The Term Structure of Equity Risk Premia*

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Abstract

We show that the term structure of dividend risk premia and discount rates implied by equity strip yields are downward sloping in recessions and upward sloping in expansions, a finding which is statistically significant and robust across the U.S., Europe, and Japan. Our results are based on the estimation of a regime-switching dividend growth model, which allows us to characterize not just the conditional but also unconditional moments. Our evidence suggests that the claim about downward sloping equity term structure is rejected from the data. This is an important finding as the standard asset pricing models are not in conflict with the new data on dividend strips. In fact, we show that the standard asset pricing models extended with regime-switching dynamics are able to reconcile these facts.

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1 Introduction

A number of recent studies, e.g., Binsbergen, Brandt, and Koijen (2012), Binsbergen, Hueskes, Koijen, and Vrugt (2013), and Binsbergen and Koijen (2017), claim that the term structure of equity risk premia is downward sloping. This fact poses a challenge to the predictions of standard macro-finance models, e.g., Campbell and Cochrane (1999), Bansal and Yaron (2004), and Gabaix (2012), and gives rise to the appearance of research that provides potential explanations (see Binsbergen and Koijen (2017) for the reference). These studies have attracted great attention in the literature due to the recognition of the importance of the equity term structure in evaluating existing asset pricing models. However, as Cochrane (2017) puts it, the "facts" about the term structure of equity risk premia are contentious because the periods over which we have data on dividend strips are short to begin with and several financial and economic turmoils have broken out during those periods which could severly affect the representativeness of the data.

We contribute to the literature by examining the robustness of these "facts" put forward by the earlier studies using similar data and establishing stylized facts about the conditional features of data that are relatively more robust to small sample issues. Most importantly, we make inference on the unconditional features of the term structure of equity risk premia by carefully taking into account the small sample features (including the representativeness) of the data and show that these data features can be completely reconciled with the implications from the standard macro-finance models.

Our analyses rely on traded equity dividend strips from the U.S., Europe, and Japan that cover the period from December 2004 to February 2017 following Binsbergen, Hueskes, Koijen, and Vrugt (2013) and Binsbergen and Koijen (2017).[1] We differ from the earlier studies on two fronts. First, we focus on hold-to-maturity returns which are much more robust to illiquidity compared with monthly holding-period returns and are the economic object of interest as they correspond to discount rates for each dividend strip. The expected hold-to-maturity returns (dividend discount rates) are simply measured by adding the observed equity yields by maturity to the expected dividend growth rates by horizon.

[1]Our proprietary data are provided from a major financial institution active in dividend strips markets. We verify the identical features of the data during periods of overlap with Binsbergen and Koijen (2017) which are from 2003 to 2014.

Next, we explicitly account for different phases of business cycle, which we believe to be a crucial element in characterizing expected growth, and consequently, return dynamics. The regime-switching dividend growth model that we estimate is able to characterize the conditional as well as the unconditional slopes of the term structure of expected growth rate, discount rate, and risk premia. Empirically speaking, the unconditional moments involve calculating the ergodic probability of the state of business cycle which requires a longer span of data. Based on the long run historical frequency of recessions in the U.S., which serves as benchmark, we impose the Markov transtion matrix to imply an ergodic recession probability of 15%. This assumption allows us to make inference about the unconditional moments and evaluate statistical uncertainty in the estimates.[2]

We find that the estimated slope of the dividend term structure varies with the state of the economy. Across all regions, a robust finding is that the term structure of discount rate (expected growth) appears to be upward (downward) sloping in expansions and downward (upward) sloping in recessions.[3] While these conditional aspects of the data are robust (statistically significant), the estimates for their sample average are insignificant and highly sensitive to the frequency of realized recessions in the sample. When recessions are overrepresented in a short sample, which is particulary true for Europe whose in-sample recession probability nearly doubles the ergodic one, the average slope of the dividend term structure can be negative. If, on the other hand, the in-sample recession probability does not differ much from the ergodic recession probability, we find the sample average slope of the dividend term structure to be positive, which is the case for the U.S. and Japan, respectively.

To learn about the unconditional data features, we weigh the conditional data features with the ergodic regime probabilities. Interestingly, we find the unconditional slope of the dividend term structure to be positive across all regions albeit statistically insignificant. We believe that the short sample issues result in a high level of parameter uncertainty and make the inference problem on the unconditional moments challeng-

[2] One nice feature of the regime-switching model is the disconnect between the ergodic and in-sample probability of recessions (which could be higher than the ergodic one in finite samples) which allows us to characterize the finite sample moments as well.

[3] Binsbergen, Hueskes, Koijen, and Vrugt (2013) also find similar patterns in the data. A recent paper by Kapp, Syrichas, and Werner (2019) also show that the term structure of equity is upward sloping in maturity in normal times but steeply downward sloping in economic slowdowns based on a large historical data set of put and call options. At-Sahalia, Karaman, and Mancini (2020) also find similar evidence.

ing. That said, this evidence is still able to reject the claim about downward sloping dividend term structure that the earlier studies argue.

We show that the standard asset pricing models extended with regime-switching dynamics are able to reconcile these stylized facts. For theoretical illustration, we consider extending the consumption-based asset pricing model of Bansal and Yaron (2004) and a reduced-form no-arbitrage model which inherits the key features of Campbell and Cochrane (1999). The important takeaway is that the theoretical model can serve as a laboratory for interpreting the finite sample properties relative to the corresponding population ones. In sum, both theoretical models are able to reproduce the observed conditional dynamics of expected dividend growth and discount rates, and generate an unconditionally upward-sloping term structure of equity risk premia. In spite of the unconditional upward slope, the finite sample (that matches the length of our estimation sample) average of the equity risk premia term structure can slope down when recessions are overrepresented. Our analysis provides a different perspective to the literature that the standard asset pricing models are not in conflict with the new data on the pricing of dividend claims.

Last but not least, we discuss the liquidity of the contracts which could severly affect returns on dividend strips. Our comprehensive dataset includes expanded information on asset liquidity, specifically bid-ask spreads, unavailable in earlier studies. We find that the transaction costs implied by bid-ask spreads are substantial which also increase significantly over maturity. In addition, these contracts have very low liquidity compared to the futures market on the underlying equity index. This liquidity difference makes the comparison both between markets (e.g., returns on the strips relative to the index) and across maturities (e.g., the slope of the term structure) extremely unreliable for the short-term holding-period returns. We show that hold-to-maturity returns are much more robust to the liquidity issues in these markets than are the short-term holding-period returns. We emphasize that this is precisely the reason why our study focuses on hold-to-maturity returns.

We add to the literature that provides evidence on the equity term structure. Binsbergen, Brandt, and Koijen (2012) and Binsbergen and Koijen (2017) use equity options and strips data, respectively, to show the evidence of a downward sloping equity term structure. Based on the cross section of equities, Weber (2018) finds that stocks with short cash flow duration earn on average higher returns than high duration stocks. However, there are more recent papers that express concern about the earlier evidence.

Boguth, Carlson, Fisher, and Simutin (2019), Song (2018), and Schulz (2016) show why inference regarding dividend strips based on equity options is suspect due to, respectively, micro-structure effects, dealer funding costs, and tax issues.[4] Mixon and Onur (2017), Klein (2018), and Gomes and Ribeiro (2018), also urge caution in interpreting short-term holding-period strip returns due to large trading costs and low trading activity. We also argue that there is no reliable inference to be drawn from monthly holding-period returns due to large trading costs and small sample issues.

Our paper is also related to a number of studies developing (equilibrium) models that can generate time varying slopes. Ai, Croce, Diercks, and Li (2018) are based on the interplay of learning about exposure to aggregate shocks, and the time-varying volatility of news regarding future productivity shocks in a production economy. Gormsen (2018) extend the single-priced risk-factor framework of Lettau and Wachter (2007) to two priced risk factors for generating cyclical variation in the term structure of equity returns.[5] Hasler, Khapko, and Marfe (2019) explain through information processing and learning in an endowment economy. Li and Xu (2019) rely on a financial intermediary-based asset pricing model in an endowment economy.

More broadly, our paper relates to Hansen (2013), Backus, Boyarchenko, and Chernov (2017), and Piazzesi, Schneider, and Tuzel (2007) which study implications of various asset pricing models for different cashflow durations.

2 Equity Yields

This section describes simple fundamental relations about equity prices, dividend yields, and dividend strip returns. These relations will be informative for our subsequent empirical analysis. Note that log-transformed variables are indicated with lower case letters.

[4]These papers question the evidence of downward sloping term structure obtained from equity options data. We also point out another potential drawback of using equity options data that it does not provide evidence on the strip curve past two years. It rather relies on comparing the index to the strips to infer the shape of the strip discount rate curve.

[5]Our evidence differs from that of Gormsen (2018) in the definition of expansions and recessions. Gormsen (2018) uses above or below median market price-to-dividend ratio to measure different phases of business cycles. It can overrepresent "bad times" by construction because it classifies 50% of the sample as bad times. Better identification of the different phases of business cycles allows for superior measurement of the conditional moments of expected returns and growth, which we show are economically sensible and consistent with standard models.

2.1 Equity as a portfolio of dividend strips

Let S_t denote the price of a claim on all future dividends. Then, S_t can be written as

$$S_t = \sum_{n=1}^{\infty} P_{n,t}, \tag{1}$$

where $P_{n,t}$ is the price of a claim on dividend at time $t+n$, D_{t+n}. Such a claim is often called "dividend strip" or "zero-coupon equity". We can write $P_{n,t}$ as

$$P_{n,t} = E_t \left[M_{t+n} D_{t+n} \right], \tag{2}$$

where M_{t+n} denotes the stochastic discount factor. The price of this claim tomorrow is $P_{n-1,t+1}$, noting that both the conditioning information and the time to maturity have changed. As a result, we can define the one-period return on the dividend strip with time to maturity n as

$$R_{n,t+1} = \frac{P_{n-1,t+1}}{P_{n,t}}. \tag{3}$$

Note that for $n = 1$, the dividend strip return is equal to $R_{1,t+1} = \frac{P_{0,t+1}}{P_{1,t}}$. The price of a claim on the current dividend is the value of the dividend itself which implies $P_{0,t+1} = D_{t+1}$. For maturities longer than one period, the dividend strip does not have a payout at $t+1$ and, therefore, its return only reflects the change in its price.

Using the no-arbitrage relation, we can always write the return on the asset, R_{t+1}, in terms of its payoff as the sum of tomorrow's dividend and the value of all the future strips divided by the purchase price. Therefore, the one-period equity return can be expressed as a weighted average of dividend strip returns where the weights are given by the fraction of the corresponding dividend strip value in the total equity value:

$$R_{t+1} = \sum_{n=1}^{\infty} \frac{P_{n-1,t+1}}{S_t} = \sum_{n=1}^{\infty} \frac{P_{n,t}}{S_t} \frac{P_{n-1,t+1}}{P_{n,t}} = \sum_{n=1}^{\infty} \frac{P_{n,t}}{S_t} R_{n,t+1} = \sum_{n=1}^{\infty} \omega_{n,t} R_{n,t+1} \tag{4}$$

where $\omega_{n,t}$ is the weight of the maturity n strip in the portfolio of all strips for the asset. This equation establishes that the asset return can be viewed as the weighted average of the strip returns, where the weights are the fraction of the value of the asset for which each strip accounts.

2.2 Relation to dividend futures

Dividend futures are agreements where, at time t, the buyer and the seller agree on a contract price of $F_{n,t}$ which the buyer will pay to the seller at $t+n$, and will receive the realized dividend D_{t+n} in exchange. Hence, the price is agreed upon at t while money changes hands at $t+n$. Let $y_{n,t}$ be the time t zero-coupon bond yield with maturity n. Then, the futures price is given by

$$F_{n,t} = P_{n,t}\ \exp(ny_{n,t}), \tag{5}$$

which can be alternatively written as $P_{n,t} = F_{n,t}\ \exp(-ny_{n,t})$. The dividend strip return then becomes the product of the change in the futures price and the return on the bond with maturity n:

$$R_{n,t+1} = \frac{F_{n-1,t+1}}{F_{n,t}} \frac{\exp(-(n-1)y_{n-1,t+1})}{\exp(-ny_{n,t})}. \tag{6}$$

Using the future price $F_{n,t}$ and current dividend D_t, it is also instructive to define the spot equity and forward equity yield for maturity n respectively as:

$$e_{n,t} = \frac{1}{n} \ln\left(\frac{D_t}{P_{n,t}}\right) \tag{7}$$

$$e^f_{n,t} = \frac{1}{n} \ln\left(\frac{D_t}{F_{n,t}}\right) = e_{n,t} - y_{n,t}. \tag{8}$$

2.3 Hold-to-maturity expected returns

What is the relationship of the strip yield to the expected returns on the strip? Note that we can always rewrite the strip return to maturity as:

$$R_{t+n} = \frac{D_{t+n}}{P_{n,t}} = \frac{D_t}{P_{n,t}} \frac{D_{t+n}}{D_t}. \tag{9}$$

This is the n-period return on the dividend strip with time to maturity n, which relies on the same expression provided in (3). For notational simplicity, R_{t+n} is used instead of $R_{n,t+n}$.

Denote the n period average log return on an n period strip as r_{t+n} and the n period average dividend growth as

$$g_{d,t,t+n} = \frac{1}{n} \ln(\frac{D_{t+n}}{D_t}). \tag{10}$$

Rearranging (9) with (7) and (10), we can re-express the average strip return as

$$r_{t+n} \equiv \frac{1}{n} \ln(R_{t+n}) = e_{n,t} + g_{d,t,t+n}. \tag{11}$$

Therefore, the average expected strip return is

$$E_t[r_{t+n}] = e_{n,t} + E_t[g_{d,t,t+n}] \tag{12}$$

which is the sum of the spot equity yield and the average expected dividend growth rates. (12) is referred to as the hold-to-maturity expected return, which is the conditional discount rate on the strip. Note that $e_{n,t}$ is an inflation neutral quantity, so using an estimate of real growth for $E_t[g_{d,t,t+n}]$ yields an estimate of real discount rate $E_t[r_{t+n}]$, which is the economic object of interest. One can also compute the premium on the hold to maturity expected return by subtracting the maturity-matched real yield, $y^r_{n,t}$, from both sides of (12)

$$E_t[rx_{t+n}] \equiv E_t[r_{t+n}] - y^r_{n,t} = e_{n,t} + E_t[g_{d,t,t+n}] - y^r_{n,t}. \tag{13}$$

We can go further in characterizing the economic informational content of the dividend yields by computing the Sharpe ratio. We can compute the variance of returns conditional on the time t information set:

$$V_t[r_{t+n}] = V_t[g_{d,t,t+n}]. \tag{14}$$

This suggests that the volatility of the contract conditional on time t information is just the expected dividend growth volatility. This allows us to write the annualized conditional Sharpe ratio of the strip conditional on the time t information set as:

$$SR_{n,t} = \frac{E_t[rx_{t+n}]}{\sqrt{V_t[g_{d,t,t+n}]}}. \tag{15}$$

Note that we are not accounting for the half variance term in defining excess returns.

3 Data

3.1 Data source

Dividend futures prices. The data set covers the period from December 2004 to February 2017 at daily frequency and is provided from the proprietary data of a major financial institution that is active in dividend strips markets. The data consists of pricing and liquidity information on dividend futures. Dividend futures contracts typically mature on or after the third Friday of December in the year they mature. On that date the buyer of the contract pays the agreed amount at the initiation of the contract (which we call "the futures price") and the contract seller pays the realized dividends of the index in the year of maturity. The data is the internal pricing information used to trade in these markets by the providing institution and the data delivered to us by the institution contains mid prices for the entire sample and bid and ask prices for a slightly shorter sample—starting in July 2008 for the Eurostoxx, in June 2010 for the Nikkei, and in January 2010 for the S&P 500. The main data set that is used to calculate equity yields and returns corresponds to mid prices on the last trading day of the month.

Daily exchange traded volume and open interest are available for Eurostoxx and Nikkei for the same period as bid and ask prices are available.[6] We also show that for the overlapping periods our data is consistent with that used in Binsbergen, Hueskes, Koijen, and Vrugt (2013) and Binsbergen and Koijen (2017) in terms of descriptive statistics. The S&P 500 contracts are only traded over the counter (OTC) and no comparable public data is available for the spreads on these contracts. Practitioner-oriented work of Mixon and Onur (2017) provide data on the volume traded and contracts outstanding in the OTC portions of each market and show qualitatively consistent evidence on the liquidity of these markets. Section 6 discusses the liquidity data for these markets in detail.

The data set is short (146 months) and it is more practical to analyze the behavior of fixed maturity contracts at monthly frequency. Therefore, we linearly interpolate between futures prices to obtain a finer grid of maturities. For example, we would like to track the futures price with maturity $n = 24$ months. However, we have contracts with maturities of 5, 17, 29, 41, ... months. To obtain a price for the 24-month

[6] For much of the post-2010 period data on exchange traded volume, open interest, bid, and ask prices are also available daily via Bloomberg as well for the Eurostoxx and Nikkei, although we use the institution provided data throughout.

contract, we linearly interpolate between $F_{17,t}$ and $F_{29,t}$, similar to the process used in Binsbergen, Hueskes, Koijen, and Vrugt (2013).[7] When we compute holding period returns we interpolate between the returns themselves, thus obtaining a portfolio return for a portfolio with the same average maturity as the desired contract. This makes our estimates of holding period returns, particularly spread adjusted returns, achievable portfolio returns as in Binsbergen and Koijen (2016).

Zero-coupon bond yields. As can be seen from (6), the calculation of a monthly return on dividend strip requires availability of both futures prices, as well as zero-coupon bond yields with maturities at monthly frequency. In order to ensure a consistent methodology is used in constructing the zero coupon interest rate curve we use the Bloomberg zero curve estimates for all three regions, for the dollar, yen, and for euro-denominated German sovereigns. To extend the data further back than these estimates exist, we use the bond yield data from Gürkaynak, Sack, and Wright (2007) available on the Federal Reserve Board's website. We obtain maturities at monthly frequency by linearly interpolating between available yields.

Dividend growth rates. We measure realized dividends from index returns. We use realized dividend data to construct dividend growth series starting in December 1979 for the U.S. and December 1994 for Europe and Japan, where the extended sample is reduced due to data availability. We provide the time series of the annualized dividend growth rates in the appendix (Figure A-1 for each region).

Recession frequency. Our most dramatic finding in the data is the stark and robust variation of return and growth term structures across the business cycle. To identify different phases of business cycles, we use the NBER recession dates for the U.S., the CEPR recession dates for Europe, and the Cabinet Office recession dates for Japan. We argue that the frequency of recessions in a given short sample can substantially affect the sample mean of the slope of the term structure of equity risk premia. We document that during the sample in which strips data are available, 2005-2017, the frequency of recessions is 14% in the U.S., 17% in Japan, and 29% in Europe. For the U.S., this is relatively close to the long run recession frequency of 14% since 1950s. In contrast, the rate for Europe is at least 50% higher than its long-run average which is about 20%.[8] This strongly suggests that the behavior of any cyclical slope in Europe will be

[7] We emphasize that our results are robust to yield interpolation instead of price interpolation method.

[8] We refer to Figure A-4 for time series evidence.

Table 1: Forward equity yields: Summary statistics

n	1y	2y	3y	4y	5y	5y-1y	(t-stat)
			Panel A: S&P 500				
Sample average	-5.08	-4.55	-4.19	-4.01	-3.88	1.20	(0.68)
Expansion average	-7.15	-5.99	-5.17	-4.77	-4.52	2.63	(2.52)
Recession average	18.19	11.68	6.80	4.54	3.33	-14.86	(-6.71)
			Panel B: Eurostoxx 50				
Sample average	1.94	3.11	2.70	2.28	1.94	-0.00	(0.00)
Expansion average	-2.74	-0.92	-0.12	0.19	0.29	3.02	(2.34)
Recession average	18.10	17.04	12.42	9.48	7.64	-10.47	(-2.94)
			Panel C: Nikkei 225				
Sample average	-1.48	-1.47	-1.86	-1.85	-1.73	-0.26	(-0.13)
Expansion average	-6.26	-5.65	-5.18	-4.65	-4.17	2.09	(2.41)
Recession average	10.47	8.97	6.46	5.14	4.35	-6.12	(-1.57)

Notes: Forward equity yields are constructed by $e^f_{n,t} = \frac{1}{n}\ln(\frac{D_t}{F_{n,t}})$ with $F_{n,t}$ the futures price and D_t the trailing sum of 12 month dividends. We provide the subsample average and standard deviation of the forward equity yields from December 2004 to February 2017 for the three markets, i.e., S&P 500, Eurostoxx 50, and Nikkei 225. We partition the sample into expansions and recessions. t-statistics are based on Newey-West standard errors. Maturities are in annual units.

substantially biased towards its recession mean. We formally address this by conducting various analyses via regime-switching models that explicitly capture different phases of business cycles and account for ergodic probability of recessions. We refer to Section 4 and 5.

3.2 The stylized facts about equity yields and dividends

Table 1 provides the summary statistics of the forward equity yields in these three markets. We first look at the average term structure of equity yields. We find that only the U.S. market seems to show the evidence of upward sloping term structure of equity yields whereas the European and Japanese markets exhibit mildly downward sloping term structure of equity yields.

We then highlight the behavior of equity yields conditional on the state of business cycle, i.e., expansions and recessions. There is remarkable consistency across these three markets. We find that the term structure of equity yields is upward (downward) sloping in expansions (recessions) in all three markets. The absolute magnitude of the spread between 5-year and 1-year maturity equity yields tends to be much larger during recessions than expansions.[9] One can easily deduce from this finding that the sample average of the term structure of equity yields heavily depends on the frequency of recession in the sample.

A longer data set for dividend growth series is available across these three markets. The data start from December 1979 for the U.S. and December 1994 for Europe and Japan, respectively. The common feature shared across three markets is that the realized dividend growth dynamics conditional on recessions are distinctively different from those conditional on expansions. Roughly speaking, the slope of the term structure of dividend growth rates is upward-sloping in recessions while it is downward-sloping or flat during expansions. These facts are robust across different subsamples (not just the sample during which the equity yields are available) particularly for the U.S. To save space, we refer to Figure A-1 for the time series evidence and summary statistics. As we explain before, the periods after 2005 are overrepresented by recessions in Europe (there was no recession identified by the CEPR during 1994-2005). This evidence suggests that the measurement of expected growth in Europe will be substantially more diffcult than other regions and likely to be biased significantly due to the overrepresentation issue. This feature of the data has an important implication for predictability.

4 Forecasting Dividend Growth Rates

We expect that the dynamics of expected dividend growth rates, and consequently, expected returns would be quite different conditional on different phases of business cycle. To show this, we develop a regime-switching model for forecasting dividend growth rates.

4.1 Regime-switching dividend growth dynamics

Suppose that there exists a monthly economic variable x_t that reflects information about the current state S_t, which can predict future dividend growth rates. We can infer the

[9]The downward sloping pattern of the term structure is most notable during the Great Recession, see Figure A-2.

following predictive regression

$$\begin{bmatrix} g_{d,t,t+n} \\ f_t \end{bmatrix} = \begin{bmatrix} \mu_d(S_t) \\ \mu_f(S_t) \end{bmatrix} + \begin{bmatrix} \frac{1}{n}\sum_{j=1}^{n} \epsilon_{d,t+j} \\ \epsilon_{f,t} \end{bmatrix} \tag{16}$$

where $\epsilon_{i,t} \sim iidN(0, \sigma_i^2)$ for $i \in \{d, f\}$. For ease of exposition, define $\varepsilon_{d,t+n} = \frac{1}{n}\sum_{j=1}^{n} \epsilon_{d,t+j}$ from which we can deduce $var(\varepsilon_{d,t+n}) = \frac{1}{n}\sigma_d^2$. A longer-horizon ($Hn$-step-ahead) average dividend growth rates can be expressed by

$$g_{d,t,t+H\cdot n} = \frac{1}{H}\sum_{h=1}^{H} g_{d,t+(h-1)n,t+hn} = \frac{1}{H}\sum_{h=1}^{H} \mu_d(S_{t+(h-1)n}) + \frac{1}{H}\sum_{h=1}^{H} \varepsilon_{d,t+hn}. \tag{17}$$

This exposition is easy to understand for $n = 12$ because H represents a calendar year. In that case, (17) expresses the H-year-ahead average dividend growth rates.

Note that S_t is a discrete Markov state variable that takes on N values with the corresponding transition matrix $\mathbb{P}$

$$\mathbb{P} = \begin{bmatrix} p_{11} & \cdots & p_{N1} \\ \vdots & \ddots & \vdots \\ p_{1N} & \cdots & p_{NN} \end{bmatrix}. \tag{18}$$

Here $p_{ji} = p(S_{t+1} = j | S_t = i)$ and $\sum_{j=1}^{N} p_{ji} = 1$.

Conditional moments. The average Hn-step-ahead conditional expectation and variance[10] can be computed

$$\begin{aligned} E(g_{d,t,t+Hn}|S_t = k) &= \frac{1}{H}\left(\mu_d(k) + \sum_{h=2}^{H} \boldsymbol{\mu_d} \cdot (\mathbb{P}^\top)^{(h-1)n-1} \cdot \mathbb{P}_k^\top\right), \\ V(g_{d,t,t+Hn}|S_t = k) &\approx \frac{1}{Hn}\sigma_d^2, \end{aligned} \tag{19}$$

where $\boldsymbol{\mu_d} = [\ \mu_d(1) \quad \ldots \quad \mu_d(N)\ ]$ and $\mathbb{P}_k = [\ p_{1k} \quad \ldots \quad p_{Nk}\ ]$. These expressions are valid for $H \geq 2$.

[10]The expression for conditional variance is not exact because we are ignoring the variance component associated with uncertainty about $\frac{1}{H}\sum_{h=1}^{H} \mu_d(S_{t+(h-1)n})$.

Unconditional moments. We show that

$$E(g_{d,t,t+Hn}) = E(E(g_{d,t,t+Hn}|S_t = k)) = \boldsymbol{\mu_d} \cdot \frac{1}{H}\big(I_N + \sum_{h=2}^{H}(\mathbb{P}^\top)^{(h-1)n}\big) \cdot \mathbb{P}^* = \boldsymbol{\mu_d} \cdot \mathbb{P}^* \quad (20)$$

where $\mathbb{P}^*$ is the ergodic probability vector derived from $\mathbb{P}^\top$. It can be deduced from (20) that the expression holds for any value of H. Thus, the spread between the average Hn-step-ahead and n-step-ahead growth rates is always zero $\forall H \; E(g_{d,t,t+Hn} - g_{d,t,t+n}) = 0$.

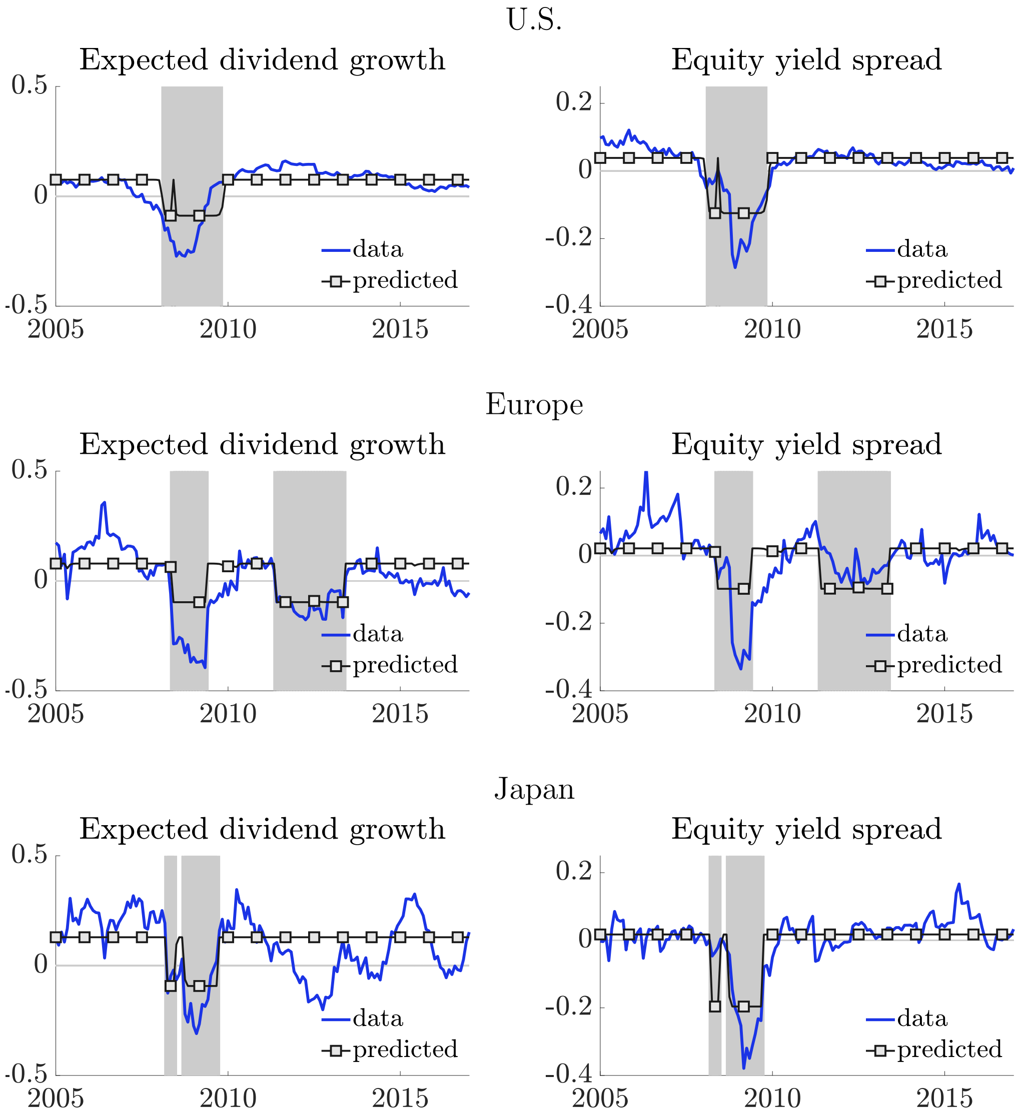
4.2 Estimation results

For parsimony, we assume that there are two states ($N = 2$) in the economy. We pick f_t in (16) to be the spread of the forward equity yield of maturity 5-year and 1-year. This is because we believe that the spread of equity yields is available to investors in real time and is known to predict turning points of business cycles.[11] The estimation sample is from December 2004 to February 2016. We set $n = 12$ which implies that the spread of equity yields today contains information about the average dividend growth rate a year later. We consider $H \in \{1, 2, 3, 4, 5\}$ for empirical illustration. We provide the prior choices and posterior distribution of parameters in the appendix. We use the Hamilton filter to evaluate the likelihood function and implement Bayesian estimation. The transition matrix is estimated with the restriction that the ergodic probability associated with the low growth regime is 15% which is roughly the steady-state probability of recessions in the U.S.

Rather than explaining the parameter estimates, we discuss the model-implied expected dividend growth rate and equity yield spread in Figure 1. The inversion of the equity yield spread strongly predicts drop in dividend growth rates a year later. This finding is consistently documented for the case of U.S. and Europe. For Japan, while the model does capture the big drop in dividend growth rates during the financial crisis, it is not able to predict the drop that happened afterwards mainly because the equity yield spread did not turn negative enough. This can be alleviated if we add more predictor variables, e.g., output or unemployment rates, in (16) to better identify the turning points.

[11] We refer to Figure A-3 for time series evidence. Binsbergen, Hueskes, Koijen, and Vrugt (2013) show that the levels of equity yields are known to predict future dividend growth rates.

Figure 1: Estimation results



Notes: We estimate $g_{d,t,t+1y} = \mu_d(S_t) + \frac{1}{n}\sum_{j=1}^{n} \epsilon_{d,t+j}$ and $f_t = \mu_f(S_t) + \epsilon_{f,t}$ using data from the U.S., Europe, and Japan. The model-predicted lines indicate $\hat{\mu}_d(\hat{S}_t)$ and $\hat{\mu}_f(\hat{S}_t)$, respectively. We identify the state $\hat{S}_t$ if the estimated regime probability is greater than 0.8. The estimation sample is from December 2004 to February 2016. We identify the regime S_t when the 5y-1y forward equity yield spread turns significantly negative. The gray bars indicate the "low growth" regime identified from the estimation. The transition matrix is estimated with the restriction that the ergodic probability associated with the low growth regime is 15%.

Table 2 provides the model-implied average expected dividend growth rates $E_t[g_{d,t,t+n}]$ ("Exp. growth"); the expected discount rate $E_t[r_{t+n}]$ ("Exp. return"); the expected excess return $E_t rx_{t+n}$ ("Premium"); the Sharpe ratio $SR_{n,t}$ ("Sharpe ratio"). We also provide the descriptive statistics for the forward equity yields $e^f_{n,t}$ ("Equity yield") and

Table 2: Model-implied expected dividend growth, prices, and returns: U.S.

	(1) Exp. growth	(2) Exp. return	(3) Premium	(4) Sharpe ratio	(5) Equity yield	(6) RYLD
			Sample average			
1y	5.28	0.33	0.87	0.13	-4.95	-0.54
2y	5.41	1.08	1.43	0.30	-4.34	-0.35
3y	5.49	1.58	1.71	0.44	-3.91	-0.12
4y	5.54	1.91	1.78	0.54	-3.64	0.12
5y	5.58	2.17	1.80	0.60	-3.41	0.37
5y-1y	0.30	1.84	0.93	0.47	1.54	0.91
			Expansion (high-growth) state			
1y	7.57	-0.49	-0.01	0.00	-8.06	-0.48
2y	6.99	0.61	0.92	0.20	-6.38	-0.32
3y	6.65	1.36	1.47	0.38	-5.29	-0.11
4y	6.45	1.75	1.62	0.49	-4.70	0.13
5y	6.31	2.02	1.67	0.56	-4.28	0.35
5y-1y	-1.26	2.52	1.68	0.56	3.78	0.83
			Recession (low-growth) state			
1y	-8.81	6.90	7.81	1.17	15.71	-0.90
2y	-4.28	4.95	5.55	1.18	9.23	-0.60
3y	-1.64	3.62	3.83	1.00	5.25	-0.22
4y	0.01	3.41	3.31	0.99	3.40	0.10
5y	1.09	3.49	2.99	1.00	2.40	0.50
5y-1y	9.90	-3.41	-4.82	-0.17	-13.31	1.40

Notes: We provide the annualized average expected dividend growth rates $E_t[g_{d,t,t+n}]$ ("Exp. growth"); the expected discount rate $E_t[r_{t+n}]$ ("Exp. return"), computed as in (12); the expected excess return $E_t rx_{t+n}$ ("Premium"), computed as in (13); the Sharpe ratio $SR_{n,t}$ ("Sharpe ratio"), computed as in (15); the forward equity yields $e^f_{n,t}$ ("Equity yield") from the data; and the real bond yields $y^f_{n,t} = y_{n,t} - E(\pi)$ ("RYLD") computed from subtracting the sample average inflation rates from the nominal yields.

the real bond yields $y_{n,t}$ ("RYLD") from the data. Table 2 is generated conditional on posterior median parameter estimates. We identify the state if the estimated regime probability is greater than 0.8 for this table. To save space, we only provide the case of U.S. and the results for the remaining regions are provided in the appendix.

We summarize the main findings as follows. The slope of the expected dividend growth is negative (positive) if the economy is in high (low) growth regime because of the mean reversion feature in the regime-switching model, see column (1) of Table 2.

Table 3: Model-implied spread between 5-year and 1-year forecasts

Country	Expected growth 50%	[5%,	95%]	Equity yield 50%	[5%,	95%]	Expected return 50%	[5%,	95%]
Unconditional mean									
U.S.	0.00	N/A		1.70	[-0.22,	3.34]	1.70	[-0.22,	3.34]
Europe	0.00	N/A		0.34	[-0.85,	1.99]	0.34	[-0.85,	1.99]
Japan	0.00	N/A		0.17	[-2.26,	1.63]	0.17	[-2.26,	1.63]
Sample mean									
U.S.	0.00	[-1.88,	1.16]	1.59	[1.03,	2.14]	1.55	[-0.35,	2.92]
Europe	0.08	[-0.82,	0.96]	0.00	[-0.70,	0.80]	-0.11	[-0.90,	0.71]
Japan	0.21	[-0.97,	1.01]	-0.14	[-0.69,	0.46]	0.01	[-1.05,	1.07]
Expansion (high-growth) state									
U.S.	-1.08*	[-1.45,	-0.52]	3.13*	[2.01,	4.66]	2.02*	[0.84,	3.84]
Europe	-1.32*	[-1.86,	-0.88]	3.09*	[1.82,	5.32]	1.86*	[0.48,	4.15]
Japan	-1.48*	[-2.50,	-0.39]	2.74*	[1.47,	4.24]	1.19*	[0.21,	2.83]
Recession (low-growth) state									
U.S.	6.11*	[2.93,	8.23]	-9.35*	[-10.08,	-7.82]	-3.12*	[-6.08,	-0.95]
Europe	7.47*	[4.99,	9.35]	-8.73*	[-9.92,	-6.71]	-1.14*	[-3.06,	-0.12]
Japan	8.37*	[2.18,	14.17]	-15.87*	[-19.76,	-11.23]	-7.08*	[-16.88,	-0.70]

Notes: We provide the posterior distributions for the averages of $E_t[g_{d,t,t+5y} - g_{d,t,t+1y}]$ and $E_t[r_{t+5y} - r_{t+1y}]$. We use $*$ to indicate the statistical significance at the 90% confidence level.

The slopes of the expected return, excess return, and sharpe ratio are positive (negative) in the high (low) growth regime, see columns (2), (3), and (4), respectively. The slopes of the entire period averages of expected return, expected excess return, and sharpe ratio are positive.

To check the statistical significance of the findings, Table 3 provides the posterior 90% credible intervals associated with the selective forecasts, expected return and growth, implied from the model.[12] In addition, we provide the estimate of the spread of equity yields (see (16)) as well. It is important to understand that these results are entirely based on the posterior estimates unlike Table 2 where the equity yields and real rates are computed from the data.[13] Here, we include the results for the U.S., Europe,

[12]Note that the median numbers in Table 3 do not coincide with those in Table 2 because they are sorted based on the entire posterior distribution of parameters and states. More importantly, Table 3 is based on posterior estimates of equity yield spread while Table 2 is based on the realized equity yield spread.

[13]The results are robust to replacing the model-implied equity yield spread with the actual spread.

and Japan. By comparing the estimation results across the three markets, we aim to examine the robustness of our findings. It is interesting to observe that all slopes of the conditional moments are statistically significantly different from zero at the 90% credible level across three regions. While the magnitudes are different, we find that the sign of the conditional slopes for the European and Japanese markets are completely consistent with the U.S. market.

Lastly, but not the least, we examine the model-implied unconditional slope of the term structure of expected return.[14] For this, we weight the respective conditional moments by the ergodic transition probabilities to produce the unconditional mean. Caution is required in interpreting the results because we believe that the short-sample issues result in a high level of parameter uncertainty and make the inference problem on the unconditional moments more difficult. That said, interestingly, we find that the unconditional average of the slope of expected return is positive across all regions albeit not statistically significant. This evidence is important because it rejects the claim about downward sloping dividend term structure that the earlier studies argue.

5 Asset Pricing Models with Regime Switching

Motivated from the previous empirical findings, we discuss the theoretical implications of the LRR model and a no-arbitrage model extended with regime switching cash flows dynamics. The no-arbitrage model inherits the key features of the Campbell and Cochrane (1999)'s habit-formation model, which is one of the leading asset pricing models. We aim to show whether the standard asset pricing models extended with regime-switching dynamics are able to produce the conditional dynamics of expected dividend growth and discount rates consistent with the data documented in the previous section. We emphasize that these regime-switching models will still preserve the core implication of the standard models that risks unconditionally increase with horizon. It is important to understand that we use the model to guide the readers how to interpret the model-implied moments when there are small sample issues.

[14] As explained in (20), the unconditional average of the slope of the expected dividend growth is zero in the model.

5.1 Cash flow dynamics

The joint dynamics of monthly consumption and dividend growth are

$$\begin{aligned}\Delta c_t &= \mu(S_t) + x_t + \sigma_c \eta_{c,t}, \quad \eta_{c,t} \sim N(0,1), \\ \Delta d_t &= \bar{\mu} + \phi(\Delta c_t - \bar{\mu}) + \sigma_d \eta_{d,t}, \quad \eta_{d,t} \sim N(0,1), \\ x_t &= \rho x_{t-1} + \sigma_x(S_t)\epsilon_t,\end{aligned} \tag{21}$$

where $\bar{\mu}$ is the unconditional mean of consumption growth, x_t is the persistent component of consumption growth, and S_t is a discrete Markov state variable that takes on two values $S_t \in \{1, 2\}$. We assume $\mu_1 > \mu_2$ without loss of generality and indicate $S_t = 1$ an expansion state and $S_t = 2$ a recession state.

The model-implied average expected dividend growth is

$$E_t[g_{d,t,t+n}] = \frac{1}{n} E\Big[\sum_{i=1}^{n} \Delta d_{t+i} | S_t\Big]. \tag{22}$$

The agent in the model observes the current regime, S_t, and makes forecast of future regime, S_{t+i}, based on the transition matrix

$$\mathbb{P} = \begin{bmatrix} p_1 & 1-p_1 \\ 1-p_2 & p_2 \end{bmatrix}. \tag{23}$$

It is easy to understand from (22) that the path of $E_t[g_{d,t,t+n}]$ significantly depends on the current state S_t. To provide a preview, the slope of the expected dividend growth is negative (positive) if the economy is in expansion (recession). This is illustrated in Figure 2. Later, we argue that the model characterizes an important aspect of the data especially when it comes to the short-horizon forecasts.

5.2 Stochastic discount factor

We consider two types of stochastic discount factors (SDF): one with preference-based and the other in reduced-form. Since the asset pricing implications from both SDFs are qualitatively identical, we use the preference-based one as our benchmark SDF.

5.2.1 The long-run risks model of Bansal and Yaron (2004)

The log stochastic discount factor is

$$m_{t+1} = \theta \ln \delta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1) r_{c,t+1}, \tag{24}$$

where δ is the time discount rate; γ is risk aversion; ψ is the intertemporal elasticity of substitution; $\theta = (1-\gamma)/(1-1/\psi)$; $z_{c,t}$ is the log price to consumption ratio; and $r_{c,t+1} = \kappa_{0,c} + \kappa_{1,c} z_{c,t+1} - z_{c,t} + \Delta c_{t+1}$ is the log return on the consumption claim. The risk-free rate is implied from the SDF.

5.2.2 A no-arbitrage model

The log stochastic discount factor follows

$$m_{t+1} = -r_{t+1} - \frac{1}{2}\lambda(S_{t+1})^2 - \lambda(S_{t+1})\epsilon_{t+1} \tag{25}$$

with an exogenously specified risk-free rate, r_{t+1}. We set $r_{t+1} = \bar{r}$ so that the risk-free rate does not depend on the state. The market price of risk depends on the state $\lambda(S_{t+1})$. We impose that $\lambda(1) < \lambda(2)$ in the spirit of Campbell and Cochrane (1999), e.g., higher risk aversion in bad times. The joint restriction of $\mu(1) > \mu(2)$ and $\lambda(1) < \lambda(2)$ allow us to match the recession and expansion dynamics of both growth and returns while preserving the implications of standard models. For example, short term risk goes up in bad states then gradually comes down over time, generating the conditional features in the data we have documented before. Unconditionally, the model produces discount rate term structures in which risks rise with horizon as implied by either the LRR or habit-formation models. The details are provided in the appendix.

5.3 Price to dividend ratio of the zero-coupon equity

The price of zero-coupon equity is $P_{n,t} = Z_{n,t} D_t$. In the economy of (21) with the SDF of (24) or (25), we can conjecture that the log price to dividend ratio of the zero-coupon equity $z_{n,t}$ depends on the regime and persistent growth component, i.e., $z_{n,t} = z_{n,0}(S_t) + z_{n,1}(S_t) x_t$. Exploiting the law of iterated expectations, we can solve for $z_{n,t} = \ln E\big(E\big[\exp(m_{t+1} + z_{n-1,t+1} + \Delta d_{t+1})|S_{t+1}\big]|S_t\big)$. The detailed derivation is provided in the appendix.

5.4 Hold-to-maturity expected excess return

Define the m-month holding period return of the n-month maturity equity by

$$R_{n,t+m} = \frac{Z_{n-m,t+m}}{Z_{n,t}} \frac{D_{t+m}}{D_t}. \tag{26}$$

The average log expected return is

$$E_t r_{n,t+m} = \frac{1}{m} E_t[z_{n-m,t+m} - z_{n,t} + \sum_{i=1}^{m} \Delta d_{t+i}]. \tag{27}$$

When $m = n$, the equation (27) becomes the hold-to-maturity expected return of the n-month maturity equity. We show in the appendix that

$$e_{t,n} = -\frac{1}{n} z_{n,t} \tag{28}$$

and how to calculate $E_t[g_{d,t,t+n}]$, $z_{n,t}$, and $y^r_{n,t}$. It is then straightforward to compute (12), (13), and (15).

5.5 The model-implied term structure of equity risk premia

We calibrate the model to match the U.S. data moments for consumption and dividend growth rates and market equity premium. The calibrated parameters for consumption and dividend growth are standard (the expansion state is associated with higher mean and lower volatility). The calibrated parameters for preferences lead to equity premium of 4.7% in the expansion and 12.2% in the recession state, respectively. We also calibrate the regime transition probabilities to match the long run frequency of recessions across regions, which is about 15%. The unconditional average equity premium implied from the model is around 6%.

Population conditional moments. Note that the population conditional moments will be function of consumption growth component x_t and the regime (discrete state) S_t. For ease of illustrating the model implications, we set $x_t = 0$ for simplicity and highlight the role of regime S_t.[15] Panel A of Figure 2 provides the model-implied (7), (10), (12), and (13). We summarize the model implications as follow:

[15]This means that the risk associated with x_t is priced, but we assume that the realization is $x_t = 0$ for graphical illustration.

1. The slope of the expected dividend growth is negative (positive) if the economy is in expansion (recession).[16]

2. The slope of the term structure of the risk-free rate is negative (positive) if the economy is in expansion (recession).[17]

3. The slope of the equity yields, dividend discount rates, and dividend risk premia is positive (negative) if the economy is in expansion (recession).

We match the key conditional expected growth rate and dividend discount rate slope features documented in the data in Tables 1, 2, and 3.[18]

Population unconditional moments. Panel B of Figure 2 provides the population unconditional moments of the term structure of the equity risk premia and expected growth in the model. The model generates unconditional term structure of discount rate and equity risk premia in which macroeconomic risks rise with horizon.

Small sample moments. Empirically speaking, the unconditional price and return moments can be hard to measure, since they involve calculating the unconditional probabilities of the state of the business cycle. The results depend on the sample over which the probabilities are calculated. The small sample bias is especially relevant in this context. Hence, there is substantial risk of misinterpreting the results if sample averages are used to estimate unconditional means without attention to the frequency of recessions.

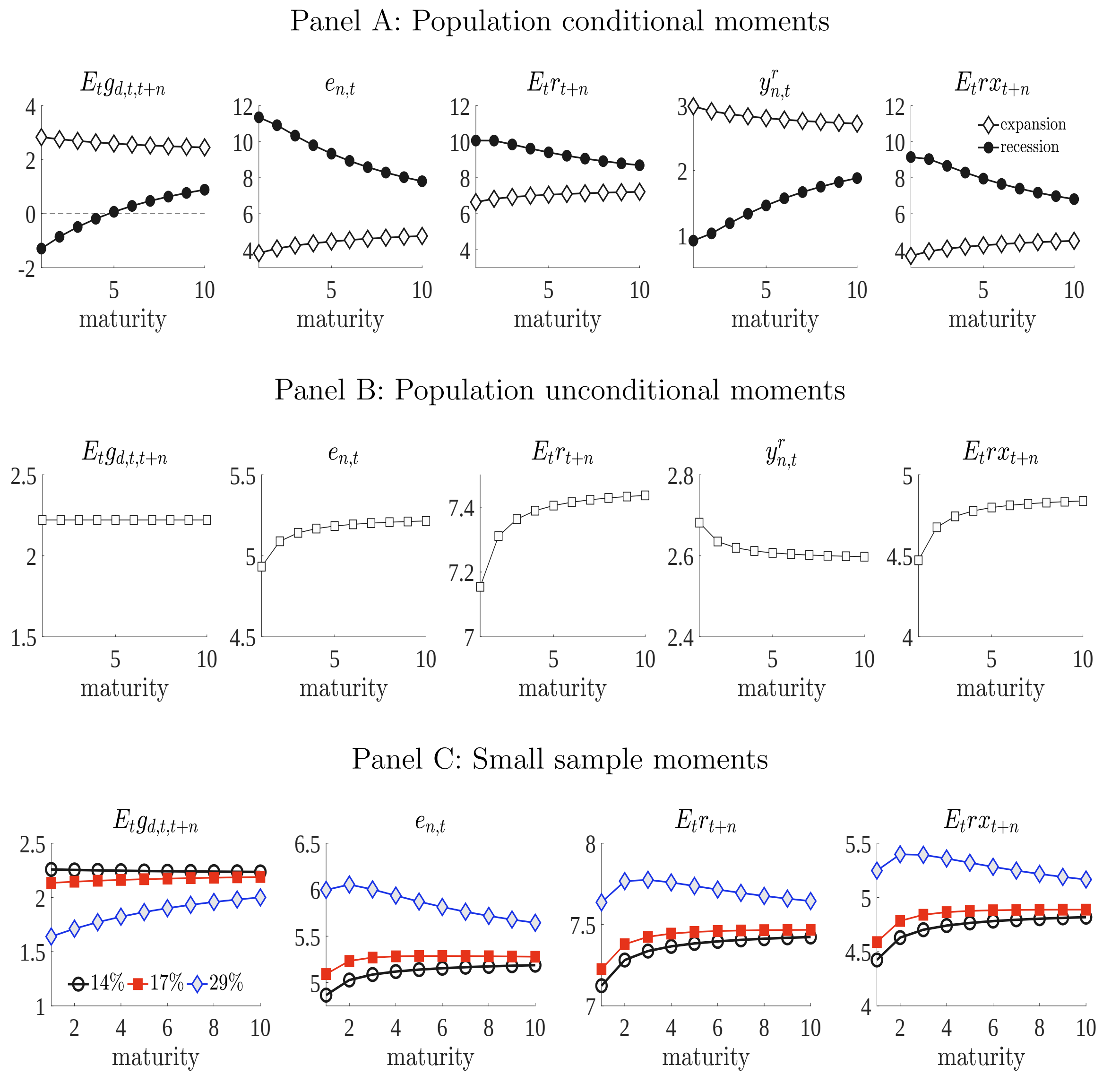
To show this, we average the conditional moments implied from the model across the two states based on recession frequency that is different from the steady state probability of recession (around 15%). We proceed with three cases of recession frequency based on the realized short sample recession frequency which are 14%, 17%, and 29% for the U.S., Japan, and Europe respectively. Panel C of Figure 2 provides the small-sample averages of the term structure of the equity risk premia and expected growth among

[16]One thing to emphasize is that the expected dividend growth rate is much lower in recession even though we calibrated $\mu_2 = 1.2\%$ (annualized). What matters is whether the current economic state is below the long-run mean $\bar{\mu}$ or not.

[17]The slope of the term structure of the risk-free rate under the reduced-form SDF in (25) is flat (by construction)

[18]We calibrated the persistence of the consumption growth component ρ to 0.6 which is lower than the value typically used in the literature. For higher values of $\rho \approx 1$, there will be no distinction between the two population conditional moments and the model implications will be qualitatively identical to those from the original Bansal and Yaron (2004). That is, the slope of the term structure of equity risk premia is always positive.

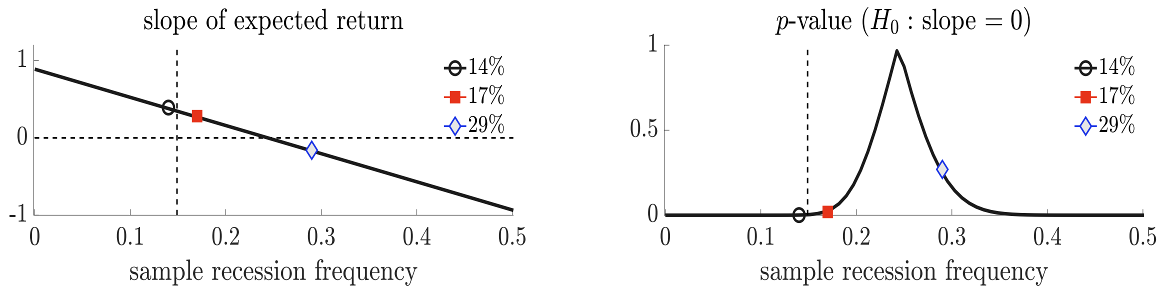
Figure 2: Model-implied moments



Notes: While we use a monthly model to compute these components, parameter calibration is reported in annualized term. Panel A and B - We examine the case of $x_t = 0$. Panel C - In the data, the sample recession frequency is 14%, 17%, and 29% of the sample from December 2004 to February 2016 for the U.S., Japan, and Europe respectively. Motivated from this, we average the moments implied from the model across the two states with the probabilities obtained from the data to compute various sample averages.

others based on three cases of recession frequency. One could clearly observe the pattern of downward sloping term structure of discount rate (equity risk premia) when recession frequency is much greater than the model steady-state recession frequency, as in the case of Europe. If the small sample recession frequency is below the model steady state

Figure 3: The 5y-1y slope of expected return from simulation



Notes: We simulate data $T = 11$ years and repeat the simulation $N = 10,000$ times. Thus, we have a panel $N \times T$ of the model-implied slope of expected return. We compute the average probability of recession for each time series. We then sort this N dimensional vector of recession probability from low to high. Starting from low to high recession probability, we report the sample average slope of expected return (first panel). We also test the null hypothesis that the slope of expected return is zero (second panel). The dashed-lines indicate the steady-state frequency of recession in the model which is around 15%. We compare three cases of recession frequency motivated from the data: The recession periods are 14%, 17%, and 29% of the sample from December 2004 to February 2016 for the U.S., Japan, and Europe respectively.

recession probability, then the term structure of discount rate and equity risk premia are strongly upward sloping, as in the case of U.S. Note that Japan is in between these two cases.

Figure 3 pursues this idea more formally. We simulate the time series of economic state (recession and expansion) from the model that matches the length of our prediction sample, which is roughly eleven years ($T = 134$ months). Conditional on the economic state at each time t, we pick the corresponding moments of expected return for the entire maturity from Panel A of Figure 2. We repeat the exercise by $N = 10,000$ times to provide variation in the realization of recession states. Thus, we have an $N \times T$ panel of the model-implied slope of expected return. Next, we compute the realized recession frequency for each simulated time series and sort the set of time series on the realized frequency. Starting from low to high recession probability, we report the sample average slope of expected return (first panel). We also test the null hypothesis that the slope of expected return is zero (second panel).[19]

We cannot reject the null hypothesis, i.e., slope is zero, if recession frequencies are around 19%-33%. In this range, the corresponding p-values are greater than 10% at least. Once the recession frequency falls below 19%, the p-value approaches zero and

[19] If the slope of the term structure of real rates is nearly flat in the model, this is equivalent to testing the slope of equity risk premia.

the model-implied slope of expected return is statistically strongly positive. In contrast, if recession frequency is greater than 33%, the opposite is true. In the data, we find that the recession periods are 14%, 17%, 29% for U.S., Japan, and Europe, respectively. From the perspective of our model, the U.S. and Japan seem to show the evidence of upward-sloping term structure of expected return (discount rate) which is statistically significant with the corresponding p-value of 0.00 and 0.04, respectively. On the opposite end of the spectrum, Europe shows the evidence of downward-sloping term structure of expected return albeit statistically insignificant with the p-value of 0.27. Overall, this simulation exercise helps rationalize the data features we document in the previous section.

5.6 Summary of the model

We examined two types of standard asset pricing models extended with regime-switching growth dynamics. These models produce conditional dynamics of expected growth and dividend discount rates consistent with the data and generate unconditional discount rate term structures in which risks rise with horizon. We show that, in spite of the unconditional upward slope, in finite samples the sample average dividend discount rate term structure can slope down when recessions are overrepresented, a key feature of the data. Based on these results we conclude that the implications of the standard asset pricing models are entirely consistent with the strip data. The key takeaway is to understand the perils of making inferences on the unconditional slope of the term structure based on a short sample in which recessions are overrepresented.

6 Liquidity in Short-Term Holding Period Returns

Some of the literature on dividend strips relies on monthly holding-period returns at the mid of bid and ask prices to estimate the term structure of dividend discount rates. We show that monthly dividend strip holding period returns are poorly measured, highly sensitive to spreads, and are smaller than average spreads almost universally. Based on these returns there are two claims (see Binsbergen and Koijen (2017))—that the holding period returns decline with maturity and are below the index (implying a downward slope) and that Sharpe ratios follow a similar pattern. In Table 4, below, we replicate this evidence from our dataset and show that these returns are measured with large standard errors at mid prices. More importantly, we show that holding-period returns are contaminated by severe illiquidity as reflected in large bid-ask spreads. Indeed, spreads are larger than monthly returns, making these holding period returns unreliable for measuring the underlying discount rates of economic interest.

6.1 Monthly holding-period returns at mid prices

First, we reproduce the evidence on average monthly holding-period returns at mid prices, the focus of earlier work. We examine whether (i) the index is above or crosses the term structure of dividend strip returns and (ii) returns and Sharpe ratios rise with maturity. Table 4 displays the point estimates for returns in excess of the index and strip Sharpe ratios for the S&P 500, Eurostoxx 50, and Nikkei 225. Table 4 shows that the point estimates of dividend strip returns are below the monthly index for the S&P 500 but are above the monthly index for Nikkei 225 and Eurostoxx 50. Strip returns slope up in the U.S. and Japan and down in Europe. But, the small t-statistics associated with mean estimates suggest that the results are largely insignificant. We find similar results for the strip Sharpe ratios. As emphasized by Cochrane (2017), there is no reliable inference to be drawn from monthly holding-period returns (at mid prices) because they are both poorly measured and are based on a short sample. The fact that average spreads are universally larger than returns in these markets casts a deeper pall on the reliability of the holding period return evidence, as we discuss next.

6.2 Illiquidity in the short-term holding-period returns

Recent work by Mixon and Onur (2017), who document the illiquidity of the strip market and its causes, and analysis in news media, e.g., Klein (2018), both suggest that these markets are highly illiquid and are dominated by liability hedging at long horizons. Motivated by these studies, the availability of our novel dataset of bid-ask spreads for the OTC S&P 500 strip market allows us to directly examine the implications of spreads in these markets for monthly holding-period and hold-to-maturity returns. Note that our bid-ask data are the spread faced by a large financial institution trading in these markets, the data provider for the remaining data. The sample of spread data is shorter for all regions than the mid price data, starting around 2008-2010 across regions.

Table 4: Dividend strip returns less market returns: One-month holding period returns

Maturity	1-M	2-M	3-M	4-M	5-M	M
Panel A: S&P 500						
Average	-4.32	-2.31	-0.94	-0.06	1.33	6.58
(t-stat)	(-1.24)	(-1.03)	(-0.40)	(-0.02)	(0.83)	
Stdev of Strip	12.07	11.39	11.53	11.87	12.55	14.10
Sharpe ratio of Strip	0.13	0.27	0.37	0.41	0.47	0.47
Panel B: Eurostoxx 50						
Average	2.70	2.90	2.10	1.89	2.05	4.28
(t-stat)	(0.66)	(0.86)	(0.62)	(0.58)	(0.65)	
Stdev of Strip	15.22	14.41	14.34	14.23	14.19	17.14
Sharpe ratio of Strip	0.46	0.31	0.25	0.23	0.24	0.25
Panel C: Nikkei 225						
Average	2.29	2.25	3.82	5.40	6.28	7.63
(t-stat)	(0.45)	(0.42)	(0.72)	(1.00)	(1.38)	
Stdev of Strip	17.40	19.86	20.17	19.54	18.99	19.79
Sharpe ratio of Strip	0.82	0.47	0.48	0.53	0.55	0.38

Notes: The time series of dividend strip returns less the market return $R_{M,t+1}$ is calculated as $R_{n,t+1} - R_{M,t+1} = \frac{F_{n-1,t+1}}{F_{n,t}} \frac{\exp(-(n-1)y_{n-1,t+1})}{\exp(-ny_{n,t})} - R_{M,t+1}$ with $F_{n,t}$ the futures price for maturity n and $y_{n,t}$ the risk free zero coupon bond yield for maturity n. Returns for maturities not currently traded are constructed from portfolios of returns on traded maturities. Means, standard deviations, and Sharpe Ratios are annualized for monthly hold periods. Results are reported for the period from January 2005 to February 2017. The asset is the monthly total return on the index used to settle the contract. t-statistics are based on Newey-West standard errors. Maturities are in annual units.

To estimate the magnitude of transaction costs relative to our historical return estimates, we compute the bid-ask spread as follows:

$$BA_{n,t} = \frac{F_{n,t}^{ask} - F_{n,t}^{bid}}{0.5 \cdot (F_{n,t}^{ask} + F_{n,t}^{bid})}. \tag{29}$$

Table 5 reports average bid-ask spreads for fixed maturity contracts. It is evident that the bid-ask spreads are very large in all three markets and strongly increase in both mean and volatility with horizon in the Eurostoxx and Nikkei markets. While short run Eurostoxx strips trade in the most liquid of these markets, the differences in liquidity by horizon are particularly large outside the U.S., increasing by a factor of between 3 and

Table 5: Dividend strip bid-ask spreads

n	1y	2y	3y	4y	5y	Asset
Panel A: S&P 500						
Sample average	1.31	1.60	1.78	2.06	2.26	0.04
Standard deviation	0.57	0.68	0.74	0.77	0.84	0.05
Panel B: Eurostoxx 50						
Sample average	0.45	0.86	1.43	2.59	3.73	0.04
Standard deviation	0.46	0.91	1.40	2.98	4.89	0.02
Panel C: Nikkei 225						
Sample average	1.42	2.39	2.98	3.41	4.63	0.56
Standard deviation	0.99	2.02	2.36	2.16	2.47	0.47

Notes: The period starts in July 2008 for the Eurostoxx, in June 2010 for the Nikkei, and in January 2010 for the S&P 500, and ends in February 2017 for all. The time series of bid-ask spreads for dividend futures is calculated as $BA_{t,n} = \frac{F^{ask}_{n,t} - F^{bid}_{n,t}}{0.5 \cdot (F^{ask}_{n,t} + F^{bid}_{n,t})}$ with $F^{ask}_{t,n}$ the dividend futures ask price for maturity n and $F^{bid}_{n,t}$ the bid. Spreads are presented in percentages (multiplied by 100). Results are reported using monthly data. The period starts in July 2008 for Eurostoxx, in June 2010 for Nikkei, and in January 2010 for S&P 500, and ends in February 2017 for all. The asset or index is the nearest to maturity Chicago Mercantile Exchange futures contract on the same index in local currency (Eurex for the Eurostoxx 50). Maturities are in annual units.

8 for spread mean and 2.5-9 for spread volatility, from 1 to 5 years. Note that strongly increasing spreads and spread volatility with horizon will particularly contaminate evidence comparing the long and short end of the term structures of expected returns and Sharpe ratios.

Importantly, bid-ask spread means are dramatically larger than monthly strip returns at all horizons, and spread variance is on the same order of magnitude as return variance for most markets at all but the shortest horizons. Further, all of these markets are substantially less liquid than the counterpart markets for short run index futures on the same indexes.[20] In comparison, index returns are relatively well measured, even accounting for spreads. Liquidity differences contaminate comparisons both between markets and across maturities. Drawing conclusions on relative index and strip returns and Sharpe ratios in the presence of such large illiquidity is highly unreliable.

The illiquidity in longer dated contracts makes it difficult to justify drawing strong

[20] Chicago Mercantile Exchange E-Mini futures for the S&P 500 and Nikkei, Eurex futures for the Eurostoxx.

conclusions about the relative economic risk of dividend strips by horizon based on the monthly holding-period returns. To show why, we estimate what the actual return would be if one were to buy the dividend strip at the ask and sell at the bid on a monthly basis. We present the results of this analysis in Table 6. Note that the bid-ask adjusted returns at the monthly horizon are negative for all three markets, and massively so for the longer maturity contracts. All of these achievable returns are well below the returns on the asset. Given that transaction costs swamp the returns at short holding horizons, the marginal investor in these contracts is unlikely to evaluate the contract at these horizons and therefore the economic information about their discount rates is not reflected in the monthly return information.[21]

One way to mitigate the impact of large transaction costs is to increase the holding period. However, we find that increasing the holding period to 12 months does not resolve these issues at any but the shortest maturities. The discrepancy between returns and returns net of transaction costs is still on the same order of magnitude as the mean return in all three markets. For longer maturity contracts, it is still difficult to justify the assumption that the marginal investor intends to give up between 30% and all of the return on the contract by trading it at a 1 year horizon.

Mixon and Onur (2017) reinforce the view that these markets are highly illiquid using trading volume and open interest information. They show that across exchanges and OTC markets, dividend futures trade in markets orders of magnitude smaller than their associated index futures, both in terms of notional and contracts outstanding.[22] Both Mixon and Onur (2017) and Klein (2018) indicate that the issuers of structured notes are long these products to reduce their exposure to dividends. This suggests buy-and-hold liability hedging could be driving a considerable volume of trade.

Given the bid-ask spreads and Mixon and Onur (2017)'s evidence, we mitigate the effects of illiquidity consistent with contracts held by hold-to-maturity investors. These investors would buy the contract at the ask price then receive the dividend growth at maturity. This strategy accurately reflects the returns achievable by investors while

[21] Investors may implement trading strategies that mitigate the impact of spreads but nevertheless these spreads reflect the considerable transaction costs that any investor would face.

[22] For instance, the Eurostoxx dividend futures market, the largest strip market, is less than 10% of the size of the associated index futures market by notional. In addition Klein (2018) claims that U.S. domiciled traders could not invest in Eurostoxx markets until 2017. Note that Mixon and Onur (2017)'s data is exclusively from 2015, when this market was relatively mature compared to the majority of the sample period, thus the liquidity of these markets was likely substantially smaller for most of the sample for which we have strip data.

Table 6: Average dividend strip spread-adjusted returns

Return		Maturity					Asset
		1y	2y	3y	4y	5y	
Panel A: S&P 500							
1-month-hold	mid-price	2.91	6.76	8.45	9.34	9.94	12.43
	bid/ask spread-adj.	-12.84	-12.61	-13.07	-15.51	-16.95	-
hold-to-maturity exp.	mid-price	-0.44	-0.38	0.11	0.48	0.74	-
	ask-price	-1.02	-0.75	-0.19	0.22	0.54	-
Panel B: Eurostoxx 50							
1-month-hold	mid-price	7.06	6.40	5.32	4.50	4.60	3.81
	bid/ask spread-adj.	1.00	-4.49	-11.35	-25.96	-38.96	-
hold-to-maturity exp.	mid-price	2.49	3.66	2.81	2.04	1.42	-
	ask-price	2.13	3.25	2.52	1.75	1.06	-
Panel C: Nikkei 225							
1-month-hold	mid-price	8.71	12.78	15.76	17.73	18.77	13.68
	bid/ask spread-adj.	-8.62	-16.26	-19.99	-23.38	-36.67	-
hold-to-maturity exp.	mid-price	0.57	2.04	2.86	3.59	4.13	-
	ask-price	-0.16	1.48	2.35	3.11	3.59	-

Notes: The period starts in July 2008 for the Eurostoxx, in June 2010 for the Nikkei, and in January 2010 for the S&P 500, and ends in February 2017 for all. The asset is the monthly total return on the index used to settle the contract, less the spread on the nearest to maturity futures contract where appropriate. Dividend strip returns are computed as in (6) and spread adjusted dividend strip returns correspond to

$$R_{h,t+k} = \left(\frac{F^{bid}_{n-k,t+k}}{F^{ask}_{n,t}} \frac{\exp(-(n-k)y_{n-k,t+k})}{\exp(-ny_{n,t})} \right)^{1/k} - 1,$$

where results are reported for maturities $n = 1, ..., 5$ years, and holding period of k=1 month. Returns for maturities not currently traded are constructed from portfolios of returns on traded maturities. Means and standard deviations are monthly annualized percentages. Maturities are in annual units.

mitigating the impact of transaction costs to the greatest extent possible. We report the hold-to-maturity expected returns, averaged over the sample with bid-ask data using the purchase price as the last price and the ask price in the last two lines of each panel of Table 6. Within the sample for which spreads are available, spread-adjusted expected returns also reflect the same qualitative and quantitative patterns as the expected returns unadjusted for transaction costs. Further, the level effect of transaction costs is small, consistent with the evidence presented and referenced above. This suggests that the economic information contained in the strip yields, which strongly supports the leading asset pricing models, is substantially more robust to the liquidity issues in these markets than is the short horizon holding period return-based evidence.

Once we have corrected for the dramatic illiquidity of the dividend futures markets and the substantial variation in liquidity by horizon, the data continue to provide strong support for the implications of standard asset pricing models of short horizon dividend claims carrying less macroeconomic risk than long horizon claims. There is no reliable evidence supporting the existing claims of the literature because monthly holding period returns are both too poorly measured and too illiquid to be useful for inference. Short holding period return estimates are heavily contaminated by spread and spread volatility that it is difficult to justify drawing economic conclusions about dividend risk, as opposed to microstructure and trading risk, from these realized returns.

7 Conclusion

Using additional asset prices to learn about risk and reward in financial markets is a welcomed endeavor. At the same time, as more esoteric markets are analyzed, any inference has to be judicious and with an eye to institutional features of such markets and the limitations of the data. Recently, several papers suggest that the term structure of dividend strip returns is downward sloping and thus poses a challenge to existing asset pricing models. In this paper, we examine the robustness of this claim put forward by earlier studies using nearly identical data, i.e., traded equity dividend strips.

We show that the term structure of dividend strip risk premia and discount rates implied by equity strip yields are downward sloping in recessions and upward sloping in expansion periods, a finding which is statistically significant and robust across regions. Our results are based on the estimates of the regime-switching dividend growth model, which allows us to characterize not just the conditional but also unconditional moments. We do this by carefully taking into account the small sample features (the representativeness of different phases of business cycle) of data acknowledging the perils of misinterpreting the finite samples moments as the unconditional moments. Our evidence suggests that the claim about downward sloping equity term structure is rejected from the data.

This is an important finding as the standard asset pricing models are not in conflict with the new data on dividend strips. In fact, we show that the standard asset pricing models when extended with regime-switching dynamics, which we believe to be a crucial element in characterizing expected growth, and consequently, return dynamics, are able to reconcile these stylized facts.

Our analyses are based on expected hold-to-maturity returns (dividend discount rates) instead of short-term holding-period returns since they are the economic object of interest and the preferred focus for statistical and institutional reasons. Based on the comprehensive data on asset liquidity, we show that short-term holding-period returns are severly affected by the dramatic illiquidity of dividend strip markets. This implies that the inference based on short-term strip returns can be highly unreliable. Instead, we find that hold-to-maturity returns are much more robust to the liquidity issues.

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Appendix: The Term Structure of Equity Risk Premia

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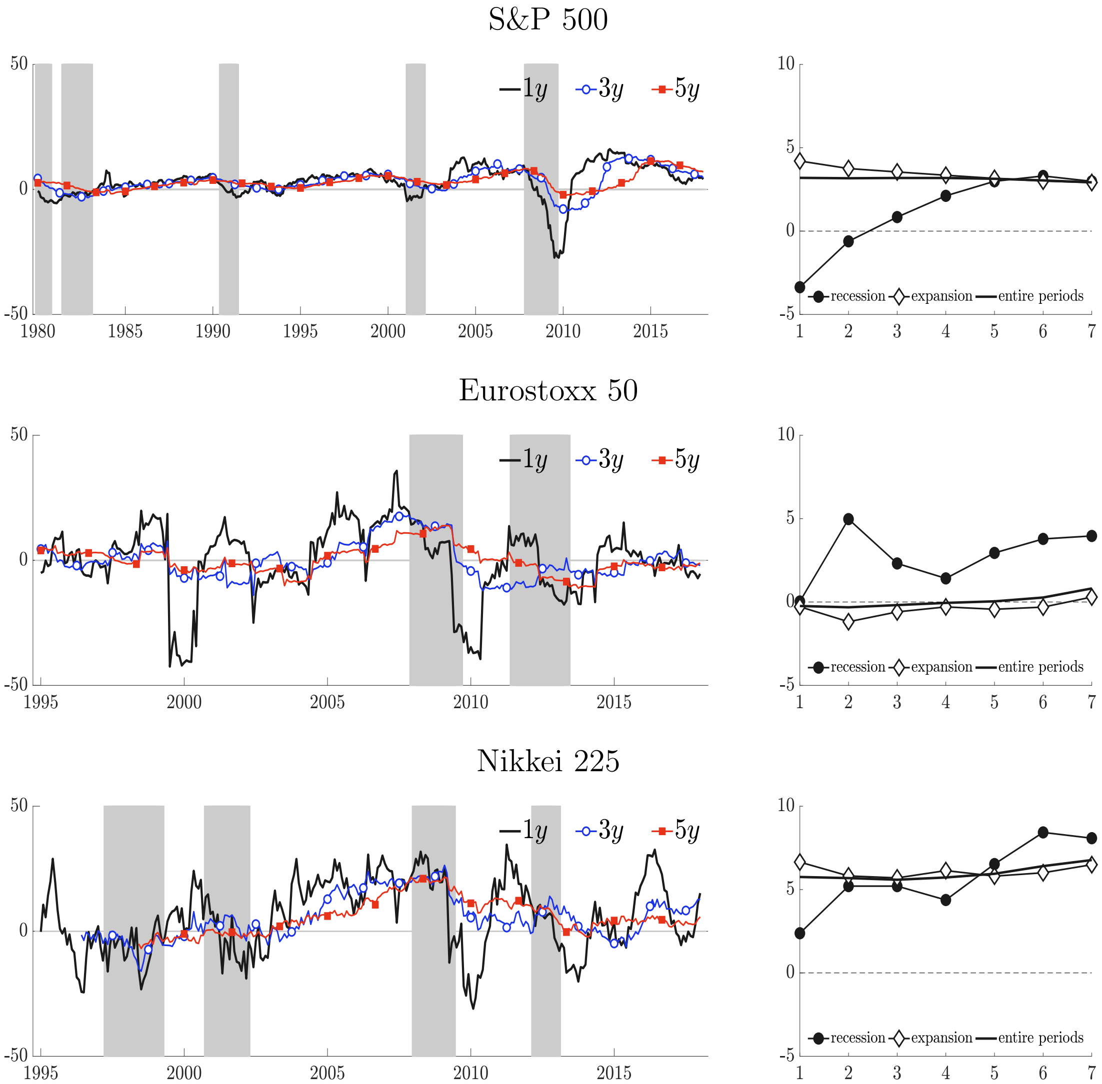
A Supplementary Figures and Tables

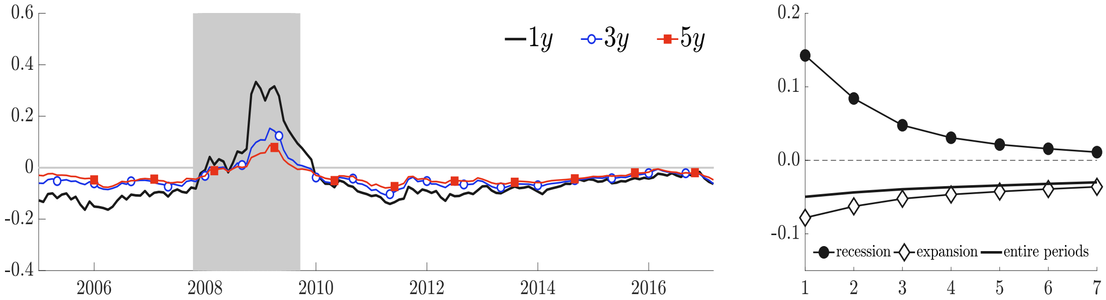
Figure A-1: Dividend growth



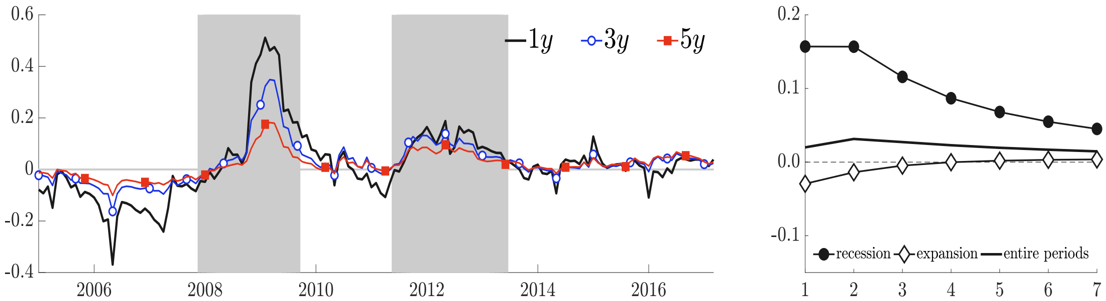
Notes: For US, dividend growth data are available from 1979:M12 to 2017:M2 for all horizons. For Europe, dividend growth data are available from 1994:M12 to 2017:M2 for all horizons. For Japan, 1-year dividend growth data are available starting from 1994:M12, 3-year dividend growth data from 1996:M5, and 5-year dividend growth from 1998:M5. Data end in 2017:M2. Shaded bars indicate recession dates. The y-axis is expressed in annualized percentage terms.

Figure A-2: Forward equity yields

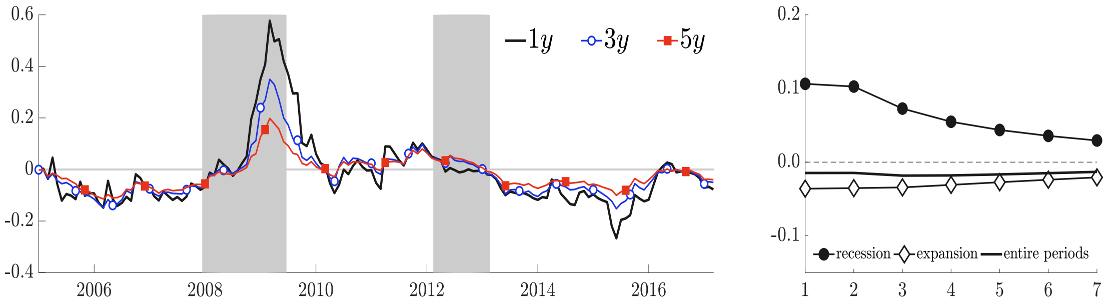
S&P 500



Eurostoxx 50

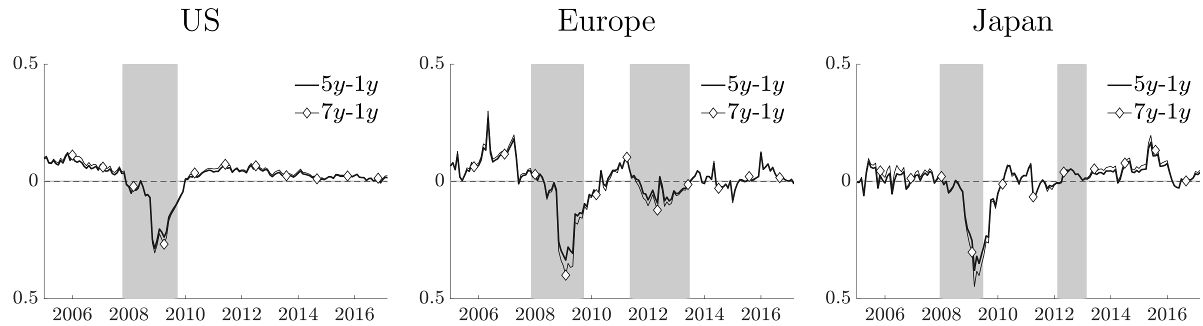


Nikkei 225



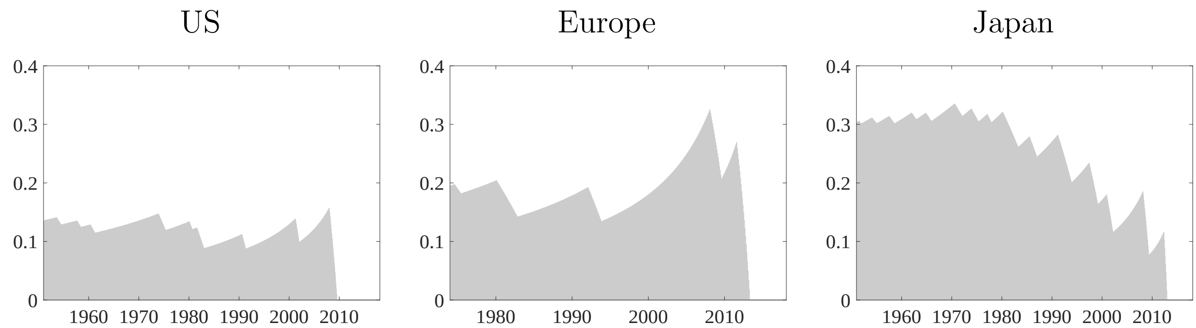
Notes: We provide the time series of the forward equity yields from 2004:M12 to 2017:M2. Equity yields are $e^{f}_{n,t} = \frac{1}{n}\ln(\frac{D_t}{F_{n,t}})$ with $F_{n,t}$ the futures price and D_t the trailing sum of 12 month dividends. Shaded bars indicate recession dates.

Figure A-3: The spread of forward equity yields



Notes: Shaded bars indicate recession dates. Negative spread coincides with recessions. The sample starts from 2004:M12 to 2017:M2.

Figure A-4: Sample recession frequency



Notes: For each time t, we fix the end point and compute the average sample recession frequency starting from t to 2017:M12. Shaded areas indicate the average of recession frequency which is zero at 2017:M12 by construction.

Table A-1: Model-implied expected dividend growth, prices, and returns: Europe

	(1) Exp. growth	(2) Exp. return	(3) Premium	(4) Sharpe ratio	(5) Equity yield	(6) RYLD
			Sample average			
1y	0.05	2.05	2.89	0.30	1.99	-0.84
2y	0.11	3.28	4.03	0.59	3.17	-0.75
3y	0.14	2.87	3.48	0.62	2.73	-0.61
4y	0.17	2.45	2.88	0.59	2.29	-0.42
5y	0.18	2.12	2.39	0.55	1.94	-0.28
5y-1y	0.13	0.07	-0.49	0.25	-0.06	0.57
			Expansion (high-growth) state			
1y	5.75	2.93	3.75	0.39	-2.82	-0.83
2y	4.15	3.28	4.04	0.59	-0.87	-0.77
3y	3.17	3.19	3.84	0.68	0.02	-0.65
4y	2.54	2.94	3.43	0.71	0.40	-0.49
5y	2.11	2.65	2.99	0.69	0.54	-0.35
5y-1y	-3.64	-0.28	-0.76	0.30	3.36	0.48
			Recession (low-growth) state			
1y	-17.00	2.01	2.89	0.30	19.00	-0.89
2y	-11.97	5.47	6.15	0.90	17.44	-0.68
3y	-8.90	3.39	3.85	0.69	12.29	-0.45
4y	-6.92	2.02	2.20	0.45	8.94	-0.18
5y	-5.59	1.29	1.31	0.30	6.88	-0.02
5y-1y	11.40	-0.71	-1.58	0.00	-12.12	0.87

Notes: We provide the annualized average expected dividend growth rates $E_t[g_{d,t,t+n}]$ ("Exp. growth"); the expected discount rate $E_t[r_{t+n}]$ ("Exp. return"), computed as in (12); the expected excess return $E_t rx_{t+n}$ ("Premium"), computed as in (13); the Sharpe ratio $SR_{n,t}$ ("Sharpe ratio"), computed as in (15); the forward equity yields $e^f_{n,t}$ ("Equity yield"); and the real bond yields $y^f_{n,t} = y_{n,t} - E(\pi)$ ("RYLD") computed from subtracting the sample average inflation rates from the nominal yields.

Table A-2: Model-implied expected dividend growth, prices, and returns: Japan

	(1) Exp. growth	(2) Exp. return	(3) Premium	(4) Sharpe ratio	(5) Equity yield	(6) RYLD
			Sample average			
1y	8.41	7.01	7.06	0.54	-1.40	-0.05
2y	8.80	7.39	7.37	0.80	-1.41	0.02
3y	9.02	7.23	7.13	0.94	-1.79	0.10
4y	9.17	7.40	7.20	1.10	-1.77	0.20
5y	9.26	7.62	7.34	1.25	-1.64	0.29
5y-1y	0.85	0.61	0.28	0.71	-0.24	0.34
			Expansion (high-growth) state			
1y	10.21	5.76	5.82	0.44	-4.45	-0.06
2y	10.05	6.10	6.09	0.66	-3.95	0.01
3y	9.95	6.41	6.33	0.84	-3.54	0.09
4y	9.89	6.82	6.64	1.01	-3.07	0.18
5y	9.85	7.20	6.94	1.18	-2.65	0.27
5y-1y	-0.37	1.44	1.11	0.74	1.80	0.32
			Recession (low-growth) state			
1y	-14.21	25.58	25.52	1.95	39.79	0.06
2y	-6.90	26.04	25.89	2.79	32.94	0.15
3y	-2.57	19.19	18.91	2.50	21.76	0.28
4y	0.14	15.83	15.41	2.35	15.69	0.42
5y	1.94	13.88	13.33	2.27	11.95	0.55
5y-1y	16.14	-11.69	-12.18	0.33	-27.84	0.49

Notes: We provide the annualized average expected dividend growth rates $E_t[g_{d,t,t+n}]$ ("Exp. growth"); the expected discount rate $E_t[r_{t+n}]$ ("Exp. return"), computed as in (12); the expected excess return $E_t rx_{t+n}$ ("Premium"), computed as in (13); the Sharpe ratio $SR_{n,t}$ ("Sharpe ratio"), computed as in (15); the forward equity yields $e^f_{n,t}$ ("Equity yield"); and the real bond yields $y^f_{n,t} = y_{n,t} - E(\pi)$ ("RYLD") computed from subtracting the sample average inflation rates from the nominal yields.

Table A-3: Posterior estimates

Parameter	Dist.	Implied	Prior 50%	[5%	95%]	Posterior 50%	[5%	95%]
				U.S.				
$\mu_d(H)$	G		0.150	[0.080	0.240]	0.042	[0.025	0.066]
$\mu_d(L)$	N		-0.150	[-0.230	-0.070]	-0.060	[-0.092	-0.045]
σ_d	IG		0.140	[0.060	0.380]	0.161	[0.137	0.196]
σ_x	IG		0.070	[0.030	0.250]	0.051	[0.043	0.062]
p_H	B		0.995	[0.732	0.999]	0.987	[0.959	0.997]
p_L		✓	0.970	[0.000	0.999]	0.924	[0.771	0.984]
$\mu_x(H)$		✓	0.115	[0.001	0.214]	0.031	[0.022	0.044]
$\mu_x(L)$		✓	-0.192	[-0.388	-0.011]	-0.093	[-0.140	-0.042]
				Europe				
$\mu_d(H)$	G		0.150	[0.080	0.240]	0.061	[0.033	0.089]
$\mu_d(L)$	N		-0.150	[-0.230	-0.070]	-0.105	[-0.165	-0.066]
σ_d	IG		0.140	[0.060	0.380]	0.382	[0.350	0.397]
σ_x	IG		0.070	[0.030	0.250]	0.078	[0.067	0.089]
p_H	B		0.995	[0.732	0.999]	0.994	[0.987	0.997]
p_L		✓	0.970	[0.000	0.999]	0.968	[0.925	0.985]
$\mu_x(H)$		✓	0.115	[0.001	0.214]	0.031	[0.026	0.049]
$\mu_x(L)$		✓	-0.192	[-0.388	-0.011]	-0.083	[-0.151	-0.043]
				Japan				
$\mu_d(H)$	G		0.150	[0.080	0.240]	0.106	[0.051	0.193]
$\mu_d(L)$	N		-0.150	[-0.230	-0.070]	-0.081	[-0.148	-0.043]
σ_d	IG		0.140	[0.060	0.380]	0.392	[0.370	0.402]
σ_x	IG		0.070	[0.030	0.250]	0.060	[0.047	0.073]
p_H	B		0.995	[0.732	0.999]	0.994	[0.968	0.999]
p_L		✓	0.970	[0.000	0.999]	0.964	[0.819	0.995]
$\mu_x(H)$		✓	0.115	[0.001	0.214]	0.027	[0.024	0.033]
$\mu_x(L)$		✓	-0.192	[-0.388	-0.011]	-0.157	[-0.302	-0.035]

Notes: The estimation sample is from 2004:M12 to 2016:M2. N, G, B, IG denote normal, gamma, beta, inverse gamma, distributions, respectively. We impose a composite prior on the spread of expected return of maturity 5 and 1 years. Specifically, we introduce a new parameter $\mu_r(S_t)$ to capture the mean component of $E_t r_{t+5y} - E_t r_{t+1y}$ where $E_t r_{t+5y} = e_{5y,t} + E_t g_{d,t,t+5y}$ and $E_t r_{t+1y} = e_{1y,t} + E_t g_{d,t,t+1y}$. Since $x_t = e_{5y,t} - e_{1y,t} = \mu_x(S_t) + \epsilon_{x,t}$, we can re-write $\mu_r(S_t) = \mu_x(S_t) + E_t g_{d,t,t+5y} - E_t g_{d,t,t+1y}$. In sum, while we are estimating $\mu_x(\cdot)$, we are imposing priors on $\mu_r(\cdot)$ instead. We indicate that $\mu_x(\cdot)$ is implied from $\mu_r(\cdot)$. We use symmetric priors of $\mu_r(H) \sim N(0.1, 0.075^2)$ and $\mu_r(L) \sim N(-0.1, 0.075^2)$. The ergodic probability vector is $\mathbb{P}^*$. We let p_L be deterministic function of p_H such that $\mathbb{P}(H)^* = 0.85$ and $\mathbb{P}(L)^* = 0.15$. It can be deduced from the expressions for $\mathbb{P}^*$ that $p_L = (1+\mathbb{P}(H)^*(p_H - 2))/(1-\mathbb{P}(H)^*)$ which is implied from p_H.

B Solving the Long-Run Risks Model

This section provides approximate analytical solutions for the equilibrium asset prices.

B.1 Exogenous dynamics

The joint dynamics of consumption and dividend growth are

$$\begin{aligned}
\Delta c_{t+1} &= \mu(S_{t+1}) + x_{t+1} + \sigma_c \eta_{c,t+1}, \quad \eta_{c,t+1} \sim N(0,1), \\
\Delta d_{t+1} &= \bar{\mu} + \phi(\Delta c_{t+1} - \bar{\mu}) + \sigma_d \eta_{d,t+1}, \quad \eta_{d,t+1} \sim N(0,1), \\
x_{t+1} &= \rho x_t + \sigma_x(S_{t+1})\epsilon_{t+1},
\end{aligned} \tag{A-1}$$

where $\bar{\mu}$ is the unconditional mean of consumption growth and x_t is the persistent component of consumption growth. Agents observe the current regime, S_t, and make forecast of future regime, S_{t+1}, based on the transition matrix below

$$\mathbb{P} = \begin{bmatrix} p_1 & 1-p_1 \\ 1-p_2 & p_2 \end{bmatrix}. \tag{A-2}$$

B.2 Derivation of the approximate analytical solutions

The Euler equation for the economy is

$$1 = E_t\left[\exp\left(m_{t+1} + r_{c,t+1}\right)\right] \tag{A-3}$$

where the log stochastic discount factor is

$$m_{t+1} = \theta \ln \delta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1) r_{c,t+1}, \tag{A-4}$$

$z_{c,t}$ is the log price to consumption ratio and $r_{c,t+1}$ is the log return on the consumption claim

$$r_{c,t+1} = \kappa_{0,c} + \kappa_{1,c} z_{c,t+1} - z_{c,t} + \Delta c_{t+1}. \tag{A-5}$$

Derivation of (A-3) follows Bansal and Zhou (2002), which make repeated use of the law of iterated expectations and log-linearization.

$$
\begin{aligned}
1 &= E\Big(E\left[\exp\left(m_{t+1} + r_{c,t+1}\right) \mid S_{t+1}\right] \mid S_t \Big) \\
&= \sum_{j=1}^{2} \mathbb{P}_{ij} E\Big(\exp\left(m_{t+1} + r_{c,t+1}\right) \mid S_{t+1} = j, S_t = i \Big) \\
0 &= \sum_{j=1}^{2} \mathbb{P}_{ij} \Big(E\big[m_{t+1} + r_{c,t+1} \mid S_{t+1} = j, S_t = i\big] + \frac{1}{2} Var\big[m_{t+1} + r_{c,t+1} \mid S_{t+1} = j, S_t = i\big] \Big).
\end{aligned}
\tag{A-6}
$$

The first line uses the law of iterated expectations; the second line uses the definition of Markov chain; and the third line relies on the log-normality assumption and applies log-linearization (i.e., $\exp(B) - 1 \approx B$).

B.3 Real consumption claim

Conjecture that the log price to consumption ratio follows

$$
z_{c,t}(S_t) = A_{0,c}(S_t) + A_{1,c} x_t. \tag{A-7}
$$

From (A-1), (A-5), and (A-7), we can express the return on consumption claim by

$$
\begin{aligned}
r_{c,t+1} &= \kappa_{0,c} + \kappa_{1,c} A_{0,c}(S_{t+1}) - A_{0,c}(S_t) + \mu(S_{t+1}) \\
&+ (\kappa_{1,c} A_{1,c}(S_{t+1})\rho - A_{1,c}(S_t) + \rho) x_t + (\kappa_{1,c} A_{1,c}(S_{t+1}) + 1)\sigma_x(S_{t+1})\epsilon_{t+1} + \sigma_c \eta_{c,t+1}.
\end{aligned}
\tag{A-8}
$$

Using (A-8), we can re-express the log SDF (A-4) by

$$
\begin{aligned}
m_{t+1} &= \theta \ln \delta + (\theta - 1)\big(\kappa_{0,c} - A_{0,c}(S_t) + \kappa_{1,c} A_{0,c}(S_{t+1})\big) - \gamma \mu(S_{t+1}) \\
&+ \big((\theta - 1)(\kappa_{1,c} A_{1,c}(S_{t+1})\rho - A_{1,c}(S_t)) - \gamma\rho\big) x_t \\
&+ \big((\theta - 1)\kappa_{1,c} A_{1,c}(S_{t+1}) - \gamma\big)\sigma_x(S_{t+1})\epsilon_{t+1} - \gamma \sigma_c \eta_{c,t+1}.
\end{aligned}
\tag{A-9}
$$

The solutions for $A_{0,c}$ and $A_{1,c}$ that describe the dynamics of the price-consumption ratio are determined from (A-6) which are

$$
\begin{bmatrix} A_{1,c}(1) \\ A_{1,c}(2) \end{bmatrix} = \begin{bmatrix} \mathbb{I}_2 - \rho \kappa_{1,c} \mathbb{P} \end{bmatrix}^{-1} \begin{bmatrix} (1 - \frac{1}{\psi})\rho \\ (1 - \frac{1}{\psi})\rho \end{bmatrix} \tag{A-10}
$$

$$
\begin{bmatrix} A_{0,c}(1) \\ A_{0,c}(2) \end{bmatrix} = \begin{bmatrix} \mathbb{I}_2 - \kappa_{1,c}\mathbb{P} \end{bmatrix}^{-1} \mathbb{P} \times \qquad \text{(A-11)}
$$

$$
\begin{bmatrix} \ln\delta + \kappa_{0,c} + (1-\frac{1}{\psi})\mu(1) + \frac{\theta}{2}(1-\frac{1}{\psi})^2\sigma_c^2 + \frac{\theta}{2}\big(\kappa_{1,c}A_{1,c}(1) + (1-\frac{1}{\psi})\big)^2\sigma_x(1)^2 \\ \ln\delta + \kappa_{0,c} + (1-\frac{1}{\psi})\mu(2) + \frac{\theta}{2}(1-\frac{1}{\psi})^2\sigma_c^2 + \frac{\theta}{2}\big(\kappa_{1,c}A_{1,c}(2) + (1-\frac{1}{\psi})\big)^2\sigma_x(2)^2 \end{bmatrix}.
$$

B.4 Linearization parameters

Let $\bar{p}_j = \frac{1-p_l}{2-p_l-p_j}$. The linearization parameters are determined endogenously by the following system of equations

$$
\begin{aligned}
\bar{z}_c &= \sum_{j=1}^{2} \bar{p}_j A_{0,c}(j) \\
\kappa_{1,c} &= \frac{\exp(\bar{z}_c)}{1+\exp(\bar{z}_c)} \\
\kappa_{0,c} &= \log(1+\exp(\bar{z}_c)) - \kappa_{1,c}\bar{z}_c.
\end{aligned}
$$

The solution is determined numerically by iteration until reaching a fixed point of $\bar{z}_c$.

B.5 Real bond prices

Conjecture that $b_{n,t}$ depends on the regime S_t and x_t,

$$
b_{n,t} = b_{n,0}(S_t) + b_{n,1}(S_t)x_t. \qquad \text{(A-12)}
$$

Exploit the law of iterated expectations

$$
b_{n,t} = \ln E_t\Big(E[\exp(m_{t+1} + b_{n-1,t+1})|S_{t+1}]\Big)
$$

and log-linearization to solve for $b_{n,t}$

$$
b_{n,t} \approx \sum_{j=1}^{2} \mathbb{P}_{ij}\Big(E[m_{t+1} + b_{n-1,t+1}|S_{t+1}] + \frac{1}{2}Var[m_{t+1} + b_{n-1,t+1}|S_{t+1}]\Big).
$$

The solution to (A-12) is

$$\begin{bmatrix} b_{n,1}(1) \\ b_{n,1}(2) \end{bmatrix} = \mathbb{P}\begin{bmatrix} b_{n-1,1}(1)\rho + (\theta-1)\kappa_{1,c}A_{1,c}(1)\rho \\ b_{n-1,1}(2)\rho + (\theta-1)\kappa_{1,c}A_{1,c}(2)\rho \end{bmatrix} - \begin{bmatrix} (\theta-1)A_{1,c}(1) + \gamma\rho \\ (\theta-1)A_{1,c}(2) + \gamma\rho \end{bmatrix} \tag{A-13}$$

$$\begin{aligned}
\begin{bmatrix} b_{n,0}(1) \\ b_{n,0}(2) \end{bmatrix} &= \begin{bmatrix} \theta\ln\delta + (\theta-1)\kappa_{0,c} - (\theta-1)A_{0,c}(1) + \frac{\gamma^2}{2}\sigma_c^2 \\ \theta\ln\delta + (\theta-1)\kappa_{0,c} - (\theta-1)A_{0,c}(2) + \frac{\gamma^2}{2}\sigma_c^2 \end{bmatrix} \\
&+ \mathbb{P}\times\begin{bmatrix} b_{n-1,0}(1) + (\theta-1)\kappa_{1,c}A_{0,c}(1) - \gamma\mu(1) \\ b_{n-1,0}(2) + (\theta-1)\kappa_{1,c}A_{0,c}(2) - \gamma\mu(2) \end{bmatrix} \\
&+ \mathbb{P}\times\begin{bmatrix} \frac{1}{2}\big((\theta-1)\kappa_{1,c}A_{1,c}(1) - \gamma + b_{n-1,1}(1)\big)^2\sigma_x^2(1) \\ \frac{1}{2}\big((\theta-1)\kappa_{1,c}A_{1,c}(2) - \gamma + b_{n-1,1}(2)\big)^2\sigma_x^2(2) \end{bmatrix}
\end{aligned}$$

with the initial condition $b_{0,0}(i) = 0$ and $b_{0,1}(i) = 0$ for $i \in \{1,2\}$. The real yield of the maturity n-period bond is $y^r_{n,t} = -\frac{1}{n}b_{n,t}$.

B.6 Price to dividend ratio of zero coupon equity

Conjecture that the log price to dividend ratio of zero coupon equity $z_{n,t}$ depends on the regime S_t and persistent component x_t,

$$z_{n,t} = z_{n,0}(S_t) + z_{n,1}(S_t)x_t. \tag{A-14}$$

Exploit the law of iterated expectations

$$Z_{n,t} = E_t\Big(E[M_{t+1}Z_{n-1,t+1}\frac{D_{t+1}}{D_t}|S_{t+1}]\Big)$$

Take log

$$z_{n,t} = \ln E_t\Big(E[\exp(m_{t+1} + z_{n-1,t+1} + \Delta d_{t+1})|S_{t+1}]\Big)$$

and log-linearization to solve for $z_{n,t}$

$$z_{n,t} \approx \sum_{j=1}^{2}\mathbb{P}_{ij}\Big(E[m_{t+1} + z_{n-1,t+1} + \Delta d_{t+1}|S_{t+1}] + \frac{1}{2}Var[m_{t+1} + z_{n-1,t+1} + \Delta d_{t+1}|S_{t+1}]\Big).$$

The solution is

$$\begin{bmatrix} z_{n,1}(1) \\ z_{n,1}(2) \end{bmatrix} = \mathbb{P} \begin{bmatrix} z_{n-1,1}(1)\rho + (\theta-1)\kappa_{1,c}A_{1,c}(1)\rho \\ z_{n-1,1}(2)\rho + (\theta-1)\kappa_{1,c}A_{1,c}(2)\rho \end{bmatrix} - \begin{bmatrix} (\theta-1)A_{1,c}(1) - (\phi-\gamma)\rho \\ (\theta-1)A_{1,c}(2) - (\phi-\gamma)\rho \end{bmatrix} \tag{A-15}$$

$$\begin{aligned} \begin{bmatrix} z_{n,0}(1) \\ z_{n,0}(2) \end{bmatrix} = & \begin{bmatrix} (1-\phi)\bar{\mu} + \theta\ln\delta + (\theta-1)\kappa_{0,c} - (\theta-1)A_{0,c}(1) + \frac{1}{2}(\phi-\gamma)^2\sigma_c^2 + \frac{1}{2}\sigma_d^2 \\ (1-\phi)\bar{\mu} + \theta\ln\delta + (\theta-1)\kappa_{0,c} - (\theta-1)A_{0,c}(2) + \frac{1}{2}(\phi-\gamma)^2\sigma_c^2 + \frac{1}{2}\sigma_d^2 \end{bmatrix} \\ & + \mathbb{P} \times \begin{bmatrix} z_{n-1,0}(1) + (\theta-1)\kappa_{1,c}A_{0,c}(1) + (\phi-\gamma)\mu(1) \\ z_{n-1,0}(2) + (\theta-1)\kappa_{1,c}A_{0,c}(2) + (\phi-\gamma)\mu(2) \end{bmatrix} \\ & + \mathbb{P} \times \begin{bmatrix} \frac{1}{2}\big((\theta-1)\kappa_{1,c}A_{1,c}(1) + (\phi-\gamma) + z_{n-1,1}(1)\big)^2\sigma_x^2(1) \\ \frac{1}{2}\big((\theta-1)\kappa_{1,c}A_{1,c}(2) + (\phi-\gamma) + z_{n-1,1}(2)\big)^2\sigma_x^2(2) \end{bmatrix} \end{aligned}$$

with the initial condition $z_{0,0}(i) = 0$ and $z_{0,1}(i) = 0$ for $i \in \{1,2\}$.

B.7 m-holding-period and hold-to-maturity expected return

The price of zero coupon equity is $P_{n,t} = Z_{n,t}D_t$. Define the m-holding period return of the n-maturity equity is

$$R_{n,t+m} = \frac{Z_{n-m,t+m}}{Z_{n,t}}\frac{D_{t+m}}{D_t}. \tag{A-16}$$

The corresponding log expected return is defined by

$$E_t[r_{n,t+m}] = \frac{1}{m}E_t\Big(z_{n-m,t+m} - z_{n,t} + \sum_{i=1}^{m}\Delta d_{t+i}\Big) \tag{A-17}$$

To compute the excess return, we subtract the real rate of the same maturity

$$E_t[r_{n,t+m}] - y^r_{m,t}. \tag{A-18}$$

We consider two cases

- $m \neq n$: This is the m-holding-period expected excess return of the n maturity equity.

$$\begin{aligned} E_t[g_{d,t+m}] &= \frac{1}{m} E_t\big(\sum_{i=1}^{m} \Delta d_{t+i}\big) \\ e_{n,m,t} &= \frac{1}{m} E_t\big(z_{n-m,t+m} - z_{n,t}\big) \\ E_t[r_{n,t+m}] &= e_{n,m,t} + E_t[g_{d,t+m}] \\ E_t[rx_{n,t+m}] &= E_t[r_{n,t+m}] - y^r_{m,t}. \end{aligned} \tag{A-19}$$

- $m = n$: This is the hold-to-maturity expected excess return of the n maturity equity. Define

$$\begin{aligned} E_t[g_{d,t+n}] &= \frac{1}{n} E_t\big(\sum_{i=1}^{n} \Delta d_{t+i}\big) \\ e_{n,t} &= \frac{1}{n} E_t\big(- z_{n,t}\big) \\ E_t[r_{t+n}] &= e_{n,t} + E_t[g_{d,t+n}] \\ E_t[rx_{t+n}] &= E_t[r_{t+n}] - y^r_{n,t}. \end{aligned} \tag{A-20}$$

B.8 Computing moments

The cumulative sum of log dividend growth rates are

$$\begin{aligned} \sum_{i=1}^{n} \Delta d_{t+i} = n(1-\phi)\bar{\mu} + \phi\big(\mu(S_{t+1}) + ... + \mu(S_{t+n})\big) + \phi\rho\left(\frac{1-\rho^n}{1-\rho}\right)x_t \\ + \phi\left(\frac{1-\rho^n}{1-\rho}\right)\sigma_x(S_{t+1})\epsilon_{t+1} + ... + \phi\left(\frac{1-\rho}{1-\rho}\right)\sigma_x(S_{t+n})\epsilon_{t+n} \\ + \phi\sigma_c(\eta_{c,t+1} + ... + \eta_{c,t+n}) + \sigma_d(\eta_{d,t+1} + ... + \eta_{d,t+n}). \end{aligned} \tag{A-21}$$

For ease of exposition, we introduce the following notations

$$\boldsymbol{\mu} = [\mu(1), \mu(2)]', \quad \boldsymbol{\sigma_x^2} = [\sigma_x(1)^2, \sigma_x(2)^2]'.$$

The first two moments of the average log dividend growth rates for the case of $S_t = k$ are

$$\begin{aligned} E_t[g_{d,t+n}] &= \frac{1}{n}E_t[\sum_{i=1}^{n}\Delta d_{t+i}] = \frac{1}{n}\boldsymbol{\mu_G}(k) \\ V_t[g_{d,t+n}] &= \frac{1}{n^2}V_t[\sum_{i=1}^{n}\Delta d_{t+i}] = \frac{1}{n^2}\boldsymbol{\sigma^2_G}(k) \end{aligned} \tag{A-22}$$

where

$$\begin{aligned} \boldsymbol{\mu_G} &= \begin{bmatrix} n(1-\phi)\bar{\mu} + \phi\rho\left(\frac{1-\rho^n}{1-\rho}\right)x_t \\ n(1-\phi)\bar{\mu} + \phi\rho\left(\frac{1-\rho^n}{1-\rho}\right)x_t \end{bmatrix} + \sum_{j=1}^{n}\phi\mathbb{P}^j\boldsymbol{\mu} \\ \boldsymbol{\sigma^2_G} &\approx \begin{bmatrix} n(\phi^2\sigma_c^2+\sigma_d^2) \\ n(\phi^2\sigma_c^2+\sigma_d^2) \end{bmatrix} + \phi^2\sum_{j=1}^{n}\left(\frac{1-\rho^{n+1-j}}{1-\rho}\right)^2\mathbb{P}^j\boldsymbol{\sigma^2_x}. \end{aligned} \tag{A-23}$$

We acknowledge that the expression for $\boldsymbol{\sigma^2_G}$ is not exact because we are ignoring the variance component associated with uncertainty about $\mu(S_{t+j})$.

The above expressions allow us to calculate the Sharpe ratio

$$SR_{n,t} = \frac{e_{n,t} + E_t[g_{d,t+n}] - y^r_{n,t}}{\sqrt{V_t[g_{d,t+n}]}} \tag{A-24}$$

In the main text, we report the case of $x_t = 0$ for ease of illustration, e.g., $E_t g_{d,t+n}|_{x_t=0}$ and $V_t g_{d,t+n}|_{x_t=0}$.

B.9 Market return and equity premium

We derive the market return via Campbell-Shiller approximation

$$\begin{aligned} r_{d,t+1} &= \kappa_{0,d} + \kappa_{1,d}A_{0,d}(S_{t+1}) - A_{0,d}(S_t) + (1-\phi)\bar{\mu} + \phi\mu(S_{t+1}) \\ &\quad +\big(\phi\rho + \kappa_{1,d}A_{1,d}(S_{t+1})\rho - A_{1,d}(S_t)\big)x_t \\ &\quad +(\phi + \kappa_{1,d}A_{1,d}(S_{t+1}))\sigma_x(S_{t+1})\epsilon_{t+1} + \phi\sigma_c\eta_{c,t+1} + \sigma_d\eta_{d,t+1} \end{aligned} \tag{A-25}$$

where the log price-dividend ratio is given by

$$z_t = A_{0,d}(S_t) + z_{1,d}(S_t)x_t. \tag{A-26}$$

We solve for the coefficients

$$\begin{bmatrix} A_{1,d}(1) \\ A_{1,d}(2) \end{bmatrix} = \begin{bmatrix} \mathbb{I}_2 - \rho\kappa_{1,d}\mathbb{P} \end{bmatrix}^{-1} \left(\begin{bmatrix} (\phi-\gamma)\rho - (\theta-1)A_{1,c}(1) \\ (\phi-\gamma)\rho - (\theta-1)A_{1,c}(2) \end{bmatrix} + \mathbb{P} \begin{bmatrix} (\theta-1)\kappa_{1,c}A_{1,c}(1)\rho \\ (\theta-1)\kappa_{1,c}A_{1,c}(2)\rho \end{bmatrix} \right) \tag{A-27}$$

$$\begin{bmatrix} A_{0,d}(1) \\ A_{0,d}(2) \end{bmatrix} = \begin{bmatrix} \mathbb{I}_2 - \kappa_{1,d}\mathbb{P} \end{bmatrix}^{-1} \times$$
$$\left(\begin{bmatrix} (1-\phi)\bar{\mu} + \kappa_{0,d} + \theta\ln\delta + (\theta-1)\kappa_{0,c} - (\theta-1)A_{0,c}(1) + 0.5(\phi-\gamma)^2\sigma_c^2 + 0.5\sigma_d^2 \\ (1-\phi)\bar{\mu} + \kappa_{0,d} + \theta\ln\delta + (\theta-1)\kappa_{0,c} - (\theta-1)A_{0,c}(2) + 0.5(\phi-\gamma)^2\sigma_c^2 + 0.5\sigma_d^2 \end{bmatrix} \right.$$
$$\left. \mathbb{P} \begin{bmatrix} (\phi-\gamma)\mu(1) + (\theta-1)\kappa_{1,c}A_{0,c}(1) + 0.5((\theta-1)\kappa_{1,c}A_{1,c}(1) + \kappa_{1,d}A_{1,d}(1) + \phi-\gamma)^2\sigma_x(1)^2 \\ (\phi-\gamma)\mu(2) + (\theta-1)\kappa_{1,c}A_{0,c}(2) + 0.5((\theta-1)\kappa_{1,c}A_{1,c}(2) + \kappa_{1,d}A_{1,d}(2) + \phi-\gamma)^2\sigma_x(2)^2 \end{bmatrix} \right).$$

The market equity premium $E_t[r_{d,t+1}] - y_{1,t}^r + \frac{1}{2}V_t[r_{d,t+1}]$ is

$$-Cov_t(r_{d,t+1}, m_{t+1}) = \phi\gamma\sigma_c^2 + \mathbb{P} \times \begin{bmatrix} (\phi + \kappa_{1,d}A_{1,d}(1))((\theta-1)\kappa_{1,c}A_{1,c}(1) - \gamma)\sigma_x(1)^2 \\ (\phi + \kappa_{1,d}A_{1,d}(2))((\theta-1)\kappa_{1,c}A_{1,c}(2) - \gamma)\sigma_x(2)^2 \end{bmatrix}. \tag{A-28}$$

The conditional variance of the market return is

$$V_t[r_{d,t+1}] \approx \begin{bmatrix} \phi^2\sigma_c^2 + \sigma_d^2 \\ \phi^2\sigma_c^2 + \sigma_d^2 \end{bmatrix} + \mathbb{P} \times \begin{bmatrix} (\phi + \kappa_{1,d}A_{1,d}(1))^2\sigma_x(1)^2 \\ (\phi + \kappa_{1,d}A_{1,d}(2))^2\sigma_x(2)^2 \end{bmatrix}. \tag{A-29}$$

The market Sharpe ratio is

$$SR_t = \frac{E_t[r_{d,t+1}] - y_{1,t}^r}{\sqrt{V_t[r_{d,t+1}]}}. \tag{A-30}$$

Here, we are not accounting for $\frac{1}{2}V_t[r_{d,t+1}]$ in the numerator.

B.10 Calibration

With this calibration, we derive the market return via Campbell-Shiller approximation and compute the expected excess return of the market. The equity premium (accounting for the Jensen's inequality term) is 4.73 and 12.22 in the expansion and recession state, respectively. The unconditional average (weighted by steady state probability) is around 5.85.

Table A-4: Calibration

Parameters

$\mu(1)$	0.0020	ρ	0.60	δ	0.998
$\mu(2)$	0.0010	$\sigma_x(1)$	0.0033	ψ	1.5
σ_c	0.0063	$\sigma_x(2)$	0.0064	γ	10
ϕ	4.0	p_1	0.9965		
σ_d	0.0224	p_2	0.98		

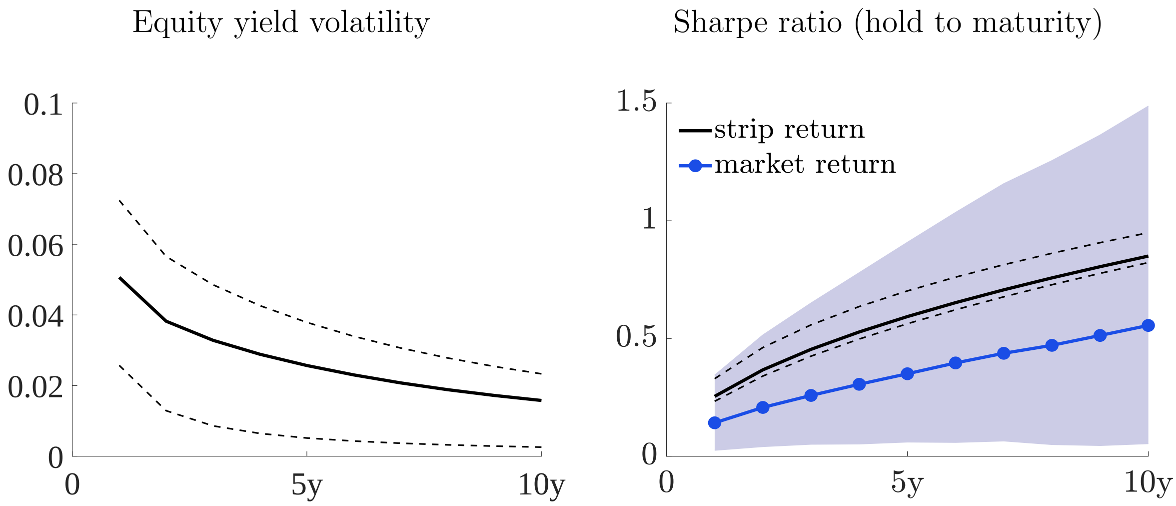
Simulated moments

	data	model		
		50%	[5%	95%]
$E(\Delta c)$	1.83	2.21	[1.25	3.15]
$\sigma(\Delta c)$	2.19	3.13	[2.53	4.08]
$\rho(\Delta c)$	0.48	0.26	[0.01	0.46]
$E(\Delta d)$	1.00	2.25	[-2.00	6.25]
$\sigma(\Delta d)$	11.15	13.78	[11.12	17.67]
$\rho(\Delta d)$	0.20	0.25	[0.01	0.44]

Notes: Top panel - The steady state probabilities for the expansion and recession states are $(1-p_2)/(2-p_1-p_2) = 0.8511$ and $(1-p_1)/(2-p_1-p_2) = 0.1489$, respectively. The steady state consumption growth mean is $\bar{\mu} = (1-p_2)/(2-p_1-p_2)\mu(1) + (1-p_1)/(2-p_1-p_2)\mu(2) = 0.0019$. Bottom panel - The table is constructed based on $T = 50$ years of simulated data which is repeated $N = 10,000$ times.

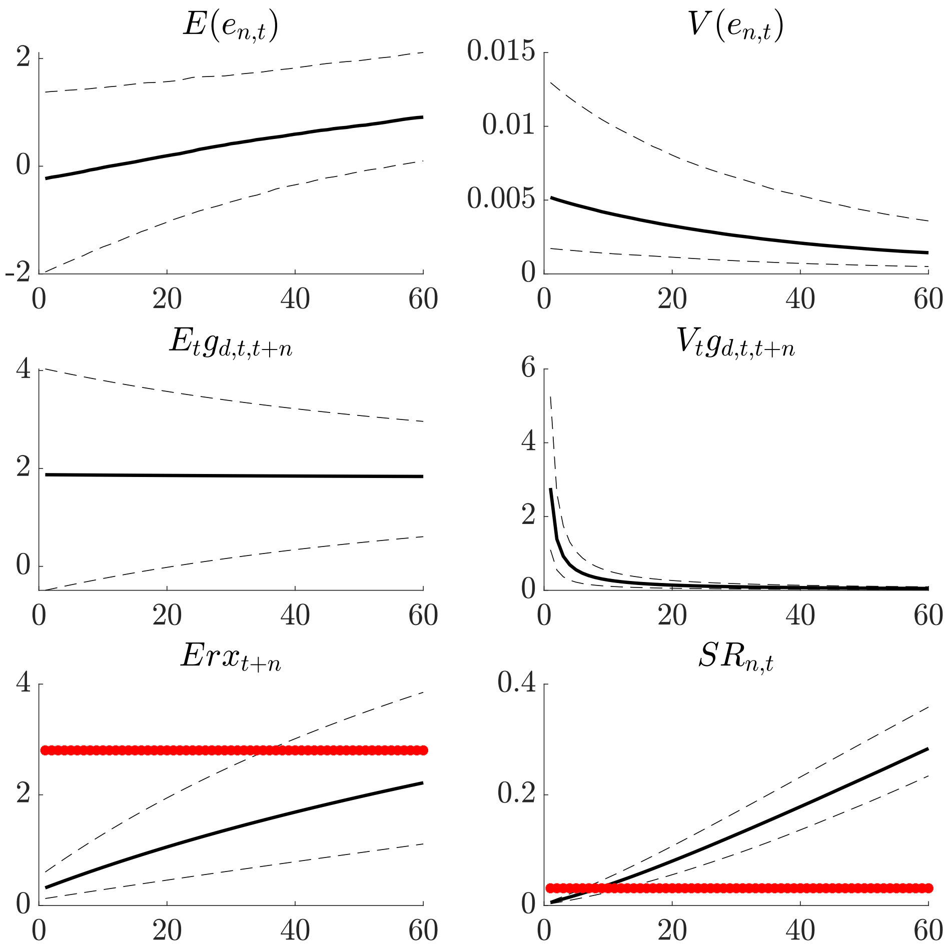
B.11 Model implication

Figure A-5: Model-implied moments



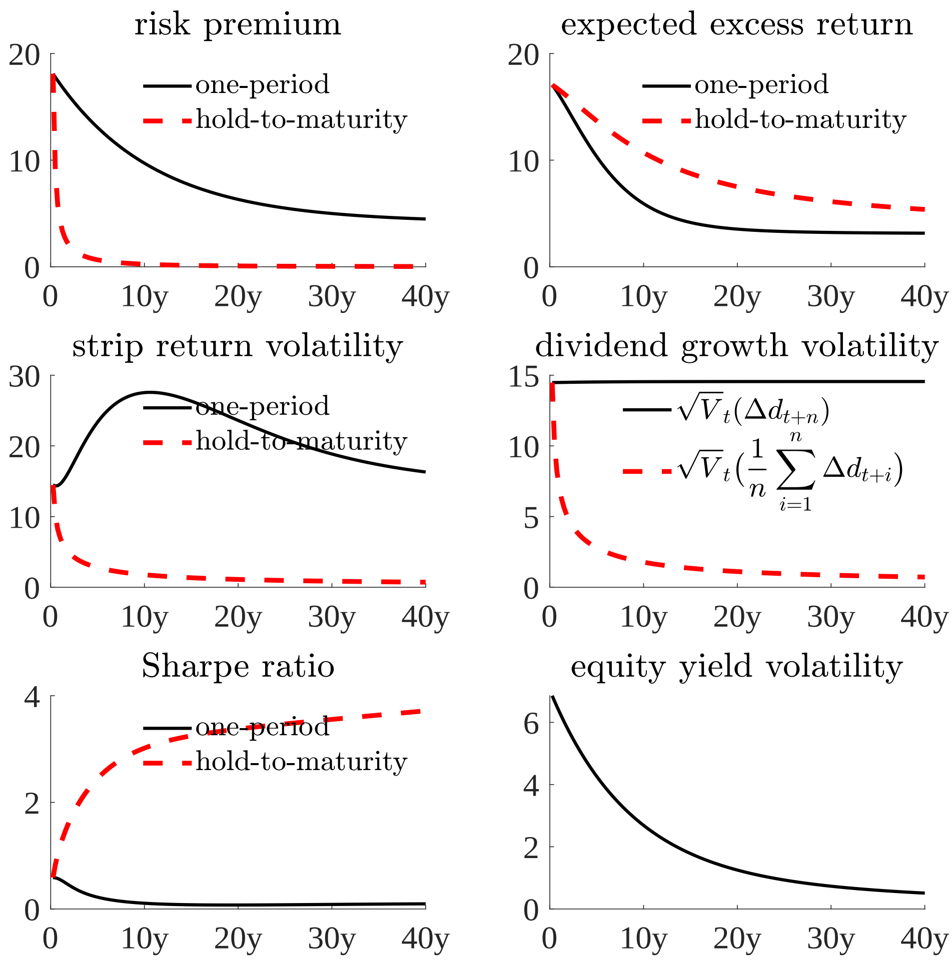
Notes: We simulate the model 10,000 times for 50 years. For each simulated data, we compute the standard deviation of the equity yields, $\text{std}(\{e_{n,t}\}_{t=1}^{T})$ and sample average of Sharpe ratio $\frac{1}{T}\sum_{t=1}^{T} SR_{n,t}$. The solid lines indicate the median value and the dashed-lines indicate the 90% interval. Equity yield volatility is multiplied by 12. For ease of comparison across different horizons, we define the average n-period-ahead return of the market as $r_{d,t+n} = \frac{1}{n}\sum_{j=1}^{n} r_{d,t+j}$. We compute its Sharpe ratio as $SR_n^d = E(\{r_{d,t+n} - y_{n,t}^r\}_{t=1}^{T-n})/\text{std}(\{r_{d,t+n}\}_{t=1}^{T-n})$. Note that while $SR_{n,t}$ is computed analytically, SR_n^d is computed via simulation. Because the realization of $r_{d,t+n} - y_{n,t}^r$ can be negative somtimes, it is possible that $SR_n^d < 0$ under particular simulation paths.

Figure A-6: Bansal and Yaron (2004) original model-implied moments



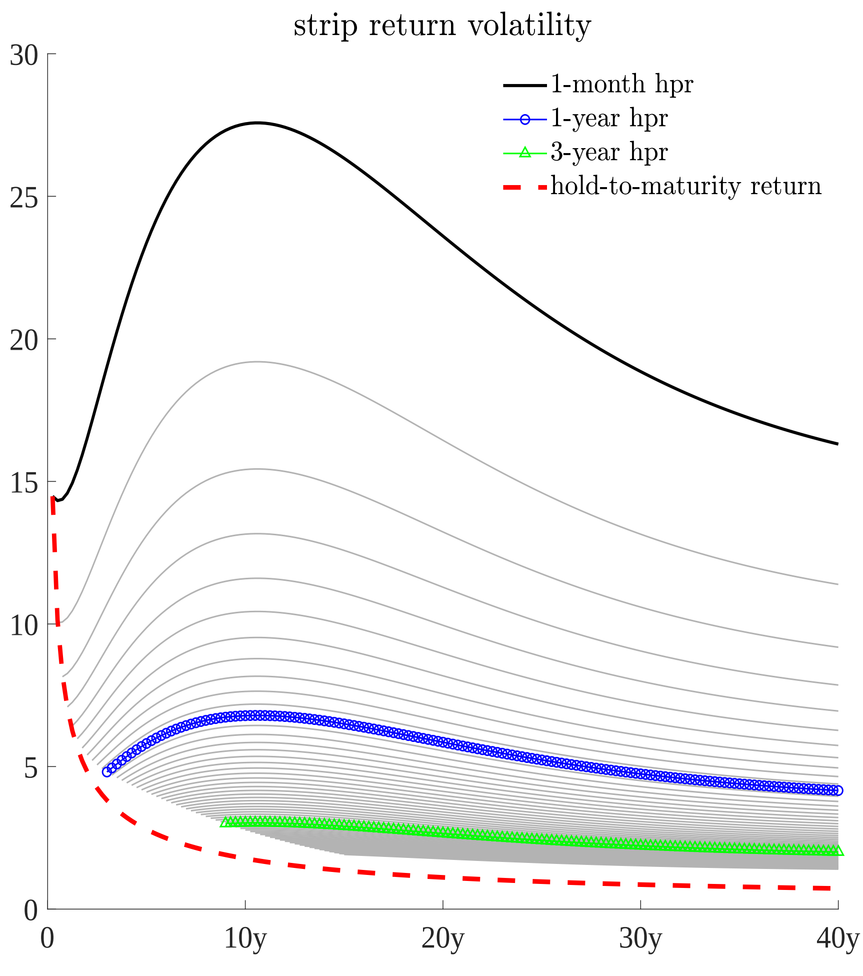
Notes: We simulate the LRR model with BKY (2012) calibration 1,000 times for 50 years. For each simulated data, we compute various moments and provide their averages. The solid lines indicate the median value and the dashed-lines indicate the 90% interval. The y-axis (except for the Sharpe ratio) is in annualized percentage terms. The red-circled-lines indicate the counterpart of the market return.

Figure A-7: Lettau and Wachter (2007) original model-implied moments



Notes: Black-solid lines reproduce Lettau and Wachter (2007). Red-dashed lines are with respect to the hold-to-maturity returns. The y-axis (except for the Sharpe ratio) is in annualized percentage terms.

Figure A-8: Lettau and Wachter (2007) original model-implied moments



Notes: Black-solid lines reproduce Lettau and Wachter (2007). Red-dashed lines are with respect to the hold-to-maturity returns. The y-axis is in annualized percentage terms.

C Solving the No-Arbitrage Model

This section provides approximate analytical solutions for the asset prices.

C.1 Exogenous dynamics

The joint dynamics of consumption and dividend growth are

$$\begin{aligned}
\Delta c_{t+1} &= \mu(S_{t+1}) + x_{t+1} + \sigma_c \eta_{c,t+1}, \quad \eta_{c,t+1} \sim N(0,1), \\
\Delta d_{t+1} &= \bar{\mu} + \phi(\Delta c_{t+1} - \bar{\mu}) + \sigma_d \eta_{d,t+1}, \quad \eta_{d,t+1} \sim N(0,1), \\
x_{t+1} &= \rho x_t + \sigma_x(S_{t+1})\epsilon_{t+1},
\end{aligned} \tag{A-31}$$

where $\bar{\mu}$ is the unconditional mean of consumption growth and x_t is the persistent component of consumption growth. Agents observe the current regime, S_t, and make forecast of future regime, S_{t+1}, based on the transition matrix below

$$\mathbb{P} = \begin{bmatrix} p_1 & 1-p_1 \\ 1-p_2 & p_2 \end{bmatrix}. \tag{A-32}$$

C.2 Stochastic discount factor

Assume that the log stochastic discount factor is

$$m_{t+1} = -r_{t+1} - \frac{1}{2}\lambda(S_{t+1})^2 - \lambda(S_{t+1})\epsilon_{t+1}. \tag{A-33}$$

The risk-free rate is exogenously defined by $r_{t+1} = r_0(S_{t+1}) + r_1(S_{t+1})x_{t+1}$. We assume that the market price of risk is $\lambda_{t+1} = \lambda(S_{t+1})$.

C.3 Real bond prices

Conjecture that $b_{n,t}$ depends on the regime S_t and x_t,

$$b_{n,t} = b_{n,0}(S_t) + b_{n,1}(S_t)x_t. \tag{A-34}$$

Exploit the law of iterated expectations

$$b_{n,t} \;\; = \;\; \ln E_t\Big(E[\exp(m_{t+1} + b_{n-1,t+1})|S_{t+1}]\Big)$$

and log-linearization to solve for $b_{n,t}$

$$b_{n,t} \;\; \approx \;\; \sum_{j=1}^{2} \mathbb{P}_{ij}\Big(E[m_{t+1} + b_{n-1,t+1}|S_{t+1}] + \frac{1}{2}Var[m_{t+1} + b_{n-1,t+1}|S_{t+1}]\Big).$$

The solution to (A-34) is

$$\begin{bmatrix} b_{n,0}(1) \\ b_{n,0}(2) \end{bmatrix} = \mathbb{P} \times \begin{bmatrix} b_{n-1,0}(1) - r_0(1) + 0.5\big(b_{n-1,1}(1) - r_1(1)\big)^2\sigma_x(1)^2 - \big(b_{n-1,1}(1) - r_1(1)\big)\sigma_x(1)\lambda(1) \\ b_{n-1,0}(2) - r_0(2) + 0.5\big(b_{n-1,1}(2) - r_1(2)\big)^2\sigma_x(2)^2 - \big(b_{n-1,1}(2) - r_1(2)\big)\sigma_x(2)\lambda(2) \end{bmatrix} \tag{A-35}$$

$$\begin{bmatrix} b_{n,1}(1) \\ b_{n,1}(2) \end{bmatrix} = \mathbb{P} \times \begin{bmatrix} \big(b_{n-1,1}(1) - r_1(1)\big)\rho \\ \big(b_{n-1,1}(1) - r_1(1)\big)\rho \end{bmatrix}$$

with the initial condition $b_{0,0}(i) = 0$ and $b_{0,1}(i) = 0$ for $i \in \{1, 2\}$. The real yield of the maturity n-period bond is $y^r_{n,t} = -\frac{1}{n}b_{n,t}$.

C.4 Price to dividend ratio of zero coupon equity

Conjecture that the log price to dividend ratio of zero coupon equity $z_{n,t}$ depends on the regime S_t and persistent component x_t,

$$z_{n,t} = z_{n,0}(S_t) + z_{n,1}(S_t)x_t. \tag{A-36}$$

Exploit the law of iterated expectations

$$Z_{n,t} \;\; = \;\; E_t\Big(E[M_{t+1}Z_{n-1,t+1}\frac{D_{t+1}}{D_t}|S_{t+1}]\Big)$$

Take log

$$z_{n,t} \;\; = \;\; \ln E_t\Big(E[\exp(m_{t+1} + z_{n-1,t+1} + \Delta d_{t+1})|S_{t+1}]\Big)$$

and log-linearization to solve for $z_{n,t}$

$$z_{n,t} \approx \sum_{j=1}^{2} \mathbb{P}_{ij}\Big(E[m_{t+1}+z_{n-1,t+1}+\Delta d_{t+1}|S_{t+1}]+\frac{1}{2}Var[m_{t+1}+z_{n-1,t+1}+\Delta d_{t+1}|S_{t+1}]\Big).$$

The solution is

$$\begin{bmatrix} z_{n,0}(1) \\ z_{n,0}(2) \end{bmatrix} = \begin{bmatrix} (1-\phi)\bar{\mu}+\frac{\phi^2}{2}\sigma_c^2+\frac{1}{2}\sigma_d^2 \\ (1-\phi)\bar{\mu}+\frac{\phi^2}{2}\sigma_c^2+\frac{1}{2}\sigma_d^2 \end{bmatrix} + \mathbb{P}\times \begin{bmatrix} z_{n-1,0}(1)+\phi\mu(1)-r_0(1)+\Xi(1) \\ z_{n-1,0}(2)+\phi\mu(2)-r_0(2)+\Xi(2) \end{bmatrix} \tag{A-37}$$

$$\Xi(j) = \frac{1}{2}\big(z_{n-1,1}(j)-r_1(j)+\phi\big)^2\sigma_x(j)^2 - \big(z_{n-1,1}(j)-r_1(j)+\phi\big)\sigma_x(j)\lambda(j)$$

$$\begin{bmatrix} z_{n,1}(1) \\ z_{n,1}(2) \end{bmatrix} = \mathbb{P}\times \begin{bmatrix} \big(z_{n-1,1}(1)-r_1(1)+\phi\big)\rho \\ \big(z_{n-1,1}(1)-r_1(1)+\phi\big)\rho \end{bmatrix}. \tag{A-38}$$

The initial condition is $z_{0,0}(i)=0$ and $z_{0,1}(i)=0$ for $i\in\{1,2\}$.

C.5 m-holding-period and hold-to-maturity expected return

The price of zero coupon equity is $P_{n,t}=Z_{n,t}D_t$. Define the m-holding period return of the n-maturity equity is

$$R_{n,t+m} = \frac{Z_{n-m,t+m}}{Z_{n,t}}\frac{D_{t+m}}{D_t}. \tag{A-39}$$

The corresponding log expected return is defined by

$$E_t[r_{n,t+m}] = \frac{1}{m}E_t\big(z_{n-m,t+m}-z_{n,t}+\sum_{i=1}^{m}\Delta d_{t+i}\big) \tag{A-40}$$

To compute the excess return, we subtract the real rate of the same maturity

$$E_t[r_{n,t+m}] - y^r_{m,t}. \tag{A-41}$$

We consider two cases

- $m\neq n$: This is the m-holding-period expected excess return of the n maturity equity.

$$\begin{aligned} E_t[g_{d,t+m}] &= \frac{1}{m} E_t\big(\sum_{i=1}^{m} \Delta d_{t+i}\big) \\ e_{n,m,t} &= \frac{1}{m} E_t\big(z_{n-m,t+m} - z_{n,t}\big) \\ E_t[r_{n,t+m}] &= e_{n,m,t} + E_t[g_{d,t+m}] \\ E_t[rx_{n,t+m}] &= E_t[r_{n,t+m}] - y^r_{m,t}. \end{aligned} \tag{A-42}$$

- $m = n$: This is the hold-to-maturity expected excess return of the n maturity equity. Define

$$\begin{aligned} E_t[g_{d,t+n}] &= \frac{1}{n} E_t\big(\sum_{i=1}^{n} \Delta d_{t+i}\big) \\ e_{n,t} &= \frac{1}{n} E_t\big(- z_{n,t}\big) \\ E_t[r_{t+n}] &= e_{n,t} + E_t[g_{d,t+n}] \\ E_t[rx_{t+n}] &= E_t[r_{t+n}] - y^r_{n,t}. \end{aligned} \tag{A-43}$$

C.6 Computing moments

The cumulative sum of log dividend growth rates are

$$\begin{aligned} \sum_{i=1}^{n} \Delta d_{t+i} = n(1-\phi)\bar{\mu} + \phi\big(\mu(S_{t+1}) + ... + \mu(S_{t+n})\big) + \phi\rho\left(\frac{1-\rho^n}{1-\rho}\right) x_t \\ + \phi\left(\frac{1-\rho^n}{1-\rho}\right)\sigma_x(S_{t+1})\epsilon_{t+1} + ... + \phi\left(\frac{1-\rho}{1-\rho}\right)\sigma_x(S_{t+n})\epsilon_{t+n} \\ + \phi\sigma_c(\eta_{c,t+1} + ... + \eta_{c,t+n}) + \sigma_d(\eta_{d,t+1} + ... + \eta_{d,t+n}). \end{aligned} \tag{A-44}$$

For ease of exposition, we introduce the following notations

$$\boldsymbol{\mu} = [\mu(1), \mu(2)]', \quad \boldsymbol{\sigma_x^2} = [\sigma_x(1)^2, \sigma_x(2)^2]'.$$

The first two moments of the average log dividend growth rates for the case of $S_t = k$ are

$$\begin{aligned} E_t[g_{d,t+n}] &= \frac{1}{n}E_t[\sum_{i=1}^{n}\Delta d_{t+i}] = \frac{1}{n}\boldsymbol{\mu_G}(k) \\ V_t[g_{d,t+n}] &= \frac{1}{n^2}V_t[\sum_{i=1}^{n}\Delta d_{t+i}] = \frac{1}{n^2}\boldsymbol{\sigma^2_G}(k) \end{aligned} \tag{A-45}$$

where

$$\begin{aligned} \boldsymbol{\mu_G} &= \begin{bmatrix} n(1-\phi)\bar{\mu} + \phi\rho\left(\frac{1-\rho^n}{1-\rho}\right)x_t \\ n(1-\phi)\bar{\mu} + \phi\rho\left(\frac{1-\rho^n}{1-\rho}\right)x_t \end{bmatrix} + \sum_{j=1}^{n}\phi\mathbb{P}^j\boldsymbol{\mu} \\ \boldsymbol{\sigma^2_G} &\approx \begin{bmatrix} n(\phi^2\sigma_c^2 + \sigma_d^2) \\ n(\phi^2\sigma_c^2 + \sigma_d^2) \end{bmatrix} + \phi^2\sum_{j=1}^{n}\left(\frac{1-\rho^{n+1-j}}{1-\rho}\right)^2\mathbb{P}^j\boldsymbol{\sigma^2_x}. \end{aligned} \tag{A-46}$$

We acknowledge that the expression for $\boldsymbol{\sigma^2_G}$ is not exact because we are ignoring the variance component associated with uncertainty about $\mu(S_{t+j})$.

The expressions in (A-45) allow us to calculate the Sharpe ratio

$$SR_{n,t} = \frac{e_{n,t} + E_t[g_{d,t+n}] - y^r_{n,t}}{\sqrt{V_t[g_{d,t+n}]}} \tag{A-47}$$

In the main text, we report the case of $x_t = 0$ for ease of illustration, e.g., $E_t g_{d,t+n}|_{x_t=0}$ and $V_t g_{d,t+n}|_{x_t=0}$.

C.7 Market return

We derive the market return via Campbell-Shiller approximation

$$\begin{aligned} r_{m,t+1} &= \kappa_0 + \kappa_1 z_{m,0}(S_{t+1}) - z_{m,0}(S_t) + \bar{\mu}(1-\phi) + \phi\mu(S_{t+1}) \\ &\quad +(\phi\rho + \kappa_1 z_{m,1}(S_{t+1})\rho - z_{m,1}(S_t))x_t \\ &\quad +(\phi + \kappa_1 z_{m,1}(S_{t+1}))\sigma_x(S_{t+1})\epsilon_{t+1} + \phi\sigma_c\eta_{c,t+1} + \sigma_d\eta_{d,t+1} \end{aligned} \tag{A-48}$$

where the log price-dividend ratio is given by

$$z_t = z_{m,0}(S_t) + z_{m,1}(S_t)x_t. \tag{A-49}$$

The market equity premium is

$$\begin{aligned} E_t[r_{m,t+1}] - y^r_{1,t} + \frac{1}{2}V_t[r_{m,t+1}] &= -Cov_t(r_{m,t+1}, m_{t+1}) \\ &= \mathbb{P} \times \begin{bmatrix} (\phi + \kappa_1 z_{m,1}(1))\sigma_x(1)\lambda(1) \\ (\phi + \kappa_1 z_{m,1}(2))\sigma_x(2)\lambda(2) \end{bmatrix}. \end{aligned} \tag{A-50}$$

The conditional variance of the market return is

$$V_t[r_{m,t+1}] \approx \begin{bmatrix} \phi^2\sigma_c^2 + \sigma_d^2 \\ \phi^2\sigma_c^2 + \sigma_d^2 \end{bmatrix} + \mathbb{P} \times \begin{bmatrix} (\phi + \kappa_1 z_{m,1}(1))^2\sigma_x(1)^2 \\ (\phi + \kappa_1 z_{m,1}(2))^2\sigma_x(2)^2 \end{bmatrix}. \tag{A-51}$$

The market Sharpe ratio is

$$SR_t = \frac{E_t[r_{m,t+1}] - y^r_{1,t}}{\sqrt{V_t[r_{m,t+1}]}}. \tag{A-52}$$

Here, we are not accounting for $\frac{1}{2}V_t[r_{m,t+1}]$ in the numerator.

C.8 Calibration

With this calibration, we derive the market return via Campbell-Shiller approximation and compute the expected excess return of the market. The equity premium is 4.13 and 18.60 in expansion and recession, respectively. The unconditional average (weighted by steady state probability) is around 6.29.

Table A-5: Calibration

Parameters

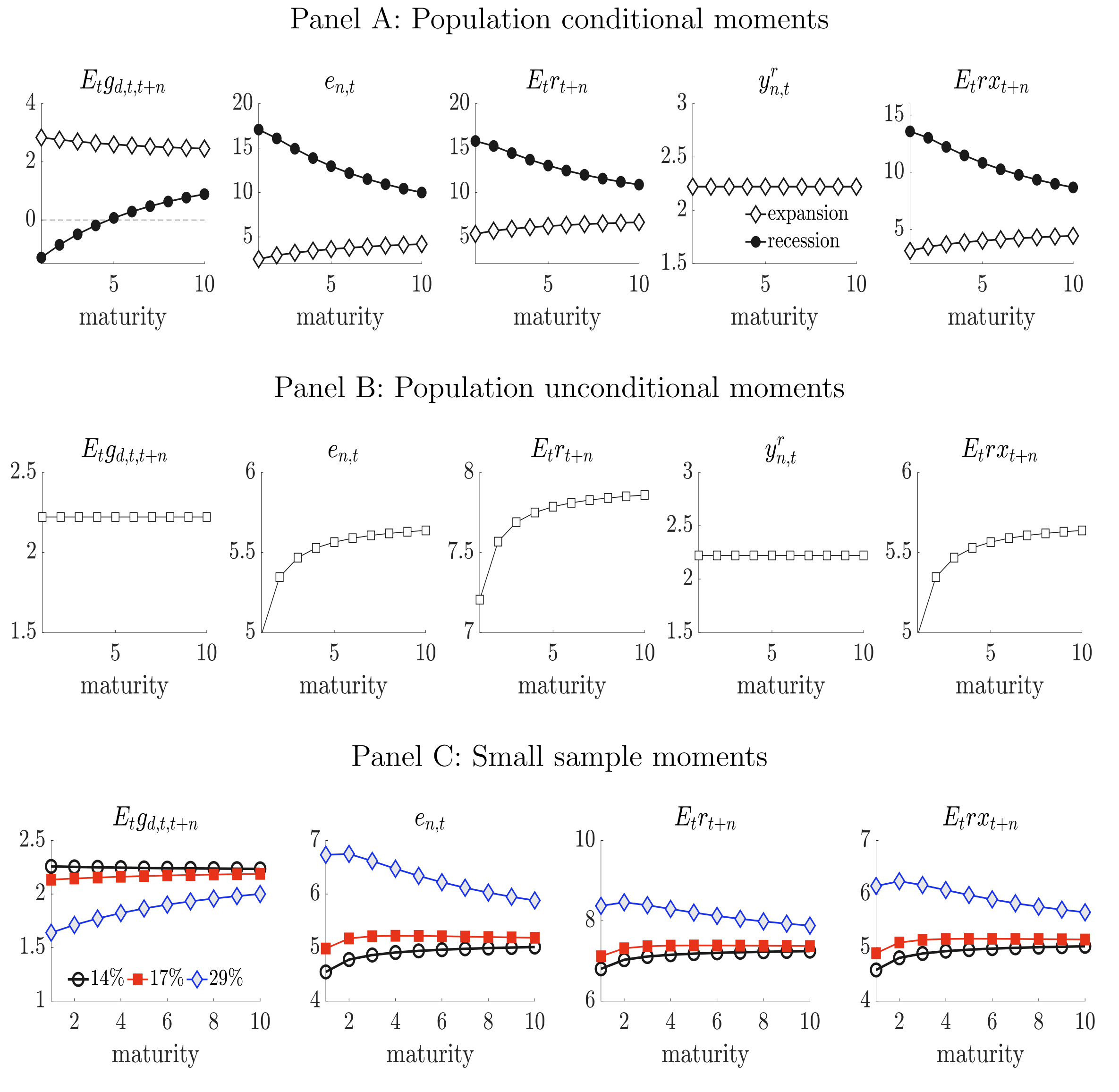
$\mu(1)$	0.0020	ρ	0.60	$\lambda(1)$	0.1315
$\mu(2)$	0.0010	$\sigma_x(1)$	0.0033	$\lambda(2)$	0.2789
σ_c	0.0063	$\sigma_x(2)$	0.0064	$r(1)$	0.0019
ϕ	4.0	p_1	0.9965	$r(2)$	0.0019
σ_d	0.0224	p_2	0.98		

Simulated moments

	data	model		
		50%	[5%	95%]
$E(\Delta c)$	1.83	2.21	[1.25	3.15]
$\sigma(\Delta c)$	2.19	3.13	[2.53	4.08]
$\rho(\Delta c)$	0.48	0.26	[0.01	0.46]
$E(\Delta d)$	1.00	2.25	[-2.00	6.25]
$\sigma(\Delta d)$	11.15	13.78	[11.12	17.67]
$\rho(\Delta d)$	0.20	0.25	[0.01	0.44]

Notes: Top panel - The steady state probabilities for the expansion and recession states are $(1-p_2)/(2-p_1-p_2) = 0.8511$ and $(1-p_1)/(2-p_1-p_2) = 0.1489$, respectively. The steady state consumption growth mean is $\bar{\mu} = (1-p_2)/(2-p_1-p_2)\mu(1) + (1-p_1)/(2-p_1-p_2)\mu(2) = 0.0019$. Risk-free rate coefficients are $r(1) = r(2) = \bar{\mu}$. Bottom panel - The table is constructed based on $T = 50$ years of simulated data which is repeated $N = 10,000$ times.

Figure A-9: Model-implied moments



Notes: We set $\mu(1) = 2.4, \mu(2) = 1.2, \sigma_c = 2.2, \rho = 0.50, \sigma_x(1) = 1.13, \sigma_x(2) = 2.41$, $p_1 = 0.9965, p_2 = 0.98$. Dividend growth dynamics are set according to $\phi = 4, \sigma_d = 6$. The market price of risk is set to $\lambda(1) = 0.13$ and $\lambda(2) = 0.28$. The risk-free rate is 2.2. While we use a monthly model to compute these components, parameter calibration is reported in annualized term. Panel A and B - We examine the case of $x_t = 0$. Panel C - In the data, the sample recession frequency is 14%, 17%, and 29% of the sample from December 2004 to February 2016 for the U.S., Japan, and Europe respectively. Motivated from this, we average the moments implied from the model across the two states with the probabilities obtained from the data to compute various sample averages.

D Forecasting Dividend Growth Rates

We expect that the dynamics of expected dividend growth rates, and consequently, expected returns, see (12), would be quite different conditional on recession and expansion. To show this, we develop a model for forecasting dividend growth rates both in sample and out-of-sample, as discussed below.

D.1 VAR-based dividend forecasts

Let $x_{A,t}$ be a vector of monthly variables that predicts dividend growth. We consider an annual first order VAR dynamics for the predictor vector

$$x_{A,t+12} = \mu_A + \Gamma_A x_{A,t} + \varepsilon_{A,t+12}. \tag{A-53}$$

This is because we are interested in the annual horizon forecasts. There are two ways of estimating the coefficients in (A-53): The first is via the direct projection method and the second is estimate a monthly first order VAR model[23]

$$x_{A,t+1} = \mu_m + \Gamma_m x_{A,t} + \varepsilon_{m,t+1} \tag{A-54}$$

and obtain

$$\mu_A \equiv \left(\sum_{i=0}^{11} \Gamma_m^i\right)\mu, \quad \gamma_A \equiv \Gamma_m^{12}, \quad \varepsilon_{A,t+12} \equiv \sum_{i=0}^{11} \Gamma_m^{12-i} \varepsilon_{m,t+i}.$$

Regressing dividend growth on lagged predictor vector gives the estimates for ψ_0 and ψ_1

$$g_{d,t+12} = \psi_0 + \psi_1 x_{A,t} + \varepsilon_{d,t+12}. \tag{A-55}$$

To recap, $g_{d,t+12} = \ln(\frac{D_{t+12}}{D_t})$ where D_t is the 12-month trailing sum dividends.

For ease of exposition, we stack (A-53) and (A-55) together and express them in an annual first order VAR model as

$$\begin{bmatrix} x_{A,t+12} \\ g_{d,t+12} \end{bmatrix} = \begin{bmatrix} \mu_A \\ \psi_0 \end{bmatrix} + \begin{bmatrix} \Gamma_A & 0 \\ \psi_1 & 0 \end{bmatrix} \begin{bmatrix} x_{A,t} \\ g_{d,t} \end{bmatrix} + \begin{bmatrix} \varepsilon_{A,t+12} \\ \varepsilon_{d,t+12} \end{bmatrix}. \tag{A-56}$$

[23]Note that Binsbergen, Hueskes, Koijen, and Vrugt (2013) follow the second approach.

From (A-56), we derive the conditional expectation of the annual dividend growth n years ahead as

$$E_t[g_{d,t+12n}] = \psi_0 + \psi_1\left(\left[\sum_{i=0}^{n-2}\Gamma_A^i\right]\mu_A + \Gamma_A^{(n-1)}x_t\right). \tag{A-57}$$

The $12n$-month-ahead dividend growth shocks and their cumulative shocks are

$$\begin{aligned} g_{d,t+12n} - E_t[g_{d,t+12n}] &= \psi_1\left(\sum_{i=0}^{n-2}\Gamma_A^i\varepsilon_{A,t+12(n-1-i)}\right) + \varepsilon_{d,t+12n} \\ \sum_{n=1}^{m}\Big(g_{d,t+12n} - E_t(g_{d,t+12n})\Big) &= \psi_1\left(\sum_{j=0}^{m-2}(I-\Gamma_A)^{-1}(I-\Gamma_A^{m-1-j})\varepsilon_{t+12(j+1)}\right) + \sum_{n=1}^{m}\varepsilon_{d,t+12n}. \end{aligned} \tag{A-58}$$

It is straightforward to compute $V_t[g_{d,t+12n}]$ and $V_t\big[\sum_{n=1}^{m} g_{d,t+12n}\big]$ from (A-58).

D.2 Bayesian inference

The sample in which we have equity yields is quite short. However, data on dividend growth rates are available much before. We have seen from Figure A-1 that we can potentially rely on the historical data to learn about the future dividend growth dynamics to the extent that dynamics have not changed substantially over time. In this section, we formally show how one could optimally use prior information (extracted from historical data) and improve forecasts.

Posterior. The first-order vector autoregression (A-56) can be always re-written as

$$y_t = \Phi x_t + \varepsilon_t, \quad \varepsilon_t \sim N(0,\Sigma) \tag{A-59}$$

where $y_t = \left[x'_{A,t}, g_{d,t}\right]'$ and $x_t = \left[1, x'_{A,t-12}\right]'$. Define $Y = [y_{13},...,y_T]'$, $X = [x_{13},...,x_T]'$, and $\varepsilon = [\varepsilon_{13},...,\varepsilon_T]'$. Taking the initial 12 observations as given, if the prior is

$$\Phi|\Sigma \sim MN\big(\underline{\Phi}, \Sigma\otimes(\underline{V}_\Phi\xi)\big), \quad \Sigma \sim IW(\Psi, d) \tag{A-60}$$

then the posterior can be expressed as

$$\Phi|\Sigma \sim MN\big(\overline{\Phi}, \Sigma\otimes\overline{V}_\Phi\big), \quad \overline{\Phi} = \left(X'X + (\underline{V}_\Phi\xi)^{-1}\right)^{-1}\left(X'Y + (\underline{V}_\Phi\xi)^{-1}\underline{\Phi}\right) \tag{A-61}$$

because of the conjugacy.[24] Here, ξ is a scalar parameter controlling the tightness of the prior information.

The elicitation of prior. Suppose that we can divide the sample into the pre-sample, estimation sample, and prediction sample. We set the prior mean $\underline{\Phi}$ equal to the pre-sample OLS estimate, from the 1980-2004 sample for the U.S. and the 1995-2004 sample for Europe and Japan. Here, prior becomes more informative when $\xi \to 0$. In the limit, posterior equals the pre-sample OLS estimate, i.e., prior. In contrast, when $\xi = \infty$, then it is easy to see that $\overline{\Phi} = \hat{\Phi} = (X'X)^{-1}(X'Y)$, i.e., an OLS estimate from the estimation sample. In this case, prior does not play any role. We can optimize the scaling parameter by choosing the value that maximizes the marginal likelihood function, $\hat{\xi} = \text{argmax } p(Y|\xi)$. The closed form of the marginal likelihood function is available in the appendix. We refer to Giannone, Lenza, and Primiceri (2015) for a detailed discussion.

D.3 Model selection and estimation

The VAR expression in (A-56) can describe the approach of Binsbergen, Hueskes, Koijen, and Vrugt (2013) where the predictor vector x_A comprises the 2-year and 5-year forward equity yields. We refer to this three-variable (dividend growth plus two predictors) VAR, identical to that of BHKV, as the Short Sample Predictor (SSP) approach for simplicity. We propose a different three-variable VAR where the predictor vector x_A comprises the 5y-1y nominal bond yield spread and dividend to earnings ratio, which is referred to as the Long Sample Predictor (LSP) approach. In addition to improved forecast accuracy, the LSP will allow us to conduct out-of-sample forecasting because these predictors have a longer history than dividend yields.

Ideally, we would like to conduct both in-sample and out-of-sample forecast exercises for the SSP and LSP approaches. Unfortunately, we cannot conduct out-of-sample forecast exercise for the SSP approach since their predictor variables, the 2-year and 5-year forward equity yields, are only available from 2004:M12 according to our data vendor. The VAR coefficients in the SSP approach cannot be recursively estimated unless the prediction sample is substantially shortened. This is not ideal given the data availability. The reason we desire to use recursive updating and out-of-sample

[24] Since we are mainly interested in the conditional expectation, we omit the expression for Σ. The readers are referred to Giannone, Lenza, and Primiceri (2015).

forecasting using the LSP is to establish that our results hold in real-time forecasts. This study is the first to show the term structure of expected growth and returns in real time.

To minimize confusion, we define two estimation strategies. The in-sample estimation is carried out with data from 2004:M12 to 2017:M2 using the maximum available data. We can formally conduct model selection and compare the in-sample forecasting performance of the SSP and LSP approaches. When we generate in-sample forecasts, we allow for look-ahead bias by including all the data in the estimation at once. Here, we briefly describe the model selection result. The forecast results are discussed shortly. We choose the LSP approach over the SSP approach based on the model selection via the marginal likelihood maximization. For U.S., the log marginal likelihood values are 430 versus 374; for Europe, they are 704.4 versus 58.5; for Japan, 596 versus 135, all in favor of LSP over SSP approach.

To define the out-of-sample estimation period, we first set the prediction sample to 2005:M1 to 2013:M2. The initial out-of-sample estimation starts from 2001:M1 to 2004:M12. When we move the forecast origin from 2005:M1 to 2013:M2, the posterior VAR coefficients are also updated as we recursively increase the sample. In doing so, we optimize the scaling parameter ξ that controls the tightness of the prior.

With respect to prior information, we use the sample before the prediction sample to obtain prior for the VAR coefficients. Specifically, we use data from 1979:M12 to 2000:M12 (U.S.) and from 1994:M12 to 2000:M12 (Europe and Japan) to elicit prior information. It is possible for the LSP approach because the 5y-1y nominal bond yield spread and dividend to earnings ratio are available. Since elicitation of prior information is not possible for the SSP approach, we set the scaling parameter to $\xi = \infty$ so that (whichever specified) prior does not play any role and the posterior mean is identical to the OLS estimate. Thus, we can rely on the same expression (A-61) to generate posterior forecasts for both the SSP and LSP approaches.

D.4 Forecast results

The dividend growth rate forecasts are generated up to 5-year-out to maximize the data availability.[25] Table A-6 summarizes the root mean squared errors (RMSEs) of

[25] One could generate upto 7-year-out horizon which results in shortening the prediction sample to 2005:M1-2011:M2 instead of 2005:M1-2013:M2.

Table A-6: Root mean squared errors for the dividend growth rate forecasts

	Panel A: U.S.			Panel B: Europe			Panel C: Japan		
	SSP	LSP	LSP	SSP	LSP	LSP	SSP	LSP	LSP
n			o.o.s.			o.o.s.			o.o.s.
1y	8.41	9.82	10.10	8.46	7.59	20.14	11.62	12.79	18.32
2y	9.51	8.60	9.43	10.24	7.10	15.73	9.91	8.70	14.98
3y	9.19	7.03	8.10	9.11	6.38	12.01	7.96	6.78	11.80
4y	8.20	5.87	6.89	6.32	4.26	10.05	5.68	5.22	9.00
5y	6.73	4.68	5.75	4.98	3.07	9.33	4.44	3.93	8.26

Notes: For the in-sample forecasts, we use data from 1979:M12 to 2004:M12 (U.S.) and from 1994:M12 to 2004:M12 (Europe and Japan) to elicit prior information. The estimation sample is from 2005:M1 to 2017:M2. Our prediction sample is from 2005:M1 to 2013:M2. For the out-of-sample forecasts, we use data from 1979:M12 to 2000:M12 (U.S.) and from 1994:M12 to 2000:M12 (Europe and Japan) to elicit prior information. The initial estimation sample is from 2001:M1 to 2004:M12. We increase the estimation sample recursively as we move the forecast origin. The root mean squared errors based on the out-of-sample forecasts are indicated with o.o.s.

the dividend growth rate forecasts for the three markets. Let us focus on Panel A: U.S. and compare the RMSEs from the out-of-sample LSP (o.o.s.) with those from the in-sample SSP approach. The results are surprising given that the LSP approach (o.o.s.) is at a clear informational disadvantage compared with the SSP approach. Except at the 1-year horizon, we find that the RMSEs for the respective horizons are much smaller than those from the SSP approach. What is interesting is the magnitude of the RMSEs, whether they are based on in-sample or out-of-sample forecasts, are similar for the LSP approach. This evidence strongly suggests the superior forecast performance of our VAR approach and the usefulness of extracting information embedded in the historical data in the form of priors.

The RMSEs from the out-of-sample forecasts are uniformly larger (roughly by a factor of two) than those of the in-sample SSP forecasts for Europe and Japan. Again, the large RMSE and apparent changes in dividend dynamics relative to the pre-sample suggests that these dividend growth events were unexpected. For in sample forecasts, our model continues to produce superior estimates to those of Binsbergen, Hueskes, Koijen, and Vrugt (2013) and dominates in marginal likelihood.

Once the expected dividend growth rates are generated, given the forward equity yields and real rates, we are able to compute the expected return (see (12)), excess return (see (13)), and hold to maturity Sharpe Ratio (see (15)) as well. We construct

Table A-7: Out-of-sample forecasts of dividend growth, prices, and returns: U.S.

	Exp. growth	Exp. return	Premium	Sharpe ratio	STRIPS	YLD
			Entire period			
1y	2.93	0.39	0.35	0.08	-4.58	2.04
2y	2.61	0.82	0.65	0.23	-3.96	2.17
3y	2.59	1.30	0.93	0.38	-3.66	2.37
4y	2.62	1.75	1.16	0.50	-3.46	2.59
5y	2.65	2.22	1.39	0.65	-3.26	2.83
5y-1y	-0.28	1.83	1.04	0.57	1.32	0.79
			Expansion period			
1y	5.03	-2.99	-3.28	-0.96	-10.31	2.29
2y	3.87	-1.38	-1.77	-0.61	-7.64	2.39
3y	3.40	-0.20	-0.73	-0.29	-6.14	2.53
4y	3.19	0.53	-0.18	-0.09	-5.37	2.72
5y	3.07	1.16	0.25	0.10	-4.83	2.92
5y-1y	-1.96	4.14	3.52	1.06	5.48	0.62
			Recession period			
1y	-3.45	10.65	11.39	3.24	12.84	1.26
2y	-1.22	7.51	8.00	2.78	7.22	1.52
3y	0.10	5.86	5.99	2.38	3.89	1.87
4y	0.88	5.45	5.25	2.32	2.36	2.20
5y	1.36	5.43	4.86	2.34	1.49	2.58
5y-1y	4.81	-5.22	-6.54	-0.90	-11.34	1.32

Notes: We provide the annualized average expected dividend growth rates $E_t[g_{t+n}]$ ("Exp. growth"); the expected discount rate $E_t[r_{t+n}]$ ("Exp. return"), computed as in (12): $E_t[r_{t+n}] = e_{n,t} + E_t[g_{d,t+n}]$; the expected excess return $E_t rx_{t+n}$ ("Premium"), computed as in (13): $E_t[rx_{t+n}] = E_t[r_{t+n}] - y^r_{n,t}$; the Sharpe ratio $SR_{n,t}$ ("Sharpe ratio"), computed as in (15): $SR_{n,t} = \frac{E_t[rx_{t+n}]}{\sqrt{V_t[g_{d,t+n}]}}$; the forward equity yields $e^f_{n,t}$ ("STRIPS"); and the nominal bond yields $y_{n,t}$ ("YLD"). Results are based on the LSP approach, a 3-variable VAR approach that includes 5y-1y nominal bond yield spread, asset dividend to earnings ratio, and dividend growth. The initial estimation sample for the LSP approach is from 2001:M1 to 2004:M12. We increase the estimation sample recursively as we move the forecast origin, i.e., 2005:M1 to 2013:M2. The sample average of inflation rates is around 2%.

the real rate proxy by subtracting the average inflation from the nominal yields.[26] To be conservative and to save space, we only show the out-of-sample forecast results for

[26]Given the relatively small variation of inflation rates, especially relative to the large movements in real growth and discount rates, by horizon within the recession and expansion subsamples, it is highly unlikely that compensation for inflation risk substantively has any bearing on our measure of risk premia. For U.S., we later replace with the Treasury Inflation-Protected Securities (TIPS) to confirm the robustness of our results.

the U.S. market. We provide the in-sample forecast results for the three markets in the appendix in Tables A-9, A-10, and A-11.

Table A-7 provides the annualized average expected dividend growth rates $E_t[g_{t+n}]$ ("Exp. growth"); the expected discount rate $E_t[r_{t+n}]$ ("Exp. return"); the expected excess return $E_t rx_{t+n}$ ("Premium"); the Sharpe ratio $SR_{n,t}$ ("Sharpe ratio"); the forward equity yields $e^f_{n,t}$ ("STRIPS"); and the nominal bond yields $y_{n,t}$ ("YLD"). We provide the corresponding averages of the entire prediction sample and the averages conditional on whether the forward equity yield spread between 5-year and 1-year is positive or negative. This is because we believe that the negative spread of forward equity yields between 5-year and 1-year closely tracks the recession dates.[27] Remember that these are the averages of real time out-of-sample forecasts. We provide the evidence for the remaining regions in the appendix. We refrain from using the recession indicators to forecast as they are determined ex post. On the other hand, the equity yields and LSP predictors are available to investors in real time.

We summarize the main findings as follows. The slope of the expected dividend growth is negative (positive) during expansions (recessions). The slopes of the expected return, excess return, and sharpe ratio are positive (negative) during expansion (recession). The slopes of the entire period averages of expected return, expected excess return, and sharpe ratio are positive. Both the in and out-of-sample hold to maturity Sharpe Ratios are either upward-sloping (U.S. and Japan) or flat (Europe in sample), however these statistics have large standard errors relative to risk premium estimates. To check the statistical significance of the findings, Table A-8 provides the 90% credible intervals associated with the selective forecasts: expected return and growth. It is interesting to observe that all slopes of the conditional moments are statistically significantly different from zero at the 90% confidence level. The findings are largely robust to the out-of-sample and in-sample forecast results. The in-sample forecast results based on the SSP approach deliver qualitatively similar message, which is provided in Table A-12. Finally, we note that while the estimates of hold to maturity Sharpe Ratios are substantially noisier, the Sharpe Ratio slope is significantly positive in the U.S., where the estimates are best and the recession frequency is in line with the long run mean, at 0.39 for 5y-1y in sample and 0.51 out-of-sample.

[27] Figure A-3 plots the two series, which appear to be highly correlated. The correlation between the equity yield spread and recession indicator is around 65% for the U.S. and Europe and 40% for Japan, respectively. The seemingly low correlation than it appears is because we are computing correlation with a dummary variable.

Table A-8: The spread between 5-year and 1-year forecasts

	In-sample						Out-of-sample					
	Exp. return			Exp. growth			Exp. return			Exp. growth		
	50%	[5%	95%]	50%	[5%	95%]	50%	[5%	95%]	50%	[5%	95%]
						Entire period						
U.S.	2.52*	[1.13,	3.85]	0.40	[-0.98,	1.73]	1.83*	[1.40,	2.25]	-0.28	[-0.72,	0.14]
Europe	-0.28	[-2.68,	2.25]	-0.45	[-2.85,	2.07]	2.21	[-0.32,	4.96]	2.21	[-0.30,	4.99]
Japan	-3.07*	[-5.27,	-0.90]	-1.06	[-3.26,	1.11]	-2.76*	[-5.54,	-0.04]	-0.62	[-3.41,	2.18]
						Expansion period						
U.S.	4.18*	[3.01,	5.32]	-1.91*	[-3.09,	-0.78]	4.14*	[3.83,	4.47]	-1.96*	[-2.28,	-1.63]
Europe	0.04	[-2.17,	2.39]	-7.23*	[-9.44,	-4.88]	9.85*	[7.23,	12.75]	2.85	[-0.47,	5.75]
Japan	1.19	[-0.85,	3.13]	-2.34*	[-4.38,	-0.44]	1.68	[-0.90,	4.23]	-1.92	[-4.49,	0.63]
						Recession period						
U.S.	-2.55*	[-4.58,	-0.64]	7.48*	[5.45,	9.38]	-5.22*	[-6.00,	-4.51]	4.81*	[4.03,	5.51]
Europe	-0.68	[-3.34,	2.06]	8.44*	[5.78,	11.19]	-7.79*	[-10.21,	-5.25]	1.37	[-1.05,	3.91]
Japan	-7.79*	[-10.18,	-5.37]	0.36	[-2.03,	2.78]	-7.48*	[-10.49,	-4.41]	0.75	[-2.25,	3.83]

Notes: We provide the results based on the in-sample and out-of-sample forecasts for $E_t[g_{d,t+5} - g_{d,t+1}]$ and $E_t[r_{t+5} - r_{t+1}]$ computed as in (12): $E_t[r_{t+n}] = e_{n,t} + E_t[g_{d,t+n}]$. For the in-sample forecasts, we use data from 1979:M12 to 2004:M12 (U.S.) and from 1994:M12 to 2004:M12 (Europe and Japan) to elicit prior information. The estimation sample is from 2005:M1 to 2017:M2. Our prediction sample is from 2005:M1 to 2013:M2. For the out-of-sample forecasts, we use data from 1979:M12 to 2000:M12 (U.S.) and from 1994:M12 to 2000:M12 (Europe and Japan) to elicit prior information. The initial estimation sample is from 2001:M1 to 2004:M12. We increase the estimation sample recursively as we move the forecast origin. We use $*$ to indicate the statistical significance at the 90% confidence level.

We find that the sign of the conditional slopes for the European and Japanese markets are broadly consistent with the U.S. market with expected returns sloping downward in recession and upward in booms. Caution is required in interpreting the sample average as an unconditional mean because the balance of recessions in the short sample is not representative for these regions. If the sample overrepresents recessions, as is the case in Europe and Japan, the behavior of the sample average expected growth and dividend discount rate slopes will be biased towards their recession means.

We have shown that the most robust feature of this data is the recession and expansion variation of the growth rate and risk premium term structures. Due to the large differences between recessions and expansions, we develop a regime-switching model in the next section that preserves the core implications of standard asset pricing models - risk unconditionally increases with horizon, both risk and expected growth vary across the cycle, and the riskfree rate is nearly constant - while matching the conditional facts on expected growth and risk premia we document via the BVAR. This model allows us to formally address issues of short sample biases and recession-expansion balance in context and show that both the conditional and sample average facts documented in this section are wholly consistent with standard models like the LRR or habits.

D.5 Bayesian linear regression

Without loss of generality, we can express any linear dynamics by

$$y_t = \Phi x_t + \varepsilon_t, \quad \varepsilon_t \sim N(0, \Sigma). \tag{A-62}$$

For ease of exposition, define $Y = [y_p, ..., y_T]'$, $X = [x_p, ..., x_T]'$, and $\varepsilon = [\varepsilon_p, ..., \varepsilon_T]'$. Assume that the initial p observations are available. Because of the conjugacy if the prior is

$$\Phi|\Sigma \sim MN\big(\underline{\Phi}, \Sigma \otimes (\underline{V}_\Phi \xi)\big), \quad \Sigma \sim IW(\Psi, d) \tag{A-63}$$

then the posterior can be expressed as $\Phi|\Sigma \sim MN\big(\overline{\Phi}, \Sigma \otimes \overline{V}_\Phi\big)$ where

$$\overline{\Phi} = \Big(X'X + (\underline{V}_\Phi \xi)^{-1}\Big)^{-1} \Big(X'Y + (\underline{V}_\Phi \xi)^{-1} \underline{\Phi}\Big), \quad \overline{V}_\Phi = \Big(X'X + (\underline{V}_\Phi \xi)^{-1}\Big)^{-1}.$$

We follow the exposition in Giannone, Lenza, and Primiceri (2015). ξ is a scalar parameter controlling the tightness of the prior information. For instance, prior becomes more informative when $\xi \to 0$. In contrast, when $\xi = \infty$, then it is easy to see that $\overline{\Phi} = \hat{\Phi}$, i.e., an OLS estimate. We can choose ξ that maximizes the marginal likelihood function (A-64), which is available in closed form

$$p(Y|\xi) = \left(\frac{1}{\pi}\right)^{\frac{n(T-p)}{2}} \frac{\Gamma_n(\frac{T-p+d}{2})}{\Gamma_n(\frac{d}{2})} |\underline{V}_\Phi \xi|^{-\frac{n}{2}} |\Psi|^{\frac{d}{2}} \left|X'X + (\underline{V}_\Phi \xi)^{-1}\right|^{-\frac{n}{2}} \tag{A-64}$$
$$\left|\Psi + \hat{\varepsilon}'\hat{\varepsilon} + (\hat{\Phi} - \underline{\Phi})'(\underline{V}_\Phi \xi)^{-1}(\hat{\Phi} - \underline{\Phi})\right|^{-\frac{T-p+d}{2}}.$$

We refer to Giannone, Lenza, and Primiceri (2015) for a detailed description.

Table A-9: The expected dividend growth rates and the expected excess returns: U.S.

	RMSE		Premium		Exp. return		Sharpe Ratio		Exp. growth		STRIPS	YLD	INF
horizon	SSP	LSP	SSP	LSP	SSP	LSP	SSP	LSP	SSP	LSP			
Entire period													
1y	8.41	9.82	1.43	1.22	1.46	1.26	0.19	0.14	4.01	3.80	-4.58	2.04	2.00
2y	9.51	8.60	2.22	1.95	2.39	2.13	0.37	0.24	4.18	3.91	-3.96	2.17	2.00
3y	9.19	7.03	2.65	2.39	3.02	2.76	0.48	0.34	4.31	4.05	-3.66	2.37	2.00
4y	8.20	5.87	2.94	2.69	3.53	3.28	0.56	0.44	4.40	4.14	-3.46	2.59	2.00
5y	6.73	4.68	3.21	2.95	4.04	3.78	0.63	0.54	4.47	4.21	-3.26	2.83	2.00
5y-1y	-	-	1.78	1.72	2.58	2.52	0.44	0.40	0.46	0.41	1.32	0.79	-
Positive strips spread													
1y	5.90	5.24	-0.60	-1.86	-0.30	-1.57	-0.08	-0.21	7.71	6.44	-10.31	2.29	2.00
2y	8.41	7.01	1.58	-0.58	1.97	-0.20	0.27	-0.07	7.21	5.05	-7.64	2.39	2.00
3y	8.65	6.72	2.69	0.53	3.23	1.06	0.49	0.08	6.83	4.66	-6.14	2.53	2.00
4y	7.55	5.90	3.16	1.19	3.88	1.91	0.60	0.20	6.53	4.56	-5.37	2.72	2.00
5y	5.82	4.66	3.47	1.71	4.39	2.62	0.68	0.31	6.30	4.53	-4.83	2.92	2.00
5y-1y	-	-	4.07	3.57	4.69	4.18	0.76	0.52	-1.41	-1.91	5.48	0.62	-
Negative strips spread													
1y	13.33	17.41	7.59	10.61	6.84	9.87	1.01	1.21	-7.25	-4.23	12.84	1.26	2.00
2y	12.12	12.16	4.17	9.67	3.69	9.19	0.70	1.19	-5.05	0.45	7.22	1.52	2.00
3y	10.51	7.84	2.52	8.06	2.39	7.93	0.46	1.15	-3.37	2.17	3.89	1.87	2.00
4y	9.77	5.71	2.28	7.24	2.48	7.44	0.43	1.18	-2.08	2.88	2.36	2.20	2.00
5y	8.85	4.66	2.41	6.73	2.99	7.31	0.47	1.20	-1.08	3.24	1.49	2.58	2.00
5y-1y	-	-	-5.17	-3.87	-3.85	-2.55	-0.54	-0.01	6.17	7.48	-11.34	1.32	-

Notes: Tables are generated from models that are estimated from 2005:M1 to 2017:M2 sample. We show two approaches: (1) SSP and (2) BMY, i.e., a 3-variable VAR approach that includes the 5y-1y nominal bond spread, asset dividend to earnings ratio, and dividend growth.

Table A-10: The expected dividend growth rates and the expected excess returns: Europe

horizon	RMSE SSP	RMSE LSP	Premium SSP	Premium LSP	Exp. return SSP	Exp. return LSP	Sharpe Ratio SSP	Sharpe Ratio LSP	Exp. growth SSP	Exp. growth LSP	STRIPS	YLD	INF
Entire period													
1y	8.46	7.59	3.70	2.76	3.56	2.62	0.47	0.38	-0.53	-1.47	2.12	1.96	2.10
2y	10.24	7.10	5.08	4.30	5.07	4.30	0.68	0.63	-0.80	-1.58	3.78	2.09	2.10
3y	9.11	6.38	4.16	3.33	4.33	3.50	0.55	0.51	-0.89	-1.72	2.95	2.27	2.10
4y	6.32	4.26	3.37	2.46	3.76	2.84	0.46	0.41	-0.90	-1.81	2.17	2.49	2.10
5y	4.98	3.07	2.83	1.83	3.37	2.37	0.40	0.33	-0.88	-1.88	1.61	2.64	2.10
5y-1y	-	-	-0.87	-0.93	-0.18	-0.24	-0.08	-0.05	-0.36	-0.42	-0.51	0.68	-
Positive strips spread													
1y	6.39	7.61	2.01	0.85	2.46	1.30	0.26	0.12	8.55	7.39	-8.64	2.55	2.10
2y	10.34	8.14	3.05	1.06	3.58	1.59	0.41	0.16	5.90	3.91	-4.94	2.63	2.10
3y	9.99	7.43	3.19	0.87	3.85	1.53	0.42	0.13	4.23	1.91	-3.14	2.77	2.10
4y	7.00	4.60	2.95	0.60	3.78	1.44	0.40	0.10	3.14	0.80	-2.29	2.93	2.10
5y	5.24	3.08	2.63	0.38	3.60	1.35	0.37	0.07	2.42	0.17	-1.89	3.06	2.10
5y-1y	-	-	0.62	-0.47	1.14	0.05	0.11	-0.05	-6.13	-7.22	6.75	0.52	-
Negative strips spread													
1y	10.49	7.48	5.90	5.25	4.99	4.34	0.76	0.72	-12.42	-13.07	16.22	1.19	2.10
2y	9.99	5.40	7.73	8.54	7.02	7.83	1.03	1.26	-9.58	-8.77	15.21	1.39	2.10
3y	7.68	4.61	5.43	6.56	4.95	6.08	0.72	1.01	-7.59	-6.46	10.92	1.62	2.10
4y	5.23	3.76	3.92	4.88	3.73	4.69	0.53	0.81	-6.20	-5.24	8.02	1.91	2.10
5y	4.56	3.05	3.08	3.72	3.08	3.72	0.43	0.67	-5.21	-4.57	6.19	2.10	2.10
5y-1y	-	-	-2.81	-1.53	-1.91	-0.62	-0.32	-0.05	7.21	8.50	-10.03	0.90	-

Notes: Tables are generated from models that are estimated from 2005:M1 to 2017:M2 sample. We show two approaches: (1) SSP and (2) BMY, i.e., a 3-variable VAR approach that includes the 5y-1y nominal bond spread, dividend to price ratio, and dividend growth.

Table A-11: The expected dividend growth rates and the expected excess returns: Japan

	RMSE		Premium		Exp. return		Sharpe Ratio		Exp. growth		STRIPS	YLD	INF
horizon	SSP	LSP	SSP	LSP	SSP	LSP	SSP	LSP	SSP	LSP			
Entire period													
1y	11.62	12.79	8.86	11.99	8.91	12.04	0.73	0.87	6.45	9.58	2.18	0.28	0.23
2y	9.91	8.70	8.88	11.25	9.02	11.39	0.94	1.27	7.07	9.44	1.57	0.38	0.23
3y	7.96	6.78	8.28	9.80	8.53	10.05	0.97	1.34	7.55	9.08	0.49	0.49	0.23
4y	5.68	5.22	8.11	8.97	8.50	9.35	1.01	1.38	7.93	8.78	-0.04	0.62	0.23
5y	4.44	3.93	8.17	8.46	8.68	8.97	1.06	1.42	8.22	8.51	-0.28	0.74	0.23
5y-1y	-	-	-0.69	-3.53	-0.24	-3.08	0.33	0.55	1.77	-1.07	-2.46	0.46	-
Positive strips spread													
1y	12.60	13.39	5.43	5.07	5.49	5.12	0.44	0.37	11.51	11.14	-6.31	0.29	0.23
2y	8.47	8.33	6.54	5.30	6.71	5.47	0.70	0.60	11.25	10.01	-4.94	0.40	0.23
3y	7.65	7.02	7.08	5.55	7.36	5.83	0.83	0.76	11.05	9.52	-4.20	0.51	0.23
4y	5.81	4.55	7.43	5.63	7.84	6.04	0.92	0.87	10.90	9.10	-3.70	0.64	0.23
5y	4.37	2.61	7.77	5.76	8.30	6.29	1.00	0.97	10.79	8.78	-3.25	0.76	0.23
5y-1y	-	-	2.34	0.70	2.81	1.16	0.56	0.60	-0.72	-2.37	3.06	0.47	-
Negative strips spread													
1y	10.62	12.33	12.67	19.67	12.71	19.71	1.04	1.42	0.83	7.84	11.61	0.27	0.23
2y	11.31	9.14	11.48	17.75	11.59	17.95	1.22	2.01	2.45	8.81	8.80	0.35	0.23
3y	8.21	6.44	9.60	14.45	9.83	14.73	1.13	1.98	3.68	8.58	5.69	0.46	0.23
4y	5.50	5.82	8.87	12.59	9.23	13.03	1.10	1.95	4.63	8.42	4.01	0.59	0.23
5y	4.47	4.96	8.62	11.46	9.09	11.93	1.11	1.92	5.37	8.21	3.01	0.71	0.23
5y-1y	-	-	-4.06	-8.22	-3.62	-7.78	0.08	0.50	4.54	0.38	-8.59	0.44	-

Notes: Tables are generated from models that are estimated from 2005:M1 to 2017:M2 sample. We show two approaches: (1) SSP and (2) BMY, i.e., a 3-variable VAR approach that includes the 5y-1y nominal bond spread, dividend to price ratio (pd12SPXD1), and dividend growth.

Table A-12: The spread between 5-year and 1-year forecasts: the SSP approach

	Exp. return			Exp. growth		
	50%	[5%	95%]	50%	[5%	95%]
			Entire period			
U.S.	2.58*	[0.80,	3.60]	0.19	[-1.23,	1.57]
Europe	-0.18	[-2.86,	1.73]	-0.36	[-2.96,	1.63]
Japan	-0.24	[-3.44,	1.78]	1.77	[-1.41,	3.81]
			Expansion period			
U.S.	4.69*	[2.16,	5.83]	-1.41*	[-2.88,	-0.22]
Europe	1.14	[-0.41,	2.80]	-6.13*	[-8.67,	-4.46]
Japan	2.81	[-0.26,	3.49]	-0.72*	[-2.79,	-0.08]
			Recession period			
U.S.	-3.85*	[-4.40,	-1.07]	6.17*	[4.72,	7.95]
Europe	-1.91	[-4.45,	0.65]	7.21*	[5.68,	10.75]
Japan	-3.62*	[-5.92,	-0.16]	4.54*	[2.42 ,	8.18]

Notes: We provide the SSP results based on the in-sample forecasts. The estimation sample is from 2005:M1 to 2017:M2. Our prediction sample is from 2005:M1 to 2013:M2. We use $*$ to indicate the statistical significance at the 90% confidence level.