Price War Risk

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Abstract

We develop a general-equilibrium asset pricing model incorporating dynamic supergames of price competition. Price wars can arise endogenously from declines in long-run consumption growth, because firms become effectively more impatient for cash flows and their incentives to undercut prices are stronger. The triggered price wars amplify the initial shocks in long-run growth by narrowing profit margins and discouraging customer base development. In the industries with a higher capacity of distinctive innovation, incentives of price undercutting are less responsive to persistent growth shocks, and thus firms are more immune to price war risks and thus long-run risks. Exploiting detailed patent, product price, and brand-perception survey data, we find evidence for price war risks, which are significantly priced. Our results shed new light on how long-run risks are priced cross-sectionally through industry competition.

Keywords: Long-run risks, Cross-sectional returns, Oligopoly, Innovation similarity, Deep habits, Financial constraints. (**JEL**: G1, G3, O3, L1)

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1 Introduction

Price war risks are vital and concern investors, partly because product markets are highly concentrated, featuring rich strategic competition among leading firms (see, e.g. Autor et al., 2017; Loecker and Eeckhout, 2017).[1] According to the U.S. Census data, the top four firms within each 4-digit SIC industry account for about 48% of the industry's total revenue on average (see Figure B.1), and the top eight firms own over 60% market shares. However, little is known about what fundamentally drives price war risks at the aggregate level and how the heterogeneous exposures to price war risks are determined across industries.

This paper answers these two questions. First, we show that persistent growth shocks (as in Bansal and Yaron, 2004) can drive price war risks. The endogenous price war risks amplify the impact of the underlying long-run-risk shocks, because price wars further narrow down profit margins and depress customer base development. Second, in the model and the data, we show that in the industry with a larger capacity of distinctive and radical innovations (see, e.g. Jaffe, 1986; Christensen, 1997; Manso, 2011; Kelly et al., 2018), firms' exposures to price war risks and thus long-run risks are lower. Importantly, our results shed new light on how long-run risks are priced in the cross section (see, e.g. Bansal, Dittmar and Lundblad, 2005; Hansen, Heaton and Li, 2008; Bansal, Dittmar and Kiku, 2009; Malloy, Moskowitz and Vissing-Jørgensen, 2009; Constantinides and Ghosh, 2011; Ai, Croce and Li, 2013; Kung and Schmid, 2015; Bansal, Kiku and Yaron, 2016; Dittmar and Lundblad, 2017).

We develop a general-equilibrium asset pricing model incorporating dynamic supergames of price competition among firms. Our baseline model deviates from the standard long-run-risk model (Bansal and Yaron, 2004) mainly in two aspects: (1) consumers have deep habits (see Ravn, Schmitt-Grohe and Uribe, 2006) over firms' products, and thus firms need to maintain their customer bases; and (2) there are a continuum of industries and each industry features dynamic Bertrand oligopoly with differentiated products and implicit price collusion (Tirole, 1988, Chapter 6).[2] We further extend the baseline model with costly financing and endogenous cash holdings similar to Bolton,

[1] There has been extensive and constant media coverage on the implications of price war risks on stock returns. We list a few of headline quotes in Appendix A.

[2] Tirole (1988) builds oligopoly models with Bertrand price competition and obtains similar price war implications as in the models of Cournot quantity competition (Green and Porter, 1984; Rotemberg and Saloner, 1986).

Chen and Wang (2011). In the extended model, the endogenous amplification effect of price war risks is enhanced by the adverse feedback loop between price war risks and financial constraints risks, generating significant asset pricing implications.

Price wars can arise endogenously from declines in long-run consumption growth, because firms become effectively more impatient for cash flows and their incentives to undercut prices become stronger. This essentially follows the key economic insight of the Folk theorem, which proposes that sub-game perfect outcomes with higher average payoffs are attainable when agents are more patient. More precisely, oligopolies tend to implicitly collude with each other on setting high product prices.[3] Given the implicit collusive price levels, a firm can boost up its short-run revenue by secretly undercutting peers on prices to attract more customers; however, deviating from the collusive price levels may reduce revenue in the long run when the price undercutting behavior is detected and punished by its peers. Following the literature (see, e.g. Green and Porter, 1984; Brock and Scheinkman, 1985; Rotemberg and Saloner, 1986), we adopt the non-collusive Nash equilibrium as the incentive-compatible punishment for deviation, which can be interpreted as the most severe price war. The implicit collusive price levels depend on firms' deviation incentives: a higher implicit collusive price can only be sustained by a lower deviation incentive, which is further shaped by firms' tradeoff between short-term and long-term cash flows. In other words, a higher collusive price becomes more difficult to sustain when the long-run growth rate is lower, because firms expect a persistent decline in aggregate consumption demand, rendering the future punishment less threatening. As a result, price wars emerge from negative long-run-growth shocks, and importantly, the triggered price wars amplify the initial shocks in long-run growth by narrowing profit margins and discouraging customer base development.

We emphasize that long-run consumption risks play an essential role in altering firms' incentives to undercut prices. A moderate temporary shock to the level of aggregate consumption demand has little impact on the potential losses caused by the punishment, and hence, it has little impact on the deviation incentive. Therefore, moderate temporary shocks cannot drive substantial price war risks in equilibrium. Only persistent shocks in long-run growth can significantly change the severity of punishment and thus firms'

[3] Even though explicit collusion is illegal in many countries including United States, Canada and most of the EU due to antitrust laws, but implicit collusion in the form of price leadership and tacit understandings still takes place. For example, Intel and AMD implicitly collude on prices of graphic cards and central processing units in the 2000s, though a price war was waged between the two companies recently in October 2018.

effective discount rates. We show that the magnitude of price war risks declines when the growth shocks become less persistent. Specifically, price war risks become negligible when there are only moderate temporary shocks in consumption growth.

In the baseline model without financial frictions, the endogenously triggered price wars force firms to experience a double whammy when long-run growth drops. Quantitatively, we show that the amplification mechanism from price war risks alone increases the industry's exposure to long-run risks by about 30% on average. More importantly, we find that financial frictions further amplify price war risks. In our extended model, by introducing costly financing and endogenous cash holdings following the literature on external financing constraints (see, e.g. Gomes, 2001; Riddick and Whited, 2009; Gomes and Schmid, 2010; Bolton, Chen and Wang, 2011; Eisfeldt and Muir, 2016), the amplification effect of price war risks is levered up to about 50%. Intuitively, firms' financial constraints are rapidly tightened as cash flow growth plunges, owing to both the negative shocks in long-run growth and the triggered price wars. The heightened marginal value of liquidity raises firms' valuation of short-term cash flows, motivating them to further undercut peers. This in turn makes implicit collusion on product prices more difficult and drag the industry into more severe price wars. Such an adverse feedback loop between price war risks and financial constraints risks dramatically amplify the asset pricing implications of long-run-risk shocks. This novel feedback channel shares the spirit of He and Xiong (2012) and Edmans, Goldstein and Jiang (2015) who emphasize frictions in financial markets, but ours is through the interaction between imperfect competition in product markets and frictions in financial markets.

Our theory sheds new light on industries' heterogeneous exposures to price war risks and thus long-run risks. In the model and the data, we show that firms in the industries with a higher capacity of distinctive innovation are more immune to price war risks. The capacity of distinctive innovation is a fundamental, persistent, and predictable industry characteristic. Intuitively, a successful distinctive innovation allows firms to radically disrupt the market and rapidly grab substantial market shares. A prominent recent example is from Apple, a company that disrupted the mobile phone market by cobbling together an amazing touch screen with user-friendly interface. Thus, in the industries with a higher capacity of distinctive innovation, the market structure is more likely to experience dramatic changes and become highly concentrated in the future. This implies that firms in such industries would find it more difficult to implicitly collude with each other, because they all rationally expect that the product market is likely to be monopolized in the future, rendering future punishment on price undercutting less threatening. As a result, these industries feature low implicit collusive prices regardless of long-run growth rates, generating much less variation in product prices over long-run growth fluctuations. By contrast, in the industries with a lower capacity of distinctive innovation, the market structure is relatively stable, making a costly future punishment more credible. As a result, firms have stronger incentives for implicit price collusion, and rationally focus on maintaining existing customer bases and profit margins. However, because firms collude on higher prices on average in these industries, the equilibrium levels of collusive prices become more sensitive to the fluctuations in firms' collusion incentives, which are fundamentally driven by long-run-risk shocks. Hence, these industries are more exposed to price war risks and long-run risks.

Particularly, our model has the following main implications on product prices and stock returns across industries with different capacities of distinctive innovation. In the industries where the capacity of distinctive innovation is higher, (1) product markups (product prices minus marginal costs) are lower and less sensitive to long-run consumption growth shocks; (2) firms are less likely to engage in price wars in response to declines in long-run consumption growth, and (3) the (risk-adjusted) expected excess returns are lower.

We test these predictions using detailed data on patents and product prices. We first construct an innovation similarity measure based on U.S. patenting activities from 1976 to 2017 to capture the capacity of distinctive innovation across industries. In light of previous studies (e.g. Jaffe, 1986; Bloom, Schankerman and Van Reenen, 2013), our innovation similarity measure is constructed based on the technology classifications of firms' patents within industries. An industry is associated with a higher innovation similarity measure, if the patents produced by firms within the industry have more similar technology classifications. Thus, intuitively, an industry with a lower innovation similarity measure should have a higher capacity of distinctive innovation. We find that industries' capacities of distinctive innovation are priced in the cross section of industry returns. In particular, industries with a higher capacity of distinctive innovation are associated with lower (risk-adjusted) expected excess returns. Importantly, we show that the industries with a higher capacity of distinctive innovation are less exposed to long-run-risk shocks; moveover, the growth rates of their sales and markups are less exposed to long-run-risk shocks than those with a lower capacity of distinctive innovation.

We further test the key economic mechanism of our model by examining the dynamics of product prices in industries with different capacities of distinctive innovation. We measure the changes in product prices using the Nielsen retail scanner data, which contain price information for more than 3.5 million products from 2006 to 2016. We find that industries with a higher capacity of distinctive innovation have less dramatic product price declines after negative shocks in long-run growth. In particular, our event-type study shows that the industries with a higher capacity of distinctive innovation were less likely to engage in price wars in response to the Lehman crash in September of 2008, a time period in which the U.S. economy experienced a prominent negative long-run-risk shock according to the estimation of Schorfheide, Song and Yaron (2018). Finally, consistent with our model, we find that the sensitivity of product prices to long-run risks becomes much more similar across industries with different innovation capacities following antitrust enforcement.

Related Literature. Our paper contributes to the literature on long-run risks (see, e.g. Bansal and Yaron, 2004; Bansal, Dittmar and Lundblad, 2005; Hansen, Heaton and Li, 2008; Malloy, Moskowitz and Vissing-Jørgensen, 2009; Ai, 2010; Chen, 2010; Constantinides and Ghosh, 2011; Bansal, Kiku and Yaron, 2012; Gârleanu, Panageas and Yu, 2012; Croce, 2014; Kung and Schmid, 2015; Bansal, Kiku and Yaron, 2016; Dittmar and Lundblad, 2017; Schorfheide, Song and Yaron, 2018). Our main contribution is to show that price war risks can endogenously arise from long-run risks, generating a novel amplification mechanism. Moreover, we shed new light on the cross-sectional implication of long-run risks based on industries' capacity of distinctive innovation.

Our paper contributes to the burgeoning literature on the intersection between industrial organization, marketing and finance (see, e.g. Phillips, 1995; Kovenock and Phillips, 1997; Allen and Phillips, 2000; Hou and Robinson, 2006; Carlin, 2009; Aguerrevere, 2009; Hoberg and Phillips, 2010; Hackbarth and Miao, 2012; Carlson et al., 2014; Hoberg, Phillips and Prabhala, 2014; Bustamante, 2015; Weber, 2015; Hoberg and Phillips, 2016; Loualiche, 2016; Bustamante and Donangelo, 2017; Corhay, 2017; Corhay, Kung and Schmid, 2017; Hackbarth and Taub, 2018; D'Acunto et al., 2018; Dou and Ji, 2018; Dou et al., 2018; Andrei and Carlin, 2018).[4] In a closely related paper, Corhay, Kung and Schmid (2017) develop a novel general equilibrium model to understand the endogenous

[4]There is also a strand of the literature that studies the asset pricing implications of imperfect competition in the market micro-structure setting (see, e.g. Christie and Schultz, 1994; Biais, Martimort and Rochet, 2000; Liu and Wang, 2018).

relation between markups and stock returns. Their model implies that industries with higher markups are associated with higher expected returns. Our model yields a similar implication through price war risks. We show that industries with a lower capacity of distinctive innovation are associated with higher markups and more exposed to price war risks and long-run risks. Theoretically, our paper pushes forward the literature by developing a general-equilibrium model incorporated with dynamic supergames, in which price war risks arise endogenously and industry competition is endogenously connected to fundamental long-run risks in consumption growth.

Our paper is also related to a growing literature that studies the implications of innovation on asset prices (see, e.g. Li, 2011; Gârleanu, Kogan and Panageas, 2012; Gârleanu, Panageas and Yu, 2012; Hirshleifer, Hsu and Li, 2013; Kung and Schmid, 2015; Kumar and Li, 2016; Hirshleifer, Hsu and Li, 2017; Kogan et al., 2017; Dou, 2017; Fitzgerald et al., 2017; Kogan, Papanikolaou and Stoffman, 2018; Kogan et al., 2018). We contribute to this literature by showing that industries with a higher capacity of distinctive innovation are less prone to price war risks and are associated with lower (risk-adjusted) expected excess returns. Importantly, as emphasized in our paper, the capacity of distinctive innovation provides forward-looking competition information, complementing the traditional static measures of competition such as HHI and the product similarity measure.

Our paper also contributes to the macroeconomics and industrial organization literature on implicit collusion and price wars in dynamic oligopoly industries (see Stigler, 1964; Green and Porter, 1984; Rotemberg and Saloner, 1986; Haltiwanger and Harrington, 1991; Rotemberg and Woodford, 1991; Staiger and Wolak, 1992; Bagwell and Staiger, 1997; Athey, Bagwell and Sanchirico, 2004; Opp, Parlour and Walden, 2014). We make several contributions to this literature. First, we analyze the asset pricing implications of price war risks. Second, we show that, in the model and the data, the exposure to price war risks varies across industries with different capacities of distinctive innovation. Third, we show that there exists an adverse feedback loop between financial constraints risks and price war risks, which further amplifies firms' exposure to long-run risks. The implication of financial constraints on product prices has been analyzed in existing literature (Chevalier and Scharfstein, 1996; Gilchrist et al., 2017; Dou and Ji, 2018). However, none of them consider dynamic supergame equilibria or analyze the interaction between financial constraints risks and price collusion incentive.

Finally, our paper lies in the cross-sectional asset pricing literature (see, e.g. Cochrane, 1991; Berk, Green and Naik, 1999; Gomes, Kogan and Zhang, 2003; Pastor and Stambaugh, 2003; Nagel, 2005; Belo and Lin, 2012; Ai and Kiku, 2013; Kogan and Papanikolaou, 2013; Belo, Lin and Bazdresch, 2014; Donangelo, 2014; Kogan and Papanikolaou, 2014; Tsai and Wachter, 2016; Koijen, Lustig and Nieuwerburgh, 2017; Kozak, Nagel and Santosh, 2017; Ai et al., 2018; Dou et al., 2018; Gomes and Schmid, 2018). A comprehensive survey is provided by Nagel (2013). We show that the exposure to price war risks varies across industries with different capacities of distinctive innovation. The price wars risks interact with financial constraints risks, further amplify firms' exposures to long-run risks. Thus, our paper is particularly related to the works investigating the cross-sectional stock return implications of firms' fundamental characteristics through intangible capital (see, e.g. Ai, Croce and Li, 2013; Eisfeldt and Papanikolaou, 2013; Belo, Lin and Vitorino, 2014; Dou et al., 2018) and through financial constraints (see, e.g. Campbell, Hilscher and Szilagyi, 2008; Livdan, Sapriza and Zhang, 2009; Gomes and Schmid, 2010; Garlappi and Yan, 2011; Belo, Lin and Yang, 2018; Dou et al., 2018).

2 The Baseline Model

The economy contains a continuum of industries indexed by $i \in \mathcal{I} \equiv [0,1]$. Each industry i is a duopoly, consisting of two all-equity firms that are indexed by $j \in \mathcal{F} \equiv \{1,2\}$. We label a generic firm by ij and its competitor in industry i by ik. All firms are owned by a continuum of atomistic homogeneous households. Firms produce differentiated goods and set prices strategically to maximize shareholder value.

2.1 Preferences

Households are homogeneous and have stochastic differential utility of Duffie and Epstein (1992*a*,*b*). This preference is a continuous-time version of the recursive preferences proposed by Kreps and Porteus (1978), Epstein and Zin (1989), and Weil (1990). The Epstein-Zin-Weil recursive preference has become a standard preference in asset pricing and macro literature to capture reasonable joint behavior of asset prices and macroeconomic quantities. More precisely, the utility is defined recursively as follows:

$$U_0 = \mathbb{E}_0 \left[\int_0^\infty f(C_t, U_t) \mathrm{d}t \right], \tag{2.1}$$

where

$$f(C_t, U_t) = \beta U_t \frac{1-\gamma}{1-1/\psi} \left[\frac{C_t^{1-1/\psi}}{[(1-\gamma)U_t]^{\frac{1-1/\psi}{1-\gamma}}} - 1 \right]. \tag{2.2}$$

The felicity function $f(C_t, U_t)$ is an aggregator over current consumption rate C_t and future utility level U_t. The coefficient β is the rate parameter of time preference, γ is the relative risk aversion parameter for one-period consumption, and ψ is the parameter of elasticity of intertemporal substitution (EIS) for deterministic consumption paths.

Utility is derived from consuming the *final consumption good* C_t, which is obtained through a two-layer aggregation. Firm-level differentiated goods $C_{ij,t}$ are aggregated into industry-level consumption composites $C_{i,t}$, which are further aggregated into C_t. This setup ensures tractable aggregation across industries, while at the same time allowing us to model firms' strategic competition within each industry. We follow the functional form of *relative deep habits* developed by Ravn, Schmitt-Grohe and Uribe (2006).[5]

In particular, the demand for the final consumption good C_t is determined by the aggregation of industry composites

$$C_t = \left(\int_0^1 M_{i,t}^{1/\epsilon} C_{i,t}^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}i \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{2.3}$$

where $\epsilon > 1$ captures the elasticity of substitution among industry composites. The weight $M_{i,t} > 0$ captures the preference for industry i's composite at time t.

Further, industry i's composite is determined by the aggregation of firm-level differentiated goods

$$C_{i,t} = \left(\sum_{j \in \mathcal{F}} m_{ij,t}^{1/\eta} C_{ij,t}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}, \tag{2.4}$$

where $\sum_{j \in \mathcal{F}} m_{ij,t} = 1$. The parameter $\eta > 0$ captures the elasticity of substitution among the goods produced in the same industry. Consistent with the empirical evidence, we assume $\eta > \epsilon$ so that goods produced within the same industry are more substitutable relative to goods produced across different industries. $m_{ij,t}$ is the *relative deep habit* of firm

[5]This specification is inspired by Abel (1990), preferences feature *catching up with the Joneses*. The key difference is that Agents form habits over individual varieties of goods as opposed to over a composite consumption good. It is referred to as *deep habit formation*.

j in industry i defined as

$$m_{ij,t} = \frac{M_{ij,t}}{M_{i,t}} = \frac{M_{ij,t}}{\sum\limits_{j' \in \mathcal{F}} M_{ij',t}}, \quad \text{with } j = 1, 2. \tag{2.5}$$

The weight $M_{ij,t}$ captures the household's preference for firm j's good in industry i at time t. Given $M_{i,t}$, a higher relative deep habit $m_{ij,t}$ means that the household prefers firm j's goods more than firm k's goods in industry i.

2.2 Consumption Risks for the Long Run

We directly model the dynamics of aggregate consumption demand C_t. We incorporate product market frictions into a Lucas-tree model (Lucas, 1978) with homogeneous agents and complete financial markets. Many other extensions of the basic homogeneous-agent complete-market Lucas-tree models have been developed in the literature. For example, Longstaff and Piazzesi (2004), Bansal and Yaron (2004), Santos and Veronesi (2006), and Wachter (2013) consider leveraged dividends and implicitly incorporate labor market frictions in the Lucas-tree model; Menzly, Santos and Veronesi (2004) and Santos and Veronesi (2006, 2010), Martin (2013), and Tsai and Wachter (2016) consider a multi-asset (or multi-sector) Lucas-tree economy. We consider a Lucas-tree economy with multiple sectors whose shares are endogenously determined in the equilibrium.[6] Specifically, we assume that C_t evolves according to

$$\frac{\mathrm{d}C_t}{C_t} = \theta_t \mathrm{d}t + \sigma_c \mathrm{d}Z_{c,t}, \tag{2.6}$$

where

$$\mathrm{d}\theta_t = \kappa(\overline{\theta} - \theta_t)\mathrm{d}t + \varphi_\theta \sigma_c \mathrm{d}Z_{\theta,t}. \tag{2.7}$$

The consumption growth rate contains a persistent predictable component θ_t, which determines the conditional expectation of consumption growth (see, e.g. Kandel and Stambaugh, 1991, for early empirical evidence). The parameter $\overline{\theta}$ captures the average

[6]The heterogenous-agent complete-market Lucas-tree models have also been developed and widely used in asset pricing literature. For example, Xiong and Yan (2010) introduced information frictions to the Lucas-tree model, and Chan and Kogan (2002) introduced heterogeneous risk aversions in the Lucas-tree model.

long-run consumption growth rate. The parameter κ determines the persistence of the expected growth rate process. The parameter φ_θ determines the exposure to long-run risks. $\mathrm{d}Z_{c,t}$ and $\mathrm{d}Z_{\theta,t}$ are independent standard Brownian motions.

2.3 Stochastic Discount Factors

The state-price density Λ_t is

$$\Lambda_t = \exp\left[\int_0^t f_U(C_s, U_s)\mathrm{d}s\right] f_C(C_t, U_t). \tag{2.8}$$

The market price of risk evolves according to

$$\frac{\mathrm{d}\Lambda_t}{\Lambda_t} = -r\mathrm{d}t - \lambda_c \mathrm{d}Z_{c,t} - \lambda_\theta \mathrm{d}Z_{\theta,t}, \tag{2.9}$$

where r is the interest rate and

$$\lambda_c = \gamma\sigma_c \quad \text{and} \quad \lambda_\theta = \frac{\gamma - 1/\psi}{h+\eta}, \tag{2.10}$$

where h is the long-run deterministic steady-state consumption-wealth ratio determined in general equilibrium (see Section 2.8). Compared to other models with long-run risks, the key feature of our model is that firm-level demand is endogenous and depends on strategic competition, which we illustrate below.

2.4 What's a Firm's Customer Base?

A firm's customer base determines the demand for the firm's goods, and it exists due to consumers' brand loyalty or their habits in consumption. Below, we derive the firm's demand curve as a function of the firm's customer base.

Demand Curves. Let $P_{i,t}$ denote the price of industry i's composite. Given $P_{i,t}$ and C_t, solving a standard expenditure minimization problem gives the demand for industry i's composite:

$$C_{i,t} = M_{i,t}\left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} C_t, \tag{2.11}$$

where the price index P_t for the final consumption good is

$$P_t = \left(\int_0^1 M_{i,t} P_{i,t}^{1-\epsilon} \mathrm{d}i \right)^{\frac{1}{1-\epsilon}}. \tag{2.12}$$

Without loss of generality, we normalize $P_t \equiv 1$ so that the final consumption good is the numeraire. Given C_t, equation (2.11) implies that demand $C_{i,t}$ increases with $M_{i,t}$ and decreases with $P_{i,t}$. Thus, we can think of the household's preference $M_{i,t}$ as capturing industry i's total customer base. Importantly, the customer base $M_{i,t}$ evolves endogenously and gradually in the equilibrium (see Section 2.6).

Next, given $C_{i,t}$ and the price of firm j's good $P_{ij,t}$, the demand for firm j's good is:

$$C_{ij,t} = m_{ij,t} \left(\frac{P_{ij,t}}{P_{i,t}} \right)^{-\eta} C_{i,t}, \quad \text{with } j = 1, 2, \tag{2.13}$$

where industry i's price index is

$$P_{i,t} = \left(\sum_{j \in \mathcal{F}} m_{ij,t} P_{ij,t}^{1-\eta} \right)^{\frac{1}{1-\eta}}. \tag{2.14}$$

In equation (2.13), households' preference share $m_{ij,t}$ reflects the relative consumption inertia towards good j in industry i. At the same time, according to equation (2.14), a higher $m_{ij,t}$ indicates that firm j has more influence on the industry's price index $P_{i,t}$. Thus, we can think of $m_{ij,t}$ as firm j's *intrinsic market share* in industry i. In the economy, firms would undercut their competitors' prices in order to gain intrinsic market shares.

There are two different elasticity coefficients: the coefficient ϵ captures the between-industry elasticity of substitution, and the coefficient η captures the within-industry elasticity of substitution. Consistent with the literature, we assume that $\eta \geq \epsilon > 1$, meaning that products are more similar to those in the same industry and thus have higher within-industry elasticity of substitution. For example, the elasticity of substitution between Apple iPhone and Samsung Galaxy is higher than the elasticity of substitution between Apple iPhone and Dell Desktop.

Effective Short-Run Elasticity. The effective elasticity of firm j in industry i is

$$\begin{aligned}\frac{\partial \ln C_{ij,t}}{\partial \ln P_{ij,t}} &= \underbrace{s_{ij,t}\frac{\partial \ln C_{i,t}}{\partial \ln P_{i,t}}}_{\text{between-industry}} + \underbrace{(1-s_{ij,t})\frac{\partial \ln(C_{i,t}/C_{i,t})}{\partial \ln(P_{ij,t}/P_{i,t})}}_{\text{within-industry}} \\ &= s_{ij,t}\epsilon + (1-s_{ij,t})\eta \end{aligned} \tag{2.15}$$

where $s_{ij,t}$ is the *revenue market share* of firm j in industry i

$$s_{ij,t} = \frac{P_{ij,t}C_{ij,t}}{P_{i,t}C_{i,t}} = \left(\frac{P_{ij,t}}{P_{i,t}}\right)^{1-\eta} m_{ij,t}. \tag{2.16}$$

Thus, equation (2.15) shows that the short-run price elasticity of demand is given by the average of the within-industry elasticity of substitution η and the across-industry elasticity of substitution ϵ weighted by the firm's revenue shares. Depending on the revenue market share $s_{ij,t}$, firm j's short-run price elasticity of demand lies in $[\epsilon, \eta]$. On the one hand, when firm j's revenue market share $s_{ij,t}$ is small, within-industry competition becomes more relevant and thus firm j's price elasticity of demand depends more on η. In the extreme case with $m_{ij,t} = 0$ and thus $s_{ij,t} = 0$, firm j becomes atomistic and takes the industry price index $P_{i,t}$ as given. As a result, firm j's short-run price elasticity of demand is entirely determined by the within-industry elasticity of substitution η. On the other hand, when firm j's intrinsic market share is large, across-industry competition becomes more relevant and thus firm j's short-run price elasticity of demand depends more on ϵ. In the extreme case with $m_{ij,t} = 1$, firm j becomes the monopoly in industry i and its short-run price elasticity of demand is entirely determined by the between-industry elasticity of substitution ϵ.

Because $\eta > \epsilon$, our model naturally implies that an industry with higher concentration has a higher markup given other industry characteristics fixed. The key reason why across-industry competition matters for the firm's price elasticity of demand is that each firm's price has a non-negligible effect on the industry's price index in the duopoly industry. The magnitude of this effect is determined by the firm's intrinsic market share $m_{ij,t}$. Thus, when setting prices, each firm internalizes the effect of its own price on the industry's price index, which in turn determines the demand for the industry's goods given the across-industry elasticity of substitution ϵ. If there exist a continuum of firms in each industry, as in standard monopolistic competition models, then each firm is atomic and has no influence on the industry's price index. As a result, across-industry competition would have no impact on firms' price elasticities of demand.

Thus, although each industry only has two firms, the modeling of endogenous intrinsic market shares allows us to simultaneously capture the pricing behavior resembling a price taker (as in a model with monopolistic competition) and the pricing behavior resembling an industry-level monopoly. As we show later, this tractable framework also allows us to analyze how collusion incentive would endogenously change due to the change in market structure caused by distinctive innovation.

Demand Intensity. Substituting equation (2.5) into the demand function (2.13), we obtain

$$C_{ij,t} = M_{ij,t}\left(\frac{P_{ij,t}}{P_{i,t}}\right)^{-\eta}\widehat{C}_{i,t}, \quad \text{with } j = 1, 2, \tag{2.17}$$

where $\widehat{C}_{i,t}$ is the demand intensity per unit of the customer base in industry i:

$$\widehat{C}_{i,t} \equiv \frac{C_{i,t}}{M_{i,t}} = P_{i,t}^{-\epsilon}C_t. \tag{2.18}$$

As shown in equation (2.17), the household's preference $M_{ij,t}$ determines their demand for good (i, j) at time t. Thus we can think of $M_{ij,t}$ as capturing firm j's *customer base* in industry i. The firm-level demand intensity $\widehat{C}_{ij,t}$ is defined as

$$\widehat{C}_{ij,t} \equiv \frac{C_{ij,t}}{M_{ij,t}} = \left(\frac{P_{ij,t}}{P_{i,t}}\right)^{-\eta}\widehat{C}_{i,t}, \quad \text{with } j = 1, 2. \tag{2.19}$$

2.5 Firm Production and Cash Flows

Firms produce differentiated goods using labor and linear technology. Over $[t, t + \mathrm{d}t]$, firm j produces a flow of goods with intensity

$$Y_{ij,t} = AN_{ij,t}. \tag{2.20}$$

Without loss of generality, we normalize $A \equiv 1$. Labor is supplied by households inelastically at wage rate $\omega > 0$, which is also the unit flow cost of production. In equilibrium, firms choose $P_{ij,t} > \omega$ for positive profit. Given $P_{ij,t}$, the demand for firm j's good $C_{ij,t}$ is determined. The market clears for each differentiated good:

$$Y_{ij,t} = C_{ij,t}. \tag{2.21}$$

Thus, firm j's operating profit over $[t, t + \mathrm{d}t]$ is given by

$$\mathrm{d}O_{ij,t} = \left(P_{ij,t} - \omega\right) C_{ij,t}\mathrm{d}t. \tag{2.22}$$

Because firms do not face financial frictions or cash flow risks, there is no incentive for firms to hoard cash. Thus, the net dividend payout $\mathrm{d}D_{ij,t}$ (or equity financing if $\mathrm{d}D_{ij,t} < 0$) is trivially pinned down by

$$\mathrm{d}D_{ij,t} = \mathrm{d}O_{ij,t}. \tag{2.23}$$

2.6 Dynamics of Customer Bases

Consumers tend to repeatedly purchase the same goods or services that they have used in the past. To capture consumers' brand loyalty or consumption inertia (Klemperer, 1995; Gourio and Rudanko, 2014), we assume that the customer base M_{ij} of firm j is a persistent state variable and long term in nature. Because the customer base determines the demand and revenue for the firm, this renders the customer base a long-term asset that every firm wants to compete for.

There are two primary ways to attract customers and hence, accumulate customer bases. One is by setting lower product prices and the other is by designing innovative features. The evolution of firm-level customer bases is determined by

$$\mathrm{d}M_{ij,t} = \text{Price channel}_{ij,t} + \text{Innovation channel}_{ij,t}, \quad \text{with } j = 1, 2. \tag{2.24}$$

Let us describe them below in turn.

Strategic Pricing. There are uncountable cases about how firms attract consumers through price undercutting or discount offering. A temporary cut in prices can have a persistent effect on increasing the firm's demand because consumers have switching costs. Attracted by lower prices, new customers buy the firm's products, feel satisfied and become loyal to the firm. Due to switching costs, these consumers become the firm's customers and keep buying the firm's products in the future. To capture this idea, we model the price channel as

$$\text{Price channel}_{ij,t} = \left[z \left(\frac{C_{ij,t}}{C_t} \right)^{\alpha} M_{ij,t}^{1-\alpha} - \rho M_{ij,t} \right] \mathrm{d}t, \tag{2.25}$$

where the parameter $z \geq 0$ determines the speed of customer base accumulation through the price channel. Intuitively, a lower price $P_{ij,t}$ increases the contemporaneous demand flow rate $C_{ij,t}$, allowing the firm to accumulate a larger customer base over $[t, t + \mathrm{d}t]$. The parameter $\alpha > 0$ captures the relative importance of contemporaneous demand in accumulating the customer base. The parameter $\rho > 0$ captures customer base depreciation due to economy-wide reasons such as the mortality of consumers.

The firm's pricing decision crucially depends on the value of z and its current intrinsic market share $m_{ij,t}$. To elaborate, if $z = 0$, the firm's pricing decision is static, chosen to maximize contemporaneous profits. If $z > 0$, the firm's pricing decision becomes dynamic, facing the tradeoff between increasing contemporaneous profits through setting higher prices and increasing future profits through setting lower prices to accumulate more customer bases. Consistent with the empirical evidence, the slow-moving customer base $M_{ij,t}$ implies that the long-run price elasticity of demand is higher than the short-run elasticity (see, e.g. Rotemberg and Woodford, 1991).

Incremental and Distinctive Innovation. Firms can conduct innovation to improve their products and attract customers from other firms. There are two types of innovation: incremental innovation and distinctive innovation, both allowing firms to expand their customer bases.

Incremental innovation adds value to customers through introducing new features to existing products or services. These small incremental changes are proved to be incrementally innovative if customers have a better experience with the product. Almost all companies engage in incremental innovation in one form or another. For example, Motorola has launched a series of Motorola Razr since 2004, based on constant improvement of previous generations.

Distinctive innovation involves applying new technology or processes to the firm's current market. It is stealthy in nature since newer tech will often be inferior to existing market technology. It is only after a few iterations that the newer tech surpasses the old and disrupts all existing companies. By then, it might be too late for other established firms to quickly compete with the newer technology. There are quite a few examples of distinctive innovation, one of the more prominent being Apple's iPhone disruption of the mobile phone market.[7]

In our model, we assume that each firm's incremental innovation and distinctive innovation succeed independently with different intensities $\phi_{i,t}\lambda_\iota$ and $(1-\phi_{i,t})\lambda_d$. The industry characteristic $\phi_{i,t}$ reflects the units of incremental R&D projects that firms have in industry i, and $1-\phi_{i,t}$ reflects the units of distinctive R&D projects, or the industry's capacity of distinctive innovation. $\phi_{i,t}$ is the only ex-ante heterogeneity across industries, evolving idiosyncratically according to a finite-state Markov chain on $\Phi = \{0 \leq \phi_1 < \phi_2 < ... < \phi_N \leq 1\}$.

Conditional on successful innovations, incremental and distinctive innovations separately create new customer bases with rate g_ι and g_d. In addition, successful innovation also snatches the peer firm's customer bases by τ_ι and τ_d fractions. We emphasize the difference in the customer base competition effect between the two types of innovation by assuming

$$0 \approx \tau_\iota << \tau_d \approx 1 \quad \text{and} \quad \lambda_\iota >> \lambda_d \geq 0, \tag{2.26}$$

so that distinctive innovation happens rarely but allows firms to capture almost the entire industry's market share. The average growth effects from the two types of innovation are assumed to be equal, i.e. $\lambda_\iota g_\iota = \lambda_d g_d$.

Thus, the innovation channel for customer base accumulation is given by

$$\begin{aligned}\text{Innovation channel}_{ij,t} = & \underbrace{\left[\tau_\iota \mathrm{d}I^\iota_{ij,t} + \tau_d \mathrm{d}I^d_{ij,t}\right] M_{ik,t}}_{j\text{'s competition effect}} + \underbrace{\left[g_\iota \mathrm{d}I^\iota_{ij,t} + g_d \mathrm{d}I^d_{ij,t}\right] M_{i,t}}_{j\text{'s growth effect}} \\ & - \underbrace{\left[\tau_\iota \mathrm{d}I^\iota_{ik,t} + \tau_d \mathrm{d}I^d_{ik,t}\right] M_{ij,t}}_{k\text{'s competition effect}},\end{aligned} \tag{2.27}$$

where $I^\iota_{ij,t}$ and $I^d_{ij,t}$ are independent Poisson processes with intensity $\phi_{i,t}\lambda_\iota$ and $(1-\phi_{i,t})\lambda_d$, capturing the success of incremental and distinctive innovation.

[7]Prior to the iPhone, most popular phones relied on buttons, keypads or scroll wheels for user input. The iPhone was the result of a technological movement that was years in making, mostly iterated by Palm Treo phones and personal digital assistants (PDAs). In order to disrupt the mobile phone market, Apple cobbled together an amazing touch screen that had a simple to use interface, and provided users access to a large assortment of built-in and third-party mobile applications.

Evolution of Customer Bases. Substituting the price channel (2.25) and the innovation channel (2.27) into equation (2.24), we derive the evolution of firm j's customer base

$$\mathrm{d}M_{ij,t} = \left[z\left(\frac{C_{ij,t}}{C_t}\right)^{\alpha} M_{ij,t}^{1-\alpha} - \rho M_{ij,t}\right]\mathrm{d}t + \left(\tau_\iota \mathrm{d}I_{ij,t}^{\iota} + \tau_d \mathrm{d}I_{ij,t}^{d}\right) M_{ik,t}$$
$$+ \left(g_\iota \mathrm{d}I_{ij,t}^{\iota} + g_d \mathrm{d}I_{ij,t}^{d}\right) M_{i,t} - \left(\tau_\iota \mathrm{d}I_{ik,t}^{\iota} + \tau_d \mathrm{d}I_{ik,t}^{d}\right) M_{ij,t}. \tag{2.28}$$

The evolution of industry i's aggregate customer base is

$$\frac{\mathrm{d}M_{i,t}}{M_{i,t}} = \left[z\left(\sum_{j\in\mathcal{F}} m_{ij,t}\widehat{C}_{ij,t}^{\alpha}\right) - \rho\right]\mathrm{d}t + \sum_{j\in\mathcal{F}}\left(g_\iota \mathrm{d}I_{ij,t}^{\iota} + g_d \mathrm{d}I_{ij,t}^{d}\right). \tag{2.29}$$

Therefore, the growth rate of each industry i's customer base is the average growth rate of the two firms' customer bases weighted by their intrinsic market shares:

$$\frac{\mathrm{d}M_{i,t}}{M_{i,t}} = \sum_{j\in\mathcal{F}} m_{ij,t}\frac{\mathrm{d}M_{ij,t}}{M_{ij,t}}. \tag{2.30}$$

2.7 Price Setting Supergames

In this section, we illustrate the rich interactions among firms in the same industry. Essentially, the two firms in the same industry play a stochastic dynamic game. As time is continuous, the stage game of setting prices are played continuously and infinitely repeated. In this setting, there exist multiple equilibria because one can apply a conditional punishment strategy to sustain some specific payoff distribution. Below, we first illustrate the non-collusive equilibrium. Then we define and characterize the collusive equilibrium that yields higher profits for both firms.

Non-Collusive Equilibrium. Substituting equation (2.17) into equation (2.23), we obtain

$$\frac{\mathrm{d}D_{ij,t}}{M_{ij,t}} = \Pi_{ij}(P_{i1,t}, P_{i2,t}, M_{i1,t}, M_{i2,t}, C_t)\mathrm{d}t, \tag{2.31}$$

where $\Pi_{ij}(P_{i1,t}, P_{i2,t}, M_{i1,t}, M_{i2,t}, C_t)$ is the conditional expected profit rate defined by[8]

$$\Pi_{ij}(P_{i1,t}, P_{i2,t}, M_{i1,t}, M_{i2,t}, C_t) \equiv \left(P_{ij,t} - \omega\right) \left(\frac{P_{ij,t}}{P_{i,t}}\right)^{-\eta} P_{i,t}^{-\epsilon} C_t. \tag{2.32}$$

Equation (2.32) shows that the (local) conditional expected profit rate of firm j depends on its peer firm k's product price $P_{ik,t}$ through the industry's price index $P_{i,t}$. This reflects the externality of firm k's decisions. For example, if firm k sets a low price $P_{ik,t}$, the price index $P_{i,t}$ will drop, and thus the demand for firm j's goods $C_{ij,t}$ will decrease. This will motivate firm j to set a lower price $P_{ij,t}$, and thus the two firms' pricing decisions exhibit strategic complementarity in equilibrium.

In the non-collusive equilibrium, firm j chooses product price $P_{ij,t}$ to maximize shareholder value $V_{ij}^{N}(M_{i1,t}, M_{i2,t}, \phi_{i,t}, C_t, \theta_t)$, conditional on its peer firm k setting the equilibrium price $P_{ik,t}^{N}$. The firm's value also depends on persistent consumption growth rate θ_t because it affects the state-price density Λ_t. Following the standard recursive formulation in dynamic games with Markov Perfect Nash Equilibrium (see, e.g. Pakes and McGuire, 1994; Ericson and Pakes, 1995; Maskin and Tirole, 2001), the optimization problem for firm 1 can be formulated recursively by an HJB equation:

$$0 = \max_{P_{i1,t}} \Lambda_t \Pi_{i1}(P_{i1,t}, P_{i2,t}^{N}, M_{i1,t}, M_{i2,t}, C_t) M_{ij,t} \mathrm{d}t + \mathbb{E}_t \left[\mathrm{d}(\Lambda_t V_{i1}^{N}(M_{i1,t}, M_{i2,t}, \phi_{i,t}, C_t, \theta_t))\right]. \tag{2.33}$$

Similarly, the optimization problem for firm 2 is given by the following HJB equation:

$$0 = \max_{P_{i2,t}} \Lambda_t \Pi_{i2}(P_{i1,t}^{N}, P_{i2,t}, M_{i1,t}, M_{i2,t}, C_t) M_{ij,t} \mathrm{d}t + \mathbb{E}_t \left[\mathrm{d}(\Lambda_t V_{i2}^{N}(M_{i1,t}, M_{i2,t}, \phi_{i,t}, C_t, \theta_t))\right] \tag{2.34}$$

The non-collusive equilibrium prices $P_{ij,t}^{N}(M_{i1,t}, M_{i2,t}, \phi_{i,t}, C_t, \theta_t)$ (with $j = 1, 2$) are determined by solving the coupled HJB equations (2.33-2.34).

A couple of points are worth mentioning. First, the expectation $\mathbb{E}_t$ is conditioning on the exogenous state variable $\phi_{i,t}$, C_t, θ_t and endogenous state variables $M_{i1,t}$ and $M_{i2,t}$. Second, The expectation is also conditioning on the endogenous control variables, $P_{ij,t}$ and $P_{ik,t}$, because they determine the evolution of endogenous state variables (see equation 2.28). Third, the Nash equilibrium considered here is non-collusive, because it does

[8] Note we define the conditional expected profit rate as $\Pi_{ij}(P_{i1,t}, P_{i2,t}, M_{i1,t}, M_{i2,t}, C_t)$ rather than $\Pi_{ij}(P_{ij,t}, P_{ik,t}, M_{ij,t}, M_{ik,t}, C_t)$ because we want to keep the order of state variables the same when we consider $j = 1$ ($k = 2$) and $j = 2$ ($k = 1$).

not depend on historical information (i.e. not using conditional punishment strategies based on the two firms' historical decisions). Such an equilibrium is called "static Nash equilibrium" by Fudenberg and Tirole (1991). Fourth, the non-collusive equilibrium is actually the equilibrium of a dynamic game because the strategic pricing decisions involve dynamic concerns. Each firm j's price $P_{ij,t}$ not only affects both firms' contemporaneous cash flows through the current price index $P_{i,t}$ but also their continuation values $\mathrm{d}V_{ij}^{N}(M_{i1,t}, M_{i2,t}, \phi_{i,t}, C_t, \theta_t)$ due to the price channel specified in equation (2.25).

Collusive Equilibrium. The non-collusive equilibrium features low product prices and low operating profits due to the lack of collusion. In a dynamic setting, the repeated interactions between the two firms within the same industry naturally motivate the formation of a cartel, in which firms collectively set higher prices to gain higher operating profits. We define the collusive equilibrium as the sub-game perfect equilibrium in which the two firms adopt prices higher than non-collusive prices.[9]

Consider a collusive equilibrium in which firms follow the collusive pricing schedule $P_{ij}^{C}(M_{i1,t}, M_{i2,t}, \phi_{i,t}, C_t, \theta_t)$ (with $j = 1,2$).[10] Firm j's value in the collusive equilibrium, denoted by $V_{ij}^{C}(M_{i1,t}, M_{i2,t}, \phi_{i,t}, C_t, \theta_t)$, is determined by

$$0 = \Lambda_t \Pi_{ij}(P_{i1}^{C}, P_{i2}^{C}, M_{i1,t}, M_{i2,t}, C_t) M_{ij,t} \mathrm{d}t + \mathbb{E}_t \left[\mathrm{d}(\Lambda_t V_{ij}^{C}(M_{i1,t}, M_{i2,t}, \phi_{i,t}, C_t, \theta_t)) \right], \text{ with } j = 1,2. \tag{2.35}$$

The sub-game perfection of the collusive equilibrium is ensured by conditional strategies with punishment on deviation. In particular, we assume that firms receive the opportunity to inspect peers' past prices up to t with Poisson intensity ς over $[t, t + \mathrm{d}t]$.[11] When a price deviation is detected by the peer firm, the peer firm will start setting the non-collusive price in the future. Conditional on the peer firm's price being non-collusive,

[9] In the industrial organization and macroeconomics literature, this equilibrium is called collusive equilibrium or collusion (see e.g. Green and Porter, 1984; Rotemberg and Saloner, 1986). Game theorists generally call it the equilibrium of repeated game (Fudenberg and Tirole, 1991) in order to distinguish its nature from the equilibrium of stage games (i.e. our non-collusive equilibrium).

[10] Fershtman and Pakes (2000) require all firms to adopt the same collusive price to maintain tractability. Our collusive pricing schedule is more general because it allows firms to set different prices based on their customer bases, the industry's characteristic, and aggregate conditions.

[11] In discrete time, a standard assumption in dynamic game-theoretic models is that deviation is observed in the next period. This assumption, however, is unrealistic in a continuous time model. If we assume that deviation is observed in the next instant, it means that information flows infinitely fast, then any average payoffs can be achieved by a sub-game perfect equilibrium. Bergin and MacLeod (1993) discuss the technical difficulties in continuous-time repeated games.

the deviating firm would also set the non-collusive price after her deviation is detected, because setting the non-collusive price is the best response to the peer firm's non-collusive price. Thus, the punishment on deviation is based on switching to the non-collusive equilibrium, which itself has sub-game perfection.[12] The collusive equilibrium is a sub-game perfect equilibrium if the collusive pricing schedule $P_{ij}^{C}(M_{i1,t}, M_{i2,t}, \phi_{i,t}, C_t, \theta_t)$ is set to ensure that there is no deviation along the equilibrium path. Formally, denote $V_{ij}^{D}(M_{i1,t}, M_{i2,t}, \phi_{i,t}, C_t, \theta_t)$ as firm j's value conditional on one-shot deviation[13], the HJB equations are:

$$
\begin{aligned}
0 = &\max_{P_{i1,t}} \Lambda_t \Pi_{i1}(P_{i1}, P_{i2}^{C}, M_{i1,t}, M_{i2,t}, C_t) M_{i1,t} \mathrm{d}t + \mathbb{E}_t \left[\mathrm{d}(\Lambda_t V_{i1}^{D}(M_{i1,t}, M_{i2,t}, \phi_{i,t}, C_t, \theta_t)) \right] \\
&+ \Lambda_t \left[V_{i1}^{N}(M_{i1,t}, M_{i2,t}, \phi_{i,t}, C_t, \theta_t) - V_{i1}^{D}(M_{i1,t}, M_{i2,t}, \phi_{i,t}, C_t, \theta_t) \right] \varsigma \mathrm{d}t, \qquad (2.36) \\
0 = &\max_{P_{i2,t}} \Lambda_t \Pi_{i2}(P_{i1}^{C}, P_{i2}, M_{i1,t}, M_{i2,t}, C_t) M_{i2,t} \mathrm{d}t + \mathbb{E}_t \left[\mathrm{d}(\Lambda_t V_{i2}^{D}(M_{i1,t}, M_{i2,t}, \phi_{i,t}, C_t, \theta_t)) \right] \\
&+ \Lambda_t \left[V_{i2}^{N}(M_{i1,t}, M_{i2,t}, \phi_{i,t}, C_t, \theta_t) - V_{i2}^{D}(M_{i1,t}, M_{i2,t}, \phi_{i,t}, C_t, \theta_t) \right] \varsigma \mathrm{d}t. \qquad (2.37)
\end{aligned}
$$

To ensure that the collusive equilibrium with the pricing schedule $P_{ij}^{C}(M_{i1,t}, M_{i2,t}, \phi_{i,t}, C_t, \theta_t)$ is a sub-game perfect equilibrium, there must be no deviation along the equilibrium path. Thus the following incentive compatibility (IC) constraints have to be satisfied:

$$
V_{ij}^{D}(M_{i1,t}, M_{i2,t}, \phi_{i,t}, C_t, \theta_t) \leq V_{ij}^{C}(M_{i1,t}, M_{i2,t}, \phi_{i,t}, C_t, \theta_t), \qquad (2.38)
$$

for $j = 1, 2$, $M_{i1,t}, M_{i2,t} > 0$, $\phi_{i,t} \in \Phi$, $C_t > 0$, and all θ_t. In fact, there exist infinitely many collusive pricing schedules $P_{ij}^{C}(M_{i1,t}, M_{i2,t}, \phi_{i,t}, C_t, \theta_t)$ that satisfy the IC constraints, and hence infinitely many collusive equilibrium. Because firms maximize profits in general equilibrium, it is reasonable for them two collude on prices as high as possible.[14] We

[12]We adopt the non-collusive equilibrium as the incentive-compatible punishment for deviation to follow the literature (see, e.g. Green and Porter, 1984; Rotemberg and Saloner, 1986). The non-collusive equilibrium can be interpreted as the most severe price war, though it never happens in equilibrium. All else equal, adopting a less stringent punishment would lower collusive prices. In a similar multi-period game, Bond and Krishnamurthy (2004) consider a lenient "debt-default" rule as a punishment for debt default, rather than a full exclusion from financial markets. Interestingly, the "debt-default" rule provides optimal repayment incentives while at the same time resembles laws governing default on debt contracts.

[13]The one-shot deviation principle indicates that there is no need to consider the case with two firms simultaneously deviating from collusive pricing to obtain a sub-game perfect equilibrium (see Fudenberg and Tirole, 1991).

[14]There are two reasons why we focus on the highest collusive pricing schedule. First, non-binding IC constraints imply that there is room to further increase both firms' values by increasing collusive prices.

thus focus on the highest collusive pricing schedule, under which the IC constraints are binding, i.e. $P^C_{ij}(M_{i1,t}, M_{i2,t}, \phi_{i,t}, C_t, \theta_t)$ are determined such that

$$V^D_{ij}(M_{i1,t}, M_{i2,t}, \phi_{i,t}, C_t, \theta_t) = V^C_{ij}(M_{i1,t}, M_{i2,t}, \phi_{i,t}, C_t, \theta_t). \tag{2.39}$$

Intuitively, a higher ς increases the likelihood of implementing the punishment strategy, dampening the incentive to deviate by reducing $V^D_{ij}(M_{i1,t}, M_{i2,t}, \phi_{i,t}, C_t, \theta_t)$. Thus, all else equal, a higher ς allows firms to collude on higher prices. Importantly, the collusion incentive also depends on the long-run growth rate θ_t and its persistence.[15] We discuss them in Section 4.

2.8 General Equilibrium Conditions

In equilibrium, the value function of the representative household is

$$U_t = \exp\left(A_0 + A_1\theta_t\right)\frac{W_t^{1-\gamma}}{1-\gamma}, \tag{2.40}$$

where A_0 is a deterministic function of model parameters, and A_1 is equal to

$$A_1 = \frac{\psi^{-1}(1-\gamma)}{h+\kappa}, \quad \text{with} \quad h = \exp\left(\overline{\ln C - \ln W}\right). \tag{2.41}$$

The equilibrium wealth-consumption ratio is

$$\frac{W_t}{C_t} = \rho^{-\psi}\exp\left[\frac{1-\psi}{1-\gamma}A_0 + \left(\frac{1-\psi^{-1}}{h+\kappa}\right)\theta_t\right]. \tag{2.42}$$

In equilibrium, the long-run deterministic steady-state consumption-wealth ratio is:

$$\ln(h) = \overline{\ln C - \ln W} = \psi\ln(\rho) - \frac{1-\psi}{1-\gamma}A_0 - \frac{1-\psi^{-1}}{h+\kappa}\bar{\theta}. \tag{2.43}$$

Given that firms collude with each other to maximize their values, it is a bit unreasonable to rule out a better collusion. Second, considering the highest collusive price allows us to conduct more disciplined comparative statics in the presence of multiple equilibria. In other words, focusing on the highest collusive price ensures that we always pick up the same equilibrium when we compare across different industries.

[15]We do not model dynamic entries and exits due to the complexity of the current setup. Introducing entries and exits will strengthen our mechanism by increasing the magnitude of price war risks. Intuitively, it becomes even harder for firms to collude during periods with low long-run growth rates because exits are more likely due to lower operating cash flows.

2.9 Model Solution

By exploiting linearity on $M_{i,t}C_t$, we reduce the model to three state variables: $m_{i1,t}$, $\phi_{i,t}$, and θ_t. We solve the normalized firm value $v_{ij}^N(m_{i1,t}, \phi_{i,t}, \theta_t)$, $v_{ij}^C(m_{i1,t}, \phi_{i,t}, \theta_t)$, and $v_{ij}^D(m_{i1,t}, \phi_{i,t}, \theta_t)$. The details of our numerical algorithm are presented in Appendix D.

We solve the model in risk-neutral measure. By Girsanov's theorem, we have

$$\mathrm{d}Z_{c,t} = -\lambda_c \mathrm{d}t + \mathrm{d}\tilde{Z}_{c,t}, \tag{2.44}$$

$$\mathrm{d}Z_{\theta,t} = -\lambda_\theta \mathrm{d}t + \mathrm{d}\tilde{Z}_{\theta,t}. \tag{2.45}$$

Under the risk-neutral measure, the dynamics of aggregate conditions are

$$\frac{\mathrm{d}C_t}{C_t} = \theta_t \mathrm{d}t + \sigma_c \mathrm{d}\tilde{Z}_{c,t}, \tag{2.46}$$

$$\mathrm{d}\theta_t = \kappa\left(\overline{\theta}^{Q} - \theta_t\right)\mathrm{d}t + \varphi_\theta \sigma_c \mathrm{d}\tilde{Z}_{\theta,t}, \tag{2.47}$$

where

$$\overline{\theta}^{Q} = \overline{\theta} - \lambda_c \sigma_c - \kappa^{-1}\lambda_\theta \varphi_\theta \sigma_c. \tag{2.48}$$

3 Illustration of Equilibrium Concepts

Our model is based on a general equilibrium framework with a continuum of industries. Within each industry, we formulate the two firms' dynamic competition using stochastic game-theoretic models. In this section, we illustrate the dynamic game-theoretic equilibrium within an industry. We start by illustrating the non-collusive equilibrium in Section 3.1. We highlight that the strategic complementarity embedded in the non-collusive equilibrium is a crucial force that generates price wars during periods with low long-run consumption growth. We then illustrate the collusive equilibrium that naturally arises from the dynamic repeated interaction between the two firms. The collusive equilibrium is a sub-game perfect equilibrium that is endogenously sustained by using the non-collusive equilibrium as punishment. In Section 3.3, we illustrate the IC constraints and the determination of collusive prices in the collusive equilibrium.

3.1 Non-Collusive Equilibrium

In the non-collusive equilibrium, the two firms simultaneously set prices, taking the other firm's price as given. Thus, the equilibrium prices are determined by the intersection of the two firms' optimal price as a function of the other firm's price. To characterize the determination of the non-collusive equilibrium, we consider an industry with $\phi = 0$ and long-run growth rate $\theta = 0.06$. Denote $\widehat{P}_{i1}^N(m_{i1}; P_{i2})$ as firm 1's optimal price as a function of its intrinsic market share m_{i1} and firm 2's price P_{i2}. Similarly, we denote $\widehat{P}_{i2}^N(m_{i1}; P_{i1})$ as firm 2's optimal price as a function of firm 1's intrinsic market share m_{i1} and price P_{i1}.

In Panel A of Figure 1, the blue solid line plots firm 1's optimal price as a function of firm 2's price P_{i2}, when the two firms have equal intrinsic market shares (i.e. $m_{i1} = 0.5$). The black dash-dotted line plots firm 2's optimal price as a function of firm 1's price P_{i1} for the same intrinsic market share. The intersection of the two curves (the blue filled circle) determines the equilibrium prices, i.e. $P_{i1}^N(0.5)$ and $P_{i2}^N(0.5)$:

$$P_{i1}^N(0.5) = \widehat{P}_{i1}^N(0.5; P_{i2}^N(0.5)) \quad \text{and} \quad P_{i2}^N(0.5) = \widehat{P}_{i2}^N(0.5; P_{i1}^N(0.5)). \tag{3.1}$$

The two firms set exactly the same prices when they have the same intrinsic market shares. Both curves are upward sloping, indicating that there exists strategic complementarity in setting prices in the non-collusive equilibrium: both firms tend to set lower prices when the other firm's price is lower. This is because when the other firm's price is lower, the price elasticity of demand endogenously increases, motivating the firm to lower its own price. Because of such strategic complementarity, the non-collusive equilibrium features low prices and hence low profit margins for both firms. To see it clearly, suppose firm 2 sets $P_{i2} = 1.6$, then firm 1's best response is to set $P_{i1} = 1.4$ (A_1). Given that firm 1's price is lower than firm 2's, firm 2 will further lower its price to $P_{i2} = 1.28$ (A_2). But then firm 2's price is lower than firm 1's, which triggers firm 1 to lower its price to $P_{i1} = 1.2$ (A_3), and so on, until the prices reach equilibrium values. Such price adjustments happen instantaneously in rational expectation equilibrium.[16]

We emphasize that the strategic complementarity in price setting is a crucial force that generates price wars from declines in long-run consumption growth. As we discuss in Section 4.1, collusive prices decrease with long-run growth rates in consumption. This is because if firms were to collude on high prices during periods with low long-run growth

[16]The dynamics of price adjustment is related to the old tradition that used Tâtonnement or Cobweb dynamics to capture the off-equilibrium adjustment of prices in Walrasian economies.

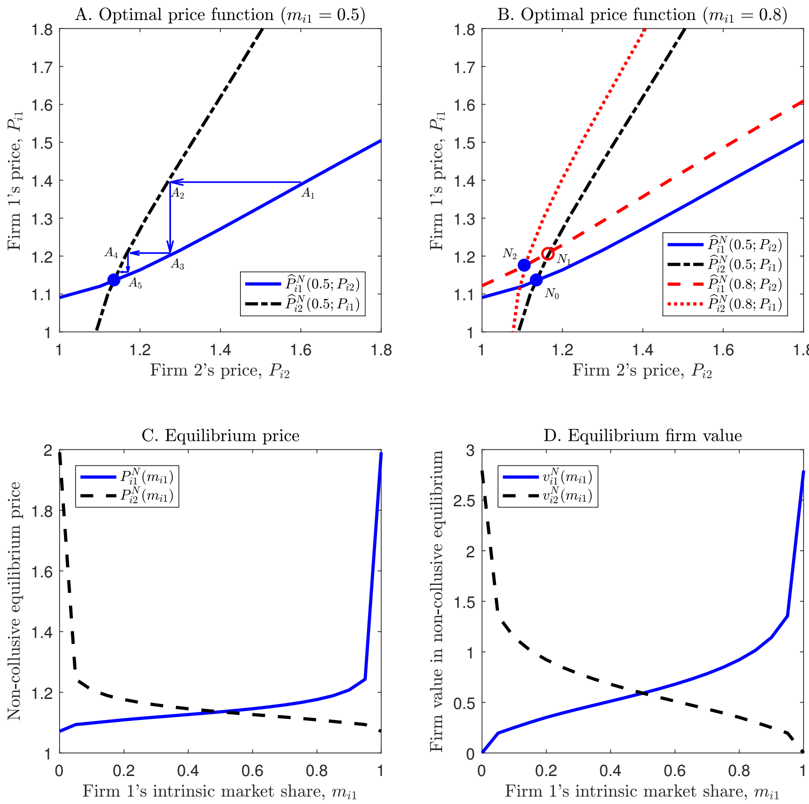


Figure 1: Prices and firm values in the non-collusive equilibrium.

rates, both firms will have the incentive to lower its prices to undercut the other firm's market share. This in turn will trigger a spiral of downward price adjustments due to strategic complementarity, eventually converging to the non-collusive equilibrium. Thus, the strategic complementarity rationalizes the use of non-collusive equilibrium as credible punishment to sustain the collusive equilibrium.

In Panel B, we investigate how firms change prices when their intrinsic market shares change. The blue solid line and black dash-dotted line represent the same benchmark case (i.e. $m_{i1} = 0.5$) as in Panel A. The red dashed and red dotted lines refer to the prices set by the two firms when firm 1's intrinsic market share m_{i1} increases from 0.5 to 0.8 (thus firm 2's intrinsic market share decreases from 0.5 to 0.2 accordingly). It is shown that firm 1's optimal price function shifts upward and firm 2's optimal price function shifts to the left, implying that both firms tend to set higher prices when their own intrinsic market shares increase. Intuitively, there are two main reasons. First, when the intrinsic market share is higher, setting low prices to further compete for the market share is relatively more costly compared to setting high prices to profit from inertial customers. Second, the firm's influence on the equilibrium price index increases with its intrinsic market share (see equation 2.14). Therefore, a higher market share increases the firm's market power and lowers the price elasticity of demand, resulting in higher prices.

Panel B also clearly illustrates the implication of strategic pricing. In the benchmark equilibrium (N_0), the prices are P_{i1,N_0} and P_{i2,N_0}. A higher market share m_{i1} shifts the equilibrium to N_2, and the new equilibrium prices satisfy $P_{i1,N_2} > P_{i1,N_0}$ and $P_{i2,N_2} < P_{i2,N_0}$. However, if firm 2 were to hold its price decisions unchanged (at the black-dashed line), the new equilibrium would be N_1, with $P_{i1,N_1} > P_{i1,N_2}$, indicating that firm 1 would raise its price more in response to the increase in its intrinsic market share m_{i1}. Therefore, firm 1's price is less responsive precisely because it anticipates that firm 2 would lower its price P_{i2} (as captured by the red dotted line). Such strategic concerns result in a smaller increase in firm 1's price P_{i1}, which helps prevent too much loss in its intrinsic market share m_{i1}.

Panel C shows that when firm 1's intrinsic market share increases, firm 1's value increases (the blue solid line) and firm 2's value decreases symmetrically (black dashed line). Moreover, both firms set higher equilibrium prices when their intrinsic market shares increase (see Panel D).

3.2 Collusive Equilibrium

We now turn to the illustration of the collusive equilibrium. In the collusive equilibrium, both firms set prices according to the collusive pricing schedule $P_{ij}(m_{i1,t}, \phi_{i,t}, \theta_t)$. We illustrate the collusive equilibrium with $\phi = 0$ and $\theta = 0.06$ below.

In Panel A of Figure 2, we compare the firm's prices in the collusive equilibrium and the non-collusive equilibrium. As the two firms are symmetric, we only focus on illustrating firm 1's price. The black dashed line plots firm 1's price in the non-collusive equilibrium (as in Panel C of Figure 1). The blue solid line plots firm 1's price in the collusive equilibrium. It is shown that due to collusion, firm 1 sets higher prices than what it would set in the non-collusive equilibrium. The prices increase monotonically with intrinsic market shares in both the collusive and the non-collusive equilibria.

Interestingly, Panel B shows that the ability to collude on higher prices, as reflected

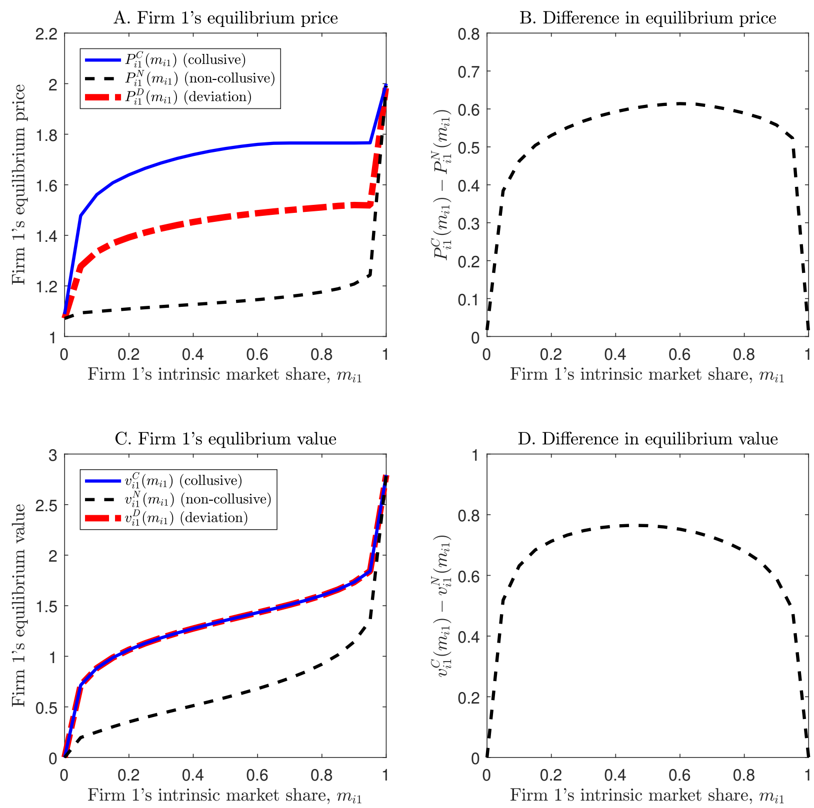


Figure 2: Comparing prices and firm values in the collusive and non-collusive equilibria.

by the difference between the collusive price and the non-collusive price exhibits an inverted-U shape. The increase in prices due to collusion is the largest when the two firms have comparable intrinsic market shares (i.e. $m_{i1} \approx 0.5$). Intuitively, collusion allows both firms to set higher prices to enjoy higher profit margins than what they would have in the non-collusive equilibrium. However, the collusive pricing schedule has to be chosen such that both firms have no incentive to deviate given their current intrinsic market shares. When firm 1 is dominating the market (i.e. with high m_{i1}), forming a collusive equilibrium would be less appealing from firm 1's perspective as it already has high market power, which allows it to set a high price in the non-collusive equilibrium any way (see the black dashed line). On the other hand, when firm 1 has low intrinsic market share m_{i1}, forming a collusive equilibrium would be less appealing from firm 2's perspective who already has high market power to set a high price in the non-collusive equilibrium. Thus, it is easier to collude on relatively higher prices when firm 1 and firm 2 have comparable intrinsic market shares.

The above intuition is more clearly seen in two extreme cases. When firm 1's intrinsic market share $m_{i1} \approx 1$, Panel A shows that it sets a price close to $\frac{\epsilon}{\epsilon-1}\omega = 2$. This is the price that firm 1 would choose facing a price elasticity of demand ϵ. In this case, firm 1 essentially acts almost as a monopoly in industry i and sets prices to compete with firms in other industries. Thus, the constant across-industry price elasticity of demand is what determines firm 1's optimal price in both the collusive and the non-collusive equilibria. By contrast, when firm 1's intrinsic market share $m_{i1} \approx 0$, Panel A shows that it sets a price close to $\frac{\eta}{\eta-1}\omega = 1.07$. This is the price that firm 1 would choose facing a price elasticity of demand η. In this case, firm 1 essentially acts almost as a price taker in industry i because it has little market power to influence the industry's price index. Thus, the constant within-industry price elasticity of demand is what determines firm 1's optimal price in both the collusive and the non-collusive equilibria.

Panel C compares firm 1's value in the collusive and the non-collusive equilibria. Colluding on higher prices increases firm 1's profit margins, leading to higher firm values. Not surprisingly, due to the inverted-U collusive prices, the difference in firm values displays a similar inverted-U shape (Panel D) when the intrinsic market share varies.

3.3 Determination of Collusive Prices

In this section, we clarify how the collusive prices are determined in equilibrium. In Panel A of Figure 2, the red line plots the optimal price that firm 1 would choose conditional on its deviation from the collusive pricing schedule.[17] It shows that the optimal deviation price is always lower than the collusive price. This is intuitive because firms collude on higher prices relative to what they would set in the non-collusive equilibrium, and thus both firms have the incentive to undercut the other firms in order to increase both contemporaneous demand and gain more intrinsic market shares. Whether firm 1 would deviate depends on what deviation value firm 1 would obtain by setting the optimal deviation price. Intuitively, there are countervailing forces that determine the gains from

[17] Here, we follow the standard game theory by considering one-shot deviation. That is, we consider what the deviation price that firm 1 would choose conditional on firm 2 not deviating from the collusive equilibrium. The one-shot deviation property ensures that no profitable one-shot deviations for every player is a necessary and sufficient condition for a strategy profile of a finite extensive-form game to form a sub-game perfect equilibrium.

deviation. If deviation is not detected by firm 2, then firm 1 would gain by stealing intrinsic market shares from firm 2 through lower prices. However, if deviation is detected by firm 2, then firm 1 will be punished by switching to the non-collusive equilibrium which features low prices and low profit margins.

Whether the collusive equilibrium can be sustained depends on the level of collusive prices. A higher collusive price increases the profits from deviation and is more difficult to be sustained in equilibrium. The collusive prices we choose are the highest prices subject to the IC constraints that both firms have no incentive to deviate in the collusive equilibrium. In Panel C of Figure 2, the red dash-dotted line plots the deviation value that firm 1 would obtain by setting the optimal deviation price (the red dash-dotted line in Panel A). It is shown that firm 1's deviation value is exactly the same as firm 1's value in the collusive equilibrium, indicating that firm 1 is indifferent between setting the collusive price or deviating from the collusive equilibrium. In other words, firm 1's IC constraints are binding. Because the collusive and deviation values are equal for any intrinsic market share, firm 2 is also indifferent about collusion and deviation.

The IC constraints are violated, if we choose collusive prices higher than the blue solid line in Panel A. We illustrate this in Figure 3. To obtain a stark comparison, we assume that the collusive price is set equal to $\frac{\epsilon}{\epsilon-1}\omega = 2$ (as shown by the blue solid line in Panel A), which is the price that maximizes the contemporaneous demand if the two firms can perfectly collude with each other and act like a monopoly.

The red dash-dotted line indicates that when firm 1's intrinsic market share m_{i1} is lower than 0.6, it would set a significantly lower price to steal firm 2's intrinsic market share. As a result, firm 1's deviation value is strictly larger than its collusion value (see the red dash-dotted line in Panel B) when $m_{i1} < 0.6$, indicating that the IC constraint is violated. Thus, requiring the two firms to collude on a higher price like what is considered here does not form a sub-game perfect equilibrium because one of the firms (or both firms) will deviate by setting a lower price.

3.4 Discussions on Elasticities

The parameter η and ϵ capture the elasticity of substitution of goods produced within the same industry and the elasticity of substitution of goods produced in different industries. In this section, we discuss the role played by the two elasticities on collusion incentives and prices. To fix ideas, we shut down the price channel for customer base accumulation

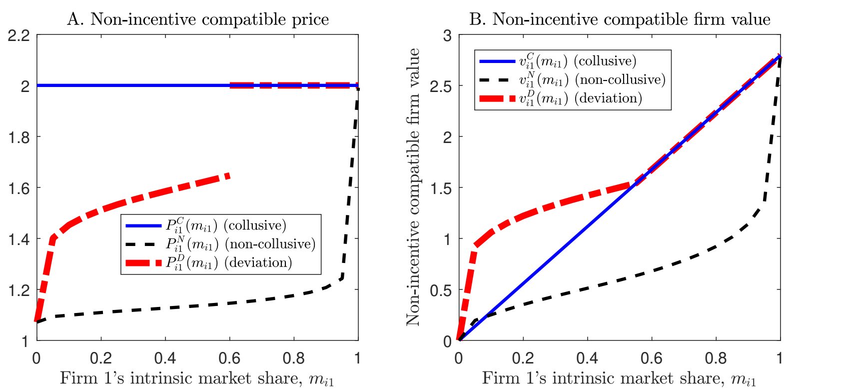


Figure 3: An illustration of non-incentive compatible collusive prices.

by setting $z = 0$.

In our baseline calibration, we set $\eta > \epsilon$ to be consistent with empirical estimates. As we vary η and ϵ, the model can capture different degrees of within- and across-industry competition. As we show in equation (2.15), the price elasticity of demand for firm 1 depends on both the within-industry elasticity η and the across-industry elasticity ϵ because firm 1 simultaneously faces within-industry competition from firm 2 as well as the across-industry competition from firms in other industries.

With $\eta > \epsilon$, within-industry competition is more fierce than across-industry competition due to the higher elasticity of substitution among goods produced in the same industry. Thus, essentially the within-industry elasticity η gives the upper bound of competition, and hence determines the lower bound of prices; whereas the across-industry elasticity ϵ gives the lower bound of competition, and hence determines the upper bound of prices.

In particular, the highest level of competition is obtained by firm 1 when it becomes atomic in industry i (i.e. $m_{i1} = 0$). In this case, firm 1 would set the lower-bound price $\frac{\eta}{\eta-1}\omega$, determined by the within-industry elasticity η. However, when firm 1 is atomic, firm 2 is essentially the monopoly in industry i, facing the minimal level of competition due to the absence of within-industry competition. Thus, firm 2 would set the upper-bound price $\frac{\epsilon}{\epsilon-1}\omega$, determined by the across-industry elasticity ϵ. Because firm 2 already sets its price equal to the upper bound, there is no incentive for firm 2 to collude with firm 1, although firm 1 wants to collude due to its low price.

Thus, the two firms have the incentive to collude with each other only when neither firm is the monopoly in industry i. In this case, collusion benefits both firms by alleviating

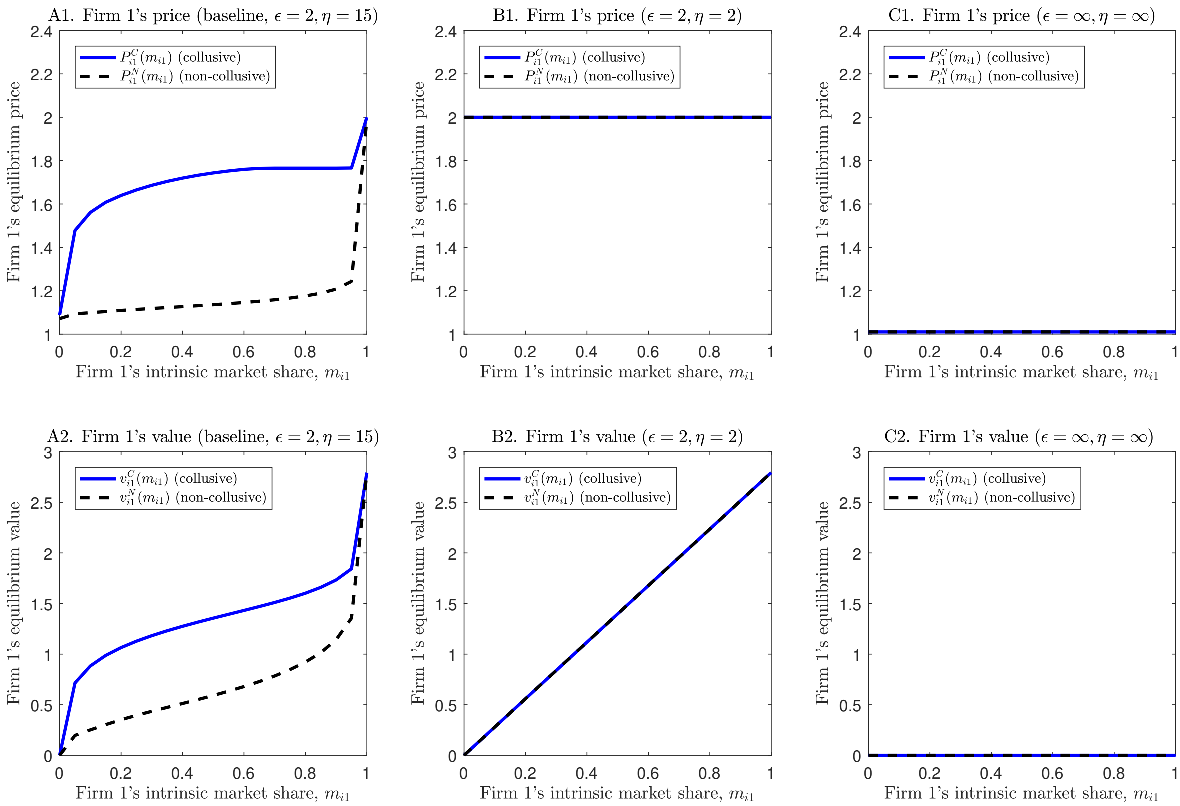


Figure 4: The role of price elasticities of demand on collusive prices.

within-industry competition so that prices become higher, more reflecting the across-industry elasticity ϵ. Therefore, the existence of collusion incentive crucially depends on the assumption that $\eta > \epsilon$. If $\eta = \epsilon$, the level of competition does not change with the intrinsic market share. And the firm would always set the upper bound price $\frac{\epsilon}{\epsilon-1}\omega$, determined by the across-industry elasticity ϵ.

Specifically, if we set $\eta = 2\ (= \epsilon)$, Panel B1 of Figure 4 shows that firm 1 always sets its price equal to $\frac{\epsilon}{\epsilon-1}\omega = 2$. In this case, achieving the collusive equilibrium does not further increase the two firms' prices because they already set the upper bound price consistent with what is implied by across-industry competition.[18] Firm 1's value increases linearly with its intrinsic market share m_{i1} (see Panel B2). In Panels C1 and C2, we further increase

[18] In fact, when the two elasticities are the same ($\eta = \epsilon$), the two layers of CES aggregation collapses to a single across-industry CES aggregation, and within-industry competition would not matter for price setting.

$\eta = \epsilon = \infty$ to mimic an economy with perfect competition. The infinite elasticity results in zero markups. Both firms set their prices equal to the marginal costs (see Panel C1) and attain zero values (see Panel C2) in equilibrium regardless of their intrinsic market shares.

4 Main Predictions

In this section, we present the main predictions of our model. First, we show that collusive prices are lower when the long-run consumption growth rate is low. The endogenous price wars during periods with low long-run growth rates amplify firms' exposure to long-run risks. Second, we show that the industries with a larger capacity of distinctive innovation are less exposed to long-run risks because there are less variations in collusive prices over long-run growth fluctuations. Thus our model implies that industries with a larger capacity of distinctive innovation are less risky. Finally, we discuss the effect of long-run risks and antitrust enforcement on our model's asset pricing implications.

4.1 Price Wars and Long-Run Growth Rates

In this section, we study the endogenous change in collusive prices and firm values over business cycles. Figure 5 plots firm 1's collusive price during periods with high long-run growth rates (i.e. $\theta_t = \theta_b = 0.06$, blue solid line) and periods with low long-run growth rates (i.e. $\theta_t = \theta_r = -0.06$, black dashed line). We choose large differences in growth rates for better visualizing the channels. It is shown that the two firms collude on significantly lower prices during periods with low long-run growth rates. Thus, our model predicts the occurrence of price wars during periods with low long-run consumption growth.

Intuitively, the incentive to collude on higher prices depends on the extent to which the two firms value future revenue relative to its current revenue. By deviating from the collusive price, firms can attain higher contemporaneous revenue and accumulate more customer bases in the short run. However, firms run into the risk of losing future revenue because once the deviation is detected by the other firm, the non-collusive equilibrium will be implemented as a punishment strategy. Switching to the non-collusive equilibrium will be more costly when the future revenue is relatively higher, which happens during periods with high long-run growth rates. In fact, firms' effective discount rate is lower during high long-run-growth periods due to the high growth rate in revenue. This makes

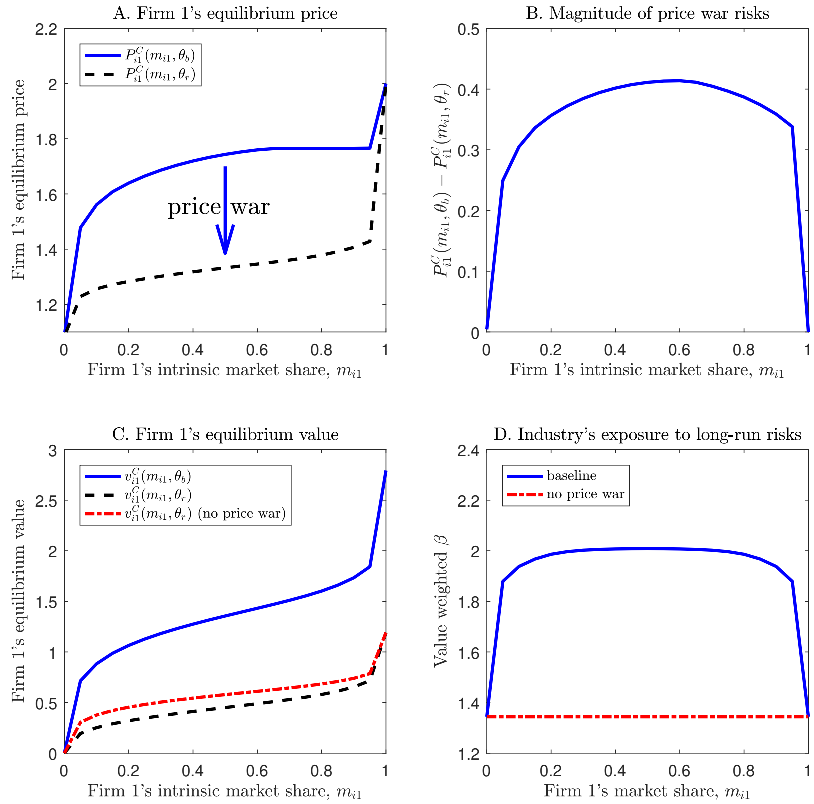


Figure 5: Price war risks and the industry's exposure to long-run risks.

the two firms more patient and value future revenue more, which in turn strengthens the incentive to collude with each other.[19]

Panel B illustrates the magnitude of price war risks by plotting the difference in collusive prices between periods with high and low long-run growth rates. It is shown that price war risks display an inverted U-shape, and the risks are the largest when the two firms have comparable intrinsic market shares (i.e. $m_{i1} \approx 0.5$). The intuition is related to what we discussed above. When one firm dominates the other firm, there is not much incentive to form a collusion with the other firm, and thus there is not much variation in

[19]The intuition is related to the Folk Theorem in game theory. The Folk Theorem says that provided players are sufficiently patient, not only can repeated interaction allow many sub-game perfect outcomes, but actually sub-game perfection can allow virtually any outcome in the sense of average payoffs. The effective discount rate is approximately given by $r - \theta_t$. Thus, the periods with low θ_t feature high discount rates and more impatience.

collusive prices when long-run growth rates change.

The time-varying collusion incentive amplifies the effect of long-run risks, making firms riskier. In Panel C, the blue solid line and the black dashed line plot firm 1's value during periods with high long-run growth rates ($\theta_t = \theta_b$) and periods with low long-run growth rates ($\theta_t = \theta_r$). Firms have significantly lower values during periods with low long-run growth rates due to low growth rates and low product prices. Different from canonical long-run-risk models, our model emphasizes that the endogenous price wars arising from declines in long-run consumption growth further reduce firms' cash flows, generating an amplification mechanism for the effect of adverse long-run growth shocks. This amplification effect is quantitatively important. The red-dashed line illustrates that the firm's value in periods with low long-run growth rates would be 20% higher on average if collusive prices were the same as those in periods with high long-run growth rates.

To illustrate the industry's exposure to long-run risks, we calculate the industry-level beta as value-weighted firm-level betas

$$\beta_i(m_{i1}) = \sum_{j=1,2} \frac{v_{ij}^C(m_{i1}, \theta_r)}{v_{i1}^C(m_{i1}, \theta_r) + v_{i2}^C(m_{i1}, \theta_r)} \beta_{ij}(m_{i1}), \tag{4.1}$$

where

$$\beta_{ij}(m_{i1}) = v_{ij}^C(m_{i1}, \theta_b) / v_{ij}^C(m_{i1}, \theta_r) - 1. \tag{4.2}$$

The blue solid line in Panel D plots industry i's beta when firm 1's market share m_{i1} varies. It shows that the industry's exposure to long-run risks displays an inverted U-shape due to the inverted-U price war risks (see Panel B). As a benchmark, the red dash-dotted line plots the industry's exposure to long-run risks in the absence of price war risks (i.e. when collusive prices do not change with long-run growth rates). It is shown that when the two firms have comparable intrinsic market shares, the price war risks significantly amplify the industry's exposure to long-run risks, increasing the value of beta by about 50%, from 1.35 to 2.

4.2 Distinctive Innovation and the Exposure to Long-Run Risks

In this section, we study the implication of distinctive innovation on collusive prices and firm values. We consider two industries different in the units of distinctive R&D projects.

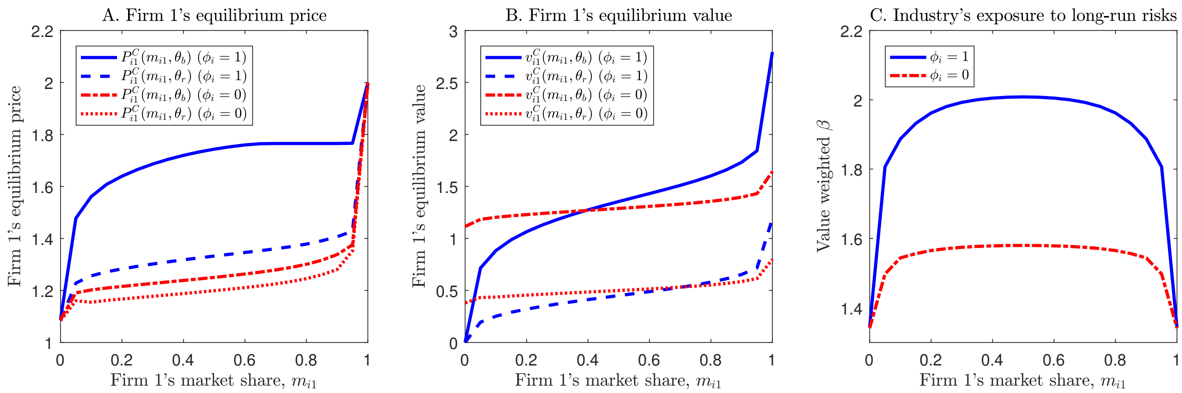


Figure 6: Comparing collusive prices, firm values, and the exposure to long-run risks across industries with different capacities of distinctive innovation.

In one industry, there are no distinctive R&D projects ($\phi_i = 1$); in the other industry, all R&D projects are distinctive ($\phi_i = 0$).

Panel A of Figure 6 plots the equilibrium collusive prices in the two industries during periods with high long-run growth rates and periods with low long-run growth rates. It shows that although firms in both industries collude on higher prices during periods with high long-run growth rates, collusive prices are much lower in the industry with distinctive innovation ($\phi_i = 0$). As we discuss in section 3.2, the incentive to collude exhibits an inverted-U shape and becomes the smallest in concentrated industries (i.e. one firm's intrinsic market share is much larger than the other's). The industry with distinctive innovation is more likely to be concentrated in future because one firm can steal its competitor's intrinsic market share and almost monopolize the industry upon the success of distinctive innovation. Thus, even if the two firms have comparable intrinsic market shares today, the possibility of having a successful distinctive innovation in future still largely dampens the incentive to collude, resulting in low collusive prices.

Not only the levels are lower, collusive prices are also less responsive to persistent growth shocks in the industry with distinctive innovation. Panel A shows that when the economy switches between periods with high and low long-run growth rates, the change in collusive prices in the industry with distinctive innovation (the difference between the red dash-dotted line and the red dotted line) is much smaller compared to that in the industry without distinctive innovation (the difference between the blue solid line and the blue dashed line). This implies that firms in the industry with distinctive innovation face smaller price war risks simply because collusion is difficult to form in the first place.

Panel B compares firm 1's value in the two industries during periods with high and low long-run growth rates. In the industry with distinctive innovation, the firm's value is less sensitive to its intrinsic market share because distinctive innovation allows the firm to possibly monopolize the industry in future regardless of the firm's current intrinsic market share. Moreover, the firm's value has less variation over long-run growth rates due to smaller price war risks.

In Panel C, we compare the two industries' exposure to long-run risks for different levels of industry concentration, as reflected by firm 1's intrinsic market share. Conditional on the same level of concentration, firms in the industry with distinctive innovation are less exposed to long-run risks. The industry-level value-weighted beta exhibits an inverted U-shape in both industries. The difference in beta across the two industries is as large as 0.55 when the two firms within the same industry have comparable intrinsic market shares ($m_{i1} = 0.5$). However, the difference in beta drops to zero, when the two industries are monopolized by one single firm ($m_{i1} = 0$ or $m_{i1} = 1$). Thus our model implies that the industries with low capacities of distinctive innovation tend to be riskier as price wars are more severe when the long-run consumption growth rate declines. Moreover, this cross-industry difference is more pronounced when the two firms have comparable intrinsic market shares.

4.3 Discussions on Long-Run Risks and Antitrust Enforcement

We emphasize that long-run risks play a crucial role in generating price war risks. In our model, firms collude more during periods with high long-run growth rates precisely because they know that the growth rate of consumption is persistent. In Figure 7, we compare the baseline calibration with a 0.49 auto-correlation of annual consumption growth rates to an economy with a 0.049 auto-correlation of annual consumption growth rates, featuring less persistent long-run risks. Panel A shows that, in the economy with less persistent long-run risks, there is almost no change in collusive prices between periods with high and low long-run growth rates, and this is true regardless of the capacity of distinctive innovation (see the red dash-dotted line and the red dotted line). Moreover, Panel B shows that the industry's exposure to long-run risks is much smaller in the economy with less persistent long-run risks. Importantly, there is virtually no difference in the exposure to long-run risks across the two industries. Thus, the model suggests that the persistence of long-run risks is crucial in generating both the high

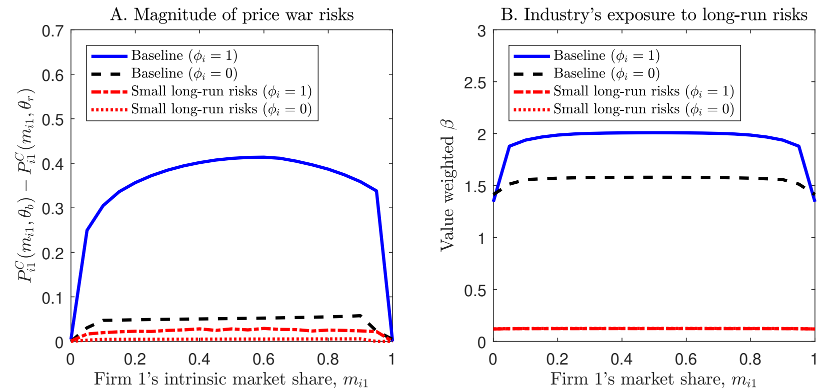


Figure 7: Illustrating the importance of long-run risks in generating price war risks.

magnitude of price war risks and the variation in the exposure to long-run risks across industries with different capacities of distinctive innovation.

Intuitively, with long-run risks in consumption, what determines the collusion incentive is not only the current level of aggregate consumption, but also the expected change in aggregate consumption in the future. Firms that expect a relative increase in aggregate consumption are able to sustain higher collusive prices now because none of the firms want to deviate and be punished later by their competitors in periods with higher aggregate consumption. On the contrary, during bad times, firms expect a relative decrease in aggregate consumption and the later punishment looks less costly. Consequently, declines in long-run consumption growth generate price wars, amplifying firms' exposure to long-run risks.

Our model predicts that antitrust enforcement reduces price war risks. Intuitively, with stronger laws against collusion, it is more difficult for firms to conduct collusive pricing, resulting in lower collusive prices and less variation in collusive prices with long-run growth rates. In our model, the parameter ς controls the ability for firms to collude with each other. A smaller ς makes it harder to implement higher collusive prices, which is equivalent to the effect of implementing more stringent antitrust enforcement. In the extreme case with $\varsigma = 0$, there is no way to detect whether a price deviation has occurred in the past, and as a result, there is no way to sustain an incentive compatible collusive equilibrium.

In Figure 8, we compare our baseline calibration with $\varsigma = 0.09$ to an economy with $\varsigma = 0.05$. The magnitude of price war risks is significantly lower in the latter economy (see Panel A). As a result, the industry's exposure to long-run risks is much smaller when

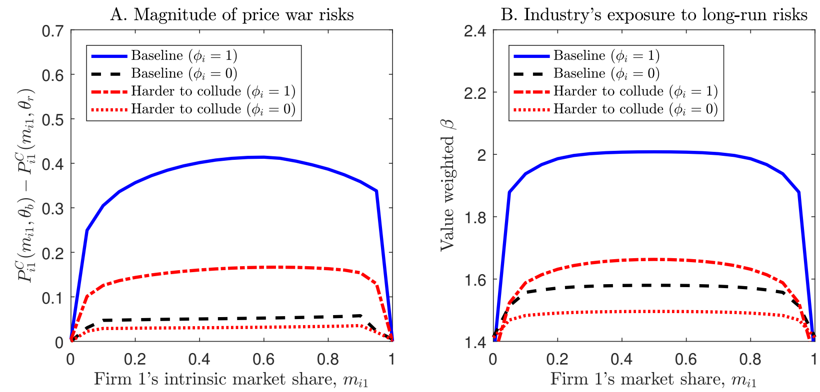


Figure 8: Antitrust enforcement, price war risks, and the exposure to long-run risks.

collusion is more difficult to implement. Across the two industries, our model implies that antitrust enforcement has larger effects in the industry with no distinctive innovation, as this is the industry with the highest collusion incentive to begin with.

5 Empirical Analyses

In this section, we empirically test the main predictions of our model. We first use patent data to construct an innovation similarity measure for the industry characteristic ϕ in our model. We find that industries with higher innovation similarity are more exposed to long-run risks, and they have higher average excess returns and risk-adjusted returns. Moreover, we find that industries with higher innovation similarity are more exposed to price war risks; and they were more likely to engage in price wars after the Lehman crash in 2008. We further show that these findings reflect the cross-sectional difference in collusion incentive by exploring the consequence of antitrust enforcement. Finally, we present additional tests for our theoretical mechanism.

5.1 Data and the Innovation Similarity Measure

In this subsection, we introduce our data and construct the innovation similarity measure.

Patent Data and Our Merged Sample. We obtain the patent issuance data from PatentView, a patent data visualization and analysis platform. PatentView contains detailed and up-to-date information on granted patents from 1976 onward. Its coverage of recent patenting activities is more comprehensive than the NBER patent data (Hall, Jaffe and Trajtenberg, 2001) and the patent data assembled by Kogan et al. (2017).[20] Patent assignees in PatentView are disambiguated and their locations and patenting activities are longitudinally tracked. PatentView categorizes patent assignees into different groups, such as corporations, individuals, and government agencies. It also provides detailed information of individual patents, including their grant dates and technology classifications.

We match patent assignees in PatentView to U.S. public firms in CRSP/Compustat, and to U.S. private firms and foreign firms in Capital IQ.[21] We drop patents granted to individuals and government agencies. We include private firms in our sample because they play an important role in industry competition (see e.g. Ali, Klasa and Yeung, 2008).

We use a fuzzy name-matching algorithm to obtain a pool of potential matches from CRSP/Compustat and Capital IQ for each patent assignee in PatentView. We then manually screen these potential matches to identify the exact matches based on patent assignees' names and addresses. In Appendix B.2, we detail our matching procedure. In total, we match $2,235,201$ patents to $10,139$ U.S. public firms, $132,100$ patents to $3,080$ U.S. private firms, $241,582$ patents to 300 foreign public firms, and $35,597$ patents to 285 foreign private firms.[22] The merged sample covers $13,804$ firms in 752 4-digit SIC industries from 1976 to 2017.[23]

Innovation Similarity Measure. We construct our innovation similarity measure (denoted as "innosimm") for the industry-level innovation similarity based on the technology classifications of an industry's patents. In light of previous studies (e.g. Jaffe, 1986; Bloom, Schankerman and Van Reenen, 2013), we measure the cosine similarity of two patents within the same industry based on their technology classification vectors.[24] Specifically,

[20]The PatentView data contain all patents granted by the U.S. Patent and Trademark Office (USPTO) from 1976 to 2017, while the NBER data and the data used by Kogan et al. (2017) only cover patents granted up to 2006 and 2010, respectively.

[21]Capital IQ is one of the most comprehensive data that include private firms and foreign firms.

[22]Our empirical results remain robust if we confine the patent data to those granted to U.S. public firms.

[23]We use 4-digit SIC codes in Compustat and Capital IQ to identify the industries of patent assignees. Both Compustat and Capital IQ are developed and maintained by S&P Global and the SIC industry classifications in these two datasets are consistent with each other. We verify the consistency by comparing the SIC codes for U.S. public firms covered by both Compustat and Capital IQ. We find that the SIC codes of these firms are virtually identical across the two data sources.

[24]PatentView provides both the Cooperative Patent Classification (CPC) and the U.S. Patent Classification (USPC), the two major classification systems for U.S. patents. We use CPC for our analyses because USPC

the similarity between two patents, a and b, is defined by:

$$\text{similarity}\ (a, b) = \frac{\mathbf{A} \cdot \mathbf{B}}{\|\mathbf{A}\|\ \|\mathbf{B}\|}, \tag{5.1}$$

where **A** and **B** are the technology vectors of patent a and patent b. If the two patents share exactly the same technology classifications, the cosine similarity attains the maximum value, 1. If the two patents are mutually exclusive in their technology classifications, the cosine similarity reaches the minimum value, 0. Because patent technology classifications are assigned according to the technical features of patents, the cosine similarity measure captures how similar the patents are in terms of their technological positions. Based on the pairwise cosine similarity of patents, we take the following steps to construct the industry-level innovation similarity measure.

First, we construct the patent-level similarity measure to capture to what extent a patent is differentiated from other patents recently developed by peer firms. In particular, for a patent granted to firm i in year t, the patent-level similarity measure is the average of the pairwise cosine similarity (defined by equation 5.1) between this patent and the other patents granted to firm i's peer firms in the same 4-digit SIC industry from year $t-5$ to year $t-1$.

Next, we aggregate patent-level similarity measures to obtain industry-level similarity measures. For example, a 4-digit SIC industry's similarity measure in year t is the average of patent-level similarity measures associated with all the patents granted to firms in the industry in year t. Because firms are not granted with patents every year, we further average the industry-level similarity measures over time to filter noise and better capture firms' ability in generating differentiated innovations. In particular, our innosimm measure in industry i and year t (i.e. innosimm_{it}) is constructed as the time-series average of industry i's similarity measures from year $t-9$ to year t.

Panel A of Figure 9 presents the time-series of several industries' innosimm measure. In the "Search, Detection, Navigation, Guidance, Aeronautical, and Nautical Systems and Instruments" industry, the innosimm measure is low throughout our sample period, suggesting that firms in this industry seem to be able to consistently generate distinctive innovation. The innosimm measure keeps increasing for the "Drilling Oil and Gas Wells"

is not available after 2015. Our results are robust to the classification based on USPC for data prior to 2015. There are 653 unique CPC classes in PatentView. The technology classification vector for a patent consists of 653 indicator variables that represent the patent's CPC classes.

industry, while it peaks in year 2000 for the "Rubber and Plastics Footwear." industry.

Validation of the Innosimm Measure. We conduct one external validation for our innosimm measure. If a higher innosimm captures a lower capacity of distinctive innovation in an industry, we expect that fewer consumers would consider the brands of high-innosimm industries as distinctive. We test this hypothesis by examining the relation between innosimm and the relative change in brand distinctiveness over time, measured using the BAV consumer survey data.[25] We standardize innosimm using its unconditional mean and the standard deviation of all industries' innosimm across all time to ease the interpretation of coefficients in our regression analyses. Column (1) of Table 1 shows that innosimm is negatively correlated with the two-year percent change in the industry-level brand distinctiveness, suggesting that industries with higher innosimm are associated with lower brand distinctiveness in future.

Our innosimm measure is conceptually different from the product similarity measure (denoted as "prodsimm") constructed by Hoberg and Phillips (2016). Innosimm captures to what extent firms in an industry can differentiate their products from peers' through innovation. Thus, it is a forward-looking measure that captures the (potential) similarity/distinctiveness of firms' businesses in the future. Product similarity, on the other hand, is derived from text analyses based on firms' current product description (Hoberg and Phillips, 2016). Therefore, it reflects the similarity of products produced by different firms as of today, rather than the potential similarity/distinctiveness of firms' products in the future. In other words, product similarity contains littile information, if at all, about firms' innovation activities, which are the necessary inputs for making products distinctive in the future. The conceptual difference between the two measures is formally confirmed by column (2) of Table 1, which shows that innosimm is unrelated with product similarity.[26] In Section 5.2 and 5.3, we further show that the product similarity measure is neither priced in the cross section nor related to industries' price war risks.

[25] The BAV database is regarded as the world's most comprehensive database of consumers' perception of brands (see, e.g. Gerzema and Lebar, 2008; Keller, 2008; Mizik and Jacobson, 2008; Aaker, 2012; Lovett, Peres and Shachar, 2014; Tavassoli, Sorescu and Chandy, 2014). The BAV brand perception survey consists of more than 870,000 respondents in total, and it is constructed to represent the U.S. population according to gender, ethnicity, age, income group, and geographic location. See Dou et al. (2018) for the details of the survey.

[26] The correlation between product similarity and innosimm is low. The Pearson correlation coefficient, the Spearman's rank correlation coefficient, and the Kendall's τ_A and τ_B coefficients between the two variables are 0.06, 0.02, 0.04, and 0.04, respectively.

Table 1: Innovation similarity, brand distinctiveness and product similarity (yearly analysis).

	(1)	(2)
	Percent changes from year t to year $t+2$ (%)	
	Brand distinctiveness	Product similarity
Innosimm_t	-0.69^{***}	0.10
	[−3.06]	[0.04]
Year FE	Yes	Yes
Observations	2466	5906
R-squared	0.298	0.002

Note: This table shows the relation of our innosimm measure with measures of brand distinctiveness and product similarity at the 4-digit SIC industry level. In column (1) , the dependent variable is the two-year percent change in brand distinctiveness. The percent change is computed as $100\times(\text{brand distinctiveness}_{t+2} - \text{brand distinctiveness}_t)$ /$\text{brand distinctiveness}_t$. At the brand level, brand distinctiveness is the fraction of consumers who consider a brand to be distinctive. We first aggregate the brand-level distinctiveness measure to the firm level, and then further aggregate it to the 4-digit SIC industry level. Product similarity comes from Hoberg and Phillips (2016), and it is derived from text analyses based on the business description in 10-K filings. We download the product similarity measure from the Hoberg and Phillips Data Library, and aggregate it to the 4-digit SIC industry level. The sample in column (1) spans from 1993 to 2017, and the sample in column (2) spans from 1996 to 2015. We include t-statistics in brackets. Standard errors are clustered by the 4-digit SIC industry and year. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels.

5.2 Asset Pricing Tests

We now test the asset pricing implications of our model. We find that industries with higher innosimm are more exposed to long-run-risk shocks, and this relationship is weaker among the industries that experience antitrust enforcement in recent years. All these findings are consistent with our model's predictions. In addition, we perform double-sort analyses to show that innosimm spreads (i.e. the spreads between high-innosimm industries and low-innosimm industries) are robust after controlling for related measures.

5.2.1 Innosimm Spreads Across Industries

We first examine whether innosimm is priced in the cross section. Panel A of Table 2 presents the value-weighted average excess returns and alphas for the 4-digit SIC industry portfolios sorted on innosimm. It shows that the portfolio consisting of high-innosimm industries (i.e. Q5) exhibits significantly higher average excess returns and alphas. The spread in average excess returns between Q1 and Q5 is 3.41% and the spreads in alphas range from 4.75% to 9.24% across different factor models.

Next, we perform the same analysis for prodsimm. We find that prodsimm is not priced in the cross section. The return difference between the high-prodsimm portfolio and the low-prodsimm portfolio is statistically insignificant (see Panel B).

Table 2: The average excess returns and alphas of portfolios sorted on innovation similarity and product similarity (monthly analysis).

	1 (Low)	2	3	4	5 (High)	5 − 1
			Panel A: Portfolios sorted on innosimm			
			Average excess returns			
$\mathbb{E}[R] - r_f$ (%)	6.13***	8.37***	7.35***	8.62***	9.54***	3.41***
	[2.74]	[3.73]	[3.01]	[4.42]	[3.17]	[2.71]
			Fama-French three-factor model (Fama and French, 1993)			
α (%)	−2.51**	0.07	−1.34	1.08	2.71**	5.22***
	[−2.48]	[0.10]	[−0.69]	[1.21]	[2.49]	[3.54]
			Carhart four-factor model (Carhart, 1997)			
α (%)	−2.47***	0.09	−1.25	1.43	2.28***	4.75***
	[−2.70]	[0.18]	[−0.78]	[1.50]	[2.63]	[4.01]
			Panel B: Portfolios sorted on prodsimm			
			Average excess returns			
$\mathbb{E}[R] - r_f$ (%)	4.94**	6.44**	8.07**	6.25*	6.19*	1.25
	[2.29]	[2.05]	[2.59]	[1.77]	[1.91]	[0.42]
			Fama-French three-factor model (Fama and French, 1993)			
α (%)	−0.89	0.03	2.21**	0.08	0.82	1.70
	[−0.48]	[0.01]	[2.53]	[0.06]	[0.86]	[0.64]
			Carhart four-factor model (Carhart, 1997)			
α (%)	−0.57	0.42	2.19**	0.69	0.70	1.26
	[−0.36]	[0.20]	[2.43]	[0.68]	[0.75]	[0.53]

Note: This table shows the value-weighted average excess returns and alphas for the 4-digit SIC industry portfolios sorted on innosimm. In June of year t, we sort the 4-digit SIC industries into five quintiles based on this industry's innosimm in year $t-1$. Once the portfolios are formed, their monthly returns are tracked from July of year t to June of year $t+1$. The sample period is from July 1988 to June 2018. The average market excess returns (mean of the "Rm - Rf" factor in the Fama-French three factor model) is 8.05% over this time period. We exclude financial firms and utility firms from the analysis. We include t-statistics in parentheses. Standard errors are computed using the Newey-West estimator allowing for serial correlation in returns. We annualize average excess returns and alphas by multiplying by 12. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels.

5.2.2 Exposure to Long-Run Risks in Consumption Growth

What drives the innosimm spreads? In this section, we show that industries with different innosimm have differential exposure to long-run risks, as suggested by our model. We estimate the exposure to long-run risks (i.e. LRR beta) following Dittmar and Lundblad (2017). In particular, we first sort all industries into quintile portfolios based on innosimm. Then, we regress the cumulative portfolio returns of each portfolio on long-run risks. The coefficients give us an estimate for LRR beta (see Table 3). We use two long-run risks measures in our analyses: the cumulative realized consumption growth and the consumption growth filtered by a Bayesian mixed-frequency approach as in Schorfheide, Song and Yaron (2018) (see Panel B of Figure 9).[27] The difference in LRR betas between Q1 and Q5 is statistically significant for both measures of consumption growth, indicating that high innosimm industries are more exposed to long-run risks.

Table 3: Long-run-risk exposures of portfolios sorted on innosimm (quarterly analysis).

Portfolios sorted on innosimm	1 (Low)	2	3	4	5 (High)	5 − 1
Panel A: Long-run risks measured by 8-quarter cumulative realized consumption growth						
LRR betas	1.78	6.55***	3.48***	5.51***	5.24***	3.46**
	[1.58]	[5.09]	[3.12]	[3.73]	[4.03]	[2.09]
Panel B: Long-run risks measured by 8-quarter cumulative filtered consumption growth						
LRR betas	−0.05	4.45***	−0.62	3.57**	4.69**	4.75**
	[−0.05]	[3.16]	[−0.52]	[2.57]	[2.41]	[2.61]

Note: This table shows the exposures to long-run risks for industry portfolios sorted on innosimm. In June of year t, we sort industries into five quintiles based on innosimm in year $t-1$. Once the portfolios are formed, their monthly returns are tracked from July of year t to June of year $t+1$. In Panel A, following Dittmar and Lundblad (2017), we regress the 8-quarter cumulative portfolio returns on the 8-quarter cumulative realized consumption growth: $\prod_{j=0}^{7} R_{i,\tau-j} = \alpha_i + \beta_i \sum_{j=0}^{7} \hat{\eta}_{\tau-j} + e_{i,\tau}$, where $\hat{\eta}_\tau$ is the consumption growth shock, measured by the difference between the log consumption growth in quarter τ and the unconditional mean of log consumption growth over 1947–2018. We measure consumption using per-capita real personal consumption expenditures on non-durable goods and services. $R_{i,\tau}$ is the gross real return of the industry portfolio i in quarter τ. Consumption and returns are deflated to real terms using the personal consumption expenditure deflator from the U.S. Bureau of Economic Analysis (BEA). The analysis is conducted at quarterly frequency for the sample period from 1988 to 2018. In Panel B, we replace realized cumulative consumption growth ($\sum_{j=0}^{7} \hat{\eta}_{\tau-j}$) in the above regression with cumulative filtered consumption growth ($\sum_{j=0}^{7} \hat{x}_{\tau-j}$) as in Schorfheide, Song and Yaron (2018). The sample period in Panel B is from 1988 to 2015 because data on the filtered consumption growth end in 2015. We exclude financial firms and utility firms from the analysis. We include t-statistics in parentheses. Standard errors are computed using the Newey-West estimator allowing for serial correlation in returns. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels.

We further examine the exposure of real dividend growth to long-run risks for the long-short portfolio sorted on innosimm. We construct real dividend growth rate following previous literature (see e.g. Campbell and Shiller, 1988; Bansal, Dittmar and Lundblad, 2005; Hansen, Heaton and Li, 2005, 2008; Bansal, Kiku and Yaron, 2016). We detail the construction method in Appendix B.3. Importantly we account for the stock entries and exits in computing the portfolio dividend growth rate. Table 4 shows that high-innosimm industries have higher dividend growth rate and their dividend growth has significantly higher exposure to long-run risks. This finding is robust to both measures of long-run risks. As a comparison, in Table 4, we also document the spreads of dividend growth and the exposure to long-run risks for the long-short firm portfolio sorted on book-to-market ratio. Consistent with the literature (see e.g. Bansal, Dittmar and Lundblad, 2005; Hansen, Heaton and Li, 2005, 2008; Bansal, Kiku and Yaron, 2016), we find that the dividend growth of value firms has higher exposure to long-run risks. In addition, we also show

[27] We are grateful to Amir Yaron for sharing data on the filtered consumption growth.

that value firms have lower dividend growth rate than growth firms.[28]

Table 4: Long-run-risk exposures of real dividend growth (quarterly analysis).

Portfolios	Long-short industry portfolio sorted on innosimm (Q5−Q1)	Long-short firm portfolio sorted on book-to-market ratio (Q5−Q1)
	Panel A: Annualized real dividend growth spreads	
Spreads of dividend growth (%)	2.09	−2.88
	[0.48]	[−1.11]
	Panel B: Exposure of real dividend growth to realized consumption growth	
LRR exposure	6.69***	2.63***
	[2.84]	[3.09]
	Panel C: Exposure of real dividend growth to filtered consumption growth	
LRR exposure	7.08***	1.24
	[3.05]	[1.16]

Note: This table shows the exposures of real dividend growth to long-run risks for the long-short industry portfolios sorted on innosimm. As a comparison, we also show results for the long-short firm portfolios sorted on book-to-market ratio. Panel A documents the annualized spreads of dividend growth. Panel B and C show the exposure of dividend growth to long-run risks. We measure long-run risks using both realized consumption growth and filtered consumption growth as in Schorfheide, Song and Yaron (2018). Specifically, in Panel B, we regress the 12-quarter cumulative dividend growth of the long-short innosimm portfolios (annualized) on the lagged 8-quarter cumulative realized consumption growth (annualized): $\sum_{j=1}^{12}(D_{Q_5,t+j} - D_{Q_1,t+j})/3 = \alpha + \beta\sum_{j=0}^{7}\hat{\eta}_{t-j}/2 + e_t$, where $\hat{\eta}_t$ is the realized consumption growth shock. We cumulate dividend growth over three years to reduce noise. In Panel C, we replace the 8-quarter realized cumulative consumption growth in the above regression ($\sum_{j=0}^{7}\hat{\eta}_{t-j}/2$) with 4-quarter cumulative filtered consumption growth ($\sum_{j=0}^{3}\hat{x}_{t-j}$). We use shorter cumulation period for filtered consumption growth because it is less noisy than realized consumption growth. We exclude financial firms and utility firms from the analysis. We include t-statistics in parentheses. Standard errors are computed using the Newey-West estimator allowing for serial correlation in returns. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels.

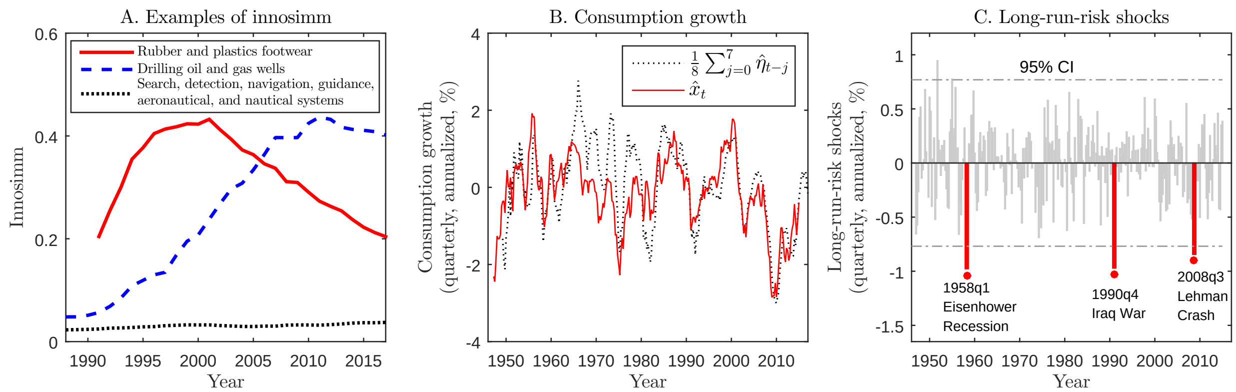
5.2.3 Double-Sort Analyses

We conduct several double-sort analyses for robustness checks. As shown in Table 5, the innosimm spreads are robust after controlling for measures of brand values, product similarity, and innovation originality, suggesting that the innovation similarity channel that our paper proposes cannot be explained by alternative measures.

5.3 Product Prices and Innovation Similarity

The key mechanism of our model is that high-innosimm industries collude on higher product prices in good times, and their prices drop more in bad times due to endogenous price wars. We test it in this subsection. Our findings suggest that high-innosimm

[28]Without the adjustment for stock entries and exits, our calculation of dividend growth would show that value firms have higher dividend growth rates than growth firms, which would be counterintuitive.



Note: Panel A plots the innosimm for three industries: rubber and plastics footwear (SIC 3021); drilling oil and gas wells (SIC 1381); and search, detection, navigation, guidance, aeronautical, and nautical systems and instruments (SIC 3812). Panel B plots consumption growth. The red solid line represents filtered consumption growth as in Schorfheide, Song and Yaron (2018). The black dotted line represents 8-quarter cumulative realized consumption growth. Panel C plots the long-run-risk shocks in the post-war period (from 1947q1 to 2015q4). We construct long-run-risk shocks from the residuals of the AR(1) model for the quarterly time series of filtered consumption growth in Schorfheide, Song and Yaron (2018). Gray dashed lines represent the 95% CI of long-run-risk shocks. Red bars highlight the three prominent negative shocks in 1958q1, 1990q4, and 2008q3, which represent the Eisenhower recession, the Iraq war, and the Lehman crash.

Figure 9: Examples of innosimm, consumption growth, and long-run-risk shocks.

Table 5: Double-sort analyses (monthly analysis).

Double-sort variables	Excess returns (%)	FF3F (%)	FF4F (%)
Brand values	2.18**	3.49**	3.20**
	[2.44]	[2.22]	[2.35]
Product similarity	2.15*	3.81***	3.72***
	[1.92]	[4.27]	[4.81]
Innovation originality	2.83***	4.31***	3.70***
	[3.36]	[3.27]	[3.59]

Note: This table shows the average excess returns and alphas from double-sort analyses. In the double-sort analyses, we first sort the 4-digit SIC industries into three groups based on measures of brand values, product similarity or innovation originality in June of year t. We then sort firms within each group into five quintiles based on innosimm in year $t-1$. Once the portfolios are formed, their monthly returns are tracked from July of year t to June of year $t+1$. Brand values are measured by BAV Stature, a metric constructed by the BAV group to quantify a firm's brand loyalty. Product similarity (Hoberg and Phillips, 2016) is derived from text analysis based on the business description in 10-K filings. Innovation originality is constructed following Hirshleifer, Hsu and Li (2017) to capture the patents' originality. In particular, we count the number of unique technology classes contained in a patent's citations/reference list. We then obtain the industry-level innovation originality measure by averaging the number of classes across all patents in a 4-digit SIC industry every year. We exclude financial firms and utility firms from the analysis. We include t-statistics in parentheses. Standard errors are computed using the Newey-West estimator allowing for serial correlation in returns. We annualize average excess returns and alphas by multiplying by 12. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels.

industries are more exposed to price war risks, and they are also more likely to engage in price wars in the periods after the Lehman crash. We further show that price war risks are owing to changes in collusion incentive. We find that after antitrust enforcement, the exposure to price war risks in high-innosimm industries becomes much weaker.

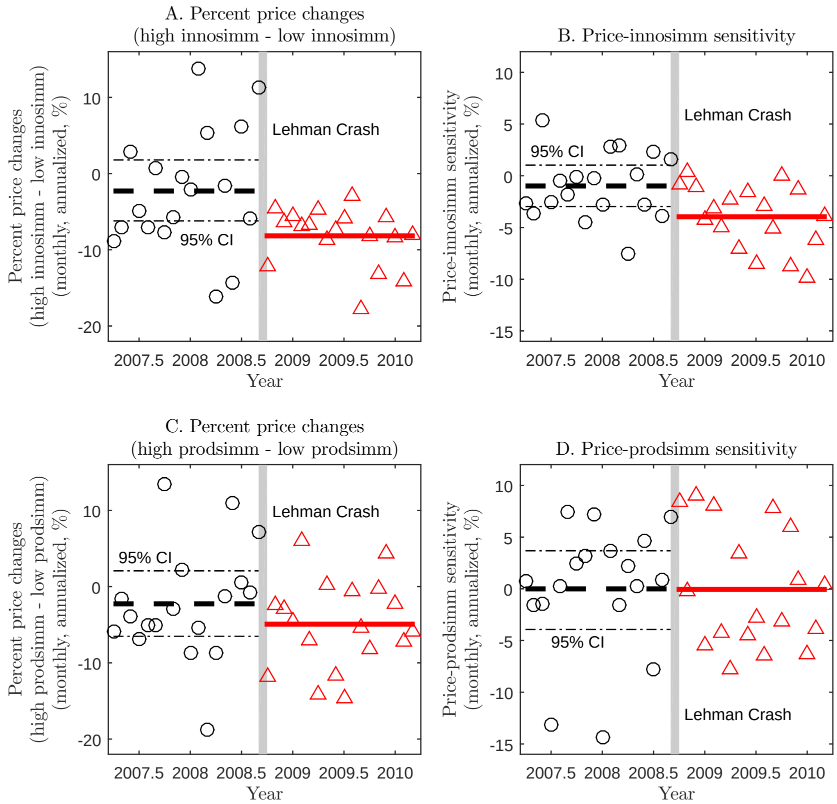
5.3.1 The Nielsen Data for Product Prices

We use the Nielsen Retail Measurement Services scanner data to measure product price changes.[29] The Nielsen data record prices and quantities of every unique product that had any sales in the $42,928$ stores of more than 90 retail chains in the U.S. market from January 2006 to December 2016. In total, the Nielsen data consist of more than 3.5 million unique products identified by the Universal Product Codes (UPCs), and the data represent 53% of all sales in grocery stores, 55% in drug stores, 32% in mass merchandisers, 2% in convenience stores, and 1% in liquor stores (see e.g. Argente, Lee and Moreira, 2018). We use the product-firm links provided by GS1, the official source of UPCs in the U.S., to match products in the Nielsen data to firms in CRSP/Compustat and Capital IQ. In Appendix B.4, we detail the matching procedure. Our merged data cover the product prices of 472 4-digit SIC industries.

5.3.2 Product Prices around the Lehman Crash

To begin, we examine the changes in product prices around the Lenman crash, the period during which the U.S. economy experienced a prominent negative long-run-risk shock (see Panel C of Figure 9). We sort all industries into tertiles based on innosimm. Table 6 quantifies the changes in product prices among high-innosimm industries (Tertile 3) relative to low-innosimm industries (Tertile 1) around the Lehman crash. In particular, we restrict the sample to industries in Tertile 1 and Tertile 3, and create a Tertile-3 indicator for the latter group. We also create a post-Lehman indicator that equals one for observations in Oct. 2008 and thereafter. We then regress the percent change in product prices on the Tertile-3 indicator, the post-Lehman indicator, and an interaction term between these two indicators. The coefficient of the interaction term is negative and statistically significant across different regression specifications (see column 1 - 4), suggesting that product prices in high-innosimm industries reduce significantly relative to those in low-innosimm industries after the Lehman crash. The difference in product prices is economically significant. Relative to low-innosimm industries, product prices decrease by 4.98% to 6.64% in high-innosimm industries after the Lehman crash.

[29] The Nielsen data are obtained from the Kilts Center for Marketing at the University of Chicago Booth School of Business (`https://www.chicagobooth.edu/research/kilts/datasets/nielsen`). The data have been widely used in the macroeconomics literature (e.g. Aguiar and Hurst, 2007; Broda and Weinstein, 2010; Hottman, Redding and Weinstein, 2016; Argente, Lee and Moreira, 2018; Jaravel, 2018).



Note: Panel A plots the difference in the percent change in product prices between high-innosimm (i.e. Tertile 3) and low-innosimm (i.e. Tertile 1) industries around the Lehman crash. The percent change in product prices is annualized from monthly data. The gray vertical bar represents the Lehman crash. The black circles and red triangles represent the difference in annualized monthly percent price changes between high-innosimm and low-innosimm industries in the 18 months before and after the Lehman crash. The black dashed and red solid lines represent the mean values of the differences before and after the Lehman crash. The 95% CI is obtained from bootstrapping the difference between the two mean values. Panel B shows the price-innosimm sensitivity around the Lehman crash. The black circles and red triangles represent the monthly estimates of the price-innosimm sensitivity in the 18 months before and after the Lehman crash. The black dashed lines and red solid lines represent the mean values of the price-innosimm sensitivity before and after the Lehman crash. The 95% CI is obtained by bootstrapping the difference between the two mean values. Panel C plots the difference in the percent change in product prices between high-prodsimm (i.e. Tertile 3) and low-prodsimm (i.e. Tertile 1) industries. Panel D shows the price-prodsimm sensitivity.

Figure 10: Product prices and price-similarity sensitivity around the Lehman crash.

Panel A of Figure 10 visualizes the difference in average product prices between low-innosimm and high-innosimm industries in the 36-month period around the Lehman crash. The plot clearly shows that product prices in high-innosimm industries increase at a much lower rate after the Lehman crash. These findings support our model's prediction that high-innosimm industries are more likely to engage in price wars following negative long-run-risk shocks.

We continue to extend our analysis to all industries. According to the theory, the

Table 6: Product prices around the Lehman crash (monthly analysis).

	(1)	(2)	(3)	(4)
	Percent change in product prices (monthly, annualized, %)			
Similarity measure	Innosimm		Prodsimm	
Tertile-3 similarity$_{t-1}$ × post Lehman crash$_t$	−4.98***	−5.12**	−2.02	−1.48
	[−2.85]	[−2.64]	[−1.33]	[−0.78]
Tertile-3 similarity$_{t-1}$	−2.32	−4.73	−2.25	−3.17
	[−1.05]	[−0.93]	[−1.07]	[−1.58]
post Lehman crash$_t$	0.40	1.03	−0.82	−0.74
	[0.19]	[0.50]	[−0.59]	[−0.44]
Industry FE	No	Yes	No	Yes
Observations	5106	5106	4809	4809
R-squared	0.003	0.053	0.001	0.041

Note: This table shows the changes in product prices around the Lehman crash. The dependent variable is the annualized monthly percent change in product prices of 4-digit SIC industries. Product prices are obtained from the Nielsen Data. To compute the monthly percent change in product prices for 4-digit SIC industries, we first compute the transaction-value weighted price for each product across all stores in each month. We then calculate the monthly percent change in prices for each product. Finally, we compute the value-weighted percent change in product prices for each 4-digit SIC industry based on the transaction values of the industry's products. In column (1) and (2), the similarity measure is innosimm. In column (3) and (4), the similarity measure is prodsimm. We consider the 36-month period around the Lehman crash. We include t-statistics in parentheses. Standard errors are clustered by the 4-digit SIC industry and month. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels.

Table 7: Price-similarity sensitivity around the Lehman crash (monthly analysis).

	(1)	(2)	(3)	(4)
	Percent change in product prices (monthly, annualized, %)			
Similarity measure	Innosimm		Prodsimm	
similarity$_{t-1}$ × post Lehman crash$_t$	−3.04***	−2.84***	−0.07	−0.35
	[−3.19]	[−2.79]	[−0.20]	[−1.15]
similarity$_{t-1}$	−1.00	−2.05	−0.01	−5.90***
	[−1.32]	[−1.45]	[−0.01]	[−3.95]
post Lehman crash$_t$	−1.61	−1.64	−2.25*	−2.35*
	[−1.44]	[−1.47]	[−1.80]	[−1.87]
Industry FE	No	Yes	No	Yes
Observations	7641	7641	7192	7192
R-squared	0.004	0.040	0.001	0.039

Note: This table shows the price-similarity sensitivity around the Lehman crash. The dependent variable is the annualized monthly percent change in product prices of 4-digit SIC industries. We consider the 36-month period around the Lehman crash. We include t-statistics in parentheses. Standard errors are clustered by the 4-digit SIC industry and month. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels.

Lehman crash brings about product price changes in all industries, but with different magnitudes presumably depending on the industry's innosimm. To understand how the percent change in product prices varies with industry-level innosimm, or what we call the price-innosimm sensitivity, we regress the percent change in product prices on innosimm, the post-Lehman indicator, and an interaction term between innosimm and the post-Lehman indicator. The regression results are presented in Table 8. The price-innosimm sensitivity before the Lehman crash is given by the coefficient on innosimm; the price-innosimm sensitivity after the Lehman crash is given by the sum of the coefficients on innosimm and the interaction term; and the change in price-innosimm sensitivity owing to the Lehman crash is given by the coefficient on the interaction term. Table 8 shows that the coefficient of the interaction term is negative, indicating that high-innosimm industries are more affected by the Lehman crash and their product prices decrease relatively more (or increase relatively more slowly) compared to low-innosimm industries. Panel B of Figure 10 visualizes the monthly price-innosimm sensitivity. It is evident that the price-innosimm sensitivity reduces significantly after the Lehman crash.

We also examine the product prices around the Lehman crash for industries with different prodsimm. We find that product prices do not move differently for high-prodsimm industries and low-prodsimm industries (Panel B of Figure 10, Columns (3) and (4) in Table 6). Moreover, we observe little change in price-prodsimm sensitivity following the Lehman crash (Panel D of Figure 10, Columns (3) and (4) in Table 7).

5.3.3 Product Prices for 2006–2016

Having tested a particular period around the Lehman crash, we now extend our analysis to the whole time series covered by the Nielsen data from 2006 to 2016. Specifically, we regress the percent change in product prices on innosimm, consumption growth, and the interaction term between innosimm and consumption growth. Columns (1) and (3) of Table 8 show that the coefficients of the interaction term are positive and statistically significant for realized and filtered consumption growth, suggesting that high-innosimm industries are associated with higher price war risks.

5.4 The Impact of Antitrust Enforcement

We now test whether the difference in the price war risks across industries with different innosimm is due to the difference in collusion incentive. We exploit the variation in collusion incentive due to antitrust enforcement, which punishes collusive behavior and

Table 8: Price-innosimm sensitivity and consumption growth (quarterly analysis).

	(1)	(2)	(3)	(4)
	Annualized percent change in product prices ($\sum_{j=1}^{12} price\_gr_{t+j}/3$)			
	$\hat{\theta}_t = \sum_{j=0}^{7} \hat{\eta}_{t-j}/2$ Annualized realized consumption growth		$\hat{\theta}_t = \sum_{j=0}^{3} \hat{x}_{t-j}$ Annualized filtered consumption growth	
$\hat{\theta}_t \times$ innosimm$_t$	0.62*	1.09**	0.98*	1.39**
	[2.00]	[3.24]	[2.44]	[2.97]
$\hat{\theta}_t \times$ innosimm$_t$ $\times$ post antitrust$_t$		−1.75**		−1.56**
		[−3.18]		[−3.25]
$\hat{\theta}_t \times$ post antitrust$_t$		1.97		1.74
		[1.87]		[1.48]
innosimm$_t$ $\times$ post antitrust$_t$		−0.09		−0.07*
		[−1.49]		[−2.17]
$\hat{\theta}_t$	−0.77	−1.53	−1.08	−1.73
	[−1.53]	[−1.75]	[−1.78]	[−1.63]
innosimm$_t$	0.07*	0.11*	0.08*	0.11*
	[2.09]	[2.24]	[2.28]	[2.27]
post antitrust$_t$		0.08		0.07**
		[1.27]		[2.43]
Industry FE	Yes	Yes	Yes	Yes
Observations	6031	6031	5312	5312
R-squared	0.433	0.439	0.549	0.553

Note: This table shows the sensitivity of percent changes in product prices to consumption growth across the 4-digit SIC industries with different innosimm. The dependent variable is the industry-level annualized percent change in product prices from quarter $t+1$ to quarter $t+12$. Long-run risks are measured by annualized cumulative realized consumption growth from quarter $t-7$ to t ($\sum_{j=0}^{7} \hat{\eta}_{t-j}/2$) in column (1) and (2), and by cumulative filtered consumption growth from quarter $t-3$ to t ($\sum_{j=0}^{3} \hat{x}_{t-j}$) in column (3) and (4). We use shorter cumulation period for filtered consumption growth because it is less noisy than realized consumption growth. The sample period of columns is from 2006 to 2016. We include t-statistics in parentheses. Standard errors are clustered by the 4-digit SIC industry and year. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels.

thus dampens firms' incentive to collude.

We start by examining the impact of antitrust enforcement on product prices across industries with different innosimm.[30] Specifically, we create a post-antitrust indicator that equals one for industries that have experienced at least one antitrust enforcement in the past ten years. We include the triple-interaction term between the post-antitrust indicator, innosimm, and consumption growth in the regression. Columns (2) and (4) of Table 8 show that the coefficient on the triple-interaction term is negative and statistically

[30] The antitrust enforcement cases are hand collected from the websites of the U.S. Department of Justice (DOJ) and the Federal Trade Commission (FTC). DOJ provides 4-digit SIC codes for the firms in some of the cases. For the rest of DOJ cases and all FTC cases, we match the firms involved in antitrust enforcement to CRSP/Compustat and Capital IQ, from which we collect the 4-digit SIC codes of these firms.

significant, suggesting that antitrust enforcement significantly reduces the price war risks in high-innosimm industries.

Next, we examine the impact of antitrust enforcement on innosimm spreads. We split all industries in each year into two groups based on whether they have experienced any antitrust enforcement in the past ten years. As shown in Table 9, the innosimm spreads are much smaller in the industries that have recently experienced antitrust enforcement, suggesting that our asset pricing findings in Table 2 are mainly driven by the difference in collusion incentive across industries with different innosimm.

Table 9: Antitrust enforcement and innosimm spreads (monthly analysis).

Excess returns (%)	FF3F (%)	FF4F (%)
Panel A: Industries with antitrust enforcement in the past 10 years		
−0.81	0.59	−0.44
[−0.33]	[0.24]	[−0.21]
Panel B: Industries without antitrust enforcement in the past 10 years		
3.27**	5.44**	5.54***
[2.01]	[2.91]	[3.00]

Note: This table presents the average excess returns and alphas (both in percent) of the value-weighted long-short 4-digit SIC industry portfolio sorted on innosimm in the sub-samples with (Panel A) and without (Panel B) antitrust enforcement in past ten years. We exclude financial firms and utility firms from the analysis. We include t-statistics in parentheses. Standard errors are computed using the Newey-West estimator allowing for serial correlation in returns. We annualize the average excess returns and the alphas by multiplying by 12. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels.

5.5 More Tests on the Mechanisms

5.5.1 Innovation Similarity, Markups, and Market Shares

We provide additional tests on the model's mechanism in this subsection. Our model implies that, all else equal, firms in high-innosimm industries endogenously have higher markups and more comparable market shares.

In Table 10, we show that both markups and the dispersion of market shares are positively associated with innosimm. The results are statistically and economically significant. According to the regression with year fixed effects (columns 2 and 4), a one standard deviation increase in innosimm is associated with a 3.35-percentage-point increase in markups and a 1.61-percentage-point decrease in the dispersion of market shares.

Table 10: Innovation similarity, markups, and market share dispersion (yearly analysis).

	(1)	(2)	(3)	(4)
	Industry-level markups (%)		The dispersion of market shares (%)	
Innosimm_t	3.45^{***}	3.35^{***}	-1.26^{***}	-1.61^{***}
	[3.86]	[3.73]	[−2.60]	[−3.26]
Year FE	No	Yes	No	Yes
Observations	2783	2783	8967	8967
R-squared	0.068	0.076	0.008	0.033

Note: This table shows the relation of the innovation similarity with markups and the dispersion of market shares. The observation in this analysis is at the 4-digit SIC industry-year level. In columns (1) and (2), the dependent variable is the industry-level markup following the definition of Allayannis and Ihrig (2001):

$$\text{Industry-level markup}_{j,t} = \frac{\text{Value of shipments}_{j,t} + \Delta\text{Inventory}_{j,t} - \text{Payroll}_{j,t} - \text{Cost of material}_{j,t}}{\text{Value of shipments}_{j,t} + \Delta\text{Inventory}_{j,t}}.$$

The markups are constructed from the NBER-CES Manufacturing Industry Database, covering manufacturing industries from 1958 to 2011. The dispersion of market shares is defined as the standard deviation of all firms' market shares shares (measured by sales) within the 4-digit SIC industry. The sample spans from 1988 to 2011 in columns (1) and (2) and from 1988 to 2017 in columns (3) and (4). Standard errors are clustered by the 4-digit SIC industry and year. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels.

5.5.2 Sensitivity of Sales and Markups to Long-Run Risks

Our model predicts that sales and markups are more exposured to long-run risks in high-innosimm industries. Consistent with the model, Table 11 shows that both the three-year percent changes in industry-level sales and markups are more positively correlated with long-run risks in high-innosimm industries.

6 Quantitative Analyses

To conduct quantitative analyses, we extend the baseline model with external financing costs, which motivate firms to endogenously hoard cash on their balance sheets. We emphasize that more financially constrained firms have more incentive to deviate, lowering collusive prices. The financial constraints risk increases endogenously during periods with low long-run growth rates, further amplifying price war risks. Calibrating the extended model, we show that the cross-sectional asset pricing patterns observed in the data are also quantitatively explained by the model.

Table 11: Sensitivity of sales and markups to long-run risks (yearly analysis).

	(1)	(2)	(3)	(4)
	Annualized percent changes in industry-level sales		Annualized percent changes in industry-level markups	
	$\hat{\theta}_t = \sum_{j=0}^{1} \hat{\eta}_{t-j}/2$	$\hat{\theta}_t = \hat{x}_t$	$\hat{\theta}_t = \sum_{j=0}^{1} \hat{\eta}_{t-j}/2$	$\hat{\theta}_t = \hat{x}_t$
$\hat{\theta}_t \times \text{innosimm}_t$	1.44**	1.37**	0.20*	0.15**
	[2.57]	[2.38]	[1.92]	[2.42]
$\hat{\theta}_t$	0.15	0.24	−0.81**	−0.53
	[0.17]	[0.28]	[−2.49]	[−1.70]
Industry FE	Yes	Yes	Yes	Yes
Observations	9137	8823	2539	2539
R-squared	0.110	0.114	0.052	0.053

Note: This table shows the sensitivity of percent changes in sales and markups to long-run risks across 4-digit SIC industries with different innosimm. The dependent variables are the annualized percent change in industry-level sales and the annualized percent change in industry-level markups from year t to year $t+3$. Sales data are obtained from Compustat. Markups are constructed from the NBER-CES Manufacturing Industry Database, covering manufacturing industries from 1958 to 2011. Long-run risks are measured by cummulative realized consumption growth from year $t-1$ to t (annualized) in column (1) and (3), and by filtered consumption growth in year t in column (2) and (4). We use shorter cumulation period for filtered consumption growth because it is less noisy than realized consumption growth. We include t-statistics in parentheses. Standard errors are clustered by the 4-digit SIC industry and year. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels.

6.1 The Extended Model

We extend the baseline model in Section 2 with external financing frictions and idiosyncratic operating cash flow shocks. Following Bolton, Chen and Wang (2011, 2013), we assume that equity financing is costly. The firm incurs a fixed cost $\mathrm{d}X_{ij,t}$ to issue an amount $\mathrm{d}H_{ij,t}$ of equity. For tractability, we assume that $\mathrm{d}H_{ij,t}$ is a constant proportion ϖ of firm size $C_t M_{ij,t}$, i.e. $\mathrm{d}H_{ij,t} = \varpi C_t M_{ij,t}$. To ensure that the firm does not grow out of financial frictions, we assume that the fixed financing cost is also proportional to firm size, i.e. $\mathrm{d}X_{ij,t} = \delta C_t M_{ij,t}$. As in Dou and Ji (2018), firms face idiosyncratic operating cash flow shocks, modeled by $\sigma C_t M_{ij,t} \mathrm{d}Z_{ij,t}$, where $Z_{ij,t}$ is a standard Brownian motion independent of $Z_{c,t}$ and $Z_{\theta,t}$. Thus, equation (2.22) is modified as:

$$\mathrm{d}O_{ij,t} = \left(P_{ij,t} - \omega\right) C_{ij,t}\mathrm{d}t + \sigma C_t M_{ij,t}\mathrm{d}Z_{ij,t}. \tag{6.1}$$

In the presence of financing frictions and cash flow risks, firms choose to endogenously hoard cash $W_{ij,t}$ on their balance sheets. Following Bolton, Chen and Wang (2011, 2013), we assume that the return from cash is the risk-free rate r minus a carry cost $\nu > 0$. The cash-carrying cost implies that the firm would pay out dividends $\mathrm{d}E_{ij,t}$ when cash holdings are high. We assume that dividend payouts are conducted in lump-sum and proportional to firm size, i.e. $\mathrm{d}E_{ij,t} = \varkappa C_t M_{ij,t}$. The net dividend is defined by

$$\mathrm{d}D_{ij,t} = \mathrm{d}E_{ij,t} - \mathrm{d}H_{ij,t}, \tag{6.2}$$

which implies that the firm's optimal financing decisions is boiled town to compare three value of $\mathrm{d}D_{ij,t}$:[31]

$$\mathrm{d}D_{ij,t} = \{\underbrace{\varkappa C_t M_{ij,t},}_{\text{payout}} \underbrace{\quad 0 \quad}_{\text{hoard cash}}, \underbrace{-\varpi C_t M_{ij,t}}_{\text{financing}}\}. \tag{6.3}$$

Firms can create new customer bases $q_{ij,t}M_{i,t}\mathrm{d}t$ by investing $q_{ij,t} \geq 0$ in customer bases at quadratic costs $\xi q_{ij,t}^2 M_{ij,t}\mathrm{d}t$ over $[t, t + \mathrm{d}t]$. Different from innovations, there is a free-rider problem in investment (Holmstrom, 1982; Battaglini, 2006; Bond and Pande, 2007). We assume that when either firm invests, the new customer base $q_{ij,t}M_{i,t}\mathrm{d}t$ will be accumulated by both firms with a fraction depending on their intrinsic market shares $m_{ij,t}$. Thus, the investment decision is not strategic because it only increases firms' customer bases without changing their market power. In the presence of financial constraints risks, firms cut investment when their cash holdings are low.

The evolution of cash holdings $W_{ij,t}$ is given by

$$\mathrm{d}W_{ij,t} = (r - \nu)W_{ij,t} + \mathrm{d}O_{ij,t} - (1 + \xi q_{ij,t})q_{ij,t}M_{ij,t}\mathrm{d}t - \mathrm{d}D_{ij,t}. \tag{6.4}$$

The dividend policy can be sufficiently characterized by payout and decision boundaries, $\underline{W}(m_{ij,t}, \phi_{i,t}, \theta_t)$ and $\overline{W}(m_{ij,t}, \phi_{i,t}, \theta_t)$. The HJB equations are detailed in Appendix D.2. Because equity financing is costly, firms issues equity only when it runs out of cash, i.e. $W_{ij,t} \leq \underline{W}(m_{ij,t}, \phi_{i,t}, \theta_t) \equiv 0$. The firm pays out dividend when $W_{ij,t} \geq \overline{W}(m_{ij,t}, \phi_{i,t}, \theta_t)$. High dividend growth occurs in periods with high long-run growth rates, both because of high consumption growth and firms' high investment $q_{ij,t}$ in customer bases motivated by high collusive prices. Thus, the endogenous price wars provide a micro foundation for the dividend-leverage ratio on expected consumption growth in the models of Abel (1999) and Bansal and Yaron (2004).

[31] In Bolton, Chen and Wang (2011, 2013), $\varkappa$ and ϖ are endogenous variables optimally decided by the firm. Allowing $\varkappa$ and ϖ to be endogenous (and hence continuous) decision variables (in addition to product prices) introduces significant computation complexity for solving the Nash equilibrium. Because the key point we hope to emphasize is the time-varying marginal value of internal funds, we adopt this simplified modeling approach for tractability. Moreover, because ϖ is a constant in our model, it is not necessary to have the variable financing cost as in Bolton, Chen and Wang (2011, 2013).

6.2 Calibration

We discipline the parameters based on both existing estimates and micro data (see Table 12) without referring to asset pricing information, and we examine whether the calibrated model can quantitatively explain the observed asset pricing patterns.

The process of aggregate consumption is calibrated following Bansal and Yaron (2004). We set the persistence of expected growth rate to be $\kappa = 0.49$, so that the autocorrelation of annual consumption growth rates is 0.49. We set $\overline{\theta} = 0.018$ and $\sigma_c = 0.029$ so that the average annual consumption growth rate is 1.8% and its standard deviation is about 2.9%. Following Bansal and Yaron (2004), we set $\varphi_\theta = 0.044$, indicating that the predictable variation in consumption growth is 4.4%. Following the standard practice, we set the subjective discount factor $\beta = 0.976$, the risk aversion parameter $\gamma = 10$, and the intertemporal elasticity of substitution $\psi = 1.5$.

We set the within-industry elasticity of substitution $\eta = 15$ and the across-industry elasticity of substitution to be $\epsilon = 2$, broadly consistent with the values of Atkeson and Burstein (2008). We set the customer base depreciation rate to be $\rho = 0.135$, within the range of 15%-25% estimated by Gourio and Rudanko (2014). We set $\alpha = 0.3$ and $z = 0.05$ to ensure that customer base is sticky and long-term in nature (Gourio and Rudanko, 2014; Gilchrist et al., 2017). The price inspection rate is set to be $\varsigma = 0.09$, implying that price changes by about 3-6% for a one-percent change in annual consumption growth rates, roughly consistent with our data.

We allow the industry characteristic $\phi_{i,t}$ to take 11 values, i.e. $\phi_{i,t} \in \Phi = \{0, 0.1, 0.2, ..., 1\}$. The characteristic $\phi_{i,t}$ remains the same unless it is hit by a Poisson shock with rate ε. Conditional on receiving the Poisson shock, a new characteristic is randomly drawn from Φ with equal probabilities of each value. We set $\varepsilon = 0.03$ to make $\phi_{i,t}$ a persistent industry characteristic. We set the $\lambda_\iota = 4$ and $\lambda_d = 0.2$, implying that on average across all industries, incremental innovation succeeds every 6 months and distinctive innovation succeeds every 10 years. We target a 1.5% annualized growth rate of firm size by setting $g_\iota = 0.1875\%$ and $g_d = 3.75\%$. The within-industry customer base transfers due to incremental and distinctive innovation are set to be $\tau_\iota = 0.10$ and $\tau_d = 0.99$. These values imply that $\lambda_\iota \tau_\iota > \lambda_d \tau_d$, indicating that the changes in market shares due to the innovation channel is more significant in high-innosimm industries, consistent with the evidence in Appendix Table C.3.

Following Bolton, Chen and Wang (2011), we set the cash-carrying cost to be $\nu = 1.5\%$,

Table 12: Calibration and parameter choice.

Parameters	Symbol	Value	Parameters	Symbol	Value
Average long-run consumption growth rate	$\bar{\theta}$	0.018	Persistence of expected growth rate	κ	0.49
Exposure to long-run growth risks	φ_θ	0.044	Volatility of consumption growth	σ_c	0.029
Customer base adjustment friction	χ	0.3	Cost of production	ω	1
Customer base accumulation rate	z	0.05	Customer base depreciation rate	ρ	0.135
Across-industry elasticity of substitution	ϵ	2	Within-industry elasticity of substitution	η	15
Success rate of incremental innovation	λ_ι	4	Success rate of distinctive innovation	λ_d	0.2
Growth effect from incremental innovation	g_ι	0.1875%	Growth effect from distinctive innovation	g_d	3.75%
Customer base transfers (incremental)	τ_ι	0.10	Customer base transfers (distinctive)	τ_d	0.99
Financing cost	δ	0.03	Cash holding cost	ν	0.015
Equity issuance ratio	ϖ	0.06	Dividend payout ratio	$\varkappa$	0.1
Price inspection rate	ς	0.09	Risk aversion	γ	10
Inter-temporal elasticity of substitution	ψ	1.5	Subjective discount factor	β	0.976
Cash flow volatility	σ	0.09	Persistence of R&D projects' composition	ε	0.03
Investment adjustment costs	ξ	5			

and the volatility of cash flow shocks to be $\sigma = 0.09$. We set the financing cost to be $\delta = 3\%$, which is consistent with the sum of fixed and variable financing cost used by Bolton, Chen and Wang (2011). We set $\omega = 1$ so that the steady-state cash-to-sales ratio is about 15%. We set $\varkappa = 0.1$, implying that the average dividend payout ratio is 50% of net income, consistent with the U.S. data. We set $\varpi = 0.06$, implying an equity issuance amount equal to 6% of firm size, consistent with the model-implied value of Bolton, Chen and Wang (2011). We set the adjustment cost $\xi = 5$ following Dou et al. (2018).

6.3 Amplification of Price War Risks Due to Financing Frictions

Our baseline model illustrates that price war risks amplify long-run risks. In this section, we show that financing frictions in our extended model further amplify price war risks. Thus, in the presence of financing frictions, the amplification mechanism provided by price war risks is significantly larger.

Panel A of Figure 11 compares the equilibrium prices with and without financing frictions for an industry with $\phi = 0$. It is shown that financing frictions push down collusive prices both in periods with high and low long-run growth rates (see the red dash-dotted line and the red dotted line), but importantly, the decrease in collusive prices is more significant during low-growth-rate periods. The magnitude of price war risks, as measured by the difference in collusive prices, is much larger in the presence of financing frictions (see Panel B). As a result, the amplification effect of price war risks is larger,

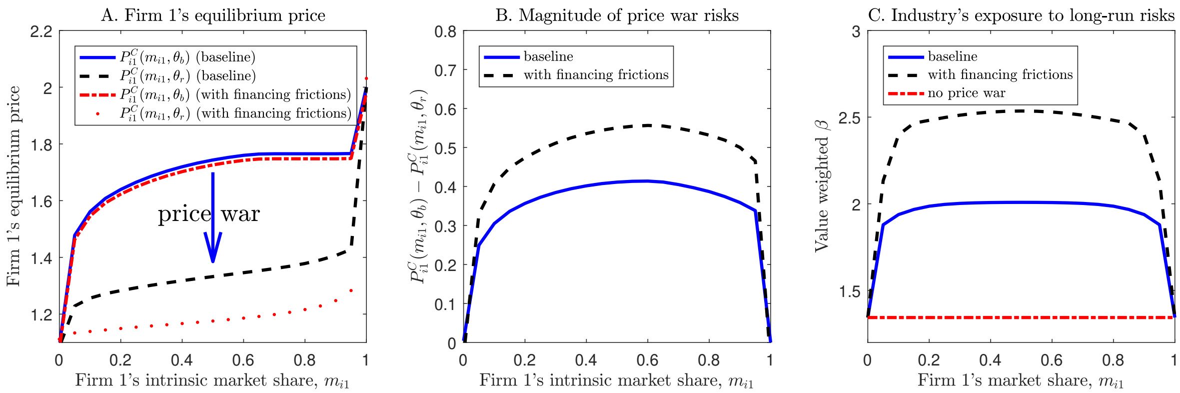


Figure 11: Amplification of price war risks due to financing frictions.

significantly increasing the industry's exposure to long-run risks (see Panel C).

Intuitively, when firms face financing costs, the marginal value of internal funds is higher, especially for financially constrained firms. This implies that firms start to value short-term cash flows more than long-term cash flows, effectively increasing firms' discount rates and making them more impatient. Because deviating from the collusive equilibrium boosts up short-term cash flows, the presence of financing frictions increases deviation incentive, making it more difficult to collude on higher prices. This explains why financing frictions result in lower collusive prices both in periods with high and low long-run growth rates.[32]

The reduction in collusive prices is more pronounced during periods with low consumption growth rates. This is because firms receive low cash flows and face high marginal value of liquidity. Thus, the presence of financial constraints would significantly change their deviation incentives, lowering collusive prices and reducing investment in customer bases. Moreover, the lower collusive prices further make firms more financially constrained, which in turn further increase their deviation incentive and lower collusive prices and investment. This feedback loop further amplifies the negative effect of financing frictions on collusive prices and investment during periods with low consumption growth rates, resulting in large price war risks. By contrast, during high long-run-growth periods, both the levels (due to high collusive prices) and growth rates (due to high long-run growth rates of consumption) of cash flows are high. Firms are not quite financially

[32]There is a countervailing force from the price channel. More financially constrained firms tend to raise prices to boost short-term revenue by profiting from currently locked-in customers. This channel is dominated given our calibrated parameters.

constrained anyway, and thus whether they face financing costs would not matter much for their pricing decisions.

Our mechanism is different from that of Gilchrist et al. (2017), who show both theoretically and empirically that financially constrained firms tend to raise their product prices relative to financially unconstrained firms. Our mechanism is about the amplification effect of financing frictions on the endogenous variation in the level of product prices over time. Thus, we emphasize more about the time-series rather than the cross-sectional implications.

In the presence of financing frictions, the dispersion in risk premia is larger across different industries. As we illustrate in Section 4.2, industries with a higher capacity of distinctive innovation are naturally immune to price war risks due to the lack of collusion. Thus, the amplification effect of financing frictions on price war risks does not change these industries exposure to long-run risks much. By contrast, in the industries with a lower capacity of distinctive innovation, firms' exposure to price war risks is larger, which is further amplified by financing frictions due to the feedback effect.

6.4 Quantitative Results

Now we check whether our model can quantitatively replicate the main asset pricing patterns presented in Table 2. In each year t, we sort the simulated firms into five quintiles based on their $\phi_{i,t}$ at the beginning of the year. We then compute the value-weighted average excess return of each quintile's portfolio.

Table 13 shows that the baseline-model-implied difference in average excess returns between Q1 and Q5 is about 2.04%. In the extended model with financing frictions, the difference increases to 3.28% due to the amplification of price war risks. These numbers are quantitatively consistent with the findings in Table 2.

Table 13: Average excess returns of portfolios in data and model.

	Data	Model	
	$\mathbb{E}[R] - r_f$	baseline	with financing frictions
Quintile 1 (%)	6.13	3.13	4.02
Quintile 5 (%)	9.54	5.17	7.30
Q5 – Q1 (%)	3.41	2.04	3.28

7 Conclusion

In this paper, we explore the the implication of price war risks. We develop a general-equilibrium asset pricing model incorporating dynamic supergames of price competition among firms. In our model, price wars can arise endogenously from declines in long-run consumption growth, since firms become effectively more impatient for cash flows and their incentives to undercut prices become stronger.

Our model implies that financial frictions amplify price war risks, which in turn largely amplify long-run risks and generate cross-sectional implications on stock returns. The exposure to price war risks reflect predictable and persistent heterogeneous industry characteristics. Firms in industries with more frequent distinctive innovation are more immune to price war risks due to the higher likelihood of creative destruction and market disruption. Exploring detailed patent and product price data, we found evidence for the existence of price war risks. Moreover, that endogenous price war risks are priced in the cross section.

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Appendix

A Headline Quotes for Price Wars and Stock Returns

We cite a few recent media headlines on how price wars can depress firms' stock returns.

- "Best Buy Co. shares plunged 11% Tuesday, after the electronics chain warned investors about price war fears." – *The Wall Street Journal* on November 20th of 2013.
- "Target shares dive as it shifts to cut-price strategy." – *Financial Times* on February 28th of 2017.
- "Price war eats into the profits of pharmaceutical wholesalers and manufacturers alike and erases billions of dollars of the market value in recent days" – *The Wall Street Journal* on August 5th of 2017.
- "Airline stocks plunge on price war fears." – *Financial Times* on January 24th of 2018.
- "Investors Purge Infinera Stock on Price War Concerns, Ignore Q1 Results." – *SDxCentral* on May 10th of 2018.
- "A fierce price war between consumer goods giants hit Unilever shares hard." – *The Wall Street Journal* on July 19th of 2018.
- "Coffee price war takes jolt out of Dunkin' results." – *Financial Times* on September 27th of 2018.

B Data

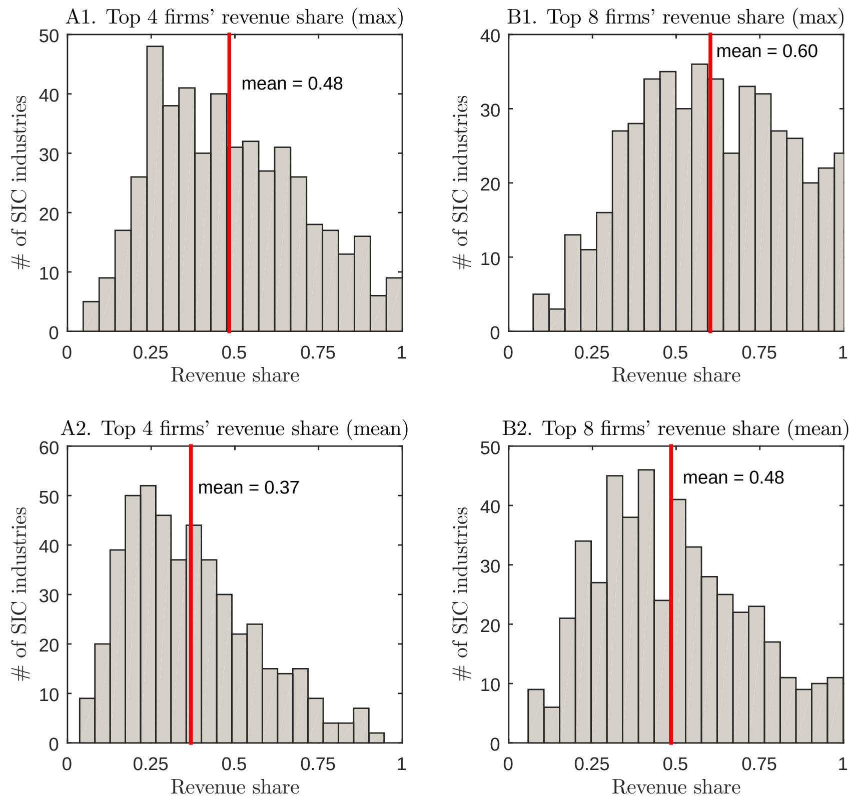
B.1 Industry Concentration Ratio

We use the U.S. Census concentration ratio data from 1987, 1992, 1997, 2002, 2007, and 2012 to compute the time-series maximal and mean revenue shares for the top 4 firms (CR4) and top 8 firms (CR8) in each 4-digit SIC industry. The concentration ratios are at the 6-digit NAICS level after 1997. We follow Ali, Klasa and Yeung (2008) and convert the ratios to 4-digit SIC levels. Figure B.1 plots the histogram of the max CR4 (Panel A1), max CR8 (Panel B1), mean CR4 (Panel A2), and mean CR8 (Panel B2) in all 4-digit SIC industries. Red vertical lines represent the cross-sectional mean values.

B.2 Match PatentView with CRSP/Compustat/Capital IQ

In this Appendix, we detail the matching procedure for the data from PatentView, CRSP/Compustat, and Capital IQ.[33] We first drop patent assignees that are classified as individuals and government agencies by PatentView, because these assignees are not associated with any particular industry. We then clean assignee names in PatentView and firm names in CRSP/Compustat and Capital IQ following the approach of Hall, Jaffe and Trajtenberg (2001). To elaborate, we remove punctuations and clean special characters. We then

[33]The PatentView data are available at http://www.patentsview.org/download/.



Note: This figure plots the histogram of the top 4 and top 8 firms' total revenue share in 4-digit SIC industries. We use the U.S. Census concentration ratio data from 1987, 1992, 1997, 2002, 2007, and 2012 to compute the time-series maximal and mean revenue shares for the top 4 firms (CR4) and top 8 firms (CR8) in each 4-digit SIC industry. The concentration ratios are at the 6-digit NAICS level after 1997. We follow Ali, Klasa and Yeung (2008) and convert the ratios to 4-digit SIC levels. We plot the histogram of the max CR4 (Panel A1), max CR8 (Panel B1), mean CR4 (Panel A2), and mean CR8 (Panel B2) in all 4-digit SIC industries. Red vertical lines represent the cross-sectional mean values.

Figure B.1: Top 4 and top 8 firms' revenue share in 4-digit SIC industries.

transform the names into upper cases and standardize them. For example, "INDUSTRY" is standardized to be "IND"; and "RESEARCH" is standardized to be "RES"; and corporate form words (e.g. "LLC" and "CORP") are dropped, etc.

Match PatentView with CRSP/Compustat. We match patent assignees in PatentView with firms in CRSP/Compustat based on standardized names. We use the fuzzy name matching algorithm (*matchit* command in Stata), which generates the matching scores (Jaccard index) for all name pairs between patent assignees in PatentView and firms in CRSP/Compustat.[34] We obtain a pool of potential matches

[34] Jaccard index measures the similarity between finite sample sets, and is defined as the size of the intersection divided by the size of the union of the sample sets. Jaccard index ranges between 0 and 1, reflecting none to perfect similarity.

based on two criteria: (1) we require the matching score to be higher than 0.6; (2) we require the first three letters of patent assignees to be the same as those of firms in CRSP/Compustat.[35] We then go through all potential matches to manually identify exact matches.[36]

As pointed out by Lerner and Seru (2017), one major challenge for linking patent data to CRSP/Compustat is that some patent assignees are subsidiaries of firms in CRSP/Compustat. For these assignees, we cannot directly match them with CRSP/Compustat based on firm names. To deal with this challenge, the NBER patent data (Hall, Jaffe and Trajtenberg, 2001) use the 1989 edition of the Who Owns Whom directory (now known as the D&B WorldBase®- Who Owns Whom) to match subsidiaries to parent companies. Kogan et al. (2017) purged the matches identified by the NBER patent data, and extended the matching between patent data and CRSP/Compustat to 2010. For those patent assignees who are subsidiaries of firms in CRSP/Compustat, we augment our matches by incorporating the data of Kogan et al. (2017) for patents granted before 2010. For patents granted after 2010, we use the subsidiary-parent link table from the 2017 snapshot of the Orbis data to match subsidiaries in PatentView to their parent firms in CRSP/Compustat.

Match PatentView with Capital IQ. We match the remaining patent assignees in PatentView with firms in Capital IQ following the same matching procedure. To keep the workload manageable, we drop firms in Capital IQ whose assets are worth less than $100 million (in 2017 dollars). Because we focus on the U.S. product market, we also drop foreign firms whose asset values are below the 90th percentile of the asset value distribution among firms in the CRSP/Compustat sample in each year, respectively. This is because small foreign firms are less likely to have a material impact on the competition environment of the U.S. product market. We match PatentView to Capital IQ directly using the information on subsidiaries provided by Capital IQ.

B.3 Construct Dividend Growth

We follow previous studies (see e.g. Campbell and Shiller, 1988; Bansal, Dittmar and Lundblad, 2005; Hansen, Heaton and Li, 2005, 2008; Bansal, Kiku and Yaron, 2016) to contrust dividend growth rates of portfolios.

Denote V_{0t} as the market value of all firms in a given portfolio. Denote the value of this portfolio at date $t+1$ to be V_{t+1}. The aggregate dividends for data $t+1$ for this portfolio is D_{t+1}. The total return on the portfolio between t and $t+1$ is:

$$R_{t+1} = \frac{V_{t+1} + D_{t+1}}{V_{0t}} = h_{t+1} + d_{t+1}. \tag{B.1}$$

where h_{t+1} is the price appreciation, which represents the ratio of the value at time $t+1$ to time t (i.e., $\frac{V_{t+1}}{V_{0t}}$), while d_{t+1} is the dividend yield, which represents the total dividends paid by at time $t+1$ divied by

[35]These two matching criteria are sufficiently conservative to ensure that exact matches are included in the pool of potential matches. For example, among all the exact matches in the first quarter of 2016, 98% of them satisfy the two matching criteria and are included in our pool of potential matches.

[36]We rely on assignee names in PatentView and firm names in CRSP/Compustat to identify matches. In addition, we use location information in both datasets to facilitate the matching process.

portfolio value at time t (i.e., $\frac{D_{t+1}}{V_{0t}}$).

Holding the portfolio composition constant (i.e., no exits and entries), the real dividend growth rate is:

$$\frac{D_{t+1}/PCE_{t+1}}{D_t/PCE_t} = \frac{d_{t+1}V_{0t}}{d_t V_{0(t-1)}} \frac{PCE_t}{PCE_{t+1}} = \frac{d_{t+1}h_t}{d_t} \frac{PCE_t}{PCE_{t+1}} = \frac{(R_{t+1} - h_{t+1})h_t}{R_t - h_t} \frac{PCE_t}{PCE_{t+1}}, \tag{B.2}$$

where PCE is the personal consumption expenditure deflator from the U.S. BEA.

Because stocks move in and out of portfolios, we account for the entries and exits following the literature (see e.g. Hansen, Heaton and Li, 2005, 2008; Bansal, Kiku and Yaron, 2016) by adding an adjustment term. Specifically, the real dividend growth rate for a portfolio is:

$$\frac{V_t}{V_{0t}} \frac{D_{t+1}/PCE_{t+1}}{D_t/PCE_t}, \tag{B.3}$$

where V_t is exit value of the portfolio at time t (i.e., the time t market value of firms in the portfolio formed at time $t-1$), and V_{0t} is the market value of firms in the new position we initiate at time t. Plug equation B.2 into B.3, the real dividend growth rate for a portfolio is:

$$\frac{V_t}{V_{0t}} \frac{D_{t+1}/PCE_{t+1}}{D_t/PCE_t} = \frac{V_t}{V_{0t}} \frac{(R_{t+1} - h_{t+1})h_t}{R_t - h_t} \frac{PCE_t}{PCE_{t+1}}. \tag{B.4}$$

We calculate portfolio R_t and h_t by computing the value-weighted RET and $RETX$ (both from CRSP) across firms within the portfolio. Since share repurchases are prevelant in our sample period, we follow Bansal, Dittmar and Lundblad (2005) and adjust the capital gain series for a given firm as following:

$$RETX^*_{t+1} = RETX_{t+1} \min\left[\left(\frac{n_{t+1}}{n_t}\right), 1\right], \tag{B.5}$$

where n_t is the number of shares after adjusting for splits, stock dividends, etc using the CRSP share adjustment factor.

B.4 Match Nielsen with CRSP/Compustat/Capital IQ

We follow previous studies (e.g. Hottman, Redding and Weinstein, 2016; Argente, Lee and Moreira, 2018; Jaravel, 2018) to find the companies that own the products in the Nielsen data using the product-firm link table in the "GS1 US Data Hub | Company" data, which is provided by GS1 – the official source of UPCs in the U.S.[37] We match 95.3% of the products in the Nielsen data with firms in the GS1 data. Our matching rate is the same as those reported by Argente, Lee and Moreira (2018) and Jaravel (2018). We further match the companies in the GS1 data to CRSP/Compustat and Capital IQ to find their SIC industry codes. The matching procedures are the same as patent matching. Our merged data cover products in 472 SIC industries.

[37]The "GS1 US Data Hub | Company" data provide the company names, company addresses, and the UPC prefixes owned by the companies. More information about the "GS1 US Data Hub | Company" is available at: https://www.gs1us.org/tools/gs1-us-data-hub/company.

C Supplementary Empirical Results

C.1 Product Prices in the 24-month Period Around the Lehman Crash

Table C.1: Product prices around the Lehman crash (monthly analysis).

	(1)	(2)
	Percent change in product prices (monthly, annualized, %)	
Tertile-3 innosimm$_{t-1}$ × post Lehman crash$_t$	-6.04^{***}	-6.64^{***}
	[−3.14]	[−3.23]
Tertile-3 innosimm$_{t-1}$	−1.44	−5.79
	[−0.57]	[−0.99]
post Lehman crash$_t$	1.50	2.62
	[0.58]	[1.08]
Industry FE	No	Yes
Observations	3398	3398
R-squared	0.003	0.055

Note: This table shows the changes in product prices around the Lehman crash. The dependent variable is the annualized monthly percent change in product prices of 4-digit SIC industries. Product prices are obtained from the Nielsen Data. To compute the monthly percent change in product prices for 4-digit SIC industries, we first compute the transaction-value weighted price for each product across all stores in each month. We then calculate the monthly percent change in prices for each product. Finally, we compute the value-weighted percent change in product prices for each 4-digit SIC industry based on the transaction values of the industry's products. We consider we consider the 24-month period around the Lehman crash. We include t-statistics in parentheses. Standard errors are clustered by the 4-digit SIC industry and year. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels.

Table C.2: Price-innosimm sensitivity around the Lehman crash (monthly analysis).

	(1)	(2)
	Percent change in product prices (monthly, annualized, %)	
innosimm$_{t-1}$ × post Lehman crash$_t$	-2.52^{**}	-2.32^{**}
	[−2.52]	[−2.27]
innosimm$_{t-1}$	−1.02	-3.02^{*}
	[−1.35]	[−2.02]
post Lehman crash$_t$	−2.09	−1.92
	[−1.49]	[−1.36]
Industry FE	No	Yes
Observations	5086	5086
R-squared	0.003	0.043

Note: This table shows the price-innosimm sensitivity around the Lehman crash. The dependent variable is the annualized monthly percent change in product prices of 4-digit SIC industries. We consider we consider the 24-month period around the Lehman crash. We include t-statistics in parentheses. Standard errors are clustered by the 4-digit SIC industry and year. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels.

C.2 The Sensitivity of Market Shares to Innovative Outputs

In this Appendix, we examine the relation between innovation similarity and the sensitivity of market shares to innovative outputs. Specifically, we regress the changes in firms' market shares on the firm's own innovative outputs, peers' innovative outputs, lagged innosimm, the interaction term between innosimm and the firm's own innovative outputs, and the interaction term between innosimm and peers' innovative outputs. Table C.3 shows that the coefficient of the interaction term between innosimm and the firm's own innovative outputs is significantly positive, while the coefficient of the interaction term between innosimm and peers' innovative outputs is significantly negative. These findings suggest that in high-innosimm industries, firms obtain more market shares from their own innovation, but also lose more market shares from their peers' innovation. These results are economically significant. Consider two firms, one in an industry with the mean value of innosimm and the other is in an industry whose innosimm is one standard deviation above the mean. According to the regression specification with year fixed effect (column 4 of Table C.3), the latter firm's market share increases by 8.3 percentage points more in two years in response to a one standard deviation increase in the firm's own innovative outputs. Moreover, the latter firm's market share decreases by 4.2 percentage points more in two years in response to a one standard deviation increase in peers' innovative outputs.

D Numerical Algorithm

In this section, we detail the numerical algorithm that solves the model. To give an overview, our algorithm proceeds in the following steps:

(1). We solve for the non-collusive equilibrium. This requires us to solve the Markov-Perfect equilibrium of the dynamic game played by two firms. The simultaneous-move dynamic game requires us to solve the intersection of the two firms' best response (i.e. optimal price) functions, which themselves are optimal solutions to coupled PDEs.

(2). We solve for the collusive equilibrium using the value functions in the non-collusive equilibrium as punishment values. Because we are interested in the highest collusive prices with binding incentive-compatibility constraints, this requires us to solve a high-dimensional fixed-points problem. We thus use an iteration method inspired by and to solve the problem (Abreu, Pearce and Stacchetti, 1986, 1990; Ericson and Pakes, 1995; Fershtman and Pakes, 2000).

(3). After solving the baseline model, we solve the extended model with endogenous cash holdings by repeating steps (1) and (2). The extended model is more challenging because it involves solving PDEs with free boundaries (due to endogenous payout boundaries). We employ the piecewise multilinear interpolation method of Weiser and Zarantonello (1988) to obtain accurate interpolants in a 3-dimensional space.

Note that standard methods for solving PDEs with free boundaries (e.g. finite difference or finite element) can easily lead to non-convergence of value functions. To mitigate such problems and obtain accurate solutions, we solve the continuous-time game using a discrete-time dynamic programming method.

Table C.3: Innovation similarity and the sensitivity of market shares to innovative outputs (yearly analysis).

	(1)	(2)	(3)	(4)
	Percent changes in firms' market shares (market share$_{t+2}$ − market share$_t$, %)			
Own innovative outputs$_t$	0.040*	0.041*	−0.008	−0.011
	[1.888]	[1.919]	[−0.356]	[−0.487]
Own innovative outputs$_t$ × innosimm$_{t-1}$			0.077***	0.083***
			[2.964]	[3.240]
Peers' innovative outputs$_t$	−0.037***	−0.072***	−0.003	−0.048**
	[−2.730]	[−4.174]	[−0.201]	[−2.413]
Peers' innovative outputs$_t$ × innosimm$_{t-1}$			−0.053***	−0.042**
			[−2.845]	[−2.217]
Innosimm$_{t-1}$	0.058**	0.015	0.056**	0.013
	[2.558]	[0.596]	[2.524]	[0.525]
lnsize$_{t-1}$	−0.012	−0.029***	−0.012	−0.029***
	[−1.233]	[−2.752]	[−1.183]	[−2.747]
lnBEME$_{t-1}$	−0.073***	−0.076***	−0.076***	−0.079***
	[−4.149]	[−4.240]	[−4.339]	[−4.454]
Year FE	No	Yes	No	Yes
Observations	23758	23758	23758	23758
R-squared	0.004	0.015	0.005	0.016

Note: This table shows the relation between innovation similarity and the sensitivity of market shares to innovative outputs. The regression is run at the firm-level. The dependent variable is the two-year percent change in a firm's market share within its 4-digit SIC industry. A firm's market share in year t is defined as the ratio of the firm's sales to the sales of its 4-digit SIC industry. The dependent variable is winsorized at the 5th and 95th percentiles of its empirical distribution. We measure the firms' own innovative outputs using the sum of patent values normalized by the firm's book asset value. Patent values are measured in dollars based on the stock market reaction to patent issuance (Kogan et al., 2017). We measure peers' innovative outputs using the sum of peer firms' patent values in the 4-digit SIC industry normalized by peer firms' book asset values. We standardize own and peers' innovative outputs to ease the interpretation of coefficients. We include lagged firm size (lnsize$_{t-1}$) and lagged book-to-market ratios (lnBEME$_{t-1}$) as firm controls. The sample spans from 1988 and 2010. We include t-statistics in brackets. Standard errors are clustered by firm. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels.

In Appendix D.1, we present the discretized recursive formulation for the baseline model, including firms' problems in non-collusive equilibrium, collusive equilibrium, and deviation. In Appendix D.2, we present the discretized recursive formulation for the extended model. In Appendix D.3, we discuss how we discretize the stochastic processes, time grids, and state variables in the model. Finally, in Appendix D.4, we discuss discuss the details on implementing our numerical algorithms, including finding the equilibrium prices in the non-collusive equilibrium and solving the optimal collusive prices.

D.1 The Baseline Model

Because firm 1 and firm 2 are symmetric, one firm's value and policy functions are obtained directly given the other firm's value and policy functions. In this section, we illustrate firm 1's problem in our baseline model. We first illustrate the non-collusive equilibrium and then we illustrate the collusive equilibrium.

D.1.1 Non-Collusive Equilibrium

Below, we present the recursive formulation for the firm's value in the non-collusive equilibrium. Then we exploit linearity to simplify the problem and present the recursive formulation for the normalized firm value. Finally, we present the conditions that determine the non-collusive (Nash) equilibrium.

Recursive Formulation for The Non-Collusive Firm Value. The industry's state is characterized by five state variables, firm 1's customer base $M_{i1,t}$, firm 2's customer base $M_{i2,t}$, the industry's R&D characteristic $\phi_{i,t}$, the aggregate consumption C_t, and the long-run growth rate θ_t. Denote the value functions in the non-collusive equilibrium as $V_{ij}^N(M_{i1,t}, M_{i2,t}, \phi_{i,t}, C_t, \theta_t)$ for $j = 1, 2$.

To characterize the equilibrium value functions, it is more convenient to introduce two off-equilibrium value functions. Let $\widehat{V}_{ij}^N(M_{i1,t}, M_{i2,t}, \phi_{i,t}, C_t, \theta_t; P_{ik,t})$ be firm $j(=1,2)$'s value when its peer firm k's price is set at any (off-equilibrium) value $P_{ik,t}$.

Firm 1 solves the following problem:

$$\begin{aligned}\widehat{V}_{i1}^N(M_{i1,t}, M_{i2,t}, \phi_{i,t}, C_t, \theta_t; P_{i2,t}) = &\max_{P_{i1,t}} (P_{i1,t} - \omega) \left(\frac{P_{i1,t}}{P_{i,t}}\right)^{-\eta} P_{i,t}^{-\epsilon} C_t M_{i1,t} \Delta t \\ &+ \mathbb{E}_t \left[\frac{\Lambda_{t+\Delta t}}{\Lambda_t} V_{i1}^N(M_{i1,t+\Delta t}, M_{i2,t+\Delta t}, \phi_{i,t+\Delta t}, C_{t+\Delta t}, \theta_{t+\Delta t})\right], \end{aligned} \tag{D.1}$$

subject to the evolution of state variables, including:

The evolution of the customer base is

$$\begin{aligned} M_{ij,t+\Delta t} = &M_{ij,t} + \left[z \left(\frac{C_{ij,t}}{C_t}\right)^{\alpha} M_{ij,t}^{1-\alpha} - \rho M_{ij,t}\right] \Delta t + M_{ik,t}(\tau_\iota \Delta I_{ij,t}^\iota + \tau_d \Delta I_{ij,t}^d) + M_{i,t}(g_\iota \Delta I_{ij,t}^\iota + g_d \Delta I_{ij,t}^d) \\ &- M_{ij,t}(\tau_\iota \Delta I_{ik,t}^\iota + \tau_d \Delta I_{ik,t}^d), \qquad \text{for } j = 1, 2 \end{aligned} \tag{D.2}$$

where firm-level demand is given by

$$C_{ij,t} = \left(\frac{P_{ij,t}}{P_{i,t}}\right)^{-\eta} P_{i,t}^{-\epsilon} C_t M_{ij,t}, \qquad \text{for } j = 1, 2 \tag{D.3}$$

and the industry's price index is given by

$$P_{i,t} = \left(\frac{M_{i1,t}}{M_{i,t}} P_{i1,t}^{1-\eta} + \frac{M_{i2,t}}{M_{i,t}} P_{i2,t}^{1-\eta}\right)^{\frac{1}{1-\eta}}. \tag{D.4}$$

The evolution of aggregate consumption C_t is

$$C_{t+\Delta t} = (1 + \theta_t \Delta t + \sigma \Delta Z_{c,t}) C_t, \tag{D.5}$$

The industry's R&D characteristic $\phi_{i,t}$ and the long-run growth rate θ_t evolves according to the discrete Markov chain specified in Appendix D.3.

Recursive Formulation for The (Normalized) Non-Collusive Firm Value. Exploiting the linearity, we normalize the firm's value by $M_{i,t}C_t$. Firm 1's intrinsic market share is $m_{i1,t} = M_{i1,t}/M_{i,t}$; firm 2's intrinsic market share is $m_{i2,t} = M_{i2,t}/M_{i,t} = 1 - m_{i1,t}$. Define

$$v_{ij}^N(m_{i1,t}, \phi_{i,t}, \theta_t) = \frac{V_{ij}^N(M_{i1,t}, M_{i2,t}, \phi_{i,t}, C_t, \theta_t)}{M_{i,t}C_t} \tag{D.6}$$

$$\widehat{v}_{ij}^N(m_{i1,t}, \phi_{i,t}, \theta_t; P_{ik,t}) = \frac{\widehat{V}_{ij}^N(M_{i1,t}, M_{i2,t}, \phi_{i,t}, C_t, \theta_t; P_{ik,t})}{M_{i,t}C_t} \tag{D.7}$$

Firm 1 solves the following normalized problem:

$$\begin{aligned}\widehat{v}_{i1}^N(m_{i1,t}, \phi_{i,t}, \theta_t; P_{i2,t}) = &\max_{P_{i1,t}} (P_{i1,t} - \omega) \left(\frac{P_{i1,t}}{P_{i,t}}\right)^{-\eta} P_{i,t}^{-\epsilon} m_{i1,t} \Delta t \\ &+ \mathbb{E}_t \left[\frac{\Lambda_{t+\Delta t}}{\Lambda_t} \frac{M_{i,t+\Delta t}}{M_{i,t}} (1 + \theta_t \Delta t + \sigma \Delta Z_{c,t}) v_{i1}^N(m_{i1,t+\Delta t}, \phi_{i,t+\Delta t}, \theta_{t+\Delta t})\right],\end{aligned} \tag{D.8}$$

subject to the evolution of state variables, including:

The evolution of firm 1's intrinsic market share is

$$\begin{aligned}m_{i1,t+\Delta t} \frac{M_{i,t+\Delta t}}{M_{i,t}} = &m_{i1,t} + \left[z \left(\frac{P_{i1,t}}{P_{i,t}}\right)^{-\eta\alpha} P_{i,t}^{-\epsilon\alpha} - \rho\right] m_{i1,t} \Delta t + (1 - m_{i1,t})(\tau_\iota \Delta I_{i1,t}^\iota + \tau_d \Delta I_{i1,t}^d) + (g_\iota \Delta I_{i1,t}^\iota + g_d \Delta I_{i1,t}^d) \\ &- m_{i1,t}(\tau_\iota \Delta I_{i2,t}^\iota + \tau_d \Delta I_{i2,t}^d),\end{aligned} \tag{D.9}$$

where the industry's price index is given by

$$P_{i,t} = \left[m_{i1,t} P_{i1,t}^{1-\eta} + (1 - m_{i1,t}) P_{i2,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}. \tag{D.10}$$

The evolution of the industry's customer base is

$$\begin{aligned}\frac{M_{i,t+\Delta t}}{M_{i,t}} = &1 + \left[z \left(\frac{P_{i1,t}}{P_{i,t}}\right)^{-\eta\alpha} P_{i,t}^{-\epsilon\alpha} - \rho\right] m_{i1,t} \Delta t + \left[z \left(\frac{P_{i2,t}}{P_{i,t}}\right)^{-\eta\alpha} P_{i,t}^{-\epsilon\alpha} - \rho\right] (1 - m_{i1,t}) \Delta t \\ &+ g_\iota \Delta I_{i1,t}^\iota + g_d \Delta I_{i1,t}^d + g_\iota \Delta I_{i2,t}^\iota + g_d \Delta I_{i2,t}^d.\end{aligned} \tag{D.11}$$

The industry's R&D characteristic $\phi_{i,t}$ and the long-run growth rate θ_t evolve according to the discrete Markov chain specified in Appendix D.3.

Non-Collusive (Nash) Equilibrium. Denote the equilibrium price functions as $P_{ij}^{N}(m_{i1,t}, \phi_{i,t}, \theta_t)$ and the off-equilibrium price functions as $\widehat{P}_{ij}^{N}(m_{i1,t}, \phi_{i,t}, \theta_t; P_{ik,t})$. Exploiting the symmetry between firm 1 and firm 2, we can obtain firm 2's off-equilibrium value and policy functions as

$$\widehat{v}_{i1}^{N}(m_{i1,t}, \phi_{i,t}, \theta_t; P_{i2,t}) = \widehat{v}_{i2}^{N}(1 - m_{i1,t}, \phi_{i,t}, \theta_t; P_{i1,t}), \tag{D.12}$$

$$\widehat{P}_{i1}^{N}(m_{i1,t}, \phi_{i,t}, \theta_t; P_{i2,t}) = \widehat{P}_{i2}^{N}(1 - m_{i1,t}, \phi_{i,t}, \theta_t; P_{i1,t}). \tag{D.13}$$

Given $j = 1, 2$'s price P_{ij}, firm k optimally sets the price P_{ik}. The non-collusive (Nash) equilibrium is derived from the fixed point—each firm's price is optimal given the other firm's optimal price:

$$P_{i1}^{N}(m_{i1,t}, \phi_{i,t}, \theta_t) = \widehat{P}_{i1}^{N}(m_{i1,t}, \phi_{i,t}, \theta_t; P_{i2}^{N}(m_{i1,t}, \phi_{i,t}, \theta_t)), \tag{D.14}$$

$$P_{i2}^{N}(m_{i1,t}, \phi_{i,t}, \theta_t) = \widehat{P}_{i2}^{N}(m_{i1,t}, \phi_{i,t}, \theta_t; P_{i1}^{N}(m_{i1,t}, \phi_{i,t}, \theta_t)). \tag{D.15}$$

The equilibrium value functions are given by

$$v_{i1}^{N}(m_{i1,t}, \phi_{i,t}, \theta_t) = \widehat{v}_{i1}^{N}(m_{i1,t}, \phi_{i,t}, \theta_t; P_{i2}^{N}(m_{i1,t}, \phi_{i,t}, \theta_t)), \tag{D.16}$$

$$v_{i2}^{N}(m_{i1,t}, \phi_{i,t}, \theta_t) = \widehat{v}_{i2}^{N}(m_{i1,t}, \phi_{i,t}, \theta_t; P_{i1}^{N}(m_{i1,t}, \phi_{i,t}, \theta_t)). \tag{D.17}$$

After solving the equilibrium value and policy functions above, we can verify that the following conditions are satisfied due to symmetry,

$$v_{i1}^{N}(m_{i1,t}, \phi_{i,t}, \theta_t) = v_{i2}^{N}(1 - m_{i1,t}, \phi_{i,t}, \theta_t), \tag{D.18}$$

$$P_{i1}^{N}(m_{i1,t}, \phi_{i,t}, \theta_t) = P_{i2}^{N}(1 - m_{i1,t}, \phi_{i,t}, \theta_t). \tag{D.19}$$

D.1.2 Collusive Equilibrium

Below, we present the recursive formulation for the firm's value in the collusive equilibrium. Then we present the recursive formulation for the firm's value when the firm deviates from the collusive equilibrium. Finally, we present the incentive compatibility constraints and the conditions that determine the optimal collusive prices.

Recursive Formulation for The Collusive Firm Value. In the collusive equilibrium, we can still exploit the linearity property and solve firms' values as a function of intrinsic market shares. Specifically, denote $\widehat{v}_{ij}^{C}(m_{i1,t}, \phi_{i,t}, \theta_t; \widehat{P}_{ij}^{C})$ as firm j's value in the collusive equilibrium with collusive prices $\widehat{P}_{ij}^{C}(m_{i1,t}, \phi_{i,t}, \theta_t)$. Note that because the two firms in the same industry are symmetric, the collusive prices satisfy $\widehat{P}_{i1}^{C}(m_{i1,t}, \phi_{i,t}, \theta_t) = \widehat{P}_{i2}^{C}(1 - m_{i1,t}, \phi_{i,t}, \theta_t)$.

Firm 1 solves the following normalized problem:

$$\widehat{v}_{i1}^{C}(m_{i1,t},\phi_{i,t},\theta_t;\widehat{P}_{ij}^{C}) = \left(\widehat{P}_{i1}^{C}(m_{i1,t},\phi_{i,t},\theta_t)-\omega\right)\left(\frac{\widehat{P}_{i1}^{C}(m_{i1,t},\phi_{i,t},\theta_t)}{P_{i,t}}\right)^{-\eta} P_{i,t}^{-\epsilon} m_{ij,t}\Delta t$$
$$+\mathbb{E}_t\left[\frac{\Lambda_{t+\Delta t}}{\Lambda_t}\frac{M_{i,t+\Delta t}}{M_{i,t}}(1+\theta_t\Delta t+\sigma\Delta Z_{c,t})\widehat{v}_{i1}^{C}(m_{i1,t+\Delta t},\phi_{i,t+\Delta t},\theta_{t+\Delta t};\widehat{P}_{ij}^{C})\right], \quad \text{(D.20)}$$

subject to the evolution of state variables, including:

The evolution of firm 1's intrinsic market share is

$$m_{i1,t+\Delta t}\frac{M_{i,t+\Delta t}}{M_{i,t}} = m_{i1,t}+\left[z\left(\frac{\widehat{P}_{i1}^{C}(m_{i1,t},\phi_{i,t},\theta_t)}{P_{i,t}}\right)^{-\eta\alpha}P_{i,t}^{-\epsilon\alpha}-\rho\right]m_{i1,t}\Delta t+(1-m_{i1,t})(\tau_\iota\Delta I_{i1,t}^{\iota}+\tau_d\Delta I_{i1,t}^{d})$$
$$+(g_\iota\Delta I_{i1,t}^{\iota}+g_d\Delta I_{i1,t}^{d})-m_{i1,t}(\tau_\iota\Delta I_{i2,t}^{\iota}+\tau_d\Delta I_{i2,t}^{d}), \quad \text{(D.21)}$$

where the industry's price index is given by

$$P_{i,t}=\left[m_{i1,t}\widehat{P}_{i1}^{C}(m_{i1,t},\phi_{i,t},\theta_t)^{1-\eta}+(1-m_{i1,t})\widehat{P}_{i2}^{C}(m_{i1,t},\phi_{i,t},\theta_t)^{1-\eta}\right]^{\frac{1}{1-\eta}}. \quad \text{(D.22)}$$

The evolution of the industry's customer base is

$$\frac{M_{i,t+\Delta t}}{M_{i,t}} = 1+\left[z\left(\frac{\widehat{P}_{i1}^{C}(m_{i1,t},\phi_{i,t},\theta_t)}{P_{i,t}}\right)^{-\eta\alpha}P_{i,t}^{-\epsilon\alpha}-\rho\right]m_{i1,t}\Delta t+\left[z\left(\frac{\widehat{P}_{i2}^{C}(m_{i1,t},\phi_{i,t},\theta_t)}{P_{i,t}}\right)^{-\eta\alpha}P_{i,t}^{-\epsilon\alpha}-\rho\right](1-m_{i1,t})\Delta t$$
$$+g_\iota\Delta I_{i1,t}^{\iota}+g_d\Delta I_{i1,t}^{d}+g_\iota\Delta I_{i2,t}^{\iota}+g_d\Delta I_{i2,t}^{d}. \quad \text{(D.23)}$$

The industry's R&D characteristic $\phi_{i,t}$ and the long-run growth rate θ_t evolve according to the discrete Markov chain specified in Appendix D.3.

Recursive Formulation for The Deviation Value. The deviation value is obtained by assuming that firm j optimally sets its price conditional on firm k following the collusive pricing rule $\widehat{P}_{ik}^{C}(m_{i1,t},\phi_{i,t},\theta_t)$. We exploit the linearity property and solve firms' deviation values as a function of intrinsic market shares. Denote $\widehat{v}_{ij}^{D}(m_{i1,t},\phi_{i,t},\theta_t;\widehat{P}_{ij}^{C})$ as firm j's deviation value.

Firm 1 solves the following normalized problem:

$$\widehat{v}_{i1}^{D}(m_{i1,t},\phi_{i,t},\theta_t;\widehat{P}_{ij}^{C}) = \max_{P_{i1,t}}(P_{i1,t}-\omega)\left(\frac{P_{i1,t}}{P_{i,t}}\right)^{-\eta}P_{i,t}^{-\epsilon}m_{ij,t}\Delta t$$
$$+\mathbb{E}_t\left[\frac{\Lambda_{t+\Delta t}}{\Lambda_t}\frac{M_{i,t+\Delta t}}{M_{i,t}}(1+\theta_t\Delta t+\sigma\Delta Z_{c,t})\left[(1-\varsigma\Delta t)\widehat{v}_{i1}^{D}(m_{i1,t+\Delta t},\phi_{i,t+\Delta t},\theta_{t+\Delta t};\widehat{P}_{ij}^{C})+\varsigma\Delta t v_{i1}^{N}(m_{i1,t+\Delta t},\phi_{i,t+\Delta t},\theta_{t+\Delta t})\right]\right], \quad \text{(D.24)}$$

subject to the evolution of state variables, including:

The evolution of firm 1's intrinsic market share is

$$m_{i1,t+\Delta t}\frac{M_{i,t+\Delta t}}{M_{i,t}} = m_{i1,t} + \left[z\left(\frac{P_{i1,t}}{P_{i,t}}\right)^{-\eta\alpha} P_{i,t}^{-\epsilon\alpha} - \rho\right] m_{i1,t}\Delta t + (1-m_{i1,t})(\tau_\iota \Delta I_{i1,t}^{\iota} + \tau_d \Delta I_{i1,t}^{d}) + (g_\iota \Delta I_{i1,t}^{\iota} + g_d \Delta I_{i1,t}^{d})$$
$$- m_{i1,t}(\tau_\iota \Delta I_{i2,t}^{\iota} + \tau_d \Delta I_{i2,t}^{d}), \tag{D.25}$$

where the industry's price index is given by

$$P_{i,t} = \left[m_{i1,t}P_{i1,t}^{1-\eta} + (1-m_{i1,t})\widehat{P}_{i2}^{C}(m_{i1,t},\phi_{i,t},\theta_t)^{1-\eta}\right]^{\frac{1}{1-\eta}}. \tag{D.26}$$

The evolution of the industry's customer base is

$$\frac{M_{i,t+\Delta t}}{M_{i,t}} = 1 + \left[z\left(\frac{P_{i1,t}}{P_{i,t}}\right)^{-\eta\alpha} P_{i,t}^{-\epsilon\alpha} - \rho\right] m_{i1,t}\Delta t + \left[z\left(\frac{\widehat{P}_{i2}^{C}(m_{i1,t},\phi_{i,t},\theta_t)}{P_{i,t}}\right)^{-\eta\alpha} P_{i,t}^{-\epsilon\alpha} - \rho\right](1-m_{i1,t})\Delta t$$
$$+ g_\iota \Delta I_{i1,t}^{\iota} + g_d \Delta I_{i1,t}^{d} + g_\iota \Delta I_{i2,t}^{\iota} + g_d \Delta I_{i2,t}^{d}. \tag{D.27}$$

The industry's R&D characteristic $\phi_{i,t}$ and the long-run growth rate θ_t evolve according to the discrete Markov chain specified in Appendix D.3.

Incentive Compatibility Constraints and Optimal Collusive Prices. The collusive equilibrium is a sub-game perfect equilibrium if and only if the collusive prices $\widehat{P}_{ij}^{C}(m_{i1,t},\phi_{i,t},\theta_t)$ satisfy the following incentive compatibility constraints:

$$\widehat{v}_{ij}^{C}(m_{i1,t},\phi_{i,t},\theta_t;\widehat{P}_{ij}^{C}) \geq \widehat{v}_{ij}^{D}(m_{i1,t},\phi_{i,t},\theta_t;\widehat{P}_{ij}^{C}), \tag{D.28}$$

for all $m_{i1,t} \in [0,1]$, $\phi_{i,t} \in \Phi$, θ_t, and $j = 1,2$.

There exist infinitely many sub-game perfect collusive equilibrium. We focus on the collusive equilibrium with the highest collusive prices (denoted by $P_{ij}^{C}(m_{i1,t},\phi_{i,t},\theta_t)$), which are obtained when all incentive compatibility constraints are binding, i.e.

$$\widehat{v}_{ij}^{C}(m_{i1,t},\phi_{i,t},\theta_t;P_{ij}^{C}) = \widehat{v}_{ij}^{D}(m_{i1,t},\phi_{i,t},\theta_t;P_{ij}^{C}), \tag{D.29}$$

for all $m_{i1,t} \in [0,1]$, $\phi_{i,t} \in \Phi$, θ_t, and $j = 1,2$. We denote $v_{ij}^{C}(m_{i1,t},\phi_{i,t},\theta_t)$ as firm j's value in the collusive equilibrium with collusive prices $P_{ij}^{C}(m_{i1,t},\phi_{i,t},\theta_t)$, thus by definition

$$v_{ij}^{C}(m_{i1,t},\phi_{i,t},\theta_t) = \widehat{v}_{ij}^{C}(m_{i1,t},\phi_{i,t},\theta_t;P_{ij}^{C}). \tag{D.30}$$

D.2 The Extended Model

In this section, we present firm 1's problem in the extended model. The main difference between our baseline model and the extended model is that firms face equity financing frictions and endogenously hoard cash on their balance sheets. The decision on equity issuance and payout is also strategic between the two firms within the same industry. Thus, the equilibrium issuance/payout decisions have to be solved together with equilibrium prices as a high-dimensional fixed-point problem. We first illustrate the non-collusive equilibrium and then we illustrate the collusive equilibrium of the extended model.

D.2.1 Non-Collusive Equilibrium

Below, we present the recursive formulation for the firm's value in the non-collusive equilibrium. Then we exploit linearity to simplify the problem and present the recursive formulation for the normalized firm value. Finally, we present the conditions that determine the non-collusive (Nash) equilibrium.

Recursive Formulation for The Non-Collusive Firm Value. The industry's state is characterized by seven state variables, firm 1's cash holdings $W_{i1,t}$, firm 2's cash holdings $W_{i2,t}$, firm 1's customer base $M_{i1,t}$, firm 2's customer base $M_{i2,t}$, the industry's R&D characteristic $\phi_{i,t}$, the aggregate consumption C_t, and the long-run growth rate θ_t. Denote the value functions in the non-collusive equilibrium as $V_{ij}^N(W_{i1,t}, W_{i2,t}, M_{i1,t}, M_{i2,t}, \phi_{i,t}, C_t, \theta_t)$ for $j = 1, 2$.

To characterize the equilibrium value functions, it is more convenient to introduce two off-equilibrium value functions. Let $\widehat{V}_{ij}^N(W_{i1,t}, W_{i2,t}, M_{i1,t}, M_{i2,t}, \phi_{i,t}, C_t, \theta_t; P_{ik,t}, \Delta D_{ik,t}, q_{ik,t})$ be firm $j(= 1, 2)$'s value when its peer firm k's price, dividend, and investment policy are set at any (off-equilibrium) values $P_{ik,t}$, $\Delta D_{ik,t}$, and $q_{ik,t}$.

Firm 1 solves the following problem:

$$\widehat{V}_{i1}^N(W_{i1,t}, W_{i2,t}, M_{i1,t}, M_{i2,t}, \phi_{i,t}, C_t, \theta_t; P_{i2,t}, \Delta D_{i2,t}, q_{i2,t}) = \max_{P_{i1,t}, \Delta D_{i1,t}, q_{i1,t}} \Delta D_{i1,t} - \varphi_D M_{i1,t} C_t \mathbb{1}_{\Delta D_{i1,t} < 0}$$
$$+ \mathbb{E}_t \left[\frac{\Lambda_{t+\Delta t}}{\Lambda_t} V_{i1}^N(W_{i1,t+\Delta t}, W_{i2,t+\Delta t}, M_{i1,t+\Delta t}, M_{i2,t+\Delta t}, \phi_{i,t+\Delta t}, C_{t+\Delta t}, \theta_{t+\Delta t}) \right], \quad \text{(D.31)}$$

subject to the evolution of state variables, including:

The evolution of cash holdings is

$$W_{ij,t+\Delta t} = (1 + r\Delta t - \rho_w \Delta t) W_{ij,t} + (P_{ij,t} - \omega) \left(\frac{P_{ij,t}}{P_{i,t}} \right)^{-\eta} P_{i,t}^{-\epsilon} C_t M_{ij,t} \Delta t + \sigma_w M_{ij,t} C_t \Delta Z_{ij,t}$$
$$- (1 + \xi q_{ij,t}) q_{ij,t} M_{ij,t} \Delta t - \Delta D_{ij,t} \qquad \text{for } j = 1, 2. \quad \text{(D.32)}$$

The evolution of the customer base is

$$M_{ij,t+\Delta t} = (1 + q_{ij,t}\Delta t + q_{ik,t}\Delta t) M_{ij,t} + \left[z \left(\frac{C_{ij,t}}{C_t} \right)^{\alpha} M_{ij,t}^{1-\alpha} - \rho M_{ij,t} \right] \Delta t + M_{ik,t}(\tau_\iota \Delta I_{ij,t}^\iota + \tau_d \Delta I_{ij,t}^d)$$
$$+ M_{i,t}(g_\iota \Delta I_{ij,t}^\iota + g_d \Delta I_{ij,t}^d) - M_{ij,t}(\tau_\iota \Delta I_{ik,t}^\iota + \tau_d \Delta I_{ik,t}^d), \qquad \text{for } j = 1, 2 \quad \text{(D.33)}$$

where firm-level demand is given by

$$C_{ij,t} = \left(\frac{P_{ij,t}}{P_{i,t}}\right)^{-\eta} P_{i,t}^{-\epsilon} C_t M_{ij,t}, \qquad \text{for } j = 1,2 \tag{D.34}$$

and the industry's price index is given by

$$P_{i,t} = \left(\frac{M_{i1,t}}{M_{i,t}} P_{i1,t}^{1-\eta} + \frac{M_{i2,t}}{M_{i,t}} P_{i2,t}^{1-\eta}\right)^{\frac{1}{1-\eta}}. \tag{D.35}$$

The evolution of aggregate consumption C_t is given by equation (D.5). The industry's R&D characteristic $\phi_{i,t}$ and the long-run growth rate θ_t evolve according to the discrete Markov chain specified in Appendix D.3.

Recursive Formulation for The (Normalized) Non-Collusive Firm Value. Exploiting the linearity, we normalize the firm's value by $M_{i,t}C_t$. Firm j's cash ratio is defined as $w_{ij,t} = W_{ij,t}/(M_{i,t}C_t)$ $(j = 1,2)$. Firm 1's intrinsic market share is $m_{i1,t} = M_{i1,t}/M_{i,t}$; firm 2's intrinsic market share is $m_{i2,t} = M_{i2,t}/M_{i,t} = 1 - m_{i1,t}$. The normalized dividend policy is defined as $\Delta d_{ij,t} = \Delta D_{ij,t}/(M_{i,t}C_t)$ $(j = 1,2)$.

Define

$$v_{ij}^N(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t) = \frac{V_{ij}^N(W_{i1,t}, W_{i2,t}, M_{i1,t}, M_{i2,t}, \phi_{i,t}, C_t, \theta_t)}{M_{i,t}C_t} \tag{D.36}$$

$$\widehat{v}_{ij}^N(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; P_{ik,t}, \Delta d_{ik,t}, q_{ik,t}) = \frac{\widehat{V}_{ij}^N(W_{i1,t}, W_{i2,t}, M_{i1,t}, M_{i2,t}, \phi_{i,t}, C_t, \theta_t; P_{ik,t}, \Delta D_{ik,t}, q_{ik,t})}{M_{i,t}C_t} \tag{D.37}$$

Firm 1 solves the following problem:

$$\begin{aligned}\widehat{v}_{i1}^N(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; P_{i2,t}, \Delta d_{i2,t}, q_{i2,t}) = &\max_{P_{i1,t}, \Delta d_{i1,t}, q_{i1,t}} \Delta d_{i1,t} - \varphi_D m_{i1,t} \mathbb{1}_{\Delta d_{i1,t}<0} \\ + \mathbb{E}_t \left[\frac{\Lambda_{t+\Delta t}}{\Lambda_t} \frac{M_{i,t+\Delta t}}{M_{i,t}} (1 + \theta_t \Delta t + \sigma \Delta Z_{c,t}) v_{i1}^N(w_{i1,t+\Delta t}, w_{i2,t+\Delta t}, m_{i1,t+\Delta t}, \phi_{i,t+\Delta t}, \theta_{t+\Delta t})\right],\end{aligned} \tag{D.38}$$

subject to the evolution of state variables, including:

The evolution of cash holdings is

$$\begin{aligned}\frac{M_{i,t+\Delta t}}{M_{i,t}}(1 + \theta_t \Delta t + \sigma \Delta Z_{c,t}) w_{ij,t+\Delta t} = &(1 + r\Delta t - \rho_w \Delta t) w_{ij,t} - (1 + \xi q_{ij,t}) q_{ij,t} \Delta t - \Delta d_{ij,t} \\ &+ \left(P_{ij,t} - \omega\right) \left(\frac{P_{ij,t}}{P_{i,t}}\right)^{-\eta} P_{i,t}^{-\epsilon} m_{ij,t} \Delta t + \sigma_w m_{ij,t} \Delta Z_{ij,t} \qquad \text{for } j = 1,2.\end{aligned} \tag{D.39}$$

The evolution of firm 1's intrinsic market share $m_{i1,t}$ is

$$m_{i1,t+\Delta t}\frac{M_{i,t+\Delta t}}{M_{i,t}} =(1+q_{ij,t}\Delta t+q_{ik,t}\Delta t)m_{i1,t}+\left[z\left(\frac{P_{i1,t}}{P_{i,t}}\right)^{-\eta\alpha}P_{i,t}^{-\epsilon\alpha}-\rho\right]m_{i1,t}\Delta t+(1-m_{i1,t})(\tau_\iota\Delta I^\iota_{i1,t}+\tau_d\Delta I^d_{i1,t})$$
$$+(g_\iota\Delta I^\iota_{i1,t}+g_d\Delta I^d_{i1,t})-m_{i1,t}(\tau_\iota\Delta I^\iota_{i2,t}+\tau_d\Delta I^d_{i2,t}), \tag{D.40}$$

where the industry's price index is given by

$$P_{i,t}=\left[m_{i1,t}P_{i1,t}^{1-\eta}+(1-m_{i1,t})P_{i2,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}. \tag{D.41}$$

The evolution of the industry's customer base is

$$\frac{M_{i,t+\Delta t}}{M_{i,t}} =1+q_{ij,t}\Delta t+q_{ik,t}\Delta t+\left[z\left(\frac{P_{i1,t}}{P_{i,t}}\right)^{-\eta\alpha}P_{i,t}^{-\epsilon\alpha}-\rho\right]m_{i1,t}\Delta t+\left[z\left(\frac{P_{i2,t}}{P_{i,t}}\right)^{-\eta\alpha}P_{i,t}^{-\epsilon\alpha}-\rho\right](1-m_{i1,t})\Delta t$$
$$+g_\iota\Delta I^\iota_{i1,t}+g_d\Delta I^d_{i1,t}+g_\iota\Delta I^\iota_{i2,t}+g_d\Delta I^d_{i2,t}. \tag{D.42}$$

The industry's R&D characteristic $\phi_{i,t}$ and the long-run growth rate θ_t evolve according to the discrete Markov chain specified in Appendix D.3.

Non-Collusive (Nash) Equilibrium. Denote the equilibrium price functions as $P^N_{ij}(w_{i1,t},w_{i2,t},m_{i1,t},\phi_{i,t},\theta_t)$, dividend policies as $\Delta d^N_{ij}(w_{i1,t},w_{i2,t},m_{i1,t},\phi_{i,t},\theta_t)$, and investment policies as $q^N_{ij}(w_{i1,t},w_{i2,t},m_{i1,t},\phi_{i,t},\theta_t)$. Denote the off-equilibrium price functions as $\widehat{P}^N_{ij}(w_{i1,t},w_{i2,t},m_{i1,t},\phi_{i,t},\theta_t;P_{ik,t},\Delta d_{ik,t},q_{ik,t})$, dividend policies as $\Delta\widehat{d}^N_{ij}(w_{i1,t},w_{i2,t},m_{i1,t},\phi_{i,t},\theta_t;P_{ik,t},\Delta d_{ik,t},q_{ik,t})$, and investment policies as $\widehat{q}^N_{ij}(w_{i1,t},w_{i2,t},m_{i1,t},\phi_{i,t},\theta_t;P_{ik,t},\Delta d_{ik,t},q_{ik,t})$. Exploiting the symmetry between firm 1 and firm 2, we can obtain firm 2's off-equilibrium value and policy functions as

$$\widehat{v}^N_{i1}(w_{i1,t},w_{i2,t},m_{i1,t},\phi_{i,t},\theta_t;P_{i2,t},\Delta d_{i2,t},q_{i2,t}) =\widehat{v}^N_{i2}(w_{i2,t},w_{i1,t},1-m_{i1,t},\phi_{i,t},\theta_t;P_{i1,t},\Delta d_{i1,t},q_{i1,t}), \tag{D.43}$$
$$\widehat{P}^N_{i1}(w_{i1,t},w_{i2,t},m_{i1,t},\phi_{i,t},\theta_t;P_{i2,t},\Delta d_{i2,t},q_{i2,t}) =\widehat{P}^N_{i2}(w_{i2,t},w_{i1,t},1-m_{i1,t},\phi_{i,t},\theta_t;P_{i1,t},\Delta d_{i1,t},q_{i1,t}), \tag{D.44}$$
$$\Delta\widehat{d}^N_{i1}(w_{i1,t},w_{i2,t},m_{i1,t},\phi_{i,t},\theta_t;P_{i2,t},\Delta d_{i2,t},q_{i2,t}) =\Delta\widehat{d}^N_{i2}(w_{i2,t},w_{i1,t},1-m_{i1,t},\phi_{i,t},\theta_t;P_{i1,t},\Delta d_{i1,t},q_{i1,t}), \tag{D.45}$$
$$\widehat{q}^N_{i1}(w_{i1,t},w_{i2,t},m_{i1,t},\phi_{i,t},\theta_t;P_{i2,t},\Delta d_{i2,t},q_{i2,t}) =\widehat{q}^N_{i2}(w_{i2,t},w_{i1,t},1-m_{i1,t},\phi_{i,t},\theta_t;P_{i1,t},\Delta d_{i1,t},q_{i1,t}). \tag{D.46}$$

Given $j=1,2$'s price P_{ij}, dividend Δd_{ij}, and investment q_{ij}, firm k optimally sets the price P_{ik}, dividend Δd_{ik}, and investment q_{ik}. The non-collusive (Nash) equilibrium is derived from the fixed point—each firm's price, dividend, and investment are optimal given the other firm's optimal price, dividend, and investment:

$$P_{i1}^N(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t) = \widehat{P}_{i1}^N(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; P_{i2}^N, \Delta d_{i2}^N, q_{i2}^N), \tag{D.47}$$
$$P_{i2}^N(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t) = \widehat{P}_{i2}^N(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; P_{i1}^N, \Delta d_{i1}^N, q_{i1}^N), \tag{D.48}$$
$$\Delta d_{i1}^N(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t) = \Delta\widehat{d}_{i1}^N(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; P_{i2}^N, \Delta d_{i2}^N, q_{i2}^N), \tag{D.49}$$
$$\Delta d_{i2}^N(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t) = \Delta\widehat{d}_{i2}^N(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; P_{i1}^N, \Delta d_{i1}^N, q_{i1}^N), \tag{D.50}$$
$$q_{i1}^N(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t) = \widehat{q}_{i1}^N(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; P_{i2}^N, \Delta d_{i2}^N, q_{i2}^N), \tag{D.51}$$
$$q_{i2}^N(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t) = \widehat{q}_{i2}^N(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; P_{i1}^N, \Delta d_{i1}^N, q_{i1}^N). \tag{D.52}$$

The equilibrium value functions are given by

$$v_{i1}^N(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t) = \widehat{v}_{i1}^N(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; P_{i2}^N, \Delta d_{i2}^N, q_{i2}^N), \tag{D.53}$$
$$v_{i2}^N(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t) = \widehat{v}_{i2}^N(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; P_{i1}^N, \Delta d_{i1}^N, q_{i1}^N). \tag{D.54}$$

After solving the equilibrium value and policy functions above, we can verify that the following conditions are satisfied due to symmetry,

$$v_{i1}^N(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t) = v_{i2}^N(w_{i2,t}, w_{i1,t}, 1 - m_{i1,t}, \phi_{i,t}, \theta_t), \tag{D.55}$$
$$P_{i1}^N(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t) = P_{i2}^N(w_{i2,t}, w_{i1,t}, 1 - m_{i1,t}, \phi_{i,t}, \theta_t), \tag{D.56}$$
$$\Delta d_{i1}^N(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t) = \Delta d_{i2}^N(w_{i2,t}, w_{i1,t}, 1 - m_{i1,t}, \phi_{i,t}, \theta_t), \tag{D.57}$$
$$q_{i1}^N(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t) = q_{i2}^N(w_{i2,t}, w_{i1,t}, 1 - m_{i1,t}, \phi_{i,t}, \theta_t). \tag{D.58}$$

D.2.2 Collusive Equilibrium

Below, we present the recursive formulation for the firm's value in the collusive equilibrium. Then we present the recursive formulation for the firm's value when the firm deviates from the collusive equilibrium. Finally, we present the incentive compatibility constraints and the conditions that determine the optimal collusive prices.

Recursive Formulation for The Collusive Firm Value. In the collusive equilibrium, we can still exploit the linearity property and solve firms' values as a function of cash ratios and intrinsic market shares. Specifically, denote $\widehat{v}_{ij}^C(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; \widehat{P}_{ij}^C)$ as firm j's value in the collusive equilibrium with collusive prices $\widehat{P}_{ij}^C(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t)$. Note that because the two firms in the same industry are symmetric, the collusive prices satisfy $\widehat{P}_{i1}^C(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t) = \widehat{P}_{i2}^C(w_{i2,t}, w_{i1,t}, 1 - m_{i1,t}, \phi_{i,t}, \theta_t)$.

Although firms collude on prices, the dividend policies are still discretionary and strategic. To characterize the equilibrium value functions, it is more convenient to introduce two off-equilibrium value functions. Let $\overline{v}_{ij}^C(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; \widehat{P}_{ij}^C, \Delta d_{ik}, q_{ik})$ be firm $j(= 1, 2)$'s value with collusive prices $\widehat{P}_{ij}^C(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t)$ when its peer firm k's dividend and investment are set at any (off-equilibrium) values $\Delta d_{ik,t}$ and $q_{ik,t}$.

Firm 1 solves the following normalized problem:

$$\overline{v}_{i1}^{C}(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; \widehat{P}_{ij}^{C}, \Delta d_{i2,t}, q_{i2,t}) = \max_{\Delta d_{i1,t}, q_{i1,t}} \Delta d_{i1,t} - \varphi_D m_{i1,t} \mathbb{1}_{\Delta d_{i1,t}<0}$$
$$+ \mathbb{E}_t \left[\frac{\Lambda_{t+\Delta t}}{\Lambda_t} \frac{M_{i,t+\Delta t}}{M_{i,t}} (1 + \theta_t \Delta t + \sigma \Delta Z_{c,t}) \widehat{v}_{i1}^{C}(w_{i1,t+\Delta t}, w_{i2,t+\Delta t}, m_{i1,t+\Delta t}, \phi_{i,t+\Delta t}, \theta_{t+\Delta t}; \widehat{P}_{ij}^{C}) \right], \tag{D.59}$$

subject to the evolution of state variables, including:

The evolution of cash ratios is

$$\frac{M_{i,t+\Delta t}}{M_{i,t}} (1 + \theta_t \Delta t + \sigma \Delta Z_{c,t}) w_{ij,t+\Delta t} = (1 + r\Delta t - \rho_w \Delta t) w_{ij,t} - (1 + \xi q_{ij,t}) q_{ij,t} \Delta t - \Delta d_{ij,t}$$
$$+ \left(\widehat{P}_{ij}^{C}(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t) - \omega \right) \left(\frac{\widehat{P}_{ij}^{C}(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t)}{P_{i,t}} \right)^{-\eta} P_{i,t}^{-\epsilon} m_{ij,t} \Delta t + \sigma_w m_{ij,t} \Delta Z_{ij,t} \qquad \text{for } j = 1, 2. \tag{D.60}$$

The evolution of firm 1's intrinsic market share $m_{i1,t}$ is

$$m_{i1,t+\Delta t} \frac{M_{i,t+\Delta t}}{M_{i,t}} = (1 + q_{i1,t} \Delta t + q_{i2,t} \Delta t) m_{i1,t} + \left[z \left(\frac{\widehat{P}_{i1}^{C}(m_{i1,t}, \phi_{i,t}, \theta_t)}{P_{i,t}} \right)^{-\eta\alpha} P_{i,t}^{-\epsilon\alpha} - \rho \right] m_{i1,t} \Delta t$$
$$+ (1 - m_{i1,t})(\tau_\iota \Delta I_{i1,t}^{\iota} + \tau_d \Delta I_{i1,t}^{d}) + (g_\iota \Delta I_{i1,t}^{\iota} + g_d \Delta I_{i1,t}^{d}) - m_{i1,t}(\tau_\iota \Delta I_{i2,t}^{\iota} + \tau_d \Delta I_{i2,t}^{d}), \tag{D.61}$$

where the industry's price index is given by

$$P_{i,t} = \left[m_{i1,t} \widehat{P}_{i1}^{C}(m_{i1,t}, \phi_{i,t}, \theta_t)^{1-\eta} + (1 - m_{i1,t}) \widehat{P}_{i2}^{C}(m_{i1,t}, \phi_{i,t}, \theta_t)^{1-\eta} \right]^{\frac{1}{1-\eta}}. \tag{D.62}$$

The evolution of the industry's customer base is

$$\frac{M_{i,t+\Delta t}}{M_{i,t}} = 1 + \left[z \left(\frac{\widehat{P}_{i1}^{C}(m_{i1,t}, \phi_{i,t}, \theta_t)}{P_{i,t}} \right)^{-\eta\alpha} P_{i,t}^{-\epsilon\alpha} - \rho \right] m_{i1,t} \Delta t + \left[z \left(\frac{\widehat{P}_{i2}^{C}(m_{i1,t}, \phi_{i,t}, \theta_t)}{P_{i,t}} \right)^{-\eta\alpha} P_{i,t}^{-\epsilon\alpha} - \rho \right] (1 - m_{i1,t}) \Delta t$$
$$+ q_{i1,t} \Delta t + q_{i2,t} \Delta t + g_\iota \Delta I_{i1,t}^{\iota} + g_d \Delta I_{i1,t}^{d} + g_\iota \Delta I_{i2,t}^{\iota} + g_d \Delta I_{i2,t}^{d}. \tag{D.63}$$

The industry's R&D characteristic $\phi_{i,t}$ and the long-run growth rate θ_t evolve according to the discrete Markov chain specified in Appendix D.3.

Next, we characterize dividend and investment policies in collusive equilibrium. Denote the equilibrium dividend policies as $\Delta \widehat{d}_{ij}^{C}(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; \widehat{P}_{ij}^{C})$ and investment policies as $\widehat{q}_{ij}^{C}(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; \widehat{P}_{ij}^{C})$. Denote the off-equilibrium dividend policies as $\Delta \overline{d}_{ij}^{C}(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; \widehat{P}_{ij}^{C}, \Delta d_{ik,t}, q_{ik,t})$ and investment policies as $\overline{q}_{ij}^{C}(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; \widehat{P}_{ij}^{C}, \Delta d_{ik,t}, q_{ik,t})$. Exploiting the symmetry between firm 1 and firm 2, we can obtain firm 2's off-equilibrium value and policy functions as

$$\overline{v}_{i1}^C(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; \widehat{P}_{ij}^C, \Delta d_{i2,t}, q_{i2,t}) = \overline{v}_{i2}^C(w_{i2,t}, w_{i1,t}, 1-m_{i1,t}, \phi_{i,t}, \theta_t; \widehat{P}_{ij}^C, \Delta d_{i1,t}, q_{i1,t}), \tag{D.64}$$

$$\Delta\overline{d}_{i1}^C(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; \widehat{P}_{ij}^C, \Delta d_{i2,t}, q_{i2,t}) = \Delta\overline{d}_{i2}^C(w_{i2,t}, w_{i1,t}, 1-m_{i1,t}, \phi_{i,t}, \theta_t; \widehat{P}_{ij}^C, \Delta d_{i1,t}, q_{i1,t}), \tag{D.65}$$

$$\overline{q}_{i1}^C(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; \widehat{P}_{ij}^C, \Delta d_{i2,t}, q_{i2,t}) = \overline{q}_{i2}^C(w_{i2,t}, w_{i1,t}, 1-m_{i1,t}, \phi_{i,t}, \theta_t; \widehat{P}_{ij}^C, \Delta d_{i1,t}, q_{i1,t}). \tag{D.66}$$

Given $j = 1, 2$'s dividend Δd_{ij} and investment q_{ij}, firm k optimally sets its dividend Δd_{ik} and investment q_{ik}. The collusive-equilibrium dividend and investment policies are derived from the fixed points—each firm's dividend and investment policies are optimal given the other firm's dividend and investment policies:

$$\Delta\widehat{d}_{i1}^C(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; \widehat{P}_{ij}^C) = \Delta\overline{d}_{i1}^C(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; \widehat{P}_{ij}^C, \Delta\widehat{d}_{i2}^C, q_{i2}^C), \tag{D.67}$$

$$\Delta\widehat{d}_{i2}^C(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; \widehat{P}_{ij}^C) = \Delta\overline{d}_{i2}^C(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; \widehat{P}_{ij}^C, \Delta\widehat{d}_{i1}^C, q_{i1}^C), \tag{D.68}$$

$$\widehat{q}_{i1}^C(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; \widehat{P}_{ij}^C) = \overline{q}_{i1}^C(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; \widehat{P}_{ij}^C, \Delta\widehat{d}_{i2}^C, q_{i2}^C), \tag{D.69}$$

$$\widehat{q}_{i2}^C(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; \widehat{P}_{ij}^C) = \overline{q}_{i2}^C(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; \widehat{P}_{ij}^C, \Delta\widehat{d}_{i1}^C, q_{i1}^C). \tag{D.70}$$

The equilibrium value functions are given by

$$\widehat{v}_{i1}^C(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; \widehat{P}_{ij}^C) = \overline{v}_{i1}^C(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; \widehat{P}_{ij}^C, \Delta\widehat{d}_{i2}^C, \widehat{q}_{i2}^C), \tag{D.71}$$

$$\widehat{v}_{i2}^C(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; \widehat{P}_{ij}^C) = \overline{v}_{i2}^C(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; \widehat{P}_{ij}^C, \Delta\widehat{d}_{i1}^C, \widehat{q}_{i1}^C). \tag{D.72}$$

After solving the equilibrium value and policy functions above, we can verify that the following conditions are satisfied due to symmetry,

$$\widehat{v}_{i1}^C(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; \widehat{P}_{ij}^C) = \widehat{v}_{i2}^C(w_{i2,t}, w_{i1,t}, 1-m_{i1,t}, \phi_{i,t}, \theta_t; \widehat{P}_{ij}^C), \tag{D.73}$$

$$\Delta\widehat{d}_{i1}^C(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; \widehat{P}_{ij}^C) = \Delta\widehat{d}_{i2}^C(w_{i2,t}, w_{i1,t}, 1-m_{i1,t}, \phi_{i,t}, \theta_t; \widehat{P}_{ij}^C), \tag{D.74}$$

$$\widehat{q}_{i1}^C(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; \widehat{P}_{ij}^C) = \widehat{q}_{i2}^C(w_{i2,t}, w_{i1,t}, 1-m_{i1,t}, \phi_{i,t}, \theta_t; \widehat{P}_{ij}^C). \tag{D.75}$$

Recursive Formulation for The Deviation Value. The deviation value is obtained by assuming that firm j optimally sets its price, dividend, and investment policies conditional on firm k not knowing that deviation has happened. Thus, firm k remains setting price $\widehat{P}_{ik}^C(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t)$, dividend $\Delta\widehat{d}_{ik}^C(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; \widehat{P}_{ij}^C)$, and investment $\widehat{q}_{ik}^C(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; \widehat{P}_{ij}^C)$ as in the collusive equilibrium. We exploit the linearity property and solve firms' deviation values as a function of intrinsic market shares. Denote $\widehat{v}_{ij}^D(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; \widehat{P}_{ij}^C)$ as firm j's deviation value.

Firm 1 solves the following problem:

$$\widehat{v}_{i1}^D(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; \widehat{P}_{ij}^C) = \max_{P_{i1,t}, \Delta d_{i1,t}, q_{i1,t}} \Delta d_{i1,t} - \varphi_D m_{i1,t} \mathbb{1}_{\Delta d_{i1,t}<0} + \mathbb{E}_t\left[\frac{\Lambda_{t+\Delta t}}{\Lambda_t}\frac{M_{i,t+\Delta t}}{M_{i,t}}(1+\theta_t\Delta t + \sigma\Delta Z_{c,t}) \right.$$
$$\left. \times \left[(1-\varsigma\Delta t)\widehat{v}_{i1}^D(w_{i1,t+\Delta t}, w_{i2,t+\Delta t}, m_{i1,t+\Delta t}, \phi_{i,t+\Delta t}, \theta_{t+\Delta t}; \widehat{P}_{ij}^C) + \varsigma\Delta t v_{i1}^N(w_{i1,t+\Delta t}, w_{i2,t+\Delta t}, m_{i1,t+\Delta t}, \phi_{i,t+\Delta t}, \theta_{t+\Delta t})\right]\right], \tag{D.76}$$

subject to the evolution of state variables, including:

The evolution of firm 1's cash holdings is

$$\frac{M_{i,t+\Delta t}}{M_{i,t}}(1+\theta_t\Delta t+\sigma\Delta Z_{c,t})w_{i1,t+\Delta t} =(1+r\Delta t-\rho_w\Delta t)w_{i1,t}-(1+\xi q_{i1,t})q_{i1,t}\Delta t-\Delta d_{i1,t} + (P_{i1,t}-\omega)\left(\frac{P_{i1,t}}{P_{i,t}}\right)^{-\eta} P_{i,t}^{-\epsilon} m_{i1,t}\Delta t+\sigma_w m_{i1,t}\Delta Z_{i1,t}. \quad \text{(D.77)}$$

The evolution of firm 2's cash holdings is

$$\frac{M_{i,t+\Delta t}}{M_{i,t}}(1+\theta_t\Delta t+\sigma\Delta Z_{c,t})w_{i2,t+\Delta t} = (1+r\Delta t-\rho_w\Delta t)w_{i2,t}-\Delta\widehat{d}_{i2}^{C}(w_{i1,t},w_{i2,t},m_{i1,t},\phi_{i,t},\theta_t;\widehat{P}_{ij}^{C}) - [1+\xi\widehat{q}_{i2}^{C}(w_{i1,t},w_{i2,t},m_{i1,t},\phi_{i,t},\theta_t;\widehat{P}_{ij}^{C})]\widehat{q}_{i2}^{C}(w_{i1,t},w_{i2,t},m_{i1,t},\phi_{i,t},\theta_t;\widehat{P}_{ij}^{C})\Delta t + \left(\widehat{P}_{i2}^{C}(w_{i1,t},w_{i2,t},m_{i1,t},\phi_{i,t},\theta_t)-\omega\right)\left(\frac{\widehat{P}_{i2}^{C}(w_{i1,t},w_{i2,t},m_{i1,t},\phi_{i,t},\theta_t)}{P_{i,t}}\right)^{-\eta} P_{i,t}^{-\epsilon}(1-m_{i1,t})\Delta t+\sigma_w(1-m_{i1,t})\Delta Z_{i2,t}. \quad \text{(D.78)}$$

The evolution of firm 1's intrinsic market share is

$$m_{i1,t+\Delta t}\frac{M_{i,t+\Delta t}}{M_{i,t}} =[1+q_{i1,t}\Delta t+\widehat{q}_{i2}^{C}(w_{i1,t},w_{i2,t},m_{i1,t},\phi_{i,t},\theta_t;\widehat{P}_{ij}^{C})\Delta t]m_{i1,t}+\left[z\left(\frac{P_{i1,t}}{P_{i,t}}\right)^{-\eta\alpha}P_{i,t}^{-\epsilon\alpha}-\rho\right]m_{i1,t}\Delta t + (1-m_{i1,t})(\tau_\iota\Delta I_{i1,t}^{\iota}+\tau_d\Delta I_{i1,t}^{d})+(g_\iota\Delta I_{i1,t}^{\iota}+g_d\Delta I_{i1,t}^{d})-m_{i1,t}(\tau_\iota\Delta I_{i2,t}^{\iota}+\tau_d\Delta I_{i2,t}^{d}), \quad \text{(D.79)}$$

where the industry's price index is given by

$$P_{i,t} = \left[m_{i1,t}P_{i1,t}^{1-\eta}+(1-m_{i1,t})\widehat{P}_{i2}^{C}(m_{i1,t},\phi_{i,t},\theta_t)^{1-\eta}\right]^{\frac{1}{1-\eta}}. \quad \text{(D.80)}$$

The evolution of the industry's customer base is

$$\frac{M_{i,t+\Delta t}}{M_{i,t}} =1+q_{i1,t}\Delta t+\widehat{q}_{i2}^{C}(w_{i1,t},w_{i2,t},m_{i1,t},\phi_{i,t},\theta_t;\widehat{P}_{ij}^{C})\Delta t+g_\iota\Delta I_{i1,t}^{\iota}+g_d\Delta I_{i1,t}^{d}+g_\iota\Delta I_{i2,t}^{\iota}+g_d\Delta I_{i2,t}^{d}. + \left[z\left(\frac{P_{i1,t}}{P_{i,t}}\right)^{-\eta\alpha}P_{i,t}^{-\epsilon\alpha}-\rho\right]m_{i1,t}\Delta t+\left[z\left(\frac{\widehat{P}_{i2}^{C}(m_{i1,t},\phi_{i,t},\theta_t)}{P_{i,t}}\right)^{-\eta\alpha}P_{i,t}^{-\epsilon\alpha}-\rho\right](1-m_{i1,t})\Delta t. \quad \text{(D.81)}$$

The industry's R&D characteristic $\phi_{i,t}$ and the long-run growth rate θ_t evolve according to the discrete Markov chain specified in Appendix D.3.

Incentive Compatibility Constraints and Optimal Collusive Prices. The collusive equilibrium is a sub-game perfect equilibrium if and only if the collusive prices $\widehat{P}_{ij}^{C}(w_{i1,t},w_{i2,t},m_{i1,t},\phi_{i,t},\theta_t)$ satisfy the following incentive compatibility constraints:

$$\widehat{v}_{ij}^{C}(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; \widehat{P}_{ij}^{C}) \geq \widehat{v}_{ij}^{D}(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; \widehat{P}_{ij}^{C}), \tag{D.82}$$

for all $w_{i1,t}, w_{i2,t} > 0$, $m_{i1,t} \in [0,1]$, $\phi_{i,t} \in \Phi$, θ_t, and $j = 1, 2$.

There exist infinitely many sub-game perfect collusive equilibrium. We focus on the collusive equilibrium with the highest collusive prices (denoted by $P_{ij}^{C}(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t)$), which are obtained when all incentive compatibility constraints are binding, i.e.

$$\widehat{v}_{ij}^{C}(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; P_{ij}^{C}) = \widehat{v}_{ij}^{D}(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; P_{ij}^{C}), \tag{D.83}$$

for all $w_{i1,t}, w_{i2,t} > 0$, $m_{i1,t} \in [0,1]$, $\phi_{i,t} \in \Phi$, θ_t, and $j = 1, 2$. We denote $v_{ij}^{C}(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t)$ as firm j's value in the collusive equilibrium with collusive prices $P_{ij}^{C}(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t)$, thus by definition

$$v_{ij}^{C}(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t) = \widehat{v}_{ij}^{C}(w_{i1,t}, w_{i2,t}, m_{i1,t}, \phi_{i,t}, \theta_t; P_{ij}^{C}). \tag{D.84}$$

D.3 Discretization

We discretize the cash flow shocks $dZ_{ij,t}$ based on n grids spanning from -3σ to 3σ, the unpredictable consumption growth shocks $dZ_{c,t}$ based on n_c grids spanning from $-3\sigma_c$ and $3\sigma_c$ using the method of Tauchen (1986). We use the method of Rouwenhorst (1995) to approximate the persistent AR(1) process of long-run risks θ_t using n_θ discrete states. The time line is discretized into intervals with length Δt.

We use collocation methods to solve each firm's problem. Let $S_m \times S_\phi \times S_\theta$ be the grid of collocation nodes for a firm's equilibrium value in the baseline model, $S_m \times S_\phi \times S_\theta \times S_p$ be the grid of collocation notes for a firm's off-equilibrium value in the baseline model, $S_w \times S_w \times S_m \times S_\phi \times S_\theta$ be the grid of collocation nodes for a firm's equilibrium value in the extended model, and $S_w \times S_w \times S_m \times S_\phi \times S_\theta \times S_p \times S_d \times S_q$ be the grid of collocation nodes for a firm's off-equilibrium value in the extended model. We have $S_w = \{w_1, w_2, ..., w_{n_w}\}$, $S_m = \{m_1, m_2, ..., m_{n_m}\}$, $S_\phi = \{\phi_1, \phi_2, ..., \phi_{n_\phi}\}$, $S_\theta = \{\theta_1, \theta_2, ..., \theta_{n_\theta}\}$, $S_p = \{p_1, p_2, ..., p_{n_p}\}$, $S_d = \{d_1, d_2, ..., d_{n_d}\}$, and $S_q = \{q_1, q_2, ..., q_{n_q}\}$.

We approximate the firm's value function $v(\cdot)$ on the grid of collocation notes using a linear spline with coefficients corresponding to each grid point. We first form a guess for the spline's coefficients, then we iterate to obtain a vector that solves the system of Bellman equations. For our extended model, because there are three continuous endogenous states, $S_w \times S_w \times S_m$, we use the piecewise multilinear interpolation method of Weiser and Zarantonello (1988) to obtain interpolant in a 3-dimensional space.

D.4 Implementation

The numerical algorithms are implemented using C++. The program is run on the server of MIT Economics Department, supply.mit.edu and demand.mit.edu, which are built on Dell PowerEdge R910 (64 cores, Intel(R) Xeon(R) CPU E7-4870, 2.4GHz) and Dell PowerEdge R920 (48 cores, Intel(R) 4 Xeon E7-8857 v2 CPUs). We use OpenMP for parallelization when iterating value functions and simulating the model.

Selection of Grids For our baseline model, we set $n_c = 11$, $n_\theta = 21$, $n_m = 21$, $\Delta t = 1/24$, $n_\phi = 11$, $n_p = 11$. The grids for long-run risks S_θ is given by the method of Rouwenhorst (1995). The grids for unpredictable consumption growth shocks S_c is given by the method of Tauchen (1986). The grid of intrinsic market share S_m is discretized into 21 nodes from $1e-7$ to $1-1e-7$ with equal spaces. We do not set S_m from 0 to 1 to avoid the indeterminacy of optimal price with $m = 0$. The time interval Δt is set to be 1/24. A higher Δt implies faster convergence for the same number of iterations but lower accuracy. We checked that the solution is accurate enough for $\Delta t = 1/24$, further reducing Δt would not change the accuracy much. With 1/24, 5000 times iterations allow us to achieve convergence in value functions. The industry characteristic grid is discretized into 11 notes from 0 to 1 with equal spaces. The price grids is discretized into 11 nodes from 1 to 2 with equal spaces. The upper bound is chosen according to $\epsilon/(\epsilon-1) \times \omega = 2$, which is the highest price a firm will ever set.

For our extended model, we set $n_c = 3$, $n_\theta = 3$, $n_m = 21$, $\Delta t = 1/365$, $n_\phi = 11$, $n_p = 11$, $n = 5$, $n_w = 26$, $n_d = 3$, $n_q = 3$. The extended model is much more difficult to compute due to endogenous cash holdings. Thus, to ensure we can solve the model within a reasonable amount of time, we reduce the number of nodes for unpredictable consumption growth shocks to 3, and reduce the number of nodes for long-run risks to 3. To ensure accuracy, we keep the number of nodes for intrinsic market shares unchanged at 21, and the number of price nodes unchanged at 11. We use $n_w = 26$ equi-spaced nodes from 0 to 0.5 to construct S_w, which is dense enough to capture the nonlinearity in marginal value of cash. We set use $n = 5$ to approximate cash flow shocks. We have tested that further reducing n would not be able to generate sufficient cash flow risks due to discretization error, although it increases the speed of computation. Note that with endogenous cash holdings, we cannot set $\Delta = 1/24$ because the discretized cash flow shock with relatively larger Δ could exceed the upper bound of our cash holding grids. Thus, there is a tradeoff. If we use more grids for S_w to have higher upper bound, we can choose relatively larger Δ to save iteration time. However, each iteration will take longer. Given our grids $n_w = 26$ and upper bound of w equal to 0.5, we have to choose $\Delta = 1/365$ in order to maintain reasonable accuracy. With $\Delta = 1/365$, 20000 iterations are needed for value function convergence.

Calculating Iterations and Searching For the Nash Equilibrium. Given the value functions from the previous iteration, we use the golden section search method to find the equilibrium prices. The computational complexity of this algorithm is at the order of $\log(n)$, much faster and more accurate than a simple grid search.

Searching for the equilibrium markup is very difficult because we have to solve a fixed-point problem (D.47-D.54) that involves both firms' simultaneous prices and net payout decisions. Our solution technique is to iteratively solve the following three steps.

First, given $v_{i1}^N(w_{i1}, w_{i2}, m_{i1}, \phi_i, \theta)$, we solve for the off-equilibrium value $\widehat{v}_{i1}^N(w_{i1}, w_{i2}, m_{i1}, \phi_i, \theta; P_{i2}, \Delta d_{i2}, q_{i2})$, the off-equilibrium policy functions $\widehat{P}_{i1}^N(w_{i1}, w_{i2}, m_{i1}, \phi_i, \theta; P_{i2}, \Delta d_{i2}, q_{i2})$, $\Delta\widehat{d}_{i1}^N(w_{i1}, w_{i2}, m_{i1}, \phi_i, \theta; P_{i2}, \Delta d_{i2}, q_{i2})$, $\widehat{q}_{i1}^N(w_{i1}, w_{i2}, m_{i1}, \phi_i, \theta; P_{i2}, \Delta d_{i2}, q_{i2})$. Exploiting symmetry, we obtain $\widehat{v}_{i2}^N(w_{i1}, w_{i2}, m_{i1}, \phi_i, \theta; P_{i1}, \Delta d_{i1}, q_{i1})$, and $\widehat{P}_{i2}^N(w_{i1}, w_{i2}, m_{i1}, \phi_i, \theta; P_{i1}, \Delta d_{i1}, q_{i1})$, $\Delta\widehat{d}_{i2}^N(w_{i1}, w_{i2}, m_{i1}, \phi_i, \theta; P_{i1}, \Delta d_{i1}, q_{i1})$, $\widehat{q}_{i2}^N(w_{i1}, w_{i2}, m_{i1}, \phi_i, \theta; P_{i1}, \Delta d_{i1}, q_{i1})$.

Second, for each $(w_{i1}, w_{i2}, m_{i1}, \phi_i, \theta) \in S_w \times S_w \times S_m \times S_\phi \times S_\theta$, we use a nonlinear solver knitro to solve equations (D.47-D.52) and obtain the equilibrium prices $P_{i1}^N(w_{i1}, w_{i2}, m_{i1}, \phi_i, \theta)$, $P_{i1}^N(w_{i2}, w_{i2}, m_{i1}, \phi_i, \theta)$, the equilibrium net payout $\mathrm{d}d_{i1}^N(w_{i1}, w_{i2}, m_{i1}, \phi_i, \theta)$, $\mathrm{d}d_{i2}^N(w_{i1}, w_{i2}, m_{i1}, \phi_i, \theta)$, and the equilibrium investment $q_{i1}^N(w_{i1}, w_{i2}, m_{i1}, \phi_i, \theta)$, $q_{i2}^N(w_{i1}, w_{i2}, m_{i1}, \phi_i, \theta)$. Third, we solve equations (D.53-D.54) and obtain equilibrium value functions $v_{i1}^N(w_{i1}, w_{i2}, m_{i1}, \phi_i, \theta)$, and $v_{i2}^N(w_{i1}, w_{i2}, m_{i1}, \phi_i, \theta)$.

Searching for the optimal net payout is also difficult because financing and payout only happen at the boundaries when time is continuous. There are two complications when solving this continuous-time model in discrete time. First, the firm starts to issue equity in advance before the cash ratio hits the zero lower bound. Second, the discretized cash flow shocks may drive the next-period cash ratio to a negative number even when the firm's current-period cash ratio is strictly positive. We deal with the first problem by choosing a fine time grid, i.e. $\Delta = 1/365$. This ensures a reasonably good approximation as the firm issues equity when cash ratio drops below 0.02, the smallest cash ratio grid we consider. To deal with the second issue, we impose a sufficiently large penalty on the firm's value (in the code, we subtract the firm's value by 10) whenever the next-period cash ratio drops below zero. This is to ensure that the firm would issue equity whenever there is a chance to have a negative cash ratio in the next period given our discretization of cash flow shocks. The large penalty is important to guarantee the convergence of the value functions. If the penalty is not large enough, then the firm may wait for a bit longer before issuing equity, and as a result, the optimal financing boundary is not correctly solved at this level of discretization.

Searching For Collusive Prices. We modify the golden section search method to find the highest collusive prices $P^C(w_{i1}, w_{i2}, m_{i1}, \phi_i, \theta)$ by iterations. Within each iteration, we solve firms' collusion value and deviation value using standard recursive methods given $\widehat{P}_{ij}^C(w_{i1}, w_{i2}, m_{i1}, \phi_i, \theta)$.

There are two key differences between our method and a standard golden section search method. First, we guess and update the collusive pricing schedule $\widehat{P}_{ij}^C(w_{i1}, w_{i2}, m_{i1}, \phi_i, \theta)$ simultaneously for all $(w_{i1}, w_{i2}, m_{i1}, \phi_i, \theta) \in S_w \times S_w \times S_m \times S_\phi \times S_\theta$, instead of doing it one by one for each state. This is to increase efficiency. But, a natural problem introduced by the simultaneous updating is that there might be overshooting. For example, if for some particular state $(w^*, w^*, m^*, \phi^*, \theta^*)$, we updated a collusive price $\widehat{P}_{ij}^C(w^*, w^*, m^*, \phi^*, \theta^*)$ too high in the previous iteration, the collusive price for some other states $(w, w, m, \phi, \theta) \neq (w^*, w^*, m^*, \phi^*, \theta^*)$ might be affected in this iteration and never achieve a binding incentive compatibility constraint. Eventually, this may lead to non-convergence.

We solve this problem by gradually updating the collusive prices. In particular, in each round of iteration, we first compute the updated collusive pricing schedule $\widehat{P}_{ij}^{C'}(w_{i1}, w_{i2}, m_{i1}, \phi_i, \theta)$ implied by the golden section search method. Then, instead of changing the upper search bound or lower search bound to $\widehat{P}_{ij}^{C'}(w_{i1}, w_{i2}, m_{i1}, \phi_i, \theta)$ directly, we change it to $(1 - adj) \times \widehat{P}_{ij}^C(w_{i1}, w_{i2}, m_{i1}, \phi_i, \theta) + adj \times \widehat{P}_{ij}^{C'}(w_{i1}, w_{i2}, m_{i1}, \phi_i, \theta)$, a weighted average of the current collusive price $\widehat{P}_{ij}^C(w_{i1}, w_{i2}, m_{i1}, \phi_i, \theta)$ and the updated collusive price $\widehat{P}_{ij}^{C'}(w_{i1}, w_{i2}, m_{i1}, \phi_i, \theta)$. For our baseline model, we set $adj = 0.3$ to ensure perfect convergence. For our extended model, due to larger nonlinearity, we change $adj = 0.1$ to obtain perfect convergence.