



Behavioral Issues – Overconfidence

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Lecture Overview

● Psychology in Finance.

- Behavioral/cognitive biases: Overconfidence, attribution bias, optimism and wishful thinking, ambiguity aversion, recency bias, loss aversion, regret aversion, etc.
- Lead to biased financial decision-making: Trading, financial investments, diversification, real investment, corporate policies, etc.

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● Psychology and Overconfidence.

- DeBondt and Thaler (1995): “[P]erhaps the most robust finding in the psychology of judgment is that people are overconfident.”
- Fischhoff, Slovic and Lichtenstein (1977), Alpert and Raiffa (1982): People tend to **overestimate the precision** of their knowledge and information.
- Svenson (1981), Taylor and Brown (1988): Most individuals **overestimate their abilities**, see themselves as **better than average**, and see themselves better than others see them.

Lecture Overview (cont'd)

● Who's Impacted?

- Griffin and Tversky (1992): Experts tend to be more overconfident than relatively inexperienced individuals.
- Einhorn (1980): Overconfidence is more severe for tasks with delayed and vaguer feedback.
- Many fields: investment bankers (Staël von Holstein, 1972), engineers (Kidd, 1970), entrepreneurs (Cooper, Woo, and Dunkelberg, 1988), lawyers (Wagenaar and Keren, 1986), managers (Russo and Schoemaker, 1992).

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● Lecture Objectives.

- Development of a theoretical framework to model overconfidence.
- Study the impact of overconfidence in financial markets and firms.

Lecture Topics

1. Overconfidence in Financial Markets.

- Models of financial markets
- Modeling overconfidence.
- Effects of overconfidence in financial markets (trading volume, volatility, price patterns, price informativeness, etc.).

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- Attribution bias as the source of overconfidence in financial markets.
- Dynamic patterns of overconfidence.
- Survival/impact of overconfident traders.

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3. Overconfidence in Firms.

- Promotion tournaments as the source of overconfidence in firms.
- Impact on real investment decisions.

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4. Overconfidence and Contracting.

- Principal-agent model with overconfident agent.
- Compensation contracts for overconfident agents.
- Labor markets and welfare consequences of agent overconfidence.

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Section 1

Overconfidence in Financial Markets

Terrance Odean (1998)

“Volume, Volatility, Price, and Profit When All Traders Are Above Average”

Journal of Finance, 53(6), 1887–1934

Odean (*Journal of Finance*, 1998)

- **Objective:** Study the impact of overconfidence on financial markets.
- **Modeling Strategy.**
 - Three different models.
 - Diamond and Verrecchia (1981): Rational expectations with an infinite number of price-taking agents.
 - Kyle (1985): Strategic insider interacting with noise traders and a market-maker.
 - Grossman and Stiglitz (1980): Rational expectations model in which agents decide whether or not to acquire information.

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 - Overconfidence.
 - $\text{Signal} = \text{Truth} + \text{Noise}$.
 - Overconfident agents underestimate the variance of noise.

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 - $\text{Signal} = \text{Truth} + \text{Noise}$.
 - Overconfident agents underestimate the variance of noise.
- **Main Results.**
 - Consistent across models: $\text{OC} \uparrow \rightarrow \text{Trading Volume} \uparrow, \text{Market Depth} \uparrow, \text{Welfare} \downarrow$.
 - Effect of OC on Volatility and Price Informativeness depends on the model.

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 - Consistent across models: $\text{OC} \uparrow \rightarrow \text{Trading Volume} \uparrow, \text{Market Depth} \uparrow, \text{Welfare} \downarrow$.
 - Effect of OC on Volatility and Price Informativeness depends on the model.
- **This Lecture:** Hybrid of Diamond/Verrecchia (1981) and Grossman/Stiglitz (1980).

Model Setup

• Economy.

- One period: trading at time 0, payoffs at time 1.
- Two securities.
 - Risk-free security: $r_f = 0$, price normalized at 1, infinite supply.
 - Risky stock: End-of-period payoff of $\tilde{v} \sim N(0, h_v^{-1})$,
Price p at time 0 (to be endogenized),
Supply of $z > 0$.

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• Market Participants.

- Two traders, price-takers (or two types, each with a mass of 1).
- CARA utility over end-of-period wealth:

$$U_i(W) = -e^{-rW}, \quad i \in \{1, 2\}.$$

- Each endowed with
 - f_0 units of risk-free asset and
 - $x_0 \equiv \frac{z}{2}$ shares of risky stock.

Information and Overconfidence

• Information.

- Each trader $i \in \{1, 2\}$ gets a signal about $\tilde{v}$:

$$\tilde{y}_i = \tilde{v} + \tilde{\varepsilon}_i \quad \text{where } \tilde{\varepsilon}_i \sim N(0, h_i^{-1}).$$

- Assumptions: $\text{Cov}(\tilde{v}, \tilde{\varepsilon}_i) = 0$, $\text{Cov}(\tilde{\varepsilon}_1, \tilde{\varepsilon}_2) = 0$.
- h_i is signal **precision**: $h_i \uparrow \rightarrow \text{Var}(\tilde{y}_i) \downarrow \rightarrow \tilde{y}_i$ is “closer” to $\tilde{v}$.

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• Overconfidence.

- Trader i believes that the precision of his signal is

$$\hat{h}_i = (1 + \kappa_i)h_i, \quad \text{where } \kappa_i \geq 0 \text{ is his } \text{overconfidence}.$$

- Trader 1 correctly assesses h_2 , and vice versa (“agree to disagree”).
- Notation: Biased expectations and variances by trader i will be denoted $\hat{\mathbb{E}}_i$ and $\hat{\text{Var}}_i$.

Equilibrium Definition

• Trader Decisions.

- Trader i chooses portfolio:
 - f_i units of risk-free securities,
 - x_i shares of stock (i.e., buys $x_i - x_0$ shares).

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- Beginning-of-period wealth.
 - $W_0 = f_0 + x_0 p$ before trading.
 - $W_0 = f_i + x_i p$ after trading.
 - Budget constraint (BC): $f_i = f_0 - (x_i - x_0)p$.
- End-of-period wealth: $\tilde{W}_i = f_i + x_i \tilde{v} \stackrel{\text{(BC)}}{=} f_0 - (x_i - x_0)p + x_i \tilde{v}$.

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• Equilibrium: $\{\tilde{x}_1, \tilde{x}_2, \tilde{p}\}$ is an equilibrium if

- $\tilde{x}_i$ maximizes trader i 's (biased) expected utility conditional on all his information:

$$\tilde{x}_i = \arg \max_{x_i} \hat{\mathbb{E}}_i \left[U_i(\tilde{W}_i) \mid \tilde{y}_i, \tilde{p} \right]$$

- The market for shares of stock clears: $\tilde{x}_1 + \tilde{x}_2 = z$.

Equilibrium Derivation: Strategy

- **Conjecture.**

- Linear equilibrium.
 - Price is linear function of signals: $\tilde{p} = a_1 \tilde{y}_1 + a_2 \tilde{y}_2 + b$.
 - Demand is linear function of signal and price: $\tilde{x}_i = \alpha_i \tilde{y}_i + \beta_i \tilde{p} + \gamma_i$.
- Note: Will show existence and uniqueness of *linear* equilibrium.

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• Solution Technique.

- Linear functions of normal variables are normal.
- Max CARA utility with normal variables yields linear solutions.
- Goal: Find fixed point (i.e., solve for all the coefficients in above functions).

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• Maximization Problem.

$$\begin{aligned}
 \max_{x_i} \hat{\mathbb{E}}_i \left[U_i(\tilde{W}_i) \mid \tilde{y}_i, \tilde{p} \right] &= \hat{\mathbb{E}}_i \left(-e^{-r\tilde{W}_i} \mid \tilde{y}_i, \tilde{p} \right) \\
 &= \hat{\mathbb{E}}_i \left(-\exp \left\{ -r [f_0 - (x_i - x_0)\tilde{p} + x_i \tilde{v}] \right\} \mid \tilde{y}_i, \tilde{p} \right) \\
 &= -\exp \left\{ -r [f_0 - (x_i - x_0)\tilde{p}] \right\} \hat{\mathbb{E}}_i \left(e^{-rx_i \tilde{v}} \mid \tilde{y}_i, \tilde{p} \right)
 \end{aligned}$$

Statistics Result: Projection Theorem

- Suppose that

$$\begin{bmatrix} \tilde{X} \\ \tilde{Y} \end{bmatrix} \sim N \left(\begin{bmatrix} \mu_X \\ \mu_Y \end{bmatrix}, \begin{bmatrix} \Sigma_{XX} & \Sigma_{XY} \\ \Sigma_{YX} & \Sigma_{YY} \end{bmatrix} \right),$$

where $\tilde{X}$ is $M \times 1$, and $\tilde{Y}$ is $N \times 1$.

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where $\tilde{X}$ is $M \times 1$, and $\tilde{Y}$ is $N \times 1$.

- Then $\tilde{X}$ conditional on $\tilde{Y}$ is normally distributed with

$$\mathbb{E}[\tilde{X} \mid \tilde{Y}] = \mu_X + \Sigma_{XY} \Sigma_{YY}^{-1} (\tilde{Y} - \mu_Y), \quad \text{and}$$

$$\text{Var}[\tilde{X} \mid \tilde{Y}] = \Sigma_{XX} - \Sigma_{XY} \Sigma_{YY}^{-1} \Sigma_{YX}.$$

Equilibrium Derivation: Joint Distribution

- **Three Variables of Interest for Trader 1.** Under the linear conjecture,

$$\begin{bmatrix} \tilde{v} \\ \tilde{y}_1 \\ \tilde{p} \end{bmatrix} = \begin{bmatrix} \tilde{v} \\ \tilde{v} + \tilde{\varepsilon}_1 \\ a_1 \tilde{y}_1 + a_2 \tilde{y}_2 + b \end{bmatrix} = \begin{bmatrix} \tilde{v} \\ \tilde{v} + \tilde{\varepsilon}_1 \\ (a_1 + a_2) \tilde{v} + a_1 \tilde{\varepsilon}_1 + a_2 \tilde{\varepsilon}_2 + b \end{bmatrix}$$

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- **Joint Distribution.**

- Let $\tilde{Y}_1 \equiv [\tilde{y}_1 \quad \tilde{p}]^\top$.
- Joint distribution of $\tilde{v}$ and $\tilde{Y}_1$ from trader 1's perspective:

$$\begin{bmatrix} \tilde{v} \\ \tilde{Y}_1 \end{bmatrix} \sim N \left(\begin{bmatrix} 0 \\ \mu_1 \end{bmatrix}, \begin{bmatrix} \Sigma_{vv} & \Sigma_{v1} \\ \Sigma_{1v} & \hat{\Sigma}_{11} \end{bmatrix} \right)$$

where $\mu_1 = [0 \quad b]^\top$, $\Sigma_{vv} = h_v^{-1}$, $\Sigma_{1v} = \Sigma_{v1}^\top = [h_v^{-1} \quad (a_1 + a_2)h_v^{-1}]^\top$,

$$\text{and } \hat{\Sigma}_{11} = \begin{bmatrix} h_v^{-1} + \hat{h}_1^{-1} & (a_1 + a_2)h_v^{-1} + a_1 \hat{h}_1^{-1} \\ (a_1 + a_2)h_v^{-1} + a_1 \hat{h}_1^{-1} & (a_1 + a_2)^2 h_v^{-1} + a_1^2 \hat{h}_1^{-1} + a_2^2 h_2^{-1} \end{bmatrix}$$

Equilibrium Derivation: Traders' Posteriors

• Use Projection Theorem.

- Trader 1's posteriors: $\tilde{v}$ conditional on $\{\tilde{y}_1, \tilde{p}\}$ is normally distributed with

$$\hat{\mathbb{E}}_1[\tilde{v} \mid \tilde{y}_1, \tilde{p}] = \Sigma_{v1} \hat{\Sigma}_{11}^{-1} (\tilde{Y}_1 - \mu_1) = \left(\hat{h}_1 - \frac{a_1}{a_2} h_2 \right) \hat{\Omega}_1 \tilde{y}_1 + \frac{1}{a_2} h_2 \hat{\Omega}_1 (\tilde{p} - b)$$

$$\hat{\text{Var}}_1[\tilde{v} \mid \tilde{y}_1, \tilde{p}] = \Sigma_{vv} - \Sigma_{v1} \hat{\Sigma}_{11}^{-1} \Sigma_{1v} = \frac{1}{h_v + \hat{h}_1 + h_2} \equiv \hat{\Omega}_1.$$

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- Trader 2's posteriors: $\tilde{v}$ conditional on $\{\tilde{y}_2, \tilde{p}\}$ is normally distributed with

$$\hat{\mathbb{E}}_2[\tilde{v} \mid \tilde{y}_2, \tilde{p}] = \left(\hat{h}_2 - \frac{a_2}{a_1} h_1 \right) \hat{\Omega}_2 \tilde{y}_2 + \frac{1}{a_1} h_1 \hat{\Omega}_2 (\tilde{p} - b)$$

$$\hat{\text{Var}}_2[\tilde{v} \mid \tilde{y}_1, \tilde{p}] = \frac{1}{h_v + h_1 + \hat{h}_2} \equiv \hat{\Omega}_2.$$

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$$\hat{\text{Var}}_2[\tilde{v} \mid \tilde{y}_1, \tilde{p}] = \frac{1}{h_v + h_1 + \hat{h}_2} \equiv \hat{\Omega}_2.$$

• Observations.

- $\hat{\Omega}_1$ decreasing in $\hat{h}_1$.
- $\left(\hat{h}_1 - \frac{a_1}{a_2} h_2 \right) \hat{\Omega}_1$ increasing in $\hat{h}_1$, and $\frac{1}{a_2} h_2 \hat{\Omega}_1$ decreasing in $\hat{h}_1$.

Equilibrium Derivation: Traders' Demand

- **Another Statistics Result:** $\tilde{X} \sim N(0, \Sigma) \Rightarrow \mathbb{E}[e^{t\tilde{X}}] = e^{t\mu + \frac{1}{2}t^2\Sigma}$
- **In Trader i 's Problem.**
 - From page 14:

$$\begin{aligned}
 \max_{x_i} \hat{\mathbb{E}}_i \left[U_i(\tilde{W}_i) \mid \tilde{y}_i, \tilde{p} \right] \\
 &= -\exp \left\{ -r \left[f_0 - (x_i - x_0) \tilde{p} \right] \right\} \hat{\mathbb{E}}_i \left(e^{-rx_i \tilde{v}} \mid \tilde{y}_i, \tilde{p} \right) \\
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- This is equivalent to: $\max_{x_i} x_i \left[\hat{\mathbb{E}}_i(\tilde{v} \mid \tilde{y}_i, \tilde{p}) - p \right] - \frac{r}{2} x_i^2 \hat{\text{Var}}_i(\tilde{v} \mid \tilde{y}_i, \tilde{p}).$

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 - From page 14:

$$\begin{aligned} \max_{x_i} \hat{\mathbb{E}}_i \left[U_i(\tilde{W}_i) \mid \tilde{y}_i, \tilde{p} \right] \\ = -\exp \left\{ -r \left[f_0 - (x_i - x_0) \tilde{p} \right] \right\} \hat{\mathbb{E}}_i \left(e^{-rx_i \tilde{v}} \mid \tilde{y}_i, \tilde{p} \right) \\ = -\exp \left\{ -r \left[f_0 - (x_i - x_0) \tilde{p} + x_i \hat{\mathbb{E}}_i(\tilde{v} \mid \tilde{y}_i, \tilde{p}) - \frac{r}{2} x_i^2 \hat{\text{Var}}_i(\tilde{v} \mid \tilde{y}_i, \tilde{p}) \right] \right\} \end{aligned}$$

- This is equivalent to: $\max_{x_i} x_i \left[\hat{\mathbb{E}}_i(\tilde{v} \mid \tilde{y}_i, \tilde{p}) - p \right] - \frac{r}{2} x_i^2 \hat{\text{Var}}_i(\tilde{v} \mid \tilde{y}_i, \tilde{p}).$
- **Solution:**

$$\tilde{x}_i = \frac{\hat{\mathbb{E}}_i(\tilde{v} \mid \tilde{y}_i, \tilde{p}) - \tilde{p}}{r \hat{\text{Var}}_i(\tilde{v} \mid \tilde{y}_i, \tilde{p})}$$

Equilibrium Derivation: Market-Clearing

- **Demand = Supply:** $\tilde{x}_1 + \tilde{x}_2 = z$.

- Use demand functions from page 18:
$$\frac{\hat{\mathbb{E}}_1(\tilde{v} \mid \tilde{y}_1, \tilde{p}) - \tilde{p}}{r \hat{\text{Var}}_1(\tilde{v} \mid \tilde{y}_1, \tilde{p})} + \frac{\hat{\mathbb{E}}_2(\tilde{v} \mid \tilde{y}_2, \tilde{p}) - \tilde{p}}{r \hat{\text{Var}}_2(\tilde{v} \mid \tilde{y}_2, \tilde{p})} = z$$

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- Use trader posteriors from page 17 and solve for $\tilde{p}$:

$$\tilde{p} = \frac{\left(\hat{h}_1 - \frac{a_1}{a_2} h_2\right) \tilde{y}_1 + \left(\hat{h}_2 - \frac{a_2}{a_1} h_1\right) \tilde{y}_2 - \left(\frac{h_1}{a_1} + \frac{h_2}{a_2}\right) b - rz}{2h_v + \hat{h}_1 + \left(1 - \frac{1}{a_1}\right) h_1 + \hat{h}_2 + \left(1 - \frac{1}{a_2}\right) h_2}$$

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• Fixed Point.

- Recall conjecture: $\tilde{p} = a_1 \tilde{y}_1 + a_2 \tilde{y}_2 + b$.

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• Fixed Point.

- Recall conjecture: $\tilde{p} = a_1 \tilde{y}_1 + a_2 \tilde{y}_2 + b$.
- Match Coefficients (a_2 similar to a_1):

$$a_1 = \frac{\hat{h}_1 - \frac{a_1}{a_2} h_2}{2h_v + \hat{h}_1 + \left(1 - \frac{1}{a_1}\right) h_1 + \hat{h}_2 + \left(1 - \frac{1}{a_2}\right) h_2}$$

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$$b = \frac{-\left(\frac{h_1}{a_1} + \frac{h_2}{a_2}\right) b - rz}{2\hat{h}_v + \hat{h}_1 + \left(1 - \frac{1}{a_1}\right) \hat{h}_1 + \hat{h}_2 + \left(1 - \frac{1}{a_2}\right) \hat{h}_2}$$

Equilibrium: Price

• Solution to Fixed Point.

$$\tilde{p} = a_1 \tilde{y}_1 + a_2 \tilde{y}_2 + b \quad \text{where}$$

$$a_1 = \frac{\hat{h}_1 + h_1}{2h_v + \hat{h}_1 + h_1 + \hat{h}_2 + h_2} > 0$$

$$a_2 = \frac{\hat{h}_2 + h_2}{2h_v + \hat{h}_1 + h_1 + \hat{h}_2 + h_2} > 0$$

$$b = \frac{-rz}{2h_v + \hat{h}_1 + h_1 + \hat{h}_2 + h_2} < 0$$

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• Observations.

- Price depends more heavily on overconfident traders' info.
 - a_1 is increasing in $\hat{h}_1$ and decreasing in $\hat{h}_2$.
 - a_2 is decreasing in $\hat{h}_1$ and increasing in $\hat{h}_2$.

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 - a_1 is increasing in $\hat{h}_1$ and decreasing in $\hat{h}_2$.
 - a_2 is decreasing in $\hat{h}_1$ and increasing in $\hat{h}_2$.
- Risk premium.
 - Increasing in risk aversion and supply (i.e., $b \downarrow$ as $r \uparrow$ and $z \uparrow$).
 - Decreasing in overconfidence (i.e., $b \uparrow$ as $\hat{h}_1 \uparrow$ and $\hat{h}_2 \uparrow$).

Equilibrium: Demand

- **Equilibrium Demand of Trader 1.**

- Recall from page 18: $\tilde{x}_1 = \frac{\hat{\mathbb{E}}_1(\tilde{v} \mid \tilde{y}_1, \tilde{p}) - \tilde{p}}{r \hat{\text{Var}}_1(\tilde{v} \mid \tilde{y}_1, \tilde{p})}$

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- Use trader posteriors from page 17 and equilibrium values of a_1 , a_2 and b on page 20.

$$\tilde{x}_1 = \frac{(\hat{h}_1 - h_1)h_v + (\hat{h}_1\hat{h}_2 - h_1h_2)}{2h_v + \hat{h}_1 + h_1 + \hat{h}_2 + h_2} \tilde{y}_1 + \frac{-(\hat{h}_2 - h_2)h_v - (\hat{h}_1\hat{h}_2 - h_1h_2)}{2h_v + \hat{h}_1 + h_1 + \hat{h}_2 + h_2} \tilde{y}_2 + \frac{h_v + \hat{h}_1 + h_2}{2h_v + \hat{h}_1 + h_1 + \hat{h}_2 + h_2} z$$

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• Observations.

- Suppose that $\tilde{y}_1 = \tilde{y}_2 > 0$: Weight on $\tilde{y}_1$ is positive.
Weight on $\tilde{y}_2$ is negative.
- More generally: $\hat{h}_1 \uparrow \rightarrow$ Weight on $\tilde{y}_1 \uparrow$, Weight on $\tilde{y}_2 \downarrow$ (“agree to disagree”)
- Weight on z is greater than $\frac{1}{2}$ iff $\hat{h}_1 > \hat{h}_2$: Overconfidence makes traders more willing to absorb risky supply.

Trading Volume

• Symmetry.

- Assume that $h_1 = h_2 \equiv h_\varepsilon$. [same precision]
- Assume that $\kappa_1 = \kappa_2 \equiv \kappa \Rightarrow \hat{h}_1 = \hat{h}_2 = (1 + \kappa)h_\varepsilon$. [same overconfidence]
- Demand function from page 21 simplifies to

$$\tilde{x}_1 = \underbrace{\frac{\kappa h_v + [(1 + \kappa)^2 - 1] h_\varepsilon}{h_v + (2 + \kappa) h_\varepsilon}}_{\equiv A_\kappa \geq 0 \text{ (strict if } \kappa > 0)} \frac{h_\varepsilon}{2} \tilde{y}_1 - \underbrace{\frac{\kappa h_v + [(1 + \kappa)^2 - 1] h_\varepsilon}{h_v + (2 + \kappa) h_\varepsilon}}_{= A_\kappa} \frac{h_\varepsilon}{2} \tilde{y}_2 + \frac{z}{2}$$

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• Trading.

- In equilibrium, trader 1 buys $\tilde{x}_1 - x_0 = \tilde{x}_1 - \frac{z}{2} = A_\kappa(\tilde{y}_1 - \tilde{y}_2) = A_\kappa(\tilde{\varepsilon}_1 - \tilde{\varepsilon}_2)$ shares from trader 2.
- This random variable's (true) distribution is $N\left(0, \frac{2A_\kappa^2}{h_\varepsilon}\right)$.

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- This random variable's (true) distribution is $N\left(0, \frac{2A_\kappa^2}{h_\varepsilon}\right)$.
- Expected trading volume is $\mathbb{E}|A_\kappa(\tilde{\varepsilon}_1 - \tilde{\varepsilon}_2)| = \sqrt{\frac{2}{\pi}} \sqrt{\frac{2A_\kappa^2}{h_\varepsilon}} = \frac{2A_\kappa}{\sqrt{\pi h_\varepsilon}} \quad \uparrow \kappa, \downarrow h_\varepsilon$
 - Zero if $\kappa = 0$: Trading results purely from overconfidence and disagreement.
 - $\uparrow$ in κ and $\downarrow$ in h_ε : (Unjustified) Overconfidence leads to more trading.

Price Volatility

• Symmetry.

- Keep assumptions that $h_1 = h_2 \equiv h_\varepsilon$ and $\kappa_1 = \kappa_2 \equiv \kappa$.
- Equilibrium price from page 20 simplifies to

$$\tilde{p} = a\tilde{y}_1 + a\tilde{y}_2 + b \quad \text{where} \quad a = \frac{1}{2} \frac{(2 + \kappa)h_\varepsilon}{h_v + (2 + \kappa)h_\varepsilon} \quad \text{and} \quad b = \frac{1}{2} \frac{-rz}{h_v + (2 + \kappa)h_\varepsilon}$$

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• Volatility.

- The (squared) volatility of prices is

$$\begin{aligned} \text{Var}(\tilde{p}) &= \text{Var}[a\tilde{y}_1 + a\tilde{y}_2 + b] = \text{Var}[a(\tilde{v} + \tilde{\varepsilon}_1) + a(\tilde{v} + \tilde{\varepsilon}_2) + b] \\ &= \text{Var}(2a\tilde{v} + a\tilde{\varepsilon}_1 + a\tilde{\varepsilon}_2 + b) = 4a^2h_v^{-1} + 2a^2h_\varepsilon^{-1} \\ &= 2a^2(2h_v^{-1} + h_\varepsilon^{-1}), \end{aligned}$$

which is increasing in κ (since a is increasing in κ).

- Overconfident trader overreact to their information and “push prices too far.”

Price Informativeness

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• Measuring Price Informativeness.

- Odean (1998) argues that OC reduces price informativeness by using

$$\text{Var}(\tilde{p} - \tilde{v}) = \text{Var}[(2a - 1)\tilde{v} + a\tilde{\varepsilon}_1 + a\tilde{\varepsilon}_2 + b] = (2a - 1)^2 h_v^{-1} + 2a^2 h_\varepsilon^{-1},$$

which is increasing in κ (since a is increasing in κ).

- However, one should be careful with this conclusion as

$$\text{Corr}(\tilde{p}, \tilde{v}) = \frac{2ah_v^{-1}}{\sqrt{(4a^2 h_v^{-1} + 2a^2 h_\varepsilon^{-1})h_v^{-1}}} = \sqrt{\frac{2h_\varepsilon}{h_v + 2h_\varepsilon}}$$

does not depend on κ .

Return Correlation

• Sequence of Prices.

- Assume a trading round before traders know any information.
 - Since $\mathbb{E}(\tilde{v}) = 0$ and $\text{Var}(\tilde{v}) = h_v^{-1}$, we have $x_{10} = x_{20} = \frac{0 - p_0}{rh_v^{-1}}$.
 - Market-clearing: $x_{10} + x_{20} = z \Rightarrow p_0 = \frac{-rz}{2h_v}$

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- One could also argue that the stock's final price is $\tilde{p}_1 = \tilde{v}$.
- Thus, prices first go from p_0 to $\tilde{p}$, and then from $\tilde{p}$ to $\tilde{v}$, where

$$\tilde{p} = a\tilde{y}_1 + a\tilde{y}_2 + b \quad \text{where} \quad a = \frac{1}{2} \frac{(2 + \kappa)h_\varepsilon}{h_v + (2 + \kappa)h_\varepsilon} \quad \text{and} \quad b = \frac{1}{2} \frac{-rz}{h_v + (2 + \kappa)h_\varepsilon}$$

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• Measuring Return Correlation.

- Return correlation can be analyzed using

$$\begin{aligned} \text{Cov}(\tilde{p} - p_0, \tilde{v} - \tilde{p}) &= \text{Cov}[2a\tilde{v} + a\tilde{\varepsilon}_1 + a\tilde{\varepsilon}_2 + b, (1 - 2a)\tilde{v} - a\tilde{\varepsilon}_1 - a\tilde{\varepsilon}_2 - b] \\ &= 2a(1 - 2a)h_v^{-1} - 2a^2h_\varepsilon^{-1} = -\frac{\kappa(2 + \kappa)h_\varepsilon}{2[h_v + (2 + \kappa)h_\varepsilon]^2} \end{aligned}$$

- Negative iff $\kappa > 0$ and decreasing in κ : OC pushes prices too far, but prices eventually revert.

Conclusion

• Summary of Odean (1998).

- Models of financial markets with overconfident traders.
- Modeling overconfidence: Underestimation of noise variance in
Signal = Truth + Noise.
- Main results (with RE model): Overconfidence $\uparrow \rightarrow$ Trading Volume $\uparrow$
Volatility $\uparrow$
Price Informativeness $\downarrow$
 $\text{Cov}(\tilde{r}_{t-1}, \tilde{r}_t) < 0$

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Price Informativeness $\downarrow$
 $\text{Cov}(\tilde{r}_{t-1}, \tilde{r}_t) < 0$

• What Is Missing.

- Overconfidence is assumed $\rightarrow$ Where does it come from?
- Potential answer: it comes from learning (Gervais & Odean, 2001).

Related Papers

- **Theoretical.**

- Daniel, Hirshleifer, and Subrahmanyam (2001): Equilibrium asset-pricing model in which the overconfidence of traders about their information affects the covariance risk of risky securities and helps explain various anomalies.
- Scheinkman and Xiong (2003): Continuous-time model in which overconfidence generates disagreements among agents and leads them to treat the acquisition of a risky asset as an option to sell it to other more overconfidence agents later.

Related Papers

• Theoretical.

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• Empirical and Experimental.

- Barber and Odean (2000): The overconfidence of individual investors leads them to trade individual stocks despite the fact that their after-fee performance is poor.
- Barber and Odean (2001): Psychological research shows that men are more overconfident than women; this paper documents that men trade more than women and, because of the fees they incur, their performance is worse than that of women.
- Biais et al. (2005): Consistent with overconfidence, in experimental markets, investors overestimate the precision of their signals, are more subject to the winner's curse, and perform worse in trading.
- Glaser and Weber (2007): Using survey data, this paper shows that investors who think they are above average in terms of investment skills trade more.

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Section 2

The Emergence of Overconfidence

Simon Gervais, and Terrance Odean (2001)
 “Learning to Be Overconfident”
Review of Financial Studies, 14(1), 1–27

Gervais and Odean (*Review of Financial Studies*, 2001)

● Observations.

- People tend to overestimate the degree to which they are responsible for their own success. [e.g. Langer & Roth (1975), Miller & Ross (1975)]
 - Successful: “I’m good.”
 - Not successful: “I was unlucky.”
- As a result, people tend to **learn about their own abilities with a bias**, and they **become overconfident**.

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● Questions.

- How is overconfidence generated in financial markets?
- What effects does it have on these markets?
- What types of traders are most likely to be overconfident?
- Can trader overconfidence be sustained in the long run?

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- What effects does it have on these markets?
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- Can trader overconfidence be sustained in the long run?

● Modeling Strategy.

- Strategic trader (Kyle, 1985) who initially does not know his own skill.
- Learns through outcomes, *but* takes too much credit for success.
- Creates patterns in (over)confidence through trader’s career.

Preview of the Results

• Learning.

- Biased traders eventually learn their ability.
- On average, they:
 - are rational at the outset;
 - become overconfident when relatively young;
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- Success does not always increase overconfidence (even for a biased learner).
- Past success may not be a good predictor of future performance.

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• Impact of Success.

- Success does not always increase overconfidence (even for a biased learner).
- Past success may not be a good predictor of future performance.

• Overconfidence Patterns

- Overconfidence can persist.
- Learned overconfidence increases volatility and volume, but decreases expected profits.

Model Setup

- **Economy.**

- T periods (where T can be ∞).
- Two assets.
 - Risk-free security: $r_f = 0$, price normalized at 1, infinite supply.
 - Risky stock: dividend $\tilde{v}_t$ at the end of period t .

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• Market Participants.

- Informed trader (or *insider*).
- Liquidity trader.
- Market maker.

Insider

• Overview.

- Risk-neutral.
- Informed about $\tilde{v}_t$ at the beginning of period t .
- Chooses order $\tilde{x}_t$ to maximize expected period- t profits.

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- Drawn at the outset: $\tilde{a} = \begin{cases} H & \text{prob. } \phi_0 \\ L & \text{prob. } 1 - \phi_0 \end{cases}$
- Unknown by everybody (including insider) at the outset.

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• Information.

- Receives a signal $\tilde{\theta}_t = \tilde{\delta}_t \tilde{v}_t + (1 - \tilde{\delta}_t) \tilde{\varepsilon}_t$ at the beginning of every period t .
 - $\tilde{\varepsilon}_t$ has the same (continuous) distribution as $\tilde{v}_t$, but is independent from it.
 - High ability $\rightarrow$ useful signal likely: $\tilde{\delta}_t = \begin{cases} 1 & \text{prob. } \tilde{a} \\ 0 & \text{prob. } 1 - \tilde{a} \end{cases}$

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- **Advantage of this structure:** If $\tilde{v}_t = \tilde{\theta}_t$ at end of period, the insider knows that $\tilde{\delta}_t = 1$ (and $\tilde{\delta}_t = 0$ otherwise).

Insider's Bias

• Learning Process.

- Insider has the correct priors at the outset, and learns his ability through his performance.
- Notation: $\phi_t \equiv \Pr\{\tilde{a} = H \mid \mathcal{I}_t\}$, where $\mathcal{I}_t$ is info from first t periods.

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$$\phi_t = \Pr\left\{\tilde{a} = H \mid \mathcal{I}_{t-1}, \tilde{\theta}_t = \tilde{v}_t\right\} = \frac{\gamma H \phi_{t-1}}{\underbrace{\gamma H \phi_{t-1} + L(1 - \phi_{t-1})}_{\uparrow \gamma}} > \phi_{t-1}.$$

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- $\tilde{\theta}_t \neq \tilde{v}_t \rightarrow$ failure ($\tilde{\delta}_t = 0$) $\rightarrow$ decrease belief that $\tilde{a} = H$.

$$\phi_t = \Pr\left\{\tilde{a} = H \mid \mathcal{I}_{t-1}, \tilde{\theta}_t \neq \tilde{v}_t\right\} = \frac{(1 - H)\phi_{t-1}}{(1 - H)\phi_{t-1} + (1 - L)(1 - \phi_{t-1})} < \phi_{t-1}.$$

Ability Updating

• Information at t .

- Given information structure, the information available to the insider after t periods can be summarized by

$$\tilde{s}_t \equiv \sum_{\tau=1}^t \tilde{\delta}_{\tau}$$

- This is the **number of successes** by the insider **in the first t periods**.

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• Ability Update at t .

- Because of his learning bias, the insider's (biased) update about his ability is

$$\hat{\phi}_t(s) \equiv \hat{\Pr}\{\tilde{a} = H \mid \tilde{s}_t = s\} = \frac{(\gamma H)^s (1-H)^{t-s} \phi_0}{(\gamma H)^s (1-H)^{t-s} \phi_0 + L^s (1-L)^{t-s} (1-\phi_0)}$$

- Insider's updated expected ability:

$$\hat{\mu}_t(s) \equiv \hat{\mathbb{E}}[\tilde{a} \mid \tilde{s}_t = s] = H \hat{\phi}_t(s) + L[1 - \hat{\phi}_t(s)]$$

Other Market Participants

- **Liquidity Trader.**

- Trades for exogenous liquidity reasons.
- Order in period t : $\tilde{z}_t$.

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- Absorbs the net order flow $\tilde{\omega}_t = \tilde{x}_t + \tilde{z}_t$ at a price of

$$\tilde{p}_t = \mathbb{E}[\tilde{v}_t \mid \tilde{s}_{t-1}, \tilde{\omega}_t]$$

Sequence of Events

• At the Outset.

- The insider ability $\tilde{a}$ is drawn randomly.
- It is not observed by anyone.

• In Each Period.

1. $\tilde{v}_t$ is drawn but not observed by anybody.
2. Insider observes his information $\tilde{\theta}_t$.
3. Insider and liquidity trader simultaneously send their orders to MM.
4. Net order flow $\tilde{\omega}_t$ reaches the MM, who clears it at competitive price.
5. $\tilde{v}_t$ is announced and profits for the period are realized.
6. $\tilde{\theta}_t$ is announced, and both insider and MM update beliefs about insider's ability.

Equilibrium Derivation: Overview

• Assumptions.

- Probability distributions:

$$\begin{bmatrix} \tilde{v}_t \\ \tilde{\varepsilon}_t \\ \tilde{z}_t \end{bmatrix} \stackrel{\text{i.i.d.}}{\sim} N \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Sigma & 0 & 0 \\ 0 & \Sigma & 0 \\ 0 & 0 & \Omega \end{bmatrix} \right), \quad t = 1, 2, \dots$$

- $H \leq 2L$: Otherwise, the insider can become so biased that he may take positions that yield negative expected profits (problem with competitive MM).

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• Conjecture.

- Linear equilibrium.
 - Demand is linear function of signal: $\tilde{x}_t = X_t(\tilde{\theta}_t, \tilde{s}_{t-1}) = \beta_t(\tilde{s}_{t-1}) \tilde{\theta}_t$.
 - Price is linear function of order flow: $\tilde{p}_t = P_t(\tilde{\omega}_t, \tilde{s}_{t-1}) = \lambda_t(\tilde{s}_{t-1}) \tilde{\omega}_t$.
- Note: Will show existence and uniqueness of *linear* equilibrium.

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• Solution Technique.

- Linear functions of normal variables are normal.
- Max expected profits with normal variables yields linear solutions.
- Goal: Find fixed point (i.e., solve for all the coefficients in above functions).

Insider's Demand

Insider's Profits.

- Profit in period t : $\tilde{\pi}_t = x_t(\tilde{v}_t - \tilde{p}_t)$.
- Insert page 40's conjecture: $\tilde{\pi}_t = x_t[\tilde{v}_t - \lambda_t(\tilde{s}_{t-1})\tilde{\omega}_t] = x_t[\tilde{v}_t - \lambda_t(\tilde{s}_{t-1})(x_t + \tilde{z}_t)]$.

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$$\max_{x_t} \hat{\mathbb{E}}(\tilde{\pi}_t | \tilde{s}_{t-1}, \tilde{\theta}_t) = x_t \left[\hat{\mathbb{E}}(\tilde{v}_t | \tilde{s}_{t-1}, \tilde{\theta}_t) - \lambda_t(\tilde{s}_{t-1})x_t \right] \Rightarrow \tilde{x}_t = \frac{\hat{\mathbb{E}}(\tilde{v}_t | \tilde{s}_{t-1}, \tilde{\theta}_t)}{2\lambda_t(\tilde{s}_{t-1})}$$

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- Since $\hat{\Pr}\{\tilde{\delta}_t = 1 | \tilde{s}_{t-1}\} = \hat{\mathbb{E}}(\tilde{a} | \tilde{s}_{t-1}) = \hat{\mu}_t(\tilde{s}_{t-1})$, we have

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- Thus we have

$$\tilde{x}_t = \beta_t(\tilde{s}_{t-1}) \tilde{\theta}_t \quad \text{with} \quad \beta_t(s) = \frac{\hat{\mu}_t(s)}{2\lambda_t(s)} \quad (1)$$

Market Maker's Price

• Order Flow.

- Order flow in period t : $\tilde{\omega}_t = \tilde{x}_t + \tilde{z}_t$.
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• MM's Updating.

- Probability $\mu_t(\tilde{s}_{t-1})$ that $\tilde{\theta}_t = \tilde{v}_t$. In that event, $\tilde{\omega}_t = \beta_t(\tilde{s}_{t-1})\tilde{v}_t + \tilde{z}_t$.

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- Probability $1 - \mu_t(\tilde{s}_{t-1})$ that $\tilde{\theta}_t = \tilde{\varepsilon}_t$. In that event, $\tilde{\omega}_t = \beta_t(\tilde{s}_{t-1})\tilde{\varepsilon}_t + \tilde{z}_t$, which is uncorrelated with $\tilde{v} \rightarrow \mathbb{E}(\tilde{v}_t \mid \tilde{s}_{t-1}, \tilde{\theta}_t = \tilde{\varepsilon}_t, \tilde{\omega}_t) = 0$.

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• Price.

- Competitive MM: $\tilde{p}_t = \mathbb{E}(\tilde{v}_t \mid \tilde{s}_{t-1}, \tilde{\omega}_t) = \mu_t(\tilde{s}_{t-1}) \frac{\beta_t(\tilde{s}_{t-1})\Sigma}{\beta_t^2(\tilde{s}_{t-1})\Sigma + \Omega} \tilde{\omega}_t$
- Thus we have

$$\tilde{p}_t = \lambda_t(\tilde{s}_{t-1}) \tilde{\omega}_t \quad \text{with} \quad \lambda_t(s) = \mu_t(s) \frac{\beta_t(s)\Sigma}{\beta_t^2(s)\Sigma + \Omega} \quad (2)$$

Equilibrium

• Fixed Point.

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• Some Observations.

- $\beta_t(s)$ is $\uparrow$ in $\hat{\mu}_{t-1}(s)$: Insider is more aggressive when he thinks he is skilled.
- $\beta_t(s)$ is $\uparrow$ in Ω : Insider is more aggressive when he is “camouflaged” by noise.

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- $\lambda_t(s)$ is $\curvearrowright$ in $\hat{\mu}_{t-1}(s)$: MM does not fear insider when he is unskilled or when he has too high a view of himself.
- $\lambda_t(s)$ is $\uparrow$ in Σ : MM fears the insider when his info advantage is great.

Learning and Convergence

• Convergence of Beliefs (Proposition 2).

- If $\tilde{a} = H$, then $\hat{\phi}_t(\tilde{s}_t) \equiv \hat{\Pr}\{\tilde{a} = H \mid \tilde{s}_t\} \xrightarrow{\text{a.s.}} 1$ as $t \rightarrow \infty$.
- If $\tilde{a} = L$, then $\hat{\phi}_t(\tilde{s}_t) \equiv \hat{\Pr}\{\tilde{a} = H \mid \tilde{s}_t\} \xrightarrow{\text{a.s.}} \begin{cases} 0, & \text{if } \gamma < \gamma^* \\ \phi_0, & \text{if } \gamma = \gamma^* \\ 1, & \text{if } \gamma > \gamma^* \end{cases}$ as $t \rightarrow \infty$, where

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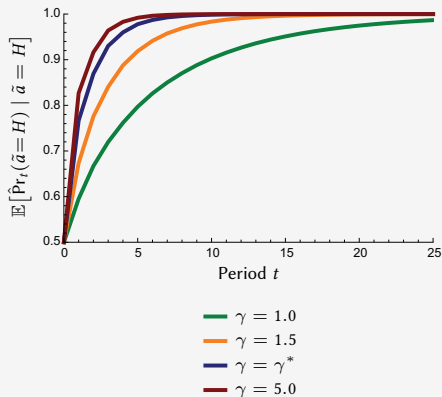
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• Intuition.

- When **skilled**, a biased insider reaches the conclusion that he is skilled **faster than a rational insider**.
- When **unskilled**,
 - an insider with a moderate bias reaches the conclusion that he is unskilled **slower than a rational insider**,
 - an insider with a (sufficiently) large bias **never learns** that he is unskilled.

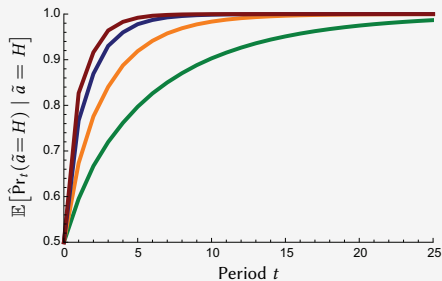
Learning and Convergence (cont'd)

Expected Beliefs of Skilled Insider



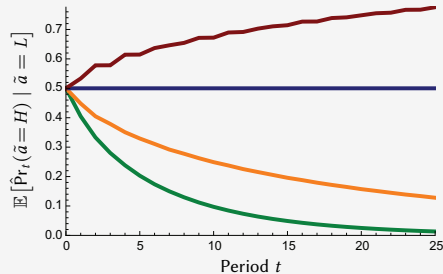
Learning and Convergence (cont'd)

Expected Beliefs of Skilled Insider



- $\gamma = 1.0$
- $\gamma = 1.5$
- $\gamma = \gamma^*$
- $\gamma = 5.0$

Expected Beliefs of Unskilled Insider



- $\gamma = 1.0$
- $\gamma = 1.5$
- $\gamma = \gamma^*$
- $\gamma = 5.0$

Overconfidence

● Insider's **Overconfidence** After t Periods:

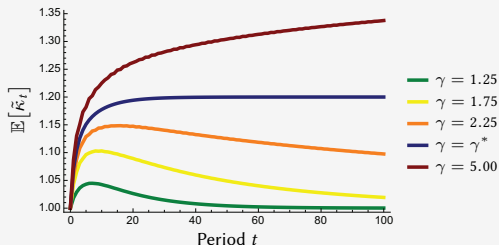
$$\tilde{\kappa}_t \equiv \frac{\text{insider's expected ability at } t}{\text{rational expected ability at } t} = \frac{\hat{\mathbb{E}}[\tilde{a} \mid \{\tilde{v}_\tau, \tilde{\theta}_\tau\}_{\tau=1, \dots, t}]}{\mathbb{E}[\tilde{a} \mid \{\tilde{v}_\tau, \tilde{\theta}_\tau\}_{\tau=1, \dots, t}]} \geq 1$$

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Expected Evolution of Overconfidence.

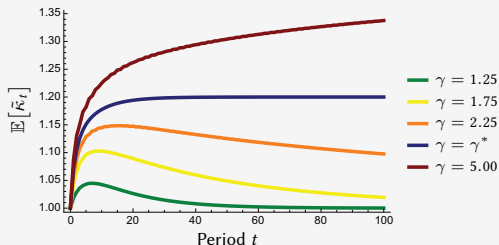


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Expected Evolution of Overconfidence.



Speed of Learning.

- Slow when the insider's bias is large (large γ).
- Slow when the insider's ability is hard to detect (small $H - L$).

Changes in Overconfidence

- **Effect of an Additional Success.**

1. A young insider's OC increases more than that of an older insider.
2. An unsuccessful insider's OC increases more than that of a successful insider.

- **Why?**

1. The statistics weigh heavier for an older trader.
2. The insider *truly is good*.

- **Note:** Additional success may make an old successful insider *less* overconfident.

Changes in Overconfidence

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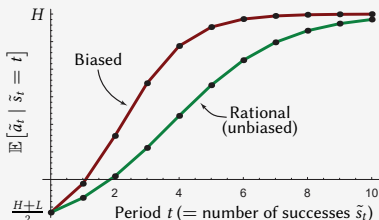
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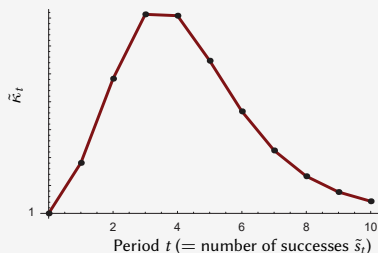
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Expected Skill With Success



Overconfidence With Success



Expected Profits

• Past Success May Not Predict Future Profits.

- On the one hand, **past success indicates greater probable ability**.
→ expected profits ↑.
- On the other hand, **past success can also indicate greater overconfidence**, and suboptimal future decision-making.
→ expected profits ↓.
- In some cases, the second effect more than offsets the first effect.

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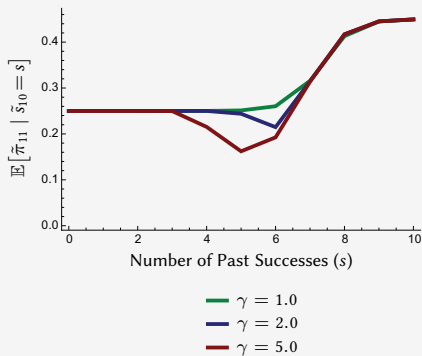
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• Implications for Money Managers.

- It may not be correct to choose a money manager based on his past record.
- Past record indicative of both ability and overconfidence.
- Young successful traders are prone to this overconfidence effect.

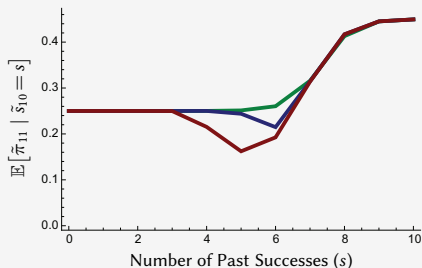
Learning and Convergence (cont'd)

Expected Future Profits Conditional on Past Success



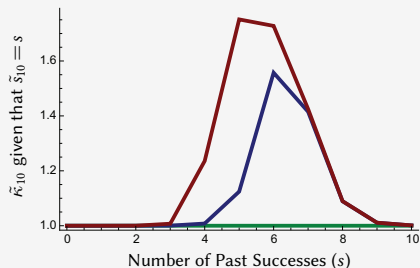
Learning and Convergence (cont'd)

Expected Future Profits Conditional on Past Success



— $\gamma = 1.0$
— $\gamma = 2.0$
— $\gamma = 5.0$

Overconfidence as a Function of Past Success



— $\gamma = 1.0$
— $\gamma = 2.0$
— $\gamma = 5.0$

Other Results

• Survival of Overconfidence.

- In this model, overconfident traders make suboptimal decisions in the future.
→ Natural selection: they will disappear. [Alchian (1950), Friedman (1953)]
- However, these traders have to have been successful in the past to become overconfident.
→ They are wealthy and thus have a non-negligible weight in the economy.
- In short, **overconfidence does not make traders wealthier, but the process of becoming wealthier can make traders overconfident.**

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● Volume and Volatility.

- Overconfidence increases both trading volume and price volatility because
 - informed traders rely more heavily on their information, so
 - they trade more aggressively with that information, and
 - such actions have bigger effects on prices. [Odean (1998)]
- The dynamic evolution of (endogenous) overconfidence in this model will imply similar dynamic evolutions of trading volume and price volatility.

Summary

● Main Ingredient.

- Attribution bias: People take too much credit for their success.
- Trader overconfidence results from biased learning about ability.
- Implement in a (repeated) Kyle (1985) model.

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- On average, traders are rational, then overconfident, and then rational again.
- Additional success $\nrightarrow$ greater overconfidence.

● Effects on Traders/Economy.

- Past success $\nrightarrow$ greater future profits.
- Overconfidence can be sustained in the long run, since overconfident traders are wealthy.
- Dynamic overconfidence patterns translate into dynamic patterns in trading volume and price volatility.

Related Papers

• Theoretical.

- Daniel, Hirshleifer, and Subrahmanyam (1998): Dynamic trading model in overconfidence evolves from a self-attribution bias and leads to negative autocorrelations, excess volatility and public-event-based return predictability.
- Bernardo and Welch (2001): The overconfidence of some entrepreneurs has a socially desirable aspect in that it helps disrupt informational cascades.

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• Empirical.

- Barber and Odean (2002): Consistent with bias in self-attribution, the trading activity of individual investors increases after they experience high returns.
- Statman et al. (2006): This paper tests the trading volume predictions of formal overconfidence models and finds that (market-wide and individual security) share turnover is positively related to lagged returns for several months.
- Griffin et al. (2007): Using data from 46 different financial markets, this paper shows that the finding that trading activity increases after good returns is a robust one.
- Billett and Qian (2008): This paper documents that CEOs are also prone to a self-attribution bias by showing that successful acquisitions tend to be followed by rapid value-destroying acquisitions.

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Section 3

Overconfidence in Firms

Anand M. Goel, and Anjan V. Thakor (2008)
“Overconfidence, CEO Selection, and Corporate Governance”
Journal of Finance, 63(6), 2737–2784

Goel and Thakor (*Journal of Finance*, 2008)

• Observations.

- Malmendier and Tate (2005, 2008): Overconfident CEOs make suboptimal investment/acquisition decisions.
- Ben-David et al. (2013), and Graham et al. (2013): Executives are miscalibrated, and this affects their corporate decisions.

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• Two Main Questions.

- If CEO overconfidence is (potentially) detrimental to firms, why is it the case that firms pick overconfident individuals to be CEOs?
- Given an overconfident CEO, how can/should the firm realign his incentives?

Overview of the Results

• Two Models.

- Model 1: CEOs are promoted through internal tournaments.
 - Overconfidence leads to more risk-taking and more extreme outcomes.
 - Because skill also leads to more extreme (good) outcomes, the use of tournaments to select CEOs produces skilled and overconfident CEOs.

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- Model 2: Contract with CEO who is risk-averse, skilled, and overconfident.
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• This Lecture.

- Slight adaptation of Goel and Thakor's Model 1.
 - Tournament with 3 managers and specific distributions.
 - Show ex post correlation between skill and overconfidence.
 - More intuitive.
- Model 2 is similar to Gervais et al. (2011) → Cover latter model in next section.

Model Setup

• **Model 1: Tournament.**

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- Project payoff: $\tilde{x}_i \sim F_x(x; R_i, \tilde{A}_i)$, where R_i is manager i choice of project risk.
 - SOSD effect of risk: $R'_i > R_i \Rightarrow F_x(x; R'_i, A_i) > F_x(x; R_i, A_i)$ for $x < 0$ and $F_x(x; R'_i, A_i) < F_x(x; R_i, A_i)$ for $x > 0$.
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• Payoffs.

- Each manager i receives $\tilde{x}_i$ (or, equivalently, a fixed fraction of $\tilde{x}_i$).
- Manager i^* , where $i^* \equiv \arg \max_i \tilde{x}_i$, gets $B > 0$ for promotion to CEO.

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- The model of sections I-III drops the impact of skills.
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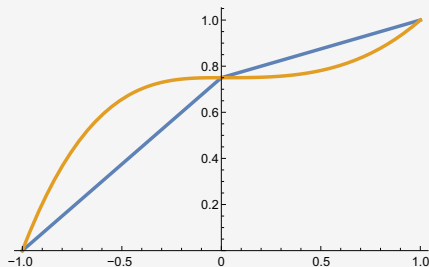
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• (Ir)rationality.

- Manager i understands that other managers are rational ($\tilde{\kappa}_j = 1$) or overconfident ($\tilde{\kappa}_j = C$) with equal probabilities.
- Doesn't correct for own irrationality.

Role of Risk and Skill

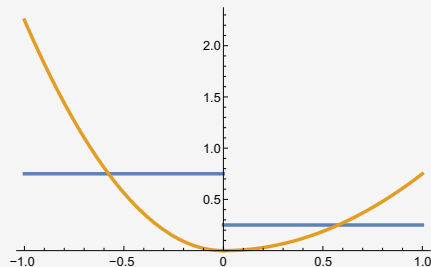
CDF of $\tilde{x}_i$ with $A_i = 0.25$



— $F_x(x; R_i = 1, A_i = 0.25)$

— $F_x(x; R_i = 3, A_i = 0.25)$

PDF of $\tilde{x}_i$ with $A_i = 0.25$

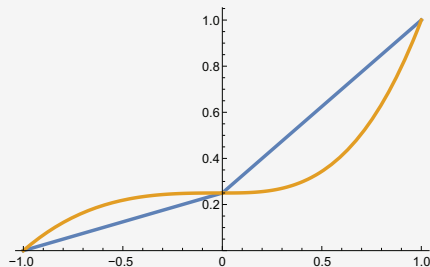


— $f_x(x; R_i = 1, A_i = 0.25)$

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Role of Risk and Skill

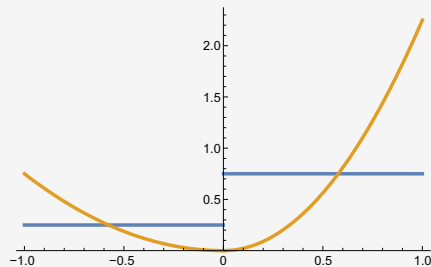
CDF of $\tilde{x}_i$ with $A_i = 0.75$



— $F_x(x; R_i = 1, A_i = 0.75)$

— $F_x(x; R_i = 3, A_i = 0.75)$

PDF of $\tilde{x}_i$ with $A_i = 0.75$



— $f_x(x; R_i = 1, A_i = 0.75)$

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Expected Payoffs

- **Payoff from the Project.**

- Irrelevant: $\hat{\mathbb{E}}_i[\tilde{x}_i] = \hat{\Pr}_i\{\tilde{A}_i = 1\}\hat{\mathbb{E}}_i[\tilde{z}_i] + \hat{\Pr}_i\{\tilde{A}_i = 0\}(-\hat{\mathbb{E}}_i[\tilde{z}_i]) = 0$
- Focus on tournament.

Expected Payoffs

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- Focus on tournament.

- **Payoff from the Tournament.**

- Assume that manager $j \in \{2, 3\}$ chose $R_j \geq 0$.
- Consider manager 1's choice of R_1 .
 - He knows that, when manager $j \in \{2, 3\}$ is overconfident, he mistakenly chooses CR_j (instead of the R_j that he meant to choose).
 - He chooses R_1 as if rational (but he too makes mistakes).

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 - He knows that, when manager $j \in \{2, 3\}$ is overconfident, he mistakenly chooses CR_j (instead of the R_j that he meant to choose).
 - He chooses R_1 as if rational (but he too makes mistakes).
- Calculate $\hat{\Pr}_1\{\text{Manager 1 wins tournament}\}$.
 - Look at all combinations of $\{\tilde{A}_1, \tilde{A}_2, \tilde{A}_3\}$ and $\{\tilde{\kappa}_2, \tilde{\kappa}_3\}$.
 - Equal probabilities under assumptions from page 59.

Probability of Winning the Tournament

$$\tilde{A}_1 = 1, \tilde{A}_2 = 1, \tilde{A}_3 = 1 : \int_0^1 \frac{1}{4} \sum_{\substack{\kappa_2 \in \{1, C\} \\ \kappa_3 \in \{1, C\}}} F_z(z; \kappa_2 R_2) F_z(z; \kappa_3 R_3) f_z(z; R_1) dz$$

$$\tilde{A}_1 = 1, \tilde{A}_2 = 1, \tilde{A}_3 = 0 : \int_0^1 \frac{1}{2} \sum_{\kappa_2 \in \{1, C\}} F_z(z; \kappa_2 R_2) f_z(z; R_1) dz$$

$$\tilde{A}_1 = 1, \tilde{A}_2 = 0, \tilde{A}_3 = 1 : \int_0^1 \frac{1}{2} \sum_{\kappa_3 \in \{1, C\}} F_z(z; \kappa_3 R_3) f_z(z; R_1) dz$$

$$\tilde{A}_1 = 0, \tilde{A}_2 = 1, \tilde{A}_3 = 1 : 0$$

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$$\tilde{A}_1 = 0, \tilde{A}_2 = 0, \tilde{A}_3 = 0 : \int_0^1 \frac{1}{4} \sum_{\substack{\kappa_2 \in \{1, C\} \\ \kappa_3 \in \{1, C\}}} [1 - F_z(z; \kappa_2 R_2)] [1 - F_z(z; \kappa_3 R_3)] f_z(z; R_1) dz$$

Probability of Winning the Tournament (cont'd)

• Probability that Manager 1 Wins:

$$\begin{aligned}
 & \hat{Pr}_1\{\tilde{x}_1 > \tilde{x}_2, \tilde{x}_1 > \tilde{x}_3\} \\
 &= \frac{1}{8} \left[\frac{1}{4} \sum_{\substack{\kappa_2 \in \{1, C\} \\ \kappa_3 \in \{1, C\}}} \frac{R_1}{R_1 + \kappa_2 R_2 + \kappa_3 R_3} + \frac{1}{2} \sum_{\kappa_2 \in \{1, C\}} \frac{R_1}{R_1 + \kappa_2 R_2} + \frac{1}{2} \sum_{\kappa_3 \in \{1, C\}} \frac{R_1}{R_1 + \kappa_3 R_3} \right. \\
 &\quad \left. + 1 + \frac{1}{4} \sum_{\substack{\kappa_2 \in \{1, C\} \\ \kappa_3 \in \{1, C\}}} \left(1 - \frac{R_1}{R_1 + \kappa_2 R_2} - \frac{R_1}{R_1 + \kappa_3 R_3} + \frac{R_1}{R_1 + \kappa_2 R_2 + \kappa_3 R_3} \right) \right] \\
 &= \frac{1}{4} \left(1 + \frac{1}{4} \sum_{\substack{\kappa_2 \in \{1, C\} \\ \kappa_3 \in \{1, C\}}} \frac{R_1}{R_1 + \kappa_2 R_2 + \kappa_3 R_3} \right)
 \end{aligned}$$

Probability of Winning the Tournament (cont'd)

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 \end{aligned}$$

• Manager 1's Problem:

$$\max_{R_1} B \cdot \hat{Pr}_1\{\tilde{x}_1 > \tilde{x}_2, \tilde{x}_1 > \tilde{x}_3\} - k R_1 \Rightarrow \text{FOC: } \frac{B}{16} \sum_{\substack{\kappa_2 \in \{1, C\} \\ \kappa_3 \in \{1, C\}}} \frac{\kappa_2 R_2 + \kappa_3 R_3}{(R_1 + \kappa_2 R_2 + \kappa_3 R_3)^2} = k$$

FOC – Some Intuition

- **FOC from page 63:**
$$\frac{B}{16} \sum_{\substack{\kappa_2 \in \{1, C\} \\ \kappa_3 \in \{1, C\}}} \frac{\kappa_2 R_2 + \kappa_3 R_3}{(R_1 + \kappa_2 R_2 + \kappa_3 R_3)^2} = k$$

- **Some Observations.**

- R_1 increasing in B : Future pay (as CEO) incentivizes risk-taking.
- R_1 decreasing in k : Take less risk as risk aversion $\uparrow$.

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• **Some Observations.**

- R_1 increasing in B : Future pay (as CEO) incentivizes risk-taking.
- R_1 decreasing in k : Take less risk as risk aversion $\uparrow$.
- The fact that $\frac{\kappa_2 R_2 + \kappa_3 R_3}{(R_1 + \kappa_2 R_2 + \kappa_3 R_3)^2} \uparrow$ in R_j and κ_j for $j \in \{2, 3\}$ implies that
 - R_1 increases in R_j : Increase risk if others take a lot of risk.
 - R_1 increases in C : Increase risk when overconfidence leads (other) managers to underestimate the risk they take.

Equilibrium

• Equilibrium Risk.

- Managers are identical ex ante (before any information about $\tilde{A}_i$ and $\tilde{\kappa}_i$ becomes public through $\tilde{x}_i$'s).
- This implies that, in equilibrium, $R_1 = R_2 = R_3 \equiv R$.
- In FOC from page 63:

$$\frac{B}{16} \sum_{\substack{\kappa_2 \in \{1, C\} \\ \kappa_3 \in \{1, C\}}} \frac{\kappa_2 R + \kappa_3 R}{(R + \kappa_2 R + \kappa_3 R)^2} = k \quad \Rightarrow \quad R = \frac{B}{16k} \sum_{\substack{\kappa_2 \in \{1, C\} \\ \kappa_3 \in \{1, C\}}} \frac{\kappa_2 + \kappa_3}{(1 + \kappa_2 + \kappa_3)^2}$$

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• Impact of Overconfidence.

- R increasing in C .
- Overconfidence commits every manager to take on more risk.
- No impact on equilibrium probability of winning:

$$\text{Identical Managers} \quad \Rightarrow \quad \Pr\{\text{Manager } i \text{ Wins}\} = \frac{1}{3}.$$

Results About Skills

• Role of Tournament.

- Goel & Thakor (2008) argue that firms use tournaments to select CEOs amongst their teams of managers, but never show why (in Propositions 1-3).
- Answer: The expected skill of tournament winner is greater than that of other managers.

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• Skill of Winner.

- We are interested in

$$\Pr\{\tilde{A}_1 = 1 \mid \tilde{x}_1 > \tilde{x}_2, \tilde{x}_1 > \tilde{x}_3\} = \frac{\Pr\{\tilde{A}_1 = 1, \tilde{x}_1 > \tilde{x}_2, \tilde{x}_1 > \tilde{x}_3\}}{\Pr\{\tilde{x}_1 > \tilde{x}_2, \tilde{x}_1 > \tilde{x}_3\}}$$

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- Solution:

$$\Pr\{\tilde{A}_i = 1 \mid \text{Manager } i \text{ finishes } 1^{\text{st}}\} = \frac{7}{8}$$

$$\Pr\{\tilde{A}_i = 1 \mid \text{Manager } i \text{ finishes } 2^{\text{nd}}\} = \frac{1}{2} = \Pr\{\tilde{A}_i = 1\}$$

$$\Pr\{\tilde{A}_i = 1 \mid \text{Manager } i \text{ finishes } 3^{\text{rd}}\} = \frac{1}{8}$$

- Intuition.

- Skilled mngrs draw from $[0, 1]$, whereas unskilled mngrs draw from $[-1, 0]$.
- In fact, $\frac{7}{8}$ is the probability that the winner draws from $[0, 1]$.

Results About Overconfidence

• Role of Tournament.

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- Solution:

$$\Pr\{\tilde{\kappa}_i = C \mid \text{Manager } i \text{ finishes 1}^{\text{st}}\} = \frac{1}{2} + \frac{3}{16} \frac{C^2 - 1}{(C + 2)(2C + 1)} \quad \uparrow C$$

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- **Intuition:** $C \uparrow \rightarrow \text{Risk} \uparrow \rightarrow \Pr\{\text{win}\} \uparrow$.

$C \uparrow \rightarrow \text{Prob } \underline{\text{and}} \text{ extent of winner OC} \uparrow$.

Investment Decisions – Model

• Model 2.

- One period (i.e., second period, now that CEO is likely overconfident).
- Principal-Agent model of firm investment with overconfident CEO.
- Principal = Board: Chooses agent's compensation to maximize firm value.

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• Project.

- The quality of the project is $\tilde{p} \sim U[0, 1]$.
- End-of-period payoff $\tilde{x}_0$.
 - If undertakes and effort: $\tilde{x}_0 = \begin{cases} h & \text{prob. } \tilde{p} \\ \ell & \text{prob. } 1 - \tilde{p} \end{cases}$
 - If undertakes and no effort: $\tilde{x}_0 = \ell$ (never optimal).
 - If drops: $\tilde{x}_0 = r \in (\ell, h)$.

Investment Decisions – Model (cont'd)

- **Agent = CEO.**

- Risk-averse.
- From tournament: Skilled (A) and Overconfident (thinks that A is AC , with $C \geq 1$)
- Agent receives a private signal about $\tilde{p}$:

$$\tilde{s} = \tilde{\delta}\tilde{p} + (1 - \tilde{\delta})\tilde{\varepsilon}, \text{ where } \tilde{\varepsilon} \stackrel{\text{indep.}}{\sim} \tilde{p} \text{ and } \tilde{\delta} = \begin{cases} 1 & \text{prob. } A \\ 0 & \text{prob. } 1 - A \end{cases}$$

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- Choices.
 - Undertake the project (risk) or not (no risk).
 - If project undertaken, exert effort (cost c) or not (no cost).

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• Equilibrium.

- CEO undertakes project and exerts effort if $\tilde{s} > \hat{s}$.
- Compensation contract effectively pins down $\hat{s}$.

Investment Decisions – Results

- **Some Overconfidence $\uparrow$ Firm Value.**

- Risk-averse $\rightarrow$ conservative (drops some positive-NPV projects, i.e., $\hat{s} > \hat{s}_{FB}$).
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• Some Remarks.

- Overconfident agents are worse off (too much risk).
- Firm may benefit, depending on extent of overconfidence.

Some Remaining Questions

• Overconfidence Detrimental to Firm Value.

- Why pick the winner of tournament as CEO?
 - May be optimal to sacrifice (expected) skill in order to reduce (expected) overconfidence.
 - General model: $\mathbb{E}[\tilde{A}_i \mid n^{\text{th}} \text{ of } N]$ and $\mathbb{E}[\tilde{\kappa}_i \mid n^{\text{th}} \text{ of } N]$ are $\downarrow$ in $n \rightarrow$ pick optimal rank n (but may change incentives...).

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- Why not also use information in project outcomes $(\tilde{x}_1, \dots, \tilde{x}_N)$?
 - If $\tilde{x}_1$ is way larger than $\tilde{x}_2, \dots, \tilde{x}_{N-1}$, and $\tilde{x}_N$, then manager 1 may be very overconfident (he took way too much risk).

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• Overconfidence Detrimental to Agent Welfare.

- Can agents learn?
- Labor market: Firms compete for labor, so winner of tournament may be stolen by competing firm [see Gervais et al. (2011)].

Related Papers

● Theoretical.

- Stein (1996): Capital budgeting by a rational manager when outside investors are irrationally optimistic about the prospects for the firm's real assets.
- Gervais and Goldstein (2007): Positive (commitment) role of overconfidence when a firm's agents work in teams that potentially benefit from synergies across teammates.

Related Papers

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● Empirical.

- Malmendier and Tate (2005): Proxy the overconfidence of CEOs by the number of deep-in-the money options that they hold, and show that this measure predicts investment distortions (e.g., overinvestment) by their firm.
- Ben-David et al. (2013): Use a survey of CFOs to document that firm executives are miscalibrated regarding future market prospects, and that miscalibrated executives follow more aggressive corporate policies.
- Graham et al. (2013): Use psychometric tests on senior executives to document that CEOs are significantly more optimistic and risk-tolerant than the lay population, and show that CEOs' behavioral traits are related to corporate financial policies.

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Section 4

Overconfidence and Contracting

Simon Gervais, J. B. Heaton, and Terrance Odean (2011)
 “Overconfidence, Compensation Contracts, and Capital Budgeting”
Journal of Finance, 66(5), 1735–1777

Gervais, Heaton, and Odean (*Journal of Finance*, 2011)

● Observations.

- Psychology: Individuals tend to be overconfident, i.e., they believe their knowledge is more precise than it actually is.
 - Own skill bias: Langer & Roth (1975), Taylor & Brown (1988).
 - Precision bias: Fischhoff et al. (1977), and Alpert & Raiffa (1982).
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Gervais, Heaton, and Odean (*Journal of Finance*, 2011)

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● Question: Does this have any effect on the economy?

- Capital markets: yes (volume, volatility) or no (market efficiency).
- Firms (or internal capital markets):
 - Irrationality more likely to have permanent effects, since these effects are more difficult to arbitrage.
 - Firms seem content with overconfident CEOs. [Malmendier & Tate, 2005; Sautner & Weber, 2009; Ben-David, Graham & Harvey, 2013]
- Labor markets:
 - Biased workers may be attractive assets.
 - Bidding for their services has unclear effects on welfare.

Background and Overview

● Agency Problems.

- Risk-taking. [Treynor & Black, 1976]
 - Stockholders hold a diversified portfolio of firms, but firm managers do not hold diversified portfolios of employers.
 - As a result, stockholders are less concerned about the risk of a new project than the manager.
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● Implications.

- Risk-taking and effort are naturally facilitated.
- Compensation is cheaper, but can be flatter or steeper.
- Firms and/or agents better off (i.e., more efficient).

Model Setup

• The Firm.

- One period.
- An all-equity firm starts with \$1 in cash.

• Investment Opportunity.

- A risky project becomes available at the start of the period.
 - The project costs \$1 to undertake.
 - Its end-of-period payoff is $\tilde{v} = \begin{cases} \sigma, & \text{prob. } \phi \\ 0, & \text{prob. } 1 - \phi. \end{cases}$
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• Assumptions.

- Discount rate is zero.
- NPV of risky project $= \sigma\phi - 1 < 0$.
 - Competition in product market eliminates large $\sigma\phi$.
 - Larger number/measure of negative-NPV projects.
 - Need managerial skill and risk-taking to undertake a project.
- Thus, by default, the firm drops the project, and the firm is worth \$1 at the outset.

Manager's Information and Overconfidence

- **Information.** The potential value from the risky project comes from the possibility of a skilled manager acquiring information about it.
 - Signal: $\tilde{s} = \tilde{\delta}\tilde{v} + (1 - \tilde{\delta})\tilde{\eta}$, where $\tilde{\eta} \overset{\text{indep.}}{\sim} \tilde{v}$ and $\tilde{\delta} = \begin{cases} 1 & \text{prob. } a \\ 0 & \text{prob. } 1 - a \end{cases}$
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- **Overconfidence.**
 - Manager thinks that his ability is $a + b \geq a$, where $b \in [0, \frac{1}{2}]$.
 - Biased updating:

$$\hat{\phi}_U \equiv \hat{\Pr}\{\tilde{v} = \sigma \mid \tilde{s} = \sigma\} = \phi + (a + b)(1 - \phi) > \phi,$$

$$\hat{\phi}_D \equiv \hat{\Pr}\{\tilde{v} = \sigma \mid \tilde{s} = 0\} = (1 - a - b)\phi < \phi.$$
 - $\uparrow b \rightarrow \hat{\phi}_U \uparrow$ and $\hat{\phi}_D \downarrow$ (too much weight on information).

Manager's Risk Aversion and Compensation

- **Compensation Contract:** $\{0, \delta_M, \delta_M + \delta_H\}$ for $\{0, 1, \sigma\}$.
 - Zero in low state: Firm's limited liability.

Investment policy affects compensation risk.
 - Interpretation of contract $\{\delta_M, \delta_H\}$.
 - Flat wage δ_M : paid as long as the firm operates (and not fired).
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- **Manager's Utility:** risk aversion $r \in [0, 1)$.

State	Outcome	Compensation	Utility
Low	Failed Project	0	0
Medium	No Project	δ_M	δ_M
High	Successful Project	$\delta_M + \delta_H$	$\delta_M + (1 - r)\delta_H$

First-Best

- **Unbiased Updating:**

$$\Pr\{\tilde{v} = \sigma \mid \tilde{s} = \sigma\} = \phi + a(1 - \phi) \equiv \phi_u > \phi,$$

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- **Value Maximization (First-Best):** Undertake project iff expected CF exceeds initial investment of \$1.
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- When $a > a^{\text{FB}}$:
 - $F^{\text{FB}} = 1 + \phi(\sigma\phi_u - 1) = 1 - \phi + \sigma\phi[\phi + a(1 - \phi)]$.
 - Increasing in a .

The Manager's Investment Decisions

- **Good Signal ($\tilde{s} = \sigma$).**

- Undertake: $\mathbb{E}[\tilde{u} \mid \tilde{s} = \sigma] = \hat{\phi}_U [\delta_M + (1 - r)\delta_H] + (1 - \hat{\phi}_U)(0)$.
- Drop: δ_M .
- Undertake iff $\frac{\delta_H}{\delta_M} \geq \frac{1 - \hat{\phi}_U}{\hat{\phi}_U(1 - r)} \equiv \hat{\kappa}_U$.

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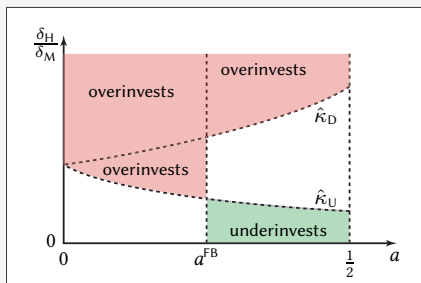
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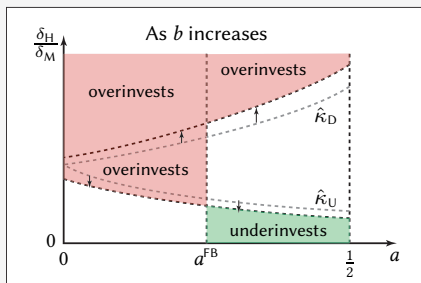
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The Firm's Problem

- From now on, assume that $a > a^{\text{FB}}$.
- **Incentive compatibility:** invest with $\tilde{s} = \sigma$, drop with $\tilde{s} = 0$.

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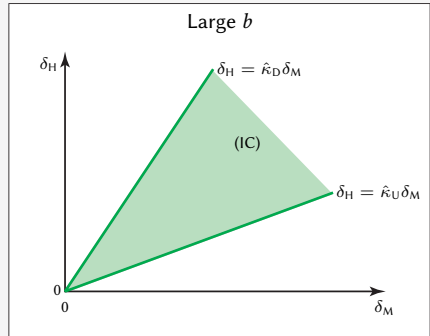
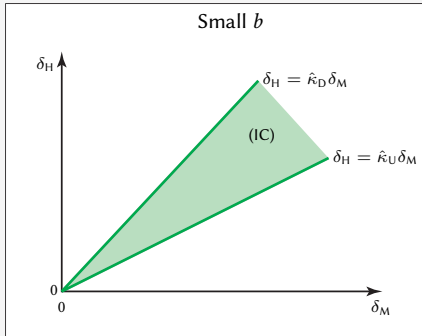
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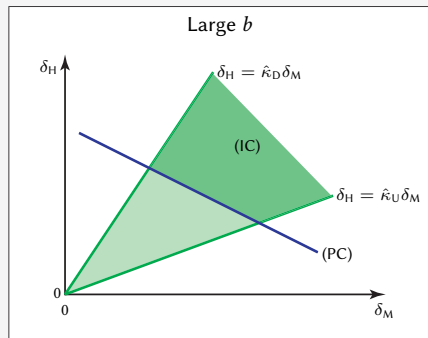
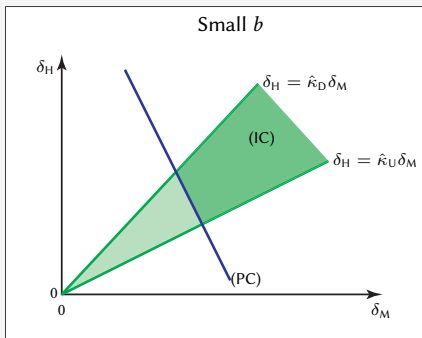
- Firm's maximization problem:**

$$\max_{\{\delta_M, \delta_H\}} \mathbb{E}[\tilde{\pi}] = \mathbb{E}[\tilde{\rho} - \tilde{w}] \quad \text{subject to (IC) and (PC).}$$

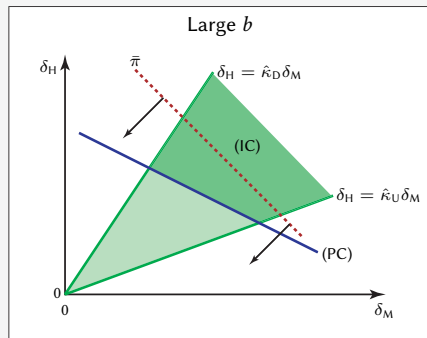
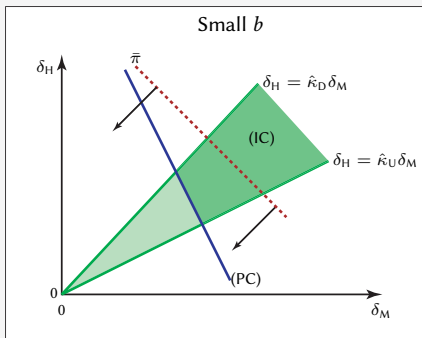
The Firm's Problem (cont'd)



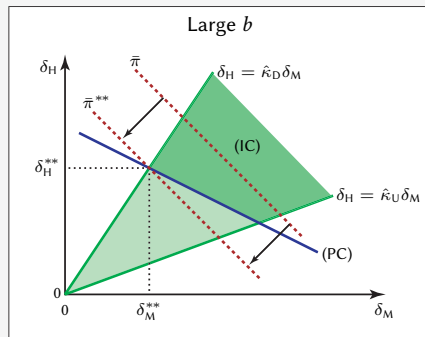
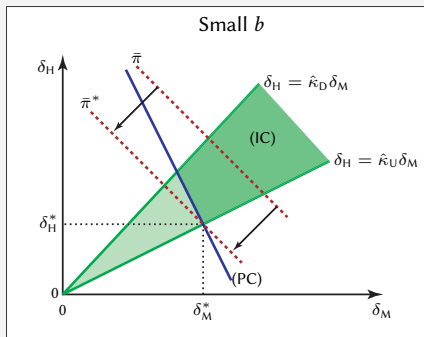
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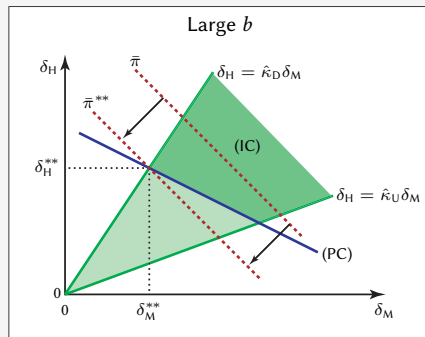
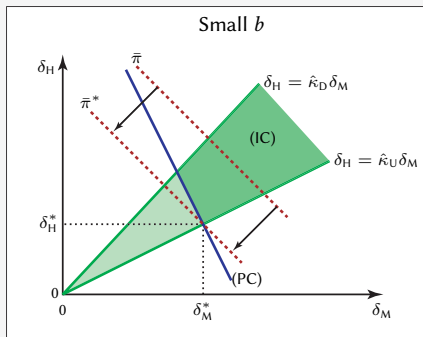
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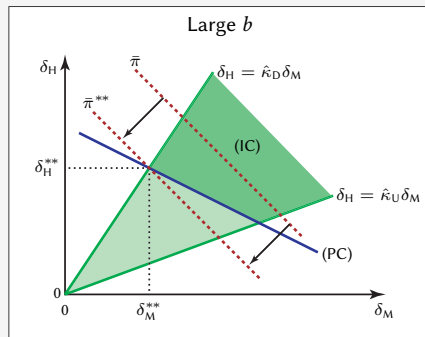
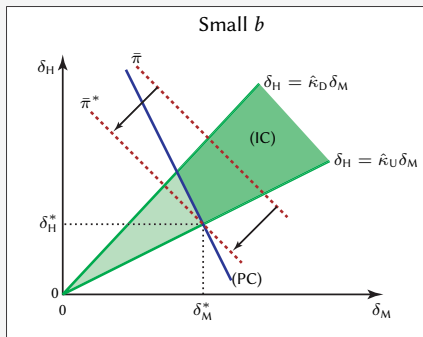


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- Small b : $\delta_M^* = \bar{u}$, $\delta_H^* = \frac{(1-\hat{\phi}_U)\bar{u}}{\hat{\phi}_U(1-r)}$.
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 - Less convexity as b increases.
- Large b : $\delta_M^{**} = \frac{\phi_D \bar{u}}{\phi} < \delta_M^*$, $\delta_H^{**} = \frac{(1-\phi_D)\bar{u}}{\phi(1-r)} > \delta_H^*$.
 - Manager thinks he can make high state likely $\rightarrow \delta_H$ cheaper for firm.
 - More convexity as b increases.

Remarks about Optimal Contract

• Some Predictions.

- Gain in firm value from overconfidence larger for small a (incentives are expensive in that case).
 - When the impact of the manager on outcome is limited.
 - When the link between decision and outcome is noisy.
- Gain in firm value from overconfidence larger for small ϕ .
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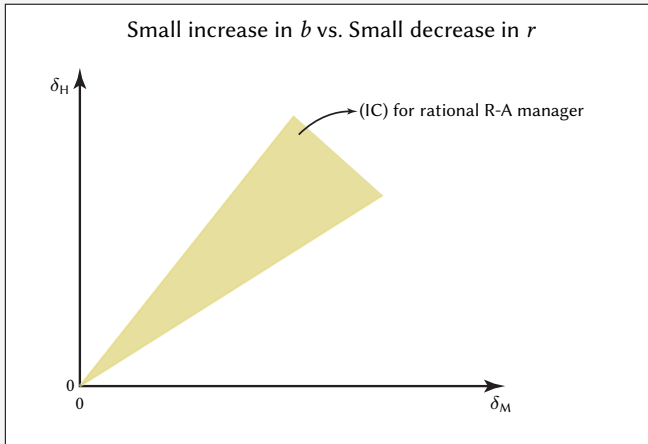
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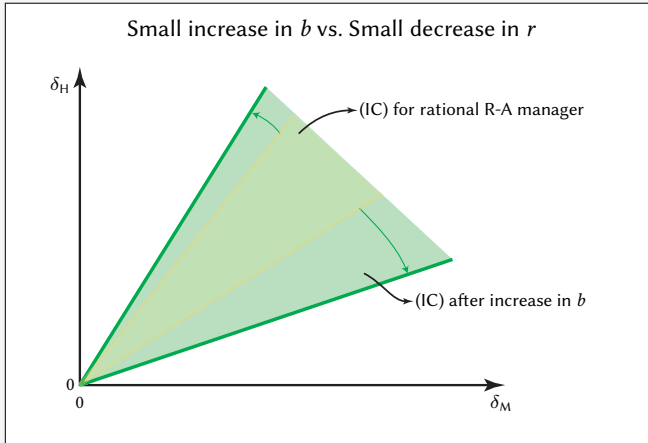
• Risk Aversion and Overconfidence: There is a b^* such that iso-utility and iso-profit curves are parallel.

- Ex ante, such an overconfident, risk-averse manager values state-contingent claims like a rational, risk-neutral manager/firm.
- However, his (IC) set is strictly larger (commitment value).
 - Clearly better for the firm.
 - More performance-based compensation than a rational, risk-loving manager for $b > b^*$.

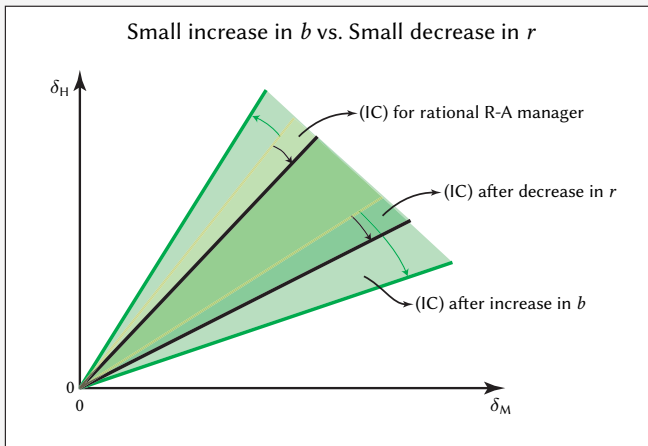
Overconfidence vs. Risk Aversion



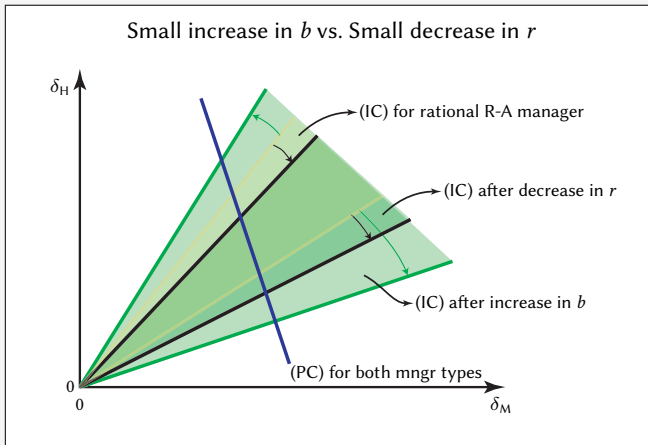
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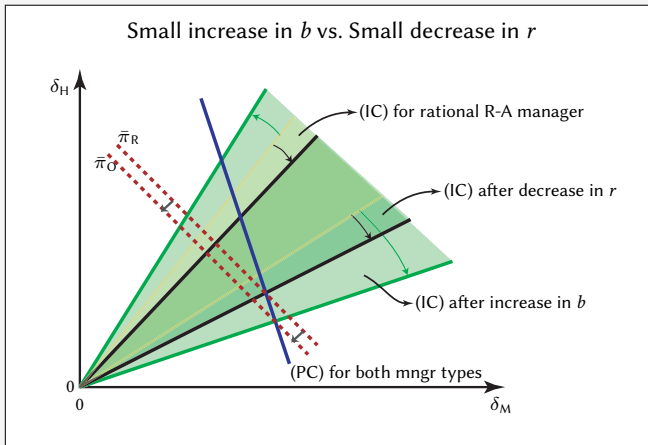
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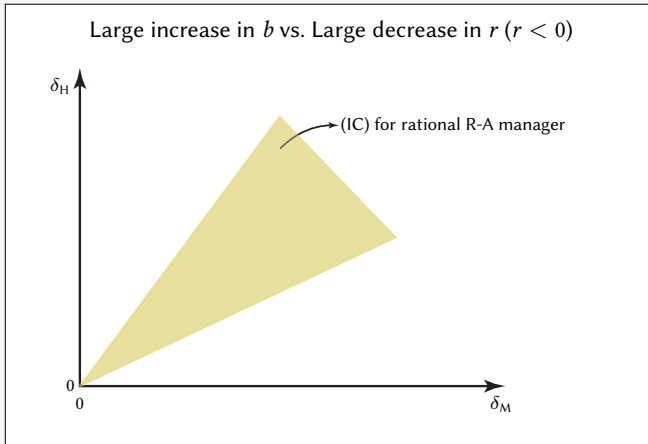
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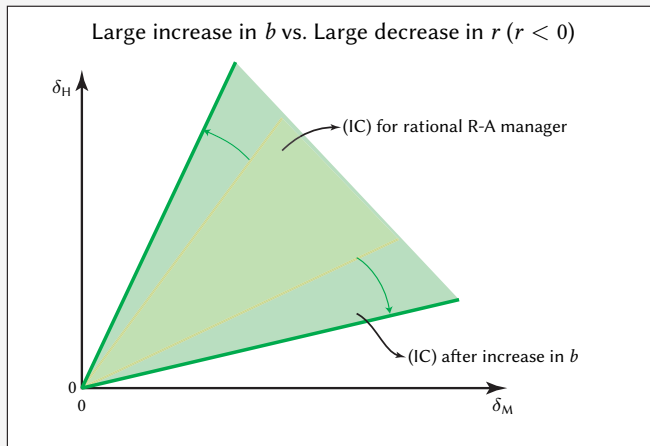
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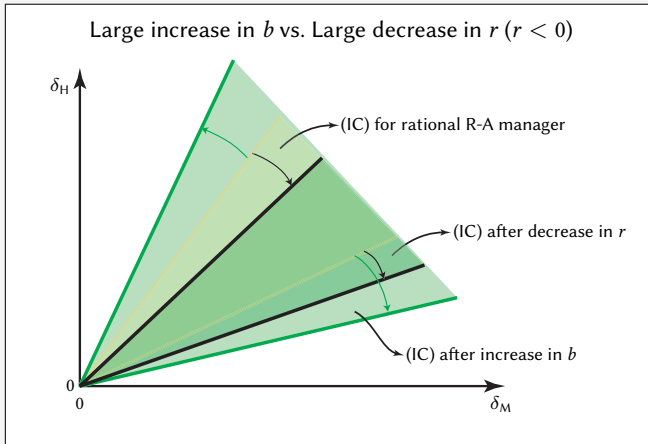
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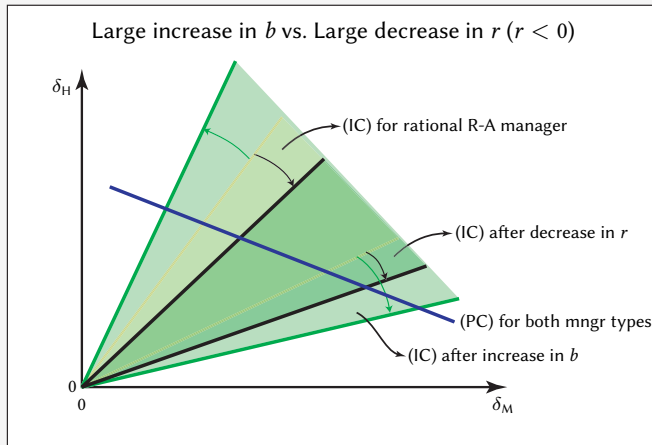
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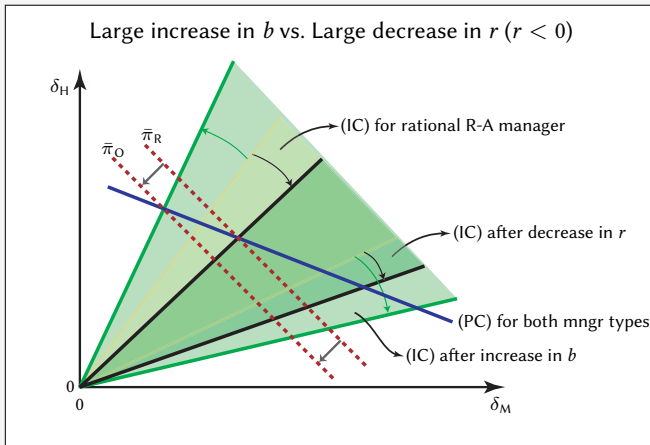
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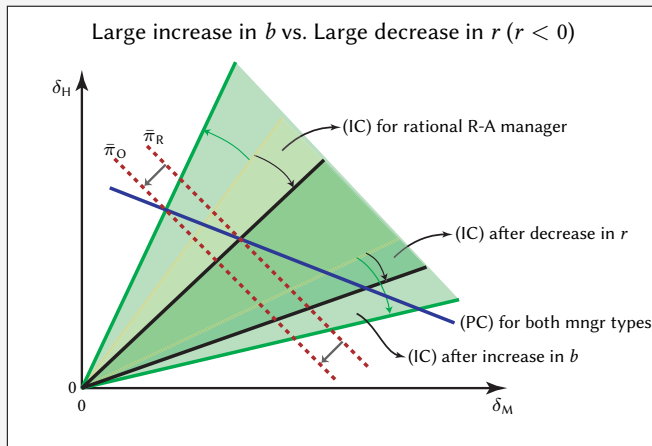
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Overconfidence vs. Risk Aversion



- Increasing b and decreasing r have a similar effect ex ante, but not conditional on information.
- Larger (IC) set $\rightarrow$ firm value $\uparrow$ by **commitment value of overconfidence**.

Problem with Traditional Setup

- **Effect of Overconfidence:** (IC) and (PC) are both easier/cheaper to satisfy when the manager is overconfident.
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 - Competition for skilled manager.
 - $\bar{u}$ is what the manager *thinks* he can get from a competing firm.

Labor Market Competition

- **Setup.**

- Suppose that two identical firms compete to hire one manager.
- Manager picks the higher expected utility.
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• In Equilibrium.

- No firm will offer a contract that is not (IC).
 - Otherwise, the manager is a deadweight loss.
- No firm makes a profit: $F_1 = F_2 = 1$.
 - Otherwise, the other could offer a slightly better compensation and steal the agent.

Labor Market Competition

• Setup.

- Suppose that two identical firms compete to hire one manager.
- Manager picks the higher expected utility.
- Managerial skill is a scarce resource $\rightarrow a = 0$ for firm without manager.

• In Equilibrium.

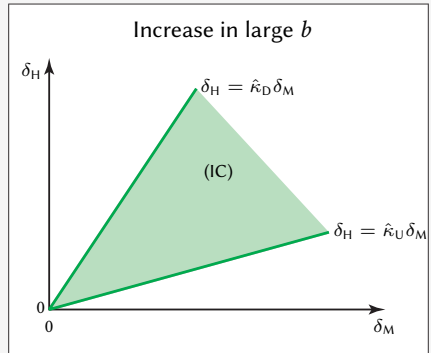
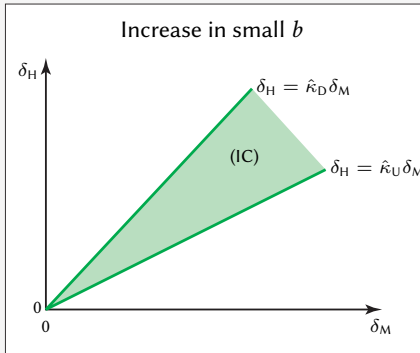
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• Maximization Problem:

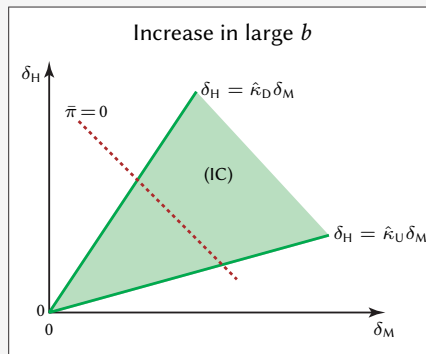
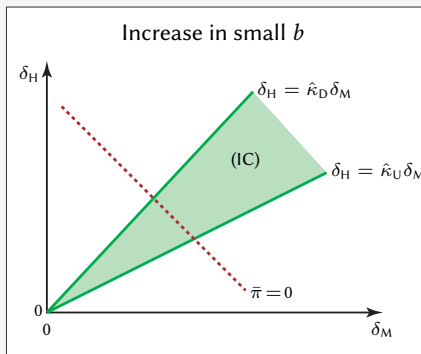
$$\max_{\{\delta_M, \delta_H\}} \hat{\mathbb{E}}[\tilde{u}] \quad \text{subject to (IC) and } \mathbb{E}[\tilde{\pi}] = 0.$$

- Biased expected utility ($\hat{\mathbb{E}}[\tilde{u}]$, not $\mathbb{E}[\tilde{u}]$).
- $\bar{u}$ = expected utility with other firm (credible bargaining).

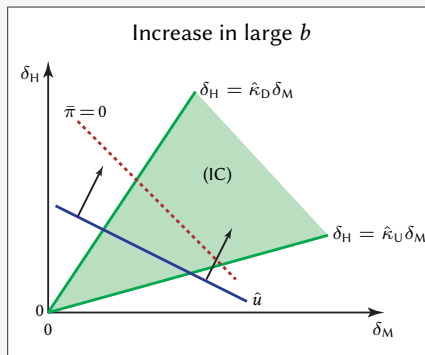
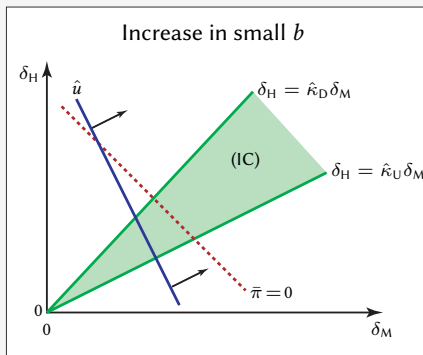
Labor Market Competition (cont'd)



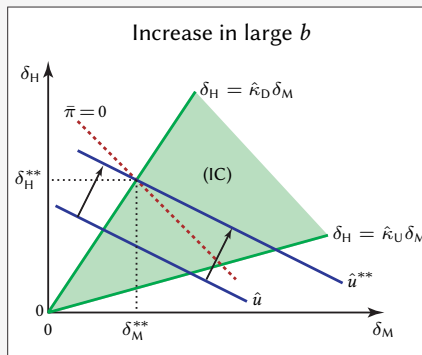
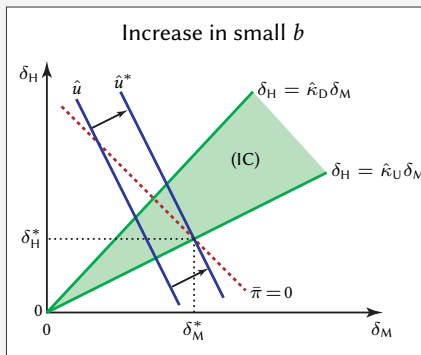
Labor Market Competition (cont'd)



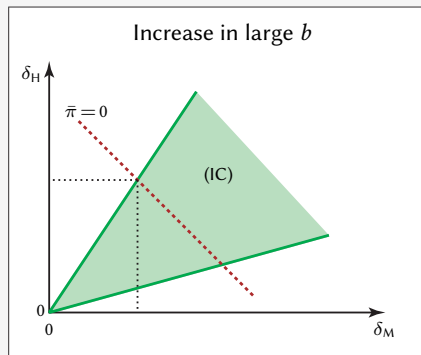
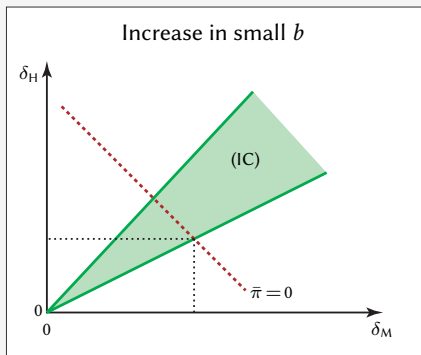
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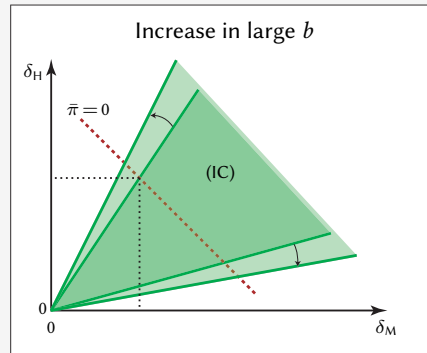
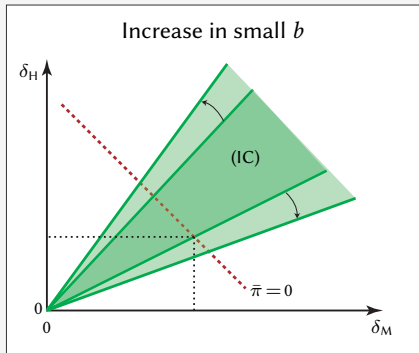
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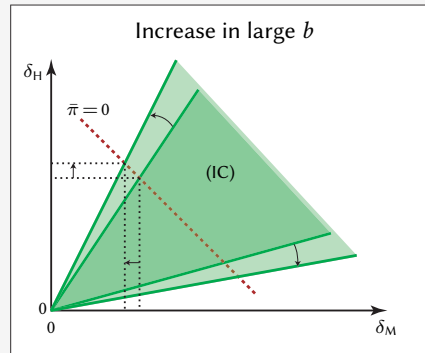
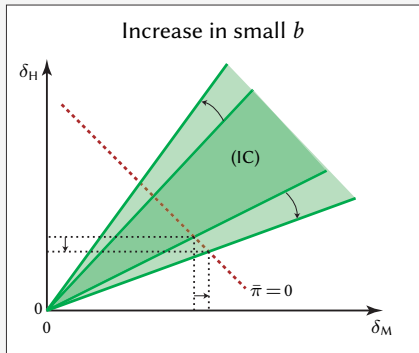
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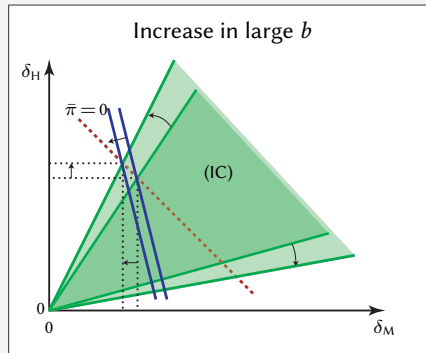
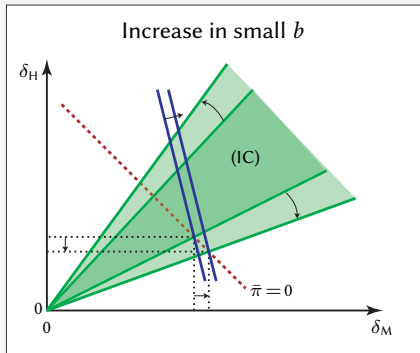
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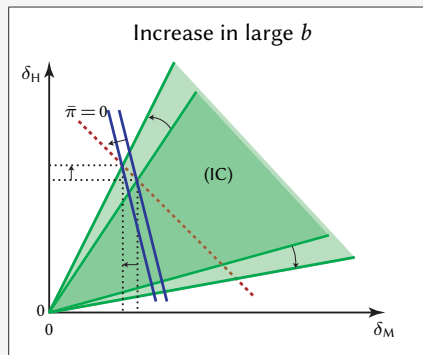
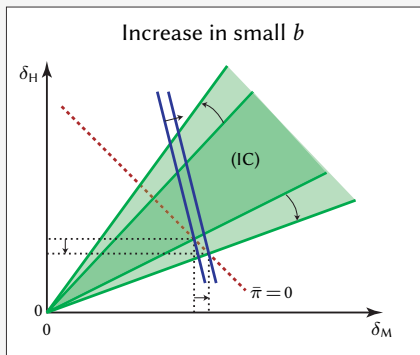
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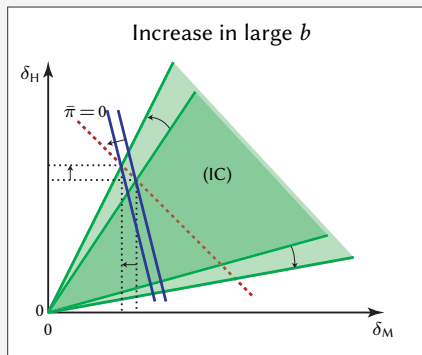
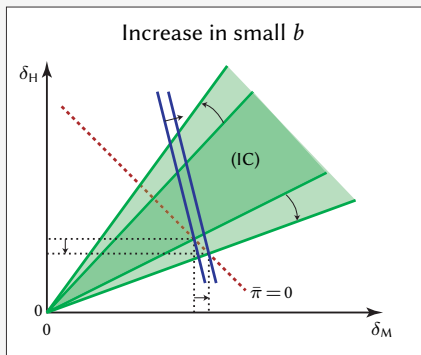
Labor Market Competition (cont'd)



• Small b .

- Increase in b : $\delta_M \uparrow$, $\delta_H \downarrow$, $\mathbb{E}[\tilde{u}] \uparrow$.
- Larger (IC) $\rightarrow$ better risk-sharing $\rightarrow$ competition on δ_M .

Labor Market Competition (cont'd)



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• Large b .

- Increase in b : $\delta_M \downarrow$, $\delta_H \uparrow$, $\mathbb{E}[\tilde{u}] \downarrow$.
- Appetite for options $\rightarrow$ competition on $\delta_H \rightarrow$ worse risk-sharing.

Competition Across industries

• Setup.

- Suppose that firm 2 has $A_2 > 1$ in assets in place and $\sigma_2 < \sigma_1$.
- More cash (safe firm) or other operations (diversified firm).
- Firm 2 can offer compensation that is risk-free *regardless of the manager's investment policy*: $\{\delta_L, \delta_L + \delta_M, \delta_L + \delta_M + \delta_H\}$.

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- $F_1^{FB} = 1 + \phi(\sigma_1\phi_U - 1) \rightarrow MB_1 \equiv \frac{F_1^{FB}}{1} = 1 + \frac{\phi(\sigma_1\phi_U - 1)}{1}$.
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• In Equilibrium.

- One firm makes a contractual offer that the other can't match.
- In general: one firm is profitable, the other breaks even.

Competition Across industries (cont'd)

● Results.

- $0 \leq b \leq b^{**}$: Firm 2 wins and $\delta_L > 0$; $\mathbb{E}[\tilde{u}] \uparrow$, $\mathbb{E}[\tilde{\pi}_2] \downarrow$ as $b \uparrow$.
- $b^{**} < b \leq b^*$: Firm 1 wins and $\delta_L = 0$; $\delta_M \uparrow$, $\mathbb{E}[\tilde{u}] \uparrow$, $\mathbb{E}[\tilde{\pi}_1] \downarrow$ as $b \uparrow$.
- $b^* < b \leq \frac{1}{2}$: Firm 1 wins and $\delta_L = 0$; $\delta_H \uparrow$, $\mathbb{E}[\tilde{u}] \downarrow$, $\mathbb{E}[\tilde{\pi}_1] \uparrow$ as $b \uparrow$.

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● Some Predictions.

- Competition $\uparrow$ in an industry $\rightarrow$ performance-based compensation of managers with high overconfidence increases more.
- Portable, general, non-industry-specific skills.
 - Low OC: flat compensation in safe, diversified, value firms.
 - High OC: convex compensation in risky, focused, growth firms; more likely to switch job.
- Dispersion of market-to-book ratios across firms is positively related to dispersion in compensation convexity across their managers.

Summary

- **Context and Questions.** Managers cannot diversify their human capital and their actions are not perfectly observable (e.g., effort) → agency costs within the firm.
 - What are the effects of overconfidence on these problems?
 - How does overconfidence interact with contracts?

Summary

- **Context and Questions.** Managers cannot diversify their human capital and their actions are not perfectly observable (e.g., effort) → agency costs within the firm.
 - What are the effects of overconfidence on these problems?
 - How does overconfidence interact with contracts?
- **Main Results.**
 - Changes in overconfidence come with changes in contracts.
 - Realigning the manager's risk-taking incentives (through compensation) is easier/cheaper with overconfident managers.
 - Commitment to use information.
 - Allows for more compensation to come from flat wage.
 - Firm more profitable and/or manager better off (low b).
 - Overconfidence also fosters effort (commitment to gather info), which makes the manager "hireable."
 - Managerial optimism also helps with risk-taking incentives, but can have perverse effects on effort incentives.

Related Papers

- **Theoretical.**

- De la Rosa, Leonidas E. (2011): Principal-agent model that studies the effects of overconfidence on incentive contracts in a moral hazard framework. Agent overconfidence can lead to low- or high-powered contractual incentives, depending on the extent of this overconfidence.
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• Empirical.

- Otto (2014): This paper documents that CEOs whose option exercise behavior and earnings forecasts are indicative of optimistic beliefs receive smaller stock option grants, fewer bonus payments, and less total compensation than their peers.
- Humphery-Jenner et al. (2016): This paper documents that firms offer incentive-heavy compensation contracts to overconfident CEOs to exploit their positively biased views of firm prospects.
- Page (2018): Builds and estimates a dynamic model of CEO compensation and effort provision, and finds that variation in CEO attributes explains the majority of variation in compensation (equity and total) but little of the variation in firm value.

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- Effects: In financial markets and in firms.

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● Overconfidence in Finance.

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- Effects: In financial markets and in firms.

● Four Sections/Papers.

1. Odean (*Journal of Finance*, 1998).
 - Overconfidence in 3 different models of financial markets.
 - $OC \uparrow \rightarrow \text{Trading Volume and Volatility} \uparrow, \text{Price Info} \downarrow, \text{Cov}(\tilde{r}_{t-1}, \tilde{r}_t) < 0$.
2. Gervais and Odean (*Review of Financial Studies*, 2001).
 - Attribution bias (take too much credit for success) leads to OC.
 - Patterns in beliefs/OC $\rightarrow$ patterns in trading volume, volatility, etc.
3. Goel and Thakor (*Journal of Finance*, 2008).
 - Promotion tournaments lead to CEO/executive overconfidence.
 - Biased investment decisions can help/hurt the firm.
4. Gervais, Heaton, and Odean (*Journal of Finance*, 2011).
 - Compensation contracts to realign incentives of overconfident managers.
 - OC is valuable for the firm and/or the agent (depends on market power).

Potential Directions

● Money Management.

- Success can be the sign of skill (Berk & Green, 2004, JPE), *but* it can also prompt overconfidence (Gervais & Odean, 2001, RFS).
- Questions:
 - Are both effects at work?
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● Leadership and Firm Culture.

- If overconfident CEOs make (mostly) negative-NPV decisions, they should be fired.
- Questions: – Since firms keep them, what/where is their value?
– Do they make better leaders or create a better culture?