Leverage Dynamics without Commitment

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Leverage dynamics is at the heart of dynamic corporate finance

- Static trade-off (maximizing firm value) differs from equity’s dynamic optimization
- Challenging, as debt prices interact with future equilibrium leverage polices

Existing literature relies on some ad hoc “commitment” of future debt policies

- Refinance to keep outstanding debt face value constant (Leland 1994 1998)
- Whenever adjusting debt, the firm has to retire the existing debt first, with some transaction costs (Fischer, Heinkel, and Zechner 1989; Goldstein, Leland, Ju 2001)
  - Abrupt adjustment to “target” leverage
- **Empirically counterfactual**: firms actively manage their debt, often incrementally
Introduction (2)

[Graphs showing the change in book debt and book debt plus market equity for American Airlines and United Airlines from 1986Q4 to 2011Q4.]
The firm cannot commit to future debt policies

- Otherwise, standard trade-off setting (tax shield vs bankruptcy cost) with stochastic asset growth; no transaction cost
- No commitment at all: say, no covenants
- A more endogenous “friction”, rather than exogenous frictions to adjust leverage

Assumption on seniority and dilution

- Zero recovery \(\Rightarrow\) seniority structure irrelevant. **Indirect** dilution: issuing more debt hurts default probability
- Positive recovery: pari-passu debt, **direct** dilution in recovery (not in this presentation)

Leverage may go down via asset growth and debt maturing, but equity never reduces debt voluntarily

- Repurchase debt is never optimal—**leverage ratchet effect** (Admati DeMarzo Hellwig Pfleiderer, 2018)
- Our setting is more canonical
This Paper (2)

- A general method to solve this class of models
  - A result reminiscent of Coase conjecture

- Closed-form solutions for work-horse log-normal cash-flow setting

- History-dependent leverage dynamics: issue more (less) following good (bad) shocks
  - Leverage dynamics tend to be mean-reverting; no immediate adjustment to leverage “target”

- Dynamic trade-off of equity value \(\neq\) Static trade-off of firm value
  - Two leverage/maturity dynamics drastically different, but both are optimal
  - Lemmon, Roberts, and Zender (2008)
General Model: Environment

Preferences
- Risk-neutral world, with common discount rate $r$

Assets
- Assets in place generate operating income (could allow for jumps):
  \[ dY_t = \mu (Y_t) \, dt + \sigma (Y_t) \, dZ_t \]
- Focus on zero recovery now (debt seniority irrelevant); can be relaxed

Debt contract: aggregate face value $F_t$ (endogenous)
- Each debt with coupon rate $c$, face value 1
- Exponentially retiring (Poisson maturing) with rate $\xi$

Corporate tax: $\pi (Y_t - cF_t)$
Debt Issuance/Repurchase and Default

Evolution of debt

- Sell/buyback debt $d\Gamma_t$, so aggregate debt face value evolves as

$$dF_t = -\zeta F_t dt + d\Gamma_t$$

contractual debt maturing  active debt managment

Timing within $[t, t + dt]$ & lack of commitment

- Cash flow realizes; either default or pay coupon/principal; announce $d\Gamma_t$; debt price set (and trade); next period
- Unable to commit on future $d\Gamma_{t+s}$ for $s > 0$

Focusing on “smooth equilibrium”: $d\Gamma_t = G_t dt$

- Equity could adjust debt discretely, but not optimal in such an equilibrium
- Other equilibria with jumps? In general, yes (more later)

Equity default at endogenous stopping time $\tau_b$
Equity Value

State variables (Markov Perfect Equilibrium)

- Exogenous cash-flows $Y_t$, and endogenous debt obligation $F_t$

Equity's problem, taking debt prices $p$ as given

- Equity receives cash-flows (if negative, covered by issuing equity)

$$ Y_t - \pi(Y_t - cF_t) - (c + \xi)F_t + p_t G_t $$

  - cash-flows
  - corporate taxes
  - interest & principal
  - issuance/repurchase

- Endogenous debt price $p_t$ determined later

- Given $Y_t = Y$ and $F_t = F$, equity is solving

$$ V(Y, F) \equiv \max_{\{G_s, \tau_b\}} \mathbb{E}_t \left\{ \int_t^{\tau_b} e^{-r(s-t)} \left[ Y_s - \pi(Y_s - cF_s) - (c + \xi)F_s + p_s G_s \right] ds \right\} $$

- Controlling 1) debt evolution $dF_t = F_t dt + G_t dt$; and 2) when to default
Debt Price

Debt price

▶ Competitive risk neutral debt investors price debt rationally
▶ Given equity default decision $\tau_b$, equilibrium debt price

$$p(Y, F) \equiv \mathbb{E}_t \left\{ \int_t^{\tau_b} e^{-(r+\xi)(s-t)} (c + \xi) \, ds \mid Y_t = Y, F_t = F \right\}$$

Why does commitment matter?

▶ $p_t$ depends on equilibrium default time $\tau_b$
▶ $\tau_b$ depends on firm’s future debt policy—the more the future debt, the more likely the default
Value Equivalence of No-Issuance (1)

- Hamilton-Jacobi-Bellman equation for equity

\[ rV(Y, F) = \max_G \left[ Gp(Y, F) + (G - \xi F) V_F(Y, F) \right] \]

\[ = \left( Y - \pi (Y - cF) - (c + \xi) F + \mu(Y) V_Y(Y, F) + \frac{\sigma^2(Y)}{2} V_{YY}(Y, F) \right) \]

- Objective linear in \( G \). Optimal \( G \) ⇒ First-Order Condition

\[ p(Y, F) + V_F(Y, F) = 0 \]

- MB of issuance + MC on future value

- Under FOC, equity indifferent at any \( G \) (given equilibrium \( p \))

  - Linear control with interior solution (smooth policy \( G_t dt \))

  - Equity value can be solved by setting \( G = 0 \) always
Value Equivalence of No-Issuance (2)

- Equity value can be solved by setting $G = 0$ always

$$rV = \xi F V_F + Y - \pi (Y - cF) - (c + \xi) F + \mu (Y) V_Y + \frac{\sigma^2 (Y)}{2} V_{YY}$$

- No gain in equilibrium by debt issuance/repurchase
  - Any potential tax shield gain is dissipated by bankruptcy cost caused by future excessive leverage
  - Reminiscent of Coase conjecture; DeMarzo and Urosevic (2006)

- Get equity value $V(Y, F)$ without knowing debt price
Equilibrium Policies

Basic idea

▶ Debt price $p(Y,F)$ must satisfy the valuation equation

$$p(Y,F) = \mathbb{E}_t \left\{ \int_t^{\tau_b} e^{-(r+\xi)(s-t)} (c + \xi) \, ds \right\}$$

▶ $V(Y,F)$ gives $-V_F(V,F) = p(Y,F)$ using equity’s FOC

▶ How to make both match? Via debt management $G(Y,F)$

▶ ODE for $V_F(V,F)$ (HJB for $V$) does not depend on $G$...

▶ while HJB for $p$, which depends on $G$
Equilibrium Policies

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- How to make both match? Via debt management \( G(Y, F) \)
  - ODE for \( V_F(V, F) \) (HJB for \( V \)) does not depend on \( G \)...
  - while HJB for \( p \), which depends on \( G \)

Equilibrium debt issuance policy

\[
G^*(Y, F) = \frac{c \cdot \pi'(Y - cF)}{-p_F(Y, F)}
\]

- \( \pi'(Y - cF) \geq 0 \), tax benefit \( \Rightarrow \) always issuing debt

- Recall \( -p_F(Y, F) = V_{FF}(Y, F) > 0 \), capturing the price impact
Strict Optimality in Discrete Time

- Taking the value function at $t + h$ as given, consider equity’s problem at $t$, where time interval $h > 0$
- Denote debt issuance by $\Delta$. Equity is maximizing
  \[
  \max_{\Delta} \left[ - (1 - \pi) \cdot \Delta c \cdot h + \Delta \left[ c \cdot h + p(Y, F + \Delta) \right] + V(F + \Delta, Y) \right]
  \]
  after-tax interest payment  
ext new debt proceeds  
  future equity Value
- First-order condition w.r.t $\Delta$
  \[
  0 = \pi c \cdot h + p(Y, F + \Delta) + \Delta \cdot p_F(Y, F + \Delta) + V_F(F + \Delta, Y)
  \]
  which implies that
  \[
  \Delta = \frac{\pi c \cdot h + p + V_F}{-p_F} = \frac{\pi c}{-p_F} \cdot h
  \]
- One can easily check the global optimality
Sufficiency of Local FOC

**Proposition 1:** Global optimality of local FOC holds if debt price
\[ p(Y, F) = -V_F(V, F) \]
is non-increasing in debt \( F \).

- Debt price decreasing in \( F \) is equivalent to the equity value function being convex in \( F \) (option value of default).
- Buyback, paying a higher price; selling too much hurts price too.
Leverage Ratchet Effect

- What is the impact of debt repurchase on equity value?
  - Often the intuition is through firm value...
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Reducing debt today alleviates future default ⇒ higher firm value

But does equity benefit strictly from this effect? No. (Do not forget existing debt holders!)

Equity optimizes default decision ex post already ⇒ zero indirect impact on equity value today (envelope theorem)
Leverage Ratchet Effect

- What is the impact of debt repurchase on **equity value**?
  - Often the intuition is through **firm value**...

- Reducing debt today alleviates future default $\Rightarrow$ higher firm value
  - But does equity benefit strictly from this effect? **No.** (Do not forget existing debt holders!)
  - Equity optimizes default decision ex post already $\Rightarrow$ zero indirect impact on equity value today (**envelope theorem**)

- **Tax** saving benefit always tempting...leverage ratcheting in ADHP
  - This paper: a more canonical setting
  - Same logic to debt overhang—equity is optimizing investment decisions ex post
Summary of General Model

1. Solve for equity value $V(Y, F)$ by setting $G(Y, F) = 0$

2. Set the equilibrium debt price $p(Y, F) = -V_F(Y, F)$

3. Check the equity holders’ global optimality condition
   ▶ Verifying $p(Y, F)$ is non-increasing in $F$ (or $V(Y, F)$ is convex in $F$)

4. Equilibrium debt issuance $G^*(Y, F) = \frac{\pi'(Y-cF)\cdot c}{-p_F(Y,F)} > 0$
Log-Normal Cash-flows Model

- Scale-invariance, cash-flows $dY_t / Y_t = \mu dt + \sigma dZ_t$
  - The work-horse model of dynamic corporate finance

- One-dimensional state variable: scaled cash-flow $y_t \equiv Y_t / F_t$
  - Equity value $V(Y, F) = F \cdot v(y)$, debt price $p(Y, F) = p(y)$; closed-form solutions
  - **Strong Markov property** (we can prove the uniqueness of such equilibria)

- Let $g^*(y_t) \equiv G^*(Y_t, F_t) / F_t$, then

\[
\frac{dy_t}{y_t} = \left( \mu + \zeta - g^*_t \right) dt + \sigma dZ_t
\]

- CF growth + debt maturing - debt issuance

- Debt growth rate $g^*_t - \zeta$; endogenous $g^*_t = \frac{(r+\zeta)\pi c}{c(1-\pi)+\zeta} \frac{1}{\gamma} \left( \frac{y}{y_b} \right) > 0$
  - $\gamma$ is a constant depending on parameters
  - Increasing in $y$, i.e., more debt issuance after good fundamental
Net Debt Issuance $g^*(y) - \bar{\xi}$, Debt Maturity
Two Benchmarks with Commitment

No future debt issuance:

⚠️ The firm commits to set $g_t = 0$ always (superscript 0)

⚠️ Equity value is the same (so does $y_b$), debt price is higher (by the tax shield)

\[
p^0(y) = p(y) + \frac{\pi c}{r + \xi} \left(1 - \left(\frac{y}{y_b}\right)^{-\gamma}\right)
\]

⚠️ Less debt ⇒ less likely to default (same $y_b$ but $y$ has a higher drift)

Fixed future debt:

⚠️ The firm commits to set $g_t = \xi$ always; Leland 1998
Model Comparisons: Debt Prices and Credit Spreads

Implication of credit spreads: $y \to \infty$ i.e. zero current leverage

- $p^\xi(y)$ and $p^0(y) \to \frac{c + \xi}{r + \xi}$, with zero credit spread
- $p(y) \to \frac{c(1 - \pi) + \xi}{r + \xi}$, non-zero credit spreads (high future excessive leverage!)
Equilibrium Debt Dynamics

- Different from static trade-off setting, it is optimal to set $F_0^* = 0$
  - Knowing the future temptation of overborrowing....

- **Proposition.** Given cash-flow history $\{ Y_s : 0 \leq s \leq t \}$, time-$t$ debt is ($\hat{y}_\xi$ is a constant depending on parameters)

$$F_t = \frac{1}{\hat{y}_\xi} \left[ \int_0^t \gamma \xi Y_s \gamma e^{-\gamma \xi (s-t)} \, ds \right]^{1/\gamma}$$

- Start from $t = 0$ debt grows at the order of $t^{1/\gamma}$
- Outstanding debt is average past earnings, with decaying weights $\gamma \xi$

- High mean-reverting speed, or more aggressive in adding leverage given high cash flows, when
  - Shorter debt maturity (higher $\xi$)
Optimal Debt Maturity Structure?

- So far the debt maturity structure $\xi$ is taken as a parameter

- Say the firm gets a one-time chance to set $\xi$ optimally for future debt issuance

- **Proposition**: Equity holders are *indifferent* at any $\xi$
  - Why? Because equity value is as if there is no future debt issuance...

- This indifference result holds more generally
Long-term vs. Short-term Debt

- Two firms start with zero debt, with different debt maturities (both being optimal)—but have different leverage dynamics/target
  - Lemmon, Roberts, and Zender (2008)
Long-term vs. Short-term Debt

- Two firms start with zero debt, with different debt maturities (both being optimal)—but have different leverage dynamics/target
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- With flexibility of shorter-term debt, the firm borrows more for higher debt tax shield
  - But tax shield is a transfer from social perspective—so long-term debt is preferred to minimize bankruptcy cost
Investment

- Special case of log-normal process. Capital $K_t$ evolves as

$$\frac{dK_t}{K_t} = (i_t - \delta) \, dt + \sigma \, dZ_t$$

with quadratic investment cost $\frac{\kappa i_t^2}{2} K_t$, and output $Y_t = AK_t$

- Leverage ratchet effect prevails despite debt overhang considerations

- Equity issues debt more aggressively when controlling investment endogenously, compared to exogenous investment
  - Endogenous investment offers equity more protection later
Conclusion and Future Work

What we have done

▶ A general methodology solving dynamic corporate finance model without commitment

▶ Leverage policy depending on the entire earnings history, new insight on debt maturity and investment

▶ Slow initial adoption of leverage, but leads ultimately to excess

Future extensions

▶ DeMarzo, 2019 AFA presidential address: importance of exlusivity in collateralized borrowing

▶ Modeling sovereign debt and default (DeMarzo, He, and Tourre, 2019)
  ▶ Covenant of no debt issuance once in distress (say for $y < \hat{y}$)
  ▶ Discrete debt issuance (jump to $\hat{y}$) in equilibrium, counter-productive

▶ Internal cash with liquidity-driven default?