Housing Prices and Real Investment: Collateral vs. Crowding-Out Effects

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Abstract

There is active debate about whether high real estate prices benefit or hurt investment. In this paper, we propose a theoretical framework to evaluate policies that target real estate prices and investment. Our model incorporates two documented empirical effects that real estate prices can have on investment — a collateral channel and a crowding-out channel. Optimal policy depends on the relative magnitude of these two effects and requires opposing interventions in demand and supply, i.e. subsidizing one while taxing the other. The distribution of collateral, rather than its aggregate level, determines the net effect of real estate prices on surplus.

Keywords: Collateral, crowding-out, investment, housing taxes and subsidies.

JEL Classifications: D25, E22, G11, H21, R21, R31, R38.

1. Introduction

Real estate prices are often at the center of debate in economic policy. One of the key goals following the collapse of the housing market in 2008 was to support housing prices, so that, through a higher collateral value, households and firms could obtain funds to invest. This has been stated explicitly in a 2013 policy brief from the White House: "Housing wealth is growing again, with owners' equity up $2.8 trillion since hitting a low at the beginning of 2009. This in turn has contributed to increased economic activity through consumer spending, small business investment, and more."[1] On the other hand, some argue that policies should be taken to moderate housing booms voicing concerns that booming housing prices crowd out other types of investment and spending.[2]

Empirical research on the relationship between real estate prices and investment has also pointed to different channels in opposite directions making the net effect of real estate prices on investment ambiguous. On the one hand, Gan (2007) and Chaney, Sraer, and Thesmar (2012) find a positive effect of increases in collateral value on the investment of firms which own real estate assets. On the other hand, Chakraborty, Goldstein, and MacKinlay (2018), Hau and Ouyang (2020) and Martín, Moral-Benito, and Schmitz (2021) find evidence that real estate price booms caused banks to increase housing credit while simultaneously decreasing commercial credit thereby crowding out real investment.

Motivated by this tension in policy goals and the heterogeneous set of empirical evidence, we provide in this paper a simple, yet comprehensive, theoretical framework to understand the effects of real estate prices on investment and to help guide policymakers in designing policies targeting real estate prices. Is there a need for such policies? What is the direction in which real estate prices should be pushed under different circumstances? What form should such policies take? The model is based on the unique nature of real estate assets, making the level of prices a policy goal in and of itself. This is very different from standard subsidization/taxation policies that typically only target

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[1]See https://obamawhitehouse.archives.gov/the-press-office/2013/08/05/fact-sheet-better-bargain-middle-class-housing.

[2]See https://www.reuters.com/article/us-china-economy-homeprices-idUSKBN26K11H and https://www.nytimes.com/2021/07/13/opinion/covid-housing-phoenix-austin.html.

the quantities of the goods in question.[3]

Our benchmark model features a representative firm that borrows from a representative bank to make two types of investment: real estate investments and investments in its own production process. Real estate prices are determined by the demand from the representative firm and the supply from a perfectly competitive construction sector. In the absence of financial constraints, first-best is achieved: investments in both real estate and the firm's production process are at their optimal level, and no intervention in real estate prices is needed. This is consistent with traditional economic theory, whereby externalities through prices — known as pecuniary externalities — do not interfere with welfare and require no policy intervention.

We then introduce financial constraints. The representative firm is unable to commit to repaying loans using its future income and therefore loans need to be collateralized. Real estate then serves a dual purpose as an asset in the model — generating investment returns and also relaxing the firm's borrowing constraint. When the firm cannot invest in all positive NPV opportunities, the equilibrium fails to achieve the first-best allocation and the market price of real estate generates externalities that call for intervention. We identify two effects. The first is a *collateral effect*. An increase in real estate prices increases the value of the firm's collateral, generating a positive externality on the firm's investment. The second is a *crowding-out effect*. While an increase in real estate prices relaxes borrowing constraints, it also increases the amount that has to be spent on new real estate purchases. This additional cost reduces the funds available for investment in the firm's own projects, generating a negative externality on firm investment.

When the crowding-out effect is stronger than the collateral effect, the equilibrium in our model simultaneously features too much demand for and too little supply of real estate assets. In this case, real estate prices are too high. A social planner, who faces the same financial constraints as the firm but internalizes the externality from housing prices, would then want to push down real estate prices — increasing the supply of and decreasing the demand for real estate. The opposite happens when

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[3]In other markets in which subsidies are common, such as staple foods or health-care, the final policy goal is typically broad access to the good. Prices matter to the extent that they affect access to the good. They do not matter in and of themselves.

the collateral effect is stronger than the crowding-out effect.

Using the tools of taxes and subsidies on the demand for and the supply of real estate, we study how the first-best allocation can be restored with the optimal mix of policies. We show that it can be achieved by intervening in both sides of the market in opposite ways. When the crowding-out effect is stronger than the collateral effect, the first-best is achieved through combining expansionary supply-side policies, such as subsidies to construction companies, with contractionary demand-side policies, such as taxes on real estate purchases. The converse is true when the collateral effect is stronger than the crowding-out effect. Intuitively, by simultaneously taxing one side of the market and subsidizing the other, it is possible to achieve any price-quantity combination in the real estate market. One curve targets the optimal quantity of real estate while the other curve targets an optimal price such that the externalities on financial constraints will lead to the first-best allocation.

This form of intervention contrasts with that coming out of standard economic models with one-sided distortions in quantities — there is either too much or too little of a good. Correcting the distortion therefore typically requires increasing (decreasing) both the supply and demand of the good when there is too little (much) of it, as both measures increase (decrease) quantity. In contrast, in our framework the fundamental inefficiency arises because equilibrium real estate prices are not at the optimal level. Increasing supply (demand) and decreasing demand (supply) both lower (increase) real estate prices, and so such mixed combinations are needed and depend on the direction of distortion in prices. Traditional economic theory also suggests that it typically does not matter whether we subsidize supply or demand from a welfare perspective. However, in our model, subsidizing the supply and demand sides of the real estate market have distinct effects since they move real estate prices in different directions.

We extend the benchmark model to incorporate heterogeneity across firms in their production functions and their levels of real estate holdings. While in the representative-firm model, an increase in aggregate collateral unambiguously increases total surplus, the same is not true when firms have heterogenous real estate holdings. In such a case, the distribution of collateral is critical, and aggregate investment might decrease when collateral becomes overall more abundant. Intuitively, if

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a firm increases its demand for real estate assets when its borrowing constraint loosens it will cause an increase in equilibrium real estate prices. Although collateral-rich firms will benefit from this increase in prices, collateral-poor firms will experience a crowding out of their investment. If the latter effect dominates the former, an increase in aggregate collateral can lead to a decrease in total surplus. Our predictions are supported by empirical findings in Doerr (2020) that real estate booms lead to a misallocation of capital towards firms which have more collateral leading to a decline in industry-level productivity.

Extending this line of reasoning, when firms have heterogeneous real estate holdings and production functions, the joint distribution of collateral and firm productivity is critical for determining optimal policy. Our model delivers a simple sufficient statistic, based on observable and measurable quantities, that enables us to determine the relative magnitude of the collateral and crowding-out effects and consequently whether real estate prices are too high or too low. For each firm, we multiply its marginal productivity of capital by a statistic that is increasing in its new real estate purchases, decreasing in its inventory of real estate assets and decreasing in the extent to which real estate can be pledged as collateral (i.e., the loan-to-value ratio). We then sum up these quantities across firms in a region. If the sum is positive, the crowding-out effect dominates and subsidizing the supply of real estate and taxing the demand for real estate should increase total surplus. If the sum is negative, the collateral effect dominates and taxing the supply of real estate and subsidizing the demand for real estate should increase total surplus. Intuitively, the statistic accounts for how an increase or decrease in real estate prices affects the investment of each firm in the economy and then weights firms with higher marginal productivity relatively more since an additional unit of capital increases their investment output by more than firms with lower marginal productivity.

The analysis of the heterogeneous-firm model also provides some nuance when thinking about empirical results on the effect of an increase in real estate prices on different firms.[4] The differences in behavior by firms with different levels of collateral might not result purely from their different collateral holdings, but rather from further equilibrium effects in the real estate market. If land-

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[4]See Gan (2007) and Chaney et al. (2012).

owning firms further increase their investment in real estate assets when their collateral constraint relaxes, they can crowd out investment that non land-owning firms can undertake. These spillovers will cause firms with different amounts of real estate assets to react differently to changes in local real estate prices beyond what the direct effect of owning land would imply.

Another extension of the model we pursue involves the introduction of banks' constraints. We show that the crowding-out effect is more severe when firms borrow from more constrained banks. Intuitively, when banks are constrained, they may not have the funds necessary to lend to a firm even if the firm has the collateral required to secure a larger loan. This effect reduces the relative magnitude of the collateral effect. Our results are in line with Chakraborty et al. (2018) and Martín et al. (2021) who document that housing booms cause a greater crowding out of loans to firms who borrow from smaller banks.

Overall, our paper highlights the interaction of opposite effects of real estate prices on investment, and the implications for policymakers when thinking how to intervene. We outline a sufficient statistic approach that can be used to determine whether housing prices should be pushed up or down, and so what combination of subsidies and taxes on the supply and the demand of real estate is warranted. We further show how the results change in the presence of heterogeneity. While our model is cast in the context of corporate investments, we outline how it can be reinterpreted to model investment by households and how it can be used to evaluate housing policy interventions relevant to household finance.

Related Literature: Our paper relates to studies on the inefficiencies that can arise from asset price booms when agents do not internalize their effect on the price of assets. Lorenzoni (2008) develops a model in which credit booms lead to inefficient borrowing because atomistic entrepreneurs do not internalize fire sale externalities. In this case, once a crisis hits there is excessive contraction in investment and asset prices. Bianchi and Mendoza (2010) propose a model in which agents over-borrow when they do not internalize how their borrowing decisions affect the price of an asset that is also used as collateral. Private agents do not internalize that fire sale of assets can cause

a debt-deflation spiral leading to a large decline is asset prices and a shrinking of the economy's ability to borrow. In these papers, the negative effects of agents' actions on investment are not observed during the asset price boom. Rather, the negative effects on investment are realized when a bust occurs due to fire sale externalities. In contrast, in our model, the negative effects of price booms on investment are realized during the boom phase itself because investment in firm projects can be crowded out due to high real estate prices.

More generally, there is a literature that studies how asset price booms crowd out productive investment. Tirole (1985) builds a model in which asset prices bubbles crowd out productive real investment by raising interest rates and reducing firm incentives to invest. In a similar vein, Farhi and Tirole (2012) show that a rise in interest rates during a bubble may further restrict credit availability for financially constrained firms. In our paper, we do not require a bubble to produce the negative real effects accompanying a price boom. We simply require that firms are financially constrained.

Our paper also contributes to the macro literature which tries to understand the role of asset prices for the real economy and how price changes amplify shocks to investment. Seminal papers in this field such as Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) discuss the amplification of negative shocks to asset prices when these assets also serve as collateral for financially constrained agents. Our proposed mechanism suggests that asset price booms may also cause negative shocks to investment.

Our paper also relates to a small literature emphasizing the importance of the distribution of collateral as opposed to its aggregate level. Di Maggio and Tahbaz-Salehi (2015) develop a model in which intermediation capacity depends on the distribution of collateral rather than simply the aggregate amount of collateral. Donaldson, Gromb, and Piacentino (2020) present a theory in which there can be a collateral overhang problem. Policies aimed at increasing the aggregate supply of collateral are not necessarily welfare improving as they can trigger a collateral rat race. In our framework, increasing collateral is not necessarily welfare improving as in Donaldson et al. (2020) but the channel is different. In our mechanism, the distribution of collateral determines the effect of aggregate real estate prices on investment as collateral-rich firms can crowd out the investment of

collateral-poor firms.

The rest of the paper is arranged as follows. Section 2. outlines the benchmark model. Section 3. discusses the equilibrium of the benchmark model. Section 4. extends the benchmark model to incorporate firm heterogeneity. Section 5. discusses other extensions to the benchmark model. Section 6. discusses some of the key insights of our framework. The last section concludes. All proofs are in the appendix.

2. Benchmark Model

There are two dates $(1,2)$, a firm which invests in real estate and its own projects, a representative bank which makes loans to the firm, a construction company which builds new real estate, and a representative household who is the final owner of the construction company, the bank and the firm. All agents are risk-neutral.

At $t = 1$, the firm owns liquid funds, ω, and an existing stock of real estate assets, B. The firm can use this stock along with any new real estate purchases, x_m, as collateral to borrow an amount, l, from banks. The firm can use its liquid capital and the bank loan to invest in its own projects or real estate. It additionally has access to a storage technology with a gross return of 1 between $t = 1$ and $t = 2$.

At $t = 1$, a representative bank can make a loan, l, to the firm at an interest rate, r. Loans need to be collateralized because of moral hazard in the repayment of loans. Specifically, the firm can use its real estate stock as collateral and can commit to repaying a portion of the value of this stock, $\phi(B + x_m)P$, at $t = 2$, which depends on the equilibrium price of real estate, P, at $t = 1$ and the degree of pledgeability of collateral, ϕ. We assume that $0 < \phi < 1$. This formulation of the collateral constraint is similar to that in Gertler and Karadi (2011) and in Gertler and Kiyotaki (2015).[5] Banks also have access to the storage technology that has a gross return of 1.

Investing in its own projects gives the firm a return of $r_f(x_f)$ at $t = 2$ for every x_f units invested

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[5]A micro-foundation for this formulation is that the firm can abscond immediately with the funds it borrows. In this case the bank is able to recover a fraction ϕ of the firm's real estate stock. For the bank to lend to the firm therefore, $l \leq \phi(B + x_m)P$.

at $t = 1$. The function r_f has the following standard properties: $r_f'(x_f) > 0$ and $r_f''(x_f) < 0 \ \forall x_f$, $lim_{x_f \to 0} r_f'(x_f) = \infty$ and $lim_{x_f \to \infty} r_f'(x_f) = 0$. Similarly, investing in real estate gives the firm a return of $r_m(x_m)$ at $t = 2$ for every x_m units invested at $t = 1$. The function r_m has the following standard properties: $r_m'(x_m) > 0$ and $lim_{x_m \to 0} r_m'(x_m) = \infty$ and $lim_{x_m \to \infty} r_f'(x_m) = 0$. The price per-unit of firm project investment is normalized to 1. The price per-unit of real estate, P, is determined by demand and supply in the real estate market. The representative construction firm takes the price of real estate as given and has a strictly increasing and convex cost of production given by $K(x_m)$. The function K has the following standard properties: $K'(x_m) > 0$ and $K''(x_m) > 0 \ \forall x_m$.

The return from real estate can be interpreted as the output the firm produces from the use of these assets in its production or sales activities. It can also reflect beneficial tax treatment of owning rather than renting property. Alternatively, the return from real estate can be viewed as innovations in rental income which take the form of savings for the firm when it owns its real estate assets rather than leases them (Sinai and Souleles, 2005).

All profits are rebated to the household and all consumption takes place at $t = 2$.[6] Importantly, all agents are price-takers in the economy. They therefore do not internalize their effect on the price of real estate, P, when making decisions.

The Firm: The firm borrows an amount l from banks and maximizes its output, π_f. At $t = 1$, it solves the following portfolio allocation problem, where x_m, x_f and x_s are the units of real estate, firm projects and storage it demands

$$\max \pi_f = \max_{(x_f, x_m, x_s, l) \geq 0} r_f(x_f) + r_m(x_m) + x_s - l(1+r) \tag{1}$$

subject to

$$x_f + P x_m + x_s \leq \omega + l, \tag{2}$$

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[6]Note that by assuming that the profits from the construction company are rebated to the firm at $t = 2$ and not at $t = 1$, we are assuming that resources cannot be shuffled across the construction company and the firm via the household in the same period. Intuitively this means that when a firm purchases real estate assets, it does not immediately receive back the funds it just spent on that purchase.

$$l \leq \phi \left(B + x_m \right) P. \tag{3}$$

The first three terms in the firm's $t = 2$ output represent the return from its investments and the last term is the repayment that the firm must make at $t = 2$ to the bank.[7] The first constraint is the firm's budget constraint while the second constraint is its borrowing constraint.

The Construction Company: At $t = 1$, the construction company decides how much real estate to produce, x_m, to maximize its profits, π_c. It solves the following maximization problem

$$\max \pi_c = \max_{x_m \geq 0} \quad P x_m - K(x_m). \tag{4}$$

The Bank: The representative bank competitively sets interest rates on loans, r, at $t = 1$. The bank can either lend to the firm or put its money in storage. In the benchmark analysis, we assume the bank is unconstrained and can lend out as many funds as is profitable. In an extension, we consider the case when the bank only has a limited supply of funds to lend out. The extension allows us to explore the effect of bank size on our results.

The Household: The household is the final holder of the firm, the construction company and the bank in the model. The household's final utility is given by the sum of the profits of the firm, π_f, the bank, π_b, and the construction company, π_c.

Remark on Investment Risk: We assume no investment risk in the model without loss of generality. Since all agents are risk-neutral, we can write an equivalent formulation of the model with investment risk. All of our results remain unchanged.[8] Under risk-aversion, we would get additional effects driven by the riskiness of investment. Since these effects are not the focus of our

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[7]Since the firm faces no investment risk in our model, its $t = 2$ portfolio and repayment to the bank are deterministic.

[8]We can equivalently model investment as follows: with probability p_j, $j \in \{m, f\}$, investment gives a high return of $r_j^h(x_j)$ at $t = 2$ and with probability $1 - p_j$ the investment has a low return of $r_j^l(x_j)$, where $r_j^h(x_j) > r_j^l(x_j) \ \forall x_j$. The functions r_j^h and r_j^l are increasing, concave and satisfy the Inada conditions. Since the sum of two strictly concave functions is itself concave, we can define a function $r_j(x_j) = p_j r_j^h(x_j) + (1 - p_j) r_j^l(x_j)$ which is also strictly concave and get an identical maximization problem as in the benchmark specification. When banks are unconstrained, the equilibrium interest rate can be written as an expectation, $E[r]$. This will be equal to the equilibrium interest rate when investment is deterministic.

model, we assume risk neutrality to illustrate the mechanism as clearly as possible.

2.1. Equilibrium

An equilibrium of this economy is given by, (i) the firm's investment in real estate, x_m, its own projects, x_f, and the storage technology, x_s, and its loan amount, l, given the price of real estate, P, and interest rate on bank loans, r, (ii) the construction company's real estate production, x_m, given the price of real estate, P, (iii) the price of real estate, P, such that the real estate market clears, (iv) the interest rate per dollar of lending, r, such that the credit market clears.

3. Equilibrium Analysis

We begin the analysis of the equilibrium by determining the first-best level of investment in the economy. Next, we describe a benchmark case in which the firm does not face any financial constraints. In this case, investment in the decentralized equilibrium is first-best. We then discuss the equilibrium when the firm is financially constrained and show how real estate prices generate externalities on investment.

3.1. First-Best

The first-best level of investment in this economy maximizes the total resources available to the household for consumption at $t = 2$

$$\max_{(x_f, x_m) \geq 0} \quad r_f(x_f) + r_m(x_m) - x_f - K(x_m). \tag{5}$$

The optimal investment is when the marginal return from investing is equal to the marginal cost of undertaking the investment. We define x_m^* and x_f^* as the first-best levels of investment in real estate and the firm's projects, i.e. $\frac{r_m'(x_m^*)}{K'(x_m^*)} = 1$ and $r_f'(x_f^*) = 1$.

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3.2.   Benchmark Equilibrium without Financial Constraints

In this subsection, we discuss a benchmark case in which the firm is not financially constrained. In the model, this is equivalent to the firm being able to borrow from the bank without posting collateral. The firm therefore maximizes (1) only subject to (2).

The presence of the storage technology ensures the firm never invests in real estate or its own projects if the gross investment return is below 1. Assuming firms do not borrow simply to store (i.e. they have a weak preference for not taking a loan), storage is used in equilibrium if $\omega \geq Px_m + x_f$. If $\omega < Px_m + x_f$, then the firm will borrow an amount $l = Px_m + x_f - \omega$ to fund additional investment. The first order conditions yield the following demand for real estate and firm project investment in the economy

$$r'_f(x_f) = 1 + r; \qquad \frac{r'_m(x_m)}{P} = 1 + r. \tag{6}$$

Since the banking sector is unconstrained and competitive and the returns to investment are deterministic, the interest rate on loans equals the bank's outside option of storage, i.e. $r = 0$.

The first order condition for the construction company determines the equilibrium quantity of real estate supplied

$$K'(x_m) = P. \tag{7}$$

Market clearing then implies that a firm that is not financially constrained invests in x_m^* units of real estate and invests x_f^* in its own projects.

To achieve the first-best level of investment the unconstrained firm will borrow

$$l = max\{0, K'(x_m^*)x_m^* + x_f^* - \omega\}. \tag{8}$$

In the presence of financial constraints which require that the firm post collateral, the borrowing constraint on the firm allows it to borrow a maximum of $\phi K'(x_m)(x_m + B)$. Therefore, if the firm faces borrowing and budget constraints, it will achieve the first-best level of investment if

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$$K'(x_m^*)x_m^* + x_f^* - \omega \leq \phi K'(x_m^*)(x_m^* + B). \tag{9}$$

When the above inequality holds, the firm is able to borrow sufficient funds to undertake all productive investment opportunities and achieve the first best level of investment. When the above inequality does not hold, the firm has more positive NPV investment opportunities than the funds necessary to invest in these opportunities. This is the pertinent case for when the price of real estate generates externalities on firm investment. Formally, we define the firm as having "Limited Funds" when the following condition is satisfied

$$K'(x_m^*)x_m^* + x_f^* > \phi K'(x_m^*)(x_m^* + B) + \omega. \tag{Limited Funds}$$

The above inequality implies that the firm is financially constrained and cannot invest in all positive NPV investment opportunities even if it borrows to its full capacity. Therefore, the firm's borrowing constraint will bind in equilibrium. This is the key case for when real estate prices distort investment. Henceforth, we will assume that the Limited Funds assumption holds.

3.3. Equilibrium with Financial Constraints

We now consider the equilibrium when the firm is financially constrained. At $t = 1$, the firm maximizes (1) subject to (2) and (3).

As before, since the representative bank is unconstrained and the investment outcome is deterministic, competition will drive the interest rate, r, to 0. When the Limited Funds assumption is satisfied, the firm's borrowing constraint binds in equilibrium and $l = \phi(B + x_m)P$. In this case, the firm will never invest in the storage technology since it has unexploited projects at $t = 1$ that yield an investment return strictly greater than 1. The firm's maximization problem can therefore be simplified to

$$\max_{(x_f, x_m) \geq 0} \quad r_f(x_f) + r_m(x_m) - \phi(B + x_m)P \tag{10}$$

subject to

$$x_f + (1 - \phi)Px_m \leq \omega + \phi BP. \tag{11}$$

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The first order conditions of the firm yield the following investment demand given the price of real estate

$$r'_m(x_m) = P(r'_f(x_f)(1-\phi) + \phi). \tag{12}$$

Market clearing requires that $P = K'(x_m)$.

Constrained Social Planner Allocation: To establish the inefficiencies in the decentralized equilibrium, we solve the problem of a social planner who is constrained by the firm's financial constraints but takes into account the effect real estate demand and supply have on aggregate prices. The constrained social planner chooses the optimal levels of real estate and firm project investment to maximize the representative household's final wealth. In particular, the planner takes into account how the firm's demand for and the construction company's supply of real estate affect real estate prices which in turn affect the financial constraints of the firm. We assume that the planner faces the same borrowing and budget constraints as the firm. Since the banking sector is competitive, $\pi_b = 0$. The social planner solves

$$\max_{(x_f,x_m,x_s,l) \geq 0} \quad \pi_f(x_m,x_f,x_s,l) + \pi_c(x_m) \tag{13}$$

subject to

$$x_f + P(x_m)x_m + x_s \leq \omega + l \tag{14}$$

$$l \leq \phi (B + x_m) P(x_m). \tag{15}$$

Note that the constrained social planner's maximization problem above differs slightly from the firm's in equation (1) as it incorporates the construction company's profits in the maximand. This captures the fact that the social planner is jointly maximizing the firm's and the construction company's profits.

When the Limited Funds assumption is satisfied, the social planner will pick a loan amount, l, such that the borrowing constraint binds and $x_s = 0$. The first order conditions with respect to x_f and x_m yield the following relationship

$$r'_m(x_m) - P(r'_f(x_f)(1-\phi)+\phi)+P-K'(x_m) = P'x_m(1-\phi)(r'(x_f)-1)-P'\phi B(r'(x_f)-1). \quad (16)$$

In the decentralized equilibrium, $r'_m(x_m) = P(r'_f(x_f)(1-\phi)+\phi)$ and $P = K'(x_m)$. Therefore, at the decentralized equilibrium levels of investment, the LHS of the above equation will be zero. When $r'(x_f) = 1$, the RHS of the above equation will also be zero and the constrained social planner's allocation will be the same as that of the decentralized equilibrium.[9] This is the case whenever the Limited Funds assumption does not hold. When the Limited Funds assumption is satisfied, at the decentralized equilibrium levels of investment $r'(x_f) > 1$. In this case, the constrained social planner's optimal investment will differ from that in the decentralized equilibrium. Therefore, the first welfare theorem does not hold.

3.3.1. *Price Externalities*

The two terms on the right hand side of (16) represent the magnitude of the inefficiencies in the decentralized equilibrium. The term $P'\phi B(r'(x_f)-1)$ captures the *collateral effect*. An increase in real estate prices increases the value of the firm's collateral, generating a positive externality on the firm's investment. The term $P'x_m(1-\phi)(r'(x_f)-1)$ captures the *crowding-out effect*. While an increase in real estate prices relaxes borrowing constraints, it also increases the amount that has to be spent on new real estate purchases. This additional cost reduces the funds available for investment in the firm's own projects, generating a negative externality on firm investment.

Using the budget constraint of the firm we can express investment in the firm's projects in terms of the firm's investment in real estate

$$x_f = \omega + P(\phi B - (1-\phi)x_m). \quad (17)$$

We define x_m^e as the decentralized equilibrium investment in real estate. It is clear from the expression above that if the crowding-out effect is stronger than the collateral effect, i.e. if $\frac{x_m^e}{B} > \frac{\phi}{1-\phi}$, a reduction in real estate prices, keeping the level of real estate investment fixed, will increase

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[9]Note that the presence of the storage technology ensures that $r'_f(x_f) \geq 1$.
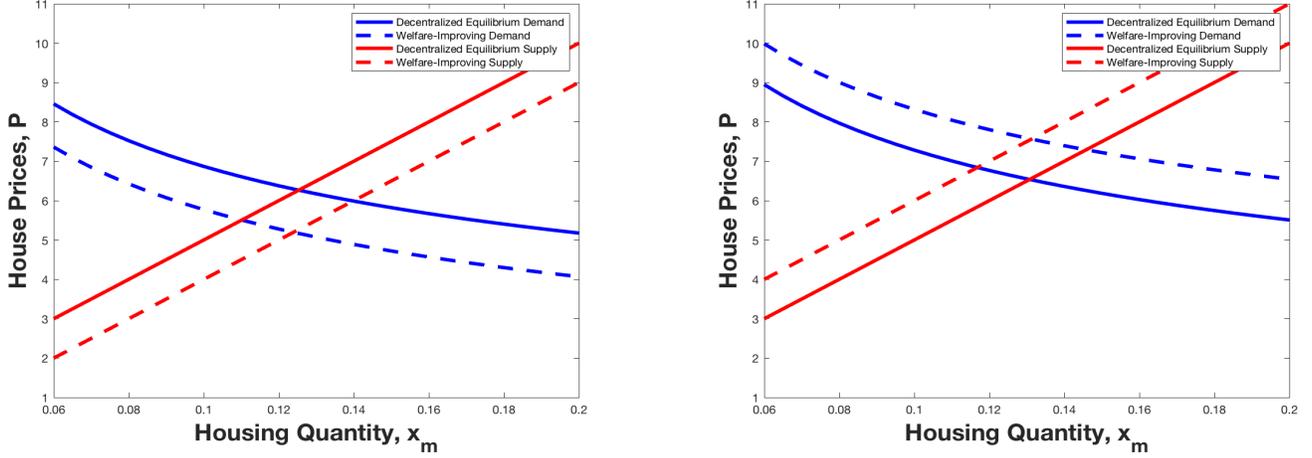
Figure 1: The figures above plot the demand and supply curves for real estate under the decentralized equilibrium (solid line) and under a welfare-improving shift of the demand and supply curves (dashed line) such that investment in real estate is the same as in the decentralized equilibrium. The parametrization is as follows: $r_f(x_f) = 5x_f^{\frac{3}{5}}$, $r_m(x_m) = 5x_m^{\frac{3}{5}}$, $K(x_m) = 25x_m^2$, $\omega = 10$, $B = .1$. In the left panel, $\phi = 0.5$ while in the right panel, $\phi = 0.8$.

firm project investment. In this case, if the social planner could achieve an allocation such that the investment in real estate was equal to the decentralized equilibrium level, but the price of real estate was lower, he would always prefer such an allocation as it would increase the firm's investment in its projects. The social planner can achieve this by lowering the firm's demand for real estate and increasing the construction company's supply of real estate. This is illustrated in the left panel of Figure 1.

Conversely, when $\frac{x_m^e}{B} < \frac{\phi}{1-\phi}$, an increase in real estate prices, keeping the level of real estate investment fixed, will lead to more investment in firm projects. In this case, the relative magnitude of the collateral effect is larger than the crowding-out effect. If the social planner could achieve an allocation in which the price of real estate was higher, keeping investment in real estate at the same level, he would always prefer such an allocation. The social planner can achieve this by increasing the firm's demand for real estate and reducing the construction company's supply of real estate. Such a case is illustrated in the right panel of Figure 1.

Formally, we can establish the following proposition.

Proposition 1 (***Demand and Supply Distortions***) *If* $\frac{x_m^e}{B} > \frac{\phi}{1-\phi}$, *the social planner can improve upon the decentralized equilibrium welfare by reducing demand for real estate and increasing the*

supply of real estate.

Conversely, if $\frac{x_m^e}{B} < \frac{\phi}{1-\phi}$, the social planner can improve upon the decentralized equilibrium welfare by increasing demand for real estate and reducing the supply of real estate.

The constrained social planner will want to decrease the equilibrium price of real estate when the crowding-out effect is stronger than the collateral effect. This can be achieved by lowering the demand for real estate investment and by increasing the supply of real estate. Therefore, in this case, the decentralized equilibrium simultaneously features an *over-demand* for and an *under-supply* of real estate. Conversely, when the collateral effect is stronger than the crowding-out effect, the decentralized equilibrium simultaneously features an *under-demand* for and an *over-supply* of real estate.

Figure 2 shows how real estate prices affect the borrowing and budget constraints of the firm as a function of the collateralizability parameter, ϕ. When ϕ is high, an increase in real estate prices loosens the borrowing constraint of the firm more than it tightens the budget constraint. Therefore, the social planner in this region wants to increase house prices. Conversely, when ϕ is low, a decrease in house prices tightens the borrowing constraint of the firm less than it loosens the budget constraint. Therefore, the social planner in this region wants to decrease house prices.

In the benchmark model, an increase in aggregate collateral always has a positive impact on total surplus. Formally, we can establish the following corollary to Proposition 1.

Corollary 1 (Collateral and Total Surplus) *An increase in aggregate collateral — i.e. an increase in existing real estate assets, B, or the collateralizability of real estate, ϕ — increases total surplus, $r_m(x_m) + r_f(x_f)$.*

An increase in existing real estate assets, *B*, or an increase in the collateralizability of real estate assets, ϕ, both loosen the firm's borrowing constraint. The firm therefore has more funds to invest in new real estate investments and its own projects leading to an increase in total surplus. In the case of a representative firm, it is therefore always optimal to increase aggregate collateral in the economy.
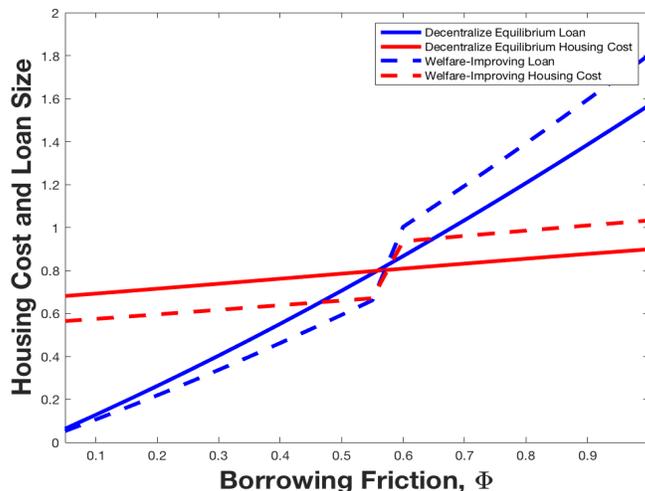
Figure 2: The figure above plots the amount the firm spends on new real estate investment and the amount it can borrow from the bank, as a function of borrowing frictions. The dotted lines plot a welfare-improvement in which equilibrium real estate investment stays the same. The solid lines show the decentralized equilibrium solution given by equation 12. The parametrization is as follows: $r_f(x_f) = 5x_f^{\frac{3}{5}}$, $r_m(x_m) = 5x_m^{\frac{3}{5}}$, $K(x_m) = 25x_m^2$, $\omega = 10$, $B = .1$.

3.3.2. *Optimal Policy*

We have thus far shown that when firms are financially constrained, the decentralized equilibrium investment in real estate is inefficient. A social planner who faces the same financial constraints as the firm but takes into account the effect of real estate demand and supply on prices, would like to lower equilibrium real estate prices when the crowding-out effect is stronger than the collateral effect and would like to increase equilibrium real estate prices when the converse is true. In this section, we explore how a government can design optimal policy to correct for these inefficiencies. To implement shifts of the demand and supply of real estate in practice, governments commonly use taxes and subsidies (e.g., the mortgage interest rate tax deduction in the United States and the foreign buyers' tax in Vancouver). We model these common policy tools in our setting and demonstrate that optimally using supply and demand taxes/subsidies can lead to the first-best level of investment even when the firm is financially constrained.

We model a demand subsidy (tax) which increases (decreases) the $t = 2$ per-unit return on real estate by an amount r_g. Additionally, we model supply subsidies (taxes) in the form of a per-unit subsidy (tax), b, that the government can give to construction companies. The government must

have a balanced budget and households are taxed an amount τ at $t = 2$ to cover the cost of any subsidies. We assume that all subsidy and taxation payments are made at $t = 2$ and that construction companies can operate at a loss between $t = 1$ and $t = 2$ without any additional costs.

A demand-side intervention, r_g, changes the firm maximization problem to

$$\max_{(x_f, x_m, x_s, l) \geq 0} \quad r_f(x_f) + r_m(x_m) + r_g x_m + x_s - l(1 + r) \tag{18}$$

subject to

$$x_f + P x_m + x_s \leq \omega + l \tag{19}$$

$$l \leq \phi (B + x_m) P. \tag{20}$$

If the Limited Funds assumption is satisfied, equilibrium demand given the price of real estate is determined by the following equation

$$r_m'(x_m) + r_g = P(r_f'(x_f)(1 - \phi) + \phi). \tag{21}$$

A supply-side intervention, b, changes the construction company's maximization problem to

$$\max_{x_m \geq 0} \quad (P + b) x_m - K(x_m). \tag{22}$$

Market clearing is given by $P + b = K'(x_m)$.

We showed previously that it is possible to keep equilibrium investment in real estate the same as in the decentralized case but increase investment in firm projects by adjusting the demand and supply of real estate to affect its price. We can push this insight further and show that a combination of demand- and supply-side interventions restore the first-best level of investment even in the presence of financial constraints. Formally, we can establish the following proposition,

Proposition 2 (*Optimal Policy*) *The first-best level of investment can be achieved by a subsidy pair*

$$\{r_g, b\} = \left\{ \frac{x_f^* - \omega}{\phi B - (1 - \phi) x_m^*} - r_m'(x_m^*), r'(x_m^*) - \frac{x_f^* - \omega}{\phi B - (1 - \phi) x_m^*} \right\}. \tag{23}$$

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This proposition yields a surprising insight. In the presence of price externalities from real estate, the optimal way to build wealth in the economy is to combine expansionary supply subsidies with contractionary demand taxes and vice-versa. Intuitively, since price distortions rather than quantity distortions drive the inefficiency in our model, the optimal policy needs to target price movements. This requires taxing one side of the market while subsidizing the other since both actions move prices in the same direction. In the optimal subsidy scheme $r_g = -b$. This feature is due to the linearity of demand and supply subsidies. In equilibrium, at the first-best level of investment $r'_f(x^*_f) = 1$. Therefore, it has to be the case that $K'(x^*_m) - b = P = r'_m(x^*_m) + r_g$. Since at the optimal level of real estate investment $K'(x^*_m) = r'_m(x^*_m)$, this implies that $-b = r_g$.

The optimal policy is able to restore the first-best level of investment even though the firm is financially constrained. By targeting price movements, the optimal policy exploits positive externalities from the level of real estate prices to alleviate the firm's financial constraints. Intuitively, intervening in only the demand (*supply*) side of the market would limit equilibrium real estate quantity and price to combinations that are on the supply (*demand*) curve. At the optimal level of real estate investment, x^*_m, the corresponding price will generally not affect the firm's financial constraints in a way that would also allow for the optimal level of firm project investment. However, intervening in both sides of the market allows for any combination of real estate quantity and price. Therefore, it is possible pick a price level such that the externalities allow for the optimal level of firm project investment. Whenever the collateral effect is stronger than the crowding-out effect, the government taxes supply and subsidizes demand, both of which put upward pressure on the price of real estate. The price is increased until the collateral value of real estate is high enough such that the firm can borrow enough to fund the optimal level of investment in its own projects. Conversely, when the crowding-out effect is stronger than the collateral effect, the government needs to put downward pressure on the price of real estate to relax the firm's budget constraint and provide it with enough funds to undertake the optimal level of investment in its own projects. In this case, the government taxes demand and subsidizes supply, both of which put downward pressure on the price of real estate.

The benchmark model highlights that the optimal level of real estate prices depends on the relative magnitude of the collateral versus the crowding-out effect. The optimal policy targets the level of real estate prices and requires opposite interventions in the demand and supply sides of the market — a departure from traditional economic theory. While the benchmark model explains the key forces through which real estate prices have an effect on investment, it is not suitable for policy prescriptions as it is based on a representative firm. In particular, when $\phi > .5$ which corresponds to LTV ratios in practice, Proposition 1 implies that if aggregate collateral is higher than new investment in real estate, i.e. $\frac{x_m^e}{B} < 1$, the collateral effect will dominate, and a social planner can increase total surplus by taking steps to increase real estate prices. Given that the value of the stock of real estate is much higher that the flow of real estate purchases in any given period, in a representative-firm economy, our model would then imply that increasing the price of real estate should always increase welfare which is at odds with empirical evidence showing a more nuanced effect of real estate prices on total surplus. In practice, firms have heterogeneous amounts of collateral and vary in their production functions which may alter optimal policy. In the following section, we extend the benchmark model to incorporate such firm heterogeneity.

4. Firm Heterogeneity

In this section, we incorporate firm heterogeneity into our framework. We start by introducing heterogeneity only in firm collateral holdings to clearly illustrate why the distribution of collateral, rather than its aggregate level, determines the effect of real estate prices on investment. We also obtain a specific tax and subsidy scheme that can restore first-best in this special case in which firms have similar production functions but differ in collateral holdings. In Section 4.3., we additionally allow for heterogeneity in firm production technology. Based on the insights of this model with heterogeneity in both firm production functions and their real estate holdings, we develop a sufficient statistic to guide policy.

4.1. Heterogeneity in Firm Collateral Holdings

We assume that there are N firms with heterogeneous collateral holdings that borrow from the representative bank. Similar to the benchmark model, all firms can invest in firm-specific projects and real estate. The firms are identical except that they differ in their existing stock of real estate assets, B^i where $i \in \{1, 2, ..., N\}$. We assume without loss of generality that $B^{i+1} > B^i$. Therefore B^1 is the lower bound of the collateral distribution and B^N is the upper bound.

We define x_f^i, x_m^i and x_s^i as representing the portfolio allocation of firm $i \in \{1, 2, ..., N\}$. Each firm i maximizes

$$\max \pi_f = \max_{(x_f^i, x_m^i, x_s^i, l^i) \geq 0} r_f(x_f^i) + r_m(x_m^i) + x_s^i - l^i(1+r) \tag{24}$$

subject to

$$x_f^i + P x_m^i + x_s^i \leq \omega + l^i \tag{25}$$

$$l \leq \phi \left(B^i + x_m^i\right) P. \tag{26}$$

The equilibrium real estate price, P, is now given by $K'(\sum_{i=1}^{N} x_i)$. In the absence of financial constraints, the first best level of investment in the economy is

$$r_f'(x_f^i) = r_f'(x_f^*) = 1 \ \forall i \in \{1, 2..., N\}; \qquad r_m'(x_m^i) = r_m'(x_m^*) = K'(N x_m^*) \ \forall i \in \{1, 2..., N\}. \tag{27}$$

When the Limited Funds assumption is satisfied, the decentralized equilibrium allocation of firm $i \in \{1, 2..., N\}$ is,[10]

$$r_m(x_m^i) = P(r_f'(x_f^i)(1 - \phi) + \phi); \qquad x_f^i = \omega + B^i \phi P - (1 - \phi) P x_m^i. \tag{28}$$

Since $B^i > B^{i-1}$, for any level of aggregate real estate prices, firm i faces a looser borrowing

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[10]In this extension, we require that the Limited Funds assumption must be satisfied for each firm $i \in \{1, 2, ..., N\}$. If the Limited Funds assumption is satisfied for firm N it is also satisfied for all other firms,

$$K'(N x_m^*) x_m^* + x_f^* > \phi(B^N + x_m^*) K'(N x_m^*) + \omega.$$

constraint than firm $i - 1$ and therefore will undertake more investment in both real estate and its own projects.

Constrained Social Planner Allocation: The constrained social planner will choose the optimal level of real estate and project investment for each firm in the economy to maximize the representative household's final wealth. As before, the planner faces the same constraints as individual firms but takes into account how the demand and supply of real estate affects prices. The planner solves

$$\max_{(x_f^i, x_m^i, x_s^i, l^i) \geq 0 \ \forall i \in \{1,2,...,N\}} \sum_{i=1}^{N} \pi_f(x_m^i, x_f^i, x_s^i, l^i) + \pi_c(x_m^1, x_m^2, ..., x_m^N) \tag{29}$$

subject to

$$x_f^i + P x_m^i + x_s^i \leq \omega + l^i \ \ \forall i \in \{1, 2, ..., N\} \tag{30}$$

$$l^i \leq \phi \left(B^i + x_m^i \right) P \ \ \forall i \in \{1, 2, ..., N\}. \tag{31}$$

The social planner faces a total of $2N$ constraints — a budget constraint and borrowing constraint for each firm. Importantly, the real estate demand of each firm affects the constraints of all other firms through its effect on real estate prices. The social planner's optimal allocation for a firm i is

$$r_m'(x_m^i) - P(r_f'(x_f^i)(1 - \phi) + \phi) + P - K' = P' \sum_{j=1}^{N} (r'(x_f^j) - 1)((1 - \phi)x_m^j - \phi B^j) \tag{32}$$

As before, if the Limited Funds assumption does not hold for all firms, then for all $i \in \{1, ..., N\}$, $r'(x_f^i) - 1 = 0$ and equilibrium investment will equal first-best. In this case, the constrained social planner allocation will correspond to the decentralized equilibrium allocation. However, when the Limited Funds assumption is satisfied, the constrained social planner allocation will differ from that of the decentralized equilibrium. The right hand side of the above equation captures the aggregate crowding-out effect and the aggregate collateral effect in the economy. This is the sum of the net collateral and crowding-out effect of each firm, weighted by each firm's opportunity cost of investment.

If $\sum_{i=1}^{N} (r'(x_f^i) - 1)((1 - \phi)x_m^i - \phi B^i) > 0$, the aggregate crowding-out effect will be stronger

than the aggregate collateral effect and the social planner will want all firms to reduce their demand for real estate and the construction company to increase its supply of real estate to push prices down. The converse is true when $\sum_{i=1}^{N}(r'(x_f^i)-1)((1-\phi)x_m^i - \phi B^i) < 0$. We define x_m^{ie} as the demand for real estate by firm i in the decentralized equilibrium. Then, we can establish an analogous proposition to Propostion 1 in the benchmark case.

Proposition 3 (***Demand and Supply Distortions with Heterogeneous Collateral***) *If the aggregate crowding-out effect is greater than the aggregate collateral effect, i.e. $\sum_{i=1}^{N}(r'(x_f^i)-1)((1-\phi)x_m^{ie} - \phi B^i) > 0$, the social planner can improve upon the decentralized equilibrium welfare by reducing the demand for real estate and increasing the supply of real estate.*

Conversely, if the aggregate collateral effect is greater than the aggregate crowding-out effect, i.e. $\sum_{i=1}^{N}(r'(x_f^i)-1)((1-\phi)x_m^{ie} - \phi B^i) < 0$, the social planner can improve upon the decentralized equilibrium welfare by increasing the demand for real estate and reducing the supply of real estate.

Importantly, when firms have heterogeneous collateral holdings, the distribution of collateral, rather than just its aggregate level, determines the effect of real estate prices on aggregate investment. In equilibrium, firms with higher levels of collateral, have a relatively stronger collateral effect and a weaker crowding-out effect. These firms are also weighed less by the social planner when determining aggregate collateral and crowding-out effects since their opportunity cost of investment is lower than that of firms with less collateral due to decreasing returns to scale.

In the benchmark model, an increase in collateral is always beneficial for investment. When firms have heterogeneous amounts of collateral, this is no longer true. In particular, if collateral-poor firms in the economy have a relatively stronger crowding-out effect, an increase in the collateral holdings of any one firm can actually decrease total surplus. Intuitively, a firm which experiences an increase in collateral will increase its equilibrium investment in real estate and its own projects. The increased demand for real estate will lead to higher real estate prices which will affect the investment of other firms. While these higher prices will increase the equilibrium investment of firms with a relatively strong collateral effect, they will decrease the equilibrium investment of firms with a relatively strong crowding-out effect. If the decrease in gross investment returns of the latter

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group of firms outweighs the increase in gross investment returns of the former, total surplus will decline. Formally, we establish the following corollary to Proposition 3.

Corollary 2 (Collateral and Total Surplus with Heterogeneous Firms) *An increase in aggregate collateral — i.e. an increase in existing real estate assets, $\sum_{i=1}^{N} B^i$, or the collateralizability of real estate, ϕ — can decrease total surplus, $\sum_{i=1}^{N}(r_m(x_m^i) + r_f(x_f^i))$.*

With heterogeneous firms, the distribution of collateral across firms determines the aggregate effect of real estate prices on investment. If the aggregate amount of collateral in the economy increases, the effect on aggregate investment is ambiguous and will depend on the entire distribution of collateral in the economy before and after the change.

The left panel of Figure 3 plots an example with two firms in which an increase in aggregate collateral leads to a decrease in total investment output. In the example, the first firm has more collateral than the second firm. Aggregate collateral is increased by increasing firm 1's collateral endowment. As firm 1's borrowing constraint loosens it invests more in real estate, leading to an increase in real estate prices which crowd out firm 2's investment. The right panel of Figure 3 plots each firm's aggregate investment output as a function of firm 1's collateral. Although firm 1 increases investment output following an increase in its collateral holdings, firm 2 decreases its investment output. Note that the figure plots investment output rather than the dollar amount invested in real estate and firm projects. In the example, firm 2 will have more funds to invest because of a loosening of collateral constraints due to increased real estate prices but will undertake less real investment due to the crowding-out effect.

Our theoretical results are in line with empirical findings by Doerr (2020) who documents that real estate price booms lead to an inefficient allocation of capital towards firms that own real estate by relaxing their collateral constraints. He finds that firms with a higher share of real estate assets over fixed assets exhibit persistently lower levels of productivity. Real estate price booms lead to lower levels of industry productivity when the industry has wide variation in collateral. In particular, he estimates that a 10% increase in average real estate values leads to a .6% decline in average industry total factor productivity.
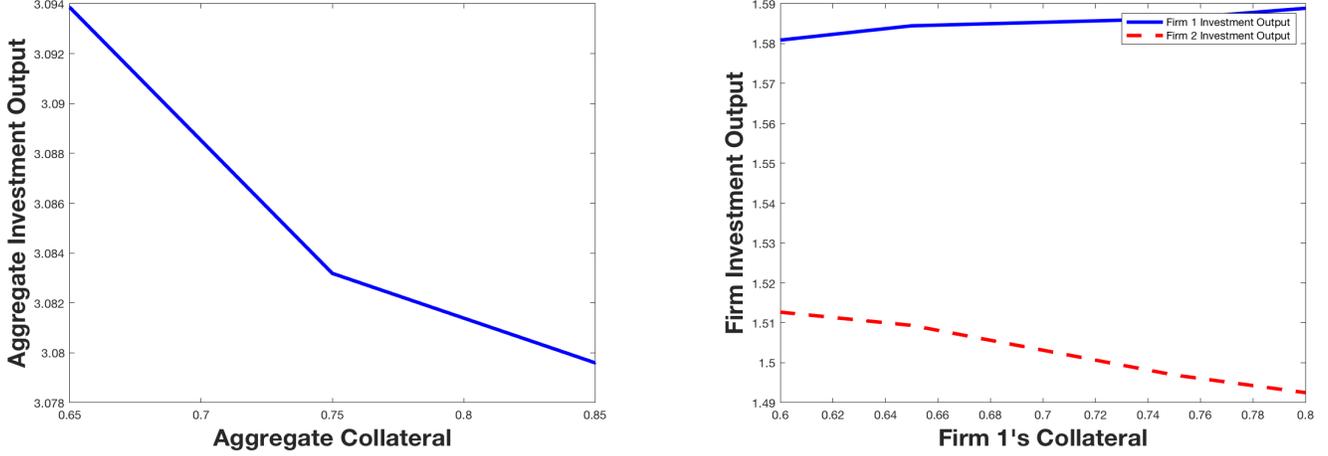
Figure 3: The left panel of the figure above plots aggregate investment output as a function of the aggregate collateral in an economy with $N = 2$ firms. The right panel plots each firm's investment output as a function of the collateral of firm 1. The parametrization is as follows: $r_f(x_f) = 10x_f^{\frac{3}{5}}$, $r_m(x_m) = 10x_m^{\frac{3}{5}}$, $K(x_m) = 12.5(x_m^1 + x_m^2)^2$, $\omega = .01$, $\phi = 0.1$, $B^2 = .05$, B^1 varies between 0.6 and 0.8.

4.2. Optimal Policy with Heterogeneity in Firm Collateral Holdings

When firms have heterogeneous real estate holdings, the distribution of collateral across firms affects the policy required to achieve the first-best level of investment. Surprisingly, even though we do not impose any restriction of the distribution of collateral across firms, the optimal policy only depends on the lower bound of the collateral distribution. In particular, the optimal tax and subsidy schemes depend on the real estate holdings of the firm with the *least* amount of collateral in the economy. We establish the following proposition,

Proposition 4 (Optimal Policy: Heterogeneity in Firm Collateral Holdings) *The first-best level of investment can be achieved by a subsidy pair*

$$\{r_g, b\} = \left\{ \frac{x_f^* - \omega}{\phi B^i - (1 - \phi)x_m^*} - r_m^{'}(x_m^*), r^{'}(x_m^*) - \frac{x_f^* - \omega}{\phi B^i - (1 - \phi)x_m^*} \right\} \tag{33}$$

where $B^i = \min\{B^1, B^2, ..., B^N\}$.

Intuitively, when demand and supply subsidies are set to ensure that the most financially-constrained firm can achieve the optimal level of investment, any firm that is less constrained can

also achieve that investment level. Importantly, if policy is set based on the total or average collateral in the economy, collateral-poor firms will not have their financial constraints relaxed adequately to achieve optimal investment.

Although the above result seems quite general, it is useful to note that it is derived under the assumption that all firms have access to similar production technologies, $r_f(x_f)$ and $r_m(x_m)$. In practice, it is therefore likely to apply to regions in which most firms plausibly satisfy this requirement — for example, firms that operate within the same industry and/or firms that are of a similar size. When firms additionally differ in production technology, if the most collateral-poor firm's financial constraints are relaxed it may not allow all other firms to achieve their optimal levels of investment. For example, if productivity is correlated with collateral holdings, then a collateral-poor firm is relatively unproductive. In this case, its optimal investment in its own projects and real estate will be below the level of those of collateral-rich firms. If the firm with the least amount of collateral has adequate funds to achieve its optimal investment, it will not necessarily imply that more productive collateral-rich firms will have adequate funds to do so as well. In the following section, we incorporate heterogeneity in firm production functions and show the robustness of our main insights. Although, in this case, there is no single policy which can guarantee the first-best allocation, we develop a sufficient statistic that can guide policy on how to tax and subsidize real estate markets to increase total surplus.

4.3. Heterogeneity in Firm Production Functions

In this subsection, we additionally allow for heterogeneity in firm production functions. Specifically, firm i's production functions are given by r_f^i and r_m^i where each function is increasing, concave and satisfies the Inada conditions. The first best level of investment in the economy is now given by,

$$r_f^{i\prime}(x_f^{i*}) = 1 \ \forall i \in \{1, 2..., N\}; \qquad r_m^{i\prime}(x_m^{i*}) = K^\prime \left(\sum_{i=1}^{N} x_m^{i*} \right) \ \forall i \in \{1, 2..., N\}. \tag{34}$$

When the Limited Funds assumption is satisfied, the decentralized equilibrium allocation of

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firm $i \in \{1,2...,N\}$ is,[11]

$$r_m^{i'}(x_m^i) = P(r_f^{i'}(x_f^i)(1-\phi)+\phi); \qquad x_f^i = \omega + B^i \phi P - (1-\phi)Px_m^i. \tag{35}$$

Constrained Social Planner Allocation: The constrained social planner solves

$$\max_{(x_f^i, x_m^i, x_s^i, l^i) \geq 0 \ \forall i \in \{1,2,...,N\}} \quad \sum_{i=1}^{N} \pi_f^i(x_m^i, x_f^i, x_s^i, l^i) + \pi_c(x_m^1, x_m^2, ..., x_m^N) \tag{36}$$

subject to

$$x_f^i + Px_m^i + x_s^i \leq \omega + l^i \quad \forall i \in \{1,2,...,N\} \tag{37}$$

$$l^i \leq \phi\left(B^i + x_m^i\right)P \quad \forall i \in \{1,2,...,N\}. \tag{38}$$

The social planner's optimal allocation for a firm i is

$$r_m^{i'}(x_m^i) - P(r_f^{i'}(x_f^i)(1-\phi)+\phi) + P - K' = P' \sum_{j=1}^{N} (r^{j'}(x_f^j)-1)((1-\phi)x_m^j - \phi B^j) \tag{39}$$

As before, if the Limited Funds assumption does not hold for all firms, then for all $i \in \{1,...,N\}$, $r'(x_f^i) - 1 = 0$ and equilibrium investment will equal first-best. When the Limited Funds assumption holds, we can establish the following proposition to characterize the distortion in investment due to real estate prices.

Proposition 5 (*Demand and Supply Distortions with Heterogeneous Collateral and Productivity*)
If the aggregate crowding-out effect is greater than the aggregate collateral effect, i.e. $\sum_{i=1}^{N}(r^{i'}(x_f^i) - 1)((1-\phi)x_m^{ie} - \phi B^i) > 0$, the social planner can improve upon the decentralized equilibrium welfare by reducing the demand for real estate and increasing the supply of real estate.

Conversely, if the aggregate collateral effect is greater than the aggregate crowding-out effect, i.e. $\sum_{i=1}^{N}(r^{i'}(x_f^i) - 1)((1-\phi)x_m^{ie} - \phi B^i) < 0$, the social planner can improve upon the decentralized

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[11]We now require that the Limited Funds assumption must be satisfied for each firm $i \in \{1,2,...,N\}$,i.e. for any i,

$$K'\left(\sum_{j=1}^{N} x_m^{j*}\right)x_m^{i*} + x_f^{i*} > \phi(B^i + x_m^{i*})K'\left(\sum_{j=1}^{N} x_m^{j*}\right) + \omega.$$

equilibrium welfare by increasing the demand for real estate and reducing the supply of real estate.

Proposition 5 is similar to Proposition 3 except that the function governing the marginal productivity of each firm may differ across firms. In this case, the correlation between firm collateral and its productivity can be important in determining whether real estate prices should be increased or decreased to increase total surplus. Since in practice, different regions likely have firms with differing productivity and collateral, the full model with heterogeneity in firms along these aspects is most suitable for policy prescriptions.

4.4. Sufficient Statistics to Guide Policy

We can implement a sufficient statistics approach to guide us in the direction of policy intervention, i.e. if real estate prices are too high or too low and if consequently real estate demand and supply should be taxed or subsidized. This approach is limited in that it cannot tell us the exact magnitude of the subsidies and taxes. However, the statistics we develop have the benefit of being easily observable. This approach follows Chetty (2009) and derives formulas based on sufficient statistics to guide policy without the need to estimate deeper parameters of the model.

As shown in Section 4.2., if firms in a region are relatively similar in terms of their production functions, for example if they operate within the same industry, then the optimal policy depends on the real estate holdings of the firm with the least amount of collateral in the economy. In this case, the relative magnitude of the crowding-out effect and the collateral effect depend on whether the ratio of new real estate transactions to inventory of the firm with the lowest collateral, $\frac{x_m}{B}$, is higher or lower than $\frac{\phi}{1-\phi}$ where ϕ is the pledgeability of real estate assets, i.e., the combined LTV ratio of loans on these assets. When the ratio of new real estate transactions to inventory is relatively high, the crowding-out effect is stronger than the collateral effect and it is optimal to subsidize the supply of real estate and tax the demand for real estate. Conversely, when the ratio of new real estate transactions to inventory is relatively low, the collateral effect is stronger than the crowding-out effect and it is optimal to tax the supply of real estate and subsidize the demand for real estate.

These statistics are observable and measurable.[12]

When firms in a region are heterogeneous in terms of their production technology as the ones considered in Section 4.3., we additionally need to account for the marginal productivity of capital of each firm.[13] For each firm i, we need to multiply the marginal productivity of capital of the firm by $(1 - \phi)x_m^i - \phi B^i$, where x_m^i is the firm's observed investment in new real estate assets and B^i is the firm's existing real estate holdings. We then sum up these quantities across firms. If the sum is positive, subsidizing the supply of real estate and taxing the demand for real estate to decrease real estate prices should increase aggregate investment. If the sum is negative, the converse is true. This approach is easily tailored to the goals of policy makers. For example, if policy makers are interested in targeting a specific group of firms within a region, they can calculate this statistic for that subset of firms. Importantly, all the firms should be exposed to the same real estate prices.

5. Extensions and Robustness

In this section, we discuss two extensions to the benchmark model which generate additional insights and establish robustness of the main mechanism. In Section 5.1., we extend the model to incorporate a constraint on the assets of the representative bank. In Section 5.2., we extend the model to allow the existing real estate assets of a firm to generate investment returns.

5.1. Bank Constraint

In the benchmark model, the bank has unlimited funds to lend. We can extend the model to incorporate a constraint on the assets the bank has available to lend. This allows us to make cross-sectional predictions about how bank size affects the severity of the collateral and crowding-out effects.

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[12]Chaney et al. (2012) and Doerr (2020) estimate the value of real estate assets at a firm-level. Data on LTV ratios for residential and commercial mortgages have been used extensively in many papers (see, for example, Titman, Tompaidis, and Tsyplakov (2005)).

[13]Imrohoroglu and Tuzel (2014) estimate firm-level productivity and construct a panel of firm-level total factor productivity for publicly traded firms in the United States. Also see Olley and Pakes (1996), Levinsohn and Petrin (2003) and Wooldridge (2009) for details on how to estimate firm-level productivity.

We extend the model by assuming the representative bank can lend a maximum amount of funds, A. For simplicity, we assume that the bank cannot raise additional funds. The firm now maximizes (1) subject to (2) and a modified borrowing constraint

$$l \leq min\{\phi\left(B + x_m\right)P, A\}. \tag{40}$$

As A decreases, the bank's asset constraint starts binding and even if the firm can credibly repay its loan, the bank cannot lend it the funds to do so. In this case, increasing real estate prices does not increase the funds a firm can borrow as a bank has already lent out all available assets. A further increase in real estate prices therefore always crowds out firm investment as there is no countervailing collateral effect.

We define x_m^A as the level of real estate investment at which the asset constraint binds, i.e.

$$A = \phi\left(B + x_m^A\right)P(x_m^A). \tag{41}$$

Any increase in prices when demand in real estate is beyond x_m^A, crowds out investment in firm projects. As A decreases, the range of real estate investment, and subsequently of real estate prices, that leads to a crowding-out of firm investment increases. Formally, we can establish the following proposition.

Proposition 6 *If $x_m^e > \min\left\{x_m^A, B\frac{\phi}{1-\phi}\right\}$, the social planner can improve upon the decentralized equilibrium welfare by reducing the demand for real estate and increasing the supply of real estate.*

Conversely, if $x_m^e < \min\left\{x_m^A, B\frac{\phi}{1-\phi}\right\}$, the social planner can improve upon the decentralized equilibrium welfare by increasing the demand for real estate and reducing the supply of real estate.

Our results are in line with empirical findings by Chakraborty et al. (2018) that an increase in real estate prices causes more severe crowding-out for firms that borrow from smaller banks. Intuitively, when real estate prices increase, smaller banks need to make larger loans to facilitate the purchase of real estate but may not have enough funds to take advantage of relaxed borrowing constraints stemming from the collateral effect. Therefore, these banks will shift their portfolio

away from commercial lending to a greater extent than larger banks.

5.2.   Productivity of Existing Real Estate Assets

In the benchmark model, to illustrate the mechanism in the simplest possible way we assume that existing real estate assets do not affect the return from real estate investment. The model is robust to relaxing this assumption. We can allow the return from real estate to depend both on the existing stock of real estate and new real estate purchases, i.e. $r_m(x_m + B)$.

In this case, existing real estate assets affect the optimal level of real estate investment due to decreasing returns to scale. The first-best level of investment is given by

$$r_f'(x_f) = 1; \qquad r_m'(x_m + B) = K'(x_m). \tag{42}$$

Although the solution seems similar to the benchmark model, the comparative statics with respect to B are different. In particular, for any given B, the optimum x_m^* is lower. As a result, in equilibrium, the relative magnitude of the crowding-out effect will decrease. We can establish an analogous proposition to proposition 2 showing that a combination of demand- and supply-side subsidies can restore the first-best level of investment.

Proposition 7 *The first-best level of welfare can be achieved by a subsidy pair*

$$\{r_g, b\} = \left\{ \frac{x_f^* - \omega}{\phi B - (1 - \phi)x_m^*} - r_m'(x_m^* + B), r'(x_m^* + B) - \frac{x_f^* - \omega}{\phi B - (1 - \phi)x_m^*} \right\}.$$

While the above expression is similar to the optimal policy in the benchmark model, there are two interesting differences. First, the range of parameters over which subsidizing demand while taxing supply is optimal will increase in the productivity of existing real estate assets. Since the equilibrium x_m is lower when existing real estate assets are productive, the absolute magnitude of the crowding-out effect is lower in equilibrium. Therefore, the collateral effect is stronger than the crowding-out effect for a larger range of parameters. Second, the magnitude of the optimal tax and subsidy is lower when existing real estate assets are productive. Due to a reduction in the

crowding-out effect, the equilibrium allocation is less inefficient and therefore, a smaller correction has to be made to get to first-best.

6. Discussion

In this section, we discuss some of the key insights of our framework — two-sided quantity distortions, the distribution of collateral — and the implications of our results for identification arguments based on the assumption that land-owning and non land-owning firms will react similarly to house price changes. We also discuss the applicability of the model to regions with extreme land constraints and to real estate investment by households rather than firms.

6.1. Two-Sided Distortions in Quantities

In our model, the decentralized equilibrium features two-sided distortions in the real estate market — there is simultaneously under-demand for and an over-supply of real estate, or vice versa. This result is different from standard economic theory in which we typically think about one-sided distortions in quantities — in equilibrium, there is either too much or too little of a good. Correcting the distortion in standard models therefore requires increasing supply and demand when there is too little of a good, or decreasing supply and demand when there is too much of a good. In contrast, the fundamental inefficiency in our model arises from equilibrium real estate prices not being at the optimal level. We therefore want to adjust supply and demand in a way that increases prices when they are too low and decreases prices when they are two high.

This price distortion causes the quantity response necessary to correct the distortion to vary across the demand and supply sides of the market, creating a distinct role for supply and demand subsidies. Traditional economic theory has long established that under general conditions, it does not matter whether we subsidize (tax) supply or demand from a welfare perspective. The gains to consumers and suppliers are the same and depend only on the relative elasticities of the supply and demand curves. However in our model, we find that subsidizing the supply and demand sides of the market generate different welfare implications since they move prices in opposite directions.

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We therefore want to tax one side of the market while subsidizing the other when targeting price movements.

6.2. Distribution of Collateral

Our results on firm heterogeneity highlight that the distribution of collateral is key in determining the effect of real estate prices on investment and designing corresponding policy interventions. In particular, even if aggregate collateral in the economy increases, such an increase may not lead to more productive investment.

Optimal policy depends on the distribution of collateral. Our results complement those of Di Maggio and Tahbaz-Salehi (2015) who argue that intermediation capacity depends on the distribution of collateral rather than simply the aggregate amount of collateral and of Donaldson et al. (2020) who show that policies aimed at increasing aggregate collateral can reduce welfare.

6.3. Spillover Effects of Collateral-Rich Firms on Collateral-Poor Firms

The analysis of the heterogeneous-firm model highlights possible spillover effects that collateral-rich firms can have on collateral-poor firms. Intuitively, when collateral is dispersed, firms that own more real estate assets experience a greater loosening of their collateral constraint than firms which own less real estate assets. If some of these collateral-rich firms invest more in real estate when their collateral constraint loosens, they will fuel an increase in real estate prices. This increase can crowd-out the investment of collateral-poor firms. This mechanism provides some nuance when thinking about empirical results on the effect of an increase in real estate prices on different firms since the differences in behavior by firms with different levels of collateral might not result only from their different collateral holdings, but also from further equilibrium effects in the real estate market.[14] These spillovers will cause firms with different amounts of real estate assets to react differently to changes in local real estate prices beyond what the direct effect of owning land would imply.

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[14]See Gan (2007) and Chaney et al. (2012).

6.4. Application to Regions with Extreme Real Estate Building Constraints

In our framework, optimal policy requires adjusting both the demand and supply of real estate. Some real estate markets, such as San Francisco or Manhattan have extreme land shortages and consequently have little ability to adjust the supply of real estate. In this case, our main insights can still be applied. Aggregate investment can be improved in a "second-best" way by adjusting only the demand curve for real estate. In particular, if the crowding-out effect is stronger than the collateral effect, a tax on real estate demand improves efficiency. When the collateral effect is stronger than the crowding-out effect, subsidizing real estate demand improves efficiency.

6.5. Application to Real Estate Investment of Households

Many interventions trying to support house prices are often targeted at households rather than at firms. Our theory can also provide insight into housing market policies that affect household rather than firm investment. The firm in our model can be reinterpreted as a household where the housing return is a convenience yield from consuming services attributable to home ownership, and the investment in firm projects can be interpreted as household investment in non-real estate assets, such as the stock and bond market. Under this interpretation, we can use insights from the model to help judge the effectiveness of various interventions in the housing market.

Consistent with the model predictions, Hilber and Turner (2014) find that the mortgage interest rate tax deduction leads to increased house prices and document a negative effect of this increase on home ownership rates among down-payment-constrained households. This effect seems to be particularly strong in areas where the supply of housing is relatively inelastic. Our model can shed insight into these empirical findings as a demand-side subsidy such as the mortgage interest rate tax deduction will push up the price of housing. This negatively affects new housing investment by households who face financial constraints.[15]

Other research has looked at policies targeting home owners in the aftermath of the crisis.

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[15]Glaeser and Shapiro (2003) and Hanson (2012) also provide evidence showing that the mortgage interest rate tax deduction seems to have little effect on increasing home ownership, particularly among financially constrained households.

Agarwal, Amromin, Ben-david, Chomsisengphet, Piskorski, and Seru (2017) and Agarwal, Amromin, Chomsisengphet, Landvoigt, Piskorski, Seru, and Yao (2015) evaluate the effectiveness of the Home Affordable Modification Program and the Home Affordable Refinancing Program. They find evidence that these programs reduced house price declines, foreclosure rates and delinquencies on consumer debt. They also find evidence that these programs allowed indebted homeowners to increase spending on durable consumption. Keys, Piskorski, Seru, and Yao (2014) find that lower mortgage rates in the aftermath of the housing crisis also led to a decrease in defaults and an increase in spending on durable consumption particularly among constrained households. Since these programs were targeted at existing home owners (a high B) rather than encouraging new home ownership, our model predicts that demand-side subsidies that increase house prices should prove effective in these cases. For households with existing home equity, an increase in housing prices should relax collateral constraints allowing them to more easily pay down debt and increase spending on durable consumption.

7. Conclusion

In this paper we develop a comprehensive framework for studying the effect of housing policy on investment. We jointly study a collateral channel and a crowding-out channel. This allows us to characterize what the trade-offs are for a policy maker when designing real estate policy that targets not just the level of real estate investment but also, the price. We show that supply and demand subsidies are not equivalent in the presence of price externalities.

When firms have heterogeneous real estate assets, the distribution of collateral determines the effect of real estate prices on investment. An increase in aggregate collateral can cause a decrease in total surplus. Optimal policy should be designed based on the joint distribution of firm productivity and collateral. We hope that our framework will help better inform the design of housing policy.

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Appendix A

Proof of Proposition 1. Let D be the amount the social planner shifts real estate demand per unit and S be the amount the social planner shifts real estate supply per unit. The firm maximizes

$$\max_{(x_f, x_m, x_s, l) \geq 0} \quad r_f(x_f) + r_m(x_m) + Dx_m + x_s - l(1+r) \tag{A.1a}$$

subject to

$$x_f + Px_m + x_s \leq \omega + l \tag{A.1b}$$

$$l \leq \phi(B + x_m)P. \tag{A.1c}$$

The construction company maximizes

$$\max_{x_m \geq 0} \quad Px_m + Sx_m - K(x_m). \tag{A.2}$$

The equilibrium level of housing investment is given by

$$r'_m(x_m) + D = P(r'_f(x_f)(1 - \phi) + \phi). \tag{A.3}$$

Market-clearing implies

$$r'_m(x_m) + D = (K'(x_m) - S)(r'_f(x_f)(1 - \phi) + \phi). \tag{A.4}$$

If $D = -S(r'_f(x_f)(1 - \phi) + \phi)$, then the investment in real estate that solves the above equation is equal to x^e_m. The investment in firm projects is given by

$$x_f = \omega + (K'(x^e_m) - S)(B - (1 - \phi)x^e_m). \tag{A.5}$$

If $(1 - \phi)x^e_m > \phi B$, then the x_f that solves the above is strictly higher than x^e_f whenever $S > 0$. The household's welfare is therefore strictly higher than in the decentralized equilibrium. Since $S > 0$, $D = -S(r'_f(x_f)(1 - \phi) + \phi) < 0$. Therefore, the social planner can do strictly better than the decentralized equilibrium by shifting the supply of real estate by a positive amount S and by

shifting the demand for real estate by a negative amount D.

If $(1-\phi)x_m^e < \phi B$, then the x_f that solves the above is strictly higher than x_f^e whenever $S < 0$. The household's welfare is therefore strictly higher than in the decentralized equilibrium. Since $S < 0$, $D = -S(r_f'(x_f)(1-\phi)+\phi) > 0$. Therefore, the social planner can do strictly better than the decentralized equilibrium by shifting the supply of real estate by a negative amount S and by shifting the demand for real estate by a positive amount D. ∎

Proof of Corollary 1. The equilibrium levels of real estate and firm project are given by

$$r_m'(x_m) = P(r'(x_f)(1-\phi)+\phi), \tag{A.6a}$$

$$x_f = \omega + P(\phi B - (1-\phi)x_m). \tag{A.6b}$$

As B or ϕ increase, for any level of aggregate prices, the firm has a looser borrowing constraint. In equilibrium, for (A.6b) and (A.6a) to be jointly satisfied, the firm will increase investment in both real estate and firm projects. Therefore, aggregate investment output, $r_m(x_m) + r_f(x_f)$, will increase since $r_m' > 0$ and $r_f' > 0$. ∎

Proof of Proposition 2. At the optimal investment levels, the firm's budget constraint is

$$x_f^* = \omega + \phi B P^* - P^*(1-\phi)x_m^*. \tag{A.7}$$

Rearranging the above equation, the price that the firm pays for real estate investment at this optimal is

$$P^* = \frac{x_f^* - \omega}{\phi B - (1-\phi)x_m^*}. \tag{A.8}$$

At the optimal level of firm project investment, $r'(x_f^*) = 1$.

From the firm's FOC, to incentivize the firm to invest in x_m^* units of real estate, we require r_g^* such that

$$\frac{r_m'(x_m^*) + r_g^*}{P^*} = 1. \tag{A.9}$$

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Substituting in for P^* and rearranging the above equation, the optimal r_g^* is

$$r_g^* = \frac{x_f^* - \omega}{\phi B - (1 - \phi)x_m^*} - r_m'(x_m^*). \tag{A.10}$$

The firm's FOC implies that the optimal b^* is

$$K'(x_m^*) = \frac{x_f^* - \omega}{\phi B - (1 - \phi)x_m^*} + b^*. \tag{A.11}$$

Rearranging the above, b^* is given by

$$b^* = K'(x_m^*) - \frac{x_f^* - \omega}{\phi B - (1 - \phi)x_m^*}. \tag{A.12}$$

At x_m^*, $K'(x_m^*) = r_m'(x_m^*)$. Therefore, we can rewrite b^* as

$$b^* = r'(x_m^*) - \frac{x_f^* - \omega}{\phi B - (1 - \phi)x_m^*} = -r_g^*. \tag{A.13}$$

Therefore, expansionary supply-side policy (positive b) have to be accompanied by contractionary demand-side intervention (negative r_g) to achieve the optimum investment. b^* is positive when

$$r'(x_m^*) - \frac{x_f^* - \omega}{\phi B - (1 - \phi)x_m^*} > 0. \tag{A.14}$$

Under this policy the household's utility is given by

$$U = r_m(x_m^*) + r_f(x_f^*) + r_g^* x_m^* - \tau - l + (P^* + b^*)x_m^* - K(x_m^*). \tag{A.15}$$

where $l = \phi(B + x_m^*)P^* = P^* x_m^* + x_f^* - \omega$. For the government to have a balanced budget, $\tau = (b^* + r_g^*)x_m^*$. Substituting for τ and l into the household's utility and using the fact that $P^* = K'(x_m) - b^*$, the above equation simplifies to

$$U = r_m(x_m^*) + r_f(x_f^*) - K(x_m^*) - x_f^* + \omega. \tag{A.16}$$

The household's utility is equal to the first-best without financial constraints. ∎

Proof of Proposition 3. Let D^i be the amount the social planner shifts the real estate demand of firm $i \in \{1, ..., N\}$ and S be the amount the social planner shifts real estate supply per unit. Each firm i maximizes

$$\max_{(x_f^i, x_m^i, x_s^i, l^i) \geq 0} \quad r_f(x_f^i) + r_m(x_m^i) + D^i x_m^i + x_s^i - l^i(1+r) \tag{A.17a}$$

subject to

$$x_f^i + P x_m^i + x_s \leq \omega + l^i \tag{A.17b}$$

$$l^i \leq \phi \left(B^i + x_m^i \right) P. \tag{A.17c}$$

The construction company maximizes

$$\max_{\sum_{i=1}^N x_m^i \geq 0} \quad P \sum_{i=1}^N x_m^i + S \sum_{i=1}^N x_m^i - K \left(\sum_{i=1}^N x_m^i \right). \tag{A.18}$$

The equilibrium level of real estate investment by firm i is given by

$$r_m'(x_m^i) + D^i = P(r_f'(x_f^i)(1 - \phi) + \phi). \tag{A.19}$$

Market-clearing implies

$$r_m'(x_m^i) + D^i = \left(K' \left(\sum_{i=1}^N x_m^i \right) - S \right) (r_f'(x_f^i)(1 - \phi) + \phi). \tag{A.20}$$

If $D^i = -S(r_f'(x_f^i)(1 - \phi) + \phi)$, then the investment in real estate that solves the above equation for each firm is equal to x_m^{ie}. The investment by firm i in its own projects is given by

$$x_f^i = \omega + \left(K' \left(\sum_{i=1}^N x_m^{ie} \right) - S \right) (\phi B - (1 - \phi) x_m^{ie}). \tag{A.21}$$

The aggregate investment in firm projects is given by

$$\sum_{i=1}^{N} x_f^i = N\omega + \sum_{i=1}^{N} K' \left(\sum_{i=1}^{N} x_m^{ie} \right) (\phi B^i - (1-\phi)x_m^{ie}) - \sum_{i=1}^{N} S \left(\phi B^i - (1-\phi)x_m^{ie} \right). \qquad \text{(A.22)}$$

When $S > 0$ (and consequently $D < 0$), each firm with $\phi B^i > (1-\phi)x_m^{ie}$ decreases firm project investment and each firm with $\phi B^i < (1-\phi)x_m^{ie}$ increases firm project investment. The household's final wealth will increase when $\sum_{i=1}^{N}(r'(x_f^i) - 1)((1-\phi)x_m^{ie} - \phi B^i) > 0$. Therefore, the social planner can do strictly better than the decentralized equilibrium by change the shifting of real estate by a positive amount S and by shifting the demand for real estate by a negative amount D^i for each firm.

Conversely, when $S < 0$ (and consequently $D > 0$), each firm with $\phi B^i > (1-\phi)x_m^{ie}$ increases firm project investment and each firm with $\phi B^i < (1-\phi)x_m^{ie}$ decreases firm project investment. The household's final wealth will increase when $\sum_{i=1}^{N}(r'(x_f^i) - 1)((1-\phi)x_m^i - \phi B^i) < 0$. Therefore, the social planner can do strictly better than the decentralized equilibrium by shifting the supply of real estate by a negative amount S and by shifting the demand for real estate by a positive amount D^i for each firm. ∎

Proof of Corollary 2. Without loss of generality, consider a change to the existing real estate holdings of firms from $\{B^1, B^2, ..., B^N\}$ to $\{B^1, B^2, ..., B^{N'}\}$ such that $B^{N'} > B^N$. This implies that aggregate collateral increases, i.e. $\sum_{i=1}^{N-1} B^i + B^{N'} > \sum_{i=1}^{N-1} B^i + B^N$. Let P be the equilibrium price of real estate under the old collateral holdings and $P' > P$ be the equilibrium price of real estate under new collateral holdings. Further, let $x_m^{i'}$ and $x_f^{i'}$ represent the levels of real estate and project investment of firm i under price P'.

Then the change in aggregate collateral will lead to a decrease in investment output if

$$\sum_{i=1}^{N} (r_m(x_m^{i'}) + r_f(x_f^{i'})) > \sum_{i=1}^{N} (r_m(x_m^i) + r_f(x_f^i)). \qquad \text{(A.23)}$$

We can alternatively consider a change in collateralizability from ϕ to $\phi' > \phi$. Such a change always increases aggregate collateral in the economy.

In the main text, we plot an example in which aggregate investment output decreases when aggregate collateral increases for a case with two firms. ∎

Proof of Proposition 4. At optimal investment the firm with collateral B^1's budget constraint is

$$x_f^* = \omega + \phi B^1 P^* - P^*(1-\phi)x_m^*. \tag{A.24}$$

Rearranging the above equation, the optimal price is

$$P^* = \frac{x_f^* - \omega}{\phi B^1 - (1-\phi)x_m^*}. \tag{A.25}$$

For the firm to invest in x_m^* units of real estate given a subsidy r_g^* per unit of housing, we require that

$$\frac{r_m'(x_m^*) + r_g^*}{P^*} = 1. \tag{A.26}$$

Substituting in for P^* and rearranging the above equation, we obtain the following expression for r_g^*

$$r_g^* = \frac{x_f^* - \omega}{\phi B - (1-\phi)x_m^*} - r_m'(x_m^*). \tag{A.27}$$

The construction company's FOC when it is subsidized an amount b^*, all firms also invest in x_m^* units of real estate and the equilibrium price is P^* is

$$K'(Nx_m^*) = \frac{x_f^* - \omega}{\phi B - (1-\phi)x_m^*} + b^*. \tag{A.28}$$

Rearranging the above equation, we obtain the following expression for b^*

$$b^* = K'(Nx_m^*) - \frac{x_f^* - \omega}{\phi B - (1-\phi)x_m^*}. \tag{A.29}$$

Since $K'(Nx_m^*) = r_m'(x_m^*)$, we can rewrite b^* as

$$b^* = r'(x_m^*) - \frac{x_f^* - \omega}{\phi B - (1 - \phi)x_m^*} = -r_g^*. \tag{A.30}$$

To complete the proof, we need to check that all other firms also want to invest in x_m^* units of real estate and to invest x_f^* in firm projects. All other firms have the capacity to borrow more than firm 1 since $B_i > B_1 \ \forall i \in \{2, ..., N\}$. However, no other firm's borrowing constraint will bind since its marginal return from real estate investment beyond x_m^* and firm project investment beyond x_f^* will be less than 1. Therefore, at the above choices of r_g^* and b^*, all firms will pick $x_m^i = x_m^*$ and $x_f^i = x_f^* \ \forall i \in \{1, ..., N\}$. ∎

Proof of Proposition 5. Let D^i be the amount the social planner shifts the real estate demand of firm $i \in \{1, ..., N\}$ and S be the amount the social planner shifts real estate supply per unit. Each firm i maximizes

$$\max_{(x_f^i, x_m^i, x_s^i, l^i) \geq 0} r_f^i(x_f^i) + r_m^i(x_m^i) + D^i x_m^i + x_s^i - l^i(1 + r) \tag{A.31a}$$

subject to

$$x_f^i + P x_m^i + x_s \leq \omega + l^i \tag{A.31b}$$

$$l^i \leq \phi \left(B^i + x_m^i \right) P. \tag{A.31c}$$

The construction company maximizes

$$\max_{\sum_{i=1}^{N} x_m^i \geq 0} P \sum_{i=1}^{N} x_m^i + S \sum_{i=1}^{N} x_m^i - K \left(\sum_{i=1}^{N} x_m^i \right). \tag{A.32}$$

The equilibrium level of real estate investment by firm i is given by

$$r_m^{i'}(x_m^i) + D^i = P(r_f^{i'}(x_f^i)(1 - \phi) + \phi). \tag{A.33}$$

Market-clearing implies

$$r_m^{i'}(x_m^i) + D^i = \left(K' \left(\sum_{i=1}^{N} x_m^i \right) - S \right) (r_f^{i'}(x_f^i)(1 - \phi) + \phi). \tag{A.34}$$

If $D^i = -S(r_f^{i'}(x_f^i)(1 - \phi) + \phi)$, then the investment in real estate that solves the above equation for each firm is equal to x_m^{ie}. The investment by firm i in its own projects is given by

$$x_f^i = \omega + \left(K' \left(\sum_{i=1}^{N} x_m^{ie} \right) - S \right) \left(\phi B - (1 - \phi)x_m^{ie} \right). \tag{A.35}$$

The aggregate investment in firm projects is given by

$$\sum_{i=1}^{N} x_f^i = N\omega + \sum_{i=1}^{N} K' \left(\sum_{i=1}^{N} x_m^{ie} \right) \left(\phi B^i - (1 - \phi)x_m^{ie} \right) - \sum_{i=1}^{N} S \left(\phi B^i - (1 - \phi)x_m^{ie} \right). \tag{A.36}$$

When $S > 0$ (and consequently $D < 0$), each firm with $\phi B^i > (1 - \phi)x_m^{ie}$ decreases firm project investment and each firm with $\phi B^i < (1 - \phi)x_m^{ie}$ increases firm project investment. The household's final wealth will increase when $\sum_{i=1}^{N}(r^{i'}(x_f^i) - 1)((1 - \phi)x_m^{ie} - \phi B^i) > 0$. Therefore, the social planner can do strictly better than the decentralized equilibrium by change the shifting of real estate by a positive amount S and by shifting the demand for real estate by a negative amount D^i for each firm.

Conversely, when $S < 0$ (and consequently $D > 0$), each firm with $\phi B^i > (1 - \phi)x_m^{ie}$ increases firm project investment and each firm with $\phi B^i < (1 - \phi)x_m^{ie}$ decreases firm project investment. The household's final wealth will increase when $\sum_{i=1}^{N}(r^{i'}(x_f^i) - 1)((1 - \phi)x_m^i - \phi B^i) < 0$. Therefore, the social planner can do strictly better than the decentralized equilibrium by shifting the supply of real estate by a negative amount S and by shifting the demand for real estate by a positive amount D^i for each firm. ∎

Proof of Proposition 6. If $B\frac{\phi}{1-\phi} < x_m^A$, then the analysis is identical to the proof of Proposition 1. Let D be the amount the social planner shifts real estate demand per unit and S be the amount the social planner shifts real estate supply per unit. The firm maximizes

$$\max_{(x_f,x_m,x_s,l)\geq 0} \quad r_f(x_f)+r_m(x_m)+Dx_m+x_s-l(1+r) \tag{A.37a}$$

subject to

$$x_f+Px_m+x_s \leq \omega+l \tag{A.37b}$$

$$l \leq min\{\phi\,(B+x_m)P,A\}. \tag{A.37c}$$

The construction company maximizes

$$\max_{x_m\geq 0} \quad Px_m+Sx_m-K(x_m). \tag{A.38}$$

When $x_m^e > x_m^A$, the equilibrium level of housing investment is given by

$$r_m'(x_m)+D = Pr_f'(x_f). \tag{A.39}$$

Market-clearing implies

$$r_m'(x_m)+D = (K'(x_m)-S)r_f'(x_f). \tag{A.40}$$

If $D = -Sr_f'(x_f)$, then the investment in real estate that solves the above equation is equal to x_m^e. The investment in firm projects is given by

$$x_f = \omega+A-(K'(x_m^e)-S)x_m^e. \tag{A.41}$$

The x_f that solves the above is strictly higher than x_f^e whenever $S > 0$. The household's welfare is therefore strictly higher than in the decentralized equilibrium. Since $S > 0$, $D = -Sr_f'(x_f) < 0$. Therefore, the social planner can do strictly better than the decentralized equilibrium by shifting the supply of real estate by a positive amount S and by shifting the demand for real estate by a negative amount D.

When $x_m^e < x_m^A$, the analysis is identical to the proof of Proposition 1. ∎

Proof of Proposition 7. At the optimal investment levels the firm's budget constraint is

$$x_f^* = \omega + \phi B P^* - P^*(1-\phi)x_m^*. \tag{A.42}$$

Rearranging the above equation, the optimal P^* is

$$P^* = \frac{x_f^* - \omega}{\phi B - (1-\phi)x_m^*}. \tag{A.43}$$

From the firm's FOC, we also require that

$$\frac{r_m'(x_m^* + B) + r_g^*}{P^*} = 1. \tag{A.44}$$

Substituting in for P^* and rearranging the above equation, the optimal r_g^* is

$$r_g^* = \frac{x_f^* - \omega}{\phi B - (1-\phi)x_m^*} - r_m'(x_m^* + B). \tag{A.45}$$

From the construction company's FOC, the optimal b^* is

$$K'(x_m^*) = \frac{x_f^* - \omega}{\phi B - (1-\phi)x_m^*} + b^*. \tag{A.46}$$

Rearranging the above equation

$$b^* = K'(x_m^*) - \frac{x_f^* - \omega}{\phi B - (1-\phi)x_m^*}. \tag{A.47}$$

At optimal investment in real estate, x_m^*, $K'(x_m^*) = r_m'(x_m^* + B)$. Therefore, we can rewrite b^* as

$$b^* = r'(x_m^* + B) - \frac{x_f^* - \omega}{\phi B - (1-\phi)x_m^*} = -r_g^*. \tag{A.48}$$

b^* is positive when,

$$r^{'}(x_m^* + B) - \frac{x_f^* - \omega}{\phi B - (1 - \phi)x_m^*} > 0. \tag{A.49}$$

Under this subsidy scheme the household's utility is given by

$$U = r_m(x_m^* + B) + r_f(x_f^*) + r_g^* x_m^* - \tau - l + (P^* + b^*)x_m^* - K(x_m^*). \tag{A.50}$$

where $l = \phi(B + x_m^*)P^* = P^* x_m^* + x_f^* - \omega$. For the government to have a balanced budget, $\tau = (b^* + r_g^*)x_m^*$. Substituting this into household utility and using the fact that $P^* = K^{'}(x_m) - b^*$, the above equation simplifies to

$$U = r_m(x_m^* + B) + r_f(x_f^*) - K(x_m^*) - x_f^* + \omega. \tag{A.51}$$

The utility is equal to the first-best level. ∎