Short Term Debt and Incentives in Banks

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Short-term debt in financial intermediation

- One of the most distinct features of banks is their reliance on short-term debt
  - Deposits represent over three-quarters of funding of US commercial banks (Hanson, Shleifer, Stein, and Vishny, 2015)
  - Not limited to deposits: banks and shadow banks rely on creditors in wholesale funding markets (Adrian and Shin, 2010)
- Reliance on short-term debt makes banks and other financial institutions prone to fragility and runs
- Two lines of theories highlight different bank functions and roles of short-term debt:
  - Banks' core function is to provide liquidity to their depositors, which is amplified by government guarantees
  - Banks' short-term debt provides market discipline against risk shifting, increasing the efficiency of banks' investments
- Both lines of theories exhibit key role for incentives in shaping banks' capital structure choices raising questions about optimality of short-term debt and implications for fragility and welfare

Providing guidance using theory

- These issues involve complex equilibrium interactions
- Developing a model to evaluate the full scope of the problem requires understanding of:
  - **(a)** How runs and fragility respond to banks' choices of short-term debt
  - **(b)** Given (a), how banks detrmine short-term debt
    - For a given original motivation, such as liquidity provision, discipline, guarantees
  - **(c)** Given (a) and (b), how other conditions are determined
    - For example, government guarantees, general equilibrium behavior in banking sector, etc.
- The two models I will cover in detail provide recent analyses of this kind for the two leading approaches:
  - Allen, Carletti, Goldstein, Leonello (2018): Short-term debt is driven by liquidity provision and government guarantees
  - Eisenbach (2017): Short-term debt is driven by market discipline

Government Guarantees and Financial Stability

Allen, Carletti, Goldstein, Leonello

Journal of Economic Theory, 2018

Liquidity creation, fragility, and guarantees

- Liquidity creation, fragility, and guarantees (Diamond and Dybvig, 1983):
  - Banks provide risk sharing against early liquidity needs to depositors, by offering demandable debt, thus improving their welfare
  - But, the deposit contracts expose banks to the risk of a run as depositors may withdraw early (coordination failure)
  - Government guarantees, such as deposit insurance, have been proposed as a way to address the problem and eliminate panic
- The problem with guarantees:
  - They are costly when runs do occur
  - They encourage banks to increase short-term debt (Calomiris, 1990), fragility (Demirgüç-Kunt and Detragiache, 1998), and/or risk (Gropp, Grundl, and Guttler, 2014)
- Goal: understand equilibrium interactions, fragility, Banks' choices, and desirability of guarantees

Modelling framework

- Follow Goldstein and Pauzner (2005), where:
  - Depositors' withdrawal decisions and probability of runs are determined by the banking contract using global-games methodology
  - Banks set deposit contract to provide risk sharing against early liquidity need, taking into account the effect on fragility
- Two inefficiencies:
  - Inefficient runs destroy good investments
  - Banks scale down liquidity creation (e.g., reducing deposit rates) in the attempt of limiting runs
- Introduce different schemes of guarantees to analyze interaction between fragility, banks' choices, and guarantees
  - Previous theoretical literature (e.g., Keeley, 1990; Cooper and Ross, 2002; Keister, 2016) does not endogenize run probability, banks' choices, and guarantees at the same time

Results in a nutshell

- Guarantees against panic runs (similar to Diamond and Dybvig, 1983):
  - Can eliminate panics altogether, but induce banks to increase demandable debt
  - This increases the probability of fundamental-based runs and may increase the probability of runs overall
  - But, this is not indication of moral hazard, as guarantees are never paid in equilibrium
  - Guarantees allow banks to provide more risk sharing and liquidity, increasing welfare despite greater fragility
- Guarantees against panic runs and fundamental failures
  - More realistic and potentially more desirable
  - They are costly and so limited; reduce probability of runs but do not eliminate them
  - They distort banks' choices, since banks do not internalize the effect on cost to government
  - Usually, banks choose too little demandable debt, as they do not internalize that runs can reduce fundamental failures and reduce cost to government

Environment and Technology

- Three date ($t = 0, 1, 2$) economy with a continuum $[0, 1]$ of banks and a continuum $[0, 1]$ of consumers in every bank
- At date 0, banks raise one unit of funds from consumers in exchange for a demandable deposit contract and invest in a risky project
- The project returns 1 if liquidated at date 1 and $\widetilde{R}$ at date 2 with

$$\tilde{R} = \begin{cases} R > 1 & \text{w. p. } p(\theta) \\ 0 & \text{w. p. } (1 - p(\theta)) \end{cases}$$

- Fundamental shock: $\theta \sim U[0, 1]$ is the fundamental of the economy; realized at date 1 and become public at date 2
- Probability of success: assume $p^{'}(\theta) > 0$ and $E_{\theta}[p(\theta)]R > 1$
  - For simplicity, $p(\theta) = \theta$
- Banking sector is competitive, so that deposit contracts maximize consumers' welfare; not taking into account externalities

Preferences

- Consumers are risk-averse ($RRA > 1$ for any $c \geq 1$) and endowed with 1 unit each at date 0
- At date 0 they deposit at the bank in exchange for a deposit contract $(c_1, \tilde{c}_2)$
- Consumers are ex ante identical but each has probability λ of suffering a liquidity shock and having to consume at date 1
  - Uncertainty is resolved privately at the beginning of date 1
- Consumers derive utility both from consuming at date 1 or 2 and from enjoying a public good g

$$U(c, g) = u(c) + v(g)$$

with $u'(c) > 0$, $v'(g) > 0$, $u''(c) < 0$, $v''(g) < 0$, $u(0) = v(0) = 0$ and

$$u'(1) < v'(g) < u'(0)$$

Depositors' information

- At the beginning of date 1, each depositor receives a private signal x_i regarding the fundamental of the economy θ of the form

$$x_i = \theta + \epsilon_i,$$

  with $\epsilon_i \sim U[-\epsilon, +\epsilon]$ being i.i.d. across agents. Most of the time, we focus on ϵ very close to 0
- Based on the signal, depositors update their beliefs about the fundamental θ and the actions of the other depositors
  - Early depositors always withdraw at date 1
  - Late depositors withdraw at date 1 if they receive a low enough signal
- The bank satisfies early withdrawal demands by liquidating its investments. If proceeds are not enough, depositors receive a pro-rata share

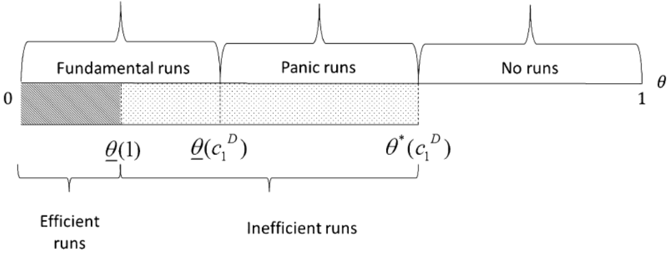
Decentralized equilibrium

- Combination of
  - Bayesian Nash equilibrium among depositors at $t = 1$
  - Competitive equilibrium among banks at $t = 0$
- At date 1:
  - Fraction of depositors who withdraw: $n \geq \lambda$
  - Depositor payoffs (depending on bank liquidity):

	liquid: $n \leq \frac{1}{c_1}$	illiquid: $n > \frac{1}{c_1}$
wait	$\frac{1-nc_1}{1-n}R$ w. p. θ	0
withdraw	c_1	$\frac{1}{n}$

- Unique equilibrium: $n = 1$ below θ^*; $n = \lambda$ above θ^*
- At date 0:
  - Banks set c_1^D to maximize expected utility of depositors

The decentralized solution: Depositors' withdrawals



- $\underline{\theta}(c_1)$ is the boundary for "fundamental runs"; determined as the indifference point assuming others don't run:

$$u(c_1) = \underline{\theta} u \left(\frac{1 - \lambda c_1}{1 - \lambda} R \right)$$

- $\theta^*(c_1)$ is the cutoff for "panic runs"; determined as the indifference point assuming uniform distribution on depositors who withdraw:

$$\int_{n=\lambda}^{\frac{1}{c_1}} \theta^* u \left(\frac{1 - nc_1}{1 - n} R \right) = \int_{n=\lambda}^{\frac{1}{c_1}} u(c_1) + \int_{n=\frac{1}{c_1}}^{1} u\left(\frac{1}{n}\right)$$

- Both thresholds $\underline{\theta}(c_1)$ and $\theta^*(c_1)$ increase in c_1

The decentralized solution: Types of crisis

- Banks fail when they are not able to repay the promised repayment
  - It can occur either at date 1 or 2
- At date 1, banks fail because of runs
  - Low fundamentals below $\underline{\theta}(c_1)-$ anticipation of low returns at date 2
  - Panic between $\underline{\theta}(c_1)$ and $\theta^*(c_1)-$ coordination failure among depositors
- At date 2, banks fail when their asset returns 0
  - Project fails with probability $(1-\theta)|\,\theta > \theta^*$

The decentralized solution: The bank's choice

- Given depositors' withdrawal decisions, at date 0 each bank chooses c_1 to maximize:

$$\int_0^{\theta^*(c_1)} u(1)\, d\theta + \int_{\theta^*(c_1)}^1 \left[\lambda u(c_1) + (1-\lambda)\theta u\left(\frac{1-\lambda c_1}{1-\lambda} R\right)\right] d\theta$$
$$+ v(g)$$

- The optimal $c_1^D > 1$ trades off:
  - Better risk sharing; transferring consumption from patient to impatient agents
  - Against higher probability of runs $\left(\frac{\partial \theta^*(c_1)}{\partial c_1} > 0\right)$
- Two inefficiencies related to panics:
  - Banks offer too little risk sharing (liquidity creation) in anticipation of the run: c_1^D is lower than first best
  - Runs lead to inefficient liquidation of bank investment for $\theta \in (\underline{\theta}(1), \theta^*(c_1^D))$
- Another inefficiency comes due to the fact that depositors are not protected against fundamental failure

Government guarantees against panics

- A natural starting point to demonstrate the effect of government guarantees is a scheme that guarantees against panic
  - This is closest to Diamond-Dybvig, except that banking contract will react to the scheme
- Specifically, depositors are guaranteed to receive $\overline{c}_s = \frac{1-\lambda c_1}{1-\lambda} R$ when the bank's project is successful at date 2, irrespective of how many depositors have withdrawn at date 1
- Panic runs are eliminated but fundamental runs remain for $\theta \in [0, \underline{\theta}(c_1)]$
- Bank chooses c_1^P to maximize

$$\int_0^{\underline{\theta}(c_1)} u(1) d\theta + \int_{\underline{\theta}(c_1)}^1 \left[\lambda u(c_1) + (1-\lambda)\theta u\left(\frac{1-\lambda c_1}{1-\lambda} R\right)\right] d\theta$$
$$+ \int_0^1 v(g) d\theta$$

Deposit contract under guarantees against panics

- Under guarantees against panic, c_1^P solves:

$$\lambda \int_{\underline{\theta}(c_1)}^{1} \left[u'(c_1) - \theta u' \left(\frac{1 - \lambda c_1}{1 - \lambda} R \right) \right] d\theta +$$
$$- \frac{\partial \underline{\theta}(c_1)}{\partial c_1} \left[\lambda u(c_1) + (1 - \lambda) \underline{\theta} u \left(\frac{1 - \lambda c_1}{1 - \lambda} R \right) - u(1) \right] = 0$$

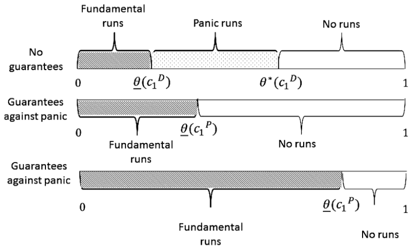
- In decentralized solution, c_1^D solves:

$$\lambda \int_{\theta^*(c_1)}^{1} \left[u'(c_1) - \theta u' \left(\frac{1 - \lambda c_1}{1 - \lambda} R \right) \right] d\theta +$$
$$- \frac{\partial \theta^*(c_1)}{\partial c_1} \left[\lambda u(c_1) + (1 - \lambda) \theta^* u \left(\frac{1 - \lambda c_1}{1 - \lambda} R \right) - u(1) \right] = 0$$

- Result: $c_1^P > c_1^D$. Thus, $\underline{\theta}(c_1^P) > \underline{\theta}(c_1^D)$ and possibly $\underline{\theta}(c_1^P) > \theta^*(c_1^D)$
- Note: No distortion in the choice of c_1^P as the guarantee entails no disbursement for the government

Runs and welfare under the guarantees against panics

- As $c_1^P > c_1^D$, guarantees
  - Increase the probability of fundamental runs and possibly runs overall
- Two scenarios depicted below:



- But, guarantees increase depositors' expected utility from the private good and increase overall welfare
  - Increased short-term debt is not evidence of moral hazard
  - It reflects better ability of banks to provide liquidity and risk sharing

Adding guarantees against bank failure at date 2

- Still keep $\overline{c}_s = \frac{1-\lambda c_1}{1-\lambda} R$ at $t = 2$ iff the project succeeds
- Introduce guarantee $\overline{c}_f \neq \overline{c}_s$ at date 2 if the bank project fails
  - $\overline{c}_f > 0$ insures agents against fundamental risk and reduces probability of fundamental runs
  - But, it is costly as bank failures can occur and the government has to reduce g to pay for the guarantee
- Questions:
  - Does the government want to set $\overline{c}_f > 0$?
  - How do banks respond?

Runs and deposit contract under additional guarantee

- Only fundamental runs occur. The threshold $\underline{\theta}$ is the solution to

$$u\left(c_1\right) = \theta u\left(\frac{1-\lambda c_1}{1-\lambda}R\right) + \left(1-\theta\right)u\left(\overline{c}_f\right),$$

- The threshold $\underline{\theta}$ increases in c_1 and decreases in $\overline{c}_f$
- Each bank sets c_1^F to maximize

$$\int_0^{\underline{\theta}} u\left(1\right)d\theta + \int_{\underline{\theta}}^1 \left[\begin{array}{c} \lambda u\left(c_1\right) + \\ \left(1-\lambda\right)\left[\begin{array}{c} \theta u\left(\frac{1-\lambda c_1}{1-\lambda}R\right) + \\ \left(1-\theta\right)u\left(\overline{c}_f\right) \end{array}\right] \end{array}\right] d\theta$$
$$+E\left[v(g, c_1^*, \overline{c}_f)\right]$$

- Results show that $\frac{dc_1^F}{d\overline{c}_f} > 0$. Thus, $c_1^F > c_1^P$ for any $\overline{c}_f > 0$
- The bank does not internalize the reduction in g for the provision of the guarantee

The government choice for additional guarantee

- Government chooses $\overline{c}_f$ to maximize depositors' overall expected utility
  - Cost of the disbursement is internalized
  - The effect on the bank's choice of c_1^F is also taken into account
- The government chooses $\overline{c}_f > 0$ when $u'(0) - v'(g) > 0$
  - The government with a sufficiently large endowment wants to provide some guarantees to reduce runs
- Interestingly, there is a reverse moral hazard: the government would choose higher short-term commitment for the bank: $c_1^G > c_1^F$
  - This is because of lower expected utility from public good if no runs occur:

$$\underline{\theta} v(g) + (1 - \underline{\theta}) v(g - (1 - \lambda)\overline{c}_f) < v(g)$$

  - This is the only thing that is not internalized by the bank in the model

Deposit insurance

- Depositors are guaranteed to receive a $\overline{c}_s = \overline{c}_f = \overline{c}$ whenever their bank is not able to repay the promised repayment
  - More realistic; similar to a standard deposit insurance scheme with $\overline{c}$ being the lowerbound on depositors' payment
  - Less desirable, because amount guaranteed is not tailored to the cause and because guarantee might also imply payment at date 1, which is never optimal
- Probability of both types of runs is reduced but both runs still occur
  - It is too costly to fully guarantee against panic when amount of guarantee is the same in all cases
- Providing guarantees is costly and the market solution is inefficient
  - Again, banks internalize the effect of their choices on the run probability, but not on the cost of providing the guarantee

Depositors' withdrawal decisions with deposit insurance

- Fundamental runs occur for $\theta < \underline{\theta}(c_1, \overline{c})$ where $\underline{\theta}(c_1, \overline{c})$ solves

$$u(c_1) = \theta u\left(\frac{1-\lambda c_1}{1-\lambda} R\right) + (1-\theta) u(\overline{c})$$

- Panic runs occur now for $\theta < \theta^*(c_1, \overline{c})$ where

$$\theta^*(c_1, \overline{c}) = \frac{\int_{n=\lambda}^{\widehat{n}} u\left(c_1\right) + \int_{n=\widehat{n}}^{1} u\left(\frac{1}{n}\right) - \int_{n=\lambda}^{1} u\left(\overline{c}\right)}{\int_{n=\lambda}^{\overline{n}} \left[u\left(\frac{1-nc_1}{1-n} R\right) - u\left(\overline{c}\right)\right]},$$

and $\overline{n} = \frac{R-\overline{c}}{Rc_1-\overline{c}}$ and $\widehat{n} = \frac{1}{c_1}$

- The thresholds $\underline{\theta}(c_1, \overline{c})$ and $\theta^*(c_1, \overline{c})$ increase with c_1 and decrease with $\overline{c}$

Bank's choice of the deposit contract under deposit insurance

- When $\overline{c} < 1$, each bank sets c_1 now to maximize

$$\int_0^{\theta^*} u(1)\, d\theta + \int_{\theta^*}^1 [\lambda u(c_1) + (1-\lambda)(\theta u\left(\frac{1-\lambda c_1}{1-\lambda} R\right) + \\ + (1-\theta)\, u(\overline{c}))] d\theta + E\left[v\left(g, c_1^*, \overline{c}\right)\right]$$

where $\theta^* = \theta^*(c_1, \overline{c})$, and

$$E[v(g, c_1^*, \overline{c})] = \int_0^{\theta^*} v(g)\, d\theta + \\ + \int_{\theta^*}^1 [\theta v(g) + (1-\theta) v(g - (1-\lambda)\overline{c})]\, d\theta$$

- The deposit contract $c_1^{DI} > c_1^D$, with $\frac{dc_1^{DI}}{d\overline{c}} > 0$ solves

$$\lambda \int_{\theta^*}^1 \left[u'(c_1) - \theta R u'\left(\frac{1-\lambda c_1}{1-\lambda} R\right)\right] d\theta +$$

$$-\frac{\partial \theta^*}{\partial c_1}\left[\lambda u(c_1) + (1-\lambda)\left(\theta^* u\left(\frac{1-\lambda c_1}{1-\lambda} R\right) + (1-\theta^*)\, u(\overline{c})\right) - u(1)\right] = 0$$

Government choice under deposit insurance

- The government has the same objective as the bank but internalizes the costs of providing the guarantee while taking c_1^{DI} as given
- It can be shown that $0 < \overline{c} < 1$ if g is not too high
- In this case, government would like to choose a $c_1^G > c_1^{DI}$ as

$$\theta^* v(g) + (1 - \theta^*) v(g - (1 - \lambda)\overline{c}) < v(g)$$

  - Liquidating banks early (e.g., via prompt corrective actions) can be optimal as it allows to minimize the costs associated with public intervention
- Despite the inefficiency of the market solution, this scheme may lead to higher welfare than the decentralized solution

Conclusions

- Government guarantees present a complicated trade-off and understanding it requires endogenizing banks' choices and depositors' behavior in response to government intervention
- Increased demandable debt and fragility may be desirable as they reflect greater liquidity provision by banks
- While banks' choices may be distorted, in many cases more demandable debt is desirable
- Theoretical framework sheds new light on empirical results and policy discussions

Rollover Risk as Market Discipline: A Two-Sided Inefficiency

Eisenbach

Journal of Financial Economics, 2017

Short-term debt and market discipline

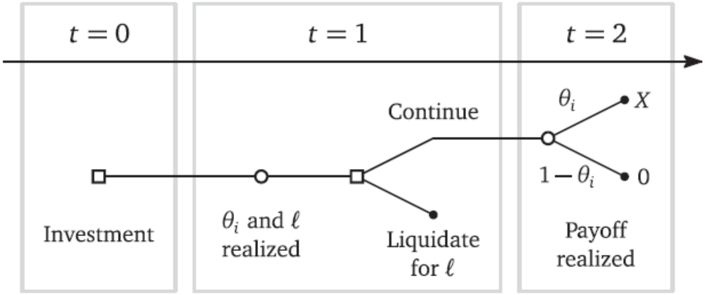
- Underlying theory (Calomiris and Kahn, 1991; Diamond and Rajan, 2001):
  - Leverage provides an incentive for bank equity holders and managers to conduct risk shifting and not liquidate bad projects
  - Demandable debt provides discipline and induces liquidation if creditors run upon receiving bad news
- Problems with market discipline:
  - Insufficient discipline in good times (e.g. Admati et al., 2010):
    - Increasing reliance on short-term funding and increasingly risky activities
  - Excessive discipline during crisis (e.g. Gorton and Metrick, 2012):
    - Large-scale withdrawal of short-term funding affecting issuers unrelated to housing

Modelling framework and key results

- Banks optimally choose debt maturity structure
  - Short term debt disciplines risk taking
- Rollover risk modeled as global game
  - Resolve multiplicity at interim stage
  - Probability of a run can be characterized for underlying parameters and banks' choices
- Embed in General equilibrium framework for amplification effects across banks
  - Excessive risk taking in good times
  - Excessive liquidation in bad times

Model

- Three periods $t = 0, 1, 2$, agents risk neutral, discount rate 0
- A continuum $[0, 1]$ of banks (i) and a continuum $[0, 1]$ of creditors (j) in every bank
- Every bank has a project:



Incentive problem

- Efficiency requires:

$$\text{Continue} \quad \Leftrightarrow \quad \theta_i X > \ell$$

- However, if bank is financed by a combination of debt and equity, risk shifting incentives emerge (Jensen and Meckling, 1976), since liquidation proceeds go mostly to creditors
  - Banker continues even if $\theta_i X < \ell$
- For simplicity, assume that bank is financed only with debt (focus on maturity choice)

Financing

- Investment at $t = 0$ funded by competitive creditors
- Each bank i has a continuum of creditors $j \in [0, 1]$
- Long-term debt:
  - Face value B_i matures at $t = 2$
- Short-term debt:
  - Face value R_i if withdrawn at $t = 1$
  - Face value R_i^2 if rolled over to $t = 2$
- Bank chooses maturity structure of debt:
  - Fraction of short-term debt α_i
  - Fraction of long-term debt $1 - \alpha_i$
- Face values B_i and R_i adjust so creditors break even

Uncertainty and information

- Idiosyncratic risk for bank i:

$$\theta_i \quad \text{drawn i.i.d. from} \quad F_s$$

- Aggregate risk state:

$$s \in \{H, L\} \quad \text{with} \quad \Pr[s = H] = p$$

- First-order stochastic dominance:

$$F_H(\theta) < F_L(\theta) \quad \text{for all} \quad \theta \in (0, 1)$$

- Information at $t = 1$:
  - Aggregate s: common knowledge
  - Idiosyncratic θ_i: creditor ji observes signal $x_{ji} = \theta_i + \sigma\varepsilon_{ji}$

Liquidation value

- Aggregate asset sales $\phi \in [0, 1]$ used in secondary sector
- Liquidation value = marginal product:

$$\ell(\phi) \quad \text{with} \quad \ell'(\phi) < 0$$

- In equilibrium:

$$\begin{aligned} \mathrm{E}_H[\theta X] &> \mathrm{E}_L[\theta X] \\ \Rightarrow \quad \phi_H &< \phi_L \\ \Rightarrow \quad \ell_H &> \ell_L \end{aligned}$$

Equilibrium

Combination of

1. Bayesian Nash equilibrium among creditors at $t = 1$
2. Competitive equilibrium among banks at $t = 0$

Creditor Coordination

- Fraction of creditors who withdraw: λ
  - Bank illiquid if $\alpha\lambda R > \ell$
- Creditor payoffs

	liquid	illiquid
roll over	θR^2	0
withdraw	R	ℓ

Complication:

- Liquidation value ℓ
  - enters payoff of all creditors at all banks
  - depends on coordination outcomes at all banks

$\longrightarrow$ All creditors at all banks are interacting

Creditor equilibrium

With symmetric banks, for $\sigma \longrightarrow 0$, the unique Bayesian Nash equilibrium is in switching strategies around a threshold $\widehat{\theta}$ given by

$$\widehat{\theta} = \frac{(1+\alpha)\, R - \ell}{R^2}$$

- For realizations $\theta_i > \hat{\theta}$:
  - All creditors *ji* roll over
  - Bank *i* is liquid and project continues
- For realizations $\theta_i < \hat{\theta}$:
  - All creditors *ji* withdraw
  - Bank *i* is illiquid and project is liquidated

Intuition

Creditor with signal $x = \hat{\theta}$ has to be indifferent:

$$\underbrace{\Pr[\text{liquid}] \times \hat{\theta} R^2}_{} = \underbrace{\Pr[\text{liquid}] \times R + \Pr[\text{illiquid}] \times \ell}_{}$$

For $\sigma \to 0$, distribution of $\lambda \mid \hat{\theta}$ becomes uniform

$$\Pr[\text{liquid}] = \Pr\left[\lambda \leq \frac{\ell}{\alpha R}\right] = \frac{\ell}{\alpha R}$$

Resulting in:

$$\frac{\ell}{\alpha R} \times \hat{\theta} R^2 = \frac{\ell}{\alpha R} \times R + \left[1 - \frac{\ell}{\alpha R}\right] \times \ell$$

$$\Rightarrow \hat{\theta} = \frac{(1+\alpha) R - \ell}{R^2}$$

Rollover risk

Ex ante rollover risk for bank i:

$$\Pr\left[\theta_i \leq \frac{(1+\alpha_i)\,R_i - \ell}{R_i^2}\right]$$

- Depends on maturity structure α_i
  - Directly $\rightarrow$ increasing
  - Indirectly through R_i
- Run is more likely for:
  1. Bad idiosyncratic news (low θ_i)
  2. Bad aggregate news (low ℓ)

No aggregate risk

- No aggregate risk, $F_H = F_L =: F$
  - $\longrightarrow$ liquidation value deterministic, $\ell_H = \ell_L =: \ell$
- Bank's problem:

$$\max_{\alpha} \int_{\hat{\theta}}^{1} \theta \left(X - \alpha R^2 - (1-\alpha) B \right) \, dF(\theta)$$

$$\text{subject to} \quad F(\hat{\theta})\, \ell + \int_{\hat{\theta}}^{1} \theta R^2 \, dF(\theta) = 1$$

$$F(\hat{\theta})\, \ell + \int_{\hat{\theta}}^{1} \theta B \, dF(\theta) = 1$$

$$\hat{\theta} = \frac{(1+\alpha)R - \ell}{R^2}$$

Above conditions implicitly define $\widehat{\theta}$ as a function of α with

$$\widehat{\theta}'(\alpha) > 0$$

Optimal maturity structure

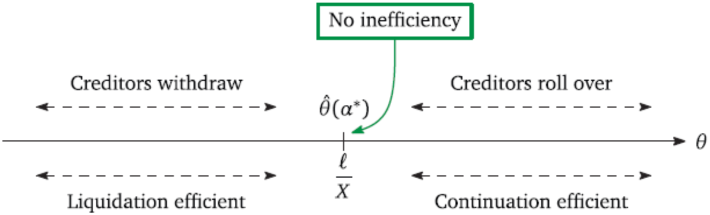
Without aggregate risk

- Bank problem becomes:

$$\max_{\alpha} F\left(\hat{\theta}(\alpha)\right)\ \ell + \int_{\hat{\theta}(\alpha)}^{1} \theta X \, dF(\theta) - 1$$

- Bank chooses efficient liquidation:

$$\hat{\theta}(\alpha^*) = \frac{\ell}{X}$$



With aggregate risk

- With aggregate risk, $F_H(\theta) < F_L(\theta)$ for all $\theta \in (0,1)$
  $\longrightarrow$ liquidation value uncertain, $\ell_H > \ell_L$
- Two opposing effects:

Efficiency: Want less liquidation in bad state

$$\frac{\ell_H}{X} > \frac{\ell_L}{X}$$

Rollover risk: Get more liquidation in bad state

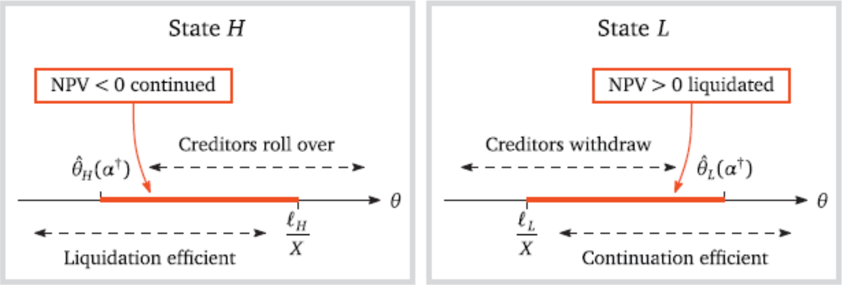
$$\frac{(1+\alpha)R - \ell_H}{R^2} < \frac{(1+\alpha)R - \ell_L}{R^2}$$

Optimal maturity structure

With aggregate risk

Bank trades off two inefficiencies:

$$\hat{\theta}_H(\alpha^*) < \frac{\ell_H}{X} \quad \text{and} \quad \hat{\theta}_L(\alpha^*) > \frac{\ell_L}{X}$$



General equilibrium

Without aggregate risk

- Conditions implicitly defining $\hat{\theta}(\alpha)$ both depend on ℓ
- Liquidation value ℓ depends on aggregate asset sales ϕ

$\longrightarrow$ Explicitly $\hat{\theta}(\alpha, \phi)$

- Competitive banks take ϕ as given
  - choose $\alpha^*(\phi)$
  - yielding $\hat{\theta}\left(\alpha^*(\phi), \phi\right)$
- Symmetry:
  mass of assets sold $=$ fraction of banks with $\theta_i < \hat{\theta}\left(\alpha^*(\phi), \phi\right)$

General equilibrium

Without aggregate risk

- Competitive equilibrium allocation:

$$\phi^{\text{CE}} = F\left(\hat{\theta}\left(\alpha^*(\phi^{\text{CE}}),\, \phi^{\text{CE}}\right)\right) \quad \text{with} \quad \hat{\theta}\left(\alpha^*(\phi),\, \phi\right) = \frac{\ell(\phi)}{X}$$

- First-best allocation:

$$\phi^{\text{FB}} = F\left(\frac{\ell(\phi^{\text{FB}})}{X}\right)$$

$\longrightarrow$ Without aggregate risk, CE achieves FB allocation

General equilibrium

With aggregate risk

- Competitive equilibrium allocation $\Phi = [\phi_H, \phi_L]$:

$$\Phi^{\mathrm{CE}} = \left[F_H\left(\hat{\theta}_H\left(\alpha^*(\Phi^{\mathrm{CE}}), \Phi^{\mathrm{CE}}\right)\right),\ F_L\left(\hat{\theta}_L\left(\alpha^*(\Phi^{\mathrm{CE}}), \Phi^{\mathrm{CE}}\right)\right)\right]$$

- First-best allocation:

$$\Phi^{\mathrm{FB}} = \left[F_H\left(\frac{\ell(\phi_H^{\mathrm{FB}})}{X}\right),\ F_L\left(\frac{\ell(\phi_L^{\mathrm{FB}})}{X}\right)\right]$$

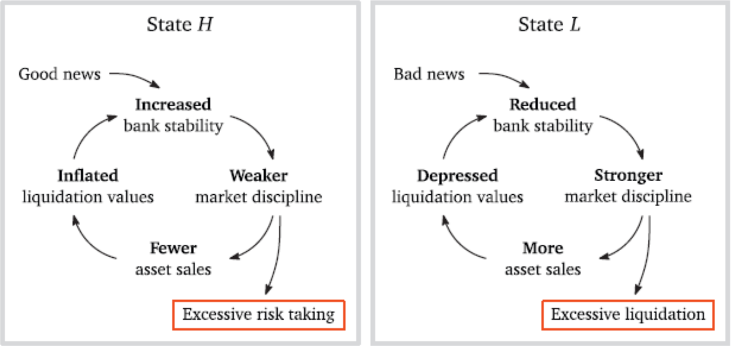
With $F_H(\theta) < F_L(\theta)$ and $F_s\left(\ell(\phi_s)/X\right)$ decreasing in ϕ_s

- Amplification:

$$\ell(\phi_H^{\mathrm{CE}}) > \ell(\phi_H^{\mathrm{FB}}) > \ell(\phi_L^{\mathrm{FB}}) > \ell(\phi_L^{\mathrm{CE}})$$

Feedback loops

With aggregate risk



Conclusions

- Individual bank stability depends on
    1. News about idiosyncratic return
    2. News about aggregate conditions
- Efficiency and market discipline diverge

$\longrightarrow$ Two-sided inefficiency, in bad and good times