Attention, Psychological Bias, and Social Interactions

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Limited Attention
Limited attention

• Environment provides
• Cognitive processing power limited
  ➔ Processing selective

Attention:

Cognitive mechanisms that determine which information processed
• More vs. less
• Especially, discarded
• Direct attention toward salient cues
General Specification of Limited Attention

Simple general framework that captures many applied models of limited attention

- Hirshleifer, Lim & Teoh (2003)
- Phrased in terms of asset valuation by investors
- Basic idea also applies to valuations managers form to make their decisions
General Specification of Limited Attention (2)

Date 1 public information set:
\( \psi = (\psi^1, \psi^2, \ldots, \psi^K) \),
E.g., date 1 earnings levels for \( K \) divisions
\( c_2 \) = terminal payoff
No payoff at date 1
Or, \( c_2 \) inclusive of any earlier payoffs
Valuations based on beliefs about \( c_2 \)
General Specification of Limited Attention (3)

Distribution of \( c_2 \) depends on \( \psi \):

\[
c_2 = H(\psi^1, \psi^2, \ldots, \psi^K; p^1, p^2, \ldots, p^N) + \nu,
\]

\[
E^\rho[\nu] = 0
\]
\( \nu \) independent of \( \psi \) and \( p \).

\( \mathbf{p} = (p^1, p^2, \ldots, p^N) \)

Vector of parameters, structure of world
General Specification of Limited Attention (4)

\( \mathbf{p} \) known to all attentive agents, by:

Direct public observation

Inference from structure of the market, strategic incentives, implied equilibrium

Rational expectation of the terminal payoff: \( S^\rho_1 \)

\[
S^\rho_1(\psi; p) \equiv E[c_2|\psi; p] = H(\psi^1, \psi^2, \ldots, \psi^K; p).
\]

E.g., stock price under risk neutrality
General Specification of Limited Attention (5)

Limited attention as simplification

• Viewing some feature of world as having specific “simple” (easy to process) or attractive value

• Two aspects:
  • Cue Neglect
  • Analytical Failure
Cue neglect

1. Cue Neglect
Viewing a realized, received signal as having specific arbitrary value
Some (probably almost all) elements of $\psi$:

$$\psi^k = (\psi^k)', \quad k = J, J + 1, \ldots, K,$$

where $(\psi^k)'$ are specific ‘simple’ values.
Which signals? What are simple values? Depends on application.
Higher salience $\rightarrow$ greater probability that attend to the signal
Cue neglect example

E.g., $\psi^2$ = a cost publicly revealed at date 1, incurred at date 2

- Nonsalient:
- Inattentive investor sets $\psi^2 = (\psi^2)' = 0$.
- Expensing of this cost (accounting rules) at date 1
- Reflected in earnings
- If earnings salient, this can correct beliefs
Analytical failure

2. Analytical Failure
Simplifying parameters of economic environment
Restricts some parameters to special values,

\[ p^i = (p^i)', \quad i = L, L + 1, \ldots, N. \]

Economizes on cognition.
Example: Costless disclosure

• Disclose truthfully vs. withhold

Rational outcomes:
• “Unravelling” ➔ full disclosure
  • Withhold ➔ Assume the worst

• Disclosure cost:
  • Threshold equilibrium, better types disclose
Inattention and voluntary disclosure

- Neglect of **nondisclosure** - Analytical Failure
  - Neglect strategic incentive for low types to withhold
  - Arbitrarily assume all types equally likely to disclose
  - Less incentive to disclose

- Attentive do draw adverse inference

- In equilibrium, nondisclosure below some cutoff

- Neglect of **disclosed signal** - Cue Neglect
  - E.g., stick to prior, or assume signal equal to ex ante mean
    - Don't update adversely
  - **Attentive** infer marginal disclosing type at **bottom** of disclosing pool (below prior)
  - So inattention **increases** incentive of marginal type to disclose
    - Disclosure threshold decreases

- Hirshleifer, Lim and Teoh (2008)
Other modeling approaches compatible with the General Limited Attention framework

- E.g., cognitive hierarchy models
  - Level-\(k\) agents think others are level-(\(k - 1\)) or below
    - Level 0 behaves randomly
  - World-parameter \(p_j\):
    - Belief about level of another agent \(j\)
  - Set to simple values (\(p_j \leq k - 1\))
Basic asset pricing application

- Mean-variance setting
- Continuum of investors
- Attentive vs. Inattentive.
  - Independent probability $f$
  - Fraction inattentive $f$
Timeline

3 dates

Date 0:
  • Prior expectations formed

Date 1:
  • Public information arrives about firm value or its components

Date 2:
  • Terminal payoff realized, firm liquidated
Asset Prices Reflect Weighted Average of Beliefs

Standard result with rational & belief-biased investors:

• Equilibrium price reflect weighted average of beliefs
  • E.g., overconfidence-based asset pricing model
    • Daniel, Hirshleifer and Subrahmanyam (2001)

• We'll focus on limited attention
Asset Prices Reflect Weighted Average of Beliefs (2)

\( \phi = \kappa, \rho \) inattentive or attentive (rational)
Mean-variance preferences

\[ E_t^\phi [C] - \frac{A}{2} var_t^\phi (C) \quad (3) \]

Now using \( C \) instead of \( c_2 \) for terminal payoff (here consumption)
1 subscript: public availability (not necessarily used) of date 1 information
\( A = ARA \)
\( W^0 = \) Initial wealth endowment (claim to terminal consumption)
\( x_0 = \) Per capita endowment of risky security
Asset Prices Reflect Weighted Average of Beliefs (3)

Date 1:
Exchange between security, ‘cash’ (claims to terminal consumption), price $S_1$.
$x =$ Position in security
$S_2 =$ terminal payoff of security

\[ C = W^0 - (x - x_0)S_1 + xS_2. \]  
(4)

Optimizing:

\[ x^* = \frac{E_1^\phi[S_2] - S_1}{\text{Avar}_1^\phi(S_2)}. \]  
(5)
Asset Prices Reflect Weighted Average of Beliefs (4)

Market clearing:

\[ f x^\kappa + (1 - f) x^\rho = x_0. \]

Substitute optimal \( x^\kappa(S_1) \) and \( x^\rho(S_1) \), solve for equilibrium price:

\[ S_1 = \kappa E^\kappa_1[S_2] + (1 - \kappa) E^\rho_1[S_2] - \frac{A x_0}{\alpha^\kappa + \alpha^\rho}, \]

where

\[ \alpha^\kappa \equiv \frac{f}{\text{var}^\kappa(S_2)}, \quad \alpha^\rho \equiv \frac{1 - f}{\text{var}^\rho_1(S_2)}, \quad \kappa \equiv \frac{\alpha^\kappa}{\alpha^\kappa + \alpha^\rho}. \]

Final term:

Risk premium for positive-net-supply asset
Asset Prices Reflect Weighted Average of Beliefs (5)

Focus is how limited attention biases beliefs, so eliminate nuisance by setting \( x_0 = 0 \):

\[
S_1 = \kappa E_1^\kappa[S_2] + (1 - \kappa) E_1^\rho[S_2].
\]  

(9)

Price reflects weighted average of the beliefs of different investors.
Weights \( \kappa \) on inattentive \( 1 - \kappa \) on attentive.
\( \alpha^\kappa \) and \( \kappa \) increasing in \( f \).
More irrationals, more mispricing.
Rational investors profit from arbitrage.
Still mispricing, since arbitrage is risky.
Valuation under signal neglect, analytic failure

Reminder:

\[ S_1 = \kappa E_1^\kappa [S_2] + (1 - \kappa) E_1^\rho [S_2]. \]  \hspace{1cm} (10)

Agents stick to special values for:

Public signals \((\psi^J)', (\psi^{J+1})', \ldots, (\psi^K)'

World parameters \((p^L)', (p^{L+1})', \ldots, (p^N)'

Expectation of inattentive agents:

\[ S_1^\kappa = E^{\kappa} [c_2 | \psi; p] = \]

\[ H (\psi^1, \psi^2, \ldots, \psi^{J-1}, (\psi^J)', (\psi^{J+1})', \ldots, (\psi^K)', p^1, p^2, \ldots, p^{L-1}, (p^L)', (p^{L+1})', \ldots, (p^N)'). \]  \hspace{1cm} (11)

Substituting these inattentive expectations, along with attentive expectations, into (10)

→ Equilibrium security price
Empirical content

- What is economic environment ($H$ function)?
- What are the limited attention simple values for signals, parameters?
Illustration: Model of *Pro Forma* Earnings Disclosure

• Between formal financial reports:
  • Informal disclosures about earnings
    • “Street” or *pro forma* earnings often exclude certain costs.
  • Purportedly to undo special transient circumstances

• Stylized fact:
  • Pro forma earnings > GAAP earnings.
  • ‘EBS releases', ‘Everything but Bad Stuff'
    • Barbash (2001)
Pro forma earnings and investor inattention

- Do investors interpret pro forma earnings naively?
  - Neglect selection bias in adjustments?
- Do firms exploit investor inattention?
- Do pro forma disclosures bias beliefs? Reduce accuracy?
Time Line

Date 0:

- Manager observes what GAAP earnings $\epsilon_1$ will be at date 1
- Publicly observable:
  State, Realization of possible earnings adjustment $a$
    - State: Whether adjustment of size $a$ appropriate
- Manager decides whether to make or not make adjustment $a$ in pro forma disclosure
- If adjust, adjustment of size $a$ publicly reported

Date 1: $\epsilon_1$ reported
Date 2: $c_2$ realized
Normal state

State \( \psi = N \) (Normal):

\[
\text{GAAP earnings}
\]

\[
\epsilon_1 = c_2 + \delta,
\]

(12)

\[
E[\delta] = 0 \text{ noise}
\]

\( c_2, \delta \) normally distributed.

\[
\text{GAAP earnings } \epsilon_1 \text{ best predictor of } c_2
\]

Adjustment inappropriate
Exceptional state

State $\psi = E$ (exceptional):

$$\epsilon_1 = c_2 - a + \delta.$$  \hfill (13)

$a$ exogenous, $E[a] = 0$, independent of $c_2$, $\delta$, realization visible to all
If don’t attend to $a$, extra noise in forecast
Adjustment appropriate:

$$E[c_2] = \epsilon_1 + a$$ \hfill (14)

Adjusting GAAP earnings by $a$ eliminates $a$ noise from forecast
Pro forma earnings adjustment

Pro forma earnings can be disclosed as either

\[ e_1 = \begin{cases} 
\epsilon_1 & \text{if state } E \\
\epsilon_1 + a & \text{otherwise}
\end{cases} \]

- Attentive investors:
  - Adjusting has no effect

- Inattentive investors
  - Ignore state, assume appropriate adjustment (iff state \( E \))
    - Neglect strategic incentives
  - Appropriate adjustment improves pro forma \( e_1 \) as forecast of \( c_2 \)
  - GAAP earnings = White noise garbling of perfectly-adjusted earnings
GAAP earnings = White noise garbling of perfectly-adjusted earnings

Appropriately adjusted *pro forma* earnings eliminate a noise:

\[ e_1 = c_2 + \delta \]

GAAP does not

\[ \epsilon_1 = c_2 + \delta + 1^{E(\psi)}a \]
Manager's objective

• Manager wants to:
  • Maintain high date 1 stock price
  • Avoid inappropriate adjustments
    • Direct preference (integrity)
    • Reputational

\[ U(\theta) = \lambda S_1 - 1^N[\psi]1^A[\theta] \]

where:
\[ \lambda > 0, \text{ weight on maintaining high stock price} \]
\[ 1^N[\psi] = \text{indicator for } \psi = N \]
\[ 1^A[\theta] = \text{indicator for } \theta = A \text{ (adjust)} \]
Safe harbor

• Manager free to stick with GAAP
  ➔ never adjust if $a < 0$
    • Even in state $E$
Threshold decision rule

If only care about current stock price ($\lambda \to \infty$):

Adjust iff $a > 0$ (both states).

If care about honesty too:

State $E$:

Adjust iff $a > 0$

State $N$:

Adjust iff $a > a^N > 0$

$a^N = \frac{1}{\lambda \kappa \omega}$, where:

$\omega$ = informativeness of earnings for $c_2$

$\kappa$ = weight of inattentive investors on $S_1$
Intuition

Higher $a$ makes adjustment more attractive. Honesty preference

→

In state $N$, don’t adjust if $a$ too close to zero. Adjust more often ($a^N$ lower) if:

- Care more about stock price (high $\lambda$)
- Investors react strongly to earnings (high $\omega$)
- Investors less attentive (high $\kappa$)
Frequency of pro forma adjustment

• Increases with $\omega$
  • Signal-to-noise ratio of (properly-adjusted) earnings
  • Market reacts more strongly to earnings information
  • More tempting to boost earnings to fool inattentive
Inattention as parameter constraints in General Attention Framework

Manager adjusts in $E$ state if $a \geq a^E$

In equilibrium, $a^E = 0$

Manager adjusts in $N$ state if $a \geq a^N$

In equilibrium, $a^N = \frac{1}{\lambda \kappa \omega}$

Inattentive expectations:

Simplifying parametric restriction

Incorrect values $a^N = \infty$, $a^E = -\infty$

Two $p^j$ parameters of general inattention framework set to special $p'^j$ values
Stock prices

By threshold rule and GAAP safe harbor, \( e_1 \geq \epsilon_1 \)

*Pro forma* disclosures sometimes boost stock price unduly:

\[
S_1 \geq S_1^\rho,
\]

with strict inequality for some realizations of state, \( a \)

So ex ante

\[
E[S_1] > E[S_1^\rho].
\]

Prices, average investor expectations biased upward.

Consistent with regulator concerns
Stock prices (2)

Mispricing rises with potential adjustment: \( \frac{dE[S_1 - S_1^p]}{da} > 0 \)
So excess pro forma earnings, \( e_1 - \epsilon_1 \) negatively predicts long-run stock returns
Evidence:
- Lougee and Marquardt (2004)
- Doyle, Lundholm and Soliman (2003)
Model predicts when relationship steeper
Broader implications

More broadly, many ways to manage investor perceptions
Key parameters here may still be relevant
Actions to exploit inattentive to boost stock price increase with
Inattention $\kappa$

Weight on short-term stock price $\lambda$

Greater informativeness $\omega$ of object of investor attention
There are empirical proxies for these parameters.
**Pro forma earnings disclosure improves beliefs: Example**

Manager places very high weight on honesty

\[ a^N \approx \infty \]

Accuracy/bias tradeoff

A noise almost eliminated from \( e_1 \)

Noise in \( e_1 \) as forecaster of \( c_2 \) almost ideal, \( \sigma_0^2 \).

\( e_1 \) much more accurate than \( e_1 \)

Social benefit of more accurate market prices

Vs. arbitrarily small upward bias in prices

Evidence: *pro forma* adjustments tend to improve accuracy

Doyle, Lundholm and Soliman (2003)

Bhattacharya et al. (2003)

Bradshaw and Sloan (2002)

More pervasive application:

Pricing of earnings, earnings components

Limited investor attention induces both under-, over-reaction to earnings components

  Hirshleifer, Lim and Teoh (2011)

Neglect implications of current earnings for future earnings
  → Post-earnings announcement drift
  Bernard and Thomas (1989)

Neglect of earnings components:
  Accruals negatively predict returns
    Evidence: Sloan (1996)
  Cash flows positively predict returns
    Evidence: Pincus, Rajgopal, and Venkatachalam (2007)
Social Transmission of Beliefs and Behaviors
Rational observational learning

- Observation only of actions of predecessors
  - Banerjee (1992), Bikhchandani, Hirshleifer & Welch (1992)
- BHW: Discrete states, actions, signals
- Herding
  - People choose same actions
- Information cascades
  - People stop using their private signals
  - Their actions become uninformative to others
    ➔ Poor information aggregation
Simple binary cascades setting

- Sequence of agents with identical choice problem
  - E.g., invest, not invest
- Agents successively choose based upon both:
  - Private signal
  - Observed choices of predecessors
Binary cascades setting (2)

States: $\omega \in \{0, 1\}$

Prior: $\Pr[\omega = 1] = \frac{1}{2}$

Private binary signals: $s_i \in \{0, 1\}$ for $I_i$

Symmetric: $\Pr(s_i = \omega | \omega) = p > 1/2$

Actions: $a \in \{0, 1\}$

Adopt, Reject

Objective:

Maximize $\Pr(a_i = \omega | \Phi_i)$

Match the state

Tie-breaking convention

If indifferent, flip a fair coin.
A = Adopt
R = Reject
H = High signal
L = Low signal
Public information pool stops growing

- Very inaccurate decisions
- Lasts indefinitely
- History dependent
  - A few early decision makers tend to dominate decisions
Information cascades and fragility

- Information cascade setting
  - People rationally understand that in equilibrium cascades aggregate little information
  - In equilibrium, low certainty

- Fragility of social outcomes
  - Even small shocks change behavior of many
    - Bikhchandani, Hirshleifer & Welch (1992)
    - “Fads”

- E.g., investment boom/busts
Models of “double counting” of signals arriving via multiple sources

• Persuasion bias
  • Updating in social network when neglect the fact that multiple signals reported by neighbors may have common original source
  • Treat each report as reflecting neighbor’s private signal
    • Level 2 thinking – think others ignore information of others

• Persuasion bias is inattentive updating
  • In general limited attention model, simplified parameter of the world:
    • $p_j =$ how much weight in updating observer believes agent $j$ placing upon observation of others
    • Simplify: $p_j = 0$
Naïve observational learning and overweighting of early signals

Eyster and Rabin (2010)
Inattentively think each predecessor acts independently

Continuous actions, signals

States: $\omega \in \{0, 1\}$
Prior: $\Pr[\omega = 1] = \frac{1}{2}$
Private signals: $s_t \in [0, 1]$, i.i.d. conditional on state
Densities $f_\omega$
Naïve observational learning, assumptions

Signals, cont.

Symmetric
$s \in [0, 1], f_0(s) = f_1(1 - s)$

Continuously differentiable monotone likelihood ratio:
$L(s) \equiv f_1(s)/f_0(s), L' > 0$

Unbounded likelihood ratio
Signals sometimes very strong

Normalize $s = \text{Pr}[\omega = 1|s]$

Social observation:
Each agent $I_t$ sees all predecessor actions
Naïve observational learning, assumptions

Actions in $[0, 1]$:

$$a_t(a_1, \ldots, a_{t-1}; s_t)$$

Rich action space $\implies$ each action reveals agent’s belief

Source of bad cascades ruled out

Belief given information set $\Phi_t$:

$$E[\omega|I_t] = \Pr[\omega = 1|\Phi_t]$$

Payoffs:

$$g_t(a; \omega) = -(a_t - \omega)^2$$

Minimized by choosing

$$a_t = E[\omega|\Phi_t]$$

Actions match beliefs
Rational benchmark

Easiest to focus on transformation of actions, signals
log odds ratios

$I_1$ posterior/action:

\[
\ln \left( \frac{a_1}{1-a_1} \right) = \ln \left( \frac{s_1}{1-s_1} \right)
\]

$I_2$ posterior:

\[
\ln \left( \frac{a_2}{1-a_2} \right) = \ln \left( \frac{a_1}{1-a_1} \right) + \ln \left( \frac{s_2}{1-s_2} \right) = \ln \left( \frac{s_1}{1-s_1} \right) + \ln \left( \frac{s_2}{1-s_2} \right)
\]

$I_k$ posterior:

\[
\ln \left( \frac{a_t}{1-a_t} \right) = \sum_{\tau \leq t} \ln \left( \frac{s_\tau}{1-s_\tau} \right)
\]
Rational benchmark (2)

Common knowledge of rationality:
   Actions fully reveal beliefs
   $\implies$ latest agent’s belief is sufficient statistic

   Each agent can update based solely on action of immediate predecessor
Beliefs/actions converge to truth
Beliefs of inattentive observers

Think each agent acts independently

$I_1$ posterior/action, as before:

$$\ln \left( \frac{a_1}{1 - a_1} \right) = \ln \left( \frac{s_1}{1 - s_1} \right)$$

$I_2$ posterior, as before:

$$\ln \left( \frac{a_2}{1 - a_2} \right) = \ln \left( \frac{a_1}{1 - a_1} \right) + \ln \left( \frac{s_2}{1 - s_2} \right) = \ln \left( \frac{s_1}{1 - s_1} \right) + \ln \left( \frac{s_2}{1 - s_2} \right)$$
Overweighting of first signal

But $I_3$ thinks $I_2$ used only own signal, so:

\[
\ln \left( \frac{a_3}{1 - a_3} \right) = \ln \left( \frac{a_1}{1 - a_1} \right) + \ln \left( \frac{a_2}{1 - a_2} \right) + \ln \left( \frac{s_3}{1 - s_3} \right)
\]

\[
= \ln \left( \frac{s_1}{1 - s_1} \right) + \left( \ln \left( \frac{s_1}{1 - s_1} \right) + \ln \left( \frac{s_2}{1 - s_2} \right) \right) + \ln \left( \frac{s_3}{1 - s_3} \right)
\]

\[
= 2 \ln \left( \frac{s_1}{1 - s_1} \right) + \ln \left( \frac{s_2}{1 - s_2} \right) + \ln \left( \frac{s_3}{1 - s_3} \right)
\]

Overweights $s_1$
Inattentive Observers (3)

Intuition:

$I_3$ ignores how $I_2$’s action depends upon $I_1$’s action/signal
So uses $I_1$’s signal twice
    Once when learning directly about $I_1$
    Again when thinks learning just about $I_2$.
Then $I_4$ will overweight $s_1$ via an $I_3$’s already-doubled weight on $s_1$
    As well as overweighing via $I_2$

Process iterates.
$I_t$: Exponentially overweights early signals
Pernicious effects of inattention

Beliefs do not converge to truth:
  Substantial chance of wrong beliefs forever
  Despite arrival of infinity of signals
  Including occasional arbitrarily informative ones
Beliefs become highly confident:
  Later agents think aggregating many independent signals correctly
Beliefs converge almost surely to 0 or 1
Comparison of naïve herding with rational cascades setting

• Information cascades model:
  • Booms fragile, small trigger can cause collapse.
  • “Fads”, e.g., boom-bust in investment

• Naive herding model:
  • Longstanding herds highly entrenched.
  • Extremely strong outcome information would be needed to break
    • E.g., people stuck for decades on idea that active managers tend to outperform?
Conversation and attraction to risk
A neglected issue in financial economics

• How investment ideas transmitted from person to person
• Biased social contagion of ideas, behaviors
  • Differential survival of cultural traits through investor populations

• Verbal communication does affect investment choices
Psychological bias affects social transmission of beliefs, behaviors

- In contrast with traditional behavioral finance
  - Some misperceptions, decision biases inherently social
- Sending biases
  - What do people like to report to others?
- Receiving biases
  - What reports do people pay attention to?
- Together, transmission bias
- Model of how transmission bias affects risk-taking
  - Han, Hirshleifer & Walden (2019)
Active vs. passive investing

Strategies:
A
• High variance
• Maybe + skew
• Maybe more engaging (conversable)

P
• Safe, routine
Social Transactions

Social transaction:
1. Pair of individuals randomly selected
2. One randomly Sender, other Receiver
3. Returns realized
4. Sender may communicate return to Receiver
5. Receiver may be transformed into Sender’s type
The Sending and Receiving Functions

In \{A, P \} pair:

- A or P Sender:
  - Return message sent with probability \( s(R_A) \) or \( s(R_P) \)

- Receiver:
  - Given message, receiver converted with probability \( r(R_A) \) or \( r(R_P) \)

Transformation

- Transformation probability:

\[ T_{AP}(R_A) = r(R_A)s(R_A) \]
Population evolution

Population shifts based on transformation probabilities, which come from sending, receiving functions

\[ f' - f = \begin{cases} 
\frac{1}{n} & \text{with probability } \frac{\lambda}{2} T_{AP}(R_A) \\
-\frac{1}{n} & \text{with probability } \frac{\lambda}{2} T_{PA}(R_P) \\
0 & \text{with probability } 1 - \left(\frac{\lambda}{2}\right) [T_{AP}(R_A) + T_{PA}(R_P)]
\end{cases} \]
SET and Sending Function

- **SET**: Sending probability increases with return performance:

\[ s(R_i) = \beta R_i + \gamma, \quad \beta, \gamma > 0, \]

- **\( \beta \)** SET-- link of self-esteem effects to return
  - Investors talk more about investment victories than defeats
- **\( \gamma \)** conversability, social interaction intensity
The Receiving Function

\[ r'(R_i) > 0 \]

- **Sender** return

- **Receiver**
  - Extrapolates from sender return
  - Limited attention (1):
    - Doesn’t fully discount for selection bias
    - E.g., set selection bias world parameter to zero

- Greater salience of extreme news (limited attention (2)):
  - Receiving function convex

\[ r(R_i) = a(R_i)^2 + bR_i + c \quad a, b, c \geq 0. \]
Convexity in conversion to a strategy as function of past returns

- Differentiate wrt $R_A$:

\[
\begin{align*}
\left(\frac{2n}{\chi}\right) \frac{\partial E[\Delta f| R_A, R_P]}{\partial R_A} &= \frac{\partial T_{AP}(R_A)}{\partial R_A} = r'(R_A)s(R_A) + r(R_A)s'(R_A) > 0 \\
\left(\frac{2n}{\chi}\right) \frac{\partial^2 E[\Delta f| R_A, R_P]}{(\partial R_A)^2} &= \frac{\partial^2 T_{AP}(R_A)}{(\partial R_A)^2} = r''(R_A)s(R_A) + 2r'(R_A)s'(R_A) > 0
\end{align*}
\]

- Higher active return favors A **convexly**
  - Multiplicative effect of greater $R_A$
    - + slopes of $s$, $r$

- Supporting evidence:
Expected Evolution toward A

• Taking expectation over returns,

\[
\left( \frac{2n}{\chi} \right) E[\Delta f] = E[T_{AP}(R_A)] - E[T_{PA}(R_P)].
\]
Unconditional evolution of population

Suppose A return more volatile, skewed

If A and P have similar expected return, on average fraction of A’s increases

Investors attracted to volatility, skewness

Why?
High Variance Causes Fraction of A’s to Increase

Attraction to high-variance strategies

- **SET**
  - Selection bias for reporting high returns stronger for A’s
  - Higher:
    - Idiosyncratic volatility
    - Factor loading
High Skewness Causes Fraction of A’s to Increase

Attraction to high-skewness strategies
• Salience of extremes
• SET
• High skew → high, influential returns
In equilibrium setting, attractive stock characteristics overpriced

- Evolutionary pressure toward $A$ increases its price
- $E[R_A]$ declines relative to $E[R_P]$
- Interior stable fraction of $A$'s
Trading, asset pricing implications

• Skewness overpriced
  • Much evidence
  • Even if no inherent preference over skewness
    • E.g., Brunnermeier & Parker (2005), Barberis & Huang (2008)
  • **Attraction** to (not preference for) skewness
    • Moths to a flame
  • **Inherently social effect**

• Beta, idiosyncratic volatility overpriced
  • Consistent with evidence on investor behavior, returns

• Greater social interaction increases attraction to skewness, beta, volatility
  • Supporting evidence, several studies
    • Empirical proxies for sociability
    • Experimental testing for better identification
Social Observation and Saving
Visibility Bias in the Transmission of Consumption Norms and Undersaving

- Savings rate in US and several OECD countries has declined sharply since 1970s
  - “The savings rate puzzle”
- New social explanation
- Learn how much to save by observing consumption of others
  - Biased observation, learning
  - Han, Hirshleifer & Walden (2019)
Social transmission bias

- Visibility bias in observation, attention
- Neglect of selection bias
Visibility bias

- **Visibility bias:**
  - Greater attention to **what is seen** than **what is unseen**
- Consumption more salient than non-consumption
  - Neighbor with boat parked in driveway
- Consumption activities engaging to talk/post about
- Consumption activities often more social
  - E.g., see others shopping, dining
  - $4 Starbucks visible, 10¢ at home not
Visibility bias
+ Neglect of selection bias

- Visibility bias
  + Neglect of selection bias

→ High estimated frequency of consumption events

- Update toward belief in high consumption (low saving) by others
  - Infer that little need to save

- So consume heavily; observed by others

- High-consumption trait spreads through population

*Self-feeding effect*
Optimal individual consumption

- 2 dates, 0 and 1, zero interest rate
- Wealth at date 1:
  - $W$ probability $p$
  - 0 probability $1 - p$  Personal disaster risk (job loss...)
    - Learning from others about this risk
- Quadratic utility: Divide expected wealth in half.
- Optimistic $\Rightarrow$ consume more today

$$c_0 = \hat{p} \left( \frac{W}{2} \right)$$
Consumption expenditures ➔ Observations

Higher consumption expenditure

Higher $Pr$(Any Given Consumption Activity)

Consumption “bins”, empty or full

- $K$ date-0 bins per person.
- See sample of others’ bins. Update.
Consumption bins

• N identical agents (except for priors)
• Date 0, each of $K$ bins empty or full: $(W/2)/K$ per bin
• All bins full $\sim$ Consume $W/2$: $\hat{p} = 1$
• All bins empty $\sim$ Consume 0: $\hat{p} = 0$
• Optimal consumption:

$$c_0 = \hat{p} \left( \frac{W}{2} \right)$$

$\Rightarrow$ Each bin full with probability $\hat{p}$

• Perceived non-disaster probability = Full-bin probability
• Informationally, seeing an empty/full bin is just like observing a disaster occur/not-occur
Observation of others’ consumption

- Observe $M$ random bins of others
  - Simultaneous
- Tilted toward full bins
  - Visibility bias
- Think random sample
  - Inattention—Neglect of selection bias (visibility bias)
- Base model -- Otherwise random
- Network model -- Sample only from neighbors
- Demographics model -- Tilt toward young or old
The population

- Many identical agents
- Identically distributed wealth disaster outcomes
- Non-disaster parameter $p$ stochastic
  - Agent-specific informative prior about $p$
  - Learn from others about it
- Large population $\Rightarrow$ Aggregate outcomes deterministic
Visibility bias

- Average fraction of bins that are full: $\bar{p}$
- $=\text{Agents’ average probability estimate for non-disaster}$
- Visibility bias:
  - Probability ratio of observing bin given full, empty: $\tau > 1$
  - Observed fraction of full bins
    - Concave transformation of actual fraction
      \[ S_\tau(\bar{p}) = \frac{\bar{p}}{\bar{p} + \frac{1-\bar{p}}{\tau}} \geq \bar{p}, \]
- All agents think $\tau = 1$
  - Selection neglect
Equilibrium

• Solve for equilibrium as fixed point
  Population-average belief
  ➔ Average consumption
  ➔ Average bin observations, update from priors
  ➔ Population-average belief

• At fixed point (exists), two effects cancel
  • Visibility-bias/selection-neglect ➔ Optimistic updating
    • Upward pull on $\hat{p}$
  • Priors
    • Downward pull on $\hat{p}$
Equilibrium condition

- Agents update based on observing $Ber(S_\tau(\bar{p}))$ distributed bins
- But think they are $Ber(\bar{p})$
- Average date 0 consumption: $\bar{p}\left(\frac{w}{2}\right)$

Equilibrium Condition:

$$\bar{p} = \frac{p + \xi S_\tau(\bar{p})}{1 + \xi}$$

**LHS:**
- Ave. Belief
  - (⇒ Ave. Consumption)

**RHS:**
- Updated ave.
  - belief given
  - this average consumption
Overconsumption

• In equilibrium, overconsumption
  • ‘Learn’ to be less thrifty

• Overconsumption increases with
  • Visibility bias, $T$
  • Intensity of social observation/interaction, $\xi$

• Rise in electronic communications since 1970s (not just internet) and visibility bias
  • Plunging call prices, cell phones, smart phones, cable TV, ...
  • Interesting to talk about trips, car purchases...
  • Vs. in-person, observe nonpurchase “events”

⇒ Greater overconsumption
Smart agents, misperception of others, and disclosure policy

- What if some `smart’ agents?
  - Rational or highly informed
  - Know true $p$
- Lower $\hat{p}$ than biased agents $\Rightarrow$ Consume less
- So for biased agents, $\bar{p}^V > \bar{p}$
  - Don’t realize others less optimistic
- Salient disclosure of $\bar{p}$ (or average consumption)
  - Biased beliefs revised downward
  - No effect on smart agents
  $\Rightarrow$ Less overconsumption
- Supporting evidence from smartphone field experiment
  - D’Acunto, Rossi & Weber (2019)
- Disclosure can also help without smart agents (e.g., network extension)
Other implications

• High network connectivity intensifies overconsumption
  • Both population-level, individual centrality
  • Stronger iterative feedback effects

• Greater wealth dispersion, more saving
  • Think others’ consumption high because they’re rich
  • Garbles/weakens inferences
  • Prediction contrasts with Veblen wealth-signaling approach
    • Overconsumption caused by information asymmetry about wealth
Summary

- Limited attention as setting environmental parameters to simple values
  - Cue neglect
  - Analytical failure
- Firms can manipulate limited investor attention toward corporate disclosure
- Social learning with full attention can be surprisingly ineffective
- Analytical failure makes social learning even worse
  - Fixated more quickly, firmly upon mistakes
- Limited attention and other individual-level biases induce social transmission bias
  - Can explain investor attraction to risky strategies, overvaluation of volatility, skewness
  - Can explain overconsumption