Cheap Thrills: the Price of Leisure and the Global Decline in Work Hours

Alexandr Kopytov  Nikolai Roussanov  Mathieu Taschereau-Dumouchel
University of Hong Kong  Wharton and NBER  Cornell University

December 22, 2020

Abstract

The real price of recreation goods and services has fallen dramatically over the last century. At the same time, hours per worker have also been on a steady decline. As recreation goods make leisure time more enjoyable, we investigate if the fall in their price has contributed to the decline in work hours. Using aggregate data from OECD countries, as well as disaggregated data from the United States, we provide evidence that the two are strongly related. To identify the effect of recreation prices on hours worked, we use variation in the bundle of recreational goods across demographic groups to instrument for the changing price of leisure faced by these groups over time. We then construct a macroeconomic model with general preferences that allows for trending relative prices and work hours along a balanced growth path. We estimate the model and find that the decline in recreation prices has been as important as the rise in wages in explaining the decline in work hours in the U.S.

JEL Classifications: E24, J22

*We thank Mark Aguiar, Mark Bils, Julieta Caunedo, Jeremy Greenwood, Joe Kaboski, and Aysegul Sahin, as well as seminar participants at Cornell, Princeton, Wharton, and the Virtual Macro Seminar (VMACS) for useful comments and suggestions.
1 Introduction

Hours worked have declined substantially over the last hundred years: American workers spend two thousand hours a year on average at work, while their 1900 counterparts worked 50% more. Over the same period, technological progress has increased labor productivity and wages, and so the decline in hours is often attributed to an income effect through which richer households choose to enjoy more leisure time. Indeed, Keynes (1930) prophesized that “the economic problem may be solved [...] within a hundred years” and that therefore there would be no need to work long hours to satisfy one’s desire for consumption.

Another important change occurred over the same period, however. New technologies such as televisions and the internet have brought a virtually unlimited trove of cheap entertainment at consumers’ fingertips. The impact of these technologies is clearly visible in the price data. For instance, the Bureau of Labor Statistics (BLS) documents that the (real and quality-adjusted) price of a television set has fallen about 1000-fold since the 1950s, while computers are about fifty times cheaper than they were in the mid-1990s. Similarly, the inflation-adjusted price of admission to a (silent, black and white) movie in 1919 is roughly equal to the current cost of a monthly subscription to a video streaming service providing essentially unlimited access to movies and television shows. Overall, the aggregate price index tracking recreational goods and services has also declined dramatically since 1900, falling by more than half in real terms. Did this large decline in the price of leisure impact the observed increase in its quantity?

In this paper, we investigate how much of the decline in hours worked can be attributed to rising wages, and how much comes from the decline in recreation prices. Answering this question has important implications for our understanding of the labor market and, in particular, for making predictions about how much people will work in the future, as well as for understanding how labor supply responds to changing wages. If the decline in hours can be mostly attributed to the income effect, as posited by Boppart and Krusell (2020), then the weakening growth in median income might lead to a slowdown in the decline in work hours (Mishel et al., 2012). If instead the movement in recreation prices is driving the downward trend in hours, as suggested by Aguiar et al. (2017), we can expect the trend to continue as new technologies keep making leisure cheaper and more enjoyable.

We begin by providing an overview of the data. For the United States, we consider three metrics in order to evaluate the decline in work hours. Using data from the Census and the BLS, we first show that hours per worker have declined at a steady pace since 1900, with the exception of large movements around the Great Depression and the Second World War. Hours per capita have also fallen over that period, although the decline is concentrated in the first part of the twentieth

---

1Similar evidence is presented in a number of studies, including Owen (1970), Lebergott (1993), Fogel (2000), Greenwood and Vandenbroucke (2005), and Boppart and Krusell (2020).
century. After 1950, the large increase in female labor force participation has kept that measure mostly flat. Finally, we plot data from the American Time Use Survey that show that self-reported leisure time has also been increasing, for both men and women, since the 1960s (Robinson and Godbey, 2010; Aguiar and Hurst, 2007b). This last piece of evidence confirms that the decline in market work hours is not simply an artifact of a reallocation toward housework. The trends observed in the U.S. are also visible in other developed countries. We look at the evolution of work hours in 38 OECD countries and find that hours per worker have declined virtually everywhere, while hours per capita have fallen in 27 countries.

This decline in work hours in the United States over the last 120 years was accompanied by a large, well-documented, increase in wages, as well as a large decline in recreation prices. We extend early work by Owen (1970) with data from the Census and the BLS to show that the real price of recreation goods and services has been steadily decreasing since 1900, at a pace of about $-0.75\%$ per year. This trend is also clearly visible in our multi-country sample. Indeed, real recreation prices have fallen in all the countries that we consider, with an average annual decline of $-1.49\%$. We conclude from these data that the decline in work hours and real recreation prices are widespread phenomena that affected a broad array of developed countries.

In order to obtain causal estimates of the impact of wages and recreation prices we turn to detailed individual-level data from the U.S. Census. While our main focus is on aggregate variables, one key advantage of using these disaggregated data is that they allow us to construct two instrumental variables to tackle potential endogeneity issues. In the spirit of Bartik (1991), we construct a first instrument, for wages, that uses location-specific industry employment shares to tease out fluctuations in local wages that are driven by national movements. We also construct a second instrument, this time for recreation prices, using variation in the type of recreation goods and services that are consumed by different demographic groups. Using data from the Consumer Expenditure Survey, we document that, for instance, individuals without a high-school diploma consume a disproportionate amount of “Audio and video” items, while those with more than a college education consume relatively more of “Other services”, which includes admissions, fees for lessons, club memberships, etc. Taking advantage of this variation, we construct our second instrument to capture how national movements in the price of different recreation items affects different demographics groups. This strategy delivers a strong instrument, as judged by high first-stage F-statistics. Using these two instruments, we find a strong positive relationship between recreation prices and hours worked. At the same time, the Bartik shocks for wages suggest a positive (rather than negative) effect of wages on hours worked. We find, however, that this is driven entirely by the decline in employment among regions and demographic groups that used to work heavily in

---

2Ramey and Francis (2009) also provide evidence that leisure time per capita has increased between 1900 and 2005. Their estimates are somewhat smaller than those of Aguiar and Hurst (2007b), mostly because of a different classification of activities. See Aguiar and Hurst (2007a) for more details.
manufacturing (e.g., non-college educated in the Midwest). Since these jobs have been dispropor-
tionately displaced by automation and globalization, controlling for the role of manufacturing is
important. More importantly, leisure time has grown the most among groups that have seen the
slowest growth in wages (e.g. the less educated), as suggested by the evidence in Aguiar and Hurst
(2009) This fact makes accounting for the role of recreation prices in driving labor supply for dif-
ferent demographic groups key to revealing the negative impact of wages on hours worked, which
is consistent with the income effect dominating the substitution effect as suggested by Boppart and
Krusell (2020).

In order to interpret this evidence, we construct a macroeconomic model in which recreation
prices and wages can affect labor supply decisions. At the heart of our analysis is a household that
values recreation time and recreation goods and services, as well as standard (i.e. non-recreation)
consumption goods. To be consistent with well-known long-run trends, we build on the standard
macroeconomic framework of balanced growth and assume that all prices and quantities in the
economy grow at constant, but potentially different, rates. Importantly, and in contrast to the
standard balanced-growth assumptions, we do not assume that hours worked remain constant over
time, but instead allow them to also grow (or decline) at a constant rate.

For our analysis to be as general as possible, we follow the approach of Boppart and Krusell
(2020) and keep the household’s preferences mostly unrestricted, only requiring that they be con-
sistent with a balanced-growth path. We characterize the general form that a utility function must
take in this setup, and show that it nests the standard balanced-growth preferences with constant
hours of King et al. (1988), as well as the more general preferences of Boppart and Krusell (2020)
that allow for hours to decline over time through the income effect of changing wages. In addi-
tion, in the class of economies we study, the growth rates of hours, recreation consumption and
non-recreation consumption are log-linearly related to those of the wage rate and the real price of
recreation items. As a result, changes in the price of recreation goods and services can affect hours
worked.

Our theoretical model has several key advantages when it comes to making contact with the
data. First, since we keep the household’s preferences quite general, our empirical strategy does not
hinge on a specific utility function, but instead remains valid under several functional forms that
have been proposed in the literature. Second, there is no need to fully specify the production sector
of the economy. We only need wages and recreation prices to grow at constant rates for our analysis
to be well-grounded. Third, the system of equations derived from the model can be estimated using
standard techniques and allows for straightforward identification of the key structural parameters
of the economy. Finally, the model provides a set of cross-equation restrictions that impose more
structure on the estimation compared to reduced-form techniques. In particular, these restrictions
allow us to use consumption data to discipline the estimation of the effect of recreation prices on
hours worked.
We estimate the structural relations implied by our model using the regional U.S. data as well as the OECD data. Once again, we find that a decline in recreation prices leads to a large and significant increase in leisure time in both of our samples. In the U.S., using our shift-share instruments in conjunction with our structural model yields a statistically and economically significant role for wages, in addition to that for recreation prices. Overall, based on this empirical analysis, we find that the fall in the price of recreation goods and services, on its own, can explain a large fraction of the decline in hours worked observed in the data.

**Literature**

Our empirical results update and extend an early analysis by Owen (1971) who finds strong evidence of complementarity between leisure time and recreational goods and services in the United States (see also Gonzalez-Chapela, 2007). Owen attributes one quarter of the decline in hours worked over the 1900-1961 period to the declining price of recreation items, and the remaining three quarters to the income effect of rising wages. In contrast, we find much less evidence in support of the income effect in our preferred specifications. An important difference with our approach is that we construct Bartik-like instruments to handle endogeneity issues. We also provide a general balanced-growth path model to guide our empirical exercises.

A weak income effect is consistent with cross-sectional evidence that higher-skilled individuals work more hours per week, especially in the more recent period (Aguiar and Hurst, 2007b). A weak income effect is also consistent with the work of Bick et al. (2018) who find that the relationship between hours and labor productivity is strongly negative across developing countries, but that it is essentially flat across individuals in developed countries, which suggests that the income effect itself might be diminishing with income. They interpret this as evidence of a subsistence level in consumption, whereby poorer households must supply more labor to purchase essential goods. Since our sample consists mostly of developed countries, our findings are consistent with this interpretation.

A subsistence level in consumption would also reconcile our findings with those of Vandenbergroucke (2009), who evaluates the impact of recreation prices in a static model with worker heterogeneity. In a calibration exercise over the 1900-1950 period, he finds that 82% of the decline in hours worked can be attributed to the income effect and only 7% to the declining price of recreation goods. With a subsistence level in consumption, one would expect a stronger income effect in the U.S. over the first half of the century, when incomes where lower, compared to the recent decades which are the focus of our analysis.

Our work is also consistent with findings from Aguiar et al. (2017) who show that the increased

---

3 Aguiar and Hurst (2009) also show that less educated men increased their leisure time over the last decades, while more educated men (whose earnings increased the most) recorded a decrease in time allocated to leisure—a finding at odds with a strong income effect.
leisure time, in particular among young men, is strongly associated with the consumption of leisure goods and services made available due to the advent of cheap new media technologies, such as online streaming and video games. In a recent paper, Fenton and Koenig (2018) argue that the introduction of televisions in the United States in the 1940s and 1950s had a substantial negative effect on labor supply decisions, especially for older men. Kopecky (2011) focuses on the reduced labor market participation of older men and argues that retirement has become more attractive due to the decline in the price of leisure.

Our main theoretical result generalizes recent work by Boppart and Krusell (2020) who characterize the class of preferences that are consistent with a balanced-growth path and declining work hours. We extend their preferences to include recreation goods that are complement with leisure time. As a result, we can investigate the importance of wages and recreation prices as drivers of the decline in work hours.

Greenwood and Vandenbroucke (2005) consider a static model of the role of technological changes in the long-run evolution of work hours through three channels: rising marginal product of labor (the income effect), the introduction of new time-saving goods (the home production channel) and the introduction of time-using goods (the leisure channel). The second effect, in particular, is important for accounting for the entry of women into the labor force, which makes the long-run decline of work hours per person (rather than hours per worker) less pronounced.

Ngai and Pissarides (2008) construct a model in which leisure time rises on a balanced growth path due to a complementarity between leisure and “capital goods” (such as entertainment durables), as well as marketization of home production. Building on this, Boppart and Ngai (2017) provide a model where both leisure time and leisure inequality increase along a balanced growth path due to the growing dispersion in labor market productivity. In recent work, Boerma and Karabarbounis (2020) argue that the rising productivity of leisure time combined with cross-sectional heterogeneity in preferences (or “non-market productivity”) is responsible for these trends. Our work departs from the existing literature in several ways. On the theoretical side, we keep the preferences of the household as general as possible. On the empirical side, we investigate the impact of recreation prices in both aggregate and disaggregated data in the U.S. as well as in a broad cross-section of countries. Most importantly, we use instruments to tease out the causal impact of recreation prices and the wage.

The next section provides an overview of the data as well as reduced-form exercises to evaluate the impact of recreation prices on hours worked in the United States and in OECD countries. We then introduce the model and provide our main theoretical result. Finally, we estimate the structural relationships derived from the model. The last section concludes.
2 Reduced-form empirical evidence

We begin by presenting the relevant data for the United States and for a cross-section of countries. We document three important trends that hold in almost all the countries in our sample over the last decades: 1) hours worked have fallen, 2) the price of recreation goods and services has declined substantially, and 3) wages have been increasing. We also present reduced-form evidence to show that the decline in work hours is strongly associated with the decline in recreations prices.\textsuperscript{4}

2.1 United States evidence

2.1.1 Aggregate trends

Figure 1 shows the evolution of work hours, wages and recreation prices in the United States. The solid blue line in panel (a) shows how hours worked per capita have evolved between 1900 and 2019. Over the whole period, hours have fallen significantly from about 1500 annual hours per person in 1900 to about 1100 hours per person today. While the figure shows an overall reduction in hours, all of the decline actually took place before 1960, with even a slight increase since then. But these aggregate statistics are somewhat misleading as they conceal substantial heterogeneity between men and women, whose hours are shown in red and green in panel (a). As the panel demonstrates, the second half of the twentieth century saw a large increase in women’s hours, presumably due to the rise in labor force participation, which clearly contributed to the stagnation of the aggregate hours worked data.\textsuperscript{5} At the same time, male hours worked have kept declining (note that between 2000 and 2019 hours declined for both men and women).

The evidence in panel (a) might suggest that women are working much more in 2020 than in 1960, but this is somewhat misleading, since the figure only reports hours worked in the marketplace. Total work hours, which also include home production, have been declining since 1960. To show this, we follow Aguiar and Hurst (2007b) and Aguiar et al. (2017) and use the American Time Use Survey to construct measures of market work, total work (including market work, home production and non-recreational childcare), and leisure for men and women between 16 and 64 years old (excluding full-time students). These series are presented in Figure 2. Between 1965 and 2017, total annual work hours have declined by 416 (8.0 hours per week) for women and by 504 (9.7 hours per week) for men. According to that metric, women work substantially less now than fifty years ago (although classifying all time spent with children, such as playing games and going to a

\textsuperscript{4}To avoid burdening the text, we keep the precise data sources and the steps taken to construct the datasets in Appendix A.

\textsuperscript{5}This increase in female labor force participation is well documented and was driven by several factors. Many women were probably kept away from market work because of discriminatory social norms. As these norms evolved, the stigma of women in the labor force faded and female participation increased. In addition, technological improvements made it easier to perform nonmarket work—mostly done by women—leaving more time for market work (Greenwood et al., 2005). Goldin and Katz (2002) also document that the adoption of contraceptives might have affected women's decisions to pursue higher education.

**Figure 1:** Hours, wages and recreation price in the U.S.
zoo, as childcare “work” rather than “leisure” moderates this trend somewhat — see discussion in Ramey and Francis, 2009 and Aguiar and Hurst, 2007a).

The decline in hours worked is also clearly visible when looking at hours per worker, instead of per capita. These data are presented in panel (b) of Figure 1. Except for large fluctuations around the Great Depression and the Second World War, that measure has been on a steady decline from more than 3000 annual hours per worker in 1900 to less than 2000 today.6

What are the drivers behind this long-run decline in hours? Clearly, people are now richer than in 1900 and it might be that at higher income levels they prefer enjoying leisure to working. Indeed, panel (c) of Figure 1 shows that real hourly wages have gone up ten-fold since 1900. Theoretically, this tremendous increase in wages could lead to an increase in labor supply, if the standard substitution effect dominates, or to its decline, if the income effect dominates instead.

Like the benefit of working, the cost of enjoying leisure has also undergone a massive change over the last century. To show this, we plot in panel (d) of Figure 1 the real price of recreation goods and services since 1900., where the price of all consumption goods and services is used as price deflator.7 Items in that category include goods and services that are associated with leisure time, such as video and audio equipment, pet products and services, sporting goods, photography, toys, games, recreational reading materials, and admission to movies, theaters, concerts, sporting events, etc.8 As we can see, these prices have experienced a steep decline, falling by about 60% in real terms since 1900. If these goods and services are complement to leisure time, a decline in their price would incentivize households to consume more leisure. As a result, they could play an important role in the decline in hours worked.

2.1.2 Using cross-household variation

So far, we have looked at the aggregate data, but in recent decades different households have experienced very different changes in their labor income and their work hours. While the large increase in earnings inequality has been extensively documented, the U.S. has also witnessed a substantial rise in leisure inequality (Aguiar and Hurst, 2009). For instance, Attanasio et al. (2012) show that low-income people have experienced a much more pronounced increase in their leisure time than their high-income counterparts. At the same time, and as we document below, the recreation prices faced by different households have also changed dramatically over the same period. In this subsection, we take advantage of these heterogeneous changes across households to evaluate the link between hours worked, wages and recreation prices from a different angle.

6Using decennial data from the Census, McGrattan et al. (2004) also find that hours per worker have declined and hours per capita have increased in the U.S. since 1950. Kendrick et al. (1961) and Whaples (1991) document a decline in work hours since 1830 (see also Figure 1 in Vandenbroucke, 2009). Kendrick et al. (1961) also show that this decline has happened in all industries.
7We use the same price deflator for all U.S. nominal variables.
8These data come from a variety of sources and their construction is detailed in Appendix A.
One key advantage of using disaggregated household-level data is that we can construct Bartik-like instruments to address potential endogeneity concerns (Bartik, 1991). To do so, we construct two instrumental variables to capture exogenous variations in wages and recreation prices. Our wage instrument relies on the differences in industrial composition across U.S. localities and across demographic groups, as is relatively standard in the literature. In contrast, our instrument for recreation prices is novel and takes advantage of differences in recreation consumption bundles across households’ demographic characteristics, such as education and age, as we describe in detail below.

### 2.1.2.1 Instrument for wages

We use initial variation in industrial composition across localities and demographic group together with nation-wide changes in sectoral wages to construct a measure of changes in wages that are driven by factors independent of regional labor market conditions, such as technological growth, etc. To be precise, we compute

$$
\Delta \log w_{gl}^{IV} = \sum_i \frac{e_{igl}^0}{\sum_j e_{jgl}^0} \Delta \log e_{ig}^{US} - \sum_i \frac{h_{igl}^0}{\sum_j h_{jgl}^0} \Delta \log h_{ig}^{US},
$$  

(1)

where $i$ denotes an industry, $g$ is a demographic group, and $l$ is a locality.\(^9\) The operator $\Delta$ denotes the total growth rate over our sample. The variable $e_{igl} = w_{igl} \times h_{igl}$ refers to labor earnings and $h_{igl}$ is total hours worked. To construct $\Delta \log w_{gl}^{IV}$, we first compute the fraction of earnings and

\(^9\)We show in Appendix C.1.1 that equation (1) can be derived from the definition of labor earnings $e_{iglt} = w_{iglt} \times h_{iglt}$ together with replacing local growth rates $\frac{x_{iglt+1}}{x_{iglt}}$, for some variable $x$, by their nation-wide equivalent $\left(\frac{x_{iglt+1}}{x_{iglt}}\right)^{US}$.
hours worked that can be attributed to an industry $i$ in a given locality-demographic unit in a base period, which we denote by the superscript $t = 0$. Since these shares provide a measure of how sensitive local earnings and hours are to aggregate changes in industry $i$, we can then compute $\Delta \log w_{gL}^{IV}$ as the growth rate in local wages that can be attributed to changes in national factors: $\Delta \log e_{ig}^{US}$ and $\Delta \log h_{ig}^{US}$. As these national factors are unlikely to be driven by local conditions, (1) provides a potential source of exogenous variation in local wages for a given demographic group.\(^{10}\)

To construct the needed measures of hours and earnings at the locality-demographic-industry level, we use data from the U.S. Census (years 1980 and 1990) and the Census’ American Community Surveys (2014-2018 five-year sample, which we refer to as 2016). The key advantage of these data is that they cover a large sample of the U.S. population, which allows us to exploit variation across the 741 commuting zones, defined as in Dorn et al. (2019).\(^{11}\) We limit our analysis to individuals between the ages of 25 and 64, and split them into 15 demographic groups based on age (25-34 years old, 35-49 years old, 50-64 years old) and education (less than high school, high school, some college, four years of college, more than college), excluding those serving in the armed forces and institutional inmates. Overall, such demographic-locality split implies 11,115 groups. We exclude groups with less than 50 individual observations, leaving us with 10,469 groups.

Our industry classification includes 34 industries.\(^{12}\) We construct initial industry shares (the base year denoted by the superscript $0$ in (1)) using the data for 1980; growth rates are then constructed by comparing 1990 outcomes to their 2016 counterparts. Importantly, the base-year shares in (1) are computed over a period that precedes the horizon over which we compute the growth rates. As a result, these shares are less likely to be affected by the future growth rates.

2.1.2.2 Instrument for recreation prices  We construct a similar instrument for recreation prices. Since these prices are not available at the local level, we cannot construct location-specific prices. Instead, we take advantage of the substantial variation in the type of goods and services that are consumed across demographic groups. Individuals of different education levels and ages consume very different types of recreation items, and the aggregate prices of these items have evolved differently over the last decades. To the extent that differences in the bundles of recreation goods consumed by different demographic groups are driven by differences in tastes, variation in the relative prices of these bundles generates exogenous variation in the implicit cost of enjoying leisure faced by different groups.

\(^{10}\)See discussion in Goldsmith-Pinkham et al. (2018) of the implicit assumptions under which the exclusion restriction is satisfied, in particular the absence of geographical spill-overs due to worker mobility, etc.

\(^{11}\)The crosswalk files to convert from the Census-provided geographic regions to commuting zones are available via David Dorn’s website, \url{https://www.ddorn.net/data.htm}.

\(^{12}\)We use the ‘IND1990’ variable. The industries are Agriculture, Forestry, and Fisheries; Mining; Construction; Manufacturing (19 subcategories); Transportation; Communications; Utilities and Sanitary Services; Wholesale Trade (2 subcategory); Retail Trade; Finance, Insurance, and Real Estate; Business and Repair Services; Personal Services; Entertainment and Recreation Services; Professional and Related Services; Public Administration.
In order to make sure that changes in the relative prices are not reflecting demand shifts, we construct a Bartik-like instrument based on the expenditure shares of different recreation goods in a pre-period interacted with aggregate changes in the relative prices of the specific goods over a subsequent period to capture a source of exogenous variation in recreation prices. Specifically, we compute

$$\Delta \log p^{IV}_g = \sum_j \frac{c_{0jg}}{\sum_i c_{0ig}} \Delta \log p^{US}_j,$$

(2)

where $c_{jg}$ denotes the nominal consumption expenditure of recreation items of type $j$ by individuals in demographic group $g$ (a combination of education and age). We use the recreation consumption shares of each demographic group during the initial period (superscript $t = 0$) as a measure of how sensitive the recreation prices they face are to nation-wide changes $\Delta \log p^{US}_j$. Since these aggregate movements are unlikely to be related to factors originating specifically from individuals in a given group, $\Delta \log p^{IV}_g$ provides a source of exogenous variation in the recreation prices affecting these individuals.\(^{13}\)

Our source of data for recreation consumption is the interview part of the Consumption Expenditure Survey (CE). We follow Aguiar and Bils (2015) in constructing and cleaning the sample. We split all recreation consumption expenditures into the seven subcategories used by the BLS to construct price indices: Audio-video, Sports, Pets, Photo, Reading, Other goods (including toys and musical instruments), Other services (including admissions, fees for lessons and instructions, club memberships, etc.). We use the pre-period 1980 and 1988 when computing the initial consumption shares.\(^{14}\) Growth rates are computed between the 1989-1991 period and the 2014-2018 period.\(^{15}\)

The strength of the price instrument (2) relies on substantial heterogeneity across the initial recreation consumption baskets of households with different demographics. We show in Figure 3 how the recreation spending of two extreme demographics is allocated, and how that spending has evolved over time. Panels (a) and (c) refer to households whose heads do not have a high school diploma and who are between the ages of 25 and 34. Panels (b) and (d) show how households whose

\(^{13}\)A similar approach is used by Acemoglu and Linn (2004) to instrument for changes in demand, rather than supply, as they interact expenditure shares of individual goods with demographic changes in order to capture shifts in market sizes over time. As shown by Goldsmith-Pinkham et al. (2018) in the context of the standard Bartik instrument, this construction is essentially equivalent to a differences-in-differences research design. We discuss the robustness of that instrument below.

\(^{14}\)We pool observations between 1980 and 1988 to construct initial consumption shares to reduce the noise because the Consumption Expenditure Survey—our source of consumption data—has on average 1484 annual observations. The results are largely unchanged if we use a shorter time series to construct initial consumption shares.

\(^{15}\)When constructing recreation consumption baskets across demographic groups in the CE data, we use demographic characteristic of reference persons. By definition, “the reference person of the consumer unit is the first member mentioned by the respondent when asked ‘What are the names of all the persons living or staying here? Start with the name of the person or one of the persons who owns or rents the home’. It is with respect to this person that the relationship of the other consumer unit members is determined”. Our measures of wages and hours from the Census are at the individual level. Our results are similar if we instead only use hours and wage data for the household heads only. See Appendix C.1.3.
heads have more than a college degree and who are between 50 and 64 allocate their recreation spending. Panels (a) and (b) refer to spending during the period 1980-1988, while panels (c) and (d) show the data between 2010 and 2018, respectively.

Focusing on the 1980-1988 period first, we see from panels (a) and (b) that consumption baskets vary substantially across demographics. In particular, households whose heads are young and less-educated spend disproportionately more on “Audio-video” items, while households whose heads are older and are more-educated spend more on “Other services”\(^{16}\). Panels (c) and (d) show that these differences remain in the most recent decade and, if anything, have become starker. Notably, the share of expenditure on “Audio-video” items by households whose heads are young and less educated has increased from about 36% to more than 50% between the two periods. Importantly, though, for our analysis, we are only using the pre-period shares as our source of exogenous variation. While expenditure shares for the same demographic groups in the recent period are somewhat different from what they looked like in the 1980-1988 period, the latter are still a good predictor of the former.

Importantly for our purpose, the aggregate price of the different items in the recreation consumption baskets have evolved very differently over the last decades. As Figure 4 shows, the real price of “Audio-video” items, disproportionately consumed by young less-educated households, has declined by 60% since 1980. In contrast, the average price of items in the “Other services” categories, mostly consumed by old highly-educated households, has increased by about 20%. As a result, the price of a recreation basket has evolved very differently across demographic groups. We use this variation in our regressions below to tease out the impact of recreation prices on hours worked.

One implicit assumption behind our identification approach is that the differences in the expenditure shares of the different recreation goods across demographic groups in the pre-period are orthogonal to the forces driving changes in the prices of the different goods categories during our period of interest, 1990-2011. This is reasonable, since the bulk of the changes over the last decades were driven by external forces such as technological innovation and globalization (e.g., development of information and computing technologies and automation driving down prices of audio-video goods and services, in particular, as well as advances in automation technologies and globalization of trade bringing down the prices of certain leisure-related manufactured goods, such as toys, musical instruments, and sporting goods).

Our instrument is able to tackle an array of endogeneity issues that can arise in standard regressions. For instance, since it relies on cross-sectional variation, it can handle aggregate shocks that might jointly affect the price of recreation items and hours worked. One example of such shock is a shock to preferences that makes leisure more enjoyable might lead to a change in demand for

\(^{16}\)Figure 8 in Appendix C.1.2 shows that education alone can account for large variations in spending habits on recreation items.
Shares of different items in total recreation consumption, constructed by pooling observations for the two periods, 1980-1988 and 2010-2018.

Source: Consumer Expenditure Survey.

**Figure 3:** Share of recreation spending across education and age groups.
recreation items, which might affect their price, while at the same time incentivizing people to work fewer hours.\textsuperscript{17} In addition, our instrument is robust to individuals changing their recreation consumption baskets in response to change in prices since we fix the consumption shares to their 1980-1988 levels, before the observed movement in prices.

One potential concern that is not addressed directly by this instrument, however, is that there might be an omitted variable that simultaneously drives changes in hours worked and in recreation prices and that affects different demographic groups differentially. Over the past decades, manufacturing jobs have been moving overseas at the same time as technological improvements have led to cheaper recreation goods. These changes might have affected different demographic groups in different ways, in particular depressing demand for less-educated labor while potentially increasing demand for highly skilled workers (e.g., Autor et al., 2006, Autor and Dorn, 2013, Bloom et al., 2019, Jaimovich and Siu, 2020). In order to account for the role of these shifts, which have been largely confined to the manufacturing sector, we add in some specifications an additional control for the share of each demographic-locality group employed in manufacturing in 1980, well before such changes occurred, thus helping to make sure that these trends are not driving our results.

\textbf{2.1.2.3 Pre-trends} Our analysis relies on the fact that there has been a substantial increase in leisure inequality among individuals of different ages and education levels since the 1980s. Using the American Time Use Survey data, Aguiar and Hurst (2009) document that, despite the fact that leisure time has been rising since at least 1965, the increase in leisure inequality has been concentrated in the post-1985 period (this finding is illustrated in panels (b) and (c) of Figure 5). Interestingly, we find evidence that the prices of different types of recreation goods and services

\textsuperscript{17}This particular type of shock would also have counterfactual implications. It would lead to an increase in demand for recreation goods, which would push up their prices, while at the same time reducing work hours.
started to diverge exactly around early 1980s. In Figure 5, we show the evolution of the real price of recreation commodities and services between 1967 and 1998. Although the data on consumption baskets and prices of finer categories of recreation goods and services is only available post-1980, it is reasonable to assume that even before 1980 highly educated and older households consumed mainly recreation services, while low educated young households consumed disproportionately more of recreation commodities. Interestingly, panel (a) of Figure 5 shows that the price of recreation commodities and services were moving down in synchronized manner between 1967 and 1980 and only started diverging around 1980. This is consistent with the finding of Aguiar and Hurst (2009): pre-1980, declining recreation prices for both commodities and services push all types of households to reduce their labor supply, while in the post-1980 period, consumers with different recreation consumption baskets are affected very differentially due to different price dynamics of the subcategories of recreation commodities and services that they consume.

### 2.1.2.4 Regression results

Now that we have defined our two instruments, we can move to estimate how the growth in hours per capita \( h \) is affected by the growth in real recreation prices \( p \) and wages \( w \). Namely, we run the regression

\[
\Delta \log h_{gl} = \beta_0 + \beta_p \Delta \log p_g + \beta_w \Delta \log w_{gl} + \gamma X_{gl} + \epsilon_{gl},
\]

where the subscripts \( g \) and \( l \) denote, respectively, demographic groups and localities. As for our instruments, the operator \( \Delta \) computes differences between 2016 and 1990 outcomes. We also allow for a set of control variables \( X_{gl} \) that we specify later. For each locality-demographic group, we are computing unique growth rates, between 1990 and 2016, for hours per capita, wages and recreation prices. As such, (3) is a purely cross-sectional regression with no time dimension. The identification comes from variations across localities and demographic groups, and aggregate trends are absorbed by the constant in the estimation.

The outcome of the estimation is presented in Table 1, where the first three columns refer to ordinary-least square regressions and the last three columns take advantage of our instruments. In all cases, the \( F \)-statistics are large, suggesting that our two instruments are strong. In columns (2)-(3) and (5)-(6) we allow for additional demographic controls. In particular, for each location-demographic group, we compute the fractions of males, whites, married and people with disabilities. We control for the 1980 values of these fractions, as well as for their growth rates between

---

18. We do not show them afterwards because these series were discontinued in 1998 due to changes in the classification scheme. But importantly, and as evident from Figure 4, the diverging trends of real prices of recreation commodities and services are also present during the two latest decades.

19. Since recreation prices are not available at the local level, we cannot construct location-specific prices. We instead construct the annual demographic-specific growth in prices according to the formula \( \frac{p_{g, t+1}}{p_{g, t}} = \sum_j \frac{c_{ij, t+1}}{c_{ij, t}} \frac{p_{j, t+1}}{p_{j, t}} \).

20. The U.S. Census does not provide a consistent disability measure throughout our sample. For 1980, we use ‘DISABWRK’ that indicates whether respondents have any lasting condition that causes difficulty working. For 1990
(a) Real prices of recreation commodities and services, 1967-1998

(b) Market work hours by education, 1965-2017

(c) Leisure hours by education, 1965-2017

Vertical black line in all panels denotes the start of the detailed consumption and price data. Panel (a): Real U.S.-wide price of recreation commodities and services. Source: BLS. Panels (b) and (c): Evolution of work and leisure annual hours for individuals with no more than high school diploma and at least four years of college. Individual between 25 and 64 years old who are not full-time students are considered. Market work includes any work-related activities, travel related to work, and job search activities. Leisure is any time not allocated to market and nonmarket work (home production, shopping, non-recreational childcare), net of time required for fulfilling biological necessities (8 hours per day). Sample includes people between 25 and 64 years old who are not full-time students. Source: ATUS, Aguiar and Hurst (2007b) and Aguiar et al. (2017).

Figure 5: Pre-trends of work hours, leisure hours, and recreation prices
1990 and 2016. Columns (3) and (6) also control for the share of manufacturing hours in each demographic group in 1980.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Δ log ( p )</td>
<td>0.43***</td>
<td>0.41***</td>
<td>0.29***</td>
<td>0.76***</td>
<td>0.74***</td>
<td>0.39***</td>
</tr>
<tr>
<td>Δ log ( w )</td>
<td>-0.05**</td>
<td>-0.09***</td>
<td>-0.09***</td>
<td>-0.71***</td>
<td>-0.58***</td>
<td>-0.55***</td>
</tr>
<tr>
<td>1980 manuf. empl.</td>
<td>-0.14**</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.32***</td>
</tr>
<tr>
<td>Locality F.E.</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Addtl. dem. cont.</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
<td>N</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>( F )-statistics</td>
<td></td>
<td></td>
<td></td>
<td>145.1</td>
<td>140.7</td>
<td>141.4</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.30</td>
<td>0.41</td>
<td>0.41</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td># observations</td>
<td>10,469</td>
<td>10,469</td>
<td>10,469</td>
<td>10,469</td>
<td>10,469</td>
<td>10,469</td>
</tr>
</tbody>
</table>

The regressions are across people sorted by locality/education-age group. Controls include manufacturing hours share in 1980, and a rich set of additional demographic controls (see text for details). Errors are clustered at location level. \( F \)-statistics are Kleibergen-Paap. *, **, *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Table 1: Regressions across people sorted by location and education-age groups: impact of wage and recreation price growth on hours worked.

We see in all cases that an increase in recreation prices is associated with an increase in work hours. The coefficients are strongly statistically and economically significant with a decline in recreation prices of 1 percent associated with a decline in hours of 0.39 percent in our preferred specification (column 6). Importantly, this effect remains strong even in the IV regressions, which are less subject to endogeneity issues.

We also find a significant effect of wages on hours worked. In all the specifications that we consider, the income effect dominates, so that an increase in wages is associated with a decline in work hours. The magnitude of this effect, however, changes markedly across specifications. In the OLS regressions (columns 1 to 3), the impact of wages is quite weak while our instrumental variable estimation finds a sizable impact of wages on hours. Controlling for demographics only has a meaningful impact on the coefficients, suggesting that the impact of rise in disability benefits on

and 2016, we use ‘DIFFCARE’ that indicates whether respondents have any lasting condition that causes difficulty to take care of their own personal needs, and ‘DIFFMOB’ that indicates whether respondents have any lasting condition that causes difficulty to perform basic activities outside the home alone.

Changes in some of demographic variables between 1990 and 2016 (such as fractions of married individuals and people with disabilities) might be affected by the treatment themselves and so might be ‘bad controls’. To address this issue, we also run the same regressions by only including the 1980 values of demographic controls. The results are largely unchanged: We obtain marginally larger estimates of \( \beta_p \) and somewhat smaller in absolute terms values of \( \beta_w \).

Recall that to construct recreation prices for different demographic groups we use the household-level CE data. Our measures of wages and hours from the Census are at the individual level. In Appendix C.1.3, we redo the same regressions using hours and wage data for all household heads and married household heads, with additional controls for the number of kids, and find very similar results.

Note that standard errors are clustered at the locality level. The results for recreation price stay significant at the 5% level if we cluster standard errors at both regional and demographic group levels. We prefer not to perform double clustering in our main analysis because we have only 15 demographic groups.
work hours might be orthogonal to that of wages and recreations prices. Adding 1980 manufacturing employment shares as a control somewhat lowers the recreation price coefficient while leaving the wage coefficient unaffected.

### 2.2 Evidence from other countries

The trends observed in the U.S. economy are also visible in international data. To show this, we gather data on real recreation prices and wages from a variety of sources, such as the OECD and national statistical agencies. The OECD tracks the price of “Recreation and culture” items which we use as our main recreation price index. This category includes items such as audio-visual and photographic equipment, musical instruments, toys, pets, admission to movies, theaters, concerts, etc. For several countries, we are able to augment these data using price series from national statistical agencies. Data on hours worked comes from the Total Economy Database of the Conference Board. Our final sample covers 38 countries and 1023 country-year observations. More information about how the dataset is constructed is provided in Appendix A.

Figure 6 shows the evolution of hours worked (both per capita and per worker), recreation prices and wages for a selected group of countries in our sample. While there is some heterogeneity across countries, the figure shows a clear overall decline in both hours and recreation prices, and an increase in real wages. Across the full sample, we find that since 1950, per capita hours have been declining at an average rate of 0.26% per year and hours per worker have been declining at 0.41% per year. At the same time, real wages have been increasing by 1.88% per year, and real recreation prices have been declining by 1.07% per year.

To show how widespread these patterns are, Table 5 in the Appendix provides the list of countries in our sample along with their individual average growth rates for hours, wages and recreation prices. We observe, first, that there has been a broad decline in hours worked throughout our sample. Hours per capita have had a negative growth rate in 27 countries out of 38, and the decline is even more pronounced when looking at hours per worker, which have declined in all but one country (Lithuania). Second, the growth in real wages $\gamma_w$ is positive for all countries except Mexico, which experienced a large decline in real wages in the 1980’s due to very high inflation rates.

---

24We compute hours per capita by dividing total hours worked by the number of persons in the population that are between 15 and 64 years of age, and similarly for hours per worker. We use the OECD’s compensation of employees divided by hours as our main measure of wages. We adjust all prices for inflation using the country-specific all-item consumer price indices, also provided by the OECD. We restrict the sample to countries with at least ten years of observations.

25See Figure 9 in Appendix C.2.1 for the same graphs with all the countries in our sample.

26Table 1 in Huberman and Minns (2007) shows that the decline in hours per worker goes back to at least 1870 in Australia, Canada, the United States and Western Europe.

27We compute these growth rates by running a pooled regression of a given variable of interest $x_{lt}$ in country $l$ at time $t$ on the time trend and a set of country fixed effects $\alpha_l$, so that $\log x_{lt} = \alpha_l + \gamma^x t + \varepsilon_{lt}$. The coefficient $\gamma^x$ therefore provides a measure of average growth rates for variable $x$ across countries.
Figure 6: Hours, wages and recreation prices for a selected group of countries.
Real recreation prices have also been declining worldwide. As the table shows, we find a negative growth rate for all countries in our sample, and these growth rates are statistically different from zero at the 1% level in all cases. The coefficients are also economically large. Even for the country with the slowest decline (Ireland), recreation prices have still gone down by 0.4% per year, a large number when compounded over a hundred years. Compared to the other countries in our sample, the United States experienced a relatively slow decline in real recreation prices (−0.7% per year). Only four countries (Ireland, Japan, Luxembourg and Norway) went through slower declines.

2.3 Discussion

We have abstracted from other potential forces that might also affect work hours in order to keep our analysis focused. One such channel has to do with the evolution of home production and, more specifically, improvements to home appliances that might have contributed to an increase in female labor force participation. To see whether these changes could explain away our results, we provide in Appendix ?? regressions in which we also include the price of durable goods as a control for the quality of appliances. We also provide an exercise in which we consider changes in the classification of recreation items by the BLS. In both cases, we still find strong and statistically significant evidence that recreation prices are positively associated with work hours.

3 Model

In the previous section, we presented reduced-form evidence for a relationship between recreation prices and work hours. But that evidence alone does not inform us about the origin of that relationship and on which features of the economy influence its strength. For instance, it is unclear whether that relationship is structural or if it could change in response to policy interventions. To gain a better understanding of the mechanisms involved, we therefore build a macroeconomic model of labor supply in which wages and recreation prices affect work hours. Our goal is to build a model that is general, microfounded and that can easily be brought to the data. We then estimate the model in the next section and use it as a tool to disentangle how economic forces affect hours worked.

3.1 Balanced-growth-path facts

Since our goal is to explain economic changes that occur over long time horizons, we adopt the standard macroeconomic framework for this type of analysis, namely that of a balanced-growth
In what follows, we therefore assume that prices and quantities grow at constant, but perhaps different, growth rates. That framework offers a good description of the evolution of the U.S. economy over the long-run, so that we can be sure that our model economy does not clash with important regularities in the data. We however make one important departure from the usual balanced-growth path assumptions: we do not impose that hours worked remain constant over time. Instead, we allow them to also decline at a constant rate.

In a recent paper, Boppart and Krusell (2020) show that — apart from hours dynamics — stylized balanced-growth facts, as outlined by Kaldor (1961), remain valid for the United States today. However, these facts do not distinguish between different types of consumption. Our modeling strategy, described below, assumes that the consumption of recreation and non-recreation items evolve in such a way that their ratio remains constant over time. Before going through the details of the model, we therefore provide some evidence to show that this assumption is justified for the United States and our sample of countries.

For the United States, we use consumption data from the NIPA tables and construct a measure of recreation consumption that includes items such as video and audio equipment, sports goods, memberships and admissions, gambling, recreational reading materials, pet products, photographic goods and services, and package tours (see Appendix A for the details of that exercise). We then compute the share of recreation consumption expenditure and plot that measure as the blue solid line in panel (a) of Figure 7. As we can see, this share has remained roughly constant over the last hundred years, moving from about six percent in 1929 to seven percent today.\(^{29}\)

When constructing our measure of recreation consumption, we follow the classification used by the BLS and exclude information processing equipment (i.e. computers), which might also be used for work or education. We however provide an alternative measure, displayed in red in panel (a) that includes all of these expenditures in the recreation category. In this case, the share of recreation expenditure increases slightly over our sample.\(^{30}\) To further emphasize that the share of recreation consumption has remained constant, we also construct expenditures on recreation goods and services using the CE data. That measure is also shown, in green, in panel (a). Although it is only available since 1980, it has remained fairly stable since then.

Since our analysis is not limited to the U.S. economy, we also compute the recreation consump-

\(^{29}\)Our finding that the share of recreation consumption has been roughly constant is in contrast with earlier work by Kopecky (2011) who uses data from Lebergott (2014) and finds an increasing recreation share over the twentieth century. Two important differences between the datasets are responsible for the different conclusions. First, our sample includes additional data from 2000 to 2019, a period over which the recreation share has declined by more than one percentage point. Second, Lebergott (2014) finds a large increase (from three to six percentage points) in the recreation share between 1900 and 1929 (see Figure 3 in Kopecky, 2011). But, unlike the rest of the time series, these data are not from NIPA, but are instead imputed from a variety of sources. For instance, adjusted sectoral wages are used as a proxy for the consumption of recreation services. While we cannot rule out a small increase in the recreation share since 1900, we view the data since 1930 as more reliable for estimating its overall trend.

\(^{30}\)Kopecky (2011) argues that up to 30% of transportation expenses are related to social and recreational trips. The transportation expenditure share has been slowly declining starting from 1980. Including transportation expenditures in the recreation consumption category would largely undo the impact of computers.
tion share in the other countries in our sample, using data from the OECD. The OECD categorizes all consumption expenditures into different baskets. We use the items in the ‘Recreation and culture’ basket as our measure of recreation consumption. This category includes items such as audio-visual and photographic equipment, musical instruments, toys, pets, admission to movies, theaters, concerts, etc.\(^{31}\) Panel (b) shows that measure for a selected group of countries, and we include the same figure but for all countries in Appendix C.2.1. While there is some variation across countries, the recreation shares stay fairly constant over time, in line with our modeling assumption.

\(\text{(a) Recreation consumption share: United States} \quad \text{(b) Recreation consumption share: International sample}\)

Panel (a): Fraction of recreation consumption in total consumption for the United States. Source: NIPA and CE Surveys. Panel (b): Fraction of recreation consumption in total consumption for a selected group of countries. Source: OECD.

**Figure 7:** Income, consumption, and recreation consumption.

### 3.2 Problem of the household

We now turn to the description of our model economy. At the heart of our analysis is a household — representative or else — that maximizes some period utility function \(u\). Our main mechanism operates through the impact of cheaper recreation goods and services on labor supply decisions. We therefore include these items, denoted by \(d\), directly into \(u\). The utility function also depends on the consumption of other goods and services \(c\), and on the amount of time worked \(h\). Since it plays a central role, we keep the utility function as general as possible, only assuming that it be consistent with a balanced-growth path — the benchmark macroeconomic framework for long-run analysis. We will show below that this assumption imposes some structure on the shape of the utility function.

Importantly for our mechanism, the utility function is free to feature some complementarity

\(^{31}\)Since the consumption categories are not as fine as the ones available from the NIPA tables, we cannot exclude information processing equipment and computers are therefore counted as recreation in this measure.
between leisure time and recreation consumption, such that, for instance, the purchase of a subscription to an online streaming service can make leisure time more enjoyable, which then pushes the household to work less. It follows that with such a complementarity a decline in recreation prices can lead to a decline in work hours.

The household maximizes its lifetime discounted utility

$$\sum_{t=0}^{\infty} \beta^t u(c_t, h_t, d_t),$$

subject to a budget constraint

$$c_t + p_{dt} d_t + a_{t+1} = w_t h_t + a_t (1 + r_t),$$

where \(w_t\) denotes the wage, \(p_{dt}\) the price of recreation goods, \(r_t\) the interest rate, and \(a_{t+1}\) the asset position of the household at the end of period \(t\).\(^{32}\) Since time worked \(h_t\) is constrained by the size of the (normalized) time endowment, we assume \(h_t \leq 1\), but we focus on interior solutions so this inequality never binds.

The household chooses \(\{c_t, d_t, h_t, a_{t+1}\}\) while taking the prices \(\{w_t, p_{dt}, r_t\}\) as given. On a balanced-growth path the prices \(\{w_t, p_{dt}\}\) grow at constant rates, and the interest rate \(r_t > 0\) remains constant. We therefore assume that \(p_{dt} = \gamma_{p_d} p_{d0}\) and \(w_t = \gamma_{w} w_0\), where \(\gamma_{p_d} > 0\) and \(\gamma_{w} > 0\) are exogenous growth rates, and \(p_{d0}\) and \(w_0\) are initial conditions. In Appendix (B), we provide a potential microfoundation for the growth rates \(\gamma_{w}\) and \(\gamma_{p_d}\) that involves the production sector of the economy.

On a balanced-growth path, \(c_t, d_t\) and \(h_t\) also grow at constant (endogenous) rates, denoted by \(g_c, g_d\) and \(g_h\). Since hours worked are naturally bounded by the time endowment we focus on the case in which \(g_h \leq 1\). These growth rates might depend, in turn, on the growth rates of the fundamentals \(\gamma_{w}\) and \(\gamma_{p_d}\), and perhaps on other features of the economy. The budget constraint of the household imposes some restrictions on these endogenous growth rates. For (5) to be satisfied in every period, each term must grow at the same rate and it must therefore be that

$$g_c = \gamma_{p_d} g_d = \gamma_{w} g_h.$$

\(^{32}\)The model uses non-recreation consumption as the numeraire. However, a price index for these items is not readily available for all the countries in our sample, so in our empirical exercises we normalize nominal terms by all-item price indices. The discrepancy between the two is unlikely to be large because recreation expenditures typically account for less than 10% of the overall consumption spending. Appendix ?? shows that the all-item and non-recreation price indices for the four U.S. Census regions are very similar to each other.
3.3 Balanced-growth path preferences

Another set of restrictions on the growth rates comes from the preferences of the household. For instance, under the utility function introduced by King et al. (1988), hours worked $h_t$ must remain constant over time which implies that consumption and the wage grow at the same rate: $g_c = \gamma_w$. Boppart and Krusell (2020) generalize these preferences to let hours worked grow on a balanced-growth path and the growth rate of consumption can take the more general form $g_c = \gamma_w^{1-\nu}$, where $\nu$ is a parameter of the utility function. In our case, the growth rate of consumption might also be affected by the growth rate of recreation prices, $\gamma_{pd}$, and we therefore consider the more general form

$$g_c = \gamma_w^{\eta} \gamma_{pd}^{\tau},$$

where $\eta$ and $\tau$ are constants that have to be determined.

We can combine equations (6) and (7) to characterize the growth rates of all the endogenous quantities in terms of the constants $\eta$ and $\tau$ such that

$$g_c = \gamma_w^{\eta} \gamma_{pd}^{\tau},$$
$$g_h = \gamma_w^{\eta - 1} \gamma_{pd}^{\tau},$$
$$g_d = \gamma_w^{\eta - 1} \gamma_{pd}^{-1}.$$  

(8)

Given these restrictions, we can formally define the properties of a utility function that is consistent with a balanced-growth path in this economy.33

Definition 1 (Balanced-growth path preferences). The utility function $u$ is consistent with a balanced-growth path if it is twice continuously differentiable and has the following properties: for any $w > 0, p > 0, c > 0, \gamma_w > 0$ and $\gamma_{pd} > 0$, there exist $h > 0, d > 0$ and $r > -1$ such that for any $t$

$$u_h \left( c \left( \gamma_w^{\eta} \gamma_{pd}^{\tau} \right)^t, h \left( \gamma_w^{\eta - 1} \gamma_{pd}^{\tau} \right)^t, d \left( \gamma_w^{\eta - 1} \gamma_{pd}^{-1} \right)^t \right) = w \gamma_w^t,$$

(9)

$$u_c \left( c \left( \gamma_w^{\eta} \gamma_{pd}^{\tau} \right)^t, h \left( \gamma_w^{\eta - 1} \gamma_{pd}^{\tau} \right)^t, d \left( \gamma_w^{\eta - 1} \gamma_{pd}^{-1} \right)^t \right) = p \gamma_{pd}^t,$$

(10)

33The following definition is a generalization of Assumption 1 in Boppart and Krusell (2020). We focus on the case $\eta > 0$ and $\tau > 0$. From (8) we see that when $\eta < 0$ higher wage growth leads to lower consumption growth. When $\tau < 0$ higher growth in the price of recreation goods leads to smaller growth in hours. We therefore focus on the parameterizations that are more empirically relevant.
and
\[
\frac{u_c \left( c \left( \gamma_{\eta \gamma_p d}^\eta \right)^t, h \left( \gamma_{\eta - 1 \gamma_p d}^\eta - \gamma_{\eta - 1 \gamma_p d}^\eta \right)^t, d \left( \gamma_{\eta \gamma_p d}^\eta - 1 \right)^t \right)}{u_c \left( c \left( \gamma_{\eta \gamma_p d}^\eta \right)^{t+1}, h \left( \gamma_{\eta - 1 \gamma_p d}^\eta - \gamma_{\eta - 1 \gamma_p d}^\eta \right)^{t+1}, d \left( \gamma_{\eta \gamma_p d}^\eta - 1 \right)^{t+1} \right)} = \beta \left( 1 + r \right),
\]
(11)
where \( \eta > 0 \) and \( \tau > 0 \).

These equations are the usual first-order conditions of the household. The first one states that the marginal rate of substitution between hours \( h_t \) and consumption \( c_t \) must equal the wage \( w_t \), the second equation states that the marginal rate of substitution between leisure goods \( d_t \) and consumption \( c_t \) must equal the price of leisure goods \( p_t \), and the third equation is the intertemporal Euler equation. Definition 1 imposes that these optimality conditions must be satisfied in every period \( t \), starting from some initial point \( \{ c, h, d, p, w \} \) and taking into account the respective growth rates of each variable provided by (8).

The following proposition describes the class of utility functions that are consistent with a balanced-growth path.

**Proposition 1.** The utility function \( u(c, h, d) \) is consistent with a balanced-growth path (Definition 1) if and only if (save for additive and multiplicative constants) it is of the form
\[
u(c, h, d) = \left( \frac{c^{1-\varepsilon} d^\varepsilon v \left( c^{1-\eta - \tau h^\eta h^\tau} d^\tau \right)^{1-\sigma} - 1}{1 - \sigma} \right),
\]
(12)
for \( \sigma \neq 1 \),
\[
u(c, h, d) = \log \left( e^{1-\varepsilon} d^\varepsilon \right) + \log \left( v \left( c^{1-\eta - \tau h^\eta h^\tau} d^\tau \right) \right),
\]
(13)
for \( \sigma = 1 \), and where \( v \) is an arbitrary twice continuously differentiable function and where \( \eta > 0 \) and \( \tau > 0 \).

**Proof.** The proof is in Appendix D. \( \square \)

This proposition establishes necessary and sufficient conditions on the shape of \( u \) so that it is consistent with a balanced-growth path. They are the only restrictions that we impose on the utility function, such that our analysis remains general and does not hinge on a particular choice of \( u \).\(^{34}\)

Of course, several utility functions that satisfy (12)–(13) make little economic sense. Additional restrictions would need to be imposed so that, for instance, \( u \) is increasing in \( c \) and decreasing in \( h \). But we do not need to explicitly specify these restrictions. For our analysis to hold, we only need that the household maximizes some version of (12)–(13), and that the first-order conditions

\(^{34}\)Proposition 1 extends Theorems 1 and 2 in Boppart and Krusell (2020). These theorems establish necessary and sufficient conditions on the shape of \( u \) for consistency with a balanced-growth path in an environment without recreation goods.
are necessary to characterize its optimal choice.

Several utility functions that have been used in the literature are nested in (12)–(13). For instance, the standard balanced-growth preferences of King et al. (1988) in which labor remains constant can be obtained by setting $\varepsilon = 0$, $\tau = 0$ and $\eta = 1$. To allow for a nonzero income effect of rising wages on the labor supply, we can instead set $\varepsilon = 0$ and $\tau = 0$ and $\eta \neq 0$ to get the preferences of Boppart and Krusell (2020).

3.4 The impact of $w$ and $p_d$

Proposition 1 also shows that the constants $\eta$ and $\tau$ introduced as placeholders in (7) can come directly from the utility function. As such, they do not depend on other (perhaps endogenous) economic variables whose presence might lead to endogeneity issues in our estimation. From (8), we therefore have a system of three equations

$$
\begin{align*}
\log g_c &= \eta \log \gamma_w + \tau \log \gamma_{p_d}, \\
\log g_d &= \eta \log \gamma_w + (\tau - 1) \log \gamma_{p_d}, \\
\log g_h &= (\eta - 1) \log \gamma_w + \tau \log \gamma_{p_d}.
\end{align*}
$$

(14)

to be estimated in the following section.

These equations show that the log of the growth rates of the endogenous variables $c_t$, $d_t$ and $h_t$ are linear relationships in the log of the growth rates of the exogenous variables $w_t$ and $p_{dt}$, and that the preference parameters $\eta$ and $\tau$ characterize these relationships. These parameters capture the intensity of standard income and substitution effects, triggered by changes in prices, that are at work in the model.

The third equation plays a central role in our exploration of the causes behind the decline in hours worked. The first term on its right-hand side captures how rising wages affect the supply of labor. When $\eta - 1 < 0$, higher wages lead to more leisure through a standard income effect: richer households substitute consumption with leisure. When instead $\eta - 1 > 0$, the substitution effect dominates and the household takes advantage of the higher wage rate to work more and earn more income. The second term on the right-hand side of the equation captures the impact of falling recreation prices on labor supply. When $\tau > 0$, a decline in the price of recreation goods

---

35 Our analysis goes through even if the utility function (12)–(13) is not concave. In this case, the first-order conditions are not sufficient to characterize a solution to the household’s optimization problem but they are still necessary. As a result, they are satisfied at the household’s optimal decision and we can use them to characterize the balanced-growth path.

36 Our preferences, however, do not nest some utility functions that have recreation goods and services as an input. For instance, the preferences used by Kopecky (2011) and Vandenbroucke (2009) do not allow for a balanced-growth path and are therefore not a special case of (12)–(13).

37 We can compute the Frisch elasticity of labor supply associated with the utility function (12)–(13) and show that it is constant along the balanced-growth path. Although it is not, in general, only a function of the parameters of the utility function.
and services incentivizes the household to enjoy more leisure and work less.

Overall, the results of this section provide a clear path to empirically evaluate the importance of the decline in recreation prices on hours worked. From (14), we know that $g_c$, $g_d$ and $g_h$ are related log-linearly to $\gamma_w$ and $\gamma_p$, so that we can estimate these relationships readily through standard techniques. Furthermore, these relationships are structural, so that we can be sure that our estimation captures deep parameters that are unaffected by changes in policy. Proposition 1 also shows that the relationship between hours worked and leisure prices is invariant to various features of the utility function, such as the function $v$ and the parameter $\varepsilon$. As a result, we can be confident that our empirical strategy is robust to a broad class of utility functions. Finally, our analysis does not hinge on a particular set of assumptions about the production sector of the economy, as long as $w_t$ and $p_{dt}$ grow at constant rates. As such, it is robust to different production technologies, market structures, etc. We nonetheless provide in Appendix B an example of a production structure that provides a microfoundation in which the constant growth rates $\gamma_w$ and $\gamma_p$ depend on underlying productivity growth in the non-recreation and recreation sectors.

4 Measuring the impact of the decline in leisure prices

We now turn to the structural estimation of the model described in the previous section. Our focus is on the system of equations (14), which relates the growth rates of hours, recreation consumption and non-recreation consumption to the growth rates of wages and recreation prices. The advantage of focusing on this system of equations is that it allows us to impose the key restrictions implied by the structural model without having to provide the complete description of the economy, i.e. the full specification of preferences, technology, etc. We estimate the model using the household-level data from the United States that we introduced in Section 2. To highlight the robustness of our results, we use various specifications and different sets of controls. In all cases, we find a strong impact of the decline of recreation prices on work hours.

4.1

4.2 Estimation

4.2.1

We estimate the system of equation

---

38 Note also that the third equation in (14) justifies our use of a linear specification in the previous section.

39 Since our theoretical model does not distinguish between the extensive and intensive margins of labor supply, we focus on total hours per capita as our main measure of work hours throughout this section. We show in Appendices ?? and ?? that our results are robust to using hours per worker instead.
\[
\begin{align*}
\Delta \log c_g &= \alpha_c + \eta \Delta \log w_{gl} + \tau \Delta \log p_g + \epsilon_g^c, \\
\Delta \log d_g &= \alpha_d + \eta \Delta \log w_{gl} + (\tau - 1) \Delta \log p_g + \epsilon_g^d, \\
\Delta \log h_{gl} &= \alpha_h + (\eta - 1) \Delta \log w_{gl} + \tau \Delta \log p_g + \epsilon_{gl}^h,
\end{align*}
\]
(15)

where \( \Delta \log x_{gl} \) denotes the log growth rate of a variable \( x \) for households in an age-education group \( g \) in location \( l \) between 1990 and 2016. As in Section 2.1.2, we instrument for the wage by using a Bartik-like instrument that captures how aggregate changes in industry-level wages affects labor earning in different location. Similarly, we take advantage of differences in the composition of recreation consumption baskets across households with different education levels and ages to construct an instrument for recreation prices.

Notice that non-recreation consumption, recreation consumption and recreation prices do not vary across locations due to limitations in the CE data. We estimate the system (15) via a two-step generalized method of moments. We do not include additional controls because otherwise the estimation procedure does not converge.

Estimating the system of equation (15) has several advantages. First, the estimation only relies on cross-sectional variation to identify the effect of recreation prices and wages on hours worked. Aggregate shocks such as technological changes, openness to trade, increased reliance on outsourcing should therefore have a limited effect on the estimation. Second, because it uses instruments, the estimation is less susceptible to endogeneity issues. Third, the system of equation (15) has been derived from a structural model which lessens worries about misspecification. The estimated coefficients are also deep parameters of the model can be thought of as exogenous. Fourth, the system (15) exploits cross-equation restrictions to further discipline the estimation. For instance, in contrast to the regressions of Table 1, consumption data helps to pin down the coefficients that relate changes in recreation prices and wages to hours work.

The estimated coefficients \( \tau \) and \( \eta - 1 \) are presented in Table 2 along with their 90% confidence interval between parenthesis. Column 1 shows the estimates without instruments while column 2 shows the outcome of the instrumental variable estimation. In both cases, we find that the \( \tau \) coefficients are significantly above zero and close to 0.3 in terms of magnitudes. From the third equation in (14), this suggests that as recreation prices decline, households prefer to enjoy more leisure and therefore hours fall, between about a quarter and a third of a percent for each one percent decrease in prices (depending on the specification). As a result, our estimation finds that the secular decline in recreation prices might have been a contributor to the decline in work hours that we documented earlier.

We also find that in both columns, the parameter \( \eta - 1 \) is estimated to be significantly negative, although its size is somewhat smaller with the instruments. Going back once again to third equation

\[40\text{Redoing the estimation without instruments delivers similar results.}\]
in (14), this implies that higher wage growth leads to smaller growth in work hours. In other words, the preferences of the household are such that the income effect dominates. Finally, in the last two rows of Table 2 we test whether the cross-equation restrictions imposed by the model are valid, and find that we cannot reject their validity at the 5% threshold when using the instruments, which is reassuring for our modeling strategy.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>τ</td>
<td>0.36</td>
<td>0.27</td>
</tr>
<tr>
<td></td>
<td>(0.29, 0.43)</td>
<td>(0.18, 0.37)</td>
</tr>
<tr>
<td>η − 1</td>
<td>−0.63</td>
<td>−0.18</td>
</tr>
<tr>
<td></td>
<td>(−0.64, −0.61)</td>
<td>(−0.34, −0.03)</td>
</tr>
<tr>
<td>αₜ</td>
<td>0.20</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>(0.17, 0.23)</td>
<td>(0.10, 0.19)</td>
</tr>
<tr>
<td>Instruments</td>
<td>N</td>
<td>Y</td>
</tr>
<tr>
<td>J-statistic</td>
<td>14.37</td>
<td>4.63</td>
</tr>
<tr>
<td>p-value</td>
<td>0.006</td>
<td>0.328</td>
</tr>
</tbody>
</table>

Estimates from a two-step GMM procedure with instrument variables (see Section ?? for the definition of the instruments). Weight matrix accounts for arbitrary correlation within education-age groups. 90% confidence intervals are reported in parentheses. In columns (1) and (3), we do not use our Bartik-like instruments for wages and recreation prices. In columns (2) and (4), they are used. In columns (3) and (4), the variables are demeaned on the regional level prior to estimation. The last two rows report results of a test of the validity of over-identifying restrictions (Hansen’s J-statistic and its p-value).

Table 2: GMM estimation of the system (15) using instruments.

We can perform some back-of-the-envelope calculations to have a sense for the magnitudes involved by the estimates of Table 2. From Table 5, we see that the annual growth rate of wages has been 1.05% in the U.S. and that the equivalent number for recreation prices is −0.7%. If we take the instrumental variables coefficients from Table 2, our results suggest that wage growth has pushed for a decline in the growth rate of hours of about 1.05% × 0.18 ≈ 0.19% per year. Similarly, the decline in recreation prices can account for a decline in the growth of hours of about 0.7% × 0.27 ≈ 0.19% per year. Based on these calculations, the recreation channel has been about as important as the income effect as a driver of the decline in work hours in the United States.

Put together, these two channels would suggest that the average annual growth rate of work hours should be about −0.38%, more than the actual annual movement in hours per capita (0.02%) observed since 1950 and reported in Table 5. What explains this discrepancy? Clearly, the intercept αₜ in 2 plays a non-trivial role, capturing for instance the entry of women into the labor force. We can filter out that effect by looking at male employment only. From Figure 1a, we see that male hours per capita have gone down by about 0.4% per year since 1962. From the CPS, we find that the median real weekly earnings for males have been essentially unchanged since 1979, so that wage growth had approximately no impact on male labor supply decisions over that period. In this case, the predicted impact of the decline in recreation prices (0.19% per year) can explain about half of the decline in male work hours.
4.3 Discussion and robustness

To better understand the role of wages for hours worked, it is important to note that the United States and most of the countries in our sample are relatively rich. Households in poorer countries, where people might need to work to purchase basic necessities, might behave differently. Bick et al. (2018) present evidence that goes in that direction. They show that average hours worked is larger in poor countries, suggesting that some income effect might indeed be at work. They also show, however, that the correlation between individual incomes and hours worked is essentially zero, or even positive, in rich countries, suggesting that this income effect might taper off as households become richer. That evidence, and ours, is consistent with a subsistence level in the utility function that pushes poor households to work extensively to afford basic food and lodging. For higher levels of income that subsistence level no longer affects decisions and the income effect vanishes. As a result, rising wages might be a powerful motivator for longer work hours in poor countries but might have only a limited impact in richer nations. In contrast, our evidence suggests that declining recreation prices remain an important driver of work hours even in richer countries. As aggregate incomes increase worldwide, it might further gain in relative importance in the future.

5 Conclusion

We analyze the role of the declining prices of recreation goods in driving the downward trend in hours worked over the recent decades, both in the U.S. and across the OECD countries. We provide a general specification of preferences that are consistent with balanced growth, and show that they imply a set of cross-equation restrictions on the growth rates of wages, recreation good prices, labor hours, and consumption of recreation as well as non-recreation goods. Taking these to the data we find that most of the decline in hours worked in the U.S. can be attributed to the falling price of recreation goods, with at best a limited impact of rising wages.

While we focus on the choice between supplying labor and enjoying leisure, the reality of household time use is surely more complex. An important branch of the literature has paid particular attention to the role of home production. Much of it argues that the increased productivity of market work relative to non-market work, as well as the reduction in the price of goods such as household appliances, have pushed towards an increase in market hours and, in particular, to the entry of women into the labor force (Greenwood and Vandenbroucke, 2005). At the same time, recent evidence points to the growing importance of spending time with children, primarily among highly-educated households (Guryan et al., 2008; Ramey and Ramey, 2010; Dotti Sani and Treas, 2016). Accounting for these mechanisms should provide a more complete picture of the forces affecting labor supply.
Finally, recent evidence by Aguiar et al. (2017) shows that young men increasingly stay at home to play video games instead of working or attending school. Our evidence together with theirs suggests that declining recreation prices might disincentivize human capital accumulation, and thus slow down the movement towards a more highly-skilled workforce. Introducing this mechanism into macroeconomic models of skill acquisition, such as Kopytov et al. (2018), might improve their performance in matching the employment data. Exploring these forces in detail is an exciting avenue for future research.
References


CARTER, S. B., S. S. GARTNER, M. R. HAINES, A. L. OLMSTEAD, R. SUTCH, G. WRIGHT,


Appendices

A Data

This appendix contains the detailed data sources and the steps that we took to construct the datasets.

A.1 United States data

A.1.1 Data used in main analyses

**Bureau of Labor Statistics**  Price data is from the Bureau of Labor Statistics (BLS). All-item Consumer Price Index (CPI) series are encoded as ‘CUUR0000SA0’. This series is used as a deflator for all nominal variables. Recreation CPI series are encoded as ‘CUUR0000SAR’ and are available starting from 1998. Before 1998, we use the price indices for the ‘Entertainment’ group, encoded as ‘MUUR0000SA6’, which are available between 1978 and 1997. For our cross-sectional analysis in Section 2.1.2, we construct price indices for seven subcategories of recreation goods and services, available at the national level. The key difficulty is that the BLS changed their classification of goods and services in 1993, which particularly affected the recreation group. We try to map pre- and post-1993 price series as close as possible to ensure consistency over time. Table 3 shows price items that we use in the pre- and post-1993 periods. For a few subcategories (Other goods, Pets, Photo, Reading, Sports), we use price series that were not changed in 1993 at all and, thus, are available for the entire sample.\(^{41}\) Despite it does not seem that there were any major changes in the “Other services” subcategory, there is no unique price series that covers the entire sample. We, therefore, smoothly past price indices ‘SE62’ (pre-1993) and ‘SERF’ (post-1993). For ‘Audio-video’ in the pre-1993 sample, we aggregate ‘SE31’ (video and audio products) with ‘SE2703’ (cable television) using corresponding consumption shares from the CE Surveys. We smoothly paste the resulting series with ‘SERA’ (post-1993) to get the price series over the entire sample.

**Annual Social and Economic Supplement**  For Figure 1, we use the ASEC dataset to construct hours. Following Cociuba et al. (2018), we compute average weighted annual hours worked using the variable ‘ahrsworkt’ within each region (variable ‘region’ is used to identify which of the four U.S. Census Bureau-designated regions agents reside). To construct hours per worker, we consider only currently employed agents that were at work last week (‘empstat’=10).

\(^{41}\)In the post-1993 period, some of these subcategories receive a few new items (for example, veterinary services were added to the ‘Pets’ subcategory, encoded by ‘SERB02’). We do not include these new additions to make price indices as comparable across the pre- and post-1993 periods as possible.
Table 3: Prices of recreation goods and services.

<table>
<thead>
<tr>
<th>Category</th>
<th>Pre-1993 code</th>
<th>Post-1993 code</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Audio-video</td>
<td>SE31 and SE2703</td>
<td>SERA</td>
<td>SE31: Video and audio products</td>
</tr>
<tr>
<td>Other goods</td>
<td>SE6101</td>
<td>SERE01</td>
<td></td>
</tr>
<tr>
<td>Other services</td>
<td>SE62</td>
<td>SERF</td>
<td></td>
</tr>
<tr>
<td>Pets</td>
<td>SE6103</td>
<td>SERB01</td>
<td></td>
</tr>
<tr>
<td>Photo</td>
<td>SE6102</td>
<td>SERD01</td>
<td></td>
</tr>
<tr>
<td>Reading</td>
<td>SE59</td>
<td>SERG</td>
<td></td>
</tr>
<tr>
<td>Sports</td>
<td>SE60</td>
<td>SERC</td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Recreation consumption subcategories.

Consumer Expenditure Survey For consumption categories, we follow Aguiar and Bils (2015) as close as possible, so we refer the reader to their data construction section for a detailed description. Relative to Aguiar and Bils (2015), an important difference in our analysis is that we construct recreation consumption for seven different subcategories. In the CE, the consumption categories are coded using Universal Classification Codes, UCCs. Table 4 shows the UCCs corresponding to the seven recreation consumption subcategories.

<table>
<thead>
<tr>
<th>Category</th>
<th>Universal Classification Codes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Audio-video</td>
<td>270310, 270311, 310110-310350, 310400, 340610, 340902, 340905, 610130, 620904, 620912, 620930, 620916-620918</td>
</tr>
<tr>
<td>Other goods</td>
<td>610110, 610140, 610120, 610130</td>
</tr>
<tr>
<td>Other services</td>
<td>610900-620111, 620121-620310, 620903</td>
</tr>
<tr>
<td>Pets</td>
<td>610320, 620410, 620420</td>
</tr>
<tr>
<td>Photo</td>
<td>610210, 620330, 620906, 610230, 620320</td>
</tr>
<tr>
<td>Reading</td>
<td>660310, 590110-590230, 590310, 590410, 690118</td>
</tr>
<tr>
<td>Sports</td>
<td>520901, 520904, 520907, 600131, 600132, 600141, 600142, 600110-600122, 600210-609999, 620906-620909, 620919-620922, 620902, 600127, 600128, 600137, 600138</td>
</tr>
</tbody>
</table>

Similarly to Aguiar and Bils (2015), we consider only households with reference persons of ages between 25 and 64 that completed 4 quarterly interviews within a year. We exclude households with extremely large expenditure shares on generally small consumption categories. We exclude households with nonzero wage and salary income (‘FSALARYX’) and zero hours (‘INC_HRS1’ multiplied by ‘INCWEEK1’ plus ‘INC_HRS2’ multiplied by ‘INCWEEK2’). We also exclude households...
with zero wage and salary income and nonzero hours. To construct consumption baskets across age-education groups, we use age and education of reference persons.

**United States Census and American Community Survey** Hours are measured as ‘UHR-SWORK’ multiplied by ‘WKSWORK1’. When ‘WKSROWK1’ is unavailable (the ASC sample of 2014-2018), we use projected values of ‘WKSWORK2’ on ‘WKSWORK1’. Measure of wage is ‘INCWAGE’. Geographic regions are constructed using cross-walk files from the David Dorn’s website (https://www.ddorn.net/data.htm). Industry classification is based on ‘IND1990’ and includes 34 industries: Agriculture, Forestry, and Fisheries; Mining; Construction; Manufacturing (19 subcategories); Transportation; Communications; Utilities and Sanitary Services; Wholesale Trade (2 subcategories); Retail Trade; Finance, Insurance, and Real Estate; Business and Repair Services; Personal Services; Entertainment and Recreation Services; Professional and Related Services; Public Administration.

**A.1.2 Data used to construct long-run series**

**Prices** Early data on real recreation prices comes from Owen (1970) (Table 4-B, pages 85-86, the data covers the period between 1901 and 1961). The data between 1935 and 1970 is from Bureau of the Census (1975) (page 210, column ‘Reading and recreation’ divided by column ‘All items’). Between 1967 and 1997, data on recreation prices comes from BLS (series ‘MUUR0000SR6’). Starting from 1993, BLS provides a new series on recreation prices, encoded as ‘CUUR0000SAR’. The BLS data is deflated using the all-item CPI series, encoded as ‘CUUR0000SA0’.

**Hours, wages and population** Early data on average weekly hours is from Bureau of the Census (1975) (series ‘D765’ and ‘D803’). For the postwar sample, the data is available from FRED of St. Louis Fed (series ‘PRS85006023’). Early data on total hours worked is from Kendrick et al. (1961) (table A-X) and Kendrick et al. (1973) (table A-10). Early data on population by age is from Carter et al. (2006) (Table Aa125-144; we focus on population of 14 years or older). Recent data on hours worked and population is from ASEC. Early data on labor productivity (wages) is from Kendrick et al. (1961) (table A-I; real gross national product, normalized by hours worked). From 1929, FRED provides data on compensation of employees (series ‘A033RC1A027NBEA’), which we normalize by total hours worked and CPI (FRED series ‘CPIAUCNS’).

**Consumption and labor income** To construct figures in Section 3.1, we use data from the NIPA tables. Consumption data is from Table 2.5.5 “Personal Consumption Expenditures by Function”. Recreation consumption is the sum of rows 75, 77, 78, 82, 90, 91, 92, 93, 94. We subtract \( \frac{\text{row 76}}{\text{row 75+row 76}} \times \text{row 77} \) to exclude a computer-related component from row 77 (“Services related to video and audio goods and computers”). Total consumption expenditures is row 1. Data
on personal income is from Table 2.1 “Personal Income and Its Disposition”. We use row 1 (total personal income) and row 2 (compensation of employees).

A.2 Cross-country data

Our final sample includes 38 countries: Australia, Austria, Belgium, Canada, Colombia, Croatia, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Israel, Italy, Japan, Korea, Latvia, Lithuania, Luxembourg, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Slovakia, Slovenia, South Africa, Spain, Sweden, Switzerland, Turkey, U.K., U.S.A.

Prices For the majority of countries, the price data is from the OECD database, category “Prices and Purchasing Power Parities”. For a few countries, longer price series are obtained from different sources. The U.S. price data is described above. For Australia, the data comes from the Australian Bureau of Statistics, Catalogue Number 6401.0. For Canada, the data comes from Statistics Canada, Table 18-10-0005-01. For a few European countries (Austria, Belgium, Croatia, Denmark, Estonia, Finland, Greece, Hungary, Iceland, Poland, Slovakia, Slovenia, Spain, Turkey), the data comes from the Eurostat’s Harmonized Index of Consumer Prices (HICP) dataset. This data usually covers slightly longer sample than the OECD database.

Other data Data on hours worked is from the Conference Board Total Economy Database. Consumption data is from the OECD. Total consumption expenditure is encoded as ‘P31DC’, recreation consumption is encoded as ‘P31CP090’. To obtain non-recreation consumption, we subtract recreation consumption from total consumption. Compensation of employees is from the OECD, series encoded as ‘D1’. GDP is from the OECD, series encoded as ‘B1_GA’. All nominal series are deflated by all-item CPIs. Population data is from the OECD (‘Demography and population’ category). We focus on population between 15 and 64 years of age.

Summary Table Table 5 provides summary statistics for our multi-country dataset.
<table>
<thead>
<tr>
<th>Country</th>
<th>Hours Growth rate [%]</th>
<th>Starting year</th>
<th>Real wages Growth rate [%]</th>
<th>Starting year</th>
<th>Real recreation prices Growth rate [%]</th>
<th>Starting year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>-0.05</td>
<td>1950</td>
<td>1.54</td>
<td>1960</td>
<td>-1.41</td>
<td>1989</td>
</tr>
<tr>
<td>Austria</td>
<td>-0.30</td>
<td>1950</td>
<td>1.61</td>
<td>1970</td>
<td>-1.20</td>
<td>1996</td>
</tr>
<tr>
<td>Belgium</td>
<td>-0.42</td>
<td>1950</td>
<td>1.54</td>
<td>1970</td>
<td>-1.19</td>
<td>1996</td>
</tr>
<tr>
<td>Canada</td>
<td>-0.14</td>
<td>1950</td>
<td>0.90</td>
<td>1970</td>
<td>-0.95</td>
<td>1950</td>
</tr>
<tr>
<td>Colombia</td>
<td>0.30</td>
<td>1960</td>
<td>2.06</td>
<td>2001</td>
<td>-1.42</td>
<td>2009</td>
</tr>
<tr>
<td>Costa Rica</td>
<td>-0.07</td>
<td>1987</td>
<td>2.31</td>
<td>1993</td>
<td>-3.56</td>
<td>1995</td>
</tr>
<tr>
<td>Czech Republic</td>
<td>-0.03</td>
<td>1993</td>
<td>2.44</td>
<td>1993</td>
<td>-1.56</td>
<td>1995</td>
</tr>
<tr>
<td>Denmark</td>
<td>-0.50</td>
<td>1950</td>
<td>1.85</td>
<td>1967</td>
<td>-1.42</td>
<td>1996</td>
</tr>
<tr>
<td>Finland</td>
<td>-0.64</td>
<td>1950</td>
<td>1.21</td>
<td>2000</td>
<td>-1.12</td>
<td>2000</td>
</tr>
<tr>
<td>France</td>
<td>-0.84</td>
<td>1950</td>
<td>3.02</td>
<td>1955</td>
<td>-1.81</td>
<td>1990</td>
</tr>
<tr>
<td>Germany</td>
<td>-0.85</td>
<td>1950</td>
<td>1.98</td>
<td>1970</td>
<td>-1.02</td>
<td>1991</td>
</tr>
<tr>
<td>Greece</td>
<td>-0.23</td>
<td>1950</td>
<td>1.70</td>
<td>1970</td>
<td>-1.31</td>
<td>1996</td>
</tr>
<tr>
<td>Hungary</td>
<td>-0.80</td>
<td>1980</td>
<td>1.84</td>
<td>1995</td>
<td>-1.72</td>
<td>1996</td>
</tr>
<tr>
<td>Iceland</td>
<td>-0.38</td>
<td>1964</td>
<td>2.45</td>
<td>1977</td>
<td>-1.23</td>
<td>1996</td>
</tr>
<tr>
<td>Ireland</td>
<td>-0.63</td>
<td>1950</td>
<td>2.71</td>
<td>1976</td>
<td>-0.40</td>
<td>1983</td>
</tr>
<tr>
<td>Israel</td>
<td>0.55</td>
<td>1981</td>
<td>1.01</td>
<td>1995</td>
<td>-1.74</td>
<td>1985</td>
</tr>
<tr>
<td>Italy</td>
<td>-0.31</td>
<td>1950</td>
<td>1.28</td>
<td>1970</td>
<td>-0.90</td>
<td>1996</td>
</tr>
<tr>
<td>Japan</td>
<td>-0.23</td>
<td>1950</td>
<td>1.78</td>
<td>1971</td>
<td>-0.57</td>
<td>1970</td>
</tr>
<tr>
<td>Korea</td>
<td>-0.12</td>
<td>1960</td>
<td>6.57</td>
<td>1972</td>
<td>-2.57</td>
<td>1985</td>
</tr>
<tr>
<td>Latvia</td>
<td>0.63</td>
<td>1996</td>
<td>5.43</td>
<td>1995</td>
<td>-2.04</td>
<td>1995</td>
</tr>
<tr>
<td>Lithuania</td>
<td>0.90</td>
<td>1995</td>
<td>4.91</td>
<td>1995</td>
<td>-2.28</td>
<td>1993</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>0.97</td>
<td>1970</td>
<td>2.18</td>
<td>1970</td>
<td>-0.47</td>
<td>1995</td>
</tr>
<tr>
<td>Mexico</td>
<td>0.33</td>
<td>1950</td>
<td>-0.86</td>
<td>1971</td>
<td>-1.30</td>
<td>2003</td>
</tr>
<tr>
<td>Netherlands</td>
<td>-0.26</td>
<td>1950</td>
<td>1.01</td>
<td>1969</td>
<td>-1.42</td>
<td>1996</td>
</tr>
<tr>
<td>New Zealand</td>
<td>-0.08</td>
<td>1970</td>
<td>0.59</td>
<td>1972</td>
<td>-2.41</td>
<td>2007</td>
</tr>
<tr>
<td>Norway</td>
<td>-0.38</td>
<td>1950</td>
<td>2.35</td>
<td>1970</td>
<td>-0.51</td>
<td>1979</td>
</tr>
<tr>
<td>Poland</td>
<td>0.37</td>
<td>1993</td>
<td>2.67</td>
<td>1993</td>
<td>-1.51</td>
<td>1996</td>
</tr>
<tr>
<td>Portugal</td>
<td>0.00</td>
<td>1950</td>
<td>1.62</td>
<td>1970</td>
<td>-1.03</td>
<td>1955</td>
</tr>
<tr>
<td>Slovak Republic</td>
<td>-0.34</td>
<td>1990</td>
<td>2.11</td>
<td>1993</td>
<td>-1.62</td>
<td>1996</td>
</tr>
<tr>
<td>Slovenia</td>
<td>0.11</td>
<td>1995</td>
<td>1.81</td>
<td>1995</td>
<td>-0.84</td>
<td>1996</td>
</tr>
<tr>
<td>South Africa</td>
<td>-0.84</td>
<td>2001</td>
<td>3.61</td>
<td>2001</td>
<td>-2.74</td>
<td>2008</td>
</tr>
<tr>
<td>Spain</td>
<td>-0.53</td>
<td>1950</td>
<td>1.73</td>
<td>1970</td>
<td>-1.94</td>
<td>1996</td>
</tr>
<tr>
<td>Sweden</td>
<td>-0.23</td>
<td>1950</td>
<td>1.72</td>
<td>1970</td>
<td>-1.68</td>
<td>1980</td>
</tr>
<tr>
<td>Switzerland</td>
<td>-0.26</td>
<td>1950</td>
<td>1.30</td>
<td>1970</td>
<td>-0.86</td>
<td>1983</td>
</tr>
<tr>
<td>Turkey</td>
<td>-0.75</td>
<td>1970</td>
<td>3.37</td>
<td>1998</td>
<td>-2.77</td>
<td>1996</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>-0.40</td>
<td>1950</td>
<td>1.90</td>
<td>1970</td>
<td>-1.64</td>
<td>1988</td>
</tr>
<tr>
<td>United States</td>
<td>0.02</td>
<td>1950</td>
<td>1.05</td>
<td>1971</td>
<td>-0.70</td>
<td>1950</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td><strong>-0.16</strong></td>
<td><strong>-0.35</strong></td>
<td><strong>2.19</strong></td>
<td></td>
<td><strong>-1.49</strong></td>
<td></td>
</tr>
</tbody>
</table>

Columns “Growth rate [%]” report log-linear trend coefficients. The series are available between the starting year given in the “Starting year” column and 2018. The earliest starting year is 1950—the first year for hours worked in the Total Economy Database.

**Table 5:** Summary statistics for multi-country sample.
B  Production side of the economy

Our empirical analysis relies on the system of equations (14). As such, it is agnostic about how
prices are determined in equilibrium as long as they grow at constant rates. In this section, we
provide one example of a production structure that delivers these constant rates, and show how
they depend on underlying productivity processes.

There are two competitive industries producing non-recreation and recreation goods \( c \) and \( d \)
using Cobb-Douglas technologies
\[
y_{jt} = A_{jt}^{\alpha}l_{jt}^{\alpha}k_{jt}^{1-\alpha},
\]
where \( j \in \{c,d\} \) denotes the industry, \( l_{jt} \) is labor, \( k_{jt} \) is capital and \( A_{jt} \) is Harrod-neutral total
factor productivity. Consistent with our balanced-growth path framework, we assume that
\( A_{jt} \) grows at an exogenous rate \( \gamma_{A_{j}} > 0 \) for \( j \in \{c,d\} \). Labor and capital are perfectly mobile across
industries and their prices are \( w_{t} \) and \( R_{t} \). Firms maximize profits
\[
\Pi_{jt} = p_{jt}y_{jt} - w_{t}l_{jt} - R_{t}k_{jt},
\]
where \( p_{jt} \) is the price of good \( j \) at time \( t \). As before, we use non-leisure consumption as the
numeraire so that \( p_{ct} = 1 \) for all \( t \), and the price of leisure goods \( p_{dt} \), the wage \( w_{t} \) and the interest
rate \( R_{t} \) are in units of non-leisure goods.

Investment goods are produced by a competitive industry using the production function \( y_{it} =
Ak_{it} \). Since these goods trade at a price \( p_{it} \), the investment sector maximizes profits
\[
\Pi_{it} = p_{it}Ak_{jt} - R_{t}k_{jt}.
\]
That sector is competitive such that \( p_{it}A = R_{t} \) in equilibrium.

Market clearing implies that the demand for leisure and non-leisure goods is equal to their
supply \( y_{jt} = c_{jt} \) for \( j \in \{c,d\} \). Similarly, the labor market clears, \( h_{t} = l_{ct} + l_{dt} \), and so does the
asset market \( a_{t} = K_{t} \). The total stock of capital \( K_{t} = k_{ct} + k_{lt} + k_{it} \) must also follow the law of
motion
\[
K_{t+1} = y_{it} + (1 - \delta)K_{t},
\]
where \( 0 < \delta < 1 \) is the depreciation rate. Finally, the market rate of returns on assets has to equal
the rental rate of capital net of depreciation, such that \( r_{t} = R_{t} - \delta \).

We can now define an equilibrium in this economy.

**Definition 2.** A dynamic competitive equilibrium, is a time path of household’s consumption,
hours worked and asset position \( \{c_{t},d_{t},h_{t},a_{t}\} \); a time path for prices, wages, returns on asset and
returns on capital \( \{p_{dt},p_{it},w_{t},r_{t},R_{t}\} \) and a time path of factor allocations \( \{l_{ct},l_{dt},k_{ct},k_{dt},k_{it}\} \) which
satisfies household and firm optimization, perfect competition, resources constraints and market
clearing.

The following proposition shows that, on a balanced-growth path, the growth rates of the leisure price $p_d$ and the wage $w_t$ are constant and linked to the growth rates of the productivity processes $A_c$ and $A_d$.

**Proposition 2.** On a balanced-growth path, the growth rates of $p_d$ and $w_t$ are

$$\log \gamma_{p_d} = \log \gamma_{A_c} - \log \gamma_{A_d},$$

$$\log \gamma_w = \alpha \log \gamma_{A_c}. \quad (17)$$

This proposition shows that, since $p_d$ is denominated in units of non-leisure goods, its growth rate captures how fast technological improvements occur in the leisure sector compared to the non-leisure sector. Similarly, productivity growth in the non-leisure sectors push wages higher.\(^{42}\)

Combining (17) with (14) provides the growth rates of $c$, $d$ and $h$ has a function of the primitives $\gamma_{A_c}$ and $\gamma_{A_d}$.

### C Additional empirical results

This appendix provides various robustness tests of the results in the body of the paper as well as several additional exercises

#### C.1 United States

**C.1.1 Derivation of equation (1)**

We show here how to derive equation (1) in Section 2.1.2. We start from the definition of wages in a locality $c$ for a demographic group $d$ at time $t$:

$$w_{glt} = \frac{\sum_i e_{igt}}{\sum_i h_{igt}}.$$

It follows that we can write the growth rate of wages as

$$\frac{w_{glt+1}}{w_{glt}} = \frac{\frac{\sum_i e_{igt+1}}{\sum_i h_{igt+1}}}{\frac{\sum_i e_{igt}}{\sum_i h_{igt}}} = \frac{\sum_i \frac{e_{igt+1}}{h_{igt+1}} \frac{e_{igt}}{h_{igt}}}{\sum_i \frac{h_{igt+1}}{h_{igt}} \frac{h_{igt}}{h_{igt}}}.$$

\(^{42}\)While $\gamma_{A_d}$ does not show up in the equation for $\gamma_w$, improvements in the leisure technology still lower $p_d$ which increases the purchasing power of each unit of the wage.
The key idea behind our instrumental strategy is to replace the local growth in earnings and hours in the equation above by their national equivalent. We therefore write, after taking the log,

\[ \Delta \log w_{IV}^{glt} = \log \left( \frac{w_{glt+1}}{w_{glt}} \right)^{IV} = \log \left( \sum_i \frac{e_{igt} - e_{igt+1}}{e_{igt}} \right) - \log \left( \sum_i \frac{h_{igt} - h_{igt+1}}{h_{igt}} \right) \]

We can also write that expression as

\[ \Delta \log w_{IV}^{glt} = \log \left( 1 + \sum_i \frac{e_{igt} - e_{igt+1}}{e_{igt}} \right) - \log \left( 1 + \sum_i \frac{h_{igt} - h_{igt+1}}{h_{igt}} \right) \approx \sum_i \frac{e_{igt}}{e_{igt+1}} \Delta \log e_{igt+1} - \sum_i \frac{h_{igt}}{h_{igt+1}} \Delta \log h_{igt+1} \]

where we have used the fact that \( \log (1 + x) \approx x \) and so

\[ \Delta \log x_{it+1} = \log x_{it+1} - \log x_{it} = \log \left( \frac{x_{it+1}}{x_{it}} \right) \approx \frac{x_{it+1} - x_{it}}{x_{it}}. \]

C.1.2 Recreation consumption share across education levels

Figure 8 shows how recreation consumption baskets vary by the level of education attainment of household heads. We do observe substantial variation, with households with low-educated heads consuming disproportionally more of “Audio-video” items, and households with highly-educated heads consuming disproportionally more of “Other services” items.

C.1.3 Analysis using household heads instead of all individuals

In the baseline analysis, our measures of wages and hours from the Census are at the individual level. The CE data, however, is at the household level, and we use the demographic characteristics of reference persons to construct demographic-specific consumption baskets. In this Appendix, we construct measures of hours and wages using the Census data on the household heads only (variable ‘RELATE’=1). To control for potentially very different consumption and labor supply choices across married and non-married household heads, we run regressions for all and married only household heads separately. Demographic controls include the 1980 shares of male, white, household heads with disabilities within each demographic-locality bin, as well as the 1990-2016 changes in these variables. In addition, we also control for the number of co-living children by computing the 1980 shares and the 1990-2016 changes in shares of household heads co-living with one, two, or more children below 18 years old. Table 6 shows the results.
The regressions are across people sorted by locality/education-age group. Controls include manufacturing hours share in 1980, and a set of additional demographic controls (see text for details). Errors are clustered at location level. F-statistics are Kleibergen-Paap. ",", "**, "***" indicate significance at the 10%, 5%, and 1% levels, respectively.

**Table 6:** Regressions across people sorted by location and education-age groups: impact of wage and recreation price growth on hours worked.
Shares of different items in total recreation consumption. Shares are constructed by pooling observations for the two periods, 1980-1988 and 2010-2018. Source: CE.

**Figure 8:** Composition of recreation consumption baskets across education groups over time.

### C.2 Cross-country sample

#### C.2.1 Figures for the entire cross-section of countries

In the main text, we present the time series of hours worked, recreation prices and wages for a selected group of countries. Figure 9 shows the same graphs for the entire cross-section of 38 countries. The bold black lines show log-linear trends, constructed by regressing the log of variables of interest on the year and a set of country fixed effects.

Figure 10 shows total consumption expenditures over labor income (panel (a)) and recreation consumption share (panel (b)) for the entire cross-section of countries. The bold black lines show log-linear trends, constructed by regressing the log of variables of interest on the year and a set of country fixed effects. The consumption-labor income ratio does not trend upwards or downwards.
Figure 9: Hours, wages and recreation price in the international sample.

(The trend coefficient is virtually 0). The recreation consumption share does exhibit a very small positive trend. However, excluding just one country—Korea—makes the trend coefficient statistically insignificant (on the 5% level). A large increase in the recreation consumption share in Korea is hardly surprising given its postwar development has started from an extremely low point, when a large fraction of population consumed only basic necessities.

C.2.2 Cointegration analysis

The model presented in Section 3 implies that long-run behaviors of hours, wages and recreation prices are related. In this Appendix, we formally evaluate this relation using cointegration analysis. To do so, we follow the approach of Pesaran and Smith (1995) who propose a method to estimate
long-run relationship in a panel setting. In particular, we estimate the following error correction model:

\[ \Delta \log h_{it} = \phi (\log h_{i,t-1} - \theta_0 - \theta_w \log w_{it} - \theta_p \log p_{it}) + \delta_w \Delta \log w_{it} + \delta_p \Delta \log p_{it} + \epsilon_{it}. \] (18)

Here \( \theta_w \) and \( \theta_p \) are coefficients of interest as they capture the long-run relations between (log of) hours, wages and recreation prices; \( \phi \) is the error-correcting speed of adjustment term; \( \delta_w \) and \( \delta_p \) describe short-run dynamics.\(^{43}\) Finally, \( \Delta x_{it} = x_{it} - x_{i,t-1} \), where \( i \) is a country identifier and \( t \) is a time subscript. All observations are annual.

We estimate 18 using hours per capita and hours per worker. We focus on countries that have at least 20 years of data on recreation prices, wages and hours. That results in a sample of 33 countries and 956 country-year observations. Table 7 shows the results. We find quite noisy estimates for the long-run relations of hours per capita with wages and recreation prices: both \( \hat{\theta}_p \) and especially \( \hat{\theta}_w \) are statistically indistinguishable from zero. This is not very surprising given that hours per capita were fairly flat over the period for which data on recreation prices is available. At the same time, we find quite tight estimates of the long-run coefficients when hours per worker is used. In particular, we obtain a significantly positive \( \hat{\theta}_p \) of 0.10 and a significantly negative \( \hat{\theta}_w \) of −0.14. These results are broadly consistent with our findings reported in Section 2.1.2: declining recreation price and raising wages are associated with declining hours worked.

\(^{43}\)Although the model, being formulated in the standard BGP framework, does not imply and short-term fluctuations, we nevertheless allow for nonzero \( \delta_w \) and \( \delta_p \) to account for any real-life short-term shocks (e.g., business cycles).
Table 7: Long-run relations between hours worked, wages and recreation prices

D Proofs

This section contains the formal results establishing restrictions on the shape of the utility function so that it be consistent with a balanced-growth path. The proofs follow mostly the same steps as Boppart and Krusell (2020) but must take care of an additional variable in the utility function.

The proof of Proposition 1 relies on the following two lemmata.

Lemma 1. If \( u(c,h,d) \) satisfies (9) and (10) for all \( t > 0, \gamma_w > 0 \) and \( \gamma_{pd} > 0 \), and for arbitrary \( c > 0, w > 0 \) and \( pd > 0 \), then its marginal rate of substitution functions, defined by \( u_h(c,h,d) / u_c(c,h,d) \) and \( u_d(c,h,d) / u_c(c,h,d) \) must be of the form

\[
\frac{u_h(c,h,d)}{u_c(c,h,d)} = \frac{c}{h} x \left( c^{1-\eta-\tau} h^{\eta} d^{\tau} \right) \tag{19}
\]

and

\[
\frac{u_d(c,h,d)}{u_c(c,h,d)} = \frac{c}{d} y \left( c^{1-\eta-\tau} h^{\eta} d^{\tau} \right) \tag{20}
\]

where \( x \) and \( y \) are arbitrary functions, and \( \eta \) and \( \tau \) are arbitrary numbers.

Proof. We beginning by showing how to derive (19). Set \( t = 0 \) in (9) to find \(-u_h(c,h,d) / u_c(c,h,d) = w \). Using that equation with (9) yields

\[
\frac{u_h(c\lambda^\eta \mu^{\tau}, h\lambda^{\eta-1} \mu^{\tau}, d\lambda^{\eta} \mu^{\tau-1})}{u_c(c\lambda^\eta \mu^{\tau}, h\lambda^{\eta-1} \mu^{\tau}, d\lambda^{\eta} \mu^{\tau-1})} = \lambda \frac{u_h(c,h,d)}{u_c(c,h,d)},
\]

where we denote \( \lambda = \gamma_w^t \) and \( \mu = \gamma_{pd}^t \) to simplify the expression. This equation must hold for every \( \lambda \) and \( \mu \).\(^{44}\) For any given \( c \) and \( h \), set \( \lambda = h/c \) and \( \mu = (c^{1-\eta} h^{\eta})^{-1/\tau} \). These imply that \( c\lambda^\eta \mu^{\tau} = 1 \),

\(^{44}\)Changing \( \mu \) and \( \lambda \) involves changing a mixture of \( t, \gamma_w \) and \( \gamma_p \). Changing \( t \) is innocuous as Definition 1 must hold for every \( t \). Changing \( \gamma_w \) or \( \gamma_p \) would affect the interest rate \( r \), but \( r \) does not show up here.
\[ h \lambda^{\eta-1} \mu^\tau = 1 \text{ and } d \lambda^\eta \mu^{\tau - 1} = dh^{\frac{\eta}{\tau}} c^{-1 + \frac{1}{\tau}(1 - \eta)}. \] From (21), we can therefore write

\[
\frac{u_h(1, 1, dh^{\frac{\eta}{\tau}} c^{-1 + \frac{1}{\tau}(1 - \eta)})}{u_c(1, 1, dh^{\frac{\eta}{\tau}} c^{-1 + \frac{1}{\tau}(1 - \eta)})} = \frac{h}{c} \frac{u_h(c, h, d)}{u_c(c, h, d)}.
\]

Now, define the function \( x(t) = \frac{u_h(1, \frac{t}{\tau}, 1)}{u_c(1, \frac{t}{\tau}, 1)} \). We can rewrite this last equation as (19) which is the result.

We now turn to (20). Set \( t = 0 \) in (10) to find

\[
\frac{u_d(c, h, d)}{u_c(c, h, d)} = \mu \frac{u_d(c, h, d)}{u_c(c, h, d)} \]

(22)

where again \( \lambda = \gamma_w^t \) and \( \mu = \gamma_d^t \). Since this most old for any \( \lambda \) and \( \mu \), Set \( \mu = d/c \) and \( \lambda = (d^{-\tau} e^{1-\tau})^{-\eta} \) to find that \( c \lambda^\eta \mu^\tau = 1 \), \( d \lambda^\eta \mu^{\tau - 1} = 1 \) and \( h \lambda^{\eta-1} \mu^\tau = hd^{\frac{\eta}{\tau}} c^{-1 + \frac{1}{\tau}(1 - \eta)} \). We can therefore write (22) as

\[
\frac{u_d(1, hd^{\frac{\eta}{\tau}} c^{-1 + \frac{1}{\tau}(1 - \eta)}, \frac{1}{\eta}, 1)}{u_c(1, hd^{\frac{\eta}{\tau}} c^{-1 + \frac{1}{\tau}(1 - \eta)}, \frac{1}{\eta}, 1)} = \mu \frac{u_d(c, h, d)}{u_c(c, h, d)} \]

Now, define the function \( y(t) = \frac{u_h(1, \frac{t}{\tau}, 1)}{u_c(1, \frac{t}{\tau}, 1)} \). We can rewrite this last equation as (20) which completes the proof.

We now turn to a Lemma that characterizes the second derivatives of \( u \).

**Lemma 2.** Under Definition 1, the second derivative of \( u \) must satisfy

\[
-\frac{cu_{cc}(c, h, d)}{u_c(c, h, d)} = z_1 \left( e^{1-\eta-\tau} h^\eta d^\tau \right) \]

(23)

\[
-\frac{hu_{ch}(c, h, d)}{u_c(c, h, d)} = z_2 \left( e^{1-\eta-\tau} h^\eta d^\tau \right)
\]

(24)

\[
-\frac{du_{cd}(c, h, d)}{u_c(c, h, d)} = z_3 \left( e^{1-\eta-\tau} h^\eta d^\tau \right)
\]

(25)

for arbitrary functions \( z_1 \), \( z_2 \) and \( z_3 \).

**Proof.** Since (11) must hold for all \( t \), we can differentiate it with respect to \( t \), divide the differentiated equation by (11) and set \( t = 0 \). Doing so we find
\[ u_{cc}(c, h, d)c \log(\gamma_{w}\gamma_{pd}^{-1}) + u_{ch}(c, h, d)h \log(\gamma_{w}^{-1}\gamma_{pd}^{\tau}) + u_{cd}(c, h, d)d \log(\gamma_{w}\gamma_{pd}^{-1}) = \]
\[ u_c(c, h, d) \]
\[ u_{cc}\left(c_{\gamma_w}^{\eta} \gamma_{pd}^{\tau}, h_{\gamma_w}^{\eta} \gamma_{pd}^{-1}, d_{\gamma_w}^{\eta} \gamma_{pd}^{-1}\right) - \eta \gamma_{w}^{\eta} \gamma_{pd}^{-1} \quad \eta \gamma_{w}^{\eta} \gamma_{pd}^{-1} \quad \eta \gamma_{w}^{\eta} \gamma_{pd}^{-1} \quad \eta \gamma_{w}^{\eta} \gamma_{pd}^{-1} \]
\[ = \frac{u_{cc}\left(c_{\gamma_w}^{\eta} \gamma_{pd}^{\tau}, h_{\gamma_w}^{\eta} \gamma_{pd}^{-1}, d_{\gamma_w}^{\eta} \gamma_{pd}^{-1}\right)}{u_c(c, h, d)} \]
\[ + \frac{u_{ch}\left(c_{\gamma_w}^{\eta} \gamma_{pd}^{\tau}, h_{\gamma_w}^{\eta} \gamma_{pd}^{-1}, d_{\gamma_w}^{\eta} \gamma_{pd}^{-1}\right) h_{\gamma_w}^{\eta} \gamma_{pd}^{-1} \log(\gamma_{w}^{-1}\gamma_{pd})}{u_c(c, h, d)} \]
\[ + \frac{u_{cd}\left(c_{\gamma_w}^{\eta} \gamma_{pd}^{\tau}, h_{\gamma_w}^{\eta} \gamma_{pd}^{-1}, d_{\gamma_w}^{\eta} \gamma_{pd}^{-1}\right) d_{\gamma_w}^{\eta} \gamma_{pd}^{-1} \log(\gamma_{w}^{-1}\gamma_{pd})}{u_c(c, h, d)}. \]

Now differentiating (19) and (20) with respect to \(c\), we find that \(h_{\gamma_w}(c,h,d)\) and \(d_{\gamma_w}(c,h,d)\) are functions of \(c^{1-\eta}h^\eta d^\tau\) and \(c, h, d\) only. We can write
\[ h \frac{u_{hc}(c, h, d)}{u_c(c, h, d)} = f_1 \left(c^{1-\eta}h^\eta d^\tau, \frac{u_{cc}(c, h, d)}{u_c(c, h, d)}c\right) \]
\[ d \frac{u_{dc}(c, h, d)}{u_c(c, h, d)} = f_2 \left(c^{1-\eta}h^\eta d^\tau, \frac{u_{cc}(c, h, d)}{u_c(c, h, d)}c\right) \]
and, since these equations holds for any \(c, h\) and \(d,\)
\[ h_{\gamma_w}^{\eta} \gamma_{pd}^{-1} \frac{u_{hc}(c_{\gamma_w}^{\eta} \gamma_{pd}^{\tau}, h_{\gamma_w}^{\eta} \gamma_{pd}^{-1}, d_{\gamma_w}^{\eta} \gamma_{pd}^{-1})}{u_c(c_{\gamma_w}^{\eta} \gamma_{pd}^{\tau}, h_{\gamma_w}^{\eta} \gamma_{pd}^{-1}, d_{\gamma_w}^{\eta} \gamma_{pd}^{-1})} = f_1 \left(c^{1-\eta}h^\eta d^\tau, \frac{u_{cc}(c_{\gamma_w}^{\eta} \gamma_{pd}^{\tau}, h_{\gamma_w}^{\eta} \gamma_{pd}^{-1}, d_{\gamma_w}^{\eta} \gamma_{pd}^{-1})}{u_c(c_{\gamma_w}^{\eta} \gamma_{pd}^{\tau}, h_{\gamma_w}^{\eta} \gamma_{pd}^{-1}, d_{\gamma_w}^{\eta} \gamma_{pd}^{-1})}c_{\gamma_w}^{\eta} \gamma_{pd}^{\tau}\right) \]
\[ d_{\gamma_w}^{\eta} \gamma_{pd}^{-1} \frac{u_{dc}(c_{\gamma_w}^{\eta} \gamma_{pd}^{\tau}, h_{\gamma_w}^{\eta} \gamma_{pd}^{-1}, d_{\gamma_w}^{\eta} \gamma_{pd}^{-1})}{u_c(c_{\gamma_w}^{\eta} \gamma_{pd}^{\tau}, h_{\gamma_w}^{\eta} \gamma_{pd}^{-1}, d_{\gamma_w}^{\eta} \gamma_{pd}^{-1})} = f_2 \left(c^{1-\eta}h^\eta d^\tau, \frac{u_{cc}(c_{\gamma_w}^{\eta} \gamma_{pd}^{\tau}, h_{\gamma_w}^{\eta} \gamma_{pd}^{-1}, d_{\gamma_w}^{\eta} \gamma_{pd}^{-1})}{u_c(c_{\gamma_w}^{\eta} \gamma_{pd}^{\tau}, h_{\gamma_w}^{\eta} \gamma_{pd}^{-1}, d_{\gamma_w}^{\eta} \gamma_{pd}^{-1})}c_{\gamma_w}^{\eta} \gamma_{pd}^{\tau}\right). \]

Plugging into (26) implies that
\[ \frac{u_{cc}(c, h, d)c}{u_c(c, h, d)} = f_3 \left(c^{1-\eta}h^\eta d^\tau, \frac{u_{cc}(c_{\gamma_w}^{\eta} \gamma_{pd}^{\tau}, h_{\gamma_w}^{\eta} \gamma_{pd}^{-1}, d_{\gamma_w}^{\eta} \gamma_{pd}^{-1})}{u_c(c_{\gamma_w}^{1-\nu} h_{\gamma_w}^{1-\nu}, \gamma_{pd}^{\gamma_{pd}}) c_{\gamma_w}^{\eta} \gamma_{pd}^{\tau}\right), \]
where \(f_3\) is an arbitrary function. This equation must hold for every \(\gamma_w\) and \(\gamma_p\) (\(r\) would also need to be adjusted, but \(r\) does not show up here). We can therefore set \(\gamma_w = 1\) and \(\gamma_p = 1\), and we find that \(\frac{u_{cc}(c, h, d)c}{u_c(c, h, d)}\) only depends on \(c^{1-\eta}h^\eta d^\tau\).

**Proposition 1.** The utility function \(u(c, h, d)\) is consistent with a balanced-growth path (Definition
if and only if (save for additive and multiplicative constants) it is of the form

\[ u(c, h, d) = \frac{(c^{1-\varepsilon}v(c^{1-\eta-\tau}h^{\eta}d^{\tau}))^{1-\sigma} - 1}{1 - \sigma} \]  

(12)

for \( \sigma \neq 1 \), or

\[ u(c, h, d) = \log (c^{1-\varepsilon}v) + \log (v(c^{1-\eta-\tau}h^{\eta}d^{\tau})) \]  

(13)

for \( \sigma = 1 \), and where \( v \) is an arbitrary twice continuously differentiable function and where \( \eta > 0 \) and \( \tau > 0 \).

**Proof.** We first consider the “if” direction of the proof and then turn to the “only if” part. Consider the case with \( 1 - \eta - \tau \neq 0 \). From Lemma 2 we have

\[ \frac{\partial \log (u_c(c, h, d))}{\partial \log (c)} = -z_1 \left( \exp ((1 - \eta - \tau) \log (c) + \eta \log (h) + \tau \log (d)) \right). \]

(28)

Integrating with respect to \( \log c \) we find that

\[ u_c(c, h, d) = f_4(c^{1-\eta-\tau}h^{\eta}d^{\tau}) m_1(h, d) \]

(29)

where \( f_4 \) is a new function of \( c^{1-\eta-\tau}h^{\eta}d^{\tau} \), and \( m_1 \) is an arbitrary function of \( h \) and \( d \).

Now we can restrict \( m_1 \) since, from Lemma 2, \( \frac{h u_{hc}(c, h, d)}{u_c(c, h, d)} \) and \( \frac{d u_{dc}(c, h, d)}{u_c(c, h, d)} \) are also only functions of \( c^{1-\eta-\tau}h^{\eta}d^{\tau} \). Taking the derivative of (28) with respect to \( h \), multiplying by \( h \) and dividing by \( u_c \) we obtain

\[ \frac{h u_{hc}(c, h, d)}{u_c(c, h, d)} = \frac{f_4'(c^{1-\eta-\tau}h^{\eta}d^{\tau}) c^{1-\eta-\tau}h^{\eta}d^{\tau} \eta}{f_4(c^{1-\eta-\tau}h^{\eta}d^{\tau})} + \frac{hm_{1h}(h, d)}{m_1(h, d)}. \]

Similarly, we can take the derivative of (28) with respect to \( d \), multiplying by \( d \) and dividing by \( u_c \) to find

\[ \frac{d u_{dc}(c, h, d)}{u_c(c, h, d)} = \frac{f_4'(c^{1-\eta-\tau}h^{\eta}d^{\tau}) c^{1-\eta-\tau}h^{\eta}d^{\tau} \tau}{f_4(c^{1-\eta-\tau}h^{\eta}d^{\tau})} + \frac{dm_{1d}(h, d)}{m_1(h, d)}. \]

So that \( \frac{h u_{hc}(c, h, d)}{u_c(c, h, d)} \) and \( \frac{d u_{dc}(c, h, d)}{u_c(c, h, d)} \) only depend on \( c^{1-\eta-\tau}h^{\eta}d^{\tau} \), it must be that \( \frac{hm_{1h}(h, d)}{m_1(h, d)} \) and \( \frac{dm_{1d}(h, d)}{m_1(h, d)} \) are constants and therefore \( m_1(h, d) = A_2 h^{\kappa}d^{\delta} \). We can rewrite (29) as

\[ u_c(c, h, d) = f_4(c^{1-\eta-\tau}h^{\eta}d^{\tau}) A_2 h^{\kappa}d^{\delta}. \]

(30)

Since \( 1 - \eta - \tau \neq 0 \) we can rewrite that equation as

\[ u_c(c, h, d) = f_5(c h^{\frac{\eta}{1-\eta-\tau}} d^{\frac{\tau}{1-\eta-\tau}}) A_2 h^{\kappa}d^{\delta}. \]

We can integrate this equation with respect to \( c \) to find
Similarly, taking the derivative of (31) with respect to \( m \) can merge the same constant. That constant can be set arbitrarily as it does not affect choices. In this case, we have

\[ u(c, h, d) = f_6 \left( ch^{\frac{\eta}{1 - \eta - \tau}} d^{\frac{\tau}{1 - \eta - \tau}} \right) h^{\kappa - \frac{\eta}{1 - \eta - \tau}} d^{\frac{\tau}{1 - \eta - \tau}} + m_2(h, d) \]  

(31)

where \( f_6 \) is another arbitrary function.

To further restrict \( m_2(h, d) \), we combine Lemma 1 together with (30) to find

\[ u_h(c, h, d) = f_7 \left( ch^{\frac{\eta}{1 - \eta - \tau}} d^{\frac{\tau}{1 - \eta - \tau}} \right) A_2 h^{\kappa - \frac{\eta}{1 - \eta - \tau}} d^{\frac{\tau}{1 - \eta - \tau}} \]  

(32)

and

\[ u_d(c, h, d) = f_8 \left( ch^{\frac{\eta}{1 - \eta - \tau}} d^{\frac{\tau}{1 - \eta - \tau}} \right) A_2 h^{\kappa - \frac{\eta}{1 - \eta - \tau}} d^{\frac{\tau}{1 - \eta - \tau}} \]  

(33)

where \( f_7 \) and \( f_8 \) are appropriately defined functions.

We can now compare the derivatives of \( u \), from (31), to these last two expressions. First, taking the derivative of (31) with respect to \( h \) we find

\[ u_h(c, h, d) = f_9 \left( ch^{\frac{\eta}{1 - \eta - \tau}} d^{\frac{\tau}{1 - \eta - \tau}} \right) h^{\kappa - \frac{\eta}{1 - \eta - \tau} - 1} d^{\frac{\tau}{1 - \eta - \tau}} \frac{\eta}{1 - \eta - \tau} + \frac{\eta}{1 - \eta - \tau} + m_{2,1}(h, d) \]

\[ = f_9 \left( ch^{\frac{\eta}{1 - \eta - \tau}} d^{\frac{\tau}{1 - \eta - \tau}} \right) h^{\kappa - \frac{\eta}{1 - \eta - \tau} - 1} d^{\frac{\tau}{1 - \eta - \tau}} + m_{2,1}(h, d) \]

For this to work with (32) for all \( c, h \) and \( d \), it must be that \( m_{2,1}(h, d) = A_3 h^{\kappa - \frac{\eta}{1 - \eta - \tau} - 1} d^{\frac{\tau}{1 - \eta - \tau}} \).

Similarly, taking the derivative of (31) with respect to \( d \) we find

\[ u_d(c, h, d) = f_{11} \left( ch^{\frac{\eta}{1 - \eta - \tau}} d^{\frac{\tau}{1 - \eta - \tau}} \right) h^{\kappa - \frac{\eta}{1 - \eta - \tau}} d^{\frac{\tau}{1 - \eta - \tau} - 1} + m_{2,2}(h, d) \]

For this to work with (33), it must be that \( m_{2,2}(h, d) = A_4 h^{\kappa - \frac{\eta}{1 - \eta - \tau}} d^{\frac{\tau}{1 - \eta - \tau} - 1} \).

We can integrate \( m_{2,1} \) and \( m_{2,2} \) to find \( m \). Let us first handle the case with \( \kappa \neq \frac{\eta}{1 - \eta - \tau} \) and \( \nu \neq \frac{\tau}{1 - \eta - \tau} \). Integrating, we find

\[ m_2(h, d) = A_5 h^{\kappa - \frac{\eta}{1 - \eta - \tau}} d^{\frac{\tau}{1 - \eta - \tau}} + g_3(d) \]  

(34)

\[ m_2(h, d) = A_6 h^{\kappa - \frac{\eta}{1 - \eta - \tau}} d^{\frac{\tau}{1 - \eta - \tau}} + g_4(h) \]

For these two equations to be jointly true it must be that \( A_5 = A_6 \), and that \( g_3 \) and \( g_4 \) are the same constant. That constant can be set arbitrarily as it does not affect choices. In this case, we can merge \( m_2 \) in (31) and find

\[ u(c, h, d) = f_{12} \left( ch^{\frac{\eta}{1 - \eta - \tau}} d^{\frac{\tau}{1 - \eta - \tau}} \right) h^{\kappa - \frac{\eta}{1 - \eta - \tau}} d^{\frac{\tau}{1 - \eta - \tau}} + A_7. \]  

(35)
Since $\eta \neq 0$, we can write
\[ u(c, h, d) = f_{13} \left( ch^{\frac{\eta}{1-\eta - \tau}} d^{\frac{\tau}{1-\eta - \tau}} \right) c^{1-\kappa} h^{\frac{\eta}{1-\eta - \tau}} d^{\frac{\tau}{1-\eta - \tau}} + A_7. \]

which is equivalent to
\[ u(c, h, d) = \left( c^{1-\varepsilon} \left( c^{1-\eta - \tau} h^{\eta} d^{\tau} \right) \right)^{1-\sigma} - 1 \]
where
\[ (1-\sigma)(1-\varepsilon) = 1 - \kappa \frac{1-\eta - \tau}{\eta} \]
\[ (1-\sigma)\varepsilon = \tau \frac{\kappa}{\eta} \]

If instead $\kappa = \frac{\eta}{1-\eta - \tau}$, integrating $m_{2,1}(h, d) = A_3 h^{\frac{\eta}{1-\eta - \tau} - 1} d^{\tau - \frac{\tau}{1-\eta - \tau}}$ yields
\[ m_2(h, d) = A_5 d^{\tau - \frac{\tau}{1-\eta - \tau}} \log h + g_3(d), \]
and if $\tau = \frac{\eta}{1-\eta - \tau}$, integrating $m_{2,2}(h, d) = A_4 h^{\frac{\eta}{1-\eta - \tau}} d^{\tau - \frac{\tau}{1-\eta - \tau} - 1}$ yields
\[ m_2(h, d) = A_6 h^{\frac{\eta}{1-\eta - \tau}} d^{\tau - \frac{\tau}{1-\eta - \tau}} \log d + g_4(h). \]

If only one of $\kappa = \frac{\eta}{1-\eta - \tau}$ or $\tau = \frac{\eta}{1-\eta - \tau}$ is true, it must be that $m_2 = A_7$, where $A_7$ is a constant. Suppose that only $\kappa = \frac{\eta}{1-\eta - \tau}$, (31) becomes
\[ u(c, h, d) = f_{14} \left( ch^{\frac{\eta}{1-\eta - \tau}} d^{\frac{\tau}{1-\eta - \tau}} \right) c^{1-\kappa} h^{\frac{\eta}{1-\eta - \tau}} d^{\frac{\tau}{1-\eta - \tau}} - \frac{\tau}{1-\eta - \tau} \kappa + A_7. \]

so we find (36) with
\[ \varepsilon = 1 \]
\[ 1 - \sigma = \tau - \frac{\tau}{1-\eta - \tau}. \]

If only $\tau = \frac{\eta}{1-\eta - \tau}$, (31) becomes
\[ u(c, h, d) = f_6 \left( ch^{\frac{\eta}{1-\eta - \tau}} d^{\frac{\tau}{1-\eta - \tau}} \right) h^{\kappa - \frac{\eta}{1-\eta - \tau}} + m_2(h, d) \]
which we can rewrite as
\[ u(c, h, d) = f_{14} \left( ch^{\frac{\eta}{1-\eta - \tau}} d^{\frac{\tau}{1-\eta - \tau}} \right) c^{1-\kappa} h^{\frac{\eta}{1-\eta - \tau}} d^{\frac{\tau}{1-\eta - \tau} - \frac{\tau}{1-\eta - \tau}} \kappa + m_2(h, d). \]
so we find (36) with

\[
(1 - \sigma)(1 - \varepsilon) = 1 - \frac{1 - \eta - \tau}{\eta} \\
(1 - \sigma)\varepsilon = \frac{\tau}{1 - \eta - \tau} - \frac{\tau}{\eta}
\]

If both \( \kappa = \frac{\eta}{1 - \eta - \tau} \) and \( \iota = \frac{\tau}{1 - \eta - \tau} \) it must be, from (37) and (38), that

\[
m_2 (h, d) = A_8 \log h + A_9 \log d + A_7,
\]

in which case we can write (31) as

\[
u (c, h, d) = f_6 (c^{1 - \eta - \tau} d^{1 - \eta - \tau}) + A_8 \log h + A_9 \log d + A_7.
\]

We can use

\[
\log (c^{1 - \eta - \tau} d^{1 - \eta - \tau}) = \log c + \frac{\eta}{1 - \eta - \tau} \log h + \frac{\tau}{1 - \eta - \tau} \log d,
\]

to write

\[
u (c, h, d) = f_{15} (c^{1 - \eta - \tau} d^{1 - \eta - \tau}) + A_8 \frac{1 - \eta - \tau}{\eta} \log c + \left(A_9 - A_8 \frac{\tau}{\eta}\right) \log d + A_7.
\]

Since the utility function is invariant to multiplication by a constant we can normalize the sum of the powers on \( c \) and \( d \) to 1, and get

\[
u (c, h, d) = (1 - \varepsilon) \log c + \varepsilon \log d + \log \left(c^{1 - \eta - \tau} h^{\eta} d^{\tau}\right).
\]

(39)

We now turn to the case in which \( 1 - \eta - \tau = 0 \).

We now turn to the case with \( 1 - \eta - \tau = 0 \). The characterization of \( u_{cc} \) in Lemma 2 can be written as

\[
\frac{\partial \log (u_c (c, h, d))}{\partial \log (c)} = -z_1 (h^{\eta} d^{\tau}).
\]

Integrating with respect to \( \log c \) we find that

\[
\log (u_c (c, h, d)) = -\log (c) z (h^{\eta} d^{\tau}) + m_3 (h, d)
\]

(40)

where \( m_3 \) is an arbitrary function of \( h \) and \( d \). Differentiating with respect to \( h \) and multiplying by \( h \) yields

\[
\frac{h u_{ch} (c, h, d)}{u_c (c, h, d)} = -\log (c) z' (h^{\eta} d^{\tau}) \eta h^{\eta} + hm_{3,1} (h, d).
\]

(41)
Similarly, differentiating with respect to \( d \) and multiplying by \( d \) yields

\[
\frac{du_{cd}}{u_c} (c, h, d) = - \log(c) z' (h^n d^r) \tau d^r + dm_{3,2} (h, d). \tag{42}
\]

From Lemma 2 we know that \( \frac{hu_{h,c}(c,h,d)}{u_c(c,h,d)} \) and \( \frac{du_{d,c}(c,h,d)}{u_c(c,h,d)} \) are only functions of \( h^n d^r \). For (41) and (42) to hold true for every \( c \) it must therefore be that \( z' (h^n d^r) = 0 \) (note that \( a \) and \( b \) cannot both be equal to 0 since \( 1 - \eta - \tau = 0 \)) so that \( z = -\sigma \) is a constant. Similarly, it must be that \( hm_{3,1} (h, d) = g_5 (h^n d^r) \) and \( dm_{3,2} (h, d) = g_6 (h^n d^r) \). Integrating, we find that \( m_3 (h, d) = f_{16} (h^n d^r) \) for some function \( f_{16} \). By exponentiating on both sides of (40), we can therefore rewrite

\[
u_c (c, h, d) = c^{-\sigma} m_4 (h^n d^r). \tag{43}\]  

We can integrate this equation with respect to \( c \) to find

\[
u (c, h, d) = \frac{(cv (h^n d^r))^{1-\sigma} - 1}{1 - \sigma} + m_5 (h, d) \tag{44}\]  

if \( \sigma \neq 1 \), or

\[
u (c, h, d) = m_4 (h^n d^r) \log (c) + \log (v (h^n d^r)) \tag{45}\]  

otherwise.

For the case with \( \sigma \neq 1 \), combine (43) with Lemma 1 that

\[
u_h (c, h, d) = \frac{1}{h} x (h^n d^r) c^{1-\sigma} m_4 (h^n d^r) \]

and

\[
u_d (c, h, d) = \frac{1}{d} y (h^n d^r) c^{1-\sigma} m_4 (h^n d^r). \]

Differentiating (44) yields

\[
u_h (c, h, d) = (cv (h^n d^r))^{-\sigma} cv' (h^n d^r) a \frac{h^n d^r}{h} + m_{5,1} (h, d) \]

and

\[
u_d (c, h, d) = (cv (h^n d^r))^{-\sigma} cv' (h^n d^r) b \frac{h^n d^r}{d} + m_{5,2} (h, d). \]

Since \( \sigma \neq 1 \) it must be that \( m_5 \) is a constant that can be set to 0 as it does not affect decisions. (44) is therefore a special case of (36).
For the case with $\sigma = 1$, we can again combine (43) with Lemma 1 to find the two equations
\[
\begin{align*}
\frac{d}{dh} u_h(c, h, d) &= \frac{1}{h} x(h^\eta d^\tau) m_4(h^\eta d^\tau) \\
\frac{d}{dh} u_d(c, h, d) &= \frac{1}{d} y(h^\eta d^\tau) m_4(h^\eta d^\tau).
\end{align*}
\]
Differentiating (45) yields
\[
\begin{align*}
\frac{d}{dh} u_h(c, h, d) &= m'_4(h^\eta d^\tau) a \frac{h^\eta d^\tau}{h} \log(c) + \frac{v'(h^\eta d^\tau)}{v(h^\eta d^\tau)} a \frac{h^\eta d^\tau}{h} \\
\frac{d}{dh} u_d(c, h, d) &= m'_4(h^\eta d^\tau) b \frac{h^\eta d^\tau}{d} \log(c) + \frac{v'(h^\eta d^\tau)}{v(h^\eta d^\tau)} b \frac{h^\eta d^\tau}{d}.
\end{align*}
\]
For these equations to be consistent it must be that $m_4$ is a constant so we find (39) again.

This completes the proofs that if $u$ satisfies Definition 1 then it must be of the form (12)--(13). We now show that if $u$ is defined as (12)--(13) then Definition 1 is also satisfied.

First notice that if we evaluate the function $c^{1-\eta-\tau} h^\eta d^\tau$ along a balanced-growth path, i.e. at a point \( \left(c_0 \left(\gamma^\eta w c^\tau_{p\hat{a}} \right)^t, h_0 \left(\gamma^\eta w \gamma^\tau_{p\hat{a}} \right)^t, d_0 \left(\gamma^\eta w \gamma^\tau_{p\hat{a}}-1 \right)^t \right) \), we get
\[
\left(c_0 \left(\gamma^\eta w c^\tau_{p\hat{a}} \right)^t \right)^{1-\eta-\tau} \left(h_0 \left(\gamma^\eta w \gamma^\tau_{p\hat{a}} \right)^t \right)^{\eta} \left(d_0 \left(\gamma^\eta w \gamma^\tau_{p\hat{a}}-1 \right)^t \right)^{\tau} = c_0^{1-\eta-\tau} h_0^\eta d_0^\tau.
\]

In other words, $c^{1-\eta-\tau} h^\eta d^\tau$ is invariant along a balanced-growth path.

The derivatives of $u$ are
\[
\begin{align*}
u_h &= (c^{1-\varepsilon} d^\varepsilon v (c^{1-\eta-\tau} h^\eta d^\tau))^{-\sigma} c^{1-\varepsilon} d^\varepsilon v' (c^{1-\eta-\tau} h^\eta d^\tau) \eta c^{1-\eta-\tau} h^\eta d^\tau \\
u_d &= (c^{1-\varepsilon} d^\varepsilon v (c^{1-\eta-\tau} h^\eta d^\tau))^{-\sigma} \left(\varepsilon c^{1-\varepsilon} d^\varepsilon d v (c^{1-\eta-\tau} h^\eta d^\tau) + c^{1-\varepsilon} d^\varepsilon v' (c^{1-\eta-\tau} h^\eta d^\tau) \tau c^{1-\eta-\tau} h^\eta d^\tau \right) \\
u_c &= (c^{1-\varepsilon} d^\varepsilon v (c^{1-\eta-\tau} h^\eta d^\tau))^{-\sigma} \times \\
&\left( (1-\varepsilon) c^{1-\varepsilon} d^\varepsilon c v (c^{1-\eta-\tau} h^\eta d^\tau) + c^{1-\varepsilon} d^\varepsilon v' (c^{1-\eta-\tau} h^\eta d^\tau) \left(1-\eta-\tau\right) c^{1-\eta-\tau} h^\eta d^\tau \right)
\end{align*}
\]

Taking the ratio of $u_h$ and $u_c$ and evaluating the expression at a point on a balanced-growth path, \( \left(c_0 \left(\gamma^\eta w c^\tau_{p\hat{a}} \right)^t, h_0 \left(\gamma^\eta w \gamma^\tau_{p\hat{a}} \right)^t, d_0 \left(\gamma^\eta w \gamma^\tau_{p\hat{a}}-1 \right)^t \right) \), we find that
\[
\frac{u_h}{u_c} = \frac{v' \left(c_0^{1-\eta-\tau} h_0^\eta d_0^\tau \right) \eta c_0^{1-\eta-\tau} h_0^\eta d_0^\tau}{(1-\varepsilon) v \left(c_0^{1-\eta-\tau} h_0^\eta d_0^\tau \right) + v' \left(c_0^{1-\eta-\tau} h_0^\eta d_0^\tau \right) \left(1-\eta-\tau\right) c_0^{1-\eta-\tau} h_0^\eta d_0^\tau} \frac{c_0 \gamma^\tau_{w}}{h_0}.
\]
so that \( u_h/u_c \) grows at rate \( \gamma_w \) and so (9) is satisfied.\(^{45}\)

Similarly, taking the ratio of \( u_d \) and \( u_c \) and evaluating the expression at
\[
\begin{pmatrix}
  c_0 \left( \gamma^\eta \gamma^\tau_{pd} \right)^t, h_0 \left( \gamma^\eta_{-1} \gamma^\tau_{pd} \right)^t, d_0 \left( \gamma^\eta \gamma^\tau_{pd}^{-1} \right)^t
\end{pmatrix}
\]
we find
\[
\frac{u_d}{u_c} = \frac{\left( \varepsilon v \left( c_0^1 - \eta - \gamma^\tau_{pd} d_0^\gamma \right) + v' \left( c_0^1 - \eta - \gamma^\tau_{pd} d_0^\gamma \right) \tau c_0^1 - \eta - \gamma^\tau_{pd} d_0^\gamma \right) c_0^t}{\left( 1 - \varepsilon \right) v \left( c_0^1 - \eta - \gamma^\tau_{pd} d_0^\gamma \right) + v' \left( c_0^1 - \eta - \gamma^\tau_{pd} d_0^\gamma \right) (1 - \eta - \tau) \left( c_0^1 - \eta - \gamma^\tau_{pd} d_0^\gamma \right)} d_0^\gamma_{pd}
\]
so that \( u_d/u_c \) grows at rate \( \gamma_{pd} \) and (10) is satisfied.

Finally, dividing \( u_c \) evaluated at \( \begin{pmatrix} c_0 \left( \gamma^\eta_{pd} \right)^t, h_0 \left( \gamma^\eta_{-1} \gamma^\tau_{pd} \right)^t, d_0 \left( \gamma^\eta \gamma^\tau_{pd}^{-1} \right)^t \end{pmatrix} \) by \( u_c \) evaluated at \( \begin{pmatrix} c_0 \left( \gamma^\eta_{pd} \right)^{t+1}, h_0 \left( \gamma^\eta_{-1} \gamma^\tau_{pd} \right)^{t+1}, d_0 \left( \gamma^\eta \gamma^\tau_{pd}^{-1} \right)^{t+1} \end{pmatrix} \) we find
\[
\frac{u_c}{u_c'} = \gamma^\eta_{pd} \gamma^{-1} (1 - \sigma)(\tau - \varepsilon)
\]
which is an expression independent of \( c, d \) and \( h \), as required by 11, and that defines \( r \).

**Proposition 2.** On a balanced-growth path, the growth rates of \( p_{dt} \) and \( w_t \) are
\[
\log \gamma_{pd} = \log \gamma_{Ac} - \log \gamma_{Ad},
\]
\[
\log \gamma_w = \alpha \log \gamma_{Ac}.
\]

**Proof.** The first-order conditions of the firms are
\[
\alpha p_{jt} y_{jt} = w_t l_{jt}
\]
and
\[
(1 - \alpha) p_{jt} y_{jt} = R_t k_{jt}
\]
so that
\[
\frac{\alpha}{1 - \alpha} R_t (k_{ct} + k_{dt}) = w_t (l_{ct} + l_{dt})
\]
and
\[
\frac{l_{ct}}{k_{ct}} = \frac{l_{dt}}{k_{dt}} = \frac{l_{ct} + l_{dt}}{k_{ct} + k_{dt}}.
\]
Combining (47) for \( j = c \) with \( p_{jt} A = R_t \), the production function (16) and using the fact that \( p_{ct} = 1 \) yields the price of investment
\[
p_{lt} = \left( 1 - \alpha \right) \frac{A}{A} \left( \frac{l_{ct} + l_{dt}}{k_{ct} + k_{dt}} \right)^\alpha.
\]

\(^{45}\)Note that by Definition 1 we can adjust \( h_0 \) to match the wage so that \( -u_h/u_c \) matches the arbitrary wage \( w \). This requires \( v' \neq 0 \), but if \( v' = 0 \) hours does not enter the utility function and the only possible wage is \( w = 0 \).
With \( p_{it} A = R_t \), this equation also pins down the interest rate

\[
R_t = (1 - \alpha) A_{ct} \left( \frac{l_{ct} + l_{dt}}{k_{ct} + k_{dt}} \right)^\alpha. \tag{50}
\]

Doing the same operations with \( j = d \) instead, and combining with (49) we find that the price of recreation goods and services, measured in units of non-recreation prices, is the ratio of sector \( c \) and sector \( d \) productivities:

\[ p_{dt} = \frac{A_{ct}}{A_{dt}}. \]

It follows that the growth rate \( \gamma_{pd} \) of \( p_{dt} \) is such that \( \log \gamma_{pd} = \log \gamma_{Ac} - \log \gamma_{Ad} \).

Combining (50) with (48) yields

\[
R_t^{1-\alpha} = (1 - \alpha) A_{ct} \left( \frac{\alpha}{1 - \alpha w_t} \right)^\alpha.
\]

Since the first-order conditions of the household imply a constant \( R_t \), this last equation yields that

\[ \log \gamma_w = \alpha \log \gamma_{Ac}, \]

which completes the proof. \( \square \)