generations of households and the financial intermediary. There are several different ways of encoding this information in three state variables. To ease the computational burden, we use the fact that market clearing implies an adding-up constraint for financial wealth:

$$(1-\delta_H)P_t\bar{H} + B_t^O + Y_t^* = \tilde{W}_t^{M,O} + \tilde{W}_t^Y + e_t,$$

where $\tilde{W}_t^{M,O} = W_t^{M,O} - p_t^M - (p_t^I + \eta e_t - I_t)$ and $\tilde{W}_t^Y = W_t^Y - p_t^Y$ are the wealth of the combined middle-aged/old and young generations, net of generation-specific assets, respectively. This adding-up constraint allows us to keep track of only two out of three endogenous state variables. We choose middle-aged/old wealth $\tilde{W}_t^{M,O}$ and intermediary equity e_t. For the benchmark case (the model without LTV or DTI constraints), the grid points in each state dimension are as follows

- Y**:** We discretize Y into a five-state Markov chain using the Rouwenhorst (1995a) method. The procedure chooses the productivity grid points $\{Y\}_{j=1}^3$, the 3×3 Markov transition matrix Π_Y between them to match the volatility and persistence of GDP growth. This yields the possible realizations for Y: $[0.9621, 0.99954, 1.0384]$.
- σ_ω**:** $[0.19, 0.32]$ (see calibration)
- $\tilde{W}^{M,O}$**:** $[1.95, 2.00, 2.05, 2.10, 2.12, 2.14, 2.16, 2.18, 2.20, 2.25, 2.30, 2.40]$
- e**:** $[0.035, 0.0456, 0.0563, 0.0669, 0.0775, 0.0881, 0.0988, 0.1094, 0.1200]$

The total state-space grid has 648 points. The boundaries and placement were readjusted for each experiment, since the ergodic distribution of the state variables depends on parameters. Finding the right values for grid boundaries and points is a matter of experimentation.

Generating an Initial Guess and Iteration Scheme. To find a good initial guess for the policy, forecasting, and transition functions, we solve the deterministic "steady-state" of the model under the assumption that the bank leverage constraint is binding, housing risk is low, and the aggregate output is at its mean. We then initialize all functions to their

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steady-state values, for all points in the state space. Note that the only role of the steady-state calculation is to generate an initial guess that enables the nonlinear equation solver to find solutions at (almost) all points during the first iteration of the solution algorithm. In our experience, this steady state delivers a good enough initial guess.

If the solver cannot find solutions for some points during the initial iterations, we revisit such points at the end of each iteration. We try to solve the system at these "failed" points using as the initial guess the solution of the closest neighboring point at which the solver was successful. This method works well to speed up convergence and eventually finds solutions at all points.

To determine convergence, we check absolute errors in the value functions of households. Out of all functions we approximate during the solution procedure, these exhibit the slowest convergence. We stop the solution algorithm when the mean absolute difference between two iterations, and for all points in the state space, falls below 1e-4.

In some cases, our grid boundaries are wider than necessary, in the sense that the simulated economy never visits the areas near the boundary on its equilibrium path. Local convergence in those areas is usually very slow, but not relevant for the equilibrium path of the economy. If the algorithm has not achieved convergence after 150 iterations, we nonetheless stop the procedure and simulate the economy. If the resulting simulation produces low relative errors (see step 3 of the solution procedure), we accept the solution. After the 150 iterations, our simulated model economies either achieve acceptable accuracy in relative errors, or not, in which case the cause is a badly configured state grid. In the latter case, we need to improve the grid and restart the solution procedure. Additional iterations beyond 150 do not change any statistics of the simulated equilibrium path for any of the simulations we report.

We implement the algorithm in MATLAB and run the code on a high-performance computing (HPC) cluster. As mentioned above, the nonlinear system of equations can be solved in parallel at each point. We parallelize across 16 CPU cores of a single HPC node. From computing the initial guess to simulating the solved model, the total running time for the benchmark calibration is about 45 minutes.

Simulation. To obtain the quantitative results, we simulate the model for 10,000 periods after a "burn-in" phase of 1,000 periods. The starting point of the simulation is the ergodic mean of the state variables. We fix the seed of the random number generator so that we use the same sequence of exogenous shock realizations for each parameter combination.

To produce impulse-response function (IRF) graphs, such as the ones in Figure 4, we

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simulate 10,000 different paths of 25 periods each. In the initial period, we set the endogenous state variables to several different values that reflect the ergodic distribution of the states. We use a clustering algorithm to represent the ergodic distribution nonparametrically. We fix the initial exogenous shock realization to mean productivity ($Y = 0.9995$) and low housing risk ($\sigma_\epsilon = 0.19$). The "impulse" in the second period is either a bad endowment shock only, or both a low endowment and a housing risk shock ($\sigma_\epsilon = 0.19$). For the remaining 23 periods, the simulation evolves according to the stochastic law of motion of the shocks. In the IRF graphs, we plot the median path across the 10,000 paths given the initial condition.

The transition dynamics for the boom-bust simulations, such as in Figures 6 and 7, are constructed similarly, with the difference that the economy does not experience an income or housing risk shock, but rather unanticipated changes in several parameters as described in Section 4.3.

Evaluating the Solution. To assess the quality and accuracy of the solution, we perform two types of checks. First, we verify that all state variable realizations along the simulated path are within the bounds of the state variable grids defined in step 1. If the simulation exceeds the grid boundaries, we expand the grid bounds in the violated dimensions and restart the procedure at step 1. Second, we compute relative errors for all equations of the system (E1) to (E20) and transition functions (T1) to (T3) along the simulated path. For equations involving expectations, this requires evaluating the transition and forecasting function as in step 2B at the current state. For each equation, we divide both sides by a sensibly chosen endogenous quantity to yield "relative" errors to make the scale of the errors economically meaningful and comparable across equations. In practice, this means that we divide both sides of each equation to normalize either the right-hand side or the left-hand side to one.

Table 5 reports the median error, the 95^{th} percentile of the error distribution, the 99^{th}, and 100^{th} percentiles during the 10,000-period simulation of the model. Median errors are very small for all equations, with even maximum errors only causing small approximation mistakes. The values reported in table 5 are for the model without LTV and DTI constraints; thus the table does not include (E17)-(E20). Errors are comparably small for all experiments we report.

These errors are small by construction when calculated at the points of the discretized state grid, since the algorithm under step 2 solved the system exactly at those points. However, the simulated path will likely visit many points that are between grid points,

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Table 5: Computational Errors for Benchmark

Equation	Percentile				
	50th	75th	95th	99th	Max
E1	0.001102	0.001144	0.001254	0.001388	0.001420
E2	0.000974	0.000997	0.001050	0.001074	0.001081
E3	0.000206	0.000306	0.000363	0.000585	0.000676
E4	0.001014	0.001030	0.001088	0.001120	0.001120
E5	1.64E-06	2.67E-06	2.97E-06	3.04E-06	6.39E-06
E6	4.63E-09	6.12E-09	8.35E-09	8.73E-09	8.89E-09
E7	0.001355	0.001705	0.006299	0.008633	0.010285
E8	0.000310	0.000591	0.002813	0.003833	0.004763
E9	0.001038	0.001325	0.003489	0.004854	0.005807
E10	7.41E-07	1.21E-06	1.35E-06	1.42E-06	2.94E-06
E11	6.79E-10	6.79E-10	6.79E-10	6.79E-10	6.79E-10
E12	0.004789	0.005896	0.035607	0.050217	0.055563
E13	0.004267	0.005540	0.033151	0.046789	0.052086
E14	0.005035	0.005987	0.036017	0.050757	0.056075
E15	0.000120	0.000151	0.001533	0.004178	0.005924
E16	1.80E-05	2.89E-05	3.22E-05	3.52E-05	7.30E-05

The table reports the median error and the 75^{th}, 95^{th}, 99^{th}, and 100^{th} percentiles of the error distribution during the 5,000-period model simulation. Each row corresponds to the equation with the same label in Appendix B.1.

at which the equilibrium functions are approximated by interpolation. Therefore, the relative errors indicate the quality of the approximation in the relevant area of the state space. We report average, median, and tail errors for all equations. If errors are too large during simulation, we investigate in which part of the state space these high errors occur. We then add additional points to the state variable grids in those areas and repeat the procedure.

C Data and Calibration

C.1 Data Sources and Construction

C.1.1 Table 1: Pre-set Parameters

1.-3. Source: Real Disposable Personal Income: Per Capita, Chained 2012 Dollars, Annual, Not Seasonally Adjusted, 1929-2017

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1. Average growth rate
2. Standard deviation of cycle component after Hodrick-Prescott filtering with parameter 10
3. First-order autocorrelation of cycle component after Hodrick-Prescott filtering with parameter 10

8. Source: 1998 Survey of Consumer Finances Extract Data. Income share of of households with age 26-45 computed based on pre-defined variable "income" in extract sample, using SCF sampling weights.

9. Source: U.S. Bureau of Economic Analysis, National Income and Product Accounts

   - Table 5.4. Current-Cost Depreciation of Residential Fixed Assets by Type of Owner, Legal Form of Organization, and Tenure Group, 1990-2017
   - Table 5.1. Current-Cost Net Stock of Residential Fixed Assets by Type of Owner, Legal Form of Organization, and Tenure Group, 1990-2017
   - Depreciation Rate calculated as depreciation of private residential fixed assets divided by stock of private residential fixed assets

15. Source: CRSP-Compustat linked data set of publicly listed companies engaging in "depository credit intermediation" (NAICS codes beginning with 5221), see Elenev, Landvoigt, and Van Nieuwerburgh (2020), Appendix D.1 for details. Dividend ratio computed as sum of dividends and share repurchases divided by book equity.

C.1.2 Table 2: Calibrated Parameters

1.-2. Source: Federal Reserve Board, Charge-off rate on single family residential mortgages, booked in domestic offices; All commercial banks (Seasonally adjusted),1991-2017

   1. Average
   2. Average, 2008-2012

3. Real interest rate computed as treasury yield net of inflation rate, both for 1998

   - Source for yield: 1-year Treasury Bill, Secondary Market Rate
   - Source for inflation rate: Consumer Price Index for All Urban Consumers: All Items, Percent Change, Annual, Not Seasonally Adjusted

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4. Source: 1998 Survey of Consumer Finances Extract Data. Housing wealth to income ratio computed as aggregate value of "houses" divided by aggregate value of "income".

6.,8. Source: 1998 Survey of Consumer Finances Extract Data. Loan to value ratio computed as aggregate "mrthel" divided by aggregate "houses", for young (ages 26-45) middle-aged (ages 46-65) respectively.

9. Prime spread computed as difference of 30-year fixed prime mortgage rate of 10-year treasury bond yield, 1990-2007.

   - Source for mortgage rate: Freddie Mac, 30-Year Fixed Rate Mortgage Average in the United States
   - Source for yield: 10-Year Treasury Constant Maturity Rate

C.1.3 Table 3: Model Fit

- Aggregate mortgage debt [1,2]: Board of Governors of the Federal Reserve System (US), Z.1 Financial Accounts of the United States, All Sectors; Total Mortgages; Asset, Level [BOGZ1FL893065005A]
- Aggregate consumption [1,8]; U.S. Bureau of Economic Analysis, Personal Consumption Expenditures [PCE]
- Housing Wealth [2]: Board of Governors of the Federal Reserve System (US), Z.1 Financial Accounts of the United States, Households and Nonprofit Organizations; Real Estate at Market Value, Market Value Levels [BOGZ1LM155035005A]
- Delinquency rate [4]: Board of Governors of the Federal Reserve System (US), Delinquency Rate on Loans Secured by Real Estate, All Commercial Banks [DRSREACBS], 1987-2020
- Charge-off rate [5]: Board of Governors of the Federal Reserve System (US), Charge-Off Rate on Loans Secured by Real Estate, All Commercial Banks [CORSREACBS], 1985-2020
- Prime mortgage rate [6]: Freddie Mac, 30-Year Fixed Rate Mortgage Average in the United States [MORTGAGE30US], 1990-2020
- Treasury yield [6,7]: Board of Governors of the Federal Reserve System (US), 10-Year Treasury Constant Maturity Rate [DGS10], 1990-2020

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- Subprime mortgage rate [7]: Average interest rate at origination for mortgages packaged in private-label MBS, 1998-2007, see Section C.1.4.
- Aggregate deposits [8]: sum of household checking deposits, saving deposits, and money market mutual fund shares, all from Board of Governors of the Federal Reserve System (US), Z.1 Financial Accounts of the United States
  - Checking: Households and Nonprofit Organizations; Checkable Deposits and Currency; Asset, Level [HNOCDCA027N]
  - Saving: Households and Nonprofit Organizations; Total Time and Savings Deposits; Asset, Level [HNOTSDA027N]
  - MMMF: Money Market Funds; Total Financial Assets, Level [MMMFFAA027N]
- Financial sector equity [9]: (assets - liabilities)/assets of a set of financial institutions, all from Board of Governors of the Federal Reserve System (US), Z.1 Financial Accounts of the United States; see details below.
- Financial sector net payout [10]: CRSP-Compustat linked data set of publicly listed companies engaging in "depository credit intermediation" (NAICS codes beginning with 5221), 1974-2018, see Elenev, Landvoigt, and Van Nieuwerburgh (2020), Appendix D.1 for details. Net payout ratio computed as sum of dividends and share repurchases minus issuances divided by book equity.
- Deposit rate [11]: Aggregate time series of average annual deposit rates constructed by Dreschler, Savov, and Schnabl (2017) using U.S. Call Reports data, 1986-2013.
- Liquidity premium [12]: Spread between yield on AAA bonds and Treasuries from Krishnamurthy and Vissing-Jorgensen (2012), 1945-2018.
- Consumption young and middle-aged [13,14,15]: From Consumer Expenditure Survey, 1984-2019. Non-durable consumption is defined as average total expenditure age minus average housing expenditure for each age group. Young are ages 25-44 while middle-aged are ages 45 to 64.
- Housing young and middle-aged [16]: 1998 Survey of Consumer Finances Extract Data. Ratio of middle-aged over young housing computed as aggregate value "houses" for 26-45 year old households divided by aggregate value of "houses" for 46-65 year old households, using SCF sampling weights.

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Financial Sector Definition for Equity Ratio. Our notion of the intermediary sector is the levered financial sector. We take book values of assets and liabilities of these sectors from the Financial Accounts of the United States (formerly Flow of Funds). We subtract holding and funding company equity investments in subsidiaries from those subsidiaries' liabilities. Below we list the table codes and sector names from the Financial Accounts that we include in the measure.

Table	Sector
L.111	U.S.-Chartered Depository Institutions
L.112	Foreign Banking Offices in U.S.
L.113	Banks in U.S.-Affiliated Areas
L.114	Credit Unions
L.125	Government-Sponsored Enterprises (GSEs)
L.126	Agency- and GSE-Backed Mortgage Pools
L.127	Issuers of Asset-Backed Securities (ABS)
L.128	Finance Companies

C.1.4 Figure 3: Credit Surface

In this section, we explain the calculations for the empirical credit surface in Figure 3. First, we briefly describe the data, then how we computed the credit surface.

Dataset. The original data set contains 35,733,974 mortgage originations from 1995-2019. We eliminate some observations based on the following criteria:

1. We use Combined Loan-To-Value (CLTV) data for our computations (for simplicity we will use the word "LTV" throughout this section). We eliminate observations with LTV ratios lower than 40% and bigger than 115%.
2. We drop observations with FICO Score less than 600.
3. We eliminate observations with origination balance bigger than 1M USD.
4. The data set distinguishes between type of mortgages rates: Fixed-Rate Mortgage, an Adjustable-Rate Mortgage, or a Hybrid-Rate Mortgage. We eliminate observations that have an unknown mortgage rate type.
5. The data distinguishes between the credit risk of the loan: Alt-A, Subprime, Prime. We eliminate observations that have an unknown credit risk.

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6. The data set distinguishes between Occupancy Type: Primary Residence, Secondary Residence, Investment. We eliminate observations that have an unknown occupancy type.

After these selections, we are left with 27,647,746 observations. Finally, since we want to understand credit pricing during the boom period, we eliminate observations before 1998 and after 2007. This leaves us with 17,253,298 observations.

Regression and Estimation of the Credit Surface. We generate 10 bins for the LTV variable and 7 bins for the FICO Score variable listed in Table 6.

Table 6: LTV and FICO Bin Definitions

Bin Number	LTV interval
1	[40,50)
2	[50,60)
3	[60,70)
4	[70,75)
5	[75,80)
6	[80,85)
7	[85,90)
8	[90,95)
9	[95,100)
10	[100,115]

Bin Number	FICO interval
1	[600,620)
2	[620,660)
3	[660,700)
4	[700,740)
5	[740,780)
6	[780,820)
7	[820,850]

To estimate the credit surface, we then run the following regression:

$$i_k = \sum_{i=1}^{n^{LTV}} \alpha_i \cdot \mathbf{1}\{LTV_k \in LTV_i^{BIN}\} + \sum_{j=1}^{n^{FICO}} \beta_j \cdot \mathbf{1}\{FICO_k \in FICO_j^{BIN}\}$$
$$+ \sum_{i}^{n^{LTV}} \sum_{j}^{n^{FICO}} \gamma_{i,j} \cdot \mathbf{1}\{LTV_k \in LTV_i^{BIN}\} \cdot \mathbf{1}\{FICO_k \in FICO_j^{BIN}\} + \zeta \cdot Control_k + \epsilon_k \quad (113)$$

Subscript k represents the observations. Subscript i is used here to denote each of the LTV bins, and subscript j is used to represent each of the FICO Score bins described above.

i_k represents the interest rate for loan k. LTV_k represents the LTV at origination observed for loan k, and $\mathbf{1}\{LTV_k \in LTV_i^{BIN}\}$ is an indicator function that equals 1 when LTV_k falls in the i^{th} LTV bin, denoted by LTV_i^{BIN}. Finally, n^{LTV} represents the number of

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LTV bins, in our case $n^{LTV} = 10$. The same notation and logic is true for the FICO Score term.

Notice that the interaction term $\mathbf{1}\{LTV_k \in LTV_i^{BIN}\} \cdot \mathbf{1}\{FICO_k \in FICO_j^{BIN}\}$ is key to our computation for the credit surface. This will equal 1 when *both* the LTV observed for loan k falls inside the i^{th} LTV bin, and the observed FICO Score for loan k falls inside the j^{th} FICO Score bin, denoted as $FICO_j^{BIN}$.

ζ denotes a vector of coefficients, and $Control_k$ is a set of control variables, each one related to its respective element in ζ. The set of controls we include are:

1. The credit risk of the loan. Each loan k can be categorized as Subprime, Alt-A, or Prime.
2. The type of interest at origination. Each loan k can be either a Fixed-Rate Mortgage, an Adjustable-Rate Mortgage, or a Hybrid-Rate Mortgage.
3. The year of origination. The years considered are between 1998 and 2007.
4. If the loan was issued by a "big" originator. A big originator in our analysis is one that issues more than 300,000 mortgages during these years.
5. Dummy variables for specific LTV ratios. Our measure for the Combined LTV exhibits large number of originations at certain values. For instance, a Conforming loan needs to have an LTV of 80% or less, not surprisingly our data shows a large number of originations at that specific value. We add dummies for loans that display LTV ratios of 70, 75, 80, 85, 90, 95, or 100 percent.

While these controls end up allowing to get a clean partial effect for credit pricing, the shape of the estimated surface is not overly sensitive to inclusion of any of the controls.

Computation of the Credit Surface. The idea is to compute *an average* interest rate for each pair of $(LTV_i^{BIN}, FICO_j^{BIN})$. In particular, after estimating equation (113), the interest rate for the (i^{th}, j^{th}) pair can be computed as:

$$\text{interest rate}_{i,j} = \alpha_i + \beta_j + \gamma_{i,j}$$

To obtain the final plot we interpolate these values using the *surf* function in Matlab. To make the rates comparable to the model interest rates and isolate the role of credit risk in spreads, we further subtract the average term spread, computed as the difference

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between the 10-year and 3-month treasury rates (4.81% for 1998-2007), and the average prime spread (1.51%).

C.2 Parameter Sensitivity Analysis

This section presents a sensitivity analysis of how the moments implied by our model vary with local changes in the model's parameters. Let $\boldsymbol{m}$ be the vector of moments presented in table 2 that we used to jointly calibrate the vector of parameters $\boldsymbol{\theta^b}$. Let $\boldsymbol{\iota}_i$ be a selector vector of the same length as $\boldsymbol{\theta}$ taking a value of 1 in the i'th position and zero elsewhere. Denote the parameter choices in the benchmark calibration by a superscript b. For each parameter θ_i, we solve the model once for $\theta^b \circ e^{\iota_i \varepsilon}$ and once for $\theta^b \circ e^{-\iota_i \varepsilon}$. We then report the symmetric finite difference:

$$\frac{\log\left(\boldsymbol{m}(\boldsymbol{\theta^b} \circ \boldsymbol{e^{\iota_i \varepsilon}})\right) - \log\left(\boldsymbol{m}(\boldsymbol{\theta^b} \circ \boldsymbol{e^{\iota_i \varepsilon}})\right)}{2\varepsilon}$$

We set $\varepsilon = 0.01$, or 1% of the benchmark parameter value. The resulting quantities are elasticities of moments to structural parameters- the percentage change in each moment from a one percentage change in each parameter.

The majority of parameters have a larger impact on their target moment than on other moments. The cost χ of issuing equity primarily impacts net payout. The cost ζ of issuing a mortgage primarily impacts the mortgage spread of the middle aged (whose mortgages are very low risk). The tax subsidy τ^m to a housing receiving a new mortgage primarily impacts the total amount of mortgage tax benefits paid out. The utility benefit ψ of holding deposits primarily impacts the convenience yield of deposits. The standard deviations of idiosyncratic housing shocks $\sigma_\epsilon^0, \sigma_\epsilon^1$ in the low and high risk states have considerably large impacts on loss/charge-off rates than on other moments. However, these two parameters significantly impact each others' target moments. Increasing the risk σ_ϵ^1 in the high housing risk state increases both crisis-specific and overall loss rates, since a significant share of losses happen during a crisis.[36] Increasing the risk the σ_ϵ^0 in the low risk state induces young households (who cause most defaults) to reduce their leverage and reduces losses in a crisis. The preference parameter θ for housing has its primary impact on house prices and a secondary impact on deposit rates and convenience yields (since the market value of the housing stock determines how much collateral is available to back

[36] To see this, note in table 2 that average losses are 3.38/.88=3.84 times larger in the high housing risk state. Based on our Markov transition matrix (5 perccent chance of entering a crisis and 20 percent chance of leaving a crisis each year), the economy spends 20 percent of its time in a housing recession.

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mortgages and deposits).

Some parameters have significant impacts on multiple moments. The discount factor β, which has an important impact on all intertemporal borrowing and investing decisions, impacts mortgage loss rates, deposit rates and house pirces, and the total tax benefit paid out (which is proportional to aggregate mortage issuance). The share δ^M of middle-aged income paid to those that have just aged from being young determines the amount of wealth the middle-aged currently need to invest in non-endowment assets. Increasing this reduces deposit rates and increases aggregate tax benefits (by increasing total mortgage issuance). An increase in the fraction λ of wealth lost in a mortgage default increases mortgage leverage for both generations and reduces mortgage loss rates. This is because a higher λ makes default more costly and therefore increases a borrower's ability to commit to repay its mortgage, resulting in lower defaults and greater credit supply ex ante.

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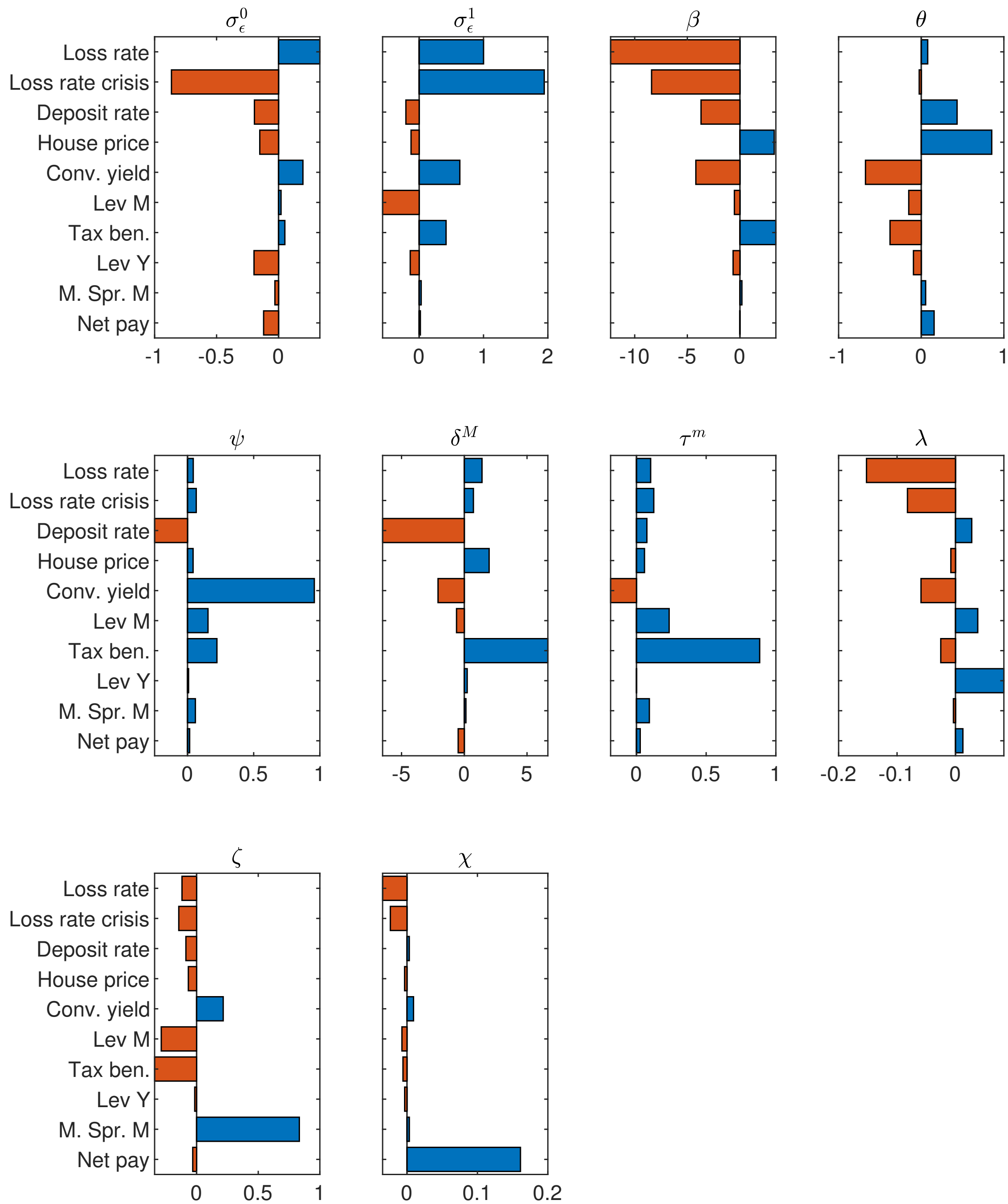
Figure 10: Parameter Sensitivity Analysis



Plots show elasticities of model moments used in the calibration with respect to all parameters calibrated jointly (Table 2). Each bar shows percentage change in model-implied moment given a marginal 1% change in the parameter.

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