Dynamic Banking with Non-Maturing Deposits

Urban Jermann and Haotian Xiang*

April 27, 2021

Abstract

Bank liabilities include debt with long-term maturities and deposits that typically are not withdrawn for extended periods. This subjects bank liabilities to debt dilution. Our analysis shows that this has major effects for how monetary policy shocks are transmitted to banks and for optimal capital regulation. Interest rate cuts produce protracted increases in bank risk which are stronger in low rate regimes. Capital regulation addresses debt dilution but is subject to a time-inconsistency problem. We compare Ramsey and Markov-perfect optimal policies and find that regulator commitment significantly impacts optimal bank capital regulation, sometimes in unexpected ways.

Keywords: Debt maturity, dilution, capital regulation, monetary policy.

JEL codes: G21, G28, E44.

1 Introduction

Macro-finance models of banks embed the view that banks create liquidity through deposits but that they are exposed to the risk of default or runs. A prominent view in corporate finance is that debt dilution is a major friction that distorts debt and investment decisions.[1] These two views are not connected in existing macro-finance models of banks. In this paper, we connect these two views and present the consequences for the transmission of monetary policy shocks and for optimal capital regulation.

In typical macro-finance banking models, deposits are modelled as one-period debt. As such, when banks issue deposits, there is no outstanding debt that can be diluted. With

*Jermann is with the Wharton School of the University of Pennsylvania and the NBER, jermann@wharton.upenn.edu. Xiang is at Guanghua School of Management, Peking University, xiang@gsm.pku.edu.cn.

[1]The related literature is discussed in detail below.

long-term debt, when new debt is issued, borrowers do not internalize the reduction in value of the outstanding debt. This gives rise to debt dilution. Recent studies have shown that dilution affects debt dynamics in major ways (Gomes, Jermann and Schmid (2016), Admati, DeMarzo, Hellwig and Pfleiderer (2018)).

U.S. banks have some debt that is explicitly long-term, but the majority of their liabilities are deposits. According to the FDIC, deposits account for 78% of banks' balance sheets at the end of 2019. Time deposits with limited maturities account for 15% of all deposits. The remaining deposits are non-maturing: they have no explicitly maturity date and are typically not withdrawn for extended periods. However, because these deposits can be withdrawn at any time, their maturity depends on market conditions.

In our model, there are frictional costs that prevent deposits from being withdrawn and repriced every period. Depositors face liquidity shocks and they withdraw only if the liquidity value net of withdrawal costs exceeds the expected future liquidity benefit of the deposit. This converts redeemable deposits into long-term debt with endogenous maturity, and thus exposes deposits to debt dilution. Banks cannot credibly commit to not dilute depositors in the future. Within this setting, we study banks' responses to interest rate shocks and optimal bank leverage policies from a regulator's perspective.

We show that banks' responses to interest rate shocks are affected in important ways by endogenous maturity. We find that an interest rate cut generates an initial reduction in bank default risk but is followed by an extended period of significantly higher default risk and higher leverage. Endogenous maturity is key for this result. Following a rate reduction, deposit withdrawals decline and lengthen the maturity which leads to stronger debt dilution incentives, driving up leverage and default rates. A second finding is that this delayed bust is more pronounced in a low interest environment than in a high interest environment. After several periods of low interest rates, banks are more levered in the model, and this increases the sensitivity of default rates to interest rate moves. Overall, our analysis suggests that accounting for endogenous deposit dilution is important for modeling the effects of monetary policy.

Banks' dilution incentive creates a role for bank capital regulation. Importantly, we show that a regulator faces a time-inconsistency problem. A regulator who can commit to future policies can impact banks' and depositors' values in a more powerful way than one without such a commitment. We derive the optimal bank leverage policies from a regulator's perspective, both for a Ramsey regulator who can commit to future policies and for a Markov-perfect regulator who cannot commit. We find that the optimal amounts of leverage and bank default risk critically depend on regulator commitment. As usual, commitment leads to better outcomes. However, this does not necessarily imply a lower leverage and fewer bank defaults on average as regulators trade off between default losses and the liquidity value of deposits. We find that endogenous maturity plays a big role in elevating the value of regulatory commitment. With endogenous maturity, the forecasted choices of future regulators affect today's withdrawal decisions. This in turn affects liquidity creation and banks' default incentives.

A Ramsey solution includes history-dependence induced by commitment that is absent in the Markov-perfect case. Despite that, we find that when optimally deleveraging from a high level of debt, possibly after a crisis, a Ramsey regulator will not necessarily delever more slowly than a Markov-perfect regulator. When considering regulators' responses to exogenous shocks, we find that commitment does lead to more persistent policy responses. In particular, in response to a sudden deterioration in asset productivity, a Ramsey regulator loosens capital requirements for a much longer time than a Markov-perfect regulator. This is because a Ramsey regulator can use future implied promises to alleviate a temporary distress situation, while a Markov-perfect regulator lacks this tool. Overall, explicitly accounting for regulator commitment significantly impacts optimal bank capital regulation, sometimes in unexpected ways even in our stripped-down setting. This suggests that properly accounting for regulator commitment is crucial for model-based policy recommendations.

One technical challenge of our analysis is the non-stationary nature of the Ramsey allocation. We show that there exists a pseudo steady state where real variables are constant but the Lagrange multipliers in the sequential planning problem are non-stationary. We develop a numerical procedure that efficiently computes dynamic responses to shocks for this type of environment.

In the rest of the paper, after reviewing the related literature, we present a parsimonious model for non-maturing deposits, in Section 2, and we study banks' responses to exogenous interest rate shocks. Section 3 presents regulators' problems and analyzes optimal policies.

1.1 Related literature

Our paper contributes to the literature on dynamically modeling banks. There are two main novel features. First, we study bank deposits when dilution is possible and with endogenous maturity. Second, when dilution is possible, we characterize optimal bank capital regulation with and without regulator commitment. Our work is related to several research areas.

There is a large and growing literature on macro-finance banking models that evaluate macroprudential policy rules, mostly bank capital requirements. For instance, Van den Heuvel (2008), Angeloni and Faia (2013), Gertler and Kiyotaki (2015), Mendicino, Nikolov, Suarez, and Supera (2018), Xiang (2018), Corbae and D'Erasmo (2019), Begenau (2020), Begenau and Landvoigt (2020), Elenev, Landvoigt and Van Nieuwerburgh (2020), and Gertler, Kiyotaki, and Prestipino (2020). Some macro-finance banking models focus on studying monetary policy rules, for instance, Gertler and Karadi (2011), Brunnermeier and Koby (2018), Porcellacchia (2020) and Bianchi and Bigio (2020). Different from these papers, our analysis features long-term debt subject to dilution, and we endogenously derive optimal macroprudential policy rules.

There are several studies that have emphasized the rich dynamics of long-term debt with dilution, for instance, Gomes, Jermann and Schmid (2016), Crouzet (2017), Admati, DeMarzo, Hellwig and Pfleider (2018), Demarzo and He (2020). Different from these, our model features endogenous maturity and we derive optimal capital regulation. Xiang (2020) shows how debt covenant violations generate endogenous maturity but does not derive optimal regulatory policies.

Optimal macroprudential regulation in dynamic models has been derived by Chari and Kehoe (2016), Davydiuk (2017), Schroth (2020), Malherbe (2020), and Van der Ghote (2021). Different from these studies, our analysis features long-term debt subject to dilution and our analysis explicitly studies the role of regulator commitment.

The view that banks engage in maturity transformation and that they offer the option to withdraw deposits early is central to a large literature building on Diamond and Dybvig (1983). In this approach, a sequential service constraint can lead to bank runs. Instead, our model features debt dilution. Dynamic models have mostly abstracted from time-varying maturity. Exceptions are several papers interested in the valuation of non-maturing deposits from a bank's perspective where maturity is exogenously specified, including Hutchison and Pennacchi (1996), Jarrow and van Deventer (1998), Nyström (2008) and Wolff (2000). Some recent macro-finance studies also emphasize the long-term nature of bank deposits. For instance, Bolton, Li, Wang and Yang (2020) present a model where banks have limited control over deposit inflows, and Drechsler, Savov and Schnabl (2020) present empirical evidence suggesting bank franchise value confers long duration to bank deposits. Different from these, we model the dilution problem associated with long-term debt and derive optimal regulatory policies.

The dynamic properties of our Ramsey allocation are reminiscent of characterizations in the optimal taxation literature where convergence to steady states cannot always be established; see for instance Chien and Wen (2017) or Straub and Werning (2020).

2 Modeling deposits and interest rate shocks

In this section, we present a parsimonious model for non-maturing deposits and study how banks respond to interest rate shocks. The model is presented in Section 2.1, followed by an illustration of the key debt dilution mechanism in Section 2.2. Results about interest rate shocks are in Section 2.3.

2.1 Setup

We start with presenting our model of deposit withdrawal, followed by bank leverage choice and the pricing of deposits.

2.1.1 Deposit withdrawals

Time is discrete and all agents are risk neutral. The economy is populated with a continuum of banks who create value by providing liquidity services to depositors. Consider an individual bank with a continuum of depositors. The liquidity value derived by depositor $i \in [0,1]$ with deposits b_i consists of two components. First, the depositor can use the bank account for regular transactions within the period, such as receiving wage bills, paying for online shopping, etc. Such convenience provides a value of μb_i each period, which reflects that banking services are more valuable for individuals with a larger amount of cash.

Second, the depositor can withdraw from the account to meet a need for cash at the period end. Specifically, depositor i encounters a liquidity shock at the end of each period. In addition to the principal redemption, there is an extra marginal benefit ν when withdrawing an additional dollar with cumulative probability density function $F(\nu)$ over support $[\underline{\nu}, \bar{\nu}]$. Withdrawal incurs a marginal cost of κ.[2] As a result, depositor i withdraws the entire deposit b_i if the shock is large enough

$$(1 + \nu - \kappa) b_i \geq q b_i, \tag{1}$$

where q is the price of the deposit. The endogenously determined price equals the present value of future liquidity services adjusted for the default risk of the bank. The depositor keeps b_i in the bank if the above condition is not satisfied. Condition (1) implies that when interest rates increase or the bank becomes riskier, both leading to a lower q, the mass of withdrawing depositors, $\lambda(q) = 1 - F(q + \kappa - 1)$, becomes larger and deposit maturity shortens. The time-varying deposit maturity is the key distinction of our model relative to

[2] It is equivalent to assume away the withdrawal benefit by setting $\nu \equiv 0$ and formulate κ into a menu cost.

typical macro-banking models which usually fix $\lambda = 1$ and thus force depositors to withdraw and re-deposit every period.

Because the withdrawing decision of depositor i does not depend on b_i, the bank's problem only depends on the total amount of deposits on the balance sheet, $b = \int_i b_i di$, rather than the whole distribution of b_i's. Summing the two components of liquidity and integrating over the optimal withdrawing behavior, the liquidity value per unit of deposit, from the banks perspective, can be written as

$$l\left(q\right) = \mu + \int_{q+\kappa-1}^{\bar{\nu}} (\nu - \kappa) dF(\nu).$$

2.1.2 Bank problem

A bank makes its leverage decision each period conditional on total outstanding deposits b and the withdrawing rule of depositors. The assets of the bank generate a per-period profit of $R+z$, where R is constant for the analysis in this section and z is a zero-mean bank-specific i.i.d. profit shock with c.d.f. (p.d.f.) $\Phi(z)$ ($\phi(z)$) over support $[-\bar{z}, \bar{z}]$. Government interest rate policy determines the discount rate for banks and depositors since treasury bills serve as the outside option for both groups. That is, the discount rate in this economy is given by $1/r$ where interest rate r follows an exogenous process $\Gamma(r'|r)$.

Bank equity value and deposit policy is given by:

$$\begin{aligned} z + v^e(r, b) = z + \max_{b'} \Bigg\{ & R - \lambda(q) b + q\{b' - [1 - \lambda(q)]b\} \\ & + \frac{1}{r}\mathbf{E}_{r'|r}\left[\int_{-v^e(r', b')}^{\bar{z}} [v^e(r', b') + z'] d\Phi(z')\right]\Bigg\}. \end{aligned} \tag{2}$$

As shown in (2), bank equity value consists of two parts. First is the current period net cash flow, which includes profits $R+z$, repayment to withdrawing depositors $\lambda(q)b$, and proceeds from new deposits $q\{b' - [1 - \lambda(q')]b\}$. Second is the continuation value, which incorporates the bank's default option tomorrow. After the realization of two shocks r' and z', the bank defaults if the equity value goes below zero, i.e. $z' + v^e(r', b') < 0$.

When typical models assume $\lambda = 1$, a bank reissues all deposits every period and thus fully internalizes the impact of its choice of b'. In contrast, with the presence of future liquidity services and the withdrawal cost, our model features deposits that can be potentially long-term. A dilution problem arises when the bank does not need to compensate non-withdrawing depositors $[1 - \lambda(q')]b$ for how its choice of b' changes the risk of default.

2.1.3 Deposit pricing

The deposit price $q(r, b')$ is pinned down by the zero profit condition of depositors. For a non-defaulting bank, the payoff to depositors in the current period consists of the liquidity value $l(q)b$, principal repayment to withdrawing depositors $\lambda(q)b$, and the value of non-withdrawing deposits $q[1 - \lambda(q)]b$. That is, depositors' value is given by:

$$v^b(r, b) = \big\{l(q) + \lambda(q) + q[1 - \lambda(q)]\big\}b,$$

which does not depend on z because its realizations do not affect equilibrium choice for b', as suggested by (2).

Our formulation of default follows Gomes, Jermann, and Schmid (2016). Upon default, depositors take over the bank and initiate a restructuring. They first collect current profits $R + z$ and then sell off the equity portion to new owners while continuing to hold their deposits. After going through the restructuring, individual depositors again decide on whether to withdraw their money or not. This means that depositors have a claim over the total bank franchise value $z + v^e(r, b) + v^b(r, b)$ in default states. However, restructuring incurs a dead-weight loss for depositors that is increasing in the amount of deposits, ξb, reflecting a larger difficulty to restructure a more levered bank. Under this formulation, we do not need to keep track of the distribution of b's when considering the aggregate economy.

To sum up, the deposit price is given by

$$\begin{aligned} qb' = \frac{1}{r}\mathbf{E}_{r'|r}\Bigg[\int_{-v^e(r',b')}^{\bar{z}} v^b(r', b')d\Phi(z') \\ + \int_{-\bar{z}}^{-v^e(r',b')} [z' + v^e(r', b') + v^b(r', b') - \xi b']d\Phi(z')\Bigg], \end{aligned} \tag{3}$$

where $v^e(r', b')$ and $v^b(r', b')$ are affected by banks' future choices, b'' and q'', because deposits have effectively a maturity that is longer than one period. This implies that future deposit dilution is priced in at the issuance stage.

Definition 1 *A Markov Perfect Equilibrium is given by (i) banks' deposit policy $b'(r, b)$ with associated equity value function $v^e(r, b)$ and default; (ii) a deposit pricing function $q(r, b')$ and associated depositors' value function $v^b(r, b)$; (iii) depositors' withdrawal policy such that (i) given $q(r, b')$ and withdrawal policy, banks' deposit policy and default decisions are optimized; (ii) given banks' decisions and withdrawal policy, $q(r, b')$ and $v^b(r, b)$ satisfy depositors' zero profit condition in* (3)*; (iii) given $q(r, b')$, withdrawal policy for depositor i, $\forall i \in [0, 1]$, is given by* (1)*.*

2.2 Long-term deposits and dilution

The key feature of our model, relative to previous work on macro-banking, is that non-maturing deposits are plausibly long-term because depositors do not withdraw and reprice deposits period by period due to future liquidity services and withdrawal cost. As a result, banks make leverage decisions with the presence of non-withdrawing deposits $[1 - \lambda(q')]b$. In this section, we illustrate how this leads to a dilution problem—that is, banks have a tendency to over-borrow and incur an excessive default risk.

First, for simplicity, we shut down the withdrawal feature and consider a fixed-maturity defaultable debt model—that is, only a fixed fraction of depositors withdraw each period, i.e. $\lambda(.) \equiv \hat{\lambda} \in [0, 1]$, who get a net benefit of $\nu - \kappa \equiv 0$. In this setting, $l(.) \equiv \mu > 0$. To further simplify notations, we assume default recovery is zero, and then some straightforward algebra gives the following first-order condition for b':

$$\frac{1}{r}\mu \mathbf{E}_{r'|r}[1 - \Phi(-v^e(r', b'))] = -[b' - (1 - \hat{\lambda})b]\frac{\partial q(r, b')}{\partial b'}, \tag{4}$$

where the price impact on the right hand side (RHS) is given by:

$$\frac{\partial q(r, b')}{\partial b'} = \frac{1}{r}\mathbf{E}_{r'|r}\Bigg[[\mu + \hat{\lambda} + (1 - \hat{\lambda})q'(r', b'')]\phi(-v^e(r', b'))\frac{\partial v^e(r', b')}{\partial b'} + [1 - \Phi(-v^e(r', b'))](1 - \hat{\lambda})\frac{\partial q'(r', b'')}{\partial b''}\frac{\partial b''}{\partial b'}\Bigg]. \tag{5}$$

Equation (4) describes the trade-off behind absorbing an additional unit of deposit. On the left hand side (LHS) is the marginal benefit—additional liquidity value μ in non-default states. On the RHS is the marginal cost. As can be verified numerically, pushing up b' reduces the debt price $q(r, b')$. This is not surprising as it reduces $v^e(r', b')$ and therefore leads to a higher default probability tomorrow, captured by the first term in (5).

Deposit dilution arises because banks only internalize the negative price impact on new deposits. Non-withdrawing deposits, $(1-\hat{\lambda})b$, also have to bear a larger default risk associated with a higher b' but are not correctly repriced. As a result, banks would like to keep absorbing new deposits even when default risk has become excessively large. By doing so, they keep capturing the additional liquidity value but do not have to pay fully for the incremental default risk. As $(1-\hat{\lambda})b$ increases, banks would choose a larger b' and dilution becomes more severe.

At the time of issuance, depositors will price in the banks' future incentive to over-borrow. Depositing an additional dollar into the bank today amplifies the conflict of interests tomorrow. This is recognized by $\partial b''/\partial b'$ in the second term in (5). That first-order condition contains the derivative of the policy function reflecting the fact that dilution is a time inconsistency problem in nature—banks would be better off if they could commit to not over-borrow in the future.

When deposits are short-term, i.e. $\hat{\lambda} = 1$, the debt dilution problem disappears. Banks have no incentive to over-borrow unless some other exogenous frictions are built in.

In addition, deposit maturity in our model is not fixed because of the withdrawal options of the depositors. This reshapes the banking dynamics relative to the fixed-maturity long-term debt model. First, the dependence of $\lambda(q(r, b'))$ on r means that interest rate shocks alter the amount of non-withdrawing deposits that banks can dilute, which in turn changes the marginal calculation in Equation (4). Banking dynamics going forward change accordingly. We will show in this section that such an effect is quantitatively important. Second, $\lambda(q(r, b'))$ also depends on b'. When $\lambda(q(r, b'))$ increases in b', meaning a higher leverage choice persuades more depositors to withdraw right away, and dilution is effectively disciplined. We will show in Section 3 the implications of this feature for capital regulation.

2.3 Interest rate shocks and boom-bust dynamics

In this section, we show how banks respond to interest rate shocks. We assume the interest rate r follows an AR(1) process and set $\Gamma(r'|r)$ accordingly—that is, $r = r^* + \exp(x) - 1$ where r^* is the long-run interest rate level and $x' = \rho_x x + \sigma_x \tilde{\epsilon}, \tilde{\epsilon} \sim \mathcal{N}(0, 1)$.

2.3.1 I.i.d. shocks

We first consider i.i.d. interest rate shocks to cleanly show the workings of endogenous deposit maturity. The parametrization aims to approximately match obvious empirical counterparts. A period is a year. For interest rate process, we set $r^* = 1/0.95, \rho_x = 0$ and $\sigma_x = 0.015$. Average profitability of bank assets is $R = 0.02$. Default loss is $\xi = 0.2$. For the zero-mean i.i.d. shocks to profitability, we set $\phi(z) = \iota_0 - \iota_1 z^2$. By imposing $\phi(\bar{z}) = 0$ and $\Phi(\bar{z}) = 1$, we can use $\bar{z}$ to pin down ι_0 and ι_1. We set $\bar{z} = 0.26$, which leads to a steady-state bank default probability of 75 basis points. Regarding liquidity parameters, we set $\mu = 0.0423$ and assume ν follows an exponential distribution, i.e. $f(\nu) = a \exp(-a\nu)$, with $a = 20$. Withdrawal cost is $\kappa = 0.1$. In steady state, bank equity ratio $1 - b_{ss}/(v^e_{ss} + v^b_{ss}) = 0.1783$ and deposit rate is $1/q_{ss} - 1 = 0.0266$.[3] We are not aware of an obvious empirical counterpart for the withdrawal frequency. Wolff (2000) argues that as a rule-of-thumb 20% of deposits are highly volatile. The parameterization implies a steady state withdrawal mass of $\lambda_{ss} = 0.2272$.

[3]We use subscript ss to denote values in steady state throughout the paper.

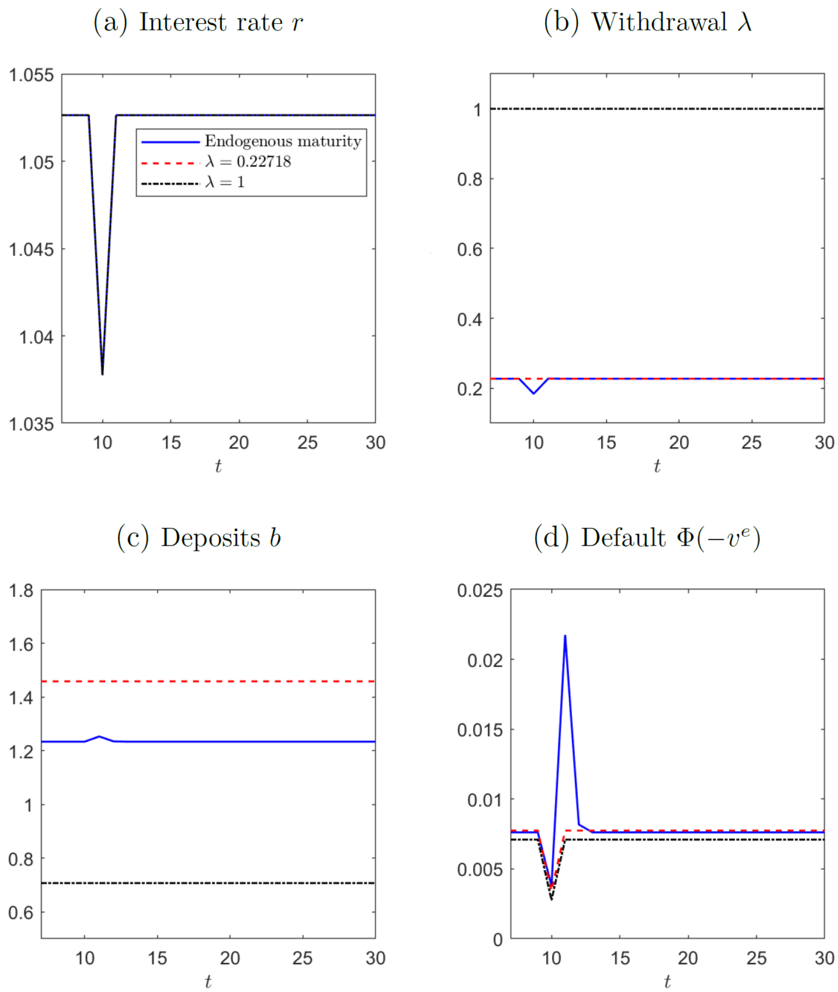

Figure 1: Banks' responses to i.i.d. interest rate shocks in laissez-faire economy. *Notes*: $r^* = 1/0.95, a = 20, \kappa = 0.1, \xi = 0.2, \mu = 0.0423,\ R = 0.02,\ \bar{z} = 0.26,\ \rho_x = 0, \sigma_x = 0.015$. For fixed-maturity models, we adjust $\mu = 0.045, \bar{z} = 0.2$ for the $\hat{\lambda} = 0.22718$ case and $\mu = 0.051, \bar{z} = 0.4$ for the $\hat{\lambda} = 1$ case for comparability.

The blue solid lines in Figure 1 describe how banks in steady state respond to a one-time interest rate cut. At $t = 10$, the interest rate is reduced and thus the discount rate becomes high, the present value of future liquidity services becomes larger. Fewer depositors choose to withdraw. Relative to the steady state, with the presence of more non-withdrawing deposits to dilute at $t = 10$, banks choose a larger b_{11}. A higher b_{11} in turn leads to a b_{12} that is still noticeably higher than b_{ss}.

The default probability exhibits a boom-bust feature. At $t = 10$, bank default probability shrinks in response to the contemporaneous rate cut. Similar to the reason behind depositors' increasing willingness to stay, the equity value of banks becomes larger due to a higher discount rate. However, going forward, as interest rate returns to normal but b_{11} and b_{12} remain to be high, the default probability becomes higher than its steady state value. In other words, an interest rate cut makes banks safer in the short run but more fragile in the long run.

Crucial for our results is the endogenous withdrawal feature. Figure 1 compares our results against those coming out of fixed-maturity models, as characterized before by Equations (4) and (5). The red dashed lines show banks' responses to an identical rate cut in a fixed-maturity long-term deposit model. We fix withdrawal mass $\hat{\lambda} = 0.22718$, which is equal to the λ_{ss} in our baseline model. We re-set $\mu = 0.045$ and $\bar{z} = 0.2$ so that thte steady state default rate is roughly the same as that in the baseline. The black dotted lines show the responses under short-term deposits, i.e. $\hat{\lambda} = 1$, where we re-set $\mu = 0.051$ and $\bar{z} = 0.4$.

In both cases with fixed maturity, i.i.d. interest rate shocks produce no impact on leverage dynamics. As is clear from the first-order conditions (4) and (5), under i.i.d. shocks, r's on the LHS and RHS cancel out. In other words, a one-time interest rate cut does not change the marginal trade-off for b'. In contrast, in our model, $\lambda(q(r, b'))$ and thus the marginal cost on the RHS change with r.

2.3.2 Persistent shocks

Now we consider persistent shocks to interest rates. We set $\rho_x = 0.8, \sigma_x = 0.005$ and adjust $\mu = 0.0435$ while keeping all the other parameter values to be same as those in the i.i.d.-shock case. Steady state moments are comparable to the i.i.d.-shock ones. The blue solid lines in Figure 2 describe how banks in the steady state respond to persistently low interest rates. Again, we consider two alternative models with fixed maturities: long-term deposits where $\hat{\lambda} = 0.2146, \mu = 0.045, \bar{z} = 0.2$ and short-term deposits where $\hat{\lambda} = 1, \mu = 0.052, \bar{z} = 0.4$.

When the interest rate cut becomes persistent, banks' deposit choice starts to respond to the rate cut even if deposit maturity is fixed. Again, by inspecting (4) and (5), one can see that $1/r$ cancels out but the conditional expectation $E_{r'|r}[.]$ does not. When expecting rates to be low in the future, banks start to absorb more deposits.

Under short-term deposits, default probability does not exhibit boom-bust dynamics even though debt remains persistently high after $t = 10$. Banks do not have a dilution incentive. They increase deposits simply because of a smaller marginal cost—that is, with an increase in discount rates, default risk becomes less sensitive to deposit absorption. As the interest rate returns to normal, banks adjust leverage downward rather quickly, and the equilibrium default risk remains moderate.

Under fixed-maturity long-term deposits, default probability becomes high for several periods after the rate cut. Compared to the short-term deposit case, debt dilution kicks in. With a large amount of deposits accumulated right after the shock, when interest rate reverts back, banks do not adjust leverage and default risk downward as quickly because the

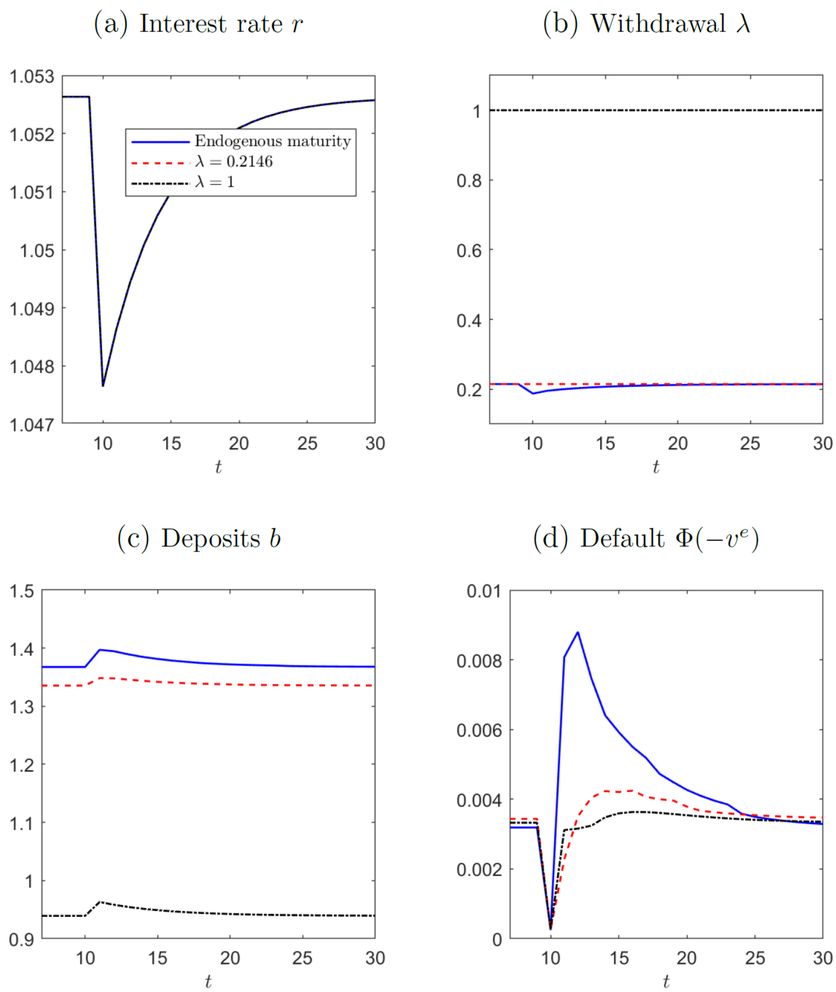

Figure 2: Banks' responses to persistent interest rate shocks in laissez-faire economy. *Notes*: $r^* = 1/0.95, a = 20, \kappa = 0.1, \xi = 0.2, \mu = 0.0435$, $R = 0.02$, $\bar{z} = 0.26$, $\rho_x = 0.8, \sigma_x = 0.005$. For fixed-maturity models, we adjust $\mu = 0.045, \bar{z} = 0.2$ for the $\lambda = 0.2146$ case and $\mu = 0.052, \bar{z} = 0.4$ for the $\lambda = 1$ case for comparability.

default loss borne by outstanding deposits is not internalized. Nonetheless, the role played by the endogenous maturity is important. The increase in non-withdrawing deposits after the rate cut greatly exacerbate the dilution problem. By comparing the blue solid and red dash lines, one can see that the surge in bank default probability arrives earlier and is much stronger in the model with endogenous maturity.

2.4 State-dependency in responses

In this section, we show that the impact of an interest rate cut is different if rates have been low for a while. Specifically, the subsequent surge in default risk following a rate cut is much stronger in a low-rate environment.

Formally, we consider a regime-switching model of interest rates. In either the high- or low-rate regime, i.e. $s \in \{H, L\}$, x follows an AR(1) process:

$$x' = (1 - \rho_x)\bar{x}(s) + \rho_x x + \sigma_x \tilde{\epsilon}$$

where $\bar{x}(H) > \bar{x}(L)$. State s shifts with probability p. In the period where the shift takes place, x is drawn from the stationary distribution of the new regime.

2.4.1 I.i.d. shocks

We first consider a case where shocks to interest rates are i.i.d. in any given regime. More specifically, we set $\rho_x = 0, \sigma_x = 0.007, \bar{x}(H) = -\bar{x}(L) = 0.015, p = 0.4$. All bank- and depositor-related parameters are set to be same as those in the i.i.d.-shock case of Section 2.3.1 without regime switches.

Figure 3 compares how banks' responses to an identical interest rate cut differ across regimes. Distances between ticks on the left and right axes are identical, meaning that magnitudes of responses are comparable. On average, in the low-rate regime (red dashed lines and right y-axes), bank leverage is high and fewer depositors withdraw their money each period due to a higher discount rate. However, a larger amount of non-withdrawing deposits worsens banks' dilution problem. Combining these two offsetting effects, average default probability turns out to be higher, which is largely consistent with our previous finding that default probability increases following interest rate cuts when deposits are endogenously long-term.

Responding to the rate cut, in the low-rate regime, banks increase leverage more aggressively and the subsequent surge in default risk is also stronger. As we have highlighted the unappealing consequence of interest rate cuts in the previous section, this result calls for central banks' exercising additional caution when the economy has been in a low-rate environment for a while and banks have a larger amount of deposits on their balance sheets. This is despite the weaker reduction in the withdrawal amounts in the low rate regime, which is driven by the stronger surge in default risk.

2.4.2 Persistent shocks

Considering persistent shocks, we set $\rho_x = 0.8, \sigma_x = 0.005$ and adjust $\mu = 0.0435$ while keeping all the other parameter values to be same as those in the i.i.d.-shock case with regime switches. Figure 4 shows that the results we find in the i.i.d.-shock case are largely preserved. Again, an interest rate cut creates a stronger surge in bank leverage and default risk in the low-rate environment.

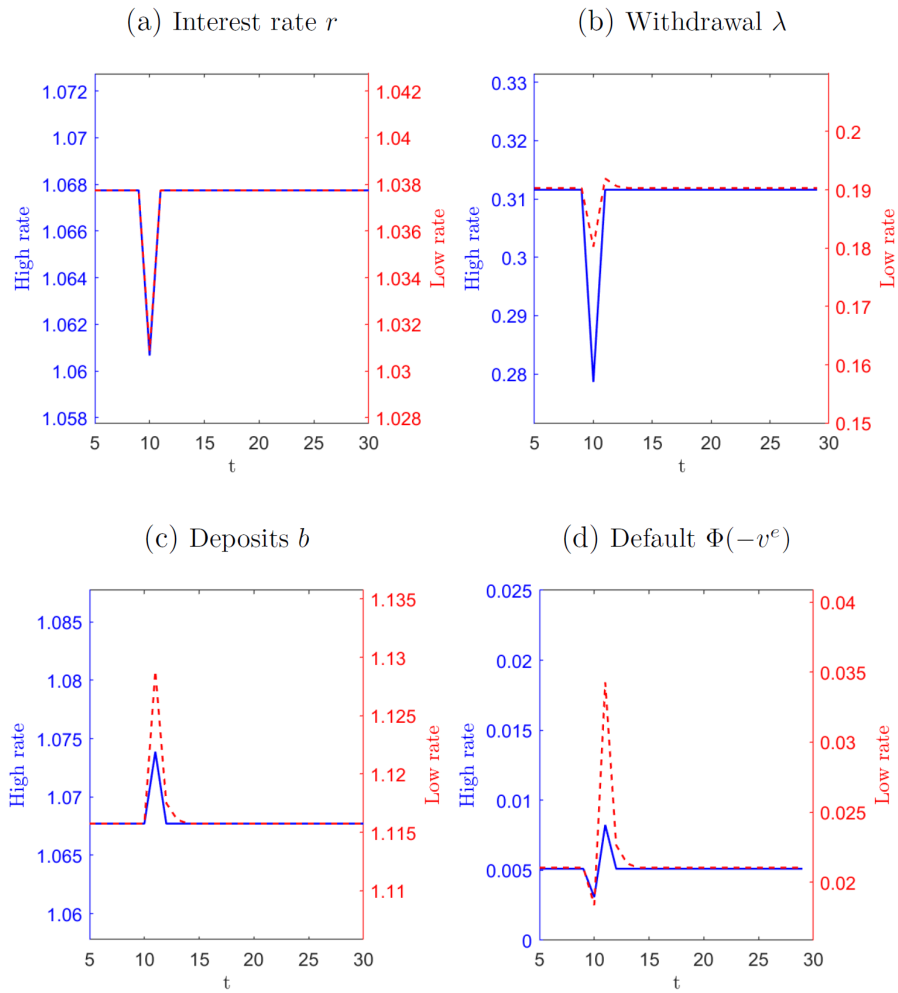

Figure 3: State dependency of banks' responses to i.i.d. interest rate shocks. *Notes*: $r^* = 1/0.95, a = 20, \kappa = 0.1, \xi = 0.2, \mu = 0.0423,\ R = 0.02,\ \bar{z} = 0.26,\ \rho_x = 0, \sigma_x = 0.007, \bar{x}(H) = 0.015, \bar{x}(L) = -0.015, p = 0.4$.

In the last two panels of Figure 4, we show the state dependency for a short-term deposit model, $\lambda = 1$, with persistent shocks. We adjust $\mu = 0.052$ and $\bar{z} = 0.4$ for comparability. We consider the same shock as in our baseline model. According to Section 2.3.1, bank leverage does not move upon i.i.d. interest rate shocks with short-term deposits, and thus there is naturally no state dependency in responses of aggregate quantities in that case. Figure 4 shows that even with persistent shocks, state dependency in b is trivial. Different from our baseline model, the average default probability of banks is lower in the low-rate regime. Also, because default probabilities are bounded by 0, default probabilities respond by less in the low-rate regime.

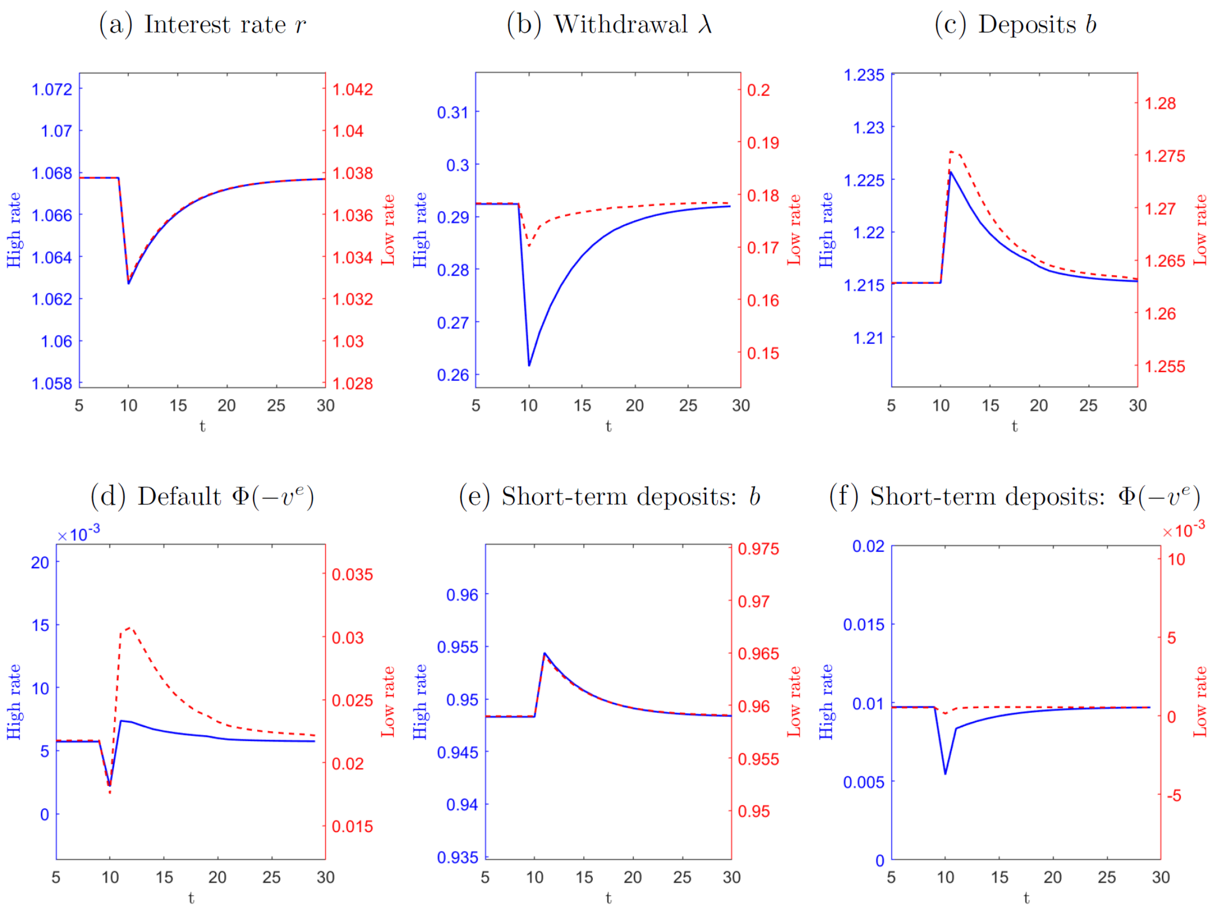

Figure 4: State dependency of banks' responses to persistent interest rate shocks. *Notes*: $r^* = 1/0.95, a = 20, \kappa = 0.1, \xi = 0.2, \mu = 0.0435$, $R = 0.02$, $\bar{z} = 0.26$, $\rho_x = 0.8, \sigma_x = 0.005, \bar{x}(H) = 0.015, \bar{x}(L) = -0.015, p = 0.4$. For the fixed-maturity model with $\hat{\lambda} = 1$, we adjust $\mu = 0.052$ and $\bar{z} = 0.4$ for comparability.

3 Optimal bank capital regulation

Banks' dilution incentive creates a role for bank capital regulation. In this section, we investigate how capital regulation addresses the inefficiency from dilution by allowing regulators to directly choose the amount of deposits banks can absorb. In analyzing this problem, we also shed light on the time inconsistency problem faced by regulators.

In this section, we fix the interest rate, $r = r^*$, and instead consider shocks to aggregate productivity R. Specifically, $R' = (1 - \rho_R)R^* + \rho_R R + \sigma_R \tilde{u}$ where R^* is the long-run productivity level and $\tilde{u} \sim \mathcal{N}(0, 1)$.

The notation of the laissez-faire economy presented in the previous section mostly carries through. As we consider aggregates, we shift to capital letters B, Q, L, V^e and V^b. Relative to the laissez-faire economy, we assume that $\mu(B)$ decreases in B quickly enough so that the regulator cannot create an infinitely large liquidity value by raising B and thus eliminate bank defaults. Such a restriction is a typical feature of deposit-in-the-utility models (e.g. Van den Heuvel, 2008)—that is, the marginal utility that households derive from holding liquidity decreases in its amount.[4]

Section 3.1 lays out the planning problem of a Ramsey regulator. Section 3.2 describes the corresponding problem of a Markov-perfect regulator without commitment, and then illustrates how capital requirements address dilution but suffer from regulator's limited commitment. Section 3.3 presents our results.

3.1 Ramsey regulator

A Ramsey regulator chooses allocations $\{V_t^e, Q_t, B_{t+1}\}_{t=0}^{\infty}$ at $t = 0$ to maximize the present value of total resources generated by this economy, taking as given banks' default rule, depositors' withdrawal rule, depositors' zero profit condition and initial condition B_0. Aggregate resources each period consist of three parts. First, bank assets provide profits R_t (i.i.d. z shocks average out). Second, bank deposits provide a liquidity value $L_t B_t$. Third, a certain fraction of banks default, which produces a total restructuring loss of $\xi B_t \Phi(-V_t^e)$. The Ramsey regulator's objective function is thus given by

$$\mathbf{E}_0 \sum_{t=0}^{\infty} \frac{1}{(r^*)^t} \Big[R_t + L_t B_t - \xi B_t \Phi(-V_t^e) \Big],$$

where the total liquidity value is given by $L_t = \mu(B_t) + \int_{Q_t+\kappa-1}^{\bar{\nu}} (\nu - \kappa) dF(\nu)$, for $t = 0, 1, ...$

For $t = 0, 1, ...$, banks' equity value and deposit price are given by

$$V_t^e = R_t - \lambda(Q_t) B_t + Q_t \{B_{t+1} - [1 - \lambda(Q_t)] B_t\} + \frac{1}{r^*} \mathbf{E}_t \left[\int_{-V_{t+1}^e}^{\bar{z}} (z + V_{t+1}^e) d\Phi(z) \right],$$

and

$$Q_t B_{t+1} = \frac{1}{r^*} \mathbf{E}_t \left[\int_{-V_{t+1}^e}^{\bar{z}} V_{t+1}^b d\Phi(z) + \int_{\underline{z}}^{-V_{t+1}^e} (z + V_{t+1}^e + V_{t+1}^b - \xi B_{t+1}) d\Phi(z) \right],$$

where the withdrawal mass is $\lambda(Q_t) = 1 - F(Q_t + \kappa - 1)$ and depositors' value is

$$V_t^b = \{L_t + \lambda(Q_t) + [1 - \lambda(Q_t)] Q_t\} B_t.$$

[4]Individual banks in the laissez-faire economy take the evolution of $\mu(B)$ as given when deciding over b'. To keep our interest rate analyses in the previous section transparent, we chose a constant μ. In this section, whenever we compute moments of the laissez-faire economy, we take into account the (b, B) problem.

Finally, there are no-Ponzi conditions: $\lim_{t\to\infty} \mathbf{E}_0 \frac{B_t}{(r^*)^t} = 0$ and $\lim_{t\to\infty} \mathbf{E}_0 \frac{V_t^e}{(r^*)^t} = 0$.

3.2 Markov-perfect regulator without commitment

We first present the problem of a Markov-perfect regulator and illustrate how capital requirements address dilution of long-term deposits by comparing it to the problem of laissez-faire banks. We then describe regulators' commitment problem by comparing Ramsey and Markov-perfect regulators. Importantly, the time inconsistency for capital requirements is also rooted in the long-term nature of deposits.

3.2.1 Time-consistent capital requirements and dilution

By rewriting the Ramsey regulator's objective recursively and assuming that the regulator optimizes B' period by period, we get the Markov-perfect regulator's problem. State variables are aggregate profit R and total deposits B. Total bank franchise value is given by:

$$H(R,B) = \max_{B'} \left\{ R + L\left(B, Q\left(R, B'\right)\right) B - \xi B \Phi\left(-V^e\left(R, B, B'\right)\right) + \frac{1}{r^*} \mathbf{E}_{R'|R} H(R', B') \right\}, \quad (6)$$

where the liquidity value is $L(B,Q) = \mu(B) + \int_{Q+\kappa-1}^{\bar{\nu}} (\nu - \kappa) dF(\nu)$.

Bank equity value, given current states and regulator's choice for B', is given by:

$$\begin{aligned} V^e(R,B,B') = R - \lambda(Q(R,B'))B + Q(R,B')\{B' - [1 - \lambda(Q(R,B'))]B\} \\ + \frac{1}{r^*} \mathbf{E}_{R'|R} \left[\int_{-V^e(R,B',B'')}^{\bar{z}} [z + V^e(R,B',B'')] d\Phi(z) \right], \quad (7) \end{aligned}$$

where $B''(R', B')$ is the optimal policy of the future regulator, which the current regulator take as given. The deposit pricing function is given by:

$$\begin{aligned} Q(R,B')B' = \frac{1}{r^*} \mathbf{E}_{R'|R} \Bigg[\int_{-V^e(R',B',B'')}^{\bar{z}} V^b(R',B',B'') d\Phi(z) \\ + \int_{\underline{z}}^{-V^e(R',B',B'')} [z + V^e\left(R',B',B''\right) + V^b\left(R',B',B''\right) - \xi B'] d\Phi(z) \Bigg], \quad (8) \end{aligned}$$

with withdrawal mass $\lambda(Q) = 1 - F(Q + \kappa - 1)$ and depositors' value is:

$$V^b(R,B,B') = \{L(B, Q(R,B')) + \lambda(Q(R,B')) + Q(R,B')[1 - \lambda(Q(R,B'))]\} B. \quad (9)$$

Now we are ready to show how capital regulation addresses deposit dilution by comparing the Markov-perfect regulator's choice for B' and what banks in the laissez-faire economy choose. Using conjecture-and-verify approach, we can straightforwardly show that:[5]

$$H(R,B) = \max_{B'} V^e(R,B,B') + V^b(R,B,B') - \xi B \Phi\left(-V^e(R,B,B')\right). \tag{10}$$

In contrast, laissez-faire banks only maximizes $V^e(R,B,B')$ or equivalently its monotone transformation $V^e(R,B,B') - \xi B\Phi\left(-V^e(R,B,B')\right)$ while ignoring the term $V^b(R,B,B')$.

First, for simplicity, we consider a fixed-maturity deposit model with a flat $\mu(B)$, i.e. $L(.) = \mu$ and $\lambda(.) = \hat{\lambda}$ and thus $V^b(R,B,B') = [\mu + \hat{\lambda} + Q(R,B')(1-\hat{\lambda})]B$. If deposits are modeled short-term, i.e. $\hat{\lambda} = 1$, the choice for B' does not affect $V^b(R,B,B')$. Markov-perfect regulator's optimal choice is then identical to that of laissez-faire banks. This is consistent with what has been shown in Section 2.2—with short-term deposits, banks do not suffer a commitment problem in deposit absorption, and thus capital requirements cannot improve efficiency. When $\hat{\lambda} > 1$, there exists a positive wedge between first-order conditions for banks and Markov-perfect regulator: $(1-\hat{\lambda})B\frac{\partial Q(R,B')}{\partial B'}$.

Furthermore, as we have mentioned in Section 2.2, endogenous withdrawals affect dilution. In our baseline model, the wedge between first-order conditions for banks and the Markov-perfect regulator is $[1-\lambda(Q(R,B'))]B\frac{\partial Q(R,B')}{\partial B'}$. When the deposit amount B is large already, increasing it on the margin, i.e. $B' > B$, triggers more withdrawals and thus reduces non-withdrawing deposits $[1-\lambda(Q(R',B''))]B'$, in contrast to the fixed-maturity case. As the wedge shrinks, dilution becomes less severe. In other words, depositors' option to withdraw disciplines banks' deposit dilution. As banks ratchet up their deposits, deposit maturity gets shortened and thus dilution severity does not grow as quickly.[6]

3.2.2 Long-term deposits and regulator's commitment

To illustrate the time inconsistency problem of the regulator, we start with the fixed-maturity long-term deposit model. The Markov-perfect regulator fully eliminates deposit dilution, the Ramsey regulator chooses not to do so. The reasons are as follows. There are two possible actions that laissez-faire banks can take to destroy value. First, they can dilute non-withdrawing depositors when choosing deposits b'. Second, they can choose not to repay depositors and default. Default, when the equity value turns negative, is optimal ex

[5] In detail, by adding up $V^e(R,B,B')$ and $V^b(R,B,B')$ in (7) and (9), we get:

$$R + L(R,B,B')B + \frac{1}{r^*}\mathbf{E}_{R'|R}\left[V^e(R',B',B'') + V^b(R',B',B'') - \xi B'\Phi\left(-V^e(R',B',B'')\right)\right].$$

Conjecture $H(R,B) = \max_{B'} V^e(R,B,B') + V^b(R,B,B') - \xi B\Phi\left(-V^e(R,B,B')\right)$, and we then get back (6).

[6] Our numerical solutions confirm that the policy function $B'(R,B)$ for laissez-faire banks becomes much flatter after incorporating withdrawals.

post but not ex ante. With the presence of the default friction, eliminating the dilution friction is no longer optimal. Instead, promising to banks some dilution opportunities in the future might be helpful in persuading them to default less today. However, a Markov-perfect regulator without commitment cannot do so because it is no longer optimal to allow dilution tomorrow after banks have decided to not default today.

Importantly, that dilution in the future is helpful in preventing default today results from the long-term nature of deposits. For simplicity, we fix R and consider a regulator with partial commitment who chooses B_1 and B_2 at time $t = 0$ but then follows Markov-perfect regulator's optimal policy for $t > 2$. The problem, for given B_0, is

$$\max_{B_1,B_2} R + \mu B_0 - \Phi(-\hat{V}_0^e)\xi B_0 + \frac{1}{r^*}\Big[V^e(B_1,B_2) + V^b(B_1,B_2) - \xi B_1 \Phi(-V^e(B_1,B_2))\Big], \quad (11)$$

where the equity value at time 0 is given by:

$$\hat{V}_0^e = R - \hat{\lambda}B_0 + [B_1 - (1-\hat{\lambda})B_0]\hat{Q}_1 + \frac{1}{r^*}\left[\int_{-V^e(B_1,B_2)}^{\bar{z}} [z + V^e(B_1,B_2)]d\Phi(z)\right],$$

and the deposit price is:

$$\hat{Q}_1 B_1 = \frac{1}{r^*}\left[\int_{-V^e(B_1,B_2)}^{\bar{z}} V^b(B_1,B_2)d\Phi(z) + \int_{-\bar{z}}^{-V^e(B_1,B_2)} [z + V^b(B_1,B_2) + V^e(B_1,B_2) - \xi B_1]d\Phi(z)\right].$$

We know from (10) that the Markov-perfect regulator picks B_2 at $t = 1$ to maximize the last term in (11) given pre-determined B_1. However, with partial commitment to B_2, the regulator internalizes how bank default loss $\Phi(-\hat{V}_0^e)\xi B_0$ today would be affected. It is easy to show that:

$$\hat{V}_0^e = R - \hat{\lambda}B_0 - (1-\hat{\lambda})B_0\hat{Q}_1 + \frac{1}{r^*}\Big[V^e(B_1,B_2) + V^B(B_1,B_2) - \xi B_1\Phi(-V^e(B_1,B_2))\Big]. \quad (12)$$

Without non-withdrawing deposits, i.e. $(1-\hat{\lambda})B_0 = 0$, Markov-perfect regulator's policy that maximizes the last term in (11) also maximizes $\hat{V}_0^e$ today. In that case, default loss is minimized and thus his policy for B_2 is also optimal from today's perspective. However, long-term deposits create a potential conflict between the optimal regulation tomorrow and what is optimal from today's perspectives—that is, committing to not fully eliminating dilution might decrease debt burden of banks $(1-\hat{\lambda})B_0\hat{Q}_1$ and thus result in fewer bank defaults today.

Endogenous withdrawals complicate the analysis. Importantly, the regulator's optimal deposit choice tomorrow might not be optimal today because of not only the default options possessed by banks but also the withdrawal options possessed by depositors. In this case, regulator with partial commitment will take into account that B_2 influences the withdrawal choice of depositors at $t = 0$, which in turn changes both $\hat{V}_0^e$ in (12) and liquidity value at $t = 0$ in (11).

3.3 Laissez-faire, Ramsey and Markov-perfect regulators

To summarize our previous discussions, when deposits are modeled to be short-term and dilution is absent, laissez-faire banks and regulators (Markov-perfect and Ramsey) adopt the same deposit policy. Capital regulation and regulator commitment are not relevant. When deposits are long-term, even with a fixed maturity, capital regulation creates value by reducing dilution and regulators face a commitment problem. In this section, we numerically solve these three models for fixed and endogenous maturities, and we compare the optimal policies.

Two issues are worth noting at this point. First, for the laissez-faire case, since we have assumed in this section that $\mu(B)$ is no longer constant in B, we take into account the (b, B) problem. For instance, when choosing b', banks forecast B and its evolution. On the equilibrium path, b and B are consistent with each other.

Second, for the Ramsey problem, we show the existence of a pseudo steady state, in some aggregate quantities. Specifically, B_t, Q_t and V_t^e, are constant. However, Lagrange multipliers keep growing at a speed under which the no-Ponzi conditions are satisfied. Such non-stationarity precludes the use of standard numerical approaches based on dynamic programming or first-order conditions. To study the responses to shocks, we reformulate the Ramsey regulator's problem recursively with the "promised equity value" to banks as an additional state variable. Details are given in the appendix.

3.3.1 Steady-state comparisons

Table 1 shows deterministic steady states for the three economies. Since capital regulation becomes more relevant when bank default risk is nontrivial, for example around financial crises, we consider in this section a parameter combination that produces a higher default risk relative to that used in Section 2.[7] For baseline models with endogenous withdrawals, we set

[7]Our results largely carry through under parameters close to those in Section 2, although quantitative magnitudes are a bit smaller since default risk is smaller.

$r^* = 1/0.9$ and $\mu(B) = 0.1245 - 0.012 \times B$, representing a crisis where safe liquidity becomes more valuable but banks are riskier. We set all bank- and depositor-related parameters not mentioned to be the same as those in Section 2.3.1. For fixed-maturity models, we choose $\hat{\lambda} = 0.3$ and re-adjust upward $\mu(B) = 0.13 - 0.012 \times B$ so that laissez-faire economies under fixed- and endogenous-maturity have similar steady-state deposits and withdrawal masses.

	Endogenous-maturity			Fixed-maturity		
Moments	Laissez-faire	Ramsey	MP	Laissez-faire	Ramsey	MP
B_{ss}	0.5594	1.1269	0.8104	0.5610	1.1045	1.1026
V^e_{ss}	0.1608	0.2112	0.2206	0.1929	0.2324	0.2337
$1 - B_{ss}/H_{ss}$	0.2652	0.2337	0.2958	0.3108	0.2494	0.2505
Q_{ss}	0.9594	1.0055	1.0328	0.9900	1.0096	1.0100
$\Phi(-V^e_{ss})$	0.0952	0.0248	0.0163	0.0457	0.0081	0.0074
λ_{ss}	0.3048	0.1213	0.0703	0.3	0.3	0.3
L_{ss}	0.1207	0.1177	0.1206	0.1233	0.1167	0.1168
H_{ss}	0.7613	1.4706	1.1508	0.8140	1.4715	1.4711

Table 1: Steady states of laissez-faire and regulated economies. *Notes*: $r^* = 1/0.9, \xi = 0.2, \kappa = 0.1, a = 20, \mu = 0.1245 - 0.012 \times B, R^* = 0.02, \rho_R = 0, \sigma_R = 0, \bar{z} = 0.26$. For the fixed-maturity model with $\hat{\lambda} = 0.3$, we adjust $\mu = 0.13 - 0.012 \times B$ for comparability between laissez-faire economies.

The table presents two main findings. The first is that in regulated economies default rates are a lot lower but the amounts of deposits are a lot higher. By addressing dilution, capital regulation actually increases the steady-state amount of deposits B_{ss} that banks absorb. Without capital requirements, banks' incentive to dilute ex post is punished heavily by a large deposit spread at the issuance stage, implying that depositors find dilution too destructive and are unwilling to put their money into banks. Capital regulation serves as a commitment device and assures depositors that their money is safe to some extent. Even though steady states of regulated economies admit more deposits, default probabilities $\Phi(-V^e_{ss})$ are much smaller. A larger liquidity value $L_{ss}B_{ss}$ that banks capture each period translates into a larger equity V^e_{ss}, which in turn reduces bank defaults.

That the amount of deposits in the laissez-faire is smaller than in the regulated economies does not imply capital requirements are not binding. For instance, in the steady states of two Markov-perfect regulated economies, we have verified that the bank equity value function $V^e(R, B, B')$ is locally increasing in B' when we evaluate at the point $(B, B') = (B_{ss}, B_{ss})$. This means that banks themselves, if having a one-shot opportunity, would like to absorb more deposits than the B_{ss} chosen by Markov-perfect regulator.

The second finding is that the differences between Markov-perfect and Ramsey regulators are significant, but only with endogenous withdrawal, not with fixed maturity. In particular, the differences are large for steady state levels of debt, B_{ss}, and the equity over asset ratios, $(H_{ss} - B_{ss})/H_{ss} = 1 - B_s/H_{ss}$. With endogenous withdrawal, the choices of future regulators affect today's withdrawal decisions, which directly affect depositors' liquidity value today. In addition, withdrawal decisions affect banks' equity values and default incentives. Therefore, endogenous maturity creates a powerful channel for commitment to matter which is absent in the fixed-maturity case.

3.3.2 Delevering an indebted economy

After the steady state properties, we consider here transition dynamics to the steady state. We are interested in how banks and regulators delever a highly indebted economy. In each model, we start with a B_0 such that $B_0 - B_{ss} > 0$ are identical across models. For the Ramsey case, we assume that the regulator is not bound by past commitments when inheriting B_0. This is a reasonable starting point for thinking about setting up a new regulatory regime, for instance after the occurrence of a crisis.

Figure 5 plots the absolute deviations from steady state in the deleveraging process, i.e. $B_t - B_{ss}$ and $\Phi(-V_t^e) - \Phi(-V_{ss}^e)$. It demonstrates that, when deposit maturity is endogenous, differences are small between regulated and non-regulated economies, as well as between the two regulated economies.

Endogenous withdrawals make a big difference for how laissez-faire banks delever. This is due to the fact that depositors' withdrawals discipline dilution. When deposit maturity is fixed, the dilution problem is severe and banks act slowly in terms of buying back debt. In contrast, banks in our baseline model end up delevering much more quickly because, otherwise, depositors would pull money out of their accounts.

Somewhat surprisingly, there is little difference between Ramsey and Markov-perfect regulators, whether withdrawals are endogenous or deposit maturity is fixed. By definition, a Ramsey solution includes history-dependence induced by commitment that is absent in the Markov-perfect case. Despite that, this does not translate into additional persistence in deleveraging. In particular, the Ramsey regulator does not delever more slowly than the Markov-perfect regulator in Figure 5.

3.3.3 Regulatory responses to shocks

This section shows the dynamics of regulated economies in response to shocks to asset productivity R in our endogenous-maturity model.[8] Different from the deleveraging experiment

[8]Dynamics within fixed-maturity models are similar, even though quantitative magnitudes are different. They are not reported here to save space.

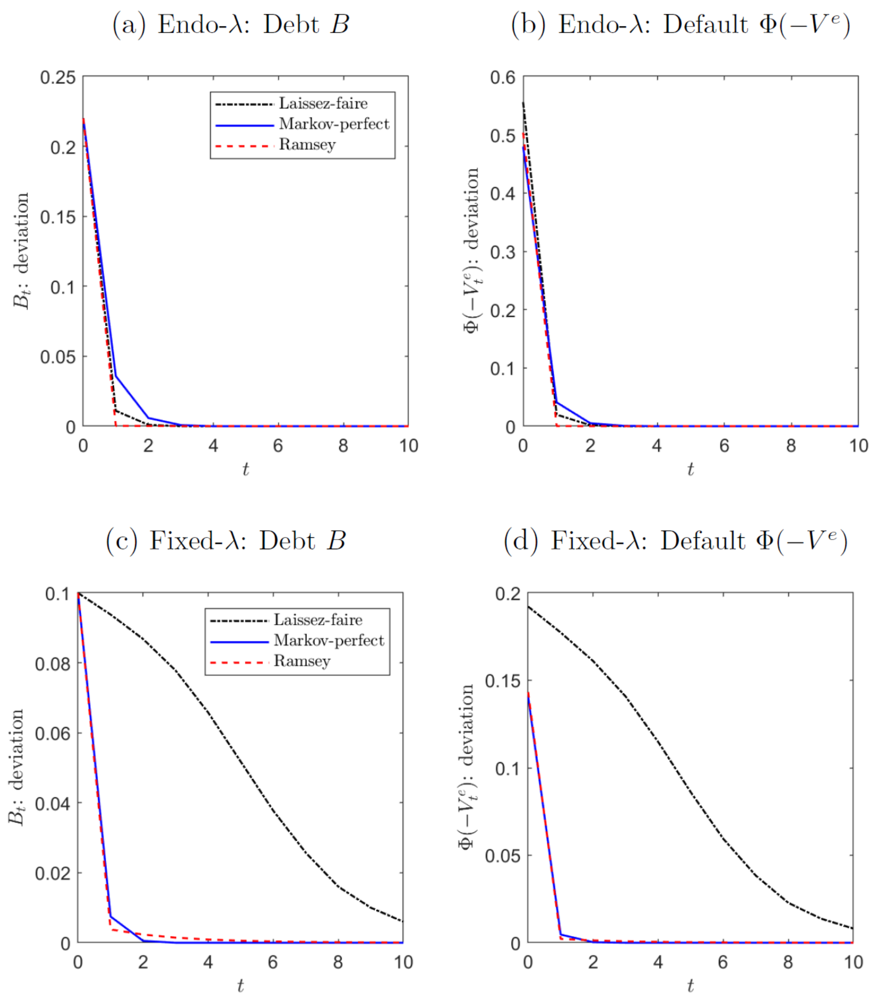

Figure 5: Delever an indebted economy. *Notes*: Parameters are identical to those in Table 1.

conducted in the previous section, the Ramsey regulator's behavior here is governed by the commitments made in the past. In other words, we are studying here the optimal capital requirements in an established policy regime. This experiment is informative about the optimal setting of a countercylical capital buffer (CCyB) as introduced in Basel III.

Figure 6 reports the impulse responses to a large i.i.d. R shock at $t = 10$, which represents a recession caused by, for example, a housing crisis or a pandemic that lasts for one year. Upon the shock, bank equity values fall and therefore bank defaults become more likely. By allowing more deposits, both regulators inflate the equity value and incentivize banks to not default.

Importantly, there is a clear difference in terms of policy persistence between the two regulators. The Markov-perfect regulator increases deposits sharply right upon the shock but then quickly delevers as R reverts to its long-run level. In contrast, the Ramsey regulator commits to allow increased bank deposits for extended periods even though it is value de-

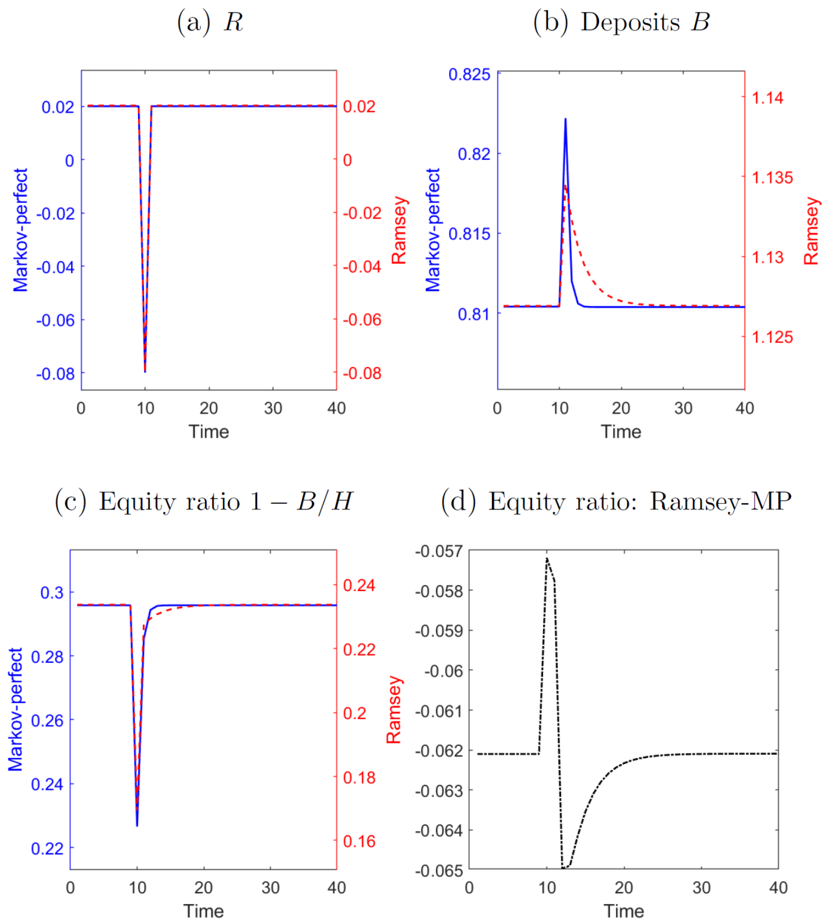

Figure 6: Regulator's commitment and IRFs to i.i.d. R shocks. *Notes*: $\rho_R = 0, \sigma_R = 0.1$, and the other parameters are identical to those in Table 1.

stroying ex post. By doing so,the Ramsey regulator is able to better reduce rollover pressures and thus prevent bank defaults today. The lower panel displays the equity ratios $1 - B/H$ in the two regulated economies and the difference between them. Relative to the Markov-perfect regulator, the Ramsey regulator reduces the equity ratio at $t = 11$ by less. After that, the Ramsey regulator keeps the equity ratio lower for several periods.

Figure 7 considers a typical business cycle shock, i.e. a small but persistent drop in R, specifically with $\rho_R = 0.9$ and $\sigma_R = 0.01$. In this case, aggregate bank deposits shrink drastically due to the long-lasting increase in default risk. So does the asset value of banks. Combining both, changes in capital requirements turn out to be not as drastic. As regulators expect the low-productivity scenario to be long-lasting, they optimally tighten the capital requirements to alleviate future default losses. The impact of commitment is comparable to that in the i.i.d. shock case. The Ramsey regulator imposes a lower equity ratio for quite a long time, relative to the Markov-perfect regulator. This again reflects the fact that the former can better alleviate the impact of the shock at impact by committing to allow a higher bank leverage in the future

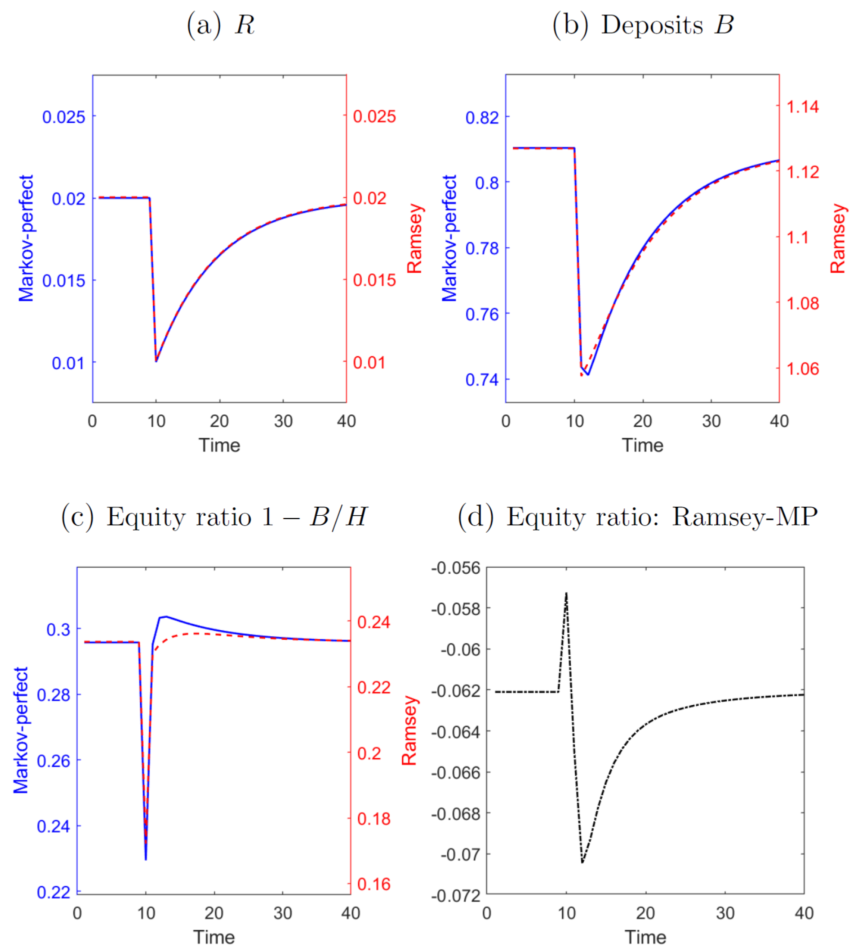

Figure 7: Regulator's commitment and IRFs to persistent R shocks. *Notes*: $\rho_R = 0.9, \sigma_R = 0.01$, and the other parameters are identical to those in Table 1.

4 Conclusions

The macro-finance literature has found it convenient to model bank liabilities as short-term debt. However, bank deposits are non-maturing, and this effectively converts deposits into long-term debt subject to dilution. In this paper, we have demonstrated that explicitly modeling the non-maturing nature of deposits significantly changes the dynamic responses of banks to interest rate shocks and therefore the impact of monetary policy shocks on the banking sector. The macro-finance literature has also found it convenient to model regulatory policies with adhoc policy rules. With deposits subject to dilution, optimal banking regulation becomes subject to a time-inconsistency problem. The results of this paper show that explicitly accounting for regulator commitment has first-order consequences for optimal macroprudential policies.

References

Admati, Anat R., Peter DeMarzo, Martin F. Hellwig, and Paul Pfleiderer, 2018, "The Leverage Ratchet Effect." *Journal of Finance*, 73.1:145-198.

Angeloni, Ignazio, and Ester Faia, 2013, "Capital Regulation and Monetary Policy with Fragile Banks." *Journal of Monetary Economics* 60.3: 311-324.

Begenau Juliane, 2020, "Capital Requirements, Risk Choice, and Liquidity Provision in a Business-Cycle Model." *Journal of Financial Economics*, 136.2: 255-378.

Begenau, Juliane, and Tim Landvoigt, 2020, "Financial Regulation in a Quantitative Model of the Modern Banking System." Working paper.

Bianchi, Javier and Saki Bigio, 2020, "Banks, Liquidity Management and Monetary Policy." accepted *Econometrica*.

Bolton, Patrick, Ye Li, Neng Wang and Jinqiang Yang, 2020, "Dynamic Banking and the Value of Deposits." NBER Working paper 28298.

Brunnermeier, Markus K., and Yann Koby, 2018, "The Reversal Interest Rate." No. w25406. National Bureau of Economic Research.

Chari, Varadarajan V. and Patrick J. Kehoe, 2016, "Bailouts, Time Inconsistency, and Optimal Regulation: A Macroeconomic View." *American Economic Review* 106(9): 2458-93.

Chien, YiLi, and Yi Wen, 2017, "Optimal Ramsey Capital Income Taxation—A Reappraisal." FRB St. Louis Working Paper.

Davydiuk, Tetiana, 2017, "Dynamic Bank Capital Requirements." Working paper.

DeMarzo, Peter M., and Zhiguo He, 2020. "Leverage Dynamics without Commitment." *Journal of Finance*, forthcoming.

Corbae, Dean and Pablo D'Erasmo, 2019, "Capital Requirements in a Quantitative Model of Banking Industry Dynamics." NBER Working Paper 25424.

Crouzet, Nicholas, 2017, "Default, Debt Maturity and Investment Dynamics." Working Paper.

Drechsler, Itamar, Alexi Savov, and Philipp Schnabl, 2020, "Banking on Deposits: Maturity Transformation without Interest Rate Risk." *Journal of Finance*, forthcoming.

Elenev, Vadim, Tim Landvoigt, and Stijn Van Nieuwerburgh, 2020, “A Macroeconomic Model with Financially Constrained Producers and Intermediaries.”accepted *Econometrica.*

Gertler, Mark, and Peter Karadi. 2011, “A Model of Unconventional Monetary Policy.” *Journal of Monetary Economics* 58.1: 17-34.

Gertler, Mark, and Nobuhiro Kiyotaki, 2015, “Banking, Liquidity, and Bank Runs in an Infinite Horizon Economy.” *American Economic Review*, 105(7): 2011-43.

Gertler, Mark, Nobuhiro Kiyotaki, and Andrea Prestipino, 2020, “Credit Booms, Banking Crises and Macroprudential Policy.” *Review of Economic Dynamics*, Special Issue: ”Frontier of Business Cycle Research”, forthcoming.

Gomes, Joao F., Urban J. Jermann, and Lukas Schmid, 2016, “Sticky Leverage.” *American Economic Review* 106(12): 3800-3828.

Hutchison, David E., and George G. Pennacchi, 1996, “Measuring Rents and Interest Rate Risk in Imperfect Financial Markets: The Case of Retail Bank Deposits.” *Journal of Financial and Quantitative Analysis*: 399-417.

Jarrow, Robert A., and Donald R. van Deventer, 1998, “The Arbitrage-Free Valuation and Hedging of Demand Deposits and Credit Card Loans.” *Journal of Banking and Finance* 22.3: 249-272.

Malherbe, Frederic. 2020. “Optimal Capital Requirements over the Business and Financial Cycles.” *American Economic Journal: Macroeconomics*, 12(3):139-74.

Mendicino, Caterina, Kalin Nikolov, Javier Suarez, and Dominik Supera, 2018, “Optimal Dynamic Capital Requirements.” *Journal of Money, Credit and Banking* 50 (6), 1271-1297.

Nyström Kaj, 2008, “On Deposit Volumes and the Valuation of Non-Maturing Liabilities.” *Journal of Economic Dynamics and Control*, 32(3): 709-756

Porcellacchia, Davide, 2020, “The Tipping Point: Low Rates and Financial Stability.”ECB Working Paper Series.

Schroth, Josef, 2020, “Macroprudential Policy with Capital Buffers.”accepted *Journal of Monetary Economics*

Straub, Ludwig and Iván Werning. 2020. “Positive Long-Run Capital Taxation: Chamley-Judd Revisited.” *American Economic Review*, 110.1: 86-119.

Van den Heuvel, Skander J., 2008, “The Welfare Cost of Bank Capital Requirements”, *Journal of Monetary Economics* 55, 298–320.

Van der Ghote, Alejandro, 2021, “Interactions and Coordination between Monetary and Macroprudential Policies.” *American Economic Journal: Macroeconomics*, 13(1):1-34.

Wolff, Alex, 2000, “Valuing Mon-Maturity Deposits: Two Alternative Approaches”, *Balance Sheet*, Vol. 8 No. 4, pp. 21-22. https://doi.org/10.1108/09657960010373437

Xiang, Haotian, 2018, “Corporate Debt Choice and Bank Capital Regulation.” Working paper.

Xiang, Haotian, 2020, “Time Inconsistency and Financial Covenants.” Working paper.

5 Appendix

5.1 Ramsey pseudo steady states

In this section we show that there is a pseudo steady state for the Ramsey regulator's allocation. Specifically, real variables B_t, Q_t and V_t^e can be at a stationary point while Lagrange multipliers are not. This property is present in both cases, with or without endogenous maturity. We check convergence from arbitrary initial states numerically with a finite horizon version of the model by increasing the number of periods.

To save space, we present only the endogenous-maturity case. The Lagrangian is given by:

$$\begin{aligned}\max_{\{V_t^e,Q_t,B_{t+1},\gamma_t,\zeta_t\}_{t=0}^{\infty}} \mathbf{E}_0 \sum_{t=0}^{\infty} \frac{1}{(r^*)^t} \Bigg\{ & R_t + L(B_t,Q_t)B_t - \xi B_t \Phi(-V_t^e) \\ & + \gamma_t \left[R_t - \lambda(Q_t)B_t + Q_t[B_{t+1} - (1-\lambda(Q_t))B_t] + \frac{1}{r^*}\mathbf{E}_t\left[\int_{-V_{t+1}^e}^{\bar{z}} (z + V_{t+1}^e) d\Phi(z)\right] - V_t^e \right] \\ & + \zeta_t \Bigg[\frac{1}{r^*}\mathbf{E}_t \Bigg[[L(B_{t+1},Q_{t+1}) + \lambda(Q_{t+1}) + (1-\lambda(Q_{t+1}))Q_{t+1}]\, B_{t+1} \\ & \qquad + \int_{\underline{z}}^{-V_{t+1}^e} (z + V_{t+1}^e - \xi B_{t+1}) d\Phi(z) \Bigg] - Q_t B_{t+1} \Bigg] \Bigg\},\end{aligned}$$

where $L(B_t,Q_t) = \mu(B_t) + \int_{Q_t+\kappa-1}^{\bar{\nu}} (\nu - \kappa) dF(\nu)$ and $\lambda(Q_t) = 1 - F(Q_t + \kappa - 1)$; γ_t and ζ_t are two Lagrange multipliers; B_0 is predetermined.

Three first-order conditions together with two constraints determine the allocation. After imposing time invariance in all real variables but not the multipliers, we get the following five steady-state equations:

$$\begin{aligned}&\frac{1}{r^*}[L_{ss} + B_{ss}L_{ss}^B - \xi\Phi(-V_{ss}^e)] + \frac{1}{r^*}\gamma_{t+1}[-\lambda_{ss} - Q_{ss}(1-\lambda_{ss})] + \gamma_t Q_{ss}, \\ &\qquad + \zeta_t \left[\frac{1}{r^*}[\lambda_{ss} + L_{ss} + B_{ss}L_{ss}^B + (1-\lambda_{ss})Q_{ss} - \xi\Phi(-V_{ss}^e)] - Q_{ss}\right] = 0, \end{aligned} \tag{13}$$

$$\xi B_{ss}\phi(-V_{ss}^e) - \gamma_t + \gamma_{t-1}[1 - \Phi(-V_{ss}^e)] + \zeta_{t-1}[\Phi(-V_{ss}^e) + \xi B_{ss}\phi(-V_{ss}^e)] = 0, \tag{14}$$

$$L_{ss}^Q + \gamma_t(-\lambda_{ss}^Q + \lambda_{ss} + Q_{ss}\lambda_{ss}^Q) - \zeta_t + \zeta_{t-1}(\lambda_{ss}^Q + L_{ss}^Q + 1 - \lambda_{ss} - \lambda_{ss}^Q Q_{ss}) = 0, \tag{15}$$

$$R^* - \lambda_{ss}B_{ss} + Q_{ss}[B_{ss} - (1-\lambda_{ss})B_{ss}] + \frac{1}{r^*}\int_{-V_{ss}^e}^{\bar{z}} (z + V_{ss}^e) d\Phi(z) - V_{ss}^e = 0,$$

$$\frac{1}{r^*}\left[[\lambda_{ss} + L_{ss} + (1-\lambda_{ss})Q_{ss}]B_{ss} + \int_{\underline{z}}^{-V_{ss}^e} (V_{ss}^e + z - \xi B_{ss}) d\Phi(z)\right] - Q_{ss}B_{ss} = 0,$$

where L^B and L^Q represent derivatives of $L(B_t, Q_t)$ with respect to B_t and Q_t respectively; λ^Q represents the derivative of $\lambda(Q_t)$ with respect to Q_t.

Define $\gamma_t^* = \gamma_t + 1$ and $\zeta_t^* = \zeta_t + 1$. Equations (13), (14) and (15) evolve into:

$$\gamma_{t+1}^* = A^0\gamma_t^* + A^1\zeta_t^*, \tag{16}$$

$$\gamma_t^* = B^0\gamma_{t-1}^* + B^1\zeta_{t-1}^*, \tag{17}$$

$$\zeta_t^* = \Omega_{ss}B^0\gamma_{t-1}^* + [\Omega_{ss}B^1 + (1 + L_{ss}^Q - \Omega_{ss})]\zeta_{t-1}^*, \tag{18}$$

where $\Omega_{ss} = \lambda_{ss} + (Q_{ss} - 1)\lambda_{ss}^Q$ and

$$\begin{aligned}
A^0 &= \frac{r^*Q_{ss}}{\lambda_{ss} + (1-\lambda_{ss})Q_{ss}}, \\
A^1 &= \frac{\lambda_{ss} + L_{ss} + B_{ss}L_{ss}^B + (1-\lambda_{ss})Q_{ss} - \xi\Phi(-V_{ss}^e) - r^*Q_{ss}}{\lambda_{ss} + (1-\lambda_{ss})Q_{ss}}, \\
B^0 &= 1 - \Phi(-V_{ss}^e), \\
B^1 &= \Phi(-V_{ss}^e) + \xi\phi(-V_{ss}^e)B_{ss}.
\end{aligned}$$

Some manipulations yield:

$$\zeta_t^* = \left[[\Omega_{ss}B^1 + (1 + L_{ss}^Q - \Omega_{ss})] - \Omega_{ss}B^0\frac{A^1 - B^1}{A^0 - B^0}\right]\zeta_{t-1}^*.$$

We know that $(A^0 - B^0)\gamma_t^* + (A^1 - B^1)\zeta_t^* = 0$, which means

$$\left[[\Omega_{ss}B^1 + (1 + L_{ss}^Q - \Omega_{ss})](A^1 - B^1) - \Omega_{ss}B^0\frac{(A^1 - B^1)^2}{A^0 - B^0} + A^1(A^0 - B^0) - A^0(A^1 - B^1)\right]\zeta_{t-1}^* = 0.$$

Setting the term in the bracket to zero gives us the restriction we need to solve for B_{ss}, Q_{ss} and V_{ss}^e. We verify numerically that $1 < [\Omega_{ss}B^1 + (1 + L_{ss}^Q - \Omega_{ss})] - \Omega_{ss}B^0\frac{A^1 - B^1}{A^0 - B^0} < r^*$—that is, multipliers are growing without violating the no-Ponzi game conditions.

5.2 Formulating Ramsey regulator with promised equity values

Since the Lagrange multipliers are non-stationary but not the real quantities, we resort to a version of the ”promised-utility” approach to solve for impulse responses of Ramsey regulator to shocks. We have verified numerically that the steady state of this recursive problem and that of the sequential problem outlined previously are indeed identical.

Denote the equity value promised by Ramsey regulator to banks, net of the i.i.d. z shocks, as v. The state space evolves into $\{R, B, v\}$ and Ramsey regulator chooses tomorrow's deposits B' and a contingent plan for promised utilities $v'(R')$ in each state.

Again, to lighten the notations here, we lay out only our formulation under endogenous maturity. It is given by:

$$H(R,B,v) = \max_{B',v'(R')} R + L(B, Q(B', v'(R'); R))B - \xi B\Phi(-v) + \frac{1}{r^*}\mathbf{E}_{R'|R} H(R', B', v'(R')),$$

subject to promise keeping:

$$\begin{aligned} v = R - \lambda(Q(B', v'(R'); R))B + Q(B', v'(R'); R)\{B' - [1 - \lambda(Q(B', v'(R'); R))]B\} \\ + \frac{1}{r^*}\mathbf{E}_{R'|R}\left[\int_{-v'(R')}^{\bar{z}} [v'(R') + z] d\Phi(z)\right], \end{aligned}$$

and a pricing constraint:

$$\begin{aligned} & Q(B', v'(R'); R)B' \\ & = \frac{1}{r^*}\mathbf{E}_{R'|R}\Bigg[\{\lambda(Q(B'', v''(R''); R')) + L(B, Q(B'', v''(R''); R')) \\ & \qquad + [1 - \lambda(Q(B'', v''(R''); R'))]Q(B'', v''(R''); R')\}B' \\ & \qquad + \int_{-\bar{z}}^{-v'(R')} [v'(R') + z - \xi B'] d\Phi(z)\Bigg]. \end{aligned}$$

It is well-acknowledged that computing models with long-term debt is non-trivial. The contingent promised equity values $v'(R')$ represent an additional challenge which precludes the use of dynamic programming. The model is solved by combining the system of first-order conditions with the steady-state solution.