Realized Semibetas: Signs of Things to Come

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Tim Bollerslev[a,*], Andrew J. Patton[b], Rogier Quaedvlieg[c]

[a] *Department of Economics, Duke University, NBER and CREATES*
[b] *Department of Economics, Duke University*
[c] *Erasmus School of Economics, Erasmus University Rotterdam*

Abstract

We propose a new decomposition of the traditional market beta into four *semi*betas depending on the signed covariation between the market and individual asset returns. Consistent with the pricing implications from a mean-*semi*variance framework, we show that higher semibetas defined by negative market and negative (positive) asset return covariation predict significantly higher (lower) future returns, while the other two semibetas do not appear to be priced. The results are robust to an array of alternative test specifications and additional controls. Rather than betting on or against beta, we conclude that it is better to bet on *and* against the "right" semibetas.

Keywords: Cross-sectional return variation; downside risk; semicovariances; semibetas.

JEL: G11, G12, C58

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[*] Corresponding author: Department of Economics, Duke University, Durham, NC 27708; 919-660-1846; boller@duke.edu.

1. Introduction

The Capital Asset Pricing Model (CAPM) reigns supreme as the most widely-studied and practically-used model for valuing speculative assets. In its basic form the model predicts a simple linear relationship between the expected excess return on an asset and the beta of that asset with respect to the aggregate market portfolio. While early empirical evidence largely corroborated this prediction (e.g., Fama, Fisher, Jensen and Roll, 1969; Blume, 1970), an extensive subsequent literature has called into question the ability of the standard market beta to satisfactorily explain the cross-sectional variation in returns, with the estimated risk premiums being too low, often insignificant, and sometimes even negative (e.g., Roll, 1977; Bhandari, 1988; Fama and French, 1992). Numerous explanations have been put forth to explain these findings, ranging from measurement errors (e.g., Shanken, 1992; Hollstein, Prokopczuk and Simen, 2019), to agency problems (Baker, Bradley and Wurgler, 2011), to the need for separate betas associated with cash-flow and discount rate news (Campbell and Vuolteenaho, 2004), to leverage constraints (Frazzini and Pedersen, 2014) and the need for separate liquidity and fundamental betas (Acharya and Pedersen, 2005), to name but a few.

These "rescue attempts" notwithstanding, another strand of literature, tracing back to the early work by Roy (1952), Markowitz (1959), Hogan and Warren (1972, 1974) and Bawa and Lindenberg (1977), posits that the mean-variance, or quadratic utility, framework underlying the basic CAPM and the resulting security market line and linear beta pricing relationship is fundamentally too simplistic. If investors are averse to volatility only when it leads to losses, not gains, then the relevant measure of risk is not (total) variance but rather the *semivariance* of negative returns.[1] Intuitively, if investors only care about downside variation, then the covariation associated with a positive aggregate market return should not be priced in equilibrium, regardless of the sign of the return on the individual asset. By contrast, assets that covary positively with the market when the latter is performing poorly exacerbate the aggregate downside return variation and so should be compensated commensurately in equilibrium. On the other hand, assets that covary negatively with the market when the latter is performing poorly mitigate the negative market return and thus should yield a lower risk premium in equilibrium.

Consistent with these ideas, Ang, Chen and Xing (2006a) find empirically that the premium for bearing downside risk is indeed significantly positive, and that the downside beta version of the CAPM does a better job than the traditional CAPM in terms of

[1]This same basic idea also underlies the notion of loss aversion and the prospect theory pioneered by Kahneman and Tversky (1979), as supported by an extensive subsequent experimental literature and other indirect empirical evidence.

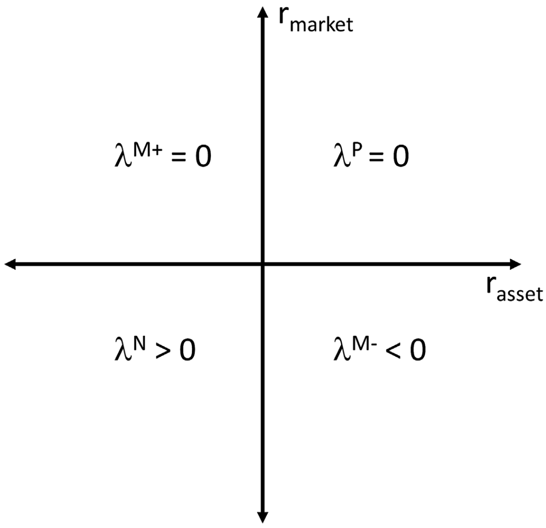
explaining the cross-sectional variation in U.S. equity returns.[2] Meanwhile, the more recent studies by Atilgan, Bali, Demirtas and Gunaydin (2018) and Atilgan, Demirtas and Gunaydin (2019) have both called question the ability of downside betas to satisfactorily explain the cross-sectional variation in international and more recent U.S. equity returns. Similarly, Levi and Welch (2019) concludes that estimated downside betas do not provide superior cross-sectional return predictions compared to those afforded by standard beta estimates, in part due to the difficulties of accurately determining downside betas from daily returns.

These studies and the downside betas analyzed therein, however, fail to fully account for the more subtle pricing implications arising from the mean-semivariance framework. Specifically, consider the four possible combinations of positive and negative returns for the individual asset and the market: both returns are positive (the "P" state), both negative ("N"), mixed sign with positive market return ("M^+"), and mixed sign with negative market return ("M^-"). The mean-semivariance framework suggests that covariation stemming from the two states where the market return is positive (P and M^+) should earn no risk premium. By contrast, any covariation associated with joint negative market and individual asset returns (N) should be positively compensated, while the covariation stemming from negative market returns and positive individual asset returns (M^-) ought to carry a negative risk premium. These richer pricing implications for the different components of the total covariation of the individual asset return with the aggregate market return are summarized in Figure 1 (adapted from Table 1 in Hogan and Warren, 1974).[3]

[2]The unpublished contemporaneous study by Post and van Vliet (2004) reached the same conclusions. Further corroborating these same general ideas, Lettau, Maggiori and Weber (2014) argued that the downside beta version of the CAPM also better explains the variation in the returns across other asset classes.

[3]These same pricing implications may also be formally justified in a setting with disappointment aversion preferences as in Gul (1991), and the generalized version thereof in Routledge and Zin (2010), as recently explored by Farago and Tedongap (2018).

Figure 1: **Mean-Semivariance risk premiums.** This figure illustrates the signs of the risk premiums (λ) associated with the four semicovariance components.



The downside version of the CAPM investigated in the studies cited above effectively combines the pricing of the latter two covariation components (states N and M^-), which have opposite predicted signs, into a single downside beta. The traditional CAPM beta, of course, does not differentiate between *any* of the four covariation components (states N, P, M^+ and M^-).

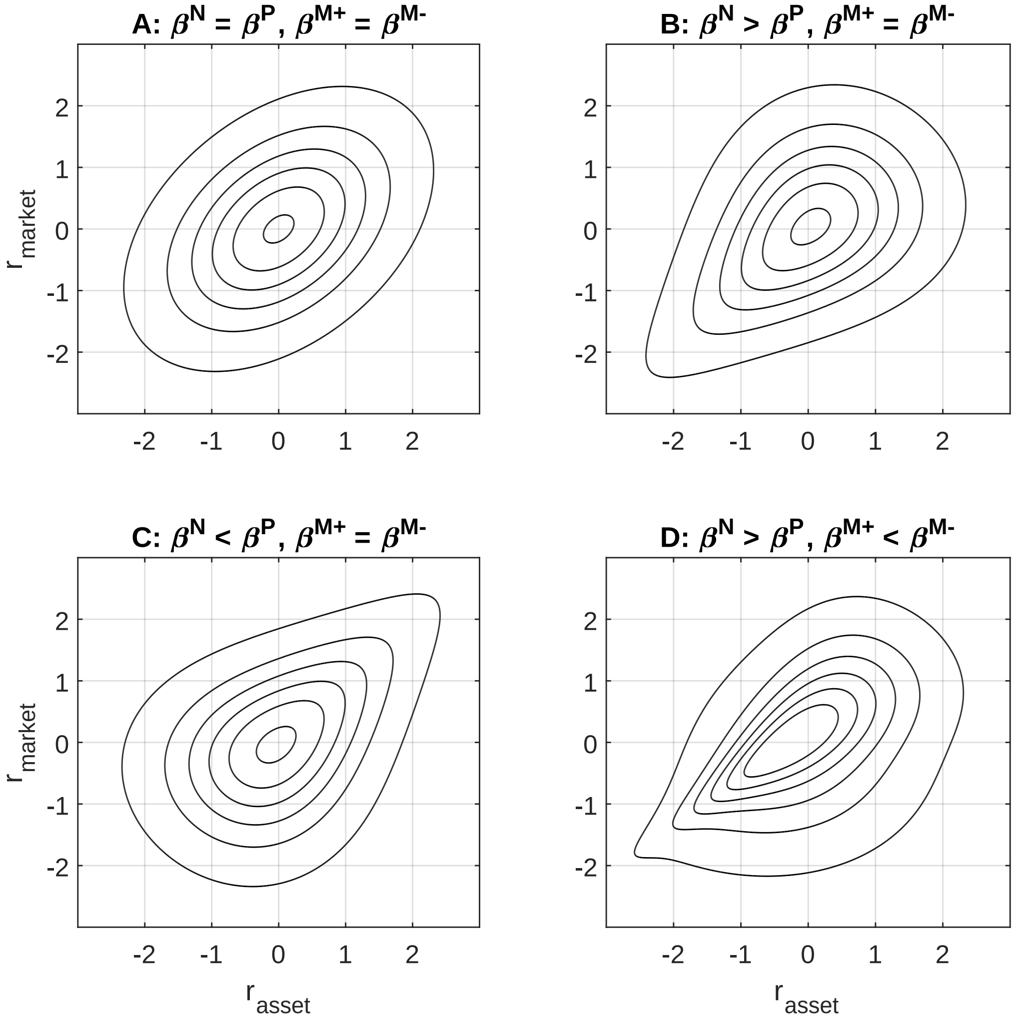
Set against this background, we propose a new four-way decomposition of the traditional market beta into four additive *semi*betas based on the corresponding *semi*covariance components (see Bollerslev, Patton, Li and Quaedvlieg, 2019b, for a formal definition of the semicovariance concept). Specifically, in Section 2 we show that the market beta can be additively decomposed into four semibetas:

$$\beta \equiv \frac{Cov(r,f)}{Var(f)} = \frac{\mathcal{N}+\mathcal{P}+\mathcal{M}^{+}+\mathcal{M}^{-}}{Var(f)} \equiv \beta^{\mathcal{N}}+\beta^{\mathcal{P}}-\beta^{\mathcal{M}^{+}}-\beta^{\mathcal{M}^{-}}. \tag{1}$$

Because the mixed-sign semicovariances, $\mathcal{M}^+$ and $\mathcal{M}^-$, are always (weakly) negative numbers, with lower values indicating stronger covariation, we define the two mixed-sign semibetas as $\beta^{\mathcal{M}^+} \equiv -\mathcal{M}^+/Var(f)$ and $\beta^{\mathcal{M}^-} \equiv -\mathcal{M}^-/Var(f)$, respectively. This sign reversal eases the interpretation of the risk premium estimates in our analysis below.

If the market and asset returns were jointly Normally distributed, or, more generally, elliptically distributed, then $\beta^{\mathcal{N}} = \beta^{\mathcal{P}}$ and $\beta^{\mathcal{M}^+} = \beta^{\mathcal{M}^-}$, and the pricing of the respective semibetas would be empirically indistinguishable from that implied by the traditional linear CAPM pricing relation and the standard market beta. However, with non-elliptically

Figure 2: **Contour plots for four distributions.** This figure presents isoprobability contours of the bivariate PDFs from four distributions, all of which have standard Normal marginal distributions and all which imply a CAPM beta of one. Different choices of copula lead to different types of dependence across the support of the variables.



distributed asset and market returns *all* of the semibetas may differ, and explicitly allowing for different risk premiums for each of them will lead to distinctly different and potentially more accurate empirical pricing relations.

To illustrate, Figure 2 presents bivariate contour plots for four hypothetical return distributions, each of them having traditional CAPM beta equal to one.[4] Since the CAPM beta is the same in all four cases, the CAPM implies identical expected returns for all

[4]The contours are generated using standard Normal marginal distributions with dependence between the two variables captured by either a Normal copula or one of three different "Clayton" copulas. Using values for market volatility and average firm volatility from our data, 0.92% and 2.26% respectively, a beta of 1 implies linear correlation of 0.41, which is used in all four panels here.

four assets. Panel A depicts the familiar bivariate Normal contours. Compared with the expected return in Panel A, the mean-semivariance framework implies a higher (lower) expected return in Panel B (Panel C), as the dependence with the market is stronger (weaker) when the market return is negative, i.e., $\beta^{\mathcal{N}} > \beta^{\mathcal{P}}$ ($\beta^{\mathcal{N}} < \beta^{\mathcal{P}}$). Panel D shows a case where the dependence is stronger in the M^- quadrant than in the M^+ quadrant, i.e. $\beta^{\mathcal{M}^-} > \beta^{\mathcal{M}^+}$. This provides a hedging benefit, and so expected returns are lower than otherwise identical assets with $\beta^{\mathcal{M}^-} \leq \beta^{\mathcal{M}^+}$.

The true semibetas, of course, are not directly observable. However, as demonstrated in the burgeoning realized volatility literature, higher-frequency intraday data may be used to construct more accurate measures of risk. In particular, as formally shown by Barndorff-Nielsen and Shephard (2004), in the absence of any market microstructure frictions, the traditional market beta of an asset may be consistently (for increasingly finer sampled data) estimated by the so-called realized beta, defined as the ratio between the realized covariance of the asset and the market and the realized variance of the market.[5] Similarly, relying on the recent study by Bollerslev, Patton, Li and Quaedvlieg (2019b) and the asymptotic results pertaining to realized semicovariances therein, semibetas may be consistently estimated by their corresponding realized semibeta counterparts, defined as the ratios of the relevant realized semicovariance components and the realized market variance. We discuss this further in Section 2.

Building on these ideas, we offer three main empirical contributions. Our first empirical investigations use daily realized semibetas based on high-frequency intraday data for all of the S&P 500 constituent stocks over the 1993-2014 sample period. The estimated semibetas clearly reveal the existence of asymmetric dependencies between the individual stocks and the market beyond those of the linear dependencies captured by the traditional market beta. More importantly, and consistent with the mean-semivariance pricing implications illustrated in Figure 1, our results strongly support the hypothesis that these non-linear dependencies are priced differently: stocks with higher $\beta^{\mathcal{N}}$ are associated with significantly higher subsequent daily returns; stocks with higher $\beta^{\mathcal{M}^-}$ are associated with significantly lower subsequent daily returns; and neither $\beta^{\mathcal{P}}$ nor $\beta^{\mathcal{M}^+}$ appear to carry a significant risk premium. The two-way decomposition of the traditional market beta into separate up and downside betas previously explored in the literature is also easily rejected in favor of the four-way semibeta decomposition proposed here. These findings for the daily realized semibetas and future daily returns carry over to longer weekly and monthly return horizons. They also remain robust to the inclusion of a long list of other return predictor variables previously analyzed in the literature, including the up and downside

[5] For additional discussion of the realized beta concept along with empirical applications, see also Andersen, Bollerslev, Diebold and Wu (2006).

betas of Ang, Chen and Xing (2006a).

This prima facie evidence notwithstanding, the requirement of intraday high-frequency data for accurately estimating the realized semibetas invariably limits the time span and number of stocks underlying our analyses. Thus, to further expand the scope of our investigations, our second empirical contribution relies on monthly semibetas constructed from daily returns for a much wider cross-section of stocks and a longer 1963-2017 sample period. Using this wider and longer sample we arrive at the same general conclusions: $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ are priced, with estimated annualized risk premiums of 10.43% and -6.42% respectively, while the estimated risk premiums for $\beta^{\mathcal{P}}$ and $\beta^{\mathcal{M}^+}$ are both statistically insignificant at conventional levels. By comparison, the estimated risk premium for the traditional market beta equals 4.10% per annum over this sample period.

Finally, we investigate whether the statistically significant differences in the compensation for the different semibetas also translate into "economically significant" differences in the performance of simple long-short portfolio strategies. We find that a long-short semibeta strategy generates average annual excess returns of 9.8%, together with an annualized Sharpe ratio of 1.05. By comparison, the standard CAPM betas and the Ang, Chen and Xing (2006a) downside betas generate excess returns of 5.0% and 7.5%, respectively, with Sharpe ratios of only 0.30 and 0.48. Relying on the four- and five-factor models of Carhart (1997) and Fama and French (1993, 2015), to further assess the risk-adjusted performance of the semivariance portfolios, results in annualized alphas of 8.4% and 9.7% respectively, and overwhelmingly significant t-statistics, while the traditional beta and the downside beta portfolios have much smaller and at best only borderline significant alphas. Hence, adding to the recent literature and debate about betting on or against beta (see, e.g., Frazzini and Pedersen, 2014; Cederburgh and O'Doherty, 2016; Bali, Brown, Murray and Tang, 2017; Novy-Marx and Velikov, 2018), we conclude that it is better to bet on *and* against the "right" semibetas.

In addition to the many previous studies on downside risk noted above, our empirical findings are also related to the vast existing literature on asymmetric dependencies in stock returns, including among others Longin and Solnik (2001), Ang and Chen (2002), Patton (2004), Hong, Tu and Zhou (2006), and Elkamhi and Stefanova (2014). They are also related to the more recent and rapidly growing literature on the pricing of downside tail, or crash, risk, including Bali, Demirtas and Levy (2009), Bollerslev and Todorov (2011), Kelly and Jiang (2014), Cremers, Halling and Weinbaum (2015) Bollerslev, Li and Todorov (2016), Chabi-Yo, Ruenzi and Weigert (2018), Farago and Tedongap (2018), Lu and Murray (2019), Bernard, Bondarenko and Vanduffel (2019), Chabi-Yo, Huggenberger and Weigert (2019), and Orlowski, Schneider and Trojani (2019). In contrast to all of these studies, however, which rely on the use of options and/or non-linear procedures for assessing the asymmetric joint tail dependencies and the pricing thereof, we maintain a simple linear pricing relationship together with a simple-to-implement additive decomposition of the traditional market beta into the four semibeta components. Our new semibeta measures are also distinctly different from, and much simpler to implement than, the entropy approach of Jiang, Wu and Zhou (2018) designed to measure asymmetries in up and downside comovements.

The semibetas, and the joint dependencies captured by them, are also related to the notion of coskewness originally proposed by Kraus and Litzenberger (1976), and the corresponding notion of cokurtosis, as investigated empirically by Harvey and Siddique (2000), Dittmar (2002), Conrad, Dittmar and Ghysels (2013) and Langlois (2019), among others. We find that the semibetas remain highly significant for explaining the cross-sectional variation controlling for coskewness and cokurtosis, while both of these co-dependency measures are rendered insignificant by the inclusion of the proposed semibeta measures. Our reliance on the new semicovariance concept for decomposing the systematic market risk and defining the semibetas also sets our analysis apart from other recent studies based on the semivariance concept for defining and empirically investigating asset specific "good" and "bad" volatility measures and the separate pricing thereof, as in, e.g., Feunou, Jahan-Parver and Okou (2018), Bollerslev, Li and Zhao (2019a) and Feunou and Okou (2019).

The remainder of the paper is structured as follows. We begin in Section 2 by discussing our construction of the daily realized semibetas, along with a brief summary of their key distributional features. Section 3 presents our empirical findings related to the pricing of the daily realized semibetas based on firm level cross-sectional regressions. Section 4 presents our results based on monthly semibetas estimated from daily data across a much broader cross-section of stocks and over a longer time span. Section 5 considers the performance of simple long-short semibeta-based portfolio strategies, along with comparisons to other similarly constructed beta-based portfolios. Section 6 concludes. Additional empirical results and robustness checks are detailed in a Supplemental Appendix.

2. Realized Semibetas

We begin by formally defining realized semibetas. We then briefly discussion the high-frequency data that we use in our main empirical investigations, followed by a summary of the salient distributional features of the resulting daily realized semibeta estimates and what they reveal about the inherent own dynamic and cross-asset joint dependencies.

2.1. Definitions

Let $r_{t,k,i}$ denote the return on asset i over the k^{th} intradaily time interval on day t, with the concurrent return for the aggregate market denoted by $f_{t,k}$. Define the signed high-frequency asset returns by $r^+_{t,k,i} \equiv \max(r_{t,k,i}, 0)$ and $r^-_{t,k,i} \equiv \min(r_{t,k,i}, 0)$, with the signed high-frequency market returns defined analogously. The realized semibetas are then defined by:

$$\begin{aligned} \widehat{\beta}^{\mathcal{N}}_{t,i} &\equiv \frac{\sum_{k=1}^{m} r^-_{t,k,i} f^-_{t,k}}{\sum_{k=1}^{m} f^2_{t,k}}, \qquad & \widehat{\beta}^{\mathcal{P}}_{t,i} &\equiv \frac{\sum_{k=1}^{m} r^+_{t,k,i} f^+_{t,k}}{\sum_{k=1}^{m} f^2_{t,k}}, \\ \widehat{\beta}^{\mathcal{M}^-}_{t,i} &\equiv \frac{-\sum_{k=1}^{m} r^+_{t,k,i} f^-_{t,k}}{\sum_{k=1}^{m} f^2_{t,k}}, \qquad & \widehat{\beta}^{\mathcal{M}^+}_{t,i} &\equiv \frac{-\sum_{k=1}^{m} r^-_{t,k,i} f^+_{t,k}}{\sum_{k=1}^{m} f^2_{t,k}}. \end{aligned} \tag{2}$$

where m denotes the number of high-frequency return intervals each day. As noted above, we purposely change the sign on the two mixed semibetas, to make them positive, thereby allowing for an easier interpretation of the corresponding risk premium estimates. The semibetas thus provide an exact four-way decomposition of the traditional realized market beta:

$$\widehat{\beta}_{t,i} \equiv \frac{\sum_{k=1}^{m} r_{t,k,i} f_{t,k}}{\sum_{k=1}^{m} f^2_{t,k}} = \widehat{\beta}^{\mathcal{N}}_{t,i} + \widehat{\beta}^{\mathcal{P}}_{t,i} - \widehat{\beta}^{\mathcal{M}^+}_{t,i} - \widehat{\beta}^{\mathcal{M}^-}_{t,i}. \tag{3}$$

Let $\mathcal{RV}_t$ and $\mathcal{COV}_{t,i}$ denote the latent true daily variation of the return on the market and the covariation between the market return and the return on the individual asset i, with the corresponding true semicovariation measures denoted by $\mathcal{P}_{t,i}$, $\mathcal{N}_{t,i}$, $\mathcal{M}^+_{t,i}$ and $\mathcal{M}^-_{t,i}$, respectively. Barndorff-Nielsen and Shephard (2004) show that, for increasingly finely-sampled high-frequency returns, or $m \to \infty$, realized betas consistently estimate true betas:

$$\widehat{\beta}_{t,i} \xrightarrow{p} \frac{\mathcal{COV}_{t,i}}{\mathcal{RV}_t}. \tag{4}$$

Similarly, the in-fill asymptotic theory in Bollerslev, Patton, Li and Quaedvlieg (2019b) pertaining to realized semicovariances imply that realized semibetas consistently estimate true semibetas:

$$\widehat{\beta}^{\mathcal{N}}_{t,i} \xrightarrow{p} \frac{\mathcal{N}_{t,i}}{\mathcal{RV}_t}, \quad \widehat{\beta}^{\mathcal{P}}_{t,i} \xrightarrow{p} \frac{\mathcal{P}_{t,i}}{\mathcal{RV}_t}, \quad \widehat{\beta}^{\mathcal{M}^+}_{t,i} \xrightarrow{p} \frac{-\mathcal{M}^+_{t,i}}{\mathcal{RV}_t}, \quad \widehat{\beta}^{\mathcal{M}^-}_{t,i} \xrightarrow{p} \frac{-\mathcal{M}^-_{t,i}}{\mathcal{RV}_t}. \tag{5}$$

For ease of notation, in the remainder we will drop subscripts and hats, and refer to these realized (semi)beta measures simply as β, $\beta^{\mathcal{N}}$, $\beta^{\mathcal{P}}$, $\beta^{\mathcal{M}^+}$ and $\beta^{\mathcal{M}^-}$.

If the market and individual asset returns are jointly Normally distributed, the conventional beta and the four semibetas convey the same information. In particular, it follows directly from the results in Bollerslev, Patton, Li and Quaedvlieg (2019b) that in this case the semibetas, betas, and variances satisfy:

$$\beta^{\mathcal{N}} = \quad \beta^{\mathcal{P}} = \quad \frac{1}{2\pi}\frac{\sigma_r^2}{\sigma_f^2}\left(\beta \arccos\left(-\frac{\sigma_r}{\sigma_f}\beta\right) + \sqrt{\frac{\sigma_r^2}{\sigma_f^2} - \beta^2}\right), \tag{6}$$

$$\beta^{\mathcal{M}^+} = \quad \beta^{\mathcal{M}^-} = \quad \frac{1}{2\pi}\frac{\sigma_r^2}{\sigma_f^2}\left(\beta \arccos\left(\frac{\sigma_r}{\sigma_f}\beta\right) - \sqrt{\frac{\sigma_r^2}{\sigma_f^2} - \beta^2}\right). \tag{7}$$

In the case of Normality, the concordant semibetas ($\beta^{\mathcal{N}}$ and $\beta^{\mathcal{P}}$) are thus equal, as are the discordant semibetas ($\beta^{\mathcal{M}^+}$ and $\beta^{\mathcal{M}^-}$). As highlighted by the contour plots in the previously discussed Figure 2, if the returns are *not* Normally distributed, the semibetas will generally differ and may convey more information than the single "total" beta.

2.2. Data and Summary Statistics

Our primary empirical investigations rely on high-frequency data obtained from the Trades and Quotes (TAQ) database. We include all of the S&P 500 constituent stocks during the January 1993 to December 2014 sample period, resulting in a total of 5,541 trading days and 1,049 unique securities. We adopt a 15-minute sampling scheme, or $m = 26$ return observations per day, in our calculations of the realized semibeta measures. This choice strikes a judicious balance between biases induced by market microstructure effects when sampling too finely versus the theoretical continuous-time arguments underlying the consistency of the realized semicovariance measures that formally hinges on increasingly finer sampled intraday returns.[6] We further match the intraday TAQ data and sample of stocks to the Center for Research in Securities Prices (CRSP) database to obtain the full-day returns for each of the stocks. All of our subsequent asset pricing investigations are based on these full-day and resulting longer weekly and monthly returns.[7] We also rely on the daily market capitalization for each of the individual stocks from the CRSP database in our construction of the high-frequency value-weighted market index.

Turning to the resulting daily realized (semi)beta estimates, the top panel of Table 1 reports the time series averages of the cross-sectional means, medians and standard deviations averaged across all of the stocks in the sample. The bottom panel gives the time series averages of the cross-sectional correlations. Consistent with on average positive dependencies between the market and each of the individual stocks, the two concordant

[6] Although a finer 5-minute sampling frequency has often been used in the realized volatility literature for the calculation of univariate realized volatility measures (see, e.g. Liu, Patton and Sheppard, 2015, and the many references therein), the market microstructure effects are further compounded in the multivariate setting by the so-called Epps (1979) effect and a downward bias in realized covariation measures stemming from asynchronous prices.

[7] As further discussed in Bollerslev, Li and Zhao (2019a), this matching of the TAQ intraday data with the daily returns from CRSP also ensures superior handling of stock splits and dividends.

Table 1: **Summary Statistics.** The top panel reports the time series averages of the cross-sectional means, medians and standard deviations of the daily realized semibetas constructed from fifteen minutes intraday returns. The bottom panel reports the time series averages of the cross-sectional correlations. The sample consists of all S&P 500 constituent stocks from January 1993 to December 2014.

	β	$\beta^{\mathcal{N}}$	$\beta^{\mathcal{P}}$	$\beta^{\mathcal{M}+}$	$\beta^{\mathcal{M}-}$
Mean	0.92	0.68	0.72	0.27	0.25
Median	0.83	0.57	0.61	0.16	0.15
St.Dev.	1.06	0.47	0.49	0.36	0.34
β	1.00	0.66	0.67	-0.33	-0.33
$\beta^{\mathcal{N}}$		1.00	0.44	0.19	0.06
$\beta^{\mathcal{P}}$			1.00	0.06	0.18
$\beta^{\mathcal{M}+}$				1.00	0.38
$\beta^{\mathcal{M}-}$					1.00

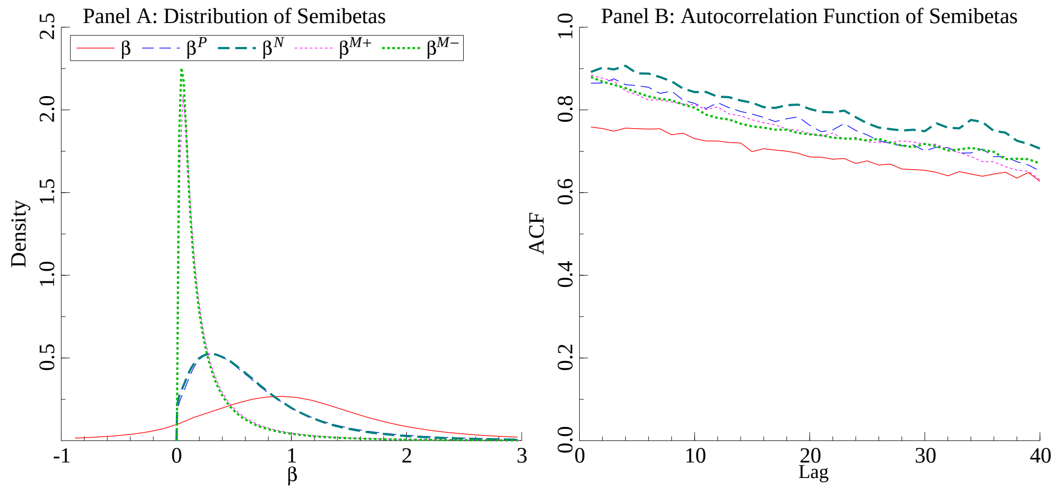
semibetas ($\beta^{\mathcal{P}}$ and $\beta^{\mathcal{N}}$) on average far exceed the two discordant semibetas ($\beta^{\mathcal{M}+}$ and $\beta^{\mathcal{M}-}$). The two concordant semibetas also correlated more strongly with the traditional market beta (β), and more so than with each other. Nonetheless, the correlations with the traditional beta are still far below unity, suggesting that the semibetas convey different, and potentially useful, information over and above that of the traditional market beta.

To further visualize some of these distributional features, Panel A of Figure 3 depicts the unconditional distributions of each of the daily realized betas and semibetas across all of the days and stocks in the sample. The distribution of the conventional betas is centered around one, as expected, and appears close to symmetric. Meanwhile, realized semibetas are all weakly positive by construction, and thus unsurprisingly their distributions are all right-skewed. Further echoing the summary statistics in Table 1, the semibeta distributions are all centered below one. Also, the unconditional distributions of the two concordant semibetas ($\beta^{\mathcal{P}}$ and $\beta^{\mathcal{N}}$) appear almost indistinguishable, as do the distributions of the two discordant semibetas ($\beta^{\mathcal{M}+}$ and $\beta^{\mathcal{M}-}$).

The average autocorrelation functions shown in Panel B of Figure 3 indicate a strong degree of persistence for the betas and semibetas, with the autocorrelations remaining in excess of 0.6 even at the 40^{th} lag.[8] Underpinning the cross-sectional return predictability regressions that we rely on in our asset pricing investigations below, the high first-order autocorrelations (close to 0.8 for the traditional beta and around 0.9 for each of the semibetas) also imply that today's realized semibetas for a given stock are an accurate prediction of tomorrow's semibetas for that stock.

[8] We rely on the instrumental variable approach of Hansen and Lunde (2014), using lags 4 through 10 as instruments, to adjust for measurement errors in the realized betas, thereby allowing for more meaningful comparisons of the autocorrelation functions across the different betas.

Figure 3: **Unconditional Distributions and Autocorrelations.** Panel A displays kernel density estimates of the unconditional distribution of the daily realized beta and semibetas averaged across time and stocks. Panel B reports the average autocorrelation functions for the daily realized beta and semibetas averaged across stocks. The sample consists of all of the S&P 500 constituent stocks from January 1993 to December 2014.



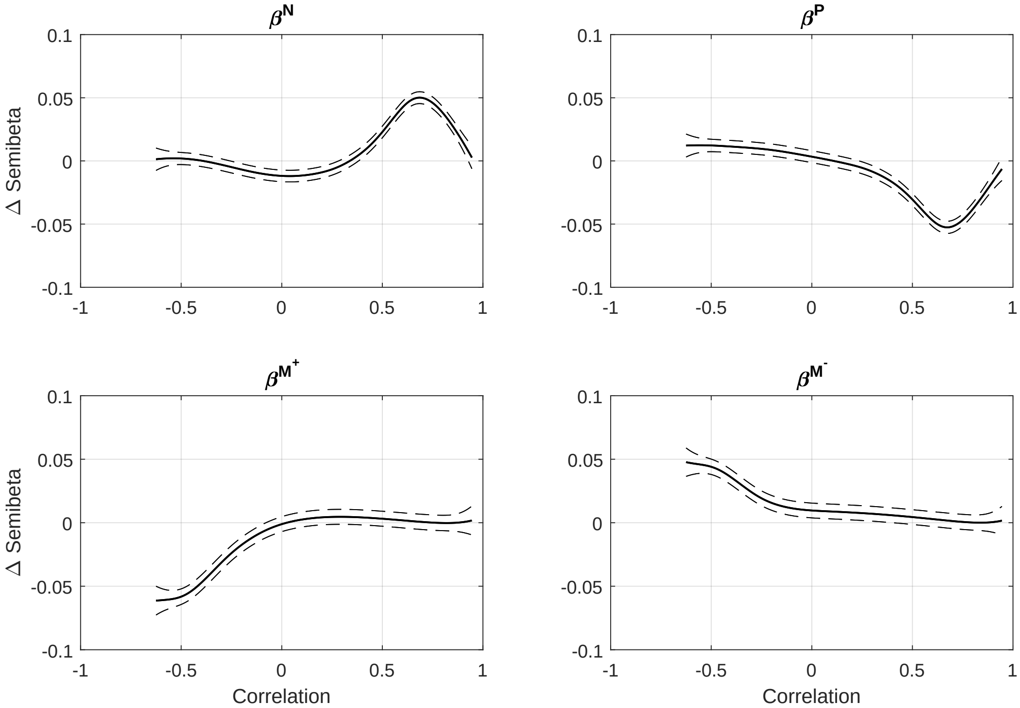
2.3. Asymmetric Dependencies

As discussed above, if the individual stock and aggregate market returns were jointly normally distributed, semibetas would not provide any information beyond that contained in the traditional market beta. In fact, since each of the realized semibetas uses fewer observations than the standard realized beta, realized semibetas would, in this scenario, yield statistically less efficient inference. Hence, in an effort to help gauge whether the semibetas do indeed convey potentially useful additional information about asymmetric dependencies, it is instructive to compare the realized semibeta estimates to the limiting values that would obtain if the individual stock and aggregate market returns were jointly normally distributed.

To that end, Figure 4 reports the differences between the observed realized semibetas and the theoretical values that would hold under joint normality, presented in equations (6) and (7) above. To facilitate the interpretation and more clearly highlight the differences, we report the results as a function of the daily realized correlations, averaged across all of the stocks and days in the sample.[9] The top panel reveals that realized negative semibetas are generally higher than would be found under joint normality, particularly for relatively highly correlated assets (e.g., for correlations between 0.4 and 0.9), where

[9]More specifically, for each day and stock in the sample, we standardize all of the intraday returns to have unit daily variance. We then compute the daily realized covariance (correlation) and the four semibetas, averaging the estimates within correlation bins of width 0.01. Finally, we use a spline to smooth the differences from their implied Gaussian values.

Figure 4: **Asymmetric Dependencies** The figure plots the deviations of the daily realized semibetas from their Gaussian limits as a function of the daily correlations between the individual stocks and the market, along with pointwise 99% confidence intervals. The estimates are averaged across all of the S&P 500 constituent stocks and days in the January 1993 to December 2014 sample.



the confidence interval clearly excludes zero. Similarly, we find that realized positive semibetas are lower than would be expected under joint normality. For the discordant semibetas, we find a similar story: β^{M-} is significantly higher than would be expected under joint normality, particularly for negatively correlated assets, while the opposite is true for β^{M+}. Taken together, these findings are consistent with the stylized fact that asset return dependence is stronger in downturns than upturns. We turn next to our main empirical analysis and the pricing of these non-linear dependencies encoded in the realized semibeta measures.

3. Semibetas and the Cross-Section of Expected Returns

We begin our empirical investigations by presenting the results from standard Fama and MacBeth (1973) type cross-sectional predictive regressions. These regressions conveniently allow for the simultaneous estimation of separate risk premiums for each of the semibetas, as implied by the mean-semivariance pricing framework. In particular, for each day $t = 1, ..., T-1$, and all of the stocks $i = 1, ..., N_t$, available on day t and $t+1$, we first estimate the day $t+1$ lambdas from the cross-sectional regression:

$$r_{t+1,i} \;=\; \lambda_{0,t+1} + \lambda_{t+1}^{\mathcal{N}} \hat{\beta}_{t,i}^{\mathcal{N}} + \lambda_{t+1}^{\mathcal{P}} \hat{\beta}_{t,i}^{\mathcal{P}} + \lambda_{t+1}^{\mathcal{M}^+} \hat{\beta}_{t,i}^{\mathcal{M}^+} + \lambda_{t+1}^{\mathcal{M}^-} \hat{\beta}_{t,i}^{\mathcal{M}^-} + \; \epsilon_{t+1,i}. \tag{8}$$

Based on these $T-1$ cross-sectional estimates, we then estimate the risk premiums associated with each of the semibetas by the time series averages of the lambdas over all of the days in the sample:

$$\hat{\lambda}^j \;=\; \frac{1}{T-1} \sum_{t=2}^{T} \hat{\lambda}_t^j \;, \qquad j = \mathcal{N}, \mathcal{P}, \mathcal{M}^+, \mathcal{M}^-. \tag{9}$$

The resulting annualized estimates, along with their t-statistics based on Newey-West robust standard errors (using 21 lags), together with the time-series average of the R^2s from the first-stage cross-sectional regressions in equation (8), are reported in the second row in Panel A of Table 2. As a benchmark, we also report in the first row of Table 2 the estimated risk premium for the traditional realized beta. Consistent with the basic mean-variance framework, the traditional beta carries a statistically significant risk premium of 4.58% per year. However, this estimated risk premium is also slightly below the average annual equity risk premium of 8.56% observed over the sample, corroborating the basic intuition underlying the "betting against beta" idea (Frazzini and Pedersen, 2014).

More interestingly, the estimated risk premiums for the semibetas reported in the second row clearly underscore the richer pricing implications derived from the mean-semivariance framework: $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ are both associated with numerically large and statistically significant risk premiums, while $\beta^{\mathcal{P}}$ and $\beta^{\mathcal{M}^+}$ do not appear to be associated with any significant differences in the returns across stocks. Further interpreting not just the statistical significance but the magnitudes of the estimated risk premiums for the semibetas, a one standard deviation increase in the value of $\beta^{\mathcal{N}}$ relative to its' cross-sectional mean is associated with a very large increase in the average annual return of 10.59%, while a one standard deviation increase in the value of $\beta^{\mathcal{M}^-}$ relative to its' cross-sectional mean *lowers* the return by 2.88% per year on average.

The cross-sectional fit, reported in the final column, rises from 2.70% when using the traditional CAPM beta to 5.43% when using the semibetas. We can formally test whether this gain in R^2 is statistically significant by noting that the semibeta-based pricing reduces to the traditional CAPM model if the following restrictions on the risk premiums hold:

$$H_{0,t}: \; \lambda_t^{\mathcal{N}} = \lambda_t^{\mathcal{P}} = -\lambda_t^{\mathcal{M}^+} = -\lambda_t^{\mathcal{M}^-}. \tag{10}$$

We find that we can reject this restriction at the 5% level for 68% of the $5,541$ days in our sample. Going one step further, we can also test an implication of this null for *average* risk premiums:

$$H_0 : \bar{\lambda}^{\mathcal{N}} = \bar{\lambda}^{\mathcal{P}} = -\bar{\lambda}^{\mathcal{M}^+} = -\bar{\lambda}^{\mathcal{M}^-}. \tag{11}$$

This hypothesis results in a p-value of less than 0.001, thus strongly rejecting the traditional one-beta model in favor of a model that exploits the additional information contained in the semibetas.

A plethora of other risk factors and firm characteristics constructed from lower frequency daily or monthly data have been put forth in the literature as significant drivers of the cross-sectional variation in equity returns; see, e.g., the recent account by Harvey, Liu and Zhu (2016). We do not attempt to include an exhaustive set of these controls here, instead we focus on a subset of the more prominent variables that have received the most attention in the literature, namely size (ME) (Banz, 1981), book-to-market (BM) (Fama and French, 1993), momentum (MOM) (Jegadeesh and Titman, 1993), return reversals (REV) (Jegadeesh, 1990), idiosyncratic volatility (IVOL) (Ang, Hodrick, Xing and Zhang, 2006b), and illiquidity (ILLIQ) (Amihud, 2002); further details concerning the construction of each of these variables are given in Appendix B.

The third row of Table 2 reports the average risk premium estimates from the cross-sectional regressions that in addition to the semibetas include ME, BM and MOM, mimicking the popular Fama-French-Carhart four factor (FFC4) model. Consistent with the extant literature, the estimated risk premiums for ME and MOM are both strongly significant, while the premium for BM is only marginally significant at conventional levels. Correspondingly, the inclusion of the three additional risk factors also increases the average cross-sectional R^2 from 5.43% for the regressions based solely on the four semibetas to 8.23% for the semibeta+FFC-based model. Importantly, the risk premiums associated with $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ remain highly statistically significant.

The bottom row of Table 2 further incorporates REV, IVOL and ILLIQ as controls, which increases the average cross-sectional R^2 to 10.32%. Again, the inclusion of the additional controls does not meaningfully alter the large and highly significant t-statistic associated with $\beta^{\mathcal{N}}$. Also, even though the t-statistic for $\beta^{\mathcal{M}^-}$ is somewhat diminished compared to some of the earlier regressions, the risk premium estimates for $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ are both remarkably similar to the estimates obtained without the inclusion of any controls reported in the second row, underscoring the robustness of the semibeta pricing.

Many other predictor variables and beta-type decompositions have, of course, been found to improve upon the traditional CAPM. Perhaps most closely related to the present analysis are the up and downside betas advocated in the widely cited study by Ang, Chen and Xing (2006a). We next turn to an empirical investigation of those.

Table 2: **Fama-Macbeth Regressions on Semibetas** The table reports the estimated annualized risk premia and Newey-West robust t-statistics from daily Fama-MacBeth cross-sectional predictive regressions. The daily semibetas are calculated from fifteen-minute intraday data. All of the control variables are measured prior to the daily returns, as detailed in Appendix B. The estimates are based on all of the S&P 500 constituent stocks and days in the January 1993 to December 2014 sample.

β	$\beta^{\mathcal{N}}$	$\beta^{\mathcal{P}}$	$\beta^{\mathcal{M}^+}$	$\beta^{\mathcal{M}^-}$	ME	BM	MOM	REV	IVOL	ILLIQ	R^2
4.58											2.70
3.04											
	22.54	-1.58	-4.29	-8.48							5.43
	5.62	-0.52	-0.86	-2.02							
	22.47	-5.67	-2.90	-12.20	-2.23	-1.77	0.11				8.23
	5.75	-2.02	-0.65	-3.14	-3.83	-1.95	3.47				
	20.36	-2.91	1.68	-6.15	-7.42	-1.65	0.09	-0.55	-3.07	-4.88	10.32
	5.44	-1.08	0.41	-1.68	-7.71	-1.87	2.55	-5.82	-3.56	-6.42	

3.1. Upside and Downside Betas

High-frequency versions of the upside and downside betas of Ang, Chen and Xing (2006a) are naturally defined as:

$$\hat{\beta}^{+}_{t,i} \equiv \frac{\sum_{k=1}^{m} r_{t,k,i} f^{+}_{t,k}}{\sum_{k=1}^{m} (f^{+}_{t,k})^2}, \qquad \hat{\beta}^{-}_{t,i} \equiv \frac{\sum_{k=1}^{m} r_{t,k,i} f^{-}_{t,k}}{\sum_{k=1}^{m} (f^{-}_{t,k})^2}. \tag{12}$$

In contrast to the semibetas proposed here, which account for joint asymmetric dependencies by conditioning the covariation on *both* the signed market and individual asset returns, the upside and downside betas condition only on the sign of the market return.

For ease of comparison, the first row in Table 3 repeats the baseline results using semibetas from Table 2. The second row in Table 3 reports the estimated average risk premiums associated with the upside and downside betas. The results are broadly consistent with the previous findings of Ang, Chen and Xing (2006a) in that only β^- carries a significant risk premium. The results are also in line with the estimated risk premiums for the semibetas presented in the top row, which show that only $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$, which account for negative market comovements, are associated with significant risk premiums.

To directly compare and contrast the pricing of the semibetas with the pricing of the up and downside betas, the third row in Table 3 reports the estimates obtained by including *all* of the six betas in the same cross-sectional regressions. Despite the relatively high correlation between the semibetas and the up/downside betas,[10] the estimated risk premium for $\beta^{\mathcal{N}}$ clearly stands out as the most significant with a t-statistic of 3.86,

[10]Correlations between all of the semibetas and the up/downside betas, along with the other controls,

followed by the premium for $\beta^{\mathcal{M}^-}$ with a t-statistic of -1.79. Meanwhile, the risk premium for β^- has a t-statistic of only 0.82, suggesting that the information contained in semibetas effectively subsumes the information in the downside beta in terms of explaining the cross-sectional variation in the returns. A joint test that all coefficients on semibetas are zero, leaving only the up and downside betas with nonzero coefficients, also rejects the null with a p-value of less than 0.001. In contrast, a joint test that both coefficients on up and downside betas are zero, leaving only the semibetas with nonzero coefficients, fails to reject the null, with a p-value of 0.13.

To facilitate a test based on risk prices of whether the semibetas provide superior cross-sectional predictions compared to the up and downside betas, notice that the latter can be obtained as a weighted sum of the former. Specifically:

$$\hat{\beta}^+_{t,i} = (\widehat{\beta}^{\mathcal{P}}_{t,i} - \widehat{\beta}^{\mathcal{M}^+}_{t,i})\frac{\sum_{k=1}^m f^2_{t,k}}{\sum_{k=1}^m (f^+_{t,k})^2}, \tag{13}$$

$$\hat{\beta}^-_{t,i} = (\widehat{\beta}^{\mathcal{N}}_{t,i} - \widehat{\beta}^{\mathcal{M}^-}_{t,i})\frac{\sum_{k=1}^m f^2_{t,k}}{\sum_{k=1}^m (f^-_{t,k})^2}. \tag{14}$$

Since the weights only involve functions of market returns, they do not vary in the cross-section, which implies that the semibeta model proposed here will have identical pricing implications to that of the up/downside beta model of Ang, Chen and Xing (2006a) if the following restrictions hold:

$$H_{0,t}: \ \lambda^{\mathcal{P}}_t = -\lambda^{\mathcal{M}^+}_t \cap \lambda^{\mathcal{N}}_t = -\lambda^{\mathcal{M}^-}_t. \tag{15}$$

We find that this restriction can be rejected at the 5% level for 58% of the 5,541 daily cross-sectional regressions. If we test the weaker restriction that these relationships between the risk premiums hold on *average*, which is of course an implication of equation (15) holding each day, we reject the null with a p-value of less than 0.001, providing strong evidence that the proposed semibeta pricing framework improves upon the up and downside beta pricing framework.

3.2. Coskewness and Cokurtosis

The semibetas explicitly account for non-normal and asymmetric systematic risks by conditioning on the signed returns. A number of other measures have been explored in the literature as a way to capture non-normal asymmetric joint return dependencies and

are presented in Appendix A. The time series averages of the cross-sectional correlations between β^+ and $\beta^{\mathcal{P}}$, and β^- and $\beta^{\mathcal{N}}$, in particular, are as high as 0.81, thus hindering a precise estimation of each of the individual risk premiums.

Table 3: **Fama-Macbeth Regressions on Other Measures** The table reports the estimated annualized risk premia and Newey-West robust t-statistics from daily Fama-MacBeth cross-sectional predictive regressions. The daily semibetas, up and downside betas, and coskewness and cokurtosis measures are calculated from fifteen-minute intraday data based on all of the S&P 500 constituent stocks and days in the January 1993 to December 2014 sample.

$\beta^{\mathcal{N}}$	$\beta^{\mathcal{P}}$	$\beta^{\mathcal{M}^+}$	$\beta^{\mathcal{M}^-}$	β^+	β^-	CSK	CKT	R^2
22.54	-1.58	-4.29	-8.48					5.43
5.62	-0.52	-0.86	-2.02					
				-1.17	6.88			3.70
				-1.11	5.54			
17.31	-8.10	-12.66	-3.86	-2.40	7.90			6.61
3.86	-0.13	0.03	-1.79	-1.67	0.82			
						-4.40	0.81	1.52
						-1.55	0.76	
30.92	-3.79	-3.89	-16.33			10.09	-3.59	6.26
6.20	-1.12	-0.76	-3.69			2.66	-3.22	

the possible pricing thereof, most notably the notion of coskewness originally proposed by Kraus and Litzenberger (1976), and analyzed more extensively by Harvey and Siddique (2000) and Christoffersen, Honarvar and Ornthanalai (2017) among others. Other studies have similarly argued that cokurtosis appears to be priced in the cross-section; see, e.g., Dittmar (2002) and Ang, Chen and Xing (2006a). Directly following these studies, we define the daily coskewness and cokurtosis measures for stock i by:

$$CSK_{t,i} = \frac{\frac{1}{m}\sum_{k=1}^{m}(r_{t,k,i}-\bar{r}_{t,i})(f_{t,k}-\bar{f}_t)^2}{\sqrt{\frac{1}{m}\sum_{k=1}^{m}(r_{t,k,i}-\bar{r}_{t,i})^2}\ \frac{1}{m}\sum_{j=1}^{m}(f_{t,k}-\bar{f}_t)^2}, \tag{16}$$

$$CKT_{t,i} = \frac{\frac{1}{m}\sum_{k=1}^{m}(r_{t,k,i}-\bar{r}_{t,i})(f_{t,k}-\bar{f}_t)^3}{\sqrt{\frac{1}{m}\sum_{k=1}^{m}(r_{t,k,i}-\bar{r}_{t,i})^2}\ \left(\frac{1}{m}\sum_{k=1}^{m}(f_{t,k}-\bar{f}_t)^2\right)^{3/2}}, \tag{17}$$

where $\bar{f}_t$ and $\bar{r}_{t,i}$ denote the average daily return on the market and stock i respectively.

Comparing the top row with the penultimate row of Table 3 we see that the semibeta model fits the data much better than the coskewness/cokurtosis model, with an average R^2 of 5.43% compared to 1.52%. Combining all six measures in a single model, as in the bottom row of Table 3, we see the fit improves slightly, to 6.26%. The relatively high contemporaneous correlation between the realized semibetas and the CSK and CKT measures (see Appendix A) makes precise estimation of the magnitudes of the risk premiums associated with each of the individual measures challenging. Nonetheless, the regression reported in the last row of Table 3, which incorporates all six measures, shows that the t-statistic associated with $\beta^{\mathcal{N}}$ is by far the largest, followed by that of $\beta^{\mathcal{M}^-}$, supporting the idea that the priced non-normal systematic risks is best captured by these two semibetas.

Interestingly, joint tests that the semibeta coefficients, or the coskewness/cokurtosis coefficients, are zero can be rejected at the 0.05 level in both cases. This indicates that while coskewness and cokurtosis have substantially less explanatory power than semibetas, as evidenced by the R^2 values in the first and fourth rows of Table 3, they do contain additional information not accounted for by the semibetas. This is perhaps unsurprising, as coskewness and cokurtosis primarily capture information about the tails, and several recent studies have argued that systematic tail risks appear to be priced differently from more "normal" risks (see, e.g., Kelly and Jiang, 2014; Bollerslev, Li and Todorov, 2016). By contrast, the semibetas rely on a simple decomposition of the standard covariation with the market and "normal" systematic risks.

3.3. Longer Investment Horizons

The strong relationship between the daily realized semibetas and the cross-sectional variation in the subsequent daily returns naturally raises the question of whether this same predictive relationship carry over to longer investment horizons. To investigate this, we rely on the identical day t realized semibetas and cross-sectional regression in (8) in which we replace the left-hand-side daily returns with the cumulative returns from day $t+1$ to day $t+h$, for $h = 5$ and $h = 20$ corresponding to a "week" and a "month," respectively.[11] The results reported in Table 4 are consistent with the daily findings reported above: the estimated risk premiums for $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ are both highly statistically significant, while neither $\beta^{\mathcal{P}}$ nor $\beta^{\mathcal{M}^+}$ appear to be priced. Meanwhile, the magnitudes of the estimated (annualized) monthly risk premiums are naturally smaller than the (annualized) weekly premiums, as the strength of the predictability afforded by the daily semibetas diminishes with the return horizon.[12]

Further corroborating the above daily findings, the inclusion of the same control variables as in Table 2 does not materially affect the weekly or monthly $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ risk premium estimates. In fact, the inclusion of the additional controls only serves to strengthen the statistical significance of the two priced semibetas.

[11]We purposely rely on overlapping return windows and appropriately adjusted standard errors and t-statistics to enhance the efficiency of our inference, but qualitatively similar findings are obtained with non-overlapping return windows.

[12]Conversely, the pairwise correlations between the concordant ($\beta^{\mathcal{N}}$ and $\beta^{\mathcal{P}}$) and discordant ($\beta^{\mathcal{M}^+}$ and $\beta^{\mathcal{M}^-}$) semibetas generally increase with the horizon over which they are calculated; for additional details see the Supplemental Appendix.

Table 4: **Weekly and Monthly Investment Horizons.** The table reports the estimated annualized risk premia and Newey-West robust t-statistics from daily Fama-MacBeth cross-sectional regressions for predicting the future weekly (5-days) and monthly (20-days) returns. The daily semibetas are calculated from fifteen-minute intraday data on the last day preceding the return window. All of the control variables are measured prior to the daily returns. The estimates are based on all of the S&P 500 constituent stocks and days in the January 1993 to December 2014 sample.

β	$\beta^{\mathcal{N}}$	$\beta^{\mathcal{P}}$	$\beta^{\mathcal{M}^+}$	$\beta^{\mathcal{M}^-}$	ME	BM	MOM	REV	IVOL	ILLIQ	R^2
Panel A: Weekly											
4.69											2.37
3.72											
	14.58	0.96	5.20	-13.80							5.07
	5.90	0.50	1.87	-3.71							
	10.85	-0.52	3.67	-13.58	-6.36	-1.94	0.08	-0.35	-1.06	-3.63	10.83
	5.92	-0.38	1.76	-5.03	-7.50	-2.30	2.54	-4.39	-1.40	-5.77	
Panel B: Monthly											
2.93											1.90
2.71											
	8.70	1.67	3.51	-3.36							4.45
	4.39	1.15	1.55	-1.30							
	4.42	-0.57	-0.65	-6.41	-4.89	-1.86	0.08	-0.25	0.98	-2.27	10.79
	3.55	-0.58	-0.44	-3.34	-7.06	-2.58	2.64	-4.15	1.56	-4.60	

4. Daily Data and Monthly Semibetas

The theory underlying the realized semibetas and the consistent estimation of the latent priced covariation components formally hinges on the use of ever finer sampled data, which motivates our analysis above based on high-frequency intraday data for the estimation of daily realized semibetas. Meanwhile, reliable high-frequency data is only available for a select set of stocks over a fairly recent sample period. In this section we extend our previous analysis, and use monthly semibetas constructed from daily data for a broader set of stocks over a longer sample period.

Specifically, we employ the CRSP daily database, expanding our sample period to January 1963 until December 2017, and include all common publicly traded stocks.[13] Armed with this expanded data set, we then calculate monthly semibetas by replicating the sums over the intraday returns in equation (2) with the corresponding sums over the

[13]Concretely, we consider all stocks with CRSP codes 10 and 11. In line with previous work we remove all "penny stocks" with prices less than five dollars to help alleviate biases arising from price discreteness; see, e.g., Harris (1994) and Amihud (2002).

Table 5: **Monthly Fama-Macbeth Regressions on Semibetas.** The table reports the estimated annualized risk premia and Newey-West robust t-statistics from overlapping monthly Fama-MacBeth cross-sectional predictive regressions. The monthly semibetas are calculated from daily data. All of the control variables are measured on the day prior to the monthly returns. The estimates are based on all of the common, non-penny, stocks in the CRSP data base from January 1963 to December 2017.

β	$\beta^{\mathcal{N}}$	$\beta^{\mathcal{P}}$	$\beta^{\mathcal{M}^+}$	$\beta^{\mathcal{M}^-}$	ME	BM	MOM	REV	IVOL	ILLIQ	R^2
4.10											2.36
3.77											
	10.43	1.40	4.15	-6.42							5.22
	4.46	0.87	1.15	-2.03							
	8.66	-0.66	5.60	-14.09	-2.55	-0.47	0.06				10.70
	3.56	-0.43	1.42	-3.72	-4.93	-0.40	2.14				
	6.59	-1.90	6.33	-15.59	-2.08	-0.75	0.07	-0.12	-1.60	2.40	13.38
	2.85	-1.06	1.50	-3.82	-4.22	-0.66	2.61	-1.97	-1.48	2.44	

daily returns within the month. All in all, this provides us with 262,308 firm-month observations. We similarly calculate monthly up and downside betas, and monthly coskewness and cokurtosis measures by replacing the intraday sums in equations (12), (16) and (17), respectively, with the corresponding daily sums.

It is well known that return distributions tend to become "more Gaussian" over longer return horizons (see, e.g., Engle, 2011). Correspondingly, one might naturally expect the use of lower frequency daily returns in the estimation of the monthly semibetas to blur some of the asymmetric dependencies evident in the daily realized semibetas discussed above. Table 5, however, shows that the relationships documented for the high-frequency-based daily semibetas are also present and significant in the monthly semibetas. In particular, in direct parallel to the second row of Table 2, only the risk premiums for $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ in the second row of Table 5 are significant. Also, similar to the earlier high-frequency-based results, the explanatory power of the semibeta model is more than double that of the traditional CAPM reported in the top row, with an average cross-sectional R^2 of 5.22% compared with 2.36%. Tests of the restriction that the risk premiums associated with the four semibetas are indeed the same, corresponding to the null hypothesis in equation (10), are rejected at the 5% level for 45% of the 659 months in the sample. A test that the risk premiums satisfy this restriction on average, corresponding to equation (11), is rejected with a p-value of less than 0.001, again directly paralleling the results in Section 3. The bottom two rows of Table 5 show that these results remain robust to the inclusion of the same set of controls used in the previous section. In short, our finding of differing risk prices for exposure to different semibetas is not specific to high frequency data in a recent sample period; it holds true more generally for a much larger sample of stocks over a much longer sample period.

Table 6 shows that the inclusion of the monthly up and downside betas and the monthly coskewness and cokurtosis measures do not affect this conclusion. Consistent with Ang, Chen and Xing (2006a), the estimates in the second row of Table 6 imply that only downside beta risk is priced. However, the inclusion of the semibetas in the cross-sectional regressions, as in the third row of the table, renders the estimated risk premiums for both β^{+} and β^{-} insignificant.[14] Further, the restrictions on the risk premiums that makes the pricing implications from the semibeta model coincide with the up and downside beta model, as in equation (15), is rejected at the 5% level for 43% of the 659 monthly cross-sectional regressions. The corresponding test that on average the risk premiums satisfy this restriction is again rejected with a p-value of less than 0.001. In line with the earlier findings of Harvey and Siddique (2000) and others, the estimated risk premiums for the monthly CSK and CKT measures, reported in the fourth row of Table 6, are both significant.[15] However, the inclusion of the semibetas, as in the last row of Table 6, substantially increases the average monthly cross-sectional R^2 from 1.69% to 6.49%. Importantly, $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ are also both strongly significant.

The next section demonstrates how these cross-sectional regression results may be translated into superior investment strategies by betting on $\beta^{\mathcal{N}}$ and betting against $\beta^{\mathcal{M}^-}$.

5. Betting On, and Against, Semibetas

In this section we investigate trading strategies based on betas and semibetas. In the mean-semivariance framework, only $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ are priced, with the former carrying a positive risk premium and the latter a negative risk premium. Given this, we implement a semibeta strategy by considering the performance of an equal-weighted combination of "betting on $\beta^{\mathcal{N}}$" and "betting against $\beta^{\mathcal{M}^-}$" portfolios. We also examine the performance of each of these portfolios separately, as well as that of a long-short strategy based on the traditional market beta for comparison.[16]

To avoid the critiques of Novy-Marx and Velikov (2018), we form the long-short portfolios using well-established methods. Firstly, we estimate betas and semibetas using

[14] The higher correlations between the monthly semibetas and the monthly controls, the up- and downside betas included, result in less stable risk premium point estimates; summary statistics for the monthly betas and controls are provided in Appendix A.

[15] The significance of the monthly CSK and CKT measures contrasts with the results in Table 3, and the lack of significance of the corresponding high-frequency-based daily measures.

[16] The Supplemental Appendix contains additional results for a portfolio which takes long positions in high $\beta^{\mathcal{N}}$ stocks and short positions in high $\beta^{\mathcal{M}^-}$ stocks, and a portfolio based on long positions in low $\beta^{\mathcal{M}^-}$ stocks and short positions in low $\beta^{\mathcal{N}}$ stocks. The performance of these two additional betting on and against semibeta portfolios are qualitatively similar to that of the semibeta portfolio presented here.

Table 6: **Monthly Fama-Macbeth Regressions and Other Measures.** The table reports the estimated annualized risk premia and Newey-West robust t-statistics from overlapping monthly Fama-MacBeth cross-sectional predictive regressions. The monthly semibetas, up and downside betas, coskewness and cokurtosis measures are calculated from daily data. The estimates are based on all of the common, non-penny, stocks in the CRSP data base from January 1963 to December 2017.

$\beta^{\mathcal{N}}$	$\beta^{\mathcal{P}}$	$\beta^{\mathcal{M}^+}$	$\beta^{\mathcal{M}^-}$	β^+	β^-	CSK	CKT	R^2
10.43	1.40	4.15	-6.42					5.22
4.46	0.87	1.15	-2.03					
				1.06	3.16			3.42
				1.61	3.74			
12.37	2.90	2.41	-7.56	-6.64	-0.90			5.57
4.97	1.03	1.20	-2.48	-0.95	-0.28			
						5.00	1.98	1.69
						2.81	2.57	
18.11	-2.27	2.87	-12.09			12.10	-2.80	6.49
4.98	-1.04	0.81	-3.40			4.26	-3.43	

standard, if modern, methods from high frequency econometrics, as described in Section 2 above. We then take a value-weighted long position in the top quintile and a value-weighted short position in the bottom quintile of stocks, rebalanced daily, to obtain zero-cost portfolios. We rely on continuously-compounded, as opposed to arithmetic, returns to facilitate the calculation of the cumulative portfolio returns over longer holding periods. We restrict the sample of stocks to the constituents of the S&P 500 index, thus explicitly excluding small and micro-cap stocks. We use the popular four-factor model of Fama and French (1993) and Carhart (1997) (FFC4) and the five-factor model of Fama and French (2015) (FF5) to assess the risk-adjusted performance of the portfolios and estimate the corresponding alphas.

The top panel of Table 7, reports the average returns, standard deviations and annualized Sharpe ratios for the long-short portfolios. The average return on the semibeta portfolio is nearly double that of the beta portfolio, while the volatility is just over half that of the beta portfolio, combining to yield a Sharpe ratio of 1.05 compared with 0.30 for the traditional beta portfolio. The latter two columns show that both the long and the short leg of the semibeta portfolio contribute to this superior performance: the $\beta^{\mathcal{N}}$ portfolio generates much higher returns and comparable volatility to the standard beta portfolio, while the $\beta^{\mathcal{M}^-}$ portfolio generates somewhat higher returns with much lower volatility.

The lower panel of Table 7 reports the estimated FFC4 and FF5 alphas and factor loadings of the different portfolios. The traditional beta strategy generates an annualized

Table 7: **Betting On and Against Semibetas.** The top panel reports annualized descriptive statistics of the betting on and against (semi)beta strategies. All of the portfolios are self-financing based on value-weighted long-short positions rebalanced daily. The bottom panel reports the time-series regression estimates and Newey-West robust t-statistics for the FFC4 and FF5 factor models, along with the corresponding alphas in annualized percentage terms. The estimates are based on all of the S&P 500 constituent stocks and days in the 1993-2014 sample.

	β		Semi β		$\beta^{\mathcal{N}}$		$\beta^{\mathcal{M}^-}$	
Avg ret	4.98		9.76		10.98		7.66	
Std dev	16.57		9.30		16.89		8.00	
Sharpe	0.30		1.05		0.65		0.96	
α	2.19	4.17	8.35	9.68	8.05	10.62	7.76	7.88
	0.87	1.77	6.32	7.38	3.36	4.58	4.17	4.19
β_{MKT}	0.59	0.51	0.30	0.25	0.61	0.52	-0.02	-0.03
	67.61	55.78	64.70	47.90	73.15	56.89	-2.34	-3.44
β_{SMB}	0.30	0.12	0.30	0.21	0.40	0.22	0.20	0.20
	18.10	7.36	33.91	22.61	25.00	13.49	16.08	14.98
β_{HML}	-0.02	0.18	-0.01	0.11	-0.08	0.13	0.05	0.08
	-1.24	10.58	-1.61	11.10	-4.75	7.57	3.82	6.08
β_{MOM}	-0.24		-0.14		-0.22		-0.07	
	-19.53		-22.46		-19.01		-7.31	
β_{RMW}		-0.50		-0.28		-0.53		-0.04
		-22.15		-22.56		-23.70		-2.28
β_{CMA}		-0.35		-0.28		-0.44		-0.13
		-13.21		-19.09		-16.74		-5.95
R^2	58.15	60.26	55.92	59.55	60.59	64.38	6.72	7.42

alpha of 4.17% according to the FF5 factor model, with a t-statistic of just 1.77. The beta strategy generates no significant alpha according to the FFC4 factor model. In contrast, the semibeta strategy, and both of its underlying components, generate large and significant alphas, according to both the FFC4 and FF5 factor models. Annualized alphas range from 7.76% to 10.62%, with the correspondig t-statistics between 3.36 and 7.38.[17] These alphas will, of course, be lower in the presence of transactions costs, and we analyze this in more detail below.

[17]To guard against potential biases in the unconditional alphas arising from temporal variation in conditional betas (see, e.g., Jagannathan and Wang (1996) and Lewellen and Nagel (2006)), we also calculate conditional alphas following the approach of Cederburgh and O'Doherty (2016) (cf. Section II.B). The same general conclusions remain true: the semibeta portfolios result in highly significant positive conditional alphas, while the conditional alphas for the standard β portfolios are always insignificant. The magnitudes of the average conditional alphas for the Semi β and $\beta^{\mathcal{N}}$ portfolios are also very sim-

Looking at the estimated factor loadings, the conventional long-short β portfolio and the $\beta^{\mathcal{N}}$ portfolio exhibit fairly similar FFC4 and FF5 systematic risk exposures. Meanwhile, the estimated factor loadings for the $\beta^{\mathcal{M}^-}$ portfolio are markedly different. In contrast to the other portfolios, the $\beta^{\mathcal{M}^-}$ portfolio is close to market neutral. The FFC4 estimates further suggest that the portfolio contains a higher proportion of value stocks than the other portfolios, while the FF5 estimates point to decidedly lower exposures to the profitability and investment factors than any of the other portfolios. The combined semi β strategy naturally reflects these different risk profiles of the $\beta^{\mathcal{N}}$ and $\beta^{\mathcal{M}^-}$ portfolios.[18]

5.1. Betting On the Competition

To further buttress the superiority of the semibeta portfolio, Table 8 reports the results from analogously constructed up and downside beta, and coskewness and cokurtosis portfolios. Given the pertinent discussion in Ang, Chen and Xing (2006a) and Harvey and Siddique (2000), we consider value-weighted long-short positions based on the top and bottom quintiles betting on β^-, against β^+, against CSK, and on CKT. In parallel to the semibeta portfolios discussed above, as a summary of the value of the two approaches we also consider equal-weighted combinations of the respective measures, denoted "$\beta^- - \beta^+$" and "$CKT - CSK$" in the table.[19]

The top panel reveals that only the β^- and the combined up and downside beta portfolios have Sharpe ratios in excess of the conventional beta sorted portfolio, equal to 0.54 and 0.48, respectively, compared to 0.30 for the traditional beta portfolio. The β^+ and the CSK and CKT portfolios all have small, or even negative Sharpe ratios. Even the two highest Sharpe ratios, however, are substantially below those for the various semibeta-based strategies, presented in Table 7, which range from 0.65 to 1.05.

The lower panel of Table 8 further shows that the FFC4 and FF5 alphas for the CSK and CKT portfolios are all small and statistically insignificant. Only the β^- portfolio

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ilar to the values reported in Table 7, while the average conditional alphas for the $\beta^{\mathcal{M}^-}$ portfolios are marginally lower. Further details of these additional results are available in the Supplemental Appendix.

[18] To further explore these differences in risk profiles, we also calculated industry concentrations. The β and $\beta^{\mathcal{N}}$ portfolios again appear fairly similar along that dimension. Most noticeably, the $\beta^{\mathcal{M}^-}$ portfolio on average invest less in HiTech firms and more in Non-durables than the other two portfolios. Moreover, it is generally less concentrated with lower overall industry exposures. Further details are available in the Supplemental Appendix.

[19] Ang, Chen and Xing (2006a) note that β^+ tends to be positively correlated with β, leading to an ambiguous prediction for the sign of the relationship between β^+ and expected returns. To overcome this, they suggest sorting on the "relative" β^+, defined as $\beta - \beta^+$. We also implemented this approach and found that the resulting portfolio did indeed have a higher Sharpe ratio than the portfolio based solely on β^+. However, the FFC4 and FF5 alphas were small and statistically insignificant. We omit these results in the interests of space.

Table 8: **Betting On the Competition.** The top panel reports annualized descriptive statistics of portfolios formed using up and downside betas, and coskewness and cokurtosis measures. All of the portfolios are self-financing based on value-weighted long-short positions rebalanced daily. The bottom panel reports the time-series regression estimates and Newey-West robust t-statistics for the FFC4 and FF5 factor models, along with the corresponding alphas in annualized percentage terms. The estimates are based on all of the S&P 500 constituent stocks and days in the 1993-2014 sample.

	β^- - β^+		β^-		β^+		CKT - CKS		CSK		CKT	
Avg ret	2.96		7.53		-3.61		-0.16		-1.21		0.54	
Std dev	5.45		15.56		14.67		6.30		7.70		9.55	
Sharpe	0.54		0.48		-0.25		-0.03		-0.16		0.06	
α	2.56	2.93	4.78	6.80	-1.67	-2.94	-0.60	-0.03	-1.44	-1.27	-0.11	0.86
	2.10	2.42	1.90	2.76	-0.69	-1.28	-0.49	-0.02	-0.86	-0.76	-0.06	0.47
β_{MKT}	0.04	0.03	0.55	0.47	-0.47	-0.41	0.13	0.11	0.02	0.01	0.24	0.21
	9.32	6.38	61.82	48.63	-54.89	-45.37	30.25	23.49	2.60	1.94	36.63	28.96
β_{SMB}	0.01	0.01	0.27	0.12	-0.25	-0.10	-0.02	-0.06	0.02	0.03	-0.06	-0.15
	1.01	1.43	16.31	7.10	-15.85	-6.02	-2.58	-7.10	1.87	2.11	-4.96	-11.23
β_{HML}	-0.02	-0.04	-0.04	0.13	-0.01	-0.20	-0.09	-0.06	-0.03	-0.06	-0.16	-0.06
	-2.53	-4.09	-2.07	7.05	-0.38	-11.98	-11.15	-6.98	-2.45	-4.85	-12.20	-4.72
β_{MOM}	0.04		-0.17		0.25		-0.01		0.04		-0.06	
	6.29		-14.09		21.19		-2.51		4.43		-7.07	
β_{RMW}		0.00		-0.43		0.42		-0.10		0.00		-0.21
		-0.30		-18.06		19.01		-9.15		0.26		-12.22
β_{CMA}		-0.01		-0.32		0.30		-0.04		0.03		-0.10
		-1.04		-11.57		11.42		-2.70		1.69		-5.05
R^2	2.67	1.93	52.93	55.64	50.17	50.32	17.43	18.91	0.89	0.58	27.91	30.36

and the combined $\beta^- - \beta^+$ portfolio result in significant alphas, ranging from 2.56% to 6.80%, with t-statistics between 1.90 and 2.76. As one might expect, the estimated risk exposures for the β^- portfolio are fairly similar to the estimates for the semibeta portfolio reported in Table 7. However, in spite of these similarities in risk profiles, the annualized FFC4 and FF5 alphas for the combined semibeta portfolio are both larger and much more strongly significant than the alphas for the β^- portfolio, again highlighting the superior performance of the betting on and against semibeta strategy.

5.2. Longer Holding Periods

The daily rebalancing of the long-short (semi)beta strategies considered in Table 7 may be difficult to implement in practice. Instead, we now consider the performance of the same portfolio strategies based on less frequent weekly and monthly rebalancing, or equivalently longer weekly and monthly holding periods.

Table 9, in particular, shows that moving to weekly rebalancing badly affects the traditional beta strategy, with the the Sharpe ratio falling markedly from 0.30 to 0.08. Moreover, the FF5 alpha that was borderline significant with daily rebalancing becomes small and insignificant. By contrast, the semibeta strategy reported in the second set of columns continues to outperform. The semibeta Sharpe ratio does fall from 1.05 to 0.63, and the annualized alphas are also somewhat lower than the alphas obtained with more frequent daily rebalacing. However, both the FFC4 and FF5 alphas remain strongly significant, with t-statistics of 3.45 and 4.35, respectively.

Table 10 presents the corresponding results based on even less frequent monthly portfolio rebalancing. The Sharpe ratio for the traditional beta strategy falls even further to -0.04, and the corresponding FFC4 and FF5 alphas are both negative, albeit not statistically significantly so. The semibeta portfolio, on the other hand, retains its appeal. The Sharpe ratio of 0.48 is obviously lower than the ratios obtained with daily and weekly rebalancing, and the annualized alphas are also both less than the daily and weekly alphas. Still, both of the alphas remain statistically significant, consistent with the analysis in Section 3.3, and the relationship between semibetas and expected returns holding true at daily, weekly, and monthly horizons.

5.3. Transaction Costs

The results above pertaining to the profitability of the betting on and against semibeta strategy did not take into account the cost of actually implementing the portfolio positions. Such costs are clearly of practical importance. Hence, in this section we explicitly consider the impact of transaction costs.

To better replicate empirical practice we focus on the semibeta portfolio with monthly rebalancing. In parallel to existing work (e.g., Han, 2006; DeMiguel, Garlappi and Uppal,

Table 9: **Betting On and Against Semibetas, Weekly Rebalancing.** The top panel reports annualized descriptive statistics of the betting on and against (semi)beta strategies. All of the portfolios are self-financing based on value-weighted long-short positions rebalanced weekly. The bottom panel reports the time-series regression estimates and Newey-West robust t-statistics for the FFC4 and FF5 factor models, along with the corresponding alphas in annualized percentage terms. The estimates are based on all of the S&P 500 constituent stocks and days in the 1993-2014 sample.

	β		Semi β		$\beta^{\mathcal{N}}$		$\beta^{\mathcal{M}^-}$	
Avg ret	1.18		5.47		7.76		2.40	
Std dev	15.30		8.64		15.50		7.52	
Sharpe	0.08		0.63		0.50		0.32	
α	-1.11	0.71	4.37	5.66	5.54	7.71	2.40	2.84
	-0.46	0.31	3.45	4.35	2.52	3.46	1.46	1.72
β_{MKT}	0.52	0.44	0.25	0.20	0.51	0.43	-0.02	-0.04
	60.65	48.93	55.60	38.70	66.56	49.31	-3.40	-5.54
β_{SMB}	0.30	0.13	0.30	0.22	0.39	0.23	0.21	0.21
	18.43	7.83	35.58	23.57	26.64	14.41	19.12	17.76
β_{HML}	-0.08	0.08	-0.06	0.02	-0.12	0.05	0.00	-0.01
	-4.68	4.54	-7.02	2.26	-8.05	2.96	-0.10	-0.55
β_{MOM}	-0.20		-0.11		-0.20		-0.01	
	-17.14		-17.93		-19.12		-1.85	
β_{RMW}		-0.47		-0.26		-0.48		-0.05
		-21.38		-21.18		-22.57		-3.05
β_{CMA}		-0.26		-0.22		-0.35		-0.08
		-9.80		-14.89		-14.07		-4.48
R^2	51.93	54.10	47.31	51.03	53.19	56.27	7.21	8.44

2009; Liu, 2009), we assume that the transaction costs are proportional to the turnover of the portfolio, with the portfolio turnover computed simply as the sum of the turnover of the long and short legs of the portfolio. Consistent with the total roundtrip transaction cost estimates for large U.S. stocks reported in the literature (see, e.g., the estimates in Novy-Marx and Velikov, 2016), we consider costs of 0.5% and 1%, with 1% providing a realistic upper bound.

Rather than simply trading all the way to the positions that would be "optimal" in the absence of transaction costs, several practically-oriented procedures have been developed in the literature to help mitigate trading costs (e.g., Bertsimas and Lo, 1998; Engle and Ferstenberg, 2007; Obizhaeva and Wang, 2013). These procedures are typically geared towards the specific setting and strategy at hand and can be difficult to realistically

Table 10: **Betting On and Against Semibetas, Monthly Rebalancing.** The top panel reports annualized descriptive statistics of the betting on and against (semi)beta strategies. All of the portfolios are self-financing based on value-weighted long-short positions rebalanced monthly. The bottom panel reports the time-series regression estimates and Newey-West robust t-statistics for the FFC4 and FF5 factor models, along with the corresponding alphas in annualized percentage terms. The estimates are based on all of the S&P 500 constituent stocks and days in the 1993-2014 sample.

	β		Semi β		$\beta^{\mathcal{N}}$		$\beta^{\mathcal{M}^-}$	
Avg ret	-0.64		4.04		3.10		4.33	
Std dev	14.43		8.39		14.72		7.18	
Sharpe	-0.04		0.48		0.21		0.60	
α	-2.48	-1.34	2.96	4.22	1.17	2.85	4.09	4.94
	-1.17	-0.64	2.30	3.24	0.55	1.34	2.83	3.42
β_{MKT}	0.46	0.41	0.23	0.19	0.46	0.39	0.00	-0.02
	62.38	50.99	51.68	36.54	62.55	47.54	0.90	-4.01
β_{SMB}	0.26	0.11	0.31	0.23	0.38	0.23	0.24	0.23
	18.67	7.27	36.10	24.59	26.93	14.85	25.18	22.62
β_{HML}	-0.08	0.05	-0.06	0.01	-0.08	0.08	-0.04	-0.06
	-5.79	3.22	-6.46	1.06	-5.48	5.15	-3.64	-5.76
β_{MOM}	-0.21		-0.10		-0.22		0.02	
	-20.65		-16.43		-21.44		2.22	
β_{RMW}		-0.42		-0.26		-0.45		-0.08
		-21.18		-21.10		-21.89		-5.86
β_{CMA}		-0.17		-0.19		-0.30		-0.08
		-7.03		-12.88		-12.57		-4.91
R^2	49.21	49.52	46.76	50.29	50.83	52.42	11.71	13.32

implement. Instead, we follow the simple-to-implement idea of Garleanu and Pedersen (2013) of only *partially* adjusting the portfolio weights each period.

Specifically, let ω_t^F denote the vector of (fully-adjusted) semibeta portfolio weights in month t. The partially-adjusted portfolio weights for month t are then obtained as:[20]

$$\omega_t^P = \lambda\omega_{t-1}^P + (1-\lambda)\omega_t^F, \tag{18}$$

where the scalar parameter $0 < \lambda < 1$ governs the adjustment rate. While such a partial-

[20]This same approach has also recently been implemented by Bollerslev, Hood, Huss and Pedersen (2018). Garleanu and Pedersen (2013) further suggest changing the "target portfolio" to one that is part-way between the currently fully-adjusted optimal portfolio and the best estimate of next period's optimal portfolio. We have not attempted to implement this additional refinement here.

adjustment approach will help mitigate turnover, it will generally also dampen the signal. As such, the benefits will depend in a complicated way on the interaction between the particular strategy and the transaction costs that are incurred, and the best choice of λ must be determined on a case-by-case basis. We will not attempt to do so here. Instead, in line with similar uses of moving average filters in other situations, volatility estimation included, we simply set $\lambda = 0.95$ and initialize the weights by setting $\omega_1^P \equiv \omega_1^F$.[21]

Table 11 summarizes the performance of the resulting partially adjusted semibeta portfolios. For ease of comparison, the left-most panel presents the results using fully-adjusted portfolio weights with zero transaction costs, corresponding to the second panel in Table 10. The second panel in Table 11 presents the results for the partially-adjusted portfolio also in the absence of transaction costs. As the numbers show, partial adjustment of the weights slightly improves the performance, even before transaction costs: the average return is slightly higher, the volatility is slightly lower, leading to an increase in the Sharpe ratio from 0.48 to 0.59. Likewise, the FFC4 and FF5 alphas are both slightly higher for the portfolio based on partially-adjusted weights. Thus, the partial adjustment not only reduces turnover, it also appears to reduce the "noise" in the semibeta estimates, thereby strengthening the signal, in turn resulting in an overall slightly better performing semibeta portfolio.

The results in the third and fourth panels show the results for the same partially-adjusted semibeta portfolio subject to round-trip transaction costs of 0.5% and 1%, respectively.[22] The addition of transaction costs naturally lowers the average returns and Sharpe ratios. However, even with 1.0% roundtrip transaction costs, the Sharpe ratio of the partially-adjusted semibeta portfolio remains as high as 0.52. The FFC4 and FF5 alphas are also both economically large and statistically significant, with t-statistics of 2.46 and 4.72, respectively.

To help further visualize the timing of the returns, and contrast the performance of the semibeta strategy with the returns based on a traditional long-short beta strategy, Figure 5 plots the cumulative returns from both. In both cases we rely on the partially-adjusted portfolio weights. The solid lines depict the cumulative returns ignoring transaction costs. The dashed lines show the returns that incorporate 1.0% roundtrip transaction costs. As the figure shows, the semibeta strategy perform well throughout most of the

[21]The Supplemental Appendix contains additional results for alternative choices of λ, highlighting the trade-off in signal retention and transaction cost reduction. It also contains additional results for alternative, more involved, procedures based on smoothing the semibeta estimates.

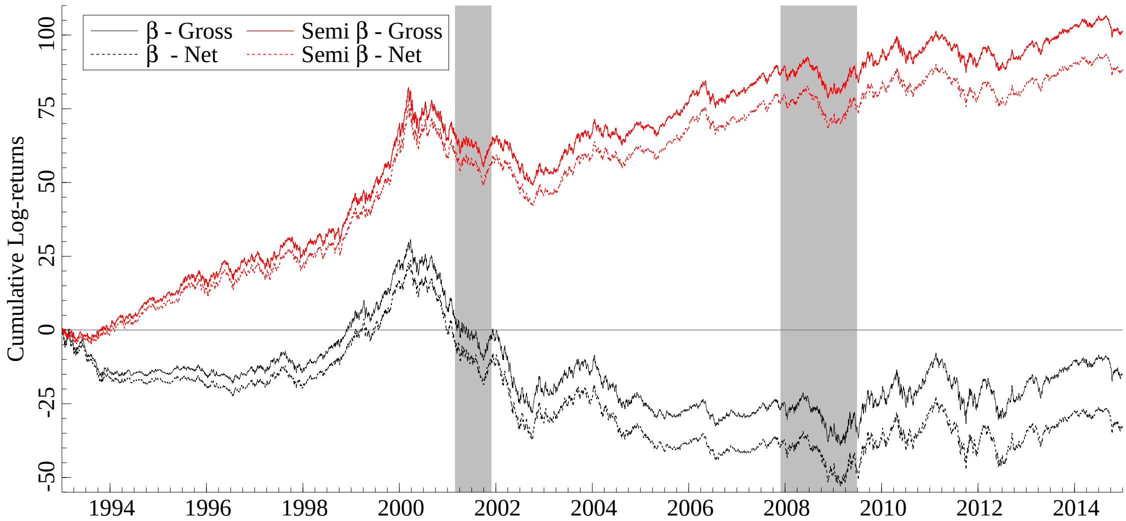
[22]The fully-adjusted semibeta portfolio performs very poorly in the presence of non-trivial transaction costs. With round-trip transaction costs of 1.0%, in particular, the Sharpe ratio equals -0.93, while the FFC4 and FF5 alphas equal -9.56% and -8.26%, respectively. Further details pertaining to this and other portfolios and transaction cost assumptions, are presented in the Supplemental Appendix.

Table 11: **Betting On and Against Semibetas with Transaction Costs.** The top panel reports annualized descriptive statistics for the semibeta portfolios. The bottom panel reports the time-series regression estimates and Newey-West robust t-statistics for the FFC4 and FF5 factor models, along with the corresponding alphas in annualized percentage terms. All of the portfolios are self-financing based on value-weighted long-short positions determined by the combined semibeta strategy rebalanced monthly. T-cost refers to the roundtrip transaction costs. The left panel is identical to the second panel in Table 10 and fully adjusted portfolio weights. The right three panels report the results based on partially-adjusted portfolio weights, as discussed in the main text. The estimates are based on all of the S&P 500 constituent stocks and days in the 1993-2014 sample.

T-cost	0%		0%		0.5%		1.0%	
Adjustment	Full		Partial		Partial		Partial	
Avg ret	4.04		4.62		4.32		4.02	
Std dev	8.39		7.77		7.77		7.77	
Sharpe	0.48		0.59		0.56		0.52	
α	2.96	4.22	3.09	5.31	2.79	5.01	2.49	4.71
	2.30	3.24	3.05	5.33	2.76	5.03	2.46	4.72
β_{MKT}	0.23	0.19	0.25	0.17	0.25	0.17	0.25	0.17
	51.68	36.54	69.25	44.87	69.23	44.85	69.18	44.81
β_{SMB}	0.31	0.23	0.27	0.20	0.27	0.20	0.27	0.20
	36.10	24.59	40.61	27.95	40.58	27.93	40.54	27.89
β_{HML}	-0.06	0.01	-0.13	-0.11	-0.13	-0.11	-0.13	-0.11
	-6.46	1.06	-18.72	-14.51	-18.69	-14.49	-18.66	-14.46
β_{MOM}	-0.10		0.01		0.01		0.01	
	-16.43		2.59		2.59		2.59	
β_{RMW}		-0.26		-0.26		-0.26		-0.26
		-21.10		-27.01		-27.01		-27.00
β_{CMA}		-0.19		-0.20		-0.20		-0.20
		-12.88		-18.16		-18.15		-18.13
R^2	46.76	50.29	52.20	58.90	52.19	58.90	52.16	58.87

sample period. The cumulative returns are also quite high at the end of the sample, even after incorporating 1% transaction costs. By contrast, and consistent with the idea of "betting against beta" advocated by Frazzini and Pedersen (2014), the traditional beta strategy performs poorly over much of the sample period, even before transaction costs are considered. However, rather than betting against beta, the semibeta strategy that bet on and against the semibetas clearly performs better.

Figure 5: **Cumulative Returns for Beta and Semibeta Long-Short Portfolio Strategies**. The figure plots the cumulative percentage returns of long-short strategies based on beta and semibeta sorted value-weighted quintile portfolios. The shaded region represents NBER recession periods. The beta estimates and portfolio returns are based on all of the S&P 500 constituent stocks and days in the 1993-2014 sample.



6. Conclusion

We propose a new additive decomposition of the traditional market beta into four *semi*betas defined by the signed covariation between the market and individual asset returns: $\beta = \beta^{\mathcal{N}} + \beta^{\mathcal{P}} + \beta^{\mathcal{M}^-} + \beta^{\mathcal{M}^+}$. Mean-semivariance asset pricing theory predicts that only two of the semibetas should be priced, with the one accounting for joint negative covariation ($\beta^{\mathcal{N}}$) carrying a positive premium, and the other associated with negative market and positive asset return variation ($\beta^{\mathcal{M}^-}$) carrying a negative premium. Summarizing the results from a variety of specifications in our main empirical analysis, we estimate the annualized risk premium for $\beta^{\mathcal{N}}$ to be around 23%, while the annualized risk premium for $\beta^{\mathcal{M}^-}$ is around -9%, compared with a risk premium for the traditional β of around 4%. These empirical findings rely on realized semibetas calculated from two decades of high-frequency intraday data for the S&P 500 stocks. However, we find similarly strong results with monthly semibetas calculated from daily data for a broader cross-section of stocks over a longer sample period.

We further establish that simple trading strategies that bet on $\beta^{\mathcal{N}}$ and against $\beta^{\mathcal{M}^-}$ leads to Sharpe ratios that are more than double that of the market. Accounting for transaction costs, these same long-short semibeta strategies also produce economically large and strongly statistically significant risk adjusted alphas. In short, do not bet on or against beta, bet on *and* against the "right" semibetas.

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Appendix A. Summary Statistics

Table A.1: **Descriptive Statistics TAQ Sample.** Panel A reports the time series averages of the cross-sectional means, medians and standard deviations. Panel B reports the time series averages of the cross-sectional correlations. The daily realized semibetas, up and donwside betas, coskewness and cokurtosis measures are all constructed from fifteen minutes intraday returns. The sample consists of all S&P 500 constituent stocks from January 1993 to December 2014.

Panel A: Cross-Sectional Summary Statistics

	β	$\beta^{\mathcal{P}}$	$\beta^{\mathcal{N}}$	$\beta^{\mathcal{M}+}$	$\beta^{\mathcal{M}-}$	β^{+}	β^{-}	CSK	CKT	ME	BM	MOM	REV	IVOL	ILLIQ
Mean	0.92	0.72	0.68	0.27	0.25	0.92	0.90	0.00	1.40	15.51	0.50	17.58	1.46	1.63	-16.94
Median	0.83	0.61	0.57	0.16	0.15	0.81	0.80	0.00	1.48	15.51	0.42	11.79	1.09	1.38	-17.06
StDev	1.06	0.49	0.47	0.36	0.34	1.32	1.40	0.40	1.28	1.34	1.09	43.05	9.90	1.01	1.61

Panel B: Cross-Sectional Correlations

	β	$\beta^{\mathcal{P}}$	$\beta^{\mathcal{N}}$	$\beta^{\mathcal{M}+}$	$\beta^{\mathcal{M}-}$	β^{+}	β^{-}	CSK	CKT	ME	BM	MOM	REV	IVOL	ILLIQ
β	1.00	0.67	0.66	-0.33	-0.33	0.78	0.77	0.00	0.65	0.05	-0.02	0.04	-0.01	0.13	-0.06
$\beta^{\mathcal{P}}$		1.00	0.44	0.06	0.18	0.82	0.29	0.30	0.32	-0.12	-0.03	0.02	0.04	0.34	0.10
$\beta^{\mathcal{N}}$			1.00	0.19	0.06	0.28	0.82	-0.30	0.32	-0.13	-0.03	0.02	-0.08	0.36	0.10
$\beta^{\mathcal{M}+}$				1.00	0.38	-0.49	-0.05	-0.16	-0.41	-0.29	-0.01	-0.03	-0.08	0.36	0.26
$\beta^{\mathcal{M}-}$					1.00	-0.05	-0.49	0.16	-0.41	-0.27	-0.01	-0.03	0.04	0.34	0.26
β^{+}						1.00	0.29	0.33	0.49	0.05	-0.02	0.03	0.07	0.11	-0.05
β^{-}							1.00	-0.34	0.49	0.03	-0.02	0.03	-0.09	0.12	-0.05
CSK								1.00	-0.01	0.00	0.00	-0.01	0.02	0.00	0.00
CKT									1.00	0.22	-0.03	0.03	0.00	-0.12	-0.20
ME										1.00	-0.09	0.07	0.03	-0.35	-0.88
BM											1.00	-0.03	0.00	-0.05	0.09
MOM												1.00	0.03	0.00	-0.11
REV													1.00	0.06	-0.03
IVOL														1.00	0.27
ILLIQ															1.00

Table A.2: **Descriptive Statistics CRSP Sample.** Panel A reports the time series averages of the cross-sectional means, medians and standard deviations. Panel B reports the time series averages of the cross-sectional correlations. The daily realized semibetas, up and donwside betas, coskewness and cokurtosis measures are all constructed from fifteen minutes intraday returns. The sample consists of all of the common, non-penny, stocks in the CRSP database from January 1963 to December 2017.

Panel A: Cross-Sectional Summary Statistics

	β	$\beta^{\mathcal{P}}$	$\beta^{\mathcal{N}}$	$\beta^{\mathcal{M}^+}$	$\beta^{\mathcal{M}^-}$	β^+	β^-	CSK	CKT	ME	BM	MOM	REV	IVOL	ILLIQ
Mean	0.98	0.75	0.60	0.21	0.16	1.00	0.96	-0.03	1.28	14.23	0.73	15.00	1.46	1.51	-3.26
Median	0.91	0.67	0.54	0.15	0.10	0.90	0.90	-0.03	1.34	14.15	0.66	11.11	0.99	1.32	-3.27
StDev	0.75	0.45	0.35	0.20	0.17	0.96	1.06	0.29	0.82	1.32	0.49	31.97	8.61	0.81	0.79

Panel B: Cross-Sectional Correlations

	β	$\beta^{\mathcal{P}}$	$\beta^{\mathcal{N}}$	$\beta^{\mathcal{M}^+}$	$\beta^{\mathcal{M}^-}$	β^+	β^-	CSK	CKT	ME	BM	MOM	REV	IVOL	ILLIQ
β	1.00	0.79	0.72	-0.30	-0.29	0.83	0.75	0.04	0.65	0.01	-0.08	0.06	-0.01	0.27	0.27
$\beta^{\mathcal{P}}$		1.00	0.43	-0.08	0.06	0.92	0.34	0.30	0.39	-0.08	-0.08	0.05	-0.04	0.48	0.39
$\beta^{\mathcal{N}}$			1.00	0.09	-0.09	0.34	0.90	-0.28	0.41	-0.11	-0.08	0.05	0.02	0.44	0.40
$\beta^{\mathcal{M}^+}$				1.00	0.23	-0.46	-0.02	-0.13	-0.44	-0.26	-0.01	-0.01	-0.01	0.48	0.35
$\beta^{\mathcal{M}^-}$					1.00	-0.04	-0.50	0.12	-0.44	-0.22	0.01	-0.01	-0.03	0.46	0.30
β^+						1.00	0.31	0.31	0.52	0.03	-0.06	0.05	-0.04	0.24	0.21
β^-							1.00	-0.30	0.54	0.00	-0.07	0.05	0.03	0.18	0.22
CSK								1.00	0.03	0.01	-0.01	-0.02	-0.03	-0.01	0.00
CKT									1.00	0.23	-0.06	0.04	0.01	-0.22	-0.12
ME										1.00	-0.22	0.07	0.02	-0.36	-0.59
BM											1.00	0.00	0.01	-0.04	0.14
MOM												1.00	-0.01	0.00	-0.18
REV													1.00	-0.03	-0.09
IVOL														1.00	0.64
ILLIQ															1.00

Appendix B. Additional Control Variables

- Size (ME). Following Fama and French (1993), a firm's size is measured by its market value of equity: the product of closing price and the number of shares outstanding. Market equity is updated daily, we use its natural logarithm to reduce skewness.

- Book-to-Market (BM). Following Fama and French (1992), Book-to-Market is computed in June of year t, as the ratio of book value of common equity in fiscal year $t-1$ to the market value of equity in December of year $t-1$. Book value of equity is defined as book value of stockholder' equity (SEQ), plus balance-sheet deferred taxes (TXDB) and investment tax credit (ITCB, if available), minus book value of preferred stock (PSTK).

- Momentum (MOM). Following Jegadeesh and Titman (1993), momentum is the compound gross return from day $t-252$ through day $t-21$; i.e. skipping the short-term reversal month. The measure is computed only if a minimum of 100 days is available.

- Reversal (REV). As in Jegadeesh (1990) and Lehmann (1990), the short-term reversal is the return on days $t-20$ to $t-1$.

- Idiosyncratic Volatility (IVOL). Following Ang, Hodrick, Xing and Zhang (2006b), this is calculated as the daily updated standard deviation of the day $t-20$ to $t-1$ residuals from the daily return regression:

$$r_{t,i} - r_t^f = \alpha_i + \beta_i(f_t - r_{t,i}^f) + \gamma_i SMB_{t,i} + \phi_i HML_{t,i} + \epsilon_{t,i},$$

  where $r_{t,i}$ and f_t denote the daily stock and market return, r_t^f denotes the risk free rate, and $SMB_{t,i}$ and $HML_{t,i}$ denote the daily size and value factors for stock i.

- Illiquidity (ILLIQ). Following Amihud (2002), illiquidity for stock i is defined as:

$$ILLIQ_{t,i} = \frac{1}{20} \sum_{j=1}^{20} \left(\frac{|r_{t-j,i}|}{volume_{t-j,i} \times price_{t-j,i}} \right)$$

  We take the natural logarithm to reduce the skewness and the impact of outliers.