Skill and Fees in Active Management

by

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Abstract

Greater skill of active investment managers can mean less fee revenue in equilibrium. Although more-skilled managers receive more revenue than less-skilled managers, greater skill for active managers overall can imply less revenue for their industry. Greater skill allows managers to identify mispriced securities more accurately and thereby make better portfolio choices. Greater skill also means, however, that active management corrects prices better and thus reduces managers' return opportunities. The latter effect can outweigh managers' better portfolio choices in a general equilibrium. Investors then rationally allocate less to active funds and more to index funds if active management is more skilled.

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1. Introduction

Active investment managers seek to outperform a passive benchmark portfolio by identifying and trading securities likely to over- or under-perform. In exchange for doing so, the managers are compensated by fees their investors pay. The fees are supported by active managers' investment profits, in excess of what investing in the benchmark would produce. Given the alternative of investing in the passive benchmark, rational investors do not consistently invest with active managers whose fees are not covered by investment profits.

Should active managers produce greater investment profits, and thus receive more fee revenue, if they are more skilled? A more-skilled manager should receive more fee revenue than a less-skilled manager, but that is not the whole story. There are many active managers. What if most of them become more skilled? Then, for example, a stock that is truly underpriced is more likely to be identified correctly as such by active managers, and that stock is more likely to be bought by them. This collective higher demand for the stock raises the price managers pay for it, reducing the investment profit they make from buying it. That is the negative effect of skill on profits, working against the positive effect of better identifying the right stocks to buy. Of course the same opposing effects of skill apply to overpriced stocks, raising the chances such stocks are correctly identified but reducing the investment profits from under-weighting or shorting them.

What is the outcome of the opposing skill effects for active managers' investment profits, and thus fee revenue? The answer can go either way, with greater skill producing either more revenue or less for the active management industry. For seemingly plausible scenarios explored here, greater skill brings lower fee revenue in a general equilibrium model that incorporates the effects of skill on stocks' prices.

The model has familiar features. Key among them is a condition of equilibrium, following Berk and Green (2004), that all of the benchmark-beating investment profit produced by active managers is captured as fee revenue. Investors are then indifferent between putting their last incremental dollar of stock-market investment into an active fund versus an index fund, as both funds offer the same expected return in equilibrium. One mechanism through which the returns are equalized involves trading costs, which are introduced in the same manner as Stambaugh (2014) and Pástor, Stambaugh, and Taylor (2018): an active fund's proportional cost of trading any stock is linear in the amount traded. These stock-level trading costs mean the fund effectively faces decreasing returns with respect to its size, as a larger fund trades larger dollar amounts of stocks and thus incurs higher proportional costs. In this respect the model of active management here differs, for example, from those of Garcia and Vanden (2009) and Gârleanu and Pedersen (2018), in which managers do not face such liquidity costs and the resulting decreasing returns to scale. Berk and Green (2004) assume decreasing returns with respect to fund size, reasoning that trading costs are convex in fund size. That mechanism in their model, also essentially at work here, allows active and passive returns to equalize in equilibrium.

Including trading costs in a model of the active management industry seems desirable, given empirical evidence of such costs' economic importance. For example, Edelen, Evans, and Kadlec (2007) conclude that trading costs present an important source of scale diseconomies for mutual funds. Edelen, Evans, and Kadlec (2013) find that mutual funds' trading costs as a fraction of net asset value are comparable in magnitude to the funds' expense ratios. The latter result is consistent with the model presented here, which implies that funds' trading costs equal their fee revenues.

Pástor and Stambaugh (2012) introduce decreasing returns with respect to the overall scale of the active management industry. They consider a setting in which competing managers do not internalize the effect of the industry's scale on investment profits. Similarly, the many managers in my model do not internalize the effect of active managers' aggregated demands for each stock on the stock's equilibrium price. These pricing effects across stocks, which in turn reduce each manager's overall profit in equilibrium, constitute the model's second dimension of decreasing returns to industry scale, in addition to that arising from the trading costs paid to intermediaries.

The model implies that the skill of an active manager relevant to fee revenue is captured by the correlation across stocks between the manager's assessed alphas and the stocks' true alphas. The correlation is defined with respect to the value-weight measure, meaning that the probability weight assigned to the stock when computing cross-sectional moments is the stock's weight in the market portfolio. Conditional on the amount of equilibrium mispricing, a more-skilled manager receives more fee revenue than a less-skilled manager. This cross-sectional relation between fee revenue and skill is consistent with the arguments and evidence of Berk and van Binsbergen (2015). Of course, conditioning on the amount of mispricing, in a partial equilibrium, does not address how the overall skill level of active managers affects mispricing and hence fee revenue.

To address the pricing effects, I specify additional features of the market and derive a general equilibrium. For active management in aggregate to beat the market benchmark, before costs and fees, other market participants must underperform that benchmark (e.g., Sharpe (1991)). The potentially underperforming segment is typically assumed to comprise "noise" traders whose asset demands deviate exogenously from those consistent with rational assessments of fundamental values. I include such noise traders in the model. The distribution of the stock-specific distortions introduced by noise traders for a large cross section of stocks is flexibly characterized by a single parameter, following Stambaugh (2014).

For the general equilibrium model, I specify two exogenous dimensions along which managers make valuation errors that impair skill. The first dimension allows stock-specific distortions reflected in noise-traders' demands to arise in the beliefs of active managers to some extent as well. The motivation here is that the noise-trader demands might reflect pervasive sentiment about individual stocks to which professional managers are also somewhat susceptible. Such a scenario seems motivated, for example, by the empirical evidence of Dasgupta, Prat, and Verardo, (2011), Akbas, Armstrong, Sorescu, and Subrahmanyam (2015), Edelen, Ince, and Kadlec (2016), and DeVault, Sias, and Starks (2018). The second dimension of valuation errors is idiosyncratic across managers, reflecting manager-specific limitations and mistakes.

Active management corrects prices less when skill is impaired by either a partial echoing of noise-trader demands or by manager-specific errors. Not surprisingly, the more that managers' active positions echo noise-trader demands, the less that establishing those positions corrects the mispricing that noise traders originate. Manager-specific errors wash out across managers and do not affect the mispricing of one asset relative to another. At the same time, however, the overall aggressiveness of an individual manager's positions is limited by trading costs. The more such positions across individual stocks reflect manager-specific errors, which wash out of aggregate active positions, the less aggressive the aggregate positions become, and the less prices get corrected.

2. Model assumptions

The model considers a single investment period. Active managers identify and exploit opportunities to outperform the market benchmark by having their weights on individual stocks deviate from those of the benchmark. The ultimate objective of the active managers is to maximize the fee revenue received from the investors they attract. There are many managers, acting competitively, each taking stock prices as given. Each manager incurs trading costs when deviating from benchmark stock weights. At the beginning of the period, manager j sets a proportional fee rate equal to $f^{(j)}$. Investors then invest in aggregate the dollar amount $W^{(j)}$ with the manager, whose fee revenue is thus $F^{(j)} = f^{(j)}W^{(j)}$.

2.1. Trading costs and decreasing returns to scale

The trading costs faced by active managers are convex in the amount traded. Specifically, for any manager, I assume the dollar cost C_i of trading dollar amount D_i of stock i is

$$C_i = c\delta_i D_i, \tag{1}$$

where δ_i is the fraction of stock i's total market capitalization represented by D_i, and c is a constant. In other words, the proportional trading cost is linear in the amount traded. These costs represent compensation to liquidity-providing intermediaries for taking short-lived positions to facilitate the ultimate market clearing in a stock between managers and noise traders. In this interpretation, the trading cost is not to be viewed as a manager-specific price impact, such that if many other managers independently produce similar price impacts, the sum of such impacts aggregates to an implausibly large total price effect. Instead, one might imagine many intermediaries accessing different sources of liquidity or acting at slightly different times. This specification of trading costs also appears in Stambaugh (2014) and Pástor, Stambaugh, and Taylor (2018).

Convex trading costs imply active managers face decreasing returns to scale. Larger funds trade larger dollar amounts, representing larger fractions of stocks' market capitalizations, so proportional trading costs increase in fund size. An assumption of fund-level decreasing returns to scale is central to the model of Berk and Green (2004).

2.2. Zero net alpha for funds

Let $\alpha_A^{(j)}$ denote active fund j's alpha net of costs and fees, which is the alpha relevant to the fund's investors. As in Berk and Green (2004), I assume investors allocate money to an active fund up to point at which additional investment in the fund, given decreasing returns to scale, would drive $\alpha_A^{(j)}$ below zero. That is, the equilibrium size of a given active fund j is such that

$$\alpha_A^{(j)} = 0. \tag{2}$$

Berk and Green (2004) impose equation (2) in analyzing a partial equilibrium, with the sources of an active fund's benchmark-beating gross returns not modeled. I begin by imposing the same zero-alpha condition in a partial equilibrium. Later, in a general equilibrium with noise traders, I link the sources of the fund's gross returns to departures of endogenously determined prices from their fundamental values.

The most natural assumption motivating $\alpha_A^{(j)} = 0$ is that investors are risk neutral. In that setting, any deviation of the fund's expected net return from that of the passive benchmark prompts a flow of investor money to or from the fund such that the deviation is removed.[1]

When investors are instead risk averse, equation (2) is better viewed as an approximation to the equilibrium. The reason is that if active managers on the whole possess skill to identify mispriced securities, then the departures of active managers' portfolios from benchmark weights are at least somewhat correlated across managers. The result of this correlation is that a non-diversifiable risk component, uncorrelated with the benchmark return, is present in active fund returns.[2] This additional risk will generally not be borne by risk-averse investors without compensation, requiring $\alpha_A^{(j)} > 0$. This point is made in a partial equilibrium by Pástor and Stambaugh (2012) and in a general equilibrium by Stambaugh (2014). Calibrations in both studies, however, suggest that violations of equation (2) are likely to be modest in economic magnitude. I impose the zero-alpha condition here for tractability, especially when considering skill differences across managers. Stambaugh (2017) provides a related analysis in a risk-averse setting that delivers the same message as this study about the effect of greater industry-level skill.

2.3. Stocks

The stock market contains N stocks, and the total supply of each stock equals one share. Stock i has share price p_i at the beginning of the investment period and value (including dividends) equal to x_i at the end of the period . The market portfolio has total end-of-period value $x_m = \sum_{i=1}^{N} x_i$. There are N shares in the market portfolio, each with price $p_m = (1/N)\sum_{i=1}^{N} p_i$. Let μ denote one plus the required discount rate, equal across stocks in the risk-neutral setting. Let $\bar{x}_i$ equal the correct expected value of x_i, in the sense that if

[1] Indeed, in an earlier version of their published study, Berk and Green (2003, p. 6) assume that investors are risk neutral.

[2] Under risk aversion, the alpha relevant to investors is the difference between the fund's expected net return and the fund's beta times the expected market benchmark return. The mean-zero tracking error accompanying this alpha is then uncorrelated with the benchmark return. In keeping with a risk-neutral motivation, the alpha in equation (2) is instead defined below as the simple difference between the fund's expected net return the expected benchmark return.

p_i, is equal to

$$\bar{p}_i = \frac{\bar{x}_i}{\mu}, \tag{3}$$

then stock i is priced at its fundamental value. I assume the overall market is fairly priced, so that

$$p_m = \frac{\bar{x}_m}{N\mu}. \tag{4}$$

That is, active managers can profit only through stock picking, not market timing or overall asset allocation. The expected difference in return between stock i and the market benchmark is therefore equal to

$$\alpha_i = \frac{\bar{x}_i}{p_i} - \mu = \mu\left(\frac{\bar{p}_i - p_i}{p_i}\right). \tag{5}$$

2.4. Fund managers' expectations

Manager j expects end-of-period value for stock i to be $\tilde{x}_i^{(j)}$ instead of $\bar{x}_i$, $i = 1, \ldots, N$. The manager does not misvalue the overall market, however, so $\sum_{i=1}^{N} \tilde{x}_i^{(j)} = \bar{x}_m$. Instead of the true alpha in equation (5), manager j's assessed alpha for stock i is therefore

$$\tilde{\alpha}_i^{(j)} = \mu\left(\frac{\tilde{p}_i^{(j)} - p_i}{p_i}\right), \tag{6}$$

with the manager's assessed fair price for the stock being

$$\tilde{p}_i^{(j)} = \frac{\tilde{x}_i^{(j)}}{\mu}. \tag{7}$$

I do not assume managers use their available information correctly in forming rational expectations about the payoffs on individual stocks. The difference between $\tilde{x}_i^{(j)}$ and $\bar{x}_i$ can reflect a failure to condition correctly on whatever information is available. In that sense, a more skilled manager simply makes smaller conditioning mistakes. While not a necessary assumption, one interpretation of $\bar{x}_i$ is that it represents the correct conditional expectation given a common signal about stock i that is observed by all of the managers, each of whom pays a cost to be a manager. Such an interpretation is somewhat analogous to having perfectly skilled managers, for whom $\tilde{x}_i^{(j)} = \bar{x}_i$, correspond to the Grossman-Stiglitz (1980) rational informed agents who pay a cost to observe a common informative signal. A common-signal scenario is motivated, for example, by the U.S. Securities and Exchange Commission's Regulation Fair Disclosure, which seeks to reduce disparities across investors in the information released by firms.

Define manager j's active weight in stock i as

$$\phi_i^{(j)} = \phi_{A,i}^{(j)} - \phi_{m,i}, \tag{8}$$

where $\phi_{A,i}^{(j)}$ is stock i's weight in the manager's portfolio, and $\phi_{m,i}$ is stock i's weight in the market portfolio. Note that

$$\sum_{i=1}^{N} \phi_i^{(j)} = 0. \tag{9}$$

Let $g^{(j)}$ denote the true gross alpha on the manager's portfolio, given by

$$g^{(j)} = \sum_{i=1}^{N} \phi_i^{(j)} \alpha_i, \tag{10}$$

and define $\tilde{g}^{(j)}$ as

$$\tilde{g}^{(j)} = \sum_{i=1}^{N} \phi_i^{(j)} \tilde{\alpha}_i, \tag{11}$$

which is the gross alpha on the portfolio implied by the $\tilde{\alpha}_i^{(j)}$'s.

2.5. Fund-level rational expectations and managers' optimization

While I do not assume managers form rational expectations about payoffs on individual stocks, I do assume both investors and managers have rational expectations about fund-level performance. That is, neither a fund's manager nor its investors are surprised on average by the fund's performance, whether gross or net of fees and costs. In this respect, although the model operates within a single period, I implicitly assume that this period is preceded by sufficiently many having the same properties, such that investors and managers come to have rational expectations about fund-level quantities. In other words, both investors and managers have realistic assessments of a manager's skill insofar as how it translates to fund performance.

The specifics of this assumption involve the following. Consider the fund's true net alpha, given by

$$\alpha_A^{(j)} = g^{(j)} - C^{(j)}/W^{(j)} - f^{(j)}, \tag{12}$$

where $C^{(j)}$ equals the fund's total trading cost,

$$C^{(j)} = c \sum_{i=1}^{N} \underbrace{\left(|\phi_i^{(j)}| W^{(j)}/p_i\right)}_{\delta_i} \underbrace{|\phi_i^{(j)}| W^{(j)}}_{D_i} = c \left(W^{(j)}\right)^2 \sum_{i=1}^{N} \left(\phi_i^{(j)}\right)^2 / p_i, \tag{13}$$

obtained by applying equation (1). I assume that investors correctly assess $\alpha_A^{(j)}$ and that they invest $W^{(j)}$ with the fund such that equation (2) is satisfied. I also assume fund managers know that their equilibrium fund sizes are determined as such, and that they are not surprised by the amount of fee revenue they receive. That revenue is determined by setting the right-hand side of equation (12) equal to zero (by equation (2)), substituting from equation (13) for $C^{(j)}$, solving for $W^{(j)}$, and then multiplying through by $f^{(j)}$, giving

$$F^{(j)} = \frac{f^{(j)}\left(g^{(j)} - f^{(j)}\right)}{c\sum_{i=1}^{N}\left(\phi_i^{(j)}\right)^2/p_i}. \tag{14}$$

This assumption about managers means that even though they incorrectly assess the true alphas of individual stocks, they correctly assess the true gross alphas of their portfolios. Specifically, manager j knows by experience the value of $\lambda^{(j)}$ such that

$$g^{(j)} = \lambda^{(j)}\tilde{g}^{(j)}. \tag{15}$$

Combining equations (11), (14), and (15) gives fee revenue as

$$F^{(j)} = \frac{f^{(j)}\left(\lambda^{(j)}\sum_{i=1}^{N}\phi_i^{(j)}\tilde{\alpha}_i - f^{(j)}\right)}{c\sum_{i=1}^{N}\left(\phi_i^{(j)}\right)^2/p_i}, \tag{16}$$

which the manager maximizes by choosing $\phi_1^{(j)}, \ldots, \phi_N^{(j)}$ and $f^{(j)}$, subject to equation (9).

3. Skill and fees in the cross section

The partial equilibrium condition in equation (2) and the additional assumptions above deliver implications about the cross section of skill versus revenue as well as the role of the fee rate.

3.1. Differences in fee revenue across managers

The following proposition reveals what drives differences in fee revenue across managers. (Proofs of all propositions are in the Appendix.)

Proposition 1. *Manager j's fee revenue in equilibrium is given by*

$$F^{(j)} = \frac{V_m}{4c}\psi\left(\rho^{(j)}\right)^2, \tag{17}$$

with

$$\psi = \text{Var}^*(\alpha) \tag{18}$$

and

$$\rho^{(j)} = \frac{\text{Cov}^*(\tilde{\alpha}^{(j)}, \alpha)}{[\text{Var}^*(\alpha)\text{Var}^*(\tilde{\alpha}^{(j)})]^{1/2}}, \tag{19}$$

where V_m is the value of the stock market (Np_m). The moments $\text{Var}^*(\cdot)$ *and* $\text{Cov}^*(\cdot,\cdot)$ *denote the cross-sectional variance and covariance of stock-specific quantities, defined across the N stocks using the stocks' market-capitalization weights (i.e., the values of p_i/V_m as probabilities). The manager's choice of the fee rate, $f^{(j)}$, does not affect the equilibrium value of $F^{(j)}$.*

We see from the above that differences in fee revenue across managers depend solely on $\rho^{(j)}$, the correlation across stocks between the manager's assessed alphas and the true alphas (with correlation defined over the value-weight measure). This correlation captures the manager's revenue-relevant skill. The higher is $\rho^{(j)}$, the more skilled is the manager in assessing relative alphas across stocks. Managers with greater skill earn more fee revenue than those with less skill. This implication is consistent with the empirical evidence of Berk and van Binsbergen (2015). They find that, in a sample of over 6,000 active equity mutual funds, greater fee revenue is collected by funds exhibiting greater skill. They measure a fund's skill as its estimated "value added," the fund's assets under management (AUM) times the fund's alpha gross of its fee rate. Fund j's value added is thus $V_A = W^{(j)}(\alpha_A^{(j)} + f^{(j)})$, which under the equilibrium condition in equation (2) is simply equal to $F^{(j)}$. Cross-sectional differences in value added, given the latter's equivalence to fee revenue in this setting, therefore perfectly correlate with managers' skills in assessing stock's true alphas. Equation (17) shows that those alphas by themselves also play a role in the manager's value added, via the quantity ψ.

The value of ψ, the true alphas' cross-sectional variance (over the value-weight measure), captures the revenue-relevant mispricing present in equilibrium. The larger is the variance of the α_i's, the greater are the relative deviations of $\bar{p}_i$ from p_i, as is evident from equation (5). Equation (17) shows that, ceteris paribus, greater equilibrium mispricing is accompanied by higher fee revenue for all managers, as ψ is not manager-specific. The equilibrium analyzed thus far is a partial one, however, in that equilibrium prices, and thus the determinants of ψ, are not addressed. One of those determinants is the collective skill of active management as an industry, as shown in the general equilibrium analyzed later.

3.2. Portfolio weights and fee rates

The following proposition reveals the role played by the manager's fee rate, $f^{(j)}$, and why it is irrelevant for equilibrium fee revenue, $F^{(j)}$.

Proposition 2. *Manager j's active weight in stock i is*

$$\phi_i^{(j)} = 2f^{(j)}\mu \left(N\lambda^{(j)}\tilde{\psi}^{(j)}\right)^{-1} \left(\frac{\tilde{p}_i^{(j)} - p_i}{p_m}\right), \tag{20}$$

with

$$\tilde{\psi}^{(j)} = \mathrm{Var}^*(\tilde{\alpha}^{(j)}). \tag{21}$$

Trading costs equal fee revenue in equilibrium,

$$C^{(j)} = F^{(j)}, \tag{22}$$

and the gross alpha is twice the fee rate,

$$g^{(j)} = 2f^{(j)}. \tag{23}$$

The value of $\lambda^{(j)}$, defined in equation (15), is given by

$$\lambda^{(j)} = \frac{\mathrm{Cov}^*(\tilde{\alpha}^{(j)}, \alpha)}{\mathrm{Var}^*(\tilde{\alpha}^{(j)})}. \tag{24}$$

The irrelevance of $f^{(j)}$ for the equilibrium $F^{(j)}$ means that raising $f^{(j)}$ simply gives the manager proportionately lower $W^{(j)}$ to manage. In managing that lower $W^{(j)}$, we see from equation (20) that the manager simply scales up all of the active weights in proportion to the higher fee rate $f^{(j)}$. The dollar sizes of the active positions, and thus total trading costs, are unaffected. The fund's portfolio becomes less diversified, however, in the sense that the portfolio weights depart more from benchmark weights. Pástor, Stambaugh, and Taylor (2018) find empirically that among active equity mutual funds, those with a higher $f^{(j)}$, measured as the expense ratio, tend to be less diversified in this sense, consistent with this implied tradeoff. That study also finds a negative correlation between $f^{(j)}$ and $W^{(j)}$, consistent with the product of those quantities, fee revenue, being more fundamentally relevant than either quantity individually. Berk and Green (2004) include a setting that also implies the fee rate's irrelevance and its inverse relation to departures from benchmark weights.

4. Skill and fees of the industry in general equilibrium

To analyze a general equilibrium, I make further assumptions, all of which are nested in the setting presented thus far. The most notable is to assume that all active managers are equally skilled, in a manner specified below. Abstracting from cross-manager differences sharpens the focus on overall industry skill, in addition to making the equilibrium solution tractable.

4.1. Additional assumptions

Noise traders own fraction h of total stock-market wealth and invest through neither the M active managers nor index funds. Investors own the remaining fraction $1-h$ of stock-market wealth. They invest in the stock market only through the M active managers and index funds. Passive managers offer a zero-cost market-index fund that invests any money the investors do not allocate to active managers. The number of active managers in the model, M, while finite, is sufficiently large to maintain the earlier assumption that each manager takes stock prices as given. Intermediaries receive the trading costs incurred by active managers but otherwise play no role.

Who are the noise traders? They can be individuals who invest directly in stocks without involving professional managers, such as in Stambaugh (2014). They can also represent professional managers with negative gross alpha who nevertheless receive money from unwitting investors similar to the "noise allocators" in Gârleanu and Pedersen (2018). Such managers are not included in the model's M active managers, as each of the latter instead has a zero net alpha in equilibrium. Also, the money placed with any negative-gross-alpha managers is included in the fraction h of stock-market wealth owned by noise-traders, not in the remaining fraction $1-h$ owned by the model's investors.

Let $\phi_{H,i}$ denote the weight on stock i in the aggregate portfolio of the noise traders, with $\phi_{H,i}$ assumed to be exogenous and non-negative (no short selling by the noise traders). The price of stock i that would arise in an equilibrium with no active management, with prices thus determined solely by the asset demands of noise traders, is equal to

$$\hat{p}_i = N p_m \phi_{H,i}. \tag{25}$$

Note that $\hat{p}_i$ does not depend on h, the fraction of the market owned by noise traders. That is, even a small presence of noise traders could distort prices significantly in the absence of active management.

I assume that manager j's assessment of the fair price for stock i obeys

$$\tilde{p}_i^{(j)} = (1 - \nu_1)\bar{p}_i + \nu_1 \hat{p}_i + \nu_2 \zeta_i^{(j)} \bar{p}_i, \tag{26}$$

with $0 \le \nu_1 < 1$ and $\nu_2 > 0$. The $\zeta_i^{(j)}$'s are purely idiosyncratic across both stocks and managers and cross-sectionally independent of both the $\bar{p}_i$'s and $\hat{p}_i$'s. For a given manager j, the $\zeta_i^{(j)}$'s have zero mean and unit variance across stocks. The overall market remains correctly valued, with

$$N p_m = \sum_{i=1}^{N} \hat{p}_i = \sum_{i=1}^{N} \tilde{p}_i. \tag{27}$$

Although managers' beliefs differ from each other to at least some degree, because $\nu_2 > 0$, managers are equally skilled in that the exogenous parameters ν_1 and ν_2 are the same across managers. The skill with which managers assess stocks' fundamental values is decreasing in both ν_1 and ν_2. The parameter ν_1 governs the extent to which noise-traders' demands also arise in the beliefs of active managers, as discussed earlier. The parameter ν_2 governs the magnitudes of manager-specific limitations and mistakes.

4.2. Equilibrium conditions

An equilibrium occurs when, simultaneously,

1. for each manager j, $j = 1, \ldots, M$, the choices of $\phi_1^{(j)}, \ldots, \phi_N^{(j)}$, and $f^{(j)}$ satisfy the manager's maximization of fee revenue, $F^{(j)}$,

2. equation (2) holds for each manager, and

3. stock prices, $\{p_1, p_2, \ldots, p_N\}$ satisfy the market-clearing condition

$$h\phi_{H,i} + (1 - h)\phi_{S,i} = \phi_{m,i}, \quad i = 1, \ldots, N, \tag{28}$$

where $\phi_{m,i} = p_i / \sum_{j=1}^{N} p_j$ is stock i's market weight, $\phi_{H,i}$ is the stock's weight in the aggregate stock portfolio of the noise traders, and $\phi_{S,i}$ is the stock's weight in investors' aggregate stock portfolio (which combines the index fund with the aggregate active portfolio).

4.3. Implications

The following proposition characterizes the stock prices and fee revenue that satisfy equilibrium.

Proposition 3. *In equilibrium, $\rho^{(j)}$ and $\tilde{\psi}^{(j)}$ are identical across managers and denoted as ρ and $\tilde{\psi}$. Fee revenue is identical across managers, given by*

$$F = \frac{V_m}{4c}\psi\rho^2, \tag{29}$$

and the ratio of aggregate fee revenue to the total value of the stock market is

$$\Pi = \frac{M}{4c}\psi\rho^2. \tag{30}$$

The price of stock i is given by

$$p_i = \bar{p}_i \ + \ \theta(\hat{p}_i - \bar{p}_i), \tag{31}$$

where θ equals the solution to

$$\theta = \frac{1 + \nu_1 q(\theta)}{1 + q(\theta)}, \tag{32}$$

with

$$q(\theta) = \left(\frac{\theta - \nu_1}{\theta}\right)\left(\frac{\psi}{\tilde{\psi}}\right)\left(\frac{M\mu}{2ch}\right), \tag{33}$$

$$\psi = \theta^2\mu^2\left(\frac{1}{N}\right)\sum_{i=1}^{N}\frac{[(\hat{p}_i - \bar{p}_i)/p_m]^2}{\bar{p}_i/p_m + \theta[(\hat{p}_i - \bar{p}_i)/p_m]}, \tag{34}$$

and

$$\tilde{\psi} = \left(\frac{\theta - \nu_1}{\theta}\right)^2\psi \ + \ \nu_2^2\mu^2\left(\frac{1}{N}\right)\sum_{i=1}^{N}\frac{(\bar{p}_i/p_m)^2}{\bar{p}_i/p_m + \theta(\hat{p}_i - \bar{p}_i)/p_m}. \tag{35}$$

The value of the skill measure, ρ, is given by

$$\rho = \left(\frac{\theta - \nu_1}{\theta}\right)\left(\frac{\psi}{\tilde{\psi}}\right)^{1/2}. \tag{36}$$

For a given quantitative specification of the noise-trader demands, as provided in the next section, the value of the scalar θ can be obtained by solving equation (32) numerically.

The general equilibrium reveals the role of skill in price correction. As shown earlier, a partial equilibrium is sufficient to show that ρ captures each manager's fee-relevant skill, conditional on a given amount of equilibrium mispricing. Of course, that property of ρ still holds in the general equilibrium. In the latter, however, the amount of mispricing depends on skill as well. Each manager, being one of many, takes prices as given when making decisions, but the skill all managers apply to those decisions affects how much mispricing survives in equilibrium. The amount of mispricing that is relevant for fee revenue is captured by ψ. Note from equation (34) that ψ depends on θ, which from equation (31) is the fraction of mispricing that survives, relative to the mispricing that would prevail in the absence of active management. The solution for θ in turn depends on both ν_1 and ν_2, the parameters that govern the skill with which managers assess fundamental value, via equation (26). The next section illustrates that the product $\psi\rho^2$ in equation (29) can either increase or decrease in response to a simultaneous increase in ρ and decrease in ψ. In other words, fee revenue need not increase with skill.

Although both ν_1 and ν_2 matter for θ, note from equation (26) that prices do not contain any of the manager-specific noise given by the last term in equation (26). That noise washes out of the ratio of one active weight to another in the aggregate portfolio of active managers, but the presence of such noise does impact the magnitudes of those weights and thus the degree of price correction.

Useful for the later quantitative analysis is a measure that summarizes the size of the active positions in the aggregate portfolio of active managers. The first step in constructing such a measure is to obtain that portfolio's active weights, provided by the following proposition.

Proposition 4. *The active weight on stock i in the aggregate portfolio of active managers is given by*

$$\phi_i = -\frac{2\bar{f}\mu\theta}{N\psi}\left(\frac{\hat{p}_i - \bar{p}_i}{p_m}\right), \tag{37}$$

where $\bar{f}$ is the harmonic mean of fee rates,

$$\bar{f} = \left(\frac{1}{M}\sum_{j=1}^{M}\frac{1}{f^{(j)}}\right)^{-1}. \tag{38}$$

Cremers and Petajisto (2009) propose active share as a measure summarizing the degree to which a portfolio's weights deviate from those of a benchmark portfolio. Their definition of active share applied to the aggregate active portfolio, with respect to the market benchmark, is given by

$$AS = (1/2)\sum_{i=1}^{N}|\phi_i|. \tag{39}$$

Substituting the expression for ϕ_i in equation (37) gives

$$AS = \frac{\bar{f}\mu\theta}{\psi}\left(\frac{1}{N}\right)\sum_{i=1}^{N}\left|\frac{\hat{p}_i - \bar{p}_i}{p_m}\right|. \tag{40}$$

Multiplying AS by the amount of money allocated to active management, W, puts the economic magnitude of active share in dollar terms, what might be termed "active position." For a given set of active portfolio weights, active management's impact on equilibrium prices is greater the larger the amount of money being deployed at those weights. Applying equation (29),

$$W = \sum_{j=1}^{M} W^{(j)} = \sum_{j=1}^{M}\frac{F^{(j)}}{f^{(j)}} = \frac{Np_m\psi\rho^2}{4c}\sum_{j=1}^{M}\frac{1}{f^{(j)}} = \frac{Np_m M\psi\rho^2}{4c}\left(\bar{f}\right)^{-1}. \tag{41}$$

Dividing $W \times AS$ by Np_m gives the active position as a fraction of the total value of the stock market,

$$AP = \frac{M\rho^2\mu\theta}{4c}\left(\frac{1}{N}\right)\sum_{i=1}^{N}\left|\frac{\hat{p}_i - \bar{p}_i}{p_m}\right|. \tag{42}$$

The value of AP, which essentially gauges the size of active management's trading footprint on the market, proves useful in understanding the amount of price correction that active management effects.

5. Quantitative analysis

5.1. Parameter specifications

The main task in parameterizing the model is specifying the exogenous noise-trader demands. The approach here follows that in Stambaugh (2014). First, the price and payoff of each stock i are normalized by expected end-of-period value, so that $\bar{x}_i = \bar{x}_m$, and thus $\bar{p}_i = p_m$. The relative pricing error $(\hat{p}_i - \bar{p}_i)/p_m$ appearing in the mispricing measure ψ in equation (34) is then given by

$$\begin{aligned}(\hat{p}_i - \bar{p}_i)/p_m &= N\phi_{H,i} - \bar{p}_i/p_m \\ &= v_i - 1\end{aligned} \tag{43}$$

with

$$v_i = N\phi_{H,i}, \tag{44}$$

using equation (25). The mispricing measure ψ in equation (34) can then be written as

$$\psi = \theta^2\mu_m^2\left(\frac{1}{N}\right)\sum_{i=1}^{N}\frac{(v_i - 1)^2}{1 + \theta(v_i - 1)}. \tag{45}$$

Similarly, equation (35) can then be written as

$$\tilde{\psi} = \left(\frac{\theta - \nu_1}{\theta}\right)^2 \psi \; + \; \nu_2^2 \mu_m^2 \left(\frac{1}{N}\right) \sum_{i=1}^{N} \frac{1}{1 + \theta(v_i - 1)}. \tag{46}$$

Next, the cross-sectional distribution of the v_i's is approximated by a continuous Weibull density for v. The density is defined for $v \geq 0$, consistent with the assumption that noise-traders do not short. The Weibull distribution has two parameters, determining the distribution's scale and shape.[3] Because $\sum_{i=1}^{N} \phi_{H,i} = 1$, the scale is determined by $\mathrm{E}(v) = 1$, so there is one free parameter k that determines the distribution's shape. As k becomes large, the density concentrates around $v = 1$, yielding the completely diversified portfolio that puts equal weights on all stocks. As k becomes small, the mass concentrates toward zero and skewness increases, yielding an undiversified portfolio that puts low weights on most stocks and large weights on a relative few. Figure 1 displays densities corresponding to alternative values of k. As Stambaugh (2014) discusses, numerous studies report evidence indicating that direct holdings of individuals are quite undiversified and exhibit significant commonality across individuals.[4] Commonality in holdings limits the extent to which the low diversification by individuals washes out when their holdings are aggregated, making the relatively low values of k plausible. In contrast, low commonality would likely result in a distribution of aggregate weights similar to the density displayed in Figure 1 for $k = 20$, corresponding to a relatively well diversified portfolio. In the quantitative analysis here, I set $k = 1$ but explore sensitivity to variation between $k = 0.5$ and $k = 2$.

The analogs of equations (45) and (46) in terms of the continuous v are

$$\psi = \theta^2 \mu_m^2 \; \mathrm{E}\left\{\frac{(v-1)^2}{1 + \theta(v - 1)}\right\} \tag{47}$$

and

$$\tilde{\psi} = \left(\frac{\theta - \nu_1}{\theta}\right)^2 \psi \; + \; \nu_2^2 \mu_m^2 \; \mathrm{E}\left\{\frac{1}{1 + \theta(v - 1)}\right\}. \tag{48}$$

Applying equation (25) and the same normalization as above in which $\bar{p}_i = p_m$ allows active position in equation (42) to be written as

$$AP \;\; = \;\; \frac{M\rho^2\mu\theta}{4c}\left(\frac{1}{N}\right)\sum_{i=1}^{N} |N\phi_{H,i} - 1|$$

[3] For a discussion of the Weibull distribution, see for example Johnson and Kotz (1970, chapter 20).

[4] Studies presenting evidence of poor diversification include Blume, Crockett, and Friend (1974), Lease, Lewellen, and Schlarbaum (1974), Blume and Friend (1975), Kelly (1995), Polkovnichenko (2005), and Goetzmann and Kumar (2008). Evidence of significant commonality in individuals' stock holdings is reported by Feng and Seasholes (2004), Dorn, Huberman, and Sengmueller (2008), Barber and Odean (2008), and Barber, Odean, and Zhu (2009).

$$= \frac{M\rho^2\mu\theta}{4c}\left(\frac{1}{N}\right)\sum_{i=1}^{N}|v_i - 1|\,, \tag{49}$$

and the analog in terms of the continuous v becomes

$$AP = \frac{M\rho^2\mu\theta}{4c}\mathrm{E}\{|v-1|\}. \tag{50}$$

The errors in managers' expectations are governed by the parameters ν_1 and ν_2 in equation (26). Recall that ν_1 is the fraction of noise-trader demands that also pervade managers' expectations. Values of ν_1 entertained below cover this parameter's entire permissible range, $0 \leq \nu_1 < 1$. The value of ν_2 governs the magnitude of managers' idiosyncratic expectation errors, and recall $\nu_2 > 0$. Because the $\zeta_i^{(j)}$'s have unit variance across stocks, ν_2 represents a manager's typical valuation error relative to correct fundamental value. For much of the analysis below, the value of ν_2 ranges from 0.01 to 0.50, corresponding to typical valuation errors between 1% and 50% of fundamental value. In one case, however, much larger values of ν_2 are entertained, in order to explore more fully the effects of this component of managers' skill.

The other parameters to be specified are the fraction of the stock market owned by noise traders, h, the trading cost parameter, c, the number of active managers, M, and one plus the discount rate for stocks, μ. I set $h = 0.25$ but explore sensitivity to variation between 0 and 0.5. Noise traders are often viewed as individuals who invest directly on their own. As discussed by Stambaugh (2014), who incorporates estimates reported by French (2008), the fraction of the equity market owned directly by individuals has trended steadily downward over the decades, from nearly 0.50 in 1980 to less than 0.20 in recent years. For trading costs, I set $c = 1$ but explore sensitivity to variation between 0.25 and 2.50. With $c = 1$, the proportional cost of trading a given amount of a stock is equal to that amount's fraction of the stock's total market capitalization. The number of managers is set at $M = 3000$, but I explore sensitivity to variation between 500 and 5000. Finally, I set $\mu = 1.065$, with 6.5% being roughly the average stock-market return over recent decades, but plausible variation in this parameter has negligible effects.

5.2. Equilibrium outcomes

5.2.1. Fee revenue, skill, and mispricing

Figure 2 illustrates the study's main result. The figure plots fee revenue versus ν_1, with revenue stated as a fraction of the stock-market's total value, given by Π in equation (30).

Recall that a higher value of ν_1 means that the beliefs of active managers contain more of the same noise present in the demands of noise traders. A higher ν_1 thus means managers have less skill in assessing stocks' fundamental values. Observe that fee revenue is hump shaped with respect to ν_1. In particular, as ν_1 increases from zero to about 0.6, fee revenue increases from less than 50 basis points of the stock-market's capitalization to over 250 basis points. Not until managers' beliefs are quite strongly in sympathy with noise-trader demands does a further decline in skill erode revenue. We also see that fee revenue is higher for the larger of the two values for ν_2. Recall that a higher ν_2 also means less skill, as it corresponds to higher variance of the idiosyncratic noise in managers' beliefs. Overall, Figure 2 strikingly illustrates that greater skill for the industry can result in less revenue.

The basic intuition behind this result is illustrated by the two panels of Figure 3. Equilibrium fee revenue in equation (29) depends on ρ and ψ, both of which are endogenous equilibrium quantities. The degree of managerial skill relevant to fee revenue is summarized by ρ, the correlation between each manager's assessed alphas on individual stocks and the stocks' true alphas. The amount of fee-relevant mispricing is summarized by ψ, which reflects the magnitudes of stocks' true alphas, as summarized their cross-sectional variance. (Recall that correlation and variance here are with respect to the value-weight measure.) Panel A of Figure 3 confirms that as ν_1 increases, skill as captured by ρ decreases. At the same time, however, more skill results in more price correction. We see in Panel B that the degree of equilibrium mispricing, captured by ψ, does indeed increase as ν_1 increases (and skill declines). These two effects of skill—more accurate alpha assessments versus lower alpha magnitudes—affect fee revenue in opposite directions. Their net effect on fee revenue depends on the product $\psi\rho^2$, as shown by the equilibrium value of F in equation (29). Over much of the range for ν_1, Figure 2 reveals that as skill decreases, the decline in ρ is outweighed by the increase in ψ, and thus fee revenue increases. That is, in the determination of equilibrium fee revenue, the weaker price correction more than offsets managers' worse assessments of alphas.

Skill's role in equilibrium price correction can be understood further from Figure 4. Panel A displays the effect of ν_1 on the quantity $1-\theta$, which from equation (31) is the fraction of noise-trader distortions of prices that active management eliminates. As ν_1 goes from zero to one, $1-\theta$ goes from almost one to a value of zero. This near-linear decline in the fraction of price correction is echoed in Panel B by a near-linear decline in active position, the value of AP in equation (50). The latter quantity drops from roughly 9% of the stock market's value at $\nu_1 = 0$ to a zero active position at $\nu_1 = 1$. As managers become less skilled, their aggregate active position declines monotonically, resulting in correspondingly less correction of noise-trader distortions of prices.

The plots in Figures 2 through 4 display results under two values of ν_2, the standard deviation of the manager-specific valuation errors in equation (7). Throughout those plots, the higher value of ν_2, 0.50, gives greater fee revenue than does the lower value, 0.01. Panel A of Figure 5 plots fee revenue for the entire range of ν_2 up to that higher value of 0.50. Throughout that range, fee revenue increases as that dimension of skill decreases, i.e, as ν_2 increases. As shown in Figure 1, for sufficiently high values of ν_1, fee revenue is decreasing in ν_1. Is the same true for ν_2? The answer is yes, as revealed in Panel B of Figure 5, but very large values of ν_2 are required before revenue becomes increasing in this dimension of skill, i.e., decreasing in ν_2. For both of the ν_1 values (0.05 and 0.10), fee revenue doesn't become decreasing in ν_2 until ν_2 reaches values of roughly 40, corresponding to a 4000% standard deviation of relative valuation errors. For practical purposes in the setting analyzed here, fee revenue increases as managers' idiosyncratic assessments of fair value become noisier.

5.2.2. Robustness to parameter specifications

Figure 6 displays plots confirming robustness to alternative values of k, h, c, and M. Recall that in Figures 2 through 5, I specify those four parameters as $k = 1$, $h = 0.25$, $c = 1$, and $M = 3000$. The four panels of Figure 6 plot fee revenue with respect to each of those parameters, over ranges extending in both directions around the above values. Each panel contains four plots, with ν_1 set to 0.05 or 0.10 and ν_2 set to 0.01 or 0.50. All of the results in Figure 6 agree with the scenario described earlier, in which fee revenue declines as skill increases along both of the dimensions governed by ν_1 and ν_2. That is, at all values of k, h, c, and M, fee revenue is greater at the higher values of both ν_1 and ν_2.

Figure 6 also illustrates the roles of k, h, c, and M in determining equilibrium fee revenue. Understanding these roles provides further insight into the model of active management presented here. Consider first the noise traders, a key component of the model. Revenue is decreasing in k, as shown in Panel A. Active management therefore makes more when noise traders place inappropriately large fractions of their stock investments in a relative few overvalued stocks. Active management also makes more, not surprisingly, when noise traders own a larger fraction, h, of the stock market, as shown in Panel B.

Trading costs, which present each fund manager with decreasing returns to scale, are another essential component of the model. When c in equation (1) is higher, the cost of trading a given amount of stock is higher. Panel C reveals that an increase in c has a negligible (barely visible) positive effect on fee revenue when $\nu_2 = 0.01$, but that positive effect is stronger when $\nu_2 = 0.50$. Although the idiosyncratic noise in managers' beliefs, governed by ν_2, washes out of the aggregate active position, higher trading costs cause each manager's positions to be less aggressive, and thereby cause the aggregate active position to be less aggressive. As a result, more of the noise-trader distortions survive in equilibrium, leaving greater mispricing. The latter means greater fee revenue at the skill levels considered.

The same trading-cost channel is at work in Panel D of Figure 6, which shows that fee revenue declines as the number of managers, M, increases. This negative effect is barely visible when $\nu_2 = 0.01$, but it is stronger when $\nu_2 = 0.50$. Managers are competitive price takers at all values of M, but when more of them compete, each essentially receives less money to manage (for a given distribution of fee rates). Trading smaller dollar amounts lowers proportional trading costs, so raising M essentially plays the same role as lowering c. The latter statement can made more precise by noting that c and M enter Π, the ratio of fee revenue to stock-market value plotted in Figure 6, only as the ratio M/c, appearing in equations (30) and (33). Given the results in Panels C and D of Figure 6, we see Π is decreasing in M/c.

5.2.3. Potential endogeneity of trading costs and the number of managers

Both the trading-cost parameter, c, and the number of active managers, M, are treated as exogenous in the equilibrium analysis. An alternative treatment could have one or both quantities depend endogenously on managers' skill. For example, greater skill could imply greater information asymmetry between managers and intermediaries, potentially increasing what an intermediary charges a manager to trade a given amount of a stock. Such a channel is somewhat similar to that analyzed by Glode, Green, and Lowery (2012), who show that an "arms race" to acquire skill by competing financial firms works to those firms' collective disadvantage due to adverse-selection concerns under information asymmetry.

As discussed above, a higher value of the trading-cost parameter, c, implies a higher amount of fee revenue for active managers. Therefore, an endogenous increase in c can weaken or reverse a drop in fee revenue that, as shown earlier, can otherwise accompany an increase in managers' skill. The magnitude of this potential effect of c does not appear to be large, however. First recall from Panel C of Figure 6 that, when the skill parameter ν_2 is only 0.01, fee revenue is virtually flat over values of c ranging from 0.25 to 2.5. When ν_2 is instead 0.50, meaning a manager's idiosyncratic valuation error is typically 50% of a stock's value, fee revenue is more steeply increasing in c. Nevertheless, observe in Panel C that if ν_1 drops from 0.10 to 0.05, thereby halving the fraction of noise-trader demands contaminating managers' beliefs, this increase in skill still implies lower fee revenue even if c increases tenfold, from 0.25 to 2.5.

The number of active managers, M, could also depend on managers' skill. If an increase in skill can bring less aggregate fee revenue, as shown earlier, that lower fee revenue could induce manager exits. Such exits could occur, for example, due to managers' fixed costs. Because aggregate fee revenue is decreasing in M, as shown in Panel D of Figure 6, an endogenous drop in M can mitigate a drop in fee revenue that can otherwise accompany an increase in managers' skill. Because the ratio of M to c is what matters for aggregate fee revenue, as noted earlier, assessing the magnitude of a potential endogenous response in M largely echoes the above discussion of a response in c. Specifically, even if a tenfold increase in M/c accompanies ν_1 dropping from 0.10 to 0.05, that increase in skill still implies lower aggregate fee revenue.

6. Conclusions

Suppose that the active management industry has become more skilled over time, as suggested by the findings of Pástor, Stambaugh, and Taylor (2015). They estimate a proxy for skill and observe that its distribution across managers trends upward over the past three decades. In other words, that study's evidence suggests the active management industry has become more skilled. The authors suggest education and technology, for example, could be part of the story. One might even construe the recent trend toward quantitatively managed "smart-beta" products as a self-proclaimed increase in the industry's skill (or at least its "smartness"). The results here show that an increase in overall skill can imply a smaller equilibrium amount of fee revenue.

If greater skill spells less revenue, an upward trend in skill represents a potential challenge for the active management industry in addition to that discussed by Stambaugh (2014): the downward trend in direct equity ownership by individuals, a potential source of noise trading. That is, if not only the presence of noise traders declines, but the mispricing they induce is more skillfully identified, then active management can face a doubly strong headwind in maintaining its presence in the money management industry. Of course an industry of competing active managers cannot decide to calm that headwind by becoming less skilled. Applying more skill is in each manager's individual interest, because more-skilled managers make more than less-skilled managers, as also implied by the model presented here.

Fee revenue can decline through a loss of AUM, a drop in the fee rate, or both. The product of those quantities, fee revenue, is what the model's equilibrium determines uniquely. In fact both the AUM market share and the typical fee rate for active management have declined over recent decades, as noted by Stambaugh (2014). In the case of equity mutual funds, for example, over the period from 2001 to 2016, active management lost 15% in its share of total AUM while reducing its fee (expense ratio) by 30 basis points.[5]

For settings in which there is a negative relation between skill and industry size, there is at least an imperfect analogy to the situation faced by any industry that gets more efficient at producing a good or service for which the capacity to consume is relatively constrained. The more efficient the industry becomes in exploiting its productive resources, the less of those resources it needs to employ. A notable example comes from agriculture, where efficiency gains play a big role in that sector's employing a much smaller share of the U.S. labor force than it once did (e.g., Dimitri, Effland, and Conklin, 2005). The capacity for consuming the active management industry's output is constrained in the sense that the industry can go no further than to drive its equilibrium net alpha to zero. Being more skilled in identifying mispriced assets can enable the industry to accomplish that job with less resources.

Even though the industry can earn less by becoming better at what it does, the model does imply that greater skill produces stronger price correction in stocks. With this positive externality, there can still be a societal benefit to having active managers be more skilled. Having prices more accurately reflect underlying fundamentals can allow more efficient resource allocations.

[5]The fraction of total equity mutual fund assets under active management went from 90% to 75%, while the asset-weighted average expense ratio of active equity funds went from 1.1% to 0.8%. (Investment Company Institute, 2017).

Appendix

Proof of Proposition 1: Define the N-element vectors α, $\tilde{\alpha}^{(j)}$, $\phi^{(j)}$ and ι, whose i-th elements equal α_i, $\tilde{\alpha}_i^{(j)}$ $\phi_i^{(j)}$, and 1, respectively. Also define the $N \times N$ matrix P with i-th diagonal element equal to p_i and all non-diagonal elements equal to zero. Manager j's equilibrium fee revenue in equation (16) can then be written as

$$F^{(j)} = \frac{f^{(j)}\left(\lambda^{(j)}\phi^{(j)\prime}\tilde{\alpha}^{(j)} - f^{(j)}\right)}{c\phi^{(j)\prime}P^{-1}\phi^{(j)}}, \tag{A1}$$

and the corresponding Lagrangian to the manager's maximization problem as described there is

$$\mathcal{L} = \frac{f^{(j)}\left(\lambda^{(j)}\phi^{(j)\prime}\tilde{\alpha}^{(j)} - f^{(j)}\right)}{c\phi^{(j)\prime}P^{-1}\phi^{(j)}} - \tilde{\xi}(\iota'\phi^{(j)}). \tag{A2}$$

Differentiating with respect to $\phi^{(j)}$ and multiplying through by $(c/f^{(j)})\left(\phi^{(j)\prime}P^{-1}\phi^{(j)}\right)$ gives

$$\lambda^{(j)}\tilde{\alpha}^{(j)} - \frac{2(\lambda^{(j)}\phi^{(j)\prime}\tilde{\alpha}^{(j)} - f^{(j)})}{\phi^{(j)\prime}P^{-1}\phi^{(j)}}P^{-1}\phi^{(j)} - \xi\iota = 0, \tag{A3}$$

where ξ is the rescaled Lagrange multiplier. Multiplying through by P and rearranging gives

$$\phi^{(j)} = \frac{\lambda^{(j)}\phi^{(j)\prime}P^{-1}\phi^{(j)}}{2\left(\lambda^{(j)}\phi^{(j)\prime}\tilde{\alpha}^{(j)} - f^{(j)}\right)}\left(P\tilde{\alpha}^{(j)} - \xi P\iota\right). \tag{A4}$$

It follows readily from (27) that the market-weighted combination of the $\tilde{\alpha}_i^{(j)}$'s is zero: $\iota'P\tilde{\alpha}^{(j)} = 0$. Therefore, since $\iota'\phi^{(j)} = 0$, multiplying both sides of equation (A4) by ι' implies $\xi = 0$, and thus

$$\phi^{(j)} = \frac{\lambda^{(j)}\phi^{(j)\prime}P^{-1}\phi^{(j)}}{2\left(\lambda^{(j)}\phi^{(j)\prime}\tilde{\alpha}^{(j)} - f^{(j)}\right)}P\tilde{\alpha}^{(j)}. \tag{A5}$$

Multiplying both sides of equation (A5) by $\phi^{(j)\prime}P^{-1}$ and simplifying gives

$$\lambda^{(j)}\phi^{(j)\prime}\tilde{\alpha}^{(j)} - 2f^{(j)} = 0, \tag{A6}$$

which is the same as the first-order condition obtained by differentiating $\mathcal{L}$ with respect to $f^{(j)}$, so the latter condition is satisfied for any positive $f^{(j)}$. That is, the choice of fee rate does not affect maximized fee revenue. Pre- and post-multiplying P^{-1} by each side of equation (A5) and then dividing through by $\phi^{(j)\prime}P^{-1}\phi^{(j)}$ gives, after rearranging,

$$\phi^{(j)\prime}P^{-1}\phi^{(j)} = \frac{4\left(\lambda^{(j)}\phi^{(j)\prime}\tilde{\alpha}^{(j)} - f^{(j)}\right)^2}{\lambda^{(j)2}\tilde{\alpha}^{(j)\prime}P\tilde{\alpha}^{(j)}} = \frac{4f^{(j)2}}{\lambda^{(j)2}\tilde{\alpha}^{(j)\prime}P\tilde{\alpha}^{(j)}}, \tag{A7}$$

with the second equality applying equation (A6). Substituting the right-hand side of that second equality for $\phi^{(j)\prime}P^{-1}\phi^{(j)}$ in equation (A1) and again applying equation (A6) gives

$$F^{(j)} = \frac{{\lambda^{(j)}}^2\tilde{\alpha}^{(j)\prime}P\tilde{\alpha}^{(j)}}{4c}. \tag{A8}$$

To obtain $\lambda^{(j)}$, first substitute the right-hand side of equation (A7) for $\phi^{(j)\prime}P^{-1}\phi^{(j)}$ in equation (A5) and simplify, again using equation (A6), to obtain

$$\phi^{(j)} = \frac{2f^{(j)}}{\lambda^{(j)}\tilde{\alpha}^{(j)\prime}P\tilde{\alpha}^{(j)}}P\tilde{\alpha}^{(j)} \tag{A9}$$

$$= \frac{\phi^{(j)\prime}\tilde{\alpha}^{(j)}}{\tilde{\alpha}^{(j)\prime}P\tilde{\alpha}^{(j)}}P\tilde{\alpha}^{(j)}. \tag{A10}$$

Multiplying both sides of equation (A10) by α' gives

$$\phi^{(j)\prime}\alpha = \left(\frac{\alpha' P\tilde{\alpha}^{(j)}}{\tilde{\alpha}^{(j)\prime}P\tilde{\alpha}^{(j)}}\right)\phi^{(j)\prime}\tilde{\alpha}^{(j)}, \tag{A11}$$

and the condition in equation (15) therefore implies

$$\lambda^{(j)} = \frac{\alpha' P\tilde{\alpha}^{(j)}}{\tilde{\alpha}^{(j)\prime}P\tilde{\alpha}^{(j)}}. \tag{A12}$$

Substituting the right-hand side of equation (A12) for $\lambda^{(j)}$ in equation (A8) gives, after rearranging,

$$F^{(j)} = \frac{1}{4c}(\alpha' P\alpha)\left(\frac{\alpha' P\tilde{\alpha}^{(j)}}{\left(\tilde{\alpha}^{(j)\prime}P\tilde{\alpha}^{(j)}\right)^{1/2}(\alpha' P\alpha)^{1/2}}\right)^2. \tag{A13}$$

The equivalance of equation (A13) to the result stated in the proposition is easily seen by noting that the value-weighted α_i's and $\tilde{\alpha}_i^{(j)}$'s are zero,

$$\sum_{i=1}^{N}\left(\frac{p_i}{Np_m}\right)\alpha_i = \sum_{i=1}^{N}\left(\frac{p_i}{Np_m}\right)\tilde{\alpha}_i^{(j)} = 0 \tag{A14}$$

and therefore

$$\alpha' P\alpha = Np_m\sum_{i=1}^{N}\left(\frac{p_i}{Np_m}\right)\alpha_i^2 = Np_m \mathrm{Var}^*(\alpha_i), \tag{A15}$$

$$\tilde{\alpha}^{(j)\prime}P\tilde{\alpha}^{(j)} = Np_m\sum_{i=1}^{N}\left(\frac{p_i}{Np_m}\right)(\tilde{\alpha}_i^{(j)})^2 = Np_m \mathrm{Var}^*(\tilde{\alpha}_i^{(j)}), \tag{A16}$$

and

$$\alpha' P\tilde{\alpha}^{(j)} = Np_m\sum_{i=1}^{N}\left(\frac{p_i}{Np_m}\right)\alpha_i\tilde{\alpha}_i^{(j)} = Np_m \mathrm{Cov}^*(\alpha_i, \tilde{\alpha}_i^{(j)}). \tag{A17}$$

Proof of Proposition 2: Equation (20) for $\phi_i^{(j)}$ follows directly as the i-th element of $\phi^{(j)}$ in equation (A9), substituting from equation (6) for $\alpha_i^{(j)}$ and from equation (A16) for $\tilde{\alpha}^{(j)\prime}P\tilde{\alpha}^{(j)}$. From equations (13), (A7) and (A8) and the relation $W^{(j)} = F^{(j)}/f^{(j)}$,

$$\begin{aligned} C^{(j)} &= c\left(W^{(j)}\right)^2 \phi^{(j)\prime}P^{-1}\phi^{(j)} \\ &= c\left(\frac{F^{(j)}}{f^{(j)}}\right)^2 \phi^{(j)\prime}P^{-1}\phi^{(j)} \\ &= c\left(\frac{F^{(j)}}{f^{(j)}}\right)\left(\frac{\lambda^{(j)2}\tilde{\alpha}^{(j)\prime}P\tilde{\alpha}^{(j)}}{4cf^{(j)}}\right)\phi^{(j)\prime}P^{-1}\phi^{(j)} \\ &= c\left(\frac{F^{(j)}}{f^{(j)}}\right)\left(\frac{\lambda^{(j)2}\tilde{\alpha}^{(j)\prime}P\tilde{\alpha}^{(j)}}{4cf^{(j)}}\right)\left(\frac{4f^{(j)2}}{\lambda^{(j)2}\tilde{\alpha}^{(j)\prime}P\tilde{\alpha}^{(j)}}\right) \\ &= F^{(j)}, \end{aligned} \tag{A18}$$

giving equation (22). If trading costs equal fee revenue, then $C^{(j)}/W^{(j)} = f^{(j)}$ in equation (12). Applying the condition in equation (2) then implies that $g^{(j)} = 2f^{(j)}$, giving equation (23). Equation (24) is equivalent to equation (A12), noting the relations in equations (A16) and (A17).

Proof of Proposition 3: I conjecture an equilibrium in which the moments in equations (A16) and (A17), and thus the quantities $\lambda^{(j)}$ and $\rho^{(j)}$, are equal across managers, and then I verify that these conditions are satisfied by the resulting equilibrium prices. Define $W = \sum_{j=1}^{M} W^{(j)}$. Let ϕ_i denote the on stock i in the aggregate active portfolio,

$$\phi_i = \sum_{j=1}^{M} \frac{W^{(j)}}{W}\phi_i^{(j)}. \tag{A19}$$

The total stock-market wealth of investors is $(1-h)Np_m$, and thus $\phi_{S,i}$ in equation (28) equals

$$\begin{aligned} \phi_{S,i} &= \left(\frac{W}{(1-h)Np_m}\right)(\phi_{m,i}+\phi_i) + \left(1 - \frac{W}{(1-h)Np_m}\right)\phi_{m,i} \\ &= \phi_{m,i} + \left(\frac{W}{(1-h)Np_m}\right)\phi_i. \end{aligned} \tag{A20}$$

Substituting into equation (28) gives

$$h\phi_{H,i} + (1-h)\left(\phi_{m,i} + \left(\frac{W}{(1-h)Np_m}\right)\phi_i\right) = \phi_{m,i}, \tag{A21}$$

or, using equation (A19),

$$h\left(\phi_{H,i} - \phi_{m,i}\right) = -\frac{W}{Np_m}\phi_i. \tag{A22}$$

Multiplying through by Np_m and using equation (A19) gives

$$h(\hat{p}_i - p_i) = -\sum_{j=1}^{M} W^{(j)} \phi_i^{(j)}. \tag{A23}$$

Combining equations (A8) and (A9), noting $W^{(j)} = F^{(j)}/f^{(j)}$, gives

$$\begin{aligned} W^{(j)}\phi_i^{(j)} &= \left(\frac{{\lambda^{(j)}}^2 \tilde{\alpha}^{(j)\prime} P \tilde{\alpha}^{(j)}}{f^{(j)} 4c} \right) \left(\frac{2f^{(j)}}{\lambda^{(j)} \tilde{\alpha}^{(j)\prime} P \tilde{\alpha}^{(j)}} P \tilde{\alpha}^{(j)} \right) \\ &= \left(\frac{\lambda^{(j)}}{2c} \right) P \tilde{\alpha}^{(j)} \\ &= \left(\frac{\lambda\mu}{2c} \right) \left(\tilde{p}_i^{(j)} - p_i \right), \end{aligned} \tag{A24}$$

where the last equality uses equation (6) and the assumption that $\lambda^{(j)} = \lambda$. Substituting into equation (A23), using equation (26), and noting that the idiosyncratic $\zeta_i^{(j)}$'s average to zero across managers,

$$\begin{aligned} h(\hat{p}_i - p_i) &= -\sum_{j=1}^{M} \left(\frac{\lambda\mu}{2c} \right) \left(\tilde{p}_i^{(j)} - p_i \right) \\ &= -\sum_{j=1}^{M} \left(\frac{\lambda\mu}{2c} \right) \left[(1-\nu_1)\bar{p}_i + \nu_1 \hat{p}_i + \nu_2 \zeta_i^{(j)} \bar{p}_i - p_i \right] \\ &= -\left(\frac{M\lambda\mu}{2c} \right) \left[(1-\nu_1)\bar{p}_i + \nu_1 \hat{p}_i + \nu_2 \bar{p}_i \left(\frac{1}{M} \sum_{j=1}^{M} \zeta_i^{(j)} \right) - p_i \right] \\ &= -\left(\frac{M\lambda\mu}{2c} \right) \left[\bar{p}_i + \nu_1(\hat{p}_i - \bar{p}_i) - p_i \right]. \end{aligned} \tag{A25}$$

Equation (A25) can then be rearranged as

$$p_i = \bar{p}_i + \theta \left(\hat{p}_i - \bar{p}_i \right), \tag{A26}$$

where

$$\theta = \frac{1 + \nu_1 q(\theta)}{1 + q(\theta)}. \tag{A27}$$

and

$$q(\theta) = \lambda \frac{M\mu}{2ch}. \tag{A28}$$

Next is to verify that with equilibrium prices of the form in equation (A26), the moments in equations (A16) and (A17), and thus the quantities $\lambda^{(j)}$ and $\rho^{(j)}$, are equal across managers, as conjectured. First note, using equation (5) and the definition of ψ in equation (18) that

$$\begin{aligned}
\psi &= \sum_{i=1}^{N} \frac{p_i}{Np_m}\mu^2 \left(\frac{\bar{p}_i - p_i}{p_i}\right)^2 \\
&= \sum_{i=1}^{N} \frac{p_i}{Np_m}\mu^2 \left(\frac{-\theta\left(\hat{p}_i - \bar{p}_i\right)}{\bar{p}_i + \theta\left(\hat{p}_i - \bar{p}_i\right)}\right)^2 \\
&= \theta^2\mu^2 \left(\frac{1}{N}\right) \sum_{i=1}^{N} \frac{[(\hat{p}_i - \bar{p}_i)/p_m]^2}{\bar{p}_i/p_m + \theta[(\hat{p}_i - \bar{p}_i)/p_m]}, \qquad \text{(A29)}
\end{aligned}$$

which is the same as equation (34). Proceeding similarly using equation (6) and the definition of $\tilde{\psi}^{(j)}$ in equation (21) gives

$$\begin{aligned}
\tilde{\psi}^{(j)} &= \sum_{i=1}^{N} \frac{p_i}{Np_m}\mu^2 \left(\frac{\tilde{p}_i^{(j)} - p_i}{p_i}\right)^2 \\
&= \frac{\mu^2}{Np_m} \sum_{i=1}^{N} \frac{\left[(\nu_1 - \theta)(\hat{p}_i - \bar{p}_i) + \nu_2\zeta_i^{(j)}\bar{p}_i\right]^2}{\bar{p}_i + \theta(\hat{p}_i - \bar{p}_i)} \\
&= (\nu_1 - \theta)^2\mu^2 \left(\frac{1}{N}\right) \sum_{i=1}^{N} \frac{[(\hat{p}_i - \bar{p}_i)/p_m]^2}{\bar{p}_i/p_m + \theta[(\hat{p}_i - \bar{p}_i)/p_m]} + \nu_2^2\frac{\mu^2}{p_m}\left(\frac{1}{N}\right) \sum_{i=1}^{N} \frac{\bar{p}_i^2}{\bar{p}_i + \theta(\hat{p}_i - \bar{p}_i)}\left(\zeta_i^{(j)}\right)^2 \\
&\quad +2(\nu_1 - \theta)\nu_2\frac{\mu^2}{p_m}\left(\frac{1}{N}\right) \sum_{i=1}^{N} \frac{(\hat{p}_i - \bar{p}_i)\bar{p}_i}{\bar{p}_i + \theta(\hat{p}_i - \bar{p}_i)}\zeta_i^{(j)} \\
&= \frac{(\nu_1 - \theta)^2}{\theta^2}\psi + \nu_2^2\frac{\mu^2}{p_m}\left[\frac{1}{N}\sum_{i=1}^{N} \frac{\bar{p}_i^2}{\bar{p}_i + \theta(\hat{p}_i - \bar{p}_i)}\right]\left[\frac{1}{N}\sum_{i=1}^{N}\left(\zeta_i^{(j)}\right)^2\right] \\
&\quad +2(\nu_1 - \theta)\nu_2\frac{\mu^2}{p_m}\left[\frac{1}{N}\sum_{i=1}^{N} \frac{(\hat{p}_i - \bar{p}_i)\bar{p}_i}{\bar{p}_i + \theta(\hat{p}_i - \bar{p}_i)}\right]\left[\frac{1}{N}\sum_{i=1}^{N}\zeta_i^{(j)}\right] \qquad \text{(A30)} \\
&= \frac{(\nu_1 - \theta)^2}{\theta^2}\psi + \nu_2^2\frac{\mu^2}{p_m}\left[\frac{1}{N}\sum_{i=1}^{N} \frac{\bar{p}_i^2}{\bar{p}_i + \theta(\hat{p}_i - \bar{p}_i)}\right][1] \\
&\quad +2(\nu_1 - \theta)\nu_2\frac{\mu^2}{p_m}\left[\frac{1}{N}\sum_{i=1}^{N} \frac{(\hat{p}_i - \bar{p}_i)\bar{p}_i}{\bar{p}_i + \theta(\hat{p}_i - \bar{p}_i)}\right][0] \\
&= \left(\frac{\theta - \nu_1}{\theta}\right)^2\psi \;+\; \nu_2^2\mu^2\left(\frac{1}{N}\right)\sum_{i=1}^{N} \frac{(\bar{p}_i/p_m)^2}{\bar{p}_i/p_m + \theta(\hat{p}_i - \bar{p}_i)/p_m}, \qquad \text{(A31)}
\end{aligned}$$

which is identical across j and the same as equation (35). The equality in (A30) follows from the assumption that the $\zeta_i^{(j)}$'s are independent of the $\bar{p}_i$'s and $\hat{p}_i$'s across i, so that the mean of the product is the product of the means. Applying again the assumed idiosyncratic properties of the $\zeta_i^{(j)}$'s along with equations (5) and (6) and the definition of $\rho^{(j)}$ in equation (19) gives

$$\left(\tilde{\psi}\psi\right)^{1/2}\rho^{(j)} = \sum_{i=1}^{N} \frac{p_i}{Np_m}\mu^2 \left(\frac{(\tilde{p}_i^{(j)} - p_i)(\bar{p}_i - p_i)}{p_i}\right)$$

$$
\begin{aligned}
&= \frac{\mu^2}{Np_m}\sum_{i=1}^{N}\frac{\left[(1-\nu_1)\bar{p}_i+\nu_1\hat{p}_i+\nu_2\zeta_i^{(j)}\bar{p}_i-\bar{p}_i-\theta(\hat{p}_i-\bar{p}_i)\right](\bar{p}_i-p_i)}{p_i} \\
&= \frac{\mu^2}{p_m}\left(\frac{1}{N}\right)\sum_{i=1}^{N}\frac{\left[(\theta-\nu_1)(\bar{p}_i-\hat{p}_i)+\nu_2\zeta_i^{(j)}\bar{p}_i\right](\bar{p}_i-p_i)}{p_i} \\
&= \frac{\mu^2}{p_m}\left(\frac{1}{N}\right)\sum_{i=1}^{N}\frac{\theta-\nu_1}{\theta}\frac{(\bar{p}_i-p_i)^2}{p_i}+\nu_2\frac{\mu^2}{p_m}\left(\frac{1}{N}\right)\sum_{i=1}^{N}\zeta_i^{(j)}\frac{\theta(\bar{p}_i-\hat{p}_i)\bar{p}_i}{\bar{p}_i+\theta\left(\hat{p}_i-\bar{p}_i\right)} \\
&= \frac{\theta-\nu_1}{\theta}\frac{\mu^2}{p_m}\left(\frac{1}{N}\right)\sum_{i=1}^{N}\frac{(\bar{p}_i-p_i)^2}{p_i}+0 \\
&= \frac{\theta-\nu_1}{\theta}\psi, \qquad \text{(A32)}
\end{aligned}
$$

which is also identical across j. Equation (36) follows immediately from equation (A32). The verification for the cross-sectional moments is thus complete, implying that $\lambda_{(j)}$ in equation (A12) is also identical across j. Also, equation (29) then follows directly from equation (17). Equation (30) directly follows, multiplying F by M and then dividing by the stock market's value, Np_m. Using equation (A12) along with equation (A32) gives

$$
\begin{aligned}
\lambda &= \frac{\alpha' P\tilde{\alpha}^{(j)}}{\tilde{\alpha}^{(j)\prime}P\tilde{\alpha}^{(j)}} \\
&= \frac{\mathrm{Cov}^*(\tilde{\alpha}_i,\alpha_i)}{\mathrm{Var}^*(\tilde{\alpha}_i)} \\
&= \frac{\rho(\psi\tilde{\psi})^{1/2}}{\tilde{\psi}} \\
&= \frac{\theta-\nu_1}{\theta}\left(\frac{\psi}{\tilde{\psi}}\right), \qquad \text{(A33)}
\end{aligned}
$$

which when substituted into equation (A28) gives equation (33).

Proof of Proposition 4: From the previous proposition, all managers earn the same equilibrium fee revenue, F, so $W^{(j)} = F/f^{(j)}$. Because $W = \sum_{j=1}^{M} W^{(j)}$,

$$
\frac{W^{(j)}}{W} = \frac{\bar{f}}{Mf^{(j)}}, \qquad \text{(A34)}
$$

with $\bar{f}$ defined in equation (38). Using these weights along with previous results gives

$$
\begin{aligned}
\phi_i &= \sum_{j=1}^{M}\frac{W^{(j)}}{W}\phi_i^{(j)} \\
&= \sum_{j=1}^{M}\frac{\bar{f}}{Mf^{(j)}}\frac{2f^{(j)}}{\tilde{\alpha}^{(j)\prime}P\alpha}P\tilde{\alpha}^{(j)} \\
&= \frac{2\bar{f}\mu}{Np_m\rho\left(\psi\tilde{\psi}\right)^{1/2}}\left(\frac{1}{M}\right)\sum_{j=1}^{M}\left(\tilde{p}_i^{(j)}-p_i\right)
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{2\bar{f}\mu}{Np_m\rho\left(\psi\tilde{\psi}\right)^{1/2}}\left(\frac{1}{M}\right)\sum_{j=1}^{M}\left[(\nu_1-\theta)(\hat{p}_i-\bar{p}_i)+\nu_2\zeta_i^{(j)}\bar{p}_i\right] \\
&= \frac{2\bar{f}\mu}{Np_m\left(\frac{\theta-\nu_1}{\theta}\right)\psi}\left(\frac{1}{M}\right)\sum_{j=1}^{M}\left[(\nu_1-\theta)(\hat{p}_i-\bar{p}_i)+\nu_2\zeta_i^{(j)}\bar{p}_i\right] \\
&= \frac{2\bar{f}\mu}{Np_m\left(\frac{\theta-\nu_1}{\theta}\right)\psi}\left[(\nu_1-\theta)(\hat{p}_i-\bar{p}_i)+\nu_2\bar{p}_i\left(\frac{1}{M}\right)\sum_{j=1}^{M}\zeta_i^{(j)}\right] \\
&= \frac{2\bar{f}\mu}{Np_m\left(\frac{\theta-\nu_1}{\theta}\right)\psi}\left[(\nu_1-\theta)(\hat{p}_i-\bar{p}_i)+0\right] \\
&= -\frac{2\bar{f}\mu\theta}{N\psi}\left(\frac{\hat{p}_i-\bar{p}_i}{p_m}\right),
\end{aligned} \tag{A35}
$$

which is equation (37).

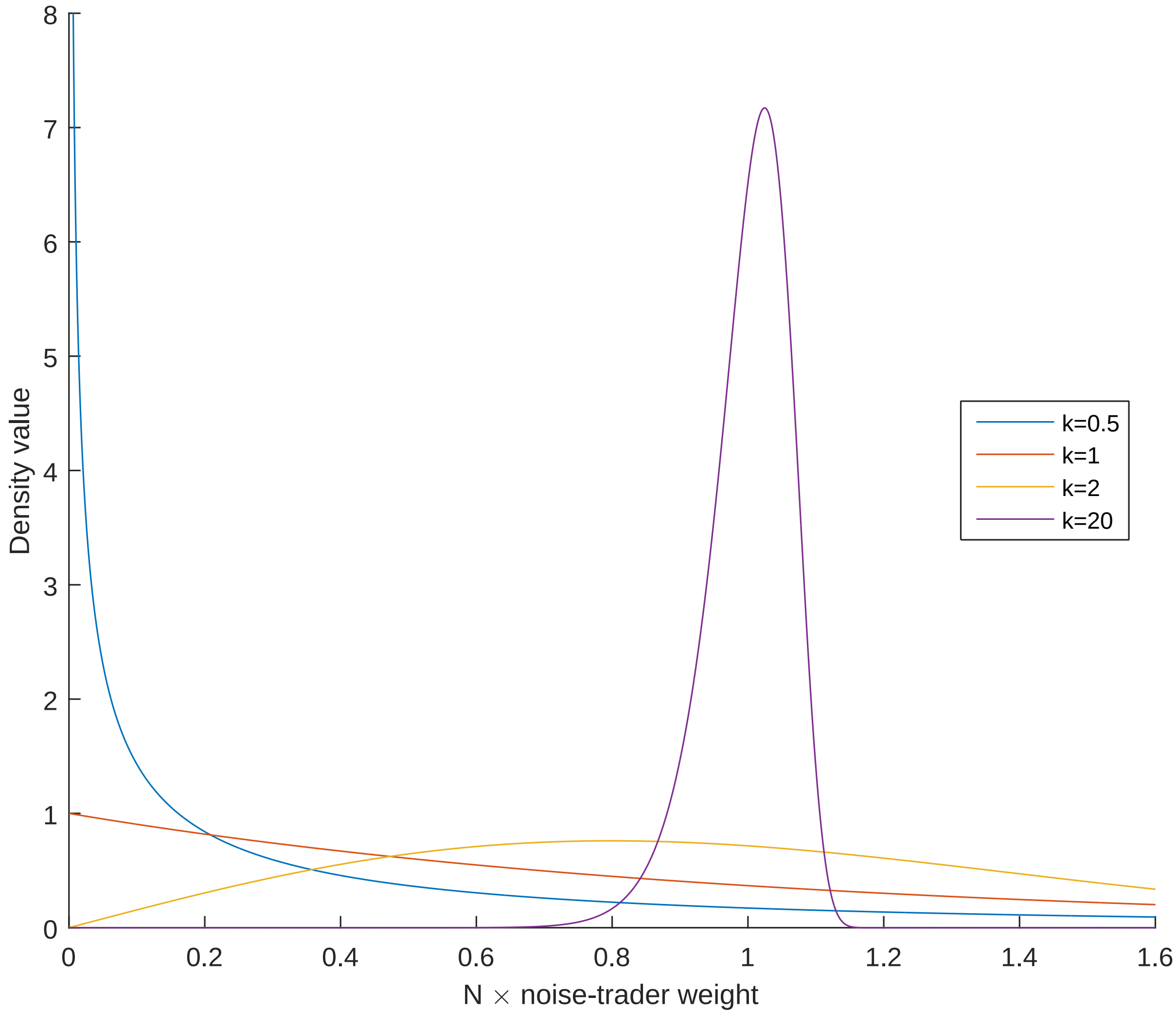

Figure 1. Noise Trading Densities. The figure plots alternative specifications of a Weibull density for approximating the cross-sectional distribution of $N\phi_{H,i}$, where N is the number of stocks in the market and $\phi_{H,i}$ is the aggregate weight that noise traders place in stock i. All densities have 1.0 as the mean and differ with respect to the shape parameter k.

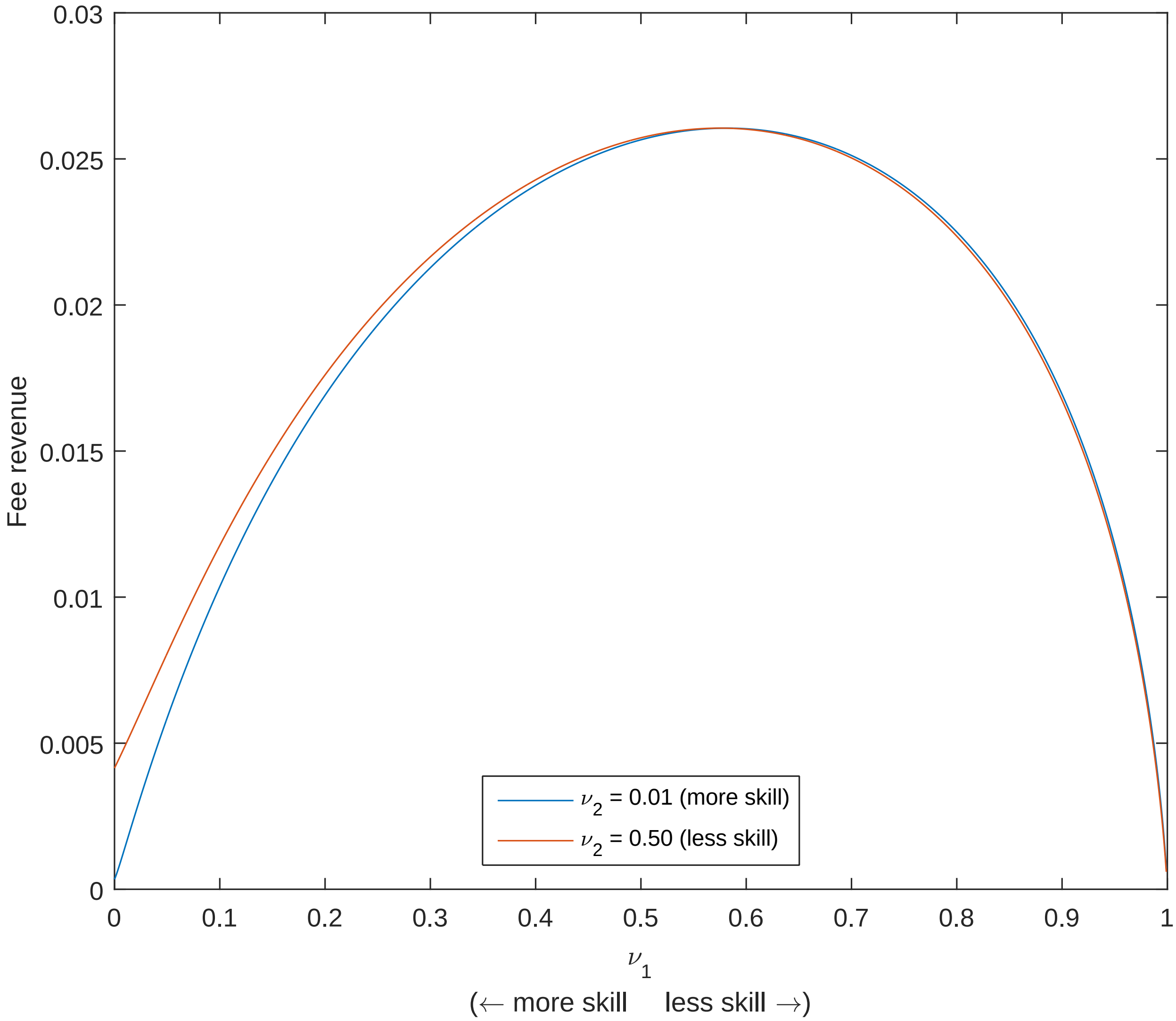

Figure 2. Fee revenue versus ν_1. The parameter ν_1 represents the fraction of noise-trader distortions present in active manager's beliefs, and ν_2 is the standard deviation of each manager's idiosyncratic deviations of beliefs relative to fundamental value. Fee revenue is stated as a fraction of the stock market's total capitalization.

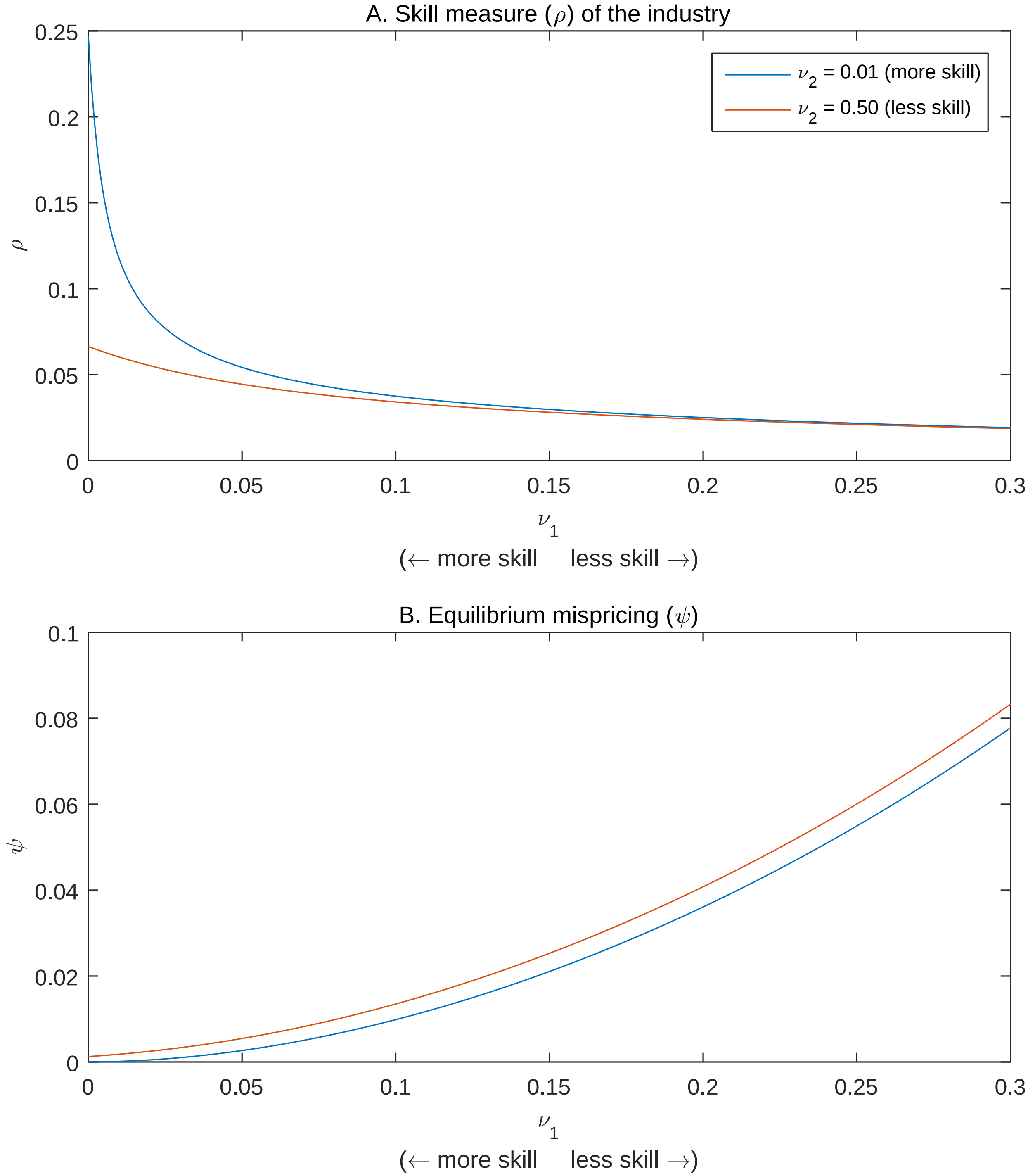

Figure 3. Fund skill and stock mispricing versus $\boldsymbol{\nu_1}$. The parameter ν_1 represents the fraction of noise-trader distortions present in active manager's beliefs, ν_2 is the standard deviation of each manager's idiosyncratic deviations of beliefs relative to fundamental value, ρ reflects skill, and ψ reflects mispricing.

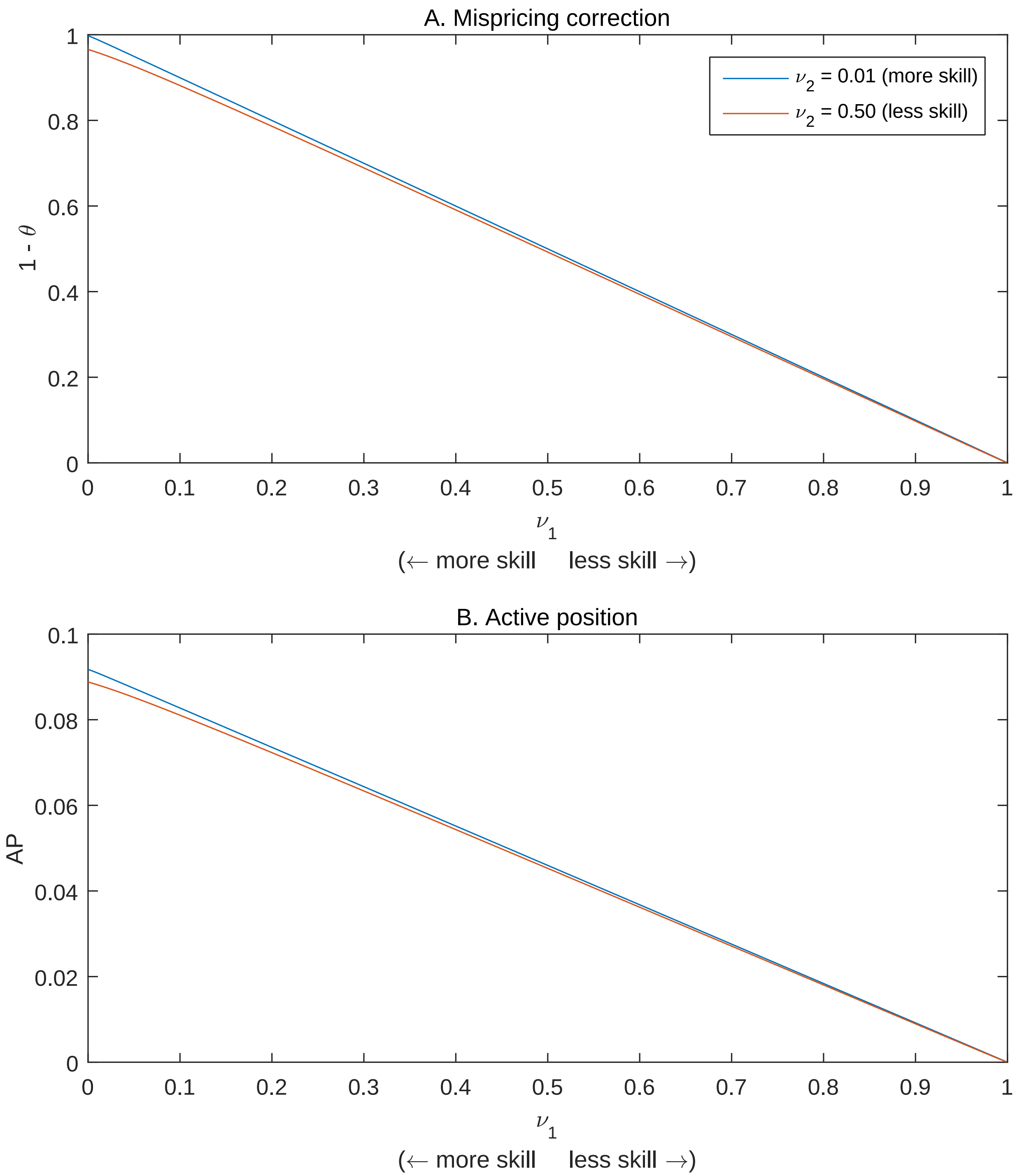

Figure 4. Mispricing correction and active position versus ν_1. The parameter ν_1 represents the fraction of noise-trader distortions present in active manager's beliefs, and ν_2 is the standard deviation of each manager's idiosyncratic deviations of beliefs relative to fundamental value. The quantity $1-\theta$, plotted in Panel A, is the fraction of noise-trader distortions eliminated by active management. Active management's active position (AP), plotted in Panel B, is stated as a fraction of the stock market's total capitalization.

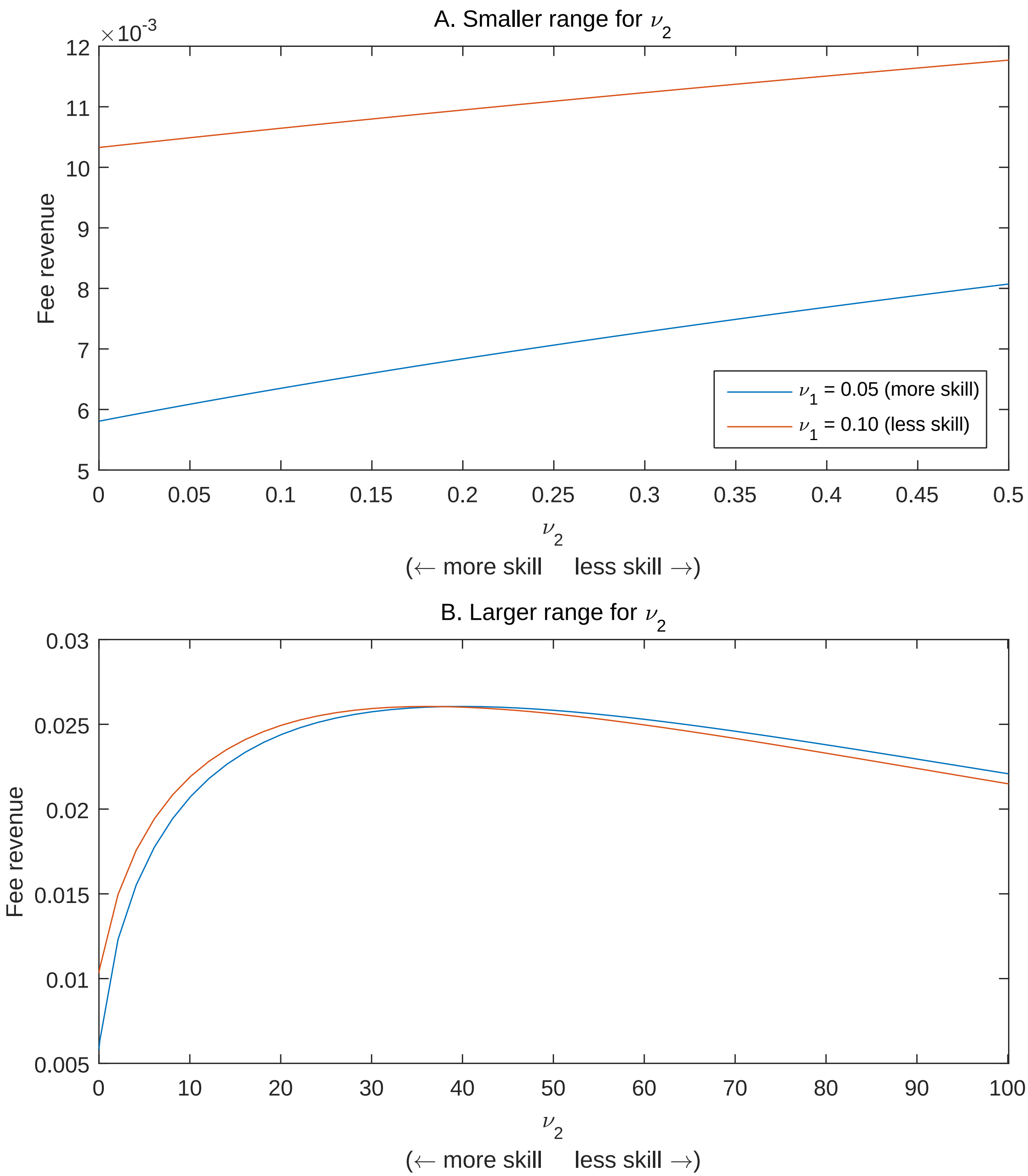

Figure 5. Fee revenue versus ν_2. The parameter ν_2 is the standard deviation of each manager's idiosyncratic deviations of beliefs relative to fundamental value, and ν_1 represents the fraction of noise-trader distortions present in active manager's beliefs. Panel B displays the relation over an expanded range relative to that in Panel A.

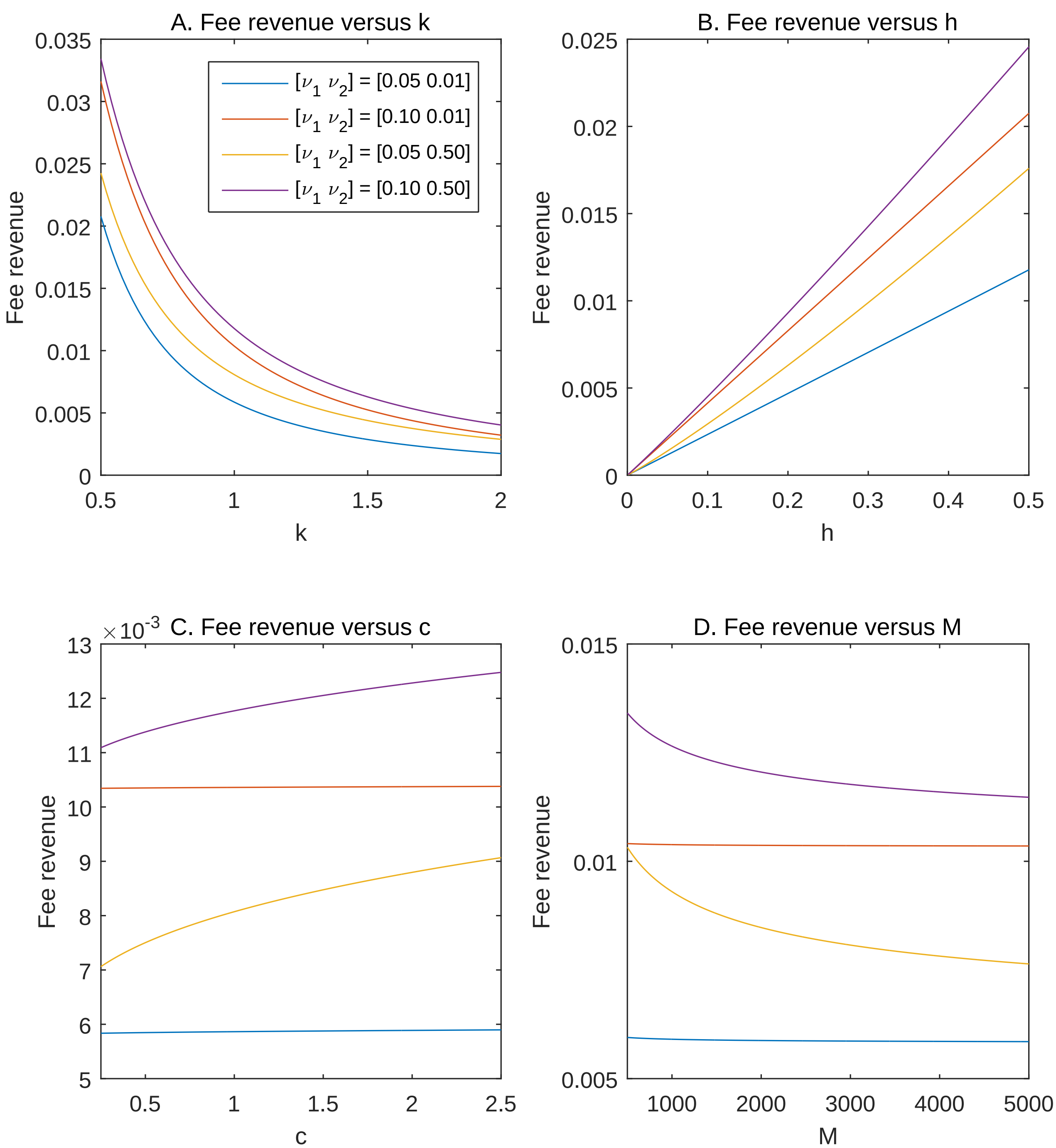

Figure 6. Fee revenue versus k, h, c, and M. The dispersion and skewness in noise-traders's demands are decreasing in the parameter k; h is the fraction of the market owned by noise traders; proportional stock-trading costs increase in the parameter c; M is the number of active managers. The parameter ν_1 represents the fraction of noise-trader distortions present in active manager's beliefs, and ν_2 is the standard deviation of each manager's idiosyncratic deviations of beliefs relative to fundamental value.

References

Akbas, Ferhat, Will J. Armstrong, Sorin Sorescu, and Avanidhar Subrahmanyam, 2015, Smart money, dumb money, and capital market anomalies, *Journal of Financial Economics* 118, 355–382.

Barber, Brad M., and Terrance Odean, 2008, All that glitters: The effect of attention and news on the buying behavior of individual and institutional investors, *Review of Financial Studies* 21, 785–818.

Barber, Brad M., Terrance Odean, and Ning Zhu, 2009, Systematic noise, *Journal of Financial Markets* 12, 547–569.

Berk, Jonathan B., and Richard C. Green, 2003, Mutual fund flows and performance in rational markets, Working paper 9275, National Bureau of Economic Research, Cambridge, MA.

Berk, Jonathan B., and Richard C. Green, 2004, Mutual fund flows and performance in rational markets, *Journal of Political Economy* 112, 1269–1295.

Berk, Jonathan, and Jules H. van Binsbergen, 2015, Measuring skill in the mutual fund industry, *Journal of Financial Economics* 118, 1–20.

Blume, Marshall E., Jean Crockett, and Irwin Friend, 1974, Stock ownership in the United States: Characteristics and trends, *Survey of Current Business* 54 (November), 16–40.

Blume, Marshall E., and Irwin Friend, 1975, The asset structure of individual portfolios and some implications for utility functions, *Journal of Finance* 30, 585–603.

Cremers, K.J. Martijn, and Antti Petajisto, 2009, How active is your fund manager? A new measure that predicts performance, *Review of Financial Studies* 22, 3329–3365.

Dasgupta, Amil, Andrea Plat, and Michela Verardo, 2011, Institutional trade persistence and long-term equity returns, *Journal of Finance* 66, 635–653.

DeVault, Luke, Richard Sias, and Laura Starks, 2018, Sentiment metrics and investor demand, *Journal of Finance*, forthcoming.

Dimitri, Carolyn, Anne Effland, and Neilson Conklin, 2005, The 20th Century transformation of U.S. agriculture and farm policy, Economic Information Bulletin Number 3, United States Department of Agriculture.

Dorn, Daniel, Gur Huberman, and Paul Sengmueller, 2008, Correlated trading and returns, *Journal of Finance* 58, 885–920.

Edelen, Roger M., Richard Evans, and Gregory B. Kadlec, 2007, Scale effects in mutual fund performance: The role of trading costs, Working paper, Echo Investment Advisors, Boston College, and Virginia Tech.

Edelen, Roger, Richard Evans, and Gregory Kadlec, 2013, Shedding light on "invisible" costs: Trading costs and mutual fund performance, *Financial Analysts Journal* 69 (January/February), 33–44.

Edelen, Roger M., Ozgur S. Ince, and Gregory B. Kadlec, 2016, Institutional investors and stock return anomalies, *Journal of Financial Economics* 119, 472–488.

Feng, Lei, and Mark S. Seasholes, 2004, Correlated trading and location, *Journal of Finance* 59, 2117–2144.

French, Kenneth R., 2008, The cost of active investing, *Journal of Finance* 63, 1537–1573.

Garcia, Diego, and Joel M. Vanden, 2009, Information acquisition and mutual funds, *Journal of Economic Theory* 144, 1965–1995.

Gârleanu, Nicolae, and Lasse Heje Pedersen, 2018, Efficiently inefficient markets for assets and asset management, *Journal of Finance* 73, 1663–1712.

Glode, Vincent, Richard C. Green, and Richard Lowery, 2012, Financial expertise as an arms race, *Journal of Finance* 67, 1723–1759.

Goetzmann, William N., and Alok Kumar, 2008, Equity portfolio diversification, *Review of Finance* 12, 433–463.

Grossman, Sanford J., and Joseph E. Stiglitz, 1980, On the impossibility of informationally efficient markets, *American Economic Review* 70, 393–408.

Investment Company Institute, 2017, *2017 Investment Company Fact Book*.

Kelly, Morgan, 1995, All their eggs in one basket: Portfolio diversification of US households, *Journal of Economic Behavior and Organization* 27, 87–96.

Lease, Ronald C., Wilbur G. Lewellen, and Gary G. Schlarbaum, 1974, The individual investor: Attributes and attitudes, *Journal of Finance* 29, 413–433.

Pástor, Ľuboš, and Robert F. Stambaugh, 2012, On the size of the active management industry, *Journal of Political Economy* 120, 740–781.

Pástor, Ľuboš, Robert F. Stambaugh, and Lucian A. Taylor, 2015, Scale and skill in active management, *Journal of Financial Economics* 116, 23–45.

Pástor, Ľuboš, Robert F. Stambaugh, and Lucian A. Taylor, 2018, Fund tradeoffs, Working paper, University of Chicago and University of Pennsylvania.

Polkovnichenko, Valery, 2005, Household portfolio diversification: A case for rank-dependent preferences, *Review of Financial Studies* 18, 1467–1502.

Sharpe, William F. 1991, The arithmetic of active management, *Financial Analysts Journal* 47 (January/February), 7–9.

Stambaugh, Robert F., 2014, Presidential address: Investment noise and trends, *Journal of Finance* 69, 1415–1453.

Stambaugh, Robert F., 2017, Noisy active management, Working paper, University of Pennsylvania.