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Ambiguity Aversion

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Plan

1. Ellsberg paradox.

2. Common theoretical approaches to ambiguity aversion.

3. Applications in finance.
   - Security design: (a) Malenko and Tsoy (2019).
     (b) Lee and Rajan (2019).

4. Conclusion.

Note: Throughout the slides and the talk, I will focus on simplified versions of models. See the original papers for the full models.
Ellsberg Paradox: Ellsberg (1961)

You win $100 if a red ball is drawn, 0 if a blue ball is drawn.
- Gamble A: Draw a ball from urn Y.
- Gamble B: Draw a ball from urn Z.
Which one do you choose?
- Modal response: $A \succ B$.

You win $100 if a blue ball is drawn, 0 if a red ball is drawn.
- Gamble C: Draw a ball from urn Y.
- Gamble D: Draw a ball from urn Z.
Which one do you choose?
- Modal response: $C \succ D$. 
The modal choices violate subjective expected utility.

They indicate a preference for gambles with known probabilities over gambles with unknown probabilities.

A situation with unknown probabilities is known as a situation with ambiguity or uncertainty, sometimes Knightian uncertainty (Knight, 1921).

Aside: Term Knightian uncertainty is likely a misnomer (Machina and Siniscalchi, 2014).

Hence the term ambiguity aversion.
Preliminaries

- State space $S$, outcome space $\mathcal{X}$.
  - In general, both are arbitrary.
  - For finance applications:
    - $S$ depends on context; e.g., project cash flows / true value of asset.
    - $\mathcal{X}$ will be monetary outcome for agent.
- Objective (roulette) lottery / gamble $P = \{(x_i, p_i)\}_{i=1}^n$.
- Subjective (horse) lottery $f = \{(x_i, E_i)\}_{i=1}^n$, where $\{E_i\}$ is some partition over $S$.
- Bernoulli utility function $u : \mathcal{X} \to \mathbb{R}$.
- Von Neumann–Morgenstern utility function: objective probabilities. $U(P) = \int_{\mathcal{X}} u(x)p(x)dx$.
- Expected utility of gamble $f$ with belief distribution $\mu$: $W(f) = \int_{S} U(f(s))d\mu(s)$.
  - Here, $f$ is in general a horse-roulette lottery; i.e., $f(s)$ is a roulette lottery.
Approach 1: Maxmin Expected Utility

- Let $\mathcal{C}$ be a closed, convex set of probability distributions on $\mathcal{S}$.
  - Each element in $\mathcal{C}$ is a prior.
  - Agent is unable to form a single prior, so considers a set of multiple priors.
- A horse-roulette lottery $f$ is evaluated as:

\[
W(f) = \min_{\mu \in \mathcal{C}} \int U(f) d\mu.
\]  
(1)

- In making a choice, the agent maximizes $W$; hence MEU.
  - Agent exhibits extreme pessimism: behaves as if the worst case scenario will occur.
Application to Ellsworth Paradox

- Let $S = \{s_b, s_r\}$, where $s_b(s_r)$ denotes draw of a blue (red) ball from urn $Z$.
  - Each belief $\mu \in \Delta S$ can be parameterized by $p(\mu)$, the probability of a blue ball.

- Let $C = \{\mu \in \Delta(S) \mid \mu(s_b) = \frac{k}{100} \text{ for } k \in \{0, 1, \ldots, 100\}; \mu(s_r) = 1 - \mu(s_b)\}$.

- Consider the value of gambles $B$ and $D$. Denote $u_1 = u(100)$ and $u_0 = u(0)$.
  - $W(B) = \min_{\mu \in C} \{p(\mu)u_1 + (1 - p(\mu))u_0\} = u_0$.
  - $W(D) = \min_{\mu \in C} \{pu_0 + (1 - p)u_1\} = u_0$.

- In each case, this is less than $0.5u_1 + 0.5u_0 = \text{value of gambles } A, C$. 
Someone must have run the portfolio experiment by now.

We’ll draw two balls with replacement from each urn.

- Ball 1: You win $100 if red, 0 if blue.
- Ball 2: You win 0 if blue, 100 if red.

Is there still a preference for urn Y? Do multiple priors have bite here?
Approach 2: Smooth Ambiguity Aversion

- The agent has a:
  - Set of multiple priors, $C$.
  - Second-order belief, $M$, over $C$.
  - Second-order utility function $\phi : \mathbb{R} \rightarrow \mathbb{R}$ that represents attitude toward uncertainty.

- A horse-roulette lottery $f$ is evaluated as:

$$W(f) = \int_C \phi \left( \int U(f) d\mu \right) dM(\mu).$$  
(2)

As usual, the agent maximizes $W$.

- Agent is ambiguity averse/neutral/loving if $\phi$ is concave/linear/convex.

- Concave $\phi$ has the same effect as overweighting pessimistic scenarios and underweighting optimistic ones.
Application to Ellsworth Paradox

Let $C = \{ \mu | \ p(\mu) = 0, 0.5, 1 \}$. Let $M$ be the uniform distribution over $C$.

Consider gamble $B$. Denote $u_1 = u(100)$ and $u_0 = u(0)$.

$$U(B, p) = pu_1 + (1 - p)u_0.$$ 

Suppose first that $\phi(x) = x$. Then,

$$W(B) = 0.5u_1 + 0.5u_0 = W(A).$$

Similarly, $W(C) = W(D) = 0.5(u_1 + u_0)$. 
Next, suppose that $u(x) \geq 0$ for all $x$, and let $\phi(x) = \sqrt{x}$.

Then, $W(A) = W(C) = \sqrt{0.5u_1 + 0.5u_0}$.

Here,

$$W(B) = W(D) = \frac{1}{3} \left( \sqrt{u_1} + \sqrt{0.5u_1 + 0.5u_0} + \sqrt{u_0} \right) < \sqrt{0.5u_1 + 0.5u_0}.$$
Set of Priors

- Where does the set of priors come from?
  - Takes the Bayesian question to another philosophical level.
  - Perhaps even more difficult, as a Bayesian prior can sometimes be obtained from past data.

- Depends on context and application.
Approach 3: Multiplier Preferences

First, consider variational preferences. Maccheroni, Marinacci and Rustichini (2006).

Suppose all beliefs in $\Delta(S)$ are permissible. Then,

$$W(f) = \min_{\mu \in \Delta(S)} \left( \int U(f) d\mu + c(\mu) \right). \quad (3)$$

Here, $c(\mu)$ is a cost associated with choosing the prior $\mu$. As usual, the agent maximizes $W$.

E.g., suppose:

$$c(\mu) = \begin{cases} 0 & \text{if } \mu \in C \\ \infty & \text{if } \mu \notin C. \end{cases}$$

Then, we recover Maxmin Expected Utility.
Approach 3: Multiplier Preferences, contd.


- Let $c(\mu) = \theta R(\mu||\mu^*)$, where $\theta \geq 0$ is a parameter and $R$ is the relative entropy (or Kullback-Leibler divergence) of $\mu$ w.r.t. reference measure $\mu^*$.

$$R(\mu||\mu^*) = \int \left( \ln \frac{d\mu}{d\mu^*} \right) d\mu$$

if $\mu$ is absolutely continuous w.r.t. $\mu^*$, and $R(\mu||\mu^*) = \infty$ otherwise.

- Interpretation: Agent has reference measure $\mu^*$ in mind. Due to uncertainty, the agent allows themselves to evaluate a gamble according to some $\mu \neq \mu^*$, but imposes a penalty on themselves for departing far from $\mu^*$. 

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Multiplier Preferences, contd.

- As $\theta$ becomes large, $\mu$ must get closer to $\mu^*$.
  - $\theta \to \infty$: We recover expected utility ($\mu = \mu^*$).
  - Finite $\theta$: agent is more pessimistic than reference measure would require.
  - $\theta \to 0$: We recover MEU with $C = \Delta(S)$.

- With the Hansen-Sargent formulation, it turns out that

$$W(f) = \min_{\mu \in \Delta(S)} \left( \int U(f) \, d\mu + \theta R(\mu || \mu^*) \right)$$

$$= -\theta \ln \left\{ \int \exp \left( - \frac{U(f)}{\theta} \right) \, d\mu^* \right\} \quad (4)$$

See Dupuis and Ellis (1997), Proposition 1.4.2.
Suppose the agent is risk-neutral, with $u(x) = \frac{x}{100}$. Then, $u_1 = 1$ and $u_0 = 0$. Hence, $W(A) = W(C) = 0.5$.

Set the reference measure $\mu^*$ to have mass 0.5 on each of $s_b, s_r$.

Then, $W(B) = W(D) = -\theta \ln\{0.5(1 + \exp^{-\frac{1}{\theta}})\} < 0.5$. 
Application 1: Investment in Risky Assets


- Investor at date 0 has cash $W$ to invest until $t = 1$.

- There are two assets:
  - A risky asset with a binary outcome.
  - A risk-free asset that has a return of zero.
  - No short-sale restrictions.

- Agent is risk-neutral but uncertain about $\pi$. Behaves according to MEU.

- Set of priors $= [\pi_1, \pi_2]$. 

\[
\begin{align*}
H & \quad \text{Price } p \\
\pi & \quad 1 - \pi \\
L & \quad t = 0 \\
& \quad t = 1
\end{align*}
\]
Non-participation

Proposition

Suppose \( \pi_1 < \frac{p-L}{H-L} < \pi_2 \). Then, an ambiguity-averse, risk-neutral agent prefers to hold the riskless asset.

Outline of proof:

▶ The agent behaves according to MEU.
▶ So, for each action they may take, find the most pessimistic belief.
▶ Suppose the agent buys 1 unit of the risky asset.
   ▶ What is the most pessimistic belief? What is the agent’s payoff?
▶ Suppose the agent sells 1 unit of the risky asset.
   ▶ What is the most pessimistic belief? What is the agent’s payoff?

Hence, we have non-participation: the agent holds only the riskless asset.
Proposition

An ambiguity-neutral agent who is either risk-averse or risk-neutral takes a non-zero position in the risky asset unless\[\pi = \frac{p-L}{H-L} .\]

**Proof:** Standard.

- Important to draw the distinction with risk-aversion. Empirically challenging to distinguish effects of ambiguity-aversion from risk-aversion.

- Ambiguity aversion creates an inertia zone, or a “status quo bias.”
  - Has been used to explain the endowment effect.
  - Perhaps explains managerial inertia w.r.t. new projects.
Participation: Portfolio Effects

  ▶ Investors uncertain about return process for asset.
  ▶ Excessive ambiguity about an asset $\rightarrow$ inertia w.r.t. that asset.
  ▶ Heterogeneous ambiguity across assets $\rightarrow$ under-diversification.

  ▶ Investors are uncertain about noisy supply in a rational expectations model.
  ▶ Value-weighted index leads to under-diversification.
  ▶ Index that depends on variance of supply shocks leads to same outcome as in model without uncertainty.

▶ Easley and O’Hara (2009): Regulation can shrink the set of priors and increase participation.
Entrepreneur with wealth $W > 0$ has a project that requires investment $K > W$ at time 0.

- Must raise $I = K - W$ from external financiers, issues a financial claim (security) to investors.

At time 1, project pays a cash flow $z \in \{z_0, z_1, z_2\}$, where $z_0 = 0 < z_1 < z_2$. Security is denoted $s = (s_0, s_1, s_2)$.

- The cash flow density is $f = (f_0, f_1, f_2)$.

- Issuer can have multiple types, each with its own $f$.

- Type is privately-known to issuer. So, choice of security can signal issuer type.

Similar setting as Myers and Majluf (1984) pecking order.

- See Nachman and Noe (1994).

- Suppose we restrict entrepreneur to debt or equity. Which one emerges depends on the likelihood ratio across states.
Ambiguity Aversion

- Malenko and Tsoy (2019).

- Investor is risk-neutral, but is uncertain about cash flow density $f = (f_0, f_1, f_2)$.

- Investors is ambiguity-averse, and behaves according to MEU.

- Investor has base density $g$ in mind. Their initial set of priors, or the “uncertainty set” is

$$B = \{ f \in \Delta(Z) \mid |f_i - g_i| \leq \nu \text{ for all } i \}.$$ 

- $B$ doubles as the set of entrepreneur types.
Uncertainty

Recall that $f_0 \in [g_0 - \nu, g_0 + \nu]$.

- Issuer designs a financial security $s = (s_0, s_1, s_2)$.
- Limited liability: $0 \leq s_i \leq z_i$ for all cash flow states $i$.
- Monotonicity: $s_i$ and $z_i - s_i$ are both weakly increasing in $i$. 
Stages in the Game

▶ Signaling game:
   1. Each type \( f \in B \) chooses whether to offer a security. If they want to offer one, they design a security \( s \).
   2. Investors update beliefs given \( s \) to some set \( B(s) \).
   3. Investors ascribe a value to security \( s \) equal to
      \[
      P(s) = \min_{f \in B(s)} E_f s \]
   4. If \( P(s) \geq I \), investors buy the security and pay \( I \) at time 0. Entrepreneur invests \( W \) of own money + \( I \) from investors, starts project.
      If \( P(s) < I \), investors do not buy the security. Project not undertaken. Entrepreneur’s payoff is \( W \), investors get 0.

▶ A critical step above is determining \( B(s) \). What is the set of beliefs investors can have given \( s \)?
Model allows securities to be very general, as long as limited liability and monotonicity are satisfied.

Given risk-neutrality of all parties, why is it not enough to look at extreme securities (debt and call)?
First Thoughts

- If beliefs can lie anywhere in $B$, we get either debt or call as the optimal security.
- Let $h = \arg \min_{f \in B} \{ f_1 + f_2 \}$.

Proposition

Suppose that $B(s) = B$ for all $s$. Then, the optimal security is debt if $\frac{f_2}{f_1} > \frac{h_2}{h_1}$ and a call option if $\frac{f_2}{f_1} < \frac{h_2}{h_1}$.

- Pick $f$ in debt region. Suppose entrepreneur deviates and offers call.
- Increase $s_1$ by $\epsilon$, reduce $s_2$ by $\frac{h_1}{h_2} \epsilon$.
- Investor is indifferent. As $\frac{f_2}{f_1} > \frac{h_2}{h_1}$, entrepreneur strictly gains.
- Argument holds for any non-debt security.
- Similar argument for call region.
Justifiable Beliefs

- Refinement akin to the Cho-Kreps Intuitive Criterion for Bayesian games.

**Definition**

Fix an equilibrium with an offered security set \( S^* \). Let \( U^*(f) \) be the utility of issuer type \( f \), where

\[
U^*(f) = \begin{cases} 
E_f - s^*(f) & \text{if } s^*(f) \in S^*(f) \\
W & \text{otherwise.}
\end{cases}
\]

For each \( s \), \( B(s) \) is justifiable if

\[
B(s) = \{ f \in B \mid E_f[z - s] \geq U^*(f) \}
\]

whenever this set is non-empty, with \( B(s) = B \) if the set is empty.

- That is, \( B(s) \) should only include those types who can weakly gain from offering \( s \) instead of \( s^*(f) \).
Lemma

If $E_f z < K$, then $f \notin B(s)$.

Proof: Suppose type $f$ issues a security which is purchased by investors.
It must be that $E_f s \geq I$.
Hence, $E_f z - E_f s < K - I = W$.
So issuer is better off holding on to their cash $W$.

- Implication: For each $s$, $B(s)$ must exclude all negative NPV types.
Restrictions on Beliefs

- Suppose $\nu$ is high, so $B$ is large.
- Suppose $K$ is in an intermediate zone: Some types have positive NPV projects, others have negative NPV ones.

**Zero-NPV line**: $f_1z_1 + f_2z_2 = K$.
- Slope $= -\frac{z_1}{z_2} > -1$.
- For some values of $K$ and $\nu$, zero-NPV line cuts through $B$.

- Observe that:
  - $E_{\phi}z = E_{\psi}z$.
  - $\phi_2 < \psi_2$, and $\psi_1 + \psi_2 < \phi_1 + \phi_2$. 

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Optimality of Equity

Define:

$\psi = \arg\min_{f \in B} \{ f_1 \mid f_1 z_1 + f_2 z_2 = K \}$, and
$\phi = \arg\max_{f \in B} \{ f_1 \mid f_1 z_1 + f_2 z_2 = K \}$.

$B_+ = \{ f \in B \mid f_1 z_1 + f_2 z_2 \geq K \}$.

Proposition

Suppose $B$ includes both positive and negative NPV types. Then, for all $f \in B_+$ such that $\frac{\phi_2}{\phi_1} < \frac{f_2}{f_1} < \frac{\psi_2}{\psi_1}$, equity is the uniquely optimal security.

Let’s go through the intuition for the proof.
Recall that $\psi_1 + \psi_2 < \phi_1 + \phi_2$.
- Hence, most pessimistic belief for a debt contract is $\psi$.
- Pick $f$ in the equity region. Suppose the entrepreneur deviates and offers debt. Reduce $s_1$ by $\epsilon$, and increase $s_2$ by $\frac{\psi_1}{\psi_2} \epsilon$.
  - Investor is indifferent, so still invests.
  - Entrepreneur is strictly better off, as $\frac{f_2}{f_1} < \frac{\psi_2}{\psi_1}$.
- Argument holds for any strictly concave security.
Recall that $\phi_2 < \psi_2$.

- Hence, the most pessimistic belief for a call option is $\phi$.

- Pick $f$ in the equity region. Suppose the entrepreneur deviates and offers a call. Reduce $s_2$ by $\epsilon$, and increase $s_1$ by $\frac{\phi_2}{\phi_1} \epsilon$.

- Investor is indifferent, so still invests.

- Entrepreneur is strictly better off, as $\frac{f_2}{f_1} > \frac{\phi_2}{\phi_1}$.

- Argument holds for any strictly convex security.

Penniless entrepreneur needs to raise $I$ from investors for a project.

Entrepreneur, investors both risk-neutral. Both protected by limited liability.

Entrepreneur can incur effort $e$ at convex cost $c(e)$.
  - Effort not contractible, so we have a moral hazard problem.

Innes (1990):
  - Optimal financial contract is “live-or-die.” Investors receive all cash below some threshold $\hat{x}$; entrepreneur receives all cash above this threshold.
  - With monotonicity, optimal financial contract is debt.

So why does practically every VC contract have an equity component?
Lee and Rajan (2019): Innes-type setting with entrepreneur, investors both ambiguity-averse.
   - Recall Knight (1921) was about entrepreneurs.

Use the Hansen-Sargent (2001) multiplier preferences approach.

Both investors and entrepreneur behave as CARA-utility maximizers.
   - Investors have parameter $\theta_I$, entrepreneur $\theta_E$. 
Contracting Problem

- **Objective Function**: Maximize [value of own stake to $E$ – effort cost]
  
  $V_E(r, a) = -\theta_E \ln \int_X e^{-\frac{x-r(x)}{\theta_E}} f(x \mid a) - \psi(a)$.

- **$E$’s IC constraint**: Given $E$’s share, action $a$ maximizes $V_E$. Assume first-order approach is valid; replace with corresponding first-order condition.

- **$I$’s IR constraint**: For any constant $z$, $V_I(z) = z$. So we can write the IR constraint as

  $V_I(r, a) = -\theta_I \ln \int_X e^{-\frac{r(x)}{\theta_I}} f(x \mid a) \geq l$. 

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Ambiguity Aversion
Contracting Problem

- Transform problem to get rid of pesky log terms.

\[
\min_{r(x), a} \quad e^{\frac{\psi'(a)}{\theta_E}} \left( \int_X e^{-\frac{x-r(x)}{\theta_E}} f(x | a) dx \right)
\]

subject to:

- (IR) \quad \int_X e^{-\frac{r(x)}{\theta_i}} f(x | a) dx \leq e^{-\frac{1}{\theta_i}}

- (IC) \quad \int_X e^{-\frac{x-r(x)}{\theta_E}} f_a(x | a) dx + \frac{\psi'(a)}{\theta_E} \int_X e^{-\frac{x-r(x)}{\theta_E}} f(x | a) dx = 0

- (LL) \quad 0 \leq r(x) \leq x \text{ for all } x.
First-best Contract

- Consider first-best outcome in which IR constraint binds. Assume there is no moral hazard. Ignore IC constraint.

- Write down the Lagrangian, solve.

**Proposition**

*In the solution to the first-best problem, the optimal security satisfies*

\[
rf(x) = \min \left\{ x, \left( \frac{\theta_I}{\theta_I + \theta_E} x + \frac{\theta_I \theta_E}{\theta_I + \theta_E} \left( \ln \frac{\lambda_f \theta_E}{\theta_I} - \ln e^{\frac{\psi(a_f)}{\theta_E}} \right) \right)^+ \right\}. \tag{5}
\]
Ambiguity Aversion or Risk Aversion?

- Results so far similar to those implied by risk aversion for $I, E$
  - Can interpret multiplier preferences with risk-neutrality as providing a foundation for CARA utility.

- But the interpretation under ambiguity aversion is quite different.

- E.g., consider a firm evolving through time. Amount of uncertainty reduces as firm grows.

- Variational preferences have another form, *constraint preferences*, in which $\theta$ is the shadow price of uncertainty faced by the agent.
  - Here, a reduction in uncertainty corresponds to a fall in $\theta$.
  - In the multiplier preference formulation, this is equivalent to a reduction in ambiguity aversion.
    - There is no particular reason for risk aversion coefficients through change over time.
Stage Financing

- Extend the model by another period.

1. Initial security issued at time 0. Entrepreneur provides effort at this point.
2. Between time 0 and time 1, more information arrives, so $\theta_E, \theta_I$ change.
3. Also assume that information about time 0 effort is revealed. (As in Hermalin and Katz, 1991).
4. $I, E$ renegotiate to new security at time 1.

- What do the time 0 and time 1 securities look like?
Increase in Information

- Initially, assume new information is acquired only by investors.

- So, $\theta_I$ increases but $\theta_E$ stays the same.

<table>
<thead>
<tr>
<th>$t = 0$</th>
<th>$t = 0.5$</th>
<th>$t = 1$</th>
</tr>
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<tbody>
<tr>
<td>Entrepreneur seeks investment $I$; offers an initial contract</td>
<td>Entrepreneurs chooses effort $a$</td>
<td>Investors observe $a$</td>
</tr>
<tr>
<td>Investors acquire more information; $\theta_I$ increases</td>
<td>Renegotiation stage; new contract may be signed</td>
<td>Project pays off; cash flow divided according to contract</td>
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Renegotiation Stage

> There are two sources of gains to trade at the renegotiation stage:

1. Usual idea that after effort is sunk, no need to provide incentives.
2. Change in uncertainty implies first-best contract has changed.

> We follow the approach in Dewatripont, Legros, Matthews (2003).

> Assume that the entrepreneur has all the bargaining power at this stage. (Consistent with objective function at time 0).
> Entrepreneur makes a take-it-or-leave-it offer to investors. Investors can reject/accept.
> Because $E$ has all the bargaining power, investors are held down to their reservation utility at the renegotiation stage.
> If renegotiation breaks down, the old contract is still valid.
Proposition

Suppose the initial contract too must satisfy limited liability. Then, the optimal initial security is risky debt with a suitably chosen face value $D^*$, so that $r_0^*(x) = \min\{x, D^*\}$. Further,

(i) At the renegotiation stage, the initial security is renegotiated to an efficient piecewise-linear ambiguity-sharing security, given $\theta_E$ and $\theta_{I_1}$.

(ii) The entrepreneur’s effort $a^*$ is strictly lower than in the first-best problem given $\theta_E$ and $\theta_{I_1}$.


After renegotiation, resulting contract has efficient ambiguity-sharing, which in our model implies a substantial equity component.
Some Other Applications of Ambiguity Aversion

  - Tournament schemes are optimal.
  - Agent's IR constraint may not bind.

- Corporate control: Dicks and Fulghieri (2015):
  - Ambiguity aversion leads to disagreement between insider and outsiders.
  - Creates need for governance.
  - Find that weakly governed firms should optimally be opaque.

- Corporate control: Garlappi, Giammarino, and Lazrak (2017):
  - Interpret multiple priors as different beliefs held by different members of (e.g.) a corporate board.
  - Group decision-making leads to dynamic inconsistency.
1. Ambiguity aversion is a robust behavioral phenomenon.
   ▶ Repeatedly demonstrated in the lab.

2. Yet, in many applications, it is hard to demonstrate that ambiguity aversion is of first-order importance.
   ▶ One problem is that often, the implications of a model with ambiguity aversion are similar to a model with risk aversion or with heterogeneous beliefs (a rather vexing identification problem).
Try to find settings in which ambiguity aversion and risk aversion have different implications.

  - Microfounds extrapolation bias.
  - Manager has an incentive to pay out and refinance at lower thresholds when ambiguity increases. An increase in risk has the opposite effect.

Try to empirically show importance of ambiguity.

- Hard (perhaps impossible?) to measure ambiguity.
- Perhaps can find situations in which we can plausibly argue that the extent of ambiguity has changed. A sort of comparative statics exercise.