FTG Summer School 2019
Ambiguity Aversion

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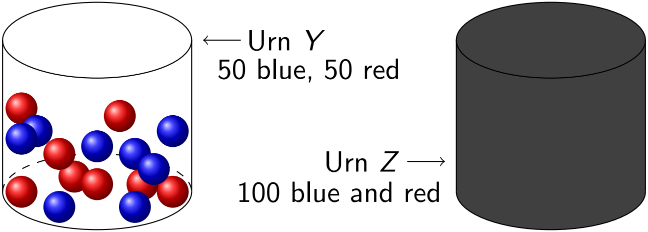
Plan

1. Ellsberg paradox.
2. Common theoretical approaches to ambiguity aversion.
   - Maxmin expected utility: Gilboa and Schmeidler (1989).
   - Smooth ambiguity aversion: Klibanoff, Marinacci, Mukherji (2005).
   - Multiplier preferences: Hansen and Sargent (2001).
3. Applications in finance.
   - Investment in risky assets: Dow and Werlang (1992).
   - Security design: (a) Malenko and Tsoy (2019).
     (b) Lee and Rajan (2019).
4. Conclusion.

Note: Throughout the slides and the talk, I will focus on simplified versions of models. See the original papers for the full models.

Ellsberg Paradox: Ellsberg (1961)



- You win \$100 if a red ball is drawn, 0 if a blue ball is drawn.
  - Gamble A: Draw a ball from urn Y.
  - Gamble B: Draw a ball from urn Z.
- Which one do you choose?
  - Modal response: $A \succ B$.

- You win \$100 if a blue ball is drawn, 0 if a red ball is drawn.
  - Gamble C: Draw a ball from urn Y.
  - Gamble D: Draw a ball from urn Z.
- Which one do you choose?
  - Modal response: $C \succ D$.

Ambiguity Aversion

- The modal choices violate subjective expected utility.
- They indicate a preference for gambles with *known* probabilities over gambles with *unknown* probabilities.
    - A situation with unknown probabilities is known as a situation with *ambiguity* or *uncertainty*, sometimes Knightian uncertainty (Knight, 1921).
        - Aside: Term Knightian uncertainty is likely a misnomer (Machina and Siniscalchi, 2014) .
    - Hence the term *ambiguity aversion*.

Preliminaries

- State space $\mathcal{S}$, outcome space $\mathcal{X}$.
  - In general, both are arbitrary.
  - For finance applications:
    - $\mathcal{S}$ depends on context; e.g., project cash flows / true value of asset.
    - $\mathcal{X}$ will be monetary outcome for agent.
- Objective (roulette) lottery / gamble $\mathbf{P} = \{(x_i, p_i)\}_{i=1}^n$.
- Subjective (horse) lottery $f = \{(x_i, E_i)\}_{i=1}^n$, where $\{E_i\}$ is some partition over $\mathcal{S}$.

- Bernoulli utility function $u : \mathcal{X} \to \mathbb{R}$.

- Von Neumann–Morgenstern utility function: objective probabilities. $U(\mathbf{P}) = \int_{\mathcal{X}} u(x)p(x)dx$.
- Expected utility of gamble f with belief distribution μ: $W(f) = \int_{\mathcal{S}} U(f(s))d\mu(s)$.
  - Here, f is in general a horse-roulette lottery; i.e., $f(s)$ is a roulette lottery.

Approach 1: Maxmin Expected Utility

- Gilboa and Schmeidler (1989).
- Let $\mathcal{C}$ be a closed, convex set of probability distributions on $\mathcal{S}$.
  - Each element in $\mathcal{C}$ is a prior.
  - Agent is unable to form a single prior, so considers a set of multiple priors.
- A horse-roulette lottery f is evaluated as:

$$W(f) = \min_{\mu \in \mathcal{C}} \int U(f) d\mu. \tag{1}$$

- In making a choice, the agent maximizes W; hence MEU.
  - Agent exhibits extreme pessimism: behaves as if the worst case scenario will occur.

Application to Ellsworth Paradox

- Let $\mathcal{S} = \{s_b, s_r\}$, where $s_b(s_r)$ denotes draw of a blue (red) ball from urn Z.
  - Each belief $\mu \in \Delta\mathcal{S}$ can be parameterized by $p(\mu)$, the probability of a blue ball.

- Let $\mathcal{C} = \{\mu \in \Delta(S) \mid \mu(s_b) = \frac{k}{100}$ for $k \in \{0, 1, \cdots, 100\};$
  $\mu(s_r) = 1 - \mu(s_b)\}$.

- Consider the value of gambles B and D.
  Denote $u_1 = u(100)$ and $u_0 = u(0)$.
  - $W(B) = \min_{\mu \in \mathcal{C}} \{p(\mu)u_1 + (1 - p(\mu))u_0\} = u_0$.
  - $W(D) = \min_{\mu \in \mathcal{C}} \{pu_0 + (1 - p)u_1\} = u_0$.

- In each case, this is less than $0.5u_1 + 0.5u_0 =$ value of gambles A, C.

Stray Thought

- Someone must have run the portfolio experiment by now.
- We'll draw two balls with replacement from each urn.
  - Ball 1: You win \$100 if red, 0 if blue.
  - Ball 2: You win 0 if blue, 100 if red.
- Is there still a preference for urn Y? Do multiple priors have bite here?

Approach 2: Smooth Ambiguity Aversion

- Klibanoff, Marinacci, and Mukherji (2005).
- The agent has a:
  - Set of multiple priors, $\mathcal{C}$.
  - Second-order belief, M, over $\mathcal{C}$.
  - Second-order utility function $\phi : \mathbb{R} \to \mathbb{R}$ that represents attitude toward uncertainty.
- A horse-roulette lottery f is evaulated as:

$$W(f) = \int_{\mathcal{C}} \phi\Big(\int U(f) d\mu \Big) dM(\mu). \qquad (2)$$

  As usual, the agent maximizes W.

- Agent is ambiguity averse/neutral/loving if ϕ is concave/linear/convex.
- Concave ϕ has the same effect as overweighting pessimistic scenarios and underweighting optimistic ones.

Application to Ellsworth Paradox

- Let $\mathcal{C} = \{\mu \mid p(\mu) = 0, 0.5, 1\}$. Let M be the uniform distribution over $\mathcal{C}$.
- Consider gamble B. Denote $u_1 = u(100)$ and $u_0 = u(0)$.

$$U(B, p) \quad = \quad pu_1 + (1 - p)u_0.$$

- Suppose first that $\phi(x) = x$. Then,

$$W(B) \quad = \quad 0.5u_1 + 0.5u_0 = W(A).$$

Similarly, $W(C) = W(D) = 0.5(u_1 + u_0)$.

Application to Ellsworth Paradox: Concave ϕ

- Next, suppose that $u(x) \geq 0$ for all x, and let $\phi(x) = \sqrt{x}$.
- Then, $W(A) = W(C) = \sqrt{0.5u_1 + 0.5u_0}$.
- Here,

$$\begin{aligned} W(B) = W(D) &= \frac{1}{3}\left(\sqrt{u_1} + \sqrt{0.5u_1 + 0.5u_0} + \sqrt{u_0}\right) \\ &< \sqrt{0.5u_1 + 0.5u_0}. \end{aligned}$$

Set of Priors

- Where does the set of priors come from?
  - Takes the Bayesian question to another philosophical level.
  - Perhaps even more difficult, as a Bayesian prior can sometimes be obtained from past data.

- Depends on context and application.

Approach 3: Multiplier Preferences

- First, consider variational preferences.
  Maccheroni, Marinacci and Rustichini (2006).
- Suppose all beliefs in $\Delta(\mathcal{S})$ are permissible. Then,

$$W(f) \quad = \quad \min_{\mu \in \Delta(\mathcal{S})} \Big(\int U(f) d\mu + c(\mu) \Big). \qquad (3)$$

  Here, $c(\mu)$ is a cost associated with choosing the prior μ.
  As usual, the agent maximizes W.

- E.g., suppose:

$$c(\mu) = \begin{cases} 0 & \text{if } \mu \in \mathcal{C} \\ \infty & \text{if } \mu \notin \mathcal{C}. \end{cases}$$

- Then, we recover Maxmin Expected Utility.

Approach 3: Multiplier Preferences, contd.

- Hansen and Sargent (2001).

- Let $c(\mu) = \theta R(\mu||\mu^*)$, where $\theta \geq 0$ is a parameter and R is the relative entropy (or Kullback-Leibler divergence) of μ w.r.t. reference measure μ^*.

$$R(\mu||\mu^*) = \int \left(\ln \frac{d\mu}{d\mu^*} \right) d\mu$$

  if μ is absolutely continuous w.r.t. μ^*, and $R(\mu||\mu^*) = \infty$ otherwise.

- Interpretation: Agent has reference measure μ^* in mind. Due to uncertainty, the agent allows themselves to evaluate a gamble according to some $\mu \neq \mu^*$, but imposes a penalty on themselves for departing far from μ^*.

Multiplier Preferences, contd.

- As θ becomes large, μ must get closer to μ^*.
  - $\theta \to \infty$: We recover expected utility ($\mu = \mu^*$).
  - Finite θ: agent is more pessimistic than reference measure would require.
  - $\theta \to 0$: We recover MEU with $\mathcal{C} = \Delta(S)$.

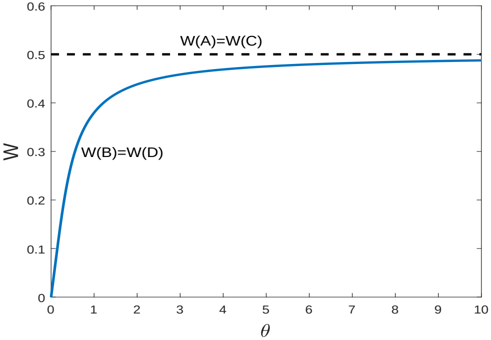
- With the Hansen-Sargent formulation, it turns out that

$$\begin{aligned} W(f) &= \min_{\mu \in \Delta(\mathcal{S})} \left(\int U(f) d\mu + \theta R(\mu || \mu^*) \right) \\ &= -\theta \ln \left\{ \int \exp\left(-\frac{U(f)}{\theta} \right) d\mu^* \right\} \end{aligned} \tag{4}$$

See Dupuis and Ellis (1997), Proposition 1.4.2.

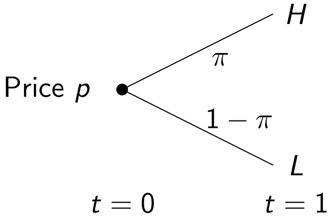
Application to Ellsworth Paradox

- Suppose the agent is risk-neutral, with $u(x) = \frac{x}{100}$.
  Then, $u_1 = 1$ and $u_0 = 0$. Hence, $W(A) = W(C) = 0.5$.
- Set the reference measure μ^* to have mass 0.5 on each of s_b, s_r.
  - Then, $W(B) = W(D) = -\theta \ln\{0.5(1 + \exp^{-\frac{1}{\theta}})\} < 0.5$.



Application 1: Investment in Risky Assets

- Dow and Werlang (1992).
- Investor at date 0 has cash W to invest until $t = 1$.
- There are two assets:
  - A risky asset with a binary outcome.
  - A risk-free asset that has a return of zero.
  - No short-sale restrictions.



- Agent is risk-neutral but uncertain about π. Behaves according to MEU.
- Set of priors $= [\pi_1, \pi_2]$.

Non-participation

Proposition

Suppose $\pi_1 < \frac{p-L}{H-L} < \pi_2$. *Then, an ambiguity-averse, risk-neutral agent prefers to hold the riskless asset.*

Outline of proof:

- The agent behaves according to MEU.
- So, for each action they may take, find the most pessimistic belief.
- Suppose the agent buys 1 unit of the risky asset.
  - What is the most pessimistic belief? What is the agent's payoff?
- Suppose the agent sells 1 unit of the risky asset.
  - What is the most pessimistic belief? What is the agent's payoff?

Hence, we have non-participation: the agent holds only the riskless asset. ∎

Risk Aversion

Proposition

An ambiguity-neutral agent who is either risk-averse or risk-neutral takes a non-zero position in the risky asset unless $\pi = \frac{p-L}{H-L}$.

Proof: Standard. ■

- Important to draw the distinction with risk-aversion. Empirically challenging to distinguish effects of ambiguity-aversion from risk-aversion.

- Ambiguity aversion creates an inertia zone, or a "status quo bias."
  - Has been used to explain the endowment effect.
  - Perhaps explains managerial inertia w.r.t. new projects.

Participation: Portfolio Effects

- Wang and Uppal (2003): Ambiguity aversion leads to optimal under-diversification.
  - Investors uncertain about return process for asset.
  - Excessive ambiguity about an asset $\longrightarrow$ inertia w.r.t. that asset.
  - Heterogeneous ambiguity across assets $\longrightarrow$ under-diversification.
- Hirshleifer, Huang, and Teoh (2019): Suitably-designed index recovers participation.
  - Investors are uncertain about noisy supply in a rational expectations model.
  - Value-weighted index leads to under-diversification.
  - Index that depends on variance of supply shocks leads to same outcome as in model without uncertainty.
- Easley and O'Hara (2009): Regulation can shrink the set of priors and increase participation.

Application 2a: Security Design with Adverse Selection

- Entrepreneur with wealth $W > 0$ has a project that requires investment $K > W$ at time 0.
  - Must raise $I = K - W$ from external financiers, issues a financial claim (security) to investors.
- At time 1, project pays a cash flow $z \in \{z_0, z_1, z_2\}$, where $z_0 = 0 < z_1 < z_2$. Security is denoted $s = (s_0, s_1, s_2)$.
- The cash flow density is $f = (f_0, f_1, f_2)$.
- Issuer can have multiple types, each with its own f.
- Type is privately-known to issuer. So, choice of security can signal issuer type.

- Similar setting as Myers and Majluf (1984) pecking order.
  - See Nachman and Noe (1994).
  - Suppose we restrict entrepreneur to debt or equity. Which one emerges depends on the likelihood ratio across states.

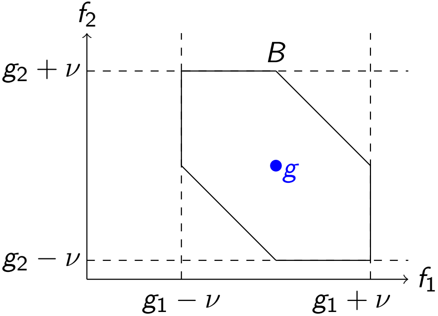
Ambiguity Aversion

- Malenko and Tsoy (2019).
- Investor is risk-neutral, but is uncertain about cash flow density $f = (f_0, f_1, f_2)$.
- Investors is ambiguity-averse, and behaves according to MEU.
- Investor has base density g in mind. Their initial set of priors, or the "uncertainty set" is

$$B = \{f \in \Delta(Z) \mid |f_i - g_i| \leq \nu \text{ for all } i\}.$$

- B doubles as the set of entrepreneur types.

Uncertainty



- Recall that $f_0 \in [g_0 - \nu, g_0 + \nu]$.

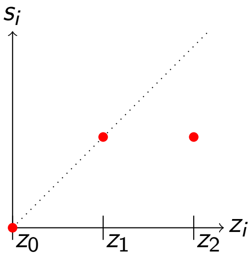
- Issuer designs a financial security $s = (s_0, s_1, s_2)$.
- Limited liability: $0 \leq s_i \leq z_i$ for all cash flow states i.
- Monotonicity: s_i and $z_i - s_i$ are both weakly increasing in i.

Stages in the Game

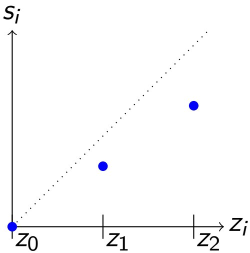
- Signaling game:
  1. Each type $f \in B$ chooses whether to offer a security. If they want to offer one, they design a security s.
  2. Investors update beliefs given s to some set $B(s)$.
  3. Investors ascribe a value to security s equal to $P(s) = \min_{f \in B(s)} E_f s$.
  4. If $P(s) \geq I$, investors buy the security and pay I at time 0. Entrepreneur invests W of own money $+ I$ from investors, starts project.
     If $P(s) < I$, investors do not buy the security. Project not undertaken. Entrepreneur's payoff is W, investors get 0.

- A critical step above is determining $B(s)$. What is the set of beliefs investors can have given s?

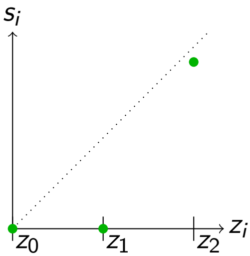
Securities



Debt



Equity



Call Option

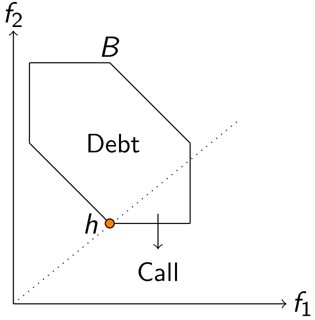
- Model allows securities to be very general, as long as limited liability and monotonicity are satisfied.
- Given risk-neutrality of all parties, why is it not enough to look at extreme securities (debt and call)?

First Thoughts

- If beliefs can lie anywhere in B, we get either debt or call as the optimal security.
- Let $h = \arg\min_{f \in B} \{f_1 + f_2\}$.

Proposition

Suppose that $B(s) = B$ for all s. Then, the optimal security is debt if $\frac{f_2}{f_1} > \frac{h_2}{h_1}$ and a call option if $\frac{f_2}{f_1} < \frac{h_2}{h_1}$.



- Pick f in debt region. Suppose entrepreneur deviates and offers call.
- Increase s_1 by ϵ, reduce s_2 by $\frac{h_1}{h_2}\epsilon$.
- Investor is indifferent. As $\frac{f_2}{f_1} > \frac{h_2}{h_1}$, entrepreneur strictly gains.
- Argument holds for any non-debt security.
- Similar argument for call region.

Justifiable Beliefs

- Refinement akin to the Cho-Kreps Intuitive Criterion for Bayesian games.

Definition

Fix an equilibrium with an offered security set $\mathcal{S}^*$. Let $U^*(f)$ be the utility of issuer type f, where

$$U^*(f) = \begin{cases} E_f - s^*(f) & \text{if } s^*(f) \in \mathcal{S}^*(f) \\ W & \text{otherwise.} \end{cases}$$

For each s, $B(s)$ is justifiable if $B(s) = \{f \in B \mid E_f[z - s] \geq U^*(f)\}$ whenever this set is non-empty, with $B(s) = B$ if the set is empty.

- That is, $B(s)$ should only include those types who can weakly gain from offering s instead of $s^*(f)$.

Ruling Out Negative NPV Projects

Lemma

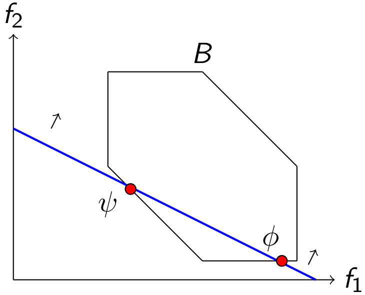
If $E_f z < K$, then $f \notin B(s)$.

Proof: Suppose type f issues a security which is purchased by investors.
It must be that $E_f s \geq I$.
Hence, $E_f z - E_f s < K - I = W$.
So issuer is better off holding on to their cash W. ∎

- Implication: For each s, $B(s)$ must exclude all negative NPV types.

Restrictions on Beliefs

- Suppose ν is high, so B is large.
- Suppose K is in an intermediate zone: Some types have positive NPV projects, others have negative NPV ones.



- Zero-NPV line: $f_1 z_1 + f_2 z_2 = K$.
  - Slope $= -\frac{z_1}{z_2} > -1$.
  - For some values of K and ν, zero-NPV line cuts through B.
- Observe that:
  - $E_\phi z = E_\psi z$.
  - $\phi_2 < \psi_2$, and $\psi_1 + \psi_2 < \phi_1 + \phi_2$.

Optimality of Equity

Define:

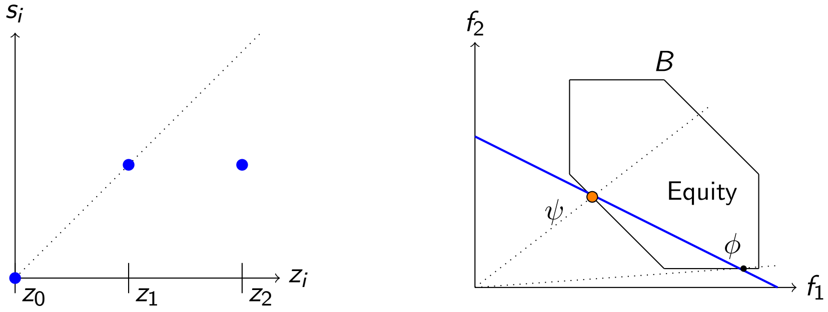
- $\psi = \arg\min_{f \in B}\{f_1 \mid f_1 z_1 + f_2 z_2 = K\}$, and $\phi = \arg\max_{f \in B}\{f_1 \mid f_1 z_1 + f_2 z_2 = K\}$.
- $B_+ = \{f \in B \mid f_1 z_1 + f_2 z_2 \geq K\}$.

Proposition

Suppose B includes both positive and negative NPV types. Then, for all $f \in B_+$ such that $\frac{\phi_2}{\phi_1} < \frac{f_2}{f_1} < \frac{\psi_2}{\psi_1}$, equity is the uniquely optimal security.

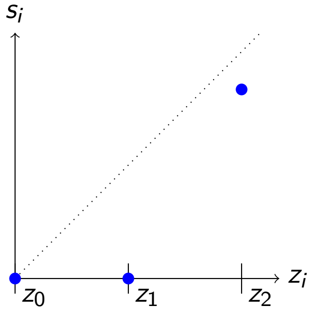
Let's go through the intuition for the proof.

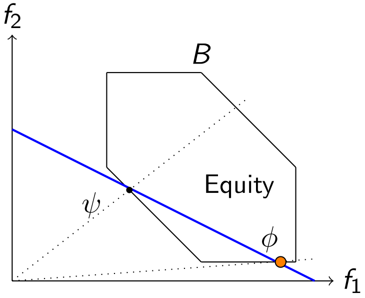
Pessimistic Beliefs: Debt



- Recall that $\psi_1 + \psi_2 < \phi_1 + \phi_2$.
  - Hence, most pessimistic belief for a debt contract is ψ.
- Pick f in the equity region. Suppose the entrepreneur deviates and offers debt. Reduce s_1 by ϵ, and increase s_2 by $\frac{\psi_1}{\psi_2}\epsilon$.
  - Investor is indifferent, so still invests.
  - Entrepreneur is strictly better off, as $\frac{f_2}{f_1} < \frac{\psi_2}{\psi_1}$.
- Argument holds for *any* strictly concave security.

Pessimistic Beliefs: Call Option





- Recall that $\phi_2 < \psi_2$.
  - Hence, most pessimistic belief for a call option is ϕ.
- Pick f in the equity region. Suppose the entrepreneur deviates and offers a call. Reduce s_2 by ϵ, and increase s_1 by $\frac{\phi_2}{\phi_1}\epsilon$.
  - Investor is indifferent, so still invests.
  - Entrepreneur is strictly better off, as $\frac{f_2}{f_1} > \frac{\phi_2}{\phi_1}$.
- Argument holds for *any* strictly convex security. ■

Application 2b: Security Design with Moral Hazard

- Recall Innes (1990).
- Penniless entrepreneur needs to raise I from investors for a project.
- Entrepreneur, investors both risk-neutral. Both protected by limited liability.
- Entrepreneur can incur effort e at convex cost $c(e)$.
  - Effort not contractible, so we have a moral hazard problem.

- Innes (1990):
  - Optimal financial contract is "live-or-die." Investors receive all cash below some threshold $\hat{x}$; entrepreneur receives all cash above this threshold.
  - With monotonicity, optimal financial contract is debt.

- So why does practically every VC contract have an equity component?

Ambiguity Aversion

- Lee and Rajan (2019): Innes-type setting with entrepreneur, investors both ambiguity-averse.
  - Recall Knight (1921) was about entrepreneurs.
- Use the Hansen-Sargent (2001) multiplier preferences approach.
- Both investors and entrepreneur behave as CARA-utility maximizers.
  - Investors have parameter θ_I, entrepreneur θ_E.

Contracting Problem

- *Objective Function*: Maximize [value of own stake to E $-$ effort cost]
  - $V_E(r, a) = -\theta_E \ln \int_X e^{-\frac{x - r(x)}{\theta_E}} f(x \mid a) - \psi(a).$
- *E's IC constraint*: Given E's share, action a maximizes V_E. Assume first-order approach is valid; replace with corresponding first-order condition.
- *I's IR constraint*: For any constant z, $V_I(z) = z$. So we can write the IR constraint as

$$V_I(r, a) = -\theta_I \ln \int_X e^{-\frac{r(x)}{\theta_I}} f(x \mid a) \geq I.$$

Contracting Problem

- Transform problem to get rid of pesky log terms.

$$\min_{r(x),a} \quad e^{\frac{\psi(a)}{\theta_E}} \left(\int_X e^{-\frac{x-r(x)}{\theta_E}} f(x \mid a) dx \right)$$

subject to: (IR) $\displaystyle \int_X e^{-\frac{r(x)}{\theta_I}} f(x \mid a) dx \ \leq \ e^{-\frac{I}{\theta_I}}$

(IC) $\displaystyle \int_X e^{-\frac{x-r(x)}{\theta_E}} f_a(x \mid a) dx + \frac{\psi'(a)}{\theta_E} \int_X e^{-\frac{x-r(x)}{\theta_E}} f(x \mid a) dx \ = \ 0$

(LL) $0 \leq r(x) \leq x$ for all x.

First-best Contract

- Consider first-best outcome in which IR constraint binds. Assume there is no moral hazard. Ignore IC constraint.
- Write down the Lagrangian, solve.

Proposition

In the solution to the first-best problem, the optimal security satisfies

$$r_f(x) = \min\left\{x,\ \left(\frac{\theta_I}{\theta_I+\theta_E}x + \frac{\theta_I\theta_E}{\theta_I+\theta_E}\left(\ln\frac{\lambda_f\theta_E}{\theta_I} - \ln e^{\frac{\psi(a_f)}{\theta_E}}\right)\right)^+\right\}. \quad (5)$$

Ambiguity Aversion or Risk Aversion?

- Results so far similar to those implied by risk aversion for I, E
  - Can interpret multiplier preferences with risk-neutrality as providing a foundation for CARA utility.
- But the interpretation under ambiguity aversion is quite different.
- E.g., consider a firm evolving through time. Amount of uncertainty reduces as firm grows.
- Variational preferences have another form, *constraint preferences*, in which θ is the shadow price of uncertainty faced by the agent.
  - Here, a reduction in uncertainty corresponds to a fall in θ.
  - In the multiplier preference formulation, this is equivalent to a reduction in ambiguity aversion.
    - There is no particular reason for risk aversion coefficients through change over time.

Stage Financing

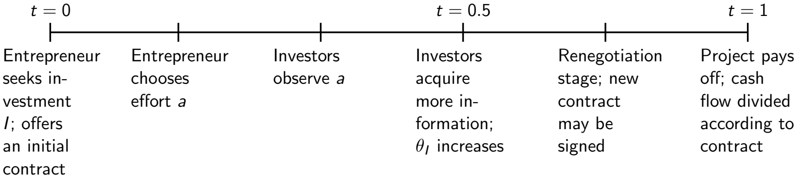
- Extend the model by another period.

  1. Initial security issued at time 0.
     Entrepreneur provides effort at this point.
  2. Between time 0 and time 1, more information arrives, so θ_E, θ_I change.
  3. Also assume that information about time 0 effort is revealed. (As in Hermalin and Katz, 1991).
  4. I, E renegotiate to new security at time 1.

- What do the time 0 and time 1 securities look like?

Increase in Information

- Initially, assume new information is acquired only by investors.
- So, θ_I increases but θ_E stays the same.



Renegotiation Stage

- There are two sources of gains to trade at the renegotiation stage:
    1. Usual idea that after effort is sunk, no need to provide incentives.
    2. Change in uncertainty implies first-best contract has changed.

- We follow the approach in Dewatripont, Legros, Matthews (2003).
    - Assume that the entrepreneur has all the bargaining power at this stage. (Consistent with objective function at time 0).
    - Entrepreneur makes a take-it-or-leave-it offer to investors. Investors can reject/accept.
    - Because E has all the bargaining power, investors are held down to their reservation utility at the renegotiation stage.
    - If renegotiation breaks down, the old contract is still valid.

Optimal Contract with Renegotiation

Proposition

Suppose the initial contract too must satisfy limited liability. Then, the optimal initial security is risky debt with a suitably chosen face value D^*, so that $r_0^*(x) = \min\{x, D^*\}$. Further,

(i) *At the renegotiation stage, the initial security is renegotiated to an efficient piecewise-linear ambiguity-sharing security, given θ_E and θ_{I1}.*

(ii) *The entrepreneur's effort a^* is strictly lower than in the first-best problem given θ_E and θ_{I1}.*

- Initial contract is risky debt. Dewatripont, Legros, and Matthews (2003).
- After renegotiation, resulting contract has efficient ambiguity-sharing, which in our model implies a substantial equity component.

Some Other Applications of Ambiguity Aversion

- Contracting: See Kellner (2015, 2017); Miao and Rivera (2016).
  - Tournament schemes are optimal.
  - Agent's IR constraint may not bind.

- Corporate control: Dicks and Fulghieri (2015):
  - Ambiguity aversion leads to disagreement between insider and outsiders.
  - Creates need for governance.
  - Find that weakly governed firms should optimally be opaque.

- Corporate control: Garlappi, Giammarino, and Lazrak (2017):
  - Interpret multiple priors as different beliefs held by different members of (e.g.) a corporate board.
  - Group decision-making leads to dynamic inconsistency.

Conclusion: Schizophrenia About Ambiguity Aversion

1. Ambiguity aversion is a robust behavioral phenomenon.
   - Repeatedly demonstrated in the lab.
2. Yet, in many applications, it is hard to demonstrate that ambiguity aversion is of first-order importance.
   - One problem is that often, the implications of a model with ambiguity aversion are similar to a model with risk aversion or with heterogenous beliefs (a rather vexing identification problem).

Future Outlook

- Try to find settings in which ambiguity aversion and risk aversion have different implications.
  - E.g., Lee and Rivera (2019): Dynamic model, with manager ambiguity-averse about firm's future cash flows.
    - Microfounds extrapolation bias.
    - Manager has an incentive to pay out and refinance at lower thresholds when ambiguity increases. An increase in risk has the opposite effect.
- Try to empirically show importance of ambiguity.
  - Hard (perhaps impossible?) to measure ambiguity.
  - Perhaps can find situations in which we can plausibly argue that the extent of ambiguity has changed. A sort of comparative statics exercise.