Non-Sovereign Stores of Value

Urban J. Jermann*

Wharton School of the University of Pennsylvania and NBER

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Abstract

Investors buy non-sovereign stores of value such as gold and bitcoin despite the absence of a yield. This paper presents an equilibrium model for studying investor adoption and the pricing of non-sovereign stores of value. The model is used for the quantitative analysis of historical gold prices and real interest rates. Since 1975, the real price of gold has been negatively related to real rates on Treasuries, but only when real rates have been low. The model is consistent with this nonlinear relation and can match quantitative properties relating real interest rates and gold prices. The model can also replicate some key properties of CME Comex gold futures prices.

Keywords: Gold, Real interest rates, Bitcoin, Futures. JEL codes: G12, G13.

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1 Introduction

Gold and bitcoin are assets that are not liabilities of anybody and are not backed by a government. A key question is how these assets should be valued. Gold has some utility as jewelry or as a conductor of electricity. Bitcoin is efficient for transferring value outside traditional financial systems. The present discounted values of these service flows represent the values of these assets to their users.

These asset are also held by investors who do not capture the utility of somebody enjoying jewelry or making payments outside traditional financial systems. From an investor's perspective neither gold nor bitcoin has a yield and the return is just the change in the price. To justify an investment, such a store of value, SOV, needs to offer a high enough expected price change relative to the returns on traditional assets or it needs attractive hedging properties.

During the 2000s, the emergence of gold ETFs lowered the costs for retail investors to trade and hold gold. It has been suggested that this additional investor adoption has contributed to a run-up in the price of gold. The recent development of enterprise-grade custody offerings has preceded the adoption of bitcoin by traditional investors. For instance, Massachusetts Mutual Life Insurance Co. recently bought \$100 million of bitcoin for its general investment account (WSJ, 2020). It is widely thought that this investor adoption has been one of the main drivers of the increase in the price of bitcoin during 2020 Q4 and 2021 Q1.

This paper presents an equilibrium model to jointly study the pricing of SOV assets and investor adoption. The model is based on the Lucas (1978) asset pricing framework. Relative to the more common representative agent structure, my model features two types of agents. There are users who derive some utility or productive use from the SOV commodity. From their perspective, the asset has a fundamental value: the present discounted value of the marginal utilities or marginal products. There are also investors who only value the SOV for the chance to resell it. The model determines whether investors hold any of the SOV and its price, which has to be consistent with both perspectives.

The model is used to investigate the connection between real Treasury rates and the price of gold since 1975.[1] Empirically, I confirm the negative relation between the real price of gold and real rates but show that this is driven by periods with low rates. When rates were high, the relation has been weak. This nonlinear relation between the price of gold and real interest rates is a natural property of my model where investors only enter the SOV market when real rates are low enough. In the model, the expected holding period of the SOV determines the sensitivity of its price to interest rates and can be seen as an alternative form of bond duration. Quantitatively, when the model is calibrated to produce real rate movements consistent with the data, the sensitivity of the price of gold to rate changes is in line with the empirical evidence. A more standard representative agent model can produce gold price movements induced by interest rates that are of similar magnitudes as observed in the data. However, the representative agent setting cannot generate the nonlinear response to interest rates. I also consider CME Comex gold futures prices and show that model-implied futures prices can replicate key properties of the data.

In the quantitative asset pricing literature, the paper is related to Barro and Misra (2015) who study long-term historical gold returns from the perspective of a Lucas-tree economy. Their favorite interpretation of the data is one where the small return to gold is mainly compensation for the utility flow explicit in the model. Huang and Kilic (2019) document that the ratio of gold to platinum prices forecasts aggregate stock returns and show that a quantitative model with recursive preferences can account for that property. Different from these two representative agent frameworks, my model features investors and users and explicitly models the interaction between the two. Also, I study the role of interest rate movements. The model of Barro and Misra (2015) features constant risk-free interest rates; Huang and Kilic (2019) do not consider the interaction between interest rates and gold prices.

Some recent empirical studies on gold prices have emphasized the link between real gold prices and real Treasury rates, in particular Johnson (2014) and Erb, Harvey and Viskanta

[1] 1975 marks the end of restrictions on US private gold investments' in place since 1933. The official US gold peg was ended in 1971.

(2020). I show that for a more extended sample period the link between real gold prices and real rates is weaker unconditionally and that it is driven by periods with low real rates.

There is a well-developed literature on heterogenous-agent asset pricing models where equilibrium quantities and prices are jointly solved for. This literature has focused on various types of heterogeneity in preferences and market access. See for instance, Basak and Cuoco (1998), Barro et al. (2020), Chabakauri (2015), Chan and Kogan (2002), Chien, Cole and Lustig (2012), Gârleanu and Panageas (2015), and Ehling et al. 2018. A unique property of my analysis is the focus on a setting where asset prices are affected by the occasional entry and exit of one type of agent.

A fast growing literature is developing models for pricing cryptocurrencies. A major focus has been on valuing the transaction services provided by cryptocurrencies, for instance, Biais et al. (2018), Cong, Li, Wang (2021), Jermann (2018), and Schilling and Uhlig (2019). My model focuses on the interaction between users who value transaction services and investors aiming to store their wealth.

The next section analyzes empirical properties of gold prices and real interest rates. Section 3 presents the model and 4 its growth trend. Calibration and quantitative model properties are in section 5 and 6, respectively. Section 7 considers gold futures. The final section concludes.

2 Gold prices and real treasury rates

This section presents empirical properties of the relation between gold prices and real interest rates. I confirm the negative relation documented in Johnson (2014) and Erb, Harvey and Viskanta (2020) for an extended sample. However, the negative relation is not very strong over the longer sample periods considered here, namely 1975-2020 and 1980-2020 for 1-year and 10-year maturities, respectively. I document a new stylized fact: the negative relation between the price of gold and real rates is produced in the periods when rates are low. When rates are high, the relation is very weak.

Average monthly prices of gold for 1975.1 to 2020.12 are computed from the daily gold fixing in the London Bullion Market. The price of gold is deflated by the CPI-U. 1-year and 10-year constant maturity Treasury rates are combined with inflation forecasts from the Survey of Professional Forecaster extended with additional data for the 10-year horizon from Blue Chip Economic Indicators for 1979.10 to 1991.9.[2] Starting with 2003.1, 10-year TIPS rates are used for the 10-year real rates.

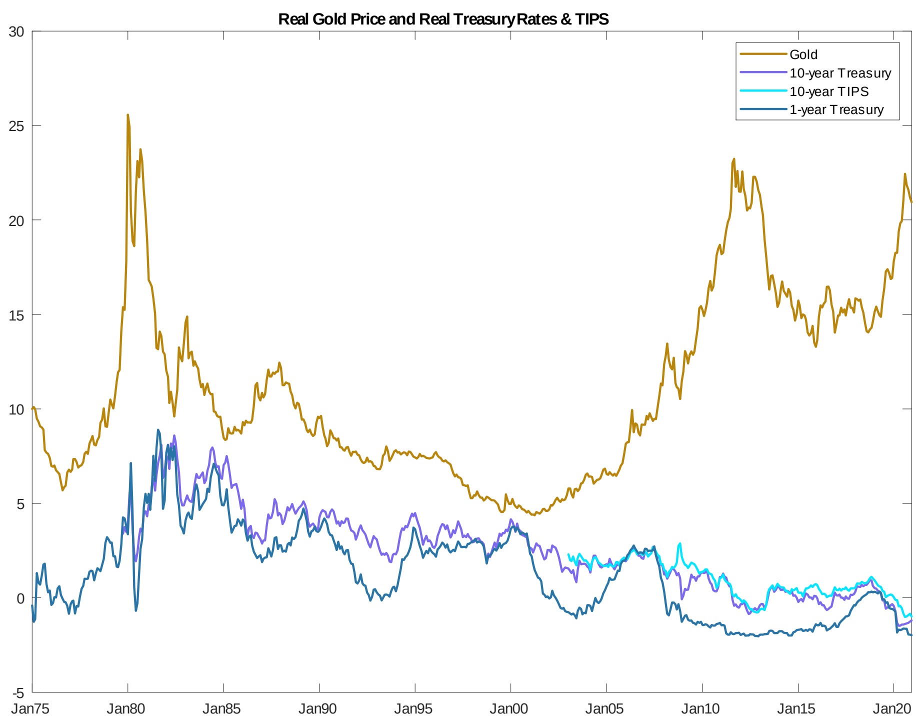

Figure 1: Gold and real government yields.

Figure 1 displays the time-series of the real price of gold (scaled to 10 at 1975.1) alongside the 1-year and 10-year real rates, including TIPS, each for their available sample periods. Yields are in general lower after 2000 and gold prices higher. But it can be visually detected that the negative relation is also present at higher frequencies.

[2]Data is from the Federal Reserve Bank of St. Louis and the Federal Reserve Bank of Philadelphia. The sample length is limited by the availability of inflation forecasts.

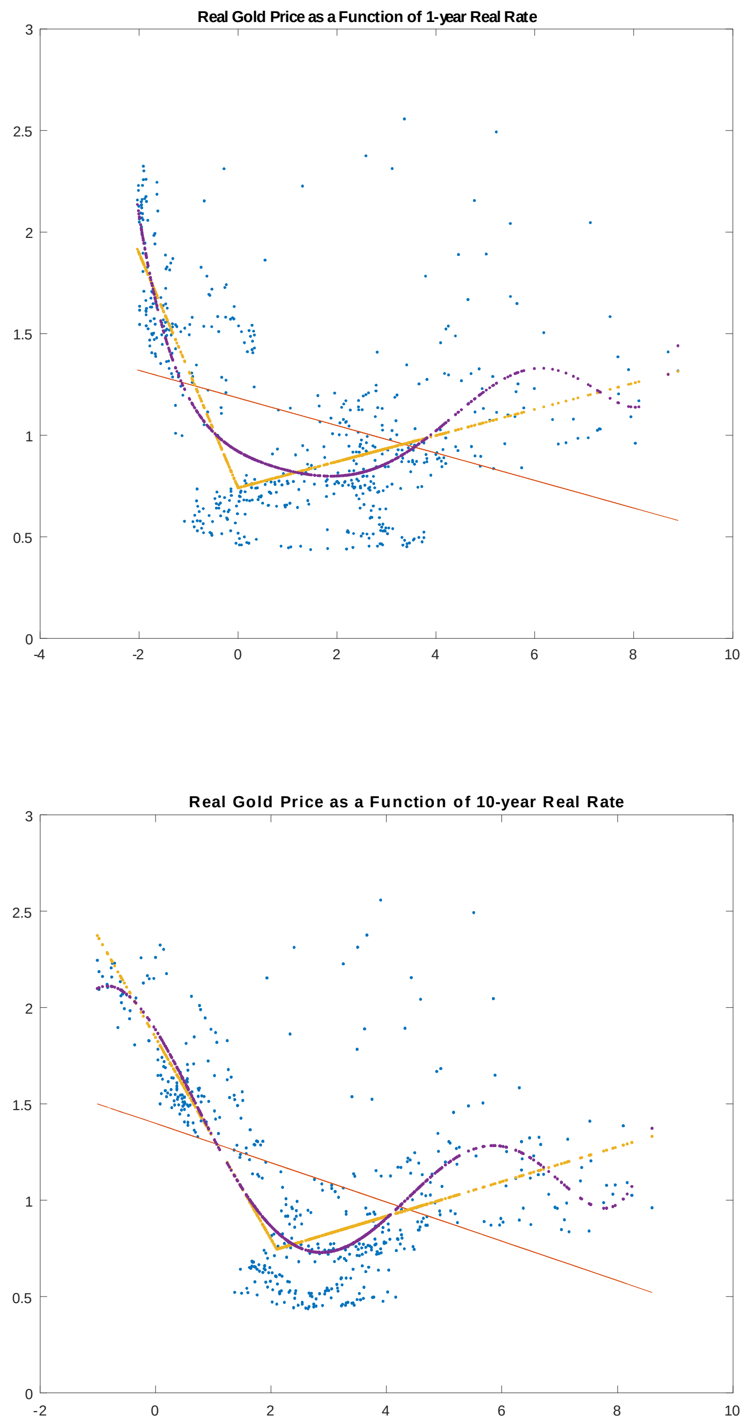

Figure 2. Real Gold Prices against Real Yields. 1975.1-2020.12 and 1979.10-2020.12 for 1-year and 10-year yields, respectively.

For instance, before and after 1985 there are periods of approximately three years each where gold and yields clearly move in opposite directions. Most recently, starting around 2019, gold and yields have sharply moved in opposite directions.

Figure 2 and 3 show the scatter plots of real gold prices (scaled to 1 at 1975.1) against 1-year and 10-year rates, respectively. For the 10-year rates, TIPS are used after 2003. From this perspective, it is clear that the negative relation between the price of gold and yields is situated in the low yield region. Linear regression lines as well as piece-wise linear and 6th-degree polynomials models are included in the plots.

The piece-wise linear regression is

$$p_t = b_0 + b_1 y_t + b_2 I_{y_t \geq y^*} \left(y_t - y^* \right),$$

with the break points chosen to maximize the adjusted R2s for each specification. These are at 0% and 2.2% for the 1-year and 10-year rates, respectively.

Table 1 shows more detailed regression results for the linear and piece-wise linear specifications with the date in natural logarithms. As suggested by the scatterplots, the R2s are significantly higher for the piece-wise linear specifications. Adjusted R2s for the linear regressions are 0.08 and 0.12 for the one-year and ten-year rates, respectively; for the piece-wise linear specifications, adjusted R2s are 0.42 and 0.55. For the piece-wise linear specification, with a slope coefficient of -45 for the 10-year rate in the low rate regime, a decline in the real yield by 1 percent corresponds to an increase in the price of gold by 45 percents (both in log percent). With a reported regression coefficient of $+56$ for the high rate region, the total effect of a 1 percent change of the 10-year yield is $+11 = -45 + 56$ percentage points.

To isolate high-frequency behavior, I run regressions of percentage changes in the gold price against changes in yields. The regressions are run separately for low-yield periods and high-yield periods. The cutoffs are unchanged at 0% for 1-year rates and 2.2% for 10-year rates. These cutoffs assign about 40% of the dates to the low rate regimes. Table 2 shows

Treasury maturity	1-year		10-year	
y_t	-5.3^{**}	-50^{***}	-8.0^{***}	-45^{***}
$I_{y_t \geq y^*}(y_t - y^*)$		$+56^{***}$		$+56^{***}$
R^2_{adj}	0.08	0.42	0.12	0.55
nobs	552	552	495	495

Table 1: Linear and piece-wise linear regressions of the natural logarithms of real gold prices on the natural logarithms of gross yields. The yield cutoffs are 0 percent for 1-year rates and 2.2 percents for 10-year rates. Significance levels are given as *** (p<.01), ** (p <.05), and * (p<.1).

Treasury maturity	1-year				10-year			
	monthly		annual		monthly		annual	
$\Delta y_t,\ y_t < y^*$	-3.0^{*}		-7.7^{***}		-11.6^{***}		-15.2^{***}	
$\Delta y_t,\ y_t \geq y^*$		-0.8^{*}		-1.2		0.7		-4.8^{**}
R^2_{adj}	.02	.006	.16	.006	.27	0	.44	.13
nobs	192	359	190	351	208	286	207	277

Table 2: Regression of log-percentage changes in real gold prices on yield changes conditional on low or high yield levels. Results for monthly and annual changes are reported. The yield cutoffs are 0 percent for 1-year rates and 2.2 percents for 10-year rates. Significance levels are given as *** (p<.01), ** (p <.05), and * (p<.1).

that, consistent with the levels, changes of gold prices are negatively related to yield changes in low-rate periods; however the coefficients are smaller (in absolute value terms). In the high-rate periods, the relation is a lot weaker. Coefficients are always smaller in absolute value terms, at a 5% level only the 10-year rate for the annual horizon is statistically different from 0, and the adjusted R2s are also considerably smaller. The overall picture that emerges is that in periods of low real interest rates gold prices were negatively related to real rates; when real interest rates were high, gold prices and real rates were at best weakly related.

3 Model

The model features two types of agents, users and investors, each valuing the SOV from their own perspective. In equilibrium, these two perspectives need to be consistent with each other. Possibly, investors chose not to hold any of the SOV in equilibrium. The model is in discrete time with an infinite horizon. Exogenous uncertainty comes from investors' discount factors which allows the model to match empirical interest rate dynamics.

3.1 Fundamental valuation

The stock of the SOV is assumed to provide some utility to its users. They have concave period utility functions separable in consumption and the SOV

$$\sum_{j=0}^{\infty} E_t \beta^j \left[u\left(C_{t+j}^U\right) + z_{t+j} v(a_{t+j}^U) \right].$$

The stock of the SOV held by users a_t^U is chosen in $t-1$; C_t^U is consumption of the numeraire good. The exogenous z_t can capture changes in the usefulness of the SOV, such as wider adoption of Bitcoin as a medium of exchange.

The flow budget constraint is

$$C_t^U + p_t a_{t+1}^U = p_t \left(a_t^U + a_{t+1} - a_t\right) + Y_t^U,$$

where Y_t^U is the endowment income, a_t is the aggregate supply (per capita of users). New SOVs are endowments of the users. Users and miners are aggregated together. If the user population grows faster than the supply, users share with their offsprings.

Intertemporal optimization requires that

$$p_t = E_t \beta \frac{u'\left(C_{t+1}^U\right)}{u'\left(C_t^U\right)} \left[\frac{z_{t+1} v'\left(a_{t+1}^U\right)}{u'\left(C_{t+1}^U\right)} + p_{t+1} \right],$$

and ruling out bubbles,

$$p_t = \sum_{j=1}^{\infty} E_t \beta^j \frac{u'\left(C_{t+j}^U\right)}{u'\left(C_t^U\right)} \frac{z_{t+j} v'\left(a_{t+j}^U\right)}{u'\left(C_{t+j}^U\right)} = \sum_{j=1}^{\infty} E_t \beta^j \frac{z_{t+j} v'\left(a_{t+j}^U\right)}{u'\left(C_t^U\right)}. \tag{1}$$

As usual in Lucas-tree models, the asset price fluctuates due to changing discount rates and changing cash flows, here marginal utility of the SOV deflated by marginal utility of consumption. It is assumed that the marginal valuation of the SOV $v'(.)$ is a decreasing function with infinite marginal value at 0. This implies that, everything else equal, the price goes up of if the users hold less of the SOV, which could be driven by investors holding a larger share of the supply. From here we can see that the users' pricing equation always holds and can be used to price the SOV if we know the processes of their holdings and consumption. These are determined in equilibrium. In a representative-agent Lucas-tree economy these quantities would be exogenous.

3.2 Investor valuation

Investors are assumed to hold a diversified portfolio of traditional assets. In equilibrium, they are getting the total payout Y_t^I, which includes other incomes. They can hold – but not short – the SOV, and they do not derive any utility from holding it.

Investors have the same utility function as users except for the absence of a utility for gold and with a time-varying discount factor. The budget constraint (omitting the holdings of other assets) is

$$C_t^I + p_t a_{t+1}^I = p_t a_t^I + Y_t^I.$$

Intertemporal optimality requires

$$u'\left(C_t^I\right) p_t = E_t \beta_t u'\left(C_{t+1}^I\right) p_{t+1} + \mu_t,$$

where $\mu_t \geq 0$ is the multiplier on the no-short sales constraint on the SOV. The discount factor β_t is subject to exogenous shocks modelled as a Markov chain, and this drives interest rates and gold prices. Iterating forward gives

$$p_t = E_t \left(\prod_{j=0}^{J-1} \beta_{t+j} \right) \left\{ \frac{u'\left(C_{t+J}^I\right)}{u'\left(C_t^I\right)} p_{t+J} \right\} + \frac{1}{u'\left(C_t^I\right)} \sum_{j=0}^{J-1} E_t \left(\prod_{k=0}^{j-1} \beta_{t+k} \right) \mu_{t+j} \tag{2}$$

Equation 2 implies that if investors hold the SOV all the time, its price is either 0 or the bubble component for $J \to \infty$ (that is the first term on the right hand side of the equation). In particular, if the holdings are never constrained by the short sales constraint, $\mu_{t+j} = 0$, then the price is the bubble component for $J \to \infty$. If the bubble component is zero, the price of the SOV is zero. In equilibrium, a zero price is ruled out by the assumption that the marginal utility of the SOV to users is infinite at 0. A finite value of the bubble component is inconsistent with investors' transversality condition for non-zero gold holdings. This implies that the short-sales constraint is occasionally binding and that there are times when investors are not holding the SOV. This also implies that the investors' Euler equation cannot be used to determine the price of the SOV alone. Still, as investors' enter the market, they affect the price of the SOV.

Rewriting investors' first-order condition for the SOV and defining the one-period real risk-free rate or yield as $Y_t^{(1)} = 1/E_t\beta_t \frac{u'\left(C_{t+1}^I\right)}{u'\left(C_t^I\right)}$,

$$Y_t^{(1)} \geq E_t \frac{p_{t+1}}{p_t} + cov_t \left(\frac{u'\left(C_{t+1}^I\right)}{E_t u'\left(C_{t+1}^I\right)}, \frac{p_{t+1}}{p_t} \right). \tag{3}$$

The equation highlights that the first-order condition holds with equality – and investors are holding the SOV – when the real rate is relatively low and/or if the covariance is positive, requiring the price to increase when consumption declines. Therefore, the SOV is appreciated by investors either in times of low real rates or as a consumption hedge. The equation also highlights that, everything else equal, a higher expected price return for the SOV makes it more likely that investors are holding it.

3.3 Equilibrium

Market clearing for the SOV requires

$$a_{t+1} = a_{t+1}^U + n^I a_{t+1}^I,$$

where n^I is the population ratio of investors to users, which is assumed to be constant. Goods market clearing requires

$$C_t^U + n^I C_t^I = Y_t^U + n^I Y_t^I,$$

which is already implied by Walras law.

An implicit assumption of the model is that asset markets other than for the SOV are segmented across investors and users. Investors are assumed to have frictionless access to the traditional assets, including government bonds. Some frictions are assumed to limit the exposure of users' marginal consumption valuations from the traditional asset markets to only the indirect impact that comes through the market of the SOV asset.

Among the assets investors have access to, I explicitly consider default-free real bonds. For numerical tractability, the maturity is represented with geometric amortization. Investors determine equilibrium prices of such bonds, p_t^λ, through

$$p_t^\lambda = E_t \beta_t \frac{u'\left(C_{t+1}^I\right)}{u'\left(C_t^I\right)} \left[c + \lambda + (1-\lambda)\, p_{t+1}^\lambda\right],$$

with λ the amortization rate, $(1/\lambda)$ is the bond's (average) maturity, and c represents the coupon rate. Iterating forward, the price of the bond can be written as a function of its implied gross yield to maturity, $Y_t^{(1/\lambda)}$, given through

$$p_t^\lambda = \frac{c+\lambda}{Y_t^{(1/\lambda)} - 1 + \lambda}.$$

4 Deterministic growth and balanced growth

The model is designed to accommodate trend growth for gold prices. To maintain numerical tractability, a restriction is required across possible drivers of growth to guarantee a balanced growth path. This section characterizes trend properties and shows that the balance growth restriction is immaterial for my quantitative analysis.

I assume three exogenously growing variables, growing at potentially different rates: the supply of the SOV (per capita), incomes of users and investors (per capita), and the utility/productivity of the SOV. For quantitative analysis, stationarity for some key model variables is a useful property. Bounded domains are also needed for computation. I start by presenting a deterministic growth path. For this to be a balanced growth path, so that the equilibrium is stationary for appropriately defined variables, we will need to impose a restriction on the exogenous income growth rates. For detrended variables, the model then has a steady state. But this steady-state is not a mid-point for a local approximation or necessarily close to model realizations. It anchors the detrended variables so that there are well-defined ranges over which to solve the model.

From the SOV market clearing

$$a_{t+1} = a_{t+1}^U + n^I a_{t+1}^I,$$

introduce a growing supply (per capita)

$$X_{t+1}^s a_0 = a_{t+1}^U + n^I a_{t+1}^I,$$

with $X_{t+1}^s = \gamma_s^{t+1}$ and $X_0^s = 1$. Incomes of the users and investors Y_t^U and Y_t^I are assumed to have a common deterministic trend X_t growing at a constant rate, γ, so that $X_t = \gamma^t$. Similarly, utility/productivity growth of the SOV is γ_z.

4.1 Deterministic growth

I conjecture a type of equilibrium growth path where variables grow at constant rates, verify it exists and solve for the equilibrium growth rate of the price of gold. This growth path is not necessarily balanced in the sense that the economy could be redefined for appropriately scaled variables that are stationary.

Assume CRRA consumption utility, $u(C_t) = \frac{1}{1-\sigma} C_t^{1-\sigma}$, with parameter $\sigma > 0$; similarly, CRRA is assumed for the SOV utility with parameter $\eta > 0$.

The conjectured equilibrium path has users hold 100% of the SOV. This implies that their consumption is growing at the same rate as their income, and their SOV holdings are growing at the exogenous supply growth rate. The price of the SOV and its growth rate can be computed analytically. For this to be an equilibrium, the investors' implied real rate has to be high enough so that they stay out of the SOV market.

Based on these assumptions, the price that is consistent with the users' first-order conditions satisfies

$$p_t = \beta \gamma^{-\sigma} \left[\frac{z_t \gamma_z \left(\gamma_s a_t^U \right)^{-\eta}}{\left(Y_t^U \gamma \right)^{-\sigma}} + p_{t+1} \right].$$

A constant price growth rate requires a constant price/payout ratio, and this requires the price to grow at the same rate as the payout, that is,

$$\frac{p_{t+1}}{p_t} = \frac{\gamma_z \gamma_s^{-\eta}}{\gamma^{-\sigma}}.$$

The price grows with the utility/productivity of the SOV deflated by marginal utility of consumption and declines with the per capita supply. Finiteness requires

$$\beta \gamma_z \gamma_s^{-\eta} < 1.$$

For this to be an equilibrium with investors holding 0 of the SOV, the investors' first-order condition has to hold with inequality

$$1 > \beta \frac{u'\left(C_{t+1}^I\right)}{u'\left(C_t^I\right)} \frac{p_{t+1}}{p_t}$$

so that

$$\frac{1}{\beta \gamma^{-\sigma}} > \frac{p_{t+1}}{p_t}.$$

For appropriately low enough discount factor β, equivalently high enough real rate $R \equiv 1/\beta\left(\gamma\right)^{-\sigma}$, this condition is satisfied.

4.2 Balanced growth

A balanced growth path is one where we can define appropriately detrended variables that are stationary. This is useful for representing a stochastic economy that has trend growth. For quantitative analysis it is convenient to have some stationarity, and numerical computation also typically requires stationary state variables. The deterministic growth path we just computed does not necessarily satisfy this property. We will impose one restriction on the three exogenous growth rates to make this into a balanced growth path.

The deterministic growth path derived in the previous subsection implies that the share of income users get from the SOV is not necessarily constant in their total income. Appropriately restricting the income growth rate guarantees that the income from the SOV remains a constant share of the total income.

Define consumption detrended by the income growth, $\hat{C}_t^U$, implicitly through

$$C_t^U = X_t \frac{C_t^U}{X_t} \equiv X_t \hat{C}_t^U,$$

and equivalently the detrended SOV position

$$p_t a_{t+1}^U \equiv \left\{X_t^p X_t^s \gamma_s\right\} \hat{p}_t \hat{a}_{t+1}^I.$$

The budget constraint can then be written as

$$X_t \hat{C}_t^U + \left\{X_t^p X_t^s \gamma_s\right\} \hat{p}_t \hat{a}_{t+1}^U = \left\{X_t^p X_t^s\right\} \hat{p}_t \left(\hat{a}_t^U + \left(\gamma_s - 1\right) a_0\right) + X_t \hat{Y}_t^U,$$

With $X_t = X_t^p X_t^s$, the trending terms can be eliminated and this implies the balanced growth restriction

$$\gamma = \gamma_p \gamma_s = \frac{\gamma_z \gamma_s^{1-\eta}}{\gamma^{-\sigma}}.$$

Given my focus on the market of the SOV, I want to be able to select γ_p and γ_s to match data for gold. The balanced growth restriction implies that γ is not a free parameter and its value may not be consistent with an empirical target for income growth. However, γ only enters the relevant (detrended) equilibrium equations jointly with the level of the discount factor β. The calibration sets the combination of these two parameters

$$\beta\gamma^{-\sigma} \equiv 1/R$$

through R which determines the level of interest rates in the model. Therefore, the value for γ is not relevant for detrended equilibrium conditions. Typical consumption-based asset pricing studies eliminate in a similar way the direct effect of the risk aversion parameter σ on the deterministic return and level of interest rates.

With the balanced growth restriction, the detrended budget constraint can be written as

$$\hat{C}_t^U + \gamma_s \hat{p}_t \hat{a}_{t+1}^U = \hat{p}_t \left(\hat{a}_t^U + (\gamma_s - 1)\, a_0 \right) + \hat{Y}_t^U, \tag{4}$$

which now depends only on stationary variables. Similarly, we can eliminate deterministic trends from all equilibrium equations.

Market clearing for the SOV holdings which are growing at the supply rate

$$X_{t+1}^s a_0 = X_{t+1}^s \hat{a}_{t+1}^U + n^I X_{t+1}^s \hat{a}_{t+1}^I,$$

becomes

$$a_0 = \hat{a}_{t+1}^U + n^I \hat{a}_{t+1}^I. \tag{5}$$

Goods market clearing with stationary variables becomes

$$\hat{C}_t^U + n^I \hat{C}_t^I = \hat{Y}_t^U + n^I \hat{Y}_t^I. \tag{6}$$

Intertemporal optimization of the users requires

$$p_t = \beta E_t \frac{\left(C_{t+1}^U\right)^{-\sigma}}{\left(C_t^U\right)^{-\sigma}} \left[z_{t+1} v'\left(a_{t+1}^U\right) + p_{t+1}\right].$$

Prices and payouts are growing at $\frac{\gamma_z (\gamma_s)^{-\eta}}{\gamma^{-\sigma}}$ per period, and the ratio of the marginal utilities by $\gamma^{-\sigma}$. Eliminating trends, we have

$$\hat{p}_t = \left\{\beta \gamma_z \gamma_s^{-\eta}\right\} E_t \frac{\left(\hat{C}_{t+1}^U\right)^{-\sigma}}{\left(\hat{C}_t^U\right)^{-\sigma}} \left[\frac{\hat{z}_{t+1} v'\left(\hat{a}_{t+1}^U\right)}{\left(\hat{C}_{t+1}^U\right)^{-\sigma}} + \hat{p}_{t+1}\right]. \tag{7}$$

Note again that the growth adjusted discount factor can also be written as

$$\beta \gamma_z \gamma_s^{-\eta} = \beta \gamma^{-\sigma} \frac{\gamma_z \gamma_s^{-\eta}}{\gamma^{-\sigma}} = \frac{1}{R} \gamma_p,$$

which shows that R and γ_p are what matters, and the calibration will fix these two parameters. Investors' pricing when $a_{t+1}^I > 0$ becomes

$$\hat{p}_t = \left\{\rho \gamma_z \gamma_s^{-\eta}\right\} E_t \frac{\left(\hat{C}_{t+1}^I\right)^{-\sigma}}{\left(\hat{C}_t^I\right)^{-\sigma}} \hat{p}_{t+1}. \tag{8}$$

The flow budget constraint for investors becomes

$$\hat{C}_t^I + \gamma_s \hat{p}_t \hat{a}_{t+1}^I = \hat{p}_t \hat{a}_t^I + \hat{Y}_t^I. \tag{9}$$

To summarize, we have 6 equations for 5 stationary variables $\hat{a}_{t+1}^I, \hat{a}_{t+1}^U, \hat{C}_t^I, \hat{C}_t^U, \hat{p}_t$ (one of the budget constraints is redundant by Walras law), together with the short-sales constraint

$$\hat{a}_{t+1}^{I} \geq 0.$$

Turning off uncertainty, this detrended economy admits a steady state with $\hat{a}^{I} = 0$, $\hat{a}^{U} = a_0$, $\hat{p} = \left[\frac{z_0\left(a_0^U\right)^{-\eta}}{\left(Y_0^U\right)^{-\sigma}}\right] \frac{\beta\gamma_z(\gamma_s)^{-\eta}}{1-\beta\gamma_z(\gamma_s)^{-\eta}}$, and consumption equals income for users and investors. When driven by exogenous Markov shocks to β_t, the detrended economy has one endogenous state variable

$$\hat{a}_t^I \in [0, a_0] \, .$$

Solving the model numerically involves finding a detrended price process $p\left(\hat{a}_t^I, \beta_t\right)$ and a policy function for investors' SOV holding (or equivalently consumption). Due to the occasionally binding constraint and the nonlinear model behavior a global solution method is used. The model solution is computed by iterating over the pricing function and the investors' policy rule with an algorithm that uses elements from Judd (1992) and Judd, Kubler and Schmedders (2002).

5 Calibration

For the quantitative analysis, the model is driven by shocks to investors' discount factors that are calibrated so that the model closely replicates averages, standard deviations and first-order serial correlation coefficients of 1-year and 10-year real Treasury yields. The trend growth rate of the price of gold plays an important role and it is set in reference to historical evidence. The rest of the parameters are less important for gold prices and their relation to interest rates.

The discount factor β is set so that the deterministic one-period rate $R \equiv \frac{\gamma^{-\sigma}}{\beta}$ equals 1.015; this determines the level of the one-period interest rate in equilibrium, which approximately matches historical data at annual frequency covering 1980-2020. Investors'

Symbol	Parameter	Value
β	Discount factor	$R \equiv \gamma^{\sigma}/\beta = 1.015$
a_β	Investor discount shock persistence	.95
σ_β	Investor discount shock innovation std	.0091
γ_p	Gold price growth	$1, 1.005$
γ_s	Gold supply growth	1
n^I	Investor population relative to users	50
Y^U, Y^I	User and investor income level	14.4
σ	Risk aversion	2
η	Gold utility/productivity curvature	0.5

Table 3: Model Parameters. The gold price growth $\gamma_p = \gamma_z \gamma_s^{-\eta}/\gamma^{-\sigma}$, with γ_z the growth rate of gold's utility flow and γ the income growth rate.

discount factor shocks are multiplicative, $R_t \equiv R \exp\left(\tilde{\beta}_t\right)$, with an AR(1) shock process for $\tilde{\beta}_t = \alpha_\beta \tilde{\beta}_{t-1} + \sigma_\beta \varepsilon_t^\beta$ that is approximated by a finite-state Markov chain. As shown in Table 4, 1-year and 10-year real rates in the model approximately replicate averages, standard deviations and first-order serial correlation coefficients of the 1-year and 10-year real Treasury yields. Differences between model versions are minimal because interest rates are mostly determined by the exogenous discount factor process. Given that the objective of the analysis is to explain gold prices as a function of interest rate movements, this approach allows me to start from realistic interest rate behavior.

As suggested by the investors' first-order condition, equation 3, a key quantitative tension comes from the comparison between the trend growth rate of the price of gold and the level of interest rates. According to Barro and Misra (2015), the annual growth of the price of gold in US inflation adjusted terms for 1836 to 2011 was 1.1% per year with a standard deviation of 13.1%, so that according to their calculations a one standard deviation band would cover $[0.1 - 2.1\%]$. Their favorite case is at the lower end of this range, as they argue that a significant portion of the return to gold (which in their view is close to the risk-free rate) comes from its utility yield. The real price of gold based on the data for 1975-2020 has a realized geometric annualized gross growth rate of 1.0162; but this is very sensitive to small changes in beginning and end dates. For annual average levels, the average percentage growth rate of the real price of gold is -0.2% . Based on these facts, I consider two baseline

	Data		$\gamma_p = 1$		$\gamma_p = 1.005$		RA	
	$Y^{(1)}$	$Y^{(10)}$	$Y^{(1)}$	$Y^{(10)}$	$Y^{(1)}$	$Y^{(10)}$	$Y^{(1)}$	$Y^{(10)}$
Mean	1.41	2.72	1.57	1.47	1.57	1.47	1.58	1.47
Std	2.43	1.99	2.61	1.74	2.59	1.74	2.63	1.75
AR(1)	.90	.95	.94	.93	.94	.94	.94	.94

Table 4: Real interest rates in the data and the model. Data are annual averages for 1980-2020 based on U.S. Treasury yields as described in Section 2. RA stands for representative agent economy. γ_p is the equilibrium trend gross growth rate of the price of gold.

cases, one with a 0.5% price growth rate, $\gamma_p = 1.005$, and a no-growth case, $\gamma_p = 1$. In the model, conditional on the other parameter values, the trend growth of the price of gold cannot be set much above 0.5% because otherwise the investors' no-short sales constraint would essentially never bind.

Barro and Misra (2015) document average per capita gold supply growth in the range $0.4 - 0.9\%$ annually for 1875-2011. They argue for some small depreciation and loss, and they take 0% as their baseline. More recent data is in line with their long sample period. Based on that, I set $\gamma_s = 1$ and use the utility/productivity growth rate γ_z to produce the target growth rate for the price of gold.

Based on World Gold Council data, the value of the global stock of gold is 11.6 trn dollars at the end of 2020. With world GDP at about 85trn dollars, the GDP to gold ratio is 7.3. The model's steady state ratio of total income to the value of gold, $\left(Y^U + n^I Y^I\right)/pa$, matches this. More specifically, with $a = 1$ as a normalization and z_0 normalized so that the detrended utility flow value of gold is $z_0 \left(\hat{a}_0\right)^{-\eta} / \left(Y^U\right)^{-\sigma} = 1$, $n^I = 50$ as an arbitrary choice (sensitivity analysis shows results robust to this parameter), the income levels $Y^U = Y^I$ are determined by this ratio for the given steady price of gold, $p = \left(\beta\gamma_z\gamma_s^{-\eta}\right) / \left(1 - \beta\gamma_z\gamma_s^{-\eta}\right)$. Risk aversion is set to a common value of 2, the gold utility/productivity curvature to 0.5. Neither of these two parameters matters very much for the relation between gold prices and interest rates.

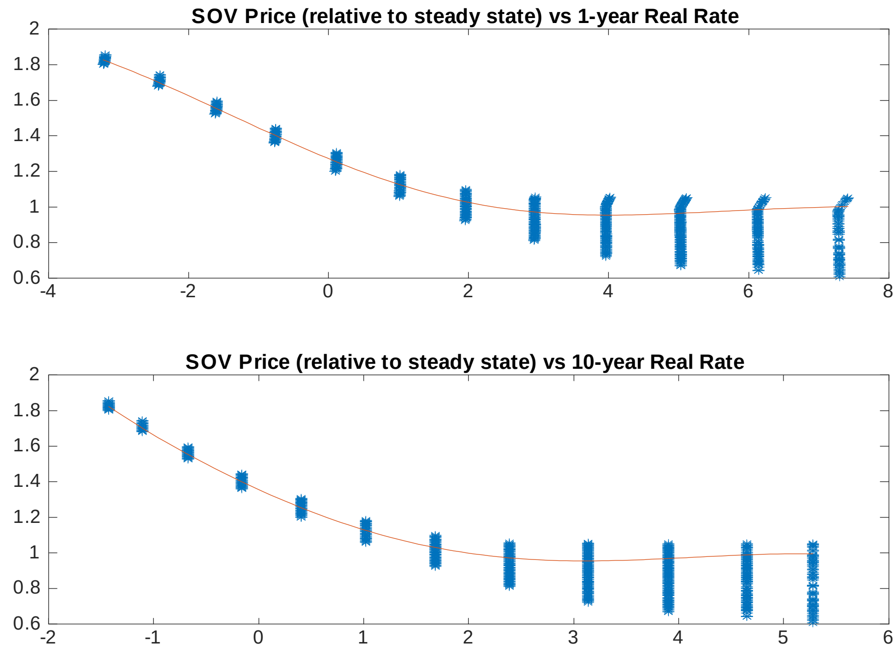

Figure 3. Price of gold versus yields. The red line is a fitted polynomial of order 6.

6 Quantitative results

Figure 3 shows that the simulated data from the model can replicate the nonlinear response pattern of gold prices to real interest rates documented empirically and displayed in Figure 2. In addition to the scatter plot, a polynomial regression line is included to summarize the data. The blue markers clearly illustrate that interest rate levels are almost exclusively determined by the realizations of the discount factor shocks represented by a Markov chain with 12 values. Gold prices are more dispersed with high interest rates as the endogenous state variable – the investors' gold position – plays a relatively more important role.

Table 5 reproduces linear and piece-linear regression coefficients from the data together with the model-implied counterparts. Quantitatively, for the baseline calibration, the elastic-

Maturity		1-year		10-year	
Data	y_t	-5.3^{**}	-49.8^{***}	-8.0^{***}	-44.8^{***}
	$I_{y_t \geq y^*}(y_t - y^*)$		$+56.5^{***}$		$+55.9^{***}$
Model	y_t	-2.3	-9.2	-3.3	-19.5; -7.4
$\gamma_p = 1$	$I_{y_t \geq y^*}(y_t - y^*)$		$+9.0$		$+18.8$; $+10.7$
$\gamma_p = 1.005$	y_t	-6.5	-14.7	-9.5	-29.6; -16.5
	$I_{y_t \geq y^*}(y_t - y^*)$		$+10.8$		$+23.3$; $+18.3$
RA Model	y_t	-13.6	-13.0	-20.4	-23.2; -21.5
$\gamma_p = 1$	$I_{y_t \geq y^*}(y_t - y^*)$		-0.9		$+3.3$; $+3.0$
$\gamma_p = 1.005$	y_t	-14.6	-13.5	-21.9	-24; -23
	$I_{y_t \geq y^*}(y_t - y^*)$		-1.4		$+2.7$; $+2.6$

Table 5: Regression of gold prices on real yields. Linear regressions and piece-wise linear regressions of the logarithm of the price of gold on log gross yields. The cutoff levels for the piecewise linear regression are at 0 percent for the one-year rate and 0 and 2.2 percents for the ten-year rate. γ_p is the equilibrium trend gross growth rate of the price of gold. RA stands for representative agent economy.

ity of the price of gold in the low-rate region is about one-third of its empirical counterpart for the 1-year rate and about one-third to two-thirds of its empirical counterpart for the 10-year rate depending on the cutoff. Consistent with the data, the slope is very different in the high-rate region. The overall response in the high-rate region is close to zero.

Table 5 also includes regression coefficients of the corresponding representative agent economy, labelled RA. In this model, there is only one type of agents who hold the entire stock of gold and get utility from it, and whose discount factors are subject to shocks. As shown in Table 4, interest behavior is essentially the same in the RA and two-agent economies. However, the RA economy cannot produce the nonlinear relation between gold prices and interest rates. As shown in Table 5 for the piece-wise linear regressions, for the RA economy the slope changes very moderately in the high-rate region.

Table 6 considers the high frequency relations between gold prices and yields. The change in the log price of gold is regressed on the log yield changes, with separate regressions for the observations in low-yield and high-yield periods. Consistent with the empirical counterparts, the model features gold price responses that are a lot stronger in periods of low rates. The regression coefficients in the model are strikingly close their empirical counterparts without

Maturity		1-year		10-year	
Data	$\Delta y_t,\ y_t < y^*$	-7.7^{***}		-15.2^{***}	
	$\Delta y_t,\ y_t \geq y^*$		-1.2		-4.8^{**}
Model	$\Delta y_t,\ y_t < y^*$	-8.3		-14; -8.9	
$\gamma_p = 1$	$\Delta y_t,\ y_t \geq y^*$		-2.4		-3.5; -1.0
$\gamma_p = 1.005$	$\Delta y_t,\ y_t < y^*$	-12		-20; -17	
	$\Delta y_t,\ y_t \geq y^*$		-7		-10; -6
RA Model	$\Delta y_t,\ y_t < y^*$	-12		-22; -21	
$\gamma_p = 1$	$\Delta y_t,\ y_t \geq y^*$		-14		-20; -19
$\gamma_p = 1.005$	$\Delta y_t,\ y_t < y^*$	-13		-23; -23	
	$\Delta y_t,\ y_t \geq y^*$		-15		-22; -21

Table 6: Regression of log-percentage changes in real gold prices on yields conditional on low or high yield levels. The cutoff levels are at 0 percent for the one-year rate and 0 and 2.2 percents for the ten-year rate. γ_p is the equilibrium trend gross growth rate of the price of gold. RA stands for representative agent economy.

this being a calibration target.

Comparing the two model versions with the different gold price trends, $\gamma_p = 1$ and $\gamma_p = 1.005$, the regressions for changes seem to favor the model without trend growth, $\gamma_p = 1$, while the regressions for levels in table 5 favor the model with growth, $\gamma_p = 1.005$. For the 10-year rate, two cutoff levels are considered, 2.2% as in the data and 0% as for the short rate. Given that the model-implied term structure does not have a term premium, the 0% cutoff is informative. Consistent with Figure 3, the regressions with the higher cutoff have flatter slopes as they shift the sample to the right in the figure.

Table 7 shows additional properties of the price of gold for the different model versions, as well as properties of the investors' gold positions. The volatility of gold prices induced by the discount rate shocks are approximately twice the size in the economy with growing gold prices, $\gamma_p = 1.005$, compared to the case with zero growth. Consistent with our earlier discussion, when the price of gold has a higher growth rate, investors's gold position are larger on average and different from zero more often. In the model with trend gold price growth, investors have zero gold holdings 17% of the time. In the economy without growth, investors have zero gold holdings 39% of the time. As shown in the table, for the $\gamma_p = 1.005$ case, investors average gold holdings amount to 17% of the total supply with a 19% standard

	Data		$\gamma_p = 1$			$\gamma_p = 1.005$		
	$\ln(p_t)$	$\ln(p_{t+1}/p_t)$	$\ln(p_t)$	$\ln(p_{t+1}/p_t)$	a^I/a	$\ln(p_t)$	$\ln(p_{t+1}/p_t)$	a^I/a
Mean %		-0.2		0	6.5		0.5	17
Std %	45.5	14.8	9.4	5.0	10	20.1	8.7	19
AR(1)	.94		.86			.91		
$\Pr(a^I = 0)$					.39			.17
			RA,$\gamma_z = 1$			RA,$\gamma_z = 1.005$		
Mean %			1.16	0		1.38	0.5	
Std %			35.6	12.7		38.3	13.6	
AR(1)			.94			.94		

Table 7: Gold prices and investors' gold positions. The price of gold is p, investors' gold position as a share of the aggregate gold supply is a_I/a. RA stands for representative agent economy, γ_p is the equilibrium trend gross growth rate of the price of gold.

deviation.

6.1 Gold's duration

This subsection documents additional model properties and introduces an alternative type of duration to provide intuition about model mechanisms.

In bond pricing, the duration measures the sensitivity of the price of the bond with respect to its yield to maturity. The duration is also defined as the value-weighted maturity of a bond. In the model, the duration of gold can be computed from the users' perspective, but its role for explaining price movements due to interest rate changes is unimportant. From the investors' perspective, the duration of gold is not well-defined. Gold has no maturity and no coupon payments, and its duration is effectively infinite. However, I can define an alternative type of duration – the duration of the investors' expected holding period. This duration is closely related to the sensitivity of the price of gold to interest rates and helps illustrate model behavior.

As a starting point, consider the price of gold from the users' perspective, equation (1), which is also the price of gold in a representative agent economy. The duration is well-defined because gold has coupon payments (the service flows). Specifically, the duration (defined as usual as the value-weighted maturity and the elasticity with respect to the per period rate) is given by $1/\left(1-\frac{\gamma_P}{Y}\right)$ with Y the per-period gross yield.[3] In the calibration with $\gamma_p = 1.005$, on the balanced growth path, the duration is $1/\left(1-\frac{1.005}{1.015}\right) = 101.5$. The yield computed at equilibrium prices is significantly lower at an average of 1.0069, because of convexity raising the price with discount rate movements. Therefore, the duration is on average about $1/\left(1-\frac{1.005}{1.0069}\right) = 530$. The effect of such a high duration on price movements depends on the movements in very long-term yields. The model matches the empirical volatility of 10-year real yields, but reliable information on longer maturities is elusive. Estimating real risk-free yields from government bonds becomes more challenging as the maturity lengthens because correcting for inflation (if using nominal bonds) and credit risk becomes more challenging.

In the quantitative analysis of the two-agent economy, the standard duration does not play an important role. From the users' perspective, the price of gold changes mostly because of changing marginal utility of gold as their position changes (the cash flow effect) and less due to discount rate effects. From the investors' perspective, gold has no coupon and gold's duration is infinite. Nevertheless, an alternative type of duration remains important for understanding the connection between the price of gold and interest rates in the model.

To illustrate the idea, consider first a two-period zero-coupon bond. The per-period gross yield is Y and the price of the bond at time t and $t+1$, respectively, is given by

$$p_t^{(2)} = \frac{p_{t+1}^{(1)}}{Y} \text{ and } p_{t+1}^{(1)} = \frac{1}{Y}.$$

This has a duration (the negative of it)

$$\frac{d\ln p_t^{(2)}}{d\ln Y} = -1 + \frac{d\ln p_{t+1}^{(1)}}{d\ln Y} = -2.$$

[3] Specifically, in equation (1), normalizing the current period coupon to 1 and assuming it grows deterministically at gross rate γ_p, the price of gold can be written as

$$p_t = 1/\left(\frac{Y_t^U}{\gamma_P} - 1\right),$$

with Y_t^U the yield to maturity implied by the price. The duration can be computed as $-\frac{\partial \ln p_t}{\partial \ln Y_t^U}$.

Now consider gold prices if investors plan to hold gold at time t for 1 period. Based on a deterministic version of the investors' pricing equation, equation (2),

$$p_t = \frac{p_{t+1}}{Y} \text{ and } p_{t+1} > \frac{p_{t+2}}{Y},$$

and the (negative of) duration

$$\frac{d \ln p_t}{d \ln Y} = -1.$$

That is, with continuity and a strict inequality, small changes in Y and p_{t+2} cannot make the equation hold with equality, and so there is no effect on p_{t+1}. Ceteris paribus, if investors plan to hold gold for 2 periods

$$\frac{d \ln p_t}{d \ln Y} = -2, \text{ etc.}$$

Based on this example, we can see that the duration of investors' holding period also measures the sensitivity of the price of gold to the yield.

In the model, there is uncertainty about the duration of the holding period and the pricing equation, equation (2), includes a covariance term. However, the link between this duration and the price volatility induced by interest rates remains. For instance, this mechanism can explain the different sensitivities to interest rate changes when comparing the model versions with gold price trends of $\gamma_p = 1$ and 1.005. Indeed, with a higher trend growth, average gold holding periods of investors are longer (Table 7), and the price of gold responds more strongly to interest rate changes (Table 5).

This type of duration can also explain the state-contingent nature of impulse responses. Figure 4 displays impulse responses to investors' discount rates that at impact lower the real interest rate from about 1% down to about 0.2%. The figure shows that the strength of the increase in the gold price depends on the initial holdings of the investors. With an initial position at 0.2 of the total supply of gold, the price of gold increases by about 12.5%. With an initial position close to 0, the price of gold increase only by about 9%. When investors have a high initial gold position, the expected duration of their holding is larger than if they have a very small position. That is because a small position could be reduced to zero with a one-time high interest realization. A large position would likely not go to zero soon. From this perspective, the time-varying duration of the expected holding period drives the sensitivity of the price of gold to interest rate movements.

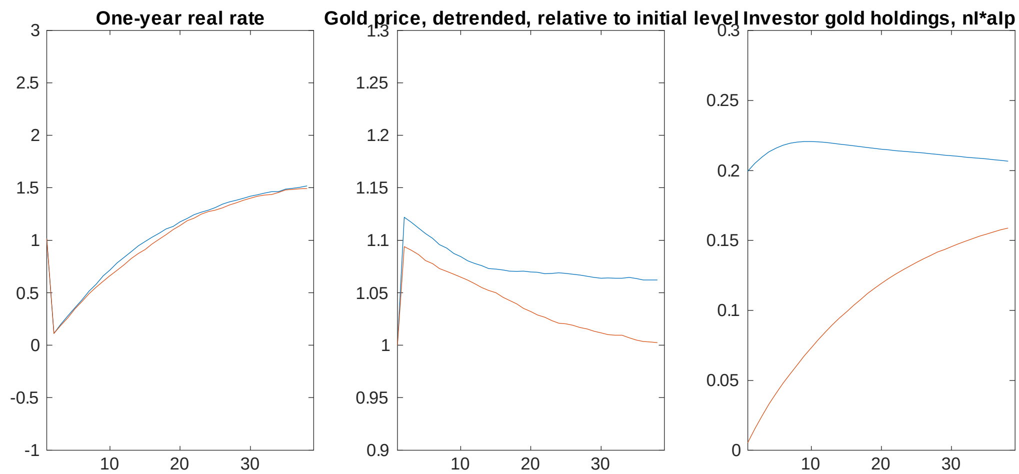

Figure 4: Impulse responses to discount rate shocks conditional on two different levels of investor gold holdings.

7 Gold futures

There is a deep market for gold futures at the CME and there is a long sample of available price data. In this section, I derive model-implied futures prices and compare these to the data. The model is shown to capture empirical properties which a representative agent model cannot.

Consider a contract to buy or sell one unit of gold at a price f_t for delivery next period $t+1$. I assume that investors trade such contracts with a zero net supply. In equilibrium, based on the investors' first-order condition, the futures price satisfies

$$f_t = \frac{E_t \left[u' \left(C^I_{t+1} \right) p_{t+1} \right]}{E_t u' \left(C^I_{t+1} \right)}. \tag{10}$$

Intuitively the futures price is the expected future spot weighted by investors marginal utilities (aka the expected future spot under risk-neutral probabilities).

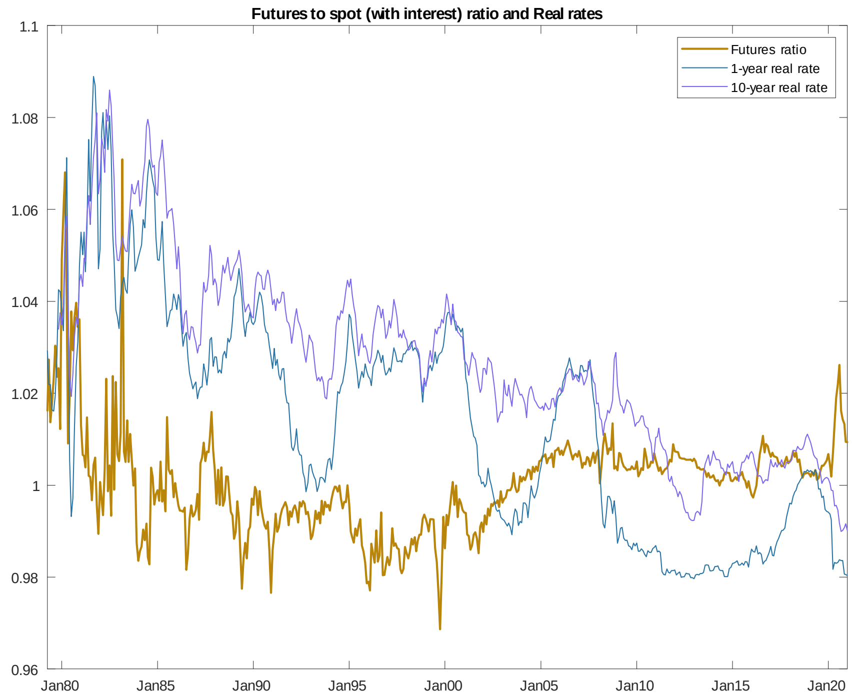

Figure 5. The futures ratio is based on the 12 or 13 months Comex CME contract relative to the contract expiring within a month, adjusted for whether the maturity difference is 11 or 12 months.

If investors are in the gold market in period t and their Euler equation for gold holds with equality, the spot price of gold is connected to the futures price through

$$p_t = \frac{E_t \beta_t \left[u' \left(C_{t+1}^I \right) p_{t+1} \right]}{u' \left(C_t^I \right)} = \frac{f_t}{Y_t^{(1)}},$$

where $Y_t^{(1)}$ is the one-period gross interest rate. If the Euler equation holds with inequality, then the spot price would be relatively higher than the right-hand side of the equation. Combining these two cases, the ratio of the futures to the spot price adjusted for interest satisfies

$$\frac{f_t}{p_t Y_t^{(1)}} \leq 1. \tag{11}$$

Intuitively, this equation represents the cash-and-carry arbitrage and the frictions in the model. Investors can always buy gold spot with borrowed funds and hedge it with a short futures, and this limits the futures price to be no higher than the cost of carrying gold, $p_t Y_t^{(1)}$. However, when investors are not in the gold market they cannot take a short spot position. In that case, the spot price including interest can be higher than the futures price. Based on our previous analysis, when interest rates are low or negative, investors are likely to be holding gold and therefore the futures ratio to spot (including interest) equals 1. With high real rates, this ratio is more likely to be smaller than 1.

For comparison, consider the representative agent economy where agents have the user's preferences and investors' market access, then

$$\frac{f_t}{p_t Y_t^{(1)}} = 1 - \frac{E_t\left[\frac{u'(C_{t+1})}{E_t u'(C_{t+1})} \frac{z_{t+1} v'(a_{t+1})}{u'(C_{t+1})}\right]}{p_t Y_t^{(1)}}. \tag{12}$$

The futures ratio would be lower than 1 by the expected utility yield for holding gold.

CME futures contracts are denominated in nominal dollar terms, the model so far has been in real terms. Assuming the consumer price level is deterministic, the model's futures to spot ratio for a dollar-denominated contract is identical to the ratio derived for contracts denominated in real terms, except that the interest is now the nominal interest rate. I compute the one-year futures to spot ratio from CME Comex futures prices buy taking the ratio of the contract with a maturity of between 12 and 13 months relative to the contract that is expiring within a month. Instead of using the spot price from the London Bullion Market this has the advantage of not introducing nonsynchronous prices. One-year interest rates are nominal Treasury rates.

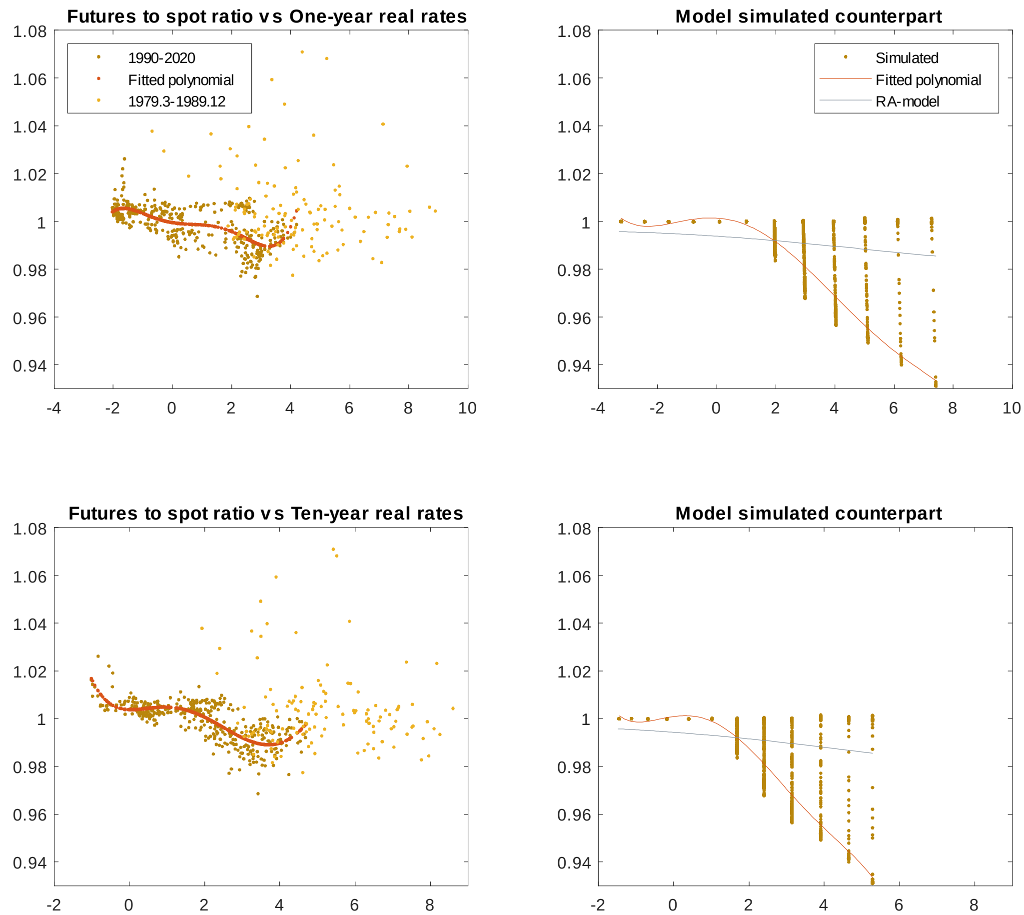

Figure 6. Futures ratio in the data and in the model.

Figure 5 displays historical futures ratios alongside one-year and ten-year real rates. Since about 2005, the futures ratio has been relatively stable at a level not far from 1. In the 15 to 20 years before 2005, the futures ratio was mostly below 1. This is qualitatively consistent with the model-implied ratio as in equation (11) and the fact that interest rates have been lower since 2005.

Figure 6 contains scatter plots of the futures ratio relative to either one-year or ten-year real interest rates. The plots in the right column are produced by model-simulated data. The model shows futures ratios consistently equal to 1 for negative and very low positive rates. As rates are higher, a range of futures prices is observed. The red line of the fitted polynomial shows that futures ratios are more often at the lower end of the range. The plots also includes the futures ratio from the corresponding representative agent economy. These show a weakly negatively slopped line. For the data plots, starting with 1990 the observations are displayed in a darker color to emphasize that they line up more closely with the model. Indeed, for low interest rates, futures ratios are around one, and futures ratio become lower at higher interest rates. Plotted against the ten-year real rate, the lower left panel suggests a nonlinear relation as in the model. Futures prices before 1990 appear to be less connected to real interest rates, which could indicate that gold futures market were less integrated with bond markets at that time.

8 Conclusion

In the model presented in this paper, rational investors can invest in store of value assets that do not have a yield despite the demand for these assets from users who enjoy some utility or service from the asset. The model is used to study the empirical relation between gold prices and real interest rates. In the model, real interest rates are closely related to investors' discount factors that drive the equilibrium price of gold. As such, lower real rates are associated with higher gold prices. The relation between investors' discount factors and the price of gold is more subtle than in a representative agent economy because investors' holdings of gold fluctuate. With larger gold holdings, the duration of investors' expected holding period increases, and the sensitivity of the price of gold with respect to real interest rates increases. The analysis in the paper shows that this mechanism can help explain the empirical relation between gold prices and real interest rates as well as the behavior of gold futures prices from qualitative and quantitative perspectives.

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