Modeling Information Revelation and Trading

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2019 FTG Summer School at Wharton
## Buy Low/ Sell High

### Earnings Report

<table>
<thead>
<tr>
<th></th>
<th>Total Company</th>
<th>Per Common Share</th>
<th>Total Company</th>
<th>Per Common Share</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net income (GAAP measure)</td>
<td>$ 1,375</td>
<td>$ 1.20</td>
<td>$ 1,325</td>
<td>$ 1.24</td>
</tr>
<tr>
<td>Net income attributable to noncontrolling interests (GAAP)</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Income attributable to participating securities (GAAP measure)</td>
<td>$ 1,375</td>
<td>$ 1.20</td>
<td>$ 1,325</td>
<td>$ 1.24</td>
</tr>
<tr>
<td>Net income attributable to CVS Health (GAAP measure)</td>
<td>$ 1,375</td>
<td>$ 1.20</td>
<td>$ 1,325</td>
<td>$ 1.24</td>
</tr>
<tr>
<td>Non-GAAP adjustments:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Amortization of intangible assets</td>
<td>936</td>
<td>0.15</td>
<td>931</td>
<td>0.15</td>
</tr>
<tr>
<td>Acquisition-related integration costs</td>
<td>199</td>
<td>0.18</td>
<td>199</td>
<td>0.18</td>
</tr>
<tr>
<td>Income tax benefit</td>
<td>506</td>
<td>(0.19)</td>
<td>506</td>
<td>(0.19)</td>
</tr>
<tr>
<td>Income attributable to participating securities, net of tax</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Adjusted net income attributable to CVS Health</td>
<td>$ 2,193</td>
<td>$ 1.96</td>
<td>$ 2,245</td>
<td>$ 1.72</td>
</tr>
</tbody>
</table>

Weighted average diluted shares outstanding: 1,264

### Additional Shares Bought

### Price
Theory

Empirical Testing

Theoretical Framework

Empirical Results

Observation

Economic Principles
Economic Principles

Theory

Empirical Testing

Understanding and Knowledge
Fully Revealing REE
Grossman

Noisy REE
Hellwig

Liquidity Trader Response
Admati/Pfleiderer

Value of Information in Noisy REE
Grossman/Stiglitz
Admati/Pfleiderer

Strategic Informed Trader
Kyle

Short Sale Restrictions
Diamond/Verrecchia

Market Order Model
Glosten/Milgrom
Rational Expectations

• Assume an economy with agents who take actions based on beliefs about the joint distribution $G(\bullet)$ of:

- Exogenous variables: $\tilde{X}$
- Endogenous variables: $\tilde{Y}$ (determined by agents' actions)
- Public Information: $\tilde{S}_p$
- Privately Observed Information: $\tilde{S}_i \ (i = 1, 2, \ldots, n)$
Rational Expectations

Agent $i$ takes action based on his private and public information and based on his assumption about the joint distribution of exogenous and endogenous variables: $\hat{G}_i (\bullet)$

\[
A_i \left( S_p, S_i \mid \hat{G}_i [\tilde{X}, \tilde{Y}, \tilde{S}_p, \tilde{S}_1, \tilde{S}_2, \ldots, \tilde{S}_n] \right)
\]

\[
A_1 \left( S_p, S_1 \mid \hat{G}_1 [\tilde{X}, \tilde{Y}, \tilde{S}_p, \tilde{S}_1, \tilde{S}_2, \ldots, \tilde{S}_n] \right)
\]
\[
A_2 \left( S_p, S_2 \mid \hat{G}_2 [\tilde{X}, \tilde{Y}, \tilde{S}_p, \tilde{S}_1, \tilde{S}_2, \ldots, \tilde{S}_n] \right)
\]
\[
A_3 \left( S_p, S_3 \mid \hat{G}_3 [\tilde{X}, \tilde{Y}, \tilde{S}_p, \tilde{S}_1, \tilde{S}_2, \ldots, \tilde{S}_n] \right)
\]
\[
A_4 \left( S_p, S_4 \mid \hat{G}_4 [\tilde{X}, \tilde{Y}, \tilde{S}_p, \tilde{S}_1, \tilde{S}_2, \ldots, \tilde{S}_n] \right)
\]
\[
\vdots
\]
\[
A_n \left( S_p, S_n \mid \hat{G}_n [\tilde{X}, \tilde{Y}, \tilde{S}_p, \tilde{S}_1, \tilde{S}_2, \ldots, \tilde{S}_n] \right)
\]

\[
G [\tilde{X}, \tilde{Y}, \tilde{S}_p, \tilde{S}_1, \tilde{S}_2, \ldots, \tilde{S}_n]
\]
Rational Expectations

\[ A_1(S_p, S_1 \mid \hat{G}_1[\bar{X}, \bar{Y}, \bar{S}_p, \bar{S}_1, \bar{S}_2, \ldots, \bar{S}_n]) \]
\[ A_2(S_p, S_2 \mid \hat{G}_2[\bar{X}, \bar{Y}, \bar{S}_p, \bar{S}_1, \bar{S}_2, \ldots, \bar{S}_n]) \]
\[ A_3(S_p, S_3 \mid \hat{G}_3[\bar{X}, \bar{Y}, \bar{S}_p, \bar{S}_1, \bar{S}_2, \ldots, \bar{S}_n]) \]
\[ A_4(S_p, S_4 \mid \hat{G}_4[\bar{X}, \bar{Y}, \bar{S}_p, \bar{S}_1, \bar{S}_2, \ldots, \bar{S}_n]) \]
\[ \vdots \]
\[ A_n(S_p, S_n \mid \hat{G}_n[\bar{X}, \bar{Y}, \bar{S}_p, \bar{S}_1, \bar{S}_2, \ldots, \bar{S}_n]) \]

\[ \hat{G}_i[\bar{X}, \bar{Y}, \bar{S}_p, \bar{S}_1, \bar{S}_2, \ldots, \bar{S}_n] = G[\bar{X}, \bar{Y}, \bar{S}_p, \bar{S}_1, \bar{S}_2, \ldots, \bar{S}_n] \quad \forall i \]
Rational Expectations

- Traders observe individual pieces of information
- Traders submit demand functions
- Walrasian Auctioneer determines price $P^*$ that clears market where $Z$ is total supply.
- $P^*$ will be public information

Information (signals) | Demand Function
---|---
$s_1$ | $D(P, s_1)$
$s_2$ | $D(P, s_2)$
$s_3$ | $D(P, s_3)$
... | ...
$s_n$ | $D(P, s_n)$

$$\sum_{i=1}^{n} D(P^*, s_i) = Z$$
Tractability: CARA and Normality

• Constant Absolute Risk Aversion (CARA) with risk tolerance $\rho$

\[ u(W) = -\exp\left(-\frac{W}{\rho}\right) \]
Tractability: CARA and Normality

- Constant Absolute Risk Aversion (CARA) with risk tolerance $\rho$

$$u(W) = -\exp\left(-\frac{W}{\rho}\right)$$

- If $W$ is normally distributed

$$E\left(u(\tilde{W})\right) = E\left[-\exp\left(-\frac{\tilde{W}}{\rho}\right)\right]$$

$$= -\exp\left[-\frac{E(\tilde{W})}{\rho} + \frac{\text{Var}(\tilde{W})}{2\rho^2}\right]$$
Conditional Expectations with A Multivariate Normal Distribution

\[
\begin{pmatrix}
    X \\
    Y
\end{pmatrix}
\sim \mathcal{N}
\begin{bmatrix}
    \begin{pmatrix}
        m_X \\
        m_Y
    \end{pmatrix},
    \begin{pmatrix}
        \Sigma_{xx} & \Sigma_{xy} \\
        \Sigma_{yx} & \Sigma_{yy}
    \end{pmatrix}
\end{bmatrix}
\]

\[
\mathbb{E}(X \mid Y = y) = m_X + \Sigma_{xy} \Sigma_{yy}^{-1} (y - m_y)
\]

\[
\text{Var}(X \mid Y = y) = \Sigma_{xx} - \Sigma_{xy} \Sigma_{yy}^{-1} \Sigma_{yx}
\]
Information = Truth plus Noise

Truth:  \( \tilde{\nu} \)

Observe noisy signal:  \( \tilde{s} = \tilde{\nu} + \tilde{\epsilon} \)

\( \tilde{\nu} \) and \( \tilde{\epsilon} \) distributed jointly normal

\( \tilde{\epsilon} \) is mean zero

\[
\begin{pmatrix}
X \\
Y
\end{pmatrix}
\sim N
\left[
\begin{pmatrix}
m_X \\
m_Y
\end{pmatrix},
\begin{pmatrix}
\Sigma_{XX} & \Sigma_{XY} \\
\Sigma_{YX} & \Sigma_{YY}
\end{pmatrix}
\right]
\]

\[
\begin{pmatrix}
\tilde{\nu} \\
\tilde{s}
\end{pmatrix}
\sim N
\left[
\begin{pmatrix}
\bar{\nu} \\
\bar{v}
\end{pmatrix},
\begin{pmatrix}
\sigma_v^2 & \sigma_v^2 \\
\sigma_v^2 & \sigma_v^2 + \sigma_\epsilon^2
\end{pmatrix}
\right]
\]
Information = Truth plus Noise

\[
\begin{pmatrix} \nu \\ s \end{pmatrix} \sim N \left( \begin{pmatrix} \bar{v} \\ \bar{v} \end{pmatrix}, \begin{pmatrix} \sigma_v^2 & \sigma_v^2 \\ \sigma_v^2 & \sigma_v^2 + \sigma^2_e \end{pmatrix} \right)
\]

\[
E(X | Y = y) = m_X + \Sigma_{XY} \Sigma_{YY}^{-1} (y - m_Y)
\]

\[
E(\nu | s) = \bar{v} + \frac{\sigma_v^2}{\sigma_v^2 + \sigma^2_e} (s - \bar{v})
\]

\[
= \left( \frac{\sigma_e^2}{\sigma_v^2 + \sigma^2_e} \right) \bar{v} + \left( \frac{\sigma_v^2}{\sigma_v^2 + \sigma^2_e} \right) s
\]
Information = Truth plus Noise

\[
\begin{pmatrix}
\nu \\
\sigma
\end{pmatrix}
\sim N
\begin{bmatrix}
\begin{pmatrix}
\nu \\
\sigma
\end{pmatrix},
\begin{pmatrix}
\sigma_v^2 & \sigma_v^2 \\
\sigma_v^2 & \sigma_v^2 + \sigma_{\epsilon}^2
\end{pmatrix}
\end{bmatrix}
\]

\[
\text{Var}(X | Y = y) = \Sigma_{xx} - \Sigma_{xy} \Sigma_{yy}^{-1} \Sigma_{yx}
\]

\[
\text{Var}(v | s) = \sigma_v^2 - \frac{\sigma_v^4}{\sigma_v^2 + \sigma_{\epsilon}^2}
\]

\[
= \frac{\sigma_v^2 \sigma_{\epsilon}^2}{\sigma_v^2 + \sigma_{\epsilon}^2}
\]

\[
= \left(\frac{1}{\sigma_v^2} + \frac{1}{\sigma_{\epsilon}^2}\right)^{-1}
\]
Simple Model

• Assume two period model (date 0 and 1)
• Consider an investor who has CARA preferences, with wealth at date 0 = \( W_0 \).
• Investor invests at date 0 and consumes at date 1
• Risk free asset (set risk free rate at 0 for simplicity).
• Risky asset
  • Terminal payoff \( \tilde{v} \sim N(\bar{v}, \sigma_v^2) \)
  • Price at date 0 is equal to \( P \)
• Investor observes truth plus noise signal \( \tilde{s} = \tilde{v} + \tilde{\epsilon} \).
• If investor buys \( x \) units of risk asset:

\[
W_1 = (W_0 - xP) + x\tilde{v}
\]
Simple Model

\[ W_1 = (W_0 - xP) + x\tilde{v} \]

• Investor will choose \( x \) to maximize expected utility:

\[
E(\tilde{W}_1) - \frac{\text{Var}(\tilde{W}_1)}{2\rho} = (W_0 - xP) + xE(\tilde{v} | \tilde{s}) - \frac{x^2\text{Var}(\tilde{v} | \tilde{s})}{2\rho}
\]

• FOC:

\[
E(\tilde{v} | \tilde{s}) - P - \frac{x\text{Var}(\tilde{v} | \tilde{s})}{\rho} = 0
\]

\[
D(P, \tilde{s}) = \rho \frac{E(\tilde{v} | \tilde{s}) - P}{\text{Var}(\tilde{v} | \tilde{s})} = a_0 + a_1\tilde{s} - a_2P
\]
Simple Model

- Now assume that there are $n$ investors.
  - All have same CARA preferences.
  - Each observes truth plus noise signal $\tilde{s}_i = \tilde{v} + \tilde{\epsilon}_i$.
  - $(\tilde{\epsilon}_1, \tilde{\epsilon}_2, \ldots, \tilde{\epsilon}_n)$ are i.i.d.

- Total supply of risky asset is $Z$.

$$
\sum_{i=1}^{n} D(P, \tilde{s}_i) = \sum_{i=1}^{n} (a_0 + a_1 \tilde{s}_i - a_2 P) = Z
$$

$$
P = \frac{a_0}{a_2} + \frac{1}{a_2} \sum_{i=1}^{n} \frac{\tilde{s}_i}{n} - \frac{Z}{na_2}
$$

$$
b_0 = \frac{a_0}{a_2} - \frac{Z}{na_2}
$$

$$
b_1 = \frac{1}{a_2}
$$
Simple Model

\[ P = b_0 + b_1 \sum_{i=1}^{n} \frac{\tilde{s}_i}{n} \]

- Since the market clearing price will contain additional information about the risky asset payoff, investors will want to use the price as well as their private signal in forming expectations.

- Demand will still be a linear function of the private signal and price, but the coefficients will be different than before:

\[ D(P, \tilde{s}_i) = \rho \frac{E(\tilde{\nu} \mid \tilde{s}_i, P) - P}{\text{Var}(\tilde{\nu} \mid \tilde{s}_i, P)} = a_0 + a_1 \tilde{s}_i - a_2 P \]
Rational Expectations Equilibrium

\[ P = b_0 + b_1 \sum_{i=1}^{n} \frac{\tilde{s}_i}{n} \]

- In order to know how to use the price in forming expectations, investors must conjecture how it is related to their signal and the risky asset payoff.
- Demand will still be a linear function of the private signal and price, but the coefficients will be different than before:
  \[ \{b_0, b_1\} \rightarrow \{a_0, a_1, a_2\} \rightarrow \{b_0^*, b_1^*\} \]
- Equilibrium is fixed point:
  \[ \{b_0^*, b_1^*\} \rightarrow \{a_0^*, a_1^*, a_2^*\} \rightarrow \{b_0^*, b_1^*\} \]
Fully Revealing REE

\[ P^* = b_0^* + b_1^* \sum_{i=1}^{n} \frac{\tilde{S}_i}{n} \]

• \( P^* \) is a fully revealing REE in the sense that it can be inverted to obtain a sufficient statistic for all the private information in the economy.

• How does this come about?????
Each period traders come to the market with another realization of \( \tilde{y} \), and another \( \bar{P}_0(y) \) is found where the auction stops. After many repetitions traders can tabulate the empirical distribution of \((\bar{P}_0, P_1)\) pairs. From this they get a good estimate of the joint distribution of \( \bar{P}_0 \) and \( P_1 \). After this joint distribution is learned, traders will have an incentive to change their bids just as the market is about to clear. This follows from the fact that if everyone observes that the market is about to clear at \( \bar{P}_0(y) \), they can condition their beliefs on \( \bar{P}_0(y) \) and learn something more about \( P_1 \). This changes their demands and thus the market will not clear at \( \bar{P}_0(y) \).

Suppose instead that the market has been clearing for a long time with prices generated by \( P_0^*(y) \), a solution to (12). Then at any particular time, given that traders come to the market with some \( y \), if the market is about to clear at \( P_0^*(y) \), and traders then realize that \( P_0^*(y) \) is the equilibrium, they will not change their bids due to the new information they get about \( P_1 \) from \( P_0^*(y) \). \( P_0^*(y) \) is a self fulfilling expectations equilibrium: when all traders think prices are generated by \( P_0^*(y) \), they will act in such a way that the market clears at \( P_0^*(y) \).

Sufficiently long time series of $(P, v)$ pairs to update distribution of $(P, v)$

New demand behavior leads to new distribution of $(P, v)$

Sufficiently long time series of $(P, v)$ pairs to update distribution of $(P, v)$

New demand behavior leads to new distribution of $(P, v)$

New demand behavior leads to REE distribution of $(P^*, v)$

DO THE UNDERLYING EXOGENOUS VARIABLES FOLLOW A STATIONARY ERGODIC PROCESS?
Learning Models


Core REE (some of the parameters are learned in equilibrium)

Learning Context:
Traders have REE about the equilibrium setting in which they learn. They know the underlying parameters.

What is needed to support the yellow is arguably more complex than what happens in the blue.

How do traders learn about the yellow? Infinite regress?
For REE Trading out of Market Clearing Equilibrium Is Very Problematic

Léon Walras
1834-1910
CHEAP Lemon Car Lot Total OR Steal? Bad Deal?
Problem: REE is fully revealing

\[ D(P^*, \tilde{s}_i) = D(P^*) \quad \forall i \]

- How does information “get into price” if all traders ignore their private signals?

The Walrasian Auctioneer can call out any price since all prices clear the market.
Problem: REE is fully revealing

Traders only Form Expectations Based on Private Signal

Fully Revealing REE: Traders only Condition on Price
Problem: REE is fully revealing

\[ D(P^*, \tilde{S}_i) = D(P^*) \quad \forall i \]

- If information is costly (in monetary or effort terms), why would any one acquire a private signal?
## REE Simplifying Assumptions

<table>
<thead>
<tr>
<th>Assumption</th>
<th>Realistic?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multivariate Normality</td>
<td>Payoffs and prices can be negative</td>
</tr>
<tr>
<td>CARA</td>
<td>Bill Gates and I invest the same amount in risky assets?</td>
</tr>
<tr>
<td>Walrasian Market Mechanism</td>
<td><strong>How do traders submit demand functions? Are actual trading protocols consistent with auction tatonnement?</strong></td>
</tr>
<tr>
<td>Price taking behavior</td>
<td>Don’t traders worry about market impact?</td>
</tr>
<tr>
<td>Learning</td>
<td>Stationarity and ergodicity; convergence time</td>
</tr>
</tbody>
</table>
The Value of Theory:

• Given all the problems with the simple, fully revealing REE model and the associated paradoxes, what contribution did Grossman’s paper make?

• What good is theory?
Efficient Markets

In an efficient market, competition among the many intelligent participants leads to a situation where, at any point in time, actual prices of individual securities already reflect the effects of information based both on events that have already occurred and on events which, as of now, the market expects to take place in the future. In other words, in an efficient market at any point in time the actual price of a security will be a good estimate of its intrinsic value.

(Fama, 1970)

- Weak Form Efficiency
- Semi-strong Form Efficiency
- Strong Form Efficiency
Grossman’s Model showed:

- The notion of a perfectly efficient market is fundamentally flawed.
  - Traders will not condition trades on private information.
  - Traders will have no incentive to gather private information.

- How information gets into prices is very delicate.
  - Trading mechanism is clearly important.
  - Traders’ demands will be based on some beliefs about what information is already in price.
    - How do they form these beliefs?
    - Are they correct?
Empirical Work is Not A Substitute for Theory

- Would unstructured empirical analysis produce insights about the nexus among
  - Traders’ beliefs about information in the price and their own information
  - Incentives to gather information
  - Trading mechanisms
  - Risk aversion
  - Degree of competition

Price
REE: Adding “Noise” to the Supply

\[
\sum_{i=1}^{n} D\left(P, \tilde{s}_i \right)
\]

\[
\sum_{i=1}^{n} D\left(P, \tilde{s}_i \hat{\ } \right)
\]

Price

<table>
<thead>
<tr>
<th>Price</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>\sum_{i=1}^{n} D\left(P, \tilde{s}_i \right)</td>
<td>Z</td>
</tr>
<tr>
<td>\sum_{i=1}^{n} D\left(P, \tilde{s}_i \hat{\ } \right)</td>
<td>Z\hat{\ }</td>
</tr>
</tbody>
</table>
REE: Adding “Noise” to the Supply

Traders will still condition on the price but will also put weight on their private signals.

Traders will potentially find it worthwhile to acquire information at a cost.
Noise: A Reasonable Assumption or a “Cheat”

Sources of noise:

• “Irrationality”
  • Animal spirits
  • Cognitive errors in processing information (overconfidence)

• Hedging demands
  • Untraded assets correlated with risky asset

• Liquidity needs
  • A little hard to motivate with CARA since in general liquidity needs will be met by trading riskless asset
Value of Information in CARA/Normal Distribution Setting

• Assume (as before):
  • CARA preference with risk tolerance $\rho$
  • Riskless asset with risk free rate $= 0$
  • Risky asset
    • Terminal payoff $\bar{v} \sim N(\bar{\nu}, \sigma_v^2)$
    • Price at date 0 is equal to $P$
  • Trader can acquire signal: $\tilde{s} = \bar{v} + \tilde{e}$
  • Otherwise the trader will form expectations just based on information in price.
Value of Information in CARA/Normal Distribution Setting

Without information

\[ W_1(D(P)) = W_0 + D(P)(\tilde{\nu} - P) \]

\[ D(P) = \rho \frac{E(\tilde{\nu} | P) - P}{\text{Var}(\tilde{\nu} | P)} \]

With information

\[ W_1(D(P, \tilde{s})) = W_0 + D(P, \tilde{s})(\tilde{\nu} - P) \]

\[ D(P, \tilde{s}) = \rho \frac{E(\tilde{\nu} | P, \tilde{s}) - P}{\text{Var}(\tilde{\nu} | P, \tilde{s})} \]

Monetary value of information = \( \varphi \)

\[ E \left[ -\exp \left( -\frac{W_1(D(P))}{\rho} \right) \right] = E \left[ -\exp \left( -\frac{W_1(D(P, \tilde{s})) - \varphi}{\rho} \right) \right] \]

Result: \( \varphi = \frac{\rho}{2} \log \left( \frac{\text{Var}(\tilde{\nu} | P)}{\text{Var}(\tilde{\nu} | \tilde{s}, P)} \right) \)
Value of Information in CARA/Normal Distribution Setting

Monetary value of information = \[ \frac{\rho}{2} \log \left( \frac{\text{var}(\tilde{v} \mid P)}{\text{var}(\tilde{v} \mid \tilde{s}, P)} \right) \]

- Note that value is increasing linearly in risk tolerance
  - The more aggressively one will use the information, the more valuable it is.

- Fixing the informativeness of the private information, the value of the private information \((\text{generally})\) decreases as the price becomes more informative.

- If \( \text{var}(\tilde{v} \mid \tilde{s}, P) = 0 \), the private information has infinite value.
  - Critical assumption: trading has no price impact and infinite positions can be taken without affecting the price.
Grossman Stiglitz: On the Impossibility of Informationally Efficient Markets

One piece of private information: $\tilde{\theta}$

$$\tilde{v} = \tilde{\theta} + \tilde{\eta}$$

$$E(\tilde{v} | \tilde{\theta}) = \tilde{\theta}$$

Equivalent to “truth plus noise”

$$\tilde{s} = \tilde{v} + \tilde{\varepsilon}$$

$$\tilde{\theta} = E(\tilde{v} | \tilde{s})$$

$$\tilde{\eta} = \tilde{v} - \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\varepsilon^2} \tilde{v} - \frac{\sigma_\varepsilon^2}{\sigma_v^2 + \sigma_\varepsilon^2} \tilde{s}$$

$$= \frac{\sigma_\varepsilon^2}{\sigma_v^2 + \sigma_\varepsilon^2} (\tilde{v} - v) - \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\varepsilon^2} \tilde{\varepsilon}$$
Grossman Stiglitz: On the Impossibility of Informationally Efficient Markets

• Assume a continuum of traders on [0,1].

• Fraction $\lambda$ will buy information (observe $\theta$).

• Continuum means all traders are infinitesimal and can justifiably be thought of as price taking agents.

• Need to be careful that continuum doesn’t support something that is not “close to” something we would get with large enough “$n$.”
Grossman Stiglitz: On the Impossibility of Informationally Efficient Markets

Key assumption: Supply Z is noisy

\[ \tilde{Z} \sim N(\bar{Z}, \sigma^2_Z) \]

Market clearing price: will solve

\[ \lambda D^{\text{informed}}(P, \tilde{\theta}) + (1 - \lambda) D^{\text{uninformed}}(P) = \tilde{Z} \]

\[ D^{\text{informed}}(P, \tilde{\theta}) = \rho \frac{\tilde{\theta} - P}{\sigma^2_\eta} \quad D^{\text{uninformed}}(P) = \rho \frac{E(v | P) - P}{\text{Var}(v | P)} \]
Grossman Stiglitz: On the Impossibility of Informationally Efficient Markets

\[ \lambda D^{\text{informed}}(P, \tilde{\theta}) + (1 - \lambda) D^{\text{uninformed}}(P) = \tilde{Z} \]

\[ \lambda \rho \left( \frac{\tilde{\theta} - P}{\sigma^2_\eta} \right) + (1 - \lambda) \rho \left( \frac{E(v | P) - P}{\text{Var}(v | P)} \right) = \tilde{Z} \]

\[ \frac{\lambda \rho}{\sigma^2_\eta} \tilde{\theta} - \tilde{Z} + \text{linear function of } P = 0 \]

• From observing \( P \) the uninformed can recover:

\[ \tilde{\theta} - \left( \frac{\sigma^2_\eta}{\lambda \rho} \right)(\tilde{Z} - \bar{Z}) \]
Grossman Stiglitz: On the Impossibility of Informationally Efficient Markets

\[ \varphi = \frac{\rho}{2} \log \left( \frac{\text{var}(\tilde{v} \mid P)}{\text{var}(\tilde{v} \mid \tilde{\theta})} \right) \]

\[ = \frac{\rho}{2} \log \left( \frac{\sigma^2 + \text{var}(\tilde{\theta})}{\sigma^2} \right) \]

\[ = \tilde{\theta} + \tilde{\eta} \]
Grossman Stiglitz: On the Impossibility of Informationally Efficient Markets

\[ \varphi = \frac{\rho}{2} \log \left( \frac{\sigma^2 + \text{var} \left( \tilde{\theta} \right) - \left( \frac{\sigma^2_\eta}{\lambda \rho} \right)(\tilde{Z} - \bar{Z})}{\sigma^2_\eta} \right) \]

Since \( \text{var} \left( \tilde{\theta} \right) - \left( \frac{\sigma^2_\eta}{\lambda \rho} \right)(\tilde{Z} - \bar{Z}) \) is decreasing in \( \lambda \),

it follows that \( \varphi \), the value of information is decreasing in \( \lambda \).
Grossman Stiglitz: On the Impossibility of Informationally Efficient Markets

\[
\varphi = \frac{\rho}{2} \log \left( \frac{\text{var}(\tilde{\nu} | P)}{\text{var}(\tilde{\nu} | \tilde{\theta})} \right)
\]

\[
\frac{\rho}{2} \log \left( \frac{\sigma^{2}_\theta + \sigma^{2}_\eta}{\sigma^{2}_\eta} \right)
\]

\(c\) (exogenously given cost)
Grossman Stiglitz: On the Impossibility of Informationally Efficient Markets

\[ \varphi = \frac{\rho}{2} \log \left( \frac{\text{var}(\tilde{v} \mid P)}{\text{var}(\tilde{v} \mid \tilde{\theta})} \right) \]

- Increase in \( \rho \)
- Increase in \( \lambda^* \)
- \( c \) (exogenously given cost)
Grossman Stiglitz: On the Impossibility of Informationally Efficient Markets

\[ \varphi = \frac{\rho}{2} \log \left( \frac{\text{var}(\bar{v} | P)}{\text{var}(\bar{v} | \bar{\theta})} \right) \]
φ = \frac{\rho}{2} \log \left( \frac{\text{var}(\tilde{v} \mid P)}{\text{var}(\tilde{v} \mid \tilde{\theta})} \right)

- Increase in $\sigma^2_\eta$
- Decrease in $\lambda^*$
- $c$ (exogenously given cost)

Grossman Stiglitz: On the Impossibility of Informationally Efficient Markets
Grossman Stiglitz: On the Impossibility of Informationally Efficient Markets

\[ \phi = \frac{\rho}{2} \log \left( \frac{\text{var}(\tilde{v} | P)}{\text{var}(\tilde{v} | \tilde{\theta})} \right) \]

increase in \( \sigma_\eta^2 \)

(cexogenously
given cost)

increase in \( \lambda^* \)

increase in \( \lambda \)
Grossman Stiglitz: On the Impossibility of Informationally Efficient Markets

\[ \varphi = \frac{\rho}{2} \log \left( \frac{\text{var}(\tilde{v} | P)}{\text{var}(\tilde{v} | \tilde{\theta})} \right) \]

increase in \( \sigma_Z^2 \)
Grossman Stiglitz: On the Impossibility of Informationally Efficient Markets

price reveals: \( \tilde{\theta} - \left( \frac{\sigma^2_\eta}{\lambda \rho} \right) (\tilde{Z} - \bar{Z}) \)  

informativeness: \( \left( \frac{\sigma^2_\eta}{\lambda \rho} \right)^2 \sigma^2_z \)

\[
\begin{bmatrix}
\rho \\
\sigma^2_\eta \\
\sigma^2_z \\
c
\end{bmatrix} \rightarrow \lambda^* \rightarrow \left( \frac{\sigma^2_\eta}{\lambda^* \rho} \right)^2 \sigma^2_z
\]

As long as we have an interior equilibrium, i.e., \( \lambda^* \in (0,1) \), a small increase in \( \sigma^2_z \) leads to an increase in \( \lambda^* \) which leaves price informativeness unchanged.
Martin Hellwig: Aggregation of Information

• Grossman-Stiglitz results apply to cases where all privately informed traders observe the **same information**, which the uninformed attempt to infer from equilibrium price.

• Hellwig looks at cases where:
  
  • Traders are endowed with different signals: \( \tilde{s}_i = \tilde{v} + \tilde{\varepsilon}_i \)

  \[ E(\tilde{\varepsilon}_i \tilde{\varepsilon}_j) = 0 \quad \forall i \neq j \]

  • Traders have potentially different levels of risk to learn: \( \rho_i \)

  • Supply \( \tilde{z} \) is noisy: \( Var(\tilde{z}) = \sigma_z^2 > 0 \)
Assume finite number \((n)\) of traders. Price will reveal:

\[
\sum_{i=1}^{n} \alpha_i \tilde{s}_i - \beta \tilde{Z}
\]

\(\alpha_i\) will depend on \(\rho_i\) and \(\text{Var}(\tilde{e}_i)\)

- Traders are assumed to be “schizophrenic”
  - When forming expectations they fully account for the correlation between the price and their signal (since the price is affected by their signal).
  - But they otherwise take the price as given and assume their trade has no effect on the price.
Assume

- “Schizophrenia”
- Noise in price becomes negligible

As \( \text{Var}(\tilde{Z}) \to 0 \)

\[
P \to A_0 + A_1 \sum_{i=1}^{n} \frac{\tilde{s}_i}{\text{Var}(\tilde{e}_i)} - A_2 \bar{Z}
\]

In the limit price perfectly aggregates information

- Weight on individual signals does not depend on risk tolerance \( \rho_i \).
- Weight precisely reflects signal quality!
Martin Hellwig: Aggregation of Information: Finite (small market) Model

- Finite number of traders / vanishing noise result is a curiosity:
  - Schizophrenia assumption is troubling.
  - Having noise is more realistic than no noise.
- Result does show that under special assumptions Grossman Fully Revealing REE can be approached as limit.
Martin Hellwig: Aggregation of Information: Large (truly competitive market) Model

Assume number of traders becomes infinitely large.

For any finite $n$, we still have the price revealing

$$\sum_{i=1}^{n} \alpha_i^{(n)} s_i - \beta^{(n)} \tilde{Z}$$

where $\alpha_i$ will depend on $\rho_i$ and $\text{Var}(\tilde{e}_i)$

By the law of large numbers

$$\lim_{n \to \infty} \frac{\sum_{i=1}^{n} \alpha_i^{(n)} s_i}{\sum_{i=1}^{n} \alpha_i^{(n)}} = \lim_{n \to \infty} \frac{\sum_{i=1}^{n} \alpha_i^{(n)} (\tilde{v} + \tilde{e}_i)}{\sum_{i=1}^{n} \alpha_i^{(n)}} = \tilde{v}$$
As we pass to the limit we can assume that the per capita supply noise is bounded away from zero.

\[ P = A_0 + A_1 \tilde{\nu} - A_2 \left( \tilde{Z} - \bar{Z} \right) \]

We obtain an equilibrium where

- Price reveals “truth” plus supply noise.
- Traders optimally condition on their own information based on variance of \( \tilde{\epsilon}_i \) and on price which is independent of \( \tilde{\epsilon}_i \).
- Traders have “infinitesimal” effects on the price and are justified in their price-taking behavior.
Market Efficiency Again

Weak Form
In the weak-form efficient market hypothesis, all historical prices of securities have already been reflected in the market prices of securities. In other words, technicians – those trading on analysis of historical trading information – should earn no abnormal returns.

Semi-strong Form
In a semi-strong-form efficient market, **prices reflect all publicly known and available information, including all historical price information.** Under this assumption, analyzing any public financial disclosures made by a company to determine a stock’s intrinsic value would be futile since every detail would be taken into account in the stock’s market price. Similarly, an investor could not earn consistent abnormal returns by acting on surprise announcements since the market would quickly react to the new information.

Strong Form
In a strong-form efficient market, **security prices fully reflect both public and private information.** Therefore, insiders could not generate abnormal returns by trading on private information because it would already figure into market prices.
Market Efficiency Again

Private Information → \texttt{PRICE}

Public Information → \texttt{PRICE}
Market Efficiency Again

*Completely Unanticipated*

Public announcement: Firm must pay a fine equal to $2.00 per share.

\[ P_t = P_{t-\delta} - $2.00 \]
Market Efficiency Again

Public announcement: Firm must pay a fine equal to $2.00 per share.

\[
P_{t} = P_{t-\delta} - ?
\]
Allocation of Trading Information

• Allocation of information is obviously important
  • What information is out there?
  • Who has it?

Three major ways to acquire information:
• Get lucky and have it fall into your lap (“endowed with information”)
• Expend personal effort to get it (a cost)
• Buy it at some cost
  • Who is selling it?
  • What do they sell?
  • What do they charge?
Monopolistic market is perhaps a reasonable place to start.  
It potentially isolates some of the key tradeoffs.  
Clearly many markets involve substantial degrees of competition.
Monopolistic Market for Information

Need to make lots of special assumptions:

1. **Seller does not trade:**
   - Clear incentive problems
   - Trading profits will be dwarfed by selling profits

2. **Information cannot be resold**
   - No problem if resale of a particular signal is limited to a set of measure zero.
   - Partial justification: must be used in trade immediately — there is no time for resale.

Model will be based on noisy REE with continuum of price-taking traders.
Continuum of traders
Risky asset payoff: \( \tilde{\nu} \)
Noisy supply (per capita): \( \tilde{Z} \)
Seller's information: \( \tilde{\nu} + \tilde{\omega} \)
Information sold to trader \( i \): \( \tilde{\nu} + \tilde{\omega} + \tilde{\varepsilon}_i \)
Let \( A \) represent the allocation of information across all traders.

Price will reveal:

\[
\tilde{\nu} + \tilde{\omega} + \tilde{\eta}(A) + G(A) \tilde{Z}
\]

where \( \tilde{\eta}(A) \) is a weighted sum of all the \( \tilde{\varepsilon}_i \)
Aggregation of the noise that is added by seller to each piece of information sold.

Contribution of noise due to noisy supply is affected by allocation of information.
Monetary value of information = \( \frac{\rho}{2} \log \left( \frac{\text{var}(\tilde{v} | P)}{\text{var}(\tilde{v} | \tilde{s}_i, P)} \right) \)

Value of \( s_i = \tilde{v} + \tilde{\omega} + \tilde{e}_i = \)

\[
\frac{\rho}{2} \log \left( \frac{\text{var}(\tilde{v} | \tilde{v} + \tilde{\omega} + \tilde{\eta}(A) + G(A)\tilde{Z})}{\text{var}(\tilde{v} | \tilde{v} + \tilde{\omega} + \tilde{e}_i, \tilde{v} + \tilde{\omega} + \tilde{\eta}(A) + G(A)\tilde{Z})} \right)
\]
Admati/Pfleiderer
Monopolistic Market for Information

• Why should the Monopolistic Seller add noise and thereby decrease the intrinsic value of what he is selling?

\[
\text{value} = \frac{\rho}{2} \log \left( \frac{\text{var}(\tilde{v} | P)}{\text{var}(\tilde{v} | \tilde{s}_i, P)} \right)
\]

• Holding the statistical properties of \(P\) constant:

- Increasing the quality of \(\tilde{s}_i\) decreases \(\text{var}(\tilde{v} | \tilde{s}_i, P)\)

- and increases \(\frac{\rho}{2} \log \left( \frac{\text{var}(\tilde{v} | P)}{\text{var}(\tilde{v} | \tilde{s}_i, P)} \right)\)
Increasing the quality of signals sold will generally **improve the value of** \( P \) **in forecasting** \( v \).

The monopolist is concerned about the ratio

\[
\frac{\text{var}(\tilde{v} \mid P)}{\text{var}(\tilde{v} \mid \tilde{s}_i, P)}
\]
Consider an extreme case:

\[ \tilde{v} + \tilde{\omega} \]

Var(\(\tilde{\omega}\)) is EXTREMELY small

Information sold to trader \(i\) : \(\tilde{v} + \tilde{\omega}\)

Even if the monopolist sells to only a small fraction of the market, the informed will trade very aggressively.

The price will reveal:

\[ \tilde{v} + \tilde{\omega} + G(A)\tilde{Z} \]

with \(G(A)\) being extremely small
Monopolistic Market for Information

\[
\text{var}(\tilde{v} | \tilde{v} + \tilde{\omega} + G(A)\tilde{Z}) \approx \text{var}(\tilde{v} | \tilde{v} + \tilde{\omega})
\]

Monetary value of information

\[
= \frac{\rho}{2} \log \left( \frac{\text{var}(\tilde{v} | P)}{\text{var}(\tilde{v} | \tilde{s}_i, P)} \right)
\]

\[
\approx 0
\]

Virtually all of the information leaks out in price. Informed are not much better off than uninformed.
Admati/Pfleiderer
Monopolistic Market for Information

- Two ways to control dilution of value of information through its (partial) revelation in $P$:
  - Reduce $\lambda$, fraction of traders who acquire and trade on information
  - Add noise to what is sold: sell $\tilde{v} + \tilde{w} + \tilde{e}$
All information buyers receive the same signal: $\tilde{v} + \tilde{\omega} + \tilde{\epsilon}$

- Quality of Monopolist’s Information Is Decreasing

- Supply Noise Hides Private Information Better

- $\lambda < 1$
  - $\sigma_\epsilon^2 = 0$

- $\lambda = 1$
  - $\sigma_\epsilon^2 = 0$

- $\sigma_Z / \rho$
All information buyers receive the personalized signal: $\tilde{v} + \tilde{ω} + \tilde{ε}_i$

All $\tilde{ε}_i$ are independently distributed

$\lambda = 1$

$\sigma^2_{ε_i} = 0$

Quality Of Monopolist’s Information Is Decreasing

Supply Noise Hides Private Information Better
Impossible in practice to submit demand functions to an auctioneer.

Market orders can be submitted to a specialist dealer.
Glosten and Milgrom: Dealer Market

• Specialist (dealer) makes a market by setting a bid (ask) price at which she is willing to buy (sell) one unit of a risky asset.

• Assume (for illustrative purposes) that risky the asset pays off $\tilde{v} \sim U(0,1)$.

• Two types of traders might show up at the specialist booth to trade:
  • Informed trader who knows $\tilde{v} \sim U(0,1)$
  • “Liquidity” trader who is uninformed but has inelastic demand to either buy or sell; equally like to want to sell or buy
Glosten and Milgrom: Dealer Market

• Specialist has no private information about risky asset payoff.
• Specialist knows that the probability that the trader arriving at her booth is informed is $\lambda$; probability that trader has liquidity motives is therefore $(1-\lambda)$.
• Specialist is assumed to be risk neutral and have no cost of making the market.
• Specialist competes in a perfectly competitive market and so bid and ask prices are set such that zero profit is earned.
  • Bertrand competition with two or more specialists
Glosten and Milgrom: Dealer Market

Assume dealer sets ask at \( a \) and bid at \( b \).

\[ \tilde{v} > a \] Trader buys

\[ b \leq \tilde{v} \leq a \] Trader neither buys nor sells

\[ \tilde{v} < b \] Trader sells

\[ \lambda \] prob = \( \frac{1}{2} \)

\[ 1 - \lambda \] prob = \( \frac{1}{2} \)
Glosten and Milgrom: Dealer Market

\[
E(\tilde{\nu} | \text{trader buys}) = \frac{\lambda \left( \frac{1+a}{2} \right)(1-a) + (1-\lambda) \frac{1}{4}}{\lambda(1-a) + (1-\lambda) \frac{1}{2}}
\]

\[
E(\tilde{\nu} | \text{trader sells}) = \frac{\lambda \left( \frac{b}{2} \right)(b) + (1-\lambda) \frac{1}{4}}{\lambda(b) + (1-\lambda) \frac{1}{2}}
\]
Equilibrium bid and ask

\[ \lambda = 0.60 \]

\[ a = \frac{\lambda \left( \frac{1+a}{2} \right) (1-a) + (1-\lambda) \frac{1}{4}}{\lambda (1-a) + (1-\lambda) \frac{1}{4}} \]

\[ b = \frac{\lambda \left( \frac{b}{2} \right) (b) + (1-\lambda) \frac{1}{4}}{\lambda (b) + (1-\lambda) \frac{1}{4}} \]

\[ a^* = 0.613 \]
\[ b^* = 0.387 \]
As is intuitive, the bid/ask spread widens as probability of informed increases.

\[ \lambda = 0.20 \]

\[ a^* = 0.53 \]
\[ b^* = 0.47 \]

\[ \lambda = 0.90 \]

\[ a^* = 0.76 \]
\[ b^* = 0.24 \]
Market can fail if Liquidity Demand is Elastic

\[ \tilde{v} > a \]

Trader buys

\[ b \leq \tilde{v} \leq a \]

Trader neither buys nor sells

\[ \tilde{v} < b \]

Trader sells

\[ \text{prob} = \frac{1}{2} (1 - a)^2 \]

Trader buys

\[ \text{prob} = 1 - \frac{1}{2} (1 - a)^2 - \frac{1}{2} b^2 \]

Trader neither buys nor sells

\[ \text{prob} = \frac{1}{2} b^2 \]

Trader sells
If liquidity demand is elastic and adverse selection is sufficiently high, no trade occurs.

\[ \lambda = 0.10 \]

\[ \lambda = 0.60 \]

\[ a^* = 0.444 \]
\[ b^* = 0.555 \]
\[ a^* = 1.000 \]
\[ b^* = 0.000 \]

Similar to Akerlof’s breakdown in used car market
Glosten and Milgrom: Binary Outcome

\[
\begin{align*}
\pi_{k+1 | \text{trader buys}} &= \frac{\lambda + (1 - \lambda) \frac{1}{2}}{\lambda \pi_k + (1 - \lambda) \frac{1}{2}} \times \pi_k \\
\pi_{k+1 | \text{trader sells}} &= \frac{(1 - \lambda) \frac{1}{2}}{\lambda (1 - \pi_k) + (1 - \lambda) \frac{1}{2}} \times \pi_k
\end{align*}
\]
Convergence in Glosten/Milgrom Setting

\[ \lambda = 0.20; \quad \tilde{v} = 1 \]

<table>
<thead>
<tr>
<th>Trade Number</th>
<th>Direction</th>
<th>Prior</th>
<th>Bid</th>
<th>Ask</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>0.500</td>
<td>0.400</td>
<td>0.600</td>
</tr>
<tr>
<td>2</td>
<td>Sell</td>
<td>0.400</td>
<td>0.308</td>
<td>0.500</td>
</tr>
<tr>
<td>3</td>
<td>Buy</td>
<td>0.500</td>
<td>0.400</td>
<td>0.600</td>
</tr>
<tr>
<td>4</td>
<td>Buy</td>
<td>0.600</td>
<td>0.500</td>
<td>0.692</td>
</tr>
<tr>
<td>5</td>
<td>Sell</td>
<td>0.500</td>
<td>0.400</td>
<td>0.600</td>
</tr>
<tr>
<td>6</td>
<td>Buy</td>
<td>0.600</td>
<td>0.500</td>
<td>0.692</td>
</tr>
<tr>
<td>7</td>
<td>Buy</td>
<td>0.692</td>
<td>0.600</td>
<td>0.771</td>
</tr>
<tr>
<td>8</td>
<td>Buy</td>
<td>0.771</td>
<td>0.692</td>
<td>0.835</td>
</tr>
<tr>
<td>9</td>
<td>Buy</td>
<td>0.835</td>
<td>0.771</td>
<td>0.884</td>
</tr>
<tr>
<td>10</td>
<td>Buy</td>
<td>0.884</td>
<td>0.835</td>
<td>0.919</td>
</tr>
</tbody>
</table>
Convergence in Glosten/Milgrom Setting

\[ \lambda = 0.05; \quad \tilde{v} = 1 \]
Diamond and Verrecchia
Introducing a Market Friction: Constraints on Short-selling

• Someone who gets positive information can easily trade on that information by taking or increasing a long position in the security.

• Someone who uncovers negative information can sell shares if he owns them, but if not must take a short position
  • In practice this is somewhat costly.
  • In some situations prohibitively costly.

• Assume short sales are prohibited or difficult
  • Does this slow the rate of private information revelation through prices?
  • Does it create an upward bias on prices?
Assume same setup as Glosten Milgrom (binary outcome)

Except:

- Informed with negative information can only sell with probability $\beta$. With probability $(1 - \beta)$ the informed trader does not own shares and is unable to short.

- In the event that the informed trader has negative information and cannot sell, there will be no trade.

- Uniformed (liquidity) traders buy with probability $\alpha (<0.5)$, sell with probability $\alpha$ and do not trade with probability $(1-2\alpha)$. 

Diamond and Verrecchia
Introducing a Market Friction: Constraints on Short-selling
Diamond and Verrecchia: Binary Outcome

\[ \pi_{k+1} \mid \text{trader buys} = \frac{\lambda + (1-\lambda)\alpha}{\lambda \pi_k + (1-\lambda)\alpha} \times \pi_k \]

\[ \pi_{k+1} \mid \text{trader sells} = \frac{(1-\lambda)\alpha}{\lambda (1-\pi_k) \beta + (1-\lambda)\alpha} \times \pi_k \]

\[ \pi_{k+1} \mid \text{no trade} = \frac{(1-\lambda)(1-2\alpha)}{\lambda (1-\pi_k)(1-\beta) + (1-\lambda)(1-2\alpha)} \times \pi_k \]
Convergence in Diamond and Verrecchia Setting

\[ \lambda = 0.20; \; \alpha = 0.4; \; \beta = 0.9; \; \tilde{\nu} = 0 \]
Convergence in Diamond and Verrecchia Setting

\[ \lambda = 0.20; \ \alpha = 0.4; \ \beta = 0.9; \ \tilde{\nu} = 0 \]

<table>
<thead>
<tr>
<th>Trade Number</th>
<th>Direction</th>
<th>Prior</th>
<th>Bid</th>
<th>Ask</th>
<th>No Trade</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>0.500</td>
<td>0.390</td>
<td>0.619</td>
<td>0.471</td>
</tr>
<tr>
<td>2</td>
<td>Buy</td>
<td>0.619</td>
<td>0.510</td>
<td>0.725</td>
<td>0.591</td>
</tr>
<tr>
<td>3</td>
<td>Sell</td>
<td>0.510</td>
<td>0.400</td>
<td>0.628</td>
<td>0.480</td>
</tr>
<tr>
<td>4</td>
<td>No Trade</td>
<td>0.480</td>
<td>0.372</td>
<td>0.600</td>
<td>0.451</td>
</tr>
<tr>
<td>5</td>
<td>No Trade</td>
<td>0.451</td>
<td>0.345</td>
<td>0.572</td>
<td>0.422</td>
</tr>
<tr>
<td>6</td>
<td>No Trade</td>
<td>0.422</td>
<td>0.319</td>
<td>0.543</td>
<td>0.394</td>
</tr>
<tr>
<td>7</td>
<td>Buy</td>
<td>0.543</td>
<td>0.432</td>
<td>0.659</td>
<td>0.513</td>
</tr>
<tr>
<td>8</td>
<td>Sell</td>
<td>0.432</td>
<td>0.327</td>
<td>0.552</td>
<td>0.403</td>
</tr>
<tr>
<td>9</td>
<td>Sell</td>
<td>0.327</td>
<td>0.237</td>
<td>0.441</td>
<td>0.302</td>
</tr>
<tr>
<td>10</td>
<td>Sell</td>
<td>0.237</td>
<td>0.166</td>
<td>0.336</td>
<td>0.217</td>
</tr>
</tbody>
</table>
Despite the censuring due to short sale constraints, prices will still be martingales.

\[
\begin{bmatrix}
\pi_{k+1} | \text{trader buys} \\
\pi_{k+1} | \text{trader sells} \\
\pi_{k+1} | \text{no trade}
\end{bmatrix}
\begin{bmatrix}
\Pr (\text{trader buys}) \\
\Pr (\text{trader sells}) \\
\Pr (\text{no trade})
\end{bmatrix}
= \\
\pi_k \times
\begin{bmatrix}
\left(\frac{\lambda + (1-\lambda) \alpha}{\lambda \pi_k + (1-\lambda) \alpha_k}\right)(\lambda \pi_k + (1-\lambda) \alpha) + \\
\frac{(1-\lambda) \alpha}{\lambda (1-\pi_k) \beta + (1-\lambda) \alpha}(\lambda (1-\pi_k) \beta + (1-\lambda) \alpha) + \\
\frac{(1-\lambda)(1-2\alpha)}{\lambda (1-\pi_k)(1-\beta) + (1-\lambda)(1-2\alpha)}(\lambda (1-\pi_k)(1-\beta) + (1-\lambda)(1-2\alpha))
\end{bmatrix}
= \pi_k
\]
Glosten/Milgrom Simplifying Assumptions

<table>
<thead>
<tr>
<th>Assumption</th>
<th>Realistic?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bertrand Competition among Dealers</td>
<td>Perhaps: if quotes are easily seen by all; ease of placing limit orders allows competition</td>
</tr>
<tr>
<td>Risk Neutrality of Dealers</td>
<td>Probably not a major concern</td>
</tr>
<tr>
<td>Market Mechanism with Only Market Orders</td>
<td>Probably not a gross distortion for many purposes</td>
</tr>
<tr>
<td>Learning by Dealers</td>
<td>Dealers through experience probably develop a sense of adverse selection risk over time. Simpler learning problem than in REE</td>
</tr>
<tr>
<td>Exogenous arrival process of traders</td>
<td>Informed are not strategic despite the fact that their trading has an affect on information revelation and future trading opportunities.</td>
</tr>
</tbody>
</table>
Kyle Model
(Continuous Auctions and Insider Trading)

- One informed trader who knows: \( \tilde{v} \)

- Noisy supply: \( \tilde{Z} \) (sum of all market orders from liquidity traders)

- Informed trader submits market order: \( \tilde{x} \)

- Market maker observes total order flow: \( \tilde{x} + \tilde{Z} \)
  - Market maker cannot see component parts of order flow.
  - Market maker sets single price at which total order flow is processed.
Kyle Model
(Continuous Auctions and Insider Trading)

• Market maker is risk neutral.

• Market is competitive so that market maker earns zero profits in expectation.
  ○ Market makers competitively bid price to process full market order flow.

• Price will therefore be: \( P = E(\tilde{v} \mid \tilde{x} + \tilde{Z}) \)

• Assume \( \tilde{v} \) and \( \tilde{Z} \) are jointly normally distributed:

\[
\begin{pmatrix}
\tilde{v} \\
\tilde{Z}
\end{pmatrix}
\sim \mathcal{N}
\begin{pmatrix}
0 \\
0
\end{pmatrix},
\begin{pmatrix}
\sigma_v^2 & 0 \\
0 & \sigma_Z^2
\end{pmatrix}
\]

(Zero means are for simplification and are w.l.o.g.)
Kyle Model
(Continuous Auctions and Insider Trading)

• Assume that \( \tilde{x} \) is linearly related to \( \tilde{\nu} \).

• Then:

\[
E\left( \tilde{\nu} \mid \tilde{x} + \tilde{Z} \right) = \frac{\text{Cov}(\tilde{\nu}, \tilde{x} + \tilde{Z})}{\text{Var}(\tilde{x} + \tilde{Z})}(\tilde{x} + \tilde{Z}) = \lambda (\tilde{x} + \tilde{Z})
\]

• Informed trader chooses \( x \) to maximize expected profits

\[
E\pi(x, \hat{P}) = E\left( x \left( \nu - \lambda (x + \tilde{Z}) \right) \right)
\]

\[
= xv - \lambda x^2
\]

\[
x^* = \beta v \quad \beta = \frac{1}{2\lambda}
\]
Kyle Model
(Continuous Auctions and Insider Trading)

\[ \lambda = \frac{\text{Cov}(\tilde{v}, \beta \tilde{v} + \tilde{Z})}{\text{Var}(\beta \tilde{v} + \tilde{Z})} = \frac{\beta \sigma_v^2}{\beta^2 \sigma_v^2 + \sigma_Z^2} \]

\[ \beta = \frac{1}{2\lambda} \]

\[ \lambda^{eq} = \frac{1}{2} \frac{\sigma_v}{\sigma_Z} \quad \beta^{eq} = \frac{\sigma_Z}{\sigma_v} \]
Kyle Model  
(Continuous Auctions and Insider Trading)

- Ex ante expected profit of informed trader = \( \frac{\sigma_v \sigma_Z}{2} \)
- Intuitively, informed's profits increase with
  - his informational edge (\( \sigma_v \))
  - the amount of noise trading (\( \sigma_Z \)) he can hide behind and take bigger positions

- Half of the information gets revealed in price:
  \[
  \text{Var}(\tilde{\nu} \mid \beta^{eq} \tilde{\nu} + \tilde{Z}) = \frac{\sigma_v^2}{2}
  \]
- Independent of the amount of noise since informed trader adjusts trade size
Kyle Model: Sequential Trading Opportunities

• “Closed form” solution to discrete time model is not obtainable but can be characterized.

  • $\tilde{\nu}$ revealed

  $\tilde{Z}_t$, $t = \{0,1,2,\ldots,19\}$ are i.i.d.

  $$\text{Var}(\tilde{Z}_t) = \frac{\sigma^2_z}{20}$$

• $\text{Var}(\tilde{\nu} | \text{order flow history})$ is declining over time.

• Total profits of informed are proportional to $\sigma_v \sigma_z$.

• Amount of information revealed is independent of $\sigma_z$. 
Kyle Model: Sequential Trading Opportunities

• Kyle analyzes equilibrium in a continuous time model
  • (He shows that the discrete time case converges to the continuous cases as the number of trading periods becomes infinite.)

\[ t \in [0,1] \]

\[ \lambda_t = \frac{\sigma_v}{\sigma_Z} \text{ (a constant!)} \]

\[ \text{var}(\tilde{v} \mid \text{order flow to time } t) = (1 - t)\sigma_v^2 \]

\[ dx(t) = \beta_t (v - p_t) dt \]

\[ \beta_t = \frac{\sigma_Z}{(1-t)\sigma_v} \]

• All information is revealed by the end of trading.
• Ex ante expected profit of informed = \( \sigma_v \sigma_Z \)
  
  ◦ Recall that in single auction it was equal to \( \frac{\sigma_v \sigma_Z}{2} \)
Admati/Pfleiderer: A Theory of Intraday Trading Patterns

Intraday Trading Patterns of Exxon Shares in 1981

<table>
<thead>
<tr>
<th></th>
<th>10 A.M. to 12 Noon</th>
<th>12 Noon to 2 P.M.</th>
<th>2 P.M. to 4 P.M.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Average Volume</strong></td>
<td>179,349</td>
<td>103,024</td>
<td>122,670</td>
</tr>
<tr>
<td><strong>SD (price changes)</strong></td>
<td>0.34959</td>
<td>0.28371</td>
<td>0.37984</td>
</tr>
</tbody>
</table>
Builds on Kyle Model

Day is broken into $T$ periods

Final payoff of risky asset $= \sum_{t=1}^{T} \tilde{v}_t$

At the end of each period $\tilde{v}_t$ becomes public
At the beginning of each period an informed trader can observe $\tilde{v}_t + \tilde{\epsilon}_t$.

• Number of informed traders in period $t = n_t$. 
Two types of liquidity traders

- nondiscretionary have immediate demands, no choice of when to trade
- discretionary can choose when during the day to trade
Admati/Pfleiderer: A Theory of Intraday Trading Patterns

\[ \lambda_t \text{ is proportional to } \frac{1}{\text{SDev}(\text{total liquidity trades})} \times \frac{\sqrt{n_t}}{n_t + 1} \]

- For fixed \( n_t \), discretionary liquidity traders will have incentive to flock together and trade at the same time (safety in numbers).

- The periods in which liquidity trading is pronounced are profitable ones for informed to trade.

- If information is costly, then \( n_t \) is potentially higher in the periods discretionary liquidity traders are trading.
  - This decreases \( \lambda_t \) reinforcing the incentives to flock together.
Things to Ponder

• Long-standing emphasis on value of
  • Prices revealing information
    • Good for consumption and production decisions
  • Liquidity
    • Ability to trade quickly with little price impact increases value of a security
The death of the corporation?
Number of publicly listed U.S. corporations*

*Excluding investment companies, mutual funds, REITS
Source: Craig Doidge, G. Andrew Karolyi, Rene Stulz, Wilshire Associates
Things to Ponder