Modeling Information Revelation and Trading

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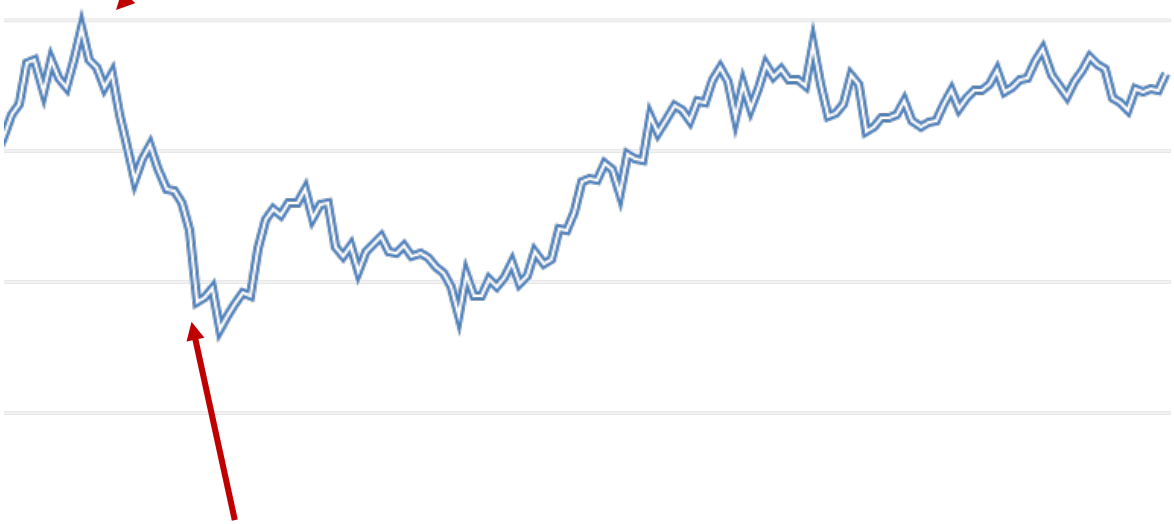
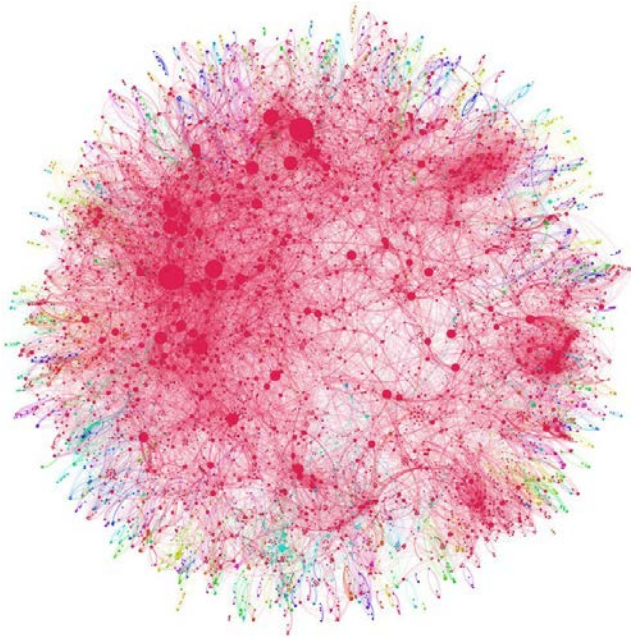
2019 FTG Summer School at Wharton

Buy Low/ Sell High

Earnings Report

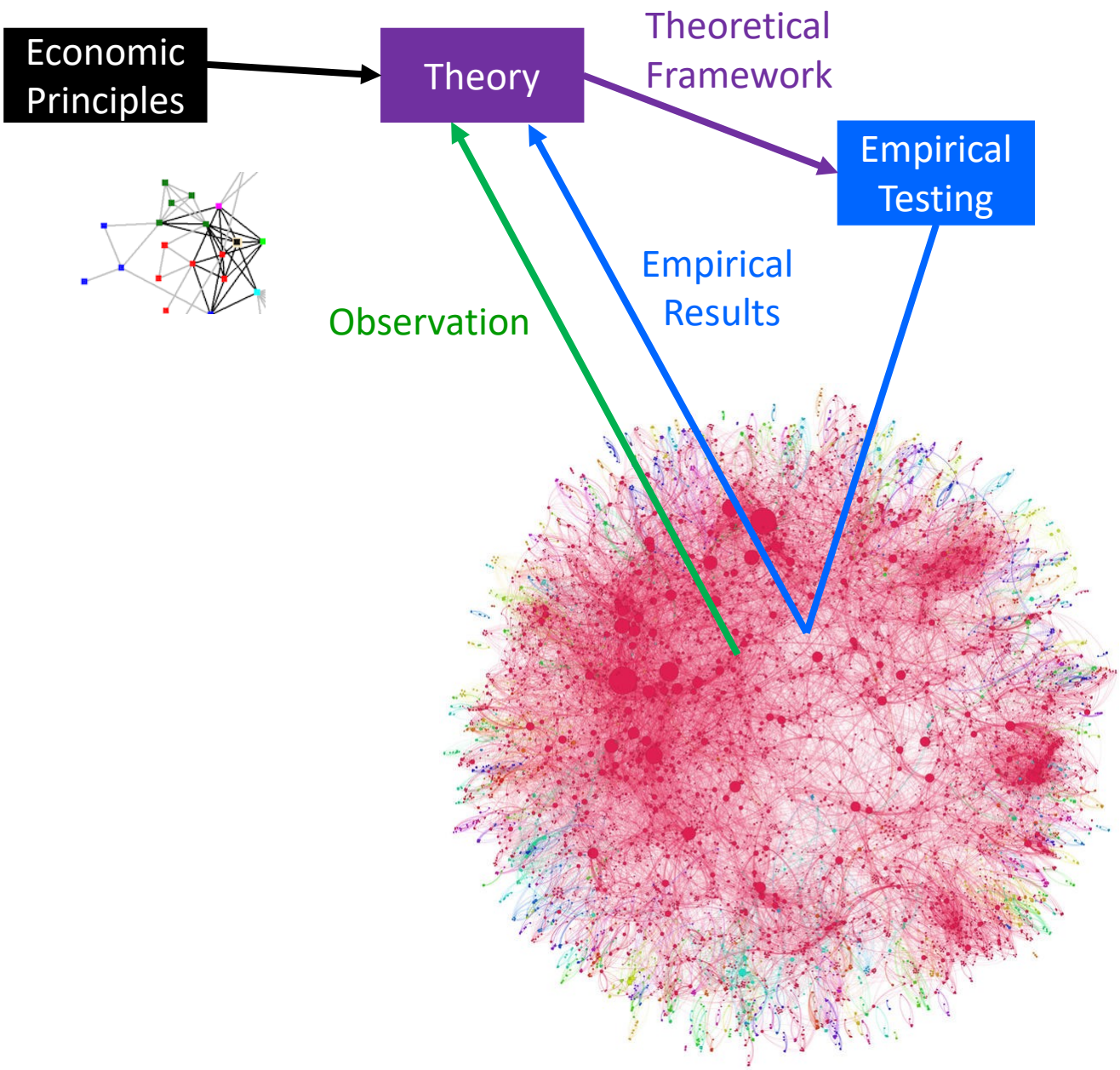
In millions, except per share amounts	Total Company	Per Common Share	Total Company	Per Common Share
Net income (GAAP measure)	$ 1,570		$ 1,620	
Net income attributable to noncontrolling interests (GAAP	—		—	
Income allocable to participating securities (GAAP measure)	—		—	
Net income attributable to CVS Health (GAAP measure)	1,570	$ 1.20	1,620	$ 1.24
Non-GAAP adjustments:				
Amortization of intangible assets	630	0.48	630	0.48
Acquisition-related integration costs	190	0.15	190	0.15
Income tax benefit	(200)	(0.15)	(200)	(0.15)
Income allocable to participating securities, net of tax	—	—	—	—
Adjusted net income attributable to CVS Health	$ 2,190	$ 1.68	$ 2,240	$ 1.72
Weighted average diluted shares outstanding		1,304		1,304

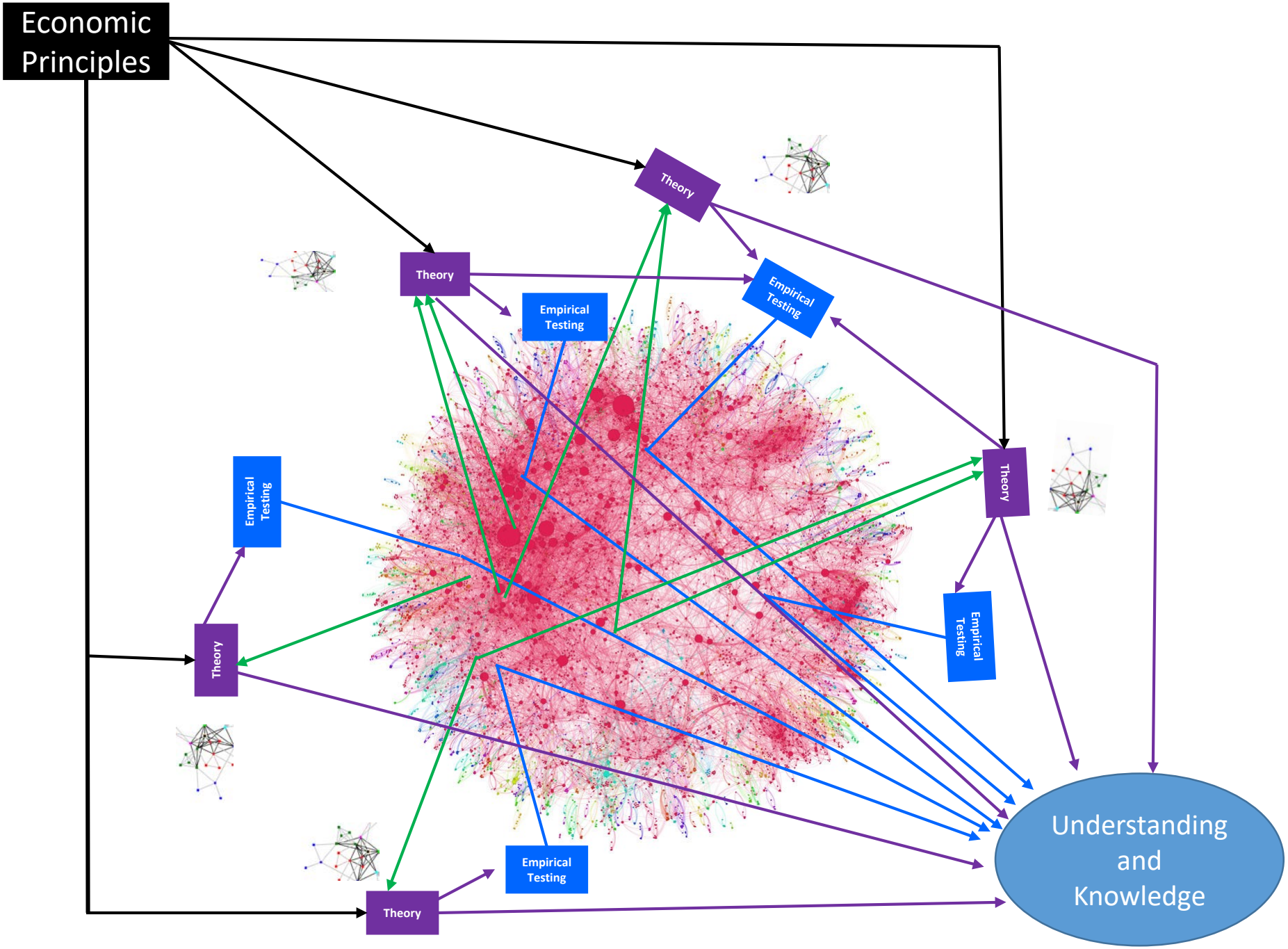




Additional Shares Bought

Price


Economic Principles
Theory
Theoretical Framework
Empirical Testing
Empirical Results
Observation


Economic Principles
Theory
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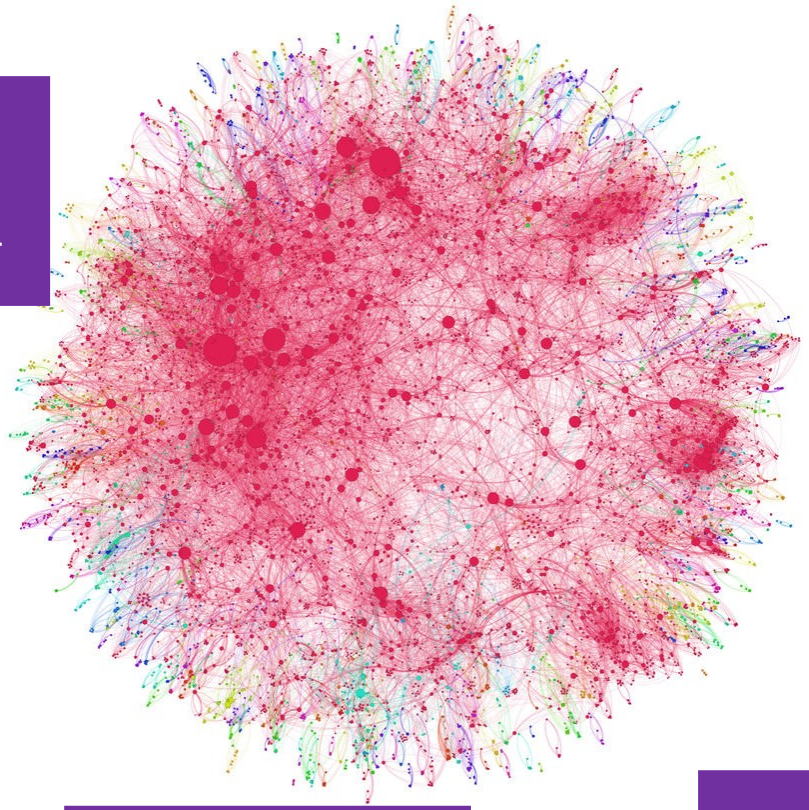
Theory
Understanding and Knowledge

Fully Revealing REE
Grossman

Noisy
REE
Hellwig

Liquidity Trader
Response
Admati/Pfleiderer



Value of
Information in
Noisy REE
Grossman/Stiglitz
Admati/Pfleiderer

Strategic Informed
Trader
Kyle

Short Sale
Restrictions
Diamond/Verrecchia

Market Order
Model
Glosten/Milgrom

Rational Expectations

- Assume an economy with agents who take actions based on beliefs about the joint distribution $G(\bullet)$ of:

  - Exogenous variables: $\tilde{X}$
  - Endogenous variables: $\tilde{Y}$ (determined by agents' actions)
  - Public Information: $\tilde{S}_P$
  - Privately Observed Information: $\tilde{S}_i \quad (i = 1, 2, \ldots, n)$

Rational Expectations

Agent *i* takes action based on his private and public information and based on his assumption about the joint distribution of exogneous and endogenous variables: $\hat{G}_i\ (\bullet)$

$$A_i\left(S_P, S_i \mid \hat{G}_i\left[\tilde{X}, \tilde{Y}, \tilde{S}_P, \tilde{S}_1, \tilde{S}_2, \ldots, \tilde{S}_n\right]\right)$$

$$A_1\left(S_P, S_1 \mid \hat{G}_1\left[\tilde{X}, \tilde{Y}, \tilde{S}_P, \tilde{S}_1, \tilde{S}_2, \ldots, \tilde{S}_n\right]\right)$$
$$A_2\left(S_P, S_2 \mid \hat{G}_2\left[\tilde{X}, \tilde{Y}, \tilde{S}_P, \tilde{S}_1, \tilde{S}_2, \ldots, \tilde{S}_n\right]\right)$$
$$A_3\left(S_P, S_3 \mid \hat{G}_3\left[\tilde{X}, \tilde{Y}, \tilde{S}_P, \tilde{S}_1, \tilde{S}_2, \ldots, \tilde{S}_n\right]\right)$$
$$A_4\left(S_P, S_4 \mid \hat{G}_4\left[\tilde{X}, \tilde{Y}, \tilde{S}_P, \tilde{S}_1, \tilde{S}_2, \ldots, \tilde{S}_n\right]\right)$$
$$\vdots$$
$$A_n\left(S_P, S_n \mid \hat{G}_n\left[\tilde{X}, \tilde{Y}, \tilde{S}_P, \tilde{S}_1, \tilde{S}_2, \ldots, \tilde{S}_n\right]\right)$$

$$\Rightarrow \quad G\left[\tilde{X}, \tilde{Y}, \tilde{S}_P, \tilde{S}_1, \tilde{S}_2, \ldots, \tilde{S}_n\right]$$

Rational Expectations

$$A_1\left(S_P, S_1 \mid \hat{G}_1\left[\tilde{X}, \tilde{Y}, \tilde{S}_P, \tilde{S}_1, \tilde{S}_2, \ldots, \tilde{S}_n\right]\right)$$
$$A_2\left(S_P, S_2 \mid \hat{G}_2\left[\tilde{X}, \tilde{Y}, \tilde{S}_P, \tilde{S}_1, \tilde{S}_2, \ldots, \tilde{S}_n\right]\right)$$
$$A_3\left(S_P, S_3 \mid \hat{G}_3\left[\tilde{X}, \tilde{Y}, \tilde{S}_P, \tilde{S}_1, \tilde{S}_2, \ldots, \tilde{S}_n\right]\right)$$
$$A_4\left(S_P, S_4 \mid \hat{G}_4\left[\tilde{X}, \tilde{Y}, \tilde{S}_P, \tilde{S}_1, \tilde{S}_2, \ldots, \tilde{S}_n\right]\right)$$
$$\vdots$$
$$A_n\left(S_P, S_n \mid \hat{G}_n\left[\tilde{X}, \tilde{Y}, \tilde{S}_P, \tilde{S}_1, \tilde{S}_2, \ldots, \tilde{S}_n\right]\right)$$

$$\Rightarrow \quad G\left[\tilde{X}, \tilde{Y}, \tilde{S}_P, \tilde{S}_1, \tilde{S}_2, \ldots, \tilde{S}_n\right]$$

$$\hat{G}_i\left[\tilde{X}, \tilde{Y}, \tilde{S}_P, \tilde{S}_1, \tilde{S}_2, \ldots, \tilde{S}_n\right] = G\left[\tilde{X}, \tilde{Y}, \tilde{S}_P, \tilde{S}_1, \tilde{S}_2, \ldots, \tilde{S}_n\right] \quad \forall i$$

Rational Expectations

- Traders observe individual pieces of information
- Traders submit demand functions
- Walrasian Auctioneer determines price *P** that clears market where *Z* is total supply.
- *P* wil*l be public information

Information (signals)		Demand Function		
s_1		$D(P, s_1)$		
s_2		$D(P, s_2)$		
s_3	$\Rightarrow$	$D(P, s_3)$	$\Rightarrow$	$\sum_{i=1}^{n} D(P^*, s_i) = Z$
$\vdots$		$\vdots$		
s_n		$D(P, s_n)$		

Tractability: CARA and Normality

- Constant Absolute Risk Aversion (CARA) with risk tolerance ρ

$$u(W) = -\exp\left(-\frac{W}{\rho}\right)$$

Tractability: CARA and Normality

- Constant Absolute Risk Aversion (CARA) with risk tolerance ρ

$$u(W) = -\exp\left(-\frac{W}{\rho}\right)$$

- If W is normally distributed

$$E\left(u\left(\tilde{W}\right)\right) = E\left[-\exp\left(-\frac{\tilde{W}}{\rho}\right)\right]$$

$$= -\exp\left[-\frac{E(\tilde{W})}{\rho} + \frac{Var\left(\tilde{W}\right)}{2\rho^2}\right]$$

Conditional Expectations with A Multivariate Normal Distribution

$$\begin{pmatrix} X \\ Y \end{pmatrix} \sim N\left[\begin{pmatrix} m_X \\ m_Y \end{pmatrix}, \begin{pmatrix} \Sigma_{XX} & \Sigma_{XY} \\ \Sigma_{YX} & \Sigma_{YY} \end{pmatrix}\right]$$

$$\mathrm{E}\left(X \mid Y = y\right) = m_X + \Sigma_{XY}\Sigma_{YY}^{-1}\left(y - m_Y\right)$$

$$\mathrm{Var}\left(X \mid Y = y\right) = \Sigma_{XX} - \Sigma_{XY}\Sigma_{YY}^{-1}\Sigma_{YX}$$

Information = Truth plus Noise

Truth: $\tilde{v}$

Observe noisy signal: $\tilde{s} = \tilde{v} + \tilde{\varepsilon}$

$\tilde{v}$ and $\tilde{\varepsilon}$ distributed jointly normal

$\tilde{\varepsilon}$ is mean zero

$$\begin{pmatrix} X \\ Y \end{pmatrix} \sim N\left[\begin{pmatrix} m_X \\ m_Y \end{pmatrix}, \begin{pmatrix} \Sigma_{XX} & \Sigma_{XY} \\ \Sigma_{YX} & \Sigma_{YY} \end{pmatrix}\right]$$

$$\begin{pmatrix} \tilde{v} \\ \tilde{s} \end{pmatrix} \sim N\left[\begin{pmatrix} \bar{v} \\ \bar{v} \end{pmatrix}, \begin{pmatrix} \sigma_v^2 & \sigma_v^2 \\ \sigma_v^2 & \sigma_v^2 + \sigma_\varepsilon^2 \end{pmatrix}\right]$$

Information = Truth plus Noise

$$\begin{pmatrix} v \\ s \end{pmatrix} \sim N\left[\begin{pmatrix} \bar{v} \\ \bar{v} \end{pmatrix}, \begin{pmatrix} \sigma_v^2 & \sigma_v^2 \\ \sigma_v^2 & \sigma_v^2 + \sigma_\varepsilon^2 \end{pmatrix}\right]$$

$$\mathrm{E}(X \mid Y = y) = m_X + \Sigma_{XY}\Sigma_{YY}^{-1}(y - m_Y)$$

$$\mathrm{E}(v \mid s) = \bar{v} + \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\varepsilon^2}(s - \bar{v})$$

$$= \left(\frac{\sigma_\varepsilon^2}{\sigma_v^2 + \sigma_\varepsilon^2}\right)\bar{v} + \left(\frac{\sigma_v^2}{\sigma_v^2 + \sigma_\varepsilon^2}\right)s$$

Information = Truth plus Noise

$$\begin{pmatrix} v \\ s \end{pmatrix} \sim N\left[\begin{pmatrix} \bar{v} \\ \bar{v} \end{pmatrix}, \begin{pmatrix} \sigma_v^2 & \sigma_v^2 \\ \sigma_v^2 & \sigma_v^2 + \sigma_\varepsilon^2 \end{pmatrix}\right]$$

$$\mathrm{Var}(v \mid s) = \sigma_v^2 - \frac{\sigma_v^4}{\sigma_v^2 + \sigma_\varepsilon^2}$$

$$= \frac{\sigma_v^2 \sigma_\varepsilon^2}{\sigma_v^2 + \sigma_\varepsilon^2}$$

$$= \left(\frac{1}{\sigma_v^2} + \frac{1}{\sigma_\varepsilon^2}\right)^{-1}$$

$$\mathrm{Var}(X \mid Y = y) = \Sigma_{XX} - \Sigma_{XY}\Sigma_{YY}^{-1}\Sigma_{YX}$$

Simple Model

- Assume two period model (date 0 and 1)
- Consider an investor who has CARA preferences, with wealth at date 0 = W_0.
- Investor invests at date 0 and consumes at date 1
- Risk free asset (set risk free rate at 0 for simplicity).
- Risky asset
  - Terminal payoff $\tilde{v} \sim N\left(\bar{v}, \sigma_v^2\right)$
  - Price at date 0 is equal to P
- Investor observes truth plus noise signal $\tilde{s} = \tilde{v} + \tilde{\varepsilon}$.
- If investor buys x units of risk asset:

$$W_1 = \left(W_0 - xP\right) + x\tilde{v}$$

Simple Model

$$W_1 = \left(W_0 - xP\right) + x\tilde{v}$$

- Investor will choose *x* to maximize expected utility:

$$E(\tilde{W}_1) - \frac{Var\left(\tilde{W}_1\right)}{2\rho} = \left(W_0 - xP\right) + xE\left(\tilde{v} \mid \tilde{s}\right) - \frac{x^2 Var\left(\tilde{v} \mid \tilde{s}\right)}{2\rho}$$

- FOC:

$$E\left(\tilde{v} \mid \tilde{s}\right) - P - \frac{xVar\left(\tilde{v} \mid \tilde{s}\right)}{\rho} = 0$$

$$D\left(P, \tilde{s}\right) = \rho \frac{E\left(\tilde{v} \mid \tilde{s}\right) - P}{Var\left(\tilde{v} \mid \tilde{s}\right)} = a_0 + a_1 \tilde{s} - a_2 P$$

Simple Model

- Now assume that there are *n* investors.
  - All have same CARA preferences.
  - Each observes truth plus noise signal $\tilde{s}_i = \tilde{v} + \tilde{\varepsilon}_i$.
  - $\left(\tilde{\varepsilon}_1, \tilde{\varepsilon}_2, \ldots \tilde{\varepsilon}_n\right)$ are *i.i.d.*

- Total supply of risky asset is *Z.*

$$\sum_{i=1}^{n} D\left(P, \tilde{s}_i\right) = \sum_{i=1}^{n}\left(a_0 + a_1 \tilde{s}_i - a_2 P\right) = Z$$

$$P = \frac{a_0}{a_2} + \frac{1}{a_2}\sum_{i=1}^{n}\frac{\tilde{s}_i}{n} - \frac{Z}{na_2}$$

$$P = b_0 + b_1 \sum_{i=1}^{n}\frac{\tilde{s}_i}{n}$$

$$b_0 = \frac{a_0}{a_2} - \frac{Z}{na_2}$$

$$b_1 = \frac{1}{a_2}$$

Simple Model

$$P = b_0 + b_1 \sum_{i=1}^{n} \frac{\tilde{s}_i}{n}$$

- Since the market clearing price will contain additional information about the risky asset payoff, investors will want to use the price as well as their private signal in forming expectations.
- Demand will still be a linear function of the private signal and price, but the coefficients will be different than before:

$$D\left(P, \tilde{s}_i\right) = \rho \frac{E\left(\tilde{v} \mid \tilde{s}_i, P\right) - P}{Var\left(\tilde{v} \mid \tilde{s}_i, P\right)} = a_0 + a_1 \tilde{s}_i - a_2 P$$

Rational Expectations Equilibrium

$$P = b_0 + b_1 \sum_{i=1}^{n} \frac{\tilde{s}_i}{n}$$

- In order to know how to use the price in forming expectations, investors must conjecture how it is related to their signal and the risky asset payoff.
- Demand will still be a linear function of the private signal and price, but the coefficients will be different than before:

$$\{b_0, b_1\} \rightarrow \{a_0, a_1, a_2\} \rightarrow \{b_0^\circ, b_1^\circ\}$$

- Equilibrium is fixed point:

$$\{b_0^*, b_1^*\} \rightarrow \{a_0^*, a_1^*, a_2^*\} \rightarrow \{b_0^*, b_1^*\}$$

Fully Revealing REE

$$P^* = b_0^* + b_1^* \sum_{i=1}^{n} \frac{\tilde{s}_i}{n}$$

- *P** is a fully revealing REE in the sense that it can be inverted to obtain a sufficient statistic for all the private information in the economy.

- How does this come about????

Each period traders come to the market with another realization of $\tilde{y}$, and another $\bar{P}_0(y)$ is found where the auction stops. After many repetitions traders can tabulate the empirical distribution of $(\bar{P}_0, P_1)$ pairs. From this they get a good estimate of the joint distribution of $\bar{P}_0$ and P_1. After this joint distribution is learned, traders will have an incentive to change their bids just as the market is about to clear. This follows from the fact that if everyone observes that the market is about to clear at $\bar{P}_0(y)$, they can condition their beliefs on $\bar{P}_0(y)$ and learn something more about P_1. This changes their demands and thus the market will not clear at $\bar{P}_0(y)$. Suppose instead that the market has been clearing for a long time with prices generated by $P_0^*(y)$, a solution to (12). Then at any particular time, given that traders come to the market with some y, if the market is about to clear at $P_0^*(y)$, and traders then realize that $P_0^*(y)$ is the equilibrium, they will not change their bids due to the new information they get about P_1 from $P_0^*(y)$. $P_0^*(y)$ is a self fulfilling expectations equilibrium: when all traders think prices are generated by $P_0^*(y)$, they will act in such a way that the market clears at $P_0^*(y)$.

Sanford Grossman, "On the Efficiency of Competitive Stock Markets Where Trades Have Diverse Information," *Journal of Finance,* Vol. 31, No. 2, (May, 1976), pp. 573-585

Sufficiently long time series of (P,v) pairs to update distribution of (P,v)

New demand behavior leads to new distribution of (P,v)

Sufficiently long time series of (P,v) pairs to update distribution of (P,v)

New demand behavior leads to new distribution of (P,v)

HOW MANY REPETIONS DOES THIS REQUIRE?

•
•
•

HOW LONG DOES THIS TAKE?

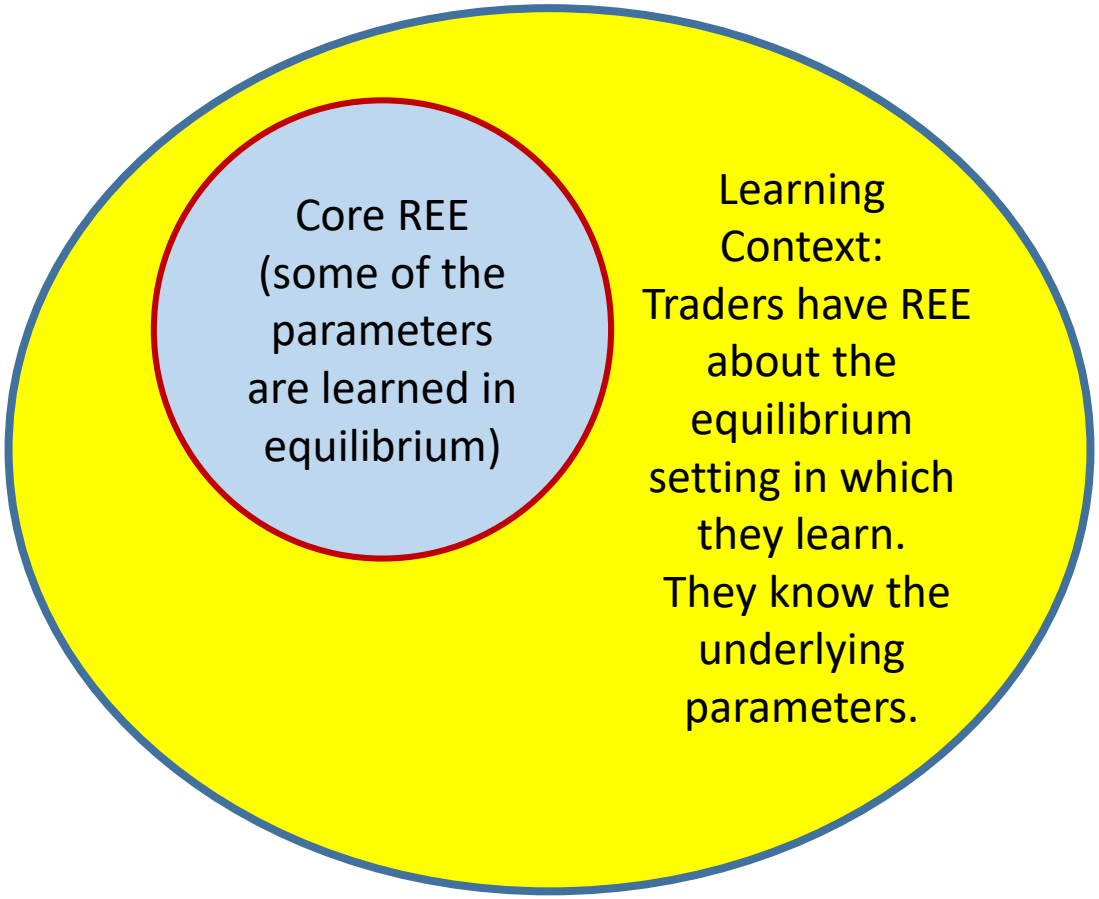
New demand behavior leads to REE distribution of (P^*,v)

DO THE UNDERLYING EXOGENOUS VARIABLES FOLLOW A STATIONARY ERGODIC PROCESS?

Learning Models

Bray, Margaret M. (1982), "Learning, Estimation, and the Stability of Rational Expectations," *Journal of Economic Theory*, 26, 318-339.

Bray, Margaret M., and David Kreps (1987), "Rational Learning and Rational Expectations," in *Arrow and the Ascent of Modern Economic Theory*, New York University Press, New York.



What is needed to support the yellow is arguably more complex than what happens in the blue.

How do traders learn about the yellow? Infinite regress?

For REE Trading out of Market Clearing Equilibrium Is Very Problematic

Léon Walras

1834-1910


CHEAP Lemon Car Lot
Total Steal?
OR
Bad Deal?

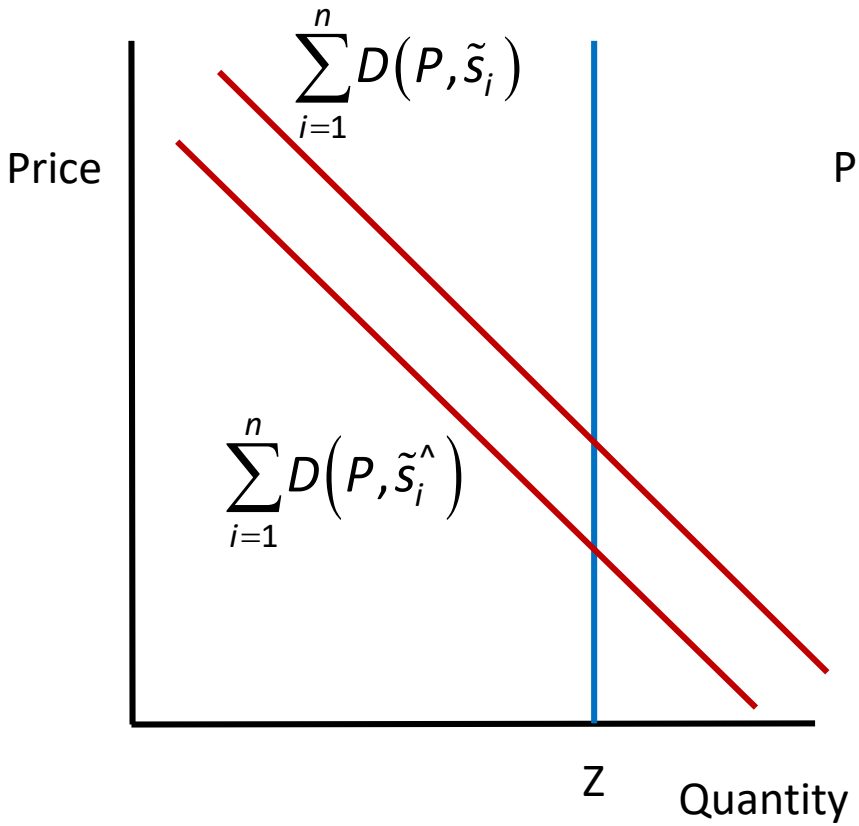
Problem: REE is fully revealing

$$D\left(P^*, \tilde{s}_i\right) = D\left(P^*\right) \qquad \forall i$$

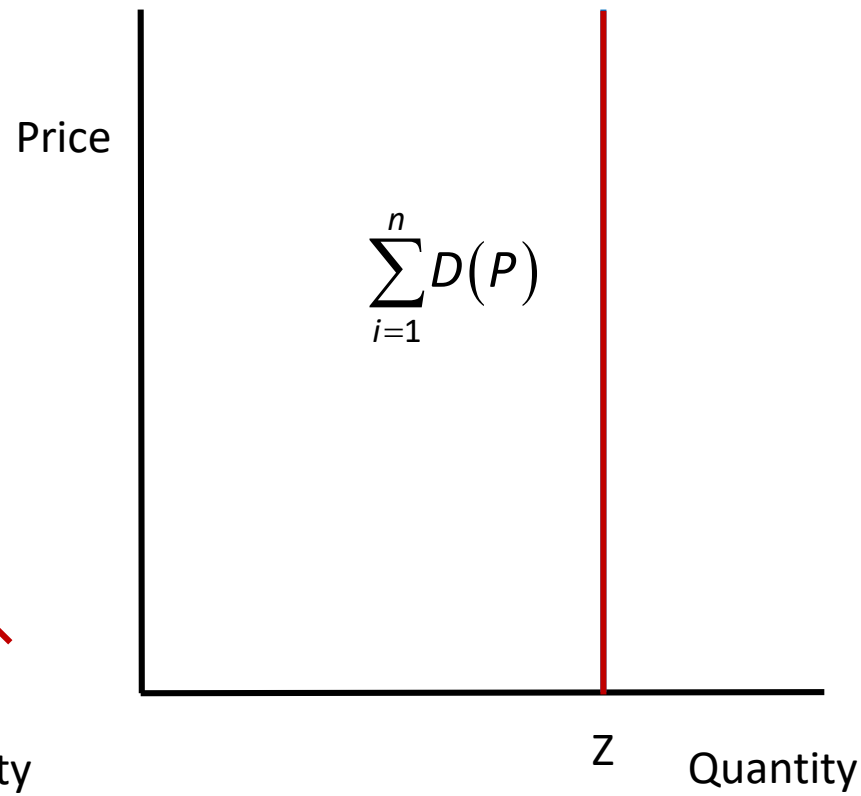
- How does information "get into price" if all traders ignore their private signals **?**

  The Walrasian Auctioneer can call out any price since all prices clear the market.

Problem: REE is fully revealing



Traders only Form Expectations Based on Private Signal



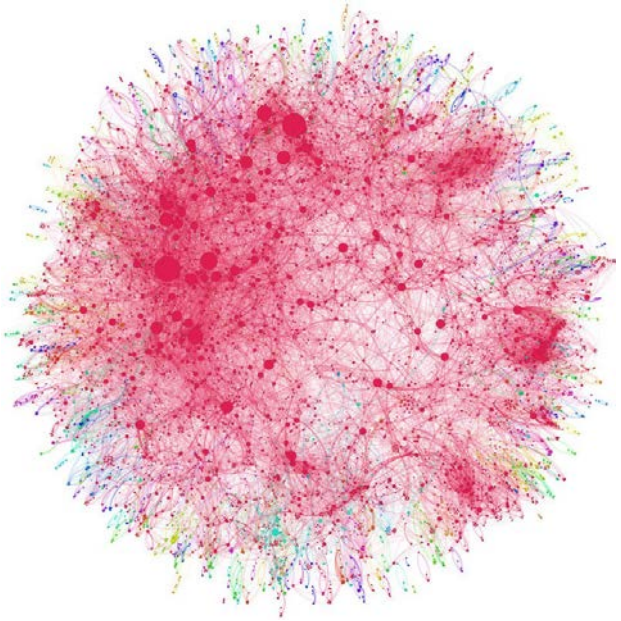
Fully Revealing REE: Traders only Condition on Price

Problem: REE is fully revealing

$$D\left(P^*, \tilde{s}_i\right) = D\left(P^*\right) \quad \forall i$$

- If information is costly (in monetary or effort terms), why would any one acquire a private signal **?**

REE Simplifying Assumptions



Price

Assumption	Realistic?
Multivariate Normality	Payoffs and prices can be negative
CARA	Bill Gates and I invest the same amount in risky assets?
Walrasian Market Mechanism	**How do traders submit demand functions? Are actual trading protocols consistent with auction tatonnement?**
Price taking behavior	**Don't traders worry about market impact?**
Learning	**Stationarity and ergodicity; convergence time**

The Value of Theory:

- Given all the problems with the simple, fully -revealing REE model and the associated paradoxes, what contribution did Grossman's paper make?

- What good is theory?

Efficient Markets

In an efficient market, competition among the many intelligent participants leads to a situation where, at any point in time, *actual prices of individual securities already reflect the effects of information based both on events that have already occurred and on events which, as of now, the market expects to take place in the future*. In other words, in an efficient market at any point in time the actual price of a security will be a good estimate of its intrinsic value.

(Fama, 1970)

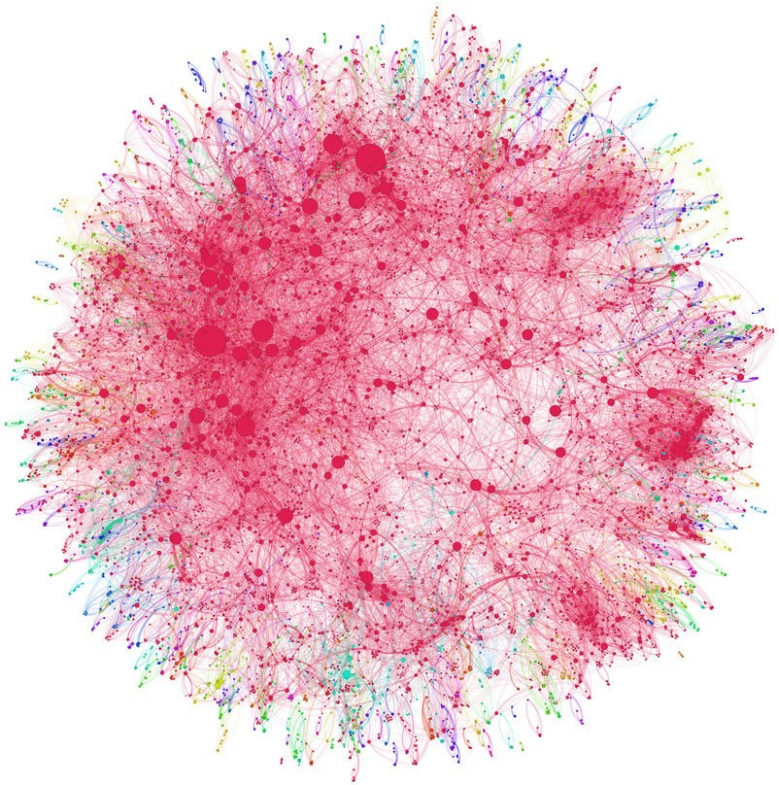
- Weak Form Efficiency
- Semi-strong Form Efficiency
- Strong Form Efficiency

Grossman's Model showed:

- The notion of a perfectly efficient market is fundamentally flawed.
  - Traders will not condition trades on private information.
  - Traders will have no incentive to gather private information.

- How information gets into prices is very delicate.
  - Trading mechanism is clearly important.
  - Traders' demands will be based on some beliefs about what information is already in price.
    - How do they form these beliefs?
    - Are they correct?

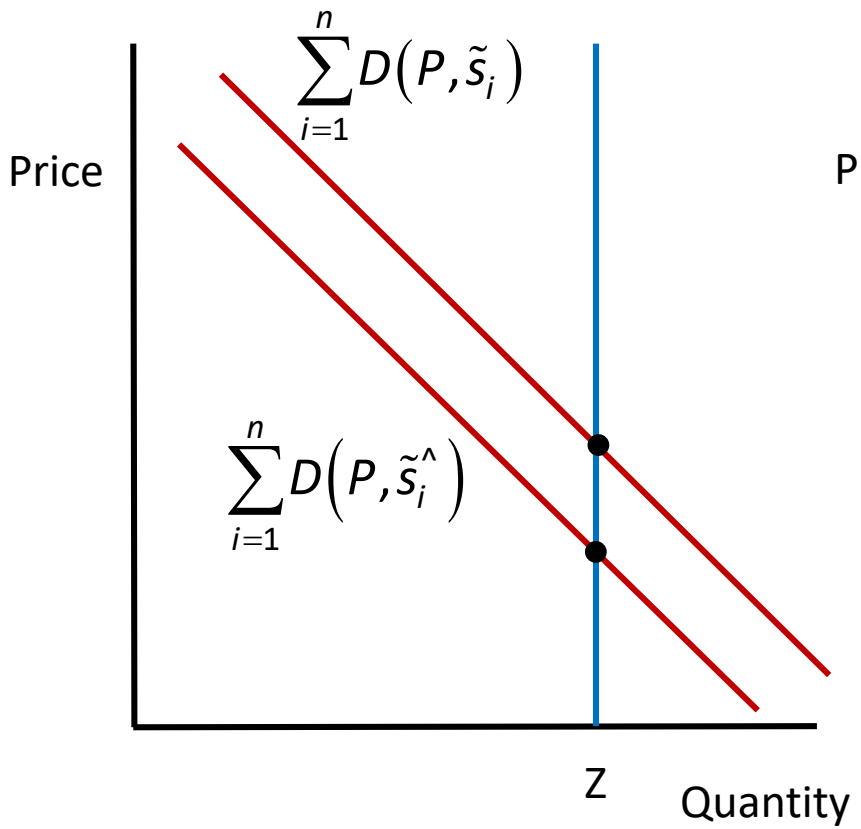
Empirical Work is Not A Substitute for Theory

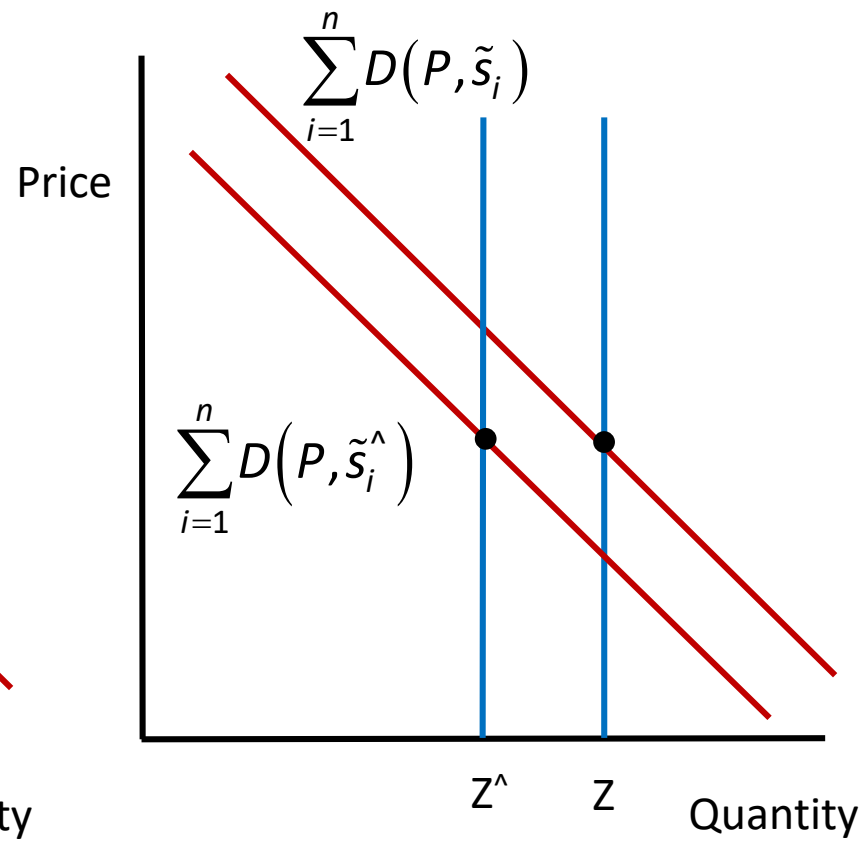


Price

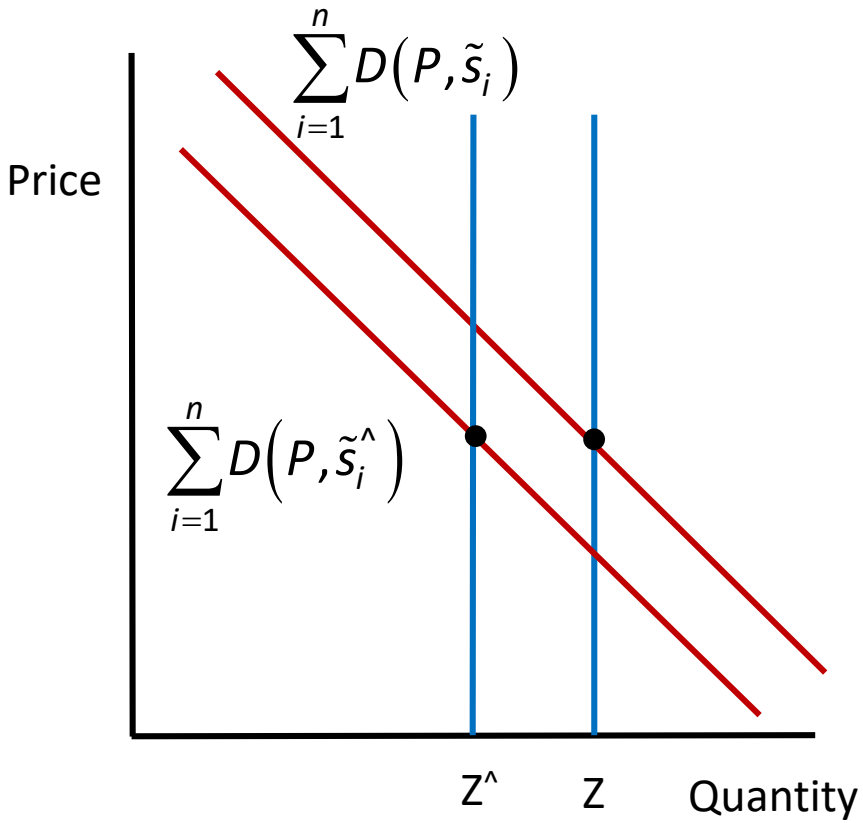
- Would *unstructured* empirical analysis produce insights about the nexus among
  - Traders' beliefs about information in the price and their own information
  - Incentives to gather information
  - Trading mechanisms
  - Risk aversion
  - Degree of competition

REE: Adding "Noise" to the Supply





REE: Adding "Noise" to the Supply



- Traders will still condition on the price but will also put weight on their private signals.
- Traders will potentially find it worthwhile to acquire information at a cost.

Noise:
A Reasonable Assumption or a "Cheat"

Sources of noise:

- "Irrationality"
  - Animal spirits
  - Cognitive errors in processing information (overconfidence)
- Hedging demands
  - Untraded assets correlated with risky asset
- Liquidity needs
  - A little hard to motivate with CARA since in general liquidity needs will be met by trading riskless asset

Value of Information in CARA/Normal Distribution Setting

- Assume (as before):
  - CARA preference with risk tolerance ρ
  - Riskless asset with risk free rate = 0
  - Risky asset
    - Terminal payoff $\tilde{v} \sim N\left(\bar{v}, \sigma_v^2\right)$
    - Price at date 0 is equal to P
  - Trader can acquire signal: $\tilde{s} = \tilde{v} + \tilde{\varepsilon}$
  - Otherwise the trader will form expectations just based on information in price.

Value of Information in CARA/Normal Distribution Setting

Without information

$$W_1\left(D(P)\right) = W_0 + D(P)\left(\tilde{v} - P\right)$$

$$D(P) = \rho \frac{E\left(\tilde{v} \mid P\right) - P}{Var\left(\tilde{v} \mid P\right)}$$

With information

$$W_1\left(D(P,\tilde{s})\right) = W_0 + D(P,\tilde{s})\left(\tilde{v} - P\right)$$

$$D(P,\tilde{s}) = \rho \frac{E\left(\tilde{v} \mid P,\tilde{s}\right) - P}{Var\left(\tilde{v} \mid P,\tilde{s}\right)}$$

Monetary value of information $= \varphi$

$$E\left[-\exp\left(-\frac{W_1\left(D(P)\right)}{\rho}\right)\right] = E\left[-\exp\left(-\frac{W_1\left(D(P,\tilde{s})\right) - \varphi}{\rho}\right)\right]$$

Result: $\varphi = \frac{\rho}{2}\log\left(\frac{\mathrm{var}\left(\tilde{v} \mid P\right)}{\mathrm{var}\left(\tilde{v} \mid \tilde{s}, P\right)}\right)$

Value of Information in CARA/Normal Distribution Setting

$$\text{Monetary value of information} = \frac{\rho}{2}\log\left(\frac{\text{var}\left(\tilde{v} \mid P\right)}{\text{var}\left(\tilde{v} \mid \tilde{s}, P\right)}\right)$$

- Note that value is increasing linearly in risk tolerance
  - The more aggressively one will use the information, the more valuable it is.
- Fixing the informativeness of the private information, the value of the private information *(generally)* decreases as the price becomes more informative.
- If $\text{var}\left(\tilde{v} \mid \tilde{s}, P\right) = 0$, the private information has infinite value.
  - Critical assumption: trading has no price impact and infinite positions can be taken without affecting the price.

Grossman Stiglitz: On the Impossibility of Informationally Efficient Markets

One piece of private information: $\tilde{\theta}$

$$\tilde{v} = \tilde{\theta} + \tilde{\eta}$$

$$E\left(\tilde{v} \mid \tilde{\theta}\right) = \tilde{\theta}$$

Equivalent to "truth plus noise"

$$\tilde{s} = \tilde{v} + \tilde{\varepsilon}$$

$$\tilde{\theta} = E\left(\tilde{v} \mid \tilde{s}\right)$$

$$= \frac{\sigma_\varepsilon^2}{\sigma_v^2 + \sigma_\varepsilon^2}\bar{v} + \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\varepsilon^2}\tilde{s}$$

$$\tilde{\eta} = \tilde{v} - \frac{\sigma_\varepsilon^2}{\sigma_v^2 + \sigma_\varepsilon^2}\bar{v} - \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\varepsilon^2}\tilde{s}$$

$$= \frac{\sigma_\varepsilon^2}{\sigma_v^2 + \sigma_\varepsilon^2}\left(\tilde{v} - \bar{v}\right) - \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\varepsilon^2}\tilde{\varepsilon}$$

Grossman Stiglitz: On the Impossibility of Informationally Efficient Markets

- Assume a continuum of traders on [0,1].
- Fraction λ will buy information (observe θ).
- Continuum means all traders are infinitesimal and can justifiably be thought of as price taking agents.
- Need to be careful that continuum doesn't support something that is not "close to" something we would get with large enough "*n*".

Grossman Stiglitz: On the Impossibility of Informationally Efficient Markets

Key assumption: Supply Z is noisy

$$\tilde{Z} \sim N\left(\bar{Z}, \sigma_Z^2\right)$$

Market clearing price: will solve

$$\lambda\, D^{\text{informed}}\left(P, \tilde{\theta}\right) + \left(1-\lambda\right) D^{\text{uninformed}}\left(P\right) = \tilde{Z}$$

$$D^{\text{informed}}\left(P, \tilde{\theta}\right) = \rho \frac{\tilde{\theta} - P}{\sigma_\eta^2} \qquad D^{\text{uninformed}}\left(P\right) = \rho \frac{E\left(v \mid P\right) - P}{Var\left(v \mid P\right)}$$

Grossman Stiglitz: On the Impossibility of Informationally Efficient Markets

$$\lambda\, D^{\text{informed}}\left(P,\tilde{\theta}\right)+\left(1-\lambda\right)\, D^{\text{uninformed}}\left(P\right)=\tilde{Z}$$

$$\lambda\rho\left(\frac{\tilde{\theta}-P}{\sigma_{\eta}^{2}}\right)+\left(1-\lambda\right)\rho\left(\frac{E\left(v\,|\,P\right)-P}{Var\left(v\,|\,P\right)}\right)=\tilde{Z}$$

$$\frac{\lambda\rho}{\sigma_{\eta}^{2}}\tilde{\theta}-\tilde{Z}+\text{linear function of } P=0$$

- From observing P the uninformed can recover:

$$\tilde{\theta}-\left(\frac{\sigma_{\eta}^{2}}{\lambda\rho}\right)\left(\tilde{Z}-\bar{Z}\right)$$

Grossman Stiglitz: On the Impossibility of Informationally Efficient Markets

$$\varphi = \frac{\rho}{2}\log\left(\frac{\mathrm{var}\left(\tilde{v}\,|\,P\right)}{\mathrm{var}\left(\tilde{v}\,|\,\tilde{\theta}\right)}\right)$$

$$= \frac{\rho}{2}\log\left(\frac{\sigma_\eta^2 + \mathrm{var}\left(\tilde{\theta}\;\middle|\;\tilde{\theta} - \left(\frac{\sigma_\eta^2}{\lambda\rho}\right)\left(\tilde{Z} - \bar{Z}\right)\right)}{\sigma_\eta^2}\right)$$

$$\tilde{v} = \tilde{\theta} + \tilde{\eta}$$

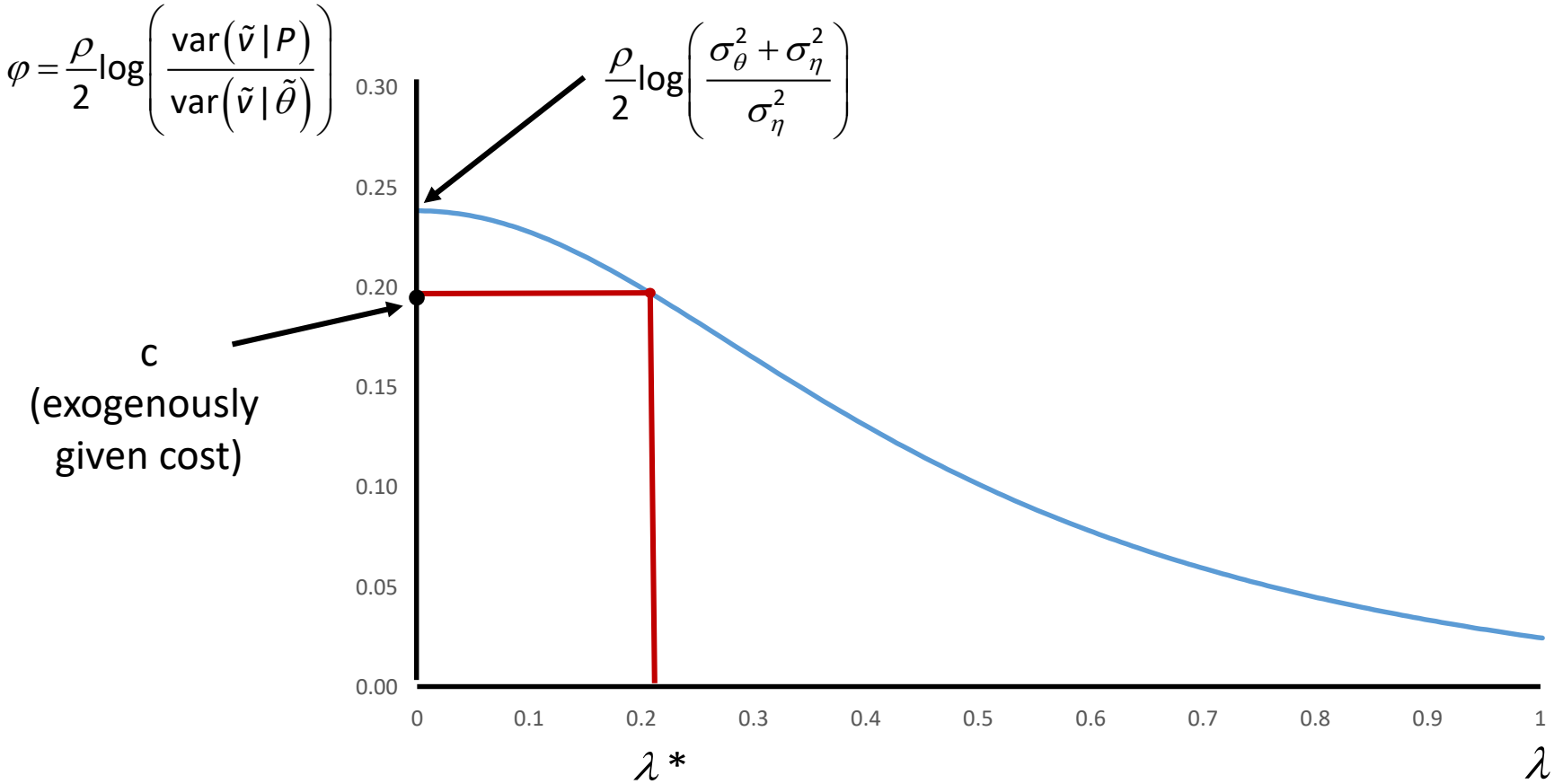
Grossman Stiglitz: On the Impossibility of Informationally Efficient Markets

$$\varphi = \frac{\rho}{2}\log\left(\frac{\sigma_\eta^2 + \text{var}\left(\tilde{\theta} \,\middle|\, \tilde{\theta} - \left(\frac{\sigma_\eta^2}{\lambda\rho}\right)\left(\tilde{Z} - \bar{Z}\right)\right)}{\sigma_\eta^2}\right)$$

Since $\text{var}\left(\tilde{\theta} \,\middle|\, \tilde{\theta} - \left(\frac{\sigma_\eta^2}{\lambda\rho}\right)\left(\tilde{Z} - \bar{Z}\right)\right)$ is decreasing in λ,

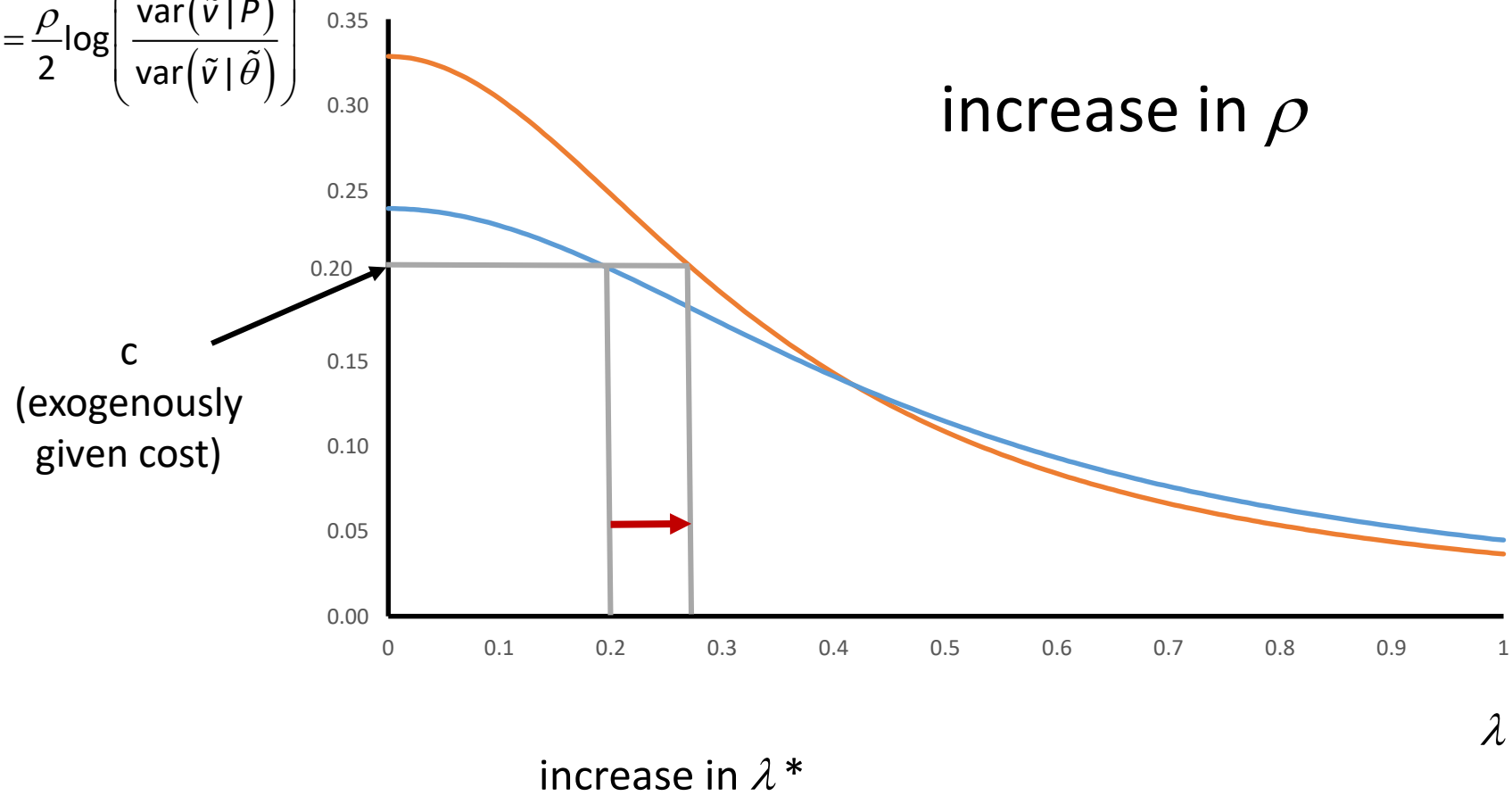
it follows that φ, the value of information is decreasing in λ .

Grossman Stiglitz: On the Impossibility of Informationally Efficient Markets



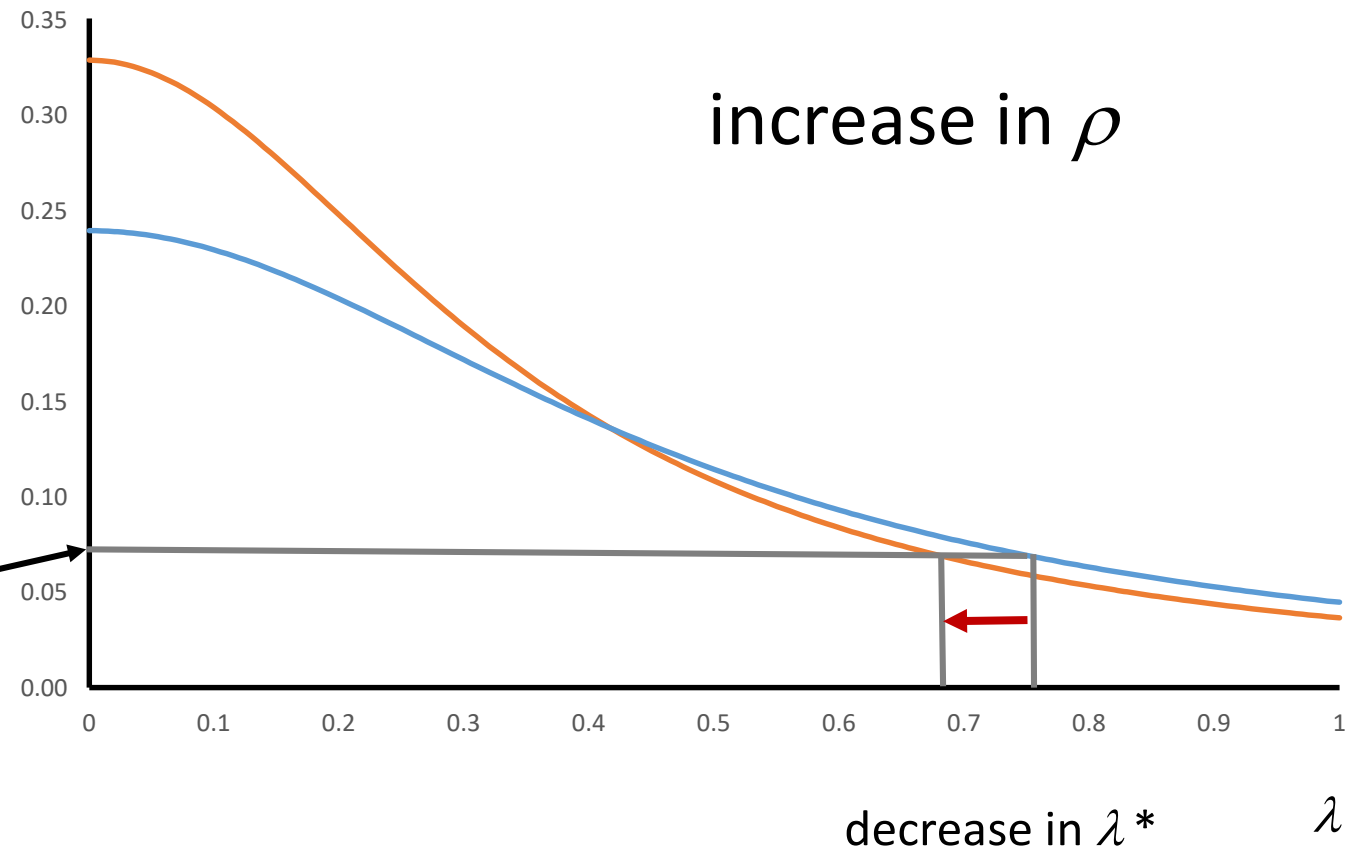
Grossman Stiglitz: On the Impossibility of Informationally Efficient Markets

$$\varphi = \frac{\rho}{2}\log\left(\frac{\mathrm{var}(\tilde{v}\,|\,P)}{\mathrm{var}(\tilde{v}\,|\,\tilde{\theta})}\right)$$



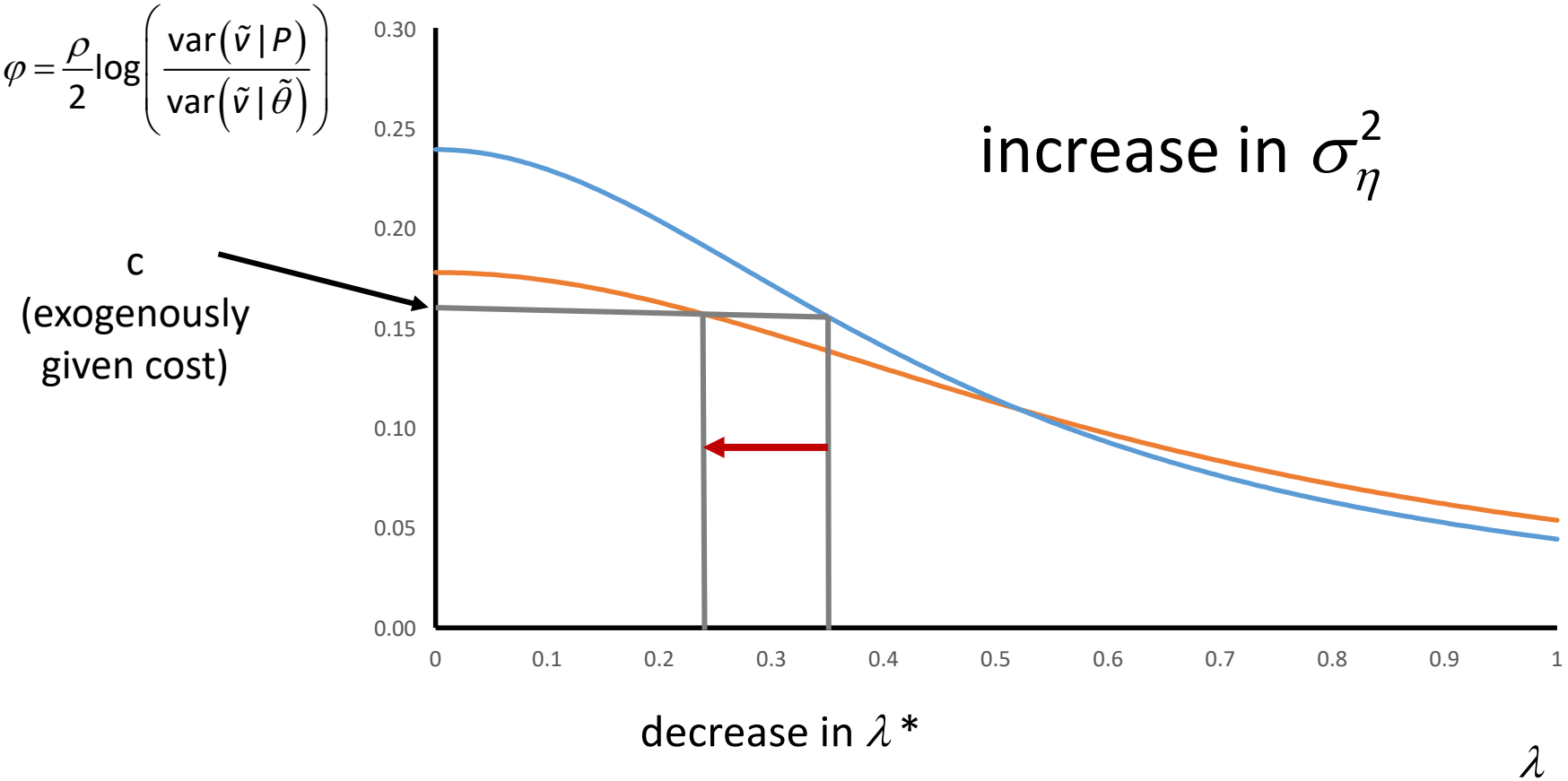
Grossman Stiglitz: On the Impossibility of Informationally Efficient Markets

$$\varphi = \frac{\rho}{2}\log\left(\frac{\mathrm{var}(\tilde{v}\,|\,P)}{\mathrm{var}(\tilde{v}\,|\,\tilde{\theta})}\right)$$



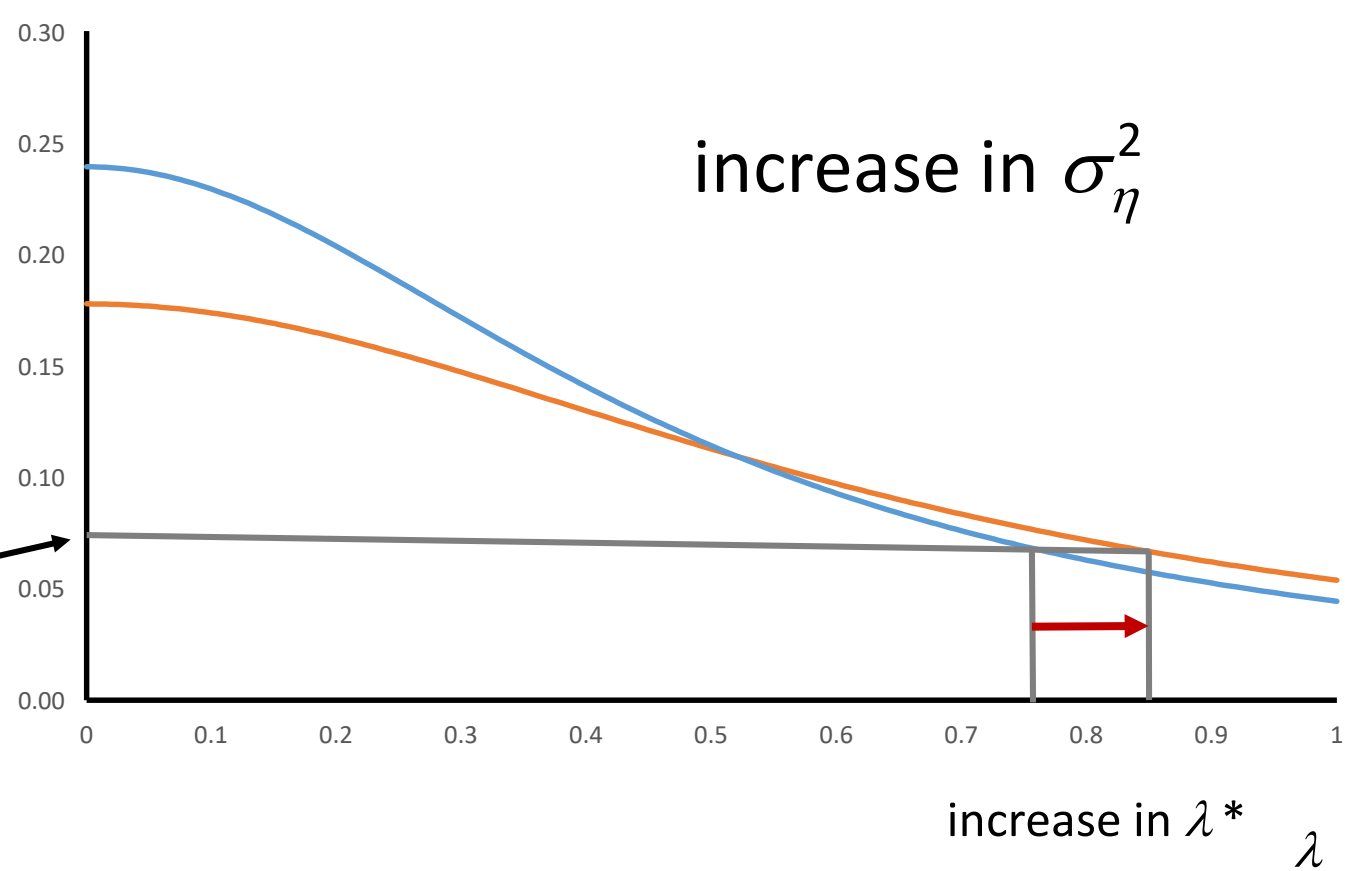
Grossman Stiglitz: On the Impossibility of Informationally Efficient Markets

$$\varphi = \frac{\rho}{2}\log\left(\frac{\mathrm{var}(\tilde{v}\,|\,P)}{\mathrm{var}(\tilde{v}\,|\,\tilde{\theta})}\right)$$



Grossman Stiglitz: On the Impossibility of Informationally Efficient Markets

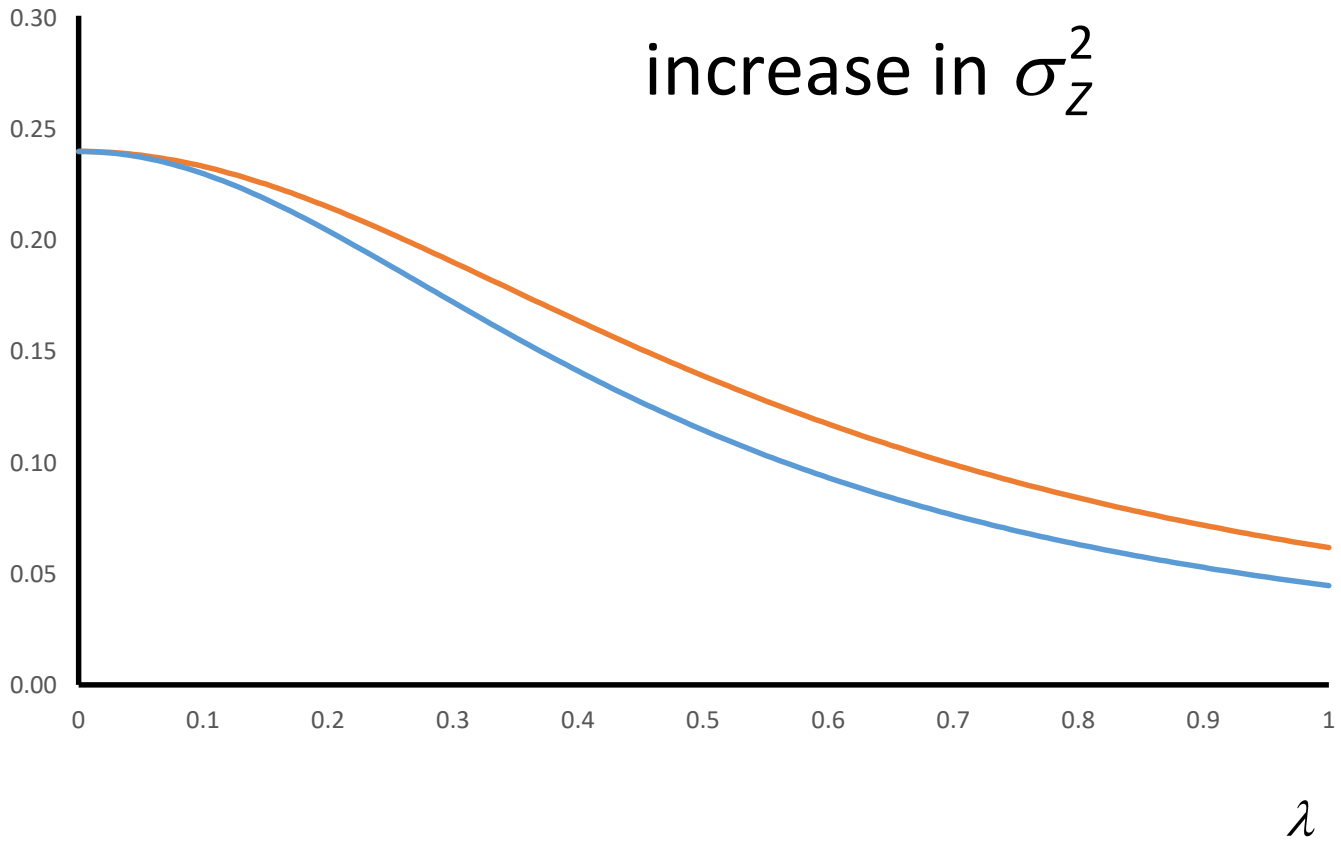
$$\varphi = \frac{\rho}{2}\log\left(\frac{\mathrm{var}(\tilde{v}\,|\,P)}{\mathrm{var}(\tilde{v}\,|\,\tilde{\theta})}\right)$$



c
(exogenously given cost)

Grossman Stiglitz: On the Impossibility of Informationally Efficient Markets

$$\varphi = \frac{\rho}{2}\log\left(\frac{\mathrm{var}(\tilde{v}\,|\,P)}{\mathrm{var}(\tilde{v}\,|\,\tilde{\theta})}\right)$$



Grossman Stiglitz: On the Impossibility of Informationally Efficient Markets

price reveals: $\tilde{\theta} - \left(\frac{\sigma_\eta^2}{\lambda\rho}\right)\left(\tilde{Z} - \bar{Z}\right)$ informativeness: $\left(\frac{\sigma_\eta^2}{\lambda\rho}\right)^2 \sigma_Z^2$

$$\left\{\begin{matrix} \rho \\ \sigma_\eta^2 \\ \sigma_Z^2 \\ c \end{matrix}\right\} \rightarrow \lambda^* \rightarrow \left(\frac{\sigma_\eta^2}{\lambda^* \rho}\right)^2 \sigma_Z^2$$

As long as we have an interior equilibrium, i.e., $\lambda^* \in (0,1)$, a small increase in σ_Z^2 leads to an increase in λ^* which leaves price informitiveness unchanged.

Martin Hellwig: Aggregation of Information

- Grossman-Stiglitz results apply to cases where all privately informed traders observe the <u>*same*</u> <u>*information*</u>, which the uninformed attempt to infer from equilibrium price.
- Hellwig looks at cases where:
  - Traders are endowed with different signals: $\tilde{s}_i = \tilde{v} + \tilde{\varepsilon}_i$

    $E\left(\tilde{\varepsilon}_i \tilde{\varepsilon}_j\right) = 0 \quad \forall i \neq j$
  - Traders have potentially different levels of risk tolearnce: ρ_i
  - Supply $\tilde{Z}$ is noisy: $Var\left(\tilde{Z}\right) = \sigma_z^2 > 0$

Martin Hellwig: Aggregation of Information: Finite (small market) Model

Assume finite number *(n)* of traders. Price will reveal:

$$\sum_{i=1}^{n} \alpha_i \tilde{s}_i - \beta \tilde{Z}$$

α_i will depend on ρ_i and $\mathrm{Var}\left(\tilde{\varepsilon}_i\right)$

- Traders are assumed to be "schizophrenic"
  - When forming expectations they fully account for the correlation between the price and their signal (since the price is affected by their signal).
  - But they otherwise take the price as given and assume their trade has no effect on the price.

Martin Hellwig: Aggregation of Information in a Finite (small market) Model

Assume

- "Schizophrenia"
- Noise in price becomes negligible

As $\mathrm{Var}\left(\tilde{Z}\right) \to 0$

$$P \to A_0 + A_1 \sum_{i=1}^{n} \frac{\tilde{s}_i}{\mathrm{Var}\left(\tilde{\varepsilon}_i\right)} - A_2 \overline{Z}$$

In the limit price perfectly aggregates information

- Weight on individual signals does not depend on risk tolerance ρ_i.
- Weight precisely reflects signal quality!

Martin Hellwig: Aggregation of Information: Finite (small market) Model

- Finite number of traders / vanishing noise result is a curiosity:
  - Schizophrenia assumption is troubling.
  - Having noise is more realistic than no noise.
- Result does show that under special assumptions Grossman Fully Revealing REE can be approached as limit.

Martin Hellwig: Aggregation of Information: Large (truly competitive market) Model

Assume number of traders becomes infinitely large.

For any finite *n*, we still have the price revealing

$$\frac{\sum_{i=1}^{n} \alpha_i^{(n)} \tilde{s}_i - \beta^{(n)} \tilde{Z}}{\sum_{i=1}^{n} \alpha_i^{(n)}} \text{ where } \alpha_i \text{ will depend on } \rho_i \text{ and } \mathrm{Var}\left(\tilde{\varepsilon}_i\right)$$

By the law of large numbers

$$\lim_{n\to\infty} \frac{\sum_{i=1}^{n} \alpha_i^{(n)} \tilde{s}_i}{\sum_{i=1}^{n} \alpha_i^{(n)}} = \lim_{n\to\infty} \frac{\sum_{i=1}^{n} \alpha_i^{(n)} \left(\tilde{v} + \tilde{\varepsilon}_i\right)}{\sum_{i=1}^{n} \alpha_i^{(n)}} = \tilde{v}$$

Martin Hellwig: Aggregation of Information: Large (truly competitive market) Model

As we pass to the limit we can assume that the per capita supply noise is bounded away from zero.

$$P = A_0 + A_1 \tilde{v} - A_2 \left(\tilde{Z} - \bar{Z} \right)$$

We obtain an equilibrium where

- Price reveals "truth" plus supply noise.
- Traders optimally condition on their own information based on variance of $\tilde{\varepsilon}_i$ and on price which is independent of $\tilde{\varepsilon}_i$.
- Traders have "infinitesimal" effects on the price and are justified in their price-taking behavior.

Market Efficiency Again

Weak Form

In the weak-form efficient market hypothesis, all historical prices of securities have already been reflected in the market prices of securities. In other words, technicians – those trading on analysis of historical trading information – should earn no abnormal returns.

Semi-strong Form

In a semi-strong-form efficient market, **prices reflect all publicly known and available information, including all historical price information.** Under this assumption, analyzing any public financial disclosures made by a company to determine a stock's intrinsic value would be futile since every detail would be taken into account in the stock's market price. Similarly, an investor could not earn consistent abnormal returns by acting on surprise announcements since the market would quickly react to the new information.

??

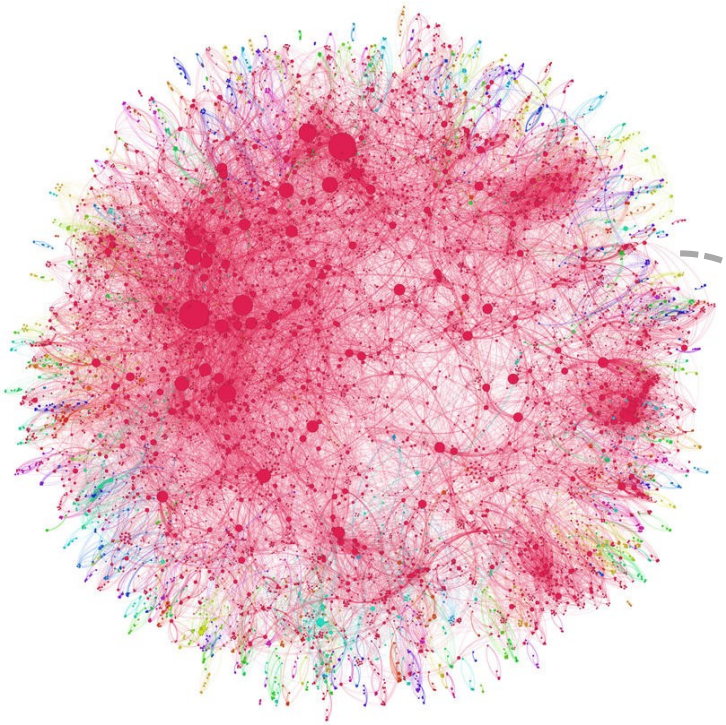
Strong Form

In a strong-form efficient market, **security prices fully reflect both public and private information**. Therefore, insiders could not generate abnormal returns by trading on private information because it would already figure into market prices.

X

Market Efficiency Again

Private Information

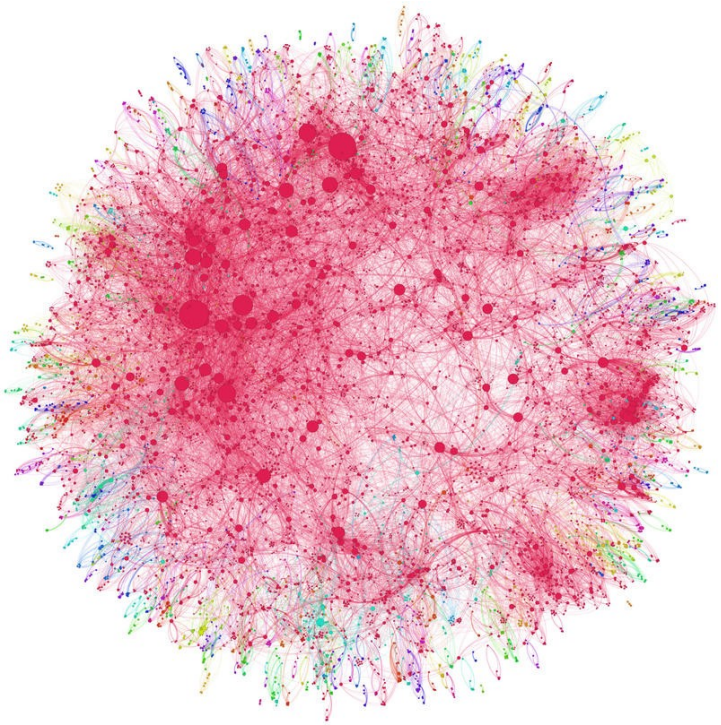


Pr*v&%
@knFF$_{or}$(^atioM

Public Information

PRICE

Market Efficiency Again

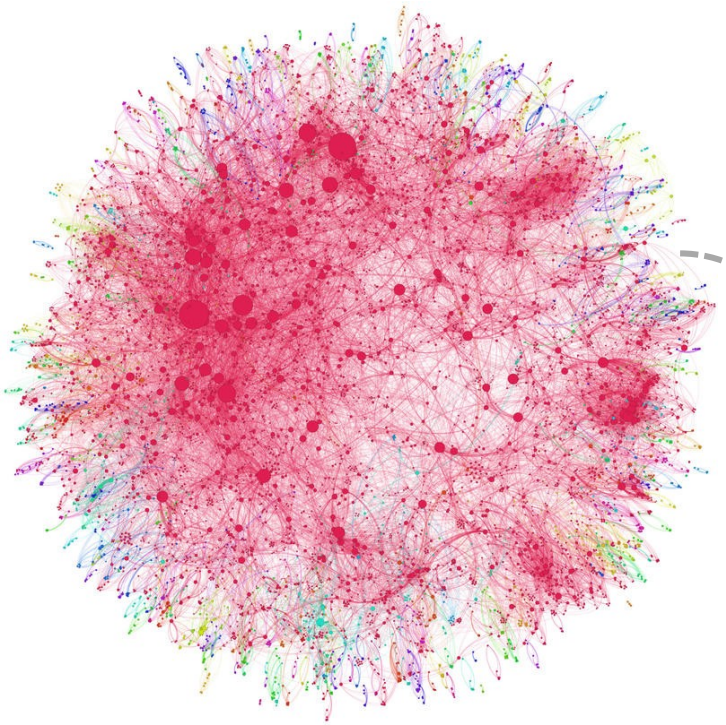


Completely Unanticipated public announcement: Firm must pay a fine equal to $2.00 per share.

$$P_t = P_{t-\delta} - \$2.00$$

Market Efficiency Again

Private Information



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Public announcement: Firm must pay a fine equal to $2.00 per share.

$$P_t = P_{t-\delta} - ?$$

Allocation of Trading Information

- Allocation of information is obviously important
  - What information is out there?
  - Who has it?

Three major ways to acquire information:

- Get lucky and have it fall into your lap ("endowed with information")
- Expend personal effort to get it (a cost)
- Buy it at some cost
  - Who is selling it?
  - What do they sell?
  - What do they charge?

Admati/Pfleiderer
Monopolistic Market for Information

Information Seller

- Monopolistic market is perhaps a reasonable place to start.
- It potentially isolates some of the key tradeoffs.
- Clearly many markets involve substantial degrees of competition.

Admati/Pfleiderer
Monopolistic Market for Information

Model will be based on noisy REE with continuum of price-taking traders.

Information Seller

Need to make lots of special assumptions:

1. Seller does not trade:
   - Clear incentive problems
   - Trading profits will be dwarfed by selling profits
2. Information cannot be resold
   - No problem if resale of a particular signal is limited to a set of measure zero.
   - Partial justification: must be used in trade immediately — there is no time for resale.

Admati/Pfleiderer
Monopolistic Market for Information

- Continuum of traders
- Risky asset payoff : $\tilde{v}$
- Noisy supply (per capita) : $\tilde{Z}$
- Seller's information : $\tilde{v} + \tilde{\omega}$
- Information sold to trader i : $\tilde{v} + \tilde{\omega} + \tilde{\varepsilon}_i$
- Let A represent the allocation of information across all traders.

Price will reveal:

$$\tilde{v} + \tilde{\omega} + \tilde{\eta}(A) + G(A)\tilde{Z}$$

where $\tilde{\eta}(A)$ is a weighted sum of all the $\tilde{\varepsilon}_i$

Admati/Pfleiderer
Monopolistic Market for Information

$$\tilde{v} + \tilde{\omega} + \tilde{\eta}(A) + G(A)\tilde{Z}$$

Aggregation of the noise that is added by seller to each piece of information sold.

Contribution of noise due to noisy supply is affected by allocation of information.

Admati/Pfleiderer
Monopolistic Market for Information

$$\text{Monetary value of information} = \frac{\rho}{2}\log\left(\frac{\text{var}\left(\tilde{v}\,|\,P\right)}{\text{var}\left(\tilde{v}\,|\,\tilde{s}_i, P\right)}\right)$$

$$\text{Value of } s_i = \tilde{v} + \tilde{\omega} + \tilde{\varepsilon}_i =$$

$$\frac{\rho}{2}\log\left(\frac{\text{var}\left(\tilde{v}\ \ |\ \ \tilde{v} + \tilde{\omega} + \tilde{\eta}\left(A\right) + G\left(A\right)\tilde{Z}\right)}{\text{var}\left(\tilde{v}\ \ |\ \ \tilde{v} + \tilde{\omega} + \tilde{\varepsilon}_i,\ \ \tilde{v} + \tilde{\omega} + \tilde{\eta}\left(A\right) + G\left(A\right)\tilde{Z}\right)}\right)$$

Admati/Pfleiderer
Monopolistic Market for Information

- Why should the Monopolistic Seller add noise and thereby decrease the intrinsic value of what he is selling?

$$\text{value}=\frac{\rho}{2}\log\left(\frac{\text{var}\left(\tilde{v}\,|\,P\right)}{\text{var}\left(\tilde{v}\,|\,\tilde{s}_i,P\right)}\right)$$

- Holding the statistical properties of *P* constant:

Increasing the quality of $\tilde{s}_i$ decreases $\text{var}\left(\tilde{v}\,|\,\tilde{s}_i,P\right)$

and increases $\frac{\rho}{2}\log\left(\frac{\text{var}\left(\tilde{v}\,|\,P\right)}{\text{var}\left(\tilde{v}\,|\,\tilde{s}_i,P\right)}\right)$

Admati/Pfleiderer
Monopolistic Market for Information

Increasing the quality of signals sold will generally *improve the value of P* in forecasting *v*

The monopolist is concerned about the ratio

$$\frac{\mathrm{var}\left(\tilde{v} \mid P\right)}{\mathrm{var}\left(\tilde{v} \mid \tilde{s}_i, P\right)}$$

Admati/Pfleiderer
Monopolistic Market for Information

Consider an extreme case:

Seller's information : $\tilde{v} + \tilde{\omega}$

$\mathrm{Var}(\tilde{\omega})$ is EXTREMELY small

Information sold to trader i : $\tilde{v} + \tilde{\omega}$

Even if the monopolist sells to only a small fraction of the market, the informed will trade very aggressively.

The price will reveal:

$$\tilde{v} + \tilde{\omega} + G(A)\tilde{Z}$$

with $G(A)$ being extremely small

Admati/Pfleiderer
Monopolistic Market for Information

$$\mathrm{var}\left(\tilde{v} \mid \tilde{v} + \tilde{\omega} + G(A)\tilde{Z}\right) \approx \mathrm{var}\left(\tilde{v} \mid \tilde{v} + \tilde{\omega}\right)$$

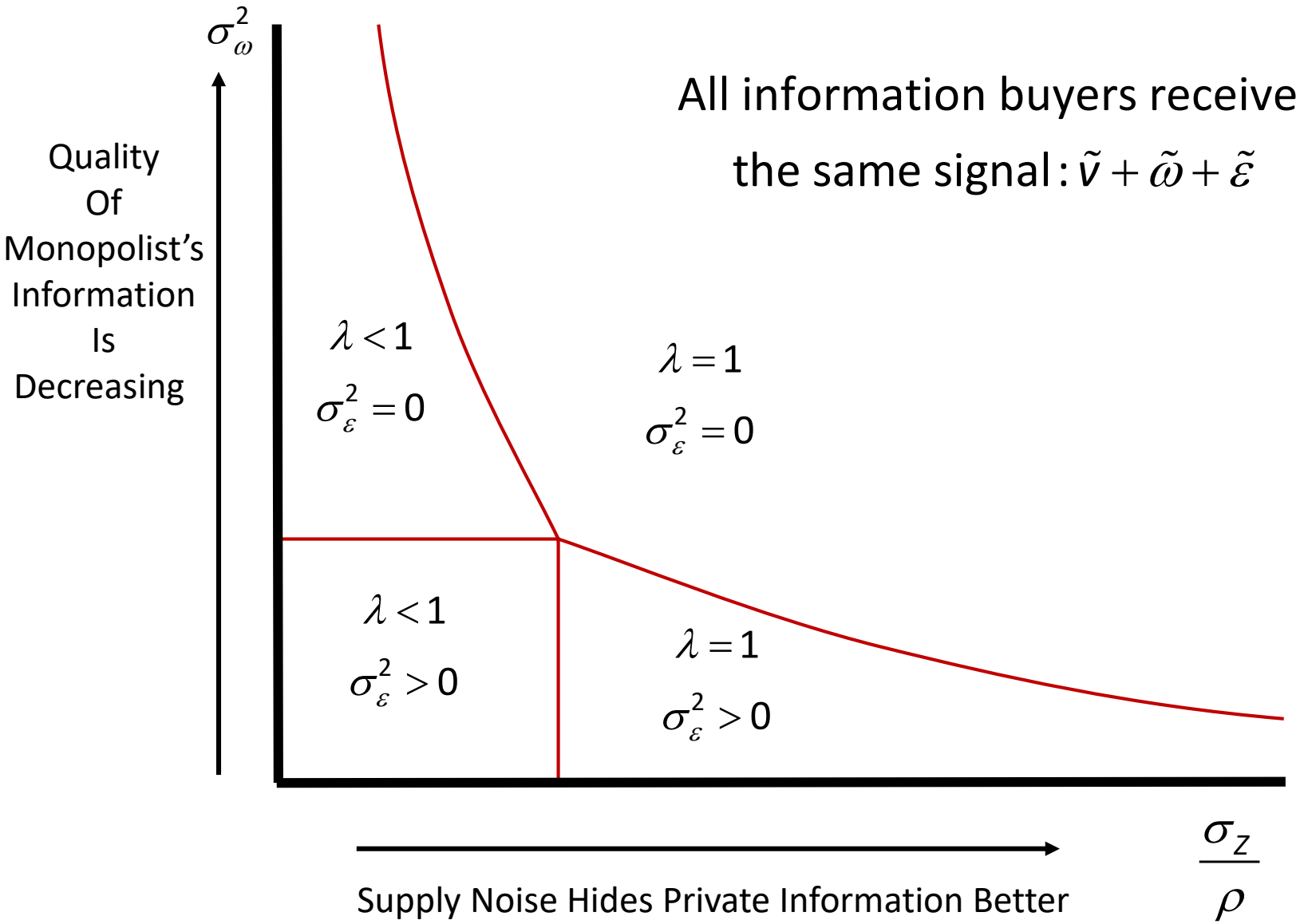
$$\text{Monetary value of information} = \frac{\rho}{2}\log\left(\frac{\mathrm{var}\left(\tilde{v} \mid P\right)}{\mathrm{var}\left(\tilde{v} \mid \tilde{s}_i, P\right)}\right)$$

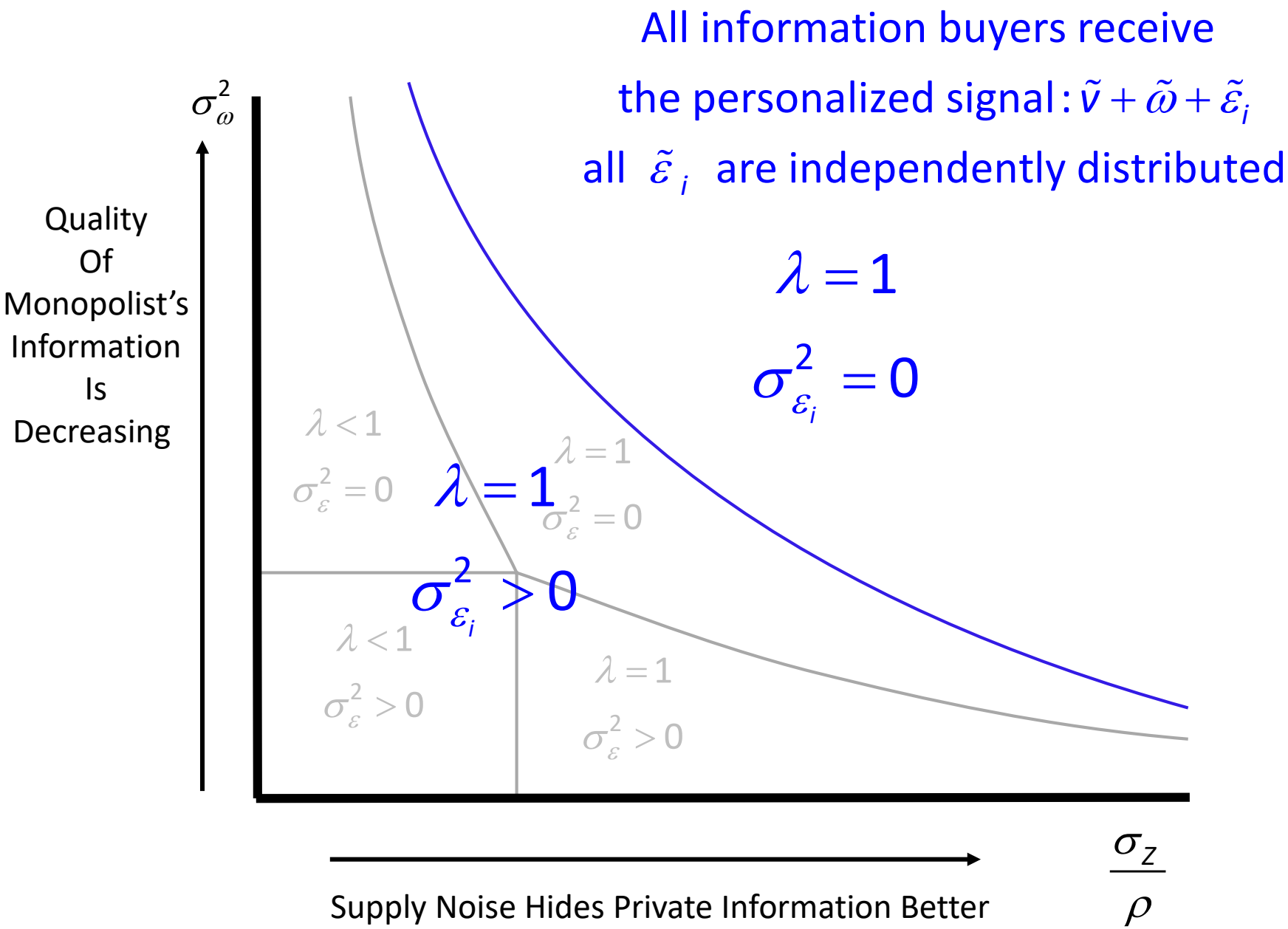
$$\approx 0$$

Virtually all of the information leaks out in price.
Informed are not much better off than uninformed.

Admati/Pfleiderer
Monopolistic Market for Information

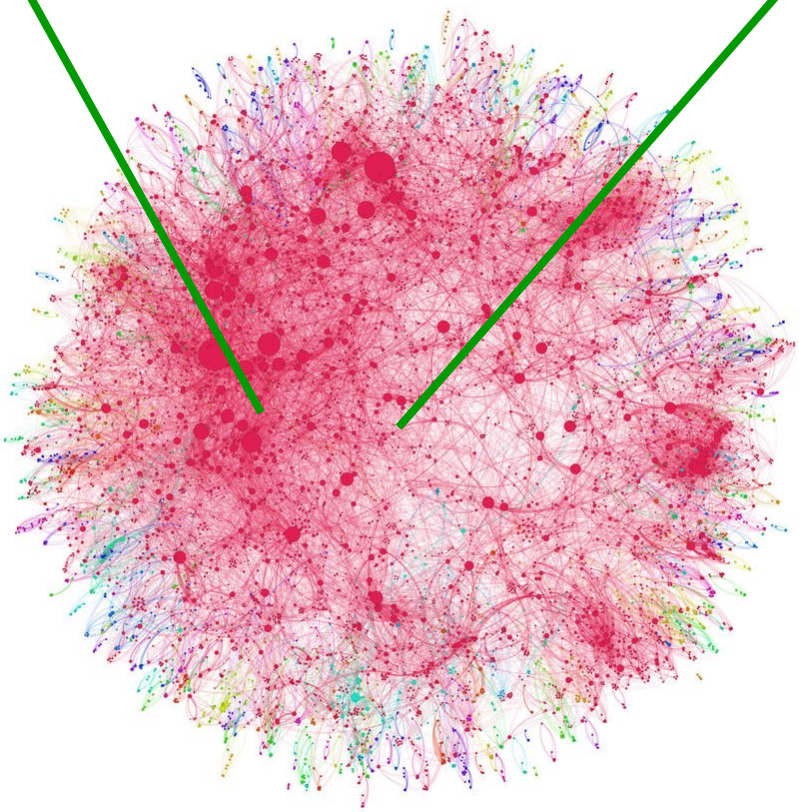
- Two ways to control dilution of value of information through its (partial) revelation in P:
  - Reduce λ, fraction of traders who acquire and trade on information
  - Add noise to what is sold: sell $\tilde{v} + \tilde{\omega} + \tilde{\varepsilon}$


σ_ω^2
Quality Of Monopolist's Information Is Decreasing
All information buyers receive the same signal: $\tilde{v} + \tilde{\omega} + \tilde{\varepsilon}$
$\lambda < 1$
$\sigma_\varepsilon^2 = 0$
$\lambda = 1$
$\sigma_\varepsilon^2 = 0$
$\lambda < 1$
$\sigma_\varepsilon^2 > 0$
$\lambda = 1$
$\sigma_\varepsilon^2 > 0$
Supply Noise Hides Private Information Better
$\frac{\sigma_Z}{\rho}$


All information buyers receive
the personalized signal: $\tilde{v} + \tilde{\omega} + \tilde{\varepsilon}_i$
all $\tilde{\varepsilon}_i$ are independently distributed
$\lambda = 1$
$\sigma^2_{\varepsilon_i} = 0$
σ^2_{ω}
Quality Of Monopolist's Information Is Decreasing
$\lambda < 1$
$\sigma^2_{\varepsilon} = 0$
$\lambda = 1$
$\sigma^2_{\varepsilon} = 0$
$\lambda = 1$
$\sigma^2_{\varepsilon_i} > 0$
$\lambda < 1$
$\sigma^2_{\varepsilon} > 0$
$\lambda = 1$
$\sigma^2_{\varepsilon} > 0$
Supply Noise Hides Private Information Better
$\frac{\sigma_Z}{\rho}$

Impossible in practice to submit demand functions to an auctioneer

Market orders can be submitted to a specialist dealer.



Glosten and Milgrom: Dealer Market

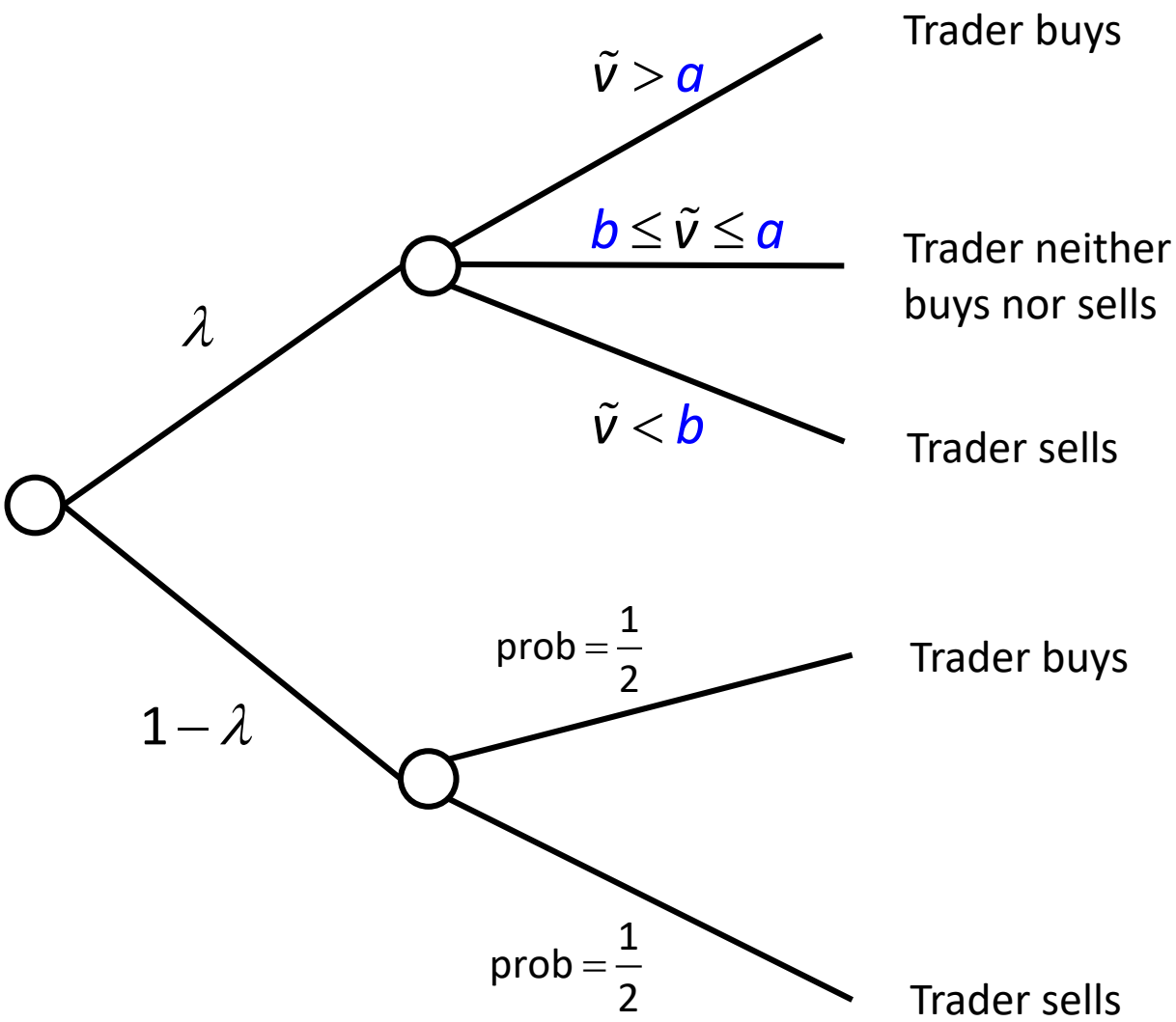
- Specialist (dealer) makes a market by setting a bid (ask) price at which she is willing to buy (sell) one unit of a risky asset.
- Assume (for illustrative purposes) that risky the asset pays off $\tilde{v} \sim U(0,1)$.
- Two types of traders might show up at the specialist booth to trade:
  - Informed trader who knows $\tilde{v} \sim U(0,1)$
  - "Liquidity" trader who is uninformed but has inelastic demand to either buy or sell; equally like to want to sell or buy

Glosten and Milgrom: Dealer Market

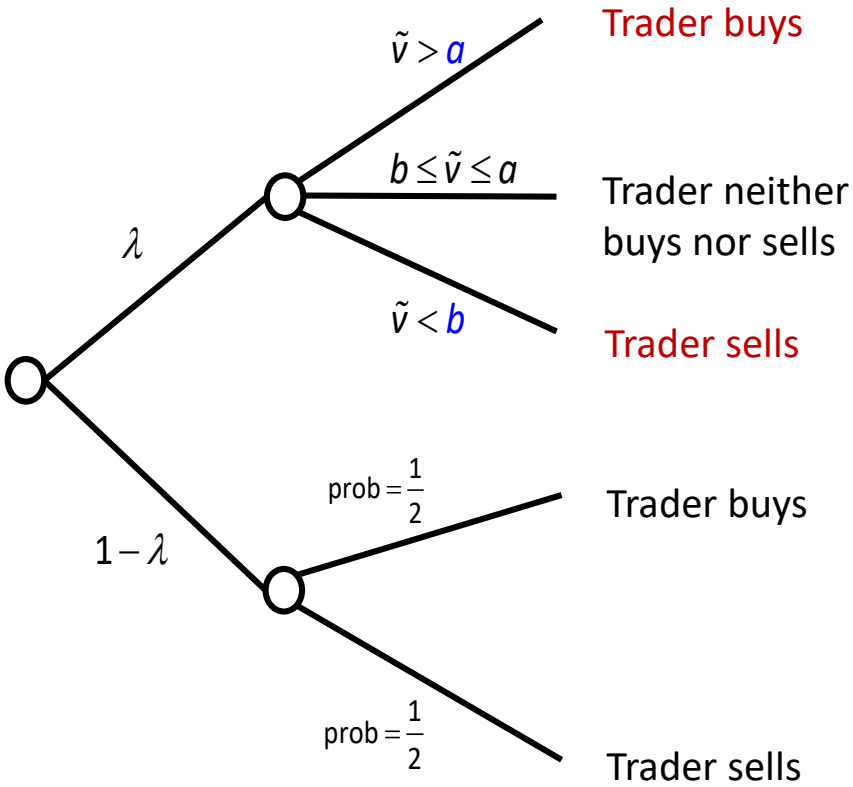
- Specialist has no private information about risky asset payoff.
- Specialist knows that the probability that the trader arriving at her booth is informed is λ; probability that trader has liquidity motives is therefore $(1-\lambda)$.
- Specialist is assumed to be risk neutral and have no cost of making the market.
- Specialist competes in a perfectly competitive market and so bid and ask prices are set such that zero profit is earned.
  - Bertrand competition with two or more specialists

Glosten and Milgrom: Dealer Market

Assume dealer sets ask at a and bid at b



Glosten and Milgrom: Dealer Market

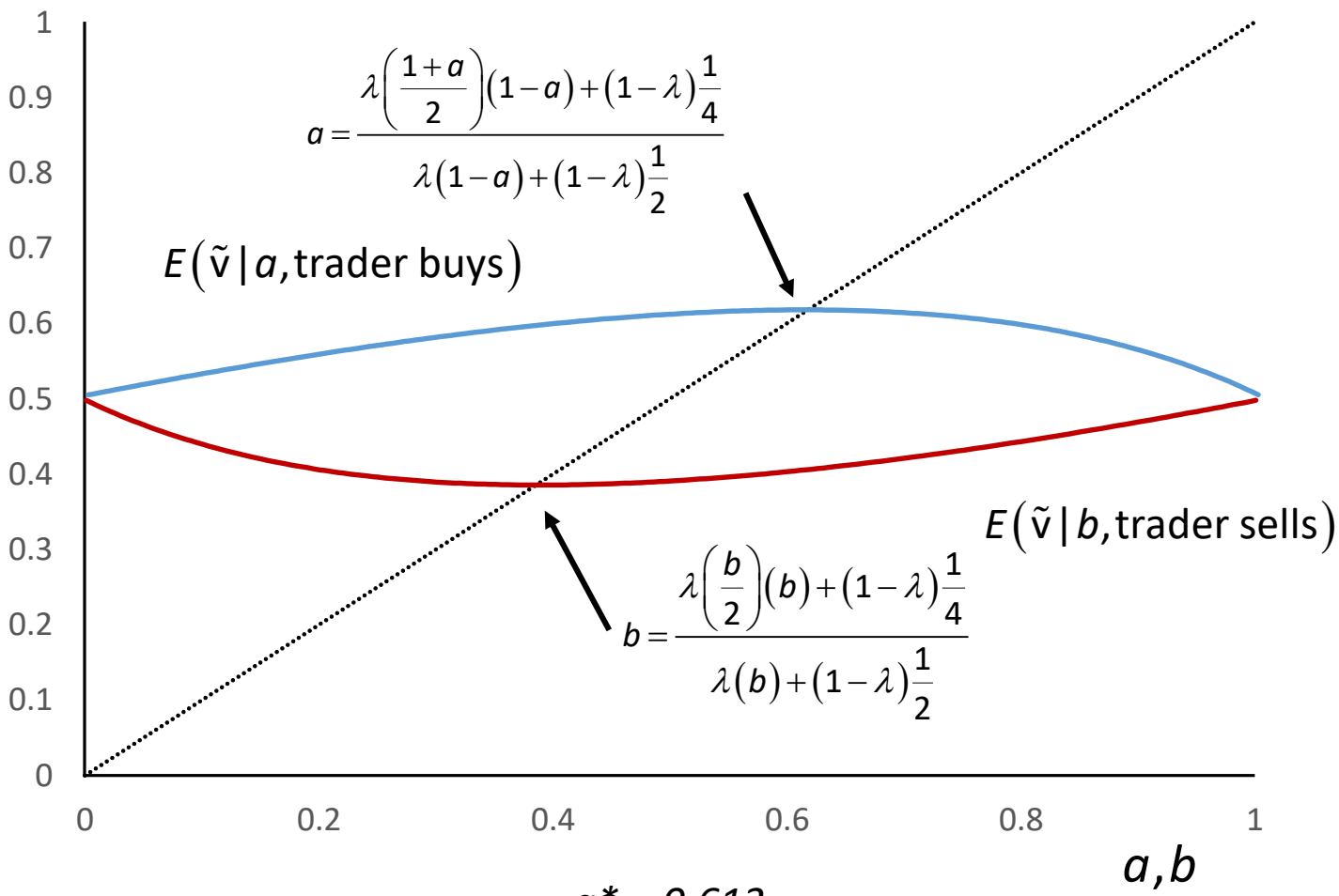


$$E(\tilde{v} \mid \text{trader buys}) = \frac{\lambda\left(\frac{1+a}{2}\right)(1-a) + (1-\lambda)\frac{1}{4}}{\lambda(1-a) + (1-\lambda)\frac{1}{2}}$$

$$E(\tilde{v} \mid \text{trader sells}) = \frac{\lambda\left(\frac{b}{2}\right)(b) + (1-\lambda)\frac{1}{4}}{\lambda(b) + (1-\lambda)\frac{1}{2}}$$

Equilibrium bid and ask

$\lambda = 0.60$

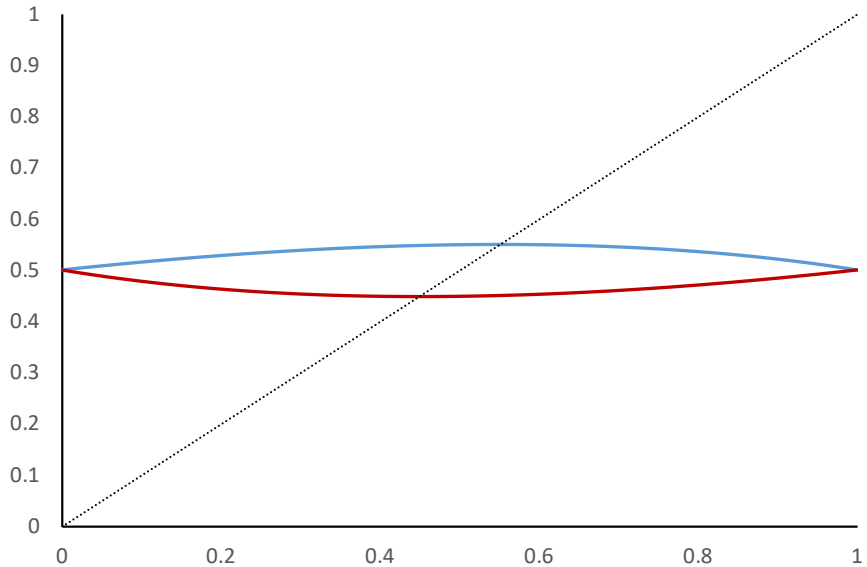


$a^* = 0.613$

$b^* = 0.387$

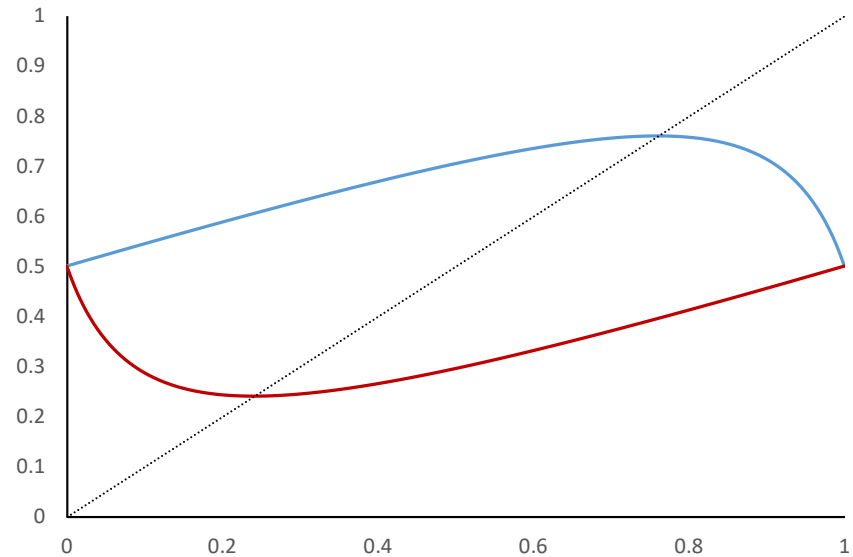
As is intuitive, the bid/ask spread widens as probability of informed increases

$\lambda = 0.20$



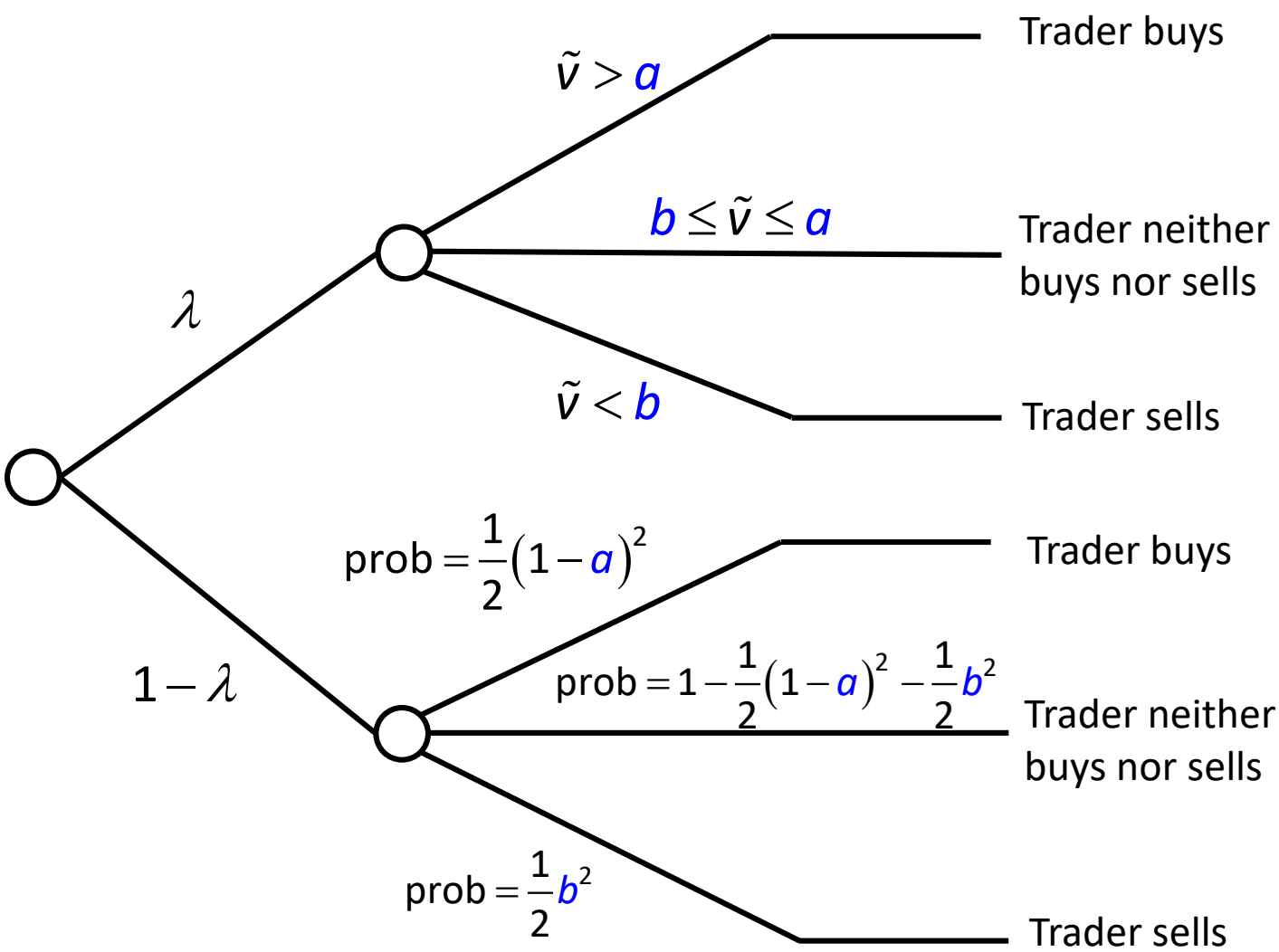
a = 0.53*
b = 0.47*

$\lambda = 0.90$



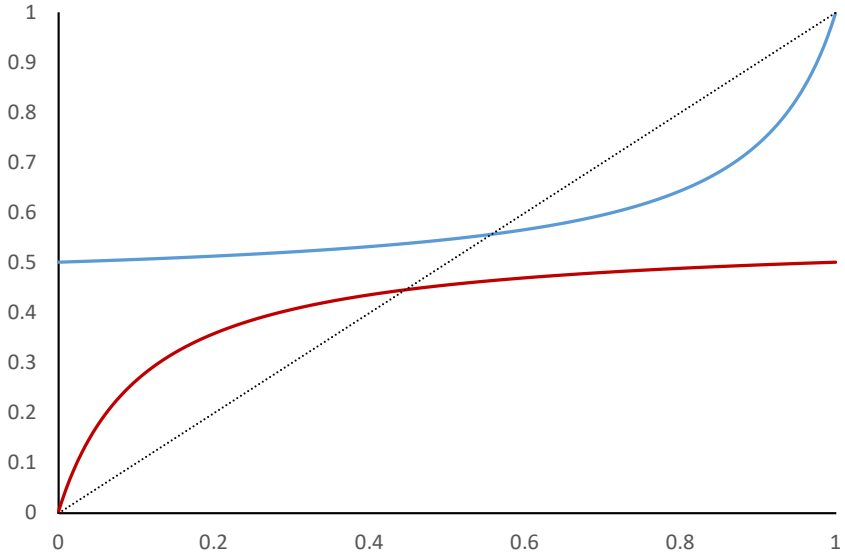
a = 0.76*
b = 0.24*

Market can fail if Liquidity Demand is Elastic



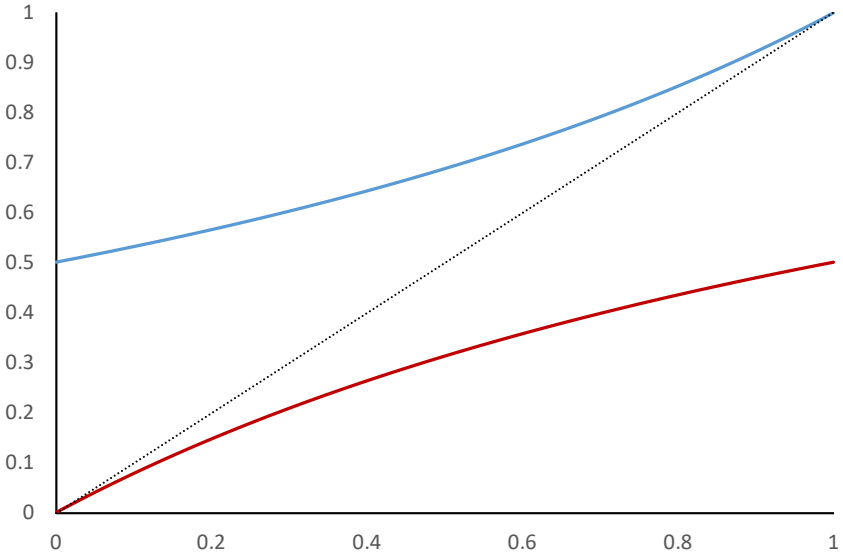
If liquidity demand is elastic and adverse selection is sufficiently high, no trade occurs.

$\lambda = 0.10$



$a^* = 0.444$
$b^* = 0.555$

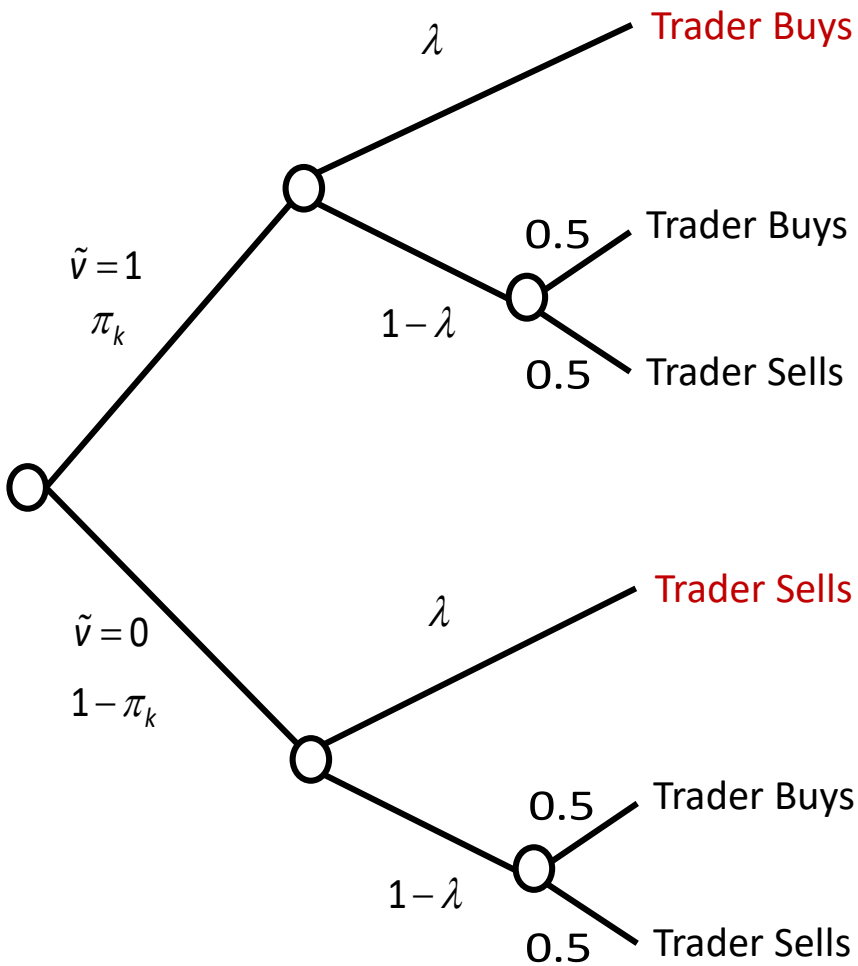
$\lambda = 0.60$



$a^* = 1.000$
$b^* = 0.000$

Similar to Akerlof's breakdown in used car market

Glosten and Milgrom: Binary Outcome



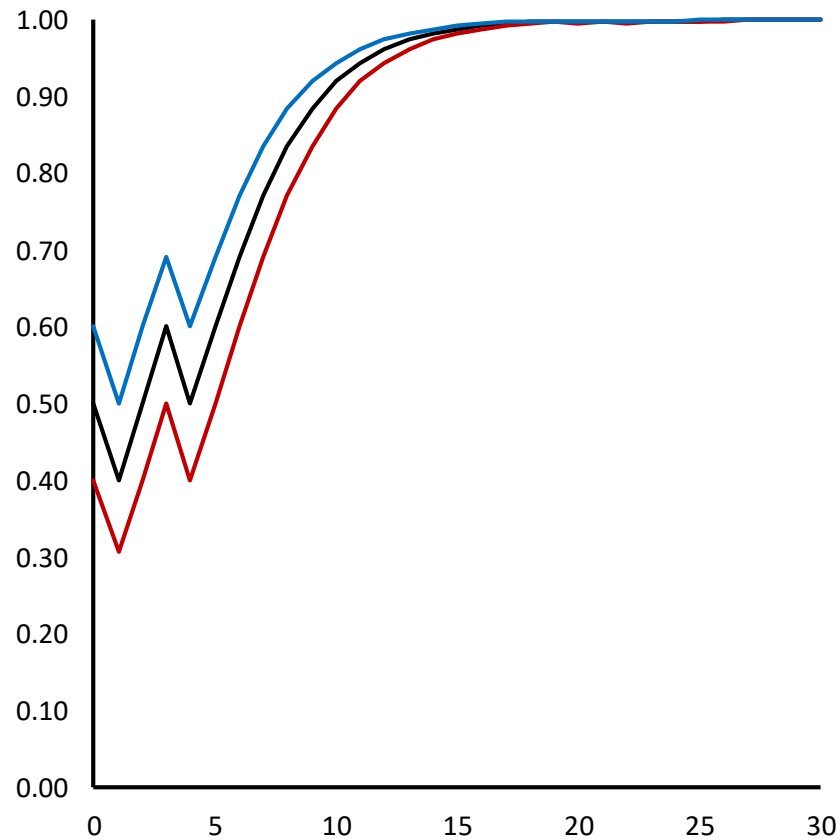
$$\left(\pi_{k+1} \mid \text{trader buys}\right) = \frac{\lambda + (1-\lambda)\frac{1}{2}}{\lambda \pi_k + (1-\lambda)\frac{1}{2}} \times \pi_k$$

$$\left(\pi_{k+1} \mid \text{trader sells}\right) = \frac{(1-\lambda)\frac{1}{2}}{\lambda(1-\pi_k) + (1-\lambda)\frac{1}{2}} \times \pi_k$$

Convergence in Glosten/Milgrom Setting

$$\lambda = 0.20; \quad \tilde{v} = 1$$

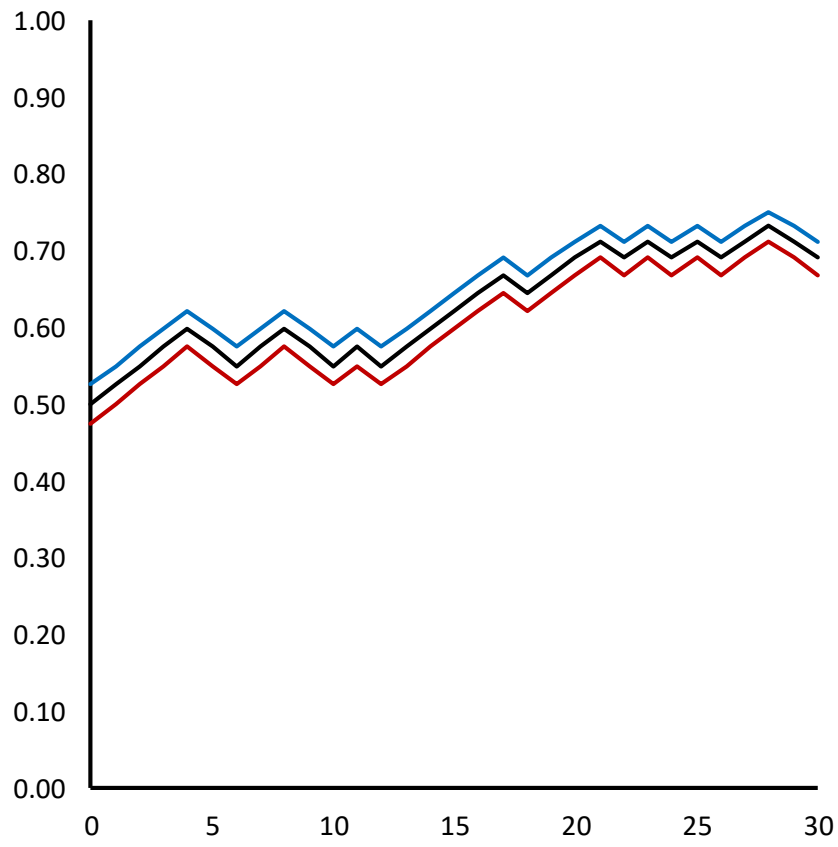
Trade Number	Direction	Prior	Bid	Ask
1		0.500	0.400	0.600
2	Sell	0.400	0.308	0.500
3	Buy	0.500	0.400	0.600
4	Buy	0.600	0.500	0.692
5	Sell	0.500	0.400	0.600
6	Buy	0.600	0.500	0.692
7	Buy	0.692	0.600	0.771
8	Buy	0.771	0.692	0.835
9	Buy	0.835	0.771	0.884
10	Buy	0.884	0.835	0.919



Convergence in Glosten/Milgrom Setting

$$\lambda = 0.05; \quad \tilde{v} = 1$$

Trade Number	Direction	Prior	Bid	Ask
1		0.500	0.475	0.525
2	Buy	0.525	0.500	0.550
3	Buy	0.550	0.525	0.575
4	Buy	0.575	0.550	0.599
5	Buy	0.599	0.575	0.623
6	Sell	0.575	0.550	0.599
7	Sell	0.550	0.525	0.575
8	Buy	0.575	0.550	0.599
9	Buy	0.599	0.575	0.623
10	Sell	0.575	0.550	0.599



Diamond and Verrecchia
Introducing a Market Friction: Constraints on Short-selling

- Someone who gets positive information can easily trade on that information by taking or increasing a long position in the security.
- Someone who uncovers negative information can sell shares if he owns them, but if not must take a short position
  - In practice this is somewhat costly.
  - In some situations prohibitively costly.

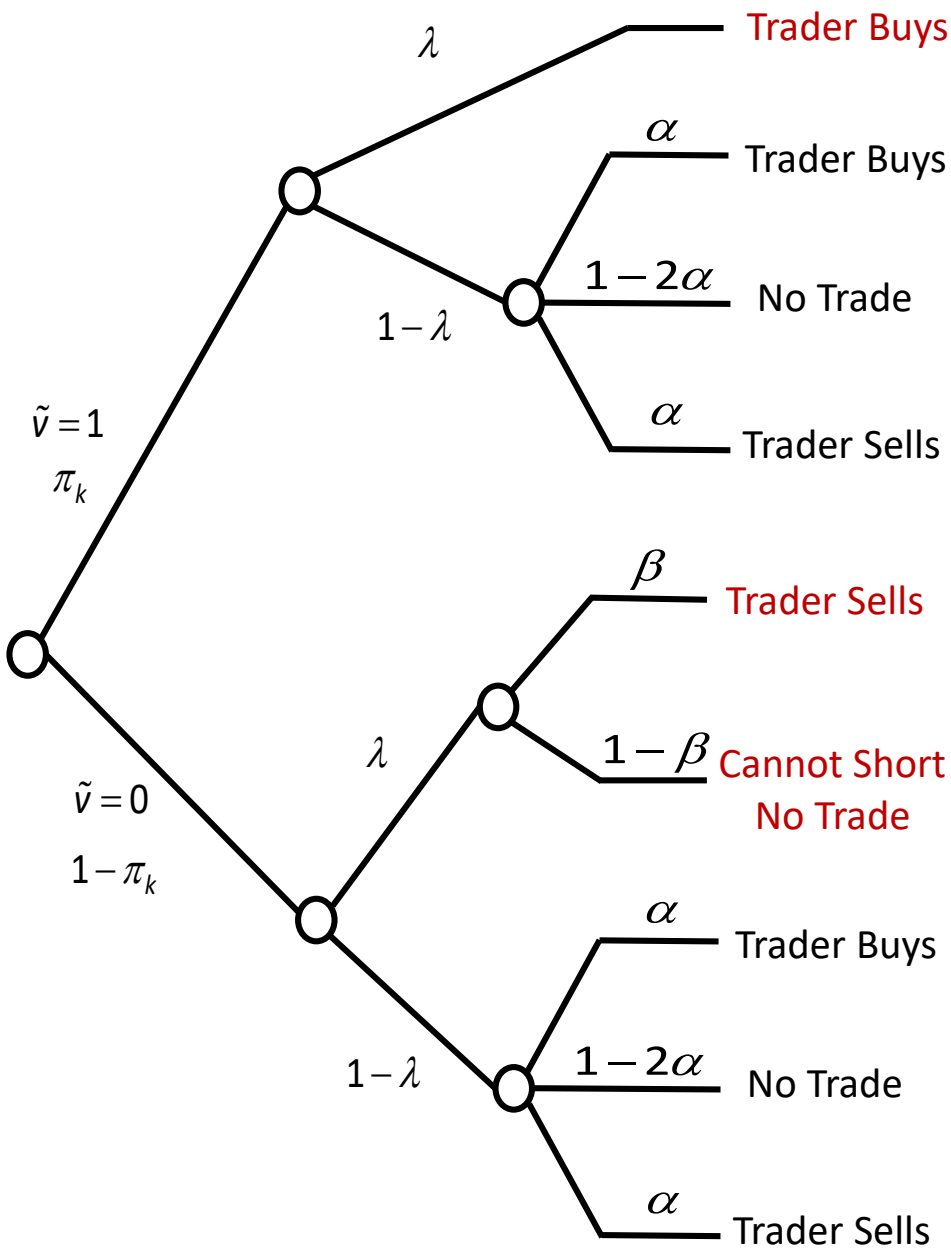
- Assume short sales are prohibited or difficult
  - Does this slow the rate of private information revelation through prices?
  - Does it create an upward bias on prices?

Diamond and Verrecchia
Introducing a Market Friction: Constraints on Short-selling

- Assume same setup as Glosten Milgrom (binary outcome)
- Except:

  - Informed with negative information can only sell with probability β. With probability $(1 - \beta)$ the informed trader does not own shares and is unable to short.
  - In the event that the informed trader has negative information and cannot sell, there will be no trade.
  - Uniformed (liquidity) traders buy with probability α (<0.5), sell with probability α and do not trade with probability $(1-2\alpha)$.

Diamond and Verrecchia: Binary Outcome



$$\left(\pi_{k+1}\,\middle|\,\begin{matrix}\text{trader}\\ \text{buys}\end{matrix}\right)=\frac{\lambda+(1-\lambda)\alpha}{\lambda\pi_k+(1-\lambda)\alpha}\times\pi_k$$

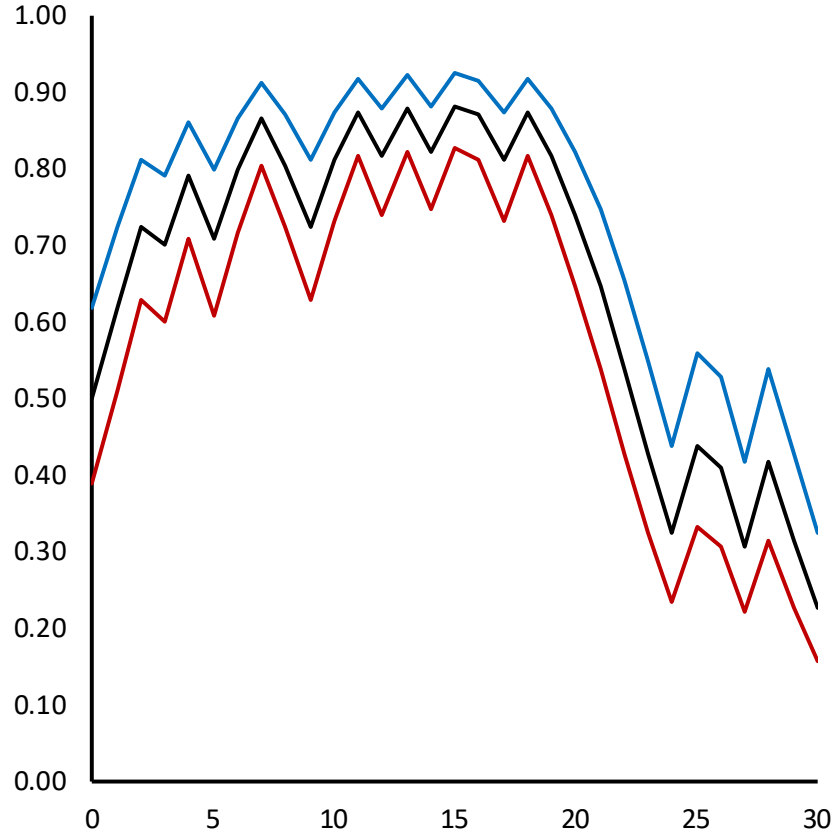
$$\left(\pi_{k+1}\,\middle|\,\begin{matrix}\text{trader}\\ \text{sells}\end{matrix}\right)=\frac{(1-\lambda)\alpha}{\lambda(1-\pi_k)\beta+(1-\lambda)\alpha}\times\pi_k$$

$$\left(\pi_{k+1}\,\middle|\,\begin{matrix}\text{no}\\ \text{trade}\end{matrix}\right)=\frac{(1-\lambda)(1-2\alpha)}{\lambda(1-\pi_k)(1-\beta)+(1-\lambda)(1-2\alpha)}\times\pi_k$$

Convergence in Diamond and Verrecchia Setting

$$\lambda = 0.20;\ \alpha = 0.4;\ \beta = 0.9;\ \tilde{v} = 0$$

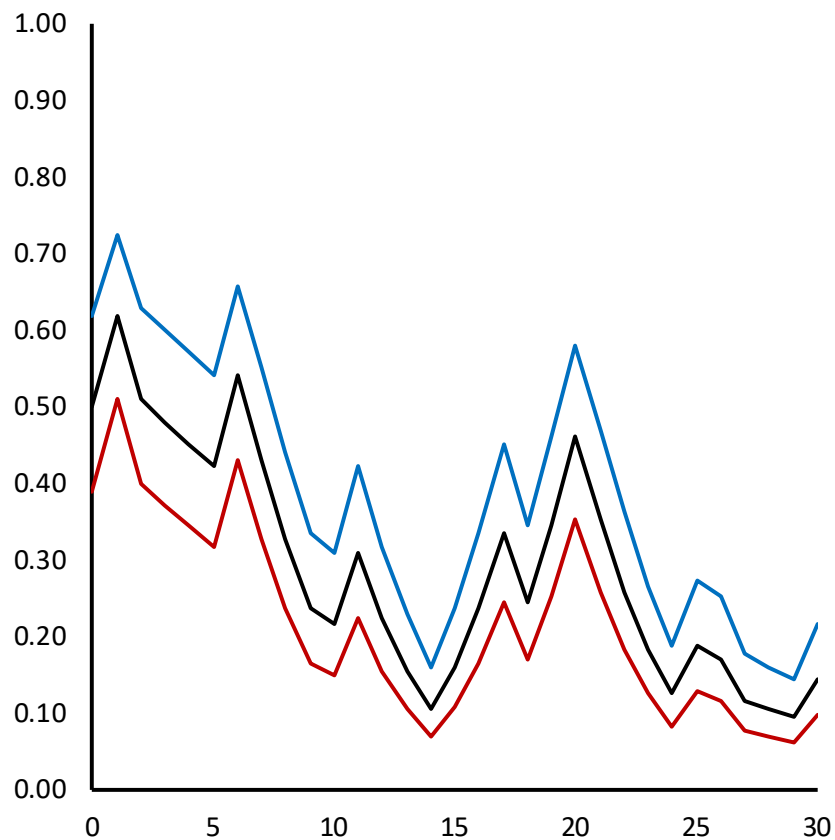
Trade Number	Direction	Prior	Bid	Ask	No Trade
1		0.500	0.390	0.619	0.471
2	Buy	0.619	0.510	0.725	0.591
3	Buy	0.725	0.628	0.811	0.701
4	No Trade	0.701	0.600	0.792	0.676
5	Buy	0.792	0.709	0.861	0.772
6	Sell	0.709	0.610	0.799	0.685
7	Buy	0.799	0.717	0.866	0.779
8	Buy	0.866	0.805	0.913	0.851
9	Sell	0.805	0.725	0.870	0.786
10	Sell	0.725	0.628	0.811	0.701



Convergence in Diamond and Verrecchia Setting

$$\lambda = 0.20;\ \alpha = 0.4;\ \beta = 0.9;\ \tilde{v} = 0$$

Trade Number	Direction	Prior	Bid	Ask	No Trade
1		0.500	0.390	0.619	0.471
2	Buy	0.619	0.510	0.725	0.591
3	Sell	0.510	0.400	0.628	0.480
4	No Trade	0.480	0.372	0.600	0.451
5	No Trade	0.451	0.345	0.572	0.422
6	No Trade	0.422	0.319	0.543	0.394
7	Buy	0.543	0.432	0.659	0.513
8	Sell	0.432	0.327	0.552	0.403
9	Sell	0.327	0.237	0.441	0.302
10	Sell	0.237	0.166	0.336	0.217



Diamond and Verrecchia

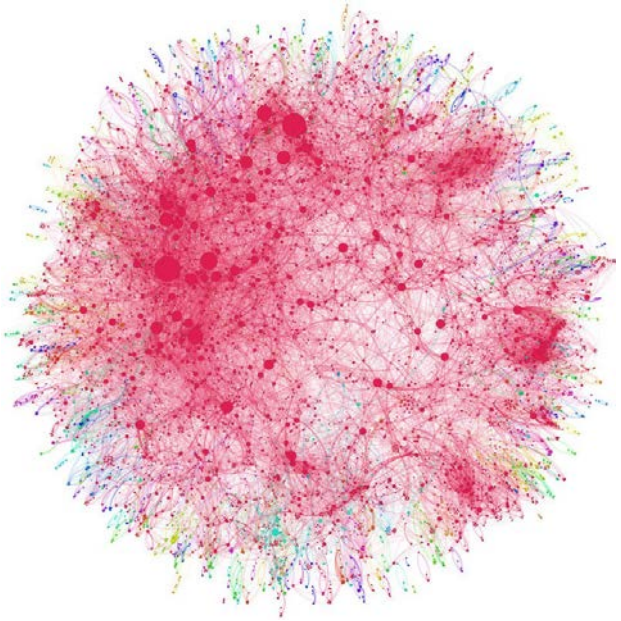
Introducing a Market Friction: Constraints on Short-selling

- Despite the censuring due to short sale constraints, prices will still be martingales

$$\left(\pi_{k+1}\middle|\begin{matrix}\text{trader}\\ \text{buys}\end{matrix}\right)\Pr\left(\begin{matrix}\text{trader}\\ \text{buys}\end{matrix}\right)+\left(\pi_{k+1}\middle|\begin{matrix}\text{trader}\\ \text{sells}\end{matrix}\right)\Pr\left(\begin{matrix}\text{trader}\\ \text{sells}\end{matrix}\right)+\left(\pi_{k+1}\middle|\begin{matrix}\text{no}\\ \text{trade}\end{matrix}\right)\Pr\left(\begin{matrix}\text{no}\\ \text{trade}\end{matrix}\right)=$$

$$\pi_k\times\left[\begin{matrix}\left(\dfrac{\lambda+(1-\lambda)\alpha}{\lambda\pi_k+(1-\lambda)\alpha_k}\right)\left(\lambda\pi_k+(1-\lambda)\alpha\right)+\\ \dfrac{(1-\lambda)\alpha}{\lambda(1-\pi_k)\beta+(1-\lambda)\alpha}\left(\lambda(1-\pi_k)\beta+(1-\lambda)\alpha\right)+\\ \dfrac{(1-\lambda)(1-2\alpha)}{\lambda(1-\pi_k)(1-\beta)+(1-\lambda)(1-2\alpha)}\left(\lambda(1-\pi_k)(1-\beta)+(1-\lambda)(1-2\alpha)\right)\end{matrix}\right]=\pi_k$$

Glosten/Milgrom Simplifying Assumptions



Price

Assumption	Realistic?
Bertrand Competition among Dealers	Perhaps: if quotes are easily seen by all; ease of placing limit orders allows competition
Risk Neutrality of Dealers	Probably not a major concern
Market Mechanism with Only Market Orders	Probably not a gross distortion for many purposes
Learning by Dealers	Dealers through experience probably develop a sense of adverse selection risk over time. Simpler learning problem than in REE
Exogenous arrival process of traders	**Informed are not strategic despite the fact that their trading has an affect on information revelation and future trading opportunities.**

Kyle Model
(Continuous Auctions and Insider Trading)

- One informed trader who knows: $\tilde{v}$

- Noisy supply: $\tilde{Z}$ (sum of all market orders from liquidity traders)

- Informed trader submits market order: $\tilde{x}$

- Market maker observes total order flow: $\tilde{x} + \tilde{Z}$
    - Market maker cannot see component parts of order flow.
    - Market maker sets single price at which total order flow is processed.

Kyle Model
(Continuous Auctions and Insider Trading)

- Market maker is risk neutral.

- Market is competitive so that market maker earns zero profits in expectation.
    - Market makers competitively bid price to process full market order flow.

- Price will therefore be : $P = E\left(\tilde{v} \mid \tilde{x} + \tilde{Z}\right)$

- Assume $\tilde{v}$ and $\tilde{Z}$ are jointly normally distributed:

$$\begin{pmatrix} \tilde{v} \\ \tilde{Z} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_v^2 & 0 \\ 0 & \sigma_Z^2 \end{pmatrix}\right)$$

(Zero means are for simplification and are w.l.o.g.)

Kyle Model
(Continuous Auctions and Insider Trading)

- Assume that $\tilde{x}$ is linearly related to $\tilde{v}$.
- Then:

$$E\left(\tilde{v} \mid \tilde{x}+\tilde{Z}\right)=\frac{\mathrm{Cov}\left(\tilde{v},\tilde{x}+\tilde{Z}\right)}{\mathrm{Var}\left(\tilde{x}+\tilde{Z}\right)}\left(\tilde{x}+\tilde{Z}\right)=\lambda\left(\tilde{x}+\tilde{Z}\right)$$

- Informed trader choses x to maximize expected profits

$$\begin{aligned}\mathrm{E}\pi\left(x,\hat{P}\right)&=E\left(x\left(v-\lambda\left(x+\tilde{Z}\right)\right)\right)\\&=xv-\lambda x^2\end{aligned}$$

$$x^*=\beta v \qquad \beta=\frac{1}{2\lambda}$$

Kyle Model
(Continuous Auctions and Insider Trading)

$$\lambda = \frac{\mathrm{Cov}\left(\tilde{v}, \beta\tilde{v} + \tilde{Z}\right)}{\mathrm{Var}\left(\beta\tilde{v} + \tilde{Z}\right)} = \frac{\beta\sigma_v^2}{\beta^2\sigma_v^2 + \sigma_Z^2}$$

$$\beta = \frac{1}{2\lambda}$$

$$\lambda^{eq} = \frac{1}{2}\frac{\sigma_v}{\sigma_Z} \qquad \beta^{eq} = \frac{\sigma_Z}{\sigma_v}$$

Kyle Model
(Continuous Auctions and Insider Trading)

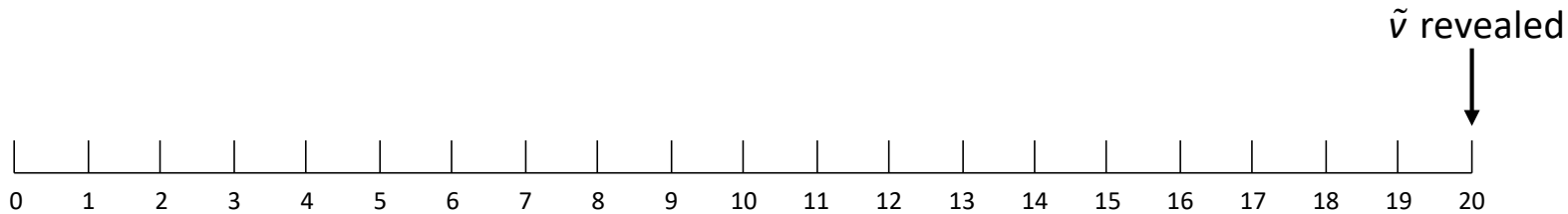
- Ex ante expected profit of informed $= \dfrac{\sigma_v \sigma_Z}{2}$
- Intuitively, informed's profits increase with
  - his informational edge (σ_v)
  - the amount of noise trading (σ_Z) he can hide behind and take bigger positions

- Half of the information gets revealed in price:

$$\mathrm{Var}\left(\tilde{v} \mid \beta^{eq}\tilde{v} + \tilde{Z}\right) = \frac{\sigma_v^2}{2}$$

- Independent of the amount of noise since informed trader adjusts trade size

Kyle Model: Sequential Trading Opportunities



$\tilde{Z}_t$, $t = \{0,1,2,\ldots,19\}$ are *i.i.d.*

$$Var\left(\tilde{Z}_t\right) = \frac{\sigma_Z^2}{20}$$

- "Closed form" solution to discrete time model is not obtainable but can be characterized.
  - Var($\tilde{v}$ |order flow history) is declining over time.
  - Total profits of informed are proportional to $\sigma_v \sigma_Z$.
  - Amount of information revealed is independent of σ_Z.

Kyle Model: Sequential Trading Opportunities

- Kyle analyzes equilibrium in a continuous time model
  - (He shows that the discrete time case converges to the continuous cases as the number of trading periods becomes infinite.)

$$t \in [0,1]$$

$$\lambda_t = \frac{\sigma_v}{\sigma_Z} \text{ (a constant!)}$$

$$\text{var}\left(\tilde{v} \,|\, \text{order flow to time } t\right) = \left(1-t\right)\sigma_v^2$$

$$\mathrm{d}x(t) = \beta_t \left(v - p_t\right)\mathrm{d}t$$

$$\beta_t = \frac{\sigma_Z}{\left(1-t\right)\sigma_v}$$

- All information is revealed by the end of trading.
- Ex ante expected profit of informed $= \sigma_v \sigma_Z$
  - Recall that in single auction it was equal to $= \dfrac{\sigma_v \sigma_Z}{2}$

Admati/Pfleiderer:
A Theory of Intraday Trading Patterns

Intraday Trading Patterns of Exxon Shares in 1981

	10 A.M. to 12 Noon	12 Noon to 2 P.M.	2 P.M. to 4 P.M.
Average Volume	179,349	103,024	122,670
SD (price changes)	0.34959	0.28371	0.37984

Admati/Pfleiderer:
A Theory of Intraday Trading Patterns

- Builds on Kyle Model
- Day is broken into T periods
- Final payoff of risky asset = $\sum_{t=1}^{T} \tilde{v}_t$
- At the end of each period $\tilde{v}_t$ becomes public

Admati/Pfleiderer:
A Theory of Intraday Trading Patterns

- At the beginning of each period

  an informed trader can observe $\tilde{v}_t + \tilde{\varepsilon}_t^i$

- Number of informed traders in period $t = n_t$

Admati/Pfleiderer:
A Theory of Intraday Trading Patterns

- Two types of liquidity traders
  - nondiscretionary have immeditae demands, no choice of when to trade
  - discretionary can choose when during the day to trade

Admati/Pfleiderer: A Theory of Intraday Trading Patterns

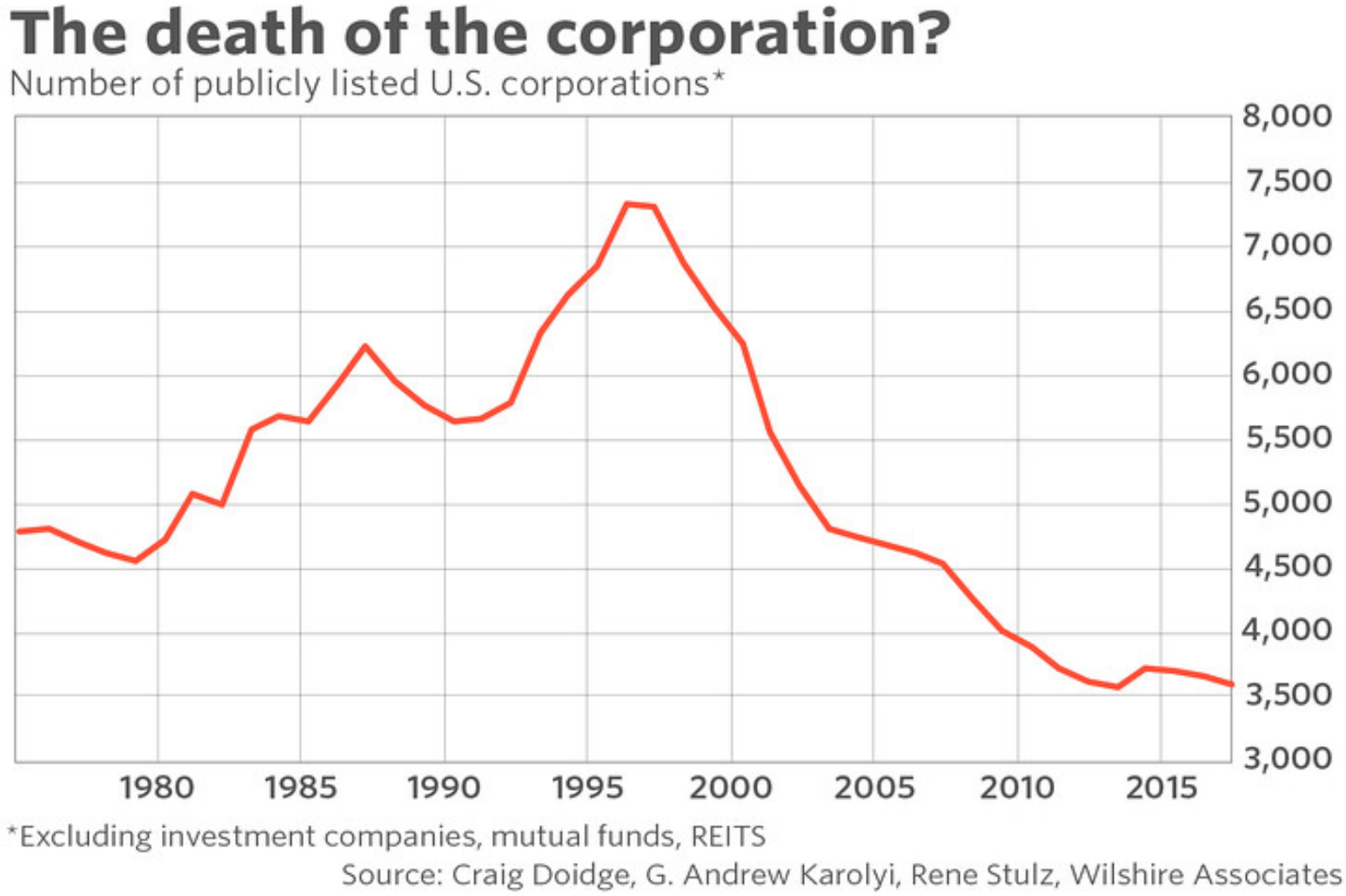
$$\lambda_t \text{ is proportional to } \frac{1}{\text{SDev}\left(\textit{total liquidity trades}\right)} \times \frac{\sqrt{n_t}}{n_t + 1}$$

- For fixed n_t, discretionary liquidity traders will have incentive to flock together and trade at the same time (safety in numbers).
- The periods in which liquidity trading is pronounced are profitable ones for informed to trade.
- If information is costly, then n_t is potentially higher in the periods discretionary liquidity traders are trading.
  - This decreases λ_t reinforcing the incentives to flock together.

Things to Ponder

- Long-standing emphasis on value of
  - Prices revealing information
    - Good for consumption and production decisions
  - Liquidity
    - Ability to trade quickly with little price impact increases value of a security

Things to Ponder



BainCapital
PRIVATE EQUITY

Blackstone

TPG

APOLLO

KKR

THE CARLYLE GROUP

Things to Ponder