Optimal dynamic financial contracting

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Issues

How can firm finance investment when there are incentive problems ?

Do incentive problems generate credit rationing ?

Or can financial contract be designed to mitigate incentive problems ?

Optimal complete contracts

Incentive problems arise because of information asymmetry:

- Adverse selection (hidden information)
- Moral hazard (hidden action) $\rightarrow$ our focus

Beyond these frictions, no ad hoc restrictions:

- All observable variables are contractible
- No ad hoc/exogenous constraints or contracts

Financial structure and contracts emerge endogenously

Literature

One period: Holmstrom Tirole (1997) [HT] $\longrightarrow$ incentives so that agent prefers effort than shirking

Infinite horizon discrete time:

- DeMarzo Fishman (2007a,b)
- Biais, Mariotti, Plantin, Rochet (2007) [BMPR]

Continuous time:

- BMPR 2007 $\longrightarrow$ continuous time limit of discrete time
- DeMarzo Sannikov (2006) $\longrightarrow$ martingale approach
- Biais, Mariotti, Rochet, Villeneuve (2010): Poisson + investment
- DeMarzo, Fishman, He, Wang(2012): Brownian + investment
- Zhu (2013): optimal contract can involve shirking

Outline

1. One period HT 1997
2. Two-period Bolton Scharfstein 1990
3. Infinite horizon discrete time BMP 2007

Clarify similarity of economic mechanisms in 3 settings, and insights specific to dynamics

Players

Principal: unlimited liability, deep pocket (investors, bank, venture capital fund)

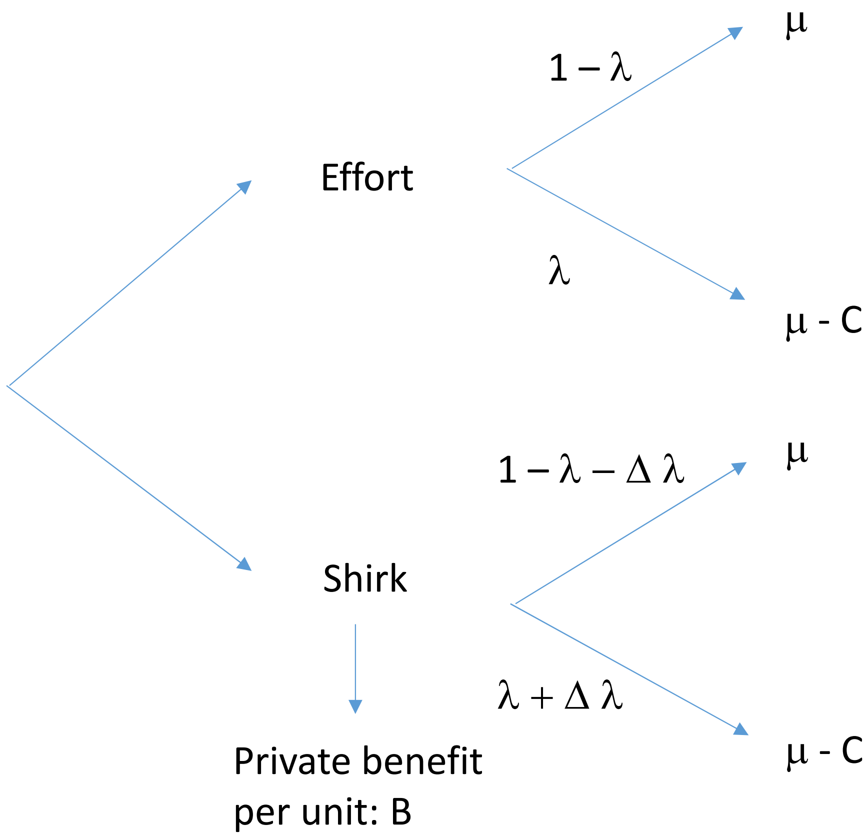
Agent: limited liability, cashless (manager, entrepreneur)

Indivisible initial project size 1 $\longrightarrow$ investment cost I

Principal invests I, hires agent to run project

All risk neutral, discount rate r

Effort and probability of success



Effort unobservable by principal + agent has limited liability -> moral hazard

First best (effort observable)

Assume high effort efficient:

$$\Delta\lambda C > B,$$

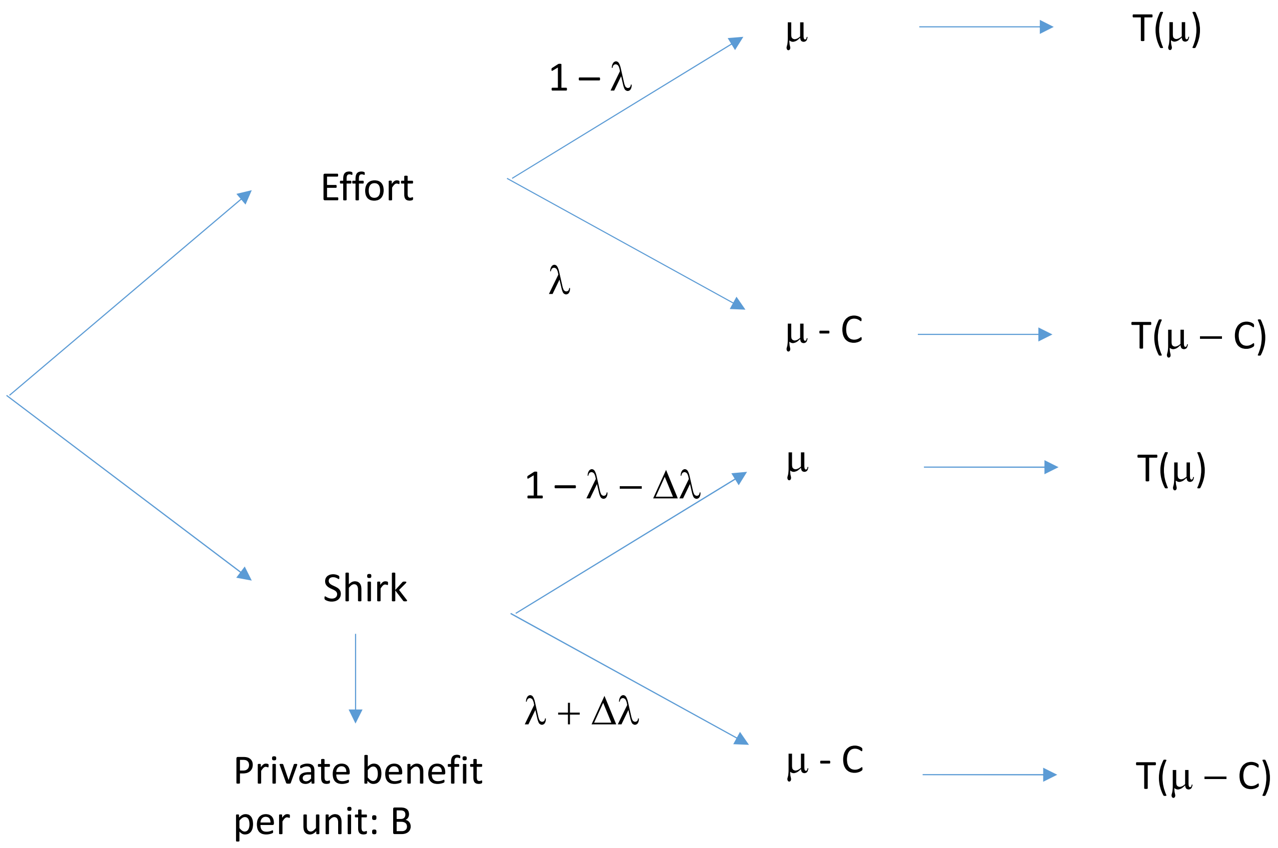
and investment efficient even if project lasts only one period:

$$\mu - \lambda C > I,$$

$\rightarrow$ under high effort project has positive net present value

First best $=$ investment, effort

One period

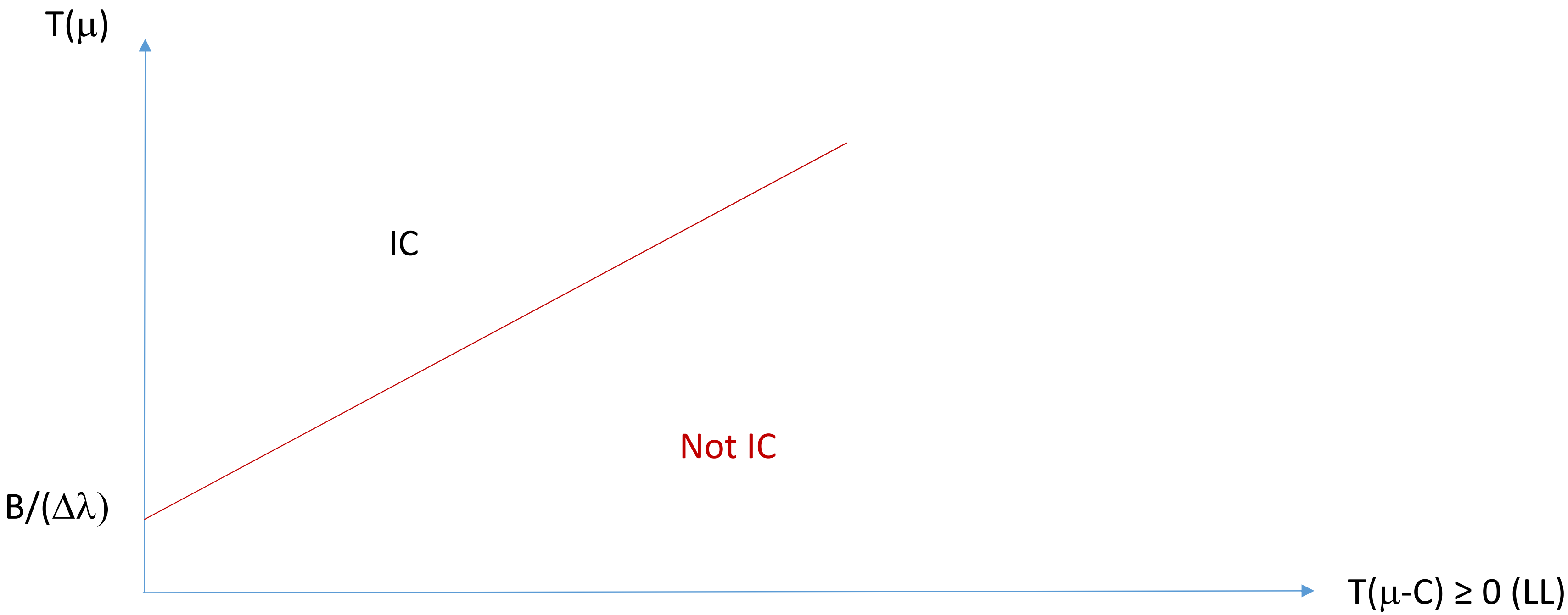


One period contract: mapping from cash flow to agent's compensation

Incentive compatibility

Agent's expected gain larger under effort than under shirking

$$T(\mu) - T(\mu\text{-}C) \geq B/(\Delta\lambda)$$

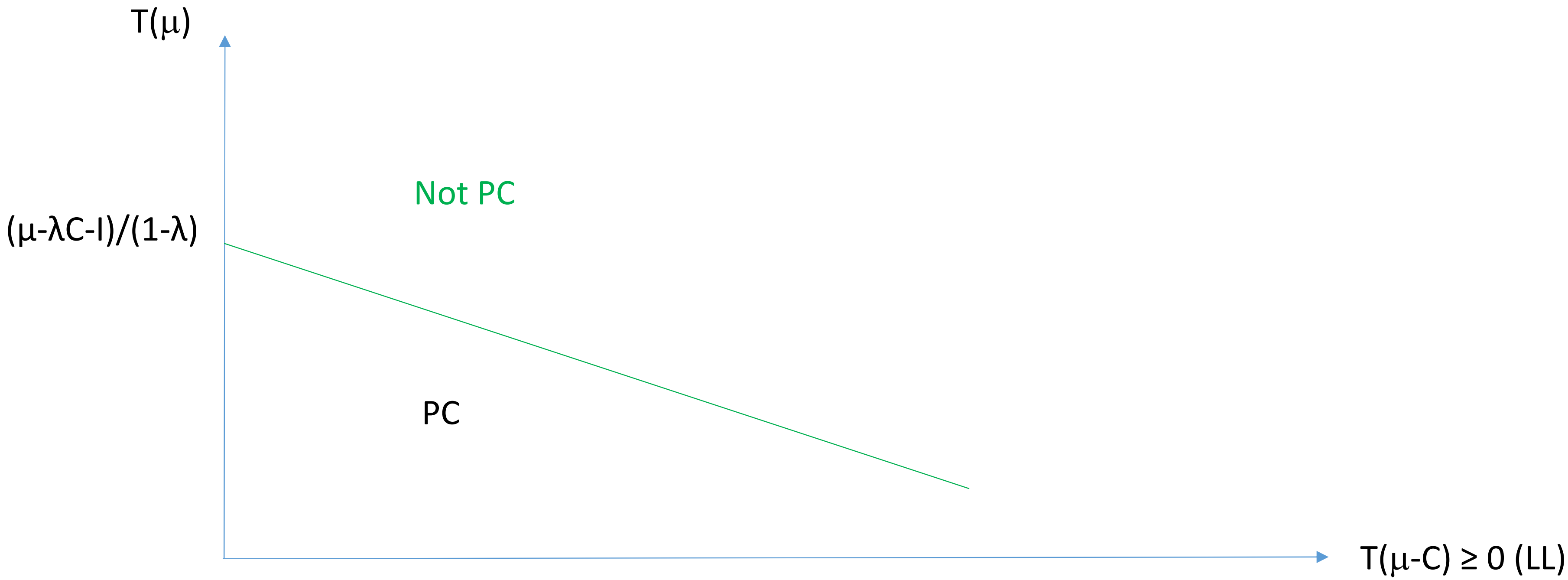


Large B → tempting to shirk → large compensation when no loss → cost of incentives
Small Δλ → tempting to shirk → cost of incentives

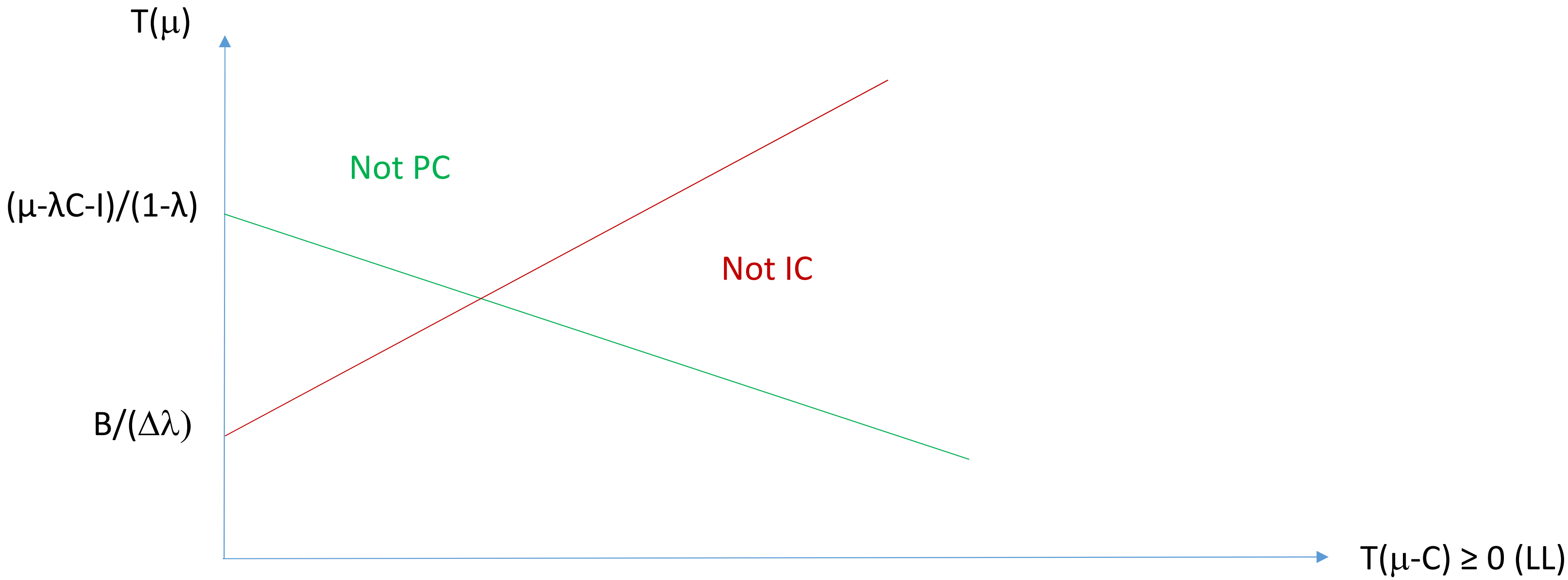
Participation constraint

Principal expected profit larger than 0

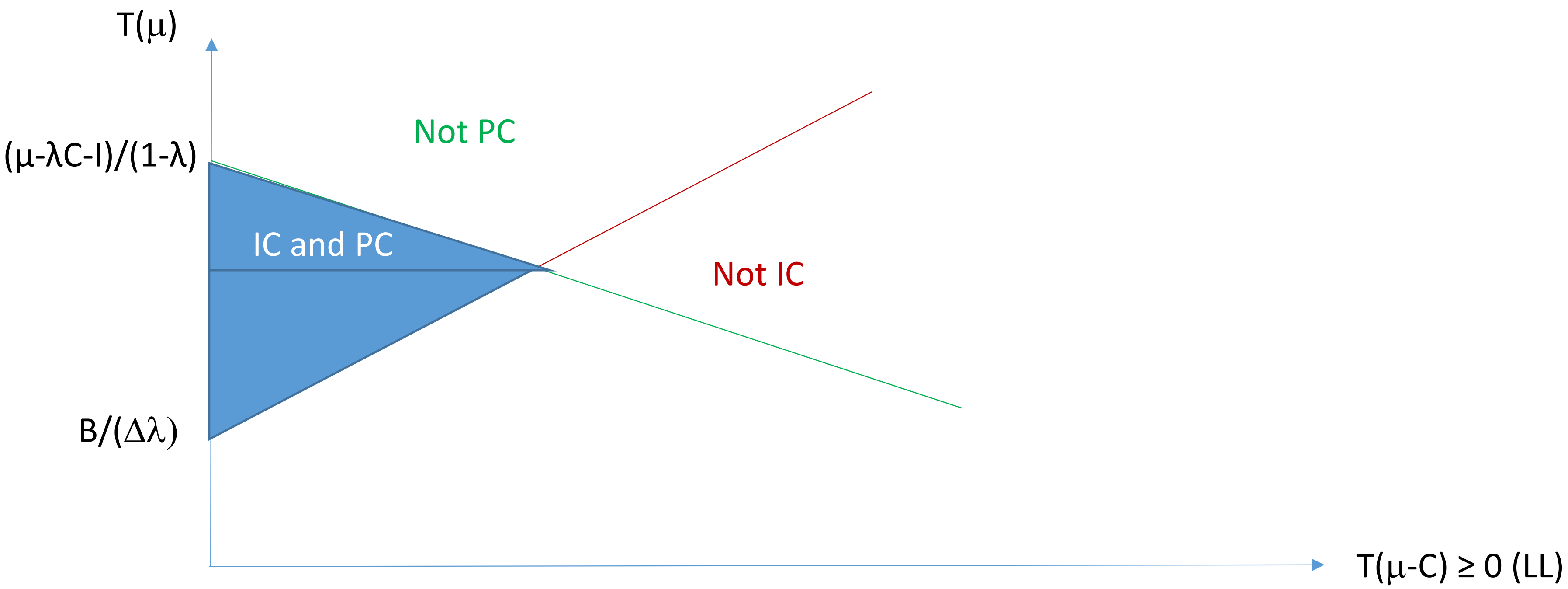
$$(1-\lambda)\, T(\mu) + \lambda\, T(\mu-C) \leq (\mu-\lambda C) - I$$



Tension between IC and PC



Incentive feasible set (Laffont Martimort, 2002)



IC set not empty if expected cash flow $\geq$ expected agency rent

$$(\mu-\lambda C) - I \geq (1-\lambda)\, B/(\Delta\lambda)$$

Pledgeable income (Tirole 2006)

$E(\text{return})$ which can be promised to investors without jeopardizing incentives of agent

$$\mathcal{P} = [\mu - \lambda C] - (1 - \lambda)\frac{B}{\Delta\lambda} \tag{1}$$

First term [in brackets] = expected cash flow with effort

Second term = expected compensation which must be left to agent for incentives (agency rent)

Incentive feasible set not empty iff

$$\mathcal{P} \geq I \tag{2}$$

Optimal contract with effort

Suppose principal has all bargaining power

$$\max(\mu - I) - \lambda[T(\mu - C) + C] - (1 - \lambda)T(\mu)$$
$$\text{s.t. IC: } T(\mu) - T(\mu - C) \geq \frac{B}{\Delta\lambda} \text{ and LL: } T \geq 0$$

$T(\mu - C) = 0$: relax IC and raise P's gains

$T(\mu) = \frac{B}{\Delta\lambda}$: bind IC to max principal's gains

Max possible gain for principal

$$(\mu - \lambda C - I) - (1 - \lambda)\frac{B}{\Delta\lambda} = \mathcal{P} - I \qquad (3)$$

Optimal contract without effort

If project operated without effort, principal pays 0 wage and gets

$$[\mu - (\lambda + \Delta\lambda)C - I] \tag{4}$$

Comparing (3) & (4), greater profits with effort iff

$$(1 - \lambda)\frac{B}{\Delta\lambda} < \Delta\lambda C. \tag{5}$$

LHS: what must be paid to agent so that effort = cost of incentives
RHS: efficiency cost of no effort = benefit of incentives

Optimal contract (from point of view of principal)

Proposition 1:

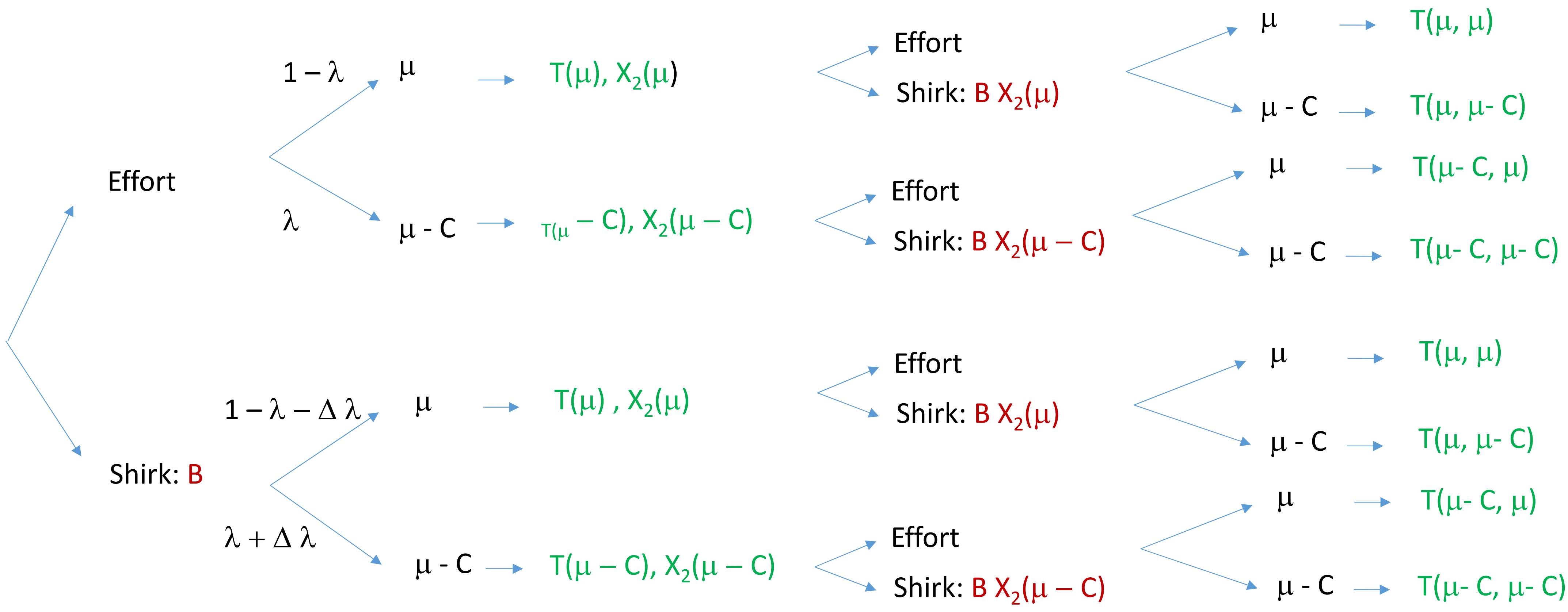
If $E(profit|\ no\ effort) \geq 0$

- *If* $B < \frac{(\Delta\lambda)^2 C}{1-\lambda}$, *pay agent* $\frac{B}{\Delta\lambda}$ *iff success* $\implies$ *effort*
- *Otherwise, don't pay agent* $\implies$ *no effort (// Zhu, 2013)*

If $E(profit|\ no\ effort) < 0$:

- *If* $\mathcal{P} \geq I$, *pay agent* $\frac{B}{\Delta\lambda}$ *iff success* $\implies$ *effort*
- *If* $\mathcal{P} < I$, *no investment (credit rationing)*

Two period



Size adjusted transfer: $T_2 = X_2\, t_2$

Agent's continuation utility

At beginning of period 2 following realization $C_1 \in \{0, C\}$

$$W_2(\mu - C_1) = E(T_2(\mu - C_1, \mu - \tilde{C}_2) | C_1)$$

At beginning of period 1 (by law of iterated expectations)

$$W_1 = E\left[T_1(\mu - \tilde{C}_1) + \frac{1}{1+r} W_2(\mu - \tilde{C}_1)\right]$$

$\rightarrow$ "promise keeping" condition

Expectations computed by agent rationally anticipating to exert effort $\rightarrow \lambda$

Principal's value function

At beginning of 2^{nd} period, value function

$$F_2 = X_2(\mu - C_1)E[\mu - \tilde{C}_2 - t_2(\mu - C_1, \mu - \tilde{C}_2)|C_1]$$

At beginning of period 1 (by law of iterated expectations)

$$F_1 = E\left[\mu - \tilde{C}_1 - T_1 + \frac{1}{1+r}F_2(\tilde{C}_1)\right]$$

Incentive compatibility

At second period, similar IC to one period

$$T_2(\mu - C_1, \mu) - T_2(\mu - C_1, \mu - C) \geq X_2(\mu - C_1)\frac{B}{\Delta\lambda} \quad (6)$$

IC $\implies$ wedge between the utility of agent after a success and after a failure, increases with $\frac{B}{\Delta\lambda}$ and X_2

At first period:

$$\left[T_1(\mu) + \frac{W_2(\mu)}{1+r}\right] - \left[T_1(\mu - C) + \frac{W_2(\mu - C)}{1+r}\right] \geq \frac{B}{\Delta\lambda} \quad (7)$$

Incentivize with current transfer T_1 and continuation utility W_2

Continuation utility dynamics

Lemma 1: *IC $\Longrightarrow$ agent's continuation utility goes down (resp. up) after loss (resp. no loss)*

Lemma 2: *No transfer after loss*

$\rightarrow$ Will also hold in infinite horizon model

Delay payment

Lemma 3: *When principal and agent equally patient, weakly optimal to postpone compensation to final period*

Because A as patient as P, delaying T (capitalizing it at rate r) generates no inefficiency, but makes incentives more effective: use late payment to reward late and early efforts

$\longrightarrow$ Will also hold in infinite horizon model

Optimal liquidation

Continuation after success: $X_2(\mu) = 1$. Moreover, if

$$\lambda(\mu - \lambda C) > (1 - \lambda)\frac{B}{\Delta\lambda}. \quad (8)$$

then continuation even after failure $X_2(\mu - C) = 1$ otherwise liquidation $X_2(\mu - C) = 0$

$X_2(\mu)$ no conflict between rents and efficiency

$X_2(\mu - C)$: rent–efficiency tradeoff

- Raising $X_2(\mu - C)$ increases productive efficiency
- but also rent after loss $\to$ cost of incentives at period 1

Funding without liquidation

If $X_2(\mu - C) = 1$, project can be funded iff

$$\left[1 + \frac{1}{1+r}\right] \mathcal{P} \geq I.$$

Without liquidation, 2 period = repetition of 1 period: $\mathcal{P}$ obtained twice

Just as in one period case, if $\mathcal{P} < 0$, project cannot be funded, no matter how small I

Funding with liquidation after loss

When $X_2(\mu - C) = 0$, project funded if

$$\hat{\mathcal{P}} = (\mu - \lambda C)\left[1 + \frac{1-\lambda}{1+r}\right] - (1-\lambda)\frac{B}{\Delta\lambda} = \mathcal{P} + \frac{1-\lambda}{1+r}(\mu - \lambda C) \geq I$$

$$\hat{\mathcal{P}} > \mathcal{P}$$

$\mathcal{P} < 0$ does not imply project cannot be funded, $\hat{\mathcal{P}}$ can still be > 0

Liquidation threat reduces cost of incentives:

→ Bolton Scharfstein 1990

Optimal contract

Proposition 1:

- *If* $\lambda(\mu - \lambda C) > (1-\lambda)\frac{B}{\Delta\lambda}$ *project funded iff*

$$\left[1 + \frac{1}{1+r}\right]\mathcal{P} \geq I$$

in which case i) there is no liquidation and ii) the compensation of the agent is

$$T_2(\mu,\mu) = [1 + \frac{1+r}{1-\lambda}]\frac{B}{\Delta\lambda},\ T_2(\mu - C, \mu) = \frac{B}{\Delta\lambda}$$

- *Otherwise, project funded iff*

$$\hat{\mathcal{P}} \geq I$$

in which case there is liquidation after failure and the agent is paid, after 2 successes only,

$$T_2(\mu,\mu) = \frac{1+r}{1-\lambda}\frac{B}{\Delta\lambda}$$

Implementing optimal contract with debt, equity and dividend threshold

Proposition 2: *Consider the case in which there is liquidation after loss, then, if*

$$\lambda < \frac{1}{2+r}, \frac{B}{\Delta\lambda C}\frac{1+r}{1-\lambda}\frac{1}{(2+r)\lambda} < 1$$

optimal contract implemented by debt, equity and dividend threshold. Agent gets

$$\alpha = \frac{B}{\Delta\lambda C}\frac{1+r}{1-\lambda}\frac{1}{(2+r)\lambda}$$

of shares, not allowed to sell (otherwise no longer incentivized)
Remaining fraction of shares and debt held by principal
Debt service at each period: $\mu - \lambda C$
Liquidation when debt cannot be served
Dividend when accumulated retained earnings reach $(2+r)\lambda C$

Timing

At period n:

- Agent's continuation utility W_n and principal's value function F_n evaluated given H_n = information available at beginning of period n = all past realizations of cash flows
- Set new size of operation X_n
- Agent privately decides exert effort or not
- Cash flow (μ or $\mu - C$) realized
- Agent receives transfer T_n

Agent's continuation utility

On equilibrium path (effort exerted)

$$W_n = E[\sum_{k=0}^{\infty} \frac{X_{n+k} t_{n+k}}{(1+r)^k} | H_n]$$

Recursively, promise keeping condition same as with 2 periods

$$W_n = E[X_n t_n + \frac{W_{n+1}}{1+r} | H_n] \tag{9}$$

Define size adjusted continuation utility $W_n = X_{n-1} w_n$, downscaling factor $x_n = \frac{X_n}{X_{n-1}}$

$$w_n = x_n E[t_n + \frac{w_{n+1}}{1+r} | H_n] \tag{10}$$

Dynamics of size adjusted continuation utility

w_n = conditional expectation

$\Longrightarrow$ changes when new info = cash flow realizations

$\Longrightarrow$ continuation utility w_n evolves with cash flow innovations

If no loss w_n goes up to $w^+(w_n)$, if loss down to $w^-(w_n)$

Size–adjusted transfer if no loss: $t^+(w_n)$, if loss: $t^-(w_n)$

PK (promise keeping) (10) rewrites as

$$w_n = x_n[\{(1-\lambda)t^+ + \lambda t^-\} + \frac{1}{1+r}\{(1-\lambda)w^+ + \lambda w^-\}] \quad (11)$$

State variable

w_n forward looking = expectation of future compensation

w_n also backward looking = up after success, down after failure $\rightarrow$ tracks performance

In principle continuation utility depends on all the information H_n

Because of constant returns to scale, it can be shown that only state variables one needs to remember are size X_n and size adjusted continuation utility w_n

Formal proof = verification theorem: conjectures on solution (e.g., only state variables are X_n and w_n), compute value function under conjectures, prove any other policy $\rightarrow$ lower value

Incentive compatibility

Lemma 4: *The incentive compatibility condition is*

$$(t^{+} + \frac{w^{+}}{1+r}) - (t^{-} + \frac{w^{-}}{1+r}) \geq \frac{B}{\Delta\lambda} \tag{12}$$

Similar to two–period case (7)

Principal's value function

$$F_n = E[\sum_{k=0}^{\infty} \frac{X_{n+k}(\mu - \tilde{C}_{n+k} - \tilde{t}_{n+k})}{(1+r)^k} | H_n]$$

Recursively (by law of iterated expectations)

$$F_n = E[X_n(\mu - \tilde{C}_n - \tilde{t}_n) + \frac{1}{1+r}\tilde{F}_{n+1} | H_n] \tag{13}$$

similar to 2–period case

Similarly to continuation utility, scale by X_{n-1}

$$f_n = x_n E[(\mu - \tilde{C}_n - \tilde{t}_n) + \frac{1}{1+r}\tilde{f}_{n+1} | H_n] \tag{14}$$

Dynamics of size adjusted principal's value function

Success $f(w_n) \rightarrow f(w^+(w_n))$. Failure $f(w_n) \rightarrow f(w^-(w_n))$

$$f(w_n) = \max_{t^+(),t^-(),x()} x(w_n)[(\mu - \lambda C) - \{(1-\lambda)t^+(w_n) + \lambda t^-(w_n)\} + \frac{1}{1+r}\{(1-\lambda)f(w^+(w_n)) + \lambda f(w^-(w_n))\}] \tag{15}$$

$\rightarrow$ Bellman equation

Heuristic derivation of optimal contract

1. Step 1: Delay pay until rid of moral hazard problem
2. Step 2: IC & PK $\longrightarrow$ dynamics of w when no payment and no liquidation
3. Step 3: Downscale if and only if you can't avoid it

Optimal pay with finite horizon

2 period model: postpone payment until period 2 (costless because P and A equally patient)

Finite horizon T and equal patience same argument

$\Longrightarrow$ delay pay until T

Optimal pay with infinite horizon

w^P = threshold at which liquidation risk eliminated because accumulated promised pay large enough to incentivize effort

How can you incentivize effort without liquidation threat?

In two period case: if no liquidation threat, dynamic contract = repetition of one period contract

Same thing with infinite horizon: to incentivise effort without liquidation threat, promise to pay, at each period, $\frac{B}{\Delta\lambda}$ after success and 0 after failure (as in one period model)

What is the expected present value of this stream of payments?

$$(1 + \frac{1}{1+r} + \frac{1}{(1+r)^2} + \ldots)(1-\lambda)\frac{B}{\Delta\lambda} = \frac{1+r}{r}(1-\lambda)\frac{B}{\Delta\lambda} = w^P$$

Step 2: IC and PK pin down w

When no transfer, binding IC and PK yields:

- If loss at period n, reduce continuation utility to

$$w^{-}(w_n) = (1+r)\left[w_n - (1-\lambda)\frac{B}{\Delta\lambda}\right]$$

- If success at period n, increase continuation utility to

$$w^{+}(w_n) = (1+r)\left[w_n + \lambda\frac{B}{\Delta\lambda}\right]$$

Step 3: when continuation utility too low impossible to incentivize using carrots only

When w_n large enough, principal can threaten to reduce continuation utility by $(1-\lambda)\frac{B}{\Delta\lambda}$ if loss

But, when w_n low, this would drive continuation utility below 0, contradicting limited liability

To relax IC, reduce X_n, i.e., downsize, to reduce private benefit from shirking $X_n B$

If w_n very low, full downsizing: liquidation

Optimal contract when P and A equally patient

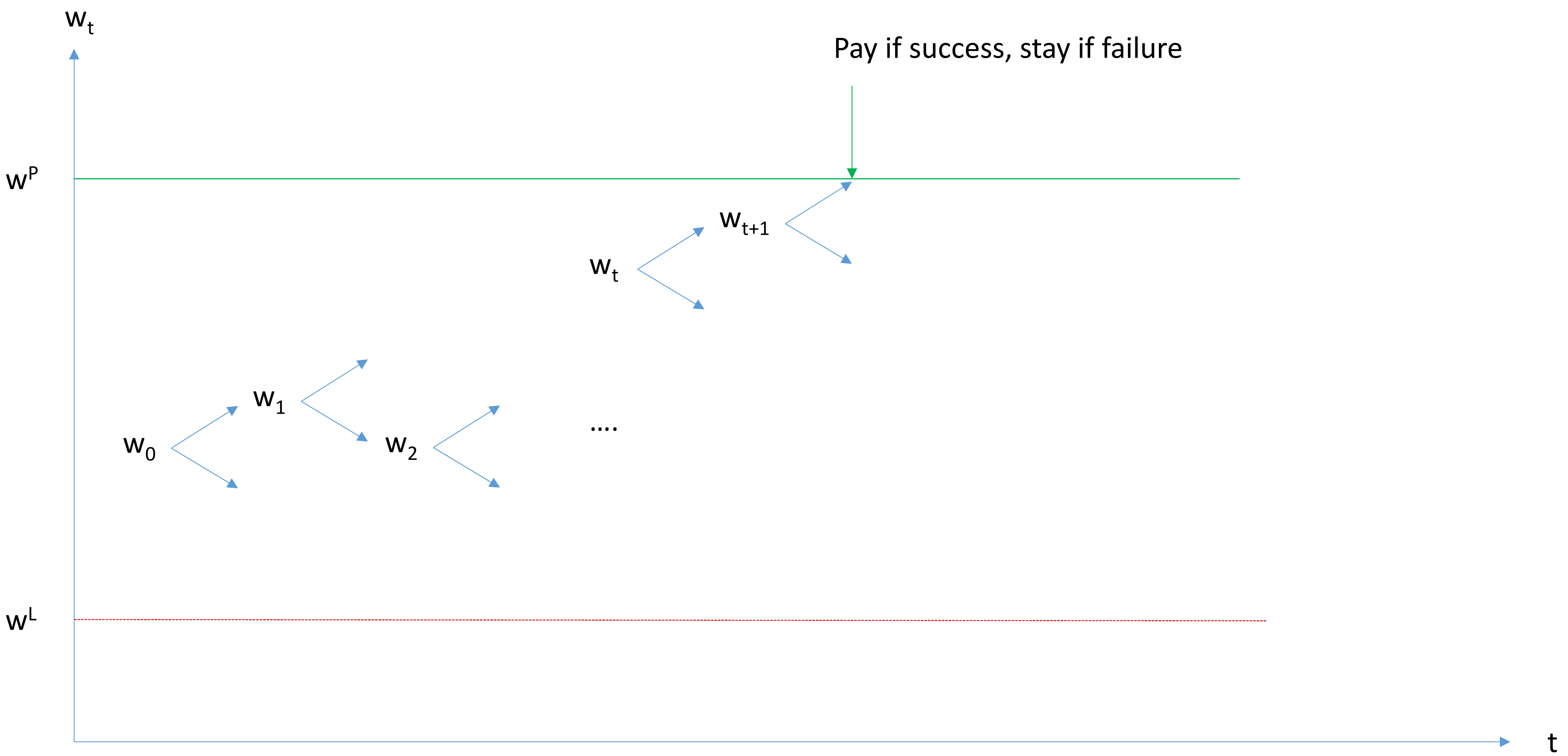
Proposition 3: *In the optimal contract, w_n evolves as discounted martingale, within w^L and w^P.*
When w_n reaches upper bound w^P:

- *if no loss, transfer ($t_n = \frac{B}{\Delta\lambda}$) and w_n stays at w^P*
- *if loss, no pay, but w_n stays at w^P (absorbed)*

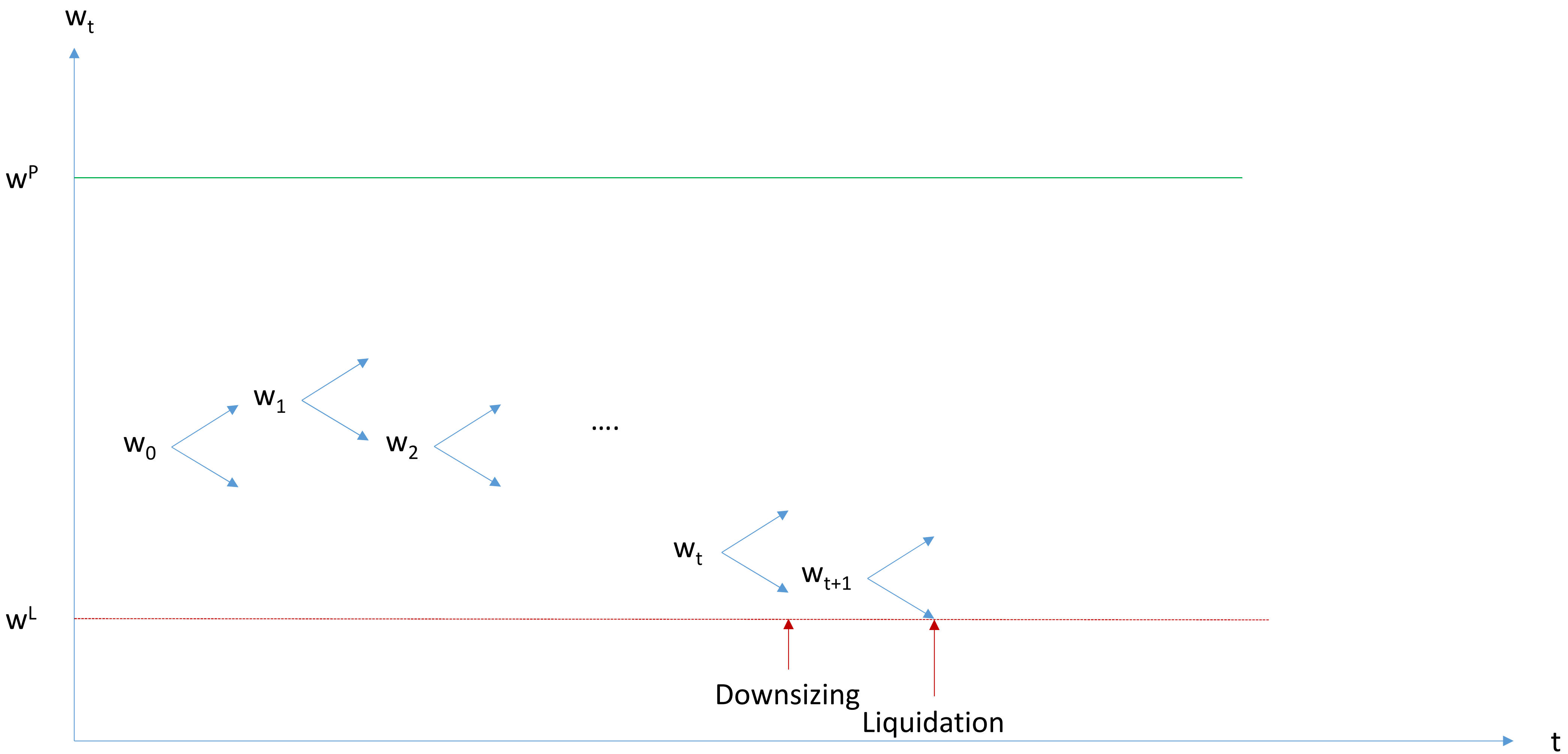
When w_n reaches lower bound:

- *if no loss, reflected upward*
- *if loss downsizing/liquidation*

Dynamics of continuation utility and pay in optimal contract



Dynamic of continuation utility and liquidation in optimal contract



Modigliani Miller (MM) and Moral Hazard (MH)

- MM: exogenous cash $\longrightarrow$ financial structure does not affect cash flow $\longrightarrow$ nor value
- MH: as long as IC holds $\longrightarrow$financial structure does not affect cash flow $\longrightarrow$ nor value

If different financial structures $\longrightarrow$ same incentives $\longrightarrow$ same value

Next, we present an intuitive and realistic implementation

DeMarzo Fishman (2007) offer another interesting implementation, with credit lines

Implementation of optimal contract

Assets (X_n)	Debt (held by principal)
Cash reserves (M_n)	Equity Inside (held by agent) Outside (held by principal)

Implementation of optimal contract

Downsized if cash at hand not enough to pay coupon	←	Assets (X_n)	Debt (held by principal)	→	Constant coupon
Increases after success, decreases after failure (cash flow from operation < coupon)	←	Cash reserves (M_n)	Equity Inside (held by agent) Outside (held by principal)	→	Dividend (if accumulated earnings = cash reserves reach milestone)

↓

Tracks performance:
Informationally equivalent to rent W_n

Conclusion

Optimal dynamic contract relies on:

- carrots: promise pay (=agency rents) if performance milestone reached
- sticks: threaten downsizing $\longrightarrow$ reduces incentives to shirk, liquidation $\longrightarrow$ no more rents, after bad performance

Dynamic incentives help cope with moral hazard: long term contracting more powerful than short term (less rents)

Dynamic optimal contract can be implemented with:

- cash reserves
- inside and outside equity + dividend threshold
- debt + downsizing/liquidation when cash $<$ debt service

$\longrightarrow$ Endogenous, optimal, financial structure