Financial Markets with Information Frictions

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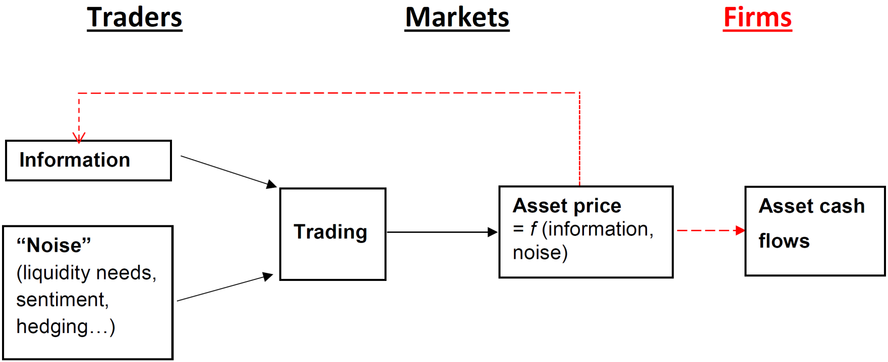
An Informational View of Financial Markets



Specifics

- How to model information?
  - Admati and Pfleiderer (1986, p. 400–401): "It is most convenient to envision information as a signal, a random variable that is jointly distributed with the state of the world."
- What kind of information?
  - Is it about fundamentals or about order flows?
- How to trade on information? (noisy-REE vs. Kyle 85, 89)
  - Market orders or limit orders (demand schedules)?
  - Price takers or strategic traders (price impact)?
  - How to form prices? Market maker? Walrasian auctioneer?
- What determine assets and their cash flows?
  - Asset pricing vs. corporate finance (separate)
  - Feedback effects (simultaneous)
  - Security design (endogenous assets/markets)

Various Applications in the Literature

- Price discovery and market liquidity (e.g., O'Hara 2003)
- Insider trading, information leakage and financial regulation (e.g.,Fishman and Hagerty 1992; Leland 1992; Indjejikian, Lu and Yang 2014; Yang and Zhu 2019)
- Disclosure and the cost of capital (e.g., Verrecchia 2001; Easley and O'Hara 2004)
- Information sales
  - Direct sales, e.g., Bloomberg and sell side analysts (Admati and Pfleiderer 1986; Veldkamp 2006; García and Sangiorgi 2011)
  - Indirect sales, e.g., mutual funds (Admati and Pfleiderer 1990; García and Vanden 2009)
- Information acquisition and complementarity, e.g., buy side analysts (Ganguli and Yang 2009; García and Strobl 2011; Goldstein and Yang 2015)
- Financial crisis and crashes (Gennotte and Leland 1990; Angeletos and Werning 2006)
- Social networks (Ozsoylev and Walden 2011; Han and Yang 2013)
- Commodity financialization (Goldstein, Li and Yang 2014; Goldstein and Yang 2019)
- ...

Roadmap of the Remaining Talk

- Applications:
  - Information disclosure in financial markets (classical, NREE)
  - Financial market feedback and commodity financialization (real effect, NREE)
  - Back running (HFT, order-flow information, dynamic Kyle)
- References:
  - Goldstein and Yang (2017), "Information Disclosure in Financial Markets," *Annual Review of Financial Economics*
  - Goldstein and Yang (2019), "Commodity Financialization and Information Transmission," Working Paper
  - Yang and Zhu (2019), "Back-Running: Seeking and Hiding Fundamental Information in Order Flows," *Review of Financial Studies*

Information Disclosure in Financial Markets

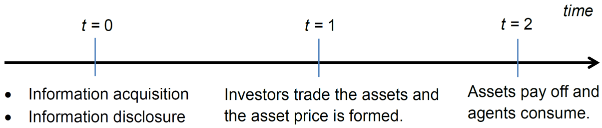
Disclosure and Financial Regulation

- Various forms of disclosure:
  - Macroeconomic statistics (interest rate, growth, inflation, unemployment); stress tests; credit ratings; mandatory corporate disclosures
- Response to crises often involves more disclosure requirement:
  - E.g., Sarbanes-Oxley and Dodd-Frank
  - Greenstone, Oyer, and Vissing-Jorgensen (2006, p. 399): "Since the passage of the Securities Act of 1933 and the Securities Exchange Act of 1934, the federal government has actively regulated U. S. equity markets. The centerpiece of these efforts is the mandated disclosure of financial information."
- "Common wisdom" among many academics and policy makers is that disclosure is a "panacea":
  - **More disclosure = Less adverse selection + More informed decisions**
- However, economic theory suggests that there are unintended costs of disclosure

Source

- This part is based on the review article, "Information Disclosure in Financial Markets" (Goldstein and Yang, *Annual Review of Financial Economics*, 2017)
- The review article provides a cohesive analytical framework to demonstrate:
  - **the conventional wisdom that disclosure improves market quality with exogenous information**
  - **that disclosure crowds out information production**
  - that disclosure affects real efficiency
  - that disclosure affects traders' welfare through changing trading opportunities and through beauty-contest motives

A CARA-Normal Model of Disclosure



- Two tradable assets in the date-1 market:
  - Risk-free asset (net risk-free rate $= 0$, numéraire)
  - Risky asset with a fixed supply $Q > 0$
    - Exogenous date-2 payoff, $\tilde{v} \sim N\left(0, \tau_v^{-1}\right)$
    - Endogenous date-1 price, $\tilde{p}$

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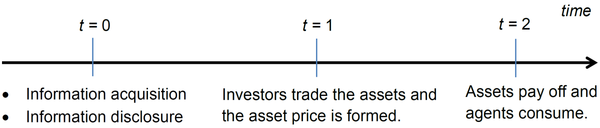

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    - Exogenous date-2 payoff, $\tilde{v} \sim N\left(0, \tau_v^{-1}\right)$
    - Endogenous date-1 price, $\tilde{p}$
- Players: a continuum of rational traders and noise traders
  - Rational trader i maximizes **CARA** preference:

    $$E\left[-\exp\left(-\gamma \tilde{W}_{i,2}\right) \middle| information_i\right]$$

    where

    $$\gamma = risk\ aversion \text{ and } \tilde{W}_{i,2} = wealth \text{ at } t = 2$$

  - Noise traders demand $\tilde{u} \sim N\left(0, \tau_u^{-1}\right)$, where $\tilde{u} \perp \tilde{v}$.

Public and Private Information

- Prior to trading, all traders observe a **public signal**

$$\tilde{y} = \tilde{v} + \tilde{\eta}, \text{ with } \tilde{\eta} \sim N\left(0, \tau_\eta^{-1}\right) \text{ and } \tau_\eta \geq 0$$

  - E.g., earnings announcements, credit ratings, stress test results
  - Better disclosure quality = a higher value of τ_η
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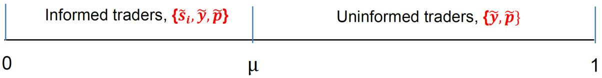
- All traders submit demand schedules and so they can condition on the price $\tilde{p}$
- Only a fraction μ of rational traders observe private information:



  - Informed trader i observes a **private signal**

$$\tilde{s}_i = \tilde{v} + \tilde{\varepsilon}_i, \text{ with } \tilde{\varepsilon}_i \sim N\left(0, \tau_\varepsilon^{-1}\right) \text{ and } \tau_\varepsilon > 0$$

  where $(\tilde{v}, \tilde{\eta}, \{\tilde{\varepsilon}_i\})$ are mutually independent

Asset Demand and Price Formation

- Demand of informed trader i :

$$D_I\left(\tilde{s}_i, \tilde{y}, \tilde{p}\right) = \frac{\overbrace{E\left(\tilde{v}|\tilde{s}_i, \tilde{y}, \tilde{p}\right) - \tilde{p}}^{\text{capital gain}}}{\underbrace{\gamma}_{\text{risk aversion}} \times \underbrace{Var\left(\tilde{v}|\tilde{s}_i, \tilde{y}, \tilde{p}\right)}_{\text{perceived risk}}}$$

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- Demand of uninformed traders:

$$D_U(\tilde{y}, \tilde{p}) = \frac{E(\tilde{v}|\tilde{y}, \tilde{p}) - \tilde{p}}{\gamma Var(\tilde{v}|\tilde{y}, \tilde{p})}$$

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- Market clearing:

$$\int_0^{\mu} D_I\left(\tilde{s}_i, \tilde{y}, \tilde{p}\right) di + \left(1 - \mu\right) D_U\left(\tilde{y}, \tilde{p}\right) + \tilde{u} = Q$$

$$\Rightarrow \tilde{p} = \tilde{p}\left(\tilde{v}, \tilde{y}, \tilde{u}\right) = p_0 + p_y \tilde{y} + p_v \tilde{v} + p_u \tilde{u},$$

where p_0, p_y, p_v and p_u are endogenous.

Price Function

Given $\mu \in [0,1]$ and $\tau_\varepsilon > 0$, there exists a unique linear noisy-REE in the date-1 market, with price function

$$\tilde{p} = p_0 + p_y \tilde{y} + p_v \tilde{v} + p_u \tilde{u},$$

where

$$\begin{aligned} p_0 &= \frac{-\gamma Q}{\mu\tau_\varepsilon + \tau_v + \tau_\eta + \rho^2\tau_u}, p_y = \frac{\tau_\eta}{\mu\tau_\varepsilon + \tau_v + \tau_\eta + \rho^2\tau_u}, \\ p_v &= \frac{\mu\tau_\varepsilon + \rho^2\tau_u}{\mu\tau_\varepsilon + \tau_v + \tau_\eta + \rho^2\tau_u}, p_u = \frac{\rho\tau_u + \gamma}{\mu\tau_\varepsilon + \tau_v + \tau_\eta + \rho^2\tau_u}, \end{aligned}$$

with

$$\rho = \frac{\mu\tau_\varepsilon}{\gamma}.$$

Measures of Market Quality

- Price efficiency and market liquidity are two important measures of market quality:
  - O'Hara (2003): "Markets have two important functions—liquidity and price discovery—and these functions are important for asset pricing."
  - SEC (1999) also highlighted that "short selling provides the market with two important benefits: market liquidity and pricing efficiency."
- Other often cited measures:
  - Cost of capital, return volatility, and price volatility

Market Efficiency

- "Market efficiency" / "price efficiency" / "informational efficiency": how well does the asset price aggregate the information that is relevant to the asset's fundamentals?
- Price efficiency promotes resource allocations (Hayek 1945, Fama and Miller 1972).
- In the model,

$$\text{market efficiency} = Corr(\tilde{v}, \tilde{p}) \propto \frac{1}{Var(\tilde{v}|\tilde{p})}$$

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- For fixed (μ, τ_ε): $\tau_\eta \uparrow \Rightarrow Corr\left(\tilde{v}, \tilde{p}\right) \uparrow$
  - Public disclosure injects more information into the price

Market Liquidity

- Market liquidity $=$ the ease of selling an asset in the market
- "Kyle's lambda":

$$\text{market } \textbf{il}\text{liquidity} \equiv \frac{\partial \tilde{p}}{\partial \tilde{u}} = p_u$$

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  - Suppose $\tilde{u} = +1 \Rightarrow$ Noise traders **buy** at an

$$Ask = E\left(\tilde{p}|\tilde{u} = +1\right) = E\left(p_0 + p_y \tilde{y} + p_v \tilde{v} + p_u \tilde{u}|\tilde{u} = +1\right) = p_0 + p_u$$

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  - Suppose $\tilde{u} = -1 \Rightarrow$ Noise traders **sell** at a

    $$Bid = E(\tilde{p}|\tilde{u} = -1) = E(p_0 + p_y\tilde{y} + p_v\tilde{v} + p_u\tilde{u}|\tilde{u} = -1) = p_0 - p_u$$

    $$\Rightarrow Ask - Bid = 2p_u$$

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- For fixed (μ, τ_ε): $\tau_\eta \uparrow \Rightarrow p_u \downarrow$

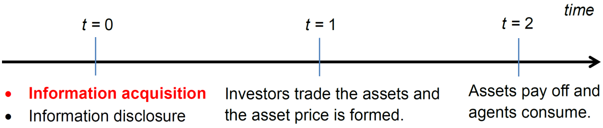
"Conventional Wisdom" Revisited

With exogenous private information (i.e., for fixed (μ, τ_ε)), increasing the precision τ_η of public information:

- improves market efficiency $Corr(\tilde{v}, \tilde{p})$
- improves market liquidity p_u^{-1}
- decreases the cost of capital $E(\tilde{v} - \tilde{p})$
- decreases return volatility $\sigma(\tilde{v} - \tilde{p})$
  $\Rightarrow$disclosure seems to be a "panacea", indeed

BUT, this only holds true for exogenous information settings (in the "short run")...

Endogenous Information Acquisition



- Trader i can spend cost $C\left(\tau_{\varepsilon_i}\right)$ to see

$$\tilde{s}_i = \tilde{v} + \tilde{\varepsilon}_i, \text{ with } \tilde{\varepsilon}_i \sim N\left(0, \tau_{\varepsilon_i}^{-1}\right) \text{ and } \tau_{\varepsilon_i} > 0,$$

  where

$$C\left(\tau_{\varepsilon_i}\right) = \underbrace{c_F}_{\text{fixed cost}} + \underbrace{c_V\left(\tau_{\varepsilon_i}\right)}_{\text{variable cost}},$$

  where $c_F > 0, c_V(0) = 0, c_V'(\cdot) > 0$, and $c_V''(\cdot) \geq 0$.
- Information equilibrium:
  - Extensive margin, μ^*: how many traders decide to become informed?
  - Intensive margin, τ_{ε}^*: how precise is the acquired information?
- The literature examines each margin separately (e.g., Verrecchia, 1982 JAR; Diamond, 1985 JF)

Interpretation of the Cost Function

- Rational traders = financial institutions
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  - Hiring one analyst costs \$**100**, generating one report:

$$\tilde{v} + \tilde{e}_{i,1}, \text{ with } \tilde{e}_{i,1} \sim N(0,1)$$

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  - Hiring two analysts costs $**200**, generating two separate reports:

$$\tilde{v} + \tilde{e}_{i,1} \text{ and } \tilde{v} + \tilde{e}_{i,2},$$

$$\text{with } \tilde{e}_{i,1} \sim N(0,1),\ \tilde{e}_{i,2} \sim N(0,1) \text{ and } \tilde{e}_{i,1} \perp \tilde{e}_{i,2}$$

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$$c_V(\tau_{\varepsilon_i}) = \$\mathbf{100} \times \tau_{\varepsilon_i} \text{ (linear)}$$

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  - ......$\Rightarrow$
  $$c_V(\tau_{\varepsilon_i}) = \$\mathbf{100} \times \tau_{\varepsilon_i} \text{ (linear)}$$
  - If hiring the second analyst costs more (wage raise or training/search costs), then $c_V(\tau_{\varepsilon_i})$ is strictly convex ($c_V(1) = \$100, c_V(2) = \$100 + \$120 = \$220$).

Information Decisions

- Given (τ_ε, μ), consider trader i:
  - The date-0 certainty equivalent of becoming informed

$$CE_I\left(\tau_{\varepsilon_i}; \tau_\varepsilon, \mu\right) \propto -\underbrace{\frac{1}{2\gamma}}_{\text{utils to \$}} \ln \underbrace{Var\left(\tilde{v} | \tilde{s}_i, \tilde{y}, \tilde{p}\right)}_{\text{perceived risk}} - C\left(\tau_{\varepsilon_i}\right)$$

$$= \frac{1}{2\gamma} \ln \left[\tau_v + \tau_\eta + \tau_{\varepsilon_i} + \left(\frac{\mu \tau_\varepsilon}{\gamma}\right)^2 \tau_u\right] - c_V\left(\tau_{\varepsilon_i}\right) - c_F$$

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    $$= \tfrac{1}{2\gamma} \ln\left[\tau_v + \tau_\eta + \tau_{\varepsilon_i} + \left(\tfrac{\mu\tau_\varepsilon}{\gamma}\right)^2 \tau_u\right] - c_V\left(\tau_{\varepsilon_i}\right) - c_F$$
  - The date-0 certainty equivalent of staying uninformed
    $$CE_U\left(\tau_\varepsilon, \mu\right) \propto -\tfrac{1}{2\gamma} \ln Var\left(\tilde{v}|\tilde{y}, \tilde{p}\right) = \tfrac{1}{2\gamma} \ln\left[\tau_v + \tau_\eta + \left(\tfrac{\mu\tau_\varepsilon}{\gamma}\right)^2 \tau_u\right]$$

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    $$= \frac{1}{2\gamma} \ln \left[\tau_v + \tau_\eta + \tau_{\varepsilon_i} + \left(\frac{\mu \tau_\varepsilon}{\gamma}\right)^2 \tau_u\right] - c_V(\tau_{\varepsilon_i}) - c_F$$
  - The date-0 certainty equivalent of staying uninformed
    $$CE_U(\tau_\varepsilon, \mu) \propto -\frac{1}{2\gamma} \ln Var(\tilde{v}|\tilde{y}, \tilde{p}) = \frac{1}{2\gamma} \ln \left[\tau_v + \tau_\eta + \left(\frac{\mu \tau_\varepsilon}{\gamma}\right)^2 \tau_u\right]$$
- The information equilibrium is determined by comparing CE_U with CE_I (the extensive margin) and by examining the FOC of CE_I (the intensive margin)

Three Types of Information Equilibrium

- $\mu^* = 1$ (Verrecchia, 1982 JAR)
  - Extensive margin: $CE_U(\tau_\varepsilon^*, 1) \leq CE_I(\tau_\varepsilon^*; \tau_\varepsilon^*, 1)$
  - Intensive margin: $\tau_\varepsilon^* = \arg\max_{\tau_{\varepsilon_i}} CE_I(\tau_{\varepsilon_i}; \tau_\varepsilon^*, 1)$
- $\mu^* \in (0,1)$ (Diamond, 1985 JF)
  - Extensive margin: $CE_U(\tau_\varepsilon^*, \mu^*) = CE_I(\tau_\varepsilon^*; \tau_\varepsilon^*, \mu^*)$
  - Intensive margin: $\tau_\varepsilon^* = \arg\max_{\tau_{\varepsilon_i}} CE_I(\tau_{\varepsilon_i}; \tau_\varepsilon^*, \mu^*)$
- $\mu^* = 0$ (Exogenous information)
  - Extensive margin: $CE_U(0,0) \geq \max_{\tau_{\varepsilon_i}} CE_I(\tau_{\varepsilon_i}; 0, 0)$
  - Intensive margin: $\mu^* = 0$

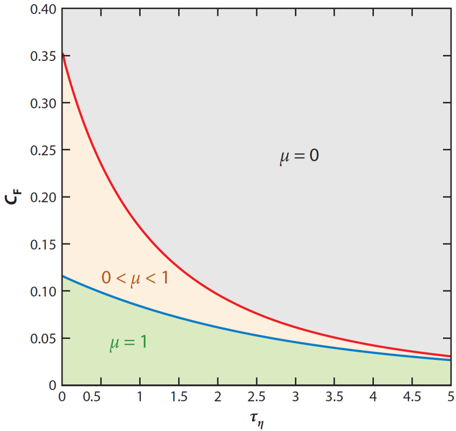


Figure 1

Equilibrium types in the economy with endogenous information acquisition. This figure plots the equilibrium types in the space of $(\tau_\eta, c_\mathrm{F})$, where parameter τ_η controls the disclosure quality and parameter c_F denotes the fixed cost of information acquisition. The variable cost of information acquisition takes a quadratic form, $c_\mathrm{V}(\tau_{\varepsilon i}) = (k/2)\tau_{\varepsilon i}^2$. The parameter values are $\tau_v = \tau_x = \gamma = Q = 1$ and $k = 0.1$.

Crowding-Out Effect of Disclosure

Disclosing public information reduces private information:

- When $\mu^* = 1$, τ_ε^* decreases with τ_η
- When $\mu^* \in (0,1)$, μ^* decreases with τ_η and τ_ε^* is independent of τ_η

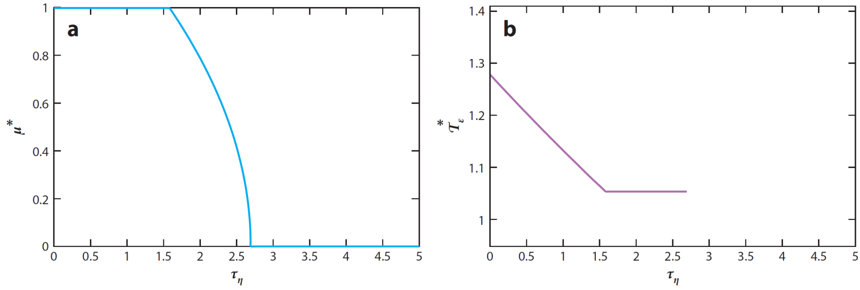


Figure 2

The crowding-out effect. This figure plots (*a*) the equilibrium fraction μ^* of informed traders and (*b*) the equilibrium precision τ_ε^* of informed traders' private information against the disclosure quality τ_η. The variable cost of information acquisition takes a quadratic form, $c_V(\tau_{\varepsilon i}) = (k/2)\tau_{\varepsilon i}^2$. The parameter values are $\tau_v = \tau_x = \gamma = Q = 1$, $c_F = 0.07$, and $k = 0.1$.

Intuitions of Crowding-Out Effect

- Suppose c_F is small so that $\mu^* = 1$.
- Intensive margin τ_ε^* is determined by:

$$\tau_\varepsilon^* = \arg\max_{\tau_{\varepsilon_i}} CE_I\left(\tau_{\varepsilon_i}; \tau_\varepsilon^*, 1\right) \Rightarrow$$

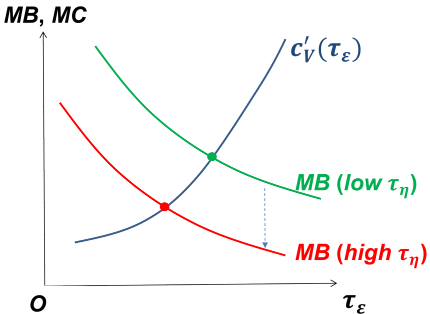
$$FOC: \underbrace{\frac{1}{2\gamma} \frac{1}{\tau_v + \tau_\eta + \tau_\varepsilon^* + \left(\frac{\tau_\varepsilon^*}{\gamma}\right)^2 \tau_u}}_{\text{marginal benefit (MB)}} = \underbrace{c_V'\left(\tau_\varepsilon^*\right)}_{\text{marginal cost (MC)}}$$

Intuitions of Crowding-Out Effect

- Suppose c_F is small so that $\mu^* = 1$.
- Intensive margin τ_ε^* is determined by:

$$\tau_\varepsilon^* = \arg\max_{\tau_{\varepsilon_i}} CE_I\left(\tau_{\varepsilon_i}; \tau_\varepsilon^*, 1\right) \Rightarrow$$

$$FOC: \underbrace{\frac{1}{2\gamma}\frac{1}{\tau_v + \tau_\eta + \tau_\varepsilon^* + \left(\frac{\tau_\varepsilon^*}{\gamma}\right)^2 \tau_u}}_{\text{marginal benefit (MB)}} = \underbrace{c_V'\left(\tau_\varepsilon^*\right)}_{\text{marginal cost (MC)}}$$



Effects on Market Quality

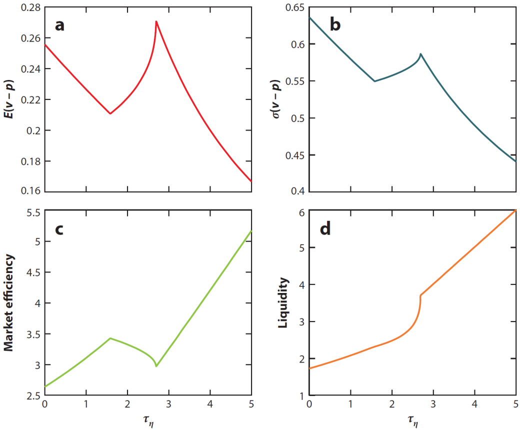


Figure 3

The effect of disclosure on an economy with endogenous information acquisition. This figure plots (*a*) the cost of capital $E(\tilde{v} - \tilde{p})$, (*b*) return volatility $\sigma(\tilde{v} - \tilde{p})$, (*c*) market efficiency $1/\mathrm{Var}(\tilde{v}|\tilde{p})$, and (*d*) market liquidity $1/p_x$ as functions of disclosure quality τ_η. The variable cost of information acquisition takes a quadratic form, $c_V(\tau_{\varepsilon i}) = (k/2)\tau_{\varepsilon i}^2$. The parameter values are $\tau_v = \tau_x = \gamma = Q = 1$, $c_F = 0.07$, and $k = 0.1$.

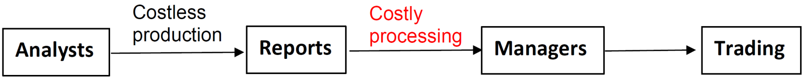
Crowding-Out vs. Crowding-In Effects: Cost Perspectives

$$\boxed{\textbf{Analysts}} \xrightarrow{\text{Costless production}} \boxed{\textbf{Reports}} \xrightarrow{\text{Costly processing}} \boxed{\textbf{Managers}} \longrightarrow \boxed{\textbf{Trading}}$$

- Interpreting data consumes capacity (Myatt-Wallace 2012):

$$\text{“mutual information”} = \frac{1}{2} \ln \frac{Var\left(\tilde{v} | \tilde{y}, \tilde{p}\right)}{Var\left(\tilde{v} | \tilde{s}_i, \tilde{y}, \tilde{p}\right)} \leq \bar{K}$$

Crowding-Out vs. Crowding-In Effects: Cost Perspectives



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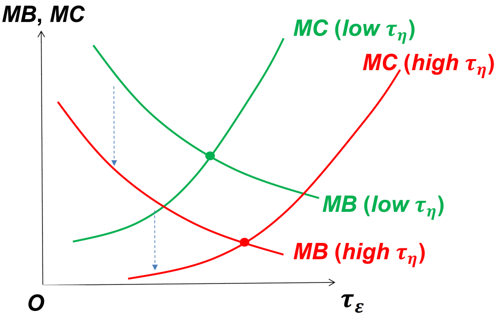
- Managers can incur a cost $c\left(\bar{K}\right)$ to expand capacity $\bar{K}$, where $c\left(\cdot\right)$ is increasing and convex
- The information acquisition cost changes to:

$$C\left(\tau_{\varepsilon_i},\tau_\eta\right) = c\left(\tfrac{1}{2}\ln\frac{Var(\tilde{v}|\tilde{y},\tilde{p})}{Var(\tilde{v}|\tilde{s}_i,\tilde{y},\tilde{p})}\right)$$

$$= C\left(1+\frac{\tau_{\varepsilon_i}}{\tau_v+\tau_\eta+\left(\frac{\tau_\varepsilon^*}{\gamma}\right)^2\tau_u}\right) \quad \left(\text{with } C\left(\cdot\right)\equiv c\left(\tfrac{1}{2}\ln\left(\cdot\right)\right)\right)$$



- $\tau_\eta \uparrow \Rightarrow MC \downarrow$:

$$\frac{\partial C\left(\tau_{\varepsilon_i}, \tau_\eta\right)}{\partial \tau_{\varepsilon_i}} = C' \left(1 + \frac{\tau_{\varepsilon_i}}{\tau_v + \tau_\eta + \left(\frac{\tau_\varepsilon^*}{\gamma}\right)^2 \tau_u}\right) \frac{1}{\tau_v + \tau_\eta + \left(\frac{\tau_\varepsilon^*}{\gamma}\right)^2 \tau_u}$$

- $\frac{1}{2\gamma} \frac{1}{\tau_v + \tau_\eta + \tau_\varepsilon^* + \left(\frac{\tau_\varepsilon^*}{\gamma}\right)^2 \tau_u} = \left.\frac{\partial C\left(\tau_{\varepsilon_i}, \tau_\eta\right)}{\partial \tau_{\varepsilon_i}}\right|_{\tau_{\varepsilon_i} = \tau_\varepsilon^*} \Rightarrow$

$$\frac{\partial \tau_\varepsilon^*}{\partial \tau_\eta} > 0 \Longleftrightarrow \gamma^2 \left(\tau_v + \tau_\eta\right) > \tau_u \tau_\varepsilon^{*2}$$

- Is it costly to digest public data? Does it consume capacity to interpret public news?
  - The best signal extracted from the public data is

$$\tilde{y} = \tilde{v} + \tilde{\eta}$$

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  - Trader i adds "receiver noise" when reading the data:

$$\tilde{y}_i = \tilde{v} + \tilde{\eta} + \tilde{\delta}_i, \text{ with } \tilde{\delta}_i \sim N(0, \tau_{\delta_i}^{-1})$$

    where $(\tilde{v}, \tilde{\eta}, \{\tilde{\delta}_i\}, \{\tilde{\varepsilon}_i\})$ are mutually independent
  - The cost function becomes:

$$C\left(\tau_{\delta_i}, \tau_{\varepsilon_i}\right) = c\left(\frac{1}{2}\ln\frac{\left|Var\left((\tilde{v}, \tilde{y})' \,|\tilde{p}\right)\right|}{\left|Var\left((\tilde{v}, \tilde{y})' \,|\,(\tilde{p}, \tilde{y}_i, \tilde{s}_i)'\right)\right|}\right)$$

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- Does it consume capacity to interpret the asset price? (Vives and Yang, 2016; Banerjee, Davis and Gondhi, 2019)

Crowding-Out vs. Crowding-In Effects: Signal Perspectives

- Multiple dimensions of uncertainty (Bond-Goldstein 2015JF; Goldstein-Yang 2015JF):

$$\tilde{v} = \tilde{v}_F + \tilde{v}_T, \text{with } \tilde{v}_F \sim N(0, \tau_F^{-1}) \text{ and } \tilde{v}_T \sim N(0, \tau_T^{-1}),$$

  where $\tilde{v}_F \perp \tilde{v}_T$ ($\tilde{v}_F$: cost; $\tilde{v}_T$: demand)

  - Disclosure is about $\tilde{v}_F$:

$$\tilde{y} = \tilde{v}_F + \tilde{\eta}, \text{ with } \tilde{\eta} \sim N\left(0, \tau_\eta^{-1}\right) \text{ and } \tau_\eta \geq 0$$

  - Traders acquire costly information about $\tilde{v}_T$:

$$\tilde{s}_i = \tilde{v}_T + \tilde{\varepsilon}_i, \text{ with } \tilde{\varepsilon}_i \sim N\left(0, \tau_{\varepsilon_i}^{-1}\right)$$

    according to a variable cost function $c_V\left(\tau_{\varepsilon_i}\right)$

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    according to a variable cost function $c_V\left(\tau_{\varepsilon_i}\right)$

- One can always redefine signals $\tilde{y}$ and $\tilde{s}_i$ in the form of $\tilde{v} + \mathit{error}$, but the error for $\tilde{y}$ and the error for $\tilde{s}_i$ will be negatively correlated (as opposed to mutually independent $(\tilde{v}_F, \tilde{v}_T, \tilde{\eta}, \{\tilde{\varepsilon}_i\})$).

- The price function:

$$\tilde{p} = p_0 + p_y \tilde{y} + p_v \tilde{v}_T + p_u \tilde{u},$$

- To traders, $\tilde{p}$ is equivalent to the following signal:

$$\tilde{s}_p \equiv \frac{\tilde{p} - p_0 - \tilde{p}_y \tilde{y}}{p_v} = \tilde{v}_T + \frac{p_u}{p_v} \tilde{u},$$

with informativeness

$$\frac{1}{Var\left(\frac{p_u}{p_v} \tilde{u}\right)} = \left(\frac{p_v}{p_u}\right)^2 \tau_u$$

- Disclosure improves price informativeness:

$$\frac{\partial \frac{p_v}{p_u}}{\partial \tau_\eta} > 0$$

because traders perceive a lower risk

$$Var\left(\tilde{v} | \tilde{s}_i, \tilde{y}, \tilde{p}\right) = {\color{red} Var\left(\tilde{v}_F | \tilde{y}\right)} + Var\left(\tilde{v}_T | \tilde{s}_p, \tilde{s}_i\right)$$

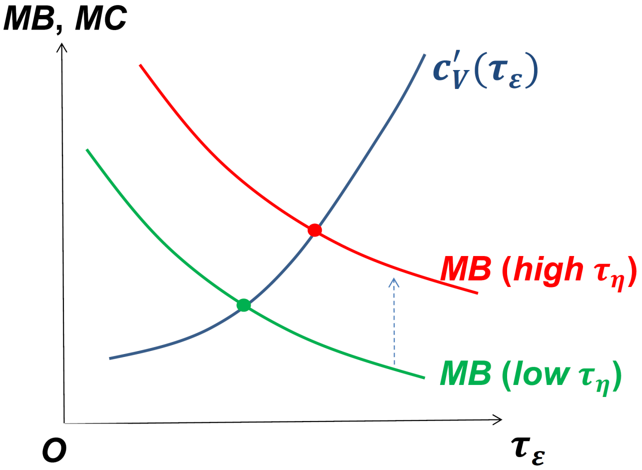
- The date-0 certainty equivalent:
$CE_I\left(\tau_{\varepsilon_i};\tau_\varepsilon\right) \propto -\frac{1}{2\gamma}\ln Var\left(\tilde{v}|\tilde{s}_i,\tilde{y},\tilde{p}\right) - c_v\left(\tau_{\varepsilon_i}\right)$
$= -\frac{1}{2\gamma}\ln\left[\frac{1}{\tau_F+\tau_\eta} + \frac{1}{\tau_T+\left(\frac{p_v}{p_u}\right)^2\tau_u+\tau_{\varepsilon_i}}\right] - c_v\left(\tau_{\varepsilon_i}\right)$
- FOC:

$$\underbrace{\frac{1}{2\gamma}\frac{1}{\frac{\tau_T+\left(\frac{p_v}{p_u}\right)^2\tau_u+\tau_\varepsilon^*}{\tau_F+\tau_\eta}+\tau_T+\left(\frac{p_v}{p_u}\right)^2\tau_u+\tau_\varepsilon^*}}_{\text{marginal benefit (MB)}} = \underbrace{c_V'\left(\tau_\varepsilon^*\right)}_{\text{marginal cost (MC)}}$$

- Increasing τ_η:
  - "Uncertainty reduction effect" (Goldstein-Yang 2015): shift upward MB via $Var\left(\tilde{v}_F|\tilde{y}\right) = \frac{1}{\tau_F+\tau_\eta}$
  - "Information leakage effect" (Grossman-Stiglitz 1980): shift downward MB via $\left(\frac{p_v}{p_u}\right)^2\tau_u$

Crowding in/out:

$$\frac{\partial \tau_\varepsilon^*}{\partial \tau_\eta} > 0 \iff \tau_T + \tau_\varepsilon^* > \left(\frac{p_v}{p_u}\right)^2 \tau_u$$



Takeaways

- The effects of disclosure depend on
  - Short run vs. long run
  - Extensive margin vs. intensive margin
  - Information acquisition cost
  - Information structure
- Anything goes $\neq$ No guidance
  - Contexts of data/players matter a lot!
  - To apply a theory, researchers have to carefully choose information technology and understand the consequences of the modeling choice

Financial Market Feedback and Commodity Financialization

Real Effects of Financial Markets

- Why care about market efficiency?
- The informativeness of prices is important since it helps facilitate the efficient allocation of resources:
  - An efficient market "has a very desirable feature. In particular, at any point in time market prices of securities provide accurate signals for resource allocation; that is, firms can make production-investment decisions..." Fama and Miller (1972)
- Who learns from the asset price?
  - Managers, creditors, regulators, customers, employees, etc.
  - As long as there is **some** information in the price that they don't know
- This literature is labeled as the "feedback effect" of financial markets (Bond, Edmans and Goldstein, 2012 ARFE)

Empirical Evidence: Financial Markets $\neq$ Side Show

- Roll (1984AER): Orange juice and weather forecast
- Luo (2005JF): Mergers are more likely to be canceled when prices react more negatively and managers are trying to learn
- Chen, Goldstein, and Jiang (2007RFS): Price informativeness affects investment sensitivity to price
- Bakke and Whited (2010RFS): Firm managers incorporate private investor information when making investment decisions
- Edmans, Goldstein, and Jiang (2012JF): Exogenous shock to price affects takeovers
- Foucault and Frésard (2012RFS): Cross listed firms exhibit stronger investment-price sensitivity
- Bond, Edmans, and Goldstein (2012ARFE): Review theoretical and empirical literature on the real effect of secondary financial markets
- Foucault and Frésard (2014JFE): Learning from peers' stock prices and corporate investment
- Dessaint, Foucault, Frésard, and Matray (2018RFS): Localized non-fundamental shocks to stock prices (noise) affect real investment
- ......

Disclosure with Feedback Effects

- Gao and Liang (2013 JAR): positive feedback effect vs. negative crowding-out effect
- Goldstein and Yang (2018 JFE): different types of information
- Yang (2019): positive feedback effect vs. negative propriety cost (leaking information to competitors)

Financialization of Commodity Markets

- Commodity futures became popular among financial investors over the last two decades
  - According to CFTC Report (2008), commodity index investments totaled 200B in 2008 (vs. 15B in 2003)
- Phenomenon known as "commodity financialization" (Cheng and Xiong, 2014)
- Economists and regulators are concerned about whether and how financialization has affected the functioning of futures and spot markets
  - Masters in his congressional testimony blamed this inflow for 2007-2008 commodity supercycle ("Masters Hypothesis," c.f. Irwin and Sanders, 2012)
  - Concerns prompted CFTC to add Commodity Index Trader (CIT) position supplement to weekly Commitments of Traders (COT) reports

Central Debates among Regulators and Academia

- G20 Report, 2011:
  - "The impact of the growing presence of financial investors on the functioning of commodity markets is a subject of ongoing debate. The discussion centers around two related questions. First, does increased financial investment alter demand for and supply of commodity futures in a way that moves prices away from fundamentals and/or increase their volatility? And second, does financial investment in commodity futures affect spot prices?"
- What happened to market quality (**spot and futures markets**)?
- What are the **real effects**?

Mixed Empirical Findings on Futures Market Quality

- Raman, Robe and Yadav (2017): U.S. crude oil futures markets
  "We provide the first detailed empirical evidence on the financialization of intraday trading activity in *the world's largest commodity market* and show that this development had a first-order *positive impact on market liquidity and pricing efficiency*."
- Brogaard, Ringgenberg and Sovich (2018 RFS): commodity index
  "Starting in 2004, there was a dramatic increase in commodity index investing, an event referred to as the financialization of commodity markets...our results suggest that index investing in financial markets *distorts the price signal* thereby generating a negative externality that impedes firms' ability to make production decisions."

Informational Role of Commodities Futures Markets

- Information incorporated in trading in futures markets may be key for **investment/production** decisions in commodities
  - Black (1976): "futures prices provide a wealth of valuable information for those who produce, store, and use commodities...The big benefit from futures markets is the side effect: the fact that *participants in the futures markets can make production, storage, and processing decisions by looking at the pattern of futures prices*, even if they don't take positions in that market."
- What would be the consequences of financialization analyzed through the lens of the **informational channel**?
- Need an informational framework to understand the various facts surrounding commodity financialization

Commodity Financialization and Information Transmission: Goldstein and Yang (2019)

- We develop an asymmetric information model where financial traders, commodity producers, and noise traders trade futures contracts
  - Financial traders inject new **information** and **noise** into the futures market
  - **Price informativeness** can either increase or decrease with commodity financialization
- Commodity producers learn information from the futures price to guide commodity production
  - **Real effects** of commodity financialization; natural framework for feedback
  - **Two types of feedback effects: primary vs. secondary**

Effects of Commodity Financialization

As the population size of financial traders increases:

- Price informativeness first increases and then decreases
  - Reconcile mixed empirical findings on market quality (Brogaard et al. 2017; Raman et al. 2017)
- Futures price bias can increase or decrease
- Market liquidity in the futures market generally increases
- Equity-commodity comovement increases due to the cross-holdings of financial traders, consistent with Büyükşahin and Robe (2013, 2014)
- Commodity producers:
  - Operating profits and price informativeness move in the same direction, consistent with Brogaard et al. (2018)
  - But producer welfare moves in the opposite direction$\Rightarrow$guidance for interpreting empirical findings

Model Setup

- Two dates: $t = 0$ (futures market), $t = 1$ (spot market)
- Date-1 spot market
  - Symmetric information; endogenous spot price $\tilde{v}$
  - Exogenous linear commodity demand
  - Endogenous commodity supply from commodity producers
- Date-0 futures market
  - Asymmetric information; endogenous futures price $\tilde{p}$
  - Players: participating commodity producers (mass λ), financial traders (mass μ), and noise traders
  - μ parameterizes commodity financialization

Timeline

t = 0 (futures market)

t = 1 (spot market)

time

- Financial traders observe private information $\tilde{\theta}$ and $\tilde{\alpha}$;
- Commodity producer *i* observes private information $\tilde{s}_i$;
- Financial traders, participating commodity producers, and noise traders trade futures contracts at price $\tilde{p}$;
- All commodity producers make production decisions;
- Financial traders make investments in the private technology.

- Spot market opens and the commodity market clears at price $\tilde{v}$;
- Cash flows are realized and all agents consume.

Date-1 Commodity Demand

- At date 1, the commodity demand is

$$y = \tilde{\theta} + \tilde{\delta} - \tilde{v},$$

  where $\tilde{v}$ is commodity spot price, and $\tilde{\theta}$ and $\tilde{\delta}$ are mutually independent demand shocks ("fundamentals")
- Demand shocks:
  - Forecastable component $\tilde{\theta} \sim N(\bar{\theta}, \tau_{\theta}^{-1})$
  - Unforecastable component $\tilde{\delta} \sim N(0, \tau_{\delta}^{-1})$

Commodity Producers

- A continuum $[0,1]$ of CARA commodity producers:
  - Producer i has private information

$$\tilde{s}_i = \tilde{\theta} + \tilde{\varepsilon}_i, \text{ with } \tilde{\varepsilon}_i \sim N(0, \tau_\varepsilon^{-1})$$

    where $(\tilde{\theta}, \tilde{\delta}, \{\tilde{\varepsilon}_i\})$ are mutually independent
  - Production is costly:

$$C(x_i) = cx_i + \frac{1}{2}x_i^2, \text{ with } c > 0$$

- Two types of producers:
  - "Participating producers" (mass λ): produce commodities + trade assets
  - "Nonparticipating producers" (mass $1-\lambda$): produce commodities

Nonparticipating Producers' Problem

- On date 0, nonparticipating producer j's problem:

$$\max_{x_j}\left[E\left(-e^{-\kappa\left[\tilde{v}x_j-\left(cx_j+\frac{1}{2}x_j^2\right)\right]}\middle|\tilde{s}_j,\tilde{p}\right)\right]$$

$$FOC \Rightarrow x_j^* = \frac{E(\tilde{v}|\tilde{s}_j,\tilde{p})-c}{1+\kappa Var(\tilde{v}|\tilde{s}_j,\tilde{p})}$$

- **Feedback effect (traditional/secondary):**
  Futures price $\tilde{p}$ has an **informational effect** on production x_j^*
- To compute x_j^*, we need to know the futures price function $\tilde{p}$

Participating Producers' Problem (Danthine, 1978)

$$\max_{x_i, d_i} \left[E\left(-e^{-\kappa\left[\tilde{v}x_i - \left(cx_i + \frac{1}{2}x_i^2\right) + (\tilde{v} - \tilde{p})d_i\right]} \middle| \tilde{s}_i, \tilde{p} \right) \right]$$

$$\Longleftrightarrow \underbrace{\max_{x_i} \left[\tilde{p}x_i - \left(cx_i + 0.5x_i^2\right)\right]}_{\text{riskless production}} +$$

$$\underbrace{\max_{x_i + d_i} \left[E\left(\tilde{v} - \tilde{p} \middle| \tilde{s}_i, \tilde{p}\right)(x_i + d_i) - 0.5\kappa\left(x_i + d_i\right)^2 Var\left(\tilde{v} - \tilde{p} \middle| \tilde{s}_i, \tilde{p}\right) \right]}_{\text{risk exposure}}$$

Participating Producers' Problem (Danthine, 1978)

$$\max_{x_i,d_i}\left[E\left(-e^{-\kappa\left[\tilde{v}x_i-\left(cx_i+\frac{1}{2}x_i^2\right)+(\tilde{v}-\tilde{p})d_i\right]}\,\middle|\,\tilde{s}_i,\tilde{p}\right)\right]$$

$$\iff \underbrace{\max_{x_i}\left[\tilde{p}x_i-\left(cx_i+0.5x_i^2\right)\right]}_{\text{riskless production}}+$$

$$\underbrace{\max_{x_i+d_i}\left[E\left(\tilde{v}-\tilde{p}\,|\,\tilde{s}_i,\tilde{p}\right)(x_i+d_i)-0.5\kappa\,(x_i+d_i)^2\,Var\left(\tilde{v}-\tilde{p}\,|\,\tilde{s}_i,\tilde{p}\right)\right]}_{\text{risk exposure}}$$

- Production:

$$FOC \Rightarrow x_i^* = \tilde{p}-c$$

Feedback effect (primary): $\tilde{p}$ has a **direct effect** on production x_i^*

Participating Producers' Problem (Danthine, 1978)

$$\max_{x_i, d_i} \left[E\left(-e^{-\kappa\left[\tilde{v}x_i - \left(cx_i + \frac{1}{2}x_i^2\right) + (\tilde{v} - \tilde{p})d_i\right]} \middle| \tilde{s}_i, \tilde{p} \right) \right]$$

$$\iff \underbrace{\max_{x_i} \left[\tilde{p}x_i - \left(cx_i + 0.5x_i^2\right) \right]}_{\text{riskless production}} +$$

$$\underbrace{\max_{x_i + d_i} \left[E\left(\tilde{v} - \tilde{p} \middle| \tilde{s}_i, \tilde{p} \right)(x_i + d_i) - 0.5\kappa\left(x_i + d_i\right)^2 Var\left(\tilde{v} - \tilde{p} \middle| \tilde{s}_i, \tilde{p} \right) \right]}_{\text{risk exposure}}$$

- Production:

$$FOC \Rightarrow x_i^* = \tilde{p} - c$$

  Feedback effect (primary): $\tilde{p}$ has a **direct effect** on production x_i^*

- Risk exposure: $FOC \Rightarrow x_i^* + d_i^* = \frac{E(\tilde{v}|\tilde{s}_i, \tilde{p}) - \tilde{p}}{\kappa Var(\tilde{v}|\tilde{s}_i, \tilde{p})}$

$$\Rightarrow d\left(\tilde{s}_i, \tilde{p}\right) = \underbrace{\frac{E\left(\tilde{v} \middle| \tilde{s}_i, \tilde{p}\right) - \tilde{p}}{\kappa Var\left(\tilde{v} \middle| \tilde{s}_i, \tilde{p}\right)}}_{\text{speculation}} - \underbrace{\left(\tilde{p} - c\right)}_{\text{hedging}}$$

A Supply Channel that Links Futures and Spot Prices

- Date-1 commodity demand:

$$y = \tilde{\theta} + \tilde{\delta} - \tilde{v}$$

- Date-1 commodity supply:

$$x(\tilde{p}, \tilde{\theta}) \equiv \underbrace{\lambda\left(\tilde{p} - c\right)}_{\text{participating producers}} + \underbrace{\int_0^{1-\lambda} \frac{E(\tilde{v}|\tilde{s}_j, \tilde{p}) - c}{1 + \kappa Var(\tilde{v}|\tilde{s}_j, \tilde{p})} dj}_{\text{nonparticipating producers}},$$

- Market clearing at date-1 spot market:

$$y = x \Rightarrow \tilde{v} = v_0 + v_\theta \tilde{\theta} + v_\delta \tilde{\delta} + v_p \tilde{p}$$

$\Rightarrow$date-0 futures price $\tilde{p}$ affects date-1 spot price $\tilde{v}$ (a supply channel)$\Rightarrow$"Yes" to Question 2 in G20 2011 Report

Financial Traders

- A mass μ of *identical* CARA financial traders who trade futures both for **speculation** and for **hedging**
- They know the demand shock $\tilde{\theta}$ $\Rightarrow$speculation
- They invest in another market (e.g., stocks, commodity linked notes), which has a net return of $\tilde{\alpha}+\tilde{\eta}$, with

$$\tilde{\alpha} \sim N(0, \tau_{\alpha}^{-1}) \text{ and } \tilde{\eta} \sim N(0, \tau_{\eta}^{-1})$$

  - $\tilde{\alpha}$ =forecastable component; $\tilde{\eta}$ =unforecastable component
  - $Corr(\tilde{\delta}, \tilde{\eta}) = \rho \in (-1, 1)$ $\Rightarrow$hedging

Financial Traders' Problem

At date 0, financial traders' problem:

$$\max_{d_F, z_F} E\left[-e^{-\gamma[(\tilde{v}-\tilde{p})d_F + (\tilde{\alpha}+\tilde{\eta})z_F]} \middle| \tilde{\theta}, \tilde{\alpha}, \tilde{p} \right]$$

FOC$\Rightarrow$

$$\begin{aligned} d_F(\tilde{\theta}, \tilde{\alpha}, \tilde{p}) &= \underbrace{\frac{\tau_\delta(v_0 + v_\theta \tilde{\theta} - (1-v_p)\tilde{p})}{\gamma(1-\rho^2)}}_{\text{speculation}} - \underbrace{\frac{\rho\sqrt{\tau_\delta \tau_\eta}}{\gamma(1-\rho^2)}\tilde{\alpha}}_{\text{hedging}} \\ &= \beta_\theta \tilde{\theta} - \beta_\alpha \tilde{\alpha} - \beta_F \tilde{p} + constant, \end{aligned}$$

where $\beta_\theta = \frac{v_\theta \tau_\delta}{\gamma(1-\rho^2)}$ and $\beta_\alpha = \frac{\rho\sqrt{\tau_\delta \tau_\eta}}{\gamma(1-\rho^2)}$.

Noise Trading and Futures Price

- In the futures market, there is also noisy demand:

$$\tilde{u} \sim N(0, \tau_u^{-1}),$$

where $\tilde{u}$ is independent of other shocks $(\tilde{\theta}, \tilde{\delta}, \tilde{\alpha}, \tilde{\eta}, \{\tilde{\varepsilon}_i\})$

- Futures price is obtained by market clearing:

$$\tilde{p} = p_0 + p_\theta \tilde{\theta} + p_\alpha \tilde{\alpha} + p_u \tilde{u},$$

where p's are endogenous constants.

Equilibrium Characterization of Futures and Spot Markets

For any given $\lambda \in (0,1)$, there exists an equilibrium with the date-1 spot-price function and the date-0 futures-price function given by equations

$$\begin{aligned} \text{Date-1 spot market:} \qquad & \tilde{v} = v_0 + v_\theta \tilde{\theta} + v_\delta \tilde{\delta} + v_p \tilde{p}, \\ \text{Date-0 futures market:} \qquad & \tilde{p} = p_0 + p_\theta \tilde{\theta} + p_\alpha \tilde{\alpha} + p_u \tilde{u}. \end{aligned}$$

The equilibrium is characterized by $v_\theta \in (0,1)$, which is determined by

$$\begin{aligned} & \frac{\tau_\delta v_\theta \left[(1-\lambda)\tau_\varepsilon - \kappa v_\theta (1-v_\theta)\right]}{\kappa + \tau_\delta (1-v_\theta)} - (\tau_\theta + \tau_\varepsilon) \\ = \quad & \frac{\left[\frac{\lambda \tau_\varepsilon (\kappa+\tau_\delta)(1-v_\theta)}{\kappa[\tau_\varepsilon(1-\lambda)+\tau_\delta v_\theta (1-v_\theta)]} + \frac{\mu \tau_\delta v_\theta}{\gamma(1-\rho^2)}\right]^2}{\frac{\mu^2 \rho^2 \tau_\delta \tau_\eta}{\gamma^2 (1-\rho^2)^2 \tau_\alpha} + \frac{1}{\tau_u}}, \end{aligned}$$

which is equivalent to a 7^{th} order polynomial of v_θ.

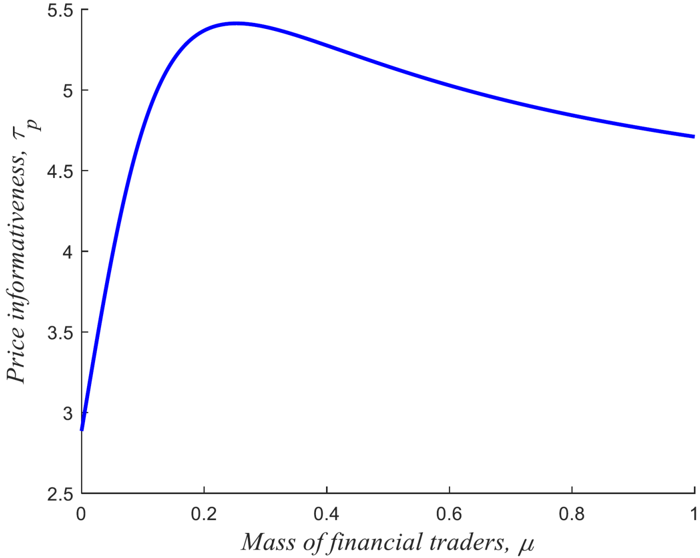
Price Informativeness

- Futures price $\tilde{p}$ is equivalent to the following signal in predicting $\tilde{\theta}$:

$$\tilde{s}_p \equiv \frac{\tilde{p} - p_0}{p_\theta} = \tilde{\theta} + \pi_\alpha \tilde{\alpha} + \pi_u \tilde{u},$$

  where $\pi_\alpha \equiv \frac{p_\alpha}{p_\theta}$ and $\pi_u \equiv \frac{p_u}{p_\theta}$.
- Price informativeness:

$$\tau_p \equiv \frac{1}{Var(\pi_\alpha \tilde{\alpha} + \pi_u \tilde{u})} = \left(\frac{\pi_\alpha^2}{\tau_\alpha} + \frac{\pi_u^2}{\tau_u} \right)^{-1}.$$



This figure plots price informativeness τ_p against the population size μ of financial traders. The other parameters are: $\tau_\theta = \tau_\delta = \tau_\varepsilon = \tau_u = \tau_\alpha = \tau_\eta = \lambda = 1$, $\gamma = \kappa = 0.1$, and $\rho = 0.5$.

Intuitions

- Demand functions:

  Commodity producers: $\int_0^\lambda d(\tilde{s}_i, \tilde{p})\, di = \phi_\theta \tilde{\theta} - \phi_p \tilde{p} + Const,$

  Financial traders: $d_F(\tilde{\theta}, \tilde{\alpha}, \tilde{p}) = \beta_\theta \tilde{\theta} - \beta_\alpha \tilde{\alpha} - \beta_p \tilde{p} + Const,$

  where ϕ_θ, β_θ and β_α are constants.
- Market clearing $\int_0^\lambda d(\tilde{s}_i, \tilde{p})\, di + \mu d_F(\tilde{\theta}, \tilde{\alpha}, \tilde{p}) + \tilde{u} = 0 \Rightarrow$

$$\underbrace{\phi_\theta \tilde{\theta}}_{\text{info from com. prod.}} + \underbrace{\mu \beta_\theta \tilde{\theta}}_{\text{info from fin. trad.}} - \underbrace{\mu \beta_\alpha \tilde{\alpha}}_{\text{noise from fin. trad.}} + \underbrace{\tilde{u}}_{\text{exog. noise}} - L(\tilde{p}) = 0,$$

where $L(\tilde{p})$ is a linear function.

- Price informativeness:

$$\tilde{\theta} - \underbrace{\frac{\mu\beta_\alpha}{\phi_\theta + \mu\beta_\theta}}_{\text{added noise by fin. trad.}}\tilde{\alpha} + \underbrace{\frac{1}{\phi_\theta + \mu\beta_\theta}}_{\text{exogenous noise trading}}\tilde{u} = \frac{L(\tilde{p})}{\phi_\theta + \mu\beta_\theta} = \tilde{s}_p$$

- Financial traders bring both "noise" ($\tilde{\alpha}$) and information ($\tilde{\theta}$) into the price
  - **Second effect dominates** only when **their mass is relatively small**
  - Initially their presence is effective to overcome noise trading ($\tilde{u}$), but as they become more prominent in the market, the additional factor they bring ($\tilde{\alpha}$) also becomes more prominent and masks $\tilde{\theta}$

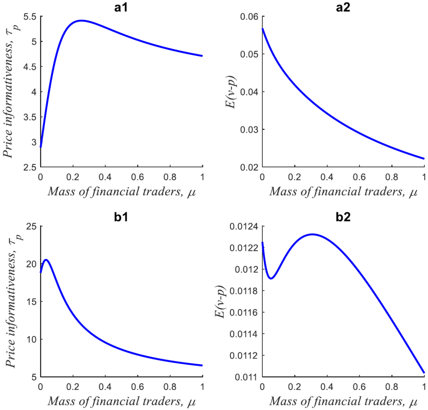
Reconcile Existing Empirical Findings

- Raman, Robe and Yadav (2017): $+$
  "We provide the first detailed empirical evidence on the financialization of intraday trading activity in *the world's largest commodity market* and show that this development had a first-order *positive impact on market liquidity and pricing efficiency*."
- Brogaard, Ringgenberg and Sovich (2018): $-$
  "Starting in 2004, there was a dramatic increase in commodity index investing, an event referred to as the financialization of commodity markets...our results suggest that index investing in financial markets *distorts the price signal* thereby generating a negative externality that impedes firms' ability to make production decisions."
- A large size of commodity futures market $\Longleftrightarrow$ a small μ

Futures Price Biases

The futures price bias $\equiv E(\tilde{v} - \tilde{p})$

$\mu \uparrow$: Risk sharing vs. price informativeness



This figure plots price informativeness τ_p and futures price biases $E(\tilde{v} - \tilde{p})$ against the population size μ of financial traders. In Panels a1 and a2, we set $\tau_\theta = \tau_\delta = \tau_\varepsilon = \tau_u = \tau_\alpha = \tau_\eta = \lambda = 1$, $\gamma = \kappa = 0.1$, $\bar{\theta} = 2$, $c = 1$, and $\rho = 0.5$. In Panels b1 and b2, we set $\tau_\theta = \tau_\varepsilon = \tau_\alpha = \lambda = 1$, $\tau_\delta = \tau_\eta = \tau_u = 5$, $\gamma = 0.5$, $\kappa = 0.1$, $\bar{\theta} = 2$, $c = 1$, and $\rho = 0.5$.

Real Effects

Thinking about the real effects is more complicated:

- On the one hand, **greater informativeness translates into greater operating profits for the producers**
  - This is in the spirit of the empirical evidence in Brogaard, Ringgenberg and Sovich (2018)
- But, there are other things operating in the opposite direction:
  - E.g., **greater informativeness hurts trading gains** for producers in futures market
- It turns out that the operating profits and producer welfare often move **in opposite directions**

- Hence, interpretation of empirical evidence might be tricky
  - Brogaard, Ringgenberg and Sovich (2018): "our results suggest that index investing in financial markets *distorts the price signal* thereby generating a negative externality that impedes firms' ability to make production decisions."
    Producer welfare?
  - Chen, Dai and Sorescu (2017): commodity trading advisors are harmed by the ongoing financialization of commodity markets, which is consistent with the model prediction on welfare of financial traders.

- Welfare of nonparticipating producers:

$$CE_N \equiv -\frac{1}{\kappa}\log\left[E\left(e^{-\kappa\left[\tilde{v}x_N(\tilde{s}_j,\tilde{p})-C(x_N(\tilde{s}_j,\tilde{p}))\right]}\right)\right]$$

- Welfare of participating producers:

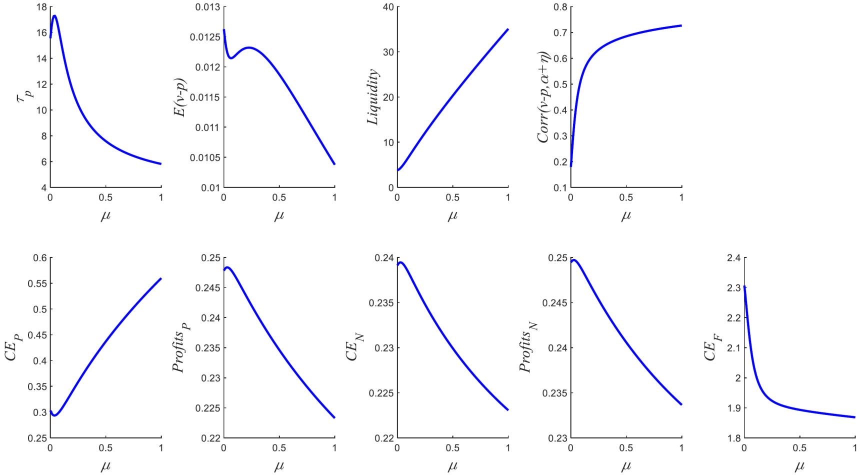
$$CE_P \equiv -\frac{1}{\kappa}\log\left[E\left(e^{-\kappa[\tilde{v}x_P(\tilde{s}_i,\tilde{p})-C(x_P(\tilde{s}_i,\tilde{p}))+(\tilde{v}-\tilde{p})d(\tilde{s}_i,\tilde{p})]}\right)\right]$$

- Operating profits of commodity producers:

$$E\left[\tilde{v}x_i^* - \left(cx_i^* + 0.5x_i^{*2}\right)\right]$$

- Welfare of financial traders:

$$CE_F \equiv -\frac{1}{\gamma}\log\left[E(e^{-\gamma\left[(\tilde{v}-\tilde{p})d_F(\tilde{\theta},\tilde{\alpha},\tilde{p})+(\tilde{\alpha}+\tilde{\eta})z_F(\tilde{\theta},\tilde{\alpha},\tilde{p})\right]})\right]$$



The parameter values are: $\tau_\theta = \tau_\varepsilon = \tau_\alpha = 1$, $\tau_\delta = \tau_\eta = \tau_u = 5$, $\gamma = 0.5$, $\kappa = 0.1$, $\bar{\theta} = 2$, $c = 1$, $\rho = 0.5$, and $\lambda = 0.8$.

Back-Running: Seeking and Hiding Fundamental Information in Order Flows (Yang and Zhu, RFS, forthcoming)

Back-Running = Trading on Past Order Flow Info

- Order-flow informed trading is strategies that first learn about other investors' order flows and then trade accordingly
- In today's market, traders have various ways to learn from *past* order flows ⇒ **Back-running**
  - Sophisticated market players (e.g., HFTs) devise algorithms to extract information from trade patterns
  - Example: Retail investors like round numbers
  - Example: Unsophisticated algorithms may split orders too regularly (200, 200, 200, ...)
  - Example: HFTs (Citadel, KCG) buy uninformed orders from retail brokers (E*Trade, Charles Schwab, TD Ameritrade)
- Front-running: directly learn about *future* order flows
  - In its original sense, front-running is illegal
  - A broker trades on his own account in the same direction as his client before he fulfills his client's order

Example of Back-Running: Order Anticipation Strategies

Order anticipation "involves any means to ascertain the existence of a large buyer (seller) that does not involve violation of a duty, misappropriation of information, or other misconduct. Examples include the employment of sophisticated pattern recognition software to ascertain from *publicly available information* the existence of a large buyer (seller), or the sophisticated use of orders to 'ping' different market centers in an attempt to locate and trade in front of large buyers and sellers." (Securities and Exchange Commission, 2010, p. 54–55)

Debate in the HFT Context

- Investors and regulators are concerned whether order anticipation harms market quality and long-term investors
  - "Do commenters believe that order anticipation significantly detracts from market quality and harms institutional investors?" (2010 SEC Concept Release)
  - "Is the U.S. stock market rigged?" (Lewis 2014)

Debate in the HFT Context

- Investors and regulators are concerned whether order anticipation harms market quality and long-term investors
  - "Do commenters believe that order anticipation significantly detracts from market quality and harms institutional investors?" (2010 SEC Concept Release)
  - "Is the U.S. stock market rigged?" (Lewis 2014)
- Common view (after Michael Lewis *Flash Boys*): defenseless institutions vs. front-running HFTs

Insitutions = Sitting Ducks

HFTs = Snipers

Our Story ...

Institutions and HFTs are more interactive:

- HFTs learn from past order flows of institutions
- Institutions defend themselves by trading more cautiously and adding noise

Overview of Results

- A model of back-running, based on Kyle (1985)
  - I fundamental investors are partly informed of the asset value
  - J back-runners try to figure out fundamental information from past order flow
  - Fundamental investors defend by adding noise to order flow
- Switch between pure strategy and mixed strategy equilibrium
- Implications:
  - Investors and back-runners strategically interact, consistent with recent empirical evidence (van Kervel and Menkveld, 2017 JF; Korajczyk and Murphy, 2018 RFS)
  - Presence of back-running affects market quality
  - Fundamental investors may benefit from more accurate back-running
  - Provide a model to quantify the value of order-flow information

Model: The Standard Parts

- A Kyle (1985) setup with two periods: $t = 1, 2$
- A risky asset pays v at the end of period 2:

$$v = p_0 + f_1 + ... + f_I,$$

  where $p_0 \in \mathbb{R}$ and

$$f_i \sim N\left(0, \sigma_f^2\right) \ \left(\text{with } \Sigma_0 \equiv I\sigma_f^2\right)$$

  is mutually independent.
- I "fundamental investors" (e.g., mutual/pension funds):
  - Fundamental investor i learns f_i at the start of the first period
  - Places market orders $x_{1,i}$ and $x_{2,i}$ in periods 1 and 2

- Prices p_1 and p_2 are set by a competitive market maker:

$$p_1 = E(v|y_1) \text{ and } p_2 = E(v|y_1, y_2),$$

  where y_1 and y_2 are total order flows in periods 1 and 2.
- Noise traders demand u_1 and U_2:

$$u_1 \sim N(0, \sigma_u^2), \text{ with } \sigma_u > 0,$$
$$U_2 \sim N(0, \sigma_U^2), \text{ with } \sigma_U > 0,$$

  where $(u_1, U_2, f_1, ..., f_I)$ are mutually independent.
  - If period 2 is much longer than period 1, we would expect $\sigma_U >> \sigma_u$.

Model: Back-Running

- There are J back-runners (e.g., HFT)
- At the beginning of period 2, back-runner j can extract a signal about the aggregate period-1 informed order flow:

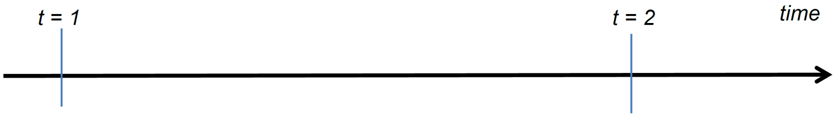
$$s_j = X_1 + \varepsilon_j,$$

  where

$$X_1 \equiv \sum_{i=1}^{I} x_{1,i}, \text{ and } \varepsilon_j \sim N\left(0, \sigma_\varepsilon^2\right) \text{ with } \sigma_\varepsilon \in [0, \infty)$$

  and $(\varepsilon_1, ..., \varepsilon_J, u_1, U_2, f_1, ..., f_I)$ are mutually independent.
- Parameter σ_ε controls the quality of order-flow information
  - a smaller $\sigma_\varepsilon \Rightarrow$more accurate information about X_1
- Back-runner j trades on s_j by placing a market order $d_{2,j}$ in the period-2 market

Timeline



- Fundamental investor i observes f_i and submits order flow $x_{1,i}$
- Noise traders submit order flow u_1
- Market maker observes total order flow $y_1 = \sum_{i=1}^{I} x_{1,i} + u_1$ and sets the price p_1

- All agents see the date-1 price p_1
- Back-runner j observes $s_j = \sum_{i=1}^{I} x_{1,i} + \varepsilon_j$ and submits order flow $d_{2,j}$
- Fundamental investor i submits order flow $x_{2,i}$
- Noise traders submit order flow U_2
- Market maker observes total order flow $y_2 = \sum_{i=1}^{I} x_{2,i} + \sum_{j=1}^{J} d_{2,j} + U_2$ and sets the price p_2
- Payoff is realized and agents consume

Linear Symmetric Equilibria

- A *linear symmetric equilibrium* is a perfect Bayesian equilibrium in which there exist constants

$$(\beta_1, \beta_2, \beta_x, \delta, \lambda_1, \lambda_2) \in \mathbb{R}^6 \quad \textit{and} \quad \sigma_z \geq 0,$$

such that

$$\begin{aligned}
x_{1,i} &= \beta_1 f_i + z_i \text{ with } z_i \sim N\left(0, \sigma_z^2\right), \\
x_{2,i} &= \beta_2 \left[f_i - E\left(f_i | y_1\right)\right] - \beta_x \left[x_{1,i} - E\left(x_{1,i} | y_1\right)\right], \\
d_{2,j} &= \delta \left[s_j - E\left(s_j | y_1\right)\right], \\
p_1 &= p_0 + \lambda_1 y_1 \text{ with } y_1 = \textstyle\sum_{i=1}^{I} x_{1,i} + u_1, \\
p_2 &= p_1 + \lambda_2 y_2 \text{ with } y_2 = \textstyle\sum_{i=1}^{I} x_{2,i} + \sum_{j=1}^{J} d_{2,j} + U_2,
\end{aligned}$$

where $(z_1, ..., z_I, \varepsilon_1, ..., \varepsilon_J, u_1, U_2, f_1, ..., f_I)$ are mutually independent.

- $\sigma_z > 0 \Longleftrightarrow$ Mixed strategy equilibrium
- $\sigma_z = 0 \Longleftrightarrow$ Pure strategy equilibrium

Market-Maker's Problems

- Period 1: $p_1 = E\left(v | y_1\right) \Rightarrow$

$$\lambda_1 = \frac{Cov\left(v, y_1\right)}{Var\left(y_1\right)} = \frac{\beta_1 \Sigma_0}{\beta_1^2 \Sigma_0 + I\sigma_z^2 + \sigma_u^2}. \qquad (1)$$

- Period 2: $p_2 = E\left(v | y_1, y_2\right) \Rightarrow$

$$\lambda_2 = \frac{\left(\beta_2 - \beta_x \beta_1 + \delta J \beta_1\right) \Sigma_0 - \frac{\beta_1 \Sigma_0 \left[\left(\beta_2 - \beta_x \beta_1 + \delta J \beta_1\right) \beta_1 \Sigma_0 + \left(\delta J - \beta_x\right) I \sigma_z^2\right]}{\beta_1^2 \Sigma_0 + I\sigma_z^2 + \sigma_u^2}}{\left[\begin{array}{c} \left(\beta_2 - \beta_x \beta_1 + \delta J \beta_1\right)^2 \Sigma_0 + \left(\delta J - \beta_x\right)^2 I \sigma_z^2 + \delta^2 J \sigma_\varepsilon^2 + \sigma_U^2 \\ - \frac{\left[\left(\beta_2 - \beta_x \beta_1 + \delta J \beta_1\right) \beta_1 \Sigma_0 + \left(\delta J - \beta_x\right) I \sigma_z^2\right]^2}{\beta_1^2 \Sigma_0 + I\sigma_z^2 + \sigma_u^2} \end{array}\right]}. \qquad (2)$$

Back-Runners' Problems

- In period 2, back-runner j:

$$\max_{d_{2,j}} E\left[d_{2,j}\left(v-p_2\right)|s_j,p_1\right]$$

- FOC$\Rightarrow d_{2,j}=\frac{E\left(v-p_1|s_j,p_1\right)}{2\lambda_2}-\frac{E\left[\sum_i x_{2,i}+\sum_{j'\neq j} d_{2,j'}|s_j,p_1\right]}{2}\Rightarrow$

$$\delta=\frac{1}{2}\frac{\sigma_\varepsilon^{-2}\left[\begin{array}{c}\frac{1}{\lambda_2}\frac{\beta_1\Sigma_0}{\beta_1^2\Sigma_0+I\sigma_z^2}\\ -\left(\beta_2\frac{\beta_1\Sigma_0}{\beta_1^2\Sigma_0+I\sigma_z^2}-\beta_x\right)-(J-1)\delta\end{array}\right]}{\left(\beta_1^2\Sigma_0+I\sigma_z^2\right)^{-1}+\sigma_\varepsilon^{-2}+\sigma_u^{-2}}. \tag{3}$$

- SOC:

$$\lambda_2>0. \tag{4}$$

Fundamental Investors' Problems

- In period 2, fundamental investor i:
  $\max_{x_{2,i}} E\left[x_{2,i}\left(v-p_2\right)|f_i, x_{1,i}, p_1\right]$
  $FOC \Rightarrow$

$$\beta_2 = \frac{1}{2\lambda_2}, \tag{5}$$

$$\beta_x = \frac{1}{2}\left[\begin{array}{c} \frac{1}{\lambda_2}\frac{\beta_1 \frac{I-1}{I}\Sigma_0}{\beta_1^2\frac{I-1}{I}\Sigma_0+(I-1)\sigma_z^2+\sigma_u^2} \\ +(I-1)\frac{-\beta_2\beta_1\frac{\Sigma_0}{I}+\beta_x\left(\beta_1^2\frac{\Sigma_0}{I}+\sigma_z^2\right)}{\beta_1^2\frac{I-1}{I}\Sigma_0+(I-1)\sigma_z^2+\sigma_u^2} \\ +J\delta\frac{\sigma_u^2}{\beta_1^2\frac{I-1}{I}\Sigma_0+(I-1)\sigma_z^2+\sigma_u^2} \end{array}\right]. \tag{6}$$

- In period 1:

$$\max_{x_{1,i}} E\left[x_{1,i}\left(v-p_1\right)+x_{2,i}^*\left(v-p_2\right)|f_i\right]$$

Mixed vs. Pure Strategies

We can compute

$$E\left(\pi_{F,1}+\pi_{F,2}|f_i\right) = -C_2 x_{1,i}^2 + C_1 f_i x_{1,i} + C_0$$

with $C_2 = \lambda_1 - \lambda_2 \left(\beta_x \frac{\beta_1^2 \frac{I-1}{I}\Sigma_0 + (I-1)\sigma_z^2 + \sigma_u^2}{\beta_1^2 \Sigma_0 + I\sigma_z^2 + \sigma_u^2} + \beta_2 \frac{\beta_1 \frac{1}{I}\Sigma_0}{\beta_1^2 \Sigma_0 + I\sigma_z^2 + \sigma_u^2}\right)^2,$

$C_1 = 1 - 2\lambda_2 \beta_2 \left(\beta_x \frac{\beta_1^2 \frac{I-1}{I}\Sigma_0 + (I-1)\sigma_z^2 + \sigma_u^2}{\beta_1^2 \Sigma_0 + I\sigma_z^2 + \sigma_u^2} + \beta_2 \frac{\beta_1 \frac{1}{I}\Sigma_0}{\beta_1^2 \Sigma_0 + I\sigma_z^2 + \sigma_u^2}\right).$

- Case 1: Mixed strategy $(\sigma_z > 0)$

$$C_2 = 0, C_1 = 0 \Rightarrow \lambda_1 = \lambda_2 \tag{7}$$

- Case 2: Pure strategy $(\sigma_z = 0)$

$$FOC : \beta_1 = \frac{C_1}{2C_2};\ SOC : C_2 > 0 \tag{8}$$

Characterization

- Mixed strategy equilibrium ($\sigma_z > 0$):
  - 7 unknowns: $(\beta_1, \sigma_z, \beta_2, \beta_x, \delta, \lambda_1, \lambda_2)$. Seven equations: (1), (2), (3), (5), (6), (7), together with one SOC (4).
  - **Proposition 1**: Simplifying the system in 3 unknowns $(\delta, \beta_1, \sigma_z)$
- Pure strategy equilibrium ($\sigma_z = 0$):
  - 6 unknowns: $(\beta_1, \beta_2, \beta_x, \delta, \lambda_1, \lambda_2)$. Six equations: (1), (2), (3), (5), (6), (8), together with two SOC's.
  - **Proposition 2**: Simplifying the system in 2 unknowns (β_1, λ_2)

Calibration

- One period = half day
- Normalize $p_0 = 1$, and set

$$\Sigma_0 = \frac{(30\%)^2}{252} = 0.00036$$

$\Rightarrow$daily return vol of 1.9% (annual return vol of 30%) for an individual stock

- Noise traders=retail investors. Set

$$\sigma_u = \sigma_U = 1 \text{ million shares}$$

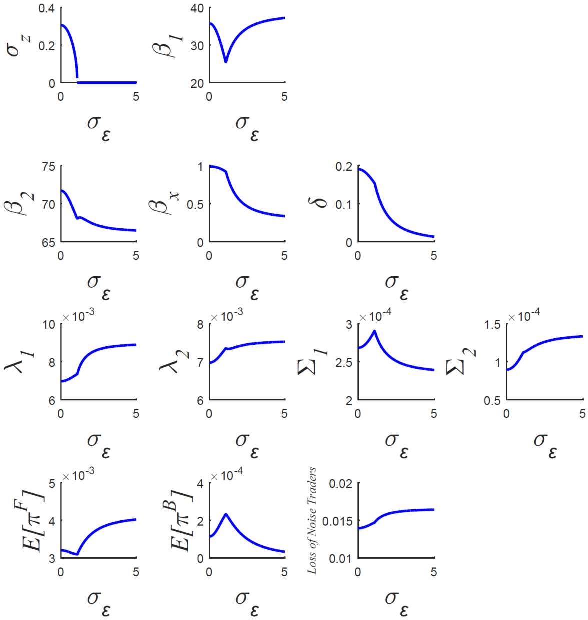
which implies a daily \$volume of retail investors:

$$E\left[|u_1 p_1| + |U_2 p_2|\right] \approx E\left[|u_1| + |U_2|\right] p_0 \approx 2\sqrt{\frac{2}{\pi}} = \$1.6 \text{ m/stock}$$

- Vary $(\sigma_\varepsilon, I, J)$

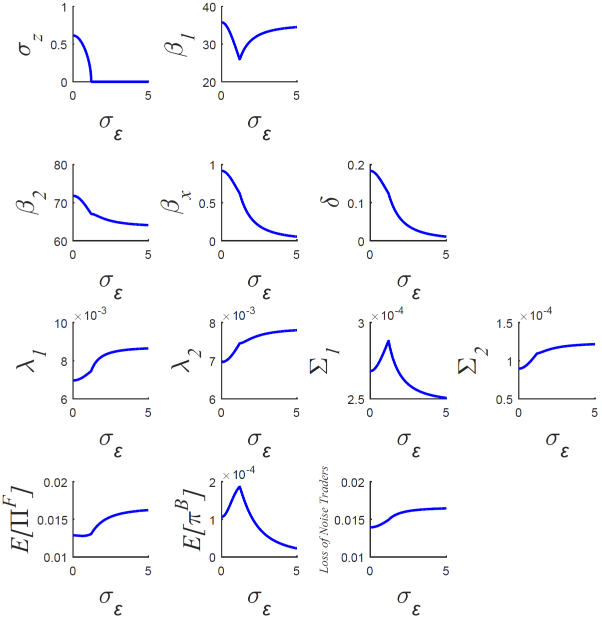
- Report three combinations of (I, J)
    1. van Kervel and Menkveld (2017 JF): $I = 4$ and $J = 10$
    2. $I = 1$ and $J = 10$
    3. $I = 1$ and $J = 1$
- Most trading and price discovery results are robust to (I, J)
- Profit results not sensitive to I, but to J

I=4 and J=10



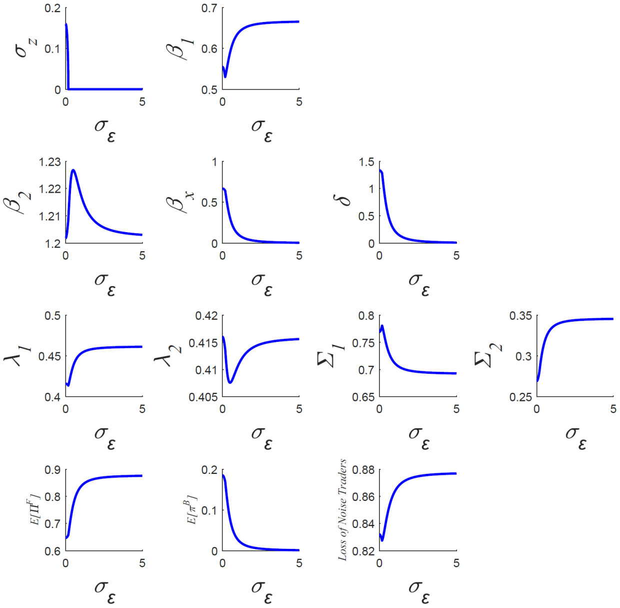
σ_z
β_1
β_2
β_x
δ
λ_1
λ_2
Σ_1
Σ_2
$E[\pi^F]$
$E[\pi^B]$
Loss of Noise Traders
σ_ε
$\times 10^{-3}$
$\times 10^{-4}$

I=1 and J=10



σ_z
β_1
β_2
β_x
δ
λ_1
λ_2
Σ_1
Σ_2
$E[\Pi^F]$
$E[\pi^B]$
Loss of Noise Traders
σ_ε
$\times 10^{-3}$
$\times 10^{-4}$

I=1 and J=1



Effects on Fundamental Investors

As σ_ε becomes smaller:

- Negative effect on $E\Pi^F$: fundamental investors face more competition from back-runners in period 2, who are now endowed with more accurate information
- Positive effect on $E\Pi^F$: fundamental investors add more noise to their period-1 order flow, and only fundamental investors know their own added noise z_i (so that they know better whether period-1 price p_1 overshoots or undershoots)
  $\Rightarrow$The second effect can dominate when J is large