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An Empirical Analysis of Limit Order Markets*

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Abstract

This paper analyzes order placement strategies in a limit order market. Traders submitting market or limit orders to the limit order book trade off the order price, the execution probability, and the winner's curse risk associated with different feasible order choices. Their optimal order strategy is characterized by a monotone function which maps the liquidity demand of the investors into their subjective execution probabilities. We provide conditions for the existence of a Markov perfect equilibria to the model whose outcomes satisfy a mixing condition. The primitives of this model are the time varying shock that is common to all valuations, as well as the probability distribution of private valuations, assumed to be a time invariant, independently and identically distributed random variable. Using data from the Stockholm Stock Exchange, we compute a semiparametric estimator of the primitives underlying the model, adapting previous work on linear index models to time dependent data. The estimated order strategies are consistent with the theoretical trade-offs. Specification tests based on the monotonicity of the optimal order strategy finds little evidence against the monotonicity restrictions. Overidentifying restrictions between equations are not rejected, and the coefficients on the linear factor structure are significant with the predicted signs. However exclusion tests find that other variables related to the limit order book and market conditions predict order choices after accounting for the trade-off between price, execution probability and winner's curse. The observed order choices are combined with estimates of the primitives to bound the potential and realized gains from trade in the market. The current trading system leads traders to sometimes take the opposite side of the market to what the social surplus maximizing outcome requires in order to capitalize on recent movements in the common shock, and traders also willingly risk failure to execute trades by submitting limit orders too far from the bid ask spread. About two thirds of these social losses, or transaction costs, are attributable to traders executing on the wrong side of the market. We estimate that the current trading mechanism achieves at least 57 percent of the potential gains from trade.

1 Introduction

A fundamental issues in economics is how the trading process works in different market institutions, to determine trading activity and market prices. For example, how does the trading process itself interact with trader preferences to determine the volume of trade and the transactions prices in the market. In this paper, we develop and empirically implement a model of optimal order placement in a particular financial market structure, a limit order market. We use data on traders' order placement choices along with the outcomes of these choices to estimate the trading opportunities in the market and the distribution of the traders' liquidity demand. The model imposes testable restrictions on the observed order strategies that we use to develop specification tests of the model. We estimate and test our model using data on order flow from the Stockholm Stock Exchange. Specification tests are conducted to determine whether traders adjust their trading strategies appropriately as the trading opportunity set changes. Our model provides information on the distribution of liquidity demand in the market, enabling us to form estimates of the realized and potential gains from trade in the market.

In a limit order market, buyers and sellers can submit an order of one of two types.[1] A market order executes immediately the most attractive price posted by previous limit orders. A limit order specifies a particular price for the order and specifies a promise to trade at that price. The limit order book is a list of all unexecuted limit orders. There is typically a trade-off between order price and execution probability which is reflected as an implicit cost for immediacy or liquidity. A limit order involves a commitment to a price, and so is generally exposed to unfavorable changes in the value of the asset. Unfavorable executions of the limit order can arise when it is matched with a more informed trader's market order or when the limit order is *ex–post* mispriced, such as after a public announcement, and is then 'picked off' which we will refer to as the winner's curse. Overall then, the order placement problem involves trade-offs between price, execution probability, and the winner's curse.

These trade–offs form the basis of much of the theoretical literature on the choice between limit and market orders.[2] Our contribution is to estimate these trade–offs directly and test whether the observed order

[1] Domowitz (1993) documents that approximately 35 financial markets in 16 different countries contain elements of limit order mechanisms in their designs.

[2] Demsetz (1968) studied the equilibrium costs of immediacy in a model where buyers and sellers arrive at different times.

placement decisions can be explained by the strategic trade–offs we consider. Furthermore, we use the observed order choices made by the traders to make inferences about their demand for liquidity. Our model is based upon a particular specification of these trade–offs and we provide empirical tests of this specification of traders' objectives.

A feature of our empirical approach is that we do not explicitly solve for the equilibrium of the model in terms of the underlying fundamentals. Instead we approach the inference problem indirectly, using the data to make inferences about the trading opportunities and to empirically estimate the optimal order strategy, for a given demand for immediacy. By combining the estimates of the optimal order strategy with the actual choices made by the traders, we can infer properties of the distribution of traders' valuations for the asset. The theoretical model imposes a monotonicity restriction on the optimal order strategies which we test in our data set. Similar approaches have been proposed by Elyakime, Laffont, Loisel and Vuong (1994), Guerre, Perrigne, and Vuong (1999) and Laffont and Vuong (1996) to analyze auction data, and by Hotz and Miller (1993) to estimate dynamic discrete choice models.

There has been previous empirical work studying limit orders. Using data on limit and market orders from the Paris Bourse, Biais, Hillion and Spatt (1995), find evidence that traders provide liquidity by submitting limit orders, when liquidity is expensive, and consume liquidity, by submitting market orders, when it is cheap to do so. Thus, in the Paris Bourse, traders follow order placement strategies that depend on the trading opportunities offered by the limit order book.[3] Our theoretical model imposes restrictions on the relationship between the available liquidity in the market, expectations of future changes in the underlying value of the asset, and the optimal order choices made by traders. We estimate and test these restrictions in our data set.

In our empirical analysis, we use data on all orders submitted in one the the most actively traded stocks on the Stockholm Stock Exchange, Ericsson. Our empirical results are as follows. Both the trade–off between order price and execution probability and the trade–off between order price and the winner's curse

Cohen, Maier, Schwartz, and Whitcomb (1981) and Kumar and Seppi (1993) theoretically analyze a trader's optimal choice between market and limit orders in different trading environments. Biais, Martimort and Rochet (1999), Foucault (1999), Glosten (1994), O'Hara and Oldfield (1986), Parlour (1998), Rock (1996), and Seppi (1997), theoretically analyze prices, trading volumes and efficiency in financial markets with limit order books.

[3]Harris and Hasbrouck (1996), and Handa and Schwartz (1996) analyze the profitability of different order placement strategies in different market conditions. Lo, MacKinlay, and Zhang (1997) estimate several econometric models of limit order execution times. Sandås (1999) estimates a structural model of competitive market making in a limit order market.

are important in explaining variation in observed order placement decisions. However, we find that these terms representing the theoretical trade–off between price, execution probability and winner's curse do not explain all of the observed variation in order submissions. We find little evidence against the monotonicity of the estimated trading strategy. We estimate that the current trading mechanism achieves at least 57 percent of the maximal gains from trade. In our model, gains from trade are foregone when traders' private incentives lead to order placement strategies that do not implement the socially optimal outcomes. Losses either occur when orders never execute, or when traders submit buy orders when they should submit sell orders to implement the socially optimal outcome, and vice versa. Unexecuted orders account for approximately 12 percent of the estimated loss, while executed buy and sell orders account for the remainder of the estimated loss.

The plan of the paper is as follows. The next section provides a brief qualitative and quantitative description of the Stockholm Stock Exchange. In section 3 we present our theoretical model, deriving its testable restrictions, and showing how to theoretically calculate the potential and attained gains from trade in this model. Section 4 contains a description of the estimation techniques that we apply and in section 5, we apply these techniques to our dataset from the Stockholm Stock Exchange. The final section concludes. All proofs are contained in an appendix.

2 The Data

This section describes the market we study, the Stockholm Stock Exchange, and the chief characteristics of our data set. In 1990 the Stockholm Stock Exchange completed the introduction of a computer based trading system inspired by the systems at the Toronto and Paris Stock Exchanges. All order prices are required to be multiples of a fixed minimum price unit, commonly referred to as the tick size. When prices are below 100 Kronor, the tick size is 1/2 Kronor and when prices are above 100 Kronor, the tick size is 1 Kronor.[4] The order size is required to be a multiple of a round lot, where a round lot is 100 shares. Traders can also choose to hide some fraction of the order quantity. Limit orders are stored in the centralized computer system and automatically executed as they cross with incoming market orders. Limit orders are prioritized by price and

[4] 6.25 Kronor $\approx$ 1 US$ during the sample period.

time of submission. At a given price all hidden orders have lower priority than the displayed orders.

All information on the status of the limit order book is instantaneously transmitted to the computer screens in the offices of the exchange members. A member of the exchange trades both as an agent for clients as a broker, and as a dealer on his own behalf. The market is very transparent, since the continuously available information includes the five best bid and ask quote levels with the corresponding buy and sell quantities. We reconstructed this information from the individual orders to create a data set that is very similar to the one available to market participants. We could not, however, decompose the order quantity into the hidden and the displayed components, only observing the sum of the two.

Our sample is representative of the electronic trading but does not cover all trading in stocks at the Swedish Stock Exchange. Transactions made in London on the SEAQ International and in the U.S. on the NASDAQ system account for a significant fraction of the turnover in some firms. In addition, block trades can be settled outside the electronic system and if this occurs during the normal trading hours these trades must be reported to the market. Our focus is strictly on the order placement decision within the electronic system.

The data set contains histories for all orders submitted to the electronic trading system during normal trading hours for Ericsson A.B. at the Stockholm Stock Exchange.[5] The sample period consists of the 59 trading days between December 3, 1991, and March 2, 1992. There are 22,128 orders to buy or sell shares of Ericsson submitted during our sample period. Table 1 gives the average price, average daily return and trading volume. The average price is 110.720 Kronor, and the tick size is 0.5 when the price is below 100 Kronor and 1 Kronor when the price is above 100 Kronor. The average daily return for the stock is 0.18%, and the stock had a 10.05% total return over the period we study. The standard deviation, maximum and minimum returns indicate that the stock returns have been reasonably volatile during this period. Thus, limit orders submitted over this period can be subject large adverse price changes. The average daily trading volume is 74.43 million Kronor. The 1991 annual report from the Stockholm Stock Exchange (1991 Stockholms Fondbörs Årsrapport) reports that over 1991, the turnover rate of the stock was 38%, so that there is a relatively large volume of trade in Ericsson during our sample period.

[5]Ericsson is traded continuously from 10:00 am to 2:30 pm.

Table 2 provides sample statistics on the order flow. The first row presents the number of market and limit orders submitted. For both the buy and sell sides most of the orders submitted are market orders. There are more buy than sell orders in our sample. The second line of the table gives information on the size of the orders submitted. The average size of a buy order is smaller than the size of a sell order both for market and limit orders, roughly offsetting the larger number of buy orders submitted. The third line of the table reports estimates of the unconditional probability of execution for different limit orders. The execution probability for market orders is by definition equal to one. The probabilities clearly show the average trade–off between execution probability and the order price, since the execution probability for limit orders drops monotonically the more favorable the submitted price is. The final row of the table gives estimates of the average time–to–execution for limit orders. These sample averages suggest that, conditioning on execution, more aggressively priced limit orders take longer to execute than less aggressively priced limit orders. Noting again the stochastic movement in trading prices, the average time–to–execution suggests that aggressively priced limit orders are more exposed to changes in the underlying value of the asset.

Once a limit order is submitted, it enters the limit order book. Table 3 contains information on the limit order book over our sample period. The first three columns give information on the size of the order queue at the first 3 buy and sell price quotes. The average market order size is roughly 2200 shares, so typically the quantities at the best quotes equal about 9 incoming orders. The median quantity in the order book is is less than the average quantity, indicating skewness in the distribution of the queue lengths. The standard deviations indicate that the size of the order book is reasonably volatile. We also report information on the price quotes in the fourth through final columns of the table. Typically the three best prices quotes on the buy and sell sides are spaced one tick apart.

Our model analyzes how the trade–offs involved in limit order submissions drive the decisions of traders. Table 4 provides further motivation for the analysis. In it, we report how various conditioning variables affect the trade–off between limit order price and execution probability. The conditioning variables are coded as Low if they are below their median values and High if they are above. The variables were chosen to characterize the limit order book, market conditions, and the size of the order. To capture variation in the limit order book, we used the number of shares at the two best bid and ask quotes, and the bid–ask spread, which equals the ask price minus bid price. Most estimates imply a trade–off between price and execution probabilities for the conditioning set under consideration. An exception to this statistical regularity appears on the sell side of the book when the bid-ask spread is below the median. For each limit order and conditioning variable, we report a chi-squared test of the hypothesis that the conditional execution probabilities are independent of the conditioning information. The hypothesis that the execution probabilities are constant across the conditioning information is typically rejected. For example, we find that the execution probabilities for limit sell orders decline as quantity at the best quote decreases. Overall, the results in this table provide evidence that the price and execution probability trade–off exhibits systematic variation.

One might anticipate that changing the trade–off between price and execution probabilities would be reflected in traders' order submission strategies. Table 5 provides estimates of the conditional probability of submitting different types of orders, conditional on the information sets described above. We calculated chi–squared test of the hypothesis that the choice probabilities are constant across different information sets. The chi–squared tests all reject the null hypothesis. Thus, traders make different decisions in different market conditions. For example, increasing the size of the limit order queue at the best ask quotes decreases the likelihood of sell limit order submissions and increases the likelihood of market sell orders. Similar results hold at the buy side.

3 A Model

This section presents and analyzes the theoretical model we apply to our data set. First, we provide assumptions on the trading environment, trader preferences, and the information available to the trader. Then we characterize the optimal trading strategy for an individual trader in this environment. We show that the optimal strategy has an important monotonicity property: Traders with higher valuations submit buy orders with higher execution probabilities and traders with lower valuations submit sell orders with higher execution probabilities. The latter parts of this section establish the existence of a Markov perfect equilibrium satisfying a uniform mixing condition and discusses its welfare properties.

3.1 Trading Rules

Agents arrive sequentially in the market with an opportunity to trade. Agents can place an order to buy or sell one unit of stock at a price chosen from the countable set

$$P \equiv \{p_1, p_2, \ldots\}.$$

Let $|P|$ represent the cardinality of the set of orders. The difference between two consecutive prices is at least $\delta > 0$, referred to as the tick size.

We use the notation t to refer both to the time period t when the order is submitted and to the agent whose turn it is to place an order at time t, where $t \in \{0, 1, \ldots\}$. Upon entering into the market, the trader decides if he would like to trade a unit of the asset, and if so, whether to buy or sell the asset and at what price. We let $\mathcal{D}$ denote the choice set of the trader. An element of $\mathcal{D}$ consists of a sequence, $\{d^i\}_{i=1}^{|P|}$, $d^i \in \{-1, 0, +1\}$, where $d^i = -1$ means that the trader submits a sell order at price p_i, $d^i = +1$ is a buy order at price p_i, and $d_i = 0$ is no order submitted at price p_i. The constraint that the order size be restricted to one unit is given by $d^i d^j = 0$, for all i, $j \neq i$. We let $d_t \in \mathcal{D}$ refer to the decision made by the agent at time t.

Once an order has been submitted, it either may trade immediately, or it may enter the queue of unfilled orders, referred to as the limit order book. Upon entry into the limit order book, one of two things eventually occur. The order may be executed, or it may be canceled. At any period t, the limit order book consists of outstanding orders to buy and sell stock at specified prices. Let h_{it} denote the number of limit orders at price p_i at the beginning of period t, where $h_{it} > 0$ indicates the number of outstanding buy orders at price i and $h_{it} < 0$ indicates the number of sell orders. The limit order book is denoted by $h_t \equiv \{h_{it}\}_{i=1}^{|P|}$.

Let a_t denote the ask price, the lowest price of existing limit orders to sell at time t,

$$a_t \equiv \begin{cases} \min\left\{p_i \in P : h_{jt} < 0\right\} & \text{if } h_{kt} < 0 \text{ for some } k, \\ \infty & \text{if } h_{kt} \geq 0 \text{ for all } k. \end{cases}$$

Symmetrically define the bid price, b_t as the highest limit price of existing limit orders to buy a unit at t, by

$$b_t \equiv \begin{cases} \max\left\{p_i \in P : h_{jt} > 0\right\} & \text{if } h_{kt} > 0 \text{ for some } k, \\ 0 & \text{if } h_{kt} \leq 0 \text{ for all } k. \end{cases}$$

The trading rules imply that $a_t > b_t$, and we refer to $a_t - b_t$ as the bid–ask spread.

If the agent places a buy order at a price which is at least as high as the ask price, that is $d_{it} = 1$ for some $p_i \geq a_t$, then the order is immediately filled by the oldest outstanding limit order at the ask price a_t. This is called a market buy order. If the agent submits a buy order less than the ask price, meaning $d_{it} = 1$ for some $p_i < a_t$, the order is called a limit buy order, and added to the queue for existing orders at that price, h_{it}. These rules imply that in the limit order book, orders are first prioritized by price and then by submission time. The rules for a sell order are symmetric.

We assume the processes governing the arrival of new orders and the cancellation of outstanding limit orders are exogenous. One order arrives per period, where the period is not necessarily of fixed length, and existing limit orders are canceled each period with some exogenous probability.

3.2 Preferences and Information

The t^{th} agent's valuation for the stock is denoted by v_t. Our model decomposes v_t into a common component, denoted y_t, plus a private component denoted u_t

$$v_t = y_t + u_t. \tag{1}$$

We assume u_t, the idiosyncratic portion of the t^{th} agent's valuation remains constant over time, and is distributed independently and identically across agents with a differentiable distribution function $G(u)$, with support $[u_l, u_h]$, $-\infty \leq u_l < u_h \leq \infty$. The private component of the valuation u_t can be interpreted as agent t's preference for liquidity; it captures his willingness to hold the stock. The common shock y_t is a stochastic process, whose realizations are observed by all traders. Since each transaction involves a market order executing a limit order, the trader placing the market order is acting on better information about the common shock than the agent who had previously submitted the limit order. Thus, agents have two motivations for trade: First is the difference in private valuations across agents, and second is the change in the common value of the asset over time.

Uncertainty and information in the market are modeled as a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ with generic element $\omega \in \Omega$. The exogenous variables in our model are all defined on this probability space, and we let $\mathcal{F}_t$ denote the sigma algebra generated by the sequence of exogenous variables up to and including time t. Then, $h_t \equiv h_t(\omega)$, $a_t \equiv a_t(\omega)$, $b_t \equiv b_t(\omega)$, and $d_t \equiv d_t(\omega)$ are all $\mathcal{F}_t$ measurable functions. Let $\mathcal{A}_t \subseteq \mathcal{F}_t$, denote the t^{th} trader's information set when making her trading decision. Clearly the trader's decision, $d_t(\omega)$ is $\mathcal{A}_t$ measurable, and we also assume that $h_t(\omega)$ is $\mathcal{A}_t$ measurable. We drop the ω and $\mathcal{F}_t$ and $\mathcal{A}_t$ notation to reduce notational burden where no confusion results.

Once an order is submitted to the limit order book, it will either be canceled or executed against an incoming market order. Let t_i^e denote the time at which an order submitted at time t and at price i is executed conditional on not being previously canceled and let t_i^w be the cancellation time of the order. Define λ_{is} as the $\mathcal{F}_s$ measurable conditional probability that a limit order at price i in the book at time s is canceled at time $s+1$. This notation implies that an order is executed if $t_i^e < t_i^w$. We define the indicator variable for execution at time $t+\tau$ as

$$1\{t+\tau = t_i^e < t_i^w\} = \begin{cases} 1, & \text{execution at time}, t+\tau \\ 0, & \text{otherwise.} \end{cases}$$

This indicator variable is measurable with respect to $\mathcal{F}_{t+\tau}$. The change in the t^{th} agent's stock holdings at period $t+\tau$ is

$$c_{it}^{\tau} \equiv d_{it} 1\{t+\tau = t_i^e < t_i^w\}.$$

An important building block for both the model and the empirical work which follows are the conditional execution probabilities for buy and sell orders at each price p_i, respectively defined as

$$\begin{aligned} \psi_{it}^b &\equiv E\left[\sum_{\tau=0}^{\infty} |c_{it}^{\tau}| \middle| \mathcal{A}_t, d_{it} = 1\right], \quad (2) \\ \psi_{it}^s &\equiv E\left[\sum_{\tau=0}^{\infty} |c_{it}^{\tau}| \middle| \mathcal{A}_t, d_{it} = -1\right]. \end{aligned}$$

If $p_i \geq a_t$ then the buy order is executed immediately, so $\psi_{it}^b = 1$. The rules governing trade imply that for a buy order less than the ask, $p_i < a_t$, the order enters the limit order book and so $\psi_{it}^b < 1$. Similar results hold for the sell side at the bid and higher.

Let Δ_τ denote the τ difference operator, so that $\Delta_\tau y_t \equiv y_{t+\tau} - y_t$. If an order at price p_i is placed at t and executed τ periods later, the net payoff to the t^{th} agent is

$$d_{it}(y_{t+\tau} + u_t - p_i) = d_{it}(y_t + u_t - p_i + \Delta_\tau y_t).$$

Summing over all possible future execution times $t+\tau$ and price choices, and integrating with respect to the random execution and cancellation times and conditioning with respect to the trader's information set,

$\mathcal{A}_t$, the expected payoff from choosing d_t is therefore

$$E\left[\sum_{\tau=0}^{\infty}\sum_{i=1}^{|P|} c_{it}^{\tau}\left(y_t + u_t - p_i + \Delta_\tau y_t\right)\middle| \mathcal{A}_t\right]. \tag{3}$$

Assuming agents are risk neutral, $d_t \in \mathcal{D}$ is chosen to maximize Equation (3).

3.3 Optimization

Let $k \in \{s, b\}$ indicate the direction of the order submitted. Then, the trader's objective function, Equation (3) can be rewritten as

$$E\left[\sum_{\tau=0}^{\infty}\sum_{i=1}^{|P|} c_{it}^{\tau}\left(y_{t+\tau} + u_t - p_i\right)\middle| \mathcal{A}_t\right] = \sum_{i=1}^{|P|}\left(d_{it}\psi_{it}^{k}\left(y_t + u_t - p_i\right) + \xi_{it}^{k}\right), \tag{4}$$

where

$$\xi_{it}^{b} \equiv E\left[\sum_{\tau=0}^{\infty} c_{it}^{\tau}\Delta_\tau y_t \middle| \mathcal{A}_t, d_{it} = 1\right], \tag{5}$$

with ξ_{it}^{s} defined similarly for the sell side. The expectations in Equations (3) through (5) are taken over the random execution time, and the associated random changes in the common value. It is in the calculation of these expectations that this order submission problem accounts for the multiperiod nature of the agent's problem. The first term in Equation (4) gives the probability of execution times the expected payoff from a sure execution at that price. This term captures the agent's trade-off between execution price and execution probability. The second term in Equation (4) measures the expected changes in the common value from time of submission until execution of the order. Thus, the second term captures the effects of receiving execution when the common value moves, and we refer to this as the winner's curse risk.

Let $d_t^o \in \mathcal{D}$ maximize the expected payoff of the trader, Equation (3). The next proposition establishes two facts. It shows that based on their valuations, agents can be split into buyers, sellers, and agents who do not submit orders, and that the optimal orders are related monotonically to valuations through their execution probabilities.

Proposition 1 *Suppose $d_{it}^o = -1$ for some p_i and let ψ_{it}^s denote the associated execution probability. If the t^{th} agent's valuation was $u < u_t$ rather than u_t, then he would set $d_{jt}^o = -1$ for some p_j satisfying $\psi_{jt}^s \geq \psi_{it}^s$. Alternatively suppose $d_{it}^o = 1$. If the t^{th} trader's valuation was u rather than u_t and $u \geq u_t$, then he would set $d_{jt}^o = 1$ for some p_j satisfying $\psi_{jt}^b \geq \psi_{it}^b$.*

Because the tick size is larger than zero, the choice variable d_t is discrete. Consequently sellers with similar, but non-identical, private valuations optimally submit the same order in similar circumstances. Proposition 1 implies that the set of valuations can be partitioned into intervals in which all agents whose valuations lie within that interval would submit the same order price. Accordingly, define the indifference or threshold valuations by $\theta_t^s(p_{it}^s, p_{jt}^s)$, the valuation of an agent who is indifferent between submitting an order to sell a unit at price p_i or an order at price p_j

$$\psi_{it}^s[p_i - \theta_t^s(p_{it}^s, p_{jt}^s)] + \xi_{it}^s = \psi_{jt}^s[p_j - \theta_t^s(p_{it}^s, p_{jt}^s)] + \xi_{jt}^s.$$

Solving for $\theta_t^s(p_{it}^s, p_{jt}^s)$ yields

$$\theta_t^s(p_{it}^s, p_{jt}^s) = p_i - \frac{\psi_{jt}^s(p_i - p_j) - (\xi_{it}^s - \xi_{jt}^s)}{\psi_{it}^s - \psi_{jt}^s}. \tag{6}$$

In principle, the choice problem can be solved in three steps. For each u, solve for the optimal buy order and sell order respectively, and calculate the expected payoffs associated with the optimal buy and sell order. The compare the maximum of the two associated expected values with zero, the expected payoffs obtained by not submitting any order. Accordingly, define $V_t^s(u)$ as the value function for submitting a sell order, and $V_t^b(u)$ and the value function for submitting a buy order,

$$V_t^s(u) = \max_{p_i \in P} \psi_{it}^s (p_i - y_t - u) + \xi_{it}^s, \tag{7}$$

and

$$V_t^b(u) = \max_{p_i \in P} \psi_{it}^b (y_t + u - p_i) + \xi_{it}^b. \tag{8}$$

First solve the subproblems embodied in Equations (7) and (8), and then, for each value of u, compare their maximum, denoted $V_t(u)$ with zero, where

$$V_t(u) \equiv \max\left(V_t^s(u), V_t^b(u)\right)$$

If $V_t(u) < 0$, the agent should decline the opportunity to participate in the market. If $V_t(u) > 0$ and $V_t^s(u) > V_t^b(u)$, the agent should submit the limit sell order at the price which solves Equation (7). Finally if $V_t(u) > 0$ and $V_t^b(u) > V_t^s(u)$, the agent should submit the limit buy order at the price which solves Equation (8). The proof of the next proposition exploits the properties of these valuation functions in extending the monotonicity result to cover the choice of whether to submit a buy order, a sell order, or not submit any order.

Proposition 2 *For each t, there is a unique solution to the equation $V_t^s(u) = V_t^b(u)$ denoted U_t.*

1. *If $V_t(U_t) \geq 0$, then $V_t^b(u) = V_t(u) > 0$ for all $y_t + u > U_t$, and $V_t^s(u) = V_t(u) > 0$ for all $y_t + u < U_t$.*

2. *If $V_t(U_t) < 0$, there exists an open interval $(\underline{u}_t, \overline{u}_t)$ on the support of $G(u)$ containing $U_t - y_t$, such that $V_t(u) < 0$ for all $u \in (\underline{u}_t, \overline{u}_t)$, and $V_t(u) > 0$ for all $u \notin [\underline{u}_t, \overline{u}_t]$.*

All that remains to complete the characterization is to explain how the subproblems embodied in Equations (7) and (8) are solved. The basic idea is to discard prices that the t^{th} agent would never pick, regardless of the private value u_t. Suppose, for example, that there are three execution probabilities, $\psi_{it}^s < \psi_{jt}^s < \psi_{kt}^s$, and an interval $(\underline{v}, \overline{v})$ such that for all $v_t \in (\underline{v}, \overline{v})$,

$$\psi_{it}^s(p_i - v_t) + \xi_{it}^s > \psi_{jt}^s(p_j - v_t) + \xi_{jt}^s,$$

and

$$\psi_{kt}^s(p_k - v_t) + \xi_{kt}^s > \psi_{jt}^s(p_j - v_t) + \xi_{jt}^s.$$

Then clearly no trader would set $d_{jt} = 1$ because p_j is dominated by the price pair p_i and p_k. Dominated prices such as p_j can be derived from comparing the threshold valuations. The inequalities above imply

$$\theta_t^s(p_i^s, p_j^s) < \theta_t^s(p_i^s, p_k^s) < \theta_t^s(p_j^s, p_k^s).$$

Thus, the procedure is to form indifference valuations for price pairs, and iteratively drop any price p_j whose indifference valuations can be nested by a pair of such inequalities involving some $\psi_i^s < \psi_j^s < \psi_k^s$. By a process of elimination we are led to the set of prices that an agent might conceivably find optimal, either as a buyer or as a seller. Denoting by P_t^b and P_t^s the two price sequences remaining in the choice set for buyers and sellers upon iteratively deleting the dominated prices, we are led to the following characterization of the optimal rule.

Proposition 3 *For each t there are two sets of undominated prices, P_t^b and P_t^s with the following properties:*

1. *Suppose* $V_t(U_t) \geq 0$*. If* $v_t > U_t$*, then* $d_{it} = 1$ *for some* $p_i \in P_t^b$*, while if* $v_t < U_t$ *then* $d_{it} = -1$ *for some* $p_i \in P_t^s$*.*

2. *Suppose* $V_t(U_t) < 0$*. If* $u_t \geq \overline{u}_t$*, then* $d_{it}^o = 1$ *for some* $p_i \in P_t^b$*, if* $u_t \leq \underline{u}_t$ *then* $d_{it}^o = -1$ *for some* $p_i \in P_t^s$*, while if* $u_t \in (\underline{u}_t, \overline{u}_t)$ *then* $d_{it}^o = 0$ *for all* $p_i \in P_t$*.*

3. *If* $(p_i, p_j, p_k) \subseteq P_t^s$ *and* $\psi_{it}^s < \psi_{jt}^s < \psi_{kt}^s$*, then* $\theta_t^s(p_{it}^s, p_{jt}^s) > \theta_t^s(p_{jt}^s, p_{kt}^s)$*. A similar result holds on the buy side.*

Proposition 3 fully characterizes the mapping from private valuations to execution probabilities, but only partially characterizes the mapping from private valuations to prices. It implies that, conditioning on the same information aside from the private valuation u_t, traders with the highest private valuation submit market buy orders, those with the lowest submit market sell orders, while the agents who stay out of the market have higher valuations than limit order sellers but lower valuations than limit order buyers. More generally, monotonicity between prices and valuations is assured if the execution probabilities for buy orders are monotonically increasing in prices, and the execution probabilities for sell orders are monotonically decreasing in prices. These conditions are satisfied if the winner's curse does not increase the closer a limit order is placed to the bid–ask spread.

Proposition 4 *Suppose* $\xi_{it}^k \leq \xi_{jt}^k$ *and* $\psi_{it}^k \leq \psi_{jt}^k$*. If* p_i *and* p_j *belong to* P_t^b*, then* $p_i \leq p_j$ *. If* p_i *and* p_j *belong to* P_t^s*, then* $p_i \geq p_j$*.*

That the winner's curse is worse for low probability speculative orders than for high probability orders close to the spread is an intuitively appealing notion, but difficult to derive from conditions on the exogenous variables, such as the stochastic process governing the common shock. There is, however, one noteworthy special case. If there are no common shocks to the system, then the execution probabilities to buy (sell) are monotone increasing (decreasing) in price. Consequently order prices are monotone increasing in private valuations in this special case.

3.4 Equilibrium

Our data set contains no information about the real costs of market participation, nor about agents who turn down opportunities to place orders. Rather than speculate about either of these considerations, our empirical work focuses on markets where participation is universal. We now provide a condition that guarantees that all traders, irrespective of their private valuations submit an order. The condition that y_t is bounded limits the maximum possible loss from the winner's curse, and therefore serves a dual purpose in our analysis. First it directly implies that universal participation is optimal. Second, the bound is used to establish the existence of a Markov perfect equilibrium, as defined in Fudenberg and Tirole (1991).

Proposition 5 *Suppose the support of the random variable y_t is bounded by a pair of real numbers $\underline{y}$ and $\bar{y}$, where $0 < \underline{y} < \bar{y} < \infty$. Then $V_t(u) \geq 0,$ for all $u \in [u_l, u_h]$.*

The preceding is robust to how agents form their subjective expectations over the future, but our empirical work assumes that agents hold rational expectations. To help motivate this assumption, we provide conditions that guarantee the existence of a Markov perfect equilibrium. Let the common value, y_t be determined by an L dimensional vector of random variables z_t via the continuous real valued function

$$y_t = \alpha(z_t). \tag{9}$$

The random variables z_t form a Markov process with a transition probability density function denoted by $f(z_{t+1}|z_t)$ and we retain the assumption that y_t is bounded, by bounding the support of z_t. We also assume there is a strictly positive probability that any outstanding limit order will be canceled each period and that t_i^w, the cancellation time for the t^{th} order is independently distributed across time, but depends on the characteristics of the order book. Accordingly, denote the probability of canceling any outstanding limit order $\lambda(h_t) \geq \underline{\lambda} > 0$. With this assumption, the probability space $(\Omega, \mathcal{F}, \mathcal{P})$ is generated by the factors, the private valuations, and the cancellation times, represented by the triple (z_t, u_t, t_i^w).

As applied to this model, a Markov perfect equilibrium is a trading strategy, $d(h_t, z_t, u_t)$ satisfying two properties. First, the subjective beliefs that traders have about the probability of execution and cancellation for each possible order are formed using Bayes' Rule. Second, the trading strategy $d(h_t, z_t, u_t)$ is sequentially rational for these beliefs.

Proposition 6 *If Equation (9) holds then a Markov perfect equilibrium to the model exists.*

The Markov property of this equilibrium is not, however, sufficiently strong for our data analysis. So, for the sake of the empirical analysis that follows, we impose two assumptions on the probability distribution functions in the model. Loosely speaking, these conditions bound the impact of events with small but strictly positive probabilities on decision making. The first assumption makes it unlikely that an unusual state of the book will endure, by assuming the probability of cancellations increases to one, as the size of the limit order book grows without bound. The other one places an analogous restriction on z_t, to prevent it from becoming stuck in a region of low probability for very long. These conditions ensure that the sequential equilibrium generates outcomes that follow a Markov process satisfying a uniform mixing condition. Let $\mathcal{M}_a^b$ denote the sigma–algebra of events generated by $X_a, ..., X_b$ and define the uniform mixing coefficients for the process $\{X_t\}$ by

$$\varphi(T) = \sup_j \sup_{A\in\mathcal{M}_1^j, B\in\mathcal{M}_{j+T}} |P(B|A) - P(B)|. \tag{10}$$

The process $\{X_t\}$ is uniformly mixing if $\varphi(T) \to 0$ as $T \to \infty$.

Proposition 7 *Let $\|h_t\|$ denote the number of orders in h_t at time t. Assume that $\ln(1 - \lambda(h_t)) = O(\|h_t\|)$ and that $f(z_{t+1}|z_t) = O(1)$. Then the outcomes generated by the Markov perfect equilibrium are uniformly mixing, with uniform mixing coefficients converging to zero at a geometric rate.*

This uniform mixing condition is used to later to establish that our estimators are consistent and asymptotically normal.

3.5 Welfare Properties

The efficiency a limit order market can be evaluated and compared with alternative arrangements for conducting trade. One benchmark is the theoretical maximum social surplus, defined as the expected gains from trade, where expectations are taken over the distribution for private valuations, $G(u)$ and the common shock process.

Let $G^{-1}(\cdot)$ denote the inverse of G, the probability distribution of u and $G^{-1}(1/2)$ the median private valuation. To achieve the maximum social surplus, limit orders would be indexed to the common value.

That is, limit prices are set so that at each point in time, traders can either place market orders at the price $y_t + G^{-1}(1/2)$ to meet existing limit sell and buy orders, or alternatively place a limit buy or sell order. Consequently any order executed at time t is priced at $y_t + G^{-1}(1/2)$. Under our assumptions on traders preferences and that the common value is known to all traders, a Pareto efficient allocation is achieved. Here traders with private valuations above the median of the private valuation distribution purchase a share and those below the median sell a share. Each order is executed at the surplus maximizing price, equal to the common value plus the median of the private valuation distribution. The maximum social surplus is given by the expression

$$\begin{aligned} E\left[\left|u_t - G^{-1}(1/2)\right|\right] &= E\left[1\left\{u_t \geq G^{-1}(1/2)\right\}\left(u_t - G^{-1}(1/2)\right) - 1\left\{u_t < G^{-1}(1/2)\right\}\left(u_t - G^{-1}(1/2)\right)\right] \\ &= E\left[1\left\{u_t \geq G^{-1}(1/2)\right\}u_t - 1\left\{u_t < G^{-1}(1/2)\right\}u_t\right] \\ &= E\left[\text{sign}\left(u_t - G^{-1}(1/2)\right)u_t\right], \qquad (11) \end{aligned}$$

where $\text{sign}(\cdot)$ is the sign function.

The maximum social surplus can be compared to the surplus under current the trading system. The current trading rules subject the traders to two potential sources of loss. First, to achieve the maximal social surplus, all agents eventually transact, while under the current mechanism, some traders' orders may not execute. The order may not execute either because the order may be canceled or the common value may move so far away from the order that the likelihood of a market order trading against the order goes to zero. Second, some agents may find it privately optimal to buy a unit of stock when they should sell to achieve the maximal social surplus, and vice versa.

We now derive the foregone surplus in the current market structure relative to the Pareto optimal allocation. We define a variable c_t which indexes whether a trader t eventually trades and the direction of his trade,

$$c_t \equiv \sum_{i=1}^{|P|} \sum_{\tau=0}^{\infty} c_{it}^{\tau}. \qquad (12)$$

Thus, $c_t = 1$ if the buy order submitted by the t^{th} trader ever executes, $c_t = -1$ if the sell order ever executes and $c_t = 0$ if the t^{th} trader's order never executes.

Let t^e denote the time that the order executes, so that $t^e \equiv \{t + \tau \ s.t. \ |c_t^{\tau}| = 1\}$. If the order does not execute, then $t^e = \infty$. Using this notation, the realized payoff of the trader is given by

$$c_t \left(u_t + y_{t^e} - p^o(h_t, z_t, u_t)\right),$$

where $p^o(h_t, z_t, u_t)$ denotes the order price optimally chosen by the trader at time t. The term $(y_{t^e} - p^o(h_t, z_t, u_t))$ denotes a transfer between buyer and seller, and so integrating over the population of traders its expected value is equal to zero. Therefore, the expected surplus under the current trading mechanism is

$$E\left[c_t u_t\right]. \tag{13}$$

The social surplus foregone by a trader whose limit order expires is equal to sign $\left(u_t - G^{-1}(1/2)\right) u_t$, and so taking expectations, the expected loss from these traders is given by

$$E\left[1\{c_t = 0\} \operatorname{sign}\left(u_t - G^{-1}(1/2)\right) u_t\right]. \tag{14}$$

The second source of loss occurs when an agent with private value less than the median of the private valuations buys the stock and vice versa for agents with private value above the median. The surplus foregone by a trader with a valuation below the median who eventually purchases the share is given by $2u_t$, with a similar expression for those traders who should be purchasing to achieve the maximum social surplus, but sell in the current mechanism. Taking expectations, the expected loss from trades in the wrong direction is given by

$$2E\left[1\{c_t = -1\} 1\left\{u_t \geq G^{-1}(1/2)\right\} u_t - 1\{c_t = 1\} 1\left\{u_t < G^{-1}(1/2)\right\} u_t\right]. \tag{15}$$

The first two indicator functions in Equation (15) pick out those traders who sell a share in the current trading system but should purchase a share to achieve the optimal surplus, and the second set of indicator functions pick out buyers who should be selling.

4 Econometrics Implementation

This section shows how we estimate our theoretical model and test its empirical restrictions. In the Markov perfect equilibria of the model, the state space can be represented by h_t and z_t and so let $w_t \equiv \{h_t, z_t\}$ be the state vector. Proposition 3 shows that at every equilibrium information set, w_t there are two finite, intersecting sets of undominated prices, now denoted $P^k(w_t)$ for $k \in \{b, s\}$ from which traders choose an order price, with generic element $p_j^k(w_t)$. We let $\theta_j^k(w_t)$ denote the indifference valuation associated with the pair of prices $\{p_j^k(w_t), p_{j+1}^k(w_t)\}$.

The key identifying assumption in our framework is that we can determine the limit orders with the highest execution probability. Our estimation strategy requires either the highest execution probability for a limit sell order always corresponds to the price tick one above the bid price, and/or the highest execution probability for a limit order to sell invariably corresponds to the price tick below the ask price. This identification condition is nested within the following null hypothesis.

Hypothesis 1 *For each $w_t \in W$, let p_i denote the ask, a_t, with $p_i > p_{i-1} > p_{i-2} > \ldots$ and and p_j denote the bid with b_t, $p_j < p_{j+1} < p_{j+2} < \ldots$. For some positive integer $I = k + l$:*

$$\psi_{i-1}^b(w_t) > \psi_{i-2}^b(w_t) > \ldots > \psi_{i-k}^b(w_t) > \max_{K>k} \psi_{i-K}^b(w_t)$$

and

$$\psi_{j+1}^s(w_t) > \psi_{j+2}^s(w_t) > \ldots > \psi_{j+l}^s(w_t) > \max_{L>l} \psi_{j+L}^s(w_t).$$

The identification condition essentially requires that the conditions of Proposition 4 hold in our data for the limit orders close to the spread. Tables 2 and 4 lend empirical support for the identifying assumption, because the estimates clearly exhibit the required monotonicity properties within two ticks of the spread. However those statistics are not conclusive, if only because they fail to fully account for all the information available to agents across conditioning sets, but may smooth over potential violations to the monotonicity requirement. For a computational standpoint, the virtue of the identification condition is that it effectively partitions all valuations by quantities that can be consistently estimated, thus facilitating the implementation of our semiparametric estimation and testing strategy.

Our estimation strategy is based on the relationship between the the probability distribution for private valuations, the observed choices of traders, and the indifference valuations which empirically link the two. The conditional probability that a trader chooses a sell order price less than $p_j^s(w_t)$ is given by

$$E\left[\sum_{\{i | p_i \leq p_j^s(w_t)\}} 1\{d_{it} = -1\} \middle| w_t\right] = \text{Prob}\left(\alpha(z_t) + u_t \leq \theta_j^k(w_t)\right) \tag{16}$$

$$= \quad G\left(\theta_j^s(w_t) - \alpha(z_t)\right).$$

The left hand side of Equation (16) gives the conditional probability that a trader chooses a sell order less than $p_j^s(w_t)$. The right hand side of the first line follows from the monotonicity of the threshold valuations and the second line follows from our assumption that the private valuation are drawn identically and independently drawn from $G(u)$. Similarly, the conditional probability that a trader either submits a sell order or a buy order at a price less than or equal to $p_j^b(w_t)$ is given by

$$E\left[1\{d_{it}=-1\} + \sum_{\{i|p_i\leq p_j^b(w_t)\}} 1\{d_{it}=1\} \middle| w_t\right] \quad = \quad G\left(\theta_j^b(w_t) - \alpha(z_t)\right). \tag{17}$$

We assume that the common value is a linear function of our factors

$$\alpha(z_t) = z_t'\alpha_0, \tag{18}$$

where α_0 is an L dimensional vector of unknown coefficients. This linear factor structure assumption allows us to put Equations (16) and (17) in the form of a linear index models, which we estimate using semiparametric methods.

Applying Equation (18), the conditional winner's curse is given by

$$\begin{aligned} \xi_i^k(w_t) \quad &\equiv \quad E\left[\sum_{\tau=0}^{\infty} c_{it}^{\tau}\Delta_{\tau}y_t \middle| w_t, |d_{it}|=1\right] \\ &= \quad E\left[\sum_{\tau=0}^{\infty} c_{it}^{\tau}\Delta_{\tau}z_t' \middle| w_t, |d_{it}|=1\right]\alpha_0 \\ &\equiv \quad \varsigma_i^k(w_t)'\alpha_0. \end{aligned} \tag{19}$$

Let x_{jkt} be an $L+1$ dimensional vector of variables specific to the t^{th} observation, defined as

$$x_{jkt} \equiv \begin{bmatrix} p_j - \frac{\psi_j^k(w_t)(p_j - p_{j+1})}{\psi_j^k(w_t) - \psi_{j+1}^k(w_t)} \\ \frac{\varsigma_j^k(w_t) - \varsigma_{j+1}^k(w_t)}{\psi_j^k(w_t) - \psi_{j+1}^k(w_t)} - z_t \end{bmatrix}, \tag{20}$$

and define β_0 by

$$\beta_0 \equiv \begin{bmatrix} 1 \\ \alpha_0 \end{bmatrix}. \tag{21}$$

Substituting Equation (19) into the expression for indifference valuations, Equation (6),

$$\begin{aligned}\theta_j^k(w_t) - \alpha(z_t) &= p_j - \frac{\psi_j^k(w_t)(p_j - p_{j+1}) - \xi_{jt}^k(w_t) - \xi_{j+1,t}^k(w_t)}{\psi_j^k(w_t) - \psi_{j+1}^k(w_t)} - z_t'\alpha_0 \\ &= p_j - \frac{\psi_j^k(w_t)(p_j - p_{j+1})}{\psi_j^k(w_t) - \psi_{j+1}^k(w_t)} + \frac{\left(\varsigma_j^k(w_t) - \varsigma_{j+1}^k(w_t)\right)'}{\psi_j^k(w_t) - \psi_{j+1}^k(w_t)}\alpha_0 - z_t'\alpha_0 \\ &= x_{jkt}'\beta_0. \end{aligned} \tag{22}$$

Therefore the linear factor structure together with our focus on Markov perfect equilibria implies that choices can be modeled as an ordered linear index model, so that for the sell side

$$E\left[\sum_{\{i|p_i \le p_j^s(w_t)\}} 1\{d_{it} = -1\} \middle| w_t\right] = G\left(x_{jst}'\beta_0\right), \tag{23}$$

with a similar expression on the buy side.

The estimation and testing of our model divides into three parts. In the first part, we form nonparametric estimates the main components of the indifference valuations. We nonparametrically estimate the execution probabilities, $\psi_j^k(w_t)$, and the contribution of the factors to the winner's curse terms, $\varsigma_j^k(w_t)$. Drawing extensively on Powell, Stock, and Stoker (1989), Robinson (1989) and Ahn (1997), we then use these estimates to form approximations of x_{jkt}, which along with the observed choices made by the traders, d_t, yields an average derivative estimator of β_0. The second part of the empirical work tests two features of our theoretical model. From the estimates of x_{jkt} and β_0 we form an approximation for $\theta_j^k(w_t)$ and test the monotonicity conditions of Proposition 3, applying the results derived by Wolak (1989), (1991). Our model also implies that the conditional choice probabilities depend only on x_{jkt}. We test this by adding in other variables to the index model and perform tests of the hypothesis that these variables are unimportant in predicting traders' order choices. The third part of our empirical work estimates bounds on the the welfare gains and losses to traders in the current trading environment. Using our consistent estimators of $x'_{jkt}\beta_0$, we obtain consistent estimates for some functions that bound $G(u)$ from both sides. These estimates are used to to derive bounds for the measures described in Section 3 that evaluate the efficiency of this market mechanism. Appendix B formally justifies our empirical approach, by establishing the asymptotic properties of our estimators and test statistics.

4.1 Semiparametric Estimation

The execution probabilities and winner's curse terms necessary to form the components of x_{jkt} are estimated nonparametrically. Denote the resulting estimators of $\psi_i^k(w_t)$ and $\varsigma_i^k(w_t)$ by $\hat{\psi}_i^k(w_t)$ and $\hat{\varsigma}_i^k(w_t)$ respectively. The vector x_{jkt} is approximated by

$$\hat{x}_{jkt} \equiv \begin{bmatrix} p_j - \frac{\hat{\psi}_j^k(w_t)(p_j - p_{j+1})}{\hat{\psi}_j^k(w_t) - \hat{\psi}_{j+1}^k(w_t)} \\ \frac{\hat{\varsigma}_j^k(w_t) - \hat{\varsigma}_{j+1}^k(w_t)}{\hat{\psi}_j^k(w_t) - \hat{\psi}_{j+1}^k(w_t)} - z_t \end{bmatrix}.$$

Let $\pi(x_{jkt})$ denote the unconditional density of x_{jkt} and let $\zeta(w_t)$ denote a trimming indicator which is equal to one when w_t lies in a compact set in the interior of the support of w_t and zero otherwise. Define $\rho(x_{jkt}) \equiv E[\zeta(w_t)|x_{jkt}]$ as the conditional expectation of this trimming indicator given x_{jkt}, and $\upsilon(x_{jkt}) \equiv \pi(x_{jkt})\rho(x_{jkt})$. For any limit sell price j ticks above the bid, we define the weighted probability that the private valuation is lower than the indifference valuation associated with that order, after netting out its common component, as

$$\phi_{js} \equiv E\left[\sum_{\{i|p_i \le p_j^s(w_t)\}} 1\{d_{it} = -1\}\,\zeta(w_t)\frac{\partial\upsilon(x_{jkt})}{\partial x_{jkt}}\right].$$

Differentiating $G\left(x'_{jkt}\beta_0\right)$ with respect to x_{jkt} and integrating by parts shows that $\phi_{jk} = \kappa_{jk}\beta_0$ where

$$\kappa_{jk} = E\left[\pi(x_{jkt})\rho^2(x_{jkt})G'\left(x'_{jkt}\beta_0\right)\right]. \tag{24}$$

See Powell, Stock, and Stoker (1989) for example. In the second step we use the $\hat{x}_{jkt}$ approximations to obtain nonparametric estimators for $\pi(x_{jkt})$ and $\rho(x_{jkt})$, differentiate $\hat{\upsilon}(x_{jkt}) = \hat{\pi}(\hat{x}_{jkt})\hat{\rho}(\hat{x}_{jkt})$ with respect to $\hat{x}_{jkt}$ and form

$$\hat{\phi}_{js} = \frac{1}{T}\sum_{t=1}^{T}\left(\sum_{\{i|p_i \le p_j^s(w_t)\}} 1\{d_{it} = -1\}\,\zeta(w_t)\frac{\partial\hat{\upsilon}(\hat{x}_{jkt})}{\partial\hat{x}_{jkt}}\right). \tag{25}$$

Similar estimators can be formed for the buy side, as well.

Noting that the first component in the vector ϕ_{jk} is κ_{jk}, an estimator of α_0 is obtained by dividing all the components of $\hat{\phi}_{jk}$ by the first component, and then picking the last L components. Our theoretical model allows us to identify α_0 since the theoretical model implies that the first component of β_0 is equal to one.

Implementation of Equation (25) can be repeated I times, for different price choices relative to the bid and ask, and in the case of any price lying on or between the spread, for both buyers and sellers. This observation motivates the third step in estimation, which combines the estimates obtained in the second step to improve asymptotic efficiency. Accordingly, let $\hat{\phi}$ denote a I dimensional vector formed from I different (j,k) combinations of $\hat{\phi}_{jk}$, let α denote the I dimensional vector of the scaling factors for the I choices and define the vector function $H(\kappa,\alpha)$ as

$$H(\kappa,\alpha) \equiv [\kappa_1, \kappa_1\alpha, \kappa_2, \kappa_2\alpha, ..., \kappa_I, \kappa_I\alpha]'.$$

We formed the minimum distance estimators

$$(\hat{\kappa}_{md}, \hat{\alpha}_{md}) \equiv \arg\min_{\kappa,\alpha} T \left[H(\kappa,\alpha) - \hat{\phi}\right]' \mathcal{C}^{-1} \left[H(\kappa,\alpha) - \hat{\phi}\right], \qquad (26)$$

for any is a positive definite I dimensional weighting matrix, $\mathcal{C}$

The asymptotic properties of $\hat{\phi}$ are established in appendix B, while the properties of the minimum distance estimator are discussed in Chamberlain (1984). Providing the ancillary assumptions made in the appendix and the null hypothesis is satisfied for $I \geq 1$, $\sqrt{T}\left(\hat{\phi} - \phi\right)$ and $\sqrt{T}\left(\hat{\alpha}_{md} - \alpha_0\right)$ converge to normal distributions with mean zero and covariances given in the appendix. In the case when a consistent estimator of the covariance matrix for $\sqrt{T}\left(\hat{\phi} - \phi\right)$ is substituted for $\mathcal{C}$, the minimized value of the objective function in Equation (26) asymptotically χ^2 distributed with $I(L+1) - I - L$ degrees of freedom under the null hypothesis that the model is correctly specified.

4.2 Asymptotic Tests

With regards to testing, suppose further that at least one of the two following conditions are satisfied. The first condition is that $k = b$ and $l = s$; the second one is that $k = l$ and $\psi_j'^k(w_t) \geq \psi_i'^l(w_t)$. Then Proposition 3 implies $\theta_j^k(w_t) \geq \theta_i^l(w_t)$, and consequently from Equation (22),

$$E\left(\theta_j^k(w_t) - \theta_i^l(w_t) \middle| w_t\right) = E\left(\left(x'_{jkt} - x'_{ilt}\right)\beta_0 \middle| w_t\right) > 0.$$

The conditional moment restriction implies the unconditional moment inequality

$$m_{ijkl}^q \equiv E\left(|q_t| \times \left(x'_{jkt} - x'_{ilt}\right)\beta_0\right) > 0 \qquad (27)$$

for any vector valued $\mathcal{A}_t$ measurable mapping q_t. Since such inequalities are not imposed in estimation, they can be tested using the estimated parameters of the model. The aim is to investigate whether positive state variables known at the time of order submission are correlated with violations of monotonicity by forming test statistics based on m^q_{ijkl}. The approximate sample moment of the left side of (27) is

$$\hat{m}^q_{ijkl} = \frac{1}{T}\sum_{t=1}^{T} |q_t| \left(\hat{x}'_{jkt} - \hat{x}'_{ilt}\right) \hat{\beta}_{md}, \tag{28}$$

where $\hat{\beta}'_{md} \equiv \left[1 : \hat{\alpha}'_{md}\right]$. Appendix B shows that $\sqrt{T}\left(\hat{m}^q_{ijkl} - m^q_{ijkl}\right)$ converges in distribution to a normal random variable with mean zero and covariance matrix Σ. Since $x'_{jkt}\beta_{md}$ is a nonlinear function of β_0, $\psi^k_i(w_t)$ and $\varsigma^k_i(w_t)$, which are the underlying parameters of our model, undertaking a global test of inequality (27) is not feasible (Wolak (1991)). However, a local test can be performed using the test statistic

$$M^q_{ijkl} = \min_{\{\mu:\mu\geq 0\}} T\left(\hat{m}^q_{ijkl} - \mu\right)' \Sigma^{-1}\left(\hat{m}^q_{ijkl} - \mu\right). \tag{29}$$

Under the null hypothesis that inequality (27) holds, Wolak (1989) shows that M^q_{ijkl} is distributed as the weighted sum of chi-squared variables:

$$\Pr\left(M^q_{ijkl} \geq r\right) = \sum_{f=0}^{dim\left(M^q_{ijkl}\right)} \Pr\left(\chi^2_f \geq r\right) W\left(dim\left(m^q_{ijkl}\right), dim\left(m^q_{ijkl}\right) - f, \Sigma\right) \tag{30}$$

where $W\left(dim\left(m^q_{ijkl}\right), dim\left(m^q_{ijkl}\right) - f, \Sigma\right)$ is a weight, which can be calculated using Monte Carlo methods.

4.3 Welfare Bounds

Because traders with similar, but non-identical, valuations submit the same order when faced with the same set of state variables, the individual private valuations are unidentified. Therefore obtaining estimators of the welfare measures and other indicators of market structure, hinges on being able to directly integrate or simulate an approximation to $G(u)$. From Equation (23), a point on $G(u)$ is identified at the index value $u = x'_{jkt}\beta_0$ if $x'_{jkt}\beta_0$ is implied by some w_t reached in equilibrium. This follows since $x'_{jkt}\beta_0$ is consistently estimated by $\hat{x}_{jkt}{}'\hat{\beta}_{md}$, and the nonparametric regression of $\sum_{\{i|\psi^s_i(w_t)_i \leq \psi^s_j(w_t)\}} 1\{d_{it} = -1\}$ on $\hat{x}_{jst}{}'\hat{\beta}_{md}$, denoted $\hat{G}\left(\hat{x}_{jks}{}'\hat{\beta}_{md}\right)$, is also a consistent estimator of $G\left(x_{jst}{}'\beta_0\right)$. We can use the index evaluated at buy prices in a similar way. Although the support of G is not necessarily covered by the range of the index function $x_{jkt}{}'\beta_0$, consistent estimates of upper and lower bounds to the welfare measures described earlier can be obtained.

The terms in Equations (14) and (15) can be bounded using the appropriate Riemann sums, taken at points where $G(u)$ is identified. Let $G_1(u)$ be the probability distribution function formed from the lower Riemann sums, and $G_2(u)$ the probability distribution function formed from the upper Riemann sums. These probability distributions have the following properties: $G_1(u) \leq G(u) \leq G_2(u)$ for all $u \in [u_l, u_h]$, $G_1(u)$ and $G_2(u)$ have the same discontinuity points, which occur on the boundaries of open intervals that are not reached by the index value for any w_t, and $G_1(u) = G(u) = G_2(u)$ in open neighborhoods covered by the index function. To obtain bounds for the welfare loss from unexecuted orders in terms of the identified probability distributions $G_1(u)$ and $G_2(u)$, denote by $u_{1t} \equiv x'_{jkt}\beta_0$ and $u_{2t} \equiv x'_{j+1kt}\beta_0$ the threshold values containing u_t, where p_j is the price associated with the t^{th} order. Applying the law of iterated expectations to Equation (14), we obtain

$$\begin{aligned} & E\left[1\{c_t = 0\}\operatorname{sign}\left(u_t - G^{-1}(1/2)\right)u_t\right] \\ = \quad & E\left[E\left[1\{c_t = 0\}\operatorname{sign}\left(u_t - G^{-1}(1/2)\right)u_t \middle| w_t, d_t\right]\right] \\ = \quad & E\left[1\{c_t = 0\}\frac{\int_{u_{1t}}^{u_{2t}}\operatorname{sign}\left(u - G^{-1}(1/2)\right)udG(u)}{G(u_{2t}) - G(u_{1t})}\right]. \end{aligned} \tag{31}$$

The respective definitions of $G_1(u)$ and $G_2(u)$ imply that

$$\int_{u_{1t}}^{u_{2t}} 1\left\{u > G_1^{-1}(1/2)\right\} udG(u) \leq \int_{u_{1t}}^{u_{2t}} 1\left\{u > G_1^{-1}(1/2)\right\} udG_2(u)$$

and also

$$\int_{u_{1t}}^{u_{2t}} 1\left\{u \leq G_1^{-1}(1/2)\right\} udG_1(u) \leq \int_{u_{1t}}^{u_{2t}} 1\left\{u \leq G_1^{-1}(1/2)\right\} udG(u).$$

Together these inequalities form the basis for the upper bounds we constructed. Making the appropriate substitutions into the third line of Equation (31), an upper bound of the welfare loss from unexecuted orders is therefore

$$E\left[\begin{array}{c} 1\{c_t = 0\}\frac{\int_{u_{1t}}^{u_{2t}} 1\left\{u > G_1^{-1}(1/2)\right\} udG_2(u)}{G(u_{2t}) - G(u_{1t})} \\ -1\{c_t = 0\}\frac{\int_{u_{1t}}^{u_{2t}} 1\left\{u \leq G_1^{-1}(1/2)\right\} udG_1(u)}{G(u_{2t}) - G(u_{1t})} \end{array}\right] \tag{32}$$

Upon setting $\hat{u}_{1t} = \hat{x}'_{jkt}\hat{\beta}$, and $\hat{u}_{2t} = \hat{x}'_{j+1kt}\hat{\beta}$, a consistent estimator for Equation (32) is

$$\frac{1}{T}\sum_{t=1}^{T}\left\{\begin{array}{l} 1\left\{c_t = 0, u_t > \hat{G}_1^{-1}(1/2)\right\}\frac{\int_{\hat{u}_{1t}}^{\hat{u}_{2t}} ud\hat{G}_2(u)}{\hat{G}(\hat{u}_{2t})-\hat{G}(\hat{u}_{1t})} \\ -1\left\{c_t = 0, u_t < \hat{G}_2^{-1}(1/2)\right\}\frac{\int_{\hat{u}_{1t}}^{\hat{u}_{2t}} ud\hat{G}_1(u)}{\hat{G}(\hat{u}_{2t})-\hat{G}(\hat{u}_{1t})} \end{array}\right\}, \tag{33}$$

where $\hat{G}_1(u)$ and $\hat{G}_2(u)$ are consistent estimators of $G_1(u)$ and $G_2(u)$. We obtained a consistent estimator of a lower bound for Equation (14) in a similar manner.

We used a similar method to bound the welfare losses arising from sellers who should purchase to maximize social surplus. For each sell order submitted at price p_j, the model implies $u_t \leq x'_{jst}\beta_0$. From equation (15) the expectation of losses from sellers with valuations higher than the median $G_1^{-1}(1/2)$ equals

$$\begin{aligned} &2E\left[1\{c_t=-1\}1\left\{u_t \geq G^{-1}(1/2)\right\}u_t\right] \\ &\quad = 2E\left[1\{c_t=-1\}1\left\{u_t \geq G^{-1}(1/2)\right\}\frac{\int_{\max\{G^{-1}(1/2),u_{1t}\}}^{u_{2t}} udG(u)}{G(u_{2t})-\max\{1/2, G(u_{1t})\}}\right]. \end{aligned} \tag{34}$$

A lower bound for Equation (34) is

$$2E\left[1\{c_t=-1\}1\left\{u_t \geq G_2^{-1}(1/2)\right\}\frac{\int_{\max\left\{G_2^{-1}(1/2),u_{1t}\right\}}^{u_{2t}} udG_1(u)}{G(u_{2t})-\max\{1/2, G_2(u_{1t})\}}\right],$$

while an upper bound is

$$2E\left[1\{c_t=-1\}1\left\{u_t \geq G_1^{-1}(1/2)\right\}\frac{\int_{\max\left\{G_1^{-1}(1/2),u_{1t}\right\}}^{u_{2t}} udG_2(u)}{G(u_{2t})-\max\{1/2, G_1(u_{1t})\}}\right].$$

The bounds for buyers who should sell to achieve the maximal gains from trade are computed in a similar manner.

Substituting consistent estimates for (u_{1t}, u_{2t}) for each limit order in the data, a consistent estimate of u_{1t} for each market sell order, a consistent estimator of u_{2t} for each market buy order, as well as consistent estimates of $G_1(u)$ and $G_2(u)$ evaluated at the appropriate points then yields a consistent estimator of a lower bound to wrong sided trades, namely

$$2\frac{1}{T}\sum_{t=1}^{T}\left\{\begin{array}{l} 1\{c_t=-1\}1\left\{u_t \geq \hat{G}_2^{-1}(1/2)\right\}\frac{\int_{\max\left\{\hat{G}_2^{-1}(1/2),\hat{u}_{1t}\right\}}^{\hat{u}_{2t}} ud\hat{G}_1(u)}{\hat{G}(u_{2t})-\max\left\{1/2,\hat{G}_2(\hat{u}_{1t})\right\}} \\ -1\{c_t=1\}\left\{u_t \leq \hat{G}_1^{-1}(1/2)\right\}\frac{\int_{\hat{u}_{1t}}^{\min\left\{\hat{G}_1^{-1}(1/2),\hat{u}_{2t}\right\}} ud\hat{G}_2(u)}{\min\left\{1/2,\hat{G}_1(\hat{u}_{2t})\right\}-\hat{G}(u_{1t})} \end{array}\right\}, \tag{35}$$

with an upper bound computed in a similar manner.

It only remains to bound the total available social surplus. The law of iterated expectations implies that (11) can be expressed as

$$E\left[\frac{\int_{u_{1t}}^{u_{2t}} \operatorname{sign}\left(u - G^{-1}(1/2)\right) u dG(u)}{G(u_{2t}) - G(u_{1t})}\right]. \tag{36}$$

Since neither u_h nor u_l are identified, $G_2(u)$ is unidentified in neighborhoods near u_h and $G_1(u)$ is unidentified around u_1. Therefore, we cannot identify an upper bound to Expression (36) in terms of consistently estimated parameters, but it is bounded below by

$$E\left[\frac{\int_{u_{1t}}^{u_{2t}} 1\left\{u > G_2^{-1}(1/2)\right\} u dG_1(u)}{G(u_{2t}) - G(u_{1t})} - \frac{\int_{u_{1t}}^{u_{2t}} 1\left\{u \leq G_1^{-1}(1/2)\right\} u dG_2(u)}{G(u_{2t}) - G(u_{1t})}\right]. \tag{37}$$

Following our treatment of the other welfare measures, a consistent estimator of Equation (37) is

$$\frac{1}{T}\sum_{t=1}^{T}\left\{\frac{\int_{\hat{u}_{1t}}^{\hat{u}_{2t}} 1\left\{u > \hat{G}_2^{-1}(1/2)\right\} u d\hat{G}_1(u)}{\hat{G}(\hat{u}_{2t}) - \hat{G}(\hat{u}_{1t})} - \frac{\int_{\hat{u}_{1t}}^{\hat{u}_{2t}} 1\left\{u \leq \hat{G}_1^{-1}(1/2\}\right) u d\hat{G}_2(u)}{\hat{G}(\hat{u}_{2t}) - \hat{G}(\hat{u}_{1t})}\right\}, \tag{38}$$

where the estimated quantities in Equation (38) are defined above.

5 Empirical Results

This section reports parameter estimates for the index model and tests of various over–identifying restrictions of the model. We provide estimates of the private valuation distributions and of the realized and potential gains from trade. Details about the implementation of the nonparametric estimators are found in Appendix C.

In our theoretical model, the conditional execution probability and winner's curse functions depend on the current limit order book and the state variable that are useful in predicting changes in the common value of the asset. As explained in the previous sections, the model leaves considerable latitude in the specification of the state variables. The following list of variables emerged from a preliminary analysis to determine the main factors effecting the execution probabilities and represents the state variables we used in our empirical specification:

- Time elapsed since the last order was submitted to the market. This variable is a measure of competition in the supply and demand for liquidity.

- Time elapsed from market open. The variable captures intra-day effects in trading opportunities.
- The current bid–ask spread. This captures variations in trading opportunities and the competition in the book. This is quoted as a percentage of the average of the bid and ask prices, or the mid–quote.
- The total quantities at the best bid quote. This variable is a measure of the quantity of orders on the buy side of the market.
- The total quantities at the best ask quote. This variable is a measure of the quantity of orders on the sell side of the market.
- The current mid-quote. This is the average of the current bid and ask quotes in the limit order book.
- The current level of the OMX market index. This is a value weighted average of the prices of the 30 most actively traded stocks on the Stockholm Stock Exchange.

We experimented with several factors that might affect the common value of Ericsson, including the Kronor-US dollar exchange rate and the short term interest rate, but found that only the OMX market index, sampled every minute had a significant affect. For this reason our empirical results below are based on the assumption that the common value of Ericsson is proportional to the OMX market index.

The first panel of Table 6 reports estimates obtained by applying Equation (25) to four different order choices, using the above variables to compute the nonparametric estimates of conditional expectations that enter into x_{jkt}. Each row refers to estimates using different order choices. The orders that we use are the limit sell order 1 tick above the market sell order, the limit sell order 2 ticks above the market sell order, the limit buy order 1 tick below the market buy order and the market buy order. The table reports the point estimates, asymptotic standard errors and asymptotic t–statistics of the scaling factor in Equation (24) and the coefficient on the market factor, β times the scaling factor. From Equation (24), the scale factor is an estimate of the expectation of the density of x_{jkt} times the density of the private value distribution evaluated at the threshold valuation. This explains the order of magnitude of these estimates. In addition, the scale factor is different for each choice equation used, so that the scale factor associated with different choices in Table 6 are estimates of different quantities. The asymptotic standard errors in Table 6 indicate that both the scale factor and the coefficient on the market index are relatively precisely estimated for the choices involving market buy or sell orders. However, for the other 2 choice equations that we consider, our estimates are less precise.

The second panel of Table 6 reports estimates obtained by applying the minimum distance estimator, Equation (26) to our model. We report the scaling coefficient for our four choice equations and the overall coefficient estimate for the market factor, plus the associated standard errors and t–statistics. We also report the minimized value of the objective function for the estimator, Equation (26), along with the associated asymptotic p–value. The coefficient on the market is estimated relatively precisely, with a point estimate of 0.378. The chi-squared statistic indicates that we cannot reject our model with this test statistic.

Together the stochastic process driving the common shock and the distribution of private valuations comprise the primitives of our model. Accordingly, Figure 1 plots the estimated common value against the midquote over the sample, while Figures 2 and 3 present nonparametric estimates of the probability distribution for private valuations at the points where it is identified. The vertical lines in Figure 1 separate the sample into 6 consecutive subsamples of roughly 2 weeks in length. Comparing the two series, we see they behave in a similar manner at low frequencies, but that at higher frequencies, there are some noticeable deviations. In our model deviations between the midquote and the common shock arise because the limit order book depends on lagged values of the common shock through limit orders placed in the past, and consequently the model would predict that the midquote series has a tendency to lag the common shock. For the most part this prediction is borne out in practice, providing some confirmation that the series estimating the common value is a reasonable approximation.

Figure 2 portrays $\hat{G}\left(\hat{x}'_{jkt}\hat{\beta}_M\right)$, the nonparametric regression of the cumulative choices at points where it is identified, namely at the thresholds estimated by $\hat{x}'_{jkt}\hat{\beta}_M$, using a normal kernel with a smoothing parameter of 3. As we remarked earlier, $G(u)$ is not identified over its entire support, and we did not estimate the bottom two deciles nor the top decile of the distribution. An alternative to our semiparametric estimation strategy would have been to impose a functional form on the probability distribution determining private valuations. Our estimate of $G(u)$ suggests that many inflection points would be missed by specifying $G(u)$ parametrically. Nevertheless the apparent decline in $G(u)$ between private values of -30 and -20 Kronor is diagnostic evidence that the model does not explain all the observed variation in choice probabilities.

We investigated this anomaly by estimating the private valuation probability distribution with data from each of the subsamples taken separately, that is given $\hat{x}_{jkt}$ and $\hat{\beta}_M$. The results, displayed in Figure 3, are noteworthy for two reasons. The second subsample contains the most egregious violation of monotonicity, and this corresponds to the second subsample in Figure 1, where differences between the midquote and the estimated common value are most pronounced. These results might indicate factors have been omitted in our specification of the common shock, and/or that the probability distribution for private valuations is not independent and identically distributed over time. The second noteworthy feature about Figure 3 is that the subsamples seem to capture different segments of the support for $G(u)$. Because the indifference valuations vary with the relation between the book and the common shock, it might not seem surprising that subsamples characterized by different sets of state variables reveal information about different parts of the $G(u)$ distribution. This second feature, therefore, might merely reflect sample size problems encountered by partitioning the data too finely, although it could also be interpreted as evidence of nonstationarity in the common shock process itself.

The results form the formal specification tests we undertook, presented in Tables 7 through 9, complement the diagnostic checks displayed in Figures 1 through 3. Table 7 presents the monotonicity test of the threshold valuations. Each row reports a one–sided t–statistic for the null hypothesis that $m^q_{ijkl} \geq 0$, and the overall monotonicity test, Equation (29) using various conditioning instruments. The first row simply takes the constant as the instrument. The next rows report on instruments that account for characteristics of the order chosen; the distance between the order price and the mid-quote and the number of shares in the order. We then experimented with instruments that capture the overall market conditions: slow trading and fast trading both measure the time–between–orders, an indicator variable for when the tick size ≤ 1 Kronor, the bid–ask spread, the absolute price change over the last 30 minutes and the trading volume over the last 30 minutes. Other instruments tried to capture the state of the order book when the order is submitted. The final row of the table reports the test performed jointly across all the state variables. The asymptotic p–values in this table never reject the monotonicity of the threshold valuations.

Our model implies that the cumulative probability of order choices depends on information set according to Equation (22). We tested this restriction with two tests. For the first test, define the vector

$$\left(f_{jkt}^{1},...,f_{jkt}^{4}\right) \equiv \left(p_{jt}, -z_t, -\frac{\psi_j^k(w_t)\left(p_j - p_{j+1}\right)}{\psi_j^k - \psi_{j+1}^k}, -\frac{\varsigma_j^k(w_t) - \varsigma_{j+1}^k(w_t)}{\psi_j^k - \psi_{j+1}^k}\right),$$

and consider the linear index model

$$E\left[\sum_{\{i|p_i \le p_j^s(w_t)\}} 1\{d_{it} = -1\} \middle| w_t\right] = G\left(\sum_{m=1} B_{jk}^m f_{jkt}^m\right), \tag{39}$$

for each choice. Equation (22) imposes the restrictions that $B_{jk}^1 = B_{jk}^3$ and $B_{jk}^2 = B_{jk}^4$, for all choices. Table 8 reports estimates from applying the average derivative estimator to Equation (39) along with asymptotic chi-squared tests of the hypothesis $B_{jk}^1 = B_{jk}^3$ and $B_{jk}^2 = B_{jk}^4$ for each equation, and also jointly across the entire system. For the choice equations involving market orders, the estimated coefficients for both the shading term and the winner's curse term are statistically significant, with signs consistent with the theoretical model. Traders do consider the trade–off between price and execution probability and winner's curse in choosing between market and limit orders, although the decisions involving the choice between 2 limit orders, Limit sell +2 ticks and limit buy 1 tick the coefficients on the trade–off terms are not statistically significant. The chi-squared tests of the restrictions of the model reject the restrictions for all choice equations except for the choice between limit sells at one and 2 ticks, and jointly across all equations.

For the second test, we estimated the model

$$E\left[\sum_{\{i|p_i \le p_j^k(w_t)\}} 1\{d_{it} = -1\} \middle| w_t\right] = G\left(\hat{x}'_{jkt}\beta + w_{it}W_{jk}^i\right), \tag{40}$$

with the average derivative estimator for each choice. Here, the scalar w_{it} is in the information set of the trader when the order is submitted. Equation (22) implies the restriction that $W_{jk}^i = 0$ for all choices. Table 9 provides estimates of Equation (40) using all of the variables w_{it} used in computing the execution probabilities and winner's curse terms in x_{jkt}.[6] These models are estimated including only one extra variable at a time to keep the dimension of the variables in the average derivate estimator manageable. For each choice equation, we computed a chi–squared test of the joint hypothesis that the variables in x_{jkt} are equal to zero. Typically, we reject this hypothesis; the trade–off is important in predicting order choices. We

[6] Variables that were not used in estimating the x_{jkt} may be important in Equation (40) through correlation with the true execution probabilities or winner's curse terms.

also computed t–statistics and joint chi–squared statistics of the hypothesis that the extra variables have coefficients equal to zero. The null hypothesis is rejected for all extra variables and choice equations. Panel B of Table 9 contains the results of adding the bid–quantity and ask–quantity on the order book as additional factors. The point estimates for the bid–quantity are negative for all choice equations. These results suggest that when the order book is thick on the buy side, traders increase the likelihood of submitting aggressive sell orders, and reduce the likelihood of submitting aggressive buy orders. The reverse holds when the order book is thick on the sell side. These results suggest that the theoretical trade-off does not capture all of the effects of competition in the order book.

Overall the empirical results in Tables 6 through 9 and Figures 1 through 3 are moderately supportive of the underlying model. The coefficient on the factor structure is positive and significant, and the common shock generated by the single factor structure leads the midquote series, as predicted by the theory. The winner's curse and execution probability terms have significant qualitative effects as predicted by the model. Neither the overidentifying restrictions between the index equations marking off different choices, nor the monotonicity restrictions between valuations and prices applied in estimation, can be rejected. The empirical results also reveal deficiencies in the model, at least in the specialization we estimate, where auxiliary assumptions guarantee that every trading opportunity produces a limit or a market order. Although the estimated distribution function is monotonically increasing throughout most of its range, there are some glaring exceptions. Also, some of the constraints restricting the way the state variables in the model should enter within equations are systematically rejected. Thus the estimates of the welfare bounds we present below should be treated with caution, as a tentative benchmark.

Table 10 presents our estimates of the bounds on the realized and potential gains from trade in our data. These estimates are formed by numerically integrating the welfare measures in Equations (33), (35) and (38) using estimates of the distribution of private valuations.[7] The unit of account is the Swedish Kronor, and the average price of a share of Ericsson is approximately 110 Kronor in our sample. At almost 22 Kronors, the expected social surplus from each order is at least 15 percent of the average closing price, implying the social surplus from optimally allocating the shares throughout the trading population is expected to yield at

[7] In the numerical integration, we forced our estimate of the density of the private valuation distribution to be positive by replacing $d\hat{G}_i(u)$ with $\max(\varepsilon, d\hat{G}_i(u))$, where $\varepsilon = 0.0005$. The numerical results are insensitive to the choice of ε in these computations.

least 44 Kronors, or 30 percent of the average price.

As explained in Section 4, an upper bound on the social surplus was not estimated, because our empirical strategy does not identify the surplus of those who have the most to gain, namely traders with valuations above the median placing market buy orders, and traders with valuations below the median placing market sell orders. This point should be stressed when contrasting the current mechanism with the losses from prohibiting trade altogether, but can be ignored when the comparison is between mechanisms that permit all individuals with extreme valuations to trade. Our finding, that there are substantial gains from optimally allocating this financial asset amongst traders, is all the more noteworthy because both buyers and sellers value the asset for its dividend stream and prospective capital gains. In our model this characteristic is captured by the stochastic component common across valuations, which has a significant empirical affect. Given the distribution of private valuations, the bigger the common component shared by buyers and sellers, the smaller are the potential gains from trade when measured as a proportion of the average transaction price.

Our definition of a transaction cost corresponds to the Marshallian concept of the expected loss in aggregate social surplus from using this market mechanism instead of an optimal one. With the exception of the loss from executed sell orders by high valuation traders, the upper and lower bounds are within five percentage points of each other, allowing us to pin down the welfare losses in this market. Estimates of the three types of expected losses sum to about 9 Kronors per order. An alternative definition of transaction cost is the bid ask spread itself, which is commonly interpreted as a gauge of market liquidity because it is the minimum cost of simultaneously placing market orders to buy and sell one unit from the existing limit orders and reduce each side of the book by one unit while retaining an asset neutral position. Referring to Table 2, the length of the bid ask spread is typically only one tick, which in our sample is 1 Kronor, except for the brief time when the midquote dropped below 100 Kronors and the tick size halved. Our estimates of social losses dwarf the bid ask spread, underlining in practice the difference between these two concepts of transaction costs.

The lower bound on the realized gains from trade, 12.50 Kronors, is found by subtracting the upper bounds to the three losses from the lower bound on the total social surplus available. We find that the current limit order system achieves at least 57 percent of the potential gains from trade. Putting this another way, aside from the gains that accrue to buyers with high valuations and sellers with low valuations placing market orders, almost half the remaining social surplus is lost to transaction costs. Of the 43 percent, 12 percent of the unrealized surplus is due to canceled orders, approximately 13 percent of the losses are due to executed buy orders and 18 percent come from executed sell orders. Within the context of our model, the losses arise from differences between the individually optimal decision rule and the socially optimal decision rule. This suggests that the private incentives in the market differ from those that implement the socially optimal outcomes, or that the transaction costs associated with this market form are quite large.

More than two thirds of the welfare losses are caused by offers that eventually trade. In our model wrong sided trades help the limit order book to re-center itself around the common value after a major shift in the common value. The size of this loss is directly attributable to the relatively short time between submission and execution indicated in Table 2, which implies almost all trading occurs between individuals who both submit their orders within the same subsample, whereas our results show that within each subsample a preponderance of the individual valuations are drawn from one side of the median or the other. For example, it is evident from Figure 3 that in the fourth subsample almost all the private valuations are positive, yet approximately 55 percent of the orders submitted are buy orders and 45 percent are sell orders.

6 Summary and Conclusion

This paper develops, estimates and tests a model of limit order markets, implementing it on the Stockholm Stock Exchange. Our theoretical framework incorporates both private and common components to valuations in order to characterize the trade-off between placing orders closer to the bid ask spread, which raises the execution probability, versus further apart, which secures better terms in the event of execution. We allow common shocks to the value of the asset that expose limit orders to the winner's curse. We derive monotonicity conditions from the individual's optimization problem which link private components of valuations to the execution probabilities of the orders.

We also establish conditions for the existence of a Markov perfect equilibrium whose outcomes satisfy a mixing condition and evaluate its welfare properties relative to the social optimum. The strategic behavior predicted in equilibrium generates two forms of distortions. Buyers with low private valuations and sellers with high valuations sometimes trade on the wrong side of the market to capitalize on unanticipated changes in the common shock and thus bring about market corrections. In addition, agents placing limit orders risk forfeiting the gains from trade in an endeavor to obtain a better price. These losses can be interpreted as transaction costs of doing business in this market.

The model's monotonicity, and uniform mixing properties along with a factor structure assumption on the common value underlie the identification of the probability distribution for private valuations from a time series on the limit order book and a series of observed factors. Our semiparametric approach to estimating a linear index model adapts methods that were developed for identically and independently distributed observations to our time series data. It applies to dependent data, in which a first step is required to nonparametrically estimate an expectations function. The estimates and test statistics from the structural estimation are of interest in their own right, but also form the foundation for identifying bounds on the welfare gains and losses in this market.

In our application, the common shock is a linear function of the stock market index, and the expectation functions estimated in the first step are used to form terms in the conditional winner's curse and execution probabilities. Thus the time dependence between observations stems from the stock market index, and also from the evolution of the limit order book itself. As illustrated in our description of the data, both expectation functions exhibit substantial variation depending on the state of the limit order book. When the restrictions of the structural model are imposed in estimation, we find the coefficient on the stock market index is positive and significant. We can reject neither the overidentifying restrictions obtained from the multiple equations that individually identify the linear index, nor the monotonicity tests which relate private valuations to prices. Market participants apparently place their orders strategically, our results suggesting that their choices reflect the trade-off between price, execution probability and the winner's curse. Indeed the estimates show that only about half of the social surplus from potential gains from trade are actually realized by this market.

Our empirical work also reveal several features of the model that are not supported by the data. Two types of misspecification are documented. First, the linear index we estimate does not satisfy all the coefficient restrictions implied by the theory. Second, the plots of the identified portions of the probability distribution of the private valuations are not monotone increasing everywhere, and also exhibit some evidence of shifting over the duration of the sample.

One interpretation of the evidence against the model is that the probability distribution determining private valuations is not identically and independently distributed, but depends on the state variables. This interpretation calls into question the assumption that the trading opportunities are exogenous. Supposing private valuations are indeed independently and identically distributed, but that the submission and withdrawals of orders are endogenous. Under this interpretation only the truncated probability distribution of those placing orders could be estimated and our theory shows that if participation is not universal, then the probability of participation, and the truncated probability distribution of participants, depends on the state variables. The fact that non-participants might be deterred from making a bid by the winner's curse is one component of welfare losses that our empirical analysis fails to identify.

A similar argument could be made if, as in our framework, everyone finds it optimal to place an order, but in contrast to our setup, traders have the opportunity to withdraw limit orders and reposition their orders. Lacking data on the identity of traders we are unable to estimate the effect of repositioning limit orders in response to changes in the aggregate shocks. Incorporating this feature would affect the way the gains from trade are computed. In this case, mistakenly attributing repositioned orders to nonexecution inflates the estimate of welfare losses.

Incomplete information is a third factor neglected in our analysis that might affect the sensitivity of our results, including the estimates of the welfare bounds. Here intuition might suggest that our welfare estimates underestimate losses for two reasons. First, the private valuations of more informed traders are partly due to advance notice about future common shocks rather than liquidity concerns arising from their own savings and consumption plans, so should not be fully included the social surplus from trade. Second, less informed traders, fearing exploitation by more informed traders through this additional source of a winner's curse, would reduce their participation in the market. Sandås (1999) is a first attempt to estimate a structural model of adverse selection and private information from data on limit order markets.

But having acknowledged the potential sensitivity of our results to the simplifying assumptions about the information structure and the nature of market participation in our model, and also indicated directions for future research, we nevertheless believe our work provides a useful benchmark.

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A Proofs

Proof of Proposition 1

Assume $d_{it}^{o} = -1$ and $u < u_t$. Then

$$\begin{aligned}
\psi_{it}^{s}(p_i - y_t - u) + \xi_{it}^{s} &> \psi_{it}^{s}(p_i - y_t - u_t) + \xi_{t}^{s} \\
&\geq \psi_{jt}^{b}(y_t + u_t - p_j) + \xi_{jt}^{b}\ \forall p_j \leq a_t \\
&> \psi_{it}^{b}(y_t + u - p_j) + \xi_{jt}^{b},\ \forall p_j \leq a_t,
\end{aligned}$$

where the first line follows because $u < u_t$, the second line follows from the optimality of choosing p_i for the agent with valuation u_t and the third line follows because $u < u_t$. Therefore it is not optimal for an agent with private valuation component $u < u_t$ to set $d_{jt}^{o} = 1$ for any j. From these inequalities we deduce that if the agent has a valuation $u < u_t$ then the optimal strategy is to set $d_{jt} = -1$ for some j. Given $d_{it} = -1$ is optimal for u_t and $d_{jt} = -1$ is optimal for u :

$$\psi_{it}^{s}(p_i - y_t - u_t) + \xi_{kit}^{s} \geq \psi_{jt}^{s}(p_j - y_t - u_t) + \xi_{jt}^{s},$$

$$\psi_{jt}^{s}(p_j - y_t - u) + \xi_{jt}^{s} \geq \psi_{it}^{s}(p_i - y_t - u) + \xi_{it}^{s}.$$

Multiplying the second inequality by -1, adding and rearranging yields:

$$\left(\psi_{it}^{s} - \psi_{jt}^{s}\right)(u - u_t) \geq 0$$

The proof for the buy side is symmetric. ■

Proof of Proposition 2

Because the maximand on the right side of Equation (7) is decreasing in u_t, the envelope theorem implies $V_t^s(u)$ is too. By monotonicity result in Proposition 1, $V_t^s(u)$ is a convex decreasing function in u_t, formed from linear segments. All valuations u_t lying on the same linear segment lead to the same optimal price tick choice, and the slope of $V_t^s(u)$ at those points gives the probability of execution. By symmetry, $V_t^b(u)$ is a piecewise convex increasing function in u_t, and an analogous interpretation applies. Since the absolute value of the gradients of both value functions are bounded by one, they must intersect once, at the value where a trader would be indifferent between buying and selling. Denote this value by U_t. Suppose that

$$\psi_{it}^{b}(U_t - p_i) + \xi_{it}^{b} \geq 0.$$

Then the monotonicity result implies $V_t(u) \geq 0$ with strict inequality holding for all $y_t + u \neq U_t$. Alternatively:

$$\psi_{it}^b (U_t - p_i) + \xi_{it}^b < 0.$$

Then there exists an open interval $(\underline{u}_t, \overline{u}_t)$ on the support of $G(u)$ containing $U_t - y_t$, such that if $u \in (\underline{u}_t, \overline{u}_t)$, then $V_t(u) < 0$. ■

Proof of Proposition 3

In the first part of the proposition, respectively define p_{1t}^b and p_{1t}^s as the prices which solve Equations (8) and (7) for U_t. In the second part, define p_{1t}^b as the price which solves Equation (8) for $\overline{u}_t$, and p_{1t}^s as the price which solves Equation (7) for $\underline{u}_t$. The remaining parts of the proposition are direct implications of Propositions 1 and 2. ■

Proof of Proposition 4

We treat the buy and sell cases separately. First, suppose $\xi_{it}^b \leq \xi_{jt}^b$ and $\psi_{it}^b \leq \psi_{jt}^b$ but $p_i > p_j$. If $p_j \in P_t^b$ then the optimality conditions of Proposition 2 imply $v_t \geq p_j \geq 0$. Consequently $p_i \notin P_t^b$ because the inequality

$$\psi_{it}^b (v_t - p_i) + \xi_{it}^b < \psi_{jt}^b (v_t - p_j) + \xi_{jt}^b$$

demonstrates that choosing $d_{jt} = 1$ (iteratively) dominates choosing $d_{it} = 1$.

Similarly suppose $\xi_{it}^s \leq \xi_{jt}^s$ and $\psi_{it}^s \leq \psi_{jt}^s$ but $p_i < p_j$. If $p_j \in P_t^s$ then by Proposition 2 $v_t < p_j$ which implies from our supposition that $\psi_{jt}^s (p_j - v_t) \geq \psi_{it}^s (p_i - v_t)$. Hence

$$\psi_{it}^s (p_i - v_t) + \xi_{it}^s < \psi_{jt}^s (p_j - v_t) + \xi_{jt}^s$$

Consequently choosing $d_{jt} = -1$ (iteratively) dominates choosing $d_{it} = -1$ and therefore $p_i \notin P_t^s$. ■

Proof of Proposition 5

Let $[\underline{y}, \overline{y}]$ denote the support of y_t. For any u_t pick $p_i > \overline{y} + u_t + 1$. Noting the value of participating in the market exceeds the expected revenue obtained from setting $d_{it} = -1$, we obtain:

$$\begin{aligned} V_t(u) &\geq E\left[\sum_{\tau=0}^{\infty} c_{it}^{\tau}\left(y_{t+\tau}+u_t-p_i\right)\middle| \mathcal{A}_t, d_{it}=-1\right] \\ &\geq E\left[\sum_{\tau=0}^{\infty} c_{it}^{\tau}\left(y_{t+\tau}-\overline{y}-1\right)\middle| \mathcal{A}_t, d_{it}=-1\right] \\ &\geq -E\left[\sum_{\tau=0}^{\infty} c_{1,t}^{\tau}\middle| \mathcal{A}_t, d_{it}=-1\right] \\ &= \psi_{it}^{s} \end{aligned}$$

Since $\psi_{it}^{s} \geq 0$, the result immediately follows. ■

Proof of Proposition 6

Let r_{it} denote the rank of a limit order at price p_i and let n_{it} denote the length of the p_i limit order queue at the beginning of the period. Denote by $|H|$ the dimension of H, and $|Z|$ the dimension of Z. Let Ψ be the set of continuous functions defined from $\{b,s\} \times \mathcal{R}^{|H|} \times \mathcal{R}^{|Z|} \times \mathcal{R}^{2}$ to $[0,1]$ satisfying the condition that

$$\psi_0^b\left(h_t, z_t, p_i, r_{it}\right) = 1$$

for all $p_i \geq a_t$ and

$$\psi_0^s\left(h_t, z_t, p_i, r_{it}\right) = 1$$

for all $p_i \leq b_t$. We interpret $\psi_0^s\left(h_t, z_t, p_i, r_{it}\right)$ as the subjective probability of executing a sell order at price p_i when the book is in state h_t, the factors driving the common shock is z_t, and the limit order is ranking r_i in the order queue at p_i. An analogous interpretation applies to $\psi_0^b\left(h_t, z_t, p_i, r_{it}\right)$. Similarly let Ξ be the space of continuous functions defined from $\{b,s\} \times \mathcal{R}^{|H|} \times \mathcal{R}^{|Z|} \times \mathcal{R}^{2}$ to $\left[\underline{y}-\overline{y}, \overline{y}-\underline{y}\right]$. Note that $H \subseteq \mathcal{R}^{|H|}$, $Z \subseteq \mathcal{R}^{|Z|}$, $P \subseteq \mathcal{R}$ and that the integers are also a subset of the real line. Finally, define for all $(k, h_t, z_t) \in \{b,s\} \times H \times Z$, the mappings

$$\psi_i^k\left(h_t, z_t\right) \equiv \psi^k\left(h_t, z_t, p_i, n_{it}+1\right)$$

and

$$\xi_i^k\left(h_t, z_t\right) \equiv \xi^k\left(h_t, z_t, p_i, n_{it}+1\right).$$

To prove the proposition, we establish that the decision rules computed using these subjective beliefs satisfy the requirements of a Markov perfect equilibrium.

Suppose that every agent t uses the same $(f,g) \in \Psi \times \Xi$ to compute the indifference valuations for determining their choices. Then the indifference valuation between buying at p_i versus p_j can be expressed as

$$\theta_t^b(p_i,p_j;f,g) \equiv p_i - \left(\frac{f^b(h_t,z_t,p_j,n_{jt}+1)(p_i-p_j)+g^b(h_t,z_t,p_i,n_{it}+1)-g^b(h_t,z_t,p_j,n_{jt}+1)}{f^b(h_t,z_t,p_i,n_{it}+1)-f^b(h_t,z_t,p_j,n_{jt}+1)}\right).$$

We denote the set of indifference valuations for buying by $\Theta_t^b(f,g)$, where the state variables (h_t,z_t) are suppressed with the use of a t subscript, and define $\Theta_t^s(f,g)$ as the corresponding set for sellers. Having computed the indifference valuations for all p_i and p_j in P, the iterative strategy described in the text is used to eliminate dominated prices to determine the optimal choices. We denote the set of undominated prices by $P_t^b(f,g)$ and $P_t^s(f,g)$ respectively.

The conditional choice probabilities characterize the population choices up to the private component of the valuation. Consider two adjacent elements in the set of undominated buy prices. That is, let $p_i \in P_t^b(f,g)$, $p_j \in P_t^b(f,g)$, where $\psi_i^b > \psi_j^b$. Then the conditional choice probability of placing a limit order to buy at price p_i is defined as:

$$\begin{aligned}\pi_i^b(h_t,z_t\,|\,f,g) &\equiv \Pr(d_{it}=1\,|\Theta_t(f,g)) \\ &= G\left(\theta_{jt}^b(f,g)\right)-G\left(\theta_{it}^b(f,g)\right).\end{aligned}$$

Notice that for all $p_k \notin P_t^b(f,g)$

$$\pi_k^b(h_t,z_t\,|\,f,g) \equiv \Pr(d_{kt}=1\,|\Theta_t(f,g)) = 0.$$

The set of conditional choice probabilities is denoted by

$$\Pi_t^b(h_t,z_t\,|\,f,g) \equiv \left\{\pi_i^b(h_t,z_t\,|\,f,g)\right\}_{i=1}^{|P|}$$

where

$$\pi_k^b(h_t,z_t\,|\,f,g)=0$$

for all $p_k > a_t$, while the set of conditional choice probabilities to sell are analogously defined by

$$\Pi_t^s(h_t,z_t\,|\,f,g) \equiv \{\pi_i^s(h_t,z_t\,|\,f,g)\}_{i=1}^{|P|}.$$

We claim $\Pi_t^b(h_t,z_t\,|\,f,g)$ and $\Pi_t^s(h_t,z_t\,|\,f,g)$ are continuous in f and g, in the sup norm. First, we define $\overline{\Theta}_t^b(f,g)$ and $\overline{\Theta}_t^s(f,g)$ by bounding the indifference valuations to lie inside the closed interval $\left[\underline{u}+\underline{y}-1,\overline{u}+\overline{y}+1\right]$.

Thus

$$\overline{\theta}_t^b\left(p_i, p_j; f, g\right) \equiv \max\left\{\theta_t^b\left(p_i, p_j; f, g\right), \underline{u} + \underline{x} - 1\right\}$$

and

$$\overline{\theta}_t^s\left(p_i, p_j; f, g\right) \equiv \min\left\{\theta_t^s\left(p_i, p_j; f, g\right), \overline{u} + \overline{x} + 1\right\}.$$

Since $u_t \in [u_l, u_h]$ and $y_t \in \left[\underline{y}, \overline{y}\right]$, basing the iterative strategy to eliminate dominated prices on the modified indifference sets $\overline{\Theta}_t^b\left(f, g\right)$ and $\overline{\Theta}_t^s\left(f, g\right)$, rather than $\Theta_t^b\left(f, g\right)$ and $\Theta_t^s\left(f, g\right)$, has no behavioral consequences. By inspection, $\overline{\Theta}_t^b\left(f, g\right)$ and $\overline{\Theta}_t^s\left(f, g\right)$ are continuous in f and g. Second, the algorithm for determining the optimal choices from the set of indifference values is made by comparing a finite number of inequalities, an operation that preserves continuity. Therefore the boundaries defining the sets which delineate the choices are continuous in f and g. Third, the vectors of choice probabilities $\Pi_t^b\left(h_t, z_t \,|\, f, g\right)$ and $\Pi_t^s\left(h_t, z_t \,|\, f, g\right)$ are composite functions, formed from applying differences in G to values taken the boundaries of the sets described above. Our assumption that G is differentiable, implies G is continuous. Finally, noting composition preserves continuity, our claim that $\Pi_t^b\left(h_t, z_t \,|\, f, g\right)$ and $\Pi_t^s\left(h_t, z_t \,|\, f, g\right)$ are continuous in f and g is justified.

When every agent uses the same $(f, g) \in \Psi \times \Xi$, the executions generated by a stationary decision rule of this form converge in distribution to a stationary probability distribution at a geometric rate. The proof of this result is based on a recursion involving the transition of the execution probabilities. At its heart is the probability that a limit order at price p_i which has rank r_{it} on period t will be executed in period $t+1$ when agents use (f, g) to form their indifference valuations.

For limit orders to sell, define the mapping

$$\zeta_{f,g}^s\left(h_t, z_t, p_i, r_{it}\right) = \left(1 - \lambda\left(h, z\right)\right)\lambda\left(h, z\right)^{r_i - 1 + \Sigma_{j=a}^{i-1}\left|h_j\right|} E\left[\pi_i^s\left(h', z' \,|\, f, g\right) |h, z\right]. \quad \text{(A1)}$$

The first term on the right side, $1 - \lambda\left(h, z\right)$, is the probability of surviving another period, the second term

$$\lambda\left(h, z\right)^{r_i - 1 + \Sigma_{j=a}^{i-1}\left|h_j\right|}$$

is the probability that all the orders having higher priority are all canceled next period, while

$$E\left[\pi_i^s\left(h', z' \,|\, f, g\right) |h, z\right]$$

is the probability that a market order for price p_i will cancel the leading limit order expected over next period's state variables. An analogous expression defines $\zeta_{f,g}^b\left(h_t, z_t, p_i, r_{it}\right)$ for limit buy orders. Clearly

$\zeta^k_{f,g}(h_t, z_t, p_i, r_{it})$ is continuous in f and g, because the vectors $\pi^s(h,z|f,g)$ and $\pi^b(h,z|f,g)$ are continuous in f and g, and the expectations operator preserves continuity.

Given $(f,g) \in \Psi \times \Xi$ as the basis for determining the indifference valuations to implement the decision rule, define the operator $\Gamma_1(f,g) : \Psi \to \Psi$ for each $\psi_0 \in \Psi$ as

$$\begin{aligned}\Gamma_1(f,g)\left[\psi^k_0(h,z,p_i,r_i)\right] &= (1-\lambda(h,z))\,\zeta^k_{f,g}(h,z,p_i,r_i) \\ &+ (1-\lambda(h,z))\left[1-\zeta^k_{f,g}(h,z,p_i,r_i)\right]E_{f,g}\left[\psi^k_0\left(h',z',p_i,r_i'\right)|h,z,p_i,r_i\right].\end{aligned} \tag{A2}$$

The first expression on the right side of Definition (A2) is the probability of executing next period, while the second expression is the probability of execution at least two periods forward. We now show $\Gamma_1(f,g)\left[\psi^k_0(h,z,p_i,r_i)\right]$ preserves monotonicity, has the contraction property, and is continuous in (f,g).

To prove continuity, observe $\pi(h,z|f,g)$ is continuous in f and g, which implies that

$$E_{f,g}\left[\psi^k_0\left(h',z',p_i,r_i'\right)|h,z,p_i,r_i\right] = E\left[\sum_{i=1}^{|P|}\pi\left(h',z'|f,g\right)\psi^k_0\left(h'+1,z',p_i,r_i'\right)\middle| h,z,p_i,r_i\right]$$

is continuous too, because the expectations and addition operators preserve continuity. The continuity of $\Gamma_1(f,g)$ in f and g now follows from the continuity of $\zeta^k_{f,g}(h,z,p_i,r_i)$.

Indeed the N stage operator $\Gamma^N_1(f,g)$ is also continuous for all finite N. To prove this property which we use below, suppose $\Gamma^{N-1}_1(f,g)$ is continuous in f and g. Using the definition of $\Gamma^N_1(f,g)$ as the composite of $\Gamma^{N-1}_1(f,g)$ and $\Gamma_1(f,g)$, the triangle inequality implies that

$$\left\|\Gamma^N_1(f+\Delta f, g+\Delta g)\left[\psi^k_0(h,z,p_i,r_i)\right] - \Gamma^N_1(f,g)\left[\psi^k_0(h,z,p_i,r_i)\right]\right\|$$

is less than the sum of

$$\left\|\begin{array}{c}\Gamma_1(f+\Delta f, g+\Delta g)\left\{\Gamma^{N-1}_1(f+\Delta f, g+\Delta g)\left[\psi^k_0(h,z,p_i,r_i)\right]\right\} \\ -\Gamma_1(f+\Delta f, g+\Delta g)\left\{\Gamma^{N-1}_1(f,g)\left[\psi^k_0(h,z,p_i,r_i)\right]\right\}\end{array}\right\|, \tag{A3}$$

and

$$\left\|\begin{array}{c}\Gamma_1(f+\Delta f, g+\Delta g)\left\{\Gamma^{N-1}_1(f,g)\left[\psi^k_0(h,z,p_i,r_i)\right]\right\} \\ -\Gamma_1(f,g)\left\{\Gamma^{N-1}_1(f,g)\left[\psi^k_0(h,z,p_i,r_i)\right]\right\}\end{array}\right\|. \tag{A4}$$

Given the premise that $\Gamma^{N-1}_1(f,g)$ is continuous, Expression (A3) is bounded above by any $\varepsilon/2$ for sufficiently small $(\Delta f, \Delta g)$ because $\Gamma_1(f,g)$ maps Ψ into itself, and the composition operator preserves continuity. Furthermore Expression (A4) is similarly bounded because $\Gamma_1(f,g)$ is continuous in f and g, as established above. Appealing to the induction principle establishes the claim at the beginning of the paragraph.

The monotonicity of $\Gamma_1(f,g)$ holds because if

$$\psi_1^k(h,z,p_i,r_i) \geq \psi_0^k(h,z,p_i,r_i),$$

then

$$\begin{aligned}
&\Gamma_1(f,g)\left[\psi_1^k(h,z,p_i,r_i)\right] - \Gamma_1(f,g)\left[\psi_0^k(h,z,p_i,r_i)\right] \\
&\quad = (1-\lambda(h,z))\left[1-\zeta_{f,g}^k(h,z,p_i,r_i)\right] \times \\
&\qquad E_{f,g}\left[\psi_1^k\left(h',z',p_i,r_i'\right) - \psi_0^k\left(h',z',p_i,r_i'\right)|h,z,p_i,r_i\right] \\
&\quad \geq 0.
\end{aligned}$$

Third, $\Gamma_1(f,g)$ satisfies the contraction property, because for all positive constants K:

$$\begin{aligned}
\Gamma_1(f,g)\left[\psi^k 0(h,z,p_i,r_i)+K\right] &= \Gamma_1(f,g)\left[\psi_0^k(h,z,p_i,r_i)\right] + K(1-\lambda(h,z))\left[1-\zeta_{f,g}^k(h,z,p_i,r_i)\right] \\
&\leq \Gamma_1(f,g)\left[\psi_0^k(h,z,p_i,r_i)\right] + (1-\underline{\lambda})K
\end{aligned}$$

and $\underline{\lambda} \in (0,1)$. Consequently $\Gamma_1(f,g)$ satisfies Blackwell's conditions, and is therefore a contraction. We denote the unique fixed point of the contraction by $\psi_{f,g}^k(h,z,p_i,r_i)$.

Following a similar proof strategy to above, we develop a recursive representation for the expected change in the common value conditional on execution for all existing limit orders in the book. Given a decision rule based on (f,g), define the operator $\Gamma_2(f,g) : \Xi \to \Xi$ for any $\xi_0 \in \Xi$ as:

$$\begin{aligned}
(1-\lambda(h,z))^{-1}\Gamma_2(f,g)\left[\xi_0^k(h,z,p_i,r_i)\right] &= \zeta_{f,g}^k(h,z,p_i,r_i)\left[E_{f,g}\Delta y_t\,|c_{t+1}=1\right] \\
&\quad + \left[\psi_{f,g}^k(h,z,p_i,r_i) - \zeta_{f,g}^k(h,z,p_i,r_i)\right] \\
&\quad \times E_{f,g}\left[\Delta_1 y_t\,|h,z,p_i,r_{i,}c_{t+1}=0\right] \\
&\quad + \left(1-\zeta_{f,g}^k(h,z,p_i,r_i)\right)E_{f,g}\left[\xi_0^k\left(h',z',p_i,r_i'\right)|h,z,p_i,r_i\right].
\end{aligned} \tag{A5}$$

Up to the factor of proportionality $(1-\lambda(h,z))$, the first expression on the right side of Equation (A5) is the contribution to the winner's curse from executing next period, the second expression arises from the one period change in y_t that affects any gain from trade if the limit order is executed after the next period, and the third term is the expected winner's curse term as computed with next period's state variables.

As in the case of $\Gamma_1(f,g)$, one can show that $\Gamma_2(f,g)$ is continuous in f and g, preserves monotonicity, has the contraction property, and is therefore a contraction mapping. Let $\xi_{f,g}^k(h,z,p_i,r_i)$ denote the unique fixed point for this operator when the decision rule is characterized by (f,g).

We now define the mapping $\mathcal{Y} : \Psi \times \Xi \to \Psi \times \Xi$ by

$$\mathcal{Y}\left(\begin{array}{c} f(h,z,p_i,r_i) \\ g(h,z,p_i,r_i) \end{array}\right) = \left(\begin{array}{c} \psi^k f,g(h,z,p_i,r_i) \\ \xi_{f,g}(h,z,p_i,r_i) \end{array}\right)$$

where, as above, $\psi^k f,g(h,z,p_i,r_i)$ and $\xi_{f,g}(h,z,p_i,r_i)$ are the respective unique fixed points of $\Gamma_1(f,g)$ and $\Gamma_2(f,g)$. The last step of the proof is to show that the conditions for Brouwer's fixed point theorem are satisfied, which together with remarks at the outset of the proof establish the existence of a Markov perfect equilibrium and thus complete the proof.

To show $\mathcal{Y}$ has a fixed point, we first note that by construction $\Psi \times \Xi$ is nonempty, convex and complete. Furthermore $\Psi \times \Xi$ is totally bounded, and thus we conclude that $\Psi \times \Xi$ is compact. This only leaves the continuity of $\mathcal{Y}$ in f and g to establish. By the triangle inequality

$$\begin{aligned} &\left\|\psi^k_{f+\Delta f,g+\Delta g}(h,z,p_i,r_i) - \psi_{f,g}(h,z,p_i,r_i)\right\| \\ \leq\ & \left\|\psi^k_{f+\Delta f,g+\Delta g}(h,z,p_i,r_i) - \Gamma_1^N(f+\Delta f,g+\Delta g)\left[\psi^k_{f,g}(h,z,p_i,r_i)\right]\right\| \\ &+\left\|\Gamma_1^N(f+\Delta f,g+\Delta g)\left[\psi_{f,g}(h,z,p_i,r_i)\right] - \psi^k_{f,g}(h,z,p_i,r_i)\right\|. \end{aligned}$$

The contraction mapping theorem implies that

$$\begin{aligned} &\left\|\psi^k_{f+\Delta f,g+\Delta g}(h,z,p_i,r_i) - \Gamma_1^N(f+\Delta f,g+\Delta g)\left[\psi^k_{f,g}(h,z,p_i,r_i)\right]\right\| \\ \leq\ & \underline{\lambda}^{-1}(1-\underline{\lambda})^N\left\|\Gamma_1(f+\Delta f,g+\Delta g)\left[\psi_{f,g}(h,z,p_i,r_i)\right] - \psi^k_{f,g}(h,z,p_i,r_i)\right\| \\ \leq\ & 2\underline{\lambda}^{-1}(1-\underline{\lambda})^N. \end{aligned}$$

Also because $\psi^k_{f,g}(h,z,p_i,r_i)$ is a fixed point for $\Gamma_1^N(f,g)$, the second expression may be written as

$$\left\|\Gamma_1^N(f+\Delta f,g+\Delta g)\left[\psi_{f,g}(h,z,p_i,r_i)\right] - \Gamma_1^N(f,g)\left[\psi^k_{f,g}(h,z,p_i,r_i)\right]\right\|.$$

Accordingly for any $\varepsilon > 0$, we fix N at any integer exceeding $(\ln\underline{\lambda} + \ln\varepsilon - \ln 4)/\ln(1-\underline{\lambda})$, which implies that

$$\left\|\psi^k_{f+\Delta f,g+\Delta g}(h,z,p_i,r_i) - \Gamma_1^N(f+\Delta f,g+\Delta g)\left[\psi^k_{f,g}(h,z,p_i,r_i)\right]\right\| \leq \frac{\varepsilon}{2}.$$

Noting that $\Gamma_1^N(f,g)$ is continuous in f and g, we choose $(\Delta f, \Delta g)$ sufficiently small so that

$$\left\|\Gamma_1^N(f+\Delta f,g+\Delta g)\left[\psi_{f,g}(h,z,p_i,r_i)\right] - \psi^k_{f,g}(h,z,p_i,r_i)\right\| \leq \frac{\varepsilon}{2}$$

to establish the continuity of $\psi^k_{f,g}(h,z,p_i,r_i)$ in f and g. The continuity of $\xi_{f,g}(h,z,p_i,r_i)$ is proved the same way.

Therefore the conditions for Brouwer's fixed point theorem are met, and thus a pair of mappings exist, respectively denoted $\psi^k(h,z,p_i,r_i) \in \Psi$ and $\xi^k(h,z,p_i,r_i) \in \Xi$, which solve

$$\mathcal{Y}\begin{pmatrix} \psi^k(h,z,p_i,r_i) \\ \xi^k(h,z,p_i,r_i) \end{pmatrix} = \begin{pmatrix} \psi^k(h,z,p_i,r_i) \\ \xi^k(h,z,p_i,r_i) \end{pmatrix}$$

as required. ■

Proof of Proposition 7

The proof is based on four properties of our model:

1. Let Z denote the support of z_t, U the support of u_t, and H the support of h_t.The probability of the book emptying through withdrawals is strictly positive, since for all $(h_t,z_t,u_t) \in H \times Z \times U$:

$$\ln \Pr[n_{t+1} = 0 | h_t, z_t] \geq \inf_{(h,z)\in\{H\times Z\}} n \ln[\lambda(h,z)] = O(1) \tag{A6}$$

2. Let Z_1^c denote the complement of Z_1 in Z. Condition M of Stokey and Lucas (1989, page 346) is satisfied by the exogenous stochastic process z_t if there exists $\varepsilon_1 > 0$ such that for all $Z_1 \subseteq Z$, either $\Pr[Z_1^c | z] \geq \varepsilon_1$ for all $z \in Z$, or $\Pr(Z_1 | z) \geq \varepsilon_1$ for all $z \in Z$. Because $f(z_{t+1} | z_t)$ is bounded above, one can show z_t satisfies Condition M.

3. Similarly since u_t is independent, it trivially satisfies Condition M.

4. The following decomposition follows from the independence of u_t and the exogeneity of z_t :

$$\Pr[h_{t+1}, z_{t+1}, u_{t+1} | h_t, z_t, u_t] = \Pr[h_{t+1} | h_t, z_t, u_t] \Pr[z_{t+1} | z_t] \Pr(u_{t+1})$$

We now demonstrate condition M carries over to the joint process (h_t,z_t,u_t) as well, by exploiting the four properties described above. More specifically we show there exists some $\varepsilon > 0$ such that for all $H_1 \times Z_1 \times U_1 \subseteq H \times Z \times U$, either $\Pr[(H_1,Z_1,U_1) | h,z,u] \geq 0$ for all $(h_t,z_t,u_t) \in H \times Z \times U$, or $\Pr[(H_1,Z_1,U_1)^c | h,z,u] \geq 0$ for all $(h_t,z_t,u_t) \in H \times Z \times U$.

There are four cases to consider, depending on whether $\Pr(Z_1 | z) \geq \varepsilon_1$ or $\Pr(Z_1^c | z) \geq \varepsilon_1$, and on whether $\Pr(U_1) \geq \varepsilon_2$ or $\Pr(U_1^c) \geq \varepsilon_2$. Each case contains two subcases depending on whether $\{h = 0\} \in H_1$ or not.

1. Suppose $\Pr(Z_1|z) \geq \varepsilon_1$ for all $z \in Z$ and $\Pr(U_1) \geq \varepsilon_2$. Then by the first property there exists some $\lambda > 0$ such that if $\{h = 0\} \in H_1$:

$$\begin{aligned} \Pr[H_1, Z_1, U_1 | h, z, u] &\geq \Pr[0 | h, z, u] \Pr[Z_1 | z] \Pr(U_1) \\ &\geq \lambda \varepsilon_1 \varepsilon_2 \end{aligned}$$

   Thus in this subcase $\Pr[(H_1, Z_1, U_1) | h, z, u] \geq 0$ for any $\varepsilon > 0$ less than $\lambda\varepsilon_1\varepsilon_2$. Alternatively, if $\{h = 0\} \notin H_1$:

$$\begin{aligned} \Pr[(H_1, Z_1, U_1)^c | h, z, u] &\geq \Pr[H_1^c, Z_1, U_1 | h, z, u] \\ &\geq \Pr[0, Z_1, U_1 | h, z, u] \\ &= \Pr[0 | h, z, u] \Pr(Z_1 | z) \Pr(U_1) \\ &\geq \lambda \varepsilon_1 \varepsilon_2 \end{aligned}$$

   Thus in this subcase $\Pr[(H_1, Z_1, U_1)^c | h, z, u] \geq 0$ for any $\varepsilon > 0$ satisfying $\varepsilon \leq \lambda\varepsilon_1\varepsilon_2$.

2. Suppose $\Pr(Z_1^c|z) \geq \varepsilon_1$ for all $z \in Z$ and $\Pr(U_1) \geq \varepsilon_2$. Substituting Z_1^c for Z_1 in the discussion above, and repeating the arguments presented there establishes the existence of an $\varepsilon > 0$ such that either $\Pr[H_1, Z_1^c, U_1 | h, z, u] \geq \varepsilon$ for all $(h, z, u) \in H$ and $H_1 \subseteq H$, or $\Pr[(H_1, Z_1^c, U_1)^c | h, z, u] \geq \varepsilon$ for all $(h, z, u) \in H$ and $H_1 \subseteq H$.

3. The third case is $\Pr[Z_1|z] \geq \varepsilon_1$ for all $z \in Z$ and $\Pr(U_1^c) \geq \varepsilon_2$.

4. The fourth case is $\Pr[Z_1^c|z] \geq \varepsilon_1$ for all $z \in Z$ and $\Pr(U_1^c) \geq \varepsilon_2$.

As in the second case, the analysis of the last two cases follow the first case, after making the appropriate substitutions. Therefore Condition M is satisfied, and consequently Theorem 11.12 on page 348 of Stokey and Lucas (1989) applies. This implies that the process is uniformly mixing with uniform mixing coefficients converging to zero at a geometric rate. ■

B Asymptotic Properties of Estimators

This appendix derives the asymptotic results we use in the discussion of our empirical results. First we provide the assumptions required to establish the asymptotic properties of our estimators and test statistics;

then we establish the propositions used in the text. The third part of this appendix provides details about the implementation of the nonparametric estimators.

B.1 Ancillary Assumptions

There are essentially two differences distinguishing the assumptions we make from those listed in Ahn (1997). First we relax his assumption that the index variables are independently and identically distributed, by assuming instead that the variables are uniform mixing. Second, we assume the intercept term on the index is known, but relax the assumption that all the index variables are drawn from a convex subset; we only require that one of the index variables has convex compact support.

A1 Uniform mixing: The data is generated by a process that satisfies a uniform mixing condition for uniform mixing coefficients that converge to zero at a geometric rate. Proposition 7 provides conditions on the primitives of the model which ensure that outcomes from the model satisfy this uniform mixing condition.

A2 Index variables:

1. At least one of the components in the $L+1$ dimensional vector of index variables, x_{jkt}, has an unconditional joint Lebesque density π. Let x denote the c dimensional subvector of x_{jkt} with this property, and assume the support of this variable is a convex subset of $\mathcal{R}^c$, with nonempty interior X. The density π is continuous in this component, so that $\pi(x) = 0$ for all $x \in \partial X$, where ∂X denotes the boundary of X. Also, π is continuously differentiable for all $x \in X$.
2. Define $F_j^s(x) \equiv E\left[\sum_{p \leq p_j^s(w_t)} |d_{it}| \middle| x\right]$, with similar definitions for the buy side and $F(x) \equiv \{F_j^k\}_{j=1,k=s,b}^{|P|}$. The functions $\{F(x), \rho(x)\}$ are continuously differentiable in all the components of $x \in \overline{X}$, where $\overline{X}$ differs from X by a set of measure 0.

A3 First Stage Lipschitz conditions:

1. Let $\upsilon(x) = \pi(x)\rho(x)$, The components of the random vector $F'(x) \equiv \frac{\partial F(x)}{\partial x}$ and the random matrix $\upsilon'(x)x \equiv \frac{\partial \upsilon(x)}{\partial x}x$ have finite unconditional second moments.
2. The following Lipschitz conditions are satisfied. For some $S(x)$

$$\left\|\upsilon'(x+e) - \upsilon'(x)\right\| < S(x)\|e\|$$

$$\left\| [\upsilon(x+e) \times F(x+e)]' - [\upsilon(x) \times F(x)]' \right\| < S(x)\|e\|$$

with $E[(2+\|x\|) \times S(x)]^2 < \infty$.

A4 Second stage bounded derivatives: The functions $\{\pi, F, \rho\}$ are all $N+2$ times continuously differentiable, with bounded $(N+2)^{nd}$ derivatives, where N is an integer satisfying $N \geq L+5$

A5 Second stage kernel: The second stage kernel $k_2 : \mathcal{R}^c \to \mathcal{R}^1$ is bounded and symmetric about zero. It is three times continuously differentiable, with bounded third derivatives, and satisfies:

1. $k_2(x) = 0$ and $k_2'(x) = 0$ for all $x \in \partial X$.
2. The kernel function is of order N:

$$\int k_2(x)dx = 1, \int k(x)(x_1^{l_1} x_2^{l_2} ... x_{L+1}^{l_{L+1}}) dx_1 dx_2 ... dx_{L+1} = 0$$

for $0 < l_1 + l_2 + ... + l_{L+1} < N$, and $\int \|x\|^N \times |k_2(x)| dx < \infty$.

A6 Second stage bandwidths: The second stage bandwidth for each index variable i is given by $constant \times T^{-\kappa_2}$, where $constant$ is a positive constant, T is the number of observations and $\kappa_2 \in (1/(2N), 1/(2[L+4]))$.

A7 First stage variables:

1. Let o denote the joint density function of w, the conditioning variables used in the first stage kernel estimates, and let O denote a compact set in the support of w. For all $w \in O, : \inf o(w) > 0$.
2. The function x, which gives the index variables as functions of the winner's curse function and conditional execution probabilities, is continuously differentiable, with bounded second derivatives. This requires that the execution probabilities must be strictly monotone functions of the price chosen.
3. Define

$$r_j^s(w,x) = \zeta(w) \left[\upsilon(x) \times F_j^{s\prime}(x) - \upsilon'(x) \times \left(\sum_{p \leq p_j^s(w)} |d_i| - F_j^s(x) \right) \right], \tag{B1}$$

and

$$\mu_{j,ik}^s(w) = \left[\frac{\partial r_j^s(x)}{\partial g_i^k(w)} \middle| w \right], \tag{B2}$$

where

$$g_i^s(w_t) = E\left[\left(\begin{array}{c}\sum_{\tau=0}^{\infty}|c_{it}^{\tau}|\\ \sum_{\tau=0}^{\infty}c_{it}^{\tau}\otimes\Delta_{\tau}z_{t+\tau}\end{array}\right)\Bigg| w_t, d_{it}=-1\right] \tag{B3}$$

is the vector of conditional expectation estimated in the first stage kernel regression conditional on choice i, with $g_s^b(w)$ defined similarly for the buy side. We make similar definitions on the buy side for $r^{jb}(w,x)$ and $\mu_{j,ik}^s(w)$. The functions $o(w), g_i^k(w)$, and $\mu_{j,ik}^s(w)$ are D times differentiable with D bounded derivatives satisfying the inequalities $D > Q/(1-2(L+4))\kappa_2$, where Q is the number of conditioning variables used in the first stage kernel estimates and κ_2 is the exponent in the second stage bandwidth, defined in assumption **A6**.

4. The variables predicted in the first stage kernel estimates, y all have bounded support.

A8 First stage kernel: The first stage kernel function, $k_1 : \mathcal{R}^Q \to \mathcal{R}^1$ is bounded and symmetric about zero. It satisfies the following conditions:

1. Lipschitz condition.

$$|k_1(w) - k_1(w')| \leq C\|w - w'\|^{\gamma},$$

where C and γ are positive constants less than ∞.

2. The kernel function is of order D:

$$\int k_1(w)dw = 1, \int k_1(w)(w_1^{l_1} w_2^{l_2} ... w_Q^{l_Q})dw_1 dw_2 ... dw_Q = 0$$

for $0 < l_1 + l_2 + ... + l_{L+1} < D$, and $\int \|m\|^D \times |k_1(w)|dw < \infty$.

A9 First stage bandwidth: The first stage bandwidth for each conditioning variable i is given by $constant \times T^{-\kappa_1}$, where $constant$ is a positive constant, T is the number of observations and $\kappa_1 \in (1/(2D), (1 - 2(L+4)\kappa_2)/2Q)$.

A10 Third stage kernel: The third stage kernel function is non negative and bounded, $\int ||u||^2 k_3(u)du < \infty$ and satisfies the Lipshitz condition

$$||k_3(u) - k_3(u')|| \leq c||u - u'||^{\gamma}$$

for some $\gamma > 0$ and $c < \infty$.

B.2 Estimating the Index Coefficients

Proposition 8 below, and its supporting Lemmas borrow heavily from Powell, Stock and Stoker (1989), Ahn (1997), Denker and Keller (1983) and Robinson (1989). Let $\mathcal{M}_a^b$ denote the sigma–algebra of events generated by $X_a, \ldots X_b$. The process $\{x_t\}$ is absolutely regular if

$$\beta(T) = E\left\{\sup_{A\in\mathcal{M}_T^{\infty}}\left|P(A|\mathcal{M}_{-\infty}^{0}) - P(A)\right|\right\} \to 0, \text{ as } T \to \infty. \tag{B4}$$

Lemma 1 provides a preliminary result about U–statistics and its projection for absolutely regular processes with sample dependent kernels. Then, we establish the asymptotic properties of $\hat{\phi}_j^k$, while Lemma 2 shows how to form a consistent estimator of the covariance matrix. Finally, Lemma 3 provides the asymptotic distribution of the statistics used in the test of monotonicity.

Lemma 1 *Suppose that $\{\xi_t\}_{t=1}^{\infty}$ is a stationary stochastic process that is absolutely regular. Assume that $\beta(T)^{\frac{2}{2+\delta}} = O\left(T^{-2+\varepsilon}\right)$ for some $\delta \geq 0$. Let $h_T(\xi_{t_1}, \xi_{t_2}, \ldots, \xi_{t_m})$ be a kernel of order m and define the U–statistic*

$$U_T \equiv \begin{pmatrix} T \\ m \end{pmatrix}^{-1} \sum_{1\leq t_1<t_2<\ldots<t_m} h_T\left(\xi_{t_1}, \xi_{t_2}, \ldots, \xi_{t_m}\right). \tag{B5}$$

Also, let

$$\hat{U}_T = \theta_T + \frac{m}{T}\sum_{t=1}^{T} h_{1T}(\xi_t), \tag{B6}$$

$$h_{1T}(\xi_t) = \int \ldots \int h_T(\xi_t, \xi_{t_2}, \xi_{t_3}, \ldots \xi_{t_m}) \prod_{i=2}^{m} dF(\xi_{t_i}), \tag{B7}$$

and

$$\theta_T = \int \ldots \int h_T(\xi_{t_1}, \xi_{t_2}, \xi_{t_3}, \ldots \xi_{t_m}) \prod_{i=1}^{m} dF(\xi_{t_i}), \tag{B8}$$

where $F(\cdot)$ is the marginal distribution of ξ_{t_i}. If

$$\sup_{1\leq t_1<\ldots<t_m\leq T}\left(E\left|h_T\left(\xi_{t_1}, \xi_{t_2}, \ldots, \xi_{t_m}\right)\right|^{2+\delta}\right)^{\frac{2}{2+\delta}} = o(T^{1-\varepsilon}), \tag{B9}$$

then

$$U_T = \theta_T + o_p(1). \tag{B10}$$

$$U_T = \hat{U}_T + o_p(T^{-\frac{1}{2}}). \tag{B11}$$

Proof: Proposition 2 of Denker and Keller (1983) implies that for a fixed kernel, $h(\xi_{t_1}, \ldots, \xi_{t_m})$ and arbitrary $\varepsilon > 0$

$$ER_T^2 \leq \Gamma_\varepsilon^2 T^{-2+\varepsilon} s_\delta^2, \tag{B12}$$

where

$$R_T \equiv U_T - \hat{U}_T,$$
$$s_\delta \equiv \sup_{1 \leq t_1 < \ldots < t_m} \left(E \left| h_T(\xi_{t_1}, \xi_{t_2}, \ldots, \xi_{t_m}) \right|^{2+\delta} | \right)^{\frac{1}{2+\delta}},$$

and Γ_ε is a constant that does not depend on the kernel. As Robinson (1989) remarks, the proof of this proposition also extends to kernels that depend on the sample size T. Therefore, Equation (B12) implies that for a kernel satisfying condition (B9),

$$ER_T^2 = 0(T^{-2+\varepsilon}) o(T^{1-\varepsilon}) = o(T^{-1}). \tag{B13}$$

The Chebyshev inequality now implies implies (B11). Equation (B10) then follows by applying the Ergodic theorem to $\hat{U}_T$. ■

Proposition 8 *Under Assumptions* **A1** *though* **A9**,

$$\sqrt{T}\left(\hat{\phi}_j^k - \phi_{j0}^k\right) = \frac{2}{\sqrt{T}} \sum_{t=1}^{T} \eta_{jt}^k + o_p(1), \tag{B14}$$

where η_t *is defined as*

$$\eta_{jt}^k \equiv \left[(r_j^k(w_t, x_{jkt}) - \phi_{k0}^j) + \sum_{l=s,b,i=0}^{|P|} \mu_{j,il}^k(w_t) \left(\frac{\varepsilon_{it}^k}{\vartheta_i(w_t)} \right) \right]$$

where $\vartheta_i(w_t)$ *is the conditional probability of choice i, conditional on information set* w_t *and*

$$\varepsilon_{it}^k = |d_{it}| \times \left[\left(\begin{array}{c} \sum_{\tau=0}^{\infty} |c_{it}^\tau| \\ \sum_{\tau=0}^{\infty} c_{it}^\tau \otimes \Delta_\tau z_{t+\tau} \end{array} \right) - g_i^k(w_t) \right]. \tag{B15}$$

is the vector of the forecast errors in the first stage conditional expectation, a $|P|+1$ *vector. Define the vector of errors across all choice equations as*

$$\eta_t' \equiv [\eta_{1t}^s, \eta_{2t}^s, \ldots \eta_{It}^b]$$

and

$$C_T \equiv 4E\left[\eta_t \eta_t'\right] + 8 \sum_{l=1}^{T} E\left[\eta_t \eta_{t-l}'\right]. \tag{B16}$$

Then,

$$C_T^{-\frac{1}{2}} \frac{1}{\sqrt{T}} \sum_{t=1}^{T} \eta_t \eta_t' \overset{d}{\rightsquigarrow} \mathcal{N}(0, I). \tag{B17}$$

Proof: By Proposition 6, **A1** implies that the outcomes of our model are uniformly strong mixing, with uniform mixing coefficients satisfying a geometric bound. Since $\beta(T) \leq \varphi(T)$, then $\beta(T)$ also satisfies a geometric bound. Thus, Lemma 1 applies to our data. The first part of the proposition then follows from applying Lemma 1 in place of Ahn's (1997) Lemma B.1 in the proof of Ahn's (1997) Lemma 2, using a similar technique to Robinson (1989), equation (7.10) to show that the kernel has the correct convergence rate, noting that by Collomb and Härdle (1986), Ahn's Lemmas C.1 and C.2 hold under our assumptions, and by applying Lemma 1 in approximating (C.24) of Ahn (1997) by its projection. Asymptotic normality follows since the process $\{\eta_t\}$ satisfies the requirements of Theorem 5.1.2, page 129 of Zhengyan and Chuanrong (1996). ■

The next Lemma demonstrates shows how we form a consistent estimator of C_T.

Lemma 2 *Assume that assumptions* **A1** *through* **A10** *hold. Let*

$$\hat{\eta}_{jt}^k \equiv \left[(\hat{r}_j^k(w_t, \hat{x}_{jkt}) - \hat{\phi}_k^j) + \sum_{l=s,b,i=0}^{|P|} \hat{\mu}_{j,il}^k(w_t) \left(\frac{\hat{\varepsilon}_{it}^k}{\hat{\vartheta}_i(w_t)} \right) \right]. \tag{B18}$$

Let

$$\hat{\eta}_t' \equiv [\hat{\eta}_{1t}^s, \hat{\eta}_{2t}^s, \ldots \hat{\eta}_{It}^b]$$

and define

$$\hat{\Gamma}(j) = \frac{1}{T-j} \sum \hat{\eta}_t \hat{\eta}_{t-j}' \tag{B19}$$

and

$$\hat{C}_T = 4\hat{\Gamma}(0) + 4 \sum_{j=1}^{LL} \left(1 - \frac{j}{LL+1}\right) \left(\Gamma\hat{(}j) + \hat{\Gamma}(j)'\right). \tag{B20}$$

Here, LL is a lag length, satisfying the condition that $LL \to \infty$ as $T \to \infty$ sufficiently slowly, $\hat{r}_t(w, \hat{x}_{jkt})$ is formed from the first and second stage kernel estimates of the quantities in Equation (B1) as in Ahn (1997), Equation (3.13), page and $\hat{\mu}_{j.ik}^s$ is are formed from third stage kernel regressions, as in Equations (3.14) and (3.15) in Ahn (1997) and $\hat{\vartheta}_i(w_t)$ are formed from a third stage kernel regression of the choices on the conditioning information. Then

$$\hat{C}_T = C_T + o_p(1). \tag{B21}$$

Proof: The follows from applying the arguments on pages 531-533 of Robinson (1989) to the estimator. ■

B.3 Testing the Monotonicity Conditions

The next lemma shows that the inequalities we aim to test are asymptotically distributed as a multivariate normal random variable.

Lemma 3

$$\sqrt{T}\left(\widehat{m}^{q}_{ijkl} - m^{q}_{ijkl}\right) \overset{d}{\rightsquigarrow} N\left(0, \Sigma\right).$$

Proof: Let $D^{q}_{ijklt} \equiv |q_t|\left(x^{'}_{jkt} - x^{'}_{ilt}\right)$ and $\hat{D}^{q}_{ijklt} \equiv |q_t|\left(\hat{x}^{'}_{jkt} - \hat{x}^{'}_{ilt}\right)$. Recalling the definition of $\hat{m}^{q}_{ijkl}$, we apply the delta method to obtain:

$$\sqrt{T}\left(\hat{m}^{q}_{ijkl} - m^{q}_{ijkl}\right) = E[D^{q}_{ijklt}]\sqrt{T}\left(\hat{\beta}_{md} - \beta_0\right) + \sqrt{T}\left(\frac{1}{T}\sum_{t=1}^{T}\hat{D}^{q}_{ijklt} - E[D^{q}_{ijklt}]\right)\beta_0 + o_p(1). \qquad \text{(B22)}$$

Following similar arguments to Ahn and Manski (1993),

$$\sqrt{T}\left(\frac{1}{T}\sum_{t=1}^{T}\hat{D}^{q}_{ijklt} - E[D^{q}_{ijklt}]\right) = \frac{1}{\sqrt{T}}\sum_{t=1}^{T}\left(D^{q}_{ijklt} - E[D^{q}_{ijklt}] + \sum_{i=0}^{J}\frac{\partial D^{q}_{ijklt}}{\partial g_i(w_t)}\frac{\varepsilon_{it}}{\vartheta_i(w_t)}\right) + o_p(1). \qquad \text{(B23)}$$

Applying Theorem 5.1.2, page 129 ofZhengyan and Chuanrong (1996) to Equation (B23) and combining Equations (B23) and (B22) we infer that $\sqrt{T}\left(\hat{m}^{q}_{ijkl} - m^{q}_{ijkl}\right)$ has an asymptotic normal distribution, since it is a combination of asymptotic normal random variables. ■

The asymptotic covariance matrix is computed in the same manner as in Lemma 2.

C Implementing the Nonparametrics

This appendix provides supplementary detail about the nonparametric estimation. As explained in the text, there are three places where nonparametric estimation is undertaken. First we estimate the conditional execution probabilities and terms that make up the conditional winner's curse risk function using first stage kernel regressions. Second, the average derivative estimators require another round of kernel regression, where the conditioning variables are formed with estimates calculated using the first stage kernel regressions. The third place nonparametric techniques are used is in estimating the asymptotic variance–covariance matrix, where the conditional choice probabilities for each choice are estimated, along with the conditional expectation of the derivative of the index variables with respect to the conditional expectations estimated in the first stage. In addition the approximation of the infinite sum with a finite one adds another dimension to the time series nature of the problem. We used standard multidimensional product Gaussian kernels for all the nonparametric estimation. Details about the bandwidth choices and trimming procedures are as follows.

First Stage

- **Bandwidths:** Assumption **A9**, the first stage bandwidth for conditioning each variable l is given by $\sigma_l T^{-\kappa_1}$, where σ_l is the sample standard deviation of the variable, T is the number of observations and $\kappa_1 \in (1/(2D), (1-2(L+4)\times\kappa_2)/2L)$, where D is the order of differentiability of the density of the conditioning variables in the first stage kernel estimates, the conditional expectation functions estimated in the first stage kernel regressions and the expected derivative of r with respect to the first stage conditional expectations, $\hat{\vartheta}_i(w_t)$ Here, κ_2 is the exponent in the second stage bandwidth, to be given below. In our application $\kappa_1 = 1/120$ satisfies the required conditions.

- **Trimming:** Let X be the vector of conditioning variables. We trim those variables in the outer 10% value of $[X-\bar{X}]\hat{\Sigma}^{-1}[X-\bar{X}]'$, where $\bar{X}$ are the sample means and $\hat{\Sigma}$ is the sample covariance matrix. We trimmed 10 percent of the observations from the sample based on the values of the conditioning variables, which reduced our sample size to 19,952.

Second Stage

- **Bandwidth:** Following assumption **A6** the second stage bandwidth for each index variable i is given by $\sigma_i T^{-\kappa_2}$, where σ_i is the sample standard deviation of the variable, T is the number of observations and $\kappa_2 \in (1/(2N), 1/(2[L+4]))$, where N is the order of differentiability of the density of the index variables, $\pi(m_{t+1})$, the expected trimming indicator, $\rho(m_{jt})$, and the conditional expected value of the decision indicator, $F(m_{jt})$. In our application $\kappa_2 = 1/9$ satisfies the required conditions.

- **Trimming:** We trimmed a further 5 percent of the observations from the trimmed sample used in the first stage, based on the lowest estimated values of the density of the index variables, leaving 18,954 observations.

Third Stage

- **Bandwidth** We used the same bandwidth as in the second stage.

- **Trimming** We used the same trimming as in the second stage, leaving us with 18,954 observations.

- **Lag Length** In implementing the estimator of the asymptotic variance covariance matrix, we set the lag length, *LL* equal to 10.

Table 1: **Price and Volume**

	Average	Std.Dev	Min	Max
Closing price (Kronor)	110.720	9.314	87.50	127.50
Daily return (percent)	0.18	3.66	-10.82	12.89
Daily volume (Millions of Kronor)	74.43	37.08	20.26	215.30
Number of orders	376	151	137	768
Number of limit orders	214	79	80	417
Number of market orders	161	75	57	374

This table reports summary statistics on price and volume for Ericsson over the period Dec 3, 1991 through March 2, 1992. There are 59 trading days in the sample.

Table 2: **Order Flow**

	Buy Orders Ticks from ask quote				Sell Orders Ticks from bid quote			
	3+	2	1	market	3+	2	1	market
Total number of orders	934	1039	4407	6422	598	879	3392	4457
Order size 1000 shares	1.50	2.98	2.53	1.84	2.38	3.72	3.65	2.69
	(0.105)	(0.138)	(0.057)	(0.045)	(0.364)	(0.153)	(0.103)	(0.066)
Unconditional execution prob.	0.196	0.363	0.696	1.000	0.187	0.332	0.673	1.000
	(0.005)	(0.007)	(0.003)		(0.006)	(0.008)	(0.004)	
Time to execution (min.)	273.6	108.7	32.4		271.1	257.9	111.2	
	(18.962)	(4.557)	(2.311)		(11.675)	(54.260)	(69.894)	

This table presents summary statistics on the order flow for Ericsson from Dec 3, 1991 through March 2, 1992. Standard errors are reported in parentheses. There are 22,128 order submissions used in the calculations.

Table 3: **Limit Order Book**

Quote	Order quantity at quote: 1000's of Shares			Ticks from midquote		
	Average	Median	Std.Dev.	Average	Median	Std.Dev
3rd Ask	16.82	10.20	17.50	2.43	2.50	0.60
2nd Ask	26.16	21.40	20.51	1.43	1.50	0.33
1st Ask	20.22	14.70	19.69	0.48	0.50	0.14
1st Bid	18.52	14.10	17.40	-0.48	-0.50	0.14
2nd Bid	24.42	19.20	21.87	-1.40	-1.50	0.32
3rd Bid	16.63	11.40	18.91	-2.35	-2.50	0.80

This table reports information on the order book in Ericsson from Dec 3, 1991 through March 2, 1992. The limit order book is computed each time an order is submitted, and there are 22,128 order submissions over the period. The mid–quote is the average of best ask and best bid prices at the time of order submission. Third best bid quote refers to the third highest price with outstanding limit buy orders.

Table 4: **Order Execution Probabilities**

Conditioning variable	Value	Execution probability: Limit Buys, Ticks from best quote 3+	2	1	Limit sells, Ticks from best quote 3+	2	1
Time of day	Low	0.214	0.403	0.743	0.229	0.383	0.709
	High	0.173	0.310	0.649	0.127	0.260	0.649
	$\chi^2(1)$	2.420	9.419	45.364	9.827	14.279	14.646
	p–value	(0.1100)	(0.0024)	(0.0000)	(0.0017)	(0.0001)	(0.0002)
Market Volatility	High	0.213	0.351	0.706	0.185	0.328	0.698
	Low	0.183	0.374	0.698	0.189	0.337	0.662
	$\chi^2(1)$	1.361	0.559	0.353	0.015	0.074	5.073
	p-value	(0.2434)	(0.4547)	(0.5524)	(0.9025)	(0.7856)	(0.0243)
Trading Volume	High	0.181	0.374	0.687	0.189	0.333	0.650
	Low	0.221	0.353	0.716	0.186	0.333	0.704
	$\chi^2(1)$	2.242	0.485	4.384	0.008	0.000	11.279
	p-value	(0.1343)	(0.4862)	(0.0363)	(0.9287)	(1.0000)	(0.0008)
Order size	Low	0.222	0.459	0.744	0.1844	0.3304	0.7500
	High	0.151	0.309	0.662	0.1899	0.3338	0.6489
	$\chi^2(1)$	7.097	23.303	34.733	0.029	0.005	35.289
	p–value	(0.0076)	(0.0000)	(0.0000)	(0.8640)	(0.9239)	(0.0000)
Ask quantity: Best quote	Low	0.206	0.383	0.639	0.2460	0.3581	0.7641
	High	0.185	0.344	0.725	0.1228	0.2929	0.5521
	$\chi^2(1)$	0.633	1.678	34.483	14.875	3.823	172.257
	p–value	(0.3999)	(0.1856)	(0.0000)	(0.0001)	(0.0462)	(0.0000)
Bid quantity: Best quote	Low	0.166	0.401	0.768	0.2117	0.3754	0.6141
	High	0.241	0.293	0.579	0.1667	0.3038	0.7129
	$\chi^2(1)$	7.915	11.861	175.104	1.976	4.613	35.518
	p–value	(0.0056)	(0.0004)	(0.0000)	(0.1598)	(0.0273)	(0.0000)
Bid–ask spread	Low	0.294	0.455	0.767	0.4364	0.3810	0.7546
	High	0.168	0.348	0.687	0.1621	0.3264	0.6690
	$\chi^2(1)$	15.972	6.031	13.541	24.687	0.952	9.328
	p–value	(0.0001)	(0.0105)	(0.0001)	(0.0000)	(0.2660)	(0.0007)

This table reports estimates of the execution probabilities conditional on different variables. We condition on whether the conditioning variables are above (High) or below (Low) their medians. The third row for each variable reports the chi-squared statistic for a test of independence, with 1 degree of freedom, with the asymptotic p–value reported in parentheses below. There are 22,128 observations.

Table 5: **Choice Probabilities**

Conditioning variable	Value	Choice probability: Buy orders, Ticks from best quote: 3+	2	1	market	Sell orders, Ticks from best quote: 3+	2	1	market	$\chi^2(7)$
Time	Low	0.048	0.053	0.200	0.268	0.032	0.047	0.154	0.198	126.93
of day	High	0.037	0.041	0.198	0.313	0.022	0.032	0.153	0.205	(.0000)
Market	Low	0.038	0.038	0.200	0.316	0.023	0.033	0.145	0.208	135.297
volatility	High	0.046	0.055	0.196	0.269	0.031	0.046	0.159	0.198	(.0000)
Trading	Low	0.052	0.052	0.191	0.308	0.026	0.036	0.142	0.192	122.165
volume	High	0.032	0.041	0.205	0.276	0.028	0.043	0.161	0.213	(.0000)
Order size	Low	0.059	0.038	0.185	0.369	0.028	0.022	0.100	0.199	1035.54
	High	0.028	0.055	0.211	0.226	0.026	0.054	0.197	0.203	(.0000)
Ask	Low	0.045	0.046	0.132	0.414	0.028	0.049	0.183	0.101	2925.56
quantity	High	0.039	0.048	0.265	0.167	0.026	0.030	0.123	0.300	(.0000)
Bid	Low	0.051	0.061	0.249	0.162	0.025	0.033	0.107	0.314	3375.35
quantity	High	0.034	0.033	0.150	0.418	0.029	0.047	0.200	0.090	(.0000)
Bid-ask	Low	0.059	0.038	0.156	0.366	0.016	0.021	0.111	0.234	271.93
spread	High	0.039	0.049	0.207	0.277	0.029	0.043	0.161	0.196	(.0000)

This table reports the empirical choice probabilities conditional on different conditioning variables. We condition on whether the conditioning variables are above (High) or below (Low) their medians. The last column reports the chi-squared statistic for a test of independence. The test has 7 degrees of freedom and the asymptotic p-value is report below the statistic. There are 22,128 observations.

Table 6: **Index Model Estimates**

Choice Equation	Coefficient	Estimate	Standard Error	t statistic
		Single Equation Estimates		
Limit sell +1 tick	scale × market factor	2.362×10^{-06}	6.065×10^{-07}	3.894
	scale	6.401×10^{-06}	1.596×10^{-06}	4.010
Limit sell +2 ticks	scale × market factor	2.841×10^{-07}	2.730×10^{-06}	0.104
	scale	1.678×10^{-06}	1.599×10^{-06}	1.049
Limit buy -1 tick	scale × market factor	1.383×10^{-06}	3.813×10^{-06}	0.363
	scale	3.316×10^{-06}	3.148×10^{-05}	0.105
Market buy	scale × market factor	3.007×10^{-06}	7.971×10^{-07}	3.773
	scale	7.481×10^{-06}	5.524×10^{-06}	1.354
		Minimum Distance Estimates		
Limit sell +1 tick	scale	6.21×10^{-06}	1.31973×10^{-07}	4.781
Limit sell + 2 ticks	scale	1.58×10^{-06}	1.35158×10^{-07}	1.171
Limit buy -1 tick	scale	3.29×10^{-06}	8.82×10^{-06}	0.373
Market buy	scale	7.83×10^{-06}	2.032×10^{-07}	3.852
	Market factor	0.378	0.097	3.894
Chi squared statistic	0.0213 (3 df)	p–value	(1.0000)	

This table reports the results of estimating the theoretical model using the market index as the factor. The first panel report the results of estimating the model for separate choices, and the second panel reports the minimum distance estimates. The final column provides the asymptotic t-statistics for each coefficient. The chi-squared statistic refers to the minimized value of the minimum distance criterion function, and the asymptotic p–value is reported in parenthesis.

Table 7: **Monotonicity Tests**

Instrument	Threshold Differences			
	1 tick sell - 2 tick sell	2 tick sell - 2 tick buy	2 tick buy - 1 tick buy	All choices
Constant	2.4980	-0.6024	0.8061	0.0001
	(0.9996)	(0.3943)	(0.7457)	(1.0000)
$\|$order price - midquote$\|$	1.8919	-0.6658	1.0122	0.0003
	(0.9925)	(0.3464)	(0.8477)	(1.0000)
Order quantity	0.9718	-0.7680	0.6471	0.0001
	(0.8307)	(0.2775)	(0.6399)	(1.0000)
Fast trading ($<$10 sec)	1.2077	-0.5913	0.9022	0.0003
	(0.9123)	(0.4030)	(0.7980)	(1.0000)
Slow trading ($>$240 sec)	0.7546	-0.4111	0.4881	0.0001
	(0.7141)	(0.5610)	(0.5100)	(1.0000)
Tick $<$ 1 Kronor	0.5817	-0.6484	0.9774	0.0001
	(0.5893)	(0.3591)	(0.8331)	(1.0000)
Absolute price change	1.7248	-0.4017	0.6038	0.0006
last 30 minutes	(0.9853)	(0.5700)	(0.6069)	(1.0000)
Trading volume	3.1947	0.8378	1.1227	0.0013
last 30 minutes	(1.0000)	(0.2361)	(0.8877)	(1.0000)
Cumulative orders	1.6060	-0.5692	0.8402	0.0008
sell side	(0.9769)	(0.4208)	(0.7652)	(1.0000)
Cumulative orders	3.3665	-0.7193	0.7947	0.0013
buy side	(1.0000)	(0.3090)	(0.7389)	(1.0000)
Depth	2.7311	-0.4768	0.5713	0.001
sell side	(0.9999)	(0.5001)	(0.5809)	(1.0000)
Depth	0.9923	-0.5350	1.0390	0.0004
buy side	(0.8395)	(0.4493)	(0.8583)	(1.0000)
All (M value)	0.0021	0.0025	0.0014	0.1245
p–value	(1.0000)	(1.0000)	(1.0000)	(1.0000)

This table reports the monotonicity tests our our model. We calculate the average of the absolute value of the instrument times differences in the thresholds for different choices. We report individual one–sided t–statistics for each choice, and the joint monotonicity test for each instrument. The asymptotic p–values are given below the statistic. For the joint test, the p–values are computed using the simulation method given in Wolak (1989).

Table 8: **Functional Form Tests**

Equation	Coefficient	Estimate	t statistic
Limit sell +1 tick	Price	3.803×10^{-08}	4.014
	Market index	6.406×10^{-09}	2.115
	Shading	2.620×10^{-07}	4.194
	Winner's curse	6.634×10^{-07}	5.036
	Price=shading $\chi^2(1)$	13.623	(0.0002)
	Common value=winner's curse $\chi^2(1)$	117.300	(0.0000)
Limit sell +2 ticks	Price	3.360×10^{-09}	3.595
	Market	7.505×10^{-10}	2.442
	Shading	-2.041×10^{-10}	-0.004
	Winner's curse	-1.200×10^{-9}	-0.014
	Price=shading $\chi^2(1)$	0.001	(0.9748)
	Common value=winner's curse $\chi^2(1)$	0.0001	(0.9920)
Limit buy -1 tick	Price	3.900×10^{-08}	4.031
	Market Index	1.307×10^{-08}	4.165
	Shading	-5.420×10^{-08}	-1.042
	Winner's curse	-4.503×10^{-08}	-0.537
	Price=shading $\chi^2(1)$	3.101	(0.0782)
	Common value=winner's curse $\chi^2(1)$	4.806	(0.0284)
Market buy	Price	1.922×10^{-08}	1.084
	Market	1.974×10^{-08}	3.047
	Shading	4.753×10^{-07}	6.000
	Winner's curse	2.970×10^{-06}	8.491
	Price=shading $\chi^2(1)$	32.99	(0.0000)
	Common value=winner's curse $\chi^2(1)$	72.160	(0.0000)
All equations	Price=shading $\chi^2(4)$	82.983	(0.0000)
	Common value=winner's curse $\chi^2(1)$	94.717	(0.0000)
	Shading unimportant $\chi^2(1)$	94.687	(0.0000)
	Winner's curse unimportant $\chi^2(1)$	94.634	(0.0000)

This table reports estimates from applying an average derivative estimator to the model in Equation (39). This model nests our theoretical model. We report asymptotic chi–squared statistics for the null hypothesis that the restrictions of the theoretical model are true. Asymptotic p–values are reported in parentheses.

Table 9: **Tests for extra factors**

Choice equation	Variable	Estimate	t statistic	Estimate	t statistic
		Panel A			
		Time-of-the-day		Time-between-orders	
Limit sell +1 tick	Extra factor	3.408×10^{-14}	4.470	-1.636×10^{-09}	-3.843
	Market index	1.316×10^{-10}	6.350	6.148×10^{-09}	6.357
	Scale	4.038×10^{-10}	6.987	2.09×10^{-08}	7.112
	trade-off = 0 $\chi^2(2)$	59.122	(0.0000)	58.722 (0.0000)	
Limit Sell +2 ticks	Extra factor	5.199×10^{-14}	6.427	-1.214×10^{-09}	-3.135
	Market index	-2.064×10^{-12}	-0.215	-1.143×10^{-09}	-2.488
	Scale	7.712×10^{-11}	2.566	3.368×10^{-09}	2.320
	trade-off = 0 $\chi^2(2)$	6.999	(0.0302)	2.369	(0.3059)
Limit Buy -1 tick	Extra factor	8.000×10^{-14}	6.825	-1.407×10^{-09}	-2.584
	Market index	6.008×10^{-11}	2.891	1.221×10^{-09}	1.258
	Scale	1.757×10^{-10}	3.119	5.64×10^{-09}	2.045
	trade-off = 0 $\chi^2(2)$	15.714	(0.0004)	5.371	(0.0.0682)
Market buy	Extra factor	4.702×10^{-14}	5.250	3.365×10^{-10}	0.781
	Market index	1.468×10^{-10}	5.910	6.98×10^{-09}	6.128
	Scale	3.736×10^{-10}	6.487	2.065×10^{-08}	7.057
	trade-off = 0 $\chi^2(2)$	51.668	(0.0000)	56.066	(0.0000)
All choices	trade-off = 0 $\chi^2(8)$	32.002	(0.0001)	25.726	(0.0012)

		Panel B			
		Bid–quantity		Ask–quantity	
Limit sell +1 tick	Extra factor	-9.57×10^{-06}	-9.021	1.022×10^{-05}	8.427
	Market index	7.448×10^{-08}	3.900	1.181×10^{-07}	4.636
	Scale	4.276×10^{-07}	6.728	2.860×10^{-07}	4.213
	trade-off = 0 $\chi^2(2)$	48.629	(0.0000)	27.024	(0.0000)
Limit Sell +2 ticks	Extra factor	-4.762×10^{-06}	-7.453	3.995×10^{-06}	6.028
	Market index	-8.638×10^{-09}	-0.677	-1.798×10^{-08}	-1.310
	Scale	1.843×10^{-07}	4.150	1.454×10^{-07}	3.169
	trade-off = 0 $\chi^2(2)$	15.635	(0.0004)	7.781	(0.0204)
Limit buy -1 tick	Extra factor	-8.294×10^{-06}	-8.037	3.426×10^{-06}	4.300
	Market index	4.554×10^{-08}	1.635	9.563×10^{-08}	3.328
	Scale	2.743×10^{-07}	3.384	2.844×10^{-07}	3.263
	trade-off $\chi^2(2) = 0$	13.415	(0.0012)	16.088	(0.0003)
Market buy	Extra factor	-1.400×10^{-05}	-9.096	9.617×10^{-06}	8.436
	Market index	1.584×10^{-08}	0.656	1.186×10^{-07}	3.946
	Scale	3.100×10^{-07}	4.700	3.417×10^{-07}	4.875
	trade-off = 0 $\chi^2(2)$	20.452	(0.0000)	28.410	(0.0000)
All choices	trade-off = 0 $\chi^2(8)$	25.380	(0.0013)	19.643	(0.0118)

This table reports estimates of the index model augmented by additional factors, Equation (40) in the text. The row marked $\chi^2(2)$: trade-off = 0 provides an asymptotic chi–squared test of the null hypothesis that the coefficients on the variables in the theoretical model are all equal to zero. Asymptotic p–values are given in parentheses.

Table 9: **Tests for extra factors** (continued)

Choice equation	Variable	Estimate	t statistic	Estimate	t-statistic
		Panel C			
		Bid-ask spread		mid-quote	
Limit sell +1 tick	Extra factor	2.09×10^{-01}	1.426	-2.703×10^{-08}	-2.615
	Market index	2.328×10^{-04}	6.162	4.046×10^{-08}	6.092
	Scale	8.761×10^{-04}	7.556	1.567×10^{-07}	8.449
	trade-off = 0 $\chi^2(2)$	63.163	(0.0000)	71.378	(0.0000)
Limit sell +2 ticks	Extra factor	-8.671×10^{-01}	-5.021	2.659×10^{-08}	2.682
	Market index	-1.935×10^{-05}	-1.182	3.767×10^{-09}	1.282
	Scale	1.932×10^{-04}	3.828	1.266×10^{-08}	2.428
	trade-off = 0 $\chi^2(2)$	11.595	(0.0030)	6.818	(0.0331)
Limit buy -1 tick	Extra factor	-7.719×10^{-01}	-4.047	3.962×10^{-08}	2.847
	Market index	4.631×10^{-05}	1.306	2.948×10^{-08}	4.126
	Scale	2.132×10^{-04}	2.226	3.175×10^{-08}	2.468
	trade-off = 0 $\chi^2(2)$	6.210	(0.0488)	14.975	0.0006)
Market buy	Extra factor	-2.877×10^{-01}	-1.536	-8.646×10^{-09}	-0.722
	Market index	1.940×10^{-01}	5.017	4.525×10^{-08}	5.845
	Scale	6.494×10^{-04}	6.562	1.349×10^{-07}	7.710
	trade-off = 0 $\chi^2(2)$	50.392	(0.000)	59.303	(0.000)
All equations	trade-off = 0 $\chi^2(8)$	29.011	(0.0003)	48.232	(0.0000)
		Panel D			
Extra Factor		Market Factor			
Limit sell +1 tick	Extra factor	-1.056×10^{-08}	-5.597		
	Market index	1.444×10^{-08}	6.834		
	Scale	6.741×10^{-08}	7.610		
	trade-off = 0 $\chi^2(2)$	63.526	(0.0000)		
Limit Sell +2 ticks	Extra factor	-4.551×10^{-09}	-3.099		
	Market index	-1.474×10^{-09}	-1.911		
	Scale	1.444×10^{-08}	3.741		
	trade-off = 0	11.407	(0.0033)		
Limit buy -1 tick	Extra factor	-1.145×10^{-08}	-5.281		
	Market index	5.774×10^{-09}	2.845		
	Scale	3.288×10^{-08}	4.023		
	trade-off = 0	20.148	(0.0000)		
Market buy	Extra factor	-9.098×10^{-09}	-4.573		
	Market index	1.673×10^{-08}	6.627		
	Scale	6.286×10^{-08}	7.158		
	trade-off = 0$\chi^2(2)$	57.072	(0.0000)		
All equations	trade-off = 0 $\chi^2(2)$	39.172	(0.0000)		

This table reports estimates of the index model augmented by additional factors, Equation (40) in the text. The row marked $\chi^2(2)$: trade-off = 0 provides an asymptotic chi–squared test of the null hypothesis that the coefficients on the variables in the theoretical model are all equal to zero. Asymptotic p–values are given in parentheses.

Table 10: **Welfare Calculations**

Panel A		
	Upper bound	Lower bound
Theoretical gains from trade	——-	21.82
Loss from cancellations	2.59	2.50
Loss from executed buy orders	3.03	3.07
Loss from executed sell orders	4.06	3.85
Lower bound on realized gains from trade		12.50

Panel B	
As a percentage of lower bound of theoretical gains from trade	
Upper bound on loss from cancellations	11.87
Upper bound on loss from executed buy orders	13.89
Upper bound on loss from executed sell orders	18.61
Lower bound on realized gains from trade	57.23

This table presents our estimates of the realized and theoretical gains from trade. Panel A provides the gains and losses in units of Swedish Kronor. The average price of Ericsson is approximately 110 over our sample period. The columns marked Lower Bounds provide the lower bounds, similarly for the columns marked Upper Bounds. We only estimate a lower bound for the maximal social surplus. The lower bound on the realized gains from trade is computed by subtracting the upper bound for the losses from the lower bound of the theoretical gains from trade. Panel B reports the lower bounds on losses and upper bounds on realized gains from trade as a percentage of the lower bound on the theoretical gains from trade. We use 16,076 observations in the calculations.

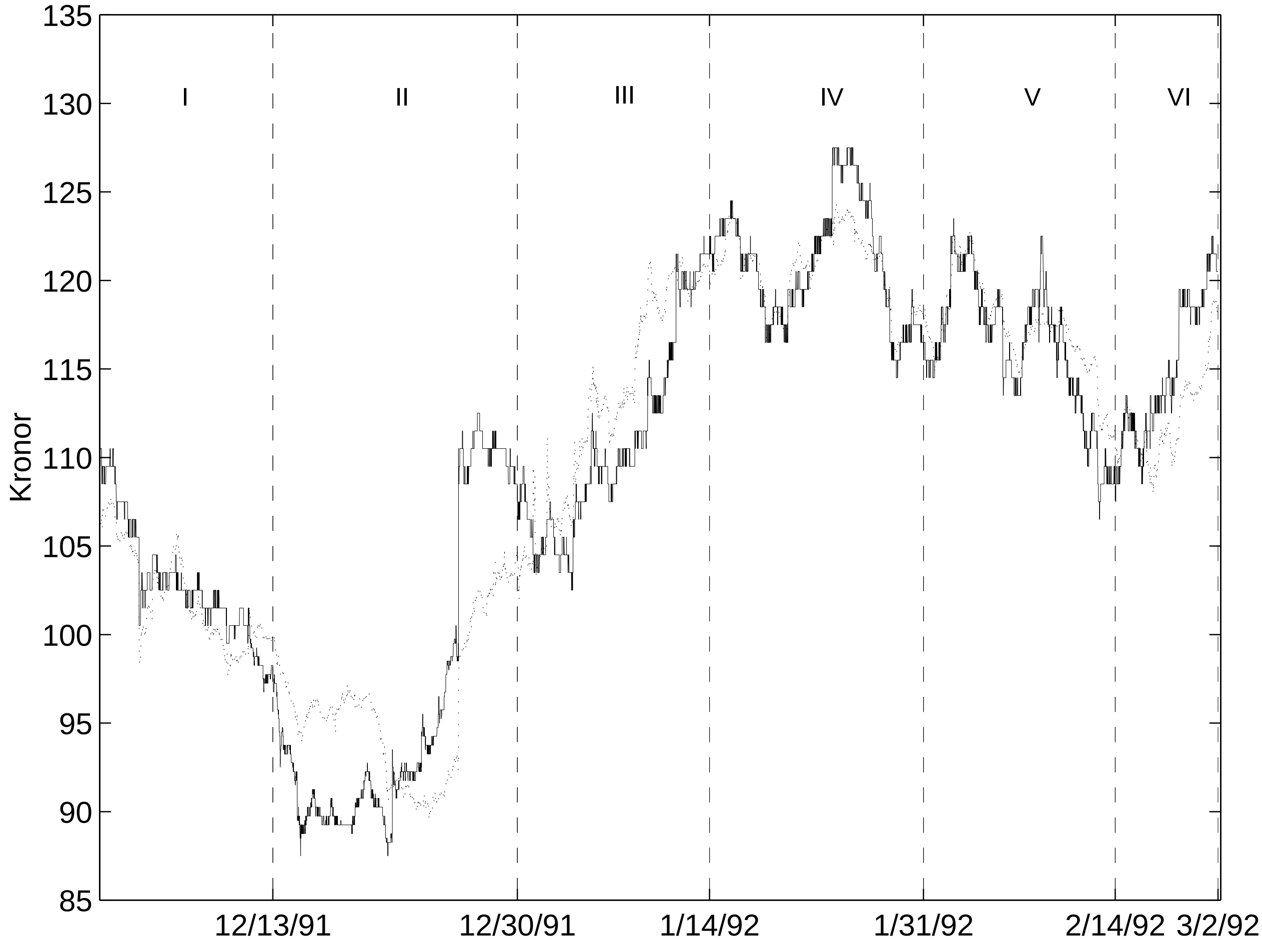

Figure 1: **Common value and mid-quote.** This figure plots estimates of the common value (- - -) and the mid-quote (—). The common value is computed using our minimum distance estimator. We divide the data into 6 sub–periods of 2 weeks in length.

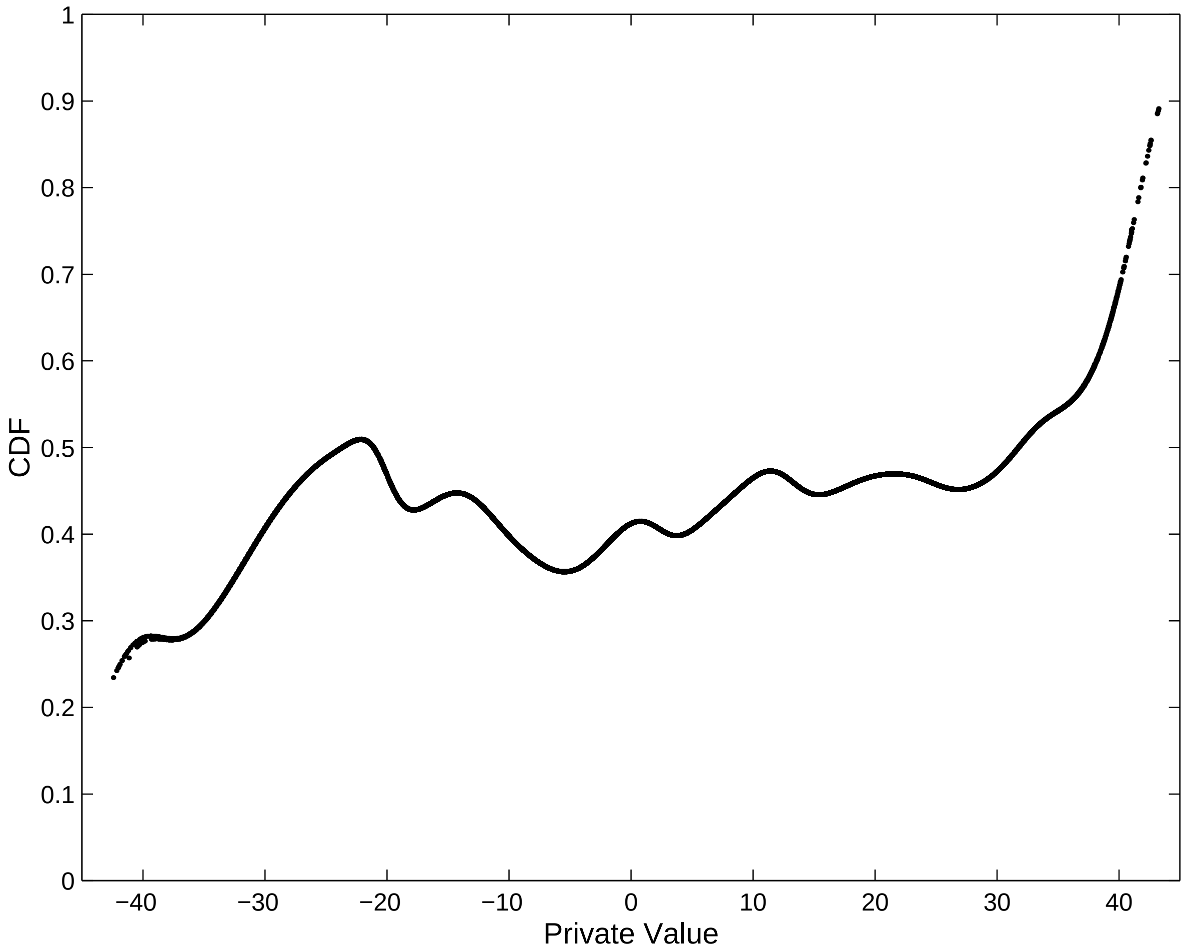

Figure 2: **Estimated Private Value Distribution** This figure plots estimates of the private value distribution. Here the CDF is obtained through a nonparametric regression of the cumulative choices on our estimates of the threshold private values, $\hat{x}'_{jkt}\hat{\beta}_{MD}$.

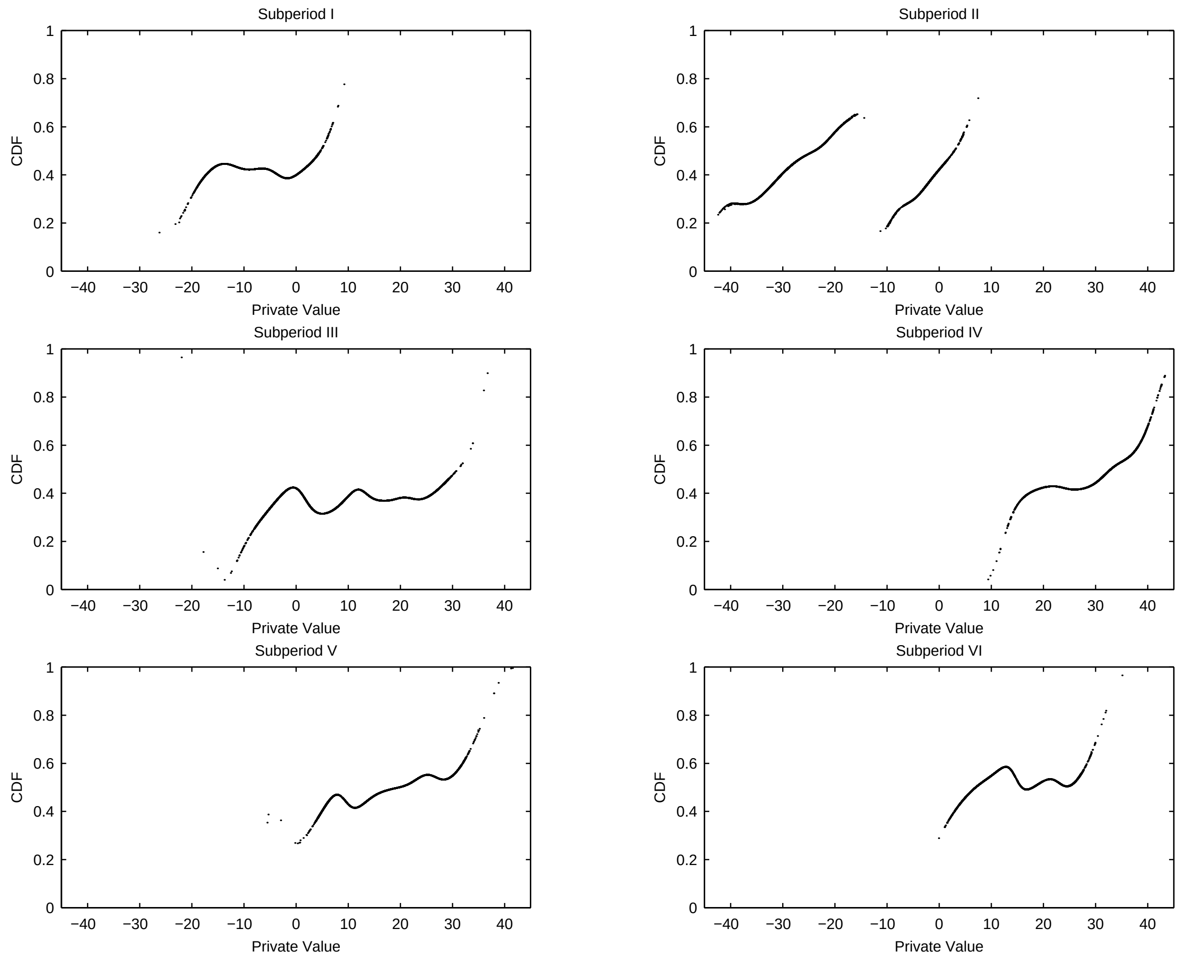

Figure 3: **Sub–period private value distribution estimates** This figure plots sub–period estimates of the private value distribution for the same 6 sub-periods of 2 weeks in length as plotted in Figure 1. Here, the CDFs are obtained through a nonparametric regression of the cumulative choices on our estimates of the threshold private values, $\hat{x}'_{jkt}\hat{\beta}_{MD}$.