Optimal contracts and equilibrium risk-sharing

Bruno Biais

HEC

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Outline

1) Introductory remarks

2) Biais, Heider, Hoerova (2016, 2019): Equilibrium efficiency and variation margins

3) Biais, Hombert, Weill (2019): Incentive constrained asset pricing and risk sharing

1) Introductory remarks

Risk sharing

Several agents with different risk preferences & endowments:

- banks, pension funds, investment funds
- assets and liabilities with random components

Endowments:

- labour income
- assets, with random future output or return

Want to share risk:

- more risk tolerant willing to bear more risk than more risk averse
- diversification

Efficient risk sharing in perfect and complete markets

First welfare theorem: Equilibrium is Pareto Optimum => optimal risk sharing

Marginal rates of substitution (MRS) between states equalised among agents

$$U'_i(c(\omega_1))/U'_i(c(\omega_2)) = U'_j(c(\omega_1))/U'_j(c(\omega_2))$$

Exchange rate between consumption in state ω_1 and ω_2 same for agent i and j

(Otherwise gains from trade left on the table, equilibrium not yet reached)

Imperfect risk sharing in imperfect markets?

If markets incomplete, risk sharing imperfect (MRS not equalised): Stiglitz 1982, Geanakoplos Polemarchakis 1986, Gromb Vayanos 2002

But why are markets incomplete? (// why contracts incomplete ?)

Exogenous incompleteness insatisfactory : Cannot rule out mutually beneficial trades, unless show why. Welfare/policy analysis impossible with exogenous constraints.

Microfoundation needed : what friction (moral hazard, adverse selection) prevents rational agents from conducting mutually beneficial trades ?

Moral hazard can make risk sharing imperfect

Different trees, held by different agents : My tree gives a lot of fruit in ω_1, yours in ω_2

Risk sharing: I give you fruits from my tree in ω_1, you give me fruits in ω_2

Agents must take unobservable actions so that their trees bear fruits

manage assets optimally, instead of shirking - diverting – gambling

no strategic default threat to obtain debt write-down

If I promise too many fruits from my tree, ruins incentives:

IC → cannot promise too many fruits: imperfect collateral pledgeability

micro-founded limits to risk-sharing/endogenous market incompleteness

collateral imperfectly pledgeable

Information constrained Pareto optimum & equilibrium

Consider given informational friction: effort costly + non observable + limited liability

Analyse equilibrium in market for risk sharing s.t. IC (agent prefers to exert effort)

Could planner do better ? Planner maximises weighted average of all utilities of all agents, s.t., resource constraint and IC

$\Rightarrow$ Information constrained Pareto optimum / second best

If there exist Pareto weights s.t. optimal planner's allocation = equilibrium allocation

$\Rightarrow$ Market implements second best

Otherwise equilibrium information constrained inefficient

If no price in IC constraint: equilibrium constrained efficient

Proof by contradiction (similar to standard proof for perfect complete markets, see, e.g., Mas Colell, Whinston, Green 1995, Chapter 16.C)

Suppose not: suppose there exists another incentive feasible allocation that Pareto dominates the equilibrium allocation

This alternative allocation must be outside the budget constraint of each agent (otherwise she would have chosen it in equilibrium since it is Pareto dominant)

Summing over all agents one can show that if the allocation is outside all the budget constraints then it violates the aggregate resource constraint → contradiction

If price in IC constraint: this reasoning no longer applies

Key initial step in previous proof: « suppose there exists another incentive feasible allocation that Pareto dominates the equilibrium allocation »

Considering alternative allocation works if prices don't enter the IC, so that the set of incentive feasible allocations is well defined independent from prices

If prices enter the IC, we cannot directly compare the original allocation (incentive feasible for certain prices) and the alternative one (incentive feasible for other prices)

→ we can no longer apply the arguments of the standard proof

If price in IC constraint: pecuniary externality

If price in IC constraint, when I sell I push price down, infinitesimally if I am atomistic, significantly if many sales

If price in IC of others, our sales affect other agents' IC: pecuniary externality

Externality → market equilibrium can be inefficient

NB: pecuniary externality via impact of my action on prices, unlike standard externalities which don't go through price (e.g., CO_2 emissions)

So is equilibrium constrained inefficient in imperfect markets with optimal contracts?

Important open question: more research needed

Stiglitz (1982), Greenwald Stigliz (1986): exogenously incomplete markets inefficient

Gromb Vayanos (2002): price in IC → pecuniary externalities → equ. inefficiency

But what if incompleteness not exogenous but endogenous, due to IC ?

Prescott Townsend (1984), Kehoe Levine (1993), Alvarez Jermann (2000): with optimal contracts/without exogenous incompleteness, equilibrium constrained efficient in spite of moral hazard

But price not in constraint

Why should we care?

Likely that in practice markets not perfect/complete due to frictions (info asy)

→ need to understand positive and normative implications

Positive analysis:

Do information asymmetries imply different patterns in contracts, trades than perfect market models and which we

Confront these implications to institutional observations or quantitative data?

Normative analysis:

If, in spite of frictions (info asy) equilibrium is constrained efficient,

no role for policy intervention.

If, in contrast, info asy implies equilibrium constrained inefficient,

calls for regulation/policy

1) Biais, Heider, Hoerova (2016, 2019): Equilibrium efficiency and optimal variation margins

Margin calls & information constrained optimality

Derivatives → risk sharing, but counterparty risk

Variation margins → reduce counterparty risk, but fire sales

Do derivatives & variation margin calls generate pecuniary externalities (price in IC)?

If so, is equilibrium is information-constrained inefficient?

3 mass-one continua of competitive agents

Bank (Société Générale)

B
Risk-averse protection buyer
endowed with one risky asset
$\theta = \underline{\theta}$ or $\overline{\theta}$ (mortgages, Greek bonds…)

Hedge fund, investment bank, broker dealer (AIG)

S
Risk-neutral protection seller
endowed with another risky asset, must exert effort at cost ψ per unit to get *R* (monitoring, transactions cost, operational risk)

I
Risk-averse investor
endowed with safe asset
(could also manage risky asset at per-unit cost $\psi_I > \psi$)

Other bank, investment fund, sovereign fund (Warren Buffet)

Timing

t=0	t=1/2	t=1
Endowments	Signal on θ (good or bad news)	Output
Contracting	Asset transfer α	Consumption
	Effort	

First best

B
Risk-averse
protection buyer
endowed with
θ

Full insurance against θ
(future, CDS, put...)

S
Risk-neutral
protection seller
endowed with
other risky asset

I not involved
in insurance,
already
efficiently
provided by S

I
Risk-averse
investor
endowed with
safe asset

No asset transfer
since *S* more
efficient than *I*

All have same MRS between consumption in different states =1

Moral hazard

Protection seller (*S*) has limited liability

If bad news: $\underline{\theta}$ likely, *S* likely to pay insurance to *B*

Likely insurance payment = liability

$\Rightarrow$ debt overhang

$\Rightarrow$ reduces incentives to exert effort

Second best

To mitigate debt overhang, reduce liability

IC: limited insurance after bad news, transfer α risky assets from *S* to *I*

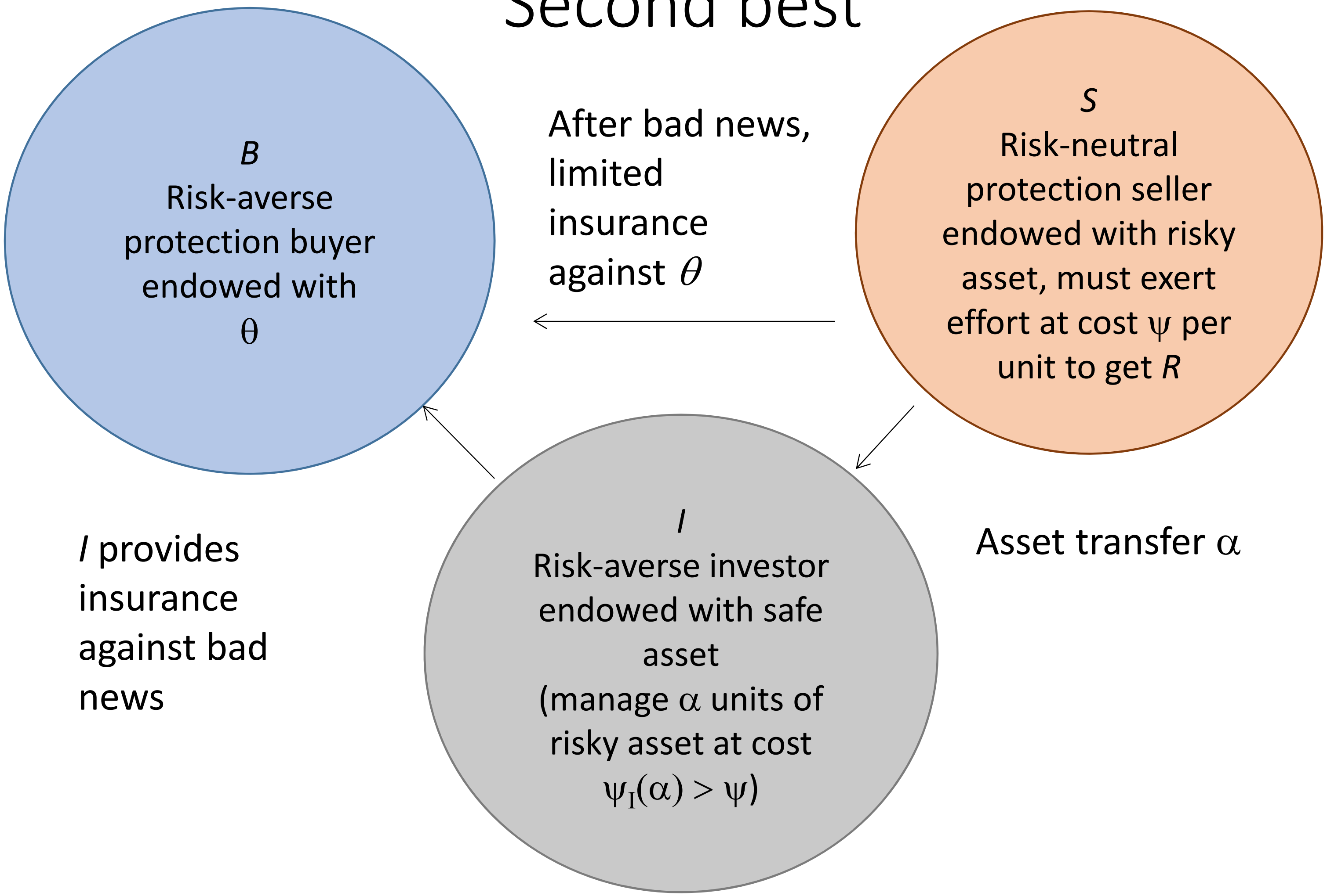
I less efficient at managing assets: $\alpha > 0 \rightarrow$ productive inefficiency

But $\alpha > 0$ relaxes IC: lower effort cost for *S*: $(1-\alpha)\ \psi$ instead of ψ
(similar to downsizing)

Optimal α : marginal incentive benefit = marginal productive inefficiency

In addition to buying assets from *S*, *I* complements limited insurance after bad news to *B*

Second best



MRS *B* = MRS *I* > MRS *S* = 1: imperfect risk sharing, because of moral hazard (no exogenous incompleteness)

Market equilibrium

Contract/securities contingent on all observable variables: θ, *signal*, *R,* s.t. IC

Privately optimal contract between *S* and *B*:

IC → limited insurance against θ: lower insurance after $\underline{s}$

Variation margin call after bad news: *S* must liquidate fraction α of assets at price *p*

Proceeds αp deposited on margin account at CCP: ring-fenced from moral hazard

I buy α assets at price *p* from *S*

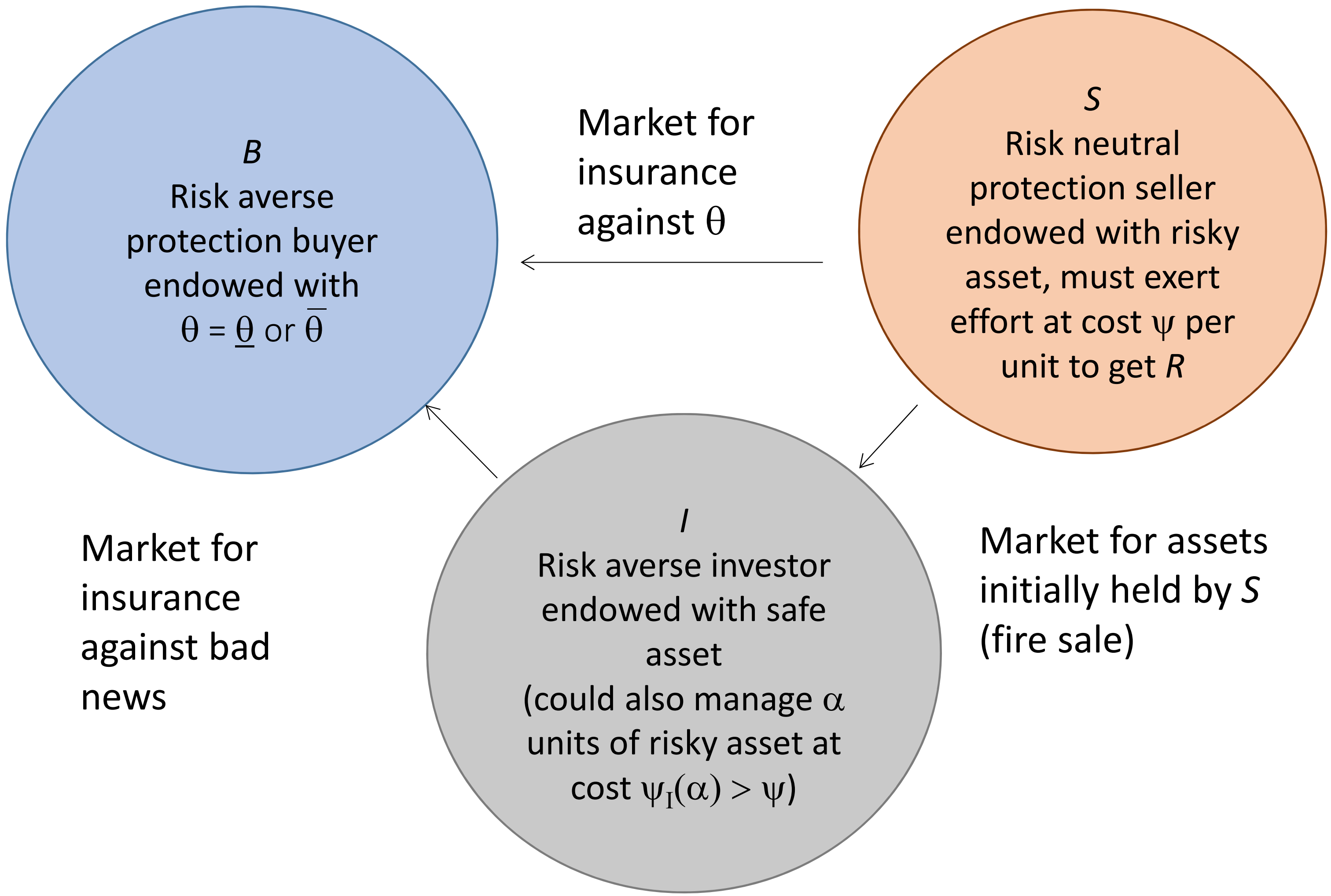
p = marginal valuation of *I* for assets: $R - (\psi_I(\alpha)+\alpha\psi_I'(\alpha)) < R - \psi$

Productive inefficiency, but raises pledgeable income from $\mathscr{P}$ to $\alpha p + (1-\alpha)\mathscr{P}$

$\mathscr{P}$ = pledgeable income on 1 unit risky asset managed by *S* (as above)

I also sell insurance against $\underline{s}$ to *B*

Market equilibrium



Pecuniary externality

When one protection buyer requests larger margin α

This reduces asset price *p*

And also pledgeable income αp + (1-α) $\mathscr{P}$ for all protection buyers

Does this negative pecuniary externality (arising because price *p* is in IC constraint) imply that equilibrium is constrained inefficient?

Market equilibrium information-constrained efficient

Bad news at t=1: negative shock for protection buyers (*B*), less insurance from S

Bad news at = 1: positive shock for investors (*I*), buy assets at low fire-sale price *p*

At t=0: *I* sell insurance against bad news to *B*

I and *B* trade until their MRS between consumption after good news and consumption after bad news are equalised -> efficient risk-sharing as in 2nd best

2) Biais, Hombert, Weill (2019): Incentive constrained asset pricing

Assets and agents

State ω realized at date 1

Assets (trees): $j \in [0,1]$, supply N_j, payoff (fruits): $d_j(\omega)$

I types, each in measure 1, endowed with trees at time 0

Concave utility over date-1 consumption

Timing

t=0

Endowments

Trade in trees and Arrow securities

t=1/2

State ω realised

Unobservable action (effort/behaves or shirk/misbehaves)

t=1

Consumption

Complete set of state-ω contingent Arrow Debreu securities → potential for risk-sharing

Investor i

At time 0, agents choose tree holdings: N_{ij}, Arrow securities: $a_i(\omega)$, to max U_i s.t.

Budget constraint: consumption = fruits of trees + payoff AD security

Incentive compatibility constraint (IC): prefers not to misbehave

State ω Arrow security sold $\leq$ pledgeable collateral $\mathscr{P}(\omega) = \delta$ state ω fruits of trees held

(If I issue a claim for more than δ fruits of trees in state ω, my IC does not hold any more in that state, because I prefer to strategically default and divert δ fruits rather than paying $|a_i(\omega)|$)

Market equilibrium

Consumption plans $c_i(\omega)$ and tree holdings N_{ij}, prices for Arrow securities $q(\omega)$ and trees p_j

Agents maximize E(utility) of time 1 consumption given price and budget and IC constraint

Markets clear:

$$\sum_i a_i(\omega)=0,$$

$$\sum_i N_{ij} = Nj$$

Endogenous segmentation

To share risk when insurance limited by IC, tilt asset allocation:

More risk averse hold safer (but still somewhat risky) assets

→ lower need to buy insurance from risk tolerant

→ by market clearing, more risk tolerant hold riskier assets

Different agents hold different portfolios of risky assets:

→ segmentation

Basis

Price of underlying asset < Price of derivative (= replicating portfolio of AD securities)

→ Deviation from Law of One Price,

Cannot be arbitraged:

To arbitrage, sell expensive AD securities → precluded by IC

Basis = shadow price of IC

(Yet, derivative and underlying equally imperfectly pledgeable)

Two premia

Expected return on asset held by agent i reflects two premia

→ Premium for covariance with consumption of i

(not aggregate consumption, because endogenous incompleteness)

→ Premium for covariance with shadow price of IC

SML flat at top, steep at bottom

IC constraints limit insurance

→ high demand for low risk assets (relative to intermediate assets) from more risk averse agents

→ relatively high price (low expected return) for very low β assets

Similarly high demand (from risk tolerant) for very high β assets → high price/low return

In contrast, relatively low demand for intermediary β assets → low price → high return

→ Expected returns concave in β

Supply effects

Assume two aggregate states, hold aggregate risk (total output in each state) constant

If many very low β and very high β assets

→ can allocate risk rather efficiently (risk averse buy low β, risk tolerant buy high β) without much need to trade derivatives

→ low shadow cost of IC

→ low basis

In contrast, low cross sectional dispersion of βs → large basis

Conclusion

Similarities BHH – BHW :

No exogenous incompleteness, but IC → endogenous incompleteness

If agent 's liability in state ω too large → misbehaves in that state
// debt overhang Myers - gambling for resurrection Jensen Meckling

In practice: misbehaviour leading to 2007 financial crisis ?

IC limits how much agents can promise to pay in state w

→ limits insurance/risk sharing → impacts price of risk/expected returns

→ contractual arrangements to mitigate this problem/ensure IC holds: margins

Differences B Heider Hoerova – B Hombert Weill

t=0
Endowments

Contracts /trades
in assets and
securities

t=1/2
Signal s or <u>s</u> on ω

Asset transfer α
Effort/behaves or
shirk/misbehaves

BHH

Variation margin

t=1
Consumption

Differences B Heider Hoerova – B Hombert Weill

t=0
Endowments

t=1/2
Signal s or $\underline{s}$ on ω

t=1/2
State ω realised

t=1
Consumption

Contracts /trades
in assets and
securities

Asset transfer α
Effort/behaves or
shirk/misbehaves

Unobservable action
(effort/behaves or
shirk/misbehaves)

BHH

BHW

Variation margin

Basis:
asset price < Arrow security
Cannot be arbitraged

Limits how much agent can
promise in state ω, i.e., sale of
state ω Arrow security

Research agenda: still much to do, you're welcome to join ☺

Broad theme: Corporate finance frictions → risk-sharing and asset pricing

Framework:

Optimal contracts: otherwise why don't agents contract problem away ?
welfare analysis and policy implications make sense only if optimal contracts

General equilibrium: interaction between contracts and markets
pecuniary externalities if price in IC → is equilibrium constrained efficient ?

Big unsolved issue: optimal dynamic contracting + GE

incentive compatible consumption smoothing through time
pecuniary externalities among generations
contracting frictions → investment and growth