Optimal dynamic financial contracting

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Issues

How can firm finance investment when there are incentive problems?

Do incentive problems generate credit rationing?

Or can financial contract be designed to mitigate incentive problems?
Optimal complete contracts

Incentive problems arise because of information asymmetry:
- Adverse selection (hidden information)
- Moral hazard (hidden action) → our focus

Beyond these frictions, no ad hoc restrictions:
- All observable variables are contractible
- No ad hoc/exogenous constraints or contracts

Financial structure and contracts emerge endogenously
Literature

One period: Holmstrom Tirole (1997) [HT] → incentives so that agent prefers effort than shirking

Infinite horizon discrete time:

- DeMarzo Fishman (2007a,b)
- Biais, Mariotti, Plantin, Rochet (2007) [BMPR]

Continuous time:

- BMPR 2007 → continuous time limit of discrete time
- DeMarzo Sannikov (2006) → martingale approach
- Biais, Mariotti, Rochet, Villeneuve (2010): Poisson + investment
- Zhu (2013): optimal contract can involve shirking
1. One period HT 1997
2. Two-period Bolton Scharfstein 1990
3. Infinite horizon discrete time BMP 2007

Clarify similarity of economic mechanisms in 3 settings, and insights specific to dynamics
Players

Principal: unlimited liability, deep pocket (investors, bank, venture capital fund)

Agent: limited liability, cashless (manager, entrepreneur)

Indivisible initial project size 1 → investment cost $I$

Principal invests $I$, hires agent to run project

All risk neutral, discount rate $r$
Effort and probability of success

Effort unobservable by principal + agent has limited liability -> moral hazard
First best (effort observable)

Assume high effort efficient:

$$\Delta \lambda C > B,$$

and investment efficient even if project lasts only one period:

$$\mu - \lambda C > I,$$

→ under high effort project has positive net present value

First best = investment, effort
One period

Effort

1 - \(\lambda\)

1 - \(\lambda - \Delta\lambda\)

\(\lambda\)

\(\lambda + \Delta\lambda\)

Shirk

Private benefit per unit: \(B\)

\(\mu\)

\(\mu - C\)

One period contract: mapping from cash flow to agent’s compensation
Incentive compatibility
Agent’s expected gain larger under effort than under shirking

\[ T(\mu) - T(\mu-C) \geq \frac{B}{(\Delta\lambda)} \]

Large \( B \) → tempting to shirk → large compensation when no loss → cost of incentives
Small \( \Delta\lambda \) → tempting to shirk → cost of incentives
Participation constraint
Principal expected profit larger than 0

\[(1-\lambda) T(\mu) + \lambda T(\mu-C) \leq (\mu-\lambda C) - I\]
Tension between IC and PC

\[ T(\mu - C) \geq 0 \ (LL) \]

Not PC \[ \frac{(\mu - \lambda C - I)}{(1 - \lambda)} \]

Not IC \[ \frac{B}{(\Delta \lambda)} \]

\[ T(\mu - C) \geq 0 \ (LL) \]
Incentive feasible set (Laffont Martimort, 2002)

IC set not empty if expected cash flow ≥ expected agency rent

\[(\mu - \lambda C - l) / (1 - \lambda) \geq (1 - \lambda) B / (\Delta \lambda)\]
Pledgeable income (Tirole 2006)

\[ E(\text{return}) \text{ which can be promised to investors without jeopardizing incentives of agent} \]

\[ P = [\mu - \lambda C] - (1 - \lambda) \frac{B}{\Delta \lambda} \]  

(1)

First term [in brackets] = expected cash flow with effort

Second term = expected compensation which must be left to agent for incentives (agency rent)

Incentive feasible set not empty iff

\[ P \geq 1 \]  

(2)
Optimal contract with effort

Suppose principal has all bargaining power

\[
\max (\mu - I) - \lambda [T(\mu - C) + C] - (1 - \lambda) T(\mu)
\]

s.t. IC: \( T(\mu) - T(\mu - C) \geq \frac{B}{\Delta \lambda} \) and LL: \( T \geq 0 \)

\( T(\mu - C) = 0 \): relax IC and raise P’s gains

\( T(\mu) = \frac{B}{\Delta \lambda} \): bind IC to max principal’s gains

Max possible gain for principal

\[
(\mu - \lambda C - I) - (1 - \lambda) \frac{B}{\Delta \lambda} = \mathcal{P} - I
\] (3)
Optimal contract without effort

If project operated without effort, principal pays 0 wage and gets

\[ [\mu - (\lambda + \Delta \lambda)C - I] \]  \hspace{1cm} (4)

Comparing (3) & (4), greater profits with effort iff

\[ (1 - \lambda) \frac{B}{\Delta \lambda} < \Delta \lambda C. \] \hspace{1cm} (5)

LHS: what must be paid to agent so that effort = cost of incentives
RHS: efficiency cost of no effort = benefit of incentives
Proposition 1:

If \( E(\text{profit} \mid \text{no effort}) \geq 0 \):

- If \( B < \frac{(\Delta \lambda)^2 C}{1-\lambda} \), pay agent \( \frac{B}{\Delta \lambda} \) iff success \( \implies \) effort
- Otherwise, don’t pay agent \( \implies \) no effort (// Zhu, 2013)

If \( E(\text{profit} \mid \text{no effort}) < 0 \):

- If \( \mathcal{P} \geq 1 \), pay agent \( \frac{B}{\Delta \lambda} \) iff success \( \implies \) effort
- If \( \mathcal{P} < 1 \), no investment (credit rationing)
Two period

Effort

\[ \begin{align*}
1 - \lambda & \quad \mu \quad \rightarrow \quad T(\mu), X_2(\mu) \\
\lambda & \quad \mu - C \quad \rightarrow \quad \tau(\mu - C), X_2(\mu - C)
\end{align*} \]

Shirk: \( B \)

\[ \begin{align*}
1 - \lambda - \Delta \lambda & \quad \mu \quad \rightarrow \quad T(\mu), X_2(\mu) \\
\lambda + \Delta \lambda & \quad \mu - C \quad \rightarrow \quad \tau(\mu - C), X_2(\mu - C)
\end{align*} \]

Effort

\[ \begin{align*}
\mu & \quad \rightarrow \quad T(\mu, \mu) \\
\mu - C & \quad \rightarrow \quad T(\mu, \mu - C)
\end{align*} \]

Shirk: \( B X_2(\mu) \)

\[ \begin{align*}
\mu & \quad \rightarrow \quad T(\mu - C, \mu) \\
\mu - C & \quad \rightarrow \quad T(\mu - C, \mu - C)
\end{align*} \]

Effort

\[ \begin{align*}
\mu & \quad \rightarrow \quad T(\mu, \mu) \\
\mu - C & \quad \rightarrow \quad T(\mu, \mu - C)
\end{align*} \]

Shirk: \( B X_2(\mu - C) \)

\[ \begin{align*}
\mu & \quad \rightarrow \quad T(\mu - C, \mu) \\
\mu - C & \quad \rightarrow \quad T(\mu - C, \mu - C)
\end{align*} \]

Size adjusted transfer: \( T_2 = X_2 t_2 \)
Agent’s continuation utility

At beginning of period 2 following realization $C_1 \in \{0, C\}$

$$W_2(\mu - C_1) = E\left( T_2(\mu - C_1, \mu - \tilde{C}_2) \mid C_1 \right)$$

At beginning of period 1 (by law of iterated expectations)

$$W_1 = E \left[ T_1(\mu - \tilde{C}_1) + \frac{1}{1 + r} W_2(\mu - \tilde{C}_1) \right]$$

→ “promise keeping” condition

Expectations computed by agent rationally anticipating to exert effort $\rightarrow \lambda$
Principal’s value function

At beginning of 2\textsuperscript{nd} period, value function

$$F_2 = X_2(\mu - C_1)E[\mu - \tilde{C}_2 - t_2(\mu - C_1, \mu - \tilde{C}_2)|C_1]$$

At beginning of period 1 (by law of iterated expectations)

$$F_1 = E\left[\mu - \tilde{C}_1 - T_1 + \frac{1}{1+r}F_2(\tilde{C}_1)\right]$$
Incentive compatibility

At second period, similar IC to one period

\[ T_2(\mu - C_1, \mu) - T_2(\mu - C_1, \mu - C) \geq X_2(\mu - C_1) \frac{B}{\Delta \lambda} \quad (6) \]

IC \( \implies \) wedge between the utility of agent after a success and after a failure, increases with \( \frac{B}{\Delta \lambda} \) and \( X_2 \)

At first period:

\[
\left[ T_1(\mu) + \frac{W_2(\mu)}{1 + r} \right] - \left[ T_1(\mu - C) + \frac{W_2(\mu - C)}{1 + r} \right] \geq \frac{B}{\Delta \lambda} \quad (7)
\]

Incentivize with current transfer \( T_1 \) and continuation utility \( W_2 \)
Continuation utility dynamics

Lemma 1: \( IC \implies agent’s continuation utility goes down (resp. up) after loss (resp. no loss) \)

Lemma 2: No transfer after loss

\( \implies \) Will also hold in infinite horizon model
Lemma 3: When principal and agent equally patient, weakly optimal to postpone compensation to final period

Because A as patient as P, delaying $T$ (capitalizing it at rate $r$) generates no inefficiency, but makes incentives more effective: use late payment to reward late and early efforts

→ Will also hold in infinite horizon model
Optimal liquidation

Continuation after success: \( X_2(\mu) = 1 \). Moreover, if

\[
\lambda(\mu - \lambda C) > (1 - \lambda) \frac{B}{\Delta \lambda}.
\]  \hspace{1cm} (8)

then continuation even after failure \( X_2(\mu - C) = 1 \) otherwise liquidation \( X_2(\mu - C) = 0 \)

\( X_2(\mu) \): no conflict between rents and efficiency

\( X_2(\mu - C) \): rent–efficiency tradeoff

- Raising \( X_2(\mu - C) \) increases productive efficiency
- but also rent after loss \( \rightarrow \) cost of incentives at period 1
Funding without liquidation

If $\mathcal{X}_2(\mu - C) = 1$, project can be funded iff

$$\left[ 1 + \frac{1}{1+r} \right] \mathcal{P} \geq I.$$ 

Without liquidation, 2 period = repetition of 1 period: $\mathcal{P}$ obtained twice

Just as in one period case, if $\mathcal{P} < 0$, project cannot be funded, no matter how small $I$. 
Funding with liquidation after loss

When $X_2(\mu - C) = 0$, project funded if

$$\hat{P} = (\mu - \lambda C) \left[ 1 + \frac{1 - \lambda}{1 + r} \right] - (1 - \lambda) \frac{B}{\Delta \lambda} = \mathcal{P} + \frac{1 - \lambda}{1 + r} (\mu - \lambda C) \geq I$$

$$\hat{P} > \mathcal{P}$$

$\mathcal{P} < 0$ does not imply project cannot be funded, $\hat{P}$ can still be $> 0$

Liquidation threat reduces cost of incentives:

$\rightarrow$ Bolton Scharfstein 1990
Optimal contract

Proposition 1:

- If $\lambda(\mu - \lambda C) > (1 - \lambda) \frac{B}{\Delta \lambda}$ project funded iff

\[
\left[1 + \frac{1}{1+r}\right] \mathcal{P} \geq 1
\]

in which case i) there is no liquidation and ii) the compensation of the agent is

\[
T_2(\mu, \mu) = \left[1 + \frac{1 + r}{1 - \lambda} \right] \frac{B}{\Delta \lambda},
\]

\[
T_2(\mu - C, \mu) = \frac{B}{\Delta \lambda}
\]

- Otherwise, project funded iff

\[
\mathcal{\hat{P}} \geq 1
\]

in which case there is liquidation after failure and the agent is paid, after 2 successes only,

\[
T_2(\mu, \mu) = \frac{1 + r}{1 - \lambda} \frac{B}{\Delta \lambda}
\]
Implementing optimal contract with debt, equity and dividend threshold

**Proposition 2:** Consider the case in which there is liquidation after loss, then, if

\[ \lambda < \frac{1}{2 + r} \cdot \frac{B}{\Delta \lambda C} \cdot \frac{1 + r}{1 - \lambda} \cdot \frac{1}{(2 + r)\lambda} < 1 \]

optimal contract implemented by debt, equity and dividend threshold. Agent gets

\[ \alpha = \frac{B}{\Delta \lambda C} \cdot \frac{1 + r}{1 - \lambda} \cdot \frac{1}{(2 + r)\lambda} \]

of shares, not allowed to sell (otherwise no longer incentivized)

Remaining fraction of shares and debt held by principal

Debt service at each period: \( \mu - \lambda C \)

Liquidation when debt cannot be served

Dividend when accumulated retained earnings reach \( (2 + r)\lambda C \)
Timing

At period $n$:

- Agent’s continuation utility $W_n$ and principal’s value function $F_n$ evaluated given $H_n = \text{information available at beginning of period } n = \text{all past realizations of cash flows}$
- Set new size of operation $X_n$
- Agent privately decides exert effort or not
- Cash flow ($\mu$ or $\mu - C$) realized
- Agent receives transfer $T_n$
Agent’s continuation utility

On equilibrium path (effort exerted)

\[ W_n = E\left[\sum_{k=0}^{\infty} \frac{X_{n+k}t_{n+k}}{(1+r)^k} | H_n\right] \]

Recursively, promise keeping condition same as with 2 periods

\[ W_n = E[X_n t_n + \frac{W_{n+1}}{1+r} | H_n] \] \hspace{1cm} (9)

Define size adjusted continuation utility \( W_n = X_{n-1} w_n \),
downscaling factor \( x_n = \frac{X_n}{X_{n-1}} \)

\[ w_n = x_n E[t_n + \frac{W_{n+1}}{1+r} | H_n] \] \hspace{1cm} (10)
Dynamics of size adjusted continuation utility

\[ w_n = \text{conditional expectation} \]

\[ \implies \text{changes when new info = cash flow realizations} \]

\[ \implies \text{continuation utility } w_n \text{ evolves with cash flow innovations} \]

If no loss, \( w_n \) goes up to \( w^+(w_n) \), if loss down to \( w^-(w_n) \)

Size-adjusted transfer if no loss: \( t^+(w_n) \), if loss: \( t^-(w_n) \)

PK (promise keeping) (10) rewrites as

\[ w_n = x_n \left[ (1 - \lambda) t^+ + \lambda t^- \right] + \frac{1}{1 + r} \left[ (1 - \lambda) w^+ + \lambda w^- \right] \] (11)
**State variable**

$w_n$ forward looking $=$ expectation of future compensation

$w_n$ also backward looking $=$ up after success, down after failure $\rightarrow$ tracks performance

In principle continuation utility depends on all the information $H_n$

Because of constant returns to scale, it can be shown that only state variables one needs to remember are size $X_n$ and size adjusted continuation utility $w_n$

Formal proof $=$ verification theorem: conjectures on solution (e.g., only state variables are $X_n$ and $w_n$), compute value function under conjectures, prove any other policy $\rightarrow$ lower value
Incentive compatibility

Lemma 4: *The incentive compatibility condition is*

\[ (t^+ + \frac{w^+}{1+r}) - (t^- + \frac{w^-}{1+r}) \geq \frac{B}{\Delta \lambda} \]  \hspace{1cm} (12)

Similar to two-period case (7)
Principal’s value function

\[ F_n = E\left[ \sum_{k=0}^{\infty} \frac{X_{n+k}(\mu - \tilde{C}_{n+k} - \tilde{t}_{n+k})}{(1 + r)^k} \right] | H_n \]

Recursively (by law of iterated expectations)

\[ F_n = E[X_n(\mu - \tilde{C}_n - \tilde{t}_n) + \frac{1}{1 + r} \tilde{F}_{n+1} | H_n] \] (13)

similar to 2–period case

Similarly to continuation utility, scale by \( X_{n-1} \)

\[ f_n = x_n E[(\mu - \tilde{C}_n - \tilde{t}_n) + \frac{1}{1 + r} \tilde{f}_{n+1} | H_n] \] (14)
Dynamics of size adjusted principal’s value function

Success $f(w_n) \rightarrow f(w^+(w_n))$. Failure $f(w_n) \rightarrow f(w^-(w_n))$

$$f(w_n) = \max_{t^+,t^-,x} x(w_n)[(\mu - \lambda C) - \{(1 - \lambda)t^+(w_n) + \lambda t^-(w_n)\}$$

$$+ \frac{1}{1 + r}\{(1 - \lambda)f(w^+(w_n)) + \lambda f(w^-(w_n))\}] \quad (15)$$

$\rightarrow$ Bellman equation
Heuristic derivation of optimal contract

1. Step 1: Delay pay until rid of moral hazard problem
2. Step 2: IC & PK $\rightarrow$ dynamics of $w$ when no payment and no liquidation
3. Step 3: Downscale if and only if you can’t avoid it
Optimal pay with finite horizon

2 period model: postpone payment until period 2 (costless because $P$ and $A$ equally patient)

Finite horizon $T$ and equal patience same argument

$\implies$ delay pay until $T$
Optimal pay with infinite horizon

\( w^P \) = threshold at which liquidation risk eliminated because accumulated promised pay large enough to incentivize effort

How can you incentivize effort without liquidation threat?

In two period case: if no liquidation threat, dynamic contract = repetition of one period contract

Same thing with infinite horizon: to incentivise effort without liquidation threat, promise to pay, at each period, \( \frac{B}{\Delta \lambda} \) after success and 0 after failure (as in one period model)

What is the expected present value of this stream of payments?

\[
(1 + \frac{1}{1+r} + \frac{1}{(1+r)^2} + \ldots)(1 - \lambda) \frac{B}{\Delta \lambda} = \frac{1+r}{r} (1 - \lambda) \frac{B}{\Delta \lambda} = w^P
\]
Step 2: IC and PK pin down $w$

When no transfer, binding IC and PK yields:

- If loss at period $n$, reduce continuation utility to
  
  $$w^-(w_n) = (1 + r) \left[ w_n - (1 - \lambda) \frac{B}{\Delta \lambda} \right]$$

- If success at period $n$, increase continuation utility to
  
  $$w^+(w_n) = (1 + r) \left[ w_n + \lambda \frac{B}{\Delta \lambda} \right]$$
Step 3: when continuation utility too low impossible to incentivize using carrots only

When $w_n$ large enough, principal can threaten to reduce continuation utility by $(1 - \lambda) \frac{B}{\Delta \lambda}$ if loss

But, when $w_n$ low, this would drive continuation utility below 0, contradicting limited liability

To relax IC, reduce $X_n$, i.e., downsize, to reduce private benefit from shirking $X_n B$

If $w_n$ very low, full downsizing: liquidation
Optimal contract when P and A equally patient

Proposition 3: In the optimal contract, \( w_n \) evolves as discounted martingale, within \( w^L \) and \( w^P \).

When \( w_n \) reaches upper bound \( w^P \):

- if no loss, transfer \( (t_n = \frac{B}{\Delta \lambda}) \) and \( w_n \) stays at \( w^P \)
- if loss, no pay, but \( w_n \) stays at \( w^P \) (absorbed)

When \( w_n \) reaches lower bound:

- if no loss, reflected upward
- if loss downsizing/liquidation
Dynamics of continuation utility and pay in optimal contract

Pay if success, stay if failure
Dynamic of continuation utility and liquidation in optimal contract
Modigliani Miller (MM) and Moral Hazard (MH)

- MM: exogenous cash → financial structure does not affect cash flow → nor value
- MH: as long as IC holds → financial structure does not affect cash flow → nor value

If different financial structures → same incentives → same value

Next, we present an intuitive and realistic implementation

DeMarzo Fishman (2007) offer another interesting implementation, with credit lines
Implementation of optimal contract

- Assets ($X_n$)
- Cash reserves ($M_n$)
- Debt (held by principal)
- Equity
  - Inside (held by agent)
  - Outside (held by principal)
Implementation of optimal contract

Downsized if cash at hand not enough to pay coupon

Increases after success, decreases after failure (cash flow from operation < coupon)

Tracks performance: Informationally equivalent to rent $W_n$

Assets ($X_n$)  $\rightarrow$ Debt (held by principal)  $\rightarrow$ Constant coupon

Cash reserves ($M_n$)  $\leftarrow$ Equity

- Inside (held by agent)
- Outside (held by principal)

Dividend (if accumulated earnings = cash reserves reach milestone)
Conclusion

Optimal dynamic contract relies on:

- **carrots**: promise pay (=agency rents) if performance milestone reached
- **sticks**: threaten downsizing → reduces incentives to shirk, liquidation → no more rents, after bad performance

Dynamic incentives help cope with moral hazard: long term contracting more powerful than short term (less rents)

Dynamic optimal contract can be implemented with:

- cash reserves
- inside and outside equity + dividend threshold
- debt + downsizing/liquidation when cash < debt service

→ Endogenous, optimal, financial structure