Macro-Finance Decoupling:
Robust Evaluations of Macro Asset Pricing Models[*]

Xu Cheng[†], Winston Wei Dou[‡], Zhipeng Liao[§]

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Abstract

This paper shows that robust inference under weak identification is important to the evaluation of many influential macro asset pricing models, including long-run risk models and (time-varying) rare-disaster risk models. Building on recent developments in the conditional inference literature, we provide a novel conditional specification test by simulating the critical value conditional on a sufficient statistic. This sufficient statistic can be intuitively interpreted as a measure capturing the macroeconomic information decoupled from the underlying content of asset pricing theories. Macro-finance decoupling is an effective way to improve the power of the specification test when asset pricing theories are difficult to refute because of a severe imbalance in the information content about the key model parameters between macroeconomic moment restrictions and asset pricing cross-equation restrictions. For empirical application, we apply the proposed conditional specification test to evaluate a time-varying rare-disaster risk model and construct data-driven robust model uncertainty sets.

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[†]Department of Economics, University of Pennsylvania; Email: xucheng@econ.upenn.edu.
[‡]Finance Department, The Wharton School, University of Pennsylvania; Email: wdou@wharton.upenn.edu.
[§]Department of Economics, University of California, Los Angeles; Email: zhipeng.liao@econ.ucla.edu.

1   Introduction

Many influential macro asset pricing models, either structural or reduced form, have been developed and widely used by researchers, practitioners, and monetary authorities.[1] However, model evaluation is challenging when many models are seemingly able to match any asset pricing and macroeconomic moments of interest, either because of severe information imbalances in structural models (Campbell, 2018; Chen, Dou, and Kogan, 2020) or because of the reduced rank with spurious factors (Kan and Zhang, 1999a; Gospodinov, Kan, and Robotti, 2017). For reduced-form factor models, Lewellen, Nagel, and Shanken (2010) argue that the existing standard specification tests overenthusiastically support a large number of such models. Particularly, Gospodinov, Kan, and Robotti (2017) show that the power of the J test (Hansen, 1982; Hansen and Singleton, 1982) could be as small as the size of the test in their model with spurious factors. These linear models with spurious factors resemble the weak instrumental variable (IV) model in many ways (as emphasized by, e.g., Kleibergen, 2009). The econometric question of how to construct a robust and efficient evaluation of macro asset pricing models, especially nonlinear structural models, is of great importance for the asset pricing literature.

To address this crucial question, this paper provides a specification test robust to information imbalances in a unified weak identification framework. In the generalized method of moments (GMM) setup, we show that the rare-disaster risk model (Rietz, 1988; Barro, 2006), the time-varying rare-disaster risk model (Gabaix, 2012; Wachter, 2013), and the long-run risk model (Bansal and Yaron, 2004) all fit in a general framework — there exists a set of baseline moments that are valid regardless of the asset-pricing theory but only provide weak identification of the key model parameters, characterized by near flatness of the baseline moments (Stock and Wright, 2000).[2] The remainder of the moments are asset-pricing moments implied by a specific macro asset pricing theory. These asset pricing moments, by design, provide tighter cross-equation restrictions, thus strongly identifying the parameters. The proposed test evaluates validity of the asset pricing moments and gauges its uncertainty by effectively exploiting the valid but noisy information in the baseline moments.

This new conditional specification test builds on the approach of conditional inference with a functional nuisance parameter by Andrews and Mikusheva (2016a). The object of interest shifts from structural parameters to model specification. The test statistic is an incremental J statistic as in the C test of Eichenbaum, Hansen, and Singleton (1988). However, the critical value is

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[1] See, e.g., Campbell (1999, 2003), Hansen (2014), Brunnermeier and Sannikov (2016), Cochrane (2017), He and Krishnamurthy (2018), and Dou, Fang, Lo, and Uhlig (2020), for reviews on macro-finance models.

[2] See, e.g., Stock, Wright, and Yogo (2002), Andrews and Stock (2007), Mikusheva (2013), and Andrews, Stock, and Sun (2020), for reviews on weak instruments and weak identification.

simulation-based and is conditional on a sufficient statistic that captures the macroeconomic information decoupled from the underlying content of asset pricing theories. It has correct asymptotic size uniformly over the identification strength in the baseline moments. In contrast, the C test, with a critical value based on the chi-square distribution, may under- or over-reject under the null because estimators for weakly identified parameters have distributions that are poorly approximated by the normal distribution even in large samples (e.g., Staiger and Stock, 1997; Stock and Wright, 2000; Andrews and Cheng, 2012).[3] The conditional specification test becomes equivalent to the optimal test in the classical scenario without weak identification (Newey, 1985). In contrast, the standard J test based on all moment conditions sacrifices power by neglecting valid information in the baseline moments.

We construct "model uncertainty sets" by relaxing the asset-pricing moments and collecting the models not rejected by the conditional specification test. In the empirical application, we evaluate the time-varying disaster risk model similar to Wachter (2013). We show that the model uncertainty set based on the conditional specification test is substantially smaller than that from the standard J test. In practice, constructing statistically valid data-driven uncertainty sets of structural models is the key step toward serious quantitative analysis accounting for model uncertainty and robustness controls (e.g., Schneider and Schweizer, 2015; Hansen and Sargent, 2020; Barnett, Brock, and Hansen, 2020) and formal econometric analysis accounting for model misspecifications (e.g., Andrews, Gentzkow, and Shapiro, 2017; Cheng, Liao, and Shi, 2019; Bonhomme and Weidner, 2020; Armstrong and Kolesar, 2021).

This paper contributes to the connection between finance and econometrics. Formalizing the first-order, prevalent asset pricing problems in advanced econometric frameworks and providing new solutions to these problems based on non-standard inferences can be extremely valuable yet highly challenging. To achieve this primary goal, our paper also makes methodological and empirical contributions to the literature from the following three aspects.

First, we study the identification issue in (time-varying) rare-disaster risk and long-run risk models. Our study shows how to coherently fit information imbalances of a macro asset pricing structural model into a formal econometric framework with both weak and strong identification through a model-specific reparameterization. The reparameterization exploits the major implications of the macro asset pricing model and paves the way for uniformly valid inference. To the best of our knowledge, the identification and robust testing problems we study here have never been explored for these macro asset pricing structural models under a formal econometric framework.

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[3]This non-standard problem under weak identification still exists even when the number of moments is large; see, e.g., Chamberlain and Imbens (2004), Han and Phillips (2006), and Newey and Windmeijer (2009), among others.

Second, we develop a new robust specification test and study its asymptotic property. The proposed conditional specification test builds on the work of Andrews and Mikusheva (2016a) and departs in two main ways from it: (i) we estimate the model parameters under the null instead of plugging in their true values under the null because the latter is not available for the purpose of evaluating models; (ii) we explore the differences in validity and information content across two sets of moments instead of studying inference based on one set of moments. Estimation under the null introduces some additional technical complications for uniformly valid asymptotic analysis.

Third, we analyze a full-blown time-varying rare-disaster risk model and provide rigorous econometric analysis. There has been little formal econometric analysis on rare-disaster risk mechanisms in the asset pricing literature, with a few exceptions (e.g., Julliard and Ghosh, 2012). Our empirical analysis fills this gap in the literature and illustrates a novel way to construct reliable and informative data-driven model uncertainty sets based on robust specification tests.

Related Literature. From the finance perspective, the feature of information imbalances is ubiquitous in macro asset pricing structural models, but the literature lacks a robust and efficient way to statistically evaluate these influential models or reliably quantify model uncertainty embedded in these potentially fragile structural models. Data-driven model uncertainty sets are crucial for robustness analysis of structural economic models (e.g., Hansen and Sargent, 2001, 2008, 2020; Cagetti, Hansen, Sargent, and Williams, 2002; Bidder and Dew-Becker, 2016).[4] Our robust specification test complements the advances in robust confidence set construction for parameters of macro asset-pricing structural models in the presence of weak identification. Examples of robust inference of an asset pricing model include Stock and Wright (2000), who study the preference parameters through nonlinear Euler equations, Yogo (2004), who estimates and makes valid inference of the elasticity of intertemporal substitution (EIS) parameter through linearized cross-equation restrictions, and Ascari, Magnusson, and Mavroeidis (2019), who study the preference parameters under different structural models featuring habits, hand-to-mouth consumers, or recursive preferences.

From the econometrics perspective, we contribute to improving upon existing specification tests and broadening the scope of weak-identification robust inference methods. Conditional inference has been successfully applied in constructing confidence sets for weakly identified parameters, following the pioneering work of Moreira (2003) for linear IV models. Kleibergen (2005) broadens its application to nonlinear GMM models. Furthermore, Andrews and Mikusheva (2016a) provide a new perspective in viewing the near-flat population moment function as a functional nuisance parameter. Standing on their shoulders, we apply the conditional inference approach to construct

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[4]In particular, model uncertainty is intrinsic to climate change and thus is an essential element in climate economics (e.g., Brock and Hansen, 2019; Barnett, Brock, and Hansen, 2020; Diebold and Rudebusch, 2021).

robust specification tests, a new setting with important applications. For model evaluations, Hahn and Hausman (2002) and Hahn, Ham, and Moon (2011) provide tests robust to weak instruments in the context of linear IV models without the information imbalances studied here. Other studies on robust inference methods for weakly identified nonlinear models include Stock and Wright (2000), Guggenberger and Smith (2005), Otsu (2006), Andrews and Cheng (2012), Qu (2014), Andrews and Mikusheva (2015, 2016b, 2020), Andrews (2016), Chen, Christensen, and Tamer (2018), Andrews and Guggenberger (2019), Han and McCloskey (2019), Cox (2020), Evdokimov and Zeleneev (2020), Antoine, Proulx, and Renault (2020), and Kaji (2021), among others.

Finally, we contribute new econometric tools to the empirical toolbox for financial economists. A growing body of literature is concerned with the efficacy of conventional methods for macro asset pricing models and the development of robust methods, such as methodological advances for linear asset pricing models (e.g., Kan and Zhang, 1999b; Kleibergen, 2009; Beaulieu, Dufour, and Khalaf, 2013, 2020; Gospodinov, Kan, and Robotti, 2014; Burnside, 2015; Kleibergen and Zhan, 2015, 2020; Anatolyev and Mikusheva, 2020). Instead of focusing on the test statistic itself, Daniel and Titman (2012), Ahn, Conrad, and Dittmar (2009), and Nagel and Singleton (2011) propose new methods for constructing informative test assets to increase the power of testing linear factor models. Under a general semiparametric framework, Chen, Dou, and Kogan (2020) formalize a measure of information imbalance that leads to low reputability and poor out-of-sample performance. Model evaluation for factor models is also considered by Bai and Ng (2006) and Penaranda and Sentana (2015), among others. Different from our model uncertainty set, the model confidence set of Hansen, Lunde, and Nason (2011) is for the selection of forecasting models. Furthermore, in predictive models of stock returns with highly persistent predictors, standard asymptotic inference can largely fail (e.g., Elliott and Stock, 1994; Stambaugh, 1999), and new valid and efficient procedures have been developed (e.g., Campbell and Yogo, 2006; Elliott, Müller, and Watson, 2015). Recently, a growing body of literature has started to apply machine learning techniques to evaluate linear asset pricing models (e.g., Kelly, Pruitt, and Su, 2019; Feng, Giglio, and Xiu, 2020; Giglio and Xiu, 2020). Nevertheless, the existing literature lacks reliable and powerful evaluation methods for structural nonlinear models in the presence of information imbalances. This paper tackles this challenging fundamental issue in the literature.

The rest of the paper is organized as follows. Section 2 provides the general GMM setup with information imbalances and two motivating examples. Section 3 describes the conditional specification test, provides the algorithm, and illustrates its finite-sample performance through Monte Carlo simulations based on the two motivating examples. Section 4 provides theoretical results on the size of the proposed test and establishes its uniform validity. Section 5 contains

an empirical application of the proposed test to time-varying rare-disaster risk models. Section 6 concludes. The Supplemental Appendix contains proofs, theoretical and simulation results on the power of the test, and details for the empirical application. Cheng, Dou, and Liao (2020), available on SSRN and the authors' personal websites, contains additional supporting materials.

2 Information Imbalances: General Setup and Examples

General Setup. Our objective is to statistically assess the validity of a macro asset pricing model by applying specification tests to a set of model-implied cross-equation restrictions. The specification test can be formulated as below:

$$H_0 : \mathbb{E}[\bar{g}_1(\theta_0)] = 0_{k_1 \times 1} \quad \text{versus} \quad H_1 : \mathbb{E}[\bar{g}_1(\theta_0)] \neq 0_{k_1 \times 1}, \tag{2.1}$$

where $\bar{g}_1(\theta) \equiv n^{-1} \sum_{i=1}^n g_{1,t}(\theta)$, $g_{1,t}(\theta) \equiv g_1(Y_t, \theta) \in \mathbb{R}^{k_1}$ depends on the data Y_t and the $d_\theta \times 1$ dimensional parameter θ, whose true value is denoted by θ_0, and some additional baseline moments are always valid under both the null and the alternative,

$$\mathbb{E}[\bar{g}_0(\theta_0)] = 0_{k_0 \times 1}, \tag{2.2}$$

where $\bar{g}_0(\theta) \equiv n^{-1} \sum_{i=1}^n g_{0,t}(\theta)$ and $g_{0,t}(\theta) \equiv g_0(Y_t, \theta) \in \mathbb{R}^{k_0}$.[5] Under the null, the full moments are $\mathbb{E}[\bar{g}(\theta_0)] = 0_{k \times 1}$, where $\bar{g}(\theta) \equiv [\bar{g}_0(\theta)', \ \bar{g}_1(\theta)']'$ is $k \equiv k_0 + k_1$ dimensional and differentiable in θ almost surely. We allow the baseline moments to depend only on a subvector of θ.

The models we consider have two key features. First, the baseline moments $\mathbb{E}[\bar{g}_0(\theta_0)] = 0_{k_0 \times 1}$ may only weakly identify θ_0, i.e., $\mathbb{E}[\bar{g}_0(\theta)]$ is nearly flat in θ. Second, once the asset pricing theory is imposed, the full moments $\mathbb{E}[\bar{g}(\theta_0)] = 0_{k \times 1}$ strongly identify θ_0 under the null, i.e., the singular values of the associated Jacobian matrix $Q \equiv \mathbb{E}[\partial \bar{g}(\theta_0)/\partial \theta']$ are bounded away from zero. For moments created with IVs, $\mathbb{E}[\bar{g}_0(\theta_0)] = 0_{k_0 \times 1}$ and $\mathbb{E}[\bar{g}_1(\theta_0)] = 0_{k_1 \times 1}$ are created with weak IVs and strong IVs, respectively.

Motivating Example 1: Disaster Risk Model for the Equity Premium. We consider a simple variant of Rietz (1988) and Barro (2006, 2009) with macroeconomic disasters characterized by extremely large consumption declines. We assume that real consumption growth follows[6]

$$\Delta c_t \equiv \ln\left(\frac{C_t}{C_{t-1}}\right) = \sigma \varepsilon_t - \zeta_t, \tag{2.1}$$

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[5]Throughout the paper, we suppress the dependence of $\bar{g}_0(\theta)$ and $\bar{g}_1(\theta)$ on n for notational simplicity.

[6]We ignore the intercept in Δc_t to maintain simplicity since it plays little role in explaining equity premia.

where C_t is real consumption per capita, the consumption shock ε_t follows a standard normal distribution, and ζ_t is a disaster variable. Particularly, the disaster variable ζ_t is characterized by

$$\zeta_t \equiv x_t(\underline{v} + J_t), \tag{2.2}$$

where the variable $x_t \sim \text{Bernoulli}(p)$ captures the occurrence of disasters, the constant $\underline{v}$ is the lower bound of the disaster size, and the variable $J_t \sim \text{Exp}(\alpha)$ is a disaster shock. The shocks $(\varepsilon_t, J_t, x_t)$ are independently and identically distributed (i.i.d.) over t and contemporaneously mutually independent. Specification of Δc_t in (2.1) provides baseline moment conditions: $\mathbb{E}[\bar{m}_0(\alpha)] = 0$, where $\bar{m}_0(\alpha) \equiv n^{-1} \sum_{t=1}^{n} m_{0,t}(\alpha)$ with

$$m_{0,t}(\alpha) \equiv \begin{bmatrix} \Delta c_t + p\mu_1(\alpha) \\ \Delta c_t^2 - \sigma^2 - p\mu_2(\alpha) \end{bmatrix} \quad \text{and} \quad \mu_j(\alpha) \equiv \mathbb{E}[(\underline{v} + J_t)^j] > 0. \tag{2.3}$$

Specifically, $\mu_1(\alpha) = \underline{v} + 1/\alpha$ and $\mu_2(\alpha) = \underline{v}^2 + 2\underline{v}/\alpha + 2/\alpha^2$. For illustrative purposes, we assume that the econometrician knows all parameters, except α, a parameter that can be only weakly identified by the moments based on Δc_t if p is close to 0.[7]

The representative agent maximizes his lifetime expected utility:

$$U_0 \equiv \mathbb{E}\left[\sum_{t=0}^{\infty} e^{-\delta t} \frac{C_t^{1-\gamma}}{1-\gamma}\right], \tag{2.4}$$

where δ is the subjective discount rate and γ is the relative risk aversion coefficient. The Euler equation for the utility maximization problem gives the following moment condition for excess log return of equity r_t^e: $\mathbb{E}[\bar{m}_1(\alpha)] = 0$, where $\bar{m}_1(\alpha) \equiv n^{-1} \sum_{t=1}^{n} m_{1,t}(\alpha)$ with

$$m_{1,t}(\alpha) \equiv r_t^e - \gamma\sigma^2 + \frac{1}{2}\sigma^2 + p\mu_1(\alpha) - \frac{p}{\alpha - \gamma}h(\alpha), \quad \text{and}$$

$$h(\alpha) \equiv \alpha\left[e^{\gamma\underline{v}} - \frac{\alpha - \gamma}{\alpha - \gamma + 1}e^{(\gamma-1)\underline{v}}\right]. \tag{2.5}$$

We call this the asset pricing moment. The model and the equilibrium condition require that $\underline{v} > 0$ and $\alpha > \gamma > 1$. The function $h(\alpha)$ is positive and finite.

The asset pricing moment (2.5) clearly demonstrates the key idea of the disaster risk model: when p and $\alpha - \gamma$ are both close to 0, the rare yet large disaster can generate a substantial equity premium as long as their ratio is a sizable loading in front of $h(\alpha)$ to match the moment of r_t^e.

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[7]In practice, one could add more moments to the lists of baseline and asset pricing moments and add more parameters to the list of the unknowns in both motivating examples. However, additional moments or unknown parameters do not change the nature of the problem, and the lists provided here sufficiently illustrate the key idea.

This ensures that the disaster risk is a meaningful economic mechanism for explaining the equity premium even if p is small. To utilize this key insight, we transform the parameter α to θ with

$$\theta \equiv \frac{p}{\alpha - \gamma} \quad \text{and} \quad \theta \in \Theta \equiv [\underline{c}, \ \overline{c}], \tag{2.6}$$

for constants $0 < \underline{c} < \overline{c}$. Our analysis allows $\alpha - \gamma$ and p to be both arbitrarily close to 0, while keeping the ratio θ bounded from above and away from zero.

To parameterize all the moments in θ, we write

$$\bar{g}_0(\theta) \equiv \bar{m}_0(\theta^{-1}p + \gamma) \quad \text{and} \quad \bar{g}_1(\theta) \equiv \bar{m}_1(\theta^{-1}p + \gamma). \tag{2.7}$$

Let $\mu_j^{(1)}(\alpha) \equiv (d/d\alpha)\mu_j(\alpha)$, where $\mu_j(\alpha)$ for $j = 1, 2$ are defined in (2.3). For the baseline moments $\bar{g}_0(\theta)$, simple calculations give

$$\mathbb{E}\left[\frac{d}{d\theta}\bar{g}_0(\theta)\right] = -(\theta^{-1}p)^2 \left[\mu_1^{(1)}(\theta^{-1}p + \gamma), \ -\mu_2^{(1)}(\theta^{-1}p + \gamma)\right]', \tag{2.8}$$

where $\mu_j^{(1)}(\theta p + \gamma)$ for $j = 1, 2$ are positive and bounded. Because θ is bounded, we have

$$\lim_{p \to 0} \mathbb{E}\left[\frac{d}{d\theta}\bar{g}_0(\theta)\right] = [0, \ 0]' \quad \text{and} \quad \lim_{p \to 0} \mathbb{E}\left[\frac{d}{d\theta}\bar{g}_1(\theta)\right] = \gamma e^{\gamma \underline{v}} \neq 0. \tag{2.9}$$

The baseline moments weakly identify θ when p is close to 0, whereas the asset pricing moment always strongly identifies θ.

Motivating Example 2: Long-Run Risk Model for the Equity Premium. We consider a simple variant of the baseline model of Bansal and Yaron (2004). As shown in the literature (e.g., Müller and Watson, 2008, 2018), the time series of U.S. real output growth exhibit a long-run (low-frequency) component, denoted by x_t. However, economists debate whether U.S. real consumption growth and U.S. real stock return are significantly loaded on the long-run component, x_t, in the real output growth (e.g., Beeler and Campbell, 2012; Bansal, Kiku, and Yaron, 2012).

The long-run component of real output growth, x_t, is latent and follows

$$x_t = \rho x_{t-1} + \varepsilon_{x,t}. \tag{2.10}$$

The representative agent's consumption has the following log growth process:

$$\Delta c_t \equiv \ln\left(\frac{C_t}{C_{t-1}}\right) = \phi x_{t-1} + \sigma_c \varepsilon_{c,t}, \tag{2.11}$$

where C_t is real consumption per capita. The shocks $(\varepsilon_{x,t}, \varepsilon_{c,t})$ follow a standard multivariate normal distribution and are i.i.d. over t. By introducing parameter ϕ in (2.11), we allow the

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expected consumption growth to be weakly dependent on or independent of the long-run component x_t as in many macro asset pricing models. Specifically, when $\phi = 0$, the consumption growth process is exactly i.i.d. as in Campbell and Cochrane (1999). When $\phi > 0$, the time series of U.S. real consumption growth share the same long-run (low-frequency) component x_t, as suggested by Kandel and Stambaugh (1991), Hansen, Heaton, and Li (2008), and Schorfheide, Song, and Yaron (2018). When ϕ is positive yet near zero, the consumption growth process is nearly i.i.d., as argued by Beeler and Campbell (2012). In the model of Bansal and Yaron (2004, Table I), the loading parameter ϕ is effectively 0.034% in the monthly frequency. The specification of Δc_t in (2.11) implies the baseline moment conditions: $\mathbb{E}[\bar{m}_0(\rho)] = 0$, where $\bar{m}_0(\rho) \equiv n^{-1} \sum_{t=1}^{n} m_{0,t}(\rho)$ with

$$
m_{0,t}(\rho) \equiv \begin{bmatrix} \Delta c_{t-1} \left(\Delta c_{t+1} - \rho \Delta c_t \right) \\ \Delta c_t \left(\Delta c_{t+1} - \rho \Delta c_t \right) + \rho \sigma_c^2 \end{bmatrix}.
\tag{2.12}
$$

For illustrative purpose, we assume that the econometrician knows all parameters except ρ, a parameter that can be only weakly identified by the moments based on Δc_t if ϕ is close to 0.

The representative agent has recursive preferences as in Epstein and Zin (1989) and Weil (1989), and the agent maximizes the following lifetime utility:

$$
V_t = \left[(1-\delta) C_t^{1-1/\psi} + \delta \left(\mathbb{E}_t \left[V_{t+1}^{1-\gamma} \right] \right)^{\frac{1-1/\psi}{1-\gamma}} \right]^{\frac{1}{1-1/\psi}},
\tag{2.13}
$$

where δ is the rate of time preference, γ is the coefficient of risk aversion for timeless gambles, and ψ is the EIS under certainty. The Euler equation as the first-order condition for the utility maximization problem requires that the equilibrium excess log return r_t^e satisfies $\mathbb{E}[\bar{m}_1(\rho)] = 0$, where $\bar{m}_1(\rho) \equiv n^{-1} \sum_{t=1}^{n} m_{1,t}(\rho)$ with

$$
m_{1,t}(\rho) \equiv r_t^e - \gamma \sigma_c^2 + \frac{1}{2} \sigma_c^2 - \frac{1}{2} \left(2\gamma - \psi^{-1} - 1 \right) \left(1 - \psi^{-1} \right) \frac{\phi^2}{(\delta^{-1} - \rho)^2}.
\tag{2.14}
$$

We call this the asset pricing moment.

The key insight of the long-run risk model can be clearly seen from (2.14): when $\gamma > 1 > \psi^{-1}$, which implies that the agent has a preference for early resolution of uncertainty and the intertemporal substitution effect dominates the income effect, the equity premium is sizable if cash flows load on the long-run component (i.e., ϕ is positive), the long-run component is persistent (i.e., ρ is close to unity), and the representative agent's rate of time preference is close to unity (i.e., δ is close to unity). This insight summarizes the central idea of the parameter calibrations in the works of Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2012), in which $\phi = 0.034\%$, $\rho = 0.975$, and $\delta = 0.9989$ in the monthly frequency. To ensure that the long-run risk is a meaningful economic

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mechanism for explaining the sizeable equity premium, $\phi/(\delta^{-1} - \rho)$ must be a positive component that is bounded away from zero and from above in order to match the moment of r_t^e. To utilize this insight, we transform ρ to θ where

$$\theta \equiv \frac{\phi}{\delta^{-1} - \rho} \quad \text{and} \quad \theta \in \Theta \equiv \left\{ \theta \in [\underline{c}, \overline{c}] \text{ and } \delta^{-1} - \theta^{-1}\phi \in [0, 1) \right\}, \tag{2.15}$$

for some constants $0 < \underline{c} < \overline{c}$. Our analysis focuses on $\rho < 1$, $0 < \delta < 1$, and $\phi > 0$. It allows ρ and δ to be both arbitrarily close to 1 and ϕ to be arbitrarily close to 0, while keeping the ratio between any pair of $\delta^{-1} - 1$, $1 - \rho$, and ϕ bounded from above and away from zero.

To parameterize all the moments in θ, plugging in $\rho = \delta^{-1} - \theta^{-1}\phi$, we obtain

$$\bar{g}_0(\theta) \equiv \bar{m}_0(\delta^{-1} - \theta^{-1}\phi) \quad \text{and} \quad \bar{g}_1(\theta) \equiv \bar{m}_1(\delta^{-1} - \theta^{-1}\phi). \tag{2.16}$$

The Jacobian matrix for the baseline moment conditions is

$$\mathbb{E}\left[\frac{d}{d\theta} \bar{g}_0(\theta) \right] = -\frac{\phi^2}{[(1 + \delta^{-1})\theta - \phi][1 + \phi^{-1}(1 - \delta^{-1})\theta]} \left[\delta^{-1} - \theta^{-1}\phi, \ 1 \right]'. \tag{2.17}$$

Thus, the baseline moment restrictions are nearly flat in θ because

$$\lim_{\phi \to 0} \mathbb{E}\left[\frac{d}{d\theta} \bar{g}_0(\theta) \right] = [0, \ 0]'. \tag{2.18}$$

However, under the reasonable calibrations in the literature (Bansal and Yaron, 2004; Beeler and Campbell, 2012; Bansal, Kiku, and Yaron, 2012), the preference parameters γ and ψ are well above 1, and thus the asset pricing moment condition has the unknown parameter θ well identified:

$$\lim_{\phi \to 0} \mathbb{E}\left[\frac{d}{d\theta} \bar{g}_1(\theta) \right] = -(2\gamma - \psi^{-1} - 1)(1 - \psi^{-1})\theta \neq 0. \tag{2.19}$$

3   Conditional Specification Test

Let $\Theta \in \mathbb{R}^{d_\theta}$ denote the parameter space that includes θ_0 as an interior point. We consider the incremental J statistic:

$$\mathcal{T} \equiv J - J_0, \quad \text{where } J \equiv \min_{\theta \in \Theta} g(\theta)'(\hat{\Omega}(\theta))^{-1} g(\theta) \text{ and } J_0 \equiv \min_{\theta \in \Theta} g_0(\theta)'(\hat{\Omega}_0(\theta))^{-1} g_0(\theta), \tag{3.1}$$

with $g_0(\theta) \equiv n^{-1/2} \sum_{t=1}^n g_{0,t}(\theta) \in \mathbb{R}^{k_0}$, $g(\theta) \equiv n^{-1/2} \sum_{t=1}^n g_t(\theta) \in \mathbb{R}^k$, $g_t(\theta) \equiv [g_{0,t}(\theta)', g_{1,t}(\theta)']'$, $\hat{\Omega}(\theta) \equiv \hat{\Omega}(\theta, \theta)$, where $\hat{\Omega}(\theta, \tilde{\theta})$ is an estimator of $\Omega(\theta, \tilde{\theta}) \equiv \lim_{n \to \infty} \text{Cov}(g(\theta), g(\tilde{\theta}))$ for any $\theta, \tilde{\theta} \in \Theta$, and $\hat{\Omega}_0(\theta)$ is the leading $k_0 \times k_0$ submatrix of $\hat{\Omega}(\theta)$.

If the baseline moments provide strong identification of θ_0, $\mathcal{T} \to_d \chi_{k_1}^2$ and a critical value from this chi-square distribution yields the C test (incremental J test) of Eichenbaum, Hansen, and Singleton (1988). Recently, Chen and Santos (2018) study this incremental J test in semi/nonparametric

settings. This test is more powerful than the standard over-identification test based on the J statistic because it exploits the validity of the baseline moments. When the baseline moments only provide weak identification, the chi-square distribution is no longer a good approximation for the finite-sample distribution of $\mathcal{T}$. We propose an alternative critical value based on the conditional inference approach.[8]

Next, we provide the algorithm to compute the critical value based on the conditional inference approach. Following Andrews and Mikusheva (2016a), we view the rescaled baseline moment function $\mathbb{E}[g_0(\theta)] = n^{1/2}\mathbb{E}[\bar{g}_0(\theta)]$ indexed by θ as a functional nuisance parameter and obtain a simulation-based critical value by conditioning on a sufficient statistic for $\mathbb{E}[g_0(\theta)]$. For the ease of presentation, we assume that the Jacobian matrix Q, the covariance function $\Omega(\theta, \tilde{\theta})$, and $\Omega \equiv \Omega(\theta_0, \theta_0)$ are all known for now. In practice, they can be replaced by consistent estimators, which are easily obtainable because the continuously updated estimator (CUE)

$$\hat{\theta} \equiv \arg\min_{\theta \in \Theta} g(\theta)'(\hat{\Omega}(\theta))^{-1}g(\theta) \tag{3.2}$$

is consistent under the null, following standard arguments (e.g., Newey and Smith, 2004).

The sufficient statistic for the rescaled baseline moments $\mathbb{E}[g_0(\theta)]$ is constructed as follows. Under the null, $g(\hat{\theta})$, the full moment function evaluated at $\hat{\theta}$, is approximately a linear function of $g(\theta_0)$:

$$g(\hat{\theta}) = \Omega^{1/2}M\upsilon + \varepsilon_n, \quad \text{where } \upsilon \equiv \Omega^{-1/2}g(\theta_0) \to_d N(0, I_k), \tag{3.3}$$

$M \equiv I_k - \Omega^{-1/2}Q(Q'\Omega^{-1}Q)^{-1}Q'\Omega^{-1/2\prime}$ is a projection matrix, and ε_n is an error term that is either zero if $g_t(\theta)$ is linear in θ or negligible in large samples if $g_t(\theta)$ is nonlinear. Consider the following decomposition

$$g_0(\theta) = m(\theta) + V(\theta)g(\hat{\theta}), \quad \text{with } m(\theta) \equiv g_0(\theta) - V(\theta)g(\hat{\theta}), \tag{3.4}$$

which is obtained by projecting $g_0(\theta)$ onto $g(\hat{\theta})$ and $V(\theta) \equiv S_0\Omega(\theta, \theta_0)\Omega^{-1}$ is the projection coefficient with $S_0 \equiv [I_{k_0}, 0_{k_0 \times k_1}]$. In large samples, the random component in $m(\theta)$, i.e., $m(\theta) - \mathbb{E}[g_0(\theta)]$, is a Gaussian process indexed by θ, and thus is independent of $g(\hat{\theta})$ by orthogonality. Conditioning on $m(\theta)$, the distribution of $g_0(\theta)$ and the test statistic do not depend on the functional nuisance parameter $\mathbb{E}[g_0(\theta)]$, which means that $m(\theta)$ is a sufficient statistic for $\mathbb{E}[g_0(\theta)]$. The sufficient statistic $m(\theta)$ contains identification information in the baseline moments and is independent of randomness in the full moments in large samples.

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[8]When the baseline moments provide only weak identification, the J test still has correct size. However, the J test suffers from low power in the presence of information imbalances, see Chen, Dou, and Kogan (2020), and it neglects the valid information in the baseline moments.

Conditioning on the sufficient statistic $m(\theta)$, we approximate the conditional distribution of the test statistic $\mathcal{T}$ by replacing $g(\hat{\theta})$ with $\Omega^{1/2}Mv$ as in (3.3) and generate independent realizations of v from the standard multivariate normal distribution. Specifically, we define

$$L(v; d_0) \equiv v'Mv - \min_{\theta \in \Theta} \left[m(\theta) + V(\theta)\Omega^{1/2}Mv \right]' (\Omega_0(\theta))^{-1} \left[m(\theta) + V(\theta)\Omega^{1/2}Mv \right], \qquad (3.5)$$

where $d_0 \equiv (m(\cdot)', \mathrm{vec}(V(\cdot))', \mathrm{vech}(\Omega)', \mathrm{vech}(\Omega_0(\cdot))', \mathrm{vech}(M)')'$ and $\Omega_0(\theta)$ denotes the leading $k_0 \times k_0$ submatrix of $\Omega(\theta) \equiv \Omega(\theta, \theta)$. This is the analog of the test statistic $\mathcal{T}$ with $g_0(\cdot)$ replaced by the decomposition in (3.4), $g(\hat{\theta})$ replaced by its linear approximation in (3.3), $\hat{\Omega}(\hat{\theta})$ replaced by Ω, and $\hat{\Omega}_0(\cdot)$ replaced by $\Omega_0(\cdot)$. To conduct the conditional specification test, we execute the following algorithm in practice.

Algorithm (Conditional Specification Test):

1. Approximate the unknown quantities $\Omega(\theta, \tilde{\theta})$, Q, $V(\theta)$, and M with their consistent estimators. Specifically, let $\hat{\Omega}(\theta, \tilde{\theta})$ be the consistent estimator of $\Omega(\theta, \tilde{\theta})$ for any $\theta, \tilde{\theta} \in \Theta$, i.e., a sample analog for i.i.d. data and a heteroskedasticity and autocorrelation consistent (HAC) estimator for time series data. Calculate the CUE $\hat{\theta}$ in (3.2) with $\hat{\Omega}(\theta) \equiv \hat{\Omega}(\theta, \theta)$. Then, $\hat{Q} \equiv \partial \bar{g}(\hat{\theta})/\partial \theta'$, $\hat{\Omega} \equiv \hat{\Omega}(\hat{\theta}, \hat{\theta})$, $\hat{M} \equiv I_k - \hat{\Omega}^{-1/2}\hat{Q}(\hat{Q}'\hat{\Omega}^{-1}\hat{Q})^{-1}\hat{Q}'\hat{\Omega}^{-1/2'}$, and $\hat{V}(\theta) \equiv S_0\hat{\Omega}(\theta, \hat{\theta})\hat{\Omega}^{-1}$.

2. Compute the test statistic $\mathcal{T}$ following (3.1).

3. Calculate the estimator for the sufficient statistic $m(\cdot)$ using the relation in (3.4) and the estimators $\hat{V}(\theta)$ and $\hat{\theta}$ obtained from step 1.

4. For $b = 1, \ldots, B$, take independent draws $v_b^* \sim N(0, I_k)$ and calculate $\mathcal{T}_b^* = L(v_b^*; \hat{d})$ using $L(v; d_0)$ defined in (3.5), where d_0 being replaced by $\hat{d}$ means that $m(\cdot)$ in d_0 is estimated in step 3 and other components are estimated in step 1.

5. Let $b_0 \equiv \lceil (1-\alpha)B \rceil$ be the smallest integer larger than or equal to $(1-\alpha)B$ and α be the nominal size of the test. The critical value $c_{B,\alpha}(\hat{d})$ is the b_0^{th} smallest value among $\{\mathcal{T}_b^*, b = 1, \ldots, B\}$.

6. Reject the null hypothesis in (2.1) if the statistic $\mathcal{T}$ in step 2 is larger than the critical value $c_{B,\alpha}(\hat{d})$ in step 5. $\qquad \square$

Besides θ, the moments may also depend on an unknown parameter ψ, which, unlike θ, cannot be strongly identified by the asset-pricing moments. In this case, we can apply the algorithm to the joint hypothesis $H_0 : \mathbb{E}[\bar{g}_1(\theta_0, \psi_0)] = 0_{k_1 \times 1}$ and $\psi_0 = \psi_c$, with ψ_c imposed in all the moments as if it were known. The original null hypothesis $H_0 : \mathbb{E}[\bar{g}_1(\theta_0, \psi_0)] = 0_{k_1 \times 1}$ is rejected if the joint hypothesis is rejected for all null values ψ_c in its parameter space. This is a projection-based subvector inference method with the weakly identified nuisance parameter ψ. In practice, we may consider different values ψ_c in the range of calibrations used in the literature and treat ψ_c as part of the model's functional-form specification being tested.

Monte Carlo Simulation. Here we conduct Monte Carlo simulations to compare the finite-sample size and power of the proposed conditional specification test, the J test, and the C test in the two motivating examples in Section 2. The models, moments, parameters, and notations are as described in Section 2.

We first conduct a simulation study for the disaster risk model. We generate Δc_t following (2.1) and (2.2), and we generate r_t^e according to the following data-generating process:

$$r_t^e = \eta + \gamma\sigma^2 - \frac{1}{2}\sigma^2 - p\mu_1(\alpha) + \frac{p}{\alpha - \gamma}h(\alpha) + \varepsilon_{d,t}^e, \tag{3.1}$$

where the error term $\varepsilon_{d,t}^e \equiv \sigma\varepsilon_t - [x_t(\underline{v} + J_t) - p\mu_1(\alpha)] + \sigma_d\varepsilon_{d,t}$, and the functions $\mu_1(\cdot)$ and $h(\cdot)$ are defined in (2.3) and (2.5), respectively.[9] Here, $\varepsilon_{d,t}$ is an i.i.d. standard normal variable capturing the measurement error and is independent of the other shocks. Under the null, $\eta = 0$, and under the alternative, we consider various values of η to compare the powers. The misspecification term η in (3.1) can be attributed to the missing risk factors or economic mechanisms if expected returns are not driven only by the disaster risk mechanism, and it can also be attributed to misspecified function forms and restrictive parametric assumptions.

The parameters are set in the yearly frequency, similar to those set by Rietz (1988), Longstaff and Piazzesi (2004), and Wachter (2013), as follows: $n = 150$, $\sigma = 2\%$, $\underline{v} = 7\%$, $\gamma = 4$, $p = 0.5\%$, and $\sigma_d = 15\%$. In accordance with the observed equity premium and its sampling uncertainty in the data, we set $\theta = p/(\alpha - \gamma) = 0.0138$ as the calibrated "true" value in the simulation experiment to match the annual equity premium of 6%, and we estimate θ using the simulated data by searching in the interval $[\underline{c}, \overline{c}]$ with $\underline{c} = 0.0055$ and $\overline{c} = 0.0217$.
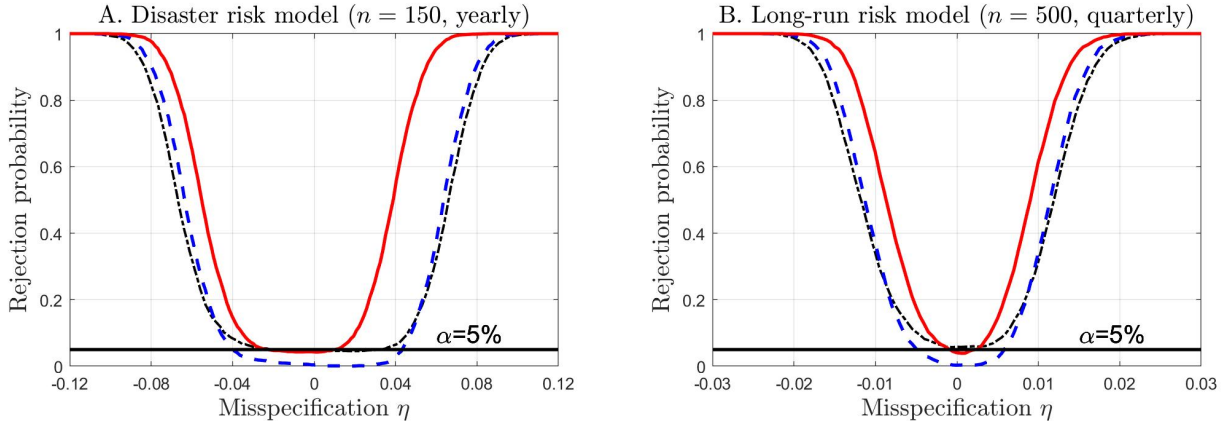
We next conduct a simulation study for the long-run risk model. We generate Δc_t following (2.10) and (2.11), and we generate r_t^e according to the following data-generating process:

$$r_t^e = \eta + \gamma\sigma_c^2 - \frac{1}{2}\sigma_c^2 + \frac{1}{2}\left(2\gamma - \psi^{-1} - 1\right)\left(1 - \psi^{-1}\right)\frac{\phi^2}{(\delta^{-1} - \rho)^2} + \varepsilon_{\ell,t}^e, \tag{3.2}$$

where the error term $\varepsilon_{\ell,t}^e \equiv \sigma_c\varepsilon_{c,t} + (1 - \psi^{-1})(\delta^{-1} - \rho)^{-1}\phi\varepsilon_{x,t} + \sigma_\ell\varepsilon_{\ell,t}$. Here, $\varepsilon_{\ell,t}$ is an i.i.d. standard normal variable capturing the measurement error and is independent of the other shocks. The misspecification term η in (3.2) has a similar interpretation to that of the disaster risk model example. The parameters are set in the quarterly frequency, close to those set by Bansal, Kiku, and Yaron (2012), as follows: $n = 500$, $\sigma_c = 0.0072 \times \sqrt{3}$, $\delta = 0.9989^3$, $\gamma = 10$, $\psi = 1.5$, $\rho = 0.975^3$, and $\sigma_\ell = 7.5\%$. In accordance with the observed equity premium and its sampling uncertainty in the data, we set $\theta = \phi/(\delta^{-1} - \rho) = 0.0665$ as the calibrated "true" value in the simulation experiment to match the annual equity premium of 6%, and we estimate θ using the simulated data by searching in the interval $[\underline{c}, \overline{c}]$ with $\underline{c} = 0.0528$ and $\overline{c} = 0.0779$.

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[9]See Section D in Cheng, Dou, and Liao (2020) for the derivation of (3.1) and (3.2).

Figure 1: A comparison of tests for the rare-disaster risk and long-run risk model.

Note: Panels A and B plot the rejection probabilities of three different specification tests for the simulation study on the rare-disaster risk and long-run risk model, respectively. In both panels, the red solid curve represents the rejection probability of the conditional specification test, with the test statistic $\mathcal{T}$ defined in (3.1) and the conditional critical value defined in (4.2); the black dash-dotted curve represents the rejection probability of the J test (Hansen, 1982); the blue dashed curve represents the rejection probability of the C test (Eichenbaum, Hansen, and Singleton, 1988); and the black solid horizontal line represents the 5% nominal size for all three specification tests.

Lastly, we discuss the simulation results for both examples, which are based on 2000 simulation repetitions and $B = 500$ random draws for the critical value of the conditional specification test in each repetition. Figure 1 reports the finite-sample rejection probabilities of the proposed conditional specification test, the J test, and the C test in the disaster risk model (panel A) and the long-run risk model (panel B). The simulation confirms that (i) the size of the conditional specification test is at the nominal level, (ii) the power of the conditional specification test is higher than that of the J test, and (iii) the size of the C test deviates from the nominal level, severely under-rejecting under the null.[10] Figure 1 demonstrates that the proposed conditional specification test offers substantial improvement over the existing tests for these two prominent macro asset pricing models.

4   Theoretical Properties of the Proposed Test

4.1   Finite-Sample Size Control in a Linear Gaussian Problem

For the test statistic and the critical value in the algorithm, there are three types of approximation error between the finite-sample distribution and the large sample distribution: (i) the linear approximation error ε_n in (3.3); (ii) the Gaussian approximation error for the distribution of $m(\cdot)$ and υ; and (iii) the estimation errors in the consistent estimators of θ_0, $\Omega(\cdot, \cdot)$, $V(\cdot)$, and M. To abstract from these approximation errors, which all disappear in large samples, below we first

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[10]It is worth pointing out that the C test may over-reject for a different data generating process.

consider a linear Gaussian statistical experiment where all types of errors are exactly zero. Let $v^* \equiv \Omega^{-1/2}\psi(\theta_0)$ and $m^*(\cdot) \equiv \mathbb{E}[g_0(\cdot)] + S_0\psi(\cdot) - V(\cdot)\Omega^{1/2}M\Omega^{-1/2}\psi(\theta_0)$ denote the Gaussian counterparts of v and $m(\cdot)$, respectively, where $\psi(\cdot)$ is a Gaussian process defined in Assumption 1 below. In the Gaussian experiment, the test statistic $\mathcal{T}$ is exactly $L(v^*; d^*)$, where d^* is defined similar to d_0 with $m(\cdot)$ in d_0 replaced by $m^*(\cdot)$. Define its conditional $1 - \alpha$ quantile as

$$c_\alpha^*(d^*) \equiv \inf\{c \in \mathbb{R} : P(L(v^*; d^*) > c \,|\, d^*) \le \alpha\}, \qquad (4.1)$$

for nominal size α, where $P(\cdot \,|\, d^*)$ denotes the conditional distribution of $L(v^*; d^*)$ given d^*.

Lemma 1. *In a linear Gaussian problem we have the following results under the null hypothesis:*
(i) *$m^*(\cdot)$ and Mv^* are independent;*
(ii) *$P(L(v^*; d^*) > c_\alpha^*(d^*)) \le \alpha$;*
(iii) *If the conditional distribution of $L(v^*; d^*)$ given d^* is continuous at its $1 - \alpha$ quantile almost surely, the size of the test equals the nominal level: $P(L(v^*; d^*) > c_\alpha^*(d^*)) = \alpha$.*

The critical value $c_\alpha^*(d^*)$ can be simulated using the marginal distribution of v^* because of the following three reasons. First, v^* enters $L(v^*; d^*)$ through Mv^*. Second, Mv^* and $m^*(\cdot)$ are independent. Finally, $m^*(\cdot)$ is the only random component in d^*. In large samples, the simulated critical value $c_{B,\alpha}(\hat{d})$ approximates $c_\alpha^*(d^*)$ with high accuracy when B is a large number.

4.2 Asymptotic Uniform Validity for Nonlinear Models

We first state the assumptions that are used to derive the asymptotic size of the test. Let $\mathbb{P}$ denote the distribution of the data $\{Y_t\}_{t=1}^n$. We allow $\mathbb{P}$ to change with the sample size n but suppress this dependence for notational simplicity. We also suppress the dependence of $\mathbb{E}[\cdot]$ and θ_0 on $\mathbb{P}$. Let $\mathcal{P}$ denote a family of distributions for which the baseline moments are valid. Let $\mathcal{P}_0$ denote a subset of $\mathcal{P}$ consistent with the null hypothesis. Both $\mathcal{P}$ and $\mathcal{P}_0$ are allowed to change with n. Let $q(\theta) \equiv \partial\bar{g}(\theta)/\partial\theta'$, then $Q(\theta) = \mathbb{E}[q(\theta)]$. For $j = 1, \ldots, d_\theta$, let $Q_j(\theta)$ denote the jth column of $Q(\theta)$ and θ_j denote the jth component in θ. Let $\lambda_{min}(A)$ denote the minimal eigenvalue of a symmetric real matrix A, and $\|\cdot\|$ denote the matrix Frobenius norm.

Assumption 1. *The following conditions hold uniformly over $\mathbb{P} \in \mathcal{P}$:*
(i) *$g(\cdot) - \mathbb{E}[g(\cdot)]$ weakly converges to a mean-zero Gaussian process $\psi(\cdot)$ with covariance $\Omega(\cdot, \cdot)$;* [11]

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[11] In the long-run risk example, the stationary component always dominates the latent non-stationary component because the loading on the latent process ϕ shrinks to 0 proportionally as $1 - \rho$ shrinks to 0. See Section E in Cheng, Dou, and Liao (2020) for details. In the disaster risk model, Assumption 1(i) holds with a non-singular covariance matrix even though the disaster occurs with a small probability because the variance of the normally distributed regular shock dominates that of the disaster shock. For the disaster risk example, alternative approximations that combine the extreme value theory and the central limit theorem are provided by Müller (2019, 2020).

(ii) $\sup_{\theta \in \Theta} \|q(\theta) - Q(\theta)\| \to_p 0$ *and* $Q(\theta)$ *is continuous;*

(iii) $\sup_{\theta \in \Theta} [\|\mathbb{E}[\bar{g}(\theta)]\| + \|Q(\theta)\| + \sum_{j=1}^{d_\theta} \|\partial Q_j(\theta)/\partial \theta'\|] \leq C_m$ *for some finite constant* C_m.

Assumption 2. *The following conditions hold uniformly over* $\mathbb{P} \in \mathcal{P}$:

(i) *There exists an estimator* $\hat{\Omega}(\cdot, \cdot)$ *of* $\Omega(\cdot, \cdot)$ *such that* $\sup_{\theta, \tilde{\theta} \in \Theta} \|\hat{\Omega}(\theta, \tilde{\theta}) - \Omega(\theta, \tilde{\theta})\| = o_p(1)$;

(ii) $\Omega(\theta, \tilde{\theta})$ *is continuous uniformly over* $(\theta, \tilde{\theta}) \in \Theta \times \Theta$;

(iii) $\sup_{\theta, \tilde{\theta} \in \Theta} \|\partial \hat{\Omega}(\theta, \tilde{\theta})/\partial \theta_j - \partial \Omega(\theta, \tilde{\theta})/\partial \theta_j\| = o_p(1)$ *for* $j = 1, \ldots, d_\theta$;

(iv) $\sup_{\theta, \tilde{\theta} \in \Theta} [\|\Omega(\theta, \tilde{\theta})\| + \sum_{j=1}^{d_\theta} \|\partial \Omega(\theta, \tilde{\theta})/\partial \theta_j\|] \leq C_\Omega$ *for some finite constant* C_Ω.

Assumption 3. *The following conditions hold uniformly over* $\mathbb{P} \in \mathcal{P}_0$:

(i) *There exists* $\theta_0 \in \Theta$ *such that* $\mathbb{E}[\bar{g}(\theta_0)] = 0_{k \times 1}$;

(ii) *for any* $\varepsilon > 0$ *and any* n, *there exists a fixed constant* $\delta_\varepsilon > 0$ *such that* $\inf_{\theta \in B_\varepsilon^c(\theta_0)} \|\mathbb{E}[\bar{g}(\theta)]\| > \delta_\varepsilon$, *where* $B_\varepsilon^c(\theta) \equiv \{\tilde{\theta} \in \Theta : \|\tilde{\theta} - \theta\| \geq \varepsilon\}$;

(iii) $\lambda_{\min}(Q'Q) \geq c_\lambda$ *and* $\inf_{\theta \in \Theta} \lambda_{\min}(\Omega(\theta)) \geq c_\lambda$ *for some positive constant* c_λ.

Let $\hat{d}$ be the analog of d, with $m(\cdot)$, $V(\cdot)$, Ω, $\Omega(\cdot, \cdot)$, and M all replaced by their consistent estimators, as in the practical algorithm. Given $\hat{d}$, we simulate independent draws $v^* \sim N(0, I_k)$ and obtain the critical value

$$c_\alpha(\hat{d}) \equiv \inf \left\{ c \in \mathbb{R} : P^*(v^* : L(v^*; \hat{d}) > c) \leq \alpha \right\}, \tag{4.2}$$

where $P^*(\cdot)$ denotes the distribution of v^*.

Theorem 1. *Suppose Assumptions 1, 2, and 3 hold. The test has correct asymptotic size, in the sense that, for any* $\varepsilon > 0$,

$$\limsup_{n \to \infty} \sup_{\mathbb{P} \in \mathcal{P}_0} \mathbb{P}\left(\mathcal{T} > c_\alpha(\hat{d}) + \varepsilon\right) \leq \alpha.$$

Theorem 1 implies that the conditional specification test controls the asymptotic size no matter whether the unknown parameter θ_0 (or its subvector) is strongly identified, weakly identified, or not identified by the baseline moments $\mathbb{E}[\bar{g}_0(\theta_0)] = 0_{k_0 \times 1}$.[12]

Next, we consider the behavior of the test statistic $\mathcal{T}$ and the conditional critical value $c_\alpha(\hat{d})$ when $\mathbb{E}[\bar{g}_0(\theta_0)] = 0_{k_0 \times 1}$ strongly identifies θ_0.

Assumption 4. *The following conditions hold uniformly over* $\mathbb{P} \in \mathcal{P}_{00} \subset \mathcal{P}$: *(i) for any* $\varepsilon > 0$, *there exists a constant* $\delta_\varepsilon > 0$ *such that* $\inf_{\theta \in B_\varepsilon^c(\theta_0)} \|\mathbb{E}[\bar{g}_0(\theta)]\| > \delta_\varepsilon$; *and (ii)* $\lambda_{\min}(Q_0'Q_0) \geq c_\lambda$ *where* $Q_0 \equiv \mathbb{E}[\partial \bar{g}_0(\theta)/\partial \theta']$.

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[12]Under additional regularity conditions on the continuity of the distribution function of the test statistic and the critical value, the test is also asymptotically similar, as discussed in Andrews and Mikusheva (2016a). See Andrews, Cheng, and Guggenberger (2020) for discussions on asymptotic similarity.

Theorem 2. *Suppose Assumptions 1, 2, 3, and 4 hold. The following results hold uniformly over* $\mathcal{P}_0 \cap \mathcal{P}_{00}$*: (i)* $\mathcal{T} \to_d \chi^2_{k_1}$*; and (ii)* $c_\alpha(\hat{d}) \to_p q_{1-\alpha}(\chi^2_{k_1})$*, where* $q_{1-\alpha}(\chi^2_{k_1})$ *denotes the* $1 - \alpha$ *quantile of a* $\chi^2_{k_1}$ *distribution.*

Theorem 2 shows that when the baseline moments $\mathbb{E}\left[\bar{g}_0(\theta_0)\right] = 0_{k_0 \times 1}$ provide strong identification of θ_0, the conditional specification test is equivalent to the C test under the null.

If the baseline moments $\mathbb{E}\left[\bar{g}_0(\theta_0)\right] = 0_{k_0 \times 1}$ only depend on a subvector $\theta_{c,0}$ of θ_0 with dimension d_c and strongly identify $\theta_{c,0}$, arguments analogous to those used to prove Theorem 2 also give

$$\mathcal{T} \to_d \chi^2_{k_1+d_c-d_\theta} \quad \text{and} \quad c_\alpha(\hat{d}) \to_p q_{1-\alpha}(\chi^2_{k_1+d_c-d_\theta}) \tag{4.3}$$

uniformly over $\mathcal{P}_0 \cap \mathcal{P}_{00}$. In this case, $k_1 \geq d_\theta - d_c$ in (4.3) because the asset pricing moments of dimension k_1 must strongly identify all the parameters not in the baseline moments.

In the presence of some additional parameter ψ that is only weakly-identified by the asset pricing moments, we can test the joint hypothesis $H_0 : \mathbb{E}[\bar{g}_1(\theta_0, \psi_0)] = 0_{k_1 \times 1}$ and $\psi_0 = \psi_c$ for some null value ψ_c as discussed in Section 3. As long as Assumptions 1 to 4 hold with ψ_0 fixed at ψ_c, results in Theorems 1 and 2 apply to the joint test. The projection-based subvector test $H_0 : \mathbb{E}[\bar{g}_1(\theta_0, \psi_0)] = 0_{k_1 \times 1}$ also has correct asymptotic size.[13] Since the projection-based test could be conservative, more efficient subvector tests are desirable (see e.g., Guggenberger, Kleibergen, Mavroeidis, and Chen, 2012; Andrews and Mikusheva, 2016b; Guggenberger, Kleibergen, and Mavroeidis, 2019; Kleibergen, 2021). Developing more efficient subvector tests for the present problem is beyond the scope of this paper.

In the Supplemental Appendix, we investigate the power of the conditional specification test. We prove that (i) the test is consistent when the asset pricing moments are globally misspecified regardless of the identification strength in the baseline moments and (ii) the conditional test shares the power optimality of the C test in standard scenarios where the baseline moments provide strong identification. The optimal test under weakly identified baseline moments is beyond the scope of the paper. However, the literature has provided several encouraging power results for various conditional tests for $H_0 : \theta = \theta_0$ (e.g., Andrews, Moreira, and Stock, 2006; Chernozhukov, Hansen, and Jansson, 2009; Mikusheva, 2010; Andrews and Mikusheva, 2016a, 2020) and we expect the conditional specification test to inherit these good properties.[14] One may also apply the generic methods of Elliott, Müller, and Watson (2015) to evaluate the efficiency of an ad hoc test with correct size. In the Supplemental Appendix, we derive some power envelopes in a Gaussian experiment as in Section 4.1 based on the observations of $g(\hat{\theta})$, under various restrictions on the alternative

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[13]See, e.g., Dufour (1997), for discussions on the projection-based inference under weak identification.

[14]Other studies on optimal tests for models with weak IVs include, e.g., Moreira and Moreira (2013, 2019), Moreira and Ridder (2017), Andrews, Marmer, and Yu (2019), and Montiel Olea (2020).

and the test. These power envelopes are akin to those in Section 3.4 of Andrews and Mikusheva (2016a). Simulation studies in the Supplemental Appendix show that the power of the conditional specification test is close to that of the infeasible uniformly most powerful unbiased test in many cases, particularly when the number of baseline moments is high and/or the baseline moments are strongly correlated with the asset-pricing moments.

5 Empirical Applications

In this Section, we consider a full-blown time-varying rare disaster risk model similar to that of Wachter (2013), a significant extension of the static rare disaster risk model in Section 2. The model is extended in a few aspects: (i) the probability of rare disasters is time-varying; (ii) the representative agent has a recursive Epstein-Zin-Weil preference; (iii) the government bill is defaultable; and (iv) the corporate dividend is modeled as the levered consumption. The model is able to generate a sizeable equity premium, a low interest rate, high volatility of equity returns, low volatility of government bill returns, and predictable excess equity returns in different prediction horizons, without generating excessive volatility for the aggregate consumption and dividend.

We consider the time-varying disaster risk model for a real-data application because it has been one of the most influential frameworks in the literature. For instance, the time-varying disaster risk mechanism has been used to explain important empirical patterns in macroeconomic quantities (Gourio, 2012, 2013), exchange rates and international capital flows (Gourio, Siemer, and Verdelhan, 2013; Martin, 2013; Farhi and Gabaix, 2015; Dou and Verdelhan, 2017; Lewis and Liu, 2017), global imbalances (Gourinchas, Rey, and Govillot, 2017), volatile unemployment flows (Kilic and Wachter, 2018; Petrosky-Nadeau, Zhang, and Kuehn, 2018), prices of derivatives (Gabaix, 2012; Farhi and Gabaix, 2015; Seo and Wachter, 2018, 2019), credit spreads (Gourio, 2013), and term structure of return volatility and risk premia (Hasler and Marfè, 2016). Tsai and Wachter (2015) and Welch (2016) provide lucid summaries of (time-varying) disaster risk mechanisms.

Model. We first describe the model. The log growth rate of consumption per capita, $\Delta c_t \equiv \ln(C_t/C_{t-1})$, evolves as follows:

$$\Delta c_t = g_c + \sigma_c \varepsilon_{c,t} - \zeta_t, \tag{5.1}$$

where C_t is real consumption per capita at time t, ζ_t is the disaster variable, g_c is the average growth rate conditional on no disaster in the next period (i.e., $\zeta_t = 0$), and $\varepsilon_{c,t}$ is the normal consumption shock that follows a standard Gaussian distribution. The disaster variable ζ_t is characterized by

$$\zeta_t = x_t(\underline{v} + J_t), \tag{5.2}$$

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where the constant $\underline{v}$ is the lower bound of the disaster size, the variable $J_t \sim \text{Exp}(\alpha)$ captures the disaster shock, and the variable $x_t = x_t^+ - x_t^-$ with x_t^+ to be a Bernoulli variable capturing the occurrence of a rare disaster with probability $\max(p_{t-1}, 0)$ and x_t^- to be a Bernoulli variable capturing the occurrence of a rare boom with probability $\max(-p_{t-1}, 0)$. The jump probability index p_{t-1} evolves according to an AR(1) process:

$$\hat{p}_t = (1 - \rho) + \rho\hat{p}_{t-1} + \sigma_p\varepsilon_{p,t}, \quad \text{with} \quad \hat{p}_{t-1} \equiv p_{t-1}/p, \quad \text{and} \quad \rho, p \in (0, 1). \tag{5.3}$$

Here, the shock $\varepsilon_{p,t}$ follows a standard Gaussian distribution. Thus, x_t follows a hidden Markovian process with the latent state variable p_{t-1}. The evolution in (5.3) says that the long-term average jump probability p_t is $\mathbb{E}[p_t] = p$ and the unconditional standard deviation of jump probability index p_{t-1} is $\mathbb{V}\text{ol}(p_{t-1}) = p\sqrt{\sigma_p^2/(1 - \rho^2)}$. Similar in spirit to our specification of (5.3), Gourio (2012) also assumes that the log transformation of the normalized jump probability index $\hat{p}_{t-1} = p_{t-1}/p$ evolves as an AR(1) process.[15]

We model the dividend D_t as the levered consumption with the log dividend growth $\Delta d_t \equiv \ln(D_t/D_{t-1})$ evolving as follows:

$$\Delta d_t = g_d + \phi\sigma_c\varepsilon_{c,t} - \phi\zeta_t, \tag{5.4}$$

similar in spirit to the works of Abel (1999) and Campbell (2003). Here, the constant g_d is the average growth rate conditional on no disaster in the next period (i.e., $\zeta_t = 0$). The shocks $(\varepsilon_{c,t}, \varepsilon_{p,t}, J_t)$ are mutually independent and i.i.d. over t. The Bernoulli variables (x_t^+, x_t^-) are independent of the contemporaneous jump probability shock $\varepsilon_{p,t}$ and its leads in the time series, but (x_t^+, x_t^-) and the lags of $\varepsilon_{p,t}$ are dependent through the jump probability index p_{t-1}. The two processes (x_t^+, x_t^-) and $(\varepsilon_{c,t}, J_t)$ are mutually independent.

Consider the government bill with a one-period maturity. Like in the works of Barro (2006) and Wachter (2013), we assume that the government bill may default only when a disaster occurs. The return on the defaultable government bill can be expressed as

$$r_{b,t} = y_{b,t-1} - x_{b,t}(\underline{v} + J_t), \tag{5.5}$$

where the variable $y_{b,t-1}$ is the yield of the government bill, and the Bernoulli variable $x_{b,t} \in \{0, 1\}$ characterizes the occurrence of a government bill default. The yield $y_{b,t-1}$ is observable in the data. The government bill defaults in period t if and only if $x_{b,t} = 1$. We assume that, in the event of disaster ($x_t = 1$), there will be a default on government liabilities with probability q. That is,

---

[15]The value of p_{t-1} can go outside the interval $[-1, 1]$ with a negligible chance under the relevant calibrations. A similar situation is encountered by Gourio (2012).

$\mathbb{P}(x_{b,t} = 1 | x_t = 0) = 0$ and $\mathbb{P}(x_{b,t} = 1 | x_t = 1) = q$. As reflected in (5.5), we follow Barro (2006) and Wachter (2013) in assuming that the percentage loss of the government bill is equal to the percentage decline in consumption in the event of default.

The representative agent has recursive preferences with unit EIS, and maximizes her utility V_t as follows:

$$\ln V_t = (1 - \delta) \ln C_t + \delta (1 - \gamma)^{-1} \ln \mathbb{E}_t \left[V_{t+1}^{1-\gamma} \right], \tag{5.6}$$

where δ is the rate of time preference, γ is the coefficient of risk aversion for timeless gambles, and $\mathbb{E}_t[\cdot]$ is the conditional expectation given the information up to the end of period t.

Equilibrium. The equilibrium log return of the government bill, denoted by $r_{b,t}$, is

$$r_{b,t} - \mathbb{E}_{t-1}[r_{b,t}] = -\left[x_{b,t}(\underline{v} + J_t) - q p_{t-1} \mu_1(\alpha) \right],$$
$$\text{with } \mathbb{E}_{t-1}[r_{b,t}] = \omega_1(\vartheta) - q \mu_1(\alpha)(p_{t-1} - p) - (1-q) h_1(\alpha, \gamma) \frac{p_{t-1} - p}{\alpha - \gamma}, \tag{5.7}$$

where $\mu_1(\alpha)$ and $\omega_1(\vartheta)$ are defined in (2.3) and (5.13), and $h_1(\alpha, \gamma) \equiv \alpha \left[e^{\underline{v}\gamma} - e^{\underline{v}(\gamma-1)} \frac{\alpha - \gamma}{\alpha - \gamma + 1} \right]$.

The equilibrium excess log return of the equity over the government bill, denoted by $r_{m,t}^e$, is

$$r_{m,t}^e - \mathbb{E}_{t-1}[r_{m,t}^e] = \phi \sigma_c \varepsilon_{c,t} + \beta_p \sigma_p \varepsilon_{p,t} - \left[(\phi x_t - x_{b,t})(\underline{v} + J_t) - (\phi - q) p_{t-1} \mu_1(\alpha) \right],$$
$$\text{with } \mathbb{E}_{t-1}[r_{m,t}^e] = \omega_3(\vartheta) + h_3(\alpha, \gamma) \frac{p_{t-1} - p}{\alpha - \gamma}, \tag{5.8}$$

where $\beta_p \equiv \frac{p\bar{\delta}}{1 - \rho\bar{\delta}} h_2(\alpha, \gamma)$ with $\bar{\delta} \equiv \delta e^{(g_d - g_c)}$ and $\lambda_p \equiv -\frac{p}{\delta^{-1} - \rho} \left[e^{\underline{v}(\gamma-1)} \frac{\alpha}{\alpha - \gamma + 1} - 1 \right]$, and $\omega_3(\vartheta)$ is defined in (5.13). Here, $h_2(\alpha, \gamma)$ and $h_3(\alpha, \gamma)$ are defined as follows:

$$h_2(\alpha, \gamma) \equiv \alpha \left[e^{\underline{v}(\gamma-\phi)} \frac{1}{\alpha - \gamma + \phi} - e^{\underline{v}(\gamma-1)} \frac{1}{\alpha - \gamma + 1} \right],$$
$$h_3(\alpha, \gamma) \equiv \alpha \left[(1-q) e^{\underline{v}\gamma} - e^{\underline{v}(\gamma-\phi)} \frac{\alpha - \gamma}{\alpha - \gamma + \phi} + q e^{\underline{v}(\gamma-1)} \frac{\alpha - \gamma}{\alpha - \gamma + 1} \right] - (\alpha - \gamma)(\phi - q) \mu_1(\alpha). \tag{5.9}$$

The equilibrium log price-dividend ratio, denoted by $z_{m,t}$, is

$$z_{m,t} = \bar{z}_m + h_2(\alpha, \gamma) \frac{p_t - p}{1 - \rho\bar{\delta}}, \quad \text{where } \bar{z}_m \equiv \ln \left[\frac{\bar{\delta}}{1 - \bar{\delta}} \right]. \tag{5.10}$$

Here, $\bar{z}_m$ is the long-run average log price-dividend ratio. To ensure the existence of the equilibrium, we require that $\bar{\delta} < 1$. As done by Barro (2009), we interpret $\bar{\delta}$ as the effective rate of time preference of the representative agent and require the effective rate of time preference to be less than 1. The detailed derivation of the equilibrium is relegated to Cheng, Dou, and Liao (2020).

Moments. We consider a set of baseline moment restrictions that summarize the key dynamic features of Δc_t and Δd_t specified in equations (5.1) – (5.4) as follows: $\mathbb{E}[\bar{m}_0(\vartheta)] = 0_{8\times 1}$, where $\bar{m}_0(\vartheta) = n^{-1}\sum_{t=1}^{n} m_{0,t}(\vartheta)$ with

$$
m_{0,t}(\vartheta) \equiv
\begin{bmatrix}
(\Delta c_t - g_c) + p\mu_1(\alpha) \\
(\Delta d_t - g_d) + \phi p\mu_1(\alpha) \\
(\Delta c_t - g_c)^2 - \sigma_c^2 - p\mu_2(\alpha) \\
(\Delta d_t - g_d)^2 - \phi^2\sigma_c^2 - p\phi^2\mu_2(\alpha) \\
\Delta c_{t-1}\left[\Delta c_{t+1} - \rho\Delta c_t + (1-\rho)(p\mu_1(\alpha) - g_c)\right] \\
\Delta d_{t-1}\left[\Delta d_{t+1} - \rho\Delta d_t + (1-\rho)(\phi p\mu_1(\alpha) - g_d)\right] \\
\Delta d_{t-1}\left[\Delta c_{t+1} - \rho\Delta c_t + (1-\rho)(p\mu_1(\alpha) - g_c)\right] \\
\Delta c_{t-1}\left[\Delta d_{t+1} - \rho\Delta d_t + (1-\rho)(\phi p\mu_1(\alpha) - g_d)\right]
\end{bmatrix},
\tag{5.11}
$$

where $\mu_j(\alpha)$ is defined in (2.3). The baseline moments depend on (α, ρ), but not on (σ_p^2, γ).

We assume that the econometrician knows all parameters except $\vartheta = (\alpha, \rho, \sigma_p^2, \gamma)$, which govern the dynamics of time-vary rare disaster risk and capture the agent's risk aversion. Other parameters $(g_c, g_d, \sigma_c^2, \phi, \underline{v}, p, q, \delta)$ are externally calibrated, and the key asset pricing implications are not sensitive to the local perturbations in these parameters (see Wachter, 2013; Chen, Dou, and Kogan, 2020, for sensitivity analysis). Specifically, we set $g_c = g_d = 0.02$, $\sigma_c^2 = 0.02^2$, $\phi = 3.5$, $\underline{v} = 0.07$, $q = 0.4$, and $\delta = 0.97$, which lie within the ballpark of the calibrations used in the literature (e.g., Bansal and Yaron, 2004; Longstaff and Piazzesi, 2004; Wachter, 2013; Farhi and Gabaix, 2015; Chen, Dou, and Kogan, 2020). We consider multiple values of $p \in \{0.3\%, 0.5\%, 0.7\%, 0.9\%, 1.1\%\}$ to focus on rare disasters following the calibrations adopted by Rietz (1988) and Longstaff and Piazzesi (2004), which are consistent with the structural estimation result from the observed equity index option prices in Backus, Chernov, and Martin (2011). The calibrated parameter values are effectively part of the functional form of the model under the examination of the specification tests, similar to Julliard and Ghosh (2012) who test the rare events hypothesis using the generalized empirical likelihood methods.

We next consider the 6 asset pricing moment restrictions targeted by Wachter (2013). The first two moments are about the low mean and low volatility of government bill returns, the third and fourth moments are about the sizeable mean and sizeable volatility of excess equity returns, and the last two moments are about the one- and two-period-ahead predictability of excess equity returns using lagged log price-dividend ratios. As demonstrated in many studies (e.g., Keim and Stambaugh,

1986; Campbell and Shiller, 1988; Fama and French, 1989; Cochrane, 1992), high price-dividend ratios predict low excess returns across various horizons. Importantly, Campbell and Yogo (2006) show that conventional tests of the predictability of stock returns can be invalid and lack power when the predictor variable is persistent and its innovations are highly correlated with returns. The asset pricing moment restrictions are $\mathbb{E}[\bar{m}_1(\vartheta)] = 0_{6\times 1}$, where $\bar{m}_1(\vartheta) = n^{-1}\sum_{t=1}^{n} m_{1,t}(\vartheta)$ with

$$
m_{1,t}(\vartheta) \equiv
\begin{bmatrix}
r_{b,t} - \omega_1(\vartheta) \\[4pt]
[r_{b,t} - \omega_1(\vartheta)]^2 - \omega_2(\vartheta) \\[4pt]
r_{m,t}^e - \omega_3(\vartheta) \\[4pt]
[r_{m,t}^e - \omega_3(\vartheta)]^2 - \omega_4(\vartheta) \\[4pt]
r_{m,t-1}^e \left[r_{m,t}^e - \omega_5(\vartheta)(z_{m,t-1} - \bar{z}_m) - \omega_3(\vartheta) \right] \\[4pt]
r_{m,t-1}^e \left[r_{m,t+1}^e - \omega_6(\vartheta)(z_{m,t-1} - \bar{z}_m) - \omega_3(\vartheta) \right]
\end{bmatrix},
\tag{5.12}
$$

where the deterministic functions $\omega_i(\vartheta)$ are described in detail as follows:

$$
\omega_1(\vartheta) \equiv -\ln\delta + g_c - \frac{1}{2}(2\gamma - 1)\sigma_c^2 - qp\mu_1(\alpha) - (1-q)h_1(\alpha,\gamma)\frac{p}{\alpha-\gamma},
$$

$$
\omega_2(\vartheta) \equiv qp\mu_2(\alpha) - q^2 p^2 \mu_1(\alpha)^2 + \left[2qp\mu_1(\alpha) + (1-q)h_1(\alpha,\gamma)\frac{p}{\alpha-\gamma} \right] \frac{(1-q)h_1(\alpha,\gamma)\sigma_p^2 p}{(1-\rho^2)(\alpha-\gamma)},
$$

$$
\omega_3(\vartheta) \equiv \phi\gamma\sigma_c^2 + \beta_p\lambda_p\sigma_p^2 - \frac{1}{2}\left(\phi^2\sigma_c^2 + \beta_p^2\sigma_p^2\right) + h_3(\alpha,\gamma)\frac{p}{\alpha-\gamma},
\tag{5.13}
$$

$$
\omega_4(\vartheta) \equiv \phi^2\sigma_c^2 + \beta_p^2\sigma_p^2 + (q - 2\phi q + \phi^2)p\mu_2(\alpha) - (q-\phi)^2\mu_1(\alpha)^2\left(p^2 + \frac{\sigma_p^2}{1-\rho^2}\right)\frac{h_3(\alpha,\gamma)^2\sigma_p^2 p^2}{(1-\rho^2)(\alpha-\gamma)^2},
$$

$$
\omega_5(\vartheta) \equiv \frac{(1-\rho\bar{\delta})}{\alpha-\gamma}h_2(\alpha,\gamma)^{-1}h_3(\alpha,\gamma), \quad \text{and} \quad \omega_6(\vartheta) \equiv \rho\omega_5(\vartheta).
$$

Reparametrization. The asset pricing moments in (5.12) clearly demonstrate the key idea of the time-varying disaster risk model to simultaneously explain the sizeable equity risk premium and sizeable equity volatility: when p, $\alpha - \gamma$, σ_p^2, and $1 - \rho^2$ are all close to 0, the rare yet severe disaster can generate a substantial equity premium and large equity volatility as long as the two ratios $\frac{p}{\alpha-\gamma}$ and $\frac{\sigma_p^2}{1-\rho^2}$ are sizable to match the equity premium and the volatility of equity excess returns. This ensures that the time-varying disaster risk is a meaningful economic mechanism for explaining the equity premium and volatility even if p is very small. To utilize this key insight, we transform the parameters α and ρ to θ_1 and θ_2, respectively, with

$$
\theta_1 \equiv \frac{p}{\alpha-\gamma}, \quad \theta_2 \equiv \frac{\sigma_p^2}{1-\rho^2}, \quad \theta_3 \equiv \rho, \quad \text{and} \quad \theta_4 \equiv \gamma,
\tag{5.14}
$$

with the stacked parameter vector $\theta \in \Theta \equiv \prod_{i=1}^{4}[\underline{c}_i, \overline{c}_i]$, for constants $0 < \underline{c}_i < \overline{c}_i$ with $i = 1, 2, 3, 4$. Our analysis allows p, $\alpha - \gamma$, σ_p^2, and $1 - \rho^2$ to be all close to 0, while keeping the ratios θ_1 and θ_2 bounded from above and below. We refer to θ_1 and θ_2 as the adjusted disaster size parameter and the adjusted disaster probability volatility parameter, respectively. To reparameterize all the moments from ϑ into θ, write

$$\bar{g}_i(\theta) \equiv \bar{m}_i(\theta_1^{-1}p + \theta_4, \theta_2(1 - \theta_3^2), \theta_3, \theta_4), \quad \text{with } i \in \{0, 1\}. \tag{5.15}$$

The asset pricing moments provide an intuitive identification structure of θ. The first and third moments on the average of (excess) returns mainly identify the adjusted disaster size parameter θ_1. The second and fourth moments on the variance of (excess) returns mainly identify the adjusted disaster probability volatility parameter θ_2 and the risk aversion parameter θ_4. The last two moments on the predictability of excess equity returns identify the persistence parameter of time-varying disaster probability θ_3.

U.S. Data, Robust Evaluations, and Model Uncertainty Sets. Based on the U.S. data of consumption, dividend, government bill returns, equity returns, and log price-dividend ratios, we compare the J test and the proposed conditional specification test, then contrast the model uncertainty sets constructed based on the two specification tests.

Ideally, a reliable empirical analysis of the time-varying rare-disaster risk model should be based on the longest possible sample. As a result, we construct a set of long time series (1871 - 2019), with the data obtained from various sources. To construct the time series of log real consumption growth rates, we use the Barro-Ursua Macroeconomic Data for 1871 - 2009 and the per-capita real personal consumption expenditure on services and nondurable goods from the National Income and Product Accounts (NIPA) for 2010 - 2019. To construct the time series of log price-dividend ratios, real log dividend growth rates, and log market returns, we obtain the data from Campbell (2003) and Robert Shiller's website for 1871 - 2012, and the Center for Research in Security Prices (CRSP) S&P Index data for 2013 - 2019. For log real returns of treasury bills, we obtain the data from Campbell (2003) and Robert Shiller's website for 1871 - 2012, and the CPI-deflated 1-year treasury bill rates from the federal reserve data program (H15) for 2013 - 2019.

Table 1 presents the results on the J test, the proposed conditional specification test, and the point estimation. In the benchmark calibration with $p = 0.7\%$, the time-varying rare-disaster risk model can easily fit the U.S. data, closely matching both the baseline and asset pricing moment restrictions through the lens of the J test (with the p-value equal to 0.434). However, severe information imbalances between the baseline and asset pricing moment restrictions lead to excessively low power of the J test in evaluating the model specification (Chen, Dou, and Kogan, 2020). The
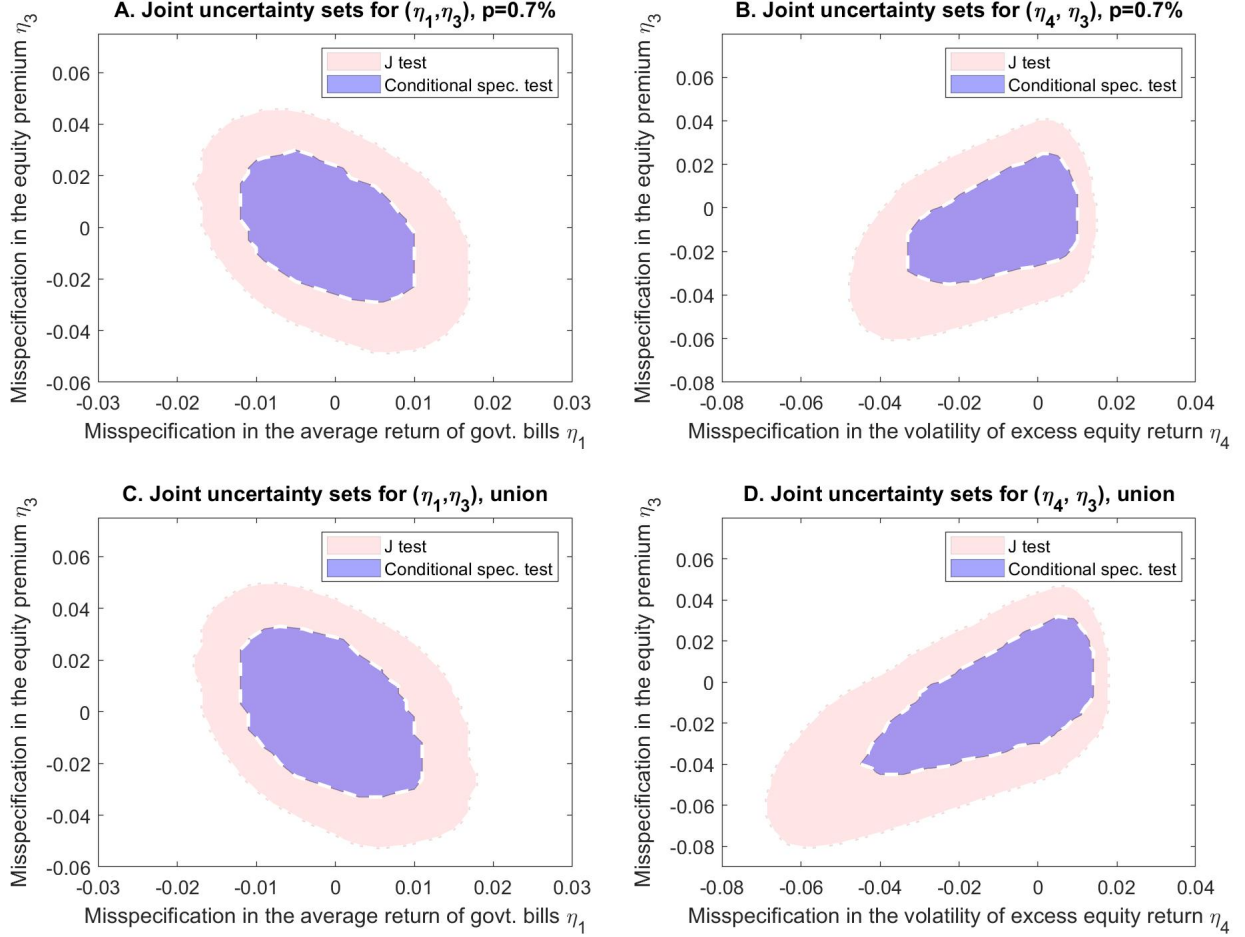
Table 1: Specification test results for time-varying rare disaster risk models with different calibrated long-run average disaster probability p.

Calibrated p	P-values of tests		Point estimates			
	J test	Cond. test	$\underline{v} + \alpha^{-1}$	$p \times \sigma_p$	ρ	γ
$p = 0.3\%$	0.120	0.017	0.321	0.205%	0.979	3.610
$p = 0.5\%$	0.400	0.127	0.279	0.140%	0.994	4.230
$p = 0.7\%$	0.434	0.145	0.242	0.115%	0.998	4.950
$p = 0.9\%$	0.411	0.126	0.224	0.101%	0.999	5.350
$p = 1.1\%$	0.254	0.042	0.212	0.116%	0.999	5.670

Note: We report the point estimates of $\underline{v} + \alpha^{-1}$ and $p \times \sigma_p$, instead of α and σ_p^2, because of the direct economic interpretations: $\underline{v} + \alpha^{-1}$ is the average disaster size in (5.1) and $p \times \sigma_p$ is the volatility of the jump probability index p_t in (5.3). The GMM estimators are computed using the continuous-updating estimator proposed by Hansen, Heaton, and Yaron (1996).

proposed conditional specification test can improve the test power by efficiently utilizing the limited information of the baseline moment restrictions, with the p-value dropping substantially from 0.434 to 0.145. Yet, it is interesting and reassuring to see that the time-varying rare-disaster risk model remains statistically consistent with the U.S. data at the 5% level, even under the stringent and robust examination of the proposed conditional specification test. When increasing (or decreasing) the long-run average disaster probability p from 0.7% to 0.9% (or 0.5%), the test and estimation results remain nearly unchanged. By contrast, when disasters occur very rarely with $p = 0.03\%$ (or fairly frequently with $p = 1.1\%$), the time-varying rare-disaster risk model is statistically rejected at the 5% level by the proposed conditional specification test based on the U.S. data with a p-value of 0.017 (or a p-value of 0.042), although it is still largely accepted according to the J test with a p-value of 0.120 (or a p-value of 0.254). This result manifests the importance of robust procedures in evaluating macro asset pricing models, and the econometric analysis addresses the irrefutability concern of economic mechanisms relying on extremely rare disasters (e.g., Campbell, 2018; Chen, Dou, and Kogan, 2020). Moreover, this result also echoes the quantitative study of Wachter (2013), showing that, unless conditioning on no disaster (i.e., the U.S. has been very lucky over the past centuries), the simulated data based on a time-varying disaster risk model with a fairly high disaster probability has a difficult time matching all the baseline and asset pricing moment restrictions in (5.11) and (5.12). Last but not least, our econometric analysis in Table 1 shows that the time-varying rare-disaster risk models indeed provide a potential explanation for the prominent asset pricing puzzles because, with a large probability ($> 99\%$) for each of the three accepted cases, the estimated risk aversion parameter γ is less than 10, the upper bound of the "reasonable" range for γ in the macroeconomics and asset pricing literature (e.g., Mehra and Prescott, 1985). There has

Figure 2: Joint model uncertainty sets based on the time-varying rare disaster risk models.

Note: Panel A plots the joint model uncertainty set for the average return of government bills (η_1) and the equity premium (η_3) by focusing on $\eta = [\eta_1, 0, \eta_3, 0, 0, 0]'$. Panel B plots the joint model uncertainty set for the volatility of excess equity return (η_4) and the equity premium (η_3) by focusing on $\eta = [0, 0, \eta_3, \eta_4, 0, 0]'$. Panels C and D are analogous to A and B, respectively, except that each uncertainty set is the union of those obtained under $p = 0.5\%$, 0.7%, 0.9%. The bigger "nearly-ellipses-shaped" areas are the joint model uncertainty sets constructed using the J test, while the smaller darker "nearly-ellipses-shaped" areas are those constructed using the proposed conditional specification test. All the sets are calculated under the 95% confidence level. We focus on these two joint model uncertainty sets because interest rates, excess equity returns, and equity return volatilities are the most important quantities in macro asset pricing theories.

been little formal econometric analysis on (time-varying) rare-disaster risk mechanisms in the asset pricing literature. As a contribution, the test and estimation results in Table 1 fill this gap.

Figure 2 shows that robust specification tests can serve as a powerful tool for constructing the model uncertainty sets. The model uncertainty set consists of the moment misspecification parameter η such that $H_0 : \mathbb{E}[\bar{g}_1(\theta)] = \eta$ cannot be rejected by a given specification test, with the asset pricing moments $\bar{g}_1(\theta)$ defined in (5.15). In fact, the specification tests in Table 1 correspond

to the null of $\eta = 0$. Similar to the intuitive interpretation of Hansen and Sargent (2001), the estimated model is viewed as an approximation of the true model, lying within a collection of alternative probabilistic models whose fit of the moment restrictions is statistically close to the estimated model.

For computational feasibility and economic interpretability, Figure 2 reports pairwise joint model uncertainty sets where only two elements of the vector η deviate from 0 for each model uncertainty set construction. Panel A shows that the joint model uncertainty set shrinks substantially when using the proposed conditional specification test. Specifically, the model uncertainty set shrinks by about 2 and 4 percentage points along the dimensions of the average government bill return and the equity risk premium, respectively, which are comparable to the the average interest rate and equity premium themselves in the data. Moreover, panel B shows that the model uncertainty set shrinks by about 3 percentage points along the dimension of the variance of equity excess returns, and the magnitude of the change is comparable to that of the variance of equity excess return itself in the data. In terms of the volume of the model uncertainty sets, the one constructed by the conditional specification test is about 40% of that by the J test. To further account for the uncertainty in the probability of rare disasters p, panels C and D display the unions of uncertainty sets for $p = 0.5\%$, 0.7%, and 0.9%, the three cases where the asset pricing moments with $\eta = 0$ are not rejected in Table 1. The model uncertainty sets naturally become larger when accounting for uncertainty of p. Nevertheless, in both cases, the uncertainty sets based on the conditional specification test remain substantially smaller than those based on the J test. Crucially, the data-driven joint model uncertainty sets on the mean and the variance of asset returns displayed in Figure 2 play a pivotal role in robust mean-variance portfolio analysis (e.g., Garlappi, Uppal, and Wang, 2007; Bertsimas, Gupta, and Paschalidis, 2012; Maccheroni, Marinacci, and Ruffino, 2013).

6   Conclusion

This paper provides a robust and powerful test to evaluate macro asset pricing models. Our test gains power by exploiting valid but noisy information in weakly identified baseline moments. To achieve robustness under weak identification, the conditional test decouples the useful macroeconomic information embedded in the baseline moment restrictions from the additional asset pricing moment restrictions. Our novel approach is particularly useful when standard over-identification tests suffer from distorted size or poor power due to information imbalances. It can help researchers, practitioners, and monetary authorities to better understand the economic mechanisms behind macro-finance models and conduct robust econometric analysis accounting for model uncertainty.

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