Sovereign default and the decline in interest rates*

Max Miller† James D. Paron‡ Jessica A. Wachter§

September 8, 2020

Abstract

Sovereign debt yields have declined dramatically over the last half-century. Standard explanations for this decline, including aging populations and increases in asset demand from abroad, encounter difficulties when confronted with the full range of evidence across asset classes. We propose instead that the decline in inflation and default risk caused falling interest rates, a phenomenon that is not unique to our century. We show that a model with investment, inventory storage, and sovereign default captures the decline in interest rates, the stability of equity valuation ratios, and the recent reduction in investment and output growth corresponding to the zero lower bound.

Keywords: Savings glut, Inflation expectations, Rare disasters, Secular stagnation
JEL codes: E31, E43, G12

*We thank Sylvain Catherine, François Gourio, Karen Lewis, Jonathan Wachter, and seminar participants at the NBER Summer Institute for helpful comments.

†Department of Finance, The Wharton School, University of Pennsylvania. Email: maxmil@wharton.upenn.edu.

‡Department of Finance, The Wharton School, University of Pennsylvania. Email: jparon@wharton.upenn.edu.

§Department of Finance, The Wharton School, University of Pennsylvania and NBER. Email: jwachter@wharton.upenn.edu.

1 Introduction

Over the last three decades, the developed world has experienced a stark decline in interest rates. This decline, together with low output growth subsequent to the Great Recession of 2009, has evoked, for some, the possibility of "secular stagnation," a term coined by Hansen (1939) to describe a persistent period of low investment, employment, and growth. Summers (2015) and Gordon (2015) argue for the relevance of Hansen's concept from two angles: demand-side — an increase in demand for savings arising from changing demographics or growing inequality — and supply-side — arising from a decline in the ideas and dynamism that have fueled the economic growth of the last half-century. A complementary idea is that of a "global savings glut" (Bernanke, 2005): that there is too great a supply of savings, mainly from patient investors outside the United States, compared to the demand, which would arise from the need to fund productive activities (ideas for which may be lacking). Caballero et al. (2008) propose that this excess supply of savings entering the U.S. is due to a combined lack of pledgeability of cash flows and increasing wealth in the developing world.

However, is a greater desire for savings in fact what lies behind the decline in interest rates? On some level, the link appears too obvious to be worth questioning. Yet any explanation based on a greater desire for savings runs into a significant problem when one also considers evidence from the U.S. stock market. A greater desire for savings should have pushed up stock prices to a similar degree as bond prices, but it did not. From the point of view of the literature on increased desire for savings, low interest rates, and low growth, the behavior of the aggregate stock market is a puzzle. For a careful explanation of this puzzle, one can look to Farhi and Gourio (2018), who jointly consider growth, interest rates, and stock valuations in a neoclassical growth model. In addition, they allow for rare disasters, of the type considered by Gourio (2012). Farhi and Gourio (2018) show that a significant increase in the risk of rare disasters is necessary to jointly reconcile the level of interest rates and stock prices.

While Farhi and Gourio (2018) succeed in jointly explaining stock prices and interest rates, basing this explanation on increased fears of a disaster runs into its own problems. The nature of rare disasters means that increased disaster fears are hard to falsify, but one would expect to find evidence in option prices. Yet options evidence suggests remarkable stability in fears of rare disasters. Second, implications of increased risk of disasters are fragile in that they depend directionally on whether the elasticity of intertemporal substitution (EIS) is above or below one. If the EIS is below, rather than above one, increased risk of rare disasters, together with low growth, require that agents must have become less patient, not more. Yet the arguments in Summers (2015) and Caballero et al. (2008) point unambiguously toward an increase in patience.

We therefore propose a different explanation, one based on a decline in the risk of inflation. There is substantial evidence for a steady decline in inflation expectations, spanning the 30 years over which interest rates have declined. More recently, evidence from options markets suggest that inflation expectations have become "anchored" — that is, investors do not fear either very high or very low inflation (Reis, 2020).[1] When one takes this evidence into account, it is not difficult to jointly explain a decline in interest rates and the stability of stock valuation ratios. Because the true real rate has not declined, valuation ratios are unchanged, and there is no need to assume a large increase in the probability of a rare disaster to explain the evidence.

One may wonder: if it is simply inflation expectations that have declined, why is it that the *measured* real rate, namely nominal rates minus ex post realized inflation, have also declined? But this apparent disconnect disappears if one accounts for inflation *risk*. Indeed, if inflation were perfectly forecastable, then a change in inflation expectations should not impact ex post real rates. But historically, inflation has on occasion come as a surprise. A decline in inflation risk will lead investors to require less of a premium to hold nominal securities. Interest rates will decline if this premium declines, even if measured in real terms ex post. This effect is more pronounced if investors fear inflation that, in sample, does not occur. From the point of view of cash flows, and given that the

[1] Further, there is evidence to suggest that the sign of the correlation of inflation on the output gap has changed since 2001, as noted by Campbell et al. (2020).

sovereign has control over the money supply, inflation risk is essentially risk of default.[2] A decline in inflation is thus a decline in the probability of default, and thus should also be expected to impact rates on securities that are said to be inflation-protected. The first contribution of our paper is to show that a model with rare disasters and a decline in inflation expectations can explain the decline in interest rates and the stability of valuation ratios. Because sovereign risk depends on institutions that have altered substantially over the centuries, this explanation could account for the striking fact that current rates are low, not just relative to the last 30 years, but to the last 300 years.

We also show that fear of rare disasters, together with low inflation risk, leads to nominal rates that are at or below zero even without the need to assume an increased desire for savings. In practice, the existence of cash creates an effective lower bound on interest rates. Such a lower bound is absent in traditional asset pricing models. Thus a second contribution of our paper is to augment a traditional asset pricing model with cash. We introduce cash in a way that does not require any change to preferences, or demand for liquidity. In our model, cash is a storage technology (inventory). When interest rates are sufficiently low, agents have an incentive to hold cash, which becomes a positive-net-supply asset.

When we consider storage of output, together with productive technologies in a general equilibrium model, we can jointly match growth, as well as valuation ratios and the riskfree rate. Including inventory in this framework allows us to obtain dynamics even within a framework with independent and identically distributed shocks.

This setup also allows us to match an additional puzzle: the decline in the investment-capital ratio over the last 40 years. In standard models with production, if interest rates are low due to an increased demand for savings, investment relative to capital should rise. This approach then requires lower productivity or high levels of risk to

[2]We say "essentially" as there is a literature that examines the possibility of outright U.S. government default. Credit default swaps (CDS) traded on U.S. government debt imply a default probability of 0.2%, a phenomenon examined in depth by Chernov et al. (2020).

offset the rise in investment created through higher demand for savings (Farhi and Gourio, 2018). We show that this is not needed in a model with riskfree storage of consumption goods. In low interest rate regimes, resources that would have been spent on capital are endogenously funneled into non-productive inventory.

This mechanism comes into play when expected returns on the risky asset do not rise fast enough as real rates fall; in the absence of attractive enough investment opportunities, investors look to hoard funds through unproductive avenues. In this way, we provide a new mechanism through which low growth can be compounded by low interest rates in a general equilibrium environment.

The remainder of this paper is organized as follows. In Section 2, we briefly summarize the evidence. In Section 3, we consider the ability of an endowment economy to match this evidence, either with changes in the probability of disaster, or changes in the probability of default. In Section 4, we solve the model with an inventory technology and show additional implications. Section 5 concludes.

2 Summary of the data

Figure 1 shows nominal government rates in a seven-century-long dataset collected by Schmelzing (2020). Interest rates are highly volatile, as Jordà et al. (2019) emphasize.[3] Periods of extreme spikes, and also low rates, occurred around the American Revolution, Napoleonic Wars, and World War II, reflecting a tension between an increase in risk of sovereign default and precautionary savings around disasters. High rates in the 1970s and 1980s clearly stand out. Nonetheless, the figure shows a steady decline. Perhaps a more dramatic demonstration comes from Figure 2, which shows the Bank of England lending rate, from the start of when the series was available. Only in the very most recent period did this rate reach a zero lower bound.

[3] Jordà et al. (2019) note that prior observations of a real rate of zero are not unusual. However, these are observations after subtracting ex post realized inflation, not ex ante yields. While it is true that both returns are zero from an investor's perspective, one was a realization of zero because of high inflation, whereas the other is an ex ante expected value of zero.

Figure 3 zooms in on the last thirty years, the focus of much of the literature. The Federal Funds Rate in the U.S. declined sharply from 10% to 2% at present (right panel). On the other hand, the price-dividend ratio has gone from around 20 to 50, implying a dividend yield of approximately 5% going to 2% — a smaller decline. Moreover, Figure 4 displays the decline in the investment-capital ratio (left panel) and real GDP growth (right panel) from 1984–2016. Investment as a percentage of the capital stock went from an average of 7.7% to 6.9%, while real GDP growth declined from an average of 3.7% to 1.9%.

Figure 5 shows a longer time series of the price-dividend ratio, and also includes the cyclically-adjusted price-earnings (CAPE) ratio and the price-dividend ratio from the United Kingdom. It shows that the price-dividend ratio shifted upward in the late 1990s. This pattern does not appear in the CAPE ratio, nor in the U.K., and therefore may reflect a use of repurchases rather than cash payments as a means of returning cash to shareholders, and not a decline in interest rates. For more information on the data and sources, please see Appendix A.

3 An endowment economy model with rare disasters

To interpret these data, we first turn to a standard endowment economy with a representative agent with Epstein and Zin (1989) preferences. We follow Farhi and Gourio (2018), and calibrate the model separately to two sample periods (1984–2000 and 2001–2016), assuming constant parameters. While this approach does mean that certain features of the data (such as volatility of prices and interest rates) will remain outside the scope of the analysis, it does allow us to consider the possibility of long-run unforeseen structural change. Farhi and Gourio assume a neoclassical growth model. We will return to such a model in the next section, but for the purpose of this section, the extra degree of complication is not necessary. As far as prices and interest rates are concerned, and in this i.i.d. growth rate setting, the production model and the endowment model yield the same predictions.

Let C_t denote the time-t endowment. Let $\Delta c_{t+1} \equiv \log(C_{t+1}/C_t)$, and assume

$$\Delta c_{t+1} = \mu - \eta_{t+1}, \tag{1}$$

where η_{t+1} is an independent and identically distributed "disaster" term of the form

$$\eta_{t+1} = \begin{cases} 0 & \text{with probability} \quad 1-p, \\ z & \text{with probability} \quad p, \end{cases} \tag{2}$$

where p represents the probability of a disaster and $z > 0$ is the magnitude of the disaster. We assume an investor with Epstein-Zin-Weil preferences (Epstein and Zin, 1989, Weil, 1990).

To evaluate the ability of the model to match the data, we make the standard assumptions that the aggregate stock market equals the claim to future consumption and that the ex post real return on the Treasury bill equals the riskfree rate. Let W_t (wealth) denote the cum-dividend value of the consumption claim and $R_{W,t+1} \equiv \frac{W_{t+1}}{W_t - C_t}$ the return on the consumption claim. Then the equilibrium condition linking consumption and returns is:

$$1 = E_t\left[\beta^\theta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}} R^\theta_{W,t+1}\right], \tag{3}$$

for patience parameter β, relative risk aversion γ, elasticity of intertemporal substitution (EIS) ψ, and $\theta \equiv (1-\gamma)/(1-\frac{1}{\psi})$.

These assumptions imply a constant price-dividend ratio $(W_t - C_t)/C_t$. Call this ratio κ. Then standard arguments (see Appendix B) imply

$$\kappa = \frac{\beta e^{(1-\frac{1}{\psi})\mu}\left[1 + p(e^{-(1-\gamma)z} - 1)\right]^{\frac{1}{\theta}}}{1 - \beta e^{(1-\frac{1}{\psi})\mu}\left[1 + p(e^{-(1-\gamma)z} - 1)\right]^{\frac{1}{\theta}}}. \tag{4}$$

Given the return on the wealth portfolio, we can use the Euler equation to price the one-period riskless bond:

$$r_f = -\log\beta + \frac{1}{\psi}\mu - \log(1 + p(e^{\gamma z} - 1)) + \left(\frac{\theta - 1}{\theta}\right)\log(1 + p(e^{-(1-\gamma)z} - 1)), \tag{5}$$

where $r_f \equiv \log R_{f,t+1}$ is the continuously-compounded riskfree rate.

Equations (4) and (5) constitute a system of two equations in two unknowns. We can in principle solve for p and β.[4] Further, combining these, we arrive at the risk premium on the wealth claim, given by

$$\log E_t[R_{W,t+1}] - r_f = \log(1+p(e^{-z}-1)) + \log(1+p(e^{\gamma z}-1)) - \log(1+p(e^{-(1-\gamma)z}-1)). \tag{6}$$

We calibrate this model assuming measured growth rates of $\mu = 0.0350$ from 1984 to 2000 and $\mu = 0.0282$ from 2000 to 2016. In what follows, we refer to μ as expected growth, even though it is in fact expected growth in the absence of disasters. Table 1 shows the results. For greatest comparability, we consider $\gamma = 12$, $\psi = 2$, and a disaster size $z = -\log 0.85$, the same parameters used by Farhi and Gourio (2018).[5] Similar to them, we find we can match the data using a discount factor of 0.967 in the early period and 0.979 in the later period, and a disaster probability of 3.4% in the early

[4] There are some parameter combinations for which this is not possible. For example, when we try to set $\beta = .967$ for both 1984–2000 and 2001–2016 and then estimate μ and p, we are not able to obtain a solution for the 2001–2016 period.

[5] Farhi and Gourio estimate the growth rate within a neoclassical growth model with rare disasters. Their growth rate is the composition of three different growth rates: namely, the growth rate of TFP, the growth rate of the population, and the growth in investment prices. We instead take this growth rate as being exogenously set at the same level. We also use identical values for the price-dividend ratio and the riskfree rate, which are reported in Panel A of Table 1.

period, going to 6.6% in the later period. We thus arrive at our first result: matching the combined stability of valuations and the decrease in riskfree rates requires a large increase in the disaster probability, even when we account for decreased growth.

One can reasonably question the robustness of this interpretation. A first question pertains to the role of growth. While we do estimate a decline in growth, the relative length of the sample period lend the possibility that one could have been lucky in the first and unlucky in the second half. What if the true μ were the same across the periods? Panel B of Table 1 shows that the disaster probability in the second period would need to be 1.3 percentage points higher to account for the failure of valuation ratios to rise. Lower expected growth helps to explain stable valuation ratios in that equalizing growth rates requires the disaster probability to do more work. On the one hand, this may seem like good news for the model; after all, data suggest a decrease in μ. However, the role of growth is sensitive to the value of the EIS.

In fact, a change in the EIS completely changes the interpretation of the data, as Panel C of Table 1 shows. For an EIS below one — say, $\psi = 1/2$ — matching (4) and (5) with p and β still requires an increase in p in the second period. However, β is now lower, implying that investors would need to have become less patient, not more, contradicting the demand-side intuition for the decline in interest rates (Summers, 2015). This result does not depend on the supply-side intuition arising from a decline in growth (Gordon, 2015). If we equalize growth rates, we again see an increase in p and a decrease in β (Panel D).

To summarize: When the EIS is greater than one, the model captures the change in the price-dividend ratio and riskfree rate with increased patience, and lower growth, provided that one assumes a higher probability of disaster. When the EIS is less than 1, the model can only capture the data with greater impatience, combined with a higher probability of disaster. These results are not sensitive to the assumed value of risk aversion; lower values of risk aversion require greater increases in the probability of disaster (Figure 7). Finally, we note that the dependence on the disaster probability increases when one looks beyond the price-dividend ratio in the United States. This ratio may be inflated by changes in the tendency of U.S. companies to pay dividends (Fama and French, 2001). If one estimates the model using the CAPE ratio, or U.K. data, one needs a much greater increase in the disaster probability to match valuation ratios (Table 2).

Interpretation In this analysis, three parameters change across the two periods and affect the price-dividend ratio and the riskfree rate: the patience parameter β, the drift term in growth μ, and the probability of a disaster p. We now drill down to find out the role of each of the three. Table 3 reports the results. We can decompose the effect of the parameters in the price-dividend ratio into a term that depends only on the riskfree rate, a term that depends on the risk premium, and a term that depends on expected growth:

$$\begin{aligned}\log\frac{\kappa}{\kappa+1} = &-\underbrace{\left(-\log\beta+\frac{1}{\psi}\mu-\log(1+p(e^{\gamma z}-1))+\left(\frac{\theta-1}{\theta}\right)\log(1+p(e^{-(1-\gamma)z}-1))\right)}_{\text{riskfree rate effect}}\\ &-\underbrace{\left(\log(1+p(e^{-z}-1))+\log(1+p(e^{\gamma z}-1))-\log(1+p(e^{-(1-\gamma)z}-1))\right)}_{\text{risk premium effect}}\\ &+\underbrace{\left(\mu+\log(1+p(e^{-z}-1))\right)}_{\text{cash flow effect}}.\end{aligned} \tag{7}$$

This is analogous to the decomposition in Campbell and Shiller (1988).

Using this decomposition, we can see that a change in β only affects the price-dividend ratio through its effect on the riskfree rate. Greater patience lowers the riskfree rate, and thus the rate at which investors discount all future cash flows. It thus raises the price-dividend ratio. Because the price-dividend ratio is a convex function of β, what appear to be small shifts in β cause massive changes in the ratio.[6] One way to

[6]Specifically, the price-dividend ratio is a convex function of $\frac{\kappa}{\kappa+1}$, meaning that, at values of $\frac{\kappa}{\kappa+1}$ close to 1 (for which the price-dividend ratio is high), a small increase in β implies a very large increase in the price-dividend ratio.

understand this result is duration: when the price-dividend ratio discounts cash flows in the distant future, their valuation will be very sensitive to small changes in rates. Thus, just to send the riskfree rate even half of the distance between the two samples would send the price-dividend ratio soaring to nearly 100. This is the fundamental problem with basing the decline in interest rates on an increased desire to save. Taking the declining growth rate into account helps to lower the price-dividend ratio, provided that the EIS is greater than one.

Note that the growth rate μ enters into the price-dividend ratio in two ways, once multiplied by $1/\psi$, representing its effect on the riskfree rate, and once multiplied by unity, representing its effect on future cash flows. A decrease in μ decreases the interest rate, following the usual consumption Euler equation intuition: the lower is expected growth, the greater the desire to save for the future, and hence the lower the riskfree rate must be. Or, in a production economy, the lower is growth, the lower the demand for borrowing, and the lower the riskfree rate. Either way, low interest rates and low growth clearly go together. However, when the EIS is above one, the effect of growth on the interest rate is small: unlike patience, the decrease in growth lowers the interest rate, but raises the price-dividend ratio, so the cash flow effect dominates the interest rate effect.

Panel A of Table 3 shows that taking both an increase in β and a decline in growth into account leads to a price-dividend ratio of about 70, not 50 as the data require. The remainder must be filled in by an increase in the risk premium (and a further decrease in expected future cash flows) through the disaster probability. Here, the model is helped by the fact that the increase in the disaster probability also causes a decline in the riskfree rate.

When the EIS is below one, any decrease in the growth rate μ will lead to an increase in the price-dividend ratio, as the interest rate effect will dominate the cash flow effect. It will also, through the channel described above, lead to a decline in interest rates, and the decline should be larger than that in the case of EIS greater than one. Suppose one starts with the demand-side view that investors have become more patient: β has risen.

As Panel A shows, any increase in β sends the price-dividend ratio soaring (this effect is not mediated by the EIS, and so is present regardless of what side of unity the EIS is on). One then needs to change growth μ and/or the disaster probability p to match the fact that the price-dividend ratio did *not* soar. However, when the EIS is below one, a decrease in μ makes the problem worse in that it raises the price-dividend ratio still further. It is then impossible to match the data with p because, again, an EIS less than one means that increasing p lowers the interest rate and *raises* the price-dividend ratio. If one tries to bring the price-dividend ratio down with lower p, the interest rate rises. For this reason, matching the price-dividend ratio and the interest rate requires that investors be less, not more patient.

To summarize: when the EIS is greater than one, one can reconcile the demand and supply intuition for lower interest rates, at the cost of needing to assume a greater probability of disaster. However, this reconciliation is fragile: it falls apart with an EIS less than one. Thus, accepting greater patience and lower growth as an explanation for the decline in interest rates requires accepting also that there is a higher probability of disaster and an EIS greater than one. While many models do assume an EIS greater than one, it is unsettling to have qualitative predictions of the model depend so heavily on a parameter falling within a certain range, for which there is little direct intuition or outside data support.

Did the equity premium rise? We return to the dependence on the value of the EIS later in the paper. We now ask whether the equity premium did in fact rise. A literature studies long-run variation in the equity premium, coming to the conclusion that the equity premium has declined over the postwar period, including from the first to the second periods that are our focus (Avdis and Wachter, 2017, van Binsbergen and Koijen, 2010, 2011, Fama and French, 2002, Lettau et al., 2008).

Options markets are another place to look for evidence of an increase in the equity premium. Virtually any explanation for an increase in the ex ante equity premium involves an increase in risk. While it is possible that such risk is not realized in sample, option prices provide market participants' ex ante measures of risk. Figure 8 shows the VIX, reported by the Chicago Board Options Exchange (CBOE). The VIX is the risk-neutral expectation of quadratic volatility, which is tightly tied to the equity premium. While the VIX is highly volatile, the average level of the VIX is remarkably stable between the two periods: equal to 21 in both. It is hard to reconcile this stability with an increase in the equity premium.

Given a model, one can say more. In Appendix C, we show how to go from the current model to a value of the VIX. A higher disaster probability implies a significantly higher VIX, not only because the ex ante volatility is higher (due to the realization of disasters), but because the risk-neutral volatility is higher still. If we ask the model to explain the level of the VIX in the earlier sample, and then modify the disaster probability as required, the VIX would need to rise from 21 to 23, rather than remain at 21 as it in fact did.[7] A test of whether the higher value is consistent with the data is rejected at the 1% level.

Inflationary default risk The typical empirical estimate of the equilibrium riskfree rate is the real return on short-term government debt; however, this return is not necessarily riskless, as the government can default either outright or through inflation. We now price this claim by including partial default that co-occurs with disasters. A decline in this partial default risk can explain the secular trends in riskfree rates and valuation ratios since 1980.

Consider a defaultable short-term government bond. Suppose that, in a disaster, the government partially defaults and pays $e^{-\zeta \eta_{t+1}}$ to its bondholders, where η_{t+1} is defined in (2). Hence, the bond pays the promised 1 dollar in the case of no default and $e^{-\zeta z}$ dollars in the case of default. Evidently, this collapses to the riskfree case

[7] In a similar way, Siriwardane (2015) and Seo and Wachter (2018) back out measures of disaster risk using options data and do not find an increase in the probability of disaster over this period.

when $\zeta = 0$. This means that the price of the one-period bond Q_t is now

$$Q_t = E_t \left[\beta^\theta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\theta}{\psi}} R_{W,t+1}^{\theta-1} e^{-\zeta \eta_{t+1}} \right]. \tag{8}$$

As shown in Appendix B.3, evaluating this expectation gives us the yield and expected return on the security. Let us define the yield on the bond $y_{b,t} \equiv -\log Q_t$ as the log of the return on the bond in the case of no default — that is, the realized return when there is no disaster and the government makes its promised payment of 1 dollar. We have that the government bond yield is given by

$$y_b = r_f + \log\left(1 + p(e^{\gamma z} - 1)\right) - \log\left(1 + p(e^{-(\zeta-\gamma)z} - 1)\right), \tag{9}$$

where r_f is the riskfree rate (5). For $\zeta > 0$, the yield is higher than the riskfree rate. Compare this to the expected return on the short-term bond, which is given by

$$\begin{aligned} \log E_t\left[R_{b,t+1}\right] &= r_f \\ &+ \underbrace{\log\left(1 + p(e^{-\zeta z} - 1)\right) + \log\left(1 + p(e^{\gamma z} - 1)\right) - \log\left(1 + p(e^{-(\zeta-\gamma)z} - 1)\right)}_{\text{risk premium}}. \end{aligned} \tag{10}$$

The last three terms of (10) are the risk premium on the short-term government bond. This premium is positive if $\zeta > 0$: the agent is compensated for the risk of partial default in a consumption disaster. Notice that the yield

$$y_b = \log E_t\left[R_{b,t+1}\right] - \underbrace{\log(1 + p(e^{-\zeta z} - 1))}_{\text{in-sample difference (no disaster)}}$$

is strictly greater than both the riskfree rate and the expected return on the bond when $\zeta > 0$. In a sample in which no disasters occur, the average ex post real return on the bond will correspond to the yield (9), not the expected return (10).

We have thus far assumed that the government defaults by failing to make part of its promised payments; a potentially more plausible way in which the government can default is through unexpected inflation. Suppose that the government bond is nominally riskfree and therefore subject to inflation in real terms. Let Π_t denote the gross inflation rate. Suppose that log inflation $\pi_{t+1} \equiv \log \Pi_{t+1}$ is subject to a standard Gaussian shock ε_t and the disaster shock given in (2):

$$\pi_{t+1} = \mu_{\pi,t} + \sigma_\pi \varepsilon_{t+1} + \zeta \eta_{t+1}. \tag{11}$$

The term $\mu_{\pi,t}$ represents some deterministic component — for example, an autoregressive process — and the coefficient ζ scales the magnitude of inflation that occurs in a disaster. Specifically, when there is a consumption disaster, there is inflation equal to ζz. In Appendix B.4, we show that there is a mapping between partial default through inflation and default through an outright failure to make payments. In particular, given the process (11), the real yield on the bond is given by

$$y_{b,t} = r_f + \log\left(1 + p(e^{\gamma z} - 1)\right) - \log\left(1 + p(e^{-(\zeta-\gamma)z} - 1)\right) - \mu_{\pi,t} + \frac{1}{2}\sigma_\pi^2, \tag{12}$$

and the expected real return on the bond is exactly equal to (10). The expression (12) is the same as (9), but now also subtracts the inflation drift terms in (11). Indeed, the real yield is simply the expected return on the bond less expected inflation:

$$y_{b,t} = \log E_t\left[R_{b,t+1}\right] - \underbrace{\left(\mu_{\pi,t} - \frac{1}{2}\sigma_\pi^2 + \log\left(1 + p(e^{-\zeta z} - 1)\right)\right)}_{\log E_t[\Pi_{t+1}]}.$$

A potential benefit of thinking of inflationary default, as opposed to outright default, is that it gives an intuitive interpretation of $\zeta < 0$. We can understand this case as corresponding to deflationary disasters, in which case the government bond becomes a hedge against disaster risk. In calibrating the model, we account for expected inflation by subtracting average inflation from the average nominal yield in each period. Because there are no disasters in the sample, this data moment is given by the right-hand side of (9).

Calibration and results We now calibrate this model in a similar way to the disaster model above, but instead of varying p and β, we keep p constant and allow ζ to vary. Table 4 shows the results. In Panel A, we match the data under the same baseline calibration as in Farhi and Gourio (2018). We see that β need not rise as much as before, because it is now needed to explain only a slight increase in the price-dividend ratio. Moreover, looking at Panels C and E, we see that the estimated changes in patience and growth in our calibration are not sensitive to whether the elasticity of intertemporal substitution (EIS) is above, below, or equal to one. Accounting for inflationary default risk allows us to match the data under a more plausible set of parameters, without assuming a counterfactually large increase in the disaster probability and without relying on a particular assumption about the EIS. Table 5 shows that these results are robust to evidence from across countries and using alternative valuation ratios. Table 6 shows that our results are not sensitive to our assumption about the disaster probability.

In all of our calibrations in Tables 4, 5, and 6, we estimate a decline in ζ from the first half of the sample to the second. In Panels D and E of Table 4 specifically, the calibrations suggest that disasters came with positive inflationary default risk in the first sample period ($\zeta = 0.501$, a 17.9% disaster inflation) and nearly no default risk in the second ($\zeta = 0.046$, a 1.6% disaster inflation). In the model, this corresponds to a positive and declining risk premium on the short-term government bond. This is similar to what Campbell et al. (2020) find, namely that inflation risk premia switched signs starting in 2001. While the magnitudes of our estimates vary across calibrations — we refer the reader to our final calibration in the next section for a quantitative estimate — the calibrations all support the conclusion that a decline in inflationary default risk is sufficient to explain the decline in real returns on short-term government debt.[8]

We have argued that a decline in the risk of unexpected inflation in a disaster is a

[8] This would also help to explain the results in van Binsbergen (2020), who shows that a stock market duration-matched portfolio of government debt has outperformed the stock market over the last 40 years. A reduction in long-run default and inflation risk would lead to unexpectedly large returns for government bond holders and would be consistent with the predictions of our model.

more plausible explanation than a large and persistent increase in the probability of a consumption disaster. But is there evidence to support our explanation? As depicted in Figure 6, expected inflation in the U.S. has declined substantially over the past four decades. It is reasonable that this stabilization in inflation expectations would coincide with a stabilization in the risk of large and unexpected inflation shocks. Indeed, we also find independent evidence of declining inflationary default risk when we compare expected and realized inflation. Consider the inflation process (11). In a sample with no disaster realizations, the difference between expected and realized inflation is given by $p\zeta z$.[9] We estimate this difference using the one-year-ahead inflation forecast from the Survey of Professional Forecasters, which is plotted in Figure 9. The horizontal dashed lines show the average difference in each of our respective samples, along with two-standard-error confidence intervals, and can be interpreted as estimates of $p\zeta z$. If we assume a disaster probability of $p = 0.03$, then the implied magnitude of inflation in the disaster state is around $\zeta z = 0.33$ from 1980 to 2000 and $\zeta z = 0$ from 2000 to present. Quantitatively, this corresponds to an inflation disaster of 33% and 0% in the first and second samples, respectively. There are other reasons for which expected and realized inflation could differ — for example, learning — but this exercise lends support to our explanation.

Our model and calibration imply that the *true* riskfree rate and equity risk premium have remained relatively constant over time. As we have discussed, this conclusion is consistent with evidence from valuation ratios, which have remained relatively flat, and

[9]For our given inflation process (11), a rational agent would predict inflation to be

$$E_t[\pi_{t+1}] = \mu_{\pi,t} + p\zeta z. \tag{13}$$

Now suppose there are no disasters in the sample, in which case the observed difference between realized and expected inflation is equal to

$$E_t[\pi_{t+1}] - \pi_{t+1} = p\zeta z - \sigma_\pi \varepsilon_{t+1}, \tag{14}$$

and its unconditional average

$$\frac{1}{T}\sum_{t=0}^{T} \left(E_t[\pi_{t+1}] - \pi_{t+1}\right) \tag{15}$$

converges to $p\zeta z$ asymptotically.

from the VIX, which suggests no substantial increase in risk. The price-dividend ratio, decomposed in (7), is unaffected by inflationary default risk. It is, however, common in the literature to use the return on the short-term government bond as a proxy for the true riskfree rate. Our calibration suggests that estimating the equity premium directly using this bond return implies an increase in the *measured* risk premium. In the model, this increase comes not from an increase in equity risk, but from a decline in the risk premium on government debt.

4 A model with production

We now move from an endowment economy into a model with a productive capital asset and a riskless inventory asset. We first solve the standard model with no inventory and show its equivalence to the standard endowment economy. Then, we introduce inventory and show that its existence imposes an endogenous zero lower bound on the equilibrium riskfree rate and, at this lower bound, has real effects on investment and risk premia.

4.1 Equilibrium with no inventory

Investment opportunities There is a productive capital asset, the quantity of which is denoted by K_t. Let δ denote the depreciation rate and A the productive capacity per unit of capital, so that output is given by a linear production function

$$Y_t = AK_t. \tag{16}$$

Following Gomes et al. (2019), we define *planned capital* to be the sum of the previous period's investment and depreciated capital stock:

$$\tilde{K}_{t+1} \equiv X_t + (1-\delta)K_t, \tag{17}$$

where X_t represents investment. As in Barro (2009), Gabaix (2011), and Gourio (2012), planned capital is subject to a capital quality shock:[10]

$$K_{t+1} = \tilde{K}_{t+1} e^{-\eta_{t+1}}, \tag{18}$$

where η_t is i.i.d. and has the distribution specified by (2). We then have that

$$R_{K,t+1} = (1 - \delta + A)e^{-\eta_{t+1}} \tag{19}$$

is the gross return on capital.

The agent can also trade in a zero-net-supply riskfree bond with gross return $R_{f,t+1}$. Because the agent faces a binary shock η_t, the market is complete. Let θ_t represent the agent's allocation, as a percentage of investable wealth $W_t - C_t$, to the capital asset.

Agent's problem Consider an investor with Epstein and Zin (1989) recursive preferences. We write the agent's consumption and investment problem as a standard infinite-horizon asset allocation problem (Samuelson, 1969), in which the agent chooses a portfolio of capital and the riskfree asset. Again, let C_t denote consumption and W_t denote wealth. The agent's value function $V(W_t)$ satisfies

$$V(W_t) = \max_{C_t,\theta_t} \left[(1-\beta)C_t^{1-1/\psi} + \beta \left(E_t \left[V(W_{t+1})^{1-\gamma} \right] \right)^{\frac{1-1/\psi}{1-\gamma}} \right]^{\frac{1}{1-1/\psi}}, \tag{20}$$

subject to the dynamic budget constraint

$$W_{t+1} = (W_t - C_t)(1 + r_f + \theta_t(r_{K,t+1} - r_f)) \equiv (W_t - C_t)R_{W,t+1}. \tag{21}$$

In the budget constraint, $r_f \equiv R_f - 1$ is the net return on the riskfree asset (this will be constant in equilibrium, so we simplify notation by omitting a time subscript) and $r_{K,t+1} \equiv R_{K,t+1} - 1$ is the net return on the capital asset from t to $t+1$. In this version

[10] In Barro (2009), this is a shock to depreciation rather than a capital quality shock, but the effects are the same.

of the model, we consider the case in which the elasticity of intertemporal substitution $\psi = 1$. Under this assumption, (20) becomes

$$V(W_t) = \max_{C_t, \theta_t} \left\{ C_t^{1-\beta} \left(E_t \left[V(W_{t+1})^{1-\gamma} \right] \right)^{\frac{\beta}{1-\gamma}} \right\}. \tag{22}$$

In the special case of relative risk aversion $\gamma = 1$, the agent has time-additive log utility.

Equilibrium The solution to this model is the standard production economy result.[11] Impose the market clearing condition $\theta = 1$, so $R_{W,t+1} = R_{K,t+1}$. The agent's first-order conditions imply a constant consumption-wealth ratio $\frac{C_t}{W_t} = (1-\beta)$. We also get that the equilibrium riskfree rate is equal to

$$\begin{aligned} R_f &= E_t \left[R_{K,t+1}^{1-\gamma} \right] E_t \left[R_{K,t+1}^{-\gamma} \right]^{-1} \\ &= (1 - \delta + A)(1 + p(e^{-(1-\gamma)z} - 1))(1 + p(e^{\gamma z} - 1))^{-1}, \end{aligned} \tag{23}$$

and the risk premium on capital is given by

$$\begin{aligned} \log E_t \left[\frac{1 + r_{K,t+1}}{1 + r_{f,t+1}} \right] &= \log \left(1 + p(e^{-z} - 1) \right) \\ &\quad + \log \left(1 + p(e^{\gamma z} - 1) \right) - \log \left(1 + p(e^{-(1-\gamma)z} - 1) \right). \end{aligned} \tag{24}$$

Moreover, for the market claim, which we define as the claim to aggregate output Y_t, the Euler equation implies that the price-dividend ratio is constant and equal to $\kappa^Y = \frac{\beta}{1-\beta}$. This is a consequence of the assumption of unit EIS.

Interpretation These results are isomorphic to the endowment economy discussed in Section 3. All that is required to equate the equilibrium prices in the two models is to set the parameters to match the consumption growth processes and returns on

[11] This model is a special case of the model with inventory that we introduce in Section 4.2. For a full derivation of the equilibrium conditions from which we obtain asset prices, see Appendix E.

wealth.[12] However, the key difference is that there are two margins of adjustment in this economy: quantities and prices. This is why β does not show up in (23): it instead acts on quantities through the investment-capital ratio, which in turn affects prices. In the standard endowment economy, quantities cannot adjust, as the representative investor must accept whatever quantity of the consumption good is produced by the Lucas tree.

In this production model, we see that a savings glut unambiguously leads to an investment boom. The equilibrium investment-capital ratio with unit EIS is given by

$$\frac{X_t}{K_t} = \beta(1 - \delta + A) - (1 - \delta), \tag{25}$$

which is strictly increasing in β. Evidently, an increased demand for savings coming from an increase in the β parameter implies an increase in the investment-capital ratio. Also note that, in the unit EIS case, risk does not affect the investment decision: lower investment relative to capital must come through either a reduction in β or the deterministic components of the return on capital A and δ. One could try to reconcile a decline in the riskfree rate with a decline in investment by arguing that productivity A or depreciation δ have declined. In order for this to match the decline in growth — a decline in μ in the endowment economy — one would need $A - \delta$ to decline as well. But even if this explanation succeeds at matching investment and interest rates, we return to the puzzle of stable valuation ratios and the dependence of results on the EIS. Furthermore, in the case where the EIS $\psi > 1$, increased risk could lead to a reduction in $\frac{X}{K}$, but this relies on scant evidence of increased risk and requires placing economically meaningful restrictions on the latent preference parameter ψ.

[12] When the representative investor has Epstein-Zin preferences with $\psi = 1$, this is achieved by setting $\beta^{-1}e^{\mu} = (1 - \delta + A)$. This restriction equates consumption growth processes and yields identical stochastic discount factors; one can verify this by comparing (5) and (23). This mapping is generally true of production and endowment economies, a result which is discussed in detail in Chapter 2 of Cochrane (2001).

Calibration and results In Panel B of Table 7, we calibrate this model to match the cyclically-adjusted price-earnings (CAPE) ratio, the short-term government bond rate, and the growth rate of the economy. Specifically, we choose β, ζ, and δ to match these moments in each sample. The calibration elucidates the puzzling nature of the reduction in the investment-capital ratio over the last four decades in light of falling interest rates and a slightly rising marginal product of capital.[13] While we are able to match rates and valuation ratios with reasonable estimates of β and ζ, the calibration implies an increase in the investment-capital ratio. We now show that accounting for the possibility of inventory allows us to explain all of the data.

4.2 Equilibrium with inventory

Investment opportunities Suppose now that, in addition to capital and a riskfree bond, the agent can invest in a zero-net-return inventory asset, the quantity of which is denoted by $I_t \geq 0$. Why would one have a positive-supply riskfree asset? Conceptually, anything that is a store of value from one period to another could be a riskfree asset, provided that it is in fact riskfree and can be frictionlessly interchanged between consumption and investment. Many consumption goods would not fit this description because they cannot easily be changed into something other than what they are. Money does fit this description provided that there is no unexpected inflation (in which case it ceases to be riskfree). To keep things simple, we will think of inventory as money.[14] Strictly speaking then, our analysis applies only to the second sample period, in which we estimate negative inflation risk. This turns out to make no difference — when the equilibrium real interest rate is greater than zero, inventory can exist but agents choose

[13]This is noted in Farhi and Gourio who proxy for the marginal product of capital using gross profitability, constructed as the ratio of (1-labor share) to the capital-output ratio. They also use the return on capital as constructed by Gomme et al. (2011). Both measures display a small increase between the two samples.

[14]This is not unlike the "social contrivance of money" as proposed by Samuelson (1958), which asserts that money can be used to obtain the socially optimal allocation in an overlapping generations framework in which the storage of consumption goods is impossible.

not to hold it.[15] Again, strictly speaking, if the asset is cash and there is non-zero expected inflation but no unexpected inflation, then we could specify a non-zero return on the inventory asset. However, expected inflation in the second sample period is small, and thus allowing for a slightly different return on inventory would make little difference. Likewise, we estimate unexpected inflation to be close to zero. Like all valuation equations, the existence of this riskfree storage is predicated on investors' (subjective) expectations about inflation. Evidence suggests (Reis, 2020) that in the later period it is reasonable to assume that investors believed inflation would be low and stable, and thus consistent with our assumptions on the existence of inventory. The interesting question of what determines inflation expectations is beyond the scope of this paper.[16]

Hence, the agent can trade in three assets: productive capital, zero-net-return inventory, and a zero-net-supply riskfree bond with gross return $R_{f,t+1}$. Because the agent faces a binary shock η_t, this investment opportunity set completes the market. As before, let θ_t represent the agent's allocation, as a percentage of investable wealth $W_t - C_t$, to the capital asset. Because inventory can be in positive supply, we will now have that θ_t can be different from unity in equilibrium. While we technically have two riskfree assets, we will take advantage of the equilibrium conditions below and think of the agent's problem as a portfolio allocation between capital and a single riskfree asset in non-negative supply and with net return $r_{f,t+1}$. The supply and return on this asset are then determined in two cases. In the first case, the riskfree rate is positive ($r_f > 0$) and the agent chooses to hold no inventory ($\theta = 1$). In the second case, the equilibrium rate is zero ($r_f = 0$) and the agent holds a positive position in inventory ($\theta < 1$). The formal statement of this intuitive breakdown is provided in Theorem 1 below; its proof, along with the technical derivation of the equilibrium conditions in our setup, is provided in Appendix E.

[15] Liquidity services could be such a reason investors choose to hold inventory in the presence of a positive riskfree interest rate, but for simplicity we do not assume these.

[16] While in this paper we consider a production economy, inventory may also exist in an endowment economy as riskfree storage of the endowment. In Appendix D, we derive results in this setting.

Agent's problem We continue to assume that the investor has Epstein and Zin (1989) recursive preferences with unit elasticity of intertemporal substitution (EIS) and solve for the agent's optimal consumption and portfolio of capital and the riskfree assets.[17] Again, let C_t denote consumption and W_t denote wealth. The agent's value function $V(W_t)$ satisfies (22), subject to the dynamic budget constraint (21) and the inventory non-negativity constraint[18]

$$\theta_t \leq 1. \tag{26}$$

Let ξ_t denote the Lagrange multiplier on this non-negativity constraint.

Equation (21) is equivalent to the market clearing condition under (16)–(19). To see this, let

$$W_t = C_t + I_t + \tilde{K}_{t+1}, \tag{27}$$

where I_t is the dollar investment in the riskless asset. Then

$$W_{t+1} = I_t(1 + r_f) + \tilde{K}_{t+1}(1 + r_{K,t+1}). \tag{28}$$

Substituting (27) into (28) implies (21), with

$$\theta_t = \frac{\tilde{K}_{t+1}}{W_t - C_t}. \tag{29}$$

Also, note that substituting the return on capital (19) into (28) implies

$$W_{t+1} = I_t(1 + r_f) + K_t(1 - \delta + A). \tag{30}$$

[17] As stated above, there are actually two riskfree assets in this economy — inventory and the bond — but the equilibrium conditions imply that either $r_f = 0$ and the agent holds inventory or $r_f > 0$ and the agent does not hold inventory.

[18] Technically, there is also a capital non-negativity constraint $\theta_t \geq 0$, but this constraint does not ever bind in our parameterization of the model, so we omit it for clarity.

Finally, note that

$$C_t + I_t + \tilde{K}_{t+1} = I_{t-1}(1 + r_f) + \tilde{K}_t(1 + r_{K,t+1}). \tag{31}$$

The left-hand side is simply (27), whereas the right-hand side is (28), but at t rather than $t+1$. Further, substituting in (17) allows us to rewrite the agent's budget constraint as

$$C_t + I_t + X_t = Y_t + I_{t-1}(1 + r_f), \tag{32}$$

which is an equivalent market clearing condition.

Characterizing equilibria In Appendix E.1, we conjecture and verify that the value function

$$V(W_t) = aW_t \tag{33}$$

for some constant a. The first-order condition with respect to consumption implies a constant consumption-wealth ratio $\frac{C_t}{W_t} = (1-\beta)$. Further, we show that for any gross return $R_{i,t+1}$ on an asset other than inventory, the Euler equation is given by

$$E_t \left[R_{W,t+1}^{1-\gamma}\right]^{-1} E_t \left[\frac{1}{(1 + r_f + \theta(r_{K,t+1} - r_f))^{\gamma}} R_{i,t+1}\right] = 1. \tag{34}$$

This implies the Euler equation for excess returns is

$$E_t \left[\frac{1}{(1 + r_f + \theta(r_{K,t+1} - r_f))^{\gamma}} (R_{i,t+1} - R_{j,t+1})\right] = 0. \tag{35}$$

For the inventory asset, which has a gross return of one, the Euler equation is

$$E_t \left[R_{W,t+1}^{1-\gamma}\right]^{-1} E_t \left[\frac{1}{(1 + r_f + \theta(r_{K,t+1} - r_f))^{\gamma}}\right] + \frac{\xi_t}{\beta} = 1, \tag{36}$$

where ξ_t is the Lagrange multiplier on the inventory non-negativity constraint (26).

We show that the model can be solved in two cases. These cases can be entirely characterized by the equilibrium riskfree rate in the production economy without inventory:

$$R^*_{f,t+1} = E_t\left[R^{1-\gamma}_{K,t+1}\right] E_t\left[R^{-\gamma}_{K,t+1}\right]^{-1}, \tag{37}$$

which is a function of the primitive model parameters only. Henceforth, we call this the *unconstrained riskfree rate*, since it is the rate that would prevail if the inventory technology did not endogenously impose a zero lower bound on the equilibrium rate. Naturally, the rate $R_{f,t+1}$ that prevails in an economy with inventory is then called the *constrained riskfree rate*. Having defined $R^*_{f,t+1}$, we get the following lemma from the agent's equilibrium conditions that we will use to characterize the equilibrium in Theorem 1:

Lemma 1. *If $\theta_t < 1$, then the gross real riskfree rate $R_{f,t+1} = 1$. If $\theta_t = 1$, then $R_{f,t+1} \geq 1$ and is equal to the real riskfree rate in a no-inventory economy $R^*_{f,t+1}$.*

Proof. See Appendix E.2. ■

The lemma tells us the relationship between the equilibrium rate and the amount of inventory held by the agent. Intuitively, the existence of a zero-net-return asset in the investment opportunity set imposes an endogenous zero lower bound on the return on any other riskfree asset. The proof can be reasoned by a no-arbitrage argument. If the return on the riskfree bond were negative ($R_f < 1$), then the agent would short the riskfree asset and purchase inventory, delivering a certain profit. If the equilibrium riskfree rate is positive, then the agent would like to short inventory and buy the riskfree bond, but this is prevented by the constraint $\theta_t \leq 1$, so the standard equilibrium rate prevails.

To pin down the equilibrium, we must then determine under what set of parameters the agent chooses a given θ_t. The following theorem shows that the equilibrium portfolio choice and riskfree return are determined solely by the unconstrained riskfree rate $R^*_{f,t+1}$:

Theorem 1. *If the unconstrained gross riskfree rate* $R^*_{f,t+1} < 1$*, then* $\theta_t < 1$ *and the constrained riskfree rate* $R_{f,t+1} = 1$*. If* $R^*_{f,t+1} \geq 1$*, then* $\theta_t = 1$ *and the equilibrium is as in a standard no-inventory production economy with* $R_{f,t+1} = R^*_{f,t+1}$*.*

Proof. See Appendix E.3. ■

The intuition is as follows. Suppose the agent does not hold inventory ($\theta = 1$) and the riskfree rate equals $R^*_{f,t+1} < 1$. Then, by the reasoning behind Lemma 1, the agent will wish to hold more inventory. However, doing so reduces the volatility of the return on the wealth portfolio and stochastic discount factor and thus increases the equilibrium riskfree rate. The agent will increase holdings of inventory until the equilibrium rate is equal to the return on inventory. Alternatively, if $R^*_{f,t+1} \geq 1$, the agent does not want to store into inventory and is unable to sell it, so the standard equilibrium prevails.

The theorem is powerful because it tells us that we can solve the model in two cases. In the first case, $R^*_f \geq 1$, so we set $\theta = 1$ and calculate $R_f = R^*_f$ from (37). In the second case, $R^*_f < 1$, so we set $R_f = 1$ and solve for θ from the Euler conditions.

Equilibrium Suppose $R^*_f \geq 1$. We solved for this case in Section 4.1: the equilibrium is as in the standard production economy with no inventory, so we set $\theta = 1$. The riskfree rate is given by (23), the risk premium by (24), and the price-dividend ratio on the output claim $\kappa^Y = \frac{\beta}{1-\beta}$. The investment-capital ratio is given by (25)

Now suppose $R^*_f < 1$. By Theorem 1, we set $R_f = 1$ and solve for the agent's optimal allocation $\theta < 1$ from the equilibrium conditions. We now have that the return on wealth $R_{W,t+1} = 1 + \theta r_{K,t+1}$, so the Euler equations (34) and (35) imply

$$E_t\left[(1+\theta r_{K,t+1})^{1-\gamma}\right]^{-1} E_t\left[\frac{1}{(1+\theta r_{K,t+1})^{\gamma}}(1+r_{i,t+1})\right] = 1, \tag{38}$$

and

$$E_t\left[\frac{1}{(1+\theta r_{K,t+1})^{\gamma}} r_{i,t+1}\right] = 0, \tag{39}$$

for some net asset return r_i. To solve for θ, let $r_{i,t+1} = r_{K,t+1}$ and evaluate the expectation in (39) to see that

$$\frac{pr_{K,z}}{(1+\theta r_{K,z})^{\gamma}} + \frac{(1-p)r_{K,0}}{(1+\theta r_{K,0})^{\gamma}} = 0, \tag{40}$$

where $r_{K,0} \equiv (1-\delta+A) - 1$ and $r_{K,z} \equiv (1-\delta+A)e^{-z} - 1$ are the net returns in the non-disaster and disaster states, respectively. We can solve this to get that

$$\theta = -\frac{((1-p)r_{K,0})^{1/\gamma} - (-pr_{K,z})^{1/\gamma}}{((1-p)r_{K,0})^{1/\gamma} r_{K,z} - (-pr_{K,z})^{1/\gamma} r_{K,0}} \tag{41}$$

is the agent's optimal allocation to capital.

The equilibrium allocation is therefore

$$\theta = \min\left\{1, -\frac{((1-p)r_{K,0})^{1/\gamma} - (-pr_{K,z})^{1/\gamma}}{((1-p)r_{K,0})^{1/\gamma} r_{K,z} - (-pr_{K,z})^{1/\gamma} r_{K,0}}\right\}. \tag{42}$$

One can verify that this is consistent with Theorem 1 by evaluating (37) and checking the conditions $R_f^* < 1$ and $R_f^* \geq 1$. For both of the cases above, we can informally write the return on the wealth portfolio $R_{W,t+1} = 1 + \theta r_{K,t+1}$. This will allow us to solve for asset prices and investment as a general function of the equilibrium θ.

In this economy, the growth rates of wealth and consumption are the same:

$$\frac{C_{t+1}}{C_t} = \frac{W_{t+1}}{W_t} = \beta(1+\theta r_{K,t+1}) = \beta\left(\theta(1-\delta+A)e^{-\eta_{t+1}} + 1 - \theta\right). \tag{43}$$

A larger allocation to inventory will lead to lower growth, but will reduce the volatility of consumption and the stochastic discount factor. The agent can mitigate the effects of disasters by storing risklessly into inventory. As a result, when the agent holds inventory, the risk premium on capital begins to *decrease* in risk. This is made clear in Figure 10, which shows comparative statics with respect to the magnitude of the disaster z. Absent inventory, the risk premium is strictly increasing in z. In the economy with inventory, the risk premium rises in z when inventory demand is zero, then declines in z when the agent begins to hold inventory.

The growth rates of capital and output are equal and given by

$$\frac{K_{t+1}}{K_t} = \frac{Y_{t+1}}{Y_t} = \beta\left(\theta(1-\delta+A)e^{-\eta_{t+1}} + (1-\theta)e^{\eta_t-\eta_{t+1}}\right), \tag{44}$$

as shown in Appendix E.4. Note that when $\theta = 1$, all of the growth rates in the economy are equal. Moreover, note from (44) that, if $\theta < 1$, the rate of capital growth depends on whether a disaster has occurred at time t. A consequence of this is that the investment-capital ratio depends on the disaster state and is given by

$$\frac{X_t}{K_t} = \beta\left(\theta(1-\delta+A) + (1-\theta)e^{\eta_t}\right) - (1-\delta). \tag{45}$$

If $\theta < 1$, this ratio is time-varying, despite the i.i.d. nature of the uncertainty and the existence of a balanced growth path. In a state in which a disaster has occurred, the agent begins with a relatively high inventory-capital ratio and reallocates inventory to capital to obtain the optimal θ. Since the initial level of capital K_t is low, investment is comparatively high. This is shown in Figure 11.

Using these results, we can solve for the price-dividend ratio of the market, defined as the claim to output Y_t. Because the distribution of output growth depends on the current realization of the disaster η_t, we conjecture that the price-dividend ratio $\kappa^Y(\eta_t)$ depends only on η_t. Thus, we have that

$$1 = E_t\left[R_{W,t+1}^{1-\gamma}\right]^{-1} E_t\left[R_{W,t+1}^{-\gamma}\left(\frac{\kappa^Y(\eta_{t+1})+1}{\kappa^Y(\eta_t)}\right)\left(\frac{Y_{t+1}}{Y_t}\right)\right]. \tag{46}$$

In Section E.5, we verify the conjecture and show that the price-divided ratio equals

$$\kappa^Y(0) = \frac{\hat{\beta}}{1-\beta}\left((1-p)(1+\theta r_{K,0})^{1-\gamma} + p(1+\theta r_{K,z})^{-\gamma}(1+\theta r_{K,0})e^{-z}\right), \tag{47}$$

$$\kappa^Y(-z) = \kappa^Y(0)\left(\frac{1+\theta r_{K,z}}{1+\theta r_{K,0}}\right)e^{z}, \tag{48}$$

where $\hat{\beta} \equiv \beta \left((1-p)(1+\theta r_{K,0})^{1-\gamma} + p(1+\theta r_{K,z})^{1-\gamma}\right)^{-1}$. In the case of $\theta = 1$, this reduces to the constant $\kappa^Y = \frac{\beta}{1-\beta}$. In the case of $\theta < 1$, the price-dividend ratio is higher following the occurrence of a disaster. This comes through the endogenous cashflow channel: in the presence of inventory, expected growth in capital and output are high when the disaster occurs and capital is destroyed. This means that, in the disaster state, the current dividend Y_t is low relative to the expected dividend next period. This can be seen in Figure 12, where the price-dividend ratio is much higher in the state where the disaster has occurred.

Finally, the price of the short-term government bond with outright default is equal to[19]

$$Q_t = \left(p(1+\theta r_{K,z})^{1-\gamma} + (1-p)(1+\theta r_{K,0})^{1-\gamma}\right)^{-1} \times \left(p(1+\theta r_{K,z})^{-\gamma} e^{-\zeta z} + (1-p)(1+\theta r_{K,0})^{-\gamma}\right). \quad (49)$$

Hence, the yield $y_b = -\log Q_t$ and the expected return

$$\log E_t[R_{b,t+1}] = \log\left(1 + p(e^{-\zeta z} - 1)\right) - \log\left(p(1+\theta r_{K,z})^{1-\gamma} + (1-p)(1+\theta r_{K,0})^{1-\gamma}\right) + \log\left(p(1+\theta r_{K,z})^{-\gamma} e^{-\zeta z} + (1-p)(1+\theta r_{K,0})^{-\gamma}\right). \quad (50)$$

When $\theta = 1$, this reduces to (10). Notice that, while the true riskfree rate cannot go below zero, the yield and expected return on the defaultable claim could be positive or negative, depending on the direction of the default risk premium.

Calibration and results We calibrate this model to match the real interest rate, price-dividend ratio, and GDP growth in the U.S., as in the sections above. To do this, we solve a system of three equations in three unknowns, where the unknowns are the

[19] The price of the bond with inflationary default, given that inflation follows (11), is equal to (49) multiplied by the expected inflation term $\exp\{-\mu_{\pi,t} + \frac{1}{2}\sigma_\pi^2\}$. We account for this term in our calibrations by subtracting average inflation from nominal yields.

calibrated parameters β, ζ, and δ. The three equations are given by

$$y_b = \log\left(p(1+\theta r_{K,z})^{1-\gamma} + (1-p)(1+\theta r_{K,0})^{1-\gamma}\right) \\ - \log\left(p(1+\theta r_{K,z})^{-\gamma} e^{-\zeta z} + (1-p)(1+\theta r_{K,0})^{-\gamma}\right) \quad (51)$$

$$\kappa^Y(0) = \frac{\hat{\beta}}{1-\beta}\left((1-p)(1+\theta r_{K,0})^{1-\gamma} + p(1+\theta r_{K,z})^{-\gamma}(1+\theta r_{K,0})e^{-z}\right) \quad (52)$$

$$\frac{Y_{t+1}(0)}{Y_t(0)} = \beta\left(\theta(1-\delta+A) + (1-\theta)\right), \quad (53)$$

where y_b is the real log yield on the defaultable claim (after accounting for non-disaster expected inflation), $\kappa^Y(0)$ is the price-dividend ratio given that a disaster has not occurred, and $\frac{Y_{t+1}(0)}{Y_t(0)}$ is output growth given no disasters at times t or $t+1$. Calibrating to these no-disaster moments is consistent with the fact that we do not observe any disaster realizations in our sample. We then find the values of the parameters of interest that make it such that the data moments match their corresponding model moments.

The results are displayed in Table 7. The model with inventory is able to match these moments with a reasonable calibration of β, ζ, and δ. Further, note that the AK production model with inventory does far better than the one without inventory. The presence of inventory allows the model to match the low growth from 2001 to 2016 with a smaller rise in depreciation than the model without it.[20] This is because inventory leads to less growth in output, as the movement of funds to inventory crowds out investment. This can be seen by the lower investment-capital ratio in the model with inventory. As a caveat, both growth and the investment-capital ratio are calculated in the states where a disaster has not occurred. Expected growth in this model is much lower than realized growth much of the time, precisely because the disaster is rare. The opposite is the case for the investment-capital ratio; the agent diverts investment

[20] This rise in depreciation is consistent with the findings of Farhi and Gourio (2018) and Barkai (2020), who also report a rise in depreciation across the two samples we examine.

from capital to inventory in the non-disaster states, anticipating that a disaster may occur. In the state where the disaster does occur, the agent invests heavily, as seen in Figure 11. This reduction in the investment-capital ratio in normal states allows the model to match the observed reduction in the data *without targeting it*. This is not possible in the model without inventory, as the higher β and increased depreciation raise the equilibrium investment-capital ratio.

5 Concluding remarks

The puzzle of low interest rates is a puzzle not only from the point of view of the last quarter century, but over a much longer horizon. It is also a joint puzzle: why have low interest rates not been accompanied by higher valuation ratios?

The purpose of this article is to argue that the most natural explanation is not an increased demand for savings, which would lower interest rates and raise valuation ratios; nor a decrease in growth, which is hardly enough on its own to account for the observed change; nor an increase in the risk premium, as there is no evidence that risk has increased by nearly the required amount. These joint phenomena have a simple explanation, which is that *the true riskfree rate has hardly changed at all*. Short-term debt claims are defaultable, and investors have come to require a lower premium for this risk of default.

Because our explanation implies that the true riskfree rate has remained roughly constant, we require a framework that allows for, first, a riskfree rate that is sufficiently low to explain nominal debt yields at zero and, second, an explanation that survives the existence of a zero lower bound. We accomplish the former using a model with a risk of rare disasters. In a rare disaster model, investors' precautionary savings pushes the riskfree rate below zero. We accomplish the latter by introducing a costless storage technology into an endowment economy. When parameters are such that the true riskfree rate is below zero, agents choose to store the consumption good until markets clear at a riskfree rate of zero.

What we do not model is the cause for the decline in investor expectations of sovereign default. Evidence suggests that this decline both has a relatively short-term component based on the history of the last 30 years and a long-term component spanning centuries, based on a growing faith over time in the stability of sovereigns. The forces determining this shift in expectations are an interesting topic for further research.

A Data appendix

We use various series to illustrate the secular decline in interest rates in the short- and long-run. To obtain interest rates from 1311-2018, we rely on data from Schmelzing (2020). The dataset contains nominal interest rate and inflation time series for several developed economies over the last eight centuries. Specifically, the data include long-term sovereign borrowing rates with an average maturity that hovers around 10-years; however, this varies over time and across countries. From these data, we plot the nominal sovereign borrowing yields for the United Kingdom, Holland, Germany, Italy, and the United States in Figure 1. The data are collected from a variety of sources, outlined in detail in the paper and online appendix. The U.K. borrowing rates come from the Calendar of State Papers and the Bank of England. Data before 1694 for the U.K. (before the founding of the Bank of England) are not used, since the data are incomplete. Data for the Netherlands come from Dormans (1991), Weeveringh (1852), the European Central Bank, and various sources from Leiden, Haarlem, Utrecht, Schiedam, and Amsterdam. German data come from various sources from several German principalities. U.S. data come from Durand and Winn (1947), Homer and Sylla (2005), the NBER Macrohistory database, and Federal Reserve Economic Data (FRED) from the Federal Reserve Bank of St. Louis.

We also report the Bank of England (BoE) short-term lending rate (series BOERUKM) from FRED. From 1694 to 1971, the "bank rate" is used; from 1972 to 1981, the minimum lending rate is used; from 1981 to 1997, the BoE base rate is used; and from 1997 to the present, the BoE Operational interest rate is used. For more information see the Bank of England research datasets webpage.

Data for U.S. interest rates from 1984 to 2016 come from FRED. Our main measure for nominal interest rates in the U.S. is the effective Federal Funds Rate (series FEDFUNDS), the rate corresponding to the median volume of overnight unsecured loans between depository institutions. This is plotted in Panel A of Figure 3. In our calibration exercises, for comparability with Farhi and Gourio (2018) we use the one-year constant maturity Treasury rate, less current inflation. Data for U.K. interest rates from 1984 to 2016 come from Jordà et al. (2019), who in turn use data from Zimmermann (2017) and the Bank of England.

Data on U.S. inflation expectations come from FRED and the Survey of Professional Forecasters. From FRED, we use the inflation expectations from the Surveys of Consumers of University of Michigan (series MICH), which covers short-term inflation expectations, and the expected 10-year-ahead inflation implied from Treasury Inflation-Indexed Constant Maturity Securities (series T10YIE). From the Survey of Professional Forecasters, we use the 10-year ahead inflation expectations. These data are shown in Figure 6. Further, we use median one-year-ahead expected inflation from the Survey of Professional Forecasters to construct the deviation of expected inflation from realized inflation, shown in Figure 9.

Growth data come from different sources. In Tables 1–6, the U.S. growth parameter μ is set to the same values as in Farhi and Gourio (2018)[21] who use a composite growth parameter obtained by combining the growth in population, investment, total factor productivity, and the Cobb-Douglas production function parameter α (which they estimate). In Figure 4 and Table 7, we use real GDP growth rates from FRED (series GDPC1) as the growth rate for the U.S. When calibrating to the U.K. data, we use the real GDP growth series from Jordà et al. (2019).

Data on investment and capital stock come from the Bureau of Economic Analysis (BEA) Fixed Assets Accounts Tables. Investment data come from Table 1.5, Line 2 and capital stock data come from Table 1.1, Line 2. In these data, investment as a fraction of capital averaged 7.7% from 1984–2000 and 6.9% from 2001–2016.

Price-dividend ratio data for the U.S. from 1984 to 2016 are from the Center for Research in Security Prices (CRSP). Specifically, we use cum-dividend returns (series VWRETD) and ex-dividend returns (series VWRETX). To calculate the price-dividend ratio, we back out prices and dividends from cum- and ex-dividend returns. This series is plotted in Panel B of Figure 3. We use this procedure to calculate our price-dividend

[21] In particular, we set μ equal to the g_T parameter in Farhi and Gourio.

ratio moments for the calibrations in Tables 1 and 4.

For the longer U.S. valuation data, we use prices and dividends on the S&P 500 from Shiller (2000). We also form the cyclically-adjusted price-earnings ratio (CAPE): the price divided by the average inflation-adjusted earnings from the previous 10 years. See (Shiller, 2000) and online data description. For the U.K. valuation data, we use data from Jordà et al. (2019). Jordà et al. aggregate total returns data from Grossman (2002) and from Barclays Equity Gilt Study.

Finally, we obtain the Volatility Index (VIX) series from the Chicago Board Options Exchange (CBOE). The CBOE calculates the risk-neutral expected 30-day quadratic variation using option prices. There are small differences in the calculation methodology over the years; see CBOE white paper.

B Derivations for Section 3: Endowment economy with rare disasters

B.1 Price-consumption ratio

The stochastic discount factor (SDF) under Epstein and Zin (1989) utility is given by

$$M_{t+1} = \beta^{\theta} \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\theta}{\psi}} R_{W,t+1}^{\theta-1}. \tag{B.1}$$

Under (B.1) the Euler equation with respect to the consumption claim is

$$1 = E_t \left[\beta^{\theta} \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\theta}{\psi}} R_{W,t+1}^{\theta} \right]. \tag{B.2}$$

Conjecture a constant price-consumption ratio

$$\kappa \equiv (W_t - C_t)/C_t. \tag{B.3}$$

Substituting (B.3) into (B.2) and using $R_{W,t+1} = W_{t+1}/(W_t - C_t)$ implies

$$1 = \beta^\theta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{\theta(1-\frac{1}{\psi})}\left(\frac{\kappa+1}{\kappa}\right)^\theta\right]. \tag{B.4}$$

Given (1)–(2),

$$\frac{\kappa}{\kappa+1} = \beta e^{(1-\frac{1}{\psi})\mu}\left[1 + p(e^{-(1-\gamma)z} - 1)\right]^{\frac{1}{\theta}}. \tag{B.5}$$

A solution exists provided that the right hand side of (B.5) is less than one. We restrict attention to parameter combinations satisfying this restriction. Finally,

$$\kappa = \frac{\beta e^{(1-\frac{1}{\psi})\mu}\left[1 + p(e^{-(1-\gamma)z} - 1)\right]^{\frac{1}{\theta}}}{1 - \beta e^{(1-\frac{1}{\psi})\mu}\left[1 + p(e^{-(1-\gamma)z} - 1)\right]^{\frac{1}{\theta}}}, \tag{B.6}$$

verifying the conjecture.

B.2 Riskfree rate

The riskfree rate is given by the Euler equation for the riskfree asset

$$R_f^{-1} = E_t\left[\beta^\theta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}} R_{W,t+1}^{\theta-1}\right]. \tag{B.7}$$

This simplifies to

$$R_f^{-1} = E_t\left[\beta^\theta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left(\frac{\kappa}{\kappa+1}\right)^{1-\theta}\right]. \tag{B.8}$$

where $\frac{\kappa}{\kappa+1}$ is given by (B.5). Solving this yields the expression for the gross riskfree rate

$$R_f = \beta^{-1} e^{\frac{1}{\psi}\mu}\left[1 + p(e^{\gamma z} - 1)\right]^{-1}\left[1 + p(e^{-(1-\gamma)z} - 1)\right]^{\frac{\theta-1}{\theta}} \tag{B.9}$$

which implies that the log riskfree rate $r_f \equiv \log R_f$ is given by

$$r_f = -\log\beta + \frac{1}{\psi}\mu - \log(1 + p(e^{\gamma z} - 1)) + \left(\frac{\theta - 1}{\theta}\right)\log(1 + p(e^{-(1-\gamma)z} - 1)). \quad \text{(B.10)}$$

B.3 Yield and expected return on defaultable claim

Consider the defaultable short-term government bond paying $e^{-\zeta\eta_{t+1}}$ dollars that is, 1 dollar in the case of no default and $e^{-\zeta z}$ dollars in the case of default. The price of the defaultable claim is obtained by solving the Euler equation

$$Q_t = E_t\left[\beta^\theta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}} R_{W,t+1}^{\theta-1} e^{-\zeta\eta_{t+1}}\right], \quad \text{(B.11)}$$

which simplifies to

$$Q_t = E_t\left[\beta^\theta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \left(\frac{\kappa}{\kappa+1}\right)^{1-\theta} e^{-\zeta\eta_{t+1}}\right], \quad \text{(B.12)}$$

where $\frac{\kappa}{\kappa+1}$ is given by (B.5). This gives the price of the defaultable claim as

$$Q_t = \beta e^{-\frac{1}{\psi}\mu}\left[1 + p(e^{-(1-\gamma)z} - 1)\right]^{\frac{1-\theta}{\theta}}\left[1 + p(e^{-(\zeta-\gamma)z} - 1)\right]. \quad \text{(B.13)}$$

The yield on the defaultable claim is defined as $y_{b,t} \equiv -\log Q_t$, and is thus equal to the constant

$$y_b = r_f + \log\left(1 + p(e^{\gamma z} - 1)\right) - \log\left(1 + p(e^{-(\zeta-\gamma)z} - 1)\right), \quad \text{(B.14)}$$

where r_f is given by (B.10). The expected return on the bond is the expected payoff divided by the price, and therefore equals

$$\begin{aligned}\log E_t\left[R_{b,t+1}\right] &= r_f \\ &+ \log\left(1 + p(e^{-\zeta z} - 1)\right) + \log\left(1 + p(e^{\gamma z} - 1)\right) - \log\left(1 + p(e^{-(\zeta-\gamma)z} - 1)\right). \quad \text{(B.15)}\end{aligned}$$

B.4 Inflationary default

We now model the defaultable short-term government bond as the nominally riskfree asset; the government partially defaults through inflation. We thus have that the real payoff on the bond is $e^{-\pi_{t+1}}$ dollars, where inflation π_{t+1} follows the process (11). The price of the defaultable claim is obtained by solving the Euler equation

$$Q_t = E_t\left[\beta^\theta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}} R_{W,t+1}^{\theta-1} e^{-\pi_{t+1}}\right], \tag{B.16}$$

which simplifies to

$$Q_t = E_t\left[\beta^\theta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \left(\frac{\kappa}{\kappa+1}\right)^{1-\theta} e^{-\pi_{t+1}}\right], \tag{B.17}$$

where $\frac{\kappa}{\kappa+1}$ is given by (B.5). This gives the price of the defaultable claim as

$$Q_t = \beta e^{-\frac{1}{\psi}\mu}\left[1 + p(e^{-(1-\gamma)z} - 1)\right]^{\frac{1-\theta}{\theta}} \left[1 + p(e^{-(\zeta-\gamma)z} - 1)\right] e^{-\mu_{\pi,t}+\frac{1}{2}\sigma_\pi^2}. \tag{B.18}$$

This price is identical to that in the case of outright default, but accounts also for expected inflation. The yield on the defaultable claim, defined as $y_{b,t} \equiv -\log Q_t$, is then equal to the constant

$$y_{b,t} = r_f + \log\left(1 + p(e^{\gamma z} - 1)\right) - \log\left(1 + p(e^{-(\zeta-\gamma)z} - 1)\right) - \mu_{\pi,t} + \frac{1}{2}\sigma_\pi^2, \tag{B.19}$$

where r_f is given by (B.10). The yield in this case is equal to that in the outright default case less the expected inflation drift. The expected return on the bond is the expected payoff divided by the price, and therefore equals

$$\begin{aligned}\log E_t\left[R_{b,t+1}\right] &= r_f \\ &+ \log\left(1 + p(e^{-\zeta z} - 1)\right) + \log\left(1 + p(e^{\gamma z} - 1)\right) - \log\left(1 + p(e^{-(\zeta-\gamma)z} - 1)\right).\end{aligned} \tag{B.20}$$

Note that this value is identical to that in the case of outright default.

C Volatility Index in a disaster economy

For tractability, we adapt the simple disaster model to continuous time, following Seo and Wachter (2019). Suppose consumption follows the jump-diffusion process

$$\frac{dC_t}{C_{t^-}} = \mu dt + \sigma dB_t + (e^{-z_t} - 1)dN_t, \tag{C.1}$$

where B_t is a standard Brownian motion, N_t is a Poisson process with constant intensity λ, and z_t has time-invariant distribution υ. As in Abel (1999) and Campbell (2003), we model dividends as levered consumption: $D_t = C_t^{\phi}$. Under both power utility and recursive preferences, it follows that the price of the claim to the dividend stream follows the process

$$\frac{dS_t}{S_{t^-}} = \mu_S dt + \phi\sigma dB_t + (e^{-\phi z_t} - 1)dN_t. \tag{C.2}$$

The quadratic variation is then given by

$$QV_{t,t+\tau} \equiv \int_t^{t+\tau} d[\log S, \log S]_s = \phi^2\sigma^2\tau + \int_t^{t+\tau} \phi^2 z_s^2 dN_s. \tag{C.3}$$

For risk-neutral measure Q, the VIX is then given by

$$\text{VIX}_t^2 \equiv E_t^Q[QV_{t,t+\tau}] = \phi^2\left(\sigma^2 + \lambda E_\upsilon\left[e^{\gamma z_t} z_t^2\right]\right)\tau, \tag{C.4}$$

where the last term follows from Girsanov's theorem:

$$E_{t^-}^Q\left[\phi^2 z_s^2 dN_s\right] = E_{t^-}\left[\frac{\pi_t}{\pi_{t^-}}\phi^2 z_s^2 dN_s\right] = \lambda\phi^2 E_\upsilon\left[e^{\gamma z_t} z_t^2\right]. \tag{C.5}$$

Note that these formulas hold for both time-additive utility and recursive preferences.

To calculate the implied VIX in the model, we choose parameters according to our calibration in Table 1: disaster size $z = -\log 0.85$, relative risk aversion coefficient $\gamma = 12$, consumption volatility $\sigma^2 = 0.02$, first sample disaster intensity $\lambda_1 = 0.03$, and second sample disaster intensity $\lambda_2 = 0.07$. These are annualized parameters, so $\tau = 1/12$ matches the time interval used to calculate the VIX. We then choose ϕ^2 such that (C.4) with λ_1 is equal to the empirically observed value 0.2056^2 in the first sample. Given this calibration — which implies $\phi^2 = 19.8$ — we calculate that the implied VIX with $\lambda_2 = 0.07$ is 23.36 compared to the empirical average of 20.66. Using Newey-West standard errors with two lags on the monthly VIX, the t-statistic on this test is 2.66.

D Endowment economy with inventory

D.1 A two-period model with inventory

There are two periods $t \in \{0, 1\}$. Let Ω represent the set of states ω at time $t = 1$. Markets are completed by a set of state-contingent claims with prices $\varphi(\omega)$, quantities $\nu(\omega)$, and unit payoffs. Let P_i denote the price of a portfolio of these claims with payoff $x_i(\omega)$. The representative agent chooses consumption $\{C_t\}$, inventory I, and allocations $\{\theta(\omega)\}$ to maximize

$$(1-\beta)\log C_0 + \beta E_0 \left[\log C_1(\omega)\right] \tag{D.1}$$

subject to[22]

$$C_0 + I + \int_{\omega \in \Omega} \varphi(\omega)\nu(\omega) d\omega = Y_0, \tag{D.2}$$

$$C_1(\omega) = Y_1(\omega) + I + \nu(\omega), \tag{D.3}$$

[22]Because marginal utility approaches infinity as consumption approaches zero, the constraint $I \leq Y_0$ will never bind, so we omit it from the list of constraints.

$$I \geq 0. \tag{D.4}$$

The contingent claims are in zero net supply, so markets clear when

$$C_0 = Y_0 - I, \quad C_1(\omega) = Y_1(\omega) + I. \tag{D.5}$$

Let λ_0, $\lambda_1(\omega)$, and ξ represent the Lagrange multipliers on (D.2), (D.3), and (D.4), respectively. When $\xi = 0$, (D.4) does not bind. We then have the first-order conditions with respect to C_0, $C_1(\omega)$, I, and $\nu(\omega)$, respectively:

$$(1-\beta)C_0^{-1} = \lambda_0, \tag{D.6}$$

$$\beta C_1(\omega)^{-1} = \lambda_1(\omega), \tag{D.7}$$

$$\lambda_0 = E_0\left[\lambda_1(\omega)\right] + \xi, \tag{D.8}$$

$$\varphi(\omega)\lambda_0 = \lambda_1(\omega). \tag{D.9}$$

Note that (D.8) holds only in expectation, since inventory is chosen before the realization of ω. Note also that combining (D.6), (D.7), and (D.9) gives us the traditional Euler equation

$$\varphi(\omega) = \frac{\beta}{1-\beta}\left(\frac{C_0}{C_1(\omega)}\right). \tag{D.10}$$

Moreover, we can combine the first-order conditions, take expectations, and impose market clearing to get two conditions (suppressing the argument of $Y_1(\omega)$):

$$1 = E_0\left[\frac{\beta}{1-\beta}\left(\frac{Y_0 - I}{Y_1 + I}\right)\right] + \xi\frac{(Y_0 - I)}{1-\beta}, \tag{D.11}$$

$$1 = E_0 \left[\frac{\beta}{1-\beta} \left(\frac{Y_0 - I}{Y_1 + I} \right) R_i \right], \tag{D.12}$$

where R_i is the gross return of any asset. Furthermore, suppose that output Y is distributed according to

$$\log Y_1 = \log Y_0 + \mu - \eta, \tag{D.13}$$

where η is defined as in equation (2) for disaster probability p and disaster magnitude z.

D.1.1 Riskfree rate

Now suppose there is a riskfree asset. Its return is characterized by

$$1 = E_0 \left[\frac{\lambda_1(\omega)}{\lambda_0} R_f \right], \tag{D.14}$$

which gives us

$$\frac{\xi}{\lambda_0} = \begin{cases} 1 - \frac{1}{R_f} & \text{if} \quad R_f > 1, \\ 0 & \text{otherwise.} \end{cases} \tag{D.15}$$

In the first case, the constraint binds, so we know that $I = 0$. In the second case, the constraint does not bind, so $I > 0$. This implies the following important result: when the constraint binds, R_f is the same as in the usual endowment economy.

The inventory asset serves as zero-return endowment storage, and thus the absence of arbitrage ensures that the riskfree rate is bounded below. Intuitively, if the unconstrained (i.e., absent inventory) gross riskfree rate $R_f < 1$, then $\exists I > 0$ such that $R_f = 1$. This intuition is formalized as follows.

Proposition D.1. *Suppose the agent has strictly positive marginal utility $u'(C_t)$. When $I > 0$, the gross riskfree rate $R_f = 1$. When $I = 0$, $R_f > 1$ and is equal to the riskfree rate in a no-inventory endowment economy.*

Proof. If $I > 0$, then $\xi = 0$ and (D.11) and (D.12) combine to give us $R_f = 1$. If $I = 0$, then $\xi > 0$ and

$$R_f = 1 + \frac{\xi}{\beta E_0 \left[u'(C_1)\right]}, \tag{D.16}$$

which is greater than 1 by the assumption that $u'(C_t) > 0$. Moreover, if $I = 0$, then the Euler equation (D.12) yields

$$R_f = E_0 \left[\frac{\beta}{1-\beta} \left(\frac{u'(Y_1)}{u'(Y_0)} \right) \right]^{-1}, \tag{D.17}$$

which is the same as in the no-inventory endowment economy. ■

Note that this result is independent of the stochastic process governing output Y. If we impose (D.13), then we can calibrate the model and do comparative statics for different disaster probabilities. Figure D.1 plots the riskfree rate as a function of the disaster probability p. The ratio of inventory to initial output $i \equiv I/Y_0$ is given implicitly by the relation

$$1 = E_0 \left[\frac{\beta}{1-\beta} \left(\frac{1-i}{e^{\mu-\eta} + i} \right) \right]. \tag{D.18}$$

when the constraint does not bind ($\xi = 0$), and $i = 0$ otherwise. Given the process (D.13), we have an explicit solution for i, which we plot along with the unconstrained riskfree rate in Figure D.1. As suggested by the above proposition, inventory imposes a lower bound on the riskfree rate. For low values of p, the equilibrium riskfree rate is positive and there is no demand for inventory; for high values of p, the agent invests in inventory such that $R_f = 1$.

D.1.2 Risk premia

We define two risky assets and solve for their expected returns and implied risk premia. The first is the consumption claim, which pays a dividend of $Y_1 + I$; the second is the market (the output claim), which pays Y_1. Given our log utility setup, the price of the consumption claim is

$$P_C = \frac{\beta}{1-\beta}(Y_0 - I). \tag{D.19}$$

This is independent of Y_1, a result of log utility. This security's return is

$$R_C = \frac{1-\beta}{\beta}\left(\frac{Y_1 + I}{Y_0 - I}\right). \tag{D.20}$$

So the return on the consumption claim equals the inverse of the agent's intertemporal marginal rate of substitution. Similarly, the price of the market claim is

$$P_M = \frac{\beta}{1-\beta}(Y_0 - I)E_0\left[\left(\frac{Y_1}{Y_1 + I}\right)\right] = P_C E_0\left[\left(\frac{Y_1}{Y_1 + I}\right)\right], \tag{D.21}$$

and the market return is thus

$$R_M = \frac{1-\beta}{\beta}\left(\frac{Y_1}{Y_0 - I}\right)E_0\left[\left(\frac{Y_1}{Y_1 + I}\right)\right]^{-1}. \tag{D.22}$$

Note that, in the absence of inventory, the consumption claim and the market are the same security. Thus, we can consider three securities: the consumption claim with inventory, the market claim with inventory, and the market claim without inventory. Figure D.2 plots the expected returns and the implied risk premia for each of these securities under the same calibration as in the previous section. Notice that the risk premia of all securities are virtually the same, but due to the lower bound on the riskfree rate, the gross returns on these securities diverge for high disaster probabilities.

D.1.3 Price-dividend ratio

We can calculate the price-dividend ratio for each of the three securities in the previous section. For the consumption claim, we retrieve the usual log utility result:

$$\frac{P_C}{C_0} = \frac{\beta}{1-\beta}. \tag{D.23}$$

The price-dividend ratio for the market is

$$\frac{P_M}{Y_0} = \frac{\beta}{1-\beta} E_0 \left[\left(\frac{Y_1}{Y_1 + I} \right) \right]. \tag{D.24}$$

Evidently, these ratios are the same when there is no inventory asset or when $I = 0$. Figure D.3 plots these ratios for each of the three securities. Note that the price-dividend ratio absent inventory equals the price-consumption ratio with inventory, and both are constant. Note also that, as inventory becomes more positive due to an increasing disaster probability, the price-dividend ratio on the market begins to decline. This can be thought of as a leverage effect of inventory.

D.2 A multi-period model with inventory

D.2.1 General theory: Real inventory

Consider an infinite horizon economy. The agent has access to an inventory technology with which he or she can store consumption across periods. It follows from this definition that inventory is a real asset. Let $\Omega_{t+1|t}$ represent the set of time-$(t+1)$ states ω_{t+1} that can be reached from state ω_t. Markets are completed by a set of state-contingent claims. Let $\varphi(\omega_{t+1}|\omega_t)$ and $\nu(\omega_{t+1}|\omega_t)$ represent the price and quantity, respectively, of the state-ω_{t+1}-contingent claim in state ω_t. Let $\mathbb{P}(\omega_{t+1}|\omega_t)$ represent the conditional probability of state ω_{t+1}. The representative agent chooses consumption $\{C(\omega_t)\}$, inventory $\{I(\omega_t)\}$, investment $\{X(\omega_t)\}$, and asset allocations $\{\nu(\omega_{t+1}|\omega_t)\}$ to maximize[23]

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t u(C_t) \right] \tag{D.25}$$

subject to

$$C_t + I_t + X_t + \int_{\Omega_{t+1|t}} \varphi_t(\omega_{t+1}) \nu_t(\omega_{t+1}) d\omega_{t+1} = Y_t + I_{t-1} + \nu_{t-1}(\omega_t), \tag{D.26}$$

[23]For notational simplicity, we suppress the argument ω_t except where necessary for clarity. For example, we write C_t instead of $C(\omega_t)$ and $\varphi_t(\omega_{t+1})$ instead of $\varphi(\omega_{t+1}|\omega_t)$.

$$I_t \geq 0, \tag{D.27}$$

$$I_t \leq Y_t + I_{t-1} - X_t. \tag{D.28}$$

Note the generality of this setup: in a production economy, output Y_t is a function of last period's investment X_{t-1} and capital stock K_{t-1}; in an endowment economy, we let Y_t follow some given process and set $X_t = 0$, $\forall t$. Now define $\Delta I_t \equiv I_t - I_{t-1}$. The contingent claims are in zero net supply, so markets clear when

$$C_t = Y_t - \Delta I_t - X_t. \tag{D.29}$$

For Lagrange multipliers $\lambda(\omega_t)$ and $\xi(\omega_t)$ on (D.26) and (D.27), respectively, we get first-order conditions

$$\beta^t u'(C_t) = \lambda_t, \tag{D.30}$$

$$E_t[\lambda_{t+1}] + \xi_t = \lambda_t, \tag{D.31}$$

$$E_t\left[\lambda_{t+1}\frac{\partial Y_{t+1}}{\partial X_t}\right] = \lambda_t, \tag{D.32}$$

$$\varphi_t(\omega_{t+1})\lambda_t = \mathbb{P}_t(\omega_{t+1})\lambda_{t+1}. \tag{D.33}$$

Note that (D.31) and (D.32) hold only in expectation, since inventory I_t and investment X_t are chosen before the realization of state ω_{t+1}. Note also that combining (D.30) and (D.33) gives us the traditional Euler equation

$$\varphi(\omega_{t+1}|\omega_t) = \mathbb{P}(\omega_{t+1}|\omega_t)\beta\frac{u'(C(\omega_{t+1}))}{u'(C(\omega_t))}. \tag{D.34}$$

Moreover, we can combine the first-order conditions and take expectations to get two conditions:

$$E_t\left[\beta\frac{u'(C_{t+1})}{u'(C_t)}\right]+\frac{\xi_t}{\beta^t u'(C_t)}=1, \tag{D.35}$$

$$E_t\left[\beta\frac{u'(C_{t+1})}{u'(C_t)}R_{i,t+1}\right]=1, \tag{D.36}$$

where $R_{i,t+1}$ is the gross return of any asset, including the return on investment $R_{X,t+1}\equiv\frac{\partial Y_{t+1}}{\partial X_t}$. We can use these equilibrium results to prove the following proposition.

Proposition D.2. *Suppose the agent has strictly positive marginal utility $u'(C_t)$ and inventory is a real asset. When $I_t>0$, the gross real riskfree rate $R_{f,t+1}=1$. When $I_t=0$, $R_{f,t+1}>1$ and is equal to the real riskfree rate in a no-inventory economy.*

Proof. If $I_t>0$, then $\xi_t=0$ and (D.35) and (D.36) combine to give us $R_{f,t+1}=1$. If $I_t=0$, then $\xi_t>0$ and

$$R_{f,t+1}=1+\frac{\xi_t}{\beta^{t+1}E_t\left[u'(C_{t+1})\right]}, \tag{D.37}$$

which is greater than 1 by the assumption that $u'(C_t)>0$. Moreover, if $I_t=0$, then the Euler equation (D.36) yields

$$R_{f,t+1}=E_t\left[\beta\frac{u'(Y_{t+1}-X_{t+1})}{u'(Y_t-X_t)}\right]^{-1}, \tag{D.38}$$

which is the same as in the no-inventory economy. ■

D.2.2 General theory: Nominal inventory

Consider the same setup as above, but now we have a price level P_t and assume that inventory is exposed to inflation $\Pi(\omega_{t+1}|\omega_t)\equiv\frac{P(\omega_{t+1})}{P(\omega_t)}$. Under this definition, we can think of inventory as cash when there is a lot of inflation risk. The only difference is that the budget constraint (D.26) now becomes

$$C_t + I_t + X_t + \int_{\Omega_{t+1|t}} \varphi_t(\omega_{t+1})\nu_t(\omega_{t+1})d\omega_{t+1} = Y_t + \Pi(\omega_t|\omega_{t-1})^{-1}I_{t-1} + \nu_{t-1}(\omega_t). \quad \text{(D.39)}$$

The defining equilibrium conditions then become

$$E_t\left[\beta\frac{u'(C_{t+1})}{u'(C_t)}\Pi_{t+1}^{-1}\right] + \frac{\xi_t}{\beta^t u'(C_t)} = 1, \quad \text{(D.40)}$$

$$E_t\left[\beta\frac{u'(C_{t+1})}{u'(C_t)}R_{i,t+1}\right] = 1, \quad \text{(D.41)}$$

where $R_{i,t+1}$ is a real return. As we state in the following proposition, we now get that the effective lower bound is on the nominal riskfree rate, not the real rate.

Proposition D.3. *Suppose the agent has strictly positive marginal utility $u'(C_t)$ and inventory is a nominal asset. When $I_t > 0$, the gross nominal riskfree rate $R^{\$}_{f,t+1} = 1$. When $I_t = 0$, $R^{\$}_{f,t+1} > 1$ and is equal to the riskfree rate in a no-inventory economy.*

Proof. The real return on a nominally riskfree asset is $\Pi_{t+1}^{-1}R^{\$}_{f,t+1}$. Given this fact, we can apply the proof method for Proposition D.2 to the equilibrium conditions (D.40) and (D.41) directly. ∎

Something worth noting is that the market clearing condition in a nominal inventory economy now becomes

$$C_t = Y_t - I_t + \Pi_t^{-1}I_{t-1} - X_t. \quad \text{(D.42)}$$

Thus, for an agent with positive inventory supply, inflation has immediate real effects on consumption.

E Production model with inventory

E.1 Solution for recursive utility with unit EIS

We restate the agent's problem for a more general set of investment opportunities. The only assumptions here are that the agent has recursive preferences with unit EIS, and that the capital and inventory assets are the only non-negative net supply assets.

The agent can invest in an inventory asset with net return $r_I = 0$ and a risky capital asset with net return $r_{K,t+1}$. Markets are completed by a set of (non-redundant) securities with net returns $r_{i,t+1}$, $i \in \mathcal{I}$, including inventory and capital. Inventory and capital are the only securities in positive net supply; furthermore, we restrict inventory to be in non-negative supply ($I_t \geq 0$). The number of securities needed to complete the market depends on the set of shocks. In our model with a binary shock η_{t+1}, we need two assets (in addition to the inventory asset, which cannot complete the market because of the restriction $I_t \geq 0$) — a natural choice is capital and a zero-net-supply riskfree bond with return $r_{f,t+1}$. Note, however, that this derivation makes no assumptions about the stochastic shocks. Let $\theta_{i,t}$ denote the agent's allocation, as a percentage of investable wealth $W_t - C_t$, to asset i; thus, we have that the return on wealth $R_{W,t+1} = \sum_{i \in \mathcal{I}} \theta_{i,t}(1 + r_{i,t+1})$ and that $\sum_{i \in \mathcal{I}} \theta_{i,t} = 1$.

Suppose that the agent has Epstein-Zin utility with unit EIS. The agent's optimization problem is therefore

$$V(W_t) = \max_{C_t, \{\theta_{i,t}\}_{i \in \mathcal{I}}} \left\{ C_t^{1-\beta} \left(E_t \left[V(W_{t+1})^{1-\gamma} \right] \right)^{\frac{\beta}{1-\gamma}} \right\}. \tag{E.1}$$

subject to the dynamic budget constraint

$$W_{t+1} = (W_t - C_t) R_{W,t+1} = (W_t - C_t) \sum_{i \in \mathcal{I}} \theta_{i,t}(1 + r_{i,t+1}), \tag{E.2}$$

the portfolio weight restriction

$$\sum_{i \in \mathcal{I}} \theta_{i,t} = 1, \tag{E.3}$$

and the inventory non-negativity constraint

$$\theta_{I,t} \geq 0. \tag{E.4}$$

Let λ_t and ξ_t denote the Lagrange multipliers on the constraints (E.3) and (E.4), respectively.

To solve the model, conjecture that the value function takes the form

$$V(W_t) = aW_t, \tag{E.5}$$

for some constant $a > 0$. Substituting this conjecture and the budget constraint (21) into (22), then taking logs, we get that

$$(1-\beta)\log a + \log W_t = \max_{C_t,\theta_t} \left\{ (1-\beta)\log C_t + \beta \log(W_t - C_t) + \frac{\beta}{1-\gamma} \log\left(E_t\left[R_{W,t+1}^{1-\gamma} \right] \right) \right\}. \tag{E.6}$$

The first-order condition with respect to consumption implies a constant consumption-wealth ratio:

$$C_t = (1-\beta)W_t. \tag{E.7}$$

The first-order condition with respect to asset allocation $\theta_{i,t}$, $i \neq I$, is

$$\beta E_t\left[R_{W,t+1}^{1-\gamma} \right]^{-1} E_t\left[R_{W,t+1}^{-\gamma}(1 + r_{i,t+1}) \right] = \lambda_t, \tag{E.8}$$

and the first-order condition with respect to the inventory allocation $\theta_{I,t}$ is

$$\beta E_t\left[R_{W,t+1}^{1-\gamma} \right]^{-1} E_t\left[R_{W,t+1}^{-\gamma} \right] + \xi_t = \lambda_t. \tag{E.9}$$

Multiply both sides of (E.8) by $\theta_{i,t}$, take the sum over $i \in \mathcal{I} \setminus \{I\}$, and substitute in

(E.9) to see that

$$\lambda_t = \beta + \xi_t \theta_{I,t} = \beta, \tag{E.10}$$

by complementary slackness. This implies the Euler equation for gross returns

$$E_t \left[R_{W,t+1}^{1-\gamma} \right]^{-1} E_t \left[R_{W,t+1}^{-\gamma} R_{i,t+1} \right] = 1 \tag{E.11}$$

and the Euler equation for inventory

$$E_t \left[R_{W,t+1}^{1-\gamma} \right]^{-1} E_t \left[R_{W,t+1}^{-\gamma} \right] + \frac{\xi_t}{\beta} = 1. \tag{E.12}$$

Note the market clearing condition $\theta_{I,t} = 1 - \theta_{K,t}$, where $\theta_{K,t}$ is simply denoted θ_t in our setup in the main text. We thus have that $\xi_t > 0$ if and only if $\theta_t < 1$.

Conjecture that $\theta_t = \theta$ (this is verified in the main text) and substitute these optimality conditions back into (E.6) to verify the conjecture for the value function and solve for the constant:

$$a = (1 - \beta)\beta^{\frac{\beta}{1-\beta}} E_t \left[R_{W,t+1}^{1-\gamma} \right]^{\left(\frac{\beta}{1-\beta}\right)\frac{1}{1-\gamma}}, \tag{E.13}$$

which is indeed constant because θ is constant and returns are i.i.d.

E.2 Proof of Lemma 1

If $\theta_{I,t} > 0$, then $\xi_t = 0$ and (E.11) and (E.9) combine to give us $R_{f,t+1} = 1$. If $\theta_{I,t} = 0$, then $\xi_t \geq 0$ and

$$R_{f,t+1} = \frac{\beta}{\beta - \xi_t}, \tag{E.14}$$

which is greater than or equal to 1. Moreover, if $\theta_{I,t} = 0$, then market clearing implies $R_{W,t+1} = R_{K,t+1}$ and the Euler equation (E.11) yields

$$R_{f,t+1} = E_t \left[R_{K,t+1}^{1-\gamma} \right] E_t \left[R_{K,t+1}^{-\gamma} \right]^{-1}, \tag{E.15}$$

which is the same as the riskfree rate $R^*_{f,t+1}$ in the no-inventory economy.

E.3 Proof of Theorem 1

We will prove the theorem by contradiction using Lemma 1.

Suppose $R^*_{f,t+1} < 1$ and $\theta_{I,t} = 0$. Then $R_{f,t+1} = R^*_{f,t+1} < 1$, which contradicts Lemma 1. It must therefore be the case that $R^*_{f,t+1} < 1$ implies $\theta_{I,t} > 0$, which implies $R_{f,t+1} = 1$.

Now suppose $R^*_{f,t+1} > 1$ and $\theta_{I,t} > 0$. Then $R_{f,t+1} = 1 < R^*_{f,t+1}$, which contradicts Lemma 1. Moreover, in the knife-edge case $R^*_{f,t+1} = 1$, the equilibrium conditions (E.11) and (E.9) imply $\xi_t = 0$, which implies that $\theta_{I,t} = 0$ and $R_{f,t+1} = R^*_{f,t+1} = 1$. Thus, it must be that $R^*_{f,t+1} \geq 1$ implies $\theta_{I,t} = 0$, which implies $R_{f,t+1} = R^*_{f,t+1} \geq 1$.

E.4 Output growth and the investment-capital ratio

Under $Y_t = AK_t$, we have that

$$\frac{K_{t+1}}{K_t} = \frac{Y_{t+1}}{Y_t}. \tag{E.16}$$

To find an explicit expression for these two growth rates, note that the definition of θ implies that $\tilde{K}_{t+1} = \theta(W_t - C_t) = \beta\theta W_t = \beta\theta(\tilde{K}_t(1+r) + I_{t-1})$. Dividing by K_t yields

$$\frac{\tilde{K}_{t+1}}{K_t} = \beta\theta\left((1-\delta+A) + \frac{I_t}{K_t}\right). \tag{E.17}$$

Next, note that

$$\frac{I_{t+1}}{K_{t+1}} = \frac{(1-\theta)(W_t - C_t)}{\theta(W_t - C_t)} e^{\eta_{t+1}} = \frac{(1-\theta)}{\theta} e^{\eta_{t+1}}, \tag{E.18}$$

which implies that

$$\frac{\tilde{K}_{t+1}}{K_t} = \beta\left(\theta(1-\delta+A) + (1-\theta)e^{\eta_t}\right). \tag{E.19}$$

Finally, this can be used to understand the growth rate of capital in the economy

$$\frac{K_{t+1}}{K_t} = \beta \left(\theta(1-\delta+A)e^{-\eta_{t+1}} + (1-\theta)e^{\eta_t - \eta_{t+1}}\right). \tag{E.20}$$

E.5 Price-dividend ratio

We conjecture that the price-dividend ratio depends only on the current state η_t (i.e. whether the disaster occurred or not). The intuition for this is that output growth $\frac{Y_{t+1}}{Y_t}$ is a function of η_t. Thus,

$$1 = E_t\left[R_{W,t+1}^{1-\gamma}\right]^{-1} E_t\left[R_{W,t+1}^{-\gamma}\left(\frac{\kappa^Y(\eta_{t+1})+1}{\kappa^Y(\eta_t)}\right)\left(\frac{Y_{t+1}}{Y_t}\right)\right]. \tag{E.21}$$

This implies that we have two equations, one for the non-disaster state,

$$\begin{aligned}\kappa^Y(0) = \hat{\beta}\bigg[&(1-p)(1+\theta r_{K,0})^{1-\gamma}(\kappa^Y(0)+1) \\ &+ p(1+\theta r_{K,z})^{-\gamma}(\kappa^Y(-z)+1)e^{-z}(1+\theta r_{K,0})\bigg],\end{aligned} \tag{E.22}$$

and one for the disaster state,

$$\begin{aligned}\kappa^Y(-z) = \hat{\beta}\bigg[&(1-p)(1+\theta r_{K,0})^{-\gamma}(\kappa^Y(0)+1)e^{z}(1+\theta r_{K,z}) \\ &+ p(1+\theta r_{K,z})^{1-\gamma}(\kappa^Y(-z)+1)\bigg].\end{aligned} \tag{E.23}$$

In these equations, $\hat{\beta} \equiv \beta\left[(1-p)(1+\theta r_{K,0})^{1-\gamma} + p(1+\theta r_{K,z})^{1-\gamma}\right]^{-1}$, $r_{K,0} \equiv (1-\delta+A)-1$, and $r_{K,z} \equiv (1-\delta+A)e^{-z}-1$. The solution to this system is

$$\kappa^Y(0) = \frac{\hat{\beta}}{1-\beta}\left[(1-p)(1+\theta r_{K,0})^{1-\gamma} + p(1+\theta r_{K,z})^{-\gamma}e^{-z}(1+\theta r_{K,0})\right], \tag{E.24}$$

$$\kappa^Y(-z) = \kappa^Y(0)e^{z}\frac{1+\theta r_{K,z}}{1+\theta r_{K,0}}. \tag{E.25}$$

References

Abel, Andrew B., "Risk premia and term premia in general equilibrium," *Journal of Monetary Economics*, 1999, *43* (1), 3 – 33.

Avdis, Efstathios and Jessica A. Wachter, "Maximum likelihood estimation of the equity premium," *Journal of Financial Economics*, 2017, *125* (3), 589–609.

Barkai, Simcha, "Declining Labor and Capital Shares," 2020. Forthcoming, *Journal of Finance.*

Barro, Robert J., "Rare Disasters, Asset Prices, and Welfare Costs," *American Economic Review*, 2009, *99* (1), 243–264.

Bernanke, Ben, "The global savings glut and the U.S. current account deficit," March 2005. Sandridge Lecture, Virginia Association of Economics, Richmond, VA.

Binsbergen, Jules H. van, "Duration-Based Stock Valuation," 2020. National Bureau of Economic Research Working Paper No. 27367.

_ and Ralph S. J. Koijen, "Likelihood-Based Estimation of Exactly-Solved Present-Value Models," *Journal of Finance*, 2010, *65* (4), 1439–1471.

_ and _ , "Likelihood-Based Estimation of Exactly-Solved Present-Value Models," 2011. Working paper, University of Pennsylvania, University of Chicago.

Caballero, Ricardo J., Emmanuel Farhi, and Pierre Olivier Gourinchas, "An equilibrium model of global imbalances and low interest rates," *American Economic Review*, 2008, *98* (1), 358–393.

Campbell, John Y., "Consumption-based asset pricing," in "Handbook of the Economics of Finance," Vol. 1, Elsevier, 2003, pp. 803 – 887.

— **and Robert J. Shiller**, "The dividend-price ratio and expectations of future dividends and discount factors," *Review of Financial Studies*, Fall 1988, *1* (3), 195–228.

— **, Carolin Pflueger, and Luis M. Viceira**, "Macroeconomic Drivers of Bond and Equity Risks," *Journal of Political Economy*, 2020, *Forthcoming.*

Chernov, Mikhail, Lukas Schmid, and Andres Schneider, "A Macrofinance View of U.S. Sovereign CDS Premiums," *Journal of Finance*, 2020, *Forthcoming.*

Cochrane, John H., *Asset Pricing*, Princeton, NJ: Princeton University Press, 2001.

Dormans, Eduard H.M, "Het tekort. Staatsschuld in de tijd der Republiek." PhD dissertation, Tilburg University 1991.

Durand, David and Willis J. Winn, *Basic Yields of Bonds, 1926-1947: Their Measurement and Patterns*, Vol. 6, Cambridge, MA: National Bureau of Economic Research, 1947.

Epstein, Larry and Stan Zin, "Substitution, risk aversion and the temporal behavior of consumption and asset returns: A theoretical framework," *Econometrica*, 1989, *57*, 937–969.

Fama, Eugene F and Kenneth R French, "Disappearing dividends: changing firm characteristics or lower propensity to pay?," *Journal of Financial economics*, 2001, *60* (1), 3–43.

Fama, Eugene F. and Kenneth R. French, "The Equity Premium," *Journal of Finance*, 2002, *57* (2), 637–659.

Farhi, Emmanuel and François Gourio, "Accounting for Macro-Finance Trends: Market Power, Intangibles, and Risk Premia," *Brookings Papers on Economic Activity*, Fall 2018, pp. 147–250.

Gabaix, Xavier, "Disasterization: A simple way to fix the asset pricing properties of macroeconomic models," *American Economic Review: Papers & Proceedings*, 2011, *101* (3), 406–409.

Gomes, João F, Marco Grotteria, and Jessica A Wachter, "Cyclical Dispersion in Expected Defaults," *Review of Financial Studies*, April 2019, *32* (4), 1275–1308.

Gomme, Paul, B. Ravikumar, and Peter Rupert, "The Return to Capital and the Business Cycle," *Review of Economic Dynamics*, 2011, *14* (2).

Gordon, Robert J, "Secular Stagnation: A Supply-Side View," *American Economic Review: Papers & Proceedings*, 2015, *105* (5), 54–59.

Gourio, François, "Disaster Risk and Business Cycles," *American Economic Review*, 2012, *102* (6), 2734–2766.

Grossman, Richard S., "New Indices of British Equity Prices," *Journal of Economic History*, 2002, *62* (1).

Hansen, Alvin H, "Economic Progress and Declining Population Growth," *American Economic Review*, 1939, *29* (1), 1–15.

Homer, Sidney and Richard E. Sylla, *A History of Interest Rates*, New Brunswick, NJ: Rutgers University Press, 2005.

Lettau, Martin, Sydney C. Ludvigson, and Jessica A. Wachter, "The declining equity premium: What role does macroeconomic risk play?," *Review of Financial Studies*, 2008, *21*, 1653–1687.

Newey, Whitney K. and Kenneth D. West, "A Simple, Positive Semi-definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix," *Econometrica*, 1987, *55* (3).

Òscar Jordà, Katharina Knoll, Dmitry Kuvshinov, Moritz Schularick, and Alan M Taylor, "The Rate of Return on Everything, 1870–2015," *The Quarterly Journal of Economics*, 2019, *134* (3), 1225–1298.

Reis, Ricardo, "The anchoring of long-run inflation expectations today," March 2020. Inflation dynamics in Asian and the Pacific, BIS Papers 111.

Samuelson, Paul A., "An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money," *Journal of Political Economy*, 1958, *66* (6).

— , "Lifetime Portfolio Selection by Dynamic Stochastic Programming," *The Review of Economics and Statistics*, 1969, *51* (3).

Schmelzing, Paul, "Eight centuries of global real interest rates, $R - G$, and the 'suprasecular' decline, 1311–2018," *Bank of England Staff Working Paper No. 845*, 2020, pp. 1–110.

Seo, Sang Byung and Jessica A. Wachter, "Do Rare Events Explain CDX Tranche Spreads?," *The Journal of Finance*, August 2018, *73* (5), 2343–2383.

— **and** — , "Option Prices in a Model with Stochastic Disaster Risk," *Management Science*, 2019, *65* (8), 3449–3469.

Shiller, Robert J., *Irrational Exuberance*, Princeton University Press, 2000.

Siriwardane, Emil, "The Probability of Rare Disasters: Estimation and Implications," October 2015. Harvard Business School Working Paper, No. 16-061.

Summers, Lawrence H, "Demand Side Secular Stagnation," *American Economic Review: Papers & Proceedings*, 2015, *105* (5), 60–65.

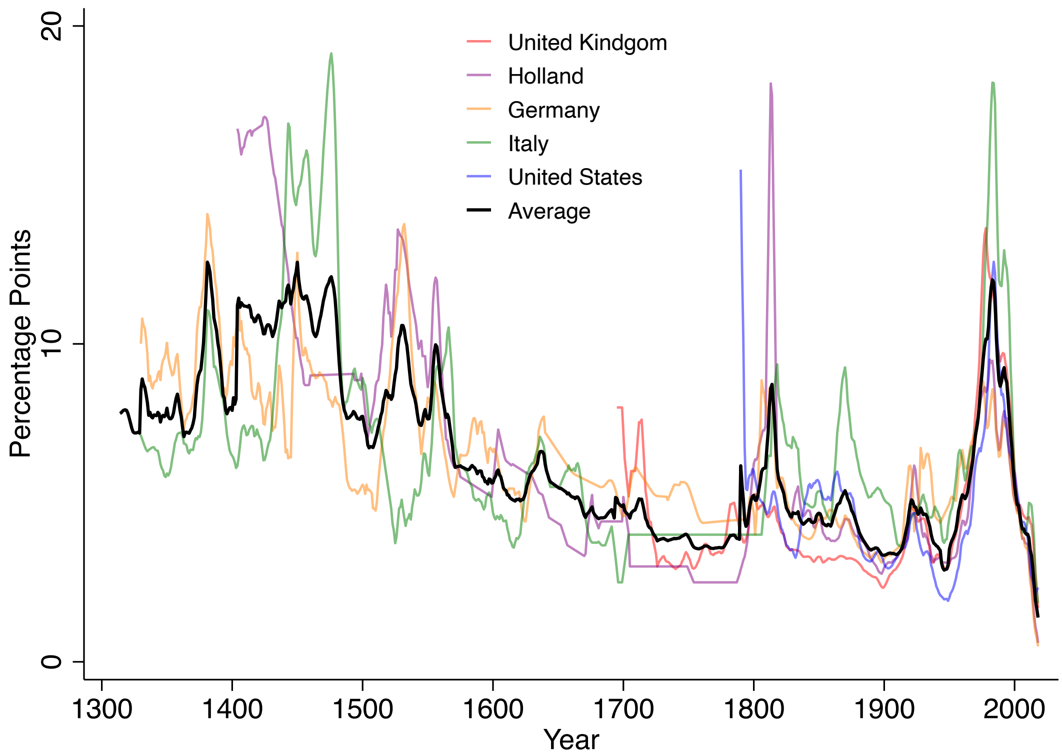
Weeveringh, J.J., *Handleiding tot de Geschiedenis der Staatsschulden*, Amsterdam: Gebroeders Kraay, 1852.

Weil, Philippe, "Nonexpected utility in macroeconomics," *Quarterly Journal of Economics*, February 1990, *105*, 29–42.

Zimmermann, Kaspar, "Breaking Banks? Monetary Policy and Bank Profitability," September 2017. Working paper, University of Bonn.

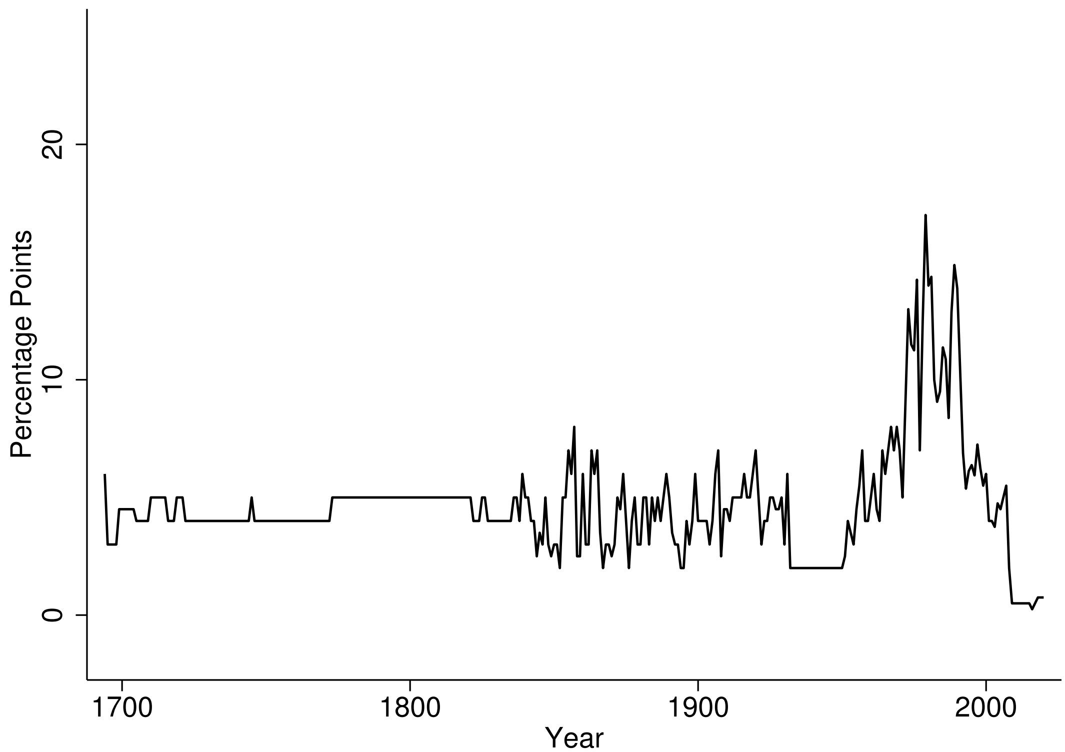
Tables and figures

Figure 1: Nominal government rates



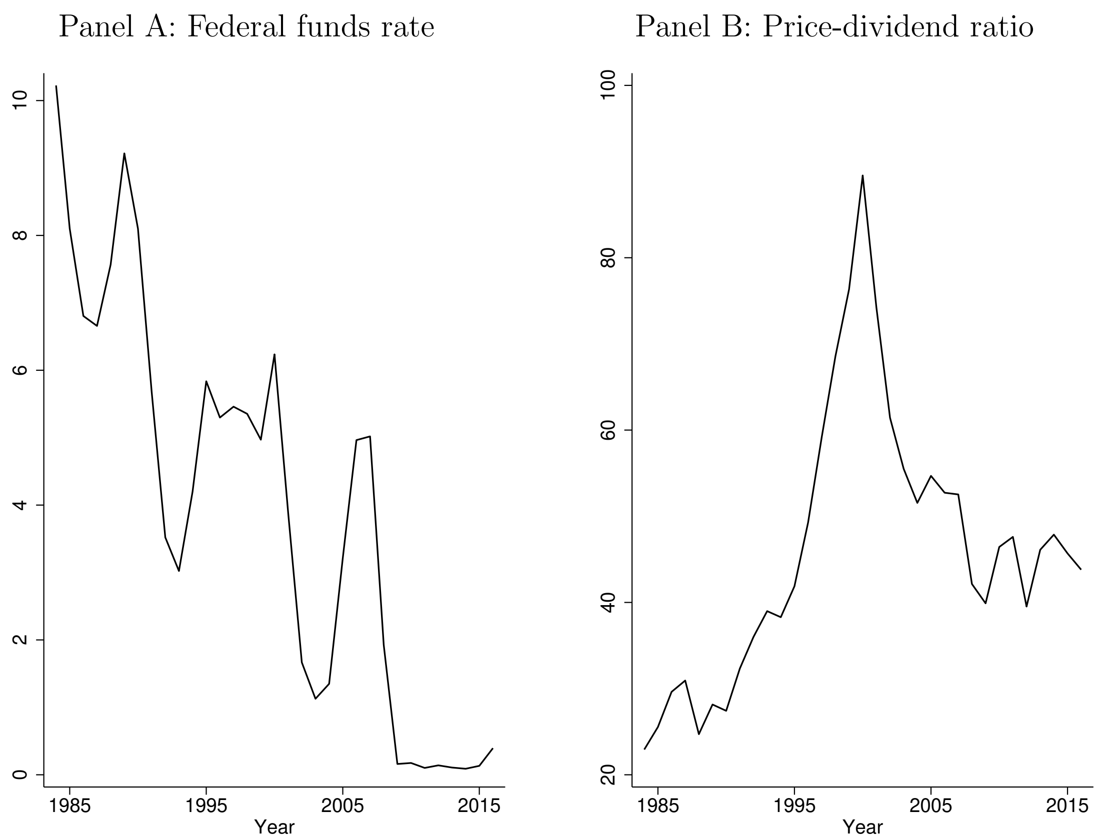
NOTES: The figure shows a five-year moving average of long-term nominal sovereign yields in the United Kingdom, Holland, Germany, Italy, and the United States from 1311–2018. The solid black line represents an average of all of the plotted series. Yields are from Schmelzing (2020) and are in annual terms. Yields come from a variety of archival, primary, and secondary sources.

Figure 2: Bank of England lending rate



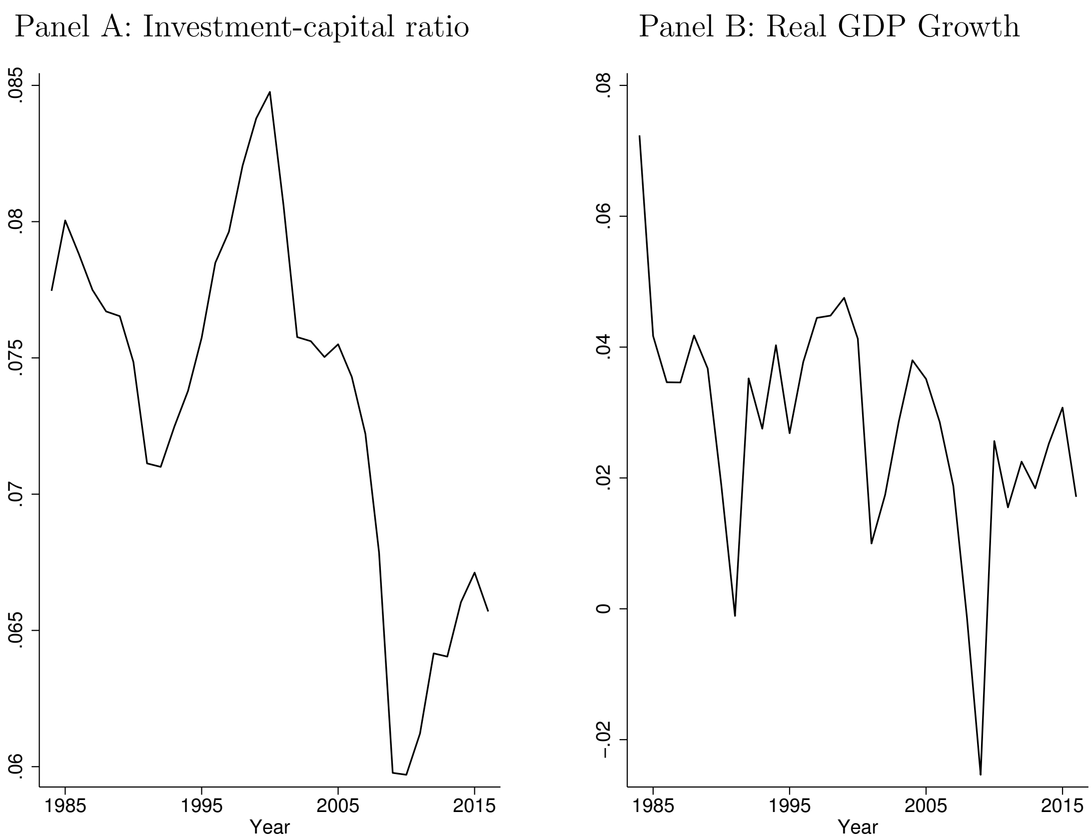
NOTES: The figure shows the nominal lending rate for the Bank of England expressed in annual terms.

Figure 3: Riskfree rate and price-dividend ratio in the United States from 1984–2018



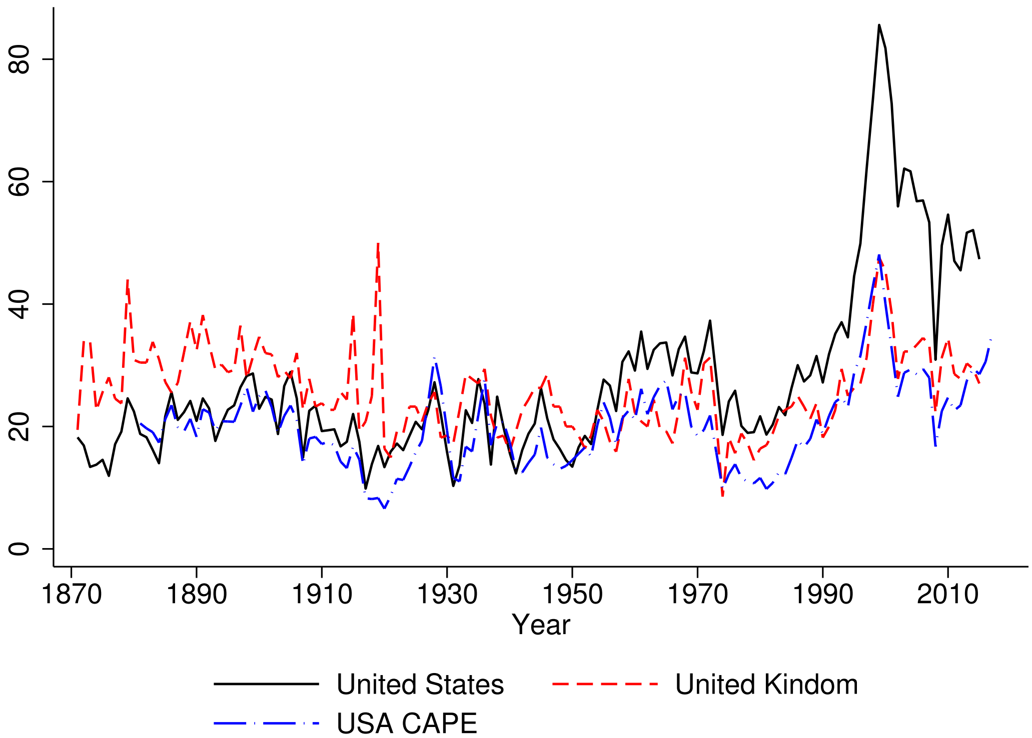
NOTES: The figure shows the effective federal funds rate (shown in annual percentage points) and the annual price-dividend ratio for the United States on the value-weighted CRSP index.

Figure 4: Investment-capital ratio and real GDP growth in the United States from 1984–2018



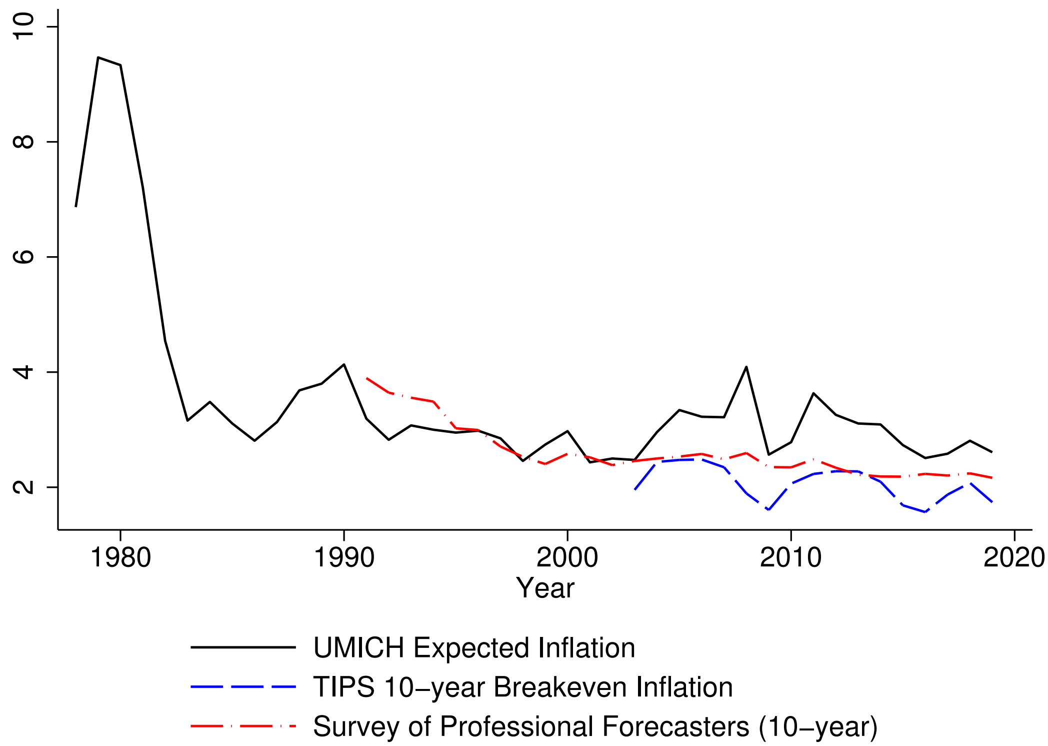
Notes: The figure shows the investment-capital ratio and the annual real GDP growth rate for the United States.

Figure 5: Price-dividend and price-earning ratios: United States and United Kingdom



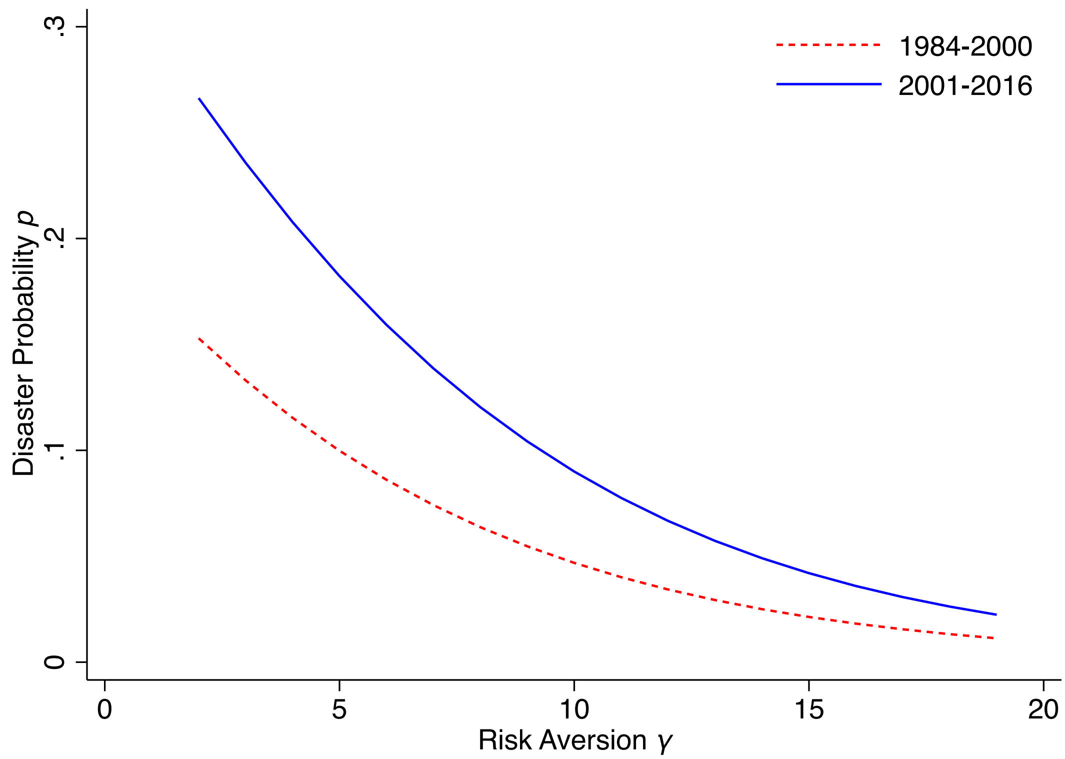
Notes: The figure shows the price-dividend ratio for the United States and United Kingdom since 1870 and the U.S. cyclically-adjusted price-earnings (CAPE) ratio. The black, solid line shows data for the United States price-dividend ratio and the red, solid line shows data for the price-dividend ratio of the United Kingdom. Price-dividend ratios are the end of year price divided by the aggregate dividends from the preceding year. The blue dashed-dotted line shows the CAPE ratio.

Figure 6: Expected inflation in the United States



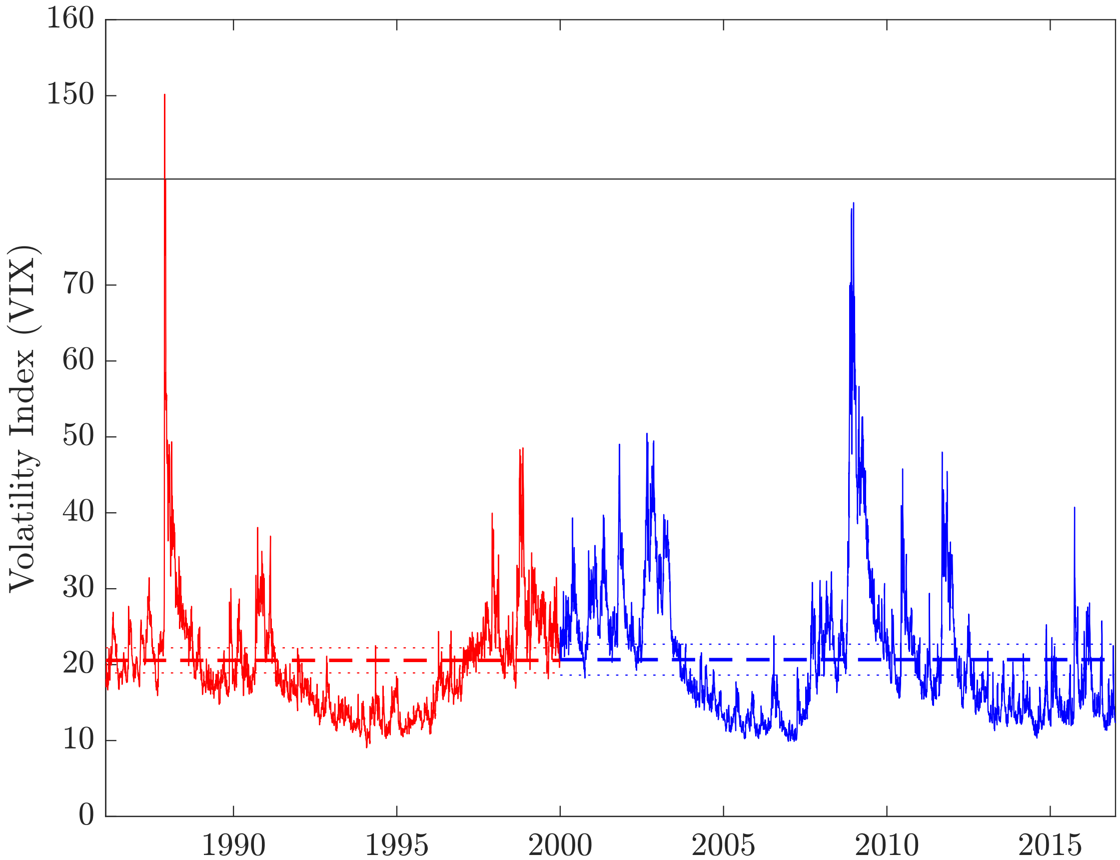
NOTES: The solid black line shows expected inflation from the Surveys of Consumers of University of Michigan. The dashed blue line shows the 10-year breakeven inflation rate computed from Treasury Inflation-Indexed Constant Maturity Securities. The dashed-dotted red line shows 10-year expected inflation from the Survey of Professional Forecasters.

Figure 7: Disaster probability under different risk aversion



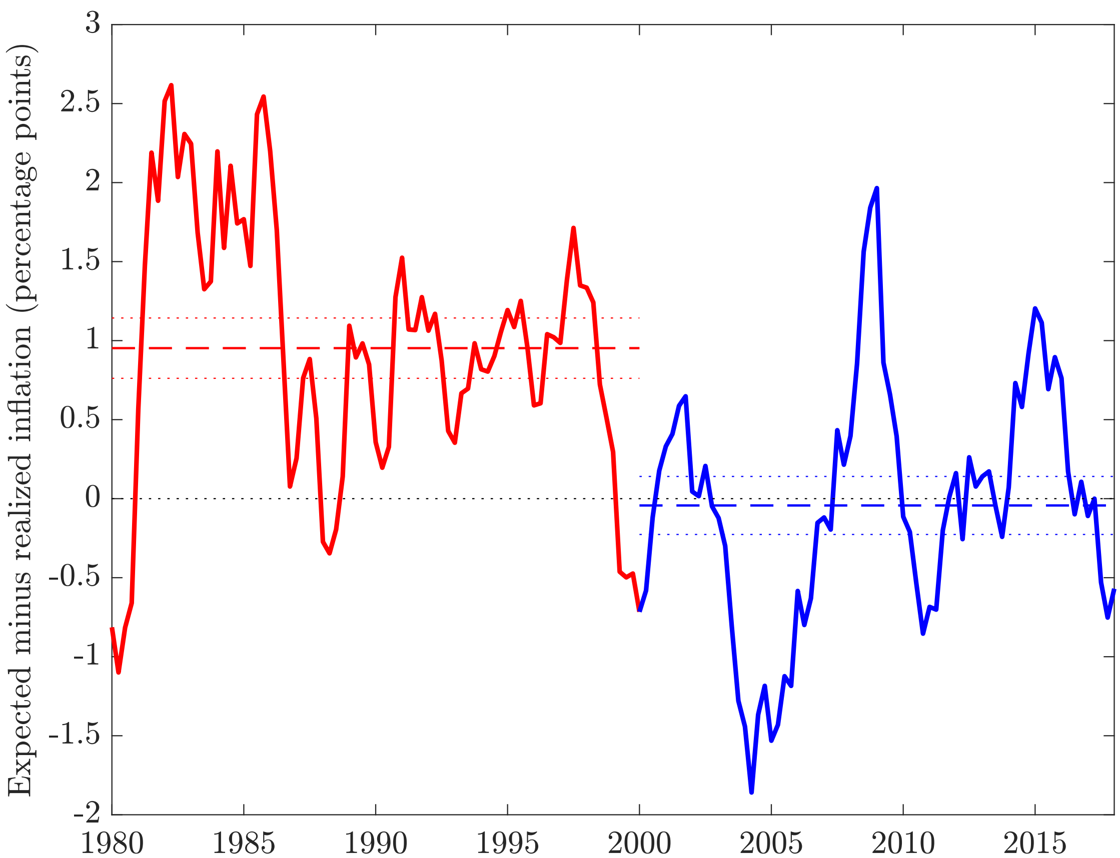
NOTES:For a given risk aversion, we find the disaster probability necessary to match the price-dividend ratio and the riskfree rate in the data. We fix the EIS ($\psi = 2$) and the disaster size ($z = -\log 0.85$). We use the growth rates as estimated in the data (3.5% and 2.8%, respectively). We let $\beta = 0.967$ in the first period and let $\beta = 0.979$ in the second period. The figure shows the disaster probability necessary to match the data for a given coefficient of relative risk aversion. A risk aversion coefficient of 12 is used in Table 1.

Figure 8: CBOE Volatility Index (VIX)



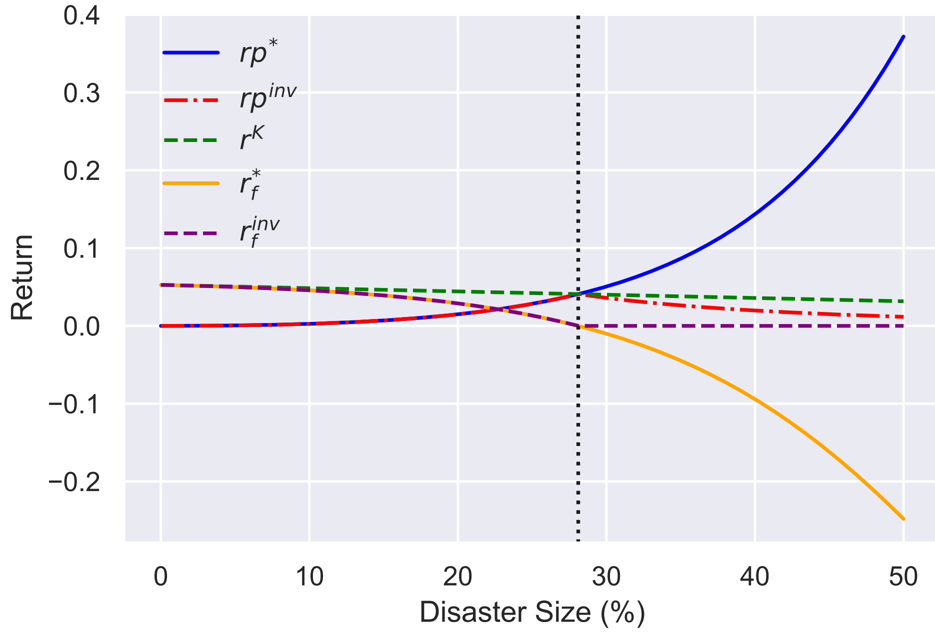
NOTES: The figure plots the VIX series from 1986 to 2020 from the Chicago Board Options Exchange (CBOE). The long dashed red line is the average VIX from the beginning of the series to the end of the year 2000. The long dashed blue line shows the average VIX from the beginning of 2001 to 2016. Estimated averages in both samples are plotted with a two-standard-error confidence interval where standard errors are adjusted for heteroskedasticity and autocorrelation (Newey and West, 1987) with two lags on the monthly VIX.

Figure 9: Expected versus realized one-year inflation



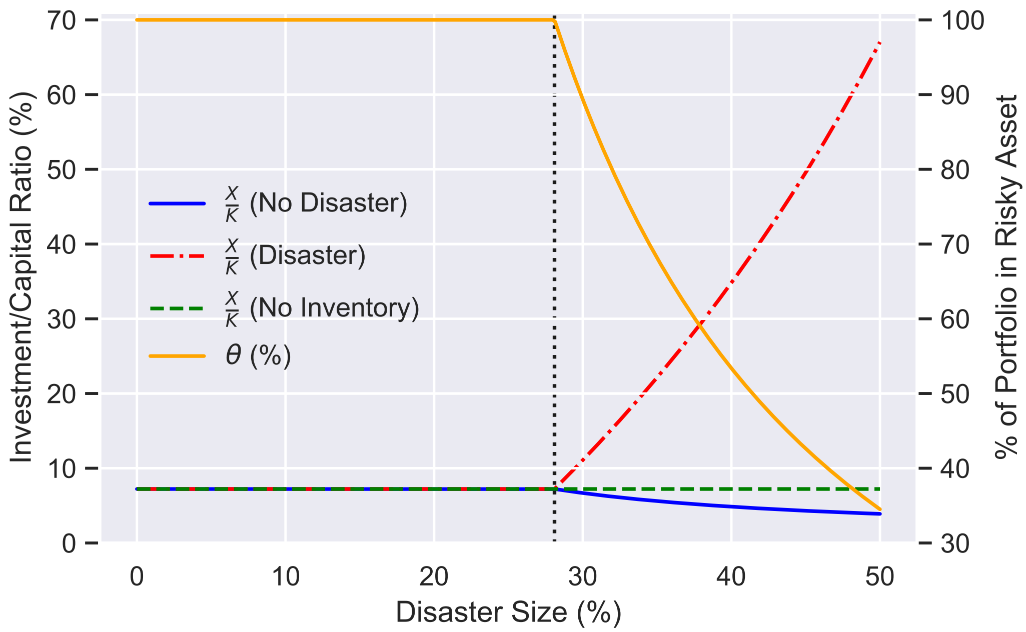
NOTES: The figure plots the difference between expected and realized one-year inflation, where expectations are taken from the Survey of Professional Forecasters. The horizontal dashed lines show the average difference in each of our respective samples along with two-standard-error confidence intervals. These averages could be interpreted as estimates of $p\zeta z$ in our model, where p is the probability that a disaster occurs, and ζz is the size of the inflation shock when a disaster occurs.

Figure 10: Risk premia and riskfree rate in the model



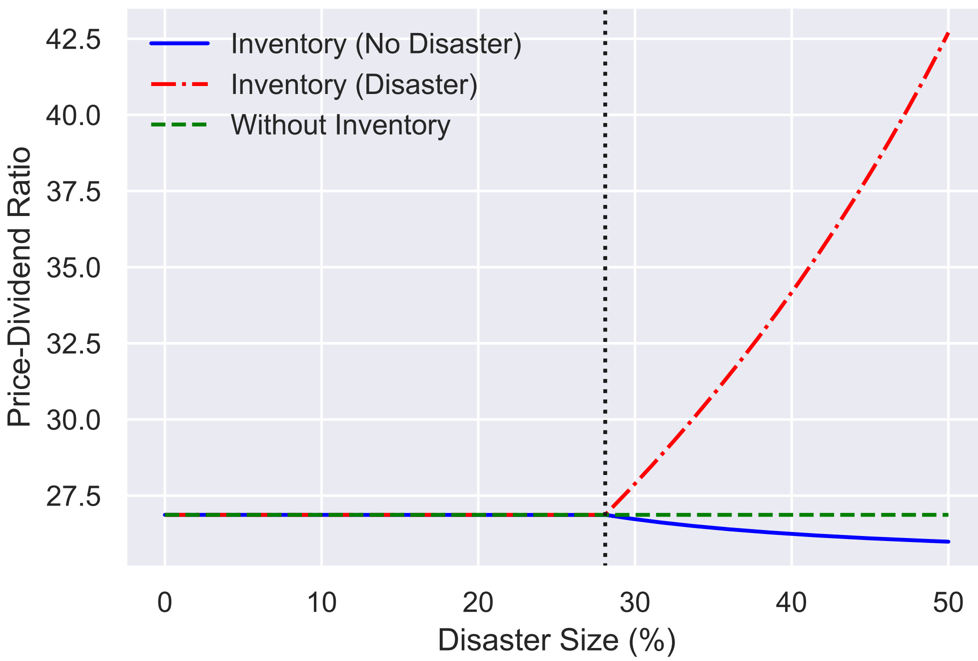
NOTES: This figure shows the riskfree rate and risk premium for the AK model with Epstein-Zin (EZ) utility with $\psi = 1$ and $\gamma = 6$ for varying levels of the disaster shock. The solid blue line represents the risk premium when $\theta = 1$ is imposed, defined as rp^*, where the star denotes that this quantity is from the standard AK model with EZ utility and capital quality shocks as solved in Section 4. The dashed and dotted red line represents the risk premium on the output claim in the model with inventory rp^{inv}. The dashed green line shows the return on capital, r^K, which, to the right of the dotted black line, is the risk premium on capital in the model with inventory. The solid orange line shows the riskfree rate in the model where the capital share of invested wealth $\theta = 1$ is imposed r_f^*, and the dashed purple line is the riskfree rate when the agent chooses to hold inventory, r_f^{inv}. For this diagram, we set patience parameter $\beta = 0.964$, depreciation $\delta = 0.057$, the probability of disaster $p = 0.0343$, and the marginal product of capital $A = 0.12$. The dotted black line represents the point at which the riskfree rate is equal to 0 in the model without inventory.

Figure 11: Investment capital ratio in the model



NOTES: This figure shows how capital investment varies in the AK model with Epstein-Zin utility with $\psi = 1$ and $\gamma = 6$ for different levels of the disaster shock. The solid blue line shows the investment-capital ratio when there is no disaster, while the dashed and dotted red line shows the price-dividend ratio in the state where there is a disaster. The dashed green line shows the investment-capital ratio in the model without inventory. Finally, the solid orange line shows θ, the fraction of capital invested wealth over invested wealth, in percentage terms. For this diagram, we set patience parameter $\beta = 0.964$, depreciation $\delta = 0.057$, the probability of disaster $p = 0.0343$, and the marginal product of capital $A = 0.12$. The dotted black line represents the point at which the riskfree rate is equal to 0 in the model without inventory.

Figure 12: Price-dividend ratio in the model



Notes: This figure shows how the price-dividend ratio varies in the AK model with Epstein-Zin utility with $\psi = 1$ and $\gamma = 6$ for different levels of the disaster shock. The solid blue line shows the price-dividend ratio when there is no disaster, while the dashed and dotted red line shows the price-dividend ratio in the state where there is a disaster. The dashed green line price dividend ratio in the model without inventory (here when $\theta = 1$). For this diagram, we set patience parameter $\beta = 0.964$, depreciation $\delta = 0.057$, the probability of disaster $p = 0.0343$, and the marginal product of capital $A = 0.12$. The dotted black line represents the point at which the riskfree rate is equal to 0 in the model without inventory.

Table 1: Accounting for the data with a change in disaster probability

	Values	
	1984–2000	2001–2016
Panel A: Moments in the data		
Price-dividend ratio κ	42.34	50.11
Riskfree rate r_f	0.0279	-0.0035
Panel B: Baseline model parameters		
μ	0.0350	0.0282
β	0.967	0.979
p	0.0343	0.0667
Panel C: No growth change		
μ	0.0350	0.0282
β	0.967	0.978
p	0.0343	0.0793
Panel D: EIS = 0.5		
μ	0.0350	0.0282
β	0.997	0.983
p	0.0343	0.0667
Panel E: No growth change, EIS = 0.5		
μ	0.0350	0.0282
β	0.997	0.985
p	0.0343	0.0793

Notes: Unless otherwise noted, risk aversion $\gamma = 12$, EIS (ψ) = 2, and disaster size (decline in log consumption in the event of a disaster) $z = -\log 0.85$. In Panel B, consumption growth μ is taken from the data and the disaster probability p and the patience parameter β are calibrated to match the data moments. In Panel C, growth is held constant, and p and β are calibrated to match the data moments. In Panel D, β is held constant, and we attempt to match the data moments given μ and p. Panel E repeats the exercise of Panel B, except with EIS = 0.5. All parameters and returns are annual.

Table 2: Accounting for the data: Alternative measures for valuations and rates

	Values	
	1984–2000	2001–2016
Panel A: Moments in the data		
U.S. CAPE ratio κ_{PE}^{US}	25.97	26.73
U.K. price-dividend ratio κ^{UK}	27.78	30.86
U.S. bill rate y_b^{US}	0.0279	-0.0035
U.K. bill rate y_b^{UK}	0.0500	0.0040
Panel B: U.S. moments, CAPE ratio		
μ	0.0350	0.0282
β	0.957	0.968
p	0.0556	0.101
Panel C: U.K. moments, PD ratio		
μ	0.0278	0.0156
β	0.955	0.971
p	0.0134	0.0533

Notes: Unless otherwise noted, risk aversion $\gamma = 12$, EIS = 2, and disaster size (decline in log consumption in the event of a disaster) $z = -\log 0.85$. The patience parameter β and the probability of disaster p are calibrated to match the data moments, and the growth parameter μ is varied exogenously across the two samples. Panel B is calibrated to the US riskfree rate and the cyclically adjusted price-to-earnings ratio. Panel C is calibrated to match the price-dividend ratio and real riskfree rate (average nominal rate less average inflation) in United Kingdom. All parameters and returns are annual. More information about the calibrating data is available in Appendix A.

Table 3: Contribution of each parameter

Panel A: EIS = 2					
	Parameter values			Data moments	
	β	μ	p	κ	r_f
Baseline calibration (1984–2000)	0.967	0.0350	0.0343	42.34	0.0279
Higher β	0.979	0.0350	0.0343	94.74	0.0151
Higher β & lower μ	0.979	0.0282	0.0343	71.44	0.0117
Baseline calibration (2001–2016)	0.979	0.0282	0.0667	50.11	-0.0035

Panel B: EIS = 0.5					
	Parameter values			Data moments	
	β	μ	p	κ	r_f
Baseline calibration (1984–2000)	0.997	0.0350	0.0343	42.34	0.0279
Higher β	0.983	0.0350	0.0343	25.63	0.0428
Higher β & lower μ	0.983	0.0282	0.0343	31.27	0.0292
Baseline calibration (2001–2016)	0.983	0.0282	0.0667	50.11	-0.0035

Notes: Risk aversion $\gamma = 12$ and disaster size (decline in log consumption in the event of a disaster) $z = -\log 0.85$. The first panel shows what happens what parameters are changed sequentially with $\psi = 2$ in Panel A and $\psi = 0.5$ in Panel B. The first row in each panel represents the calibration set to match the US riskfree rate and price-dividend ratio from 1984–2000. The second row in each panel uses the same calibration, but move the patience parameter β to its 2001–2016 calibrated level. The third in each panel row has the same calibration as the second, aside from setting consumption growth μ to it's 2001–2016 level, moving it from $\mu = 0.0350$ to $\mu = 0.0282$. Finally, the fourth row in each panel sets β, μ, and the disaster probability p, such that they match the US riskfree rate and price-dividend ratio from 2001 – 2016.

Table 4: Accounting for the data with inflationary default risk

	Values	
	1984–2000	2001–2016
Panel A: Moments in the data		
Price-dividend ratio κ	42.34	50.11
U.S. bill rate y_b^{US}	0.0279	-0.0035
Panel B: Baseline model parameters		
μ	0.0350	0.0282
β	0.967	0.974
ζ	-0.001	-0.746
Panel C: EIS $= 0.5$		
μ	0.0350	0.0282
β	0.997	0.994
ζ	-0.001	-0.746
Panel D: $\gamma = 5$, $z = -\log 0.7$		
μ	0.0350	0.0282
β	0.972	0.979
ζ	0.501	0.046
Panel E: EIS $= 1$, $\gamma = 5$, & $z = -\log 0.7$		
μ	0.0350	0.0282
β	0.977	0.980
ζ	0.501	0.046

Notes: Unless otherwise noted, risk aversion $\gamma = 12$, EIS $= 2$, the disaster probability $p = 0.0343$, and disaster size (decline in log consumption in the event of a disaster) $z = -\log 0.85$. In each Panel, growth is taken from the data, while the relative bond payoff in disasters ζ and the patience parameter β are calibrated to match the data moments. More information about the calibrating data is available in Appendix A. All parameters and returns are annual.

Table 5: Calibration for inflation default risk under alternative moments

	Values	
	1984–2000	2001–2016
Panel A: Moments in the data		
U.S. CAPE ratio κ_{PE}^{US}	25.97	26.73
U.K. price-dividend ratio κ^{UK}	27.78	30.86
U.S. bill rate y_b^{US}	0.0279	-0.0035
U.K. bill rate y_b^{UK}	0.0500	0.0040
Panel B: U.S. moments, CAPE ratio		
μ	0.0350	0.0282
β	0.963	0.964
ζ	0.182	-0.278
Panel C: U.K. moments, PD ratio		
μ	0.0278	0.0156
β	0.965	0.969
ζ	0.925	0.212

Notes: Unless otherwise noted, risk aversion $\gamma = 5$, EIS $= 1$, the disaster probability $p = 0.0343$, and disaster size (decline in log consumption in the event of a disaster) $z = -\log 0.7$. The patience parameter β and the relative bond payoff in disasters ζ are calibrated to match the data moments; the growth parameter μ is varied exogenously across the two samples. Panel B is calibrated to the US riskfree rate and the cyclically adjusted price-to-earnings ratio. Panel C is calibrated to match the price-dividend ratio and real riskfree rate (average nominal rate less average inflation) in United Kingdom. All parameters and returns are annual. More information about the calibrating data is available in Appendix A.

Table 6: Inflationary default under different disaster probabilities

	Values	
	1984–2000	2001–2016
Panel A: Moments in the data		
U.S. CAPE ratio κ_{PE}^{US}	25.97	26.73
U.S. bill rate y_b^{US}	0.0279	-0.0035
Panel B: $p = 0.03$		
β	0.963	0.964
ζ	0.065	-0.430
Panel C: $p = 0.04$		
β	0.963	0.964
ζ	0.305	-0.118
Panel D: $p = 0.05$		
β	0.963	0.964
ζ	0.463	0.090

Notes: Unless otherwise noted, risk aversion $\gamma = 5$, EIS = 1, and disaster size (decline in log consumption in the event of a disaster) $z = -\log 0.7$. The patience parameter β and the relative bond payoff in disasters ζ are calibrated to match the data moments; the growth parameter μ is varied exogenously across the two samples. In each panel, the 1984–2000 period has $\mu = 0.0350$ and the 2001–2016 period has $\mu = 0.0282$. All parameters and returns are annual. More information about the calibrating data is available in Appendix A.

Table 7: Inventory and inflationary default in a model with production

	Values	
	1984–2000	2001–2016
Panel A: Moments in the data		
U.S. CAPE ratio κ_{PE}^{US}	25.97	26.73
U.S. bill rate y_b^{US}	0.0279	-0.0035
GDP growth	0.0368	0.0191
Panel B: Without inventory		
β	0.963	0.964
ζ	0.316	0.158
δ	0.043	0.063
Panel C: With inventory		
β	0.963	0.964
ζ	0.316	-0.05
δ	0.043	0.057

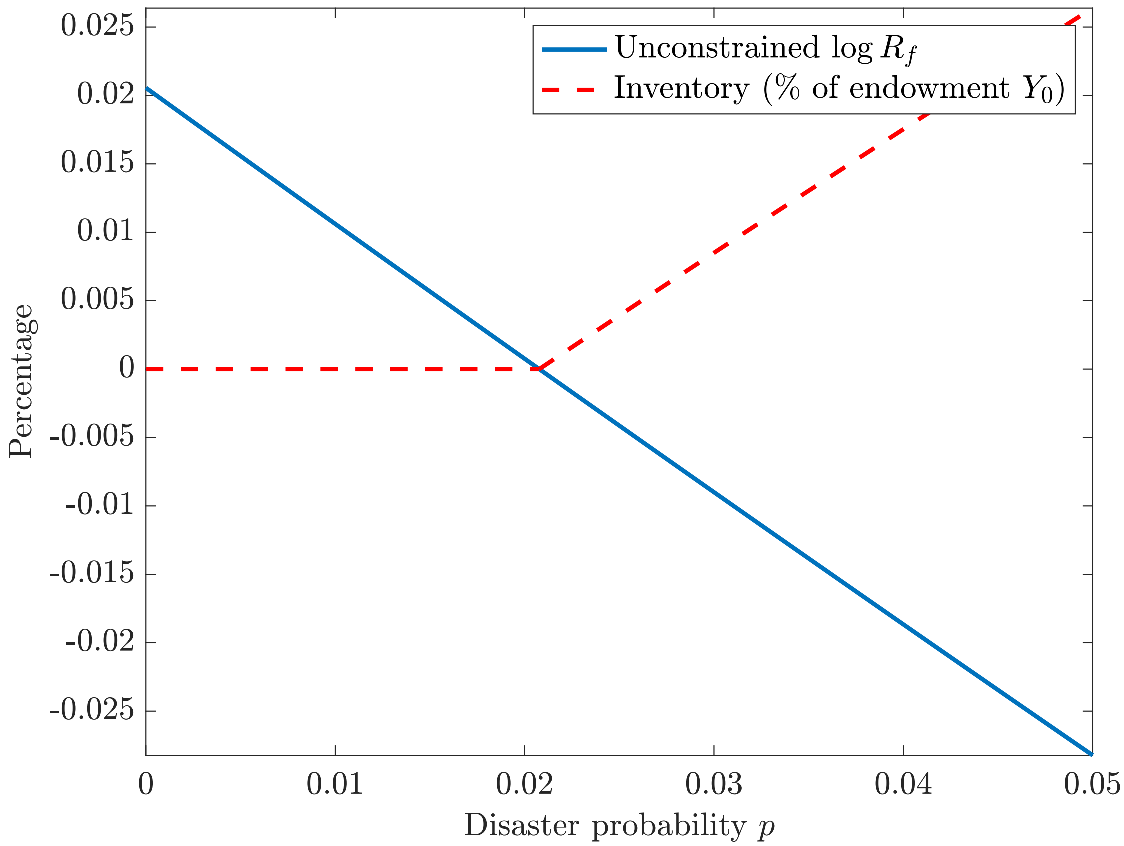
Notes: The model is solved under Epstein-Zin utility with risk aversion $\gamma = 6$ and EIS $= 1$. The disaster size (decline in log consumption in the event of a disaster) $z = -\log 0.7$, the probability of disaster $p = 0.0343$, and the marginal product of capital $A = 0.12$. The patience parameter β, the relative bond payoff in disasters ζ, and depreciation δ are calibrated to match the data moments shown in Panel A. More information about the calibrating data is available in Appendix A. Panel B shows the model solved assuming the agent can hold inventory, and Panel C assumes this is not the case.

Table 8: Inventory and inflationary default with production: untargeted moments

	Values	
	1984–2000	2001–2016
Panel A: Without inventory		
Risky capital share θ	1.000	1.000
Investment-capital ratio $\frac{X}{K}$	0.080	0.082
Unconstrained riskfree rate r_f^*	0.002	-0.016
Panel B: With inventory		
Risky capital share θ	1.000	0.912
Investment-capital ratio $\frac{X}{K}$	0.080	0.077
Unconstrained riskfree rate r_f^*	0.002	-0.011

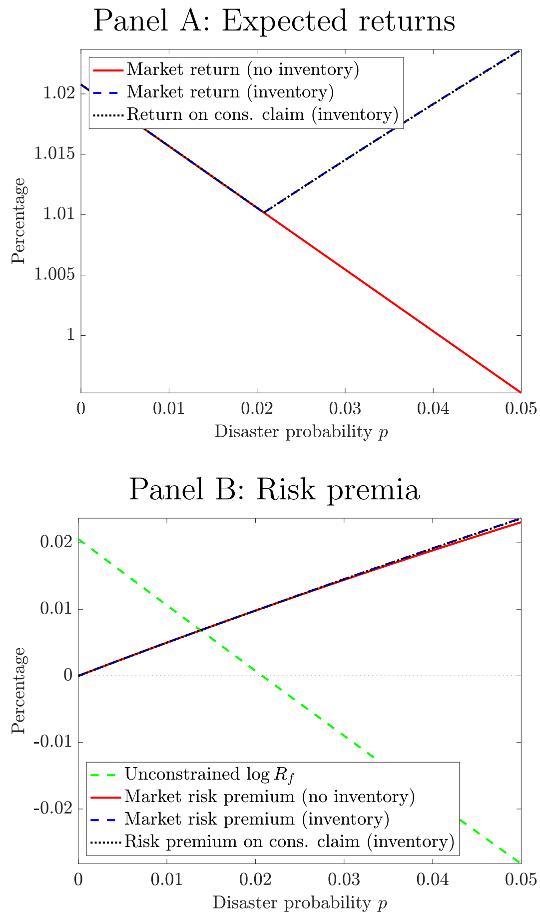
Notes: The model is solved under Epstein-Zin utility with risk aversion $\gamma = 6$ and EIS $= 1$. The disaster size (decline in log consumption in the event of a disaster) $z = -\log 0.7$, the probability of disaster $p = 0.0343$, and the marginal product of capital $A = 0.12$. The calibrated parameters from Table 7 are used for β, ζ, and δ. Panel A reports moments for the model without inventory, and Panel B reports moments for the model with inventory. In each Panel, we report the capital share of invested wealth θ, the investment-capital ratio $\frac{X}{K}$, and the unconstrained riskfree rate r_f^*.

Figure D.1: Riskfree rate with and without inventory



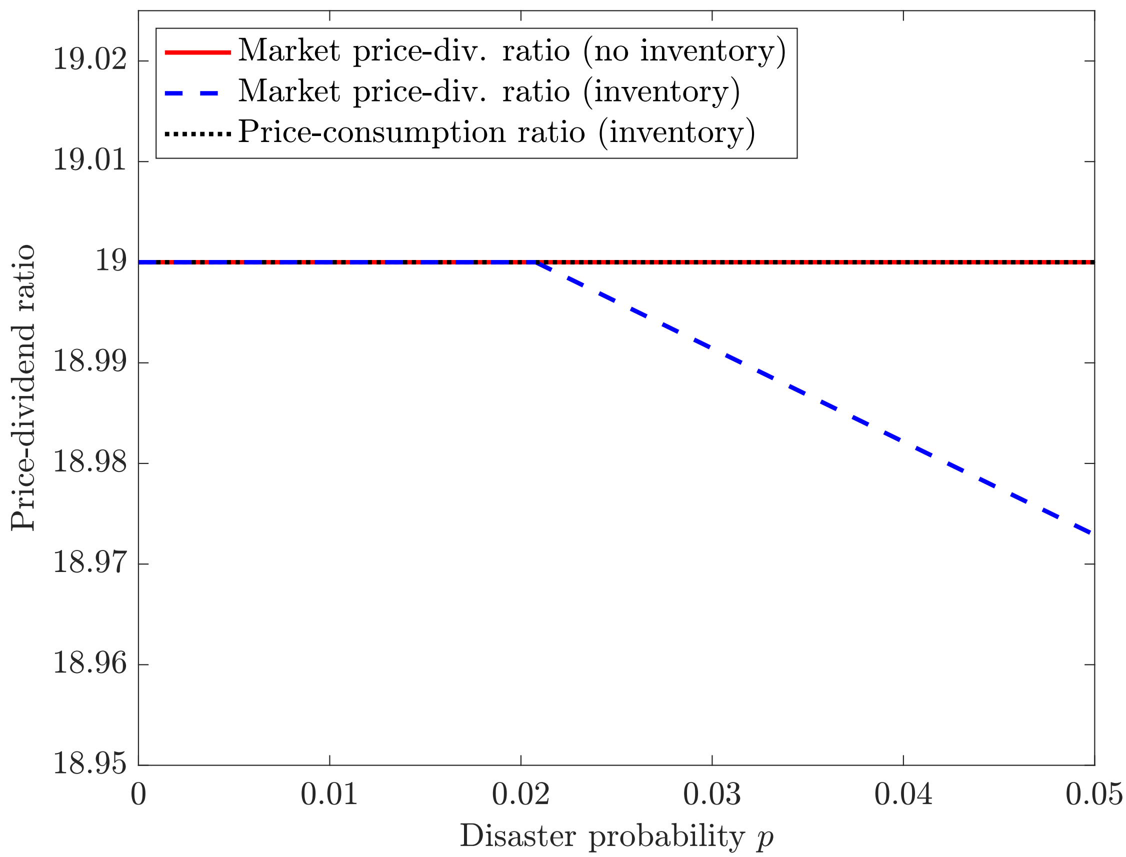
NOTES: The figure shows the unconstrained riskfree rate and inventory demand implied by our two-period model. The unconstrained riskfree rate is defined as the riskfree rate in an economy with no inventory asset. The inventory demand is represented by the ratio of inventory to the initial endowment I/Y_0 in an economy with the inventory asset. Both values are calculated under the same calibration with log utility.

Figure D.2: Risky asset returns with and without inventory



NOTES: The figure shows the expected returns (Panel A) and risk premia (Panel B) of three risky assets in our two-period model. Specifically, we consider two economies: one with and one without the inventory asset. For each economy, we consider the consumption claim, which pays a dividend of $Y_1 + I$, and the market (the output claim), which pays Y_1. The consumption and output claims are the same asset in the economy with no inventory. Panel B also shows the unconstrained riskfree rate, defined as the riskfree rate in an economy with no inventory asset. All values are calculated under the same calibration with log utility.

Figure D.3: Price-dividend ratios



NOTES: The figure shows the price-dividend ratio of three risky assets in our two-period model. Specifically, we consider two economies: one with and one without the inventory asset. For each economy, we consider the consumption claim, which pays a dividend of $Y_1 + I$, and the market (the output claim), which pays Y_1. The consumption and output claims are the same asset in the economy with no inventory. The price-consumption ratio with inventory and the market price-dividend ratio with no inventory are both constant and equal to $\beta/(1-\beta)$. All values are calculated under the same calibration with log utility.