Bank Debt versus Mutual Fund Equity in Liquidity Provision*

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Abstract

We propose a unified framework to compare and quantify liquidity provision by debt-issuing banks and equity-issuing mutual funds. We show that both types of financial intermediaries provide liquidity by insuring against idiosyncratic liquidity risks. However, they are subject to distinct constraints depending on the contractual form of their liabilities. Banks issue demandable debt, which exposes them to the possibility of panic runs, whereas funds issue demandable equity, which leads to fundamentals-driven outflows. Based on our theoretical framework, we develop the first empirical measure of liquidity provision that can be generally applied across demandable-debt- and demandable-equity-issuing financial institutions: the Liquidity Provision Index (LPI). We find that a dollar invested in bond mutual funds provides 4.8 cents of liquidity, which is economically significant at one-quarter of the liquidity provided by uninsured bank deposits at the end of 2017. The gap between bank and fund liquidity provision has continuously narrowed over time, suggesting a migration of liquidity provision away from the deposit-taking banking sector to equity-funded non-banks. We find Quantitative Easing and post-crisis liquidity regulation to be contributing factors for this trend. Finally, we exploit the 2016 Money Market Reform, in which institutional prime Money Market Funds (MMF) switched from a fixed to a floating share value, to corroborate the effect of contractual forms on liquidity provision.

1 Introduction

A key function of financial intermediaries is liquidity provision. Banks provide liquidity by issuing demandable debt backed by a portfolio of illiquid assets. Their liquidity provision is achieved by pooling investors' resources at the intermediary level so that idiosyncratic liquidity shocks are insured against, and more illiquid assets can be funded using liquid short-term debt (Diamond and Dybvig, 1983).

While the literature has primarily attributed liquidity provision to banks, financial intermediation has increasingly moved beyond the traditional debt-issuing banking sector. In particular, intermediaries issuing demandable equity have played an increasingly important role in the economy. For example, US fixed-income mutual funds saw their total assets increase from $0.5 trillion in 1995 to $4.5 trillion in 2019, which amounts to 35% of the banking sector's deposits in 2019 (Figure 1). These trends beg the question of whether liquidity provision is still only confined to debt-issuing intermediaries. After all, open-end mutual funds also invest in significant amounts of illiquid assets like corporate bonds and loans, and issue fund shares redeemable at short notice. Further, if equity-issuing intermediaries provide liquidity to investors, what are the underlying limitations and financial stability implications? How do they compare to those by debt-issuing intermediaries?[1]

We study these questions in a unified framework for liquidity provision by debt- and equity-issuing intermediaries. Our contribution is three-fold. First, we theoretically show that debt is not a prerequisite for liquidity provision. Bond mutual funds that issue demandable shares also provide liquidity because they allow investors' idiosyncratic liquidity risks to be shared. Second, we shed light on how the choice of contractual forms determines the constraints for and financial stability implications of liquidity provision. We find that the adjustable share value of fund equity can mitigate panic runs that result from the use of demandable bank debt in liquidity provision. Nevertheless, fund equity also renders investor redemptions and fund asset liquidations continuously sensitive to fluctuations in economic fundamentals. Third, we develop the Liquidity Provision Index (LPI) as the first empirical measure of liquidity provision that can be applied to both debt- and equity-issuing intermediaries. Taking the LPI to the data, we show that in the US, bond mutual funds have increasingly contributed to liquidity provision relative

[1] To be exact, we consider open-end equity contracts redeemable at short notice. Thus, although banks also issue equity, bank equity does not directly fall into our consideration of liquidity provision.

to traditional banks over the last decade. We further highlight the role of Quantitative Easing and post-crisis liquidity regulation in spurring the migration of liquidity provision away from the banking sector.

We begin by jointly analyzing liquidity provision by bank debt and fund equity under a common theoretical framework. Like Diamond and Dybvig (1983), investors in our model are subject to idiosyncratic liquidity shocks in an incomplete market. Investment decisions have to be made before the realization of liquidity shocks with a tradeoff between liquid cash and a risky illiquid long-term project. The latter incurs a higher liquidation cost as in Shleifer and Vishny (1992) when sold prematurely but can potentially yield a higher long-run return depending on the realization of future economic fundamentals. Investors can pool their resources and jointly invest through an intermediary bank, which issues claims with a fixed promised payment, or an intermediary mutual fund, which issues equity claims at a proportional share of its total asset value. When liquidity shocks realize, investors in need of immediate consumption always redeem their claims from the intermediary. In contrast, those without immediate consumption needs consider whether or not to redeem depending on a noisy private signal they receive about the illiquid project's long-run return.

We show that both banks and funds provide liquidity. That is, investors expect to obtain more from redeeming early with their intermediary than by prematurely selling the underlying assets themselves. The reason is that both types of intermediaries pool investors' resources and first use liquid cash to meet redemptions, which allows more of the illiquid project to be held until maturity and realize its long-run return. The issuance of debt contracts is therefore not a prerequisite for liquidity provision.

Nevertheless, the difference in contractual forms leads to different endogenous constraints in liquidity provision. Debt-issuing intermediaries are susceptible to *panic runs* when fundamentals deteriorate past a threshold.[2] This is because the fixed face value of debt promised to withdrawing depositors imposes all losses from premature asset liquidations on depositors remaining in the bank, which creates a first-mover advantage to withdraw. In contrast, the value of demandable equity flexibly adjusts with the number of redemptions and liquidations, which allows losses from premature liquidations to be proportionally born by redeeming and non-redeeming shareholders and thereby removes the first-mover advantage. In other words, panic runs are not induced

[2]Panic runs are of high relevance today. Notable examples of modern commercial bank runs include those of Northern Rock and Washington Mutual (WaMu).

by liquidity provision alone—the disproportionate distribution of premature liquidation losses is also a necessary condition.

While equity resolves the first-mover advantage created by debt, it comes with the disadvantage of rendering liquidity provision more sensitive to fluctuations in fundamentals.[3] When equity investors receive private news about project returns, they decide whether to stay in the fund to earn the long-run return or to redeem their shares and store cash on their own. Flexible NAVs cause redemption decisions to be continuously revised with changes in fundamentals. This results in volatile *fundamentals-driven flows*, which ultimately constrain liquidity provision by the fund. The presence of fundamentals-driven outflows implies that even with flexibly adjusting NAVs (e.g., through swing pricing), large and systematic fund outflows and asset liquidations may still occur in response to deteriorations in economic fundamentals. In contrast, the fixed payment promised by bank debt insulates depositors from fluctuations in fundamentals as long as the fundamentals do not deteriorate past the threshold for panic runs.

Since liquidity provision by debt- and equity-funded intermediaries is subject to different constraints, we derive a unified empirical measure for liquidity provision, the Liquidity Provision Index (LPI), to compare bank and fund liquidity provision capacity in practice. The LPI is defined as the difference between an intermediary's expected contract payment to investors withdrawing at short notice and the direct liquidation value of the underlying asset portfolio per dollar invested. It is the first measure of liquidity provision that is generally applicable to financial institutions regardless of the contractual form of their liabilities.

Calculating the LPI involves three steps. First, we determine the contract payment of demandable shares (demandable debt) for any given proportion of early redemptions (withdrawals). When outflows are very small, funds can first use cash to meet redemptions so the contract payment is 100% of the initial NAV. As outflows increase, funds resort to selling increasingly illiquid assets with higher haircuts, continuously decreasing the NAV. For bank debt, losses incurred from asset liquidations also increase with withdrawals, but depositors receive a fixed value unless the bank defaults. Second, we calculate the expected contract payment by integrating over the

[3] Our baseline model highlights that the use of debt exposes banks to panic runs, whereas the use of redeemable equity with perfectly adjusting values induces volatile flow-to-fundamentals relationship in funds. In reality, institutional differences between banks and mutual funds could induce fundamentals-driven, non-panic-based runs in banks (e.g., Allen and Gale, 1998) and panic-based runs in mutual funds (Chen, Goldstein and Jiang, 2010, Goldstein, Jiang and Ng, 2017, Zeng, 2017). While our theory presents a benchmark case focusing on the most salient difference between debt and equity contracts, our empirical analysis will consider their implications for our findings.

distribution of outflows at the fund (bank). Using observed outflows, which are an equilibrium outcome from fluctuations in economic fundamentals, we sidestep the difficult task of measuring the distribution of economic fundamentals. It also incorporates the influence of regulations and frictions outside of the model, which allows the LPI to be a realistic measure of liquidity provision. Finally, we deduct the direct liquidation value of the underlying portfolio from the expected contract payment to get the intermediary's contribution to liquidity provision.

We first apply the LPI to a laboratory of US commercial banks and bond mutual funds from 2011 to 2017. We find that fund shares provide a significant amount of liquidity relative to uninsured bank deposits. As of 2017, a dollar invested in redeemable bond shares provides 4.8 cents more than under direct liquidation, while a dollar invested in uninsured bank deposits provides 19.0 cents more. In other words, the liquidity provided by bond mutual fund shares amounts to one-quarter of the liquidity provided by bank deposits. In the cross-section, more liquidity is provided by funds with outflows less sensitive to economic fluctuations and by banks with a higher fraction of insured deposits. In the time series, the gap in liquidity production capacity between banks and funds has increasingly narrowed from 2011 to 2017. We find evidence that the decline in the bank-fund LPI gap is driven by an expansion in central bank reserves following Quantitative Easing and the implementation of the Liquidity Coverage Ratio. Both policies increase the proportion of liquid assets on bank balance sheets, which raises the asset portfolio's direct liquidation value and shrinks the capacity for liquidity provision.

We further explore the drivers of the bank-fund LPI gap. The LPI is designed to serve as a consistent measure of liquidity provision in practice, where financial institutions' outflows and portfolio choices may be influenced by the regulatory environment. Thus, we perform a number of additional checks to isolate the impact of contractual forms on liquidity provision. First, we already focus on uninsured deposits only in calculating bank LPIs, which is consistent with the demandable debt in our model.[4] Nevertheless, explicit and implicit guarantees may still indirectly affect the estimated LPI of uninsured deposits through equilibrium flows and portfolio choice. To this end, we further use cross-sectional variation in banks' liability structure to project that a hypothetical bank without any deposit insurance or non-deposit liabilities provides two-thirds of the estimated average bank LPI. These results imply that the bulk of the bank-fund LPI gap arises from the use of demandable debt versus equity.

[4] Uninsured deposits are economically important and of high empirical relevance. Egan, Hortacsu and Matvos (2017) estimate that uninsured deposits comprise half of all consumer deposits in large US commercial banks and show that they are subject to runs.

Finally, we identify the effect of demandable-debt versus demandable-equity funding on liquidity provision by applying the LPI to MMFs around the 2016 Money Market Reform. The reform required institutional prime MMFs to switch from fixed to floating NAVs, representing a transition from debt to equity funding.[5] At the same time, retail prime MMFs were exempt from this requirement and provide a natural control group. Our difference-in-differences estimates confirm that 81% of liquidity provision is preserved after the reform, corroborating that liquidity provision by demandable equity is significant but relatively lower than that by demandable debt.

Related Literature. The theoretical literature on liquidity provision has mostly centered around deposit-issuing banks as in Diamond and Dybvig (1983), Diamond and Rajan (2001), Kashyap, Rajan and Stein (2002), and Goldstein and Pauzner (2005). More recently, Hanson, Shleifer, Stein and Vishny (2015) consider debt claims issued by traditional banks versus shadow banks in liquidity provision. We contribute to this literature by showing that demandable equity issued by mutual funds can also provide liquidity. Our focus on demandable claims is fundamentally different from Jacklin (1987), who focuses on liquidity provision by tradable but non-demandable claims that resemble shares of close-end mutual funds rather than open-end mutual funds. Furthermore, Jacklin (1987) does not consider aggregate risks and thus cannot accommodate the notion of debt versus equity contracts.

Our paper also contributes to the empirical literature on liquidity provision. A long-standing question is how to measure liquidity provision by financial intermediaries. Prior work has focused on the banking sector (Berger and Bouwman, 2009, Brunnermeier, Gorton and Krishnamurthy, 2012, Bai, Krishnamurthy and Weymuller, 2018), as the theoretical foundation of liquidity provision by non-bank intermediaries has not yet been laid out. We apply our model to develop the LPI as the first of liquidity provision that can be applied to both demandable-debt- and demandable-equity-issuing financial institutions. A related strand of the empirical literature has analyzed the features of liquidity provision by debt-issuing banks and shadow banks (e.g., Kacperczyk and Schnabl, 2013, Sunderam, 2015, Drechsler, Savov and Schnabl, 2017).[6] We complement their findings by quantifying liquidity provision by bond mutual funds that issue demandable equity rather than debt. Sunderam (2015) shows that the demand for liquidity was

[5] Institutional prime MMFs were indeed subject to panic runs before the reform, notable examples including the Reserve Primary Fund, the first MMF that "broke the buck," and the Putnam Fund, the first MMF that suddenly closed, both due to severe runs in 2008 (Kacperczyk and Schnabl, 2013).

[6] Other papers on this topic include Gorton and Metrick (2010), Stein (2012), Nagel (2016), Li, Ma and Zhao (2020) and Xiao (2019).

a significant driver of shadow banks' growth pre-crisis. We further document a migration of liquidity provision from traditional banks to bond mutual funds in the post-crisis period.

Our focus on the role of equity-issuing financial intermediaries in liquidity provision speaks to the growing literature on mutual fund liquidity transformation and the financial stability implications. Most closely related is Chernenko and Sunderam (2017), who empirically show that mutual funds accommodate redemptions using cash rather than transacting in the underlying portfolio assets. We show that this liquidity management practice emerges in our model, where mutual funds insure investors from idiosyncratic liquidity shocks and allow more illiquid assets to be held to maturity. Chen, Goldstein and Jiang (2010) and Goldstein, Jiang and Ng (2017) find that panic runs may occur in mutual funds holding illiquid assets when adjustments in fund NAVs are imperfect. Our theory analyzes a fund with flexibly adjusting NAVs, which serves as a benchmark for comparing equity with debt contracts in liquidity provision. This comparison reveals that panic runs are not induced by fund equity in liquidity provision but by the frictions that render fund equity debt-like (e.g., stale NAVs). Our result is in line with the empirical finding that swing-pricing alleviates fund outflows (Jin, Kacperczyk, Kahraman and Suntheim, 2020). Our framework further implies that even with flexible NAV adjustments (e.g., through swing pricing), funds may still experience systematic outflows and asset liquidations in response to fundamental deterioration.

The remainder of the paper is organized as follows. Section 2 lays out the theoretical framework for liquidity provision by debt- and equity-issuing intermediaries. Section 3 explains the LPI construction and Section 4 presents the estimation results for liquidity provision by bond mutual funds, commercial banks, and MMFs. Section 5 concludes.

2 Theoretical Framework

We first build a theoretical framework to formulate the notion of liquidity provision by a financial intermediary issuing demandable claims, including debt-issuing banks and equity-issuing mutual funds. The goal of the model is to conceptualize how liquidity is provided by both debt- and equity-issuing financial intermediaries and what the underlying constraints are. We leave the measurement and quantitative analysis of liquidity provision to Section 3, which builds on the theory to develop an empirical Liquidity Provision Index (LPI).

2.1 Model Setting

The economy has three dates, $t = 0, 1, 2$. There is a $[0, 1]$ continuum of ex-ante identical agents, each of whom is endowed with one unit of a consumption good at $t = 0$, called "cash", which serves as the numeraire. Each agent is uncertain about her preferences over consumption at $t = 1$ and $t = 2$. At the beginning of $t = 1$, an agent learns her preferences privately: with probability $\pi > 0$ she is an early-type and gets utility $u(c_1)$ from date-1 consumption only while with probability $1 - \pi$ she is a late-type and gets utility $u(c_2)$ from date-2 consumption only. Let the primitive flow utility function, $u(c)$, be increasing, concave, and satisfy the Inada conditions. The consumption good, cash, can be directly consumed at any given date or transferred to the next date via one of two technologies: 1) a long-term, risky, and illiquid investment project denoted as "project", and 2) a short-term, riskless, and liquid asset denoted as "storage".

The long-term project is risky, illiquid, and available for investment at $t = 0$ only. One unit of cash invested in the project at $t = 0$ yields R units of cash at $t = 2$, where R is a random variable that follows a distribution of $G(\cdot)$ with support $[0, +\infty)$. Denote R as the fundamentals of the economy. Since R is uncertain, the economy entails aggregate risks. Only the distribution of R, $G(\cdot)$, is common knowledge to agents at $t = 0$, and we assume $E[R] > 1$ so that the project generates a higher long-run expected return than stored cash. Importantly, the project is illiquid at $t = 1$ in two aspects. First, at $t = 1$, the project has not yet come to fruition and yields a normalized value of one if not traded. Further, if the project is prematurely liquidated at $t = 1$, a liquidation discount will be incurred in the spirit of Shleifer and Vishny (1992). Specifically, denote the cash value obtainable when l of the project is liquidated at $t = 1$ as:

$$C(l) = l - \frac{\phi}{2} l^2 , \qquad (2.1)$$

where $0 < \phi \leq 1$ captures the extent of project illiquidity.[7]

We note that the premature project value captures general aspects of selling illiquid assets in secondary markets that are common to both corporate loans and bonds.[8] In reality, few loans or bonds have a putable option that can force the borrower to repay or repurchase the security

[7] This parametric form (2.1) can be micro-founded by a downward-sloping demand for the illiquid project: the more of the project is prematurely liquidated, the lower the marginal liquidation value.

[8] For this reason, our model does not apply to intermediaries that invest in liquid assets only and do not engage in liquidity transformation, such as an S&P 500 equity mutual fund.

before maturity.[9] Rather, the liquidation of loans and bonds on secondary markets can depress asset prices and generate negative real impacts by affecting the cost and capacity of corporates rolling over their debt (e.g., He and Xiong, 2012). Moreover, both loans and bonds are hard-to-price in secondary markets before maturity so that their market prices adjust primarily when actual trading happens (e.g., Duffie, 2010).

The order of play is as follows. At $t = 0$, all agents pool their endowments to collectively form a representative intermediary. The intermediary allocates the pool of endowments into the two assets and offers a demandable contract to agents in the form of either debt or equity, which we will elaborate below. Then, every agent i receives a private and non-contractable signal of R at the beginning of $t = 1$: $s_i = \theta(R) + \varepsilon_i$, where $\theta(R) \in [0, 1)$ is strictly increasing in R, and ε_i is i.i.d. and arbitrarily small.[10] Since this information is dispersed and private, it is neither contractable nor available to the intermediary at $t = 1$. An early agent always leaves the intermediary at $t = 1$ regardless of her signal, while a late agent chooses whether to leave the intermediary depending on her signal and the intermediary's contract payment. We denote by w the number of late agents and $\lambda = \pi + w$ the total number of agents who leave the intermediary at $t = 1$. When late agents redeem from the intermediary at $t = 1$, they can obtain γ at $t = 2$ for each unit stored. Specifically, $\gamma = 1 - \kappa w$, where $0 < \kappa \leq 1$, captures potentially decreasing returns to scale of the storage.[11] For example, agents may find it costly to physically store or invest large amounts of cash without an intermediary. Finally, at $t = 2$, the project matures, fundamentals R are realized, and the remaining proceeds are paid out according to the contracts.

To uncover the similarities and differences between debt and equity in liquidity provision, we compare two scenarios with different intermediary arrangements at $t = 0$ given the same underlying economy. To focus on the effect of contractual differences on liquidity provision, we consider banks and funds in parallel. The competition between banks and funds is an interesting but distinct dimension that we leave for future research.

In both scenarios, the representative intermediary makes portfolio choices $(x_{k,0}, y_{k,0})$ at $t = 0$ on agents' behalf, where $x_{k,0}$ is the amount of cash and $y_{k,0}$ the amount of projects. The

[9]For example, most bank loans cannot be recalled at the lender's discretion but only when borrowers violate certain loan covenants.

[10]This information structure is similar to that in Goldstein and Pauzner (2005), as we detail below.

[11]The model mechanism and predictions still carry through if the parameter κ is arbitrarily close to 0. In other words, κ does not determine the presence of liquidity provision. The purpose of allowing for decreasing returns to scale is to have one more degree of freedom to capture the flows-to-fundamentals relationship in mutual funds.

subscript $k \in \{b, f\}$ denotes the intermediary type: a bank or a fund. Since the intermediary is representative, it maximizes agents' utility and breaks even in equilibrium.

In the first scenario, the representative bank offers a standard demandable debt contract $(c_{b,1}, c_{b,2})$ to agents at $t = 0$. The cash payment at $t = 1$, $c_{b,1}$, is fixed, and subject to a standard sequential service constraint in the event of default.[12]

In the second scenario, the representative open-end mutual fund offers an NAV-based, pro-rata equity contract, $(c_{f,1}(\lambda), c_{f,2}(\lambda))$, where the cash payments are the end-of-date net asset values (NAVs). Notably, the cash payment $c_{f,1}(\lambda)$ is flexible (as opposed to $c_{b,1}$ which is fixed) in the sense that it is contingent on the number of agents redeeming at $t = 1$, λ, which is in turn determined by economic fundamentals R in equilibrium. It thus represents an equity contract. Looking forward, we will provide the explicit expressions of $c_{f,1}(\lambda)$ and $c_{f,2}(\lambda)$ based on the NAV calculation. Note that the demandable equity contract, which captures shares of open-end mutual funds in reality, is fundamentally different from Jacklin (1987) and the broader literature that explores how non-demandable but tradable contracts in financial markets provides liquidity different from banks (e.g., Allen and Gale, 2004, Farhi, Golosov and Tsyvinski, 2009, Dang, Gorton, Holmström, and Ordoñez, 2017).

We derive optimal debt and equity contracts that are most closely aligned with those in practice, i.e., demandable deposits and open-end mutual fund shares with flexible NAV. In this sense, we solve for constrained optimal debt and equity contracts rather than consider a mechanism design problem over a general contract space.[13]

Before proceeding, we also note the following straightforward result regarding equilibrium portfolio choice at $t = 0$, which is useful below when we establish the main results:

Lemma 1. *In any equilibrium, there must be $x^*_{k,0} \geq \pi c^*_{k,1}(\pi)$ for either $k \in \{b, f\}$.*

[12] Our model insights will not change if we instead assume a pro-rata rule for agents to share the liquidation value in a bank default event following, for example, Allen and Gale (1998). We do not consider alternative versions of the sequential service constraint that are different from that in Diamond and Dybvig (1983), for example, that in Green and Lin (2003).

[13] Another reason we do not consider a generally optimal contract is because such a contract is not observed in reality. For an unconstrained optimal contract, notice that a social planner who can verify agents' types, once realized, and who has the same almost perfect signal about fundamentals R as the agents have, would set t-1 consumption level c^*_1 of the early agents to maximize the ex-ante expected welfare. This implies that the early agent's marginal utility at c^*_1 will be equal to the marginal utility of late agents at (the endogenously determined) c^*_2 at any given realized R, which means that the unconstrained optimal contract must depend on the specific utility function.

Lemma 1 stems from the fact that π early agents always leave at $t = 1$ regardless of the fundamentals or the contractual setting. Thus, the intermediary always holds enough cash to pay them. Otherwise, the intermediary may either increase cash holdings or reduce t-1 contract payments to reduce the liquidation cost of the illiquid project. The proof follows from a perturbation argument based on the intuition described above and we omit it for simplicity.

2.2 Liquidation Value and Liquidity Provision

We first define a benchmark liquidation value of how much short-term consumption an agent can obtain by liquidating a given portfolio at short notice in the absence of an intermediary. Note that the liquidation value is general because it can be defined on any portfolio and thus is a function of (x_0, y_0); it does not necessarily rely on any equilibrium concept.

Definition 1. *Given any t-0 portfolio (x_0, y_0), its liquidation value at $t = 1$ is given by*

$$c_1(x_0, y_0) \doteq x_0 + y_0 - \frac{\phi}{2} y_0^2 = 1 - \frac{\phi}{2} y_0^2 \,. \tag{2.2}$$

As (2.2) indicates, the portfolio's liquidation value is lower than 1 due to the liquidation discount specified in (2.1). The more illiquid a portfolio is, the lower its liquidation value.

We then define a unified notion of liquidity provision as the difference between the expected intermediary contract payment and the liquidation value of the underlying portfolio.

Definition 2. *For a dollar invested in claims issued by an intermediary $k \in \{b, f\}$, the amount of liquidity provision is defined as*

$$E[c^*_{k,1}(\lambda(R))] - c_1(x^*_{k,0}, y^*_{k,0}) \,, \tag{2.3}$$

where the contract payment $c^*_{k,1}(\lambda(R))$ and intermediary portfolio holdings $(x^*_{k,0}, y^*_{k,0})$ are equilibrium outcomes given the intermediary contractual arrangement (bank or fund) and the underlying economy, the expectation $E[\cdot]$ is taken over fundamentals R, and $c_1(\cdot, \cdot)$ is the liquidation value function as defined in Definition 1.

Definition 2 indicates that the amount of liquidity provided to an agent subject to a liquidity shock is affected by both sides of the intermediary's balance sheet. Liquidity provision is the difference between the expected contract payment (liability side of the intermediary) and the liquidation value of the underlying portfolio (asset side of the intermediary). It is positive if the claims issued by the intermediary are more liquid, i.e., have a higher value upon liquidation at short notice, than the underlying assets it holds. Our definition of liquidity provision has two advantages. Theoretically, it is consistent with the notion of liquidity provision in Diamond and Dybvig (1983) for banks and allows for a unified comparison across debt- and equity-issuing financial intermediaries.[14] Empirically, all essential inputs for calculating the magnitude of liquidity provision are observable, which allows for a direct application of the model to measure liquidity provision in practice. Finally, note that Definition 2 focuses on consumption by early agents that are subject to liquidity shocks, which is consistent with our focus on liquidity provision. It does not represent the overall welfare provided, which is an interesting but separate question beyond the scope of this paper.

2.3 Bank Debt

At $t = 0$, the representative bank offers a demandable debt contract $(c_{b,1}, c_{b,2})$, which is subject to a sequential service constraint at $t = 1$ in the event of bank default, to agents. Given our focus on liquidity provision, we focus on the short-term contact payment $c_{b,1}$.[15] At $t = 0$, the bank also chooses the optimal portfolio $(x_{b,0}, y_{b,0})$ to maximize ex-ante expected agent utility.

Although the face value of debt, $c_{b,1}$, is fixed and independent of withdrawals when the bank is solvent, the sequential service constraint implies that the actual payments of $(c_{b,1}, c_{b,2})$ implicitly depend on λ_b given any initial bank portfolio choice $(x_{b,0}, y_{b,0})$ as shown in Table 1:

Table 1: Ex-post bank debt payments

t-1 withdrawal	$\lambda_b \leq \dfrac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}$	$\lambda_b > \dfrac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}$
t-1 payment	$c_{b,1}$	$c_{b,1}$ with probability $q(\lambda_b)$
t-2 payment	$\dfrac{x_{b,1}+y_{b,1}R}{1-\lambda_b}$	0

[14] In Appendix A, we further show that the notion of liquidation value is tightly linked to the equilibrium outcome in autarky, which serves as a direct welfare benchmark but is not empirically observable.

[15] Note that $c_{b,1}$ is sufficient to capture the contract offered to agents because the bank breaks even and $c_{b,2}$ is determined once the face value of debt $c_{b,1}$ is chosen.

where

$$q(\lambda_b) = \frac{1 - \frac{\phi}{2} y_{b,0}^2}{\lambda_b c_{b,1}} \tag{2.4}$$

is the probability of withdrawing agents to be served when the bank fails and the sequential service constraint is binding, and $(x_{b,1}, y_{b,1})$ is the bank's remaining portfolio after meeting date-1 withdrawals:

$$x_{b,1} = \max(x_{b,0} - \lambda_b c_{b,1}, 0)\,,$$

and

$$y_{b,1} = \begin{cases} y_{b,0} & \text{if} \quad x_{b,0} \geqslant \lambda_b c_{b,1}\,, \\ y_{b,0} - l_b & \text{if} \quad x_{b,0} < \lambda_b c_{b,1}\,, \end{cases} \tag{2.5}$$

where l_b, the unit of projects that the bank has to liquidate at $t = 1$, is determined by

$$\lambda_b c_{b,1} = x_{b,0} + l_b - \frac{\phi}{2} l_b^2\,, \tag{2.6}$$

if the bank is solvent at $t = 1$, and clearly $l_b = y_{b,0}$ if the bank fails.

The debt payment schedule shown in Table 1 suggests that the debt-issuing bank is vulnerable to potential panic runs. The intuition is as follows: by Lemma 1, the bank always has enough cash to meet withdrawals by the π population of early agents. If a late agent expects all other late agents to wait until $t = 2$, it is her best response to wait to obtain a strictly positive t-2 consumption level. Nevertheless, if she expects all other late agents to withdraw at $t = 1$, nothing would remain at $t = 2$ so that her best response would be to withdraw as well. Thus, the design of $(c_{b,1}, c_{b,2})$ must take the probability of panic runs at $t = 1$ into account.

We work backwards by analyzing late agents' run decisions, the bank's optimal choice of debt face value $c_{b,1}$ and the portfolio choice problem. In doing so, we apply the global games technique following Goldstein and Pauzner (2005) to pin down the probability of panic runs. The advantage of using global games is to link bank-run probability to fundamentals while preserving the panic-based nature of runs when interpreting the result. In this analysis, we also contribute to the global-games-based bank-run literature by considering the bank's initial portfolio choice and showing its effect on the unique run-threshold.

To formulate the analysis, we assume θ to be uniformly distributed, and that ε_i, the i.i.d. noise term, is uniformly distributed over $[-\varepsilon, \varepsilon]$, where ε is arbitrarily small. Under these mild distributional assumptions regarding agents' private signal s_i, we show that there exists a unique t-1 equilibrium in which a panic run occurs when R is below a threshold R^*:[16]

Proposition 1. *Given the date-0 fund position* $(x_{b,0}, y_{b,0})$*, the promised debt value* $c_{b,1}$*, and the signal* s_i *received by agents at* $t = 1$*, there exists a unique run threshold* $R^* > \underline{R}$*, where* $\underline{R}$ *is given by* (D.2) *in the proof.*

Proposition 1 shows that the use of demandable debt in liquidity provision leads to panic runs that are coordinated by fundamentals R. When $R < R^*$ and a run happens, the ex-post contract payment discontinuously drops from $c_{b,1}$ to 0 for agents who are not serviced, suggesting lower liquidity provision in bank runs (see Table 1).

To further show how panic runs constrain ex-ante liquidity provision by bank debt, we further analyze how bank contract design and portfolio choice affect the magnitude of runs:

Proposition 2. *The run threshold* $R^*(c_{b,1}, x_{b,0})$ *is increasing in* $c_{b,1}$ *and decreasing in* $x_{b,0}$*.*

Proposition 2 shows that a higher promised debt value $c_{b,1}$ or a lower level of stored cash $x_{b,0}$ renders panic runs more likely. A higher $c_{b,1}$ indicates a higher payment if one withdraws early at $t = 1$, and a lower $x_{b,0}$ implies a higher loss if one withdraws late at $t = 2$ while others withdraw early. Both forces give rise to a stronger first-mover advantage. However, recall that a lower $c_{b,1}$ or a larger $x_{b,0}$, which helps to reduce runs, would reduce the amount of ex-ante liquidity provision at the same time, as specified in Definition 2. Therefore, the potential for panic runs constrains bank liquidity provision.

Finally, we ensure that the bank indeed provides liquidity ex-ante. Taking bank runs into account, the representative bank solves the optimal face value of debt, $c_{b,1}$, and portfolio allocation, $(x_{b,0}, y_{b,0})$, at $t = 0$ to maximize the expected utility of agents:

$$\max_{c_{b,1}, x_{b,0}, y_{b,0}} \int_0^{R^*} \left(\pi u(c_{b,1}) + (1-\pi) u(c_{b,1}(1-\kappa(1-\pi)))\right) q(1) dG(R) + \int_{R^*}^{+\infty} \left(\pi u(c_{b,1}) + (1-\pi) u(c_{b,2})\right) dG(R) \tag{2.7}$$

[16] More formally and technically, we need to assume an "upper dominance region," which means that late agents never run when $R \to +\infty$, to ensure the existence of a threshold run equilibrium. This is a technical assumption to ensure that the global-games technique can be appied. Since this is a well understood technical point in the literature that is not crucial to our economic mechanism, we omit the details and refer interested readers to Goldstein and Pauzner (2005) for the economic motivation of the upper dominance region.

where the function q is given by (2.4) and R^* is determined according to Proposition 1. Specifically, the first integral in (2.7) captures the expected agent utility conditional on a bank run, while the second integral captures that conditional on a run not happening. To confirm that the bank can provide liquidity, it then suffices to identify a set of sufficient conditions that ensures $E[c^*_{b,1}(R)] > c_1(x^*_{b,0}, y^*_{b,0})$ without fully solving (2.7), as standard in the literature.[17] To give one simple example of such sufficient conditions, note that when the distribution of $G(R)$ reflects sufficiently good fundamentals (e.g., $G(R)$ follows an exponential distribution with a sufficiently low rate), we have $x^*_{b,0} \to 1$ and $c^*_{b,1} \to \frac{1}{\pi} > 1$ whereas $c_1(x^*_{b,0}, y^*_{b,0}) < 1$. Thus, by standard continuity argument, there exists a distribution $G(R)$ that ensures the bank to provide liquidity in our model. For any distribution $G'(R)$ that first-order stochastic dominates $G(R)$, the bank provides liquidity ex-ante.[18]

2.4 Fund Equity

Different from the bank, the open-end mutual fund offers an NAV-based, pro-rata equity contract $(c_{f,1}(\lambda_f), c_{f,2}(\lambda_f)) = (NAV_1(\lambda_f), NAV_2(\lambda_f))$, which is demandable at the end of each date with the end-of-day NAVs as payment. Similar to $c_{b,1}$ for bank debt, $c_{f,1}(\lambda_f)$ is relevant for liquidity provision and can sufficiently capture the equity contract. $c_{f,1}(\lambda_f)$ varies with the number of redeeming agents λ_f because the more redemption requests there are, the more assets have to be liquidated and the larger the liquidation costs incurred.[19] In equilibrium, λ_f is determined by fundamentals R, and thus the equity payment at $t = 1$ takes the form of $c_{f,1}(\lambda_f(R))$. The fund also chooses the optimal portfolio $(x_{f,0}, y_{f,0})$ at $t = 0$ to maximize expected agent utility.

To capture the essence of demandable equity, our model considers fund shares with flexible NAVs that fully incorporates liquidation costs in NAV_1. In other words, liquidation costs are proportionally borne by agents that redeem and agents that stay in the fund at $t = 1$. This is in contrast to the fixed payment promised by debt contacts, which imposes all liquidation costs

[17] For example, see Theorem 3 in Goldstein and Pauzner (2005), which provides a sufficient condition based on the lower dominance region.

[18] We again note that the sufficient condition here just serves as a transparent example to show that bank debt can indeed provide liquidity in our framework. We are able to provide a tighter but more technical sufficient condition using the lower dominance region as well. We can also show that, under similar sufficient conditions, the bank provides a higher expected t-1 consumption than the autarky. See Appendix A for a formal analysis.

[19] In practice, open-end mutual funds can achieve this by "striking the NAV" – a standard industry practice during the trading day to form the estimated amount of redemption requests, perform the necessary asset transactions, and pre-calculate the end-of-day NAVs.

to agents that remain with the bank. In practice, fund NAV may not be fully flexible because some same-day liquidation costs only realize at future trading dates. In this aspect, our equity contract can be interpreted as mutual fund shares with swing pricing, which allows same-day liquidation costs to be accurately incorporated in NAVs.

We start analyzing the equilibrium by calculating NAV_1. Note that in our two-period model, the fund optimally follows a pecking order of liquidation to meet redemptions. Specifically, the representative fund that maximizes expected agent utility will first use its stored cash to meet redemptions at $t = 1$. Doing so avoids premature liquidation of the project, which has a higher expected return when held until maturity. If the cash stored no longer suffices to pay all redeeming agents, the fund will resort to liquidating the project prematurely, raise cash, adjust the end-of-day NAV downwards, and pay the resulting NAV.

Hence, if the fund has enough stored cash to meet redemptions at the initial NAV, that is, if $x_{f,0} \geq \lambda_f$, no liquidations will occur, and consequently, the end-of-day NAV_1 will be unchanged at its initial value:

$$NAV_1(\lambda_f) = x_{f,0} + y_{f,0} = 1\,, \text{ if } x_{f,0} \geq \lambda_f\,, \tag{2.8}$$

where the per-unit value of stored cash is 1 and that of the illiquid project is also 1.

Instead, if the fund does not have enough stored cash in the sense that $x_{f,0} < \lambda_f$, it has to liquidate $l_f > 0$ unit of the illiquid project to help meet redemptions. This liquidation process implies two different approaches to calculate NAV_1 when $x_{f,0} < \lambda_f$, which jointly pin down NAV_1 and l_f as functions of λ_f. First, the flexibility of NAVs implies that the liquidation cost $\frac{\phi}{2}l_f^2$ will be incorporated by NAV_1 and proportionally borne by all agents:

$$NAV_1(\lambda_f) = 1 - \frac{\phi}{2}l_f^2(\lambda_f)\,, \text{ if } x_{f,0} < \lambda_f\,. \tag{2.9}$$

in which l_f depends on λ_f in equilibrium.

Second, because liquidation is costly, the fund liquidates just enough of the illiquid project to meet redemptions requests at NAV_1. In other words, the total amount of cash distributed to λ_f redeeming agents is the sum of stored cash and that raised from costly liquidations:

$$\lambda_f NAV_1(\lambda_f) = x_{f,0} + l_f - \frac{\phi}{2}l_f^2\,, \text{ if } x_{f,0} < \lambda_f\,. \tag{2.10}$$

Without fully solving for the equilibrium, now can now show that fund equity provides liquidity in any generic equilibrium:

Proposition 3. *Given any fund portfolio* $(x_{f,0}, y_{f,0})$*, the fund provides liquidity ex-ante as defined in Definition 2, that is,*

$$E[c_{f,1}(\lambda_f(R))] > c_1(x_{f,0}, y_{f,0})\,, \tag{2.11}$$

where the expectation is taken over fundamentals R*. However, the consumption promised to early agents is decreasing in the number of redeeming agents, that is,*

$$\frac{\partial c_{f,1}(\lambda_f)}{\partial \lambda_f} \leq 0\,, \tag{2.12}$$

which takes a strict form if $l_f > 0$*.*

The intuition behind inequality (2.11) is that by pooling resources among early and late agents at the fund level and following the pecking order of liquidation, the fund can provide redeeming agents with an NAV that is higher than the liquidation value of the fund portfolio. This represents an insurance against early agents' liquidity risks in the spirit of Diamond and Dybvig (1983).[20] Notably, as shown by (2.9), the fund equity is flexible in the sense that liquidation costs are fully incorporated at $t = 1$, which suggests that a debt contract is not a necessary condition for liquidity provision. Instead, the key is to share idiosyncratic liquidity risks at the intermediary level, which can be a bank or a fund.

Nevertheless, the characteristics of liquidity provision by bank debt and fund equity differ. This is formally illustrated by inequality (2.12): by issuing equity, funds can flexibly adjust their actual, ex-post liquidity provision downwards at $t = 1$ to incorporate any realized liquidation loss induced by redemptions. In contrast, banks would have to honor the fixed debt value $c_{b,1}$ to redeeming agents unless the bank fails.

Having analyzed NAV_1 and its implications on liquidity provision, NAV_2 is subsequently determined by

$$NAV_2(\lambda_f) = \frac{1}{1-\lambda_f}\left(x_{f,1} + y_{f,1}R\right)\,, \tag{2.13}$$

where $x_{f,1} = \max(x_{f,0} - \lambda_f c_{f,1}(\lambda_f), 0)$ and $y_{f,1} = y_{f,0} - l_f(\lambda_f)$.

[20] Similarly to the analysis of the bank above, the ability of fund equity to provide liquidity is general and holds qualitatively if we instead compare the optimally determined $E[c^*_{f,1}(\lambda_f)]$ to the autarky outcome $c^*_{a,1}$ in equilibrium under certain sufficient conditions. See Appendix A for a formal analysis.

Taken together, the NAV rules (2.8), (2.9), (2.10), and (2.13) lead to the following important result regarding the comparison between NAV_1 and NAV_2:

Proposition 4. *The relationship between* $NAV_2(\lambda_f)$ *and* $NAV_1(\lambda_f)$ *is characterized by*

$$NAV_2(\lambda_f) = \begin{cases} NAV_1(\lambda_f) + \dfrac{y_{f,0}}{1-\lambda_f}(R-1) & \text{if} \quad x_{f,0} \geqslant \lambda_f \,, \\ NAV_1(\lambda_f)R & \text{if} \quad x_{f,0} < \lambda_f \,, \end{cases} \tag{2.14}$$

where in both cases $NAV_1(\lambda_f) > NAV_2(\lambda_f)$ *if and only if* $R < 1$.

Proposition 4 implies that the comparison between NAV_1 and NAV_2 is uniquely determined by the fundamentals R: NAV_1 is larger if and only if R is smaller than the benchmark return 1. It does not depend on strategic motives among redeeming agents. In contrast, the comparison between $c_{b,1}$ and $c_{b,2}$ in the bank scenario, as illustrated by Table 1, depends on whether a bank run occurs, which is in turn driven by agents' strategic motives. This observation immediately suggests that the benchmark fund equity is not subject to panic runs in liquidity provision.

The key underlying Proposition 4 is that the use of demandable equity with flexible NAVs proportionally assigns losses from premature liquidations to redeeming and non-redeeming agents. Therefore, fully flexible equity contracts eliminate first-mover advantages. On the contrary, the fixed face value of debt promised to withdrawing depositors imposes all losses from premature asset liquidations on staying depositors, and thereby induces a first-mover advantage to withdraw. Note that Proposition 4 can be reconciled with the literature that highlights the potential for mutual fund runs when NAVs are not fully flexible, which are partly captured by our Proposition 1. In other words, it is the debt-like imperfections of NAVs in reality rather than liquidity provision per se that gives rise to runs.

Proposition 4 helps us solve for late agents' optimal redemption decision at $t = 1$. Suppose that after observing s_i, $w_f \in [0, 1-\pi]$ late agents choose to redeem at $t = 1$ and store the cash for consumption at $t = 2$. Then the total portion of redeeming agents will be $\lambda_f = \pi + w_f$ since early agents always redeem. Consequently, the number of late agents w_f who choose to redeem at $t = 1$ is determined by

$$\begin{cases} w_f = 0 & \text{if} \quad u(NAV_1(\lambda_f)) < E[u(NAV_2(\lambda_f))|s_i] \,, \\ w_f \in (0, 1-\pi) & \text{if} \quad u(NAV_1(\lambda_f)(1-\kappa w_f)) = E[u(NAV_2(\lambda_f))|s_i] \,, \\ w_f = 1-\pi & \text{if} \quad u(NAV_1(\lambda_f)(1-\kappa w_f))) > E[u(NAV_2(\lambda_f))|s_i] \,. \end{cases} \tag{2.15}$$

Notice that late agents take expectations over NAV_2 as if they know R due to their almost perfect signal s_i. Solving (2.15) using equations (2.14) in Proposition 4 and Lemma 1 yields the late agents' optimal redemption decision at $t = 1$:

Proposition 5. *Given date-0 fund position* $(x_{f,0}, y_{f,0})$, $x_{f,0} \geq \pi$, *and signal* s_i *received by agents at* $t = 1$, *late agents redeeming from the fund amount to*

$$w_f^* = \begin{cases} 0 & \text{if} \quad R \geq 1\,, \\ \frac{1}{2}\left(1 - \pi - \sqrt{\frac{\kappa(1-\pi)^2 - 4(1-x_{f,0})(1-R)}{\kappa}}\right) & \text{if} \quad 1 - \kappa(x_{f,0} - \pi) \leq R < 1\,, \\ \frac{1-R}{\kappa} & \text{if} \quad 1 - \kappa(1-\pi) \leq R < 1 - \kappa(x_{f,0} - \pi)\,, \\ 1 - \pi & \text{if} \quad R < 1 - \kappa(1-\pi)\,. \end{cases} \tag{2.16}$$

Proposition 5 shows that w_f^* is decreasing in R, that is, fund outflows gradually increase as economic fundamentals deteriorate. Intuitively, late agents do not redeem when fundamentals are high ($R \geq 1$) because NAV_2 is always higher than NAV_1 in this case as shown by Proposition 4. When R falls below the premature project value of 1, late agents are better off to redeem at $t = 1$ because the long-term fundamentals only fully materialize at $t = 2$ and are not yet fully reflected in the short-term t-1 value of the project. At the same time, as more late households redeem, NAV_1 decreases by incorporating the resulting liquidation costs, and the storage efficiency also declines, which mitigate late households' incentive to redeem. Taken together, late agents redeem more when and only when R falls continuously, leading to the negative relationship between outflows and fundamentals, i.e., the flows-to-fundamentals relationship.[21] Combined with inequality (2.12) in Proposition 3, the flows-to-fundamentals relationship implies that the fund provides a lower contract payment $c_{f,1}^*(R)$ at $t = 1$ as R declines.

One important observation from Proposition 5 is that equilibrium redemption decisions at funds are directly and continuously linked to the economic fundamentals R. In contrast, for a bank, withdrawal decisions are indirectly and discontinuously linked to fundamentals, which serve as a coordination device for bank runs as in Proposition 1. Therefore, fund equity is subject to a different constraint in liquidity provision than panic runs. For equity, changes in

[21] Further, w_f^* is also increasing in $1 - \pi$ and ϕ while decreasing in $x_{f,0}$ and κ.

economic fundamentals are continuously reflected in the NAV_1, whereas for debt, a fixed payment is promised so that agents are insulated against fluctuations in fundamentals unless in bank runs.

Another nice feature of Proposition 1 and Proposition 5 is that they both link equilibrium outflows to economic fundamentals. This allows us to compare liquidity provision by bank debt and fund equity in the same economic environment. Specifically, recall that liquidity provision is given by $E[c^*_{k,1}(R)] - c_1(x^*_{k,0}, y^*_{k,0})$ (Definition 2). Since the direct liquidation value $c_1(x^*_{k,0}, y^*_{k,0})$ does not depend on R, comparing bank and fund liquidity provision essentially requires us to compare the contract payment at funds $c^*_{f,1}(R)$ with the contract payment at banks $c^*_{b,1}(R)$ state by state before taking expectations across states. We graphically summarize the relationship between equilibrium outflows and economic fundamentals for banks and funds in Figure 2. We consider a bank-run threshold that satisfies $R^* < 1$.[22] When $R > 1$, there are no outflows and $c^*_{k,1}(R)$ is insensitive to R for both bank debt and fund equity. When fundamentals R decline to an intermediate region between R^* and 1, there is not yet a bank run so banks still do not suffer outflows and $c^*_{b,1}(R)$ remains insensitive to R (see Proposition 1). However, the fund is already subject to fundamentals-driven outflows (see Proposition 5). The resulting premature project liquidations in turn lower $c^*_{f,1}(R)$ (see Proposition 3). When fundamentals deteriorate past R^*, a bank run occurs, resulting in a discontinuous drop in $c^*_{b,1}(R)$ for agents not being serviced. Instead, the fund is still subject to the continuous fundamentals-driven flows and provides a relatively higher $c^*_{f,1}(R)$.

Finally, we close the model by solving the fund's optimal portfolio allocation $(x_{f,0}, y_{f,0})$ at $t = 0$ that maximizes the expected utility of all agents:

$$\max_{x_{f,0}, y_{f,0}} E\left[\pi u(c_{f,1}(\lambda_f)) + (\lambda_f - \pi) u(c_{f,1}(\lambda_f)(1 - \kappa(\lambda_f - \pi))) + (1 - \lambda_f) u(c_{f,2}(\lambda_f))\right] \tag{2.17}$$

subject to (2.8), (2.9), (2.10), (2.13), and (2.16), where the three terms in (2.17) denote the utilities for early agents, late agents who redeem at $t = 1$, and late agents who stay until $t = 2$, respectively. Thanks to Proposition 3, we know that fund equity provides liquidity ex-ante in any equilibrium without fully solving (2.17).

[22] As shown in the proof of Proposition 1, R^* can be either larger or smaller than 1 because the upper bound of the lower dominance region, $\underline{R}$, can be either larger or smaller than 1. This feature arises because the liquidation value $1 - \frac{\phi}{2} y^2_{k,0}$ is smaller than 1. Without further parametric or distributional assumptions, however, it is not analytically tractable to give exact sufficient conditions for when R^* is larger or smaller than 1, because the Laplacian equation (D.1) cannot be solved analytically.

Given our unified conceptualization of liquidity provision, a natural question is how much liquidity is actually provided by banks and funds in reality. Analytically calculating and comparing ex-ante liquidity provision would involve a number of parametric and distributional assumptions. Therefore, we map our theoretical framework to the data to quantify liquidity provision by debt- and equity-issuing financial intermediaries in practice.

3 Liquidity Provision Index (LPI)

This section develops an empirical measure for liquidity provision. The LPI captures how much the expected contract payment by an intermediary exceeds the direct liquidation value of the underlying assets per dollar invested. In other words, it is the empirical proxy for the expected improvement in short-term consumption $E[c^*_{k,1}(R)] - c_1(x^*_{k,0}, y^*_{k,0})$ as in Definition 2.[23]

In this section, we first illustrate the construction of the LPI for debt- and equity-funded intermediaries with a simple example in Section 3.1. Then we provide a formal step-by-step explanation of the construction and discuss the connection to the theory in Section 3.2. The LPI is generally applicable, but for the ease of exposition, we will henceforth use banks and funds interchangeably with demandable-debt- and demandable-equity-issuing intermediaries.

3.1 An Example

Consider a hypothetical mutual fund that holds 10% cash and 90% of corporate bonds, where the corporate bonds are illiquid and can only be converted to 70% of their fair value upon early redemption. When all investors demand to redeem their shares at short notice, the per-unit contract payment would only be $1 \times 0.1 + 0.7 \times 0.9 = 0.73$ because all illiquid assets would be prematurely liquidated and incur a discount. Notice that an investor directly holding 0.1 of cash and 0.9 of the illiquid assets receives the same 0.73 if she has to liquidate at short notice because she would also have to sell her entire portfolio. In this sense, holding a share at the fund does not improve the consumption value when all other fund investors also redeem because all liquidity risk is systematic and liquidity insurance is ineffective.

[23] The LPI remains agnostic about the long-run consumption (i..e, $c^*_{k,2}$ in the model), which is an important but separate question.

However, as long as not all investors redeem early, idiosyncratic liquidity risk is pooled at the fund level, and holding fund shares reduces the average discount suffered and improves the amount of cash obtainable upon short notice. For example, if total outflows amount to less than 10% of the funds' assets, they can be met by just using the fund's cash so that no liquidation discounts are incurred. If redemptions exceed 10%, more illiquid assets would have to be liquidated, and the contract payment would decrease until it reaches 0.73 at 100% outflows. This trend is depicted graphically by the blue line in the upper panel of Figure 3. The dotted line on the same graph indicates the liquidation value of the underlying portfolio, which is 0.73.

The difference between the solid blue line and the dotted blue line represents the fund's contribution to liquidity for a given level of outflows. Hence, we can integrate over the distribution of outflows to calculate expected liquidity provision. Note that Figure 3 does not imply that all outflow volumes will occur with equal probability. Rather, we let the data speak by taking the empirical distribution of fund flows as the equilibrium outcome of early redemptions, capturing the flows-to-fundamentals relationship as the constraint to fund liquidity provision (see Proposition 5). This approach allows us to calculate the LPI for each fund as the difference between the expected contract payment and the direct liquidation value per dollar investment.

The LPI for banks can be constructed in a very similar way. Consider a hypothetical bank with 10% cash and 90% corporate loans, where the corporate loans can only be converted to 60% of their fair value upon early redemption. The direct liquidation value of the portfolio is $1 \times 0.1 + 0.6 \times 0.9 = 0.64$. The bank can provide a higher contract payment than the direct liquidation value as long as not all depositors prematurely withdraw. However, the liquidation value does not continuously decrease with the proportion of outflows because the face value of deposit contracts is fixed until the default threshold is crossed. The contract payment and direct liquidation value for our hypothetical bank are indicated by the solid red line and the dotted red line in the lower panel of Figure 3. Finally, we also use the bank's empirical distribution of deposit flows, capturing the potential for panic runs as the constraint to bank liquidity provision (see Proposition 1), to calculate by how much the expected contract payment exceeds the direct liquidation value of assets, which is the bank LPI.

3.2 Construction

More formally, we can generalize the LPI construction into three steps. Recall that our goal is to measure how much the expected contract payment by an intermediary exceeds the direct liquidation value of the underlying assets per dollar invested, i.e., the empirical proxy of $E[c^*_{k,1}(R)] - c_1(x^*_{k,0}, y^*_{k,0})$.

Step 1: The first step is to calculate the contract payment by outflows as in Figure 3. This requires knowing how much of each asset is liquidated to meet a given proportion of redemptions and at what cost.

Let $\hat{\lambda}_i$ be the amount of outflows at bank (fund) i as a percentage of total assets. $\hat{\lambda}_i$ is the empirical counterpart to the number of investors that withdraw in the model, λ_k. We also extend the two-asset world of the model to allow for a complete portfolio of assets. Let vector Y_{it} be bank (fund) i's asset portfolio composition at time t with Y_{ijt} denoting the value weight of asset $j \in J$. The asset index, j, is ranked in increasing order with the haircut of the asset h_{jt}, i.e., $h_{jt} \leq h_{j't}$ for any $j \leq j'$. In other words, more liquid assets have smaller indexes. The ranking of liquidation haircuts and the composition of assets determine which assets are liquidated for a given outflow as banks (funds) meet redemption requests by selling more liquid assets first. To illustrate, recall the above example where a bank (fund) holding 10% of cash can meet outflows of up to 10% without losses. As outflows increase, banks (funds) use increasingly more illiquid assets to meet their redemption requests until even the most illiquid assets are liquidated.

While the contract payment of fund shares follows that of fund assets due to flexible NAVs, bank deposits do not lose value until there is not enough left to pay the face value. We consider liquidity provision by uninsured deposits throughout our calculations, for which the contract payment will fall below the face value if the proceeds from asset liquidations are insufficient. Nevertheless, the LPI framework can accommodate the presence of insured deposits and their effect on uninsured deposits (see Appendix B for details). Denoting by $1 - H_t(\hat{\lambda}_i, Y_{it})$ the contract payment for investors withdrawing (redeeming) a dollar of deposits (fund shares) when outflows are $\hat{\lambda}_i$ under bank (fund) asset composition Y_{it}, we have for bank i:

$$H_t(\hat{\lambda}_i, Y_{it}) = \begin{cases} 0 & \text{if the bank survives} \\ \sum_{j=1}^{J} Y_{ijt} h_{jt} & \text{if the bank fails}, \end{cases} \tag{3.1}$$

and for fund i:

$$H_t(\hat{\lambda}_i, Y_{it}) = \sum_{j=1}^{J_{\hat{\lambda}}-1} Y_{ijt} h_{jt} + \widehat{Y_{iJ_{\hat{\lambda}}t}} h_{J_{\hat{\lambda}}t}\,, \tag{3.2}$$

where $\widehat{Y_{iJ_{\hat{\lambda}}t}}$ is the actual position of the least liquid asset being liquidated to meet $\hat{\lambda}_i$ while assets more liquid than $J_{\hat{\lambda}}$ are fully liquidated.

Step 2: Since contract payments vary with outflows, the expected contract payment depends on the distribution of outflows. Our model predicts how early redemptions vary with economic fundamentals for a given asset portfolio of a bank (fund) due to panic runs (fundamentals-driven flows). Since economic fundamentals are difficult to quantify, our empirical approach directly measures the realized flows, which is an equilibrium outcome. In reality, institutional features like implicit guarantees at banks and sticky NAVs at mutual funds may also influence investor flows. These effects are incorporated in the observed outflows we use, which renders the LPI an accurate measure of liquidity provision in practice. Nevertheless, we will introduce further tests based on the LPI in order to isolate the pure effect of debt versus equity in Section 4.

We estimate the distribution of observed outflows for each fund and bank, $F(\hat{\lambda}_i)$, and obtain their expected contract payment as

$$\int (1 - H_t(\hat{\lambda}_i, Y_{it}))dF(\hat{\lambda}_i)\,. \tag{3.3}$$

Viewed through the lens of the model, this expected contract payment of bank deposits (fund shares) proxies for the expected consumption available to bank depositors (fund shareholders) withdrawing prematurely at the end of the first period, $E[c^*_{k,1}(\lambda)]$. Conceptually, the difference between $t = 0$ and $t = 1$ in Diamond and Dybvig (1983)-type models captures an infinitely short time period. In the same vein, our empirical construction of the contract payment as in (3.3) also reflects a very short time in which the net return on assets is negligible.[24]

Step 3: Finally, we calculate the direct liquidation value of the bank's (fund's) assets and subtract it from the expected contract payment by the bank (fund). Notice that the direct

[24]This is consistent with our model that between $t = 0$ and 1, the investment project has not yet come to fruition and thus retains a value of 1, and thus the fund's NAV is 1 at the beginning of $t = 1$. For a bank, although the level of the optimal deposit value at $t = 1$ can be theoretically different from 1 in a static setting, the effective short-term net asset return is 0.

liquidation value of the bank (funds) is equivalent to the contract payment of the bank (fund), when all investors withdraw (redeem) and all assets are sold off at short notice, i.e., $1 - H_t(1, Y_{it})$.

Taken together, the LPI of bank (fund) i at time t is its expected contract payment relative to the direct liquidation value of its assets:

$$LPI_{it} = \int (1 - H_t(\hat{\lambda}_i, Y_{it}))dF_i(\hat{\lambda}_i) - (1 - H_t(1, Y_{it})), \tag{3.4}$$

where $F_i(\hat{\lambda}_i)$ is the estimated distribution of bank deposit (fund) outflow $\hat{\lambda}_i$ and $H_t(\hat{\lambda}_i, Y_{it})$ is given by (3.1) and (3.2). To help illustrate the link between theory and empirics, we also note that LPI reflects the following close mapping between equilibrium outcomes and empirical counterparts:

Table 2: Mapping between equilibrium outcomes and empirical counterparts

	Outflows	Asset holdings	Contract Payment	Liquidation Value
Theory	λ_k	$(x^*_{k,0}, y^*_{k,0})$	$E[c^*_{k,1}(\lambda_k)]$	$c_1(x^*_{k,0}, y^*_{k,0})$
Empirics	$\hat{\lambda}_i$	Y_{it}	$\int (1 - H_t(\hat{\lambda}_i, Y_{it}))dF(\hat{\lambda}_i)$	$1 - H_t(1, Y_{it})$

The LPI is designed to quantify liquidity provision by both demandable debt and demandable equity issuing financial institutions in practice. In other words, the measurement of liquidity provision remains consistent in the presence of regulations and frictions that may influence investors' redemption decisions and intermediaries' portfolio choice in practice. The key lies in directly using the observed distribution of outflows to calculate expected contract payments. This method is consistent with the theory because fundamentals R affect liquidity provision through determining investors' outflows as shown in Proposition 1 for bank debt and Proposition 5 for fund equity. At the same time, observed flows already reflect the equilibrium outcome so that we can remain agnostic about the measurement of fundamentals R and its empirical mapping to investor decisions, in which factors outside of our model (e.g., regulation) may also play a role. We also use the actual portfolio holdings of banks and funds, which may arise from the contractual form of liabilities as well as other institutional features. In this sense, the LPI on its own is not meant as an identification strategy for the pure effect of debt versus equity in our model. To this end, we will apply the LPI in additional tests in Section 4.

4 Liquidity Provision by Bank Debt and Fund Equity

In this section, we apply the LPI to compare and contrast the magnitude of liquidity provision by debt- and equity-issuing financial institutions. Section 4.1 introduces the data, while Section 4.2 presents the baseline estimates of liquidity provision by commercial banks and bond mutual funds.

To isolate the effect of debt versus equity conceptualized in our model, we conduct a number of tests to purge out the effect of deposit insurance and non-deposit bank liabilities in Section 4.3. In Section 4.4, we further apply the LPI to MMFs. The difference-in-difference analysis we perform around the 2016 Money Market Reform further identifies the effect of debt versus equity funding on liquidity provision.

We proceed to analyze the determinants of bank and fund LPI variations in Section 4.5. Finally, Section 4.6 examines liquidity provision in the time series and provides evidence for post-crisis liquidity regulation and Quantitative Easing to have narrowed the bank-fund LPI gap from 2011 to 2017.

4.1 Data

We use bank call reports and mutual fund holdings from the CRSP database to obtain the composition of assets Y_{it} for banks and funds, respectively.[25] We remove exchange-traded funds and index funds to obtain a sample of actively-managed open-end fixed-income funds. Table 3 provides summary statistics of the portfolio holding and total asset size of banks and funds in our sample.

For haircuts, we use collateral haircuts in repo markets and discounts on loan sales in secondary markets similar to Bai, Krishnamurthy and Weymuller (2018). We collect securities haircuts from the New York Fed's repo data series, commercial loan haircuts from the Loan Syndications and Trading Association, and real estate loan haircuts from the Federal Home Loan Banks. Also following Bai, Krishnamurthy and Weymuller (2018), we smooth the haircut series

[25]Please refer to Table A1 in the Appendix for a detailed mapping between balance sheet variables and our asset categories.

using principal components to remove outliers.[26] Figure 4 plots the haircut series for different asset categories over our sample period. As expected, more liquid assets such as Treasuries have smaller haircuts, whereas relatively illiquid assets such as loans have higher haircuts. We use average market haircuts in our baseline calculation. In Appendix C, we repeat our LPI calculation with price-impact adjusted haircuts to account for the potential price pressure from bank and fund asset liquidations and show that our results are robust.

Regarding flows, we calculate uninsured deposit flows as a percentage change in total uninsured deposits using bank call reports from 2010 to 2017. We augment the resulting flow distribution with data on bank failures. For each mutual fund, flows are calculated using its portfolio-level total net asset changes using the CRSP database. For positive flows, we assume that no assets are prematurely liquidated so that the effective contract payment is equal to one.

4.2 Baseline LPI Estimates for Banks and Bond Mutual Funds

We follow the procedure described in Section 3.2 to construct the quarterly LPI for each bank and fund in our sample.

We first construct the bank- and fund-level contract payments as a function of investor outflows. We report the aggregate result in Figure 5. This aggregate result can be thought of as a representative bank holding the average portfolio of the banking sector and a representative fund holding the average portfolio of the bond mutual fund sector. Similar to the example in Section 3.1, the contract payment for funds continuously declines with early redemptions because the value of fund shares adjusts to reflect the incurred haircuts when increasingly illiquid assets are liquidated. Banks also sell more illiquid assets as deposit withdrawals increase, but the debt contract guarantees a constant contract payment until the bank defaults, i.e., when deposit withdrawals cannot be met at their promised value.

When outflows reach 100%, an intermediary has to fully liquidate the underlying portfolio, and the contract payment of uninsured bank deposits and mutual fund shares are 78 and 95 cents per dollar as shown in Figure 5. These are also the direct liquidation values of the representative bank and fund portfolio, showing that bank assets are relatively more illiquid than fund assets. One rationale provided by our theory is that the use of debt versus equity contracts induces

[26] Specifically, we first extract the first principal component PC_t based on the panel of haircuts. Then, we regress each raw haircut time-series on PC_t and use the predicted value of the regression as the smoothed haircut.

different incentives for holding liquid assets. Deposit-funded banks hold liquid assets to reduce the incidence of panic runs, while equity-funded funds hold liquid assets to alleviate flows to fundamentals. The aggregate result is consistent with the latter concern being more pronounced in our sample period.

We then estimate the flow distribution for each bank and fund in our sample. Figure 6 plots the overall distribution of bank and fund flows from 2011 to 2017 in our sample of banks and funds, respectively. As explained before, the empirical flow distribution captures the effect of economic fundamentals on investor flows in equilibrium. In practice, imperfections in the adjustment of fund NAVs could also have influenced the observed outflows. Since the observed outflows would be more pronounced than under perfectly flexible NAVs, the fund LPI we obtain can be seen as a lower bound to fund LPI with frictionless NAVs.[27]

Finally, we use the estimated flow distribution and contract payment curve for each bank and fund to calculate the LPI for each intermediary-quarter. Figure 7 plots the distribution of the average bank- and fund-level LPIs. It shows that from 2011 to 2017, the average dollar invested in funds and banks provide 4.6 cents and 21.9 cents of liquidity, respectively. The results are robust to the use of price-impact adjusted haircuts as shown in Appendix C.

These LPI estimates are realistic in the sense that they can accommodate the influence of both the constraints underlying bank and fund liquidity provision present in our model as well as other relevant institutional and regulatory features. As explained in Appendix B, deposit insurance and other regulatory policies do not affect the validity of the LPI construction. This feature of the LPI is very valuable for quantifying liquidity provision in practice.

4.3 Effect of Deposit Insurance and Non-deposit Liabilities

To further determine the pure effect of debt versus equity funding on liquidity provision, we conduct a number of additional tests. First, our LPI calculation only considers the uninsured portion of bank deposits. Nevertheless, regulatory features such as deposit insurances and non-deposit liabilities may indirectly affect the LPI magnitude through bank portfolio choice as well.

[27]Going forward, the introduction of swing pricing could potentially alleviate frictions in the adjustment of NAVs and improve the liquidity provision capacity by funds. Evidence for outflows becoming less pronounced following the introduction of swing pricing has been established in the UK market by Jin, Kacperczyk, Kahraman and Suntheim (2020).

Since the ideal experiment of the same bank operating with and without deposit insurance does not exist, we perform two tests to show that regulation can only explain a limited portion of the difference in liquidity provision by debt- versus equity-funded intermediaries.

We first relate bank-level LPIs to the ratio of insured deposits in the data and project the LPI that would have applied to a hypothetical bank without insured deposits in Table 4. The constant term in column (1), which is statistically significant, indicates that without deposit insurance, the LPI for uninsured deposits at an average bank in our sample would be 0.164, compared to the baseline bank LPI of 0.219. Similarly, column (2) indicates that a hypothetical bank without insured deposits or non-deposit liabilities would have an LPI of 0.147 for its deposits. In other words, when the effect of deposit insurance and non-deposit liabilities at banks is removed, fund liquidity provision becomes more significant relative to bank liquidity provision, where the bank-fund LPI drops from one-quarter to one-third.

4.4 Application: MMF LPI around the Money Market Reform

There may be remaining concerns that the LPI is influenced by regulatory differences other than deposit insurance such as implicit guarantees. To this end, we apply the LPI to a laboratory of MMFs before and after the Money Market Reform to isolate the effect of debt versus equity on liquidity provision. In October 2016, the Securities and Exchange Commission implemented the Money Market Reform. Among other changes, institutional prime MMFs were required to switch from reporting a $1 fixed share price to a floating NAV, which effectively corresponds to a switch from demandable debt to demandable equity in our framework. Importantly, among other dimensions of the reform, only the floating NAV rule did not apply to retail prime funds who continued reporting a $1 fixed share price.[28] This setting naturally lends itself to a difference-in-differences analysis to study the difference in liquidity provision by debt- and equity-funded intermediaries with institutional and retail prime funds as treatment and control groups, respectively.

Figure 8 plots the LPI of institutional and retail prime MMFs from three years before to three years after the reform.[29] The lower panel repeats the analysis with the subset of funds that

[28]Other regulations introduced in the 2016 Money Market Reform include liquidity fees and gates, which apply to all MMFs. See "Money Market Fund Reform; Amendments to Form PF" https://www.sec.gov/rules/final/2014/33-9616.pdf

[29]To account for changes in the sensitivity of fund flows, the flow distribution for the LPI calculation are constructed separately for the pre and post periods, respectively.

appeared in both pre- and post-reform periods. For both samples, institutional and retail prime fund LPI largely followed the same pattern before October 2016, consistent with the parallel trends assumption. Since the reform, institutional MMFs experienced a significantly larger drop in LPI than retail funds.

We corroborate our findings with a formal difference-in-differences test in Table 5. The coefficients on the interaction term reflect the change in LPI due to a switch from debt to equity contracts. Using the full sample of funds, the baseline result in column (1) shows that the LPI drops by 40/(225-13)=19% when institutional MMFs switch from fixed to floating NAVs. This result confirms that demandable equity provides a significant amount of liquidity, amounting to 81% of the liquidity provided by demandable debt. To ensure that flows during the implementation period do not drive the results, columns (3) and (4) repeat the analysis excluding flows from August to October 2016. Columns (5) and (6) shift the treatment date a year back to account for potential anticipation effects. Even-numbered columns focus on the subsample of funds appearing in both the pre- and post-reform periods, respectively. The coefficients remain statistically significant and similar in magnitude.

4.5 Liquidity Provision in the Cross-section

We proceed with analyzing variations in LPI in the cross-section of banks and funds. We show that they are consistent with our model predictions. As seen in Table 4, deposit insurance does not fully explain but has a positive effect on bank liquidity provision. This is consistent with the theory in which a lower bank-run probability allows for more liquidity provision. The same trend is also depicted in the upper panel of Figure 9. Other factors, such as leverage ratio and the proportion of non-deposit funding, may also affect the probability of runs, and banks of different asset sizes may be subject to different regulatory constraints. The lower panel of Figure 9 shows that the relationship between LPI and deposit insurance remains robust to the inclusion of these control variables.

Regarding mutual fund LPIs, our model shows that their liquidity provision is constrained by fundamentals-driven flows. We calculate empirical fund flow sensitivities by regressing fund flows against lagged fund returns while controlling for lagged fund returns. In line with the theory, we find that the higher a fund's flow sensitivity, the lower its LPI (see Figure 10). While fund returns are used as an empirical proxy for economic fundamentals, other features of the fund, such as size, age, and expense ratio, may vary simultaneously. The lower panel of Figure 10 shows that the negative relationship between liquidity provision and fund flow sensitivity is preserved after adding these fund-level characteristics as controls.

4.6 Liquidity Provision in the Time-series

Finally, we examine how liquidity provision has evolved over time. Plotting the quarterly weighted average of bank- and fund-level LPIs, Figure 11 shows that the difference has been narrowing over time with bank LPI sharply decreasing from 0.28 in 2011 to 0.19 in 2017. In other words, within six years, the liquidity provided by an average dollar invested in funds has approximately increased from a seventh to a quarter of a dollar invested in commercial banks.

While a full characterization of the trend's determinants is beyond the scope of this paper, we highlight that changes in the regulatory landscape have had a significant impact on the capacity of liquidity provision by commercial banks.

First is the increase in central bank reserves following Quantitative Easing (QE). Excess reserves held with the Federal Reserve are liquid assets on bank balance sheets. The overall effect of more reserves on liquidity provision can go in two different ways. It could have a positive effect because, as Proposition 2 suggests, more liquid bank balance sheets are less prone to runs. This is illustrated by the rightward shift of the default threshold for outflows from $\hat{\lambda}$ to $\hat{\lambda}'$ in Figure 12. On the other hand, a portfolio with more reserves also has a higher liquidation value under direct holding by investors, which decreases the potential for liquidity provision by the intermediary. This effect is reflected by the increase in liquidation value of the bank portfolio from C to C' in Figure 12.

Empirically, we find evidence consistent with the latter effect being dominant, i.e., QE decreases the capacity of liquidity provision by banks. As shown in Figure 13, the expansion in excess reserves from $1 trillion in 2011Q3 to more than $2.5 trillion in 2014Q3 is mirrored by a corresponding sharp fall in bank LPI during the same period. Sorting banks into quartiles of reserve uptake as a proportion of balance sheet size, we observe that the LPI drops consistently more for banks in the upper quartile, which lends further support for the aggregate effect. Therefore, QE lowers bank LPI through raising the liquidation value under direct holding, which limits how much banks can contribute to liquidity provision. In the limit, narrow banks, which only hold liquid assets like reserves, provide negligible amounts of liquidity relative to traditional commercial banks.

In this context, the implementation of the Liquidity Coverage Ratio (LCR) had similar effects on bank liquidity provision as the introduction of QE. In the US, the LCR stipulates that banks with \$250 billion or more in total assets or \$10 billion or more in on-balance sheet foreign exposures are required to hold sufficient amounts of High Quality Liquid Assets (HQLA), which include cash, central bank reserves, and some agency MBS, to cover expected net cash outflows for a 30-day stress period. Banks with \$50 billion or more in consolidated assets are also subject to a less stringent LCR requirement. Similar to the QE case, the LCR requires large banks to hold a higher fraction of liquid assets on their balance sheets, which raises the default threshold in terms of outflows but also increases the liquidation value of the benchmark asset portfolio (see Figure 12).

We find evidence for an overall negative impact of the LCR requirement on bank liquidity provision within our sample period. Figure 14 shows that banks most impacted by the LCR, i.e., above \$250 billion in total assets, also experience the most pronounced decline in LPI relative to those without and with a less stringent LCR requirement. This is again consistent with the interpretation that the LCR moves commercial banks more towards a narrow-banking business model, for which the gap between the liquidation value of bank assets and the contract payment of bank liabilities (i.e., deposits) is limited and liquidity provision is diminished.

5 Conclusion

This paper demonstrates that demandable equity issued by non-bank intermediaries is able to provide liquidity just like demandable debt issued by the traditional banking sector. Liquidity provision stems from the pooling of idiosyncratic liquidity shocks at the intermediary-level, which occurs independently of the contractual form of the intermediary's liabilities as long as they are redeemable at short notice. The characteristics of liquidity provision, however, are different. Equity is not prone to panic runs as in the case of debt because flexible NAVs remove the first-mover advantage in redemptions. However, the continuous adjustment of equity's contract value also renders investor flows and liquidity provision more sensitive to fluctuations in the economy.

Based on the theory, we develop the Liquidity Provision Index (LPI) as a parsimonious measure of liquidity provision across different types of intermediaries. It captures the extra proceeds an investor expects to obtain by withdrawing a debt or equity claim from an intermediary relative to directly holding and selling the underlying portfolio of assets herself. Applied to deposit-issuing commercial banks and equity-issuing bond mutual funds, we find that bond mutual funds create 4.8 cents of liquidity per dollar invested, which is one-quarter of that by uninsured bank deposits, at the end of 2017. The LPI gap between banks and funds has been continuously narrowing over time, coinciding with an increase in liquid assets on bank balance sheets following Quantitative Easing and the LCR requirement. These results highlight a new side effect of unconventional monetary policy and post-crisis liquidity regulation.

The migration of liquidity provision away from deposit-issuing banks to equity-issuing financial institutions like bond mutual funds bears far-reaching implications. Not only will liquidity provision experience volatile flows-to-fundamentals, assets held by intermediaries will also become more prone to premature liquidations as the underlying economy deteriorates even without a major default event. This was already evident during the Covid-19 crisis, when mutual funds suffered heightened outflows, collectively liquidated their portfolios, and induced significant strains in Treasury and corporate bond markets (Ma, Xiao and Zeng, 2020). Therefore, the financial stability implications from the increased reliance on liquidity transformation by equity-issuing non-banks are important to consider going forward.

Figure 1: Size of the US Fixed-Income Mutual Fund Sector

The upper panel plots the total asset size of US fixed-income mutual funds from 1995 to 2019. Fixed-income funds include government bonds funds, corporate bond funds, loan funds, and multi-sector bond funds. The lower panel plots the ratio of fund shares relative to bank deposits in the same sample period. Data source: Morningstar and Flow of Funds.

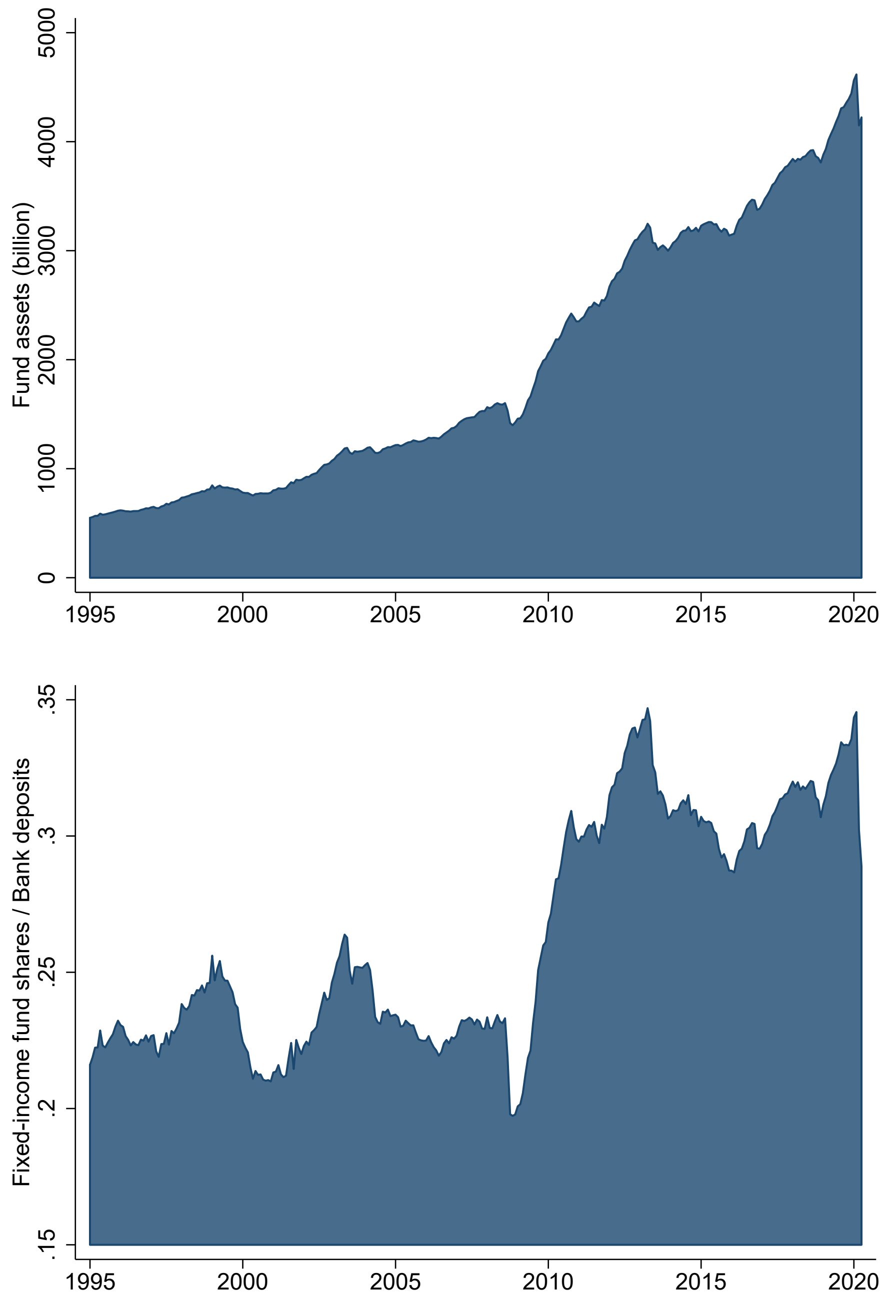


Figure 2: Investor Withdrawals and Economic Fundamentals

This graph depicts withdrawals (ω) by late agents against variation in economic fundamentals (R) under $x_{k,0} = \pi$ and $R^* < 1$. The blue line represents the equilibrium redemption decisions of fund investors, where the negative slope indicates a flows-to-fundamentals relationship for funds. The red line corresponds to withdrawal decisions of bank investors, where the steep rise in withdrawals at R^* indicates the presence of panic runs, with R^* being the run threshold.

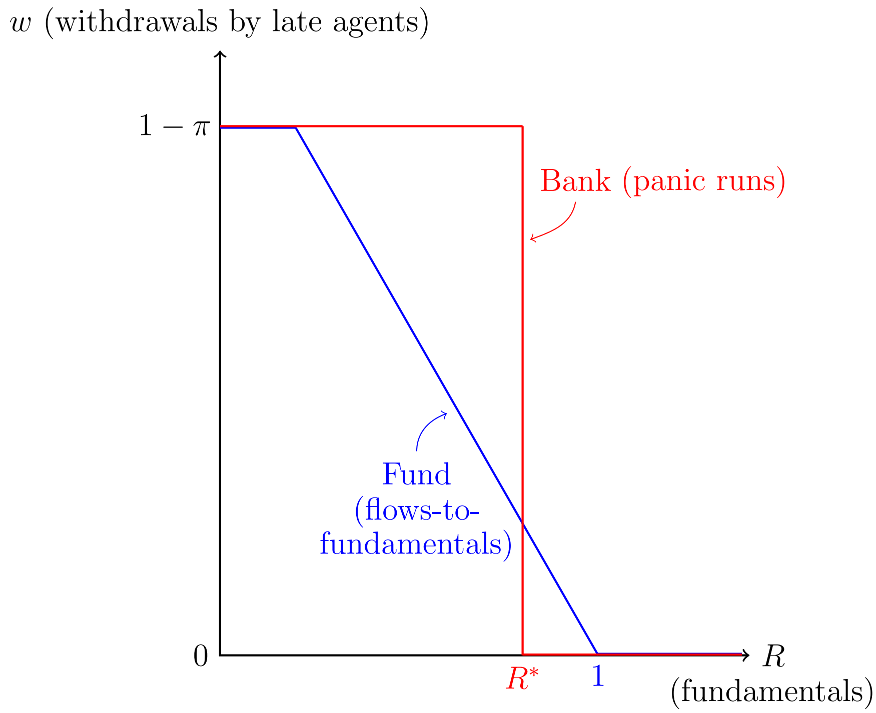


Figure 3: Contract Payment, Liquidation Value and Liquidity Provision (Example)

The upper panel plots the contract payment of a hypothetical fund for a given proportion of outflows. The fund holds 10% of cash and 90% of corporate bonds. The haircuts upon early redemption for cash and bonds are 0% and 30%, respectively. The dotted line is the liquidation value of the fund's asset portfolio when directly held and sold by an investor at short notice. The lower panel plots the contract payment of a hypothetical bank for a given proportion of outflows. The bank holds 10% of cash and 90% of corporate loans. The haircuts upon early redemption for cash and loans are 0% and 40%, respectively. The dotted line is the liquidation value of the bank's asset portfolio when directly held and sold by an investor at short notice.

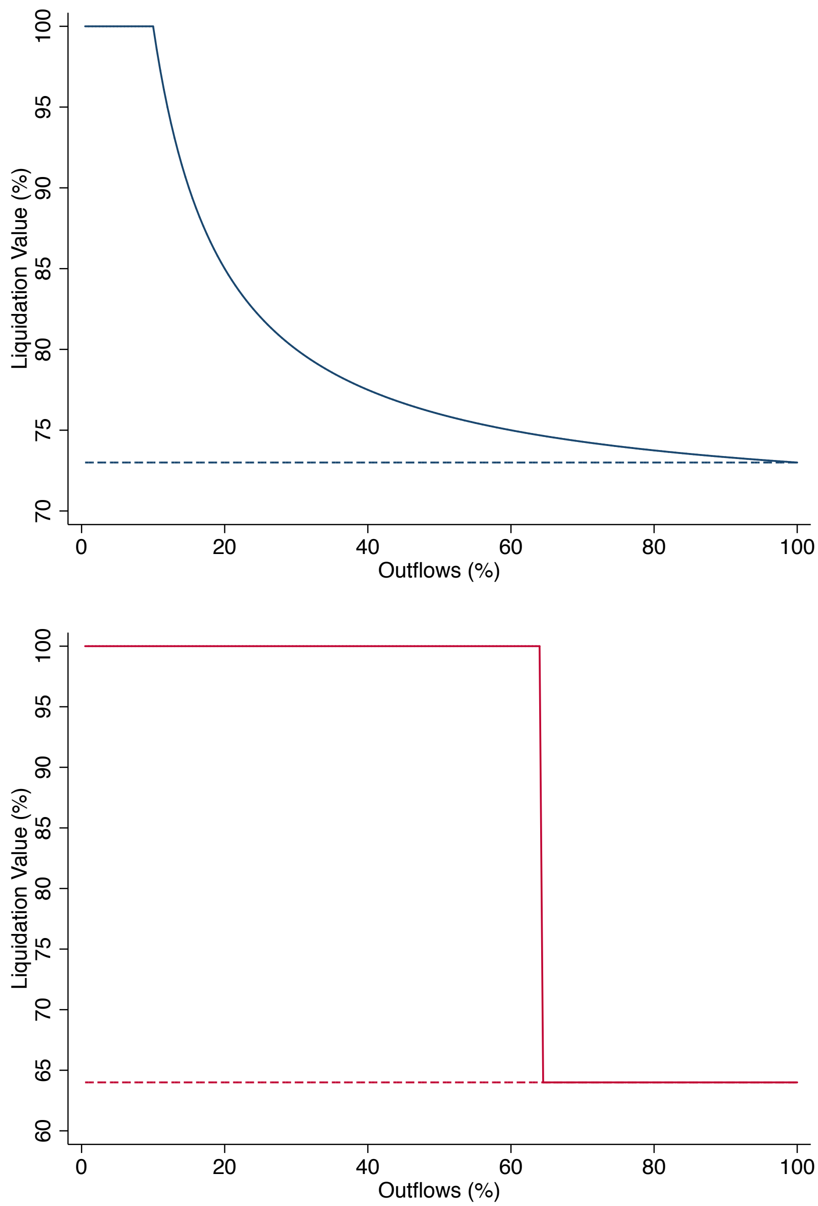


Figure 4: Haircuts

This graph plots the haircuts for different asset categories over time. Securities haircut data (upper panel) is obtained from the Federal Reserve Bank of New York's published repo series. Haircuts for commercial loans, real estate loans, and personal loans (lower panel) are from the Loan Syndications and Trading Association (LSTA), the Federal Home Loan Banks website, and the Federal Reserve, respectively. To remove outliers in the original data, we calculate the first principal component of the underlying series and plot the predicted value from the loadings regression $h_k = a_k + b_k PC_t + \epsilon_{kt}$ for each asset category k.

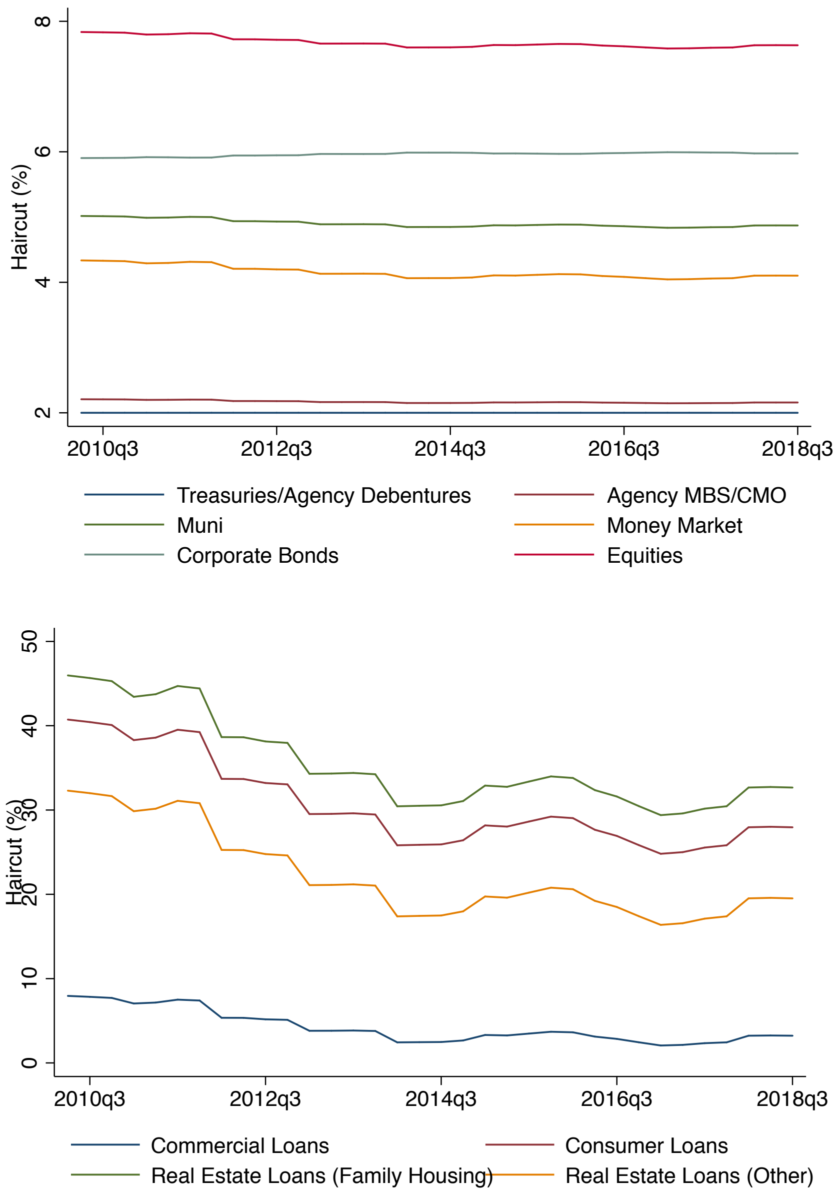


Figure 5: Contract Payment, Liquidation Value, and Liquidity Provision

The upper panel plots the contract payment of fund shares for a given proportion of outflows. The fund asset portfolio reflects the weighted average of all fixed income mutual funds' portfolios during our sample period from 2011 to 2017. The dotted lines represent the liquidation value of the portfolio of fund assets when directly held and sold by an investor at short notice. The lower panel plots the contract payment of bank deposits for a given proportion of outflows. The bank asset portfolio reflects the weighted average of all commercial banks' portfolios during our sample period from 2011 to 2017. The dotted lines represent the liquidation value of the portfolio of bank assets when directly held and sold by an investor at short notice.

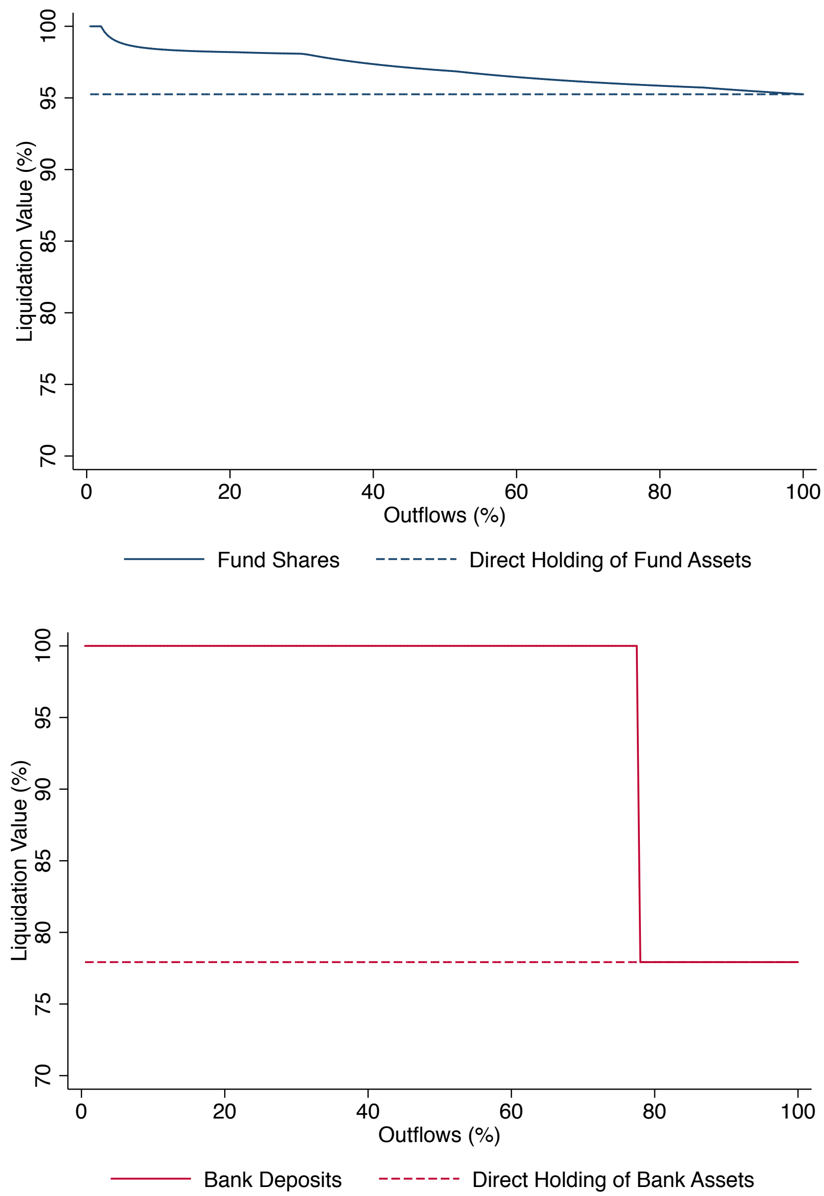


Figure 6: Distribution of Bank and Fund Flows

This graph plots the frequency distribution of flows of fund and bank liabilities in the upper and lower panel, respectively. The sample covers all commercial banks and mutual funds in our sample from 2011 to 2017. For presentation purposes, the distribution is truncated at -20% and 20%, where observations in the tail are assigned to the last bin.

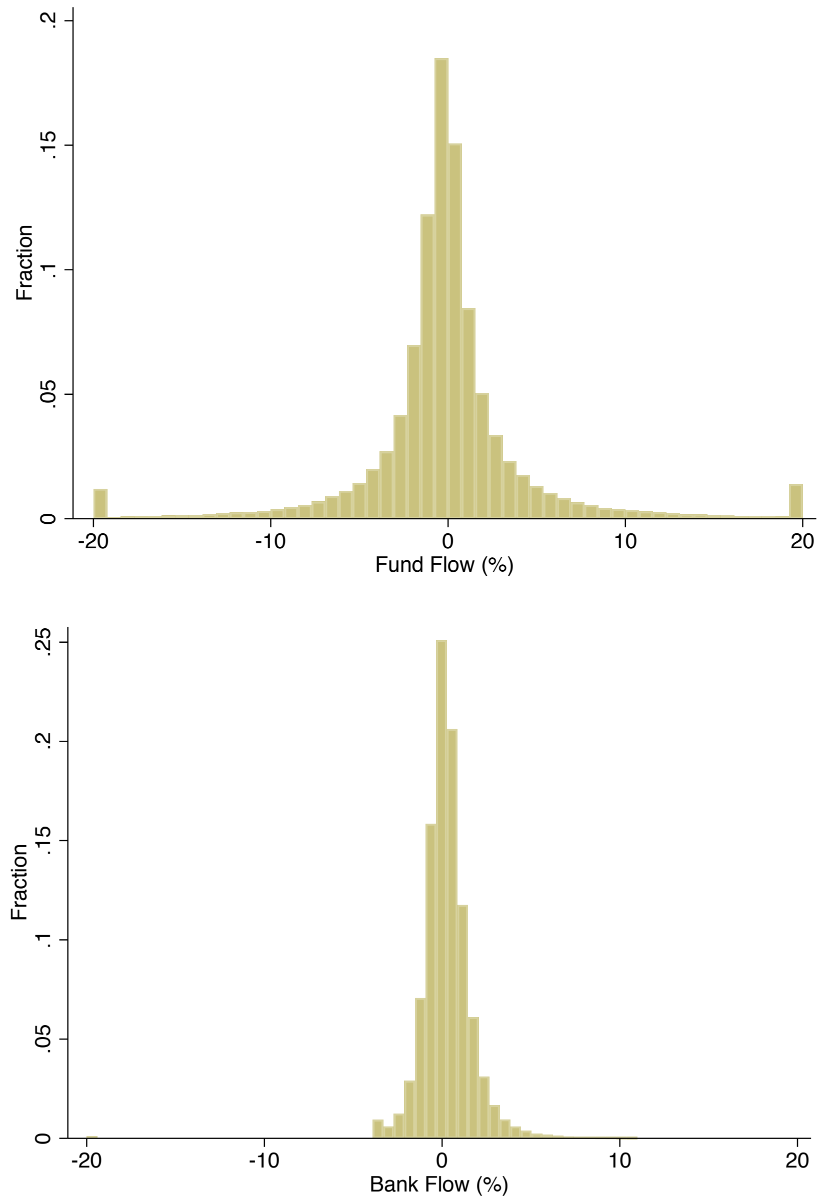


Figure 7: Cross-section of Bank and Fund Liquidity Provision

This graph plots the distribution of average commercial bank and bond mutual fund LPIs in the cross-section. The LPI for each bank and fund is calculated as the average LPI over the sample period from 2011 to 2017. For presentation purposes, the distribution is truncated at 0.4, where observations in the tail are assigned to the last bin.

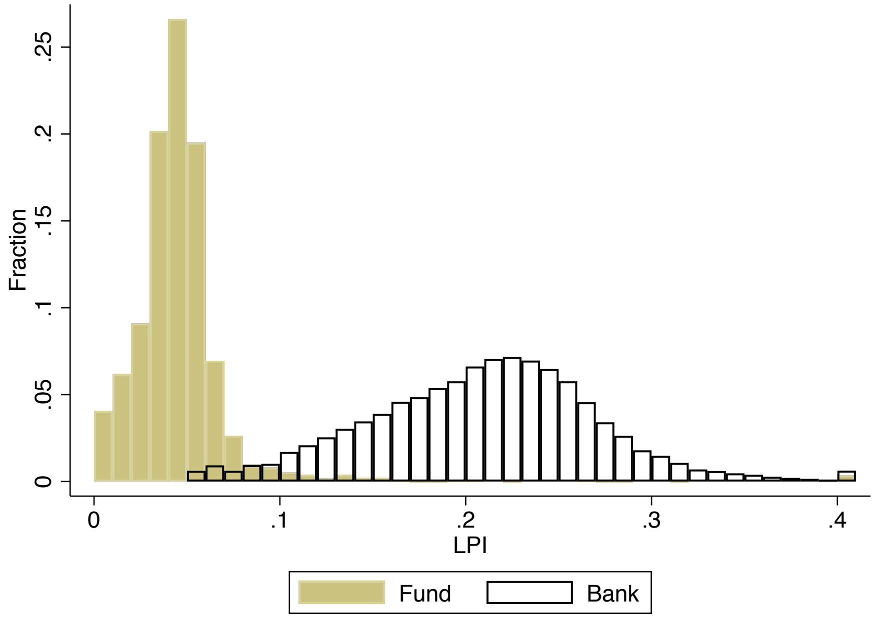
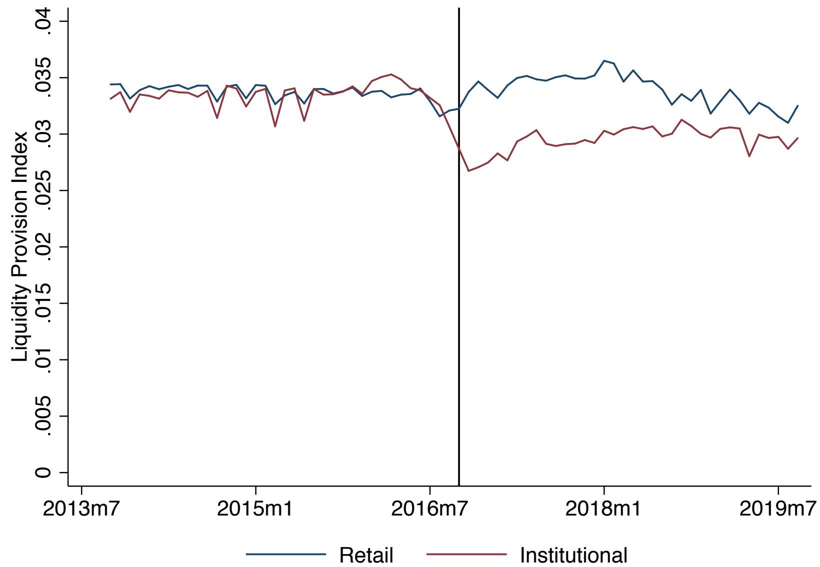


Figure 8: MMF Liquidity Provision and Floating NAVs

This graph plots the average LPIs of institutional and retail prime MMFs from September 2013 to September 2019. The vertical line marks the implementation of the Money Market Reform, which requires institutional prime funds to report floating NAVs. The upper panel covers the full sample, whereas the lower panel restricts fund share-classes that appear at least once in both the pre- and post-reform period.



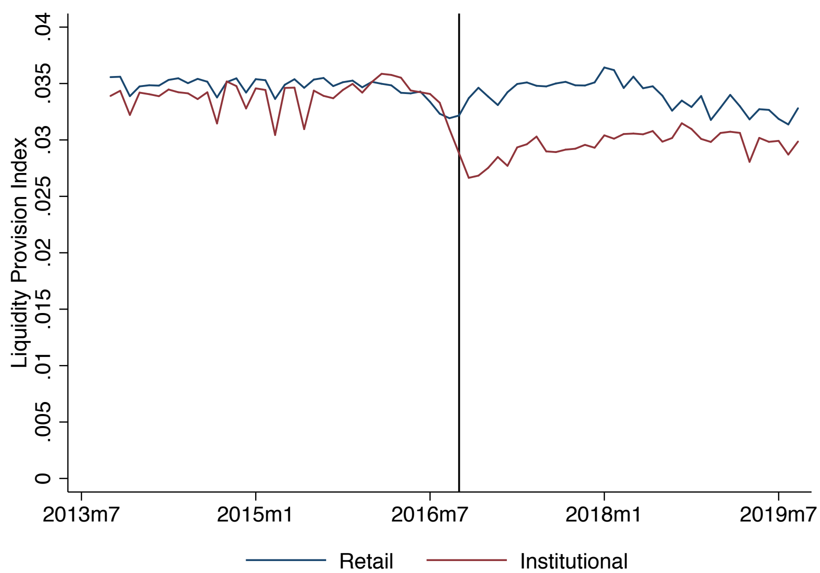


Figure 9: Sensitivity of Bank Liquidity Provision

This graph plots the average LPI of uninsured bank deposits against the ratio of insured deposits over total deposits at the bank level. Bank-level LPIs and insured deposit ratios are averaged over the sample period from 2011 to 2017. The upper panel is a univariate binned plot, while the lower panel also controls for log(assets), equity ratio, and the ratio of non-deposit liabilities over total assets.

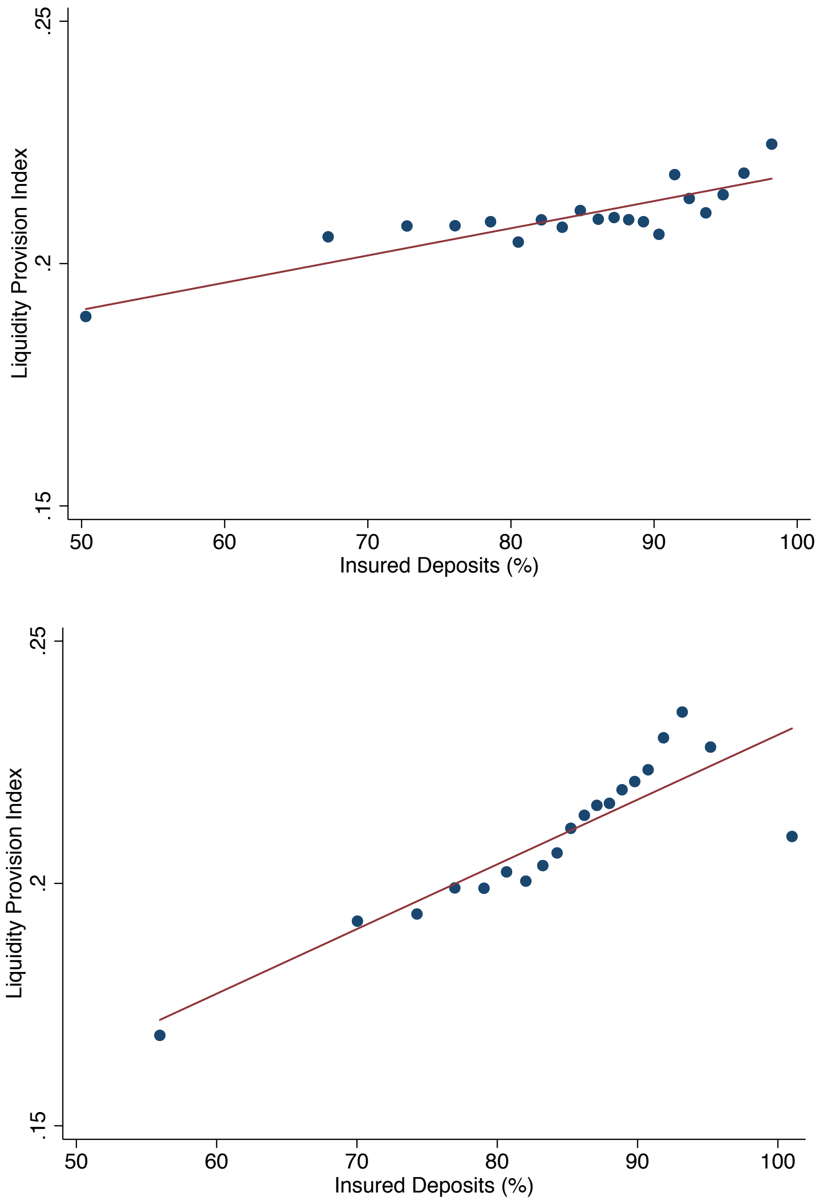
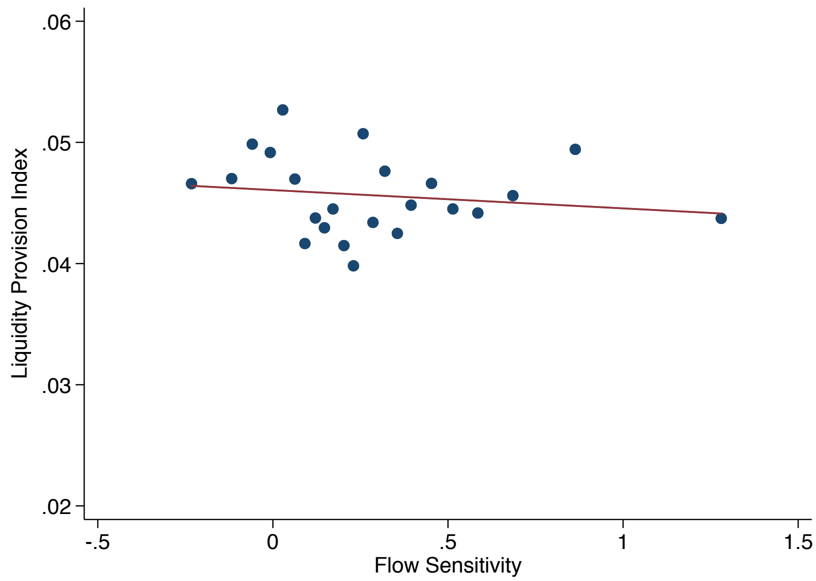


Figure 10: Sensitivity of Fund Liquidity Provision

This graph plots average LPI against fund flow sensitivity at the fund level for bond mutual funds. Fund-level LPIs are averaged over the sample period from 2011 to 2017. Fund flow sensitivity is obtained by regressing fund flows against lagged fund returns from 2011 to 2017. The upper panel is a univariate binned plot, while the lower panel also controls for log(fund age), log(assets), and expense ratio.



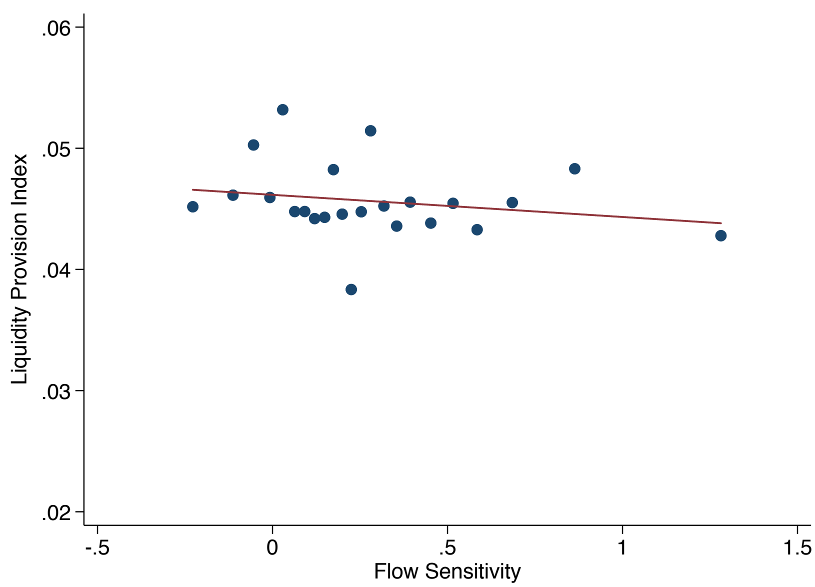


Figure 11: Bank and Fund LPI from 2011 to 2017

This graph plots the average LPI for commercial banks and bond mutual funds from 2011 to 2017. We first calculate the LPI for each bank and fund in each quarter and then plot the asset-size weighted LPI from 2011 to 2017.

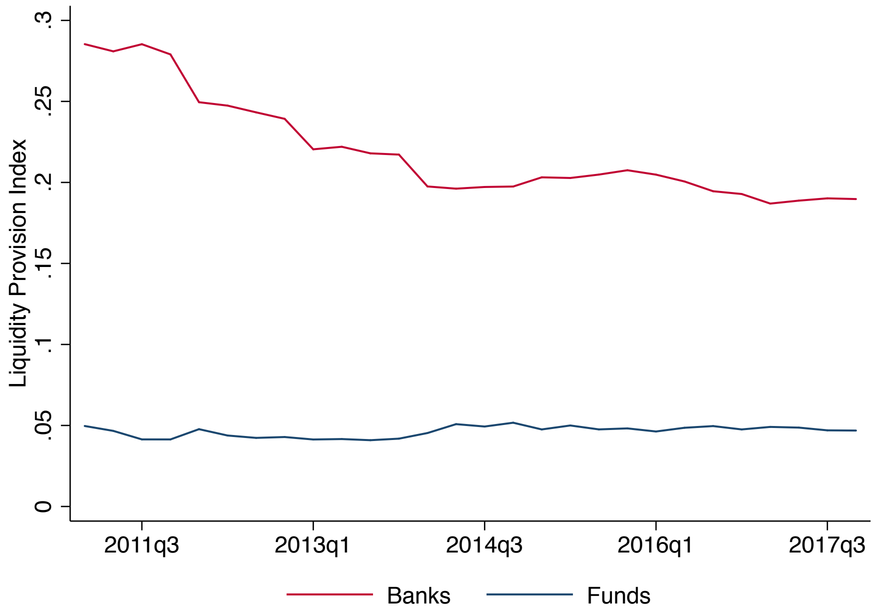


Figure 12: Liquidity Provision Pre- and Post-QE and LCR

This graph illustrates the contract payment of bank deposits when a given percentage of bank assets have been withdrawn before and after the implementation of QE (LCR). The liquidation value follows the blue line pre QE (LCR) and shifts to the red line post QE (LCR).

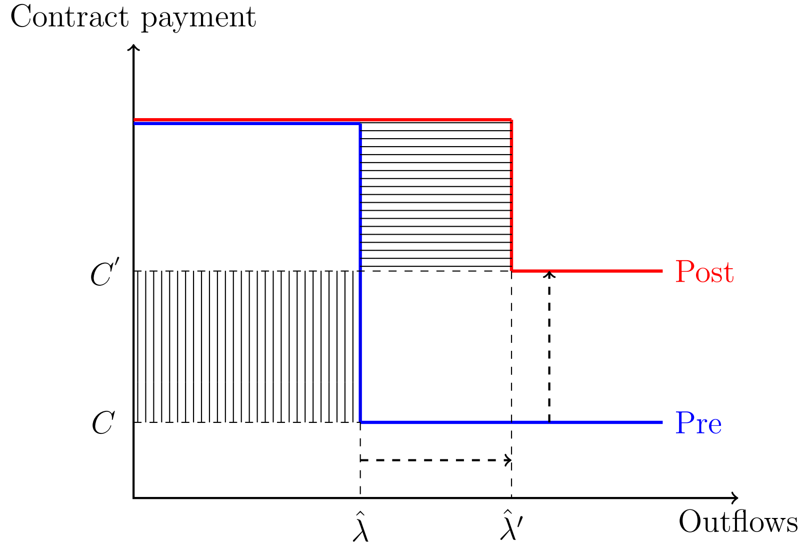


Figure 13: Bank Liquidity Provision and Excess Reserves

This graph plots the LPI for commercial banks by reserve uptake quartile (left axis) and the aggregate volume of excess reserves (right axis) from 2011 to 2017. Reserve uptake is measured as the change in reserves as a fraction of total assets from 2011Q1 to 2014Q3, when reserve levels peak. We plot the median LPI in each quartile normalized by its initial value in 2011Q1.

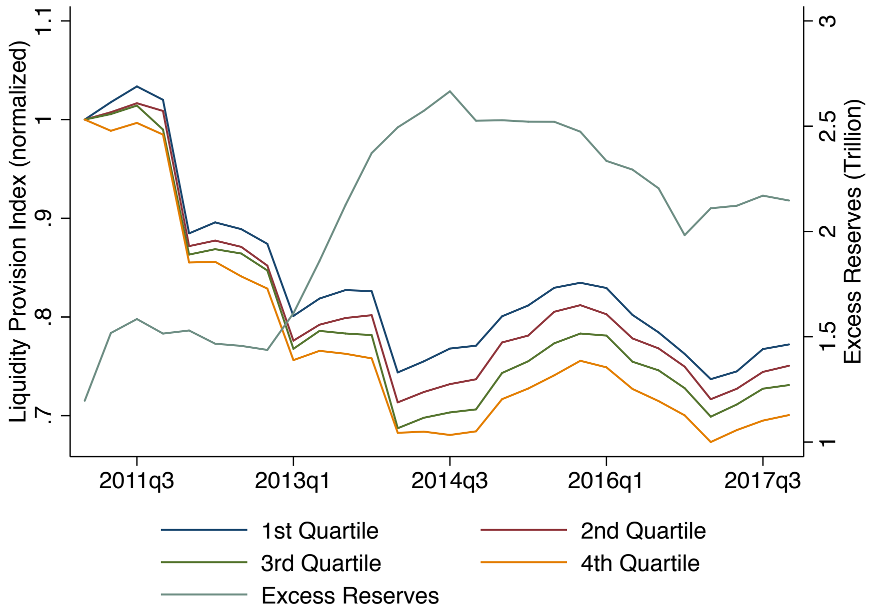


Figure 14: Bank Liquidity Provision and the Liquidity Coverage Ratio

This graph plots the LPI for commercial banks by asset size groups for the Liquidity Coverage Ratio. Banks are sorted by asset size into those above $250 billion, between $50 and $250 billion, and below $50 billion. We plot the median LPI in each asset group normalized by its initial value in 2011Q1.

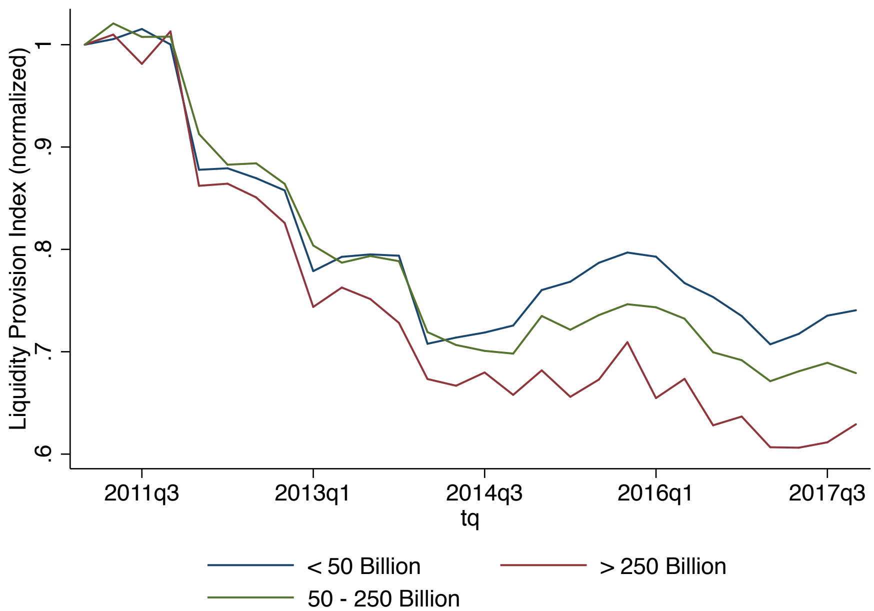


Table 3: Summary Statistics

This table provides summary statistics for the banks and fixed-income funds that appeared at least once in our sample from 2011 to 2017. Total Assets is the average asset size. The remaining rows denote portfolio holdings in percentages of total assets.

	(1) Funds	(2) Banks
Total Assets ($ billion)	1.57	2.36
Cash and Reserves	3.22	11.45
Treasury and Agency Bonds	14.61	6.50
CMO and ABS	11.31	8.18
Municipal Bonds	32.07	6.69
Corporate Bonds	32.26	0.69
Commercial Loans	0.35	12.96
Other Loans	0.00	47.69
Other	6.18	5.84

Table 4: Bank LPI and Insured Deposits Ratios

This table shows the relationship between bank LPI and the ratio of insured deposits. Control variables include the ratio of non-deposit liabilities and bank size measured by log(assets). Bank LPI and all other variables are averaged over the sample period from 2011 to 2017. Note that the constant term in column (2) represents the expected LPI of banks without insured deposits and non-deposit liabilities.

	(1) LPI	(2) LPI	(3) LPI
Insured Deposits Ratio	0.055***	0.064***	0.144***
	[0.006]	[0.006]	[0.006]
Non-deposits Ratio		0.052***	0.034***
		[0.008]	[0.008]
Log(assets)			0.017***
			[0.001]
Constant	0.164***	0.147***	-0.127***
	[0.005]	[0.006]	[0.010]
Observations	7535	7535	7535
Adj. R-squared	0.01	0.02	0.14

Table 5: The Effect of Floating NAV on MMF Liquidity Provision

This table shows the effect of the 2016 Money Market Reform on liquidity provision by institutional prime funds versus retail prime funds. *Institutional Fund* is a dummy variable for the treatment group. *Post Reform* is an indicator variable for the treatment period. For columns (1) to (2), the treatment period begins with the official implementation date of the reform in October 2016. Columns (3) and (4) repeat the analysis in columns (1) and (2) but exclude flows around the implementation period from August to October 2016. Columns (5) and (6) account for potential anticipation effects by setting the treatment period to begin one year earlier in October 2015. Columns (2), (4), and (6) restrict the sample to the set of fund share classes that appear in both the pre and post period, that is, those funds which survived the reform. The dependent variable is share-class level LPI averaged for both the pre and post period. Standard errors are clustered at the share-class level.

	(1) LPI	(2) LPI	(3) LPI	(4) LPI	(5) LPI	(6) LPI
Post Reform	-0.0003	-0.0011***	-0.0007	-0.0015***	-0.0009*	-0.0014***
	[0.0004]	[0.0004]	[0.0007]	[0.0005]	[0.0006]	[0.0003]
Inst Fund	-0.0013***	-0.0020***	-0.0002	-0.0006	-0.0006	-0.0011**
	[0.0005]	[0.0005]	[0.0006]	[0.0005]	[0.0006]	[0.0004]
Post Reform * Inst Fund	-0.0040***	-0.0032***	-0.0035***	-0.0033***	-0.0042***	-0.0044***
	[0.0007]	[0.0007]	[0.0009]	[0.0007]	[0.0007]	[0.0005]
Constant	0.0225***	0.0230***	0.0334***	0.0344***	0.0337***	0.0346***
	[0.0003]	[0.0003]	[0.0005]	[0.0004]	[0.0005]	[0.0003]
Observations	649	439	666	470	744	622
Adj. R-squared	0.17	0.24	0.07	0.13	0.15	0.27

References

Allen, Franklin and Douglas Gale (1998). "Optimal Financial Crises." *Journal of Finance*, 53: 1245-1284.

Allen, Franklin and Douglas Gale (2004). "Financial Intermediaries and Markets." *Econometrica* , 72.4: 1023-1061.

Allen, Franklin, Elena Carletti, Itay Goldstein and Agnese Leonello (2018). "Government Guarantees and Financial Stability." *Journal of Economic Theory*, 177: 518-557.

Bai, Jennie, Arvind Krishnamurthy and Charles-Henri Weymuller (2018). "Measuring Liquidity Mismatch in the Banking Sector." *Journal of Finance*, 73: 51-93.

Berger, Allen and Christa, Bouwman (2009). "Bank Liquidity Creation." *Review of Financial Studies*, 22: 3779-3837.

Brunnermeier, Markus, Gary Gorton and Arvind Krishnamurthy (2012). "Risk Topography." *NBER Macroeconomics Annual*, 26: 149-176.

Chen, Qi, Itay Goldstein and Wei Jiang (2010). "Payoff Complementarities and Financial Fragility: Evidence from Mutual Fund Outflows." *Journal of Financial Economics*, 97: 239-262.

Chernenko, Sergey and Adi Sunderam (2017). "Liquidity Transformation in Asset Management: Evidence from the Cash Holdings of Mutual Funds." Working Paper.

Dang, Tri Vi, Gary Gorton, Bengt Holmström and Guillermo Ordoñez (2017). "Banks as Secret Keepers." *American Economic Review*, 107: 1005-1029.

Diamond, Douglas and Philip Dybvig (1983). "Bank Runs, Deposit Insurance, and Liquidity." *Journal of Political Economy*, 91: 401-419.

Diamond, Douglas and Raghuram Rajan (2001). "Liquidity Risk, Liquidity Creation, and Financial Fragility: A Theory of Banking." *Journal of Political Economy*, 109: 287-327.

Drechsler, Itamar, Alexi Savov and Philipp Schnabl (2017). "The Deposits Channel of Monetary Policy." *Quarterly Journal of Economics*, 132: 1819-1876.

Duffie, Darrell (2010). "Presidential Address: Asset Price Dynamics with Slow-Moving Capital." *Journal of Finance*, 65: 1237-1267.

Egan, Mark, Ali Hortacsu and Gregor Matvos (2017). "Deposit Competition and Financial Fragility: Evidence from the US Banking Sector.." *American Economic Review*, 107: 169-216.

Farhi, Emmanuel, Mikhail Golosov and Aleh Tsyvinski (2009). “A Theory of Liquidity and Regulation of Financial Intermediation.” *Review of Economic Studies*, 76: 973-992.

Gorton, Gary and Andrew Metrick (2010). “Regulating the Shadow Banking System.” *Brookings Papers on Economic Activity*, 41: 261-312.

Goldstein, Itay and Ady Pauzner (2005). “Demand Deposit Contracts and the Probability of Bank Runs.” *Journal of Finance*, 60: 1293-1328.

Goldstein, Itay, Hao Jiang and David Ng (2017). “Investor Flows and Fragility in Corporate Bond Funds.” *Journal of Financial Economics*, 126: 592-613.

Green, Edward and Ping Lin (2003). “Implementing Efficient Allocations in A model of Financial Intermediation.” *Journal of Economic Theory*, 109: 1-23.

Hanson, Samuel, Andrei Shleifer, Jeremy Stein and Robert Vishny (2015). “Banks as patient fixed-income investors.” *Journal of Financial Economics*, 117: 449-469.

He, Zhiguo and Wei Xiong (2012). “Rollover Risk and Credit Risk.” *Journal of Finance*, 67: 391-430.

Jacklin, Charles (1987). “Demand Deposits, Trading Restrictions and Risk Sharing.” in: Prescott, E. D., Wallace, N. (Eds.), *Contractual Arrangements for Intertemporal Trade*. Minnesota: University of Minnesota Press.

Jin, Dunhong, Marcin Kacperczyk, Bige Kahraman and Felix Suntheim (2020). “Swing Pricing and Fragility in Open-end Mutual Funds.” Working Paper.

Kacperczyk, Marcin and Philipp Schnabl (2013). “How Safe Are Money Market Funds?” *Quarterly Journal of Economics*, 128: 1073-1122.

Kashyap, Anil, Raghuram Rajan and Jeremy Stein (2002). “An Explanation for the Coexistence of Lending and Deposit Taking.” *Journal of Finance*, 57: 33-73.

Li, Wenhao, Yiming Ma and Yang Zhao (2020). “The Passthrough of Treasury Supply to Bank Deposit Funding.” Working Paper.

Ma, Yiming, Kairong Xiao and Yao Zeng (2020). “Mutual Fund Liquidity Transformation and Reverse Flight to Liquidity.” Working Paper.

Nagel, Stefan (2016). “The Liquidity Premium of Near-Money Assets.” *Quarterly Journal of Economics*, 131: 1927-1971.

Shleifer, Andrei and Robert Vishny (1992). "Liquidation Values and Debt Capacity: A Market Equilibrium Approach." *Journal of Finance*, 47: 1343-1366.

Stein, Jeremy (2012). "Monetary Policy as Financial-Stability Regulation." *Quarterly Journal of Economics*, 127: 57-95.

Sunderam, Adi (2015). "Money Creation and the Shadow Banking System." *Review of Financial Studies*, 28: 939-977.

Xiao, Kairong (2019). "Monetary Transmission through Shadow Banks." *Review of Financial Studies*, forthcoming.

Zeng, Yao (2017). "A Dynamic Theory of Mutual Fund Runs and Liquidity Management." Working Paper.

Appendix

A Autarky, Liquidation Value, and Liquidity Provision

In this appendix, we first characterize an autarky outcome and show that it is tightly linked to the definition of liquidation value in Definition 1. Then we show that our definition of liquidity provision, that is, Definition 2, is tightly related but preferable to an alternative definition based on autarky.

Consider agents that live in autarky without access to financial intermediaries. They choose their portfolio $(x_{a,0}, y_{a,0})$ at $t = 0$, where $x_{a,0}$ is the amount of cash stored in the storage and $y_{a,0}$ the amount of projects invested before knowing their types.

At the beginning of $t = 1$, the agent learns her type and receives the signal s_i. An early agent always liquidates all her projects regardless of s_i and consumes the cash:

$$\begin{aligned} c_{a,1} &= x_{a,0} + y_{a,0} - \frac{\phi}{2} y_{a,0}^2 \\ &= 1 - \frac{\phi}{2} y_{a,0}^2 \,, \end{aligned} \tag{A.1}$$

where $(x_{a,0}, y_{a,0})$ represents her portfolio choice at $t = 0$. An important observation from (A.1) is that what an early agent obtains in autarky is the liquidation value of her portfolio, as in Definition 1.

Late agents, however, will only liquidate the project and store the proceeds as cash if they expect a sufficiently poor performance after observing s_i. Formally, conditional on s_i, a late agent's optimal portfolio choice problem at $t = 1$ is given by

$$\max_{l_a \geqslant 0} E\left[u(c_{a,2}) | s_i\right] \,,$$

where

$$c_{a,2} = x_{a,1}(1 - \kappa) + y_{a,1} R \,, \tag{A.2}$$

where in turn

$$\begin{cases} x_{a,1} = x_{a,0} + l_a - \frac{\phi}{2} l_a^2 \,, \\ y_{a,1} = y_{a,0} - l_a \end{cases}$$

are the late agent's position at the end of $t = 1$.[30] It is easy to show that, given the date-0 position $(x_{a,0}, y_{a,0})$ and the signal R received at $t = 1$, an early agent always liquidates all her illiquid asset

[30] One unit of cash invested in the storage yields $1 - \kappa$ units of cash since one unit of late agents is using the storage.

holdings $y_{a,0}$ regardless of R, while a late agent optimally liquidates

$$l_a = \frac{1}{\phi}\left(1 - \frac{R}{1-\kappa}\right) \tag{A.3}$$

units of project at $t = 1$, subject to $l_a \geq 0$ and $l_a \leq y_{a,0}$.

Taking the decisions at $t = 1$ into account, agents optimally choose their portfolios at $t = 0$ before knowing their type:

$$\max_{x_{a,0}} E[\pi u(c_{a,1}) + (1-\pi)u(c_{a,2})] ,$$

subject to conditions (A.1), (A.2), (A.3). Without fully solving for the equilibrium, it is straightforward to see that the optimal $c^*_{a,1}$ is strictly lower than 1 when the agent invests in a positive amount of projects at $t = 0$, that is, $y^*_{a,0} > 0$, and hence, $x^*_{a,0} < 1$.

The analysis above about the autarky allows us to link our definition of provision, as in Definition 2, to an alternative definition in which one compares the t-1 consumption level promised by an intermediary and the autarky outcome. Formally, by the Principle of Optimization, we have:

Proposition 6. *If an intermediary $k \in \{b, f\}$ provides liquidity in the sense that $E[c^*_{k,1}] - c^*_{a,1} > 0$, it must provide liquidity in the sense that $E[c^*_{k,1}] - c_1(x^*_{k,0}, y^*_{k,0}) > 0$.*

Proposition 6 suggests that liquidity provision based on Definition 2 is a necessary condition for liquidity provision under the autarky-based alternative definition. Therefore, we use Definition 2 as our preferred definition of liquidity provision because it is conceptually tightly linked to the insight of Diamond and Dybvig (1983) and because all the inputs needed to calculate it are empirically observable.

B Deposit Insurance in LPI

This appendix clarifies two points regarding the role of deposit insurance in the construction of bank LPI. First, we show that bank LPI captures the liquidity provision capacity of uninsured deposits rather than insured deposits. Second, we illustrate that the LPI construction for uninsured bank deposits is still valid if the bank is partially funded by insured deposits.

We first note that our bank LPI construction already excluded the direct effect of deposit insurance because the contract payment of bank debt drops to the liquidation value of the underlying bank asset portfolio after the bank defaults (as shown in the lower panel of Figure 5). The contract payment of insured deposits would remain at the promised deposit value because the promised deposit value is honored regardless of bank default. Hence, our LPI captures liquidity provision by uninsured rather than insured deposits.

We further show that the LPI construction for uninsured bank deposits remains valid in the presence of deposit insurance. According to the theoretical model of Allen, Carletti, Goldstein and Leonello (2018), a higher proportion of insured deposits leads to: 1) a lower run threshold (i.e., a lower run probability), and consequently, 2) a less liquid bank portfolio.[31] Intuitively, having more insured deposits renders the bank safer as a whole and encourages it to hold a less liquid portfolio. The LPI construction captures these two equilibrium outcomes by incorporating 1) the empirical distribution of outflows and 2) the actual bank asset portfolio. Thus, any spillover effects of deposit insurance on the liquidity provision by uninsured deposits will be picked up by the LPI and will not invalidate the bank LPI construction.

To illustrate the logic above, consider a hypothetical bank that is fully financed by uninsured deposits. Suppose it optimally holds 20% cash and 80% of illiquid loans, where the loans, if liquidated at short notice, can be only recovered at 60% of their fair value (i.e., the haircut is 40%). Also let the distribution of endogenous outflows be a triangular distribution with a density function of $f(\lambda) = 2\lambda$. In this setting, the liquidation value of the asset portfolio is $1 \times 0.2 + 0.6 \times 0.8 = 0.68$. Thus, a dollar invested in uninsured deposits generates an LPI of $\int_0^{0.68}(1 - 0.68)2\lambda d\lambda = 0.1479$.

Now suppose that the bank is funded by 50% insured deposits and 50% uninsured deposits instead. Consistent with the theoretical argument above, this bank would have less volatile outflows, and consequently, a less liquid asset portfolio. For example, suppose that the empirical distribution of outflows becomes uniform on [0,1] and that the optimal asset portfolio becomes 10% cash and 90% loans. In this setting, the liquidation value of the asset portfolio is $1 \times 0.1 + 0.6 \times 0.9 = 0.64$. and a dollar invested in uninsured deposits generates an LPI of $(1 - 0.64) \times 0.64 = 0.2304$. Compared to the case without insured deposits, the LPI of uninsured deposits increased by $0.2304 - 0.1479 = 0.0825$, which suggests that deposit insurance indirectly contributes 0.0825 towards the LPI of uninsured deposits. In other words, bank LPI captures liquidity provision by uninsured bank deposits in practice, which may be influenced by the presence of insured deposits

In addition to illustrating the validity of the LPI construction in the presence of deposit insurance, the example above suggests that deposit insurance indirectly improves the liquidity provision capacity of bank's uninsured deposits. To further isolate the effect of debt versus equity in liquidity provision, we perform a number of additional tests in Section 4.3 and 4.4.

[31] Similar to our baseline model, Allen, Carletti, Goldstein and Leonello (2018) build a global-games based model with a different focus on the interplay between general government guarantees (including deposit insurance) and bank runs.

C Price-Impact Adjusted Haircuts

This appendix presents LPI estimates considering intermediaries' own price impact on haircuts. In our baseline estimates, haircuts for different asset classes are taken as the average market haircuts in each quarter. Nevertheless, the liquidation of assets by funds and banks may create price pressure on the effective haircuts they incur.

We do not observe the actual liquidation discounts for each fund and bank but have some information about the distribution of haircuts for each asset class in each quarter. Therefore, we resort to assigning haircuts from the distribution to each fund depending on the size of their outflows and hence liquidations. Funds with larger liquidation volumes induce larger price pressures and are mapped to have a larger haircut from the distribution.

In the data, we observe the 10th, 50th, and 90th percentile of haircuts for each security type j in each quarter t, i.e., $h_{j,t}(10)$, $h_{j,t}(50)$, and $h_{j,t}(90)$. Please refer to Table A2 for summary statistics. We also observe the cross-sectional distribution of outflows $\lambda_{i,t}$ in each quarter. Assuming that the correspondence between the percentile of haircuts and the percentile of outflows is piece-wise linear, we can establish a correspondence between these two distributions by linear interpolation. Specifically, if fund i has an outflow ranked at the pth percentile in the distribution of outflows at time t, we estimate the effective haircut it incurs to be $\hat{h}_{i,j,t}(p)$:

$$\begin{cases} \hat{h}_{i,j,t}(p) = h_{j,t}(10) & \text{if} \quad p < 10\,, \\ \hat{h}_{i,j,t}(p) = h_{j,t}(10) + (p-10)\frac{h_{j,t}(50)-h_{j,t}(10)}{40} & \text{if} \quad 10 \leq p < 50, \\ \hat{h}_{i,j,t}(p) = h_{j,t}(50) + (p-50)\frac{h_{j,t}(90)-h_{j,t}(50)}{40} & \text{if} \quad 50 \leq p \leq 90, \\ \hat{h}_{i,j,t}(p) = h_{j,t}(90) & \text{if} \quad p > 90 \end{cases}$$

For instance, if a fund experiences a medium outflow, then the haircuts that it incurs when liquidating an asset is the medium haircut. We do not observe haircut distributions for loans and continue to apply the median haircuts as in the baseline estimate.[32]

We use the price-pressure adjusted haircuts to calculate the expected contract payment for each fund $\int(1-H_t(\hat{\lambda}_i, Y_{it}))dF(\hat{\lambda}_i)$. Then, we deduct the direct liquidation value of the fund's assets $(1-H_t(1, Y_{it}))$ to obtain the LPI. We continue to apply the median haircuts for the direct liquidation value because it

[32] Real estate loan and consumer loan haircuts are determined by regulators and are not subject to price pressure as in private markets. Commercial loans are traded in secondary markets and are subject to price pressure. Unfortunately, we were unable to obtain the relevant data series from the LSTA. This data gap mainly affects banks, for whom we will show the LPI estimates to be relatively insensitive to haircut adjustments.

represents the value obtained when investors liquidate assets at short notice absent the intermediary.[33] On average, the LPI for funds with haircut-adjustments is 3.8 cents per $1, which is slightly below the baseline estimates at 4.6 cents per $1.

For banks, haircut-adjustments affect the LPI by changing the run threshold. When haircuts are larger, a lower level of outflows will trigger the run-threshold than when haircuts are smaller. In the data, we find that the adjustment in the run-threshold of bank contract payment curves due to the new haircuts do not alter the run probability of banks in our sample. Intuitively, this is because bank outflows are distributed either below 5% or are way past the threshold in the case of default. Hence, the average bank LPI remains at 21.9 cents per $1.

Figures A2 and A3 plot the cross-section and time-series of LPI estimates with price-pressure adjusted haircuts. The depicted trends align with those of the baseline estimates, confirming the robustness of our results.

D Proofs

Proof of Proposition 1. Denote the run threshold by $R' = R(\theta')$, that is, if agent i observes a private signal $s_i < \theta'$ she runs; otherwise she stays. Then the population of agents who runs, λ_b, can be written as

$$\lambda_b(\theta, \theta') = \begin{cases} 1 & \text{if } \theta \leq \theta' - \varepsilon \\ \pi + (1-\pi)\left(\frac{\theta' - \theta + \varepsilon}{2\varepsilon}\right) & \text{if } \theta' - \varepsilon < \theta \leq \theta' + \varepsilon \\ \pi & \text{if } \theta > \theta' + \varepsilon \end{cases}.$$

Let $v(R(\theta), \lambda_b)$ be the difference of utilities between staying and running, then

$$v(R(\theta), \lambda_b) = \begin{cases} u\left(\frac{x_{b,0} - \lambda_b c_{b,1} + y_{b,0} R(\theta)}{1-\lambda_b}\right) - u(c_{b,1}(1 - \kappa(\lambda_b - \pi))) & \text{if } \pi \leq \lambda_b < \frac{x_{b,0}}{c_{b,1}} \\ u\left(\frac{(y_{b,0} - l_b(\lambda_b, c_{b,1}))R(\theta)}{1-\lambda_b}\right) - u(c_{b,1}(1 - \kappa(\lambda_b - \pi))) & \text{if } \frac{x_{b,0}}{c_{b,1}} \leq \lambda_b < \frac{1 - \frac{\phi}{2} y_{b,0}^2}{c_{b,1}} \\ -q(\lambda_b)\, u(c_{b,1}(1 - \kappa(\lambda_b - \pi))) & \text{if } \frac{1 - \frac{\phi}{2} y_{b,0}^2}{c_{b,1}} < \lambda_b \leq 1 \end{cases},$$

where $l_b(\lambda_b, c_{b,1})$ satisfies $\lambda_b c_{b,1} = x_{b,0} + l_b - \frac{\phi}{2} l_b^2$. If agent i observes signal s_i, given that other agents use the threshold strategy, she will run if $\int_{s_i-\varepsilon}^{s_i+\varepsilon} v(R(\theta), \lambda_b(\theta, \theta'))\, d\theta > 0$; or stay otherwise.

To prove that there exists a unique run threshold R^*, we need to prove that there is a unique θ^* such that if $\theta' = \theta^*$, the agent who observes signal $s_i = \theta' = \theta^*$ is indifferent between run and stay.

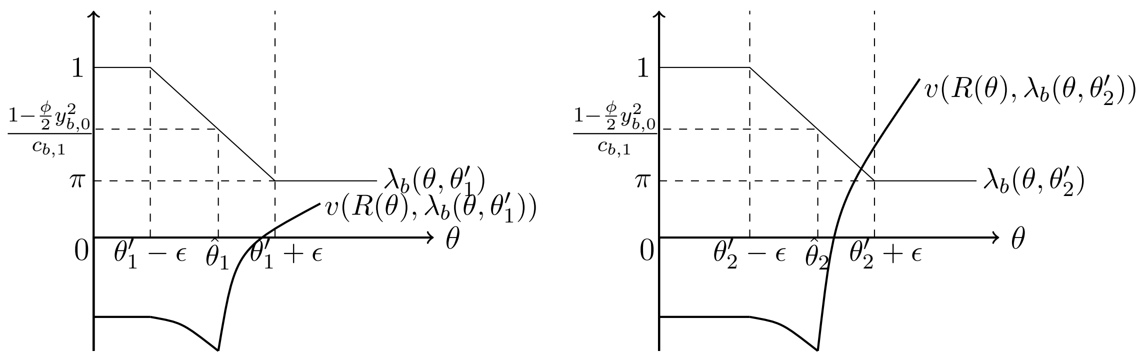
[33] Implicitly, this assumes that individual investors do not have enough volume to create price pressure.

That is,

$$V\left(\theta^*\right) \equiv \int_{\theta^*-\varepsilon}^{\theta^*+\varepsilon} v\left(R\left(\theta\right), \lambda_b\left(\theta, \theta^*\right)\right) d\theta = 0.$$

The graph of $v\left(R\left(\theta\right), \lambda_b\left(\theta, \theta'\right)\right)$ is depicted in Figure A1, where $\hat{\theta}$ satisfies $\lambda_b\left(\hat{\theta}, \theta'\right) = \frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}$. It is easy to check that v is constant on $(0, \theta' - \varepsilon)$, decreasing on $\left(\theta' - \varepsilon, \hat{\theta}\right)$, and increasing on $(\hat{\theta}, 1)$. Figure A1 also illustrates how v changes when θ' increases. As θ' increases, the integral of v on $(0, \theta' - \varepsilon)$ remains the same since v does not directly depend on θ in this interval and the length of the interval is constant given ε; on $(\hat{\theta}, \theta' + \varepsilon)$, for any given λ_b, if θ' goes up, v increases because all θ in this interval goes up. Thus the integral on $(\hat{\theta}, \theta' + \varepsilon)$ increases. In summary, $V\left(\theta'\right)$ is increasing in θ'.

Figure A1: The graph of $v(R(\theta), \lambda_b(\theta, \theta'))$



Since $G(\cdot)$ is supported on $[0, +\infty)$, $R \to +\infty$ when $\theta \to 1$. That is, $\lim_{\theta' \to 1} V\left(\theta'\right) = +\infty$. Furthermore, $V(0) < 0$. Then by the intermediate value theorem, there exists θ^* such that $V\left(\theta^*\right) = 0$. The uniqueness of then θ^* follows from the monotonicity of $V(\cdot)$ and so does R^*. Since θ^* is unique, it can then be determined by the Laplacian equation:

$$\int_0^1 v(R(\theta^*), \lambda) d\lambda = 0\,. \tag{D.1}$$

We finally note that the threshold R^* must be larger than the lower dominance region $\underline{R}$. $\underline{R}$ is pinned down by letting agents be indifferent between run and stay if there is no withdrawal and the storage is the least efficient, that is, $u\left(\frac{x_{b,1}+y_{b,1}\underline{R}}{1-\pi}\right) = u\left(c_{b,1}(1 - \kappa\left(1 - \pi\right))\right)$. This gives

$$\underline{R} = \frac{\left(\pi + \left(1 - \pi\right)\left(1 - \kappa(1 - \pi)\right)\right) c_{b,1} - x_{b,0}}{1 - x_{b,0}}\,, \tag{D.2}$$

which can be either larger or smaller than 1 because the liquidation value is strictly smaller than 1 by Lemma 1 and thus $c_{b,1}$ can be either larger or smaller than 1 in equilibrium. This completes the proof. ∎

Proof of Proposition 2. In the proof of Proposition 1, we know that $V(\cdot)$ is increasing and θ^* is pinned down by $V(\theta^*) = 0$. Thus, to prove that $\frac{d\theta^*}{dc_{b,1}} > 0$, it is sufficient to prove that $V(\theta')$ is decreasing in $c_{b,1}$. Note that when $\theta \in (\theta' - \varepsilon, \theta' + \varepsilon)$, $\pi + (1-\pi)\left(\frac{\theta' - \theta + \varepsilon}{2\varepsilon}\right) = \lambda_b$, so we have $d\theta = -\frac{2\varepsilon}{1-\pi} d\lambda_b$. Then $V(\theta')$ can be rewritten as

$$
\begin{aligned}
V(\theta') &= \int_{\pi}^{1} v\left(R\left(\theta' + \varepsilon\left(1 - 2\frac{\lambda - \pi}{1-\pi}\right)\right), \lambda_b\right) \frac{2\varepsilon}{1-\pi} d\lambda_b \\
&= \int_{\pi}^{\frac{x_{b,0}}{c_{b,1}}} \frac{2\varepsilon}{1-\pi}\left(u\left(\frac{x_{b,0} - \lambda_b c_{b,1} + y_{b,0} R\left(\theta' + \varepsilon\left(1 - 2\frac{\lambda-\pi}{1-\pi}\right)\right)}{1-\lambda_b}\right) - u\left(c_{b,1}(1 - \kappa(\lambda_b - \pi))\right)\right) d\lambda_b \\
&+ \int_{\frac{x_{b,0}}{c_{b,1}}}^{\frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}} \frac{2\varepsilon}{1-\pi}\left(u\left(\frac{(y_{b,0} - l_b(\lambda_b, c_{b,1}))\, R\left(\theta' + \varepsilon\left(1 - 2\frac{\lambda-\pi}{1-\pi}\right)\right)}{1-\lambda_b}\right) - u\left(c_{b,1}(1 - \kappa(\lambda_b - \pi))\right)\right) d\lambda_b \\
&- \int_{\frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}}^{1} \frac{2\varepsilon}{1-\pi} q(\lambda_b)\, u\left(c_{b,1}(1 - \kappa(\lambda_b - \pi))\right) d\lambda_b.
\end{aligned}
$$

Let $V = 0$, which pins down θ^* and thus R^*. And since V is continuous in ε, we can take the limit of the equation above at $\varepsilon \to 0$:

$$
\begin{aligned}
\int_{\pi}^{1} v(R(\theta^*), \lambda_b)\, d\lambda_b &= \int_{\pi}^{\frac{x_{b,0}}{c_{b,1}}} \left(u\left(\frac{x_{b,0} - \lambda_b c_{b,1} + y_{b,0} R(\theta')}{1-\lambda_b}\right) - u\left(c_{b,1}(1 - \kappa(\lambda_b - \pi))\right)\right) d\lambda_b \\
&+ \int_{\frac{x_{b,0}}{c_{b,1}}}^{\frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}} \left(u\left(\frac{(y_{b,0} - l_b(\lambda_b, c_{b,1}))\, R(\theta')}{1-\lambda_b}\right) - u\left(c_{b,1}(1 - \kappa(\lambda_b - \pi))\right)\right) d\lambda_b \quad \text{(D.3)} \\
&- \int_{\frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}}^{1} q(\lambda_b)\, u\left(c_{b,1}(1 - \kappa(\lambda_b - \pi))\right) d\lambda_b.
\end{aligned}
$$

Multiplying both sides of (D.3) with $c_{b,1}$ and taking derivative with respect to $c_{b,1}$, we have the derivative expressed by

$$
\begin{aligned}
& c_{b,1} \int_{\pi}^{\frac{x_{b,0}}{c_{b,1}}} \left(-\frac{\lambda_b}{1-\lambda_b} u'\left(\frac{x_{b,0} - \lambda_b c_{b,1} + y_{b,0} R(\theta^*)}{1-\lambda_b}\right) - (1 - \kappa(\lambda_b - \pi)) u'\left(c_{b,1}(1 - \kappa(\lambda_b - \pi))\right)\right) d\lambda_b \\
+ \quad & c_{b,1} \int_{\frac{x_{b,0}}{c_{b,1}}}^{\frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}} \left(-\frac{\frac{\partial l_b}{\partial c_{b,1}} R(\theta^*)}{1-\lambda_b} u'\left(\frac{(y_{b,0} - l_b(\lambda_b, c_{b,1}))\, R(\theta^*)}{1-\lambda_b}\right) - (1 - \kappa(\lambda_b - \pi)) u'\left(c_{b,1}(1 - \kappa(\lambda_b - \pi))\right)\right) d\lambda_b
\end{aligned}
$$

$$
\begin{aligned}
- & \int_{\frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}}^{1} \frac{1-\frac{\phi}{2}y_{b,0}^2}{\lambda_b}(1-\kappa(\lambda_b-\pi))u'\left(c_{b,1}(1-\kappa(\lambda_b-\pi))\right)d\lambda_b \\
+ & \int_{\pi}^{\frac{x_{b,0}}{c_{b,1}}} \left(u\left(\frac{x_{b,0}-\lambda_b c_{b,1}+y_{b,0}R(\theta^*)}{1-\lambda_b}\right) - u\left(c_{b,1}(1-\kappa(\lambda_b-\pi))\right)\right)d\lambda_b \\
+ & \int_{\frac{x_{b,0}}{c_{b,1}}}^{\frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}} \left(u\left(\frac{(y_{b,0}-l_b(\lambda_b,c_{b,1}))R(\theta^*)}{1-\lambda_b}\right) - u\left(c_{b,1}(1-\kappa(\lambda_b-\pi))\right)\right)d\lambda_b \\
- & \int_{\frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}}^{1} q(\lambda_b)\,u\left(c_{b,1}(1-\kappa(\lambda_b-\pi))\right)d\lambda_b\,,
\end{aligned}
$$

where $\frac{\partial l_b}{\partial c_{b,1}} = \frac{\lambda_b}{1-\phi l_b}$ and $\frac{\partial q}{\partial c_{b,1}} = -\frac{1-\phi y_{b,1}^2}{\lambda_b c^2} < 0$. Note that it suffices to prove that

$$
\begin{aligned}
& c_{b,1}\int_{\pi}^{\frac{x_{b,0}}{c_{b,1}}} -\frac{\lambda_b}{1-\lambda_b}u'\left(\frac{x_{b,0}-\lambda_b c_{b,1}+y_{b,0}R(\theta')}{1-\lambda_b}\right)d\lambda_b \\
+ \quad & c_{b,1}\int_{\frac{x_{b,0}}{c_{b,1}}}^{\frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}} -\frac{\frac{\partial l_b}{\partial c_{b,1}}R(\theta')}{1-\lambda_b}u'\left(\frac{(y_{b,0}-l_b(\lambda_b,c_{b,1}))R(\theta^*)}{1-\lambda_b}\right)d\lambda_b \\
+ \quad & \int_{\pi}^{\frac{x_{b,0}}{c_{b,1}}} u\left(\frac{x_{b,0}-\lambda_b c_{b,1}+y_{b,0}R(\theta')}{1-\lambda_b}\right)d\lambda_b \\
+ \quad & \int_{\frac{x_{b,0}}{c_{b,1}}}^{\frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}} u\left(\frac{(y_{b,0}-l_b(\lambda_b,c_{b,1}))R(\theta')}{1-\lambda_b}\right)d\lambda_b < 0\,.
\end{aligned} \tag{D.4}
$$

Integrating by parts, we have the LHS of (D.4) re-expressed as:

$$
\begin{aligned}
& -\left(\frac{c_{b,1}}{x_{b,0}-c_{b,1}+y_{b,0}R(\theta')}\frac{x_{b,0}}{c_{b,1}}\left(1-\frac{x_{b,0}}{c_{b,1}}\right)u\left(\frac{y_{b,0}R(\theta')}{1-\frac{x_{b,0}}{c_{b,1}}}\right)\right) \\
+ \quad & \frac{c_{b,1}}{x_{b,0}-c_{b,1}+y_{b,0}R(\theta')}\pi(1-\pi)\,u\left(\frac{x_{b,0}-\pi c_{b,1}+y_{b,0}R(\theta')}{1-\pi}\right) \\
+ \quad & \frac{1}{x_{b,0}-c_{b,1}+y_{b,0}R(\theta')}\int_{\pi}^{\frac{x_{b,0}}{c_{b,1}}}\left(x_{b,0}+y_{b,0}R(\theta')-2\lambda_b c_{b,1}\right)u\left(\frac{x_{b,0}-\lambda_b c_{b,1}+y_{b,0}R(\theta')}{1-\lambda_b}\right)d\lambda_b \\
+ \quad & c_{b,1}\frac{\left(1-\frac{x_{b,0}}{c_{b,1}}\right)\frac{x_{b,0}}{c_{b,1}}}{1-c_{b,1}}u\left(\frac{y_{b,0}R(\theta')}{1-\frac{x_{b,0}}{c_{b,1}}}\right) \\
+ \quad & \int_{\frac{x_{b,0}}{c_{b,1}}}^{\frac{1-\frac{\phi}{2}y_{b,0}^2}{c_{b,1}}}\left(c_{b,1}g_{\lambda_b}(c_{b,1},\lambda_b)+1\right)u\left(\frac{(y_{b,0}-l_b(\lambda_b,c_{b,1}))R(\theta')}{1-\lambda_b}\right)d\lambda_b,
\end{aligned} \tag{D.5}
$$

where $g(c_{b,1},\lambda_b) = \frac{(1-\lambda_b)\lambda_b}{1-c_{b,1}-\phi y_{b,0} l_b(\lambda_b,c_{b,1})}$.

We first consider the first three terms of (D.5). Since $u\left(\frac{x_{b,0}-\lambda_b c_{b,1}+y_{b,0}R(\theta^*)}{1-\lambda_b}\right)$ is decreasing in λ_b and $x_{b,0}-c_{b,1}+y_{b,0}R(\theta')<0$ for $c_{b,1}>1$, the sum of these three terms is less than:

$$\begin{aligned}
& -\left(\frac{c_{b,1}}{x_{b,0}-c_{b,1}+y_{b,0}R(\theta')}\frac{x_{b,0}}{c_{b,1}}\left(1-\frac{x_{b,0}}{c_{b,1}}\right)u\left(\frac{y_{b,0}R(\theta')}{1-\frac{x_{b,0}}{c_{b,1}}}\right)\right) \\
& +\frac{c_{b,1}}{x_{b,0}-c_{b,1}+y_{b,0}R(\theta')}\pi(1-\pi)u\left(\frac{x_{b,0}-\pi c_{b,1}+y_{b,0}R(\theta')}{1-\pi}\right) \\
& +\frac{1}{x_{b,0}-c_{b,1}+y_{b,0}R(\theta')}\int_{\pi}^{\frac{x_{b,0}}{c_{b,1}}}\left(x_{b,0}+y_{b,0}R(\theta')-2c_{b,1}\frac{\frac{x_{b,0}}{c_{b,1}}+\pi}{2}\right)u\left(\frac{x_{b,0}-\lambda_b c_{b,1}+y_{b,0}R(\theta')}{1-\lambda_b}\right)d\lambda_b \\
< \; & -\left(\frac{c_{b,1}}{x_{b,0}-c_{b,1}+y_{b,0}R(\theta')}\frac{x_{b,0}}{c_{b,1}}\left(1-\frac{x_{b,0}}{c_{b,1}}\right)u\left(\frac{y_{b,0}R(\theta')}{1-\frac{x_{b,0}}{c_{b,1}}}\right)\right) \\
& +\frac{c_{b,1}}{x_{b,0}-c_{b,1}+y_{b,0}R(\theta')}\pi(1-\pi)u\left(\frac{x_{b,0}-\pi c_{b,1}+y_{b,0}R(\theta')}{1-\pi}\right) \\
& +\frac{1}{x_{b,0}-c_{b,1}+y_{b,0}R(\theta')}\left(y_{b,0}R(\theta')-c_{b,1}\pi\right)\left(\frac{x_{b,0}}{c_{b,1}}-\pi\right)u\left(\frac{x_{b,0}-\pi c_{b,1}+y_{b,0}R(\theta')}{1-\pi}\right) \\
< \; & -\left(\frac{c_{b,1}}{x_{b,0}-c_{b,1}+y_{b,0}R(\theta')}\frac{x_{b,0}}{c_{b,1}}\left(1-\frac{x_{b,0}}{c_{b,1}}\right)u\left(\frac{y_{b,0}R(\theta')}{1-\frac{x_{b,0}}{c_{b,1}}}\right)\right) \\
& +\frac{R(\theta')y_{b,0}x_{b,0}}{c_{b,1}\left(x_{b,0}-c_{b,1}+y_{b,0}R(\theta')\right)}u\left(\frac{y_{b,0}R(\theta')}{1-\frac{x_{b,0}}{c_{b,1}}}\right) \\
& -\pi u\left(\frac{x_{b,0}-\pi c_{b,1}+y_{b,0}R(\theta')}{1-\pi}\right) \\
< \; & -\frac{x_{b,0}}{c_{b,1}}u\left(\frac{y_{b,0}R(\theta')}{1-\frac{x_{b,0}}{c_{b,1}}}\right)-\pi u\left(\frac{x_{b,0}-\pi c_{b,1}+y_{b,0}R(\theta')}{1-\pi}\right)<0\,.
\end{aligned}$$

We then consider the last two terms of (D.5), which decrease in ϕ. When $\phi=0$, we have $l_b=\pi c_{b,1}-x_{b,0}$, and $g_{\lambda_b}(c_{b,1},\lambda_b)=\frac{1-2\lambda_b}{1-c_{b,1}}$. Then the sum of these two terms becomes

$$\begin{aligned}
& c_{b,1}\frac{\left(1-\frac{x_{b,0}}{c_{b,1}}\right)\frac{x_{b,0}}{c_{b,1}}}{1-c_{b,1}}u\left(\frac{y_{b,0}R(\theta')}{1-\frac{x_{b,0}}{c_{b,1}}}\right)+\int_{\frac{x_{b,0}}{c_{b,1}}}^{\frac{1}{c_{b,1}}}\left(1+\frac{c_{b,1}(1-2\lambda_b)}{1-c_{b,1}}\right)u\left(\frac{(1-\lambda_b c_{b,1})R(\theta')}{1-\lambda_b}\right)d\lambda_b \\
< \; & c_{b,1}\frac{\left(1-\frac{x_{b,0}}{c_{b,1}}\right)\frac{x_{b,0}}{c_{b,1}}}{1-c_{b,1}}u\left(\frac{y_{b,0}R(\theta')}{1-\frac{x_{b,0}}{c_{b,1}}}\right)+\int_{\frac{x_{b,0}}{c_{b,1}}}^{\frac{1}{c_{b,1}}}\left(1+\frac{c_{b,1}\left(1-2\frac{1+x_{b,0}}{2c_{b,1}}\right)}{1-c_{b,1}}\right)u\left(\frac{(1-\lambda_b c_{b,1})R(\theta')}{1-\lambda_b}\right)d\lambda_b \\
= \; & c_{b,1}\frac{\left(1-\frac{x_{b,0}}{c_{b,1}}\right)\frac{x_{b,0}}{c_{b,1}}}{1-c_{b,1}}u\left(\frac{y_{b,0}R(\theta')}{1-\frac{x_{b,0}}{c_{b,1}}}\right)+\int_{\frac{x_{b,0}}{c_{b,1}}}^{\frac{1}{c_{b,1}}}-\frac{x_{b,0}}{1-c_{b,1}}u\left(\frac{(1-\lambda_b c_{b,1})R(\theta')}{1-\lambda_b}\right)d\lambda_b \\
< \; & c_{b,1}\frac{\left(1-\frac{x_{b,0}}{c_{b,1}}\right)\frac{x_{b,0}}{c_{b,1}}}{1-c_{b,1}}u\left(\frac{y_{b,0}R(\theta')}{1-\frac{x_{b,0}}{c_{b,1}}}\right)-\frac{x_{b,0}}{1-c_{b,1}}\frac{1-x_{b,0}}{c_{b,1}}u\left(\frac{y_{b,0}R(\theta')}{1-\frac{x_{b,0}}{c_{b,1}}}\right)
\end{aligned}$$

$$= \quad -\frac{x_{b,0}}{c_{b,1}} u\left(\frac{y_{b,0} R\left(\theta'\right)}{1-\frac{x_{b,0}}{c_{b,1}}}\right) < 0\,.$$

In summary, V is decreasing in $c_{b,1}$ and thus $\frac{\partial \theta^*}{\partial c_{b,1}} > 0$. That is, R^* is increasing in $c_{b,1}$.

To see why θ^* is decreasing in $x_{b,0}$, take the derivative of (D.3) with respect to $x_{b,0}$ and note that $y_{b,0} = 1 - x_{b,0}$, we have

$$\begin{aligned}
& \int_{\pi}^{\frac{x_{b,0}}{c_{b,1}}} -\frac{R\left(\theta'\right)-1}{1-\lambda_b} u'\left(\frac{x_{b,0}-\lambda_b c_{b,1}+y_{b,0} R\left(\theta'\right)}{1-\lambda_b}\right) d\lambda_b \\
& + \int_{\frac{x_{b,0}}{c_{b,1}}}^{\frac{1-\frac{\phi}{2} y_{b,0}^2}{c_{b,1}}} \frac{\left(\frac{1}{1-\phi l_b}-1\right) R\left(\theta'\right)}{1-\lambda_b} u'\left(\frac{\left(y_{b,0}-l_b\left(\lambda_b, c_{b,1}\right)\right) R\left(\theta'\right)}{1-\lambda_b}\right) d\lambda_b \\
= \; & \frac{R\left(\theta'\right)-1}{R\left(\theta'\right)-c_{b,1}-\left(R\left(\theta'\right)-1\right) x_{b,0}} \left((1-\pi)\, u\left(\frac{x_{b,0}-c_{b,1}\pi+y_{b,0} R\left(\theta'\right)}{1-\pi}\right) - \left(1-\frac{x_{b,0}}{c_{b,1}}\right) u\left(\frac{y_{b,1} R\left(\theta'\right)}{1-\frac{x_{b,0}}{c_{b,1}}}\right) \right. \\
& \left. + \int_{\pi}^{\frac{x_{b,0}}{c_{b,1}}} u\left(\frac{x_{b,0}-\lambda_b c_{b,1}+y_{b,0} R\left(\theta'\right)}{1-\lambda_b}\right) d\lambda_b \right) \\
& + \int_{\frac{x_{b,0}}{c_{b,1}}}^{\frac{1-\frac{\phi}{2} y_{b,0}^2}{c_{b,1}}} \frac{\left(\frac{1}{1-\phi l_b}-1\right) R\left(\theta'\right)}{1-\lambda_b} u'\left(\frac{\left(y_{b,0}-l_b\left(\lambda_b, c_{b,1}\right)\right) R\left(\theta'\right)}{1-\lambda_b}\right) d\lambda_b .
\end{aligned}$$

The first term in the equation above is positive according to the same argument as for the first three terms in (D.5). Furthermore, note that $\phi l_b = 1 - \sqrt{1-2\phi\left(\lambda_b c_{b,1} - x\right)} < 1$, the second them is also positive. Following the same logic, V is increasing in $x_{b,0}$, and thus θ^* and R^* are decreasing in $x_{b,0}$. This completes the proof. ∎

Proof of Proposition 3. Denote by $y_{f,1} = y_{f,0} - l_f$ the fund's remaining position in the illiquid project after meeting date-1 redemptions, which is also a function of λ_f in equilibrium. Then it follows from the NAV rules (2.8) and (2.9) that

$$c_{f,1}(\lambda_f) = NAV_1(\lambda_f) = 1 - \frac{\phi}{2}\left(y_{f,0} - y_{f,1}(\lambda_f)\right)^2 . \tag{D.6}$$

Directly comparing (D.6) and the liquidation value (2.2) as defined in Definition 1 shows that $c_{f,1}(\lambda_f(R)) \geq c_1(x_{f,0}, y_{f,0})$, which takes a strict form if $y_{f,1} > 0$. It means that unless the fund is fully liquidated, NAV_1 is strictly higher than the liquidation value of the underlying fund portfolio. Notice that this statement is true regardless of how many agents actually redeem, and thus is also true regardless of the fundamentals. Taking expectations with respect to the fundamentals thus yields inequality (2.11). Then again combining the NAV rules (2.8), (2.9), (2.10) and taking derivative with respect to λ_f yields inequality (2.12). This completes the proof. ∎

Proof of Proposition 4. Let $C(l_f)$ be the amount of cash raised by prematurely liquidating l_f project at $t = 1$. Assume that $C(0) = 0$ and for any $l_f > 0$, $0 < C(l_f) < l_f$. Note that the parametric form of $C(l_f)$ in our baseline model satisfies these conditions. We consider two cases below.

Case 1. We consider the case of $x_{f,0} \geq \lambda_f$, which implies that $l_f = 0$, $NAV_1(\lambda_f) = 1$, and

$$\begin{aligned} NAV_2(\lambda_f) &= \frac{x_{f,0} - \lambda_f + y_{f,0}R}{1 - \lambda_f} \\ &= 1 + \frac{y_{f,0}}{1 - \lambda_f}(R - 1) \\ &= NAV_1(\lambda_f) + \frac{y_{f,0}}{1 - \lambda_f}(R - 1)\,, \end{aligned} \tag{D.7}$$

implying that $NAV_2(\lambda_f) > NAV_1(\lambda_f)$ if and only if $R > 1$ when $l_f = 0$.

Case 2. Then, consider the case of $x_{f,0} < \lambda_f$, which implies that $l_f > 0$. Note that equations (2.9) and (2.10) give two ways to calculate NAV_1:

$$NAV_1(\lambda_f) = 1 - l_f + C(l_f) \tag{D.8}$$

$$= \frac{x_{f,0} + C(l_f)}{\lambda_f}\,. \tag{D.9}$$

Solving (D.9) as an equation of λ_f yields:

$$\lambda_f = \frac{x_{f,0} + C(l_f)}{1 - l_f + C(l_f)}\,. \tag{D.10}$$

Plugging (D.10) into the expression of NAV_2 (2.13):

$$\begin{aligned} NAV_2(\lambda_f) &= \frac{y_{f,0} - l_f}{1 - \lambda_f}R \\ &= \frac{1 - l_f + C(l_f)}{1 - x_{f,0} - l_f}(y_{f,0} - l_f)R\,, \end{aligned}$$

which by (D.8) and the fact that $x_{f,0} + y_{f,0} = 1$ immediately leads to

$$NAV_2(\lambda_f) = NAV_1(\lambda_f)R\,. \tag{D.11}$$

Hence, $NAV_2(\lambda_f) > NAV_1(\lambda_f)$ if and only if $R > 1$ when $l_f > 0$. This completes the proof. ∎

Proof of Proposition 5. Because ε_i is arbitrarily small, there is no fundamental uncertainty between $t = 1$ and 2. Thus, late agents' problem reduces to:

$$\begin{cases} w_f = 0 & \text{if} \quad NAV_1(\lambda_f) < NAV_2(\lambda_f)\,, \\ w_f \in (0, 1-\pi) & \text{if} \quad NAV_1(\lambda_f)(1-\kappa w_f) = NAV_2(\lambda_f)\,, \\ w_f = 1-\pi & \text{if} \quad NAV_1(\lambda_f)(1-\kappa w_f)) > NAV_2(\lambda_f)\,. \end{cases} \tag{D.12}$$

We consider four cases below, which build upon the proof of Proposition 4 above.

Case 1. When $R \geq 1$, Proposition 4 implies that $NAV_1(\lambda_f) \leq NAV_2(\lambda_f)$, which immediately implies that $w_f^* = 0$.

Case 2. By Lemma 1, it must be that $x_{f,0} \geq \pi$. Thus, when $R < 1$ the fund must first use stored cash to meet redemptions, falling into Case 1 in the proof of Proposition 4. Plugging (D.7) into (D.12) and solving the resulting quadratic equation yields:

$$w_f^* = \frac{1}{2}\left(1 - \pi - \sqrt{\frac{\kappa(1-\pi)^2 - 4(1-x_{f,0})(1-R)}{\kappa}}\right), \tag{D.13}$$

which is decreasing in R.

Case 3. When the fund starts to liquidate the project to meet redemptions, the economy falls into Case 2 in the proof of Proposition 4. Thus, plugging (D.11) into (D.12) directly yields:

$$w_f^* = \frac{1-R}{\kappa}, \tag{D.14}$$

which is also decreasing in R.

Combining equations (D.13) and (D.14) and solving as a function of R yields

$$R = 1 - \kappa(x_{f,0} - \pi)\,,$$

implying that the solution of w_f^* switches from (D.13) to (D.14) at $R = 1 - \kappa(x_{f,0} - \pi)$, and w_f^* is indeed continuous in R at this point.

Case 4. Finally, when $R < 1 - \kappa(1-\pi)$, by (D.14) we immediately have $NAV_1(\lambda_f)(1-\kappa w_f)) > NAV_2(\lambda_f)$, which implies that $w_f^* = 1 - \pi$. This concludes the proof. ∎

Figure A2: Cross-section of Bank and Fund Liquidity Provision (Price-Impact Adjusted Haircuts)

This graph plots the distribution of average commercial bank and bond mutual fund LPIs in the cross-section. The LPI for each bank and fund is calculated as the average LPI over the sample period from 2011 to 2017. Haircuts are adjusted for the price impact of liquidations by each fund and bank.

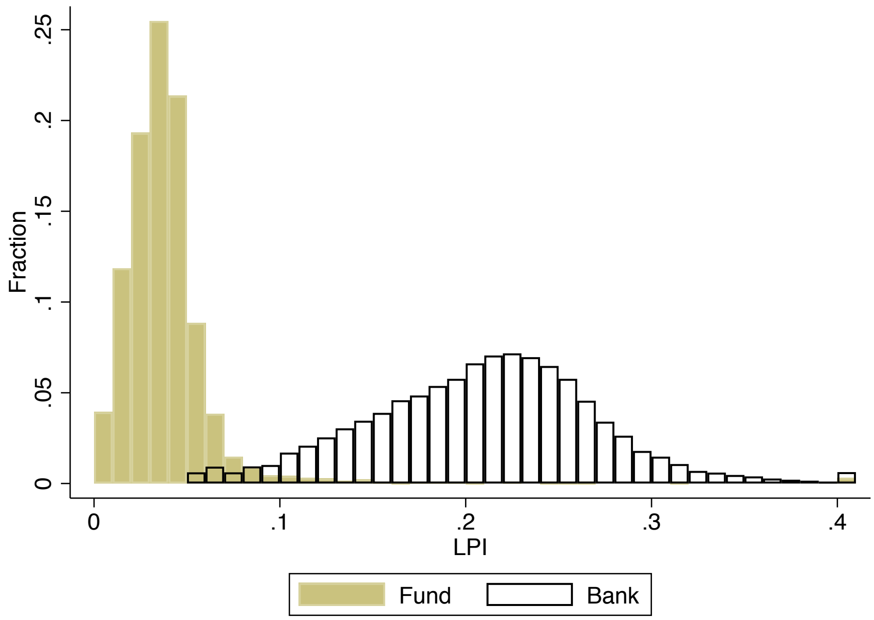


Figure A3: Liquidity Provision Index from 2011 to 2017 (Price-Impact Adjusted Haircuts)

This graph plots the average LPI for commercial banks and bond mutual funds from 2011 to 2017. We first calculate the LPI for each bank and fund in each quarter and then plot the asset-size weighted LPI from 2011 to 2017. Haircuts are adjusted for the price impact of liquidations by each fund and bank.

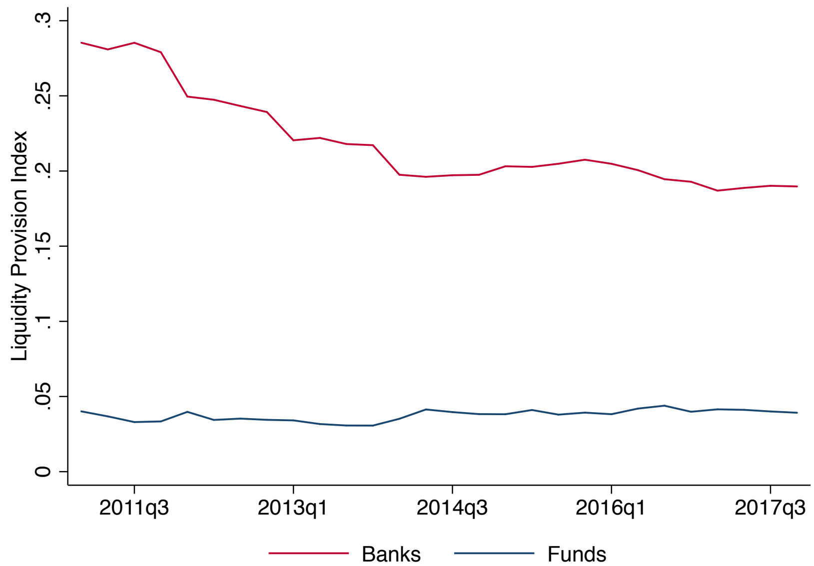


Table A1: Asset Category Sources

This table shows the sources for banks and funds for each asset class used in the LPI calculation. Bank asset holdings are obtained using bank balance sheet data from call reports. The bank holdings variables all come from RCFD (for example, the corresponding cash variable is RCFD0010) except for real estate loans which also take variables from RCON. Mutual fund holdings data is obtained from the CRSP database, and fund cash holdings are taken from CRSP mutual funds summary data. For fund holdings, all asset classes except cash holdings are categorized directly from securities-level holdings using the mapping shown.

Category	Bank Source (RCFD)	Fund Holdings Mapping
1. Treasuries &	3531, 0213, 1287	US Government & Agency Bills, Bonds,
Agency Debentures	3532, 1290, 1293, 1295, 1298	Notes, Strips, Trust Certificates
2. Agency MBS & CMO	G301, G303, G305, G307, G313, G315, G317, G319	Agency MBS, TBA MBS, CMO,
	K143, K145, K151, K153, G379, G380, K197	Pass Through CTF, REMIC, ARM
3. Commercial Loans	F610, F614, F615, F616, K199, K210, F618	Syndicated Loans, Term Deposits,
	$(2122 - 3123) - 1975 - 1410$ if ≥ 0	Term Loans
4. Money Market		Money Market, CDs, Corporate Paper
5. Municipal	8499, 8497, 3533	Municipality Debt
6. Corporate Bonds	G386, 1738, 1741, 1743, 1746	Bonds, MTN, Foreign Gov'ts & Agencies
7. Equities	A511	Equities, Funds, Convertible bonds
8. Private ABS & CMO	G309, G311, G321, G323, K147, K149, K155, K157	ABS, CMO, CDO, CLO, Covered Bonds
9. Consumer Loans	1975	
10. Real Estate Loans (Family)	$1410 * (RCON3465/RCON3385)$	
11. Real Estate Loans (Other)	$1410 * (RCON3466/RCON3385)$	
12. Cash	0010	CRSP Mutual Funds summary Cash %
13. Fixed Assets	3541, 3543, total assets - sum of above variables	

Table A2: Distribution of Haircuts

This table shows the average 10^{th}, 50^{th}, and 90^{th} of haircuts for security categories obtained from the Federal Reserve Bank of New York's repo series from 2011 to 2017.

	(1) p10 Haircut	(2) p50 Haircut	(3) p90 Haircut
Treasuries	0.9	2.0	2.7
Agency Debentures	1.9	2.0	3.6
Agency MBS/CMOs	2.0	2.2	3.9
CMO/ABS	3.0	7.5	16.4
Money Market	1.9	4.2	5.0
Municipal Bonds	2.0	4.9	10.1
Corporate Bond	3.0	6.0	10.9
Equity	5.0	7.7	15.0