Equilibrium Asset Pricing with Price War Risks

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December 17, 2018

Abstract

We develop a general-equilibrium asset pricing model incorporating dynamic supergames of price competition. Price war risks can rise endogenously due to declines in long-run consumption growth, because firms become effectively more impatient for cash flows and their incentives to undercut prices are stronger. The triggered price war risks amplify the initial shocks in long-run growth by narrowing profit margins and discouraging customer base development. In the industries with a higher capacity of distinctive innovation, incentives of price undercutting are less responsive to persistent growth shocks, and thus firms are more immune to price war risks and thus long-run risks. Exploiting detailed patent, product price, and brand-perception survey data, we find evidence for price war risks, which are significantly priced. Our results shed new light on how long-run risks are priced cross-sectionally through industry competition.

Keywords: Long-run risks, Cross-sectional returns, Oligopoly, Innovation similarity, Deep habits, Forward-looking competition. (JEL: G1, G3, O3, L1)

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1 Introduction

A price war refers to the situation in which rival firms fiercely undercut prices to gain market shares. Price war risks are vital and concern investors, partly because product markets are highly concentrated, featuring rich strategic competition among leading firms (see, e.g. Autor et al., 2017; Loecker and Eeckhout, 2017). According to the U.S. Census data, the top four firms within each 4-digit SIC industry account for about 48% of the industry’s total revenue (see Figure B.1), and the top eight firms own over 60% market shares. However, little is known about how price war risks affect asset prices.

We document three stylized facts that motivate our study. First, there is a significant comovement between aggregate profit margins of firms and long-run consumption growth rates (Panel A of Figure 1). Second, concerns about price wars seem to spike up during periods with low long-run consumption growth rates (Panel B of Figure 1). Third, the fact that low long-run consumption growth is coupled with low percent change in profit margins is more pronounced in industries where rival firms conduct more similar innovation (Panels C and D of Figure 1). These empirical findings raise three relevant questions. What fundamentally drives price war risks at the aggregate level? How the heterogeneous exposures to price war risks are determined across industries? To what extent the equity premium is attributed to firms’ exposure to price war risks?

Our paper takes the first step to answer these three questions. First, we develop a model which implies that persistent consumption growth shocks (as in Bansal and Yaron, 2004) can drive price war risks. Second, in the model and the data, we show that firms’ exposures to price war risks and thus long-run risks are higher if they are in the industry with a lower capacity of radical innovations (see, e.g. Jaffe, 1986; Christensen, 1997; Manso, 2011; Kelly et al., 2018). Our results shed new light on how long-run risks are priced in the cross section (see, e.g. Bansal, Dittmar and Lundblad, 2005; Hansen, Heaton and Li, 2008; Bansal, Dittmar and Kiku, 2009; Malloy, Moskowitz and Vissing-Jørgensen, 2009; Constantinides and Ghosh, 2011; Ai, Croce and Li, 2013; Kung and Schmid, 2015; Bansal, Kiku and Yaron, 2016; Dittmar and Lundblad, 2017). Third, we use the calibrated model to quantify the impact of price war risks. The model implies that about 30%-40% of the equity premium explained by long-run risks is attributed to price war risks.

Theoretically, the key feature of a price war is the endogenous decline in product

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1There has been extensive and constant media coverage on the implications of price war risks on stock returns. We list a few of headline quotes in Appendix A.
Panel A plots the yearly time series of aggregate profit margins (simple average across 4-digit SIC industries) and long-run growth rates of consumption, filtered by an HP filter with the smoothing parameter equal to 6.25 (Ravn and Uhlig, 2002). Panel B plots the long-run growth rates of consumption and the price war news index. Panels C and D plot long-run growth rates (annualized based on filtered consumption growth in the last quarter of the year) against the one-year ahead percent change in Compustat-based profit margins in industries with low (i.e. bottom tertile) and high (i.e. top tertile) values of innosimm. Innosimm is a measure of industry-level innovation similarity constructed by the technology classifications of firms’ patents (see Section 4.1 for details).

Figure 1: Profit margins, price war news index, and long-run growth rates.

prices and profit margins due to self-fulfilling declines in market power, not just due to initial declines in demand (see, e.g. Friedman, 1971; Green and Porter, 1984; Porter, 1985; Abreu, Pearce and Stacchetti, 1986; Rotemberg and Saloner, 1986; Lambson, 1987; Haltiwanger and Harrington, 1991; Athey, Bagwell and Sanchirico, 2004; Sannikov and Skrzypacz, 2007). With large declines in profit margins, a full-blown price war may occur, featuring a regime change from collusive price competition to non-collusive competition.
Thus in some sense, price war risks are also inherently related to the jump risks from the collusion regime to the non-collusion regime. We study price war risks by developing a general-equilibrium asset pricing model incorporating dynamic supergames of price competition among firms. Our baseline model deviates from the standard long-run-risk model (Bansal and Yaron, 2004) mainly in two aspects: (1) consumers have deep habits (see Ravn, Schmitt-Grohe and Uribe, 2006) over firms’ products, and thus firms need to maintain their customer bases; and (2) there are a continuum of industries and each industry features dynamic Bertrand oligopoly with differentiated products and implicit price collusion (Tirole, 1988, Chapter 6).

In our model, oligopolies can implicitly collude with each other on setting high product prices at some cost. Given the implicit collusive prices, a firm can boost up its short-run revenue by secretly undercutting prices to attract more customers; however, deviating from collusive prices may reduce revenue in the long run if the price undercutting behavior is detected and punished by the firm’s peers. Following the literature (see, e.g. Green and Porter, 1984; Brock and Scheinkman, 1985; Rotemberg and Saloner, 1986), we adopt the non-collusive Nash equilibrium as the incentive-compatible punishment for deviation. The implicit collusive price levels depend on firms’ deviation incentives: a higher implicit collusive price can only be sustained by a lower deviation incentive, which is further shaped by firms’ tradeoff between short-term and long-term cash flows. In other words, higher collusive prices are more difficult to sustain when the long-run growth rate is lower, because firms expect a persistent decline in aggregate consumption demand, rendering future punishment less threatening. As a result, collusive prices decline following negative long-run-growth shocks, increasing the risk of running into a full-blown price war. With large negative long-run-growth shocks, collusive prices decline significantly, making the benefit of collusion exceed its cost. As a result, firms optimally abandon collusion and the industry falls into the non-collusive equilibrium—a full-blown price war occurs. Importantly, price wars amplify the initial shocks in long-run growth by further narrowing down profit margins and discouraging customer base development.

2Tirole (1988, Chapter 6) builds oligopoly models with Bertrand price competition and obtains similar price war implications as in the models of Cournot quantity competition (Green and Porter, 1984; Rotemberg and Saloner, 1986).

3Even though explicit collusion is illegal in many countries including United States, Canada and most of the EU due to antitrust laws, but implicit collusion in the form of price leadership and tacit understandings still takes place. For example, Intel and AMD implicitly collude on prices of graphic cards and central processing units in the 2000s, though a price war was waged between the two companies recently in October 2018.
Our theory sheds new light on industries’ heterogeneous exposures to price war risks and thus long-run risks. In the model and the data, we show that firms in the industries with a higher capacity of distinctive innovation are more immune to price war risks. The capacity of distinctive innovation is a fundamental, persistent, and predictable industry characteristic. Intuitively, a successful distinctive innovation allows firms to radically disrupt the market and rapidly grab substantial market shares. A prominent recent example is from Apple, a company that disrupted the mobile phone market by cobbling together an amazing touch screen with user-friendly interface. Thus, in the industries with a higher capacity of distinctive innovation, the market structure is more likely to experience dramatic changes and become highly concentrated in the future. This implies that firms in such industries would find it more difficult to implicitly collude with each other, because they all rationally expect that the product market is likely to be monopolized in the future, rendering future punishment on price undercutting less threatening. As a result, these industries feature low implicit collusive prices regardless of long-run growth rates, generating much less variation in product prices over long-run growth fluctuations. By contrast, in the industries with a lower capacity of distinctive innovation, the market structure is relatively stable, making a costly future punishment more credible. As a result, firms have stronger incentives for implicit price collusion, and rationally focus on maintaining existing customer bases and profit margins. However, because firms collude on higher prices in these industries, the equilibrium collusive prices become more sensitive to the fluctuations in firms’ collusion incentives, which are fundamentally driven by long-run-risk shocks. Hence, these industries are more exposed to price war risks and long-run risks.

Particularly, our model has the following main implications on product prices and stock returns across industries with different capacities of distinctive innovation. In the industries where the capacity of distinctive innovation is higher, (1) profit margins are lower and less sensitive to long-run consumption growth shocks; (2) firms are less likely to engage in price wars in response to declines in long-run consumption growth, and (3) the (risk-adjusted) expected excess returns are lower.

We test these predictions using detailed data on patents and product prices. We first construct an innovation similarity measure based on U.S. patenting activities from 1976 to 2017 to capture the capacity of distinctive innovation across industries. In light of previous studies (e.g. Jaffe, 1986; Bloom, Schankerman and Van Reenen, 2013), our innovation similarity measure is constructed based on the technology classifications
of firms’ patents within industries. An industry is associated with a higher innovation similarity measure, if the patents produced by firms within the industry have more similar technology classifications. Thus, intuitively, an industry with a lower innovation similarity measure should have a higher capacity of distinctive innovation. We find that industries’ capacities of distinctive innovation are priced in the cross section of industry returns. In particular, industries with a higher capacity of distinctive innovation are associated with lower (risk-adjusted) expected excess returns. Importantly, we show that the industries with a higher capacity of distinctive innovation are less exposed to long-run-risk shocks. Moreover, their profit margins and sales are less exposed to long-run risks than those with a lower capacity of distinctive innovation.

We further test the key economic mechanism of our model by examining the dynamics of product prices in industries with different capacities of distinctive innovation. We measure the changes in product prices using the Nielsen retail scanner data, which contain price information for more than 3.5 million products from 2006 to 2016. We find that industries with a higher capacity of distinctive innovation have less dramatic product price declines after negative shocks in long-run growth. In particular, our event-type study shows that the industries with a higher capacity of distinctive innovation were less likely to engage in price wars in response to the Lehman crash in September of 2008, a time period in which the U.S. economy experienced a prominent negative long-run-risk shock according to the estimation of Schorfheide, Song and Yaron (2018).

Related Literature. Our paper contributes to the literature on long-run risks (see, e.g. Bansal and Yaron, 2004; Bansal, Dittmar and Lundblad, 2005; Hansen, Heaton and Li, 2008; Malloy, Moskowitz and Vissing-Jørgensen, 2009; Ai, 2010; Chen, 2010; Constantinides and Ghosh, 2011; Bansal, Kiku and Yaron, 2012; Gárleanu, Panageas and Yu, 2012; Croce, 2014; Kung and Schmid, 2015; Bansal, Kiku and Yaron, 2016; Dittmar and Lundblad, 2017; Schorfheide, Song and Yaron, 2018). Our main contribution is to show that price war risks can endogenously arise from long-run risks, generating a novel amplification mechanism. Moreover, we shed new light on the cross-sectional implication of long-run risks based on industries’ capacity of distinctive innovation.

Our paper contributes to the burgeoning literature on the intersection between industrial organization, marketing and finance (see, e.g. Phillips, 1995; Kovenock and Phillips, 1997; Allen and Phillips, 2000; Hou and Robinson, 2006; Carlin, 2009; Aguerrevere, 2009; Hoberg and Phillips, 2010; Hackbarth and Miao, 2012; Carlson et al., 2014; Hoberg,
Phillips and Prabhala, 2014; Bustamante, 2015; Weber, 2015; Hoberg and Phillips, 2016; Loualiche, 2016; Bustamante and Donangelo, 2017; Corhay, 2017; Corhay, Kung and Schmid, 2017; Andrei and Carlin, 2018; D’Acunto et al., 2018; Dong, Massa and Zaldokas, 2018; Dou and Ji, 2018; Dou et al., 2018; Hackbarth and Taub, 2018). In a closely related paper, Corhay, Kung and Schmid (2017) develop a novel general equilibrium model to understand the endogenous relation between markups and stock returns. Their model implies that industries with higher markups are associated with higher expected returns. Our model yields a similar implication through price war risks. We show that industries with a lower capacity of distinctive innovation have higher equilibrium product prices and are more exposed to price war risks and long-run risks. Theoretically, our paper pushes forward the literature by developing a general-equilibrium model incorporated with dynamic supergames, in which price war risks arise endogenously and industry competition is endogenously connected to fundamental long-run risks in consumption growth.

Our paper is also related to a growing literature that studies the implications of innovation on asset prices (see, e.g. Li, 2011; Gârleanu, Kogan and Panageas, 2012; Gârleanu, Panageas and Yu, 2012; Hirshleifer, Hsu and Li, 2013; Kung and Schmid, 2015; Kumar and Li, 2016; Hirshleifer, Hsu and Li, 2017; Kogan et al., 2017; Dou, 2017; Fitzgerald et al., 2017; Kogan, Papanikolaou and Stoffman, 2018; Kogan et al., 2018). We contribute to this literature by showing that industries with a higher capacity of distinctive innovation are less exposed to price war risks and associated with lower (risk-adjusted) expected excess returns. Importantly, we emphasize that the capacity of distinctive innovation provides forward-looking information on the degree of competition in the product market, complementing the traditional static measures of competition such as Herfindahl-Hirschman Index (HHI) and the product similarity measure.

Our paper also contributes to the macroeconomics and industrial organization literature on implicit collusion and price wars in dynamic oligopoly industries (see Stigler, 1964; Green and Porter, 1984; Rotemberg and Saloner, 1986; Haltiwanger and Harrington, 1991; Rotemberg and Woodford, 1991; Staiger and Wolak, 1992; Bagwell and Staiger, 1997; Athey, Bagwell and Sanchirico, 2004; Opp, Parlour and Walden, 2014). We make several contributions to this literature. First, we analyze the asset pricing implications of price

There is also a strand of the literature that studies the asset pricing implications of imperfect competition in the market micro-structure setting (see, e.g. Christie and Schultz, 1994; Biais, Martimort and Rochet, 2000; Liu and Wang, 2018).
war risks. Second, we show that, in the model and the data, the exposure to price war risks varies across industries with different capacities of distinctive innovation.

Finally, our paper lies in the cross-sectional asset pricing literature (see, e.g. Cochrane, 1991; Berk, Green and Naik, 1999; Gomes, Kogan and Zhang, 2003; Pastor and Stambaugh, 2003; Nagel, 2005; Campbell, Hilscher and Szilagyi, 2008; Livdan, Sapricz and Zhang, 2009; Gomes and Schmid, 2010; Garlappi and Yan, 2011; Belo and Lin, 2012; Ai and Kiku, 2013; Kogan and Papanikolaou, 2013; Belo, Lin and Bazdresch, 2014; Donangelo, 2014; Kogan and Papanikolaou, 2014; Tsai and Wachter, 2016; Kojen, Lustig and Nieuwerburgh, 2017; Kozak, Nagel and Santosh, 2017; Ai et al., 2018; Belo, Lin and Yang, 2018; Gomes and Schmid, 2018). A comprehensive survey is provided by Nagel (2013). We show that the exposure to price war risks varies across industries with different capacities of distinctive innovation. Our paper is particularly related to the works investigating the cross-sectional stock return implications of firms’ fundamental characteristics through intangible capital (see, e.g. Ai, Croce and Li, 2013; Eisfeldt and Papanikolaou, 2013; Belo, Lin and Vitorino, 2014; Dou et al., 2018).

2 The Baseline Model

The economy contains a continuum of industries indexed by \( i \in I \equiv [0, 1] \). Each industry \( i \) is a duopoly, consisting of two all-equity firms that are indexed by \( j \in F \equiv \{1, 2\} \). We label a generic firm by \( ij \) and its competitor in industry \( i \) by \( i\bar{j} \). All firms are owned by a continuum of atomistic homogeneous households. Firms produce differentiated goods and set prices strategically to maximize shareholder value.

2.1 Preferences

Households are homogeneous and have stochastic differential utility of Duffie and Epstein (1992a,b), defined recursively as follows:

\[
U_0 = \mathbb{E}_0 \left[ \int_0^\infty f(C_t, U_t) dt \right], \tag{2.1}
\]
where

\[ f(C_t, U_t) = \beta U_t \frac{1 - \gamma}{1 - 1/\psi} \left[ \frac{C_t^{1-1/\psi}}{[(1 - \gamma)U_t]^{1-1/\psi}} - 1 \right]. \quad (2.2) \]

This preference is a continuous-time version of the recursive preferences proposed by Kreps and Porteus (1978), Epstein and Zin (1989), and Weil (1990). The felicity function \( f(C_t, U_t) \) is an aggregator over current consumption rate \( C_t \) of the final consumption good and future utility level \( U_t \). The coefficient \( \beta \) is the rate parameter of time preference, \( \gamma \) is the relative risk aversion parameter for one-period consumption, and \( \psi \) is the parameter of elasticity of intertemporal substitution (EIS) for deterministic consumption paths.

The final consumption good \( C_t \) is obtained through a two-layer aggregation. Following the functional form of relative deep habits developed by Ravn, Schmitt-Grohe and Uribe (2006)\(^5\), industry \( i \)'s composite is determined by the aggregation of firm-level differentiated goods

\[ C_{i,t} = \left[ \sum_{j \in F} \left( \frac{M_{ij,t}}{M_{i,t}} \right)^{\frac{1}{\eta}} C_{ij,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \quad (2.3) \]

where the parameter \( \eta > 1 \) captures the elasticity of substitution among the goods produced in the same industry. \( M_{ij,t}/M_{i,t} \) captures the relative deep habits of firm \( j \) in industry \( i \), where \( M_{i,t} \) is defined as \( M_{i,t} = \sum_{j \in F} M_{ij,t} \).

Further, the demand for the final consumption good \( C_t \) is determined by the aggregation of industry composites

\[ C_t = \left( \int_0^1 M_{i,t}^{\frac{1}{\epsilon}} C_{i,t}^{\frac{\epsilon-1}{\epsilon}} \, di \right)^{\frac{\epsilon}{\epsilon-1}}, \quad (2.4) \]

where the parameter \( \epsilon > 1 \) captures the elasticity of substitution among industry composites. Consistent with the literature, we assume that \( \eta \geq \epsilon > 1 \), meaning that products are more similar to those in the same industry and thus have higher within-industry elasticity of substitution. For example, the elasticity of substitution between Apple iPhone and Samsung Galaxy is higher than the elasticity of substitution between Apple iPhone and Samsung Galaxy.

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\(^5\)The specification is inspired by Abel (1990), preferences feature catching up with the Joneses. By focusing on the pecuniary externality generated from the competition over scarce resources, DeMarzo, Kaniel and Kremer (2007, 2008) show that such preferences can arise endogenously. The key difference of deep habit formation is that agents form habits over individual varieties of goods as opposed to over a composite consumption good.
and Dell Desktop.

### 2.2 Customer Base and Its Dynamics

A firm’s customer base determines the demand for the firm’s goods, and it exists due to consumers’ habits in consumption. Below, we make the connection between customer base and habits clear by deriving the firm’s demand curve.

**Demand Curves.** Let $P_{i,t}$ denote the price of industry $i$’s composite. Given $P_{i,t}$ and $C_t$, solving a standard expenditure minimization problem gives the demand for industry $i$’s composite:

\[ C_{i,t} = \left( \frac{P_{i,t}}{P_t} \right)^{-\epsilon} C_t M_{i,t}, \tag{2.5} \]

where $P_t$ is the price index for the final consumption good, given by

\[ P_t = \left( \int_0^1 M_{i,t} P_{1,t}^{-\epsilon} di \right)^{\frac{1}{1-\epsilon}}. \tag{2.6} \]

Without loss of generality, we normalize $P_t \equiv 1$ so that the final consumption good is the numeraire. Next, given $C_{i,t}$, the demand for firm $j$’s good is:

\[ C_{ij,t} = \left( \frac{P_{ij,t}}{P_{i,t}} \right)^{-\eta} P_{ij,t}^{-\epsilon} C_t M_{ij,t}, \quad \text{with } j = 1, 2. \tag{2.7} \]

where $P_{i,t}$ is the price index for industry $i$’s composite, given by

\[ P_{i,t} = \left[ \sum_{j \in F} \left( \frac{M_{ij,t}}{M_{i,t}} \right) P_{ij,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}. \tag{2.8} \]

In equation (2.7), the demand for firm $j$’s goods increases with $M_{ij,t}$. Thus, we can think of $M_{ij,t}$ as capturing firm $j$’s customer base in industry $i$ and $M_{i,t}$ as capturing industry $i$’s total customer base. Moreover, according to equation (2.8), firm $j$ has more influence on the industry’s price index $P_{i,t}$ when there are higher relative deep habits $M_{ij,t}/M_{i,t}$ toward firm $j$’s goods. Thus, in our model, firm $j$ would naturally have the incentive to accumulate the customer base $M_{ij,t}$ in order to increase demand and gain market power.
Effective Short-Run Elasticity. The effective elasticity of firm \( j \) in industry \( i \) is

\[
\frac{\partial \ln C_{ij,t}}{\partial \ln P_{ij,t}} = s_{ij,t} \frac{\partial \ln C_{ij,t}}{\partial \ln P_{i,t}} + (1 - s_{ij,t}) \frac{\partial \ln (C_{ij,t}/C_{i,t})}{\partial \ln (P_{ij,t}/P_{i,t})}
\]

between-industry

\[
= s_{ij,t} \epsilon + (1 - s_{ij,t}) \eta \tag{2.9}
\]

within-industry

where \( s_{ij,t} \) is the revenue market share of firm \( j \) in industry \( i \)

\[
s_{ij,t} = \frac{P_{ij,t} C_{ij,t}}{P_{i,t} C_{i,t}} = \left( \frac{P_{ij,t}}{P_{i,t}} \right)^{1-\eta} \frac{M_{ij,t}}{M_{i,t}} \tag{2.10}
\]

Thus, equation (2.9) shows that the short-run price elasticity of demand is given by the average of the within-industry elasticity of substitution \( \eta \) and the between-industry elasticity of substitution \( \epsilon \) weighted by the firm’s revenue shares. Depending on the revenue market share \( s_{ij,t} \), firm \( j \)’s short-run price elasticity of demand lies in \([\epsilon, \eta]\). On the one hand, when firm \( j \)’s revenue market share \( s_{ij,t} \) is small, within-industry competition becomes more relevant and thus firm \( j \)’s price elasticity of demand depends more on \( \eta \). In the extreme case with \( s_{ij,t} = 0 \), firm \( j \) becomes atomistic and takes the industry price index \( P_{i,t} \) as given. As a result, firm \( j \)’s short-run price elasticity of demand is entirely determined by the within-industry elasticity of substitution \( \eta \). On the other hand, when firm \( j \)’s revenue market share \( s_{ij,t} \) is large, between-industry competition becomes more relevant and thus firm \( j \)’s short-run price elasticity of demand depends more on \( \epsilon \). In the extreme case with \( s_{ij,t} = 1 \), firm \( j \) becomes the monopoly in industry \( i \) and its short-run price elasticity of demand is entirely determined by the between-industry elasticity of substitution \( \epsilon \).

The key reason why between-industry competition matters for the firm’s price elasticity of demand is that each firm’s price has a non-negligible effect on the industry’s price index in the duopoly industry. The magnitude of this effect is determined by the firm’s revenue market share \( s_{ij,t} \). Thus, when setting prices, each firm internalizes the effect of its own price on the industry’s price index, which in turn determines the demand for the industry’s goods given the between-industry elasticity of substitution \( \epsilon \). If there exist a continuum of firms in each industry, as in standard monopolistic competition models, then each firm is atomistic and has no influence on the industry’s price index. As a result, between-industry competition would have no impact on firms’ price elasticities.
of demand.

Thus, although each industry only has two firms, the modeling of endogenous customer bases allows us to simultaneously capture the pricing behavior resembling a price taker (as in a model with monopolistic competition) and the pricing behavior resembling an industry-level monopoly. As we show in Section 3, this tractable framework also allows us to analyze how collusion incentive would endogenously change due to the change in market structure caused by distinctive innovation.

Dynamics of Customer Bases. There are uncountable cases about how firms attract consumers through price undercutting or discount offering. A temporary cut in prices can have a persistent effect on increasing the firm’s demand because consumers have switching costs. Attracted by lower prices, new customers buy the firm’s products, feel satisfied and become loyal to the firm. Due to switching costs, these consumers become the firm’s customers and keep buying the firm’s products in the future. To capture this idea, following Phelps and Winter (1970) and Ravn, Schmitt-Grohe and Uribe (2006), we model the evolution of firm $j$’s customer base as

$$dM_{ij,t} = -\rho M_{ij,t}dt + z\frac{C_{ij,t}}{C_t}dt,$$

where the parameter $z \geq 0$ determines the speed of customer base accumulation. Intuitively, a lower price $P_{ij,t}$ increases the contemporaneous demand flow rate $C_{ij,t}$, allowing the firm to accumulate a larger customer base over $[t, t + dt]$. The parameter $\rho > 0$ captures customer base depreciation due to economy-wide reasons such as the mortality of consumers.

The firm’s pricing decision crucially depends on the value of $z$ and its customer base $M_{ij,t}$. To elaborate, if $z = 0$, the firm’s pricing decision is static, chosen to maximize contemporaneous profits. If $z > 0$, the firm’s pricing decision becomes dynamic, facing the tradeoff between increasing contemporaneous profits through setting higher prices to exploit existing customer base $M_{ij,t}$ and increasing future profits through setting lower prices to accumulate more customer bases (Chevalier and Scharfstein, 1996; Gilchrist et al., 2017). Consistent with the empirical evidence, the slow-moving customer base $M_{ij,t}$ implies that the long-run price elasticity of demand is higher than the short-run elasticity (see, e.g. Rotemberg and Woodford, 1991).
## 2.3 Consumption Risks for the Long Run

We directly model the dynamics of aggregate consumption demand \( C_t \). Thus, in fact, we incorporate product market frictions into a Lucas-tree model (Lucas, 1978) with homogeneous agents and complete financial markets. Many other extensions of the basic homogeneous-agent complete-market Lucas-tree models have been developed in the literature. For example, Longstaff and Piazzesi (2004), Bansal and Yaron (2004), Santos and Veronesi (2006), and Wachter (2013) consider leveraged dividends and implicitly incorporate labor market frictions in the Lucas-tree model; Menzly, Santos and Veronesi (2004) and Santos and Veronesi (2006, 2010), Martin (2013), and Tsai and Wachter (2016) consider a multi-asset (or multi-sector) Lucas-tree economy. We consider a Lucas-tree economy with multiple sectors whose shares are endogenously determined in the equilibrium.\(^6\) Specifically, we assume that \( C_t \) evolves according to

\[
\frac{dC_t}{C_t} = \theta_t dt + \sigma_c dZ_{c,t}, \tag{2.12}
\]

where

\[
d\theta_t = \kappa(\bar{\theta} - \theta_t) dt + \varphi_\theta \sigma_c dZ_{\theta,t}. \tag{2.13}
\]

The consumption growth rate contains a persistent predictable component \( \theta_t \), which determines the conditional expectation of consumption growth (see, e.g. Kandel and Stambaugh, 1991, for early empirical evidence). The parameter \( \bar{\theta} \) captures the average long-run consumption growth rate. The parameter \( \kappa \) determines the persistence of the expected growth rate process. The parameter \( \varphi_\theta \) determines the exposure to long-run risks. \( dZ_{c,t} \) and \( dZ_{\theta,t} \) are independent standard Brownian motions. Compared to other models with long-run risks, the key feature of our model is that firm-level demand is endogenous and depends on strategic competition.

### Stochastic Discount Factors

The state-price density \( \Lambda_t \) is

\[
\Lambda_t = \exp \left[ \int_0^t f_U(C_s, U_s) ds \right] f_C(C_t, U_t). \tag{2.14}
\]

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\(^6\)The heterogeneous-agent complete-market Lucas-tree models have also been developed and widely used in asset pricing literature. For example, Chan and Kogan (2002) introduced heterogeneous risk aversions in the Lucas-tree model, Xiong and Yan (2010) introduced information frictions to the Lucas-tree model, and Kaniel and Kondor (2013) introduces delegation into a standard Lucas exchange economy.
The market price of risk evolves according to

$$\frac{d\Lambda_t}{\Lambda_t} = -rdt - \gamma \sigma_c dZ_{c,t} - \frac{\gamma - 1/\psi}{h + \eta} dZ_{\theta,t},$$  \hspace{1cm} (2.15)$$

where $r$ is the interest rate and $h$ is the long-run deterministic steady-state consumption-wealth ratio determined in general equilibrium (see Section 2.6).

2.4 Firm Production and Cash Flows

Firms produce differentiated goods using a linear technology. Over $[t, t+dt]$, firm $j$ produces a flow of goods $Y_{ij,t}dt$ with costs $\omega Y_{ij,t}dt$, where $\omega$ is the per unit cost of production, paid to households. In equilibrium, the firm finds it optimal to choose $P_{ij,t} > \omega$ and the market clears for each differentiated good:

$$Y_{ij,t} = C_{ij,t}. \hspace{1cm} (2.16)$$

Thus, firm $j$'s operating profit over $[t, t+dt]$ is given by

$$dE_{ij,t} = (P_{ij,t} - \omega) C_{ij,t}dt. \hspace{1cm} (2.17)$$

Because firms do not face financial frictions or cash flow risks, there is no incentive for firms to hoard cash. All the operating profits are paid out as dividends (or equity financing if $dE_{ij,t} < 0$).

2.5 Price Setting Supergames

The two firms in the same industry play a dynamic game, in which the stage games of setting prices are played continuously and infinitely repeated.\(^7\) There exist a non-collusive equilibrium and multiple collusive equilibria sustained by conditional punishment strategies. Below, we first illustrate the non-collusive equilibrium. Then we define and characterize the collusive equilibrium that yields higher profits for both firms.

\(^7\)We do not model dynamic entries and exits because most entry and exits in the data are associated with small firms while our model focuses on the main players in a market. Moreover, empirical findings indicate that entry and exit are not a systematic factor that drive asset pricing implications.
Non-Collusive Equilibrium. Substituting equation (2.7) into equation (2.17), we obtain

\[
\frac{dE_{ij,t}}{M_{ij,t}} = \Pi_{ij}(P_{i1,t}, P_{i2,t}) dt, \tag{2.18}
\]

where \( \Pi_{ij}(P_{i1,t}, P_{i2,t}) \) is the conditional expected profit rate defined by

\[
\Pi_{ij}(P_{i1,t}, P_{i2,t}) \equiv (P_{ij,t} - \omega) \left( \frac{P_{ij,t}}{P_{i,t}} \right)^{-\eta} P_{i,t}^{-\epsilon} C_t. \tag{2.19}
\]

Equation (2.19) shows that the (local) conditional expected profit rate of firm \( j \) depends on its peer firm \( j' \)'s product price \( P_{ij,t} \) through the industry’s price index \( P_{i,t} \). This reflects the externality of firm \( j' \)'s decisions. For example, if firm \( j' \) sets a low price \( P_{ij,t} \), the price index \( P_{i,t} \) will drop, and thus the demand for firm \( j' \)'s goods \( C_{ij,t} \) will decrease. This will motivate firm \( j \) to set a lower price \( P_{ij,t} \), and thus the two firms’ pricing decisions exhibit strategic complementarity in equilibrium.

In the non-collusive equilibrium, firm \( j \) chooses product price \( P_{ij,t} \) to maximize shareholder value \( V^N_{ij}(M_{i1,t}, M_{i2,t}, C_t, \theta_t) \), conditional on its peer firm \( j' \) setting the equilibrium price \( P^N_{ij,t} \). Following the standard recursive formulation in dynamic games with Markov Perfect Nash equilibrium (see, e.g. Pakes and McGuire, 1994; Ericson and Pakes, 1995; Maskin and Tirole, 2001), the optimization problems can be formulated recursively by HJB equations:

\[
0 = \max_{P_{i1,t}} \Lambda_t \Pi_{i1}(P_{i1,t}, P^N_{i2,t}) M_{ij,t} dt + \mathbb{E}_t \left[ d(\Lambda_t V^N_{i1}(M_{i1,t}, M_{i2,t}, C_t, \theta_t)) \right], \tag{2.20}
\]

\[
0 = \max_{P_{i2,t}} \Lambda_t \Pi_{i2}(P^N_{i1,t}, P_{i2,t}) M_{ij,t} dt + \mathbb{E}_t \left[ d(\Lambda_t V^N_{i2}(M_{i1,t}, M_{i2,t}, C_t, \theta_t)) \right]. \tag{2.21}
\]

The non-collusive equilibrium prices \( P^N_{ij}(M_{i1,t}, M_{i2,t}, C_t, \theta_t) \) (with \( j = 1, 2 \)) are determined by the coupled HJB equations (2.20). The Nash equilibrium considered here is non-collusive, because it does not depend on historical information (i.e. not using conditional punishment strategies based on the two firms’ historical decisions). In the non-collusive equilibrium, firms set prices independently taking the best actions of the other firms as given.
**Collusive Equilibrium.** In the collusive equilibrium, firms collectively set higher prices to gain higher profit margins and values.\(^8\)

Consider a generic collusive equilibrium in which firms follow the collusive pricing schedule \(P^C_{ij}(M_{i1,t}, M_{i2,t}, C_t, \theta_t)\) (with \(j = 1, 2\)).\(^9\) The two firms apply conditional punishment strategies to ensure the collusive price schedule is sustained in equilibrium. In particular, when a price deviation is detected by the peer firm, the peer firm will start setting the non-collusive price in the future. Conditional on the peer firm’s price being non-collusive, the deviating firm would also set the non-collusive price after her deviation is detected, because setting the non-collusive price is the best response to the peer firm’s non-collusive price. Thus, the punishment on deviation is based on switching to the non-collusive equilibrium, which itself has sub-game perfection.\(^10\)

There is imperfect monitoring on peer firms’ product prices in industry \(i\). Each firm has to incur a flow cost of \(\omega dt\) over \([t, t + dt]\) to monitor the other firm’s past prices up to \(t\), an opportunity that arrives following a firm-specific Poisson process \(N_{ij,t}\) with intensity \(\phi\). Whether firms monitor each other is common knowledge. Thus if either firm chooses not to monitor, both firms would set the non-collusive prices \(P^N_{ij}(M_{i1,t}, M_{i2,t}, C_t, \theta_t)\), or in other words, both firms have to pay the monitoring costs \(\omega dt\) in order to sustain the collusive pricing schedule \(P^C_{ij}(M_{i1,t}, M_{i2,t}, C_t, \theta_t)\) as an equilibrium outcome. Firm \(j\)’s value in the collusive equilibrium, denoted by \(V^C_{ij}(M_{i1,t}, M_{i2,t}, C_t, \theta_t)\), is determined by

\[
0 = \max \left\{ \Lambda_t \Pi_{ij} (P^C_{i1}, P^C_{i2}) M_{ij,t} dt - \omega dt + \mathbb{E}_t \left[ d(\Lambda_t V^C_{ij}(M_{i1,t}, M_{i2,t}, C_t, \theta_t)) \right],
\Lambda_t \Pi_{ij} (P^N_{i1}, P^N_{i2}) M_{ij,t} dt + \mathbb{E}_t \left[ d(\Lambda_t V^C_{ij}(M_{i1,t}, M_{i2,t}, C_t, \theta_t)) \right] \right\} \quad \text{with} \quad j = 1, 2. \tag{2.22}
\]

The collusive equilibrium is sub-game perfect if conditional on monitoring, there is no

\(^8\)In the industrial organization and macroeconomics literature, this equilibrium is called collusive equilibrium or collusion (see e.g. Green and Porter, 1984; Rotemberg and Saloner, 1986). Game theorists generally call it the equilibrium of repeated game (Fudenberg and Tirole, 1991) in order to distinguish its nature from the static equilibrium (i.e. our non-collusive equilibrium).

\(^9\)Fershtman and Pakes (2000) require all firms to adopt the same collusive price to maintain tractability. Our collusive pricing schedule is more general because it allows firms to set different prices based on their customer bases and aggregate conditions.

\(^10\)We adopt the non-collusive equilibrium as the incentive-compatible punishment for deviation to follow the literature (see, e.g. Green and Porter, 1984; Rotemberg and Saloner, 1986). All else equal, adopting a less stringent punishment would lower collusive prices. In a related multi-period game, Bond and Krishnamurthy (2004) consider a lenient “debt-default” rule as a punishment for debt default, rather than a full exclusion from financial markets. Interestingly, the “debt-default” rule provides optimal repayment incentives.
deviation from \( P_{ij}^C(M_{1,t}, M_{2,t}, C_t, \theta_t) \). Formally, denote \( V_{ij}^D(M_{1,t}, M_{2,t}, C_t, \theta_t) \) as firm \( j \)'s value for one-shot deviation conditional on monitoring.\(^{11}\) the HJB equations are:

\[
0 = \max_{P_{1,t}} \Lambda_t \Pi_1 \left( P_{1,t}, P_{2,t}^C \right) M_{1,t} \mathrm{d}t - \bar{\omega} \mathrm{d}t + \mathbb{E}_t \left[ \mathrm{d} \left( \Lambda_t V_{1,t}^D \left( M_{1,t}, M_{2,t}, C_t, \theta_t \right) \right) \right]
\]

\[+ \Lambda_t \left[ V_{1,t}^N \left( M_{1,t}, M_{2,t}, C_t, \theta_t \right) - V_{1,t}^D \left( M_{1,t}, M_{2,t}, C_t, \theta_t \right) \right] \mathrm{d}N_{1,t}, \quad (2.23)\]

\[
0 = \max_{P_{2,t}} \Lambda_t \Pi_2 \left( P_{1,t}, P_{2,t}^C \right) M_{2,t} \mathrm{d}t - \bar{\omega} \mathrm{d}t + \mathbb{E}_t \left[ \mathrm{d} \left( \Lambda_t V_{2,t}^D \left( M_{1,t}, M_{2,t}, C_t, \theta_t \right) \right) \right]
\]

\[+ \Lambda_t \left[ V_{2,t}^N \left( M_{1,t}, M_{2,t}, C_t, \theta_t \right) - V_{2,t}^D \left( M_{1,t}, M_{2,t}, C_t, \theta_t \right) \right] \mathrm{d}N_{2,t}, \quad (2.24)\]

The incentive compatibility (IC) constraints that ensure non-deviation are given by:

\[ V_{ij}^D \left( M_{1,t}, M_{2,t}, C_t, \theta_t \right) \leq V_{ij}^C \left( M_{1,t}, M_{2,t}, C_t, \theta_t \right), \quad (2.25) \]

for \( j = 1, 2 \) and all monitoring states. In fact, there exist infinitely many collusive pricing schedules \( P_{ij}^C(M_{1,t}, M_{2,t}, C_t, \theta_t) \) that satisfy the IC constraints, and hence infinitely many collusive equilibrium. Because firms maximize profits in general equilibrium, it is reasonable for them two collude on prices as high as possible.\(^{12}\) We thus focus on the highest collusive pricing schedule, under which the IC constraints are binding for all monitoring states, i.e. \( P_{ij}^C(M_{1,t}, M_{2,t}, C_t, \theta_t) \) are determined such that

\[ V_{ij}^D \left( M_{1,t}, M_{2,t}, C_t, \theta_t \right) = V_{ij}^C \left( M_{1,t}, M_{2,t}, C_t, \theta_t \right). \quad (2.26) \]

### 2.6 General Equilibrium Conditions

In equilibrium, the value function of the representative household is

\[ U_t = \exp \left( A_0 + A_1 \theta_t \right) \frac{W_t^{1-\gamma}}{1-\gamma}, \quad (2.27) \]

\(^{11}\)The one-shot deviation principle indicates that focusing on no one-shot-deviation is necessary and sufficient to attain a sub-game perfect equilibrium (see Fudenberg and Tirole, 1991).

\(^{12}\)There are two reasons why we focus on the highest collusive pricing schedule. First, non-binding IC constraints imply that there is room to further increase both firms' values by increasing collusive prices. Given that firms collude with each other to maximize their values, it is a bit unreasonable to rule out a better collusion. Second, considering the highest collusive price allows us to conduct more disciplined comparative statics in the presence of multiple equilibria. In other words, focusing on the highest collusive price ensures that we always pick up the same equilibrium when we compare across different industries.
where \( A_0 \) is a deterministic function of model parameters, and \( A_1 \) is equal to

\[
A_1 = \frac{\psi^{-1}(1 - \gamma)}{h + \kappa}, \quad \text{with} \quad h = \exp \left( \ln C - \ln W \right).
\] (2.28)

The equilibrium wealth-consumption ratio is

\[
\frac{W_t}{C_t} = \rho^{-\psi} \exp \left( \frac{1 - \psi}{1 - \gamma} A_0 + \left( \frac{1 - \psi^{-1}}{h + \kappa} \right) \theta_t \right). \quad (2.29)
\]

In equilibrium, the long-run deterministic steady-state consumption-wealth ratio is:

\[
\ln(h) = \ln C - \ln W = \psi \ln(\rho) - \frac{1 - \psi}{1 - \gamma} A_0 - \frac{1 - \psi^{-1}}{h + \kappa} \theta.
\] (2.30)

### 2.7 Price Wars and Long-Run Growth Rates

In our model, price war risks endogenously arise from long-run risks. When the long-run consumption growth rate \( \theta_t \) declines, profit margins decline due to endogenous declines in collusive prices \( P_{ij}^C \), increasing the risks of running into a price war. With significant declines in long-run consumption growth rate, a full-blown price war occurs.

Intuitively, the incentive to collude on higher prices depends on the extent to which the two firms value future revenue relative to its current revenue. By deviating from the collusive price, firms can attain higher contemporaneous revenue and accumulate more customer bases in the short run. However, firms run into the risk of losing future revenue because once the deviation is detected by the other firm, the non-collusive equilibrium will be implemented as a punishment strategy. During periods with low long-run growth rates, firms expect a relative decrease in aggregate consumption and the later punishment looks less costly. This makes firms more impatient for cash flows and attain stronger incentives to undercut peers’ prices.\(^\text{13}\) Therefore, a decline in long-run consumption growth has the potential to trigger price wars. When long-run consumption growth declines significantly, firms optimally choose not to collude with each other because the costs paid to monitor peers’ prices exceed the potential gains from collusion. When this

\(^\text{13}\)The intuition is related to the Folk Theorem in game theory. The Folk Theorem says that provided players are sufficiently patient, not only can repeated interaction allow many sub-game perfect outcomes, but actually sub-game perfection can allow virtually any outcome in the sense of average payoffs. The effective discount rate is approximately given by \( r - \theta_t \). Thus, the periods with low \( \theta_t \) feature high discount rates and more impatience.
happens, the industry enters into a full-blown price war, in which both firms set the non-collusive prices $P_{ij}^N$.

Thus, in our model, a full-blown price war is a regime shift from the collusive equilibrium to the non-collusive equilibrium, which is driven by the fluctuation in long-run consumption growth. A moderate decline in long-run consumption growth reduces equilibrium product prices and profit margins, increasing the risk of running into a full-blown price war. Importantly, we emphasize that in our model, the decrease in profit margins is owing to intensified competition and reduced market power (as in Green and Porter, 1984; Rotemberg and Saloner, 1986; Haltiwanger and Harrington, 1991) caused by lower long-run consumption growth, in addition to the immediate downward shift in demand curve.

We illustrate price war risks numerically in Figure 2. By exploiting linearity on $M_{i,t}C_t$, we reduce the model to two state variables: $M_{i1,t}/M_{i,t}$ and $\theta_t$. We solve the normalized firm value $v_{ij}^N(M_{i1,t}/M_{i,t}, \theta_t)$ in non-collusive equilibrium and $v_{ij}^C(M_{i1,t}/M_{i,t}, \theta_t)$ in collusive equilibrium.

Panel A and B plot firm 1’s equilibrium price in periods with different long-run growth rates. The blue solid line plots the firm’s price during periods with high long-run growth rates (i.e. $\theta_b$). In Panel B, we consider a moderate decrease in the long-run growth rate to $\theta_m < \theta_b$, and it shows that the firm’s equilibrium prices are lower. The industry features narrowed profit margins for both firms, increasing the risk of running into a full-blown price war. Panel B shows that when the long-run growth rate further drops to $\theta_r < \theta_m$, the two firms choose not to collude with each other, and the equilibrium prices are significantly lower and equal to non-collusive prices. The industry enters into a full-blown price war.

Panel C illustrates the magnitude of price war risks by plotting the difference in collusive prices between periods with high and low long-run growth rates. It is shown that price war risks display an inverted U-shape, and the risks are the largest when the two firms have comparable customer base shares (i.e. $M_{i1}/M_i \approx 0.5$). Intuitively, collusion allows both firms to set higher prices to enjoy higher profit margins than what they would have in the non-collusive equilibrium. However, the collusive pricing schedule has to be chosen such that both firms have no incentive to deviate given their current

---

14 Firm 2’s decision is symmetric. We choose large differences in growth rates for better visualizing the channels. The equilibrium property of our model is discussed in Appendix C. The details of our numerical algorithm are presented in Appendix F.
customer base shares. When firm 1 is dominating the market (i.e. with high $M_{i1}/M_i$), forming a collusive equilibrium would be less appealing from firm 1’s perspective as it already has high market power, which allows it to set a high price in the non-collusive equilibrium any way (see the red dash-dotted line in Panel A). On the other hand, when firm 1 has low customer base share $M_{i1}/M_i$, forming a collusive equilibrium would be less appealing from firm 2’s perspective, who already has high market power to set a high price in the non-collusive equilibrium. In other words, when one firm dominates the other firm in customer base share, there is not much incentive to form a collusion in the industry; and as a result, there is not much variation in collusive prices when long-run growth rates change.

The time-varying collusion incentive amplifies the effect of long-run risks because during periods with low long-run consumption growth, firms not only face low demand but also low profit margins. To illustrate this amplification mechanism, we calculate the
industry-level beta as value-weighted firm-level betas

\[ \beta_i(M_{i1}/M_i) = \sum_{j=1,2} \frac{v_{ij}^C(M_{i1}/M_i, \theta_r)}{v_{i1}^C(M_{i1}/M_i, \theta_r) + v_{i2}^C(M_{i1}/M_i, \theta_r)} \left[ \frac{v_{ij}^C(M_{i1}/M_i, \theta_b)}{v_{ij}^C(M_{i1}/M_i, \theta_r)} - 1 \right]. \] (2.31)

Panel D shows that the industry’s beta displays an inverted U-shape (see the blue solid line) due to the inverted-U price war risks. As a benchmark, the red dash-dotted line plots the industry’s beta in the absence of price war risks (i.e. when collusive prices do not change with long-run growth rates). When the two firms have comparable customer base shares, price war risks significantly amplify the industry’s exposure to long-run risks.

3 The Model with Distinctive Innovation

In this section, we extend the baseline model by allowing firms to conduct innovations. There are two main reasons why we emphasize innovation activities. First, product innovation is an important channel through which firms develop customer bases, on top of strategic pricing. We show that whether firms collude with each other crucially depends on the extent to which they have the ability to conduct radical innovations, which determines the future market structure (i.e. the distribution of customer bases). Second, introducing innovations expands the scope of testing our asset pricing theory of price war risks. Because collusion incentive determines the level of product prices, naturally firms collude more during periods with high long-run consumption growth are more exposed to price war risks. Our model yields new cross-sectional predictions, which suggest that industries with a lower capacity of distinctive innovation are more exposed to price war risks, and thus long-run risks.

3.1 Modeling Distinctive Innovation

Firms conduct innovations, which succeed independently across firms at constant rate \( \mu \). A successful innovation allows the innovating firm to snatch a fraction \( \tau_{ij,t} \) of the peer
firm’s customer base, where $\tau_{ij,t}$ takes two values,

$$
\tau_{ij,t} = \begin{cases} 
\tau_i & \text{with probability } \lambda_{i,t}, \\
\tau_d & \text{with probability } 1 - \lambda_{i,t}.
\end{cases}
$$

(3.1)

We assume $0 \approx \tau_i << \tau_d \approx 1$ to parsimoniously capture two different types of innovation. The event of snatching a small fraction $\tau_i$ of customer base captures a successful incremental innovation which adds value to customers through incrementally introducing new features to existing products. For example, Motorola has launched a series of Motorola Razr since 2004, based on constant improvement of previous generations. The event of snatching a large fraction $\tau_d$ of customer base captures a radical or distinctive innovation that creates newer tech to surpass the old and disrupt existing companies. There are quite a few examples of distinctive innovation, one of the more prominent being Apple’s iPhone disruption of the mobile phone market. In reality, firms’ innovations also drive the growth of aggregate consumption $C_t$, which is exogenously specified by equation (2.12). We abstract away from this aggregate growth effect to focus on the strategic customer base stealing effect of innovations.

The industry characteristic $\lambda_{i,t}$ is the only ex-ante heterogeneity across industries, evolving idiosyncratically according to a Markov chain on $\lambda = \{0 << \lambda_1 < \ldots < \lambda_N \leq 1\}$. We require $\lambda_1 >> 0$ so that in general, distinctive innovations succeed with much lower rates. The expected success rates of incremental and distinctive innovations are $\mu\lambda_{i,t}$ and $\mu(1 - \lambda_{i,t})$, indicating that a lower $\lambda_{i,t}$ captures the industry with a higher capacity of distinctive innovation.

In the presence of innovation, the dynamics of customer base (2.11) is modified as

$$
dM_{ij,t} = -\rho M_{ij,t} dt + z \frac{C_{ij,t}}{C_t} dt + \tau_{ij,t} M_{ij,t} dI_{ij,t} - \tau_{ij,t} M_{ij,t} dI_{ij,t}
$$

(3.2)

where $I_{ij,t}$ and $I_{ij,t}$ are independent Poisson processes capturing the success of firm $j$ and $j'$’s innovations.

### 3.2 Price War Risks Across Industries

We now study the implication of innovation characteristics on the industry’s exposure to price war risks. To fix ideas, consider two industries different in the capacity of distinctive
innovation. One industry has a low capacity of distinctive innovation (high $\lambda_i$) and innovation succeeds on average once a year for each firm. The other industry has a high capacity of distinctive innovation (low $\lambda_i$) and innovation succeeds on average every twenty years for each firm.

Panel A of Figure 3 plots the equilibrium collusive prices in the two industries during periods with high and low long-run growth rates, respectively. It shows that although firms in both industries collude on higher prices during periods with high long-run growth rates, collusive prices are much lower in the industry with a high capacity of distinctive innovation (low $\lambda_i$). As we discuss in section 2.7, the incentive to collude exhibits an inverted-U shape and becomes the smallest in concentrated industries (i.e. one firm’s customer base share is much larger than the other’s). The industry with a high capacity of distinctive innovation is more likely to be concentrated in future because one firm can steal its competitor’s customer base and almost monopolize the industry upon the success of distinctive innovation. Thus, even if the two firms have comparable customer base shares today, the possibility of having a successful distinctive innovation in future still largely dampens the incentive to collude, resulting in low collusive prices.

Not only the levels are lower, collusive prices are also less responsive to persistent growth shocks in the industry with a high capacity of distinctive innovation. Panel A shows that when the economy switches between periods with high and low long-run growth rates, the change in collusive prices in the industry with a high capacity of distinctive innovation (the difference between the red dash-dotted line and the red dotted line) is much smaller compared to that in the industry with a low capacity of distinctive

Figure 3: Comparing collusive prices and the exposure to long-run risks across industries with different capacities of distinctive innovation.
innovation (the difference between the blue solid line and the blue dashed line). This implies that firms in the industry with a high capacity of distinctive innovation face smaller price war risks simply because collusion is difficult to form in the first place.

In Panel B, we compare the two industries’ exposure to long-run risks for different levels of industry concentration, as reflected by firm 1’s customer base share. Conditional on the same level of concentration, firms in the industry with a high capacity of distinctive innovation are less exposed to long-run risks. The industry-level value-weighted beta exhibits an inverted U-shape in both industries. The difference in beta across the two industries is large when the two firms within the same industry have comparable customer base shares ($M_{i1}/M_i = 0.5$). Thus our model implies that the industries with low capacities of distinctive innovation tend to be riskier as price decreases more when the long-run consumption growth rate declines.

4 Empirical Analyses

In this section, we empirically test the main predictions of our model. We first use patent data to construct an innovation similarity measure for the industry characteristic $\lambda$ in our model. We find that industries with higher innovation similarity have higher average excess returns and risk-adjusted returns. The stock returns, cash flows, profit margins, and sales of these industries are more exposed to long-run risks. Moreover, we find that industries with higher innovation similarity are more exposed to price war risks. In particular, these industries were more likely to engage in price wars after the Lehman crash in 2008.

4.1 Data and the Innovation Similarity Measure

In this subsection, we introduce our data and construct the innovation similarity measure.

**Patent Data and Our Merged Sample.** We obtain the patent issuance data from PatentView, a patent data visualization and analysis platform. PatentView contains detailed and up-to-date information on granted patents from 1976 onward. Its coverage of recent patenting activities is more comprehensive than the NBER patent data (Hall,
Jaffe and Trajtenberg, 2001) and the patent data assembled by Kogan et al. (2017). Patent assignees in PatentView are disambiguated and their locations and patenting activities are longitudinally tracked. PatentView categorizes patent assignees into different groups, such as corporations, individuals, and government agencies. It also provides detailed information of individual patents, including their grant dates and technology classifications. We drop patents granted to individuals and government agencies.

We match patent assignees in PatentView to U.S. public firms in CRSP/Compustat, and to U.S. private firms and foreign firms in Capital IQ. Private firms are included in our sample because they play an important role in industry competition (see e.g. Ali, Klasa and Yeung, 2008). We use a fuzzy name-matching algorithm to obtain a pool of potential matches from CRSP/Compustat and Capital IQ for each patent assignee in PatentView. We then manually screen these potential matches to identify the exact matches based on patent assignees’ names and addresses. In Appendix B.2, we detail our matching procedure. In total, we match 2,235,201 patents to 10,139 U.S. public firms, 132,100 patents to 3,080 U.S. private firms, 241,582 patents to 300 foreign public firms, and 35,597 patents to 285 foreign private firms. The merged sample covers 13,804 firms in 752 4-digit SIC industries from 1976 to 2017.

Innovation Similarity Measure. We construct our innovation similarity measure (denoted as “innosimm”) for the industry-level innovation similarity based on the technology classifications of an industry’s patents. In light of previous studies (e.g. Jaffe, 1986; Bloom, Schankerman and Van Reenen, 2013), we measure the cosine similarity of two patents within the same industry based on their technology classification vectors. Specifically,

---

15The PatentView data contain all patents granted by the U.S. Patent and Trademark Office (USPTO) from 1976 to 2017, while the NBER data and the data used by Kogan et al. (2017) only cover patents granted up to 2006 and 2010, respectively.
16Capital IQ is one of the most comprehensive data that include private firms and foreign firms. Our empirical results remain robust if we confine the patent data to those granted to U.S. public firms.
17We use 4-digit SIC codes in Compustat and Capital IQ to identify the industries of patent assignees. Both Compustat and Capital IQ are developed and maintained by S&P Global and the SIC industry classifications in these two datasets are consistent with each other. We verify the consistency by comparing the SIC codes for U.S. public firms covered by both Compustat and Capital IQ. We find that the SIC codes of these firms are virtually identical across the two data sources.
18PatentView provides both the Cooperative Patent Classification (CPC) and the U.S. Patent Classification (USPC), the two major classification systems for U.S. patents. We use CPC for our analyses because USPC is not available after 2015. Our results are robust to the classification based on USPC for data prior to 2015. There are 653 unique CPC classes in PatentView. The technology classification vector for a patent consists of 653 indicator variables that represent the patent’s CPC classes.
the similarity between two patents, \(a\) and \(b\), is defined by:

\[
similarity(a, b) = \frac{A \cdot B}{\|A\| \|B\|'}
\]  

(4.1)

where \(A\) and \(B\) are the technology vectors of patent \(a\) and patent \(b\). If the two patents share exactly the same technology classifications, the cosine similarity attains the maximum value, 1. If the two patents are mutually exclusive in their technology classifications, the cosine similarity reaches the minimum value, 0. Because patent technology classifications are assigned according to the technical features of patents, the cosine similarity measure captures how similar the patents are in terms of their technological positions. Based on the pairwise cosine similarity of patents, we take the following steps to construct the industry-level innovation similarity measure.

First, we construct the patent-level similarity measure to capture to what extent a patent is differentiated from other patents recently developed by peer firms. In particular, for a patent granted to firm \(i\) in year \(t\), the patent-level similarity measure is the average of the pairwise cosine similarity (defined by equation 4.1) between this patent and the other patents granted to firm \(i\)’s peer firms in the same 4-digit SIC industry from year \(t - 5\) to year \(t - 1\).

Next, we aggregate patent-level similarity measures to obtain industry-level similarity measures. For example, a 4-digit SIC industry’s similarity measure in year \(t\) is the average of patent-level similarity measures associated with all the patents granted to firms in the industry in year \(t\). Because firms are not granted with patents every year, we further average the industry-level similarity measures over time to filter noise and better capture firms’ ability in generating differentiated innovations. In particular, our innosimm measure in industry \(i\) and year \(t\) (i.e. innosimm\(_{it}\)) is constructed as the time-series average of industry \(i\)’s similarity measures from year \(t - 9\) to year \(t\).

Panel A of Figure 4 presents the time-series of several industries’ innosimm measure. In the “Search, Detection, Navigation, Guidance, Aeronautical, and Nautical Systems and Instruments” industry, the innosimm measure is low throughout our sample period, suggesting that firms in this industry seem to be able to consistently generate distinctive innovation. The innosimm measure keeps increasing for the “Drilling Oil and Gas Wells” industry, while it peaks in early 2000s for the “Rubber and Plastics Footwear.” industry.
Validation of the Innosimm Measure. We conduct one external validation for our innosimm measure. If a higher innosimm captures a lower capacity of distinctive innovation in an industry, we expect that fewer consumers would consider the brands of high-innosimm industries as distinctive. We test this hypothesis by examining the relation between innosimm and the relative change in brand distinctiveness over time, measured using the BAV consumer survey data. We standardize innosimm using its unconditional mean and the standard deviation of all industries’ innosimm across all time to ease the interpretation of coefficients in our regression analyses. Column (1) of Table 1 shows that innosimm is negatively correlated with the two-year percent change in the industry-level brand distinctiveness, suggesting that industries with higher innosimm are associated with lower brand distinctiveness in future.

Our innosimm measure is conceptually different from the product similarity measure (denoted as “prodsimm”) constructed by Hoberg and Phillips (2016). Innosimm captures to what extent firms in an industry can differentiate their products from peers’ through innovation. Thus, it is a forward-looking measure that captures the (potential) similarity/distinctiveness of firms’ businesses in the future. Product similarity, on the other hand, is derived from text analyses based on firms’ current product description (Hoberg and Phillips, 2016). Therefore, it reflects the similarity of products produced by different firms as of today, rather than the potential similarity/distinctiveness of firms’ products in the future. In other words, product similarity contains little information, if at all, about firms’ innovation activities, which are the necessary inputs for making products distinctive in the future. The conceptual difference between the two measures is formally confirmed by column (2) of Table 1, which shows that innosimm is unrelated with product similarity.

In Section 4.2 and 4.4, we further show that the product similarity measure is neither priced in the cross section nor related to industries’ price war risks.

---

19 The BAV database is regarded as the world’s most comprehensive database of consumers’ perception of brands (see, e.g. Gerzema and Lebar, 2008; Keller, 2008; Mizik and Jacobson, 2008; Aaker, 2012; Lovett, Peres and Shachar, 2014; Tavassoli, Sorescu and Chandy, 2014). The BAV brand perception survey consists of more than 870,000 respondents in total, and it is constructed to represent the U.S. population according to gender, ethnicity, age, income group, and geographic location. See Dou et al. (2018) for the details of the survey.

20 The correlation between product similarity and innosimm is low. The Pearson correlation coefficient, the Spearman’s rank correlation coefficient, and the Kendall’s $\tau_A$ and $\tau_B$ coefficients between the two variables are 0.06, 0.02, 0.04, and 0.04, respectively.
Table 1: Innovation similarity, brand distinctiveness and product similarity (yearly analysis).

<table>
<thead>
<tr>
<th></th>
<th>(1) Percent changes from year $t$ to year $t+2$ (%)</th>
<th>(2) Product similarity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brand distinctiveness</td>
<td>$-0.69^{***}$</td>
<td>$0.10$</td>
</tr>
<tr>
<td></td>
<td>$[-3.06]$</td>
<td>$[0.04]$</td>
</tr>
<tr>
<td>Year FE</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>2466</td>
<td>5906</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.298</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Note: This table shows the relation of our innovsim measure with measures of brand distinctiveness and product similarity at the 4-digit SIC industry level. In column (1), the dependent variable is the two-year percent change in brand distinctiveness. The percent change is computed as $100 \times (\text{brand distinctiveness}_{t+2} - \text{brand distinctiveness}_t) / \text{brand distinctiveness}_t$. At the brand level, brand distinctiveness is the fraction of consumers who consider a brand to be distinctive. We first aggregate the brand-level distinctiveness measure to the firm level, and then further aggregate it to the 4-digit SIC industry level. Product similarity comes from Hoberg and Phillips (2016), and it is derived from text analyses based on the business description in 10-K filings. We download the product similarity measure from the Hoberg and Phillips Data Library, and aggregate it to the 4-digit SIC industry level. The sample in column (1) spans from 1993 to 2017, and the sample in column (2) spans from 1996 to 2015. We include t-statistics in brackets. Standard errors are clustered by the 4-digit SIC industry and year. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels.

4.2 Asset Pricing Tests

We now test the asset pricing implications of our model. We find that industries with higher innovsim have higher average excess returns and risk-adjusted returns, and this relationship is weaker among the industries that experience antitrust enforcement in recent years. Furthermore, the stock returns and dividends of the high-innovsim industries are more exposed to long-run risks. In addition, we show that innovsim spreads (i.e. the spreads between high-innovsim industries and low-innovsim industries) remain robust after controlling for various related measures.

4.2.1 Innovsim Spreads Across Industries

We first examine whether innovsim is priced in the cross section. Panel A of Table 2 presents the value-weighted average excess returns and alphas for the 4-digit SIC industry portfolios sorted on innovsim. It shows that the portfolio consisting of high-innovsim industries (i.e. Q5) exhibits significantly higher average excess returns and alphas. The spread in average excess returns between Q1 and Q5 is 3.41% and the spreads in alphas are 5.22% and 4.75% for the Fama-French three-factor model and the Carhart four-factor model.

Next, we perform the same analysis for prodsimm. We find that prodsimm is not
priced in the cross section. The return difference between the high-prodsimm portfolio and the low-prodsimm portfolio is statistically insignificant (see Panel B).

Table 2: The average excess returns and alphas of portfolios sorted on innovation similarity and product similarity (monthly analysis).

<table>
<thead>
<tr>
<th></th>
<th>1 (Low)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5 (High)</th>
<th>5 – 1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Portfolios sorted on inno-simm</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average excess returns</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mathbb{E}[R] - r_f$ (%)</td>
<td>6.13***</td>
<td>8.37***</td>
<td>7.35***</td>
<td>8.62***</td>
<td>9.54***</td>
<td>3.41***</td>
</tr>
<tr>
<td>[2.74]</td>
<td>[3.73]</td>
<td>[3.01]</td>
<td>[4.42]</td>
<td>[3.17]</td>
<td>[2.71]</td>
<td></td>
</tr>
<tr>
<td>Fama-French three-factor model (Fama and French, 1993)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$ (%)</td>
<td>$-2.51^{**}$</td>
<td>0.07</td>
<td>$-1.34$</td>
<td>1.08</td>
<td>2.71**</td>
<td>5.22***</td>
</tr>
<tr>
<td>[-2.48]</td>
<td>[0.10]</td>
<td>[-0.69]</td>
<td>[1.21]</td>
<td>[2.49]</td>
<td>[3.54]</td>
<td></td>
</tr>
<tr>
<td>Carhart four-factor model (Carhart, 1997)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$ (%)</td>
<td>$-2.47^{***}$</td>
<td>0.99</td>
<td>$-1.25$</td>
<td>1.43</td>
<td>2.28***</td>
<td>4.75***</td>
</tr>
<tr>
<td>[-2.70]</td>
<td>[0.18]</td>
<td>[-0.78]</td>
<td>[1.50]</td>
<td>[2.63]</td>
<td>[4.01]</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>1 (Low)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5 (High)</th>
<th>5 – 1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel B: Portfolios sorted on prod-simm</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average excess returns</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mathbb{E}[R] - r_f$ (%)</td>
<td>4.94**</td>
<td>6.44**</td>
<td>8.07**</td>
<td>6.25*</td>
<td>6.19*</td>
<td>1.25</td>
</tr>
<tr>
<td>[2.29]</td>
<td>[2.05]</td>
<td>[2.59]</td>
<td>[1.77]</td>
<td>[1.91]</td>
<td>[0.42]</td>
<td></td>
</tr>
<tr>
<td>Fama-French three-factor model (Fama and French, 1993)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$ (%)</td>
<td>$-0.89$</td>
<td>0.03</td>
<td>2.21**</td>
<td>0.08</td>
<td>0.82</td>
<td>1.70</td>
</tr>
<tr>
<td>[-0.48]</td>
<td>[0.01]</td>
<td>[2.53]</td>
<td>[0.06]</td>
<td>[0.86]</td>
<td>[0.64]</td>
<td></td>
</tr>
<tr>
<td>Carhart four-factor model (Carhart, 1997)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$ (%)</td>
<td>$-0.57$</td>
<td>0.42</td>
<td>2.19**</td>
<td>0.69</td>
<td>0.70</td>
<td>1.26</td>
</tr>
<tr>
<td>[-0.36]</td>
<td>[0.20]</td>
<td>[2.43]</td>
<td>[0.68]</td>
<td>[0.75]</td>
<td>[0.53]</td>
<td></td>
</tr>
</tbody>
</table>

Note: This table shows the value-weighted average excess returns and alphas for the 4-digit SIC industry portfolios sorted on inno-simm. In June of year $t$, we sort the 4-digit SIC industries into five quintiles based on this industry’s inno-simm in year $t - 1$. Once the portfolios are formed, their monthly returns are tracked from July of year $t$ to June of year $t + 1$. The sample period is from July 1988 to June 2018. We exclude financial firms and utility firms from the analysis. We include t-statistics in parentheses. Standard errors are computed using the Newey-West estimator allowing for serial correlation in returns. We annualize average excess returns and alphas by multiplying by 12. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels.

4.2.2 Exposure to Long-Run Risks in Consumption Growth

What drives the inno-simm spreads? In this section, we show that industries with different inno-simm have differential exposure to long-run risks, as suggested by our model. We estimate the exposure to long-run risks (i.e. LRR beta) following Dittmar and Lundblad (2017). In particular, we first sort all industries into quintile portfolios based on inno-simm. Then, we regress the cumulative portfolio returns of each portfolio on long-run risks. The coefficients give us an estimate for LRR beta (see Table 3). We use two long-run risks measures in our analyses: the cumulative realized consumption growth and the
consumption growth filtered by a Bayesian mixed-frequency approach as in Schorfheide, Song and Yaron (2018) (see Panel B of Figure 4). The difference in LRR betas between Q1 and Q5 is positive and statistically significant. This result is robust for both the cumulative realized consumption growth and the filtered consumption growth, indicating that high-innosimm industries are more exposed to long-run risks.

Table 3: Long-run-risk exposures of portfolios sorted on innosimm (quarterly analysis).

<table>
<thead>
<tr>
<th>Portfolios sorted on innosimm</th>
<th>1 (Low)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5 (High)</th>
<th>5 – 1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Long-run risks measured by 8-quarter cumulative realized consumption growth</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LRR betas</td>
<td>1.78***</td>
<td>6.55***</td>
<td>3.48***</td>
<td>5.51***</td>
<td>5.24***</td>
<td>3.46**</td>
</tr>
<tr>
<td></td>
<td>[1.58]</td>
<td>[5.09]</td>
<td>[3.12]</td>
<td>[3.73]</td>
<td>[4.03]</td>
<td>[2.09]</td>
</tr>
<tr>
<td><strong>Panel B: Long-run risks measured by 8-quarter cumulative filtered consumption growth</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LRR betas</td>
<td>–0.05</td>
<td>4.45***</td>
<td>–0.62</td>
<td>3.57**</td>
<td>4.69**</td>
<td>4.75**</td>
</tr>
<tr>
<td></td>
<td>[–0.05]</td>
<td>[3.16]</td>
<td>[–0.52]</td>
<td>[2.57]</td>
<td>[2.41]</td>
<td>[2.61]</td>
</tr>
</tbody>
</table>

Note: This table shows the exposures to long-run risks for industry portfolios sorted on innosimm. In June of year t, we sort industries into five quintiles based on innosimm in year t – 1. Once the portfolios are formed, their monthly returns are tracked from July of year t to June of year t + 1. In Panel A, following Dittmar and Lundblad (2017), we regress the 8-quarter cumulative portfolio returns on the 8-quarter cumulative realized consumption growth: $\prod_{j=0}^{7} R_{i, \tau-j} = \alpha_i + \beta_i \sum_{j=0}^{\tau} \hat{\eta}_{\tau-j} + e_{i, \tau}$, where $\hat{\eta}$ is the consumption growth shock, measured by the difference between the log consumption growth in quarter $\tau$ and the unconditional mean of log consumption growth over 1947–2018. We measure consumption using per-capita real personal consumption expenditures on non-durable goods and services. $R_{i, \tau}$ is the gross real return of the industry portfolio $i$ in quarter $\tau$. Consumption and returns are deflated to real terms using the personal consumption expenditure deflator from the U.S. Bureau of Economic Analysis (BEA). The analysis is conducted at quarterly frequency for the sample period from 1988 to 2018. In Panel B, we replace realized cumulative consumption growth ($\sum_{j=0}^{\tau} \hat{\eta}_{\tau-j}$) in the above regression with cumulative filtered consumption growth ($\sum_{j=0}^{\tau} \hat{x}_{\tau-j}$) as in Schorfheide, Song and Yaron (2018). The sample period in Panel B is from 1988 to 2015 because data on the filtered consumption growth end in 2015. We exclude financial firms and utility firms from the analysis. We include t-statistics in parentheses. Standard errors are computed using the Newey-West estimator allowing for serial correlation in returns. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels.

We further examine the exposure of real dividend growth to long-run risks for the long-short portfolio sorted on innosimm. We construct real dividend growth rate following previous literature (see e.g. Campbell and Shiller, 1988; Bansal, Dittmar and Lundblad, 2005; Hansen, Heaton and Li, 2005, 2008; Bansal, Kiku and Yaron, 2016). We detail the construction method in Appendix B.3. Importantly we account for stock entries and exits when computing a portfolio’s dividend growth rate. Table 4 shows that for both measures of long-run risks, high-innosimm industries have higher dividend growth rates, which are also more exposed to long-run risks.

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21We are grateful to Amir Yaron for sharing data on the filtered consumption growth.
Table 4: Long-run-risk exposures of real dividend growth for the long-short industry portfolio sorted on innosimm (Q5 − Q1) (quarterly analysis).

<table>
<thead>
<tr>
<th>Spreads of dividend growth (annualized, %)</th>
<th>2.09</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exposure to LRR measured by realized consumption growth</td>
<td>7.68***</td>
</tr>
<tr>
<td>Exposure to LRR measured by filtered consumption growth</td>
<td>6.23***</td>
</tr>
</tbody>
</table>

Note: This table shows the exposure of real dividend growth to long-run risks for the long-short industry portfolios sorted on innosimm. We measure long-run risks using both realized consumption growth and filtered consumption growth as in Schorfheide, Song and Yaron (2018). For the first LRR measure, we regress the 4-quarter cumulative dividend growth of the long-short innosimm portfolios on the lagged 8-quarter cumulative realized consumption growth (annualized): \[ \sum_{j=1}^{4} (D_{Q5,t+j} - D_{Q1,t+j}) = \alpha + \beta \sum_{j=0}^{7} \hat{\eta}_{t-j}/2 + \epsilon_t, \] where \( \hat{\eta}_t \) is the realized consumption growth shock. For the second LRR measure, we replace the 8-quarter realized cumulative consumption growth in the above regression \( \sum_{j=0}^{7} \hat{\eta}_{t-j}/2 \) with annualized quarterly filtered consumption growth. We exclude financial firms and utility firms from the analysis. We include t-statistics in parentheses. Standard errors are computed using the Newey-West estimator allowing for serial correlation in returns. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels.

4.2.3 Double-Sort Analyses

We conduct a number of double-sort analyses for robustness checks. As shown in Table 5, the innosimm spreads are robust after controlling for measures of product similarity, innovation originality, asset growth rate, income elasticity of demand, and the durability of firms’ outputs, suggesting that the innovation similarity channel that our paper proposes...
cannot be explained by these alternative measures.

Table 5: Double-sort analyses (monthly analysis).

<table>
<thead>
<tr>
<th>Double-sort variables</th>
<th>Excess returns (%)</th>
<th>FF3F (%)</th>
<th>FF4F (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Product similarity</td>
<td>2.15**</td>
<td>3.81***</td>
<td>3.72***</td>
</tr>
<tr>
<td></td>
<td>[1.92]</td>
<td>[4.27]</td>
<td>[4.81]</td>
</tr>
<tr>
<td>Innovation originality</td>
<td>2.83***</td>
<td>4.31***</td>
<td>3.70***</td>
</tr>
<tr>
<td></td>
<td>[3.36]</td>
<td>[3.27]</td>
<td>[3.59]</td>
</tr>
<tr>
<td>Asset growth rate</td>
<td>3.37***</td>
<td>4.91**</td>
<td>4.54***</td>
</tr>
<tr>
<td></td>
<td>[2.60]</td>
<td>[3.75]</td>
<td>[4.12]</td>
</tr>
<tr>
<td>Income elasticity of demand</td>
<td>3.76***</td>
<td>5.45***</td>
<td>4.78***</td>
</tr>
<tr>
<td></td>
<td>[2.98]</td>
<td>[3.20]</td>
<td>[3.29]</td>
</tr>
<tr>
<td>Durability of firms’ outputs</td>
<td>3.66**</td>
<td>3.96**</td>
<td>3.69**</td>
</tr>
<tr>
<td></td>
<td>[2.27]</td>
<td>[2.07]</td>
<td>[2.48]</td>
</tr>
</tbody>
</table>

Note: This table shows the average excess returns and alphas from double-sort analyses. In the double-sort analyses, we first sort the 4-digit SIC industries into three groups based on measures of product similarity, innovation originality or durability of firms’ outputs in June of year t. We then sort firms within each group into five quintiles based on innoSimm in year t – 1. Once the portfolios are formed, their monthly returns are tracked from July of year t to June of year t + 1. Product similarity (Hoberg and Phillips, 2016) is derived from text analysis based on the business description in 10-K filings. Innovation originality is constructed following Hirshleifer, Hsu and Li (2017) to capture the patents’ originality. In particular, we count the number of unique technology classes contained in a patent’s citations/reference list. We then obtain the industry-level innovation originality measure by averaging the number of classes across all patents in a 4-digit SIC industry every year. Asset growth rate is the growth rate of the total asset. We obtain the industry-level asset growth rate by averaging the firm-level asset growth rate in a 4-digit SIC industry every year. We estimate the industry-level income elasticity of demand based on the representative consumer’s income and expenditures on different products (see Appendix B.5 for details). The durability of firms’ output comes from Gomes, Kogan and Yogo (2009), who classify each SIC industry into six categories (durables, non-durables, services, private domestic investment, government, and net exports) based on its contributions to final demand. We exclude financial firms and utility firms from the analysis. We include t-statistics in parentheses. Standard errors are computed using the Newey-West estimator allowing for serial correlation in returns. We annualize average excess returns and alphas by multiplying by 12. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels.

4.2.4 The Impact of Antitrust Enforcement

We now test whether the return spreads across industries with different innoSimm is related to the difference in their collusion incentive. We exploit the variation in collusion incentive due to antitrust enforcement, which punishes collusive behavior and thus dampens firms’ incentive to collude.

We examine the impact of antitrust enforcement on innoSimm spreads. We split all industries in each year into two groups based on whether they have experienced any antitrust enforcement in the past ten years.\(^\text{22}\) As shown in Table 6, the innoSimm spreads

\(^{22}\) The antitrust enforcement cases are hand collected from the websites of the U.S. Department of Justice (DOJ) and the Federal Trade Commission (FTC). DOJ provides 4-digit SIC codes for the firms in some of the cases. For the rest of DOJ cases and all FTC cases, we match the firms involved in antitrust enforcement to CRSP/Compustat and Capital IQ, from which we collect the 4-digit SIC codes of these firms.
are much smaller in the industries that have recently experienced antitrust enforcement, suggesting that our asset pricing findings in Table 2 are mainly driven by the difference in collusion incentive across industries with different innosimm.

Table 6: Antitrust enforcement and innosimm spreads (monthly analysis).

<table>
<thead>
<tr>
<th>Excess returns (%)</th>
<th>FF3F (%)</th>
<th>FF4F (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: Industries with antitrust enforcement in the past 10 years</td>
<td></td>
<td></td>
</tr>
<tr>
<td>−0.81</td>
<td>0.59</td>
<td>−0.44</td>
</tr>
<tr>
<td>[−0.33]</td>
<td>[0.24]</td>
<td>[−0.21]</td>
</tr>
<tr>
<td>Panel B: Industries without antitrust enforcement in the past 10 years</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.27**</td>
<td>5.44**</td>
<td>5.54***</td>
</tr>
<tr>
<td>[2.01]</td>
<td>[2.91]</td>
<td>[3.00]</td>
</tr>
</tbody>
</table>

Note: This table presents the average excess returns and alphas (both in percent) of the value-weighted long-short 4-digit SIC industry portfolio sorted on innosimm in the sub-samples with (Panel A) and without (Panel B) antitrust enforcement in past ten years. We exclude financial firms and utility firms from the analysis. We include t-statistics in parentheses. Standard errors are computed using the Newey-West estimator allowing for serial correlation in returns. We annualize the average excess returns and the alphas by multiplying by 12. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels.

4.3 Sensitivity of Profit Margins and Sales to Long-Run Risks

We test our model’s mechanism in this subsection. Our model implies that, all else equal, firms in high-innosimm industries endogenously have higher profit margins and more comparable market shares. We construct two measures of the industry-level profit margins. The first measure is based on the sales and costs of goods sold in the Compustat data. The second measure is derived from the NBER-CES Manufacturing Industry Database. Both data have their own advantages. The Compustat data cover public firms from all industries, while the NBER-CES data cover both public firms and private firms in the manufacturing industry. In Table 7, we show that profit margins are positively associated with innosimm. This relation is robust for both measures of profit margins. The coefficient of innosimm is economically significant and comparable across the two measures of profit margins. According to the regression with year fixed effects (columns 2 and 4), a one standard deviation increase in innosimm is associated with a 2.47-percentage-point increase in the Compustat-based profit margins and an 3.19-percentage-point increase in the NBER-CES-based profit margin. In addition, consistent with our model, we find that the dispersion of market shares is negatively associated with innosimm.

Our model predicts that profit margins and sales are more exposed to long-run
Table 7: Innovation similarity, profit margins, and market share dispersion (yearly analysis).

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Profit margins, Compustat (%)</td>
<td>Profit margins, NBER-CES (%)</td>
<td>The dispersion of market shares (%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Innosimm&lt;sub&gt;t&lt;/sub&gt;</td>
<td>2.57***</td>
<td>2.47***</td>
<td>3.29***</td>
<td>3.19***</td>
<td>−1.26***</td>
<td>−1.61***</td>
</tr>
<tr>
<td>[4.31]</td>
<td>[4.08]</td>
<td>[3.65]</td>
<td>[3.51]</td>
<td>[−2.60]</td>
<td>[-3.26]</td>
<td></td>
</tr>
<tr>
<td>Year FE</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>9212</td>
<td>9212</td>
<td>2787</td>
<td>2787</td>
<td>8967</td>
<td>8967</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.017</td>
<td>0.019</td>
<td>0.063</td>
<td>0.072</td>
<td>0.008</td>
<td>0.033</td>
</tr>
</tbody>
</table>

Note: This table shows the relation of the innovation similarity with profit margins and the dispersion of market shares. The observation in this analysis is at the 4-digit SIC industry-year level. In columns (1) and (2), the dependent variables are the industry-level Compustat-based profit margins. In columns (3) and (4), the dependent variables are the industry-level NBER-CES-based profit margins. Profit margins are computed as in Figure 1. The dispersion of market shares is defined as the standard deviation of all firms’ market shares (measured by sales) within the 4-digit SIC industry. The sample spans from 1988 to 2017 in columns (1), (2), (5), and (6), spans from 1988 to 2011 in columns (3) and (4). Standard errors are clustered by the 4-digit SIC industry and year. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels.

Table 8: Sensitivity of profit margins and sales to long-run risks (yearly analysis).

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>\ln (Profit margins&lt;sub&gt;t+1&lt;/sub&gt;/Profit margins&lt;sub&gt;t&lt;/sub&gt;)</td>
<td>\ln (Sales&lt;sub&gt;t+1&lt;/sub&gt;/Sales&lt;sub&gt;t&lt;/sub&gt;)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>\hat{x}_t \times \text{innosimm}_t</td>
<td>0.24***</td>
<td>0.25*</td>
<td>0.27*</td>
<td>0.25***</td>
<td>0.28**</td>
<td>0.29*</td>
<td>0.29*</td>
<td>0.28**</td>
</tr>
<tr>
<td>\hat{x}_t \times \text{income elasticity of demand}_t</td>
<td>−0.30*</td>
<td>0.11</td>
<td>[−1.72]</td>
<td>[0.83]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>\hat{x}_t \times \text{luxury industries}_t</td>
<td>−0.64*</td>
<td>0.15</td>
<td>[−1.95]</td>
<td>[0.62]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>\hat{x}_t \times \text{durable industries}_t</td>
<td>−0.64*</td>
<td>0.00</td>
<td>[−1.70]</td>
<td>[0.00]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Industry FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>8843</td>
<td>6978</td>
<td>6978</td>
<td>8843</td>
<td>8867</td>
<td>6994</td>
<td>6994</td>
<td>8867</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.029</td>
<td>0.034</td>
<td>0.034</td>
<td>0.029</td>
<td>0.078</td>
<td>0.084</td>
<td>0.084</td>
<td>0.078</td>
</tr>
</tbody>
</table>

Note: This table shows the sensitivity of industry-level profit margins and sales to long-run risks. Profit margins are computed based on Compustat data as explained in Figure 1. Long-run risks are measured by the annualized filtered consumption growth in the last quarter of year \(t\) (\(\hat{x}_t\)). The sample spans from 1988 to 2015. We include t-statistics in parentheses. Standard errors are clustered by the 4-digit SIC industry and year. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels.
4.4 Product Prices and Innovation Similarity

The key mechanism of our model is that high-innosimm industries collude on higher product prices in good times, and their prices drop more in bad times due to endogenous price wars. We test this mechanism in this subsection. In particular, we study the changes of product prices using comprehensive product-level data, which allow us to examine the pricing behavior across industries with different innosimm. Our findings suggest that high-innosimm industries are more exposed to price war risks, and they are also more likely to engage in price wars in the periods after the Lehman crash.

4.4.1 The Nielsen Data for Product Prices

We use the Nielsen Retail Measurement Services scanner data to measure product price changes.\footnote{The Nielsen data are obtained from the Kilts Center for Marketing at the University of Chicago Booth School of Business (https://www.chicagobooth.edu/research/kilts/datasets/nielsen). The data have been widely used in the macroeconomics literature (e.g. Aguiar and Hurst, 2007; Broda and Weinstein, 2010; Hottman, Redding and Weinstein, 2016; Argente, Lee and Moreira, 2018; Jaravel, 2018).} The Nielsen data record prices and quantities of every unique product that had any sales in the 42,928 stores of more than 90 retail chains in the U.S. market from January 2006 to December 2016. In total, the Nielsen data consist of more than 3.5 million unique products identified by the Universal Product Codes (UPCs), and the data represent 53% of all sales in grocery stores, 55% in drug stores, 32% in mass merchandisers, 2% in convenience stores, and 1% in liquor stores (see e.g. Argente, Lee and Moreira, 2018). We use the product-firm links provided by GS1, the official source of UPCs in the U.S., to match products in the Nielsen data to firms in CRSP/Compustat and Capital IQ. In Appendix B.4, we detail the matching procedure. Our merged data cover the product prices of 472 4-digit SIC industries.

4.4.2 Product Prices around the Lehman Crash

To begin, we examine the changes in media coverage about price war around the Lehman crash, the period during which the U.S. economy experienced a prominent negative long-run-risk shock (see Panel C of Figure 4). Because the Nielsen data mainly cover consumer goods sold by retailers and wholesalers, we focus on media coverage of the consumer goods industry sector and the retail/wholesale industry sector. Panel A of Figure 5 shows that, after the Lehman crash, the number of articles covering price wars...
increased dramatically. This pattern remains robust when we normalize the number of articles with “price wars” using the total number of articles in these newspapers (see Panel B of Figure 5).

Figure 5: Changes of price war news index and product prices around the Lehman crash.

Next, we examine the changes in product prices around the Lehman crash. We sort all industries into tertiles based on innoisimm. Table 9 quantifies the changes in product prices among high-innoisimm industries (Tertile 3) relative to low-innoisimm industries (Tertile 1) around the Lehman crash. In particular, we restrict the sample to
industries in Tertile 1 and Tertile 3, and create a Tertile-3 indicator for the latter group. We also create a post-Lehman indicator that equals one for observations in Oct. 2008 and thereafter. We then regress the percent change in product prices on the Tertile-3 indicator, the post-Lehman indicator, and an interaction term between these two indicators. The coefficient of the interaction term is negative and statistically significant across different regression specifications (see column 1 - 4), suggesting that product prices in high-innosimm industries reduce significantly relative to those in low-innosimm industries after the Lehman crash. The difference in product prices is economically significant. According to the regression without industry fixed effects (column 1), product prices decrease by 4.98% in high-innosimm industries after the Lehman crash, relative to those of low-innosimm industries.

Panel C of Figure 5 visualizes the difference in average product prices between low-innosimm and high-innosimm industries in the 36-month period around the Lehman crash. The plot clearly shows that product prices in high-innosimm industries increase at a much lower rate after the Lehman crash. These findings support our model’s prediction that high-innosimm industries are more likely to engage in price wars following negative long-run-risk shocks.

Table 9: Product prices around the Lehman crash (monthly analysis).

<table>
<thead>
<tr>
<th>Similarity measure</th>
<th>Innosimm</th>
<th>Prodsimm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tertile-3 similarity, ( t \times ) post Lehman crash, ( t )</td>
<td>-4.98***</td>
<td>-5.12**</td>
</tr>
<tr>
<td></td>
<td>[-2.85]</td>
<td>[-2.64]</td>
</tr>
<tr>
<td>Tertile-3 similarity, ( t )</td>
<td>-2.32</td>
<td>-4.73</td>
</tr>
<tr>
<td></td>
<td>[-1.05]</td>
<td>[-0.93]</td>
</tr>
<tr>
<td>post Lehman crash, ( t )</td>
<td>0.40</td>
<td>1.03</td>
</tr>
<tr>
<td></td>
<td>[0.19]</td>
<td>[0.50]</td>
</tr>
<tr>
<td>Industry FE</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>5106</td>
<td>5106</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.003</td>
<td>0.053</td>
</tr>
</tbody>
</table>

Note: This table shows the changes in product prices around the Lehman crash. The dependent variable is the annualized monthly percent change in product prices of 4-digit SIC industries. Product prices are obtained from the Nielsen Data. To compute the monthly percent change in product prices for 4-digit SIC industries, we first compute the transaction-value weighted price for each product across all stores in each month. We then calculate the monthly percent change in prices for each product. Finally, we compute the value-weighted percent change in product prices for each 4-digit SIC industry based on the transaction values of the industry’s products. In column (1) and (2), the similarity measure is innosimm. In column (3) and (4), the similarity measure is prodsimm. We consider the 36-month period around the Lehman crash. We include t-statistics in parentheses. Standard errors are clustered by the 4-digit SIC industry and month. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels.
We continue to extend our analysis to all industries. According to the theory, the Lehman crash brings about product price changes in all industries, but with different magnitudes presumably depending on the industry’s innosimm. To understand how the percent change in product prices varies with industry-level innosimm, or what we call the price-innosimm sensitivity, we regress the percent change in product prices on innosimm, the post-Lehman indicator, and an interaction term between innosimm and the post-Lehman indicator. The regression results are presented in Table 11. The price-innosimm sensitivity before the Lehman crash is given by the coefficient on innosimm; the price-innosimm sensitivity after the Lehman crash is given by the sum of the coefficients on innosimm and the interaction term; and the change in price-innosimm sensitivity owing to the Lehman crash is given by the coefficient on the interaction term. Table 11 shows that the coefficient of the interaction term is negative, indicating that high-innosimm industries are more affected by the Lehman crash and their product prices decrease relatively more (or increase relatively more slowly) compared to low-innosimm industries. Panel D of Figure 5 visualizes the monthly price-innosimm sensitivity. It is evident that the price-innosimm sensitivity reduces significantly after the Lehman crash.

We also examine the product prices around the Lehman crash for industries with different prodsimm. We find that product prices do not move differently for high-prodsimm industries and low-prodsimm industries (Panel E of Figure 5, columns 3 and 4).
Moreover, we observe little change in price-prodsimm sensitivity following the Lehman crash (Panel F of Figure 5, columns 3 and 4 in Table 10).

4.4.3 Product Prices, Innosimm, and Long-Run Risks

Having tested a particular period around the Lehman crash, we now extend our analysis to the whole time series covered by the Nielsen data from 2006 to 2016. Specifically, we regress the percent change in product prices on innosimm, long-run growth rate, and the interaction term between innosimm and the long-run growth rate. Table 11 shows that the coefficients of the interaction term are positive and statistically significant, suggesting that high-innosimm industries are associated with higher price war risks.

In addition, we interact the long-run growth rate with the income elasticity of demand, indicator variables for industries producing luxury goods and durable goods. We find that the coefficient for the interaction term between innosimm and long-run growth rate remains positive and significant after including these control variables, suggesting that the positive association between innosimm and price war risks cannot be explained by other industry characteristics.

5 Quantitative Analyses

In this section, we conduct quantitative analyses. We first extend the baseline model with additional ingredients. Then, we calibrate the extended model’s parameters and examine whether our model can replicate the main asset pricing patterns from the data.

5.1 The Extended Model

To conduct quantitative analyses, we introduce additional ingredients to the model with distinctive innovation developed in Section 3.

In our baseline model, the success rate of innovations is exogenous, but in reality, firms can invest more in R&D to increase the success rates of innovations. We endogenize firms’ innovation decisions by assuming that innovation succeeds at rate $\mu_i s_{ij,t}$ if firm $j$ installs $s_{ij,t} d\ell$ units of R&D projects over $[t, t + d\ell]$. The flow cost of buying R&D projects is convex, given by $(s_{ij,t} + \xi s_{ij,t}^2) M_{ij,t} d\ell$. We assume that innovation costs are proportional to the peer firm’s customer base $M_{ij,t}$ because the benefit from snatching the peer firm’s
Table 11: Price-innosimm sensitivity and long-run growth rate (quarterly analysis).

<table>
<thead>
<tr>
<th></th>
<th>Column 1</th>
<th>Column 2</th>
<th>Column 3</th>
<th>Column 4</th>
<th>Column 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>One-year ahead percent change in product prices ((\sum_{j=1}^{4} price_{gr,t+j}))</td>
<td>(\hat{x}_t \times \text{innosimm}_t)</td>
<td>1.17**</td>
<td>1.32**</td>
<td>1.30**</td>
<td>1.32**</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[2.37]</td>
<td>[2.48]</td>
<td>[2.43]</td>
<td>[2.47]</td>
</tr>
<tr>
<td></td>
<td>(\hat{x}_t \times \text{income elasticity of demand}_t)</td>
<td>-0.22</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>[-0.49]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(\hat{x}_t \times \text{luxury industries}_t)</td>
<td>-0.07</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>[-0.24]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(\hat{x}_t \times \text{durable industries}_t)</td>
<td>0.48</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>[1.54]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(\hat{x}_t)</td>
<td>-0.20</td>
<td>0.16</td>
<td>0.44</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[-0.13]</td>
<td>[0.12]</td>
<td>[0.31]</td>
<td>[0.13]</td>
</tr>
<tr>
<td></td>
<td>(\text{innosimm}_t)</td>
<td>0.00</td>
<td>0.08</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[0.17]</td>
<td>[1.57]</td>
<td>[1.56]</td>
<td>[1.56]</td>
</tr>
<tr>
<td>Industry FE</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>7338</td>
<td>7338</td>
<td>7335</td>
<td>7335</td>
<td>7335</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.002</td>
<td>0.167</td>
<td>0.167</td>
<td>0.167</td>
<td>0.167</td>
</tr>
</tbody>
</table>

Note: This table shows the sensitivity of percent changes in product prices to consumption growth across the 4-digit SIC industries with different innosimm. The dependent variable is the industry-level annualized percent change in product prices from quarter \(t + 1\) to quarter \(t + 4\). Long-run risks are measured by annualized filtered consumption growth in quarter \(t\) (denoted as \(\hat{x}_t\)). We estimate the income elasticity of demand based on the representative consumer’s income and expenditures on different products (see Appendix B.5 for details). We define luxury industries as the industries with income elasticity of demand larger than one. The durability of firms’ output comes from Gomes, Kogan and Yogo (2009). The sample period of columns is from 2006 to 2016. We include t-statistics in parentheses. Standard errors are clustered by the 4-digit SIC industry and year. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels.

customer base is proportional to \(M_{ij,t}\). Like setting prices, innovation decisions are also strategic and have to be solved as a Nash-equilibrium outcome.

Moreover, we allow firms to make investment in customer bases to endogenize dividend growth. In particular, by investing \(q_{ij,t}dt\) over \([t, t + dt]\), firm \(j\) can create new customer bases \(\mu_q q_{ij,t}M_{ij,t}dt\) for itself and \(\mu_q q_{ij,t}M_{ij,t}dt\) for the peer firm \(\tilde{j}\). The flow cost of investment is quadratic, given by \((q_{ij,t} + \xi_q q_{ij,t}^2)M_{ij,t}dt\). Different from innovations, there is a free-rider problem in investment (Holmstrom, 1982; Battaglini, 2006; Bond and Pande, 2007), because one firm’s investment also increases the other firm’s customer base. Thus, the investment decision is not strategic because both firms’ customer bases increase by the same proportion, leaving customer base shares and market power unchanged. High dividend growth occurs in periods with high long-run growth rates, both because of high consumption growth and firms’ high investment \(q_{ij,t}\) in customer bases motivated by high collusive prices. Thus, the endogenous price wars provide a micro foundation.
for the dividend-leverage ratio on expected consumption growth in the models of Abel (1999) and Bansal and Yaron (2004).

The dynamics of customer base (3.2) is modified as

\[ \text{d} M_{ij,t} = -\rho M_{ij,t} \text{d}t + z \frac{C_{ij,t}}{C_t} \text{d}t + \tau_{ij,t} M_{ij,t} \text{d}I_{ij,t} - \tau_{ij,t} M_{ij,t} \text{d}I_{ij,t} + \mu_q q_{ij,t} M_{ij,t} \text{d}t, \]  

(5.1)

where \( I_{ij,t} \) and \( I_{ij,t} \) are independent Poisson processes with intensities \( \mu_{s,ij,t} \) and \( \mu_{s,j,t} \), capturing the success of firm \( j \) and \( j' \)'s innovations.

5.2 Calibration

We discipline the parameters based on both existing estimates and micro data (see Table 12) without referring to asset pricing information, and we examine whether the calibrated model can quantitatively explain the observed asset pricing patterns.

The process of aggregate consumption is calibrated following Bansal and Yaron (2004). We set the persistence of expected growth rate to be \( \kappa = 0.49 \), so that the autocorrelation of annual consumption growth rates is 0.49. We set \( \theta = 0.018 \) and \( \sigma_c = 0.029 \) so that the average annual consumption growth rate is 1.8% and its standard deviation is about 2.9%. Following Bansal and Yaron (2004), we set \( \phi = 0.044 \), indicating that the predictable variation in consumption growth is 4.4%. Following the standard practice, we set the subjective discount factor \( \beta = 0.976 \), the risk aversion parameter \( \gamma = 10 \), and the inter-temporal elasticity of substitution \( \psi = 1.5 \).

We set the within-industry elasticity of substitution \( \eta = 10 \) and the across-industry elasticity of substitution to be \( \epsilon = 2 \), broadly consistent with the values of Atkeson and Burstein (2008). The unit flow cost of production \( \omega \) is normalized to be one. We set the investment cost \( \zeta_i = 0.5 \) and the innovation cost \( \zeta_s = 2 \). The success rate of innovation is set to be \( \mu_s = 3 \) and the efficiency of investment is set to be \( \mu_q = 0.3 \). We set the customer base depreciation rate to be \( \rho = 0.15 \), within the range of 15%-25% estimated by Gourio and Rudanko (2014). We set \( z = 0.05 \) to ensure that customer base is sticky and long-term in nature (Gourio and Rudanko, 2014; Gilchrist et al., 2017). The price inspection rate is set to be \( \phi = 0.15 \), implying that price changes by about 3-6% for a one-percent change in annual consumption growth rates, roughly consistent with our data.

We allow the industry characteristic \( \lambda_{i,t} \) to take 11 values, i.e. \( \lambda_{i,t} \in \{0.9, 0.91, 0.92, ..., 1\} \). The characteristic \( \lambda_{i,t} \) remains the same unless it is hit by a Poisson shock with rate \( \epsilon \).
Table 12: Calibration and parameter choice.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbol</th>
<th>Value</th>
<th>Parameters</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average long-run consumption growth rate</td>
<td>( \bar{\theta} )</td>
<td>0.018</td>
<td>Persistence of expected growth rate</td>
<td>( \kappa )</td>
<td>0.49</td>
</tr>
<tr>
<td>Exposure to long-run growth risks</td>
<td>( \varphi_{\theta} )</td>
<td>0.044</td>
<td>Volatility of consumption growth</td>
<td>( \sigma_c )</td>
<td>0.029</td>
</tr>
<tr>
<td>Customer base adjustment friction</td>
<td>( \chi )</td>
<td>0.3</td>
<td>Cost of production</td>
<td>( \omega )</td>
<td>1</td>
</tr>
<tr>
<td>Customer base accumulation rate</td>
<td>( \zeta )</td>
<td>0.05</td>
<td>Customer base depreciation rate</td>
<td>( \rho )</td>
<td>0.15</td>
</tr>
<tr>
<td>Across-industry elasticity of substitution</td>
<td>( \epsilon )</td>
<td>2</td>
<td>Within-industry elasticity of substitution</td>
<td>( \eta )</td>
<td>10</td>
</tr>
<tr>
<td>Customer base stealing (incremental)</td>
<td>( \tau_i )</td>
<td>0.05</td>
<td>Customer base stealing (distinctive)</td>
<td>( \tau_d )</td>
<td>0.90</td>
</tr>
<tr>
<td>Price inspection rate</td>
<td>( \phi )</td>
<td>0.15</td>
<td>Risk aversion</td>
<td>( \gamma )</td>
<td>10</td>
</tr>
<tr>
<td>Inter-temporal elasticity of substitution</td>
<td>( \psi )</td>
<td>1.5</td>
<td>Subjective discount factor</td>
<td>( \beta )</td>
<td>0.976</td>
</tr>
<tr>
<td>Investment costs</td>
<td>( \zeta_q )</td>
<td>0.5</td>
<td>Investment efficiency</td>
<td>( \mu_q )</td>
<td>0.3</td>
</tr>
<tr>
<td>Innovation costs</td>
<td>( \zeta_s )</td>
<td>2</td>
<td>Innovation efficiency</td>
<td>( \mu_s )</td>
<td>3</td>
</tr>
<tr>
<td>Persistence of industry characteristics</td>
<td>( \varepsilon )</td>
<td>0.03</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Conditional on receiving the Poisson shock, a new characteristic is randomly drawn with equal probabilities of each value, implying that on average across all industries, incremental innovation succeeds every 6 months and distinctive innovation succeeds every 10 years. We set \( \varepsilon = 0.05 \) to make \( \lambda_{i,t} \) a persistent industry characteristic. The within-industry customer base stealing due to incremental and distinctive innovation are set to be \( \tau_i = 0.05 \) and \( \tau_d = 0.90 \).

### 5.3 Quantitative Results

Now we check whether our model can quantitatively replicate the main asset pricing patterns presented in Table 2. In each year \( t \), we sort the simulated firms into five quintiles based on their \( \lambda_{i,t} \) at the beginning of the year. We then compute the value-weighted average excess return of each quintile’s portfolio. Table 13 shows that the model-implied difference in average excess returns between Q1 and Q5 is about 3.28%. These numbers are quantitatively consistent with the findings in Table 2. If we simulate the model without price war risks (by focusing on constant collusive prices), the average risk premium decreases by about 33%, and the model-implied spread largely is largely reduced to 0.43%.

### 6 Conclusion

In this paper, we explore the implication of price war risks. We develop a general-equilibrium asset pricing model incorporating dynamic supergames of price competition.
Table 13: Average excess returns of portfolios in data and model.

<table>
<thead>
<tr>
<th>E[R] − rf</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Baseline</td>
</tr>
<tr>
<td>Quintile 1 (%)</td>
<td>6.13</td>
<td>4.02</td>
</tr>
<tr>
<td>Quintile 5 (%)</td>
<td>9.54</td>
<td>7.30</td>
</tr>
<tr>
<td>Q5 – Q1 (%)</td>
<td>3.41</td>
<td>3.28</td>
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</table>

among firms. In our model, price wars can arise endogenously from declines in long-run consumption growth, since firms become effectively more impatient for cash flows and their incentives to undercut prices become stronger. The exposure to price war risks reflect predictable and persistent heterogeneous industry characteristics. Firms in industries with a higher capacity of distinctive innovation are more immune to price war risks due to the higher likelihood of creative destruction and market disruption. Exploring detailed patent and product price data, we found evidence for the existence of price war risks. Moreover, that endogenous price war risks are priced in the cross section.

References


Appendix

A Headline Quotes for Price Wars and Stock Returns

We cite a few recent media headlines on how price wars can depress firms’ stock returns.

- “Best Buy Co. shares plunged 11% Tuesday, after the electronics chain warned investors about price war fears.” – The Wall Street Journal on November 20th of 2013.
- “Target shares dive as it shifts to cut-price strategy.” – Financial Times on February 28th of 2017.
- “Price war eats into the profits of pharmaceutical wholesalers and manufacturers alike and erases billions of dollars of the market value in recent days” – The Wall Street Journal on August 5th of 2017.
- “Investors Purge Infinera Stock on Price War Concerns, Ignore Q1 Results.” – SDxCentral on May 10th of 2018.
- “Coffee price war takes jolt out of Dunkin’ results.” – Financial Times on September 27th of 2018.

B Data

B.1 Industry Concentration Ratio

We use the U.S. Census concentration ratio data from 1987, 1992, 1997, 2002, 2007, and 2012 to compute the time-series maximal and mean revenue shares for the top 4 firms (CR4) and top 8 firms (CR8) in each 4-digit SIC industry. The concentration ratios are at the 6-digit NAICS level after 1997. We follow Ali, Klasa and Yeung (2008) and convert the ratios to 4-digit SIC levels. Figure B.1 plots the histogram of the max CR4 (Panel A1), max CR8 (Panel B1), mean CR4 (Panel A2), and mean CR8 (Panel B2) in all 4-digit SIC industries. Red vertical lines represent the cross-sectional mean values.

B.2 Match PatentView with CRSP/Compustat/Capital IQ

In this Appendix, we detail the matching procedure for the data from PatentView, CRSP/Compustat, and Capital IQ. We first drop patent assignees that are classified as individuals and government agencies by PatentView, because these assignees are not associated with any particular industry. We then clean assignee names in PatentView and firm names in CRSP/Compustat and Capital IQ following the approach of Hall, Jaffe and Trajtenberg (2001). To elaborate, we remove punctuations and clean special characters. We then

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Note: This figure plots the histogram of the top 4 and top 8 firms’ total revenue share in 4-digit SIC industries. We use the U.S. Census concentration ratio data from 1987, 1992, 1997, 2002, 2007, and 2012 to compute the time-series maximal and mean revenue shares for the top 4 firms (CR4) and top 8 firms (CR8) in each 4-digit SIC industry. The concentration ratios are at the 6-digit NAICS level after 1997. We follow Ali, Klasa and Yeung (2008) and convert the ratios to 4-digit SIC levels. We plot the histogram of the max CR4 (Panel A1), max CR8 (Panel B1), mean CR4 (Panel A2), and mean CR8 (Panel B2) in all 4-digit SIC industries. Red vertical lines represent the cross-sectional mean values.

Figure B.1: Top 4 and top 8 firms’ revenue share in 4-digit SIC industries.

transform the names into upper cases and standardize them. For example, “INDUSTRY” is standardized to be “IND”; and “RESEARCH” is standardized to be “RES”; and corporate form words (e.g. “LLC” and “CORP”) are dropped, etc.

Match PatentView with CRSP/Compustat. We match patent assignees in PatentView with firms in CRSP/Compustat based on standardized names. We use the fuzzy name matching algorithm (matchit command in Stata), which generates the matching scores (Jaccard index) for all name pairs between patent assignees in PatentView and firms in CRSP/Compustat.\textsuperscript{25} We obtain a pool of potential matches\textsuperscript{25}Jaccard index measures the similarity between finite sample sets, and is defined as the size of the intersection divided by the size of the union of the sample sets. Jaccard index ranges between 0 and 1, reflecting none to perfect similarity.
based on two criteria: (1) we require the matching score to be higher than 0.6; (2) we require the first three letters of patent assignees to be the same as those of firms in CRSP/Compustat.\textsuperscript{26} We then go through all potential matches to manually identify exact matches.\textsuperscript{27}

As pointed out by Lerner and Seru (2017), one major challenge for linking patent data to CRSP/Compustat is that some patent assignees are subsidiaries of firms in CRSP/Compustat. For these assignees, we cannot directly match them with CRSP/Compustat based on firm names. To deal with this challenge, the NBER patent data (Hall, Jaffe and Trajtenberg, 2001) use the 1989 edition of the Who Owns Whom directory (now known as the D&B WorldBase®- Who Owns Whom) to match subsidiaries to parent companies. Kogan et al. (2017) purged the matches identified by the NBER patent data, and extended the matching between patent data and CRSP/Compustat to 2010. For those patent assignees who are subsidiaries of firms in CRSP/Compustat, we augment our matches by incorporating the data of Kogan et al. (2017) for patents granted before 2010. For patents granted after 2010, we use the subsidiary-parent link table from the 2017 snapshot of the Orbis data to match subsidiaries in PatentView to their parent firms in CRSP/Compustat.

**Match PatentView with Capital IQ.** We match the remaining patent assignees in PatentView with firms in Capital IQ following the same matching procedure. To keep the workload manageable, we drop firms in Capital IQ whose assets are worth less than $100 million (in 2017 dollars). Because we focus on the U.S. product market, we also drop foreign firms whose asset values are below the 90th percentile of the asset value distribution among firms in the CRSP/Compustat sample in each year, respectively. This is because small foreign firms are less likely to have a material impact on the competition environment of the U.S. product market. We match PatentView to Capital IQ directly using the information on subsidiaries provided by Capital IQ.

**B.3 Construct Dividend Growth**

We follow previous studies (see e.g. Campbell and Shiller, 1988; Bansal, Dittmar and Lundblad, 2005; Hansen, Heaton and Li, 2005, 2008; Bansal, Kiku and Yaron, 2016) to construct dividend growth rates of portfolios.

Denote $V_{0t}$ as the market value of all firms in a given portfolio. Denote the value of this portfolio at date $t+1$ to be $V_{t+1}$. The aggregate dividends for data $t+1$ for this portfolio is $D_{t+1}$. The total return on the portfolio between $t$ and $t+1$ is:

$$R_{t+1} = \frac{V_{t+1} + D_{t+1}}{V_{0t}} = h_{t+1} + d_{t+1}. \quad (B.1)$$

where $h_{t+1}$ is the price appreciation, which represents the ratio of the value at time $t+1$ to time $t$ (i.e., $\frac{V_{t+1}}{V_{0t}}$), while $d_{t+1}$ is the dividend yield, which represents the total dividends paid by at time $t+1$ divied by

\textsuperscript{26}These two matching criteria are sufficiently conservative to ensure that exact matches are included in the pool of potential matches. For example, among all the exact matches in the first quarter of 2016, 98% of them satisfy the two matching criteria and are included in our pool of potential matches.

\textsuperscript{27}We rely on assignee names in PatentView and firm names in CRSP/Compustat to identify matches. In addition, we use location information in both datasets to facilitate the matching process.
portfolio value at time $t$ (i.e., $\frac{D_{t+1}}{V_{0t}}$).

Holding the portfolio composition constant (i.e., no exits and entries), the real dividend growth rate is:

$$\frac{D_{t+1}/PCE_{t+1}}{D_t/PCE_t} = \frac{d_{t+1}V_{0t}}{d_tV_{0(t-1)}} \frac{PCE_t}{PCE_{t+1}} = \frac{d_{t+1}h_t}{d_t} \frac{PCE_t}{PCE_{t+1}} = \frac{(R_{t+1} - h_{t+1})h_t}{R_t - h_t} \frac{PCE_t}{PCE_{t+1}}, \quad (B.2)$$

where $PCE$ is the personal consumption expenditure deflator from the U.S. BEA.

Because stocks move in and out of portfolios, we account for the entries and exits following the literature (see e.g. Hansen, Heaton and Li, 2005, 2008; Bansal, Kiku and Yaron, 2016) by adding an adjustment term. Specifically, the real dividend growth rate for a portfolio is:

$$\frac{V_t D_{t+1}/PCE_{t+1}}{V_{0t} D_t/PCE_t}, \quad (B.3)$$

where $V_t$ is exit value of the portfolio at time $t$ (i.e., the time $t$ market value of firms in the portfolio formed at time $t - 1$), and $V_{0t}$ is the market value of firms in the new position we initiate at time $t$. Plug equation B.2 into B.3, the real dividend growth rate for a portfolio is:

$$\frac{V_t D_{t+1}/PCE_{t+1}}{V_{0t} D_t/PCE_t} = \frac{V_t (R_{t+1} - h_{t+1})h_t}{V_{0t} R_t - h_t} \frac{PCE_t}{PCE_{t+1}}. \quad (B.4)$$

We calculate portfolio $R_t$ and $h_t$ by computing the value-weighted $RET$ and $RETX$ (both from CRSP) across firms within the portfolio. Since share repurchases are prevalent in our sample period, we follow Bansal, Dittmar and Lundblad (2005) and adjust the capital gain series for a given firm as following:

$$RETX^*_{t+1} = RETX_{t+1} \min\left(\left(\frac{H_{t+1}}{n_t}\right), 1\right), \quad (B.5)$$

where $n_t$ is the number of shares after adjusting for splits, stock dividends, etc using the CRSP share adjustment factor.

**B.4 Match Nielsen with CRSP/Compustat/Capital IQ**

We follow previous studies (e.g. Hottman, Redding and Weinstein, 2016; Argente, Lee and Moreira, 2018; Jaravel, 2018) to find the companies that own the products in the Nielsen data using the product-firm link table in the “GS1 US Data Hub | Company” data, which is provided by GS1 – the official source of UPCs in the U.S.\(^\text{28}\) We match 95.3% of the products in the Nielsen data with firms in the GS1 data. Our matching rate is the same as those reported by Argente, Lee and Moreira (2018) and Jaravel (2018). We further match the companies in the GS1 data to CRSP/Compustat and Capital IQ to find their SIC industry codes. The matching procedures are the same as patent matching. Our merged data cover products in 472 SIC industries.

\(^{28}\)The “GS1 US Data Hub | Company” data provide the company names, company addresses, and the UPC prefixes owned by the companies. More information about the “GS1 US Data Hub | Company” is available at: https://www.gs1us.org/tools/gs1-us-data-hub/company.
B.5 Income Elasticity of Demand

We estimate the income elasticity of demand at the 4-digit SIC industry level using the Consumer Expenditure Surveys from the Bureau of Labor Statistics (BLS) and the Nielsen data. We first obtain the representative consumer’s income and annual expenditures on various products (such as bread, ice cream, etc.) from the All CU Prepublication Means, Variances, and Percent reporting Tables. We calculate the income elasticity of demand (i.e. the percentage change in expenditure divided by the percentage change in income) for each product category. Next, we merge the product-category level income elasticity data from the BLS survey with the Nielsen data based on product descriptions, which allows us to link product categories to SIC industries. Finally, we compute the income elasticity of demand at the 4-digit SIC industry level by averaging the elasticities across all product categories in the same industries. Because the product-level Consumer Expenditure Surveys data are only available from 2013 to 2017, we make the assumption that the industry-level income elasticity of demand is a constant overtime.

C Illustration of Equilibrium Concepts

Our model is based on a general equilibrium framework with a continuum of industries. Within each industry, we formulate the two firms’ dynamic competition using stochastic game-theoretic models. In this Appendix, we illustrate the dynamic game-theoretic equilibrium within an industry in our baseline model. We start by illustrating the non-collusive equilibrium in Section C.1. We highlight that the strategic complementarity embedded in the non-collusive equilibrium is a crucial force that generates price wars during periods with low long-run consumption growth. We then illustrate the collusive equilibrium that naturally arises from the dynamic repeated interaction between the two firms. The collusive equilibrium is a sub-game perfect equilibrium that is endogenously sustained by using the non-collusive equilibrium as punishment. In Section C.3, we illustrate the IC constraints and the determination of collusive prices in the collusive equilibrium.

C.1 Non-Collusive Equilibrium

In the non-collusive equilibrium, the two firms simultaneously set prices, taking the other firm’s price as given. Thus, the equilibrium prices are determined by the intersection of the two firms’ optimal price as a function of the other firm’s price. Denote \( \hat{P}_{i1}^N (M_{i1}/M_i; P_{i2}) \) as firm 1’s optimal price as a function of its customer base share \( M_{i1}/M_i \) and firm 2’s price \( P_{i2} \). Similarly, we denote \( \hat{P}_{i2}^N (M_{i1}/M_i; P_{i1}) \) as firm 2’s optimal price as a function of firm 1’s customer base share \( M_{i1}/M_i \) and price \( P_{i1} \).

In Panel A of Figure C.2, the blue solid line plots firm 1’s optimal price as a function of firm 2’s price \( P_{i2} \), when the two firms have equal customer base shares (i.e. \( M_{i1}/M_i = 0.5 \)). The black dash-dotted line plots firm 2’s optimal price as a function of firm 1’s price \( P_{i1} \) for the same customer base share. The intersection of the two curves (the blue filled circle) determines the equilibrium prices, i.e. \( P_{i1}^N (0.5) \) and \( P_{i2}^N (0.5) \):

\[
P_{i1}^N (0.5) = \hat{P}_{i1}^N (0.5; P_{i2}^N (0.5)) \quad \text{and} \quad P_{i2}^N (0.5) = \hat{P}_{i2}^N (0.5; P_{i1}^N (0.5)).
\] (C.1)
The two firms set exactly the same prices when they have the same customer base shares. Both curves are upward sloping, indicating that there exists strategic complementarity in setting prices in the non-collusive equilibrium: both firms tend to set lower prices when the other firm’s price is lower. This is because when the other firm’s price is lower, the price elasticity of demand endogenously increases, motivating the firm to lower its own price. Because of such strategic complementarity, the non-collusive equilibrium features low prices and hence low profit margins for both firms. To see it clearly, suppose firm 2 sets $P_{i2} = 1.6$, then firm 1’s best response is to set $P_{i1} = 1.4$ ($A_1$). Given that firm 1’s price is lower than firm 2’s, firm 2 will further lower its price to $P_{i2} = 1.28$ ($A_2$). But then firm 2’s price is lower than firm 1’s, which triggers firm 1 to lower its price to $P_{i1} = 1.23$ ($A_3$), and so on, until the prices reach equilibrium values. Such price adjustments happen instantaneously in rational expectation equilibrium.\footnote{The dynamics of price adjustment is related to the old tradition that used Tâtonnement or Cobweb dynamics to capture the off-equilibrium adjustment of prices in Walrasian economies.}

In Panel B, we investigate how firms change prices when their customer base shares change. The blue solid line and black dash-dotted line represent the same benchmark case (i.e. $M_{i1}/M_i = 0.5$) as in Panel A. The red dashed and red dotted lines refer to the prices set by the two firms when firm 1’s customer
base share $M_{i1}/M_i$ increases from 0.5 to 0.8 (thus firm 2’s customer base share decreases from 0.5 to 0.2 accordingly). It is shown that firm 1’s optimal price function shifts upward and firm 2’s optimal price function shifts to the left, implying that both firms tend to set higher prices when their own customer base shares increase. Intuitively, there are two main reasons. First, when the customer base share is higher, setting low prices to further compete for customer bases is relatively more costly compared to setting high prices to profit from inertial customers. Second, the firm’s influence on the equilibrium price index increases with its customer base share (see equation 2.8). Therefore, a higher customer base share increases the firm’s market power and lowers the price elasticity of demand, resulting in higher prices.

Panel B also clearly illustrates the implication of strategic pricing. In the benchmark equilibrium ($N_0$), the prices are $P_{1,N_0}$ and $P_{2,N_0}$. A higher customer base share $M_{i1}/M_i$ shifts the equilibrium to $N_2$, and the new equilibrium prices satisfy $P_{1,N_2} > P_{1,N_0}$ and $P_{2,N_2} < P_{2,N_0}$. However, if firm 2 were to hold its price decisions unchanged (at the black-dashed line), the new equilibrium would be $N_1$, with $P_{1,N_1} > P_{1,N_2}$, indicating that firm 1 would raise its price more in response to the increase in its customer base share $M_{i1}/M_i$. Therefore, firm 1’s price is less responsive precisely because it anticipates that firm 2 would lower its price $P_{2}$ (as captured by the red dotted line). Such strategic concerns result in a smaller increase in firm 1’s price $P_{1}$, which helps prevent too much loss in its customer base share $M_{i1}/M_i$.

Panel C shows that when firm 1’s customer base share increases, firm 1’s value increases (the blue solid line) and firm 2’s value decreases symmetrically (black dashed line). Moreover, both firms set higher equilibrium prices when their customer base shares increase (see Panel D).

### C.2 Collusive Equilibrium

We now turn to the illustration of the collusive equilibrium. In the collusive equilibrium, both firms set prices according to the collusive pricing schedule $P_{ij}(M_{i1}/M_i, \theta_t)$.

In Panel A of Figure C.3, we compare the firm’s prices in the collusive equilibrium and the non-collusive equilibrium. As the two firms are symmetric, we only focus on illustrating firm 1’s price. The black dashed line plots firm 1’s price in the non-collusive equilibrium (as in Panel C of Figure C.2). The blue solid line plots firm 1’s price in the collusive equilibrium. It is shown that due to collusion, firm 1 sets higher prices than what it would set in the non-collusive equilibrium. The prices increase monotonically with customer base shares in both the collusive and the non-collusive equilibria.

Interestingly, Panel B shows that the ability to collude on higher prices, as reflected by the difference between the collusive price and the non-collusive price exhibits an inverted-U shape. The increase in prices due to collusion is the largest when the two firms have comparable customer base shares (i.e. $M_{i1}/M_i \approx 0.5$). Intuitively, collusion allows both firms to set higher prices to enjoy higher profit margins than what they would have in the non-collusive equilibrium. However, the collusive pricing schedule has to be chosen such that both firms have no incentive to deviate given their current customer base shares. When firm 1 is dominating the market (i.e. with high $M_{i1}/M_i$), forming a collusive equilibrium would be less appealing from firm 1’s perspective as it already has high market power, which allows it to set a high price in the non-collusive equilibrium any way (see the black dashed line). On the other hand, when firm 1 has low customer base share $M_{i1}/M_i$, forming a collusive equilibrium would be less appealing from firm 2’s perspective which already has high market power to set a high price in the non-collusive equilibrium.
Figure C.3: Comparing prices and firm values in the collusive and non-collusive equilibria.

Thus, it is easier to collude on relatively higher prices when firm 1 and firm 2 have comparable customer base shares.

The above intuition is more clearly seen in two extreme cases. When firm 1’s customer base share $M_{1i}/M_i \approx 1$, Panel A shows that it sets a price close to $\frac{\epsilon}{\omega} = 2$. This is the price that firm 1 would choose facing a price elasticity of demand $\epsilon$. In this case, firm 1 essentially acts almost as a monopoly in industry $i$ and sets prices to compete with firms in other industries. Thus, the constant across-industry price elasticity of demand is what determines firm 1’s optimal price in both the collusive and the non-collusive equilibria.

By contrast, when firm 1’s customer base share $M_{1i}/M_i \approx 0$, Panel A shows that it sets a price close to $\frac{\eta}{\omega} = 1.11$. This is the price that firm 1 would choose facing a price elasticity of demand $\eta$. In this case, firm 1 essentially acts almost as a price taker in industry $i$ because it has little market power to influence the industry’s price index. Thus, the constant within-industry price elasticity of demand is what determines firm 1’s optimal price in both the collusive and the non-collusive equilibria.

Panel C compares firm 1’s value in the collusive and the non-collusive equilibria. Colluding on higher prices increases firm 1’s profit margins, leading to higher firm values. Not surprisingly, due to the inverted-U collusive prices, the difference in firm values displays a similar inverted-U shape (Panel D) when the
C.3 Determination of Collusive Prices

In this section, we clarify how the collusive prices are determined in equilibrium. In Panel A of Figure C.3, the red line plots the optimal price that firm 1 would choose conditional on its deviation from the collusive pricing schedule. It shows that the optimal deviation price is always lower than the collusive price. This is intuitive because firms collude on higher prices relative to what they would set in the non-collusive equilibrium, and thus both firms have the incentive to undercut the other firms in order to increase both contemporaneous demand and gain more customer bases. Whether firm 1 would deviate depends on what deviation value firm 1 would obtain by setting the optimal deviation price. Intuitively, there are countervailing forces that determine the gains from deviation. If deviation is not detected by firm 2, then firm 1 would gain by stealing customer bases from firm 2 through lower prices. However, if deviation is detected by firm 2, then firm 1 will be punished by switching to the non-collusive equilibrium which features low prices and low profit margins.

Whether the collusive equilibrium can be sustained depends on the level of collusive prices. A higher collusive price increases the profits from deviation and is more difficult to be sustained in equilibrium. The collusive prices we choose are the highest prices subject to the IC constraints that both firms have no incentive to deviate in the collusive equilibrium. In Panel C of Figure C.3, the red dash-dotted line plots the deviation value that firm 1 would obtain by setting the optimal deviation price (the red dash-dotted line in Panel A). It is shown that firm 1’s deviation value is exactly the same as firm 1’s value in the collusive equilibrium, indicating that firm 1 is indifferent between setting the collusive price or deviating from the collusive equilibrium. In other words, firm 1’s IC constraints are binding. Because the collusive and deviation values are equal for any customer base share, firm 2 is also indifferent about collusion and deviation.

The IC constraints are violated, if we choose collusive prices higher than the blue solid line in Panel A. We illustrate this in Figure C.4. To obtain a stark comparison, we assume that the collusive price is set equal to $\tilde{e}(1-\omega) = 2$ (as shown by the blue solid line in Panel A), which is the price that maximizes the contemporaneous demand if the two firms can perfectly collude with each other and act like a monopoly. The red dash-dotted line indicates that when firm 1’s customer base share $M_{i1}/M_i$ is lower than 0.6, it would set a significantly lower price to steal firm 2’s customer base share. As a result, firm 1’s deviation value is strictly larger than its collusion value (see the red dash-dotted line in Panel B) when $M_{i1}/M_i < 0.6$, indicating that the IC constraint is violated. Thus, requiring the two firms to collude on a higher price like what is considered here does not form a sub-game perfect equilibrium because one of the firms (or both firms) will deviate by setting a lower price.

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30Here, we follow the standard game theory by considering one-shot deviation. That is, we consider what the deviation price that firm 1 would choose conditional on firm 2 not deviating from the collusive equilibrium. The one-shot deviation property ensures that no profitable one-shot deviations for every player is a necessary and sufficient condition for a strategy profile of a finite extensive-form game to form a sub-game perfect equilibrium.
D Discussions on Model Ingredients

In this Appendix, we discuss the role played by within- and between- elasticities, long-run risks, and antitrust enforcement on our model’s implications.

D.1 Discussions on Elasticities

The parameter $\eta$ and $\epsilon$ capture the elasticity of substitution of goods produced within the same industry and the elasticity of substitution of goods produced in different industries. In this section, we discuss the role played by the two elasticities on collusion incentives and prices. To fix ideas, we shut down the price channel for customer base accumulation by setting $z = 0$.

In our baseline calibration, we set $\eta > \epsilon$ to be consistent with empirical estimates. As we vary $\eta$ and $\epsilon$, the model can capture different degrees of within- and between-industry competition. As we show in equation (2.9), the price elasticity of demand for firm 1 depends on both the within-industry elasticity $\eta$ and the between-industry elasticity $\epsilon$ because firm 1 simultaneously faces within-industry competition from firm 2 as well as the between-industry competition from firms in other industries.

With $\eta > \epsilon$, within-industry competition is more fierce than between-industry competition due to the higher elasticity of substitution among goods produced in the same industry. Thus, essentially the within-industry elasticity $\eta$ gives the upper bound of competition, and hence determines the lower bound of prices; whereas the between-industry elasticity $\epsilon$ gives the lower bound of competition, and hence determines the upper bound of prices.

In particular, the highest level of competition is obtained by firm 1 when it becomes atomic in industry $i$ (i.e. $M_1 / M_i = 0$). In this case, firm 1 would set the lower-bound price $\frac{\eta}{\eta + \omega}$, determined by the within-industry elasticity $\eta$. However, when firm 1 is atomic, firm 2 is essentially the monopoly in industry $i$, facing the minimal level of competition due to the absence of within-industry competition. Thus, firm 2 would set the upper-bound price $\frac{\epsilon}{\epsilon + \omega}$, determined by the between-industry elasticity $\epsilon$. Because firm 2 already sets its price equal to the upper bound, there is no incentive for firm 2 to collude with firm 1, although firm 1 wants to collude due to its low price.
Thus, the two firms have the incentive to collude with each other only when neither firm is the monopoly in industry $i$. In this case, collusion benefits both firms by alleviating within-industry competition so that prices become higher, more reflecting the between-industry elasticity $\epsilon$. Therefore, the existence of collusion incentive crucially depends on the assumption that $\eta > \epsilon$. If $\eta = \epsilon$, the level of competition does not change with the customer base share. And the firm would always set the upper bound price $\frac{\epsilon}{\epsilon-\omega}$, determined by the between-industry elasticity $\epsilon$.

Specifically, if we set $\eta = 2$ ($= \epsilon$), Panel B1 of Figure D.5 shows that firm 1 always sets its price equal to $\frac{\epsilon}{\epsilon-\omega} = 2$. In this case, achieving the collusive equilibrium does not further increase the two firms’ prices because they already set the upper bound price consistent with what is implied by between-industry competition.\footnote{In fact, when the two elasticities are the same ($\eta = \epsilon$), the two layers of CES aggregation collapses to a single between-industry CES aggregation, and within-industry competition would not matter for price setting.}

Firm 1’s value increases linearly with its customer base share $M_{i1}/M_i$ (see Panel B2). In Panels C1 and C2, we further increase $\eta = \epsilon = \infty$ to mimic an economy with perfect competition. The infinite elasticity results in zero profit margins. Both firms set their prices equal to the marginal costs (see

Figure D.5: The role of price elasticities of demand on collusive prices.
D.2 Discussions on Long-Run Risks

We emphasize that long-run risks play a crucial role in generating price war risks. In our model, firms collude more during periods with high long-run growth rates precisely because they know that the growth rate of consumption is persistent. In Figure D.6, we compare the baseline calibration with a 0.49 auto-correlation of annual consumption growth rates to an economy with a 0.049 auto-correlation of annual consumption growth rates, featuring less persistent long-run risks. Panel A shows that, in the economy with less persistent long-run risks, there is almost no change in collusive prices between periods with high and low long-run growth rates, and this is true regardless of the capacity of distinctive innovation (see the red dash-dotted line and the red dotted line). Moreover, Panel B shows that the industry’s exposure to long-run risks is much smaller in the economy with less persistent long-run risks. Importantly, there is virtually no difference in the exposure to long-run risks across the two industries. Thus, the model suggests that the persistence of long-run risks is crucial in generating both the high magnitude of price war risks and the variation in the exposure to long-run risks across industries with different capacities of distinctive innovation.

Intuitively, with long-run risks in consumption, what determines the collusion incentive is not only the current level of aggregate consumption, but also the expected change in aggregate consumption in the future. Firms that expect a relative increase in aggregate consumption are able to sustain higher collusive prices now because none of the firms want to deviate and be punished later by their competitors in periods with higher aggregate consumption. On the contrary, during bad times, firms expect a relative decrease in aggregate consumption and the later punishment looks less costly. Consequently, declines in long-run consumption growth generate price wars, amplifying firms’ exposure to long-run risks.
D.3 Discussions on Antitrust Enforcement

Our model predicts that antitrust enforcement reduces price war risks. Intuitively, with stronger laws against collusion, it is more difficult for firms to conduct collusive pricing, resulting in lower collusive prices and less variation in collusive prices with long-run growth rates. In our model, the parameter $\phi$ controls the ability for firms to collude with each other. A smaller $\phi$ makes it harder to implement higher collusive prices, which is equivalent to the effect of implementing more stringent antitrust enforcement. In the extreme case with $\phi = 0$, there is no way to detect whether a price deviation has occurred in the past, and as a result, there is no way to sustain an incentive compatible collusive equilibrium.

In Figure D.7, we compare our baseline calibration with $\phi = 0.15$ to an economy with $\phi = 0.05$. The magnitude of price war risks is significantly lower in the latter economy (see Panel A). As a result, the industry’s exposure to long-run risks is much smaller when collusion is more difficult to implement. Across the two industries, our model implies that antitrust enforcement has larger effects in the industry with no distinctive innovation, as this is the industry with the highest collusion incentive to begin with.

E Supplementary Empirical Results

E.1 Product Prices in the 24-month Period Around the Lehman Crash

F Numerical Algorithm

In this section, we detail the numerical algorithm that solves the model. We solve the model in risk-neutral measure. By Girsanov’s theorem, we have

$$dZ_{c,t} = -\lambda_c dt + d\tilde{Z}_{c,t},$$

$$dZ_{\theta,t} = -\lambda_{\theta} dt + d\tilde{Z}_{\theta,t}.$$
Table E.1: Product prices around the Lehman crash (monthly analysis).

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent change in product prices (monthly, annualized, %)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tertile-3 innosimm_{t-1} × post Lehman crash_{t}</td>
<td>-6.04***[-3.14]</td>
<td>-6.64***[-3.23]</td>
</tr>
<tr>
<td>Tertile-3 innosimm_{t-1}</td>
<td>-1.44</td>
<td>-5.79</td>
</tr>
<tr>
<td>post Lehman crash_{t}</td>
<td>1.50</td>
<td>2.62</td>
</tr>
<tr>
<td>Industry FE</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>3398</td>
<td>3398</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.003</td>
<td>0.055</td>
</tr>
</tbody>
</table>

Note: This table shows the changes in product prices around the Lehman crash. The dependent variable is the annualized monthly percent change in product prices of 4-digit SIC industries. Product prices are obtained from the Nielsen Data. To compute the monthly percent change in product prices for 4-digit SIC industries, we first compute the transaction-value weighted price for each product across all stores in each month. We then calculate the monthly percent change in prices for each product. Finally, we compute the value-weighted percent change in product prices for each 4-digit SIC industry based on the transaction values of the industry’s products. We consider we consider the 24-month period around the Lehman crash. We include t-statistics in parentheses. Standard errors are clustered by the 4-digit SIC industry and year. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels.

Table E.2: Price-innosimm sensitivity around the Lehman crash (monthly analysis).

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Percent change in product prices (monthly, annualized, %)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>innosimm_{t-1} × post Lehman crash_{t}</td>
<td>-2.52**[-2.52]</td>
<td>-2.32**[-2.27]</td>
</tr>
<tr>
<td>innosimm_{t-1}</td>
<td>-1.02</td>
<td>-3.02*</td>
</tr>
<tr>
<td>post Lehman crash_{t}</td>
<td>-2.09</td>
<td>-1.92</td>
</tr>
<tr>
<td>Industry FE</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>5086</td>
<td>5086</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.003</td>
<td>0.043</td>
</tr>
</tbody>
</table>

Note: This table shows the price-innosimm sensitivity around the Lehman crash. The dependent variable is the annualized monthly percent change in product prices of 4-digit SIC industries. We consider we consider the 24-month period around the Lehman crash. We include t-statistics in parentheses. Standard errors are clustered by the 4-digit SIC industry and year. *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels.

Under the risk-neutral measure, the dynamics of aggregate conditions are

\[
\frac{dC_t}{C_t} = \theta_t dt + \sigma_c d\tilde{Z}_{c,t}, \quad (F.3)
\]

\[
d\theta_t = \kappa \left( \theta^Q - \theta_t \right) dt + \varphi \sigma_c d\tilde{Z}_{\theta,t}, \quad (F.4)
\]
where
\[ \theta^Q = \theta - \lambda_c \sigma_c - \kappa^{-1} \lambda \phi \theta \sigma_c. \] (F.5)

To give an overview, our algorithm proceeds in the following steps:

1. We solve for the non-collusive equilibrium. This requires us to solve the Markov-Perfect equilibrium of the dynamic game played by two firms. The simultaneous-move dynamic game requires us to solve the intersection of the two firms’ best response (i.e. optimal price) functions, which themselves are optimal solutions to coupled PDEs.

2. We solve for the collusive equilibrium using the value functions in the non-collusive equilibrium as punishment values. Because we are interested in the highest collusive prices with binding incentive-compatibility constraints, this requires us to solve a high-dimensional fixed-points problem. We thus use an iteration method inspired by and to solve the problem (Abreu, Pearce and Stacchetti, 1986, 1990; Ericson and Pakes, 1995; Fershtman and Pakes, 2000).

3. After solving the baseline model, we solve the extended model with endogenous cash holdings by repeating steps (1) and (2). The extended model is more challenging because it involves solving PDEs with free boundaries (due to endogenous payout boundaries). We employ the piecewise multilinear interpolation method of Weiser and Zarantonello (1988) to obtain accurate interpolants in a 3-dimensional space.

Note that standard methods for solving PDEs with free boundaries (e.g. finite difference or finite element) can easily lead to non-convergence of value functions. To mitigate such problems and obtain accurate solutions, we solve the continuous-time game using a discrete-time dynamic programming method. In Appendix F.1, we present the discretized recursive formulation for the baseline model, including firms’ problems in non-collusive equilibrium, collusive equilibrium, and deviation. In Appendix F.3, we discuss how we discretize the stochastic processes, time grids, and state variables in the model. Finally, in Appendix F.4, we discuss the details on implementing our numerical algorithms, including finding the equilibrium prices in the non-collusive equilibrium and solving the optimal collusive prices.

**F.1 The Baseline Model**

Because firm 1 and firm 2 are symmetric, one firm’s value and policy functions are obtained directly given the other firm’s value and policy functions. In this section, we illustrate firm 1’s problem in our baseline model. We first illustrate the non-collusive equilibrium and then we illustrate the collusive equilibrium.

**F.1.1 Non-Collusive Equilibrium**

Below, we present the recursive formulation for the firm’s value in the non-collusive equilibrium. Then we exploit linearity to simplify the problem and present the recursive formulation for the normalized firm value. Finally, we present the conditions that determine the non-collusive (Nash) equilibrium.
Recursive Formulation for The Non-Collusive Firm Value. The industry’s state is characterized by four state variables, firm 1’s customer base $M_{i1,t}$, firm 2’s customer base $M_{i2,t}$, the aggregate consumption $C_t$, and the long-run growth rate $\theta_t$. Denote the value functions in the non-collusive equilibrium as $V_{ij}^N(M_{i1,t}, M_{i2,t}, C_t, \theta_t)$ for $j = 1, 2$.

To characterize the equilibrium value functions, it is more convenient to introduce two off-equilibrium value functions. Let $\tilde{V}_{ij}^N(M_{i1,t}, M_{i2,t}, C_t, \theta_t; P_{ik,t})$ be firm $j(=1,2)$’s value when its peer firm $k$’s price is set at any (off-equilibrium) value $P_{ik,t}$.

Firm 1 solves the following problem:

$$\tilde{V}_{11}^N(M_{i1,t}, M_{i2,t}, C_t, \theta_t; P_{21,t}) = \max_{P_{11,t}} (P_{11,t} - \omega) \left( \frac{P_{11,t}}{P_{11,t}} \right)^{-\eta} P_{11,t}^{-\epsilon} C_t M_{i1,t} \Delta t + \mathbb{E}_t \left[ \Lambda_{t+\Delta t} v_{11}^N(M_{i1,t+\Delta t}, M_{i2,t+\Delta t}, C_{t+\Delta t}, \theta_{t+\Delta t}) \right], \quad (F.6)$$

subject to the evolution of state variables, including:

The evolution of the customer base is

$$M_{ij,t+\Delta t} = M_{ij,t} + z \left( \frac{C_{ij,t}}{C_t} \right)^{\alpha} M_{ij,t}^{1-\alpha} - \rho M_{ij,t} \Delta t, \quad \text{for } j = 1, 2 \quad (F.7)$$

where firm-level demand is given by

$$C_{ij,t} = \left( \frac{P_{ij,t}}{P_{i1,t}} \right)^{-\eta} P_{ij,t}^{-\epsilon} C_t M_{ij,t}, \quad \text{for } j = 1, 2 \quad (F.8)$$

and the industry’s price index is given by

$$P_{i,t} = \left( \frac{M_{i1,t}}{M_{i,t}} P_{11,t}^{1-\eta} + \frac{M_{i2,t}}{M_{i,t}} P_{21,t}^{1-\eta} \right)^{1-\eta}. \quad (F.9)$$

The evolution of aggregate consumption $C_t$ is

$$C_{t+\Delta t} = (1 + \theta_t \Delta t + \sigma \Delta Z_{\epsilon,t}) C_t. \quad (F.10)$$

The long-run growth rate $\theta_t$ evolves according to the discrete Markov chain specified in Appendix F.3.

Recursive Formulation for The (Normalized) Non-Collusive Firm Value. Exploiting the linearity, we normalize the firm’s value by $M_{i,t} C_t$. Firm 1’s customer base share is $m_{i1,t} = M_{i1,t}/M_{i,t}$; firm 2’s customer base share is $m_{i2,t} = M_{i2,t}/M_{i,t} = 1 - m_{i1,t}$. Define

$$v_{ij}^N(m_{i1,t}, \theta_t) = \frac{V_{ij}^N(M_{i1,t}, M_{i2,t}, C_t, \theta_t)}{M_{i,t} C_t}, \quad (F.11)$$

$$\tilde{v}_{ij}^N(m_{i1,t}, \theta_t; P_{ik,t}) = \frac{\tilde{V}_{ij}^N(M_{i1,t}, M_{i2,t}, C_t, \theta_t; P_{ik,t})}{M_{i,t} C_t}, \quad (F.12)$$

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Firm 1 solves the following normalized problem:

\[
\hat{v}^N_{i1}(m_{i1,t}, \theta_t; P_{2,t}) = \max_{\hat{P}_{i,t}} (P_{i,t} - \omega) \left( \frac{P_{i,t}}{\hat{P}_{i,t}} \right)^{-\eta} \hat{P}_{i,t}^{-\epsilon} m_{i1,t} \Delta t
\]

\[
+ \mathbb{E}_t \left[ \frac{M_{i1,t+\Delta t}}{M_{i1,t}} \left( 1 + \theta_t \Delta t + c \Delta Z_{c,t} \right) v^N_{i1}(m_{i1,t+\Delta t}, \theta_{t+\Delta t}) \right],
\]  

subject to the evolution of state variables, including:

The equilibrium value functions are given by

\[
v^N_{i1}(m_{i1,t}, \theta_t; P_{2,t}) = \tilde{v}^N_{i1}(1 - m_{i1,t}, \theta_t; P_{i,t}),
\]

where the industry’s price index is given by

\[
P_{i,t} = \left[ m_{i1,t} P_{i,t}^{1-\eta} + (1 - m_{i1,t}) P_{i1,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}.
\]  

The evolution of the industry’s customer base is

\[
\frac{M_{i1,t+\Delta t}}{M_{i1,t}} = 1 + \left[ \frac{z}{P_{i,t}}^{1-\eta} P_{i,t}^{-e\alpha} - \rho \right] m_{i1,t} \Delta t + \left[ \frac{z}{P_{i1,t}}^{1-\eta} P_{i1,t}^{-e\alpha} - \rho \right] (1 - m_{i1,t}) \Delta t.
\]  

The long-run growth rate \( \theta_t \) evolve according to the discrete Markov chain specified in Appendix F.3.

**Non-Collusive (Nash) Equilibrium.** Denote the equilibrium price functions as \( P^N_{i,t}(m_{i1,t}, \theta_t) \) and the off-equilibrium price functions as \( \bar{P}^N_{ij}(m_{i1,t}, \theta_t; P_{k,t}) \). Exploiting the symmetry between firm 1 and firm 2, we can obtain firm 2’s off-equilibrium value and policy functions as

\[
\hat{v}^N_{i1}(m_{i1,t}, \theta_t; P_{2,t}) = \tilde{v}^N_{i1}(1 - m_{i1,t}, \theta_t; P_{i,t}),
\]

\[
\bar{P}^N_{i1}(m_{i1,t}, \theta_t; P_{2,t}) = \tilde{P}^N_{i2}(1 - m_{i1,t}, \theta_t; P_{i1,t}).
\]  

Given \( j = 1, 2 \)’s price \( P_{ij} \), firm \( k \) optimally sets the price \( P_{ik} \). The non-collusive (Nash) equilibrium is derived from the fixed point—each firm’s price is optimal given the other firm’s optimal price:

\[
P^N_{i1}(m_{i1,t}, \theta_t) = \tilde{P}^N_{i2}(m_{i1,t}, \theta_t; P^N_{i1}(m_{i1,t}, \theta_t)),
\]

\[
P^N_{i2}(m_{i1,t}, \theta_t) = \tilde{P}^N_{i1}(m_{i1,t}, \theta_t; P^N_{i2}(m_{i1,t}, \theta_t)).
\]  

The equilibrium value functions are given by

\[
v^N_{i1}(m_{i1,t}, \theta_t) = \tilde{v}^N_{i1}(m_{i1,t}, \theta_t; P^N_{i2}(m_{i1,t}, \theta_t)),
\]

\[
v^N_{i2}(m_{i1,t}, \theta_t) = \tilde{v}^N_{i2}(m_{i1,t}, \theta_t; P^N_{i1}(m_{i1,t}, \theta_t)).
\]  

After solving the equilibrium value and policy functions above, we can verify that the following
conditions are satisfied due to symmetry,
\[ v^N_{1i}(m_{1i,t}, \theta_t) = v^N_{i2}(1 - m_{1i,t}, \theta_t), \] (F.23)
\[ P^N_{1i}(m_{1i,t}, \theta_t) = P^N_{i2}(1 - m_{1i,t}, \theta_t). \] (F.24)

**F.1.2 Collusive Equilibrium**

Below, we present the recursive formulation for the firm’s value in the collusive equilibrium. Then we present the recursive formulation for the firm’s value when the firm deviates from the collusive equilibrium. Finally, we present the incentive compatibility constraints and the conditions that determine the optimal collusive prices.

**Recursive Formulation for The Collusive Firm Value.** In the collusive equilibrium, we can still exploit the linearity property and solve firms’ values as a function of customer base shares. Specifically, denote \( \hat{v}_i^C(m_{1i,t}, \theta_t; \hat{P}_i^c) \) as firm \( i \)’s value in the collusive equilibrium with collusive prices \( \hat{P}_i^c(m_{1i,t}, \theta_t) \). Note that because the two firms in the same industry are symmetric, the collusive prices satisfy \( \hat{P}_i^c(m_{1i,t}, \theta_t) = \hat{P}_{i2}(1 - m_{1i,t}, \theta_t) \).

Firm 1 solves the following normalized problem:
\[
\hat{v}_1^C(m_{1i,t}, \theta_t; \hat{P}_i^c) = \left( \frac{\hat{P}_i^c(m_{1i,t}, \theta_t)}{P_{i,t}} \right)^{-\eta} P_{i,t}^{\epsilon} m_{i,t}\Delta t + E_t \left[ \frac{\Lambda_{i,t+\Delta t}}{\Lambda_t} \frac{M_{i,t+\Delta t}}{M_{i,t}} (1 + \theta_{1t}\Delta t + \sigma \Delta Z_{ct}) \hat{v}_1^C(m_{1i,t+\Delta t}, \theta_{1t+\Delta t}; \hat{P}_i^c) \right],
\] (F.25)

subject to the evolution of state variables, including:
The evolution of firm 1’s customer base share is
\[
m_{1i,t+\Delta t} M_{i,t+\Delta t} = m_{1i,t} + \left[ z \left( \frac{\hat{P}_i^c(m_{1i,t}, \theta_t)}{P_{i,t}} \right)^{-\eta} P_{i,t}^{\epsilon} - \rho \right] m_{1i,t}\Delta t,
\] (F.26)
where the industry’s price index is given by
\[
P_{i,t} = \left[ m_{1i,t} \hat{P}_i^c(m_{1i,t}, \theta_t) \right]^{1-\eta} + (1 - m_{1i,t}) \hat{P}_{i2}(m_{1i,t}, \theta_t) \right]^{1-\eta}. \] (F.27)

The evolution of the industry’s customer base is
\[
\frac{M_{i,t+\Delta t}}{M_{i,t}} = 1 + \left[ z \left( \frac{\hat{P}_i^c(m_{1i,t}, \theta_t)}{P_{i,t}} \right)^{-\eta} P_{i,t}^{\epsilon} - \rho \right] m_{1i,t}\Delta t + \left[ z \left( \frac{\hat{P}_i^c(m_{1i,t}, \theta_t)}{P_{i,t}} \right)^{-\eta} P_{i,t}^{\epsilon} - \rho \right] (1 - m_{1i,t})\Delta t.
\] (F.28)

The long-run growth rate \( \theta_t \) evolve according to the discrete Markov chain specified in Appendix F.3.
Recursive Formulation for The Deviation Value. The deviation value is obtained by assuming that firm $j$ optimally sets its price conditional on firm $k$ following the collusive pricing rule $\hat{P}^C_{ik}(m_{i1,t}, \theta_i)$. We exploit the linearity property and solve firms’ deviation values as a function of customer base shares. Denote $\hat{v}^D_{ij}(m_{i1,t}, \theta_i; \hat{P}^C_{ij})$ as firm $j$’s deviation value.

Firm 1 solves the following normalized problem:

$$
\hat{v}^D_{i1}(m_{i1,t}, \theta_i; \hat{P}^C_{ij}) = \max_{P_{i,t}} (P_{i,t} - \omega) \left( \frac{P_{i,t}}{\hat{P}^C_{i,t}} \right)^{-\eta} \rho P_{i,t} \Delta t
$$

subject to the evolution of state variables, including:

\[
\begin{align*}
M_{i,t+\Delta t} & = m_{i,t} + \left[ z \left( \frac{P_{i,t}}{\hat{P}^C_{i,t}} \right)^{-\eta} \rho - \eta \right] m_{i,t} \Delta t, \\
M_{i,t+\Delta t} & = 1 + \left[ z \left( \frac{\hat{P}^C_{i,t}}{\hat{P}^C_{i,t}} \right)^{-\eta} \rho - \eta \right] m_{i,t} \Delta t + \left[ \phi \Delta t \hat{v}^D_{ij}(m_{i1,t+\Delta t}, \theta_i; \hat{P}^C_{ij}) + \phi \Delta t v^N_{ij}(m_{i1,t+\Delta t}, \theta_i+\Delta \theta) \right],
\end{align*}
\]

(F.29)

where the industry’s price index is given by

$$
P_{i,t} = \left[ m_{i1,t} P_{i,t}^{1-\eta} + (1 - m_{i1,t}) \hat{P}^C_{i1}(m_{i1,t}, \theta_i)^{1-\eta} \right]^{1/\eta}.
$$

(F.30)

The evolution of firm 1’s customer base share is

$$
m_{i1,t+\Delta t} = m_{i1,t} + \left[ z \left( \frac{P_{i,t}}{\hat{P}^C_{i,t}} \right)^{-\eta} \rho - \eta \right] m_{i,t} \Delta t,
$$

(F.31)

The evolution of the industry’s customer base share is

$$
\frac{M_{i,t+\Delta t}}{M_{i,t}} = 1 + \left[ z \left( \frac{P_{i,t}}{\hat{P}^C_{i,t}} \right)^{-\eta} \rho - \eta \right] m_{i,t} \Delta t + \left[ \phi \Delta t \hat{v}^D_{ij}(m_{i1,t+\Delta t}, \theta_i; \hat{P}^C_{ij}) + \phi \Delta t v^N_{ij}(m_{i1,t+\Delta t}, \theta_i+\Delta \theta) \right] (1 - m_{i1,t}) \Delta t.
$$

(F.32)

The long-run growth rate $\theta_i$ evolve according to the discrete Markov chain specified in Appendix F.3.

Incentive Compatibility Constraints and Optimal Collusive Prices. The collusive equilibrium is a sub-game perfect equilibrium if and only if the collusive prices $\hat{P}^C_{ij}(m_{i1,t}, \theta_i)$ satisfy the following incentive compatibility constraints:

$$
\hat{v}^D_{ij}(m_{i1,t}, \theta_i; \hat{P}^C_{ij}) \geq \hat{v}^D_{ij}(m_{i1,t}, \theta_i; \hat{P}^C_{ij}),
$$

(F.33)

for all $m_{i1,t} \in [0, 1]$, $\theta_i$, and $j = 1, 2$.

There exist infinitely many sub-game perfect collusive equilibrium. We focus on the collusive equilibrium with the highest collusive prices (denoted by $P^C_{ij}(m_{i1,t}, \theta_i)$), which are obtained when all incentive compatibility constraints are binding, i.e.

$$
\hat{v}^D_{ij}(m_{i1,t}, \theta_i; P^C_{ij}) = \hat{v}^D_{ij}(m_{i1,t}, \theta_i; \hat{P}^C_{ij}),
$$

(F.34)

for all $m_{i1,t} \in [0, 1]$, $\theta_i$, and $j = 1, 2$. We denote $\hat{v}^C_{ij}(m_{i1,t}, \theta_i)$ as firm $j$’s value in the collusive equilibrium
with collusive prices $P_i^C(m_{i1,t}, \theta_t)$, thus by definition

$$v_{ij}^C(m_{i1,t}, \theta_t) = \hat{v}_{ij}^C(m_{i1,t}, \theta_t; P_i^C).$$  \hfill (F.35)

F.2  The Extended Model

TBA

F.3  Discretization

We discretize the unpredictable consumption growth shocks $dZ_{ct}$ based on $n_c$ grids spanning from $-3\sigma_c$ and $3\sigma_c$ using the method of Tauchen (1986). We use the method of Rouwenhorst (1995) to approximate the persistent AR(1) process of long-run risks $\theta_t$ using $n_{\theta}$ discrete states. The time line is discretized into intervals with length $\Delta t$.

We use collocation methods to solve each firm’s problem. Let $S_m \times S_\theta$ be the grid of collocation nodes for a firm’s equilibrium value, and $S_m \times S_\theta \times S_p$ be the grid of collocation notes for a firm’s off-equilibrium value. We have $S_m = \{m_1, m_2, ..., m_{n_m}\}$, $S_\theta = \{\theta_1, \theta_2, ..., \theta_{n_\theta}\}$, $S_p = \{p_1, p_2, ..., p_{n_p}\}$.

We approximate the firm’s value function $v(\cdot)$ on the grid of collocation notes using a linear spline with coefficients corresponding to each grid point. We first form a guess for the spline’s coefficients, then we iterate to obtain a vector that solves the system of Bellman equations.

F.4  Implementation

The numerical algorithms are implemented using C++. The program is run on the server of MIT Economics Department, supply.mit.edu and demand.mit.edu, which are built on Dell PowerEdge R910 (64 cores, Intel(R) Xeon(R) CPU E7-4870, 2.4GHz) and Dell PowerEdge R920 (48 cores, Intel(R) 4 Xeon E7-8857 v2 CPUs). We use OpenMP for parallelization when iterating value functions and simulating the model.

Selection of Grids  We set $n_c = 11$, $n_m = 21$, $\Delta t = 1/24$, $n_p = 11$. The grids for long-run risks $S_\theta$ is given by the method of Rouwenhorst (1995). The grids for unpredictable consumption growth shocks $S_c$ is given by the method of Tauchen (1986). The grid of customer base share $S_m$ is discretized into 21 nodes from $1e^{-7}$ to $1 - 1e^{-7}$ with equal spaces. We do not set $S_m$ from 0 to 1 to avoid the indeterminacy of optimal price with $m = 0$. The time interval $\Delta t$ is set to be $1/24$. A higher $\Delta t$ implies faster convergence for the same number of iterations but lower accuracy. We checked that the solution is accurate enough for $\Delta t = 1/24$, further reducing $\Delta t$ would not change the accuracy much. With $1/24$, 5000 times iterations allow us to achieve convergence in value functions. The industry characteristic grid is discretized into 11 notes from 0 to 1 with equal spaces. The price grids is discretized into 11 nodes from 1 to 2 with equal spaces. The upper bound is chosen according to $\epsilon / (\epsilon - 1) \times \omega = 2$, which is the highest price a firm will ever set.
Calculating Iterations and Searching For the Nash Equilibrium. Given the value functions from the previous iteration, we use the golden section search method to find the equilibrium prices. The computational complexity of this algorithm is at the order of \(\log(n)\), much faster and more accurate than a simple grid search.

Searching for the equilibrium price is very difficult because we have to solve a fixed-point problem (equations F.19-F.22) that involves both firms’ simultaneous prices decisions. Our solution technique is to iteratively solve the following three steps.

First, given \(v^N_{i1}(m_{i1}, \theta)\), we solve for the off-equilibrium value \(\hat{v}^N_{i1}(m_{i1}, \theta; P_{i2})\) and the off-equilibrium policy function \(\hat{P}^N_{i1}(m_{i1}, \theta; P_{i2})\). Exploiting symmetry, we obtain \(\hat{v}^N_{i2}(m_{i1}, \theta; P_{i1})\) and \(\hat{P}^N_{i2}(m_{i1}, \theta; P_{i1})\). Second, for each \((m_{i1}, \theta) \in S_m \times S_\theta\), we use a nonlinear solver \texttt{knitro} to solve equations (F.19-F.20) and obtain the equilibrium prices \(P^N_{i1}(m_{i1}, \theta), P^N_{i2}(m_{i1}, \theta)\). Third, we solve equations (F.21-F.22) and obtain equilibrium value functions \(v^N_{i1}(m_{i1}, \theta)\) and \(v^N_{i2}(m_{i1}, \theta)\).

Searching For Collusive Prices. We modify the golden section search method to find the highest collusive prices \(P^C_{i1}(m_{i1}, \theta)\) by iterations. Within each iteration, we solve firms’ collusion value and deviation value using standard recursive methods given \(\hat{P}^C_{ij}(m_{i1}, \theta)\).

There are two key differences between our method and a standard golden section search method. First, we guess and update the collusive pricing schedule \(\hat{P}^C_{ij}(m_{i1}, \theta)\) simultaneously for all \((m_{i1}, \theta) \in S_m \times S_\theta\), instead of doing it one by one for each state. This is to increase efficiency. But, a natural problem introduced by the simultaneous updating is that there might be overshooting. For example, if for some particular state \((m^*, \theta^*)\), we updated a collusive price \(\hat{P}^C_{ij}(m^*, \theta^*)\) too high in the previous iteration, the collusive price for some other states \((m, \theta) \neq (m^*, \theta^*)\) might be affected in this iteration and never achieve a binding incentive compatibility constraint. Eventually, this may lead to non-convergence.

We solve this problem by gradually updating the collusive prices. In particular, in each round of iteration, we first compute the updated collusive pricing schedule \(\hat{P}^C_{ij}(m_{i1}, \theta)\) implied by the golden section search method. Then, instead of changing the upper search bound or lower search bound to \(\hat{P}^C_{ij}(m_{i1}, \theta)\) directly, we change it to \((1 - adj) \times \hat{P}^C_{ij}(m_{i1}, \theta) + adj \times \hat{P}^C_{ij}(m_{i1}, \theta)\), a weighted average of the current collusive price \(\hat{P}^C_{ij}(m_{i1}, \theta)\) and the updated collusive price \(\hat{P}^C_{ij}(m_{i1}, \theta)\). For our baseline model, we set \(adj = 0.15\) to ensure perfect convergence.