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Multiple Large Shareholders in Corporate Governance

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First Draft: July 30, 1998

This Draft: February 17, 1999

Abstract

Large shareholders of firms with majority blocks are often at the helm of their companies and do not necessarily have the same interests as minority shareholders. We show that bargaining problems led by the presence of multiple controlling shareholders protect minority shareholders. The same bargaining problems, however, prevent efficient decisions. By solving this trade-off we find that i) multiple controlling shareholders should be present in firms with large costs of diluting minority shareholders and in firms with large financing requirements, ii) an optimal ownership structure requires the presence of a class of shareholders - the minority shareholders - with no control over corporate decisions. Evidence on the ownership structure of close corporations in the U.S. is consistent with our model.

JEL classification: G32, G34

*We would like to thank Chris Geczy, Gary Gorton, Dennis Gromb, Matthias Kahl, and seminar participants in the 1999 AFE meeting in New York for valuable comments and Yuan-Sheng Zhang for helpful research assistance. Gomes also thanks the support of the Rodney L. White Center for Financial Research. An earlier version of this paper was entitled "The Optimal Number of Larger Shareholders." E-mail: gomes@wharton.upenn.edu and novaes@u.washington.edu.

Since Berle and Means (1932), an extensive literature in corporate finance has investigated a variety of governance topics in widely held corporations. In particular, takeover threats and monitoring by large shareholders have been singled out as important mechanisms to protect the investment of minority shareholders. Takeover threats are unlikely to be effective, however, in firms with majority blocks. Moreover, as La Porta, Lopez-de-Silanes, and Shleifer (1998) show, large shareholders often participate in the management.[1] Hence, independent monitoring by large shareholders who are also in control is, at best, questionable. So, if neither takeover threats nor monitoring effectively disciplines the controlling shareholders of firms with majority blocks, then which other governance mechanisms protect minority shareholders in these firms?

In this paper we argue that the presence of multiple controlling shareholders protects minority shareholders of close corporations and public firms with large blockholders. In a nutshell, bargaining problems among controlling shareholders may prevent decisions that, although in their collective interest, harm minority shareholders. The same bargaining problems, though, may prevent efficient decisions. By solving this trade-off we find that the presence of multiple controlling shareholders should be pervasive in firms where the incentives for diluting minority shareholders are high and/or in firms with large financing requirements. Somewhat surprisingly, we also find that excluding minority shareholders from the ownership structure does not ensure efficiency. On the contrary, an optimal ownership structure requires the presence of some minority shareholders.

In our model, an all equity firm is initially owned by a single shareholder who, to undertake a project, has to attract outside investors. We allow for three forms of financing: i) a sale of equity to a class of shareholders - *the minority shareholders* - who cannot interfere with the business decisions, ii) a sale of equity to a small number of shareholders - *the controlling shareholders* - with whom the initial shareholder will share the firm's control, and iii) a combination of i) and ii), that is, equity sales to both minority and controlling shareholders.[2]

[1]In a sample of large public firms in 27 countries, La Porta et al. (1998) find that the largest shareholder participates in the management in 71% of the firms whose largest shareholder has at least 20% of the shares.

[2]We rule out debt financing and the sale of the firm to a single investor. Ownership by a single investor may not be possible because of risk aversion, credit constraint, or some firm-specific human capital owned by the initial shareholder.

In the first form of financing, the initial shareholder remains as the single decision maker, with full discretion on the firm's operations. Too much discretion, though, may hurt the initial shareholder, who may be tempted to inefficiently dilute the minority shareholders. Since the price of the minority shares should reflect the likely dilution, the initial shareholder has incentives to limit his/her own discretion.

A key insight of this paper is that, by sharing control with other large shareholders, the initial shareholder limits the dilution that will be imposed on the minority shareholders. Sharing control protects minority shareholders for two reasons. First, selling shares to controlling shareholders implies that fewer shares have to be sold to minority shareholders to finance the investment. Of course, the lower minority stake implies a larger controlling stake, which, in turn, makes the controlling group internalize the firm value to a greater extent. It then follows that there are fewer incentives to inefficiently dilute minority shareholders.

A second and more original effect arises from the bargaining problems led by the presence of multiple controlling shareholders. A controlling shareholder will not voluntarily agree to sacrifice efficiency because of the private benefits that another controlling shareholder might enjoy. More importantly, asymmetric information on the gains of diluting the minority shareholders may prevent a set of side payments that would allow the controlling shareholders to implement decisions that, although in their collective interest, would harm the minority shareholders. Disagreement among controlling shareholders, thus, protects minority shareholders.

It turns out that the same *ex-post bargaining problems* that protect minority shareholders may lead to an excessive disagreement over questions related to business policy, creating the risk of corporate paralysis. Thus, there is a trade-off on the number of large shareholders. On one hand, the ex-post bargaining problems associated with the presence of multiple large shareholders enhance value by preventing some of the inefficient dilution that a single controlling shareholder would impose on the minority shareholders. On the other hand, the same bargaining problems may prevent efficient decisions. We characterize this trade-off and provide sufficient conditions for the presence of multiple large shareholders to be optimal.

In describing the trade-off on the number of controlling shareholders, we took for granted the presence of some minority shareholders. One might think, however, that efficiency can be achieved by simply excluding minority shareholders from the ownership structure. For instance, there would be no minority shareholder to dilute if the initial shareholder gave veto power to all shareholders. As we have previously argued, though, incomplete information may prevent efficient bargaining. As a result, giving veto power to all shareholders does not eliminate the disagreement costs. Indeed, we shall actually prove that sharing control with all shareholders maximizes the disagreement costs, and that an optimal ownership structure requires the presence of a class of shareholders - the minority shareholders - with no control over the firm's decisions.

Intuitively, diluting minority shareholders provides a goal that, ex-post, all controlling shareholders should agree with. Hence, the presence of minority shareholders decreases the disagreement costs. To be sure, the existence of minority shareholders will also give rise to dilution costs. If the minority stake is sufficiently small, though, these costs are of second order. The reason that dilution costs do not increase much is due to a new effect introduced in this paper - *the benefits of disagreement* - which implies that some of the disagreement that the presence of minority shareholders aims to reduce protects those minority shareholders against inefficient dilution.

In summary, solving the optimal ownership structure tells us when and why an initial entrepreneur should share control with other shareholders. In addition, when we later discuss the implementation of the optimal ownership structure, we shall see that it also provides some new insights on commonly used corporate governance mechanisms such as shareholders' agreements and supermajority rules.

A formal test of the model's implications is beyond the scope of this paper. Nonetheless, preliminary work on the database of the National Survey of Small Business Finances (NSSBF) of 1992 gives some support to our model. In the NSSBF database, 86.9% of the close corporations with annual sales above $10 million have at least one large shareholder. Interestingly, conditioned on the existence of a large shareholder, the probability that multiple large shareholders exist is 36.1% for firms where the manager is not a shareholder, and 74.0% for firms where the manager is also a shareholder. If the conflicts of interest between minority and large shareholders are less severe in firms where the manager is not a large shareholder, then the results are consistent with the implication of our model that multiple large shareholders are more often present in firms where the conflicts are larger.

Our paper builds on a vast literature that analyzes the conflicts of interest between minority and majority shareholders. In Winton (1993) and Bolton and Von Thadden (1998), large shareholders monitor managers on behalf of all shareholders. Winton (1993) points out that multiple large shareholders face a free-rider problem in monitoring managers. As a result, an ownership structure with multiple large shareholders is optimal only if the individual cost of monitoring is sufficiently convex. Bolton and Von Thadden (1998) focus on a trade-off between liquidity and control. In their paper, large shareholders improve control over the management, but decrease the liquidity of the firm's stock. The incentives to minimize liquidity costs make it optimal to have at most one large shareholder per firm. Contrary to these papers, we do not consider monitoring by large shareholders because they are assumed to be in control.

Burkart, Gromb, and Panunzi (1997) argue that excessive monitoring by large shareholders may destroy ex-ante incentives to invest in firm-specific assets. In a way, their excessive monitoring parallels our disagreement costs and their allocation control problem can be mapped to our ownership structure decision. One of the contributions of our paper is then to show how ex-post bargaining problems affect the firm's optimal control structure. In particular, we provide an efficiency reason for the presence of minority shareholders.

Closer to our paper, Pagano and Roell (1998) identify the controlling group with a large shareholder. Still, they allow for other large shareholders to monitor the controlling shareholder on behalf of all shareholders. In Pagano and Roell, the presence of multiple large shareholders enhances value because it creates free rider problems in the monitoring decision, curbing excessive monitoring on the controlling shareholder. In contrast, the presence of multiple large shareholders imposes a constraint on the controlling group in our paper. Thus, the two papers have opposite implications for how the optimal number of large shareholders responds to the benefits of constraining the controlling group.[3]

Bennedsen and Wolfenzon (1998) focus on the impact that large shareholders have on the

[3]Pagano and Roell discuss the possibility of a collusion between the controlling shareholder and their monitors, arguing that under-monitoring would prevail. Unlike in our paper, the optimal ownership structure would then exclude the presence of minority shareholders.

equity stake of the controlling group of a close corporation.[4] In their paper, the presence of a large shareholder forces a group of shareholders to amass a larger equity stake to retain control. The larger equity stake enhances value because it makes the controlling group internalize more of the firm value. In our paper, we focus on the bargaining problems among controlling shareholders and ignore the coalition games that have established the controlling group. In part, our modelling choice is due to our belief that controlling groups make use of corporate governance mechanisms to limit the formation of new coalitions. For instance, we later document that a substantial number of public firms in Italy have shareholders' agreements that restrict the buyout of shares (*buyout agreements*) and force the participants to vote with the group (*pooling agreements*). Although we do not have data on the use of shareholders' agreements in the U.S., corporate law books (e.g., O'Neal and Thompson, 1992) report that they are often present in close corporations.

Finally, Zwiebel (1995) studies the allocation of large shareholders in a general equilibrium framework. In his model, wealthy investors avoid competing for private benefits by choosing to hold large blocks in different companies. An equilibrium with one large block-holder per firm thus exists. In our paper, we allow for a trade-off between private benefits and public cash-flows. Multiple block-holders coexist in a firm when the extraction of private benefits imposes a large sacrifice of public cash-flows.

The remainder of the paper is organized as follows. Section 1 describes the model. Section 2 relates ownership structure to firm value. Section 3 describes the benefits and costs associated with the presence of multiple controlling shareholders and provides sufficient conditions for an ownership structure with multiple controlling shareholders to be optimal. Section 4 discusses the implementation of the optimal ownership structure. Section 5 presents the empirical evidence, and the conclusion follows. Proofs of all propositions can be found in the appendix.

[4]Harris and Raviv (1988) and Stulz (1988) also focus on the equity stake of a controlling group. In these papers, however, the goal is to investigate the capital structure implications of a controlling group's attempt to defeat a control contest.

1 Framework

Our starting point is an all equity firm whose assets in place have a value V_0. For simplicity, all agents are risk-neutral and the risk-free rate is zero. The firm is initially owned by a single shareholder, who has exhausted his/her debt capacity. Undertaking any new project thus requires attracting outside investors.

1.1 Timing

The model has one production period and four dates. At time $t = 0$, an investment opportunity becomes available to the firm. If the initial shareholder does not find the project worthwhile, then the game ends and he/she collects the cash-flows generated by the existing assets at time $t = 3$. Otherwise, the shareholder has to attract outside investors to finance the project. If financing is obtained, the firm's ownership structure changes, possibly forcing the initial shareholder to share control with other investors. Additional information on the project's payoff is released at time $t = 1$, and the final decision on the investment is made at time $t = 2$. Cash-flows realize at $t = 3$, when the firm is liquidated. Figure 1 below summarizes the timing and the main events of the model.

Figure 1: Timing of events

1.2 Project Financing and Ownership Structure

As mentioned in the previous sub-section, the initial shareholder has to attract investors at $t = 0$ to keep alive the option of undertaking the new project. We allow for three forms of equity financing: (i) a sale of equity to minority shareholders who will have no say in the firm's operations, (ii) a sale of equity to a small number of shareholders, the *controlling shareholders*, with whom the initial shareholder will have to share control, and (iii) a combination of the two previous forms of financing, where a fraction of the firm's equity is sold to minority shareholders and a fraction is sold to controlling shareholders. In all three cases, we assume that the initial shareholder raises at least the amount I that is needed to finance the project, and we rule out the sale of the firm to a single investor.[5]

To differentiate the three forms of financing in a more precise way, we focus on the allocation of control over the firm's investment decision. By selling equity to minority shareholders, the initial shareholder retains full control over the investment decision. After the new information is released at $t = 1$, the initial shareholder decides whether to undertake the project according to his/her own interests. In contrast, if new controlling shareholders are brought to the firm, the initial shareholder will have to bargain with them about whether to undertake the new investment. We assume that all controlling shareholders have veto power on the investment decision.[6] If some controlling shareholder exerts his/her veto power, then the project is not undertaken.

Let $\alpha_0 \in [0, 1)$ be the fraction of equity sold to minority shareholders and α_i, $i \in \{2, \cdots, n\}$, be the fraction of shares sold to controlling shareholder i. Thus, α_1 is the fraction of shares held by the initial shareholder after the funds are raised, and the fraction in the hands of the minority shareholders is $\alpha_0 = 1 - \sum_{i=1}^{n} \alpha_i$. It then follows that the ownership structure induced by any of the three forms of financing can be described by a vector $\alpha(n) = (\alpha_0, \alpha_1, \cdots, \alpha_n)$, where n is the number of controlling shareholders.

In this paper, we are mainly interested in the incentive effects of the presence of multiple

[5]Ruling out the sale of the firm to a single investor can be justified by risk aversion, some degree of credit constraint on the outside investors, and/or the importance of the human capital of the initial entrepreneur to the firm's operations.

[6]Section 4 discusses which corporate governance mechanisms can implement the veto power.

controlling shareholders. Thus, to simplify the analysis we restrict attention to two polar ownership structures. Either the initial shareholder chooses to remain as the single decision maker, or he/she will share control with $n - 1 > 0$ controlling shareholders. In other words, we investigate when and why the presence of multiple controlling shareholders maximizes firm value, but we do not look for the optimal number of controlling shareholders.

1.3 Cash-Flows

In describing the link between ownership structure and project financing, we emphasized the allocation of control on the firm's investment decision. Control on investment decisions is only relevant, though, if shareholders have different incentives to undertake projects. As in the modern literature on the theory of firm (e.g., Grossman and Hart (1986) and Hart and Moore (1990)), we obtain conflicting incentives on the investment decision by introducing non-verifiable cash-flows, which we assume to be fully captured by the controlling shareholders. Thanks to the non-verifiable cash-flows, controlling and minority shareholders value the project in different ways. Following the literature, from now on we shall call these non-verifiable cash-flows the *private benefits of control*.[7]

We decompose the project's cash-flows in two parts: the verifiable cash-flow, $I + y$, which will be divided among all shareholders in proportion to their equity holdings, and the private benefit component, b. From the perspective of the minority shareholders, the project's net present value is y. In contrast, the project's net present value for the controlling shareholders is $y + b$.

The private benefits of control and the verifiable cash-flow component y are random when the ownership structure is chosen at $t = 0$. We assume that their probability distributions are not affected by the ownership structure choice, though. More precisely, we assume that, regardless of the ownership structure, y has a continuous density $g(y)$ with support in the interval $[\underline{y}, \bar{y}]$, where $\underline{y} < 0$ and $\bar{y} > 0$. Likewise, for any ownership structure, the total private

[7] Private benefits of control can be non-pecuniary or pecuniary. Pecuniary benefits include the ability to siphon off earnings by high compensation, leases and loans at favorable terms, appropriation of corporate assets for personal uses, etc. Non-pecuniary benefits are, for instance, perks from the job, ability to undertake pet projects that enhance the visibility of the agents in control, empire building motives, etc.

benefits from the project, b, have support in the interval $[\underline{b}, \bar{b}]$ with $\underline{b} \geq 0$ and a strictly positive density $f(b)$. Hence, for $n > 1$ controlling shareholders, if we let b_i be the private benefit of controlling shareholder i, then $\sum_{i=1}^{n} b_i = b$. The private benefits of the controlling shareholders are independently distributed, with a strictly positive density $f_i(b_i)$ in the interval $[\underline{b}_i, \bar{b}_i]$ with $\underline{b}_i \geq 0$ for any $i \in \{1, \cdots, n\}$. Last, we assume that y and b are independently distributed.[8]

1.4 Information Structure

Consider first an ownership structure where the initial shareholder keeps full control on the firm's investment decision, selling a fraction $\alpha_0 > 0$ of the equity to minority shareholders. At time $t = 1$, the single controlling shareholder learns the realization of the private benefits of control, b, and the project's verifiable cash-flow component, y. At the time that the investment decision has to be made, though, outsiders do not observe (y, b). Hence, the court does not have the necessary information to block an investment decision that is not in the best interest of the minority shareholders. Moreover, we rule out incentive mechanisms that are contingent on the verifiable cash-flow.[9]

Now, consider an information structure with multiple controlling shareholders. Once again, outsiders and minority shareholders, if any, cannot observe y and b when the investment decision has to be made. In addition, we allow the controlling shareholders to be asymmetrically informed. At $t = 1$, they all observe the realization of the verifiable cash-flow y. The level of private benefits of each controlling shareholder is private information, though. The information

[8]There are good reasons to believe that the total private benefits, b, are affected by the number of controlling shareholders. On one hand, a larger number of controlling shareholders may increase the efforts of unlocking private benefits. If so, the total private benefits should increase with the number of controlling shareholders. On the other hand, a larger number of controlling shareholders might lead to a destructive fight for private benefits. Hence, the total private benefits might decrease with the number of controlling shareholders. Since we do not have a good sense of which effect dominates, we opted for ignoring the relation between the private benefits and the number of controlling shareholders. Most of the results of our paper hold, though, if we assume a smooth function, $b(n)$, mapping the number of controlling shareholders on the total private benefits.

[9]We justify ruling out incentive contracts on two grounds. First, even if incentive contracts are written, the controlling shareholder may use his/her power to renegotiate them later on. Second, ruling out incentive contracts helps us focus the analysis on the incentive features of the ownership structure (see Allen and Phillips (1998) for evidence on the relation between incentives and ownership structure). Note, however, that the verifiable cash-flows exert some influence on the controlling shareholders' incentives through their equity holdings.

released at $t = 1$ is then represented by a vector $(y, b_1, \cdots, b_n)$, while the information received by controlling shareholder i is given by (y, b_i).[10]

2 Investment Policy and Ownership Structure

In this section, we characterize the costs and benefits of different ownership structures by comparing their implied firm value with the value implied by the first best investment policy.

In the first best investment policy, the initial shareholder raises an amount I of funds that are invested in the project if and only if the total payoffs (verifiable cash-flows plus private benefits of control) are bigger than I. Equivalently, the project is accepted if and only if $y + b \geq 0$. The firm value under the first best investment policy is then equal to

$$\bar{V} = V_0 + I + E[\max\{y + b, 0\}].$$

2.1 Investment with a Single Controlling Shareholder

Assume that, at $t = 0$, the initial entrepreneur sells a fraction α_0 of the firm's equity to minority shareholders in order to raise the investment requirement I. In the resulting ownership structure, the initial shareholder retains full discretion over the investment decision. Accordingly, the project will be undertaken if and only if it is in his/her interest.

The single controlling shareholder will not necessarily implement the first best investment policy. After all, he/she does not internalize a loss in expected cash-flows that is absorbed by the minority shareholders. In other words, an inefficient project, $y + b < 0$, may still be profitable for a controlling shareholder if the private benefits are high enough to offset his/her share of the negative verifiable cash-flow, that is, $b > -\alpha_1 y$.[11] The set of states where a single controlling shareholder over-invests is then equal to

$$\mathcal{D}(\alpha(1)) = \{(y, b) : \alpha_1 y + b \geq 0 \;\text{ and }\; y + b < 0\}.$$

[10] Incentive schemes based on the verifiable cash-flows are ruled out.

[11] Bebchuck and Zingales (1998) use a similar conflict between minority and controlling shareholders to explain why a firm's decision to go public may not be socially optimal.

The set of states $\mathcal{D}(\alpha(1))$ induces a loss in value which, ex-post, is borne by the minority shareholders. Accordingly, we call this loss of value the *cost of dilution.* To evaluate the cost of dilution, let $\mathcal{X}_{\mathcal{D}(\alpha(1))}$ be an indicator variable that takes value 1 if $(y, b) \in \mathcal{D}(\alpha(1))$ and zero otherwise. Thus, $[y + b]\mathcal{X}_{\mathcal{D}(\alpha(s))}$ is the negative payoff from undertaking the project when efficiency would dictate otherwise. The expected dilution cost given an ownership structure $\alpha(1)$ with a single controlling shareholder is then

$$D(\alpha(1)) = -E[(y + b)\mathcal{X}_{\mathcal{D}(\alpha(1))}]. \tag{1}$$

2.2 Investment with Multiple Controlling Shareholders

We now assume that the firm is run by $n > 1$ controlling shareholders. At time $t = 2$, the controlling shareholders decide whether to support the investment or to block it. In making this decision, they take into account their own private benefits of control, b_i, and their share of the project's verifiable cash-flow, $\alpha_i y$.

If two controlling shareholders disagree on the investment decision, that is, $b_i + \alpha_i y > 0$ and $b_j + \alpha_j y < 0$, then the controlling shareholder opposing the investment, j, prevails. Facing a veto from shareholder j, it is in the interest of shareholder i to offer a side payment to j to ensure the investment. Shareholders i and j will then bargain on a side payment that would make both of them at least as well off undertaking the project. Under incomplete information, though, the controlling shareholders have incentives to claim a low private benefit of control in order to maximize net transfers for not blocking the project. The bargaining game then entails the risk of an inefficient outcome where the investment would not be undertaken even if it is in the collective interest of the controlling shareholders. The firm's investment policy under an ownership structure with multiple controlling shareholders might then differ from the one that a single controlling shareholder with the same controlling stake would adopt.

For ease of exposition we will solve the investment problem with multiple controlling shareholders in two parts. In subsection 2.2.1 we use the mechanism design approach developed by Myerson and Satterthwaite (1983) to characterize the possible outcomes of the controlling shareholders' bargaining.[12] Then, in subsection 2.2.2 we describe the best (from the perspective of the controlling shareholders) investment policy that may arise in the presence of multiple asymmetrically informed controlling shareholders.

2.2.1 The Bargaining Game

The payoff of controlling shareholder i in case the project is undertaken is

$$v_i(y, b_i) = b_i + \alpha_i y.$$

For any realization of the verifiable cash-flow component, y, we define a direct mechanism as a pair $(x(y, \hat{\mathbf{b}}), t(y, \hat{\mathbf{b}}))$ that determines the probability x that the project will be undertaken and the vector of net transfers $t = (t_1, \cdots, t_n)$ as a function of the announced private benefits of control $\hat{\mathbf{b}} = (\hat{b}_1, \cdots, \hat{b}_n)$ and y. Hence, given a direct mechanism $(x(.), t(.))$, a vector of announcements $\hat{\mathbf{b}}$, actual private benefits $\mathbf{b}$, and y, we can write the expected utility of controlling shareholder i as $x(y, \hat{\mathbf{b}})v_i(y, b_i) + t_i(y, \hat{\mathbf{b}})$.

By the Revelation Principle, a direct mechanism $(x(.), t(.))$ describes the outcome of the controlling shareholders' bargaining problem if it induces truthful announcement of the private benefits and provides incentives for all controlling shareholders to participate in the mechanism. To characterize these two constraints, let $X_i(y, b_i) = E_{-i}[x(y, b_i, b_{-i})]$ be the expected probability of controlling shareholder i that the project will be undertaken conditioned on announcement b_i and y (the expectation is taken with respect to $b_{-i} \equiv (b_1, \cdots, b_{i-1}, b_{i+1}, \cdots, b_n)$).

Likewise, we define $T_i(y, b_i) = E_{-i}[t_i(y, b_i, b_{-i})]$ as the expected net transfer to shareholder i given b_i and y. Given $X_i(.)$ and $T_i(.)$, the expected utility of shareholder i conditioned on announcing $\hat{b}_i$ when his/her true private benefit is b_i is

$$U_i(y, \hat{b}_i, b_i) = v_i(y, b_i)X_i(y, \hat{b}_i) + T_i(y, \hat{b}_i).$$

We are now ready to describe the constraints that a direct mechanism must satisfy. Calling

[12] See Darrough and Stoughton (1989) for a paper that applies the mechanism design approach in a way that is similar to ours.

$U_i(y, b_i) \equiv U_i(y, b_i, b_i)$ the expected utility of truthfully announcing b_i, the incentive compatibility constraint (IC) requires that

$$(IC) \quad U_i(y, b_i) \geq U_i(y, \hat{b}_i, b_i) \quad \text{for any } i,\ b_i, \text{ and } \hat{b}_i.$$

The individual rationality constraint (IR) requires that shareholder i is at least as well off in the direct mechanism as in exercising his/her right to veto the project. Formally,

$$(IR) \quad U_i(y, b_i) \geq 0 \quad \text{for any } i \text{ and } b_i.$$

Last, we require balanced transfers, that is, for any $\hat{\mathbf{b}}$:

$$(BB) \quad \sum_{i=1}^{n} t_i(y, \hat{\mathbf{b}}) = 0.$$

We thus say that a direct mechanism $(x(.), t(.))$ implements the possible outcomes of the bargaining game played by the controlling shareholders if and only if it satisfies the IR, the IC, and the BB constraints. The following proposition characterizes the set of implementable direct mechanisms.

Proposition 1 *(Implementable investment policies) Let $x(.)$ be any investment policy. For any given y, there exists a vector of side payments $t(.)$ such that the direct mechanism $(x(.), t(.))$ satisfies the IC, IR, and the BB constraints if and only if $X_i(y, b_i)$ increases with b_i for each i and*

$$E\left[\sum_{i=1}^{n}\left(b_i + \alpha_i y - \frac{1 - F_i(b_i)}{f_i(b_i)}\right) x(y, \mathbf{b})|y\right] \geq 0. \tag{2}$$

To understand the intuition underlying Proposition 1, assume that the controlling shareholders could trust each other to truthfully announce their private benefits. In such case, given a random probability $x(.)$ that the project is undertaken, the expected joint benefit for the controlling shareholders conditioned on y is

$$E\left[\sum_{i=1}^{n}(b_i + \alpha_i y)\, x(y, \mathbf{b})|y\right].$$

Because the controlling shareholders are assumed to be self-interested, monetary incentives must be given to induce truthtelling. The monotonicity condition on $X_i(.)$ and the vector of net transfers ensure truthtelling in the announcements of the private benefits. The net transfers distort the controlling shareholders' perception of the project's payoffs by $-(\frac{1-F_i(b_i)}{f_i(b_i)})$, though. The condition for a direct mechanism to be implementable is then that the expected joint benefits from the project are positive after the adjustment for truthtelling is taken into account.

2.2.2 Optimal Investment Policy

In this section we characterize the implementable investment policy, $x(.)$, that maximizes the expected payoffs of the controlling shareholders. In other words, we assume that the rules governing the bargaining game will be chosen by the controlling shareholders in order to maximize their joint expected gains.[13] By Proposition 1, the optimal investment policy solves

$$\max_{x(.)} E[\sum_{i=1}^{n}(b_i + \alpha_i y)x(y, \mathbf{b})|y] \tag{3}$$

$$\text{s.t.} \quad E[\sum_{i=1}^{n}(b_i + \alpha_i y - \frac{1 - F_i(b_i)}{f_i(b_i)})x(y, \mathbf{b})|y] \geq 0, \tag{4}$$

$$X_i(y, b_i) \geq X_i(y, b_i') \quad \text{for any } i \text{ and } b_i \geq b_i'. \tag{5}$$

The objective function is the controlling shareholders' expected gain of investing in the project with probability $x(.)$. The two constraints ensure that the controlling shareholders are willing to participate in the mechanism for any realization of private benefits, and that they will be willing to truthfully announce their private benefits of control.

To solve Program 3, we assume that the distributions of the private benefits satisfy the monotone likelihood ratio condition.[14]

[13]The direct mechanism approach that we use may be criticized on the grounds that shareholders do not negotiate by revealing their private benefits. Models based on offers and counter-offers (see Rubinstein, 1982), for example, would be closer to an actual bargaining game. Ausubel and Deneckere (1993) show, however, that, in a game similar to ours, direct mechanisms can implement the allocation of a sequential bargaining game for fairly general distributions of valuations. Our mechanism design formulation can thus be interpreted as a way to characterize the outcome of more standard sequential bargaining games.

[14]The monotone likelihood ratio condition is a standard assumption in mechanism design problems. Roughly

Assumption 1 *For any i, the distribution of b_i satisfies the Monotone Likelihood Ratio condition, that is,*

$$\frac{d\{\frac{f_i(b_i)}{1-F_i(b_i)}\}}{db_i} \geq 0 \quad \textit{for any } i \textit{ and } b_i.$$

We thus have the following characterization of the optimal investment policy.

Proposition 2 *(Optimal investment policy) The investment policy that maximizes the expected gains of the controlling shareholders subject to the IC, IR, and BB constraints is given by*

$$x^{\lambda}(y, \mathbf{b}) = \begin{cases} 1 & \textit{if } \sum_{i=1}^{n}(b_i + \alpha_i y) \geq \lambda(y) \sum_{i=1}^{n} \frac{1-F_i(b_i)}{f_i(b_i)} \\ 0 & \textit{otherwise} \end{cases}$$

where $\lambda(y) = \frac{\mu(y)}{\mu(y)+1} \geq 0$, and $\mu(y) \geq 0$ is the Lagrange multiplier of constraint (4).

The question that we now address is whether the optimal direct mechanism, $x^{\lambda}(.)$, implements the investment rule that the controlling shareholders would choose, if they had complete information on the vector of private benefits.

In the proof of Proposition 2, we show that the monotonicity condition, constraint (5), is not binding in the optimum. Hence, if constraint (4) is not binding either, then $\lambda(y) = 0$ and the optimal investment policy is not constrained by the need to elicit truthful announcements from the controlling shareholders. The optimal mechanism then replicates the investment choice that the controlling shareholders would implement in a setting with complete information. This investment policy is the *ex-post efficient investment policy* for the controlling shareholders - invest if and only if $\sum_{i=1}^{n}(b_i + \alpha_i y) \geq 0$ -, which is also the investment rule that a single controlling shareholder who owns a fraction $\sum_{i=1}^{n} \alpha_i$ of the firm's shares would adopt. It then follows that, for a fixed fraction of shares held by the minority shareholders, the presence of multiple controlling shareholders changes the incentives to invest if and only if the incentive constraint (4) is binding.[15]

speaking, it allows us to drop the monotonicity condition (constraint 5) from Program 3.

[15] As we show in Section 3, an increase in the controlling stake reduces the incentives to dilute minority shareholders. Hence, the presence of multiple controlling shareholders changes the investment policy if it allows for a smaller fraction of shares held by the minority shareholders.

Assumption 2 *Consider an ownership structure with multiple controlling shareholders. Then it must be true that: i)* $\bar{b}_i > \underline{b}_i$ *for at least two controlling shareholders, and ii)* $\sum_{i=1}^{n} \underline{b}_i + \underline{y} < 0$.

Assumption 2 rules out the trivial case where the project is always efficient and it also requires that at least two controlling shareholders have uncertain private benefits. In the remainder of the paper, Assumptions 1 and 2 hold. We thus have,

Proposition 3 *(Ex-post inefficiency) For any given* y*, the investment policy that maximizes the ex-post utility of* $n > 1$ *controlling shareholders cannot be implemented if (and only if)* $\sum_{i=1}^{n} \underline{b}_i + \alpha_i y < 0 < \sum_{i=1}^{n} \bar{b}_i + \alpha_i y$.

Proposition 3 is a generalization of a well-known inefficiency result due to Myerson and Satterthwaite (1983). Studying a bilateral trade problem, Myerson and Satterthwaite show that, except in the trivial cases where trade is either always efficient or always inefficient, there is no incentive compatible mechanism that ensures an efficient trade of a good and gives to the seller and the buyer at least their reservation values. Proposition 3 generalizes this result to $n > 2$ agents and applies it to the investment problem that we study in this paper.[16]

3 Optimal Ownership Structure

In this section we show that the choice of the number of controlling shareholders involves a trade-off between two sources of inefficiency: the incentives to dilute minority shareholders, and the bargaining problems associated with the presence of multiple controlling shareholders. On one hand, the larger controlling stake and the ex-post bargaining problems that come with multiple controlling shareholders reduce the dilution costs that we introduced in sub-section 2.1. On the other hand, the same ex-post bargaining problems that protect minority shareholders create a *disagreement cost*, that is, the cost of inefficiently passing up the project. The optimal ownership structure minimizes the sum of the dilution and the disagreement costs.[17]

[16]Using a different framework, Mailath and Postlewaite (1990) prove that the inefficiency result of Myerson and Satterthwaite (1983) holds when the number of agents in the bargaining game goes to infinity.

[17]Grossman and Hart (1986) and Hart and Moore (1990) develop a theory of how control should be allocated based on incentives to elicit optimal ex-ante investment in human capital. They ignore the ex-post bargaining

3.1 The Benefits of Multiple Controlling Shareholders

Analogously to our analysis of the dilution costs associated with an ownership structure with a single controlling shareholder, let us define the set of states $\mathcal{D}(\alpha(n))$ where the project will be inefficiently accepted under an ownership structure $\alpha(n) = (\alpha_0, \alpha_1, \cdots, \alpha_n)$:

$$\mathcal{D}(\alpha(n)) = \{(y,b) : b + \sum_{i=1}^{n} \alpha_i y \geq \lambda(y) \sum_{i=1}^{n} \frac{1 - F_i(b_i)}{f_i(b_i)} \text{ and } b + y < 0\}.$$

Two different effects contribute to the dilution costs being lower under an ownership structure with multiple controlling shareholders. The first one is due to the larger equity stake of the controlling group.[18] As a result of selling shares with attached control rights, fewer minority shares have to be sold to finance the project. Hence, the minority stake is lower while the controlling stake is larger in the presence of multiple controlling shareholders. Their larger equity stake makes the controlling group internalize the negative cash-flow component y to a greater extent, reducing the incentives to dilute minority shareholders.

The second and more original effect arises from the coordination problems induced by the multiple number of controlling shareholders. To isolate this effect from the previous one, assume that the controlling stake is α regardless of the number of controlling shareholders.[19]

A single controlling shareholder undertakes the project if and only if the sum of the private benefits and his/her share of the verifiable cash-flow is positive (i.e., $b + \alpha y \geq 0$). In the presence of multiple controlling shareholders, the hurdle for accepting the project is at least as high. The bargaining problems of the asymmetrically informed controlling shareholders require $b + \alpha y \geq \lambda(y) \sum_{i=1}^{n} \frac{1-F_i(b_i)}{f_i(b_i)} \geq 0$ for the project to be accepted. Since the cost of dilution arises when a project with negative verifiable payoff (i.e., $y < 0$) is undertaken, then the fact that the hurdle for accepting the project is at least as high under multiple controlling shareholders

problem, which, in contrast, plays an important role in determining the optimal allocation of control in our paper.

[18]Burkart, Gromb, and Panunzi (1998) used this effect to show that a high takeover premium protects minority shareholders.

[19]Since controlling shares are more valuable, this assumption requires selling more minority shares than necessary when multiple controlling shareholders are present. Still, we adopt it here to distinguish the disagreement benefits from the benefits arising from a larger control stake.

implies that the dilution costs cannot increase when control is shared.

The first part of Proposition 4 below tells us that if the controlling stake is not too low, then the dilution costs in the presence of multiple controlling shareholders are strictly lower. The intuition is straightforward. By decreasing their equity holdings, the controlling shareholders reduce the relative importance of the verifiable cash-flows vis-á-vis the private benefits of control. As a result, for a sufficiently low controlling stake, the controlling shareholders will always agree to dilute the minority shareholders. Obviously, no reduction in the dilution cost is achieved. If the controlling stake is sufficiently high, though, the verifiable cash-flow component y will be relevant for some controlling shareholders, who will not easily accept the dilution of their (and the minority shareholders') interest. We thus conclude that both the ex-post bargaining problems (disagreement benefits) and the larger controlling stake due to the presence of multiple controlling shareholders reduce dilution costs.

Proposition 4 *(Dilution Costs) Fix the controlling stake α. Then there exists $\underline{\alpha} > 0$ such that, whenever $\alpha > \underline{\alpha}$, the bargaining problems associated with the presence of multiple controlling shareholders eliminate some of the dilution cost that would be imposed by a single controlling shareholder. Moreover, the cost of dilution is non-increasing in the percentage of shares, α, owned by the controlling shareholders.*

The second part of Proposition 4 obtains a negative relation between the dilution cost and the controlling stake. Now, given the set of states $\mathcal{D}(\alpha(n))$ where the project will be inefficiently accepted under an ownership structure $\alpha(n)$, we can describe the cost of dilution by

$$D(\alpha(n)) = -E[(y+b)\mathcal{X}_{\mathcal{D}(\alpha(n))}], \tag{6}$$

where $\mathcal{X}_{\mathcal{D}(\alpha(n))}$ is an indicator value that equals 1 when $(y,b) \in \mathcal{D}(\alpha(n))$, and 0 otherwise.

3.2 The Costs of Multiple Controlling Shareholders

Although the presence of multiple controlling shareholders reduces the dilution costs, it creates problems as well. The same bargaining problems that protect minority shareholders destroy value if the controlling shareholders cannot agree on choosing the project whenever it is efficient. More precisely, we argue that there may exist (y, b) such that $b + y > 0$ (i.e., it is efficient to undertake the project), but $b + (\sum_{i=1}^{n} \alpha_i) y < \lambda(y) \sum_{i=1}^{n} \frac{1-F_i(b_i)}{f_i(b_i)}$ (i.e., the controlling shareholders will not undertake the project). The set of states where this inefficiency happens is then

$$\mathcal{C}(\alpha(n)) = \left\{ (y, b) : b + \left(\sum_{i=1}^{n} \alpha_i \right) y < \lambda(y) \sum_{i=1}^{n} \frac{1 - F_i(b_i)}{f_i(b_i)} \text{ and } y + b > 0 \right\}.$$

If the set $\mathcal{C}(\alpha(n))$ is not empty, it induces a *cost of disagreement* that is equal to

$$C(\alpha(n)) = E[(y + b)\mathcal{X}_{\mathcal{C}(\alpha(n))}]. \tag{7}$$

Proposition 5 below shows that the cost of disagreement is strictly positive if the controlling stake is not too small. The intuition for the condition on the controlling stake is the same as the one given in Proposition 4. A too low controlling stake would make the verifiable cash-flow component y irrelevant to the controlling shareholders, who would always agree on the project selection. No disagreement cost would arise. Not surprisingly, Proposition 5 also shows that the disagreement cost is non-decreasing in the controlling stake. We thus have,

Proposition 5 *(Cost of disagreement) There exists a controlling stake $\underline{\alpha} > 0$ such that, whenever $\sum_{i=1}^{n} \alpha_i > \underline{\alpha}$, the cost of disagreement is strictly positive. Moreover, the disagreement cost is non-decreasing in the controlling stake.*

Having characterized the dilution and the disagreement costs, we now solve the optimal ownership structure as the one that minimizes the sum of the two costs. This is the task of the next sub-section.

3.3 Solving the Trade-Off

Let $s \in \{1, n\}$ be the number of controlling shareholders that the initial shareholder chooses. For $s = 1$, the initial shareholder remains as the single controlling shareholder and the corresponding ownership structure is $\alpha(1) = (\alpha_0, \alpha_1)$, where $\alpha_0 \in (0, 1)$ is the minority stake, $\alpha_1 \in (0, 1)$ is the stake of the single controlling shareholder, and $\alpha_0 + \alpha_1 = 1$. For $s = n$, the initial shareholder chooses to share control with $n - 1$ controlling shareholders and the corresponding ownership structure is $\alpha(n) = (\alpha_0, \alpha_1, \cdots, \alpha_n)$, where $\alpha_0 \in [0, 1)$ is the minority stake (possibly zero) and $\alpha_i \in (0, 1)$ for $i \in \{1, \cdots, n\}$ are the stakes of the controlling shareholders, with $\sum_{i=0}^{n} \alpha_i = 1$.

Now, let $\alpha(s)$ be a feasible ownership structure. The cost of dilution $D(\alpha(s))$ is characterized by equations (1) and (6). Of course, there is no cost of disagreement in the presence of a single controlling shareholder ($s = 1$). For multiple controlling shareholders ($s = n$), the cost of disagreement is given by equation (7). An optimal ownership structure minimizes the sum of the dilution cost, $D(\alpha(s))$, and the disagreement cost, $C(\alpha(s))$:

$$\begin{aligned} \min_{s \in \{1,n\}, \alpha(s)} \quad & D(\alpha(s)) + C(\alpha(s)) \\ \text{s.t.} \quad & P(\alpha(s)) = V_0 + I + E[x((y, \mathbf{b}), \alpha(s))y] \\ & \alpha_0 P(\alpha(s)) + \textstyle\sum_{i=2}^{s} \{\alpha_i P(\alpha(s)) + E[x((y, \mathbf{b}), \alpha(s))b_i]\} = I. \end{aligned} \tag{8}$$

The first constraint of Program 8 defines the value of the firm's verifiable cash-flows, $P(\alpha(s))$, given ownership structure $\alpha(s)$. $P(\alpha(s))$ equals the sum of the value of the existing asset, V_0, the investment requirement, I, and the project's net present value (NPV) excluding the private benefits of control. If the project's payoff were known, the NPV would be given by the product of the probability x that the project is undertaken and the project's verifiable cash-flow component, y. Nonetheless, neither y nor the vector of private benefits $\mathbf{b}$ are known at the time that the ownership structure is chosen. Hence, the project's NPV is given by $E[x((y, \mathbf{b}), \alpha(s))y]$, where the function $x((y, \mathbf{b}), \alpha(s))$ takes into account that the probability that the project is undertaken depends on the ownership structure and the realization of the project's payoffs. The second constraint in the minimization program requires that the value of the minority stake, $\alpha_0 P(\alpha(s))$, and the value of the shares sold to the new controlling shareholders add to the financing requirement I, where we adopted the convention that the sum in the left-hand side of the equation is zero if $s = 1$.

To characterize the optimal ownership structure, consider first a firm with a single controlling shareholder. In this case, the optimal controlling stake, call it $\alpha^\star(1, I)$, is entirely determined by the financing constraint, decreasing with the investment requirement I.[20] By Proposition 4, the dilution cost, $D(\alpha^\star(1, I))$, is then an increasing function of the investment requirement.

Now, consider an ownership structure with $n > 1$ controlling shareholders. The proof of Proposition 6 below shows that, in this case, the sum of the dilution and the disagreement costs depends only on the size of the controlling stake. In other words, the allocation of the controlling shares across the controlling shareholders does not affect firm value.[21] By an abuse of notation, let $\alpha^\star(n)$ be the optimal controlling stake given the requirement that $n > 1$ controlling shareholders exist.

We claim that the optimal controlling stake $\alpha^\star(n)$ does not depend on the investment requirement I. To see this, suppose that $\alpha^\star(n)$ is the optimal controlling stake given a certain I and consider an increase in the investment requirement to $I' > I$. The initial shareholder can finance I' without changing the controlling stake: simply sell more of his/her own shares to one of the new controlling shareholders. Therefore, if a controlling stake $\alpha^\star(n)$ is optimal for investment requirement I, then it must remain optimal for an investment requirement $I' > I$.

Standard arguments prove that, conditioned on the presence of $n > 1$ controlling shareholders, an optimal controlling stake, $\alpha^\star(n)$, exists. The minimum distortion to the firm value with multiple controlling shareholders is then $D(\alpha^\star(n)) + C(\alpha^\star(n))$, which does not depend on the investment requirement. For a given level I of investment requirement, the optimal ownership structure implies the presence of multiple controlling shareholders if and only if $D(\alpha^\star(n)) + C(\alpha^\star(n)) \leq D(\alpha^\star(1, I))$.

Not surprisingly, if the probability that the project is efficient is sufficiently small, then an ownership structure with multiple controlling shareholders is optimal. The intuition is clear. In this case it is unlikely that the controlling shareholders will reject value increasing projects. Hence, the disagreement costs are negligible ($C(\alpha^\star(n)) \approx 0$). Optimality for an ownership structure with multiple controlling shareholders then follows because, by selling control rights,

[20] Given the probability distribution of the project's payoffs and the value of the existing assets, the larger the investment requirement the larger the fraction of the equity that the initial shareholder will be forced to sell to finance the project.

[21] The irrelevance of the allocation of the controlling shares follows from the assumption that the distribution of the total private benefits of control does not depend on the ownership structure.

the initial shareholder sells fewer shares to minority shareholders. As a result, the controlling stake with multiple controlling shareholders is larger than the controlling stake with a single controlling shareholder ($\alpha^\star(n) > \alpha^\star(1, I)$). By Proposition 4, the dilution cost with multiple controlling shareholders is then lower, implying that $D(\alpha^\star(n)) + C(\alpha^\star(n)) \approx D(\alpha^\star(n)) < D(\alpha^\star(1, I))$.

The level of required financing provides a second (and less obvious) condition for the optimal ownership structure to include multiple controlling shareholders. To get some intuition on this result, suppose that an ownership structure with a single controlling shareholder is optimal at some level I of required financing. As we have already argued, the dilution cost, $D(\alpha^\star(1, I))$, induced by the presence of a single controlling shareholder is an increasing function of the investment requirement. In contrast, the sum of the dilution costs and the disagreement costs does not depend on I in the best ownership structure with multiple controlling shareholders. Under the assumptions of our model, for a sufficiently high level of the investment requirement, the constant sum of the dilution and disagreement costs will eventually fall below the increasing dilution cost associated with an ownership structure with a single controlling shareholder. The first part of the following proposition summarizes the two sufficient conditions for an ownership structure with multiple controlling shareholders to be optimal.

Proposition 6 *(Optimal Ownership Structure) An ownership structure where control is shared dominates an ownership structure with a single controlling shareholder if the financing requirements are large or if there is a high probability that the project is inefficient.*[22] *Moreover, an optimal ownership structure requires the presence of some minority shareholders. Equivalently, sharing control with all shareholders is sub-optimal.*

The second part of Proposition 6 tells us that some shareholders should be deprived of control. This result is somewhat surprising. After all, one might think that, if the controlling shareholders were made to collectively internalize all cash-flows, say by giving veto power to all shareholders, they would eventually achieve an efficient decision. Incomplete information (along with Assumption 2) prevents efficient bargaining, though. As a result, disagreement

[22]Formally, if there exists $\epsilon > 0$ sufficiently small such that $Prob(y + b < 0) > 1 - \epsilon$.

costs arise. The efficiency role of the presence of some minority shareholders is precisely to reduce these disagreement costs. Even though the existence of minority shareholders will give rise to dilution costs, the proof of Proposition 6 shows that this is a second-order effect. The reason that dilution costs do not increase much is due to a new effect introduced in this paper - the benefits of disagreement - which implies that some of the disagreement that the presence of minority shareholders aims to reduce protects those minority shareholders against inefficient dilution.

4 Implementing the Optimal Ownership Structure

4.1 Corporate Governance Mechanisms: Supermajority Rules and Shareholders' Agreements

In Section 3, we have characterized the optimal allocation of control without discussing its implementation. That is the task of this sub-section.

Consider first the case in which the optimal ownership structure has a single controlling shareholder with a controlling stake $\alpha^{\star}(1) \in (0, 1)$. Simple majority and one-share one-vote are clearly enough to give control to an initial shareholder who holds a fraction $\alpha^{\star}(1) > 50\%$ of the shares. If $\alpha^{\star}(1) \leq 50\%$, then the initial shareholder could finance the project while retaining control by issuing a proper number of non-voting shares.[23] More generally, the charter of a corporation can give control to the initial shareholder by assigning a number of votes to each class of shares (only one in the case of one-share one-vote) and establishing a certain supermajority threshold, $\bar{v} \geq 50\%$, for corporate decisions. The assignment of the votes among the classes of shares must be such that, if the shares of the initial shareholders receive the optimal fraction $\alpha^{\star}(1)$ of the firm's verifiable cash-flow, then a fraction $v \geq \bar{v}$ of the votes will be secured. Note, however, that given a fraction $v > 50\%$ of the votes held by the initial shareholder, any supermajority rule $50\% < \bar{v} \leq v$ will give control to the initial shareholder.

We can pin down the supermajority rule that implements the optimal ownership structure

[23] See Zingales (1994) for evidence on the use of dual-class shares in Italy.

by looking at the initial shareholder's incentives to hold $\alpha^\star(1)$ after the investment requirement is raised. Suppose that the initial shareholder owns the optimal fraction $\alpha^\star(1)$ of the firm's verifiable cash-flow and has a fraction $v > \bar{v}$ of the votes. If so, the controlling shareholder can reduce his/her controlling stake without fear of losing control. As we showed in Section 3, a lower controlling stake increases the incentives for diluting minority shareholders. The initial shareholder, thus, has incentives to reduce his/her controlling stake in order to increase private benefits of control at the expense of minority shareholders. We thus conclude that a supermajority rule $\bar{v} < v$ does not implement the optimal ownership structure. The initial shareholder would end up holding a fraction of the verifiable cash-flow that is below $\alpha^\star(1)$. The optimal ownership structure can be implemented only if the supermajority rule adopts a threshold $\bar{v}$ that is equal to the fraction of votes assigned to the optimal controlling stake. With this choice, the controlling shareholder cannot divest without bearing the risk of losing the control.

Consider now an optimal ownership structure that requires the presence of multiple controlling shareholders with a controlling stake $\alpha^\star(n)$. Exactly as in the previous case, control can be given to a group of shareholders by imposing a supermajority rule that ensures to the group a fraction of votes $v = \bar{v} \geq 50\%$ in case they hold the optimal fraction $\alpha^\star(n)$ of the rights of the verifiable cash-flows. The novelty in the case of multiple controlling shareholders stems from the mechanisms that prevent other coalitions of shareholders from amassing the threshold $\bar{v}$ that gives control. As Bennedsen and Wolfezon (1998) point out, some members of the controlling group could be co-opted to form new coalitions that would aim to defeat the remaining controlling shareholders. Also, members of the controlling group might join forces to exclude other members of the group. The winning controlling group would then be determined by these coalition games, and the optimal ownership structure would not necessarily be implemented.

Shareholders' agreements can be written, though, to prevent the exclusion of members of the controlling group as well as defections that could unravel the controlling group. For instance, *pooling or voting agreements* provide that each member of the agreement nominate a certain number of candidates for directorships, and the other members of the agreement are required to cast their votes on them. A proper choice of the supermajority provision, thus, gives effective veto power to each member of the group, avoiding their exclusion. Likewise, voting agreements prevent defections, blocking the formation of new coalitions. Finally, the cohesion of the controlling group can be further fostered by means of *buy-out agreements* that give to group members the right to veto the sale of controlling shares to undesirable investors.

4.2 Dissolving the Controlling Group

In sub-section 4.1, we have argued that supermajority rules and shareholders' agreements ensure that outsiders will not dissolve the controlling group by co-opting some of its members. In this sub-section, we discuss the possibility of an internal buy-out, that is, one of the controlling shareholders acquiring full control.

In principle, nothing prevents one of the controlling shareholders from making an offer to acquire full control in an attempt to eliminate the very same ex-post bargaining problems that protect minority shareholders. To assess this possibility, consider that there is an additional date between the time that the controlling shareholders learn their private benefits and the time that the investment decision has to be made. We ask whether there is an incentive compatible direct mechanism that allows the controlling shareholders to dissolve their partnership with probability 1. If so, ex-post bargaining problems can be solved by a buy-out, and the presence of multiple controlling shareholders would fail to protect minority shareholders.

Proposition 7 below shows that the same asymmetry of information that prevents the controlling shareholders from efficiently agreeing on the investment decision will prevent them from transferring control to one of the controlling shareholders.

Proposition 7 *(Dissolving the controlling group) There is no ex-post efficient mechanism that dissolves the controlling group after the controlling shareholders have privately learned their valuations.*

Proposition 7 departs from Cramtom, Gibbons, and Klemperer (1987), who argue that a partnership can always be efficiently dissolved if its equity holdings are evenly spread across several partners. The way we model the private benefits of control is the key to explaining the difference in the results. In Cramtom et al., the value of the firm to each controlling shareholder is proportional to the fraction of shares that they own. As a result, in an evenly distributed ownership structure, the cost of extracting a truthful announcement from any controlling shareholder decreases with the number of controlling shareholders. In our model, a controlling shareholder may have large private benefits of control in spite of an evenly distributed ownership structure.

5 Evidence and Discussions

5.1 Ownership Structure of Close Corporations in the U.S.

The main testable implication of our model is that the presence of multiple controlling shareholders should be more pervasive in firms where the conflicts of interest between minority and large shareholders are particularly severe and/or the financing requirements are high. A formal test of these implications is beyond the scope of this paper. Nonetheless, some preliminary work on the database of the National Survey of Small Business Finances (NSSBF) of 1992 gives some support to our model.

The 1992 NSSBF database provides ownership and financial data on 4,637 for-profit, non-financial businesses with less than 500 employees.[24] From the 4,637 firms that answered their surveys, NSSBF inferred the characteristics of the 4.99 million small businesses in operation at the end of 1992. Out of the 4,637 firms in the NSSBF database, we included in our sample the 310 close corporations with annual sales above $10 million that had more than one shareholder. We restrict attention to close corporations because a vast literature on corporate law describes the deadlock and dilution problems involving the presence of controlling and minority shareholders in these firms. We thus believe that they constitute a good sample to examine for the ownership structure incentives that we emphasize in this paper.

The first column of table 1 presents summary statistics of our sample. From the 310 firms, it is inferred that there existed 64,708 close corporations in the US in 1992 with more than one shareholder and annual sales above $10 million. The estimate of the average sales of these 64,708 close corporations is $23.3 million, while the average asset value and the average number of

[24]The database includes neither subsidiaries nor farm businesses.

employees are estimated at $7.5 million and 98.0, respectively. The distribution of shareholders in the close corporations is highly skewed. Although the average number of shareholders is 74.4, the median is only 3.0 (31.2% of the firms have 2.0 shareholders and 18.8% have 3.0 shareholders). Figure 2 plots the distribution of the number of shareholders in the sample.

Firms participating in the survey were asked to report, among other things, the ownership stake of the principal shareholder, who is not necessarily the largest one, and the total number of shareholders.[25] Unfortunately, the database does not provide information on the number of shares held by shareholders other than the principal one. We can obtain a lower and an upper bound, respectively, on the equity holdings of the second largest and the smallest shareholder by computing the average equity stake of the shareholders other than the principal one. We use these bounds to infer the ownership structure of the close corporations in the sample.

Following La Porta, Lopez-de-Silanes, and Shleifer (1998), we say that a firm has a large shareholder if there is a shareholder with more than 20% of the shares, and that a firm has multiple large shareholders if the largest shareholder has more than 20% of the shares while a second one has at least 10%. In implementing these criteria in our sample, we classify a firm as having multiple large shareholders if the equity stake of the principal shareholder and the average equity stake of the remaining ones are both larger than 10%, with at least one of them above 20%.[26] A firm has only one large shareholder if the maximum between the equity holdings of the principal one and the average of the other equity stakes is above 20% while the minimum is below 10%. Finally, we say that a firm has minority shareholders if there is no large shareholder or if there is a shareholder whose equity holdings are half of the equity holdings of the largest shareholder. We implement this latter condition by requiring either that the equity holdings of the principal shareholder are twice the average equity holdings, or vice versa.[27]

[25] According to the NSSBF questionnaire, the principal shareholder is "typically the owner who has the largest ownership share and has the primary authority to make financial decisions."

[26] This criterion biases the results against finding multiple large shareholders because the average equity stake may be below 10% even if there exists a shareholder other than the principal one with more than 10% of the shares.

[27] The idea here is that having at least twice the equity stake of another shareholder gives a strong bargaining power to the larger shareholder. The requirement goes both ways because the principal shareholder is not necessarily the largest one.

The last five rows of table 1 provide summary statistics on the ownership structure of the close corporations. Not surprisingly, 86.9% of the firms have at least one large shareholder. Also, 67.7% of the close corporations have minority shareholders, while 54.7% (not reported in the table) of the firms with at least one large shareholder have minority shareholders as well. These numbers suggest that conflicts of interests between large and minority shareholders are potentially relevant.[28] Contrary to the sample of La Porta et al. (1998), the majority of the firms with at least one large shareholder have other large shareholders (65.8%).[29]

Our model predicts that multiple controlling shareholders (here proxied by the large shareholders) should be more pervasive in firms with potentially large conflicts of interest between minority and controlling shareholders. Conceivably, these conflicts should be larger in firms where the large shareholders are also the managers. If so, our model predicts that multiple large shareholders should be less heavily present in the close corporations where the manager is not a large shareholder. The two last columns of table 1 show that this is indeed true. Only 24.9% of the close corporations where the manager is not a shareholder have multiple large shareholders. Conditioned on the existence of a large shareholder, the probability that other large shareholders exist is 36.1%. In contrast, 69.4% of the close corporations where the manager is also a shareholder have multiple large shareholders. Conditioned on the existence of a large shareholder, the probability that another shareholder exists is 74.0%.

The above evidence suggests that large shareholders are more often present in firms where the conflicts of interest are potentially larger. It is possible, however, that the sharp difference in the number of large shareholders in the two sub-samples (firms where the manager is a shareholder and firms where he/she is not) is due to the algorithm that we used to determine the presence of multiple large shareholders. Indeed, table 1 shows that the median number of shareholders is only 3 in firms where the manager is a shareholder, while the median number

[28]One might think that the existence of firms with no minority shareholders contradicts Proposition 6, which predicts that an optimal ownership structure should always include minority shareholders. Nonetheless, recall that Proposition 6 requires Assumption 2. In particular, there must be some uncertainty on the efficiency of the investment. If there is no uncertainty on this regard, then no disagreement among controlling shareholders will exist, making it useless to bring minority shareholders.

[29]In a sample of large public firms in 27 countries, La Porta et al. find that 25% of the firms that are controlled by a large shareholder have other large shareholders.

goes up to 10 in firms where the manager is not a shareholder. It should then be more difficult to find multiple large shareholders in the former sub-sample.[30] To address this possible bias, we computed the probability that multiple large shareholders exist conditioned on the presence of at least one large shareholder *and* the presence of minority shareholders. The last row in table 1 shows that this probability is still substantially smaller for firms where the manager is not a shareholder (20.7% compared to 55.2% for firms where the manager is also a shareholder).[31]

In summary, this very coarse look at the ownership structure of close corporations in the U.S. shows that large shareholders coexist with minority shareholders and that, on average, firms with potentially larger conflicts between large and minority shareholders are more likely to have multiple large shareholders.

5.2 Evidence from the Sports Industry

In the previous subsection we assumed that conflicts of interest between large and minority shareholders are less severe in firms where the manager is not a large shareholder, and then we looked at the probability that multiple large shareholders exist in these firms. Alternatively, one could try to pre-select industries where the conflicts of interest are severe, and then look at the number of large shareholders in the firms in these industries. We pursue this approach in this sub-section.

Demsetz and Lehn (1985) argue that private benefits of control are likely to be important in the sports industry. Indeed, in this industry the hiring of costly stars gives the controlling shareholders a degree of good publicity that is comparable only to the bad publicity following a decision to let their stars go.[32] Conflicts of interest between controlling and minority share-

[30]Nonetheless, the lower number of shareholders may also be interpreted as a response to the larger conflicts imposed by the fact that the manager is also a large shareholder. Anticipating the conflicts, the entrepreneur/manager might have opted for an ownership structure with no minority shareholders. That would be optimal in our model when information problems among the controlling shareholders are not substantial (i.e. when Assumption 2 does not hold).

[31]Since 67.6% of the firms in the sample are classified as family-controlled, the difference in the conditional probabilities could be driven by some characteristic of family businesses reflected in the presence of a manager owner. Nonetheless, the magnitude of the difference in the conditional probabilities in the two sub-samples remains unchanged when we exclude the family-controlled firms from the sample.

[32]For a recent example, the new ownership of the LA Dodgers baseball team was strongly criticized by the media after the team's poor performance in the 1998 season. The criticisms were strengthened by the

holders should be relevant, as well as the chances of overpaying for costly stars. According to our model, firms in the sports industry should then have multiple controlling shareholders.

Consistent with the implications of our model, Demsetz and Lehn show that, in a sample of 121 professional sports clubs in the U.S., the average number of shareholders holding more than 10% of the shares was 2.0 in 1979.[33] Demsetz and Lehn interpret their finding as evidence that a concentrated ownership structure helps prevent the management from destroying important private benefits of control. It is not clear, however, why a large controlling shareholder would need the assistance of other large shareholders to monitor managers of professional teams.[34] Instead, we interpret their finding as evidence that the presence of multiple controlling shareholders limits the dilution of minority shareholders.

5.3 Shareholders' Agreements

In section 4, we argued that shareholders' agreements may be used to give stability to a controlling group. Unfortunately, we do not have data on the use of shareholders' agreements in the U.S. In Italy, however, the CONSOB (the Italian equivalent of the SEC) discloses the shareholders' agreements for all public firms. Table 2 shows that, as of December 1996, 58 of the 303 Italian firms with listed shares (i.e., 19.1%) had some type of shareholder agreement. Restrictions on the sale of shares (buy-out agreements) are the most common ones (17.2%), followed by voting agreements (12.8%), and control agreements (6.6%) that establish policies for the firm. In interpreting the data, though, one should take into account that most Italian public companies have a majority shareholder. Fulghieri and Zingales (1995) report that, in 1990, 53% of the firms listed in the Milan Stock Exchange (by far Italy's largest) had a majority shareholder. Assuming that 53% of the firms in the CONSOB database also have a majority

management's decision not to agree to a multi-year $100 million compensation contract that would have retained the team's star catcher (Mike Piazza). After the season, the same management agreed to pay $105 million over 7 years to acquire a star pitcher (Kevin Brown). To put these numbers in perspective, the market value of the Dodgers when that the acquisition was announced was around $300 million, and the total annual salaries of their players increased from $70 million to $85 million.

[33] They looked at clubs in Major League Baseball, the North American Soccer League, the National Basketball Association, the National Football League, and the National Hockey League.

[34] Indeed, general managers and head coaches in the American professional leagues are known to have a "risky" and relatively (to the players) underpaid job. Owners are not shy about firing them at the first sign of trouble.

shareholder and that these firms do not have shareholders' agreements (they do not need them), then the fraction of firms with no majority shareholder where a shareholders' agreement exists would go up to 40.7%.

¿From table 2, one can also see that the median percentage of shares participating in all types of agreements is larger than 50% but less than 51%. Hence, the group participating in the agreement has the majority of the votes. Consistent with our arguments, shareholders' agreements in Italy seem to be ensuring the stability of control groups with the minimum number of votes needed to retain the control (see Section 5.1).

5.4 Ownership Structure and Law

In our framework, the private benefits and the verifiable cash-flows are uncorrelated. This is consistent, for instance, with non-pecuniary private benefits of control. In countries that offer weak legal protection to minority shareholders, though, private benefits may be financed by the firm's cash-flow. If so, the private benefits are pecuniary and they should be negatively correlated to the firm's "public" cash-flow. We can easily incorporate this negative correlation into our model.

Still assuming that b is the total private benefits of control, let $y - kb$ be the project's net present value from the perspective of the minority shareholders, where $k \in [0, 1]$. In countries that offer weak protection to minority shareholders, k is strictly positive, reflecting the possibility that private benefits are financed by public cash-flows. If so, $1 of private benefits impose a direct loss of $k to minority shareholders. In countries with strong legal protection to the minority shareholders, $k = 0$ and the private benefits of control are better interpreted as being non-pecuniary.

The major results of our paper can be extended to the case where $k > 0$. A trade-off between the disagreement and dilution costs still exists, and a very similar investment decision arises.[35] The effects of a weaker protection to minority shareholders (that is, a higher k) on the optimal ownership structure are ambiguous, though. On one hand, $k > 0$ makes private benefits more

[35] For instance, in the presence of $n > 1$ controlling shareholders, the project will be undertaken if and only if $b + \sum \alpha_i y > (1 - k\alpha_i)\, \lambda(y) \sum_{i=1}^{n} \frac{1-F_i(b_i)}{f_i(b_i)}$.

costly to minority shareholders. In the language of our model, the dilution costs are higher. Accordingly, the incentives for an ownership structure with multiple controlling shareholders increase. On the other hand, a controlling shareholder's ability to capture public cash-flows increases the side payments that another controlling shareholder might request in exchange for not blocking the project. As a result, the probability of disagreement increases when $k > 0$. The larger disagreement costs reduce the incentives for multiple controlling shareholders. The existence of two opposing effects, thus, prevents an unambiguous theoretical relation between a country's legal protection of minority shareholders (parametrized by k) and the optimal number of controlling shareholders.

An empirical study by La Porta, Lopez-de-Silanes, and Shleifer (1998), however, suggests that these two opposing effects may cancel each other. Using a sample of large firms in 27 countries, La Porta et al. find that the probability that a controlling shareholder is alone in family-controlled public firms is 71% for the countries offering weak protection to minority shareholders, while the probability increases to 78% in the countries with stronger protection to minority shareholders. Nonetheless, the difference of means is not statistically significant (t-statistics equal to -1.21).

6 Conclusion

Our paper delivers three main messages. First, minority shareholders benefit when the group in control does not always have common interests. Ex-post bargaining problems associated with the presence of multiple controlling shareholders may avoid an inefficient dilution of the rights of minority shareholders. Evidence on the presence of multiple large shareholders in close corporations in the U.S. supports this hypothesis.

Second, the presence of multiple controlling shareholders creates costs as well. The same bargaining problems that protect minority shareholders may prevent efficient decisions. Firms where a single controlling shareholder might impose large costs on minority shareholders may also bear large costs from the presence of multiple controlling shareholders. As a result, we argued that one might not find multiple controlling shareholders in countries that offer weak protection to minority shareholders. However, high probability that the minority shareholders will be inefficiently diluted and a large financing requirement are two sufficient conditions for including multiple controlling shareholders in the optimal ownership structure.

Third, the presence of minority shareholders reduces disagreement costs among controlling shareholders. As a result, an optimal ownership structure requires the presence of some minority shareholders. In order to ensure their presence, though, mechanisms must be put in place to preserve the stability of the controlling group. We argued that shareholders' agreements and supermajority rules are two such mechanisms.

We finish the paper by posing some questions about the governance of close corporations that have not yet been explored in the corporate finance literature. We have argued in this paper that giving veto power to multiple shareholders creates disagreement costs. Nevertheless, there exist contractual arrangements that reduce these costs, such as,dissolution and and arbitration provisions, both part of the daily routine of corporate lawyers. The effectiveness of these contracts is an empirical question, that may be more easily addressed if we learn more about how these arrangements affect the controlling shareholders' bargaining power.

Appendix

Proof of Proposition 1

Before proving the proposition, we state and prove the following lemma:

Lemma 1 *A direct mechanism* $(x(.), t(.))$ *satisfies the IC constraint if and only if, for any controlling shareholder* i*,*

$$
\begin{aligned}
(IC-1) \quad U_i(y,b_i) &= U_i(y,\underline{b}_i) + \int_{\underline{b}_i}^{b_i} X_i(y,b_i)db_i \qquad (9)\\
(IC-2) \quad X_i(y,b_i) &\geq X_i(y,b_i') \quad \textit{for any } b_i \geq b_i'. \qquad (10)
\end{aligned}
$$

Proof of Lemma 1

(*Necessity*) For any y and $b_i \geq b_i'$, incentive compatibility implies that $U_i(y,b_i) \geq U_i(y,b_i',b_i)$ and $U_i(y,b_i') \geq U_i(y,b_i,b_i')$, which are equivalent to

$$v_i(y,b_i)X_i(y,b_i) + T_i(y,b_i) \geq v_i(y,b_i)X_i(y,b_i') + T_i(y,b_i') \qquad (11)$$

$$v_i(y,b_i')X_i(y,b_i') + T_i(y,b_i') \geq v_i(y,b_i')X_i(y,b_i) + T_i(y,b_i) \qquad (12)$$

Adding equations (12) and (11) yields

$$X_i(y,b_i)[v_i(y,b_i) - v_i(y,b_i')] \geq X_i(y,b_i')[v_i(y,b_i) - v_i(y,b_i')]. \qquad (13)$$

Since $v_i(y,b_i) = b_i + \alpha_i y$ increases with b_i, $b_i \geq b_i' \Rightarrow v_i(y,b_i) \geq v_i(y,b_i')$. Therefore, equation (13) is satisfied if and only if $X_i(y,b_i) \geq X_i(y,b_i')$, proving that monotonicity of $X_i(y,b_i)$ with respect to b_i is implied by the IC constraints. To prove that IC also implies that $U_i(y,b_i) = U_i(y,\underline{b}_i) + \int_{\underline{b}_i}^{b_i} X_i(y,b_i)db_i$, note that, by construction,

$$U_i(y,b_i) = \max_{\hat{b}_i} U_i(y,\hat{b}_i,b_i) \quad \text{for any } b_i.$$

Using the envelope theorem in the above equation yields

$$U_i'(y,b_i) = \frac{\partial U_i(y,\hat{b}_i,b_i)}{\partial b_i}_{|\hat{b}_i=b_i} = \frac{\partial\{v_i(y,b_i)X_i(y,\hat{b}_i)+T_i(y,\hat{b}_i)\}}{\partial b_i}_{|\hat{b}_i=b_i} = X_i(y,b_i).$$

Necessity then follows by integrating $U_i'(y,b_i) = X_i(y,b_i)$ from $\underline{b}_i$ to b_i.

(*Sufficiency*) We want to prove that, for any y and b_i' and b_i satisfying equations (9) and (10), we have $U_i(y,b_i) \geq U_i(y,b_i',b_i)$. For any $b_i \geq b_i'$, equation (9) implies:

$$U_i(y,b_i) - U_i(y,b_i') = \int_{b_i'}^{b_i} X_i(y,b_i)db_i \geq X_i(y,b_i')(b_i - b_i'). \tag{14}$$

where the inequality follows from $X_i(y,b_i) \geq X_i(y,b_i')$ for $b_i \geq b_i'$. We thus have

$$U_i(y,b_i) \geq U_i(y,b_i') + X_i(y,b_i')(b_i - b_i') = U_i(y,b_i') + X_i(y,b_i')(v_i(y,b_i) - v_i(y,b_i')).$$

Sufficiency is established by noting that

$$U_i(y,b_i',b_i) = X_i(y,b_i')v_i(y,b_i) + T_i(y,b_i') = U_i(y,b_i') + X_i(y,b_i')(v_i(y,b_i) - v_i(y,b_i')).$$

This last equation and the previous inequality imply

$$U_i(y,b_i) \geq U_i(y,b_i') + X_i(y,b_i')(v_i(y,b_i) - v_i(y,b_i')) = U_i(y,b_i',b_{-i}).$$

A similar argument proves the claim for $b_i < b_i'$.

□

We now return to the proof of Proposition 1. Plugging $U_i(y,b_i) = E_{-i}[x(y,b_i,b_{-i})v_i(y,b_i)] + T_i(y,b_i)$ into constraint (IC-1) we obtain

$$E_{-i}[x(y,b_i,b_{-i})v_i(y,b_i)] + T_i(y,b_i) = U_i(y,\underline{b}_i) + \int_{\underline{b}_i}^{b_i} X_i(y,b_i)db_i. \tag{15}$$

Taking the expectation (over b_i) on equation (15) and summing over all i yields

$$E[\sum_{i=1}^{n} v_i(y,b_i)x(y,\mathbf{b})] = \sum_{i=1}^{n} U_i(y,\underline{b}_i) + \sum_{i=1}^{n} \int_{\underline{b}_i}^{\bar{b}_i} \int_{\underline{b}_i}^{b_i} X_i(y,b_i')db_i' f_i(b_i)db_i, \tag{16}$$

where we used that $\sum_{i=1}^{n} T_i(y,b_i) = 0$ (to see this, take the conditional expectation on Constraint (BB)). Integrating by parts $\int_{\underline{b}_i}^{\bar{b}_i} \int_{\underline{b}_i}^{b_i} X_i(y,b_i')db_i' f_i(b_i)db_i$ yields $\int_{\underline{b}_i}^{\bar{b}_i}(1-F_i(b_i))X_i(y,b_i)db_i$. This implies

$$E[\sum_{i=1}^{n} v_i(y,b_i)x(y,\mathbf{b})] = \sum_{i=1}^{n} U_i(y,\underline{b}_i) + \sum_{i=1}^{n} \int_{\underline{b}_i}^{\bar{b}_i} (\frac{1-F_i(b_i)}{f_i(b_i)})X_i(y,b_i)f_i(b_i)db_i. \tag{17}$$

The IR constraint implies $U_i(y,b_i) \geq 0$ for any b_i, hence $U_i(y,\underline{b}_i) \geq 0 \Rightarrow \sum_{i=1}^{n} U_i(y,\underline{b}_i) \geq 0$. Using this inequality in equation (17) gives us

$$\int_{\underline{b}_1}^{\bar{b}_1} \cdots \int_{\underline{b}_n}^{\bar{b}_n} \{\sum_{i=1}^{n} [v_i(y,b_i) - (\frac{1-F_i(b_i)}{f_i(b_i)})]x(y,\mathbf{b})\} \prod_{k=1}^{n} f_k(b_k)db_k \geq 0.$$

□

Proof of Proposition 2

For any realization of y, we solve Program 3 ignoring the monotonicity condition and show that the solution of this relaxed program satisfies the constraint that we ignored. Hence, the solution of the relaxed program is also a solution of program 3. Ignoring the monotonicity condition, program 3 can be re-written as

$$\begin{array}{ll} \max_{x(.)} & E[\sum_{i=1}^{n} (b_i + \alpha_i y)\, x(y,\mathbf{b})|y] \\ \text{s.t.} & E\left[\sum_{i=1}^{n} \left(b_i - \frac{1-F_i(b_i)}{f_i(b_i)} + \alpha_i y\right) x(y,\mathbf{b})|y\right] \geq 0, \end{array} \tag{18}$$

Define $J_i(y,b_i,\delta) = b_i - \delta\frac{1-F_i(b_i)}{f_i(b_i)} + \alpha_i y$. Note that $J_i(y,b_i,0) + \mu J_i(y,b_i,1) = (1+\mu)\, J_i(y,b_i,\frac{\mu}{1+\mu})$.

The Lagrangian of Program (18) is then

$$L = E[\{\sum_{i=1}^{n} J_i(y,b_i,0) + \mu J_i(y,b_i,1)\}x(y,\mathbf{b})|y] = (1+\mu)E[\sum_{i=1}^{n} (J_i(y,b_i,\lambda))\, x(y,\mathbf{b})|y],$$

where $\mu \geq 0$ is the Lagrangian multiplier and $\lambda = \frac{\mu}{1+\mu} \in [0,1]$. So, given λ, $x^{\lambda}(.)$ maximizes the Lagrangian if and only if

$$x^{\lambda}(y,\mathbf{b})=\begin{cases}1 & \text{if } \sum_{i=1}^{n} J_i(y,b_i,\lambda)\geq 0 \Leftrightarrow \sum_{i=1}^{n}(b_i+\alpha_i y)\geq \lambda\sum_{i=1}^{n}\frac{1-F_i(b_i)}{f_i(b_i)}\\ 0 & \text{otherwise,}\end{cases}$$

where λ is either 0, if the condition is not binding, or solves the following equation

$$\int\cdots\int\{\sum_{i=1}^{n}[(b_i+\alpha_i y-\frac{1-F_i(b_i)}{f_i(b_i)}]x(y,\mathbf{b})\}\prod_{k=1}^{n}f_k(b_k)db_k=0.$$

To finish the proof of the proposition we show that the monotonicity condition is satisfied by $x^{\lambda}(.)$, that is, $X_i(y,b_i)$ is increasing. To see this, note that $J_i(y,b_i,\lambda)$ is increasing in b_i for any $\lambda\geq 0$ because, by Assumption 1, $\frac{1-F_i(b_i)}{f_i(b_i)}$ decreases with b_i. Hence, for any y and $b_i'\geq b_i$, $J_i(y,b_i',\lambda)\geq J_i(y,b_i,\lambda)$. But this implies that $x^{\lambda}(y,b_i',b_{-i})=1$ if $x^{\lambda}(y,b_i,b_{-i})=1$. Since $x^{\lambda}(.)\in\{0,1\}$, we have just proved that $x^{\lambda}(y,b_i,b_{-i})$ increases in b_i. Monotonicity for $X_i(y,b_i)$ follows by noting that $x^{\lambda}(.)$ increases with b_i for any b_{-i}.

□

Proof of Proposition 3

(Sufficiency) Suppose that $\sum_{i=1}^{n}\underline{b}_i+\alpha_i y<0<\sum_{i=1}^{n}\overline{b}_i+\alpha_i y$ and $\overline{b}_i>\underline{b}_i$ for at least two controlling shareholders. We want to show that the ex-post optimal investment policy is not implementable. As Myerson and Satterthwaite (1983) point out, Proposition 1 makes it sufficient to prove that

$$I=\underset{\sum b_i+\alpha_i y\geq 0}{\int\ldots\int}\sum_{i=1}^{n}\left(b_i+\alpha_i y-\frac{1-F_i(b_i)}{f_i(b_i)}\right)\prod_{k=1}^{n}f(b_k)db_k<0. \tag{19}$$

To save space, we provide a proof that $I<0$ for $n=2$ only. The proof for $n>2$ follows the same line of reasoning and is available from the authors upon request.

Say that $n=2$. Consider the following change of variables: $x_1=-(b_1+\alpha_1 y)\,,x_2=b_2+\alpha_2 y$. Let the density and cumulative distribution of x_i be, respectively, f_i and F_i (by an abuse of notation) with support in the interval $[\underline{x}_i,\overline{x}_i]$ where $\underline{x}_1=-\left(\overline{b}_1+\alpha_1 y\right)$, $\overline{x}_1=-(\underline{b}_1+\alpha_1 y)$,

$\underline{x}_2 = \underline{b}_2 + \alpha_2 y$, and $\overline{x}_2 = \overline{b}_2 + \alpha_2 y$. One can easily check that the assumption of the proposition implies $\underline{x}_2 < \overline{x}_1$ and $\overline{x}_2 < \underline{x}_1$. Using the formula for the integral with a transformation of variables we have that,

$$\begin{aligned}
I &= \int_{\underline{x}_2}^{\overline{x}_2} \int_{\underline{x}_1}^{\min\{x_2,\overline{x}_1\}} ([x_2 - \frac{1-F_2(x_2)}{f_2(x_2)}] - [x_1 + \frac{F_1(x_1)}{f_1(x_1)}]) f_1(x_1) f_2(x_2) dx_1 dx_2 \\
&= \int_{\underline{x}_2}^{\overline{x}_2} \int_{\underline{x}_1}^{\min\{x_2,\overline{x}_1\}} [x_2 f_2(x_2) + F_2(x_2) - 1] f_1(x_1) dx_1 dx_2 \\
&\quad - \int_{\underline{x}_2}^{\overline{x}_2} \int_{\underline{x}_1}^{\min\{x_2,\overline{x}_1\}} [x_1 f_1(x_1) + F_1(x_1)]) dx_1 f_2(x_2) dx_2 \\
&= \int_{\underline{x}_2}^{\overline{x}_2} [x_2 f_2(x_2) + F_2(x_2) - 1] F_1(x_2) dx_2 \\
&\quad - \int_{\underline{x}_2}^{\overline{x}_2} \min\{x_2 F_1(x_2), \overline{x}_1\} f_2(x_2) dx_2 \\
&= \int_{\underline{x}_2}^{\overline{x}_2} (F_2(x_2) - 1) F_1(x_2) dx_2 + \int_{\overline{x}_1}^{\overline{x}_2} [x_2 - \overline{x}_1] f_2(x_2) dx_2 \\
&= \int_{\underline{x}_2}^{\overline{x}_2} (F_2(x_2) - 1) F_1(x_2) dx_2 - \int_{\overline{x}_1}^{\overline{x}_2} (F_2(x_2) - 1) dx_2 \\
&= \int_{\underline{x}_2}^{\overline{x}_1} (F_2(x_2) - 1) F_1(x_2) dx_2 < 0
\end{aligned}$$

because $\underline{x}_2 < \overline{x}_1$.

(Necessity) Say that $\sum_{i=1}^n \underline{b}_i + \alpha_i y \geq 0$, then the ex-post investment policy is tp invest always and $I = \sum_{i=1}^n \left(E\left[b_i + \alpha_i y\right] - \int_{\underline{b}_i}^{\overline{b}_i} (1 - F_i(b_i))\, db_i \right)$. An integration by parts yields $\int_{\underline{b}_i}^{\overline{b}_i} (1 - F_i(b_i))\, db_i = E\left[b_i\right] - \underline{b}_i$. Therefore, $I = \sum_{i=1}^n \underline{b}_i + \alpha_i y \geq 0$. By Proposition 1, the ex-post efficient investment decision is then implementable. Assume now that $\sum_{i=1}^n \overline{b}_i + \alpha_i y \leq 0$. We show that the ex-post investment policy (never invest) is implementable. If $\sum_{i=1}^n \overline{b}_i + \alpha_i y \leq 0$, then $\Pr\left(\sum_{i=1}^n b_i + \alpha_i y \geq 0 | y\right) = 0$. But this implies $I = 0$.

Finally, suppose that $\overline{b}_i = \underline{b}_i = b_i$ for $i = 2, ..., n$. Consider the following transfers

$$t_i = \begin{cases} -\left(b_i + \alpha_i y\right) & \text{if } i \neq 1, b_i + \alpha_i y < 0 \text{ and } \sum_{i=1}^n b_i + \alpha_i y \geq 0 \\ 0 & \text{if } i \neq 1, b_i + \alpha_i y \geq 0 \text{ or } \sum_{i=1}^n b_i + \alpha_i y < 0 \\ -\sum_{i=2}^n t_i & \text{for } i = 1. \end{cases}$$

One can easily check that, given the above transfers, a policy of investing if and only if $\sum b_i + \alpha_i y \geq 0$ is individually rational and induces truthtelling from shareholder $i = 1$.

Moreover, the investment policy is ex-post optimal.

□

Proof of Proposition 4

By looking at the definitions of $\mathcal{D}(\alpha(1))$ and $\mathcal{D}(\alpha(n))$, one can easily check that $\lambda(y) \geq 0$ implies $\mathcal{D}(\alpha(n)) \subset \mathcal{D}(\alpha(1))$. We now show that $\mathcal{B}(\alpha(n)) = \mathcal{D}(\alpha(1)) - \mathcal{D}(\alpha(n)) \neq \emptyset$. Let $\underline{\alpha} = -\overline{b}/\underline{y} > 0$ so that $\underline{b} + \underline{\alpha}\underline{y} = 0$. Consider any $\alpha > \underline{\alpha}$ and $y > \underline{y}$ such that $\underline{b} + \alpha y < 0 < \overline{b} + \alpha y$. Then, by Proposition 3, $\lambda(y) > 0$, which implies that the following set is open and non-empty:

$$V = \left\{ (b, y) : \underline{b} + \alpha y < 0 < \overline{b} + \alpha y \text{ and } 0 < b + \alpha y < \lambda(y) \sum_{i=1}^{n} \frac{1 - F_i(b_i)}{f_i(b_i)} \right\}.$$

Furthermore, $V \subset \mathcal{D}(\alpha(1))$ and $V \cap \mathcal{D}(\alpha(n)) = \emptyset$. Finally, since the densities of the distributions of b and y are strictly positive then $B(\alpha(n)) = -E\left[(y + b)\mathcal{X}_{\mathcal{B}(\alpha(n))}\right] > 0$.

We now prove the monotonicity of the dilution cost for the firm with one controlling shareholder. Consider first a firm with a single controlling shareholder. In this case, the fraction of shares held by minority shareholders is entirely determined by the financing requirement, I. The larger the financing requirement the larger the fraction α_0 sold to the minority shareholders and the smaller the fraction $\alpha_1 = 1 - \alpha_0$ held by the controlling shareholder. Since $\mathcal{D}(I) = \{(b, y) : \alpha_1 y + b \geq 0 \text{ and } y + b < 0\}$, we have that $\mathcal{D}(I') \supseteq \mathcal{D}(I)$, for $I' > I$, implying that $D(I') \geq D(I)$. The cost of dilution is thus increasing in I.

To prove the monotonicity of the dilution cost for multiple controlling shareholders, let $n > 1$ be the number of controlling shareholders. We model a change in the equity holdings of the minority shareholders by varying $t \in (0, 1]$ in $\alpha(t) = (1 - t, t\alpha_1, ..., t\alpha_n)$, where $\sum_{i=1}^{n} \alpha_i = 1$ and $1 - t$ is the minority stake. To simplify the notation we will omit the realization of y in what follows and we call $\lambda(t)$ the Lagrangian multiplier associated with the solution of Program 3 for a given $\alpha(t)$. The cost of dilution given $\alpha(t)$ is

$$D(t) = -\int \cdots \int_{c(t,\lambda(t))>0 \;\&\; y+b<0} (y + b) \prod_{i=1}^{n} f_i(b_i) db_i g(y) dy.$$

where

$$c(t,\lambda) = b + ty - \lambda(t)\left(\sum_{i=1}^{n}\frac{1-F_i(b_i)}{f_i(b_i)}\right).$$

By a generalization of the Leibniz rule for an integral of several variables we have that

$$\frac{dD(t)}{dt} = \int\limits_{c(t,\lambda(t))=0\ \&\ y+b<0} ... \int (y+b)\frac{-\frac{dc(t,\lambda(t))}{dt}}{\parallel grad_b c \parallel}\prod_{i=1}^{n} f_i(b_i)db_i g(y)dy. \tag{20}$$

where $\parallel grad_b c \parallel$ is the norm of the derivative of the function $c(.)$ with respect to b.

For $y \geq 0$, $\sum_{i=1}^{n} \bar{b}_i + \alpha_i(t)y \geq 0$ for any t. Thus, from Proposition 3, $\lambda(t) = 0$ for any t and $\frac{dD(t)}{dt}$ is equal to zero because $b + ty > 0$ implies that $c(t,\lambda) = 0$ cannot happen. We can then restrict attention to $y < 0$. We will show that $\lambda(t)$ is non-decreasing in t, which implies that

$$\frac{dc(t,\lambda(t))}{dt} = y - \frac{d\lambda(t)}{dt}\sum_{i=1}^{n}\frac{1-F_i(b_i)}{f_i(b_i)} \leq 0. \tag{21}$$

For given y and t, $\lambda(t)$ is defined by the unique solution of $G(t,\lambda(t)) = 0$ where

$$G(t,\lambda) = \int\limits_{c(t,\lambda)\geq 0} ... \int \left(b + ty - \sum_{i=1}^{n}\frac{1-F_i(b_i)}{f_i(b_i)}\right)\prod_{i=1}^{n} f_i(b_i)db_i.$$

Note that $\frac{\partial c(t,\lambda)}{\partial \lambda} = -\sum_{i=1}^{n}\frac{1-F_i(b_i)}{f_i(b_i)} < 0$, which implies that $\frac{\partial G(t,\lambda(t))}{\partial \lambda} > 0$ (by an application of the Leibniz rule). We now prove that $\frac{\partial G(t,\lambda)}{\partial t} < 0$. The differential $\frac{\partial G(t,\lambda)}{\partial t}$ is equal to

$$\begin{aligned}\frac{\partial G(t,\lambda)}{\partial t} &= y\int\limits_{c(t,\lambda)\geq 0} ... \int \prod_{i=1}^{n} f_i(b_i)db_i \\ &\quad + \int\limits_{c(t,\lambda)=0} ... \int \left(b + ty - \sum_{i=1}^{n}\frac{1-F_i(b_i)}{f_i(b_i)}\right)\frac{-\frac{\partial c(t,\lambda)}{\partial t}}{\parallel grad_b c \parallel}\prod_{i=1}^{n} f_i(b_i)db_i\end{aligned}$$

For $c(t,\lambda) = 0$, $\lambda(t) \in [0,1]$ implies $b + ty - \sum_{i=1}^{n}\frac{1-F_i(b_i)}{f_i(b_i)} \leq 0$. Using this last inequality and $\frac{\partial c(t,\lambda)}{\partial t} = y < 0$ we have that $\frac{\partial G(t,\lambda)}{\partial t} < 0$.

Applying the implicit function theorem on $G(t,\lambda(t)) = 0$ yields $\frac{d\lambda}{dt} = -\frac{\partial G(t,\lambda)}{\partial t}/\frac{\partial G(t,\lambda)}{\partial \lambda} \geq 0$, proving that $\lambda(t)$ is non-decreasing in t and so proving that $\frac{dc(t,\lambda(t))}{dt} \leq 0$ (see inequality 21).

Using this inequality in equation (20) gives us $\frac{dD(t)}{dt} \leq 0$.

□

Proof of Proposition 5

Obviously, the cost of disagreement for a firm with one controlling shareholder is zero. For an ownership structure with multiple large shareholders and no minority shareholders, $\sum_{i=1}^{n} \alpha_i = 1$ the set of states where the project is inefficiently rejected is

$$\mathcal{C} = \left\{ (b, y) : 0 < b + y < \lambda \sum_{i=1}^{n} \frac{1 - F_i(b_i)}{f_i(b_i)} \right\}$$

Under Assumption 2, $\lambda > 0$ which then implies that $\mathcal{C} \neq \emptyset$. The fact that the densities of b and y are strictly positive then implies that $\Pr(\mathcal{C}) > 0$. For $\alpha(n)$ such that $\alpha = \sum_{i=1}^{n} \alpha_i < 1$, the disagreement set is $\mathcal{C}(\alpha(n)) = \left\{ (y, b) : b + \alpha y < \lambda \sum_{i=1}^{n} \frac{1-F_i(b_i)}{f_i(b_i)} \text{ and } y + b > 0 \right\}$. Continuity of λ then implies that $\Pr(\mathcal{C}(\alpha(n))) > 0$ for $\alpha > \underline{\alpha}$ for an $\underline{\alpha} < 1$ close enough to 1.

We now prove the monotonicity of the cost of disagreement. The cost of disagreement given $\alpha(t) = (1 - t, t\alpha_1, ..., t\alpha_n)$ is

$$C(t) = \int_{c(t,\lambda(t))<0 \ \& \ y+b\geq 0} ... \int (y + b) \prod_{i=1}^{n} f_i(b_i) db_i g(y) dy,$$

where $c(t, \lambda) = b + ty - \lambda(t) \left(\sum_{i=1}^{n} \frac{1-F_i(b_i)}{f_i(b_i)} \right)$. Again, by a generalization of the Leibniz rule for an integral of several variables we have that

$$\frac{dC(t)}{dt} = \int_{c(t,\lambda(t))=0 \ \& \ y+b\geq 0} ... \int (y + b) \frac{-\frac{dc(t,\lambda(t))}{dt}}{\| \, grad_b c \, \|} \prod_{i=1}^{n} f_i(b_i) db_i g(y) dy. \tag{22}$$

¿From inequality (21) derived above, equation (22) implies that $\frac{dC(t)}{dt} \geq 0$.

□

Proof of Proposition 6

(Financing Requirement) For a firm with $n > 1$ controlling shareholders, we show that the cost of disagreement and the cost of dilution depend on the total control stake $\alpha = \sum \alpha_i$ but not

on how the stake is distributed across the controlling shareholders. As a result, the firm value will not depend on the financing requirement. In contrast, the value of a firm with a single controlling shareholder decreases with the financing requirement. Under a mild condition, the value of the firm with multiple controlling shareholders will then be higher than the value with a single controlling shareholder for a sufficiently large financing requirement.

So, fix the control stake α in a firm with multiple controlling shareholders. Recall that the cost of dilution and the cost of disagreement are, respectively, $D = E\left[(y+b)\,\mathcal{X}_{\mathcal{D}}\right]$ and $C = E\left[(y+b)\,\mathcal{X}_{\mathcal{C}}\right]$, where $\mathcal{C} = \left\{(b,y) : x^{\lambda} = 0 \text{ and } y+b>0\right\}$ and $\mathcal{D} = \left\{(b,y) : x^{\lambda} = 1 \text{ and } y+b<0\right\}$. Thus, given y, the costs of dilution and disagreement are determined by the optimal investment policy x^{λ} which, in turn, depends on α and λ. Since λ is given by $E[(b+\alpha y - \frac{1-F_i(b_i)}{f_i(b_i)})x^{\lambda}(y,\mathbf{b})] = 0$, we conclude that, given y, x^{λ} depends on α only. Therefore, we have just proved that the dilution and the disagreement costs depend only on the control stake.

Now, for a fixed control stake α, a larger financing requirement can be accommodated by the initial shareholder selling a larger stake to the other controlling shareholders. Hence, an increase in the financing requirement does not need to change the control stake. As a result, the solution of the minimization problem 8 does not depend on the financing requirement I. Let us call α^* the size of the control stake associated with the ownership structure that solves problem 8 conditioned on $n>1$ controlling shareholders. The value of the firm with multiple controlling shareholders is then independent of I and is given by $E\left[(b+y)\,x^{\lambda(\alpha^*)}\right]$, where $x^{\lambda(\alpha^*)}$ is the investment policy associated to the optimal control stake α^*. Note that $E\left[(b+y)\,x^{\lambda(\alpha^*)}\right] > 0$ because $E\left[(b+y)\,x^{\lambda(\alpha^*)}\right] \geq E\left[(b+y)\,x^{\lambda(1)}\right] > 0$, where $x^{\lambda(1)}$ is the investment policy that arises when 100% of the firm is given to the controlling shareholders.

Consider now a firm with a single controlling shareholder. By Proposition 4, the value of this firm (more precisely, the value subtracted from V_0+I) is decreasing in the investment level, with an upper bound of $E\left[\max(b+y,0)\right]$ when $I=0$, and a lower bound of $E\left[b+y\right]$, for a large enough I. Therefore, for a large enough financing requirement, the ownership with multiple controlling shareholders is optimal whenever $E\left[b+y\right] < E\left[(b+y)\,x^{\lambda(\alpha^*)}\right]$. This inequality holds, for example, if $E\left[b+y\right] \leq 0$, or if there is not much uncertainty with respect to the private benefits while the project may still be inefficient with some positive probability. In this latter case, the disagreement costs are close to zero regardless of the number of minority shareholders. Hence, the first best is almost obtained by giving 100% of the shares to the controlling shareholders, that is, $E\left[(b+y)\,x^{\lambda(1)}\right] \approx E\left[\max(b+y,0)\right]$. The positive probability that the project is inefficient, though, induces dilution costs when the controlling stake is small enough. We thus have $E\left[b+y\right] < E\left[\max(b+y,0)\right] \approx E\left[(b+y)\,x^{\lambda(1)}\right] \leq E\left[(b+y)\,x^{\lambda(\alpha^*)}\right]$.

(Probability of Inefficiency) We now show that an ownership structure with multiple controlling shareholders dominates an ownership structure with a single controlling shareholder if there is a high probability that the project is inefficient. Suppose that $\Pr\left(y+b>0 \mid y\right)=0$. Then an ownership structure with no minority shareholders ($\alpha=1$) achieves the first best (never invest). In contrast, an ownership structure with a single controlling shareholder requires the presence of minority shareholders. Inefficiency may then result due to the dilution costs. By continuity, an ownership structure with multiple controlling shareholders dominates an ownership structure with a single controlling shareholder for $\Pr\left(y+b>0 \mid y\right)<\epsilon$, if $\epsilon>0$ is small enough.

(Minority Shareholders) To complete the proof of the proposition, we show that a firm with multiple controlling shareholders but no minority shareholders can increase value by selling some shares to minority shareholders. In the proof of Propositions 4 and 5, we evaluated the derivatives of the disagreement costs ($\frac{dC(t)}{dt}$) and the dilution costs ($\frac{dD(t)}{dt}$) with respect to the control stake t (expressions (22) and (20), respectively). In the absence of minority shareholders, $t=1$, one can easily check that $c\left(1,\lambda\right)=0$ & $y+b<0$ is an empty set. It then follows that $\frac{dD(t)}{dt}=0$ at $t=1$, that is, the increase of dilution costs upon a small increase of minority shares is of second order if we started with no minority shareholders. In contrast, $c\left(1,\lambda\right)=0$ & $y+b\geq 0$ is a non empty set. In particular, it is non empty for $y<0$ as well. In this region, $\frac{\partial c(t,\lambda)}{\partial t}=y<0$ for any t, implying that $\frac{dC(t)}{dt}>0$ at $t=1$ if assumption 2 holds. Hence, the decrease of disagreement costs is of first order for a small sale of shares to minority shareholders. This shows that if the controlling shareholders sell a small fraction of shares to minority shareholders, then the amount of inefficiency is reduced, that is, $C\left(1-\epsilon\right)+D\left(1-\epsilon\right)<C\left(1\right)+D\left(1\right)$.

□

Proof of Proposition 7

Assume by absurd that, for any (b, y), there exists a controlling shareholder $i(b, y)$ who can successfully acquire full control paying $\{t_j(b, y)\}_{j \neq i(b,y)}$ (where we allow the payments to be contingent on the actual private benefits of all controlling shareholders as in a direct mechanism) to the other controlling shareholders. As the single controlling shareholder, he or she would internalize all of the private benefits, investing if and only if $b + \alpha y > 0$, where $\alpha \equiv \sum_{i=1}^{n} \alpha_i$. If so, the outcome of the sale can be replicated by a direct mechanism that sets $x(b, y) = 1$ if and only if $b + \alpha y > 0$, with transfers $t_j(b, y)$ for $j \neq i(b, y)$, and $t_i(b, y) = -\sum_{j \neq i(b,y)} t_j(b, y)$. But then the direct mechanism would be ex-post efficient, contradicting Proposition 3. We conclude that the controlling shareholders cannot always agree on dissolving the controlling group.

□

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Appendix to the Referee: Proof of Proposition 3

We show that, for $n \geq 2$,

$$I = \underset{\sum b_i + \alpha_i y \geq 0}{\int ... \int} \sum_{i=1}^{n} \left(b_i + \alpha_i y - \frac{1 - F_i(b_i)}{f_i(b_i)} \right) \prod_{k=1}^{n} f(b_k) db_k < 0 \tag{23}$$

if $\sum_{i=1}^{n} \underline{b}_i + \alpha_i y < 0 < \sum_{i=1}^{n} \overline{b}_i + \alpha_i y$ and $\overline{b}_i > \underline{b}_i$ for at least two controlling shareholders.

Consider the following change of variables: $x_1 = -(b_1 + \alpha_1 y)$, and $x_i = b_i + \alpha_i y$ for $i = 2, ..., n$. Let the density and cumulative distribution of x_i be, respectively, f_i and F_i (by an abuse of notation) with support in the interval $[\underline{x}_i, \overline{x}_i]$ where $\overline{x}_1 = -(\underline{b}_1 + \alpha_1 y)$, $\underline{x}_1 = -\left(\overline{b}_1 + \alpha_1 y\right)$, $\underline{x}_i = \underline{b}_i + \alpha_i y$, and $\overline{x}_i = \overline{b}_i + \alpha_i y$ for $i = 2, ..., n$.

Thus

$I = \int_{\underline{x}_n}^{\overline{x}_n} \int_{\underline{x}_{n-1}}^{\overline{x}_{n-1}} \cdots \int_{\underline{x}_1}^{\min\{x_2 + \cdots + x_n, \overline{x}_1\}} (\sum_{i=2}^{n} [x_i - \frac{1 - F_i(x_i)}{f_i(x_i)}] - [x_1 + \frac{F_1(x_1)}{f_1(x_1)}]) \prod_{k=1}^{n} f_k(x_k) dx_k$
$= \sum_{i=2}^{n} \int_{\underline{x}_n}^{\overline{x}_n} \int_{\underline{x}_{n-1}}^{\overline{x}_{n-1}} \cdots \int_{\underline{x}_1}^{\min\{x_2 + \cdots + x_n, \overline{x}_1\}} [x_i f_i(x_i) + F_i(x_i) - 1] f_1(x_1) \prod_{k=2, k \neq i}^{n} f_k(x_k) dx_k$
$- \int_{\underline{x}_n}^{\overline{x}_n} \int_{\underline{x}_{n-1}}^{\overline{x}_{n-1}} \cdots \int_{\underline{x}_1}^{\min\{x_2 + \cdots + x_n, \overline{x}_1\}} [x_1 f_1(x_1) + F_1(x_1)]) \prod_{k=2}^{n} f_k(x_k) dx_k$
$= \sum_{i=2}^{n} \int_{\underline{x}_n}^{\overline{x}_n} \int_{\underline{x}_n}^{\overline{x}_{n-1}} \cdots \int_{\underline{x}_2}^{\overline{x}_2} [F_i(x_i) - 1] F_1(x_2 + \cdots + x_n) \prod_{k=2, k \neq i}^{n} f_k(x_k) dx_k$
$+ \sum_{i=2}^{n} \int_{\underline{x}_n}^{\overline{x}_n} \int_{\underline{x}_{n-1}}^{\overline{x}_{n-1}} \cdots \int_{\underline{x}_2}^{\overline{x}_2} x_i F_1(x_2 + \cdots + x_n) \prod_{k=2}^{n} f_k(x_k) dx_k$
$- \int_{\underline{x}_n}^{\overline{x}_n} \int_{\underline{x}_{n-1}}^{\overline{x}_{n-1}} \cdots \int_{\underline{x}_2}^{\overline{x}_2} \min\{(x_2 + \cdots + x_n) F_1(x_2 + \cdots + x_n), \overline{x}_1\} \prod_{k=2}^{n} f_k(x_k) dx_k$

Without any loss of generality, we can assume that $\overline{x}_1 \geq 0$ implying that $\min\{(x_2 + \cdots + x_n) F_1(x_2 + \cdots + x_n), \overline{x}_1\} = (x_2 + \cdots + x_n) F_1(x_2 + \cdots + x_n)$ if $(x_2 + \cdots + x_n) \leq c_1$ (if $\overline{x}_1 < 0$, then the linear transformations $x_2^{'} = x_2 + |\overline{x}_1|$ and $x_1^{'} = x_1 + |\overline{x}_1|$ would not change the value of the integral I and $x_1^{'} \geq 0$).

Thus,

$I = \sum_{i=2}^{n} \int_{\underline{x}_n}^{\overline{x}_n} \int_{\underline{x}_{n-1}}^{\overline{x}_{n-1}} \cdots \int_{\underline{x}_2}^{\overline{x}_2} [F_i(x_i) - 1] F_1(x_2 + \cdots + x_n) \prod_{k=2, k \neq i}^{n} f_k(x_k) dx_k$
$+ \int_{\underline{x}_n}^{\overline{x}_n} \int_{\underline{x}_{n-1}}^{\overline{x}_{n-1}} \cdots \int_{\underline{x}_2}^{\overline{x}_2} \max\{x_2 + \cdots + x_n - \overline{x}_1, 0\} \prod_{k=2}^{n} f_k(x_k) dx_k =$
$= \sum_{i=2}^{n} \int_{\underline{x}_n}^{\overline{x}_n} \int_{\underline{x}_{n-1}}^{\overline{x}_{n-1}} \cdots \int_{\underline{x}_2}^{\overline{x}_2} [F_i(x_i) - 1] F_1(x_2 + \cdots + x_n) \prod_{k=2, k \neq i}^{n} f_k(x_k) dx_k$
$+ \underset{x_2 + \cdots + x_n \geq \overline{x}_1}{\int \cdots \int} (x_2 + \cdots + x_n - \overline{x}_1) \prod_{k=2}^{n} f_k(x_k) dx_k$

We will first show in several steps that the following equation holds:

$$\int \cdots \int_{x_2+\cdots+x_n \geq \overline{x}_1} (x_2+\cdots+x_n-\overline{x}_1) \prod_{k=2}^{n} f_k(x_k)dx_k \tag{24}$$
$$= -\int \cdots \int_{x_2+\cdots+x_n \geq \overline{x}_1} (F_2(x_2)-1)dx_2 \prod_{k=3}^{n} f_k(x_k)dx_k$$
$$+ \int \cdots \int_{x_3+\cdots+x_n \geq \overline{x}_1-\underline{x}_2} (x_3+\cdots+x_n-(\overline{x}_1-\underline{x}_2))dx_2 \prod_{k=3}^{n} f_k(x_k)dx_k$$

First note that:

$\int \cdots \int_{x_2+\cdots+x_n \geq \overline{x}_1} (x_2+\cdots+x_n-\overline{x}_1) \prod_{k=2}^{n} f_k(x_k)dx_k =$
$= \int \cdots \int_{x_3+\cdots+x_n \geq \overline{x}_1-\overline{x}_2} \int_{\overline{x}_1-(x_3+\cdots+x_n)}^{\overline{x}_2} (x_2-\overline{x}_1) \prod_{k=2}^{n} f_k(x_k)dx_k$
$+ \int \cdots \int_{x_3+\cdots+x_n \geq \overline{x}_1-\overline{x}_2} \int_{\overline{x}_1-(x_3+\cdots+x_n)}^{\overline{x}_2} (x_3+\cdots+x_n) \prod_{k=2}^{n} f_k(x_k)dx_k =$
$= \int \cdots \int_{x_3+\cdots+x_n \geq \overline{x}_1-\overline{x}_2} (\int_{\overline{x}_1-(x_3+\cdots+x_n)}^{\overline{x}_2} (x_2-\overline{x}_1)f_2(x_2)dx_2) \prod_{k=3}^{n} f_k(x_k)dx_k$
$+ \int \cdots \int_{x_3+\cdots+x_n \geq \overline{x}_1-\overline{x}_2} (\int_{\overline{x}_1-(x_3+\cdots+x_n)}^{\overline{x}_2} (x_3+\cdots+x_n)f_2(x_2)dx_2) \prod_{k=3}^{n} f_k(x_k)dx_k =$
$= \int \cdots \int_{x_3+\cdots+x_n \geq \overline{x}_1-\overline{x}_2} (x_2-\overline{x}_1)F_2(x_2)|_{\overline{x}_1-(x_3+\cdots+x_n)}^{\overline{x}_2} \prod_{k=3}^{n} f_k(x_k)dx_k$
$- \int \cdots \int_{x_2+\cdots+x_n \geq \overline{x}_1} F_2(x_2)dx_2 \prod_{k=3}^{n} f_k(x_k)dx_k$
$+ \int \cdots \int_{x_3+\cdots+x_n \geq \overline{x}_1-\overline{x}_2} (x_3+\cdots+x_n)F_2(x_2)|_{\overline{x}_1-(x_3+\cdots+x_n)}^{\overline{x}_2} \prod_{k=3}^{n} f_k(x_k)dx_k =$
$= \int \cdots \int_{x_3+\cdots+x_n \geq \overline{x}_1-\overline{x}_2} (\overline{x}_2-\overline{x}_1)+(x_3+\cdots+x_n)F_2(\overline{x}_1-(x_3+\cdots+x_n)) \prod_{k=3}^{n} f_k(x_k)dx_k$
$- \int \cdots \int_{x_2+\cdots+x_n \geq \overline{x}_1} F_2(x_2)dx_2 \prod_{k=3}^{n} f_k(x_k)dx_k$
$+ \int \cdots \int_{x_3+\cdots+x_n \geq \overline{x}_1-\overline{x}_2} (x_3+\cdots+x_n)(1-F_2(\overline{x}_1-(x_3+\cdots+x_n)) \prod_{k=3}^{n} f_k(x_k)dx_k =$
$= \int \cdots \int_{x_3+\cdots+x_n \geq \overline{x}_1-\overline{x}_2} (\overline{x}_2-\overline{x}_1+x_3+\cdots+x_n) \prod_{k=3}^{n} f_k(x_k)dx_k$
$- \int \cdots \int_{x_2+\cdots+x_n \geq \overline{x}_1} F_2(x_2)dx_2 \prod_{k=3}^{n} f_k(x_k)dx_k =$
$= \int \cdots \int_{x_3+\cdots+x_n \geq \overline{x}_1-\overline{x}_2} \int_{\overline{x}_1-(x_3+\cdots+x_n)}^{\overline{x}_2} dx_2 \prod_{k=3}^{n} f_k(x_k)dx_k$
$- \int \cdots \int_{x_2+\cdots+x_n \geq \overline{x}_1} F_2(x_2)dx_2 \prod_{k=3}^{n} f_k(x_k)dx_k$
$= \int \cdots \int_{\overline{x}_1-\underline{x}_2 \geq x_3+\cdots+x_n \geq \overline{x}_1-\overline{x}_2} \int_{\overline{x}_1-(x_3+\cdots+x_n)}^{\overline{x}_2} dx_2 \prod_{k=3}^{n} f_k(x_k)dx_k$
$+ \int \cdots \int_{x_3+\cdots+x_n \geq \overline{x}_1-\underline{x}_2} \int_{\overline{x}_1-(x_3+\cdots+x_n)}^{\overline{x}_2} dx_2 \prod_{k=3}^{n} f_k(x_k)dx_k$
$- \int \cdots \int_{x_2+\cdots+x_n \geq \overline{x}_1} F_2(x_2)dx_2 \prod_{k=3}^{n} f_k(x_k)dx_k =$
$= \int \cdots \int_{\overline{x}_1-\underline{x}_2 \geq x_3+\cdots+x_n \geq \overline{x}_1-\overline{x}_2} \int_{\overline{x}_1-(x_3+\cdots+x_n)}^{\overline{x}_2} dx_2 \prod_{k=3}^{n} f_k(x_k)dx_k$

$+ \underset{x_3+\cdots+x_n \geq \overline{x}_1-\underline{x}_2}{\int\cdots\int} \int_{\underline{x}_2}^{\overline{x}_2} dx_2 \prod_{k=3}^n f_k(x_k)dx_k$
$+ \underset{x_3+\cdots+x_n \geq \overline{x}_1-\underline{x}_2}{\int\cdots\int} \int_{\overline{x}_1-(x_3+\cdots+x_n)}^{\underline{x}_2} dx_2 \prod_{k=3}^n f_k(x_k)dx_k$
$- \underset{x_2+\cdots+x_n \geq \overline{x}_1}{\int\cdots\int} F_2(x_2)dx_2 \prod_{k=3}^n f_k(x_k)dx_k.$
But since,

$$\underset{\overline{x}_1-\underline{x}_2 \geq x_3+\cdots+x_n \geq \overline{x}_1-\overline{x}_2}{\int\cdots\int} \int_{\overline{x}_1-(x_3+\cdots+x_n)}^{\overline{x}_2} dx_2 \prod_{k=3}^n f_k(x_k)dx_k$$
$$+ \underset{x_3+\cdots+x_n \geq \overline{x}_1-\underline{x}_2}{\int\cdots\int} \int_{\underline{x}_2}^{\overline{x}_2} dx_2 \prod_{k=3}^n f_k(x_k)dx_k = \underset{x_2+\cdots+x_n \geq \overline{x}_1}{\int\cdots\int} dx_2 \prod_{k=3}^n f_k(x_k)dx_k,$$

by substituting this equation into 24 yields

$$\underset{x_2+\cdots+x_n \geq \overline{x}_1}{\int\cdots\int} (x_2+\cdots+x_n-\overline{x}_1) \prod_{k=2}^n f_k(x_k)dx_k = \underset{x_2+\cdots+x_n \geq \overline{x}_1}{\int\cdots\int} dx_2 \prod_{k=3}^n f_k(x_k)dx_k$$
$$+ \underset{x_3+\cdots+x_n \geq \overline{x}_1-\underline{x}_2}{\int\cdots\int} \int_{\overline{x}_1-(x_3+\cdots+x_n)}^{\underline{x}_2} dx_2 \prod_{k=3}^n f_k(x_k)dx_k$$
$$- \underset{x_2+\cdots+x_n \geq \overline{x}_1}{\int\cdots\int} F_2(x_2)dx_2 \prod_{k=3}^n f_k(x_k)dx_k$$
$$= - \underset{x_2+\cdots+x_n \geq \overline{x}_1}{\int\cdots\int} (F_2(x_2)-1)dx_2 \prod_{k=3}^n f_k(x_k)dx_k$$
$$+ \underset{x_3+\cdots+x_n \geq \overline{x}_1-\underline{x}_2}{\int\cdots\int} \int_{\overline{x}_1-(x_3+\cdots+x_n)}^{\underline{x}_2} dx_2 \prod_{k=3}^n f_k(x_k)dx_k$$
$$= - \underset{x_2+\cdots+x_n \geq \overline{x}_1}{\int\cdots\int} (F_2(x_2)-1)dx_2 \prod_{k=3}^n f_k(x_k)dx_k$$
$$+ \underset{x_3+\cdots+x_n \geq \overline{x}_1-\underline{x}_2}{\int\cdots\int} (\underline{x}_2-\overline{x}_1+x_3+\cdots+x_n)dx_2 \prod_{k=3}^n f_k(x_k)dx_k$$
$$= - \underset{x_2+\cdots+x_n \geq \overline{x}_1}{\int\cdots\int} (F_2(x_2)-1)dx_2 \prod_{k=3}^n f_k(x_k)dx_k$$
$$+ \underset{x_3+\cdots+x_n \geq \overline{x}_1-\underline{x}_2}{\int\cdots\int} (\underline{x}_2-\overline{x}_1+x_3+\cdots+x_n)dx_2 \prod_{k=3}^n f_k(x_k)dx_k$$
$$= - \underset{x_2+\cdots+x_n \geq \overline{x}_1}{\int\cdots\int} (F_2(x_2)-1)dx_2 \prod_{k=3}^n f_k(x_k)dx_k$$
$$+ \underset{x_3+\cdots+x_n \geq \overline{x}_1-\underline{x}_2}{\int\cdots\int} (x_3+\cdots+x_n-(\overline{x}_1-\underline{x}_2))dx_2 \prod_{k=3}^n f_k(x_k)dx_k,$$

which proves equation 24.

Notice from above that $\underset{x_2+\cdots+x_n \geq \overline{x}_1}{\int\cdots\int} (x_2+\cdots+x_n-\overline{x}_1) \prod_{k=2}^n f_k(x_k)dx_k$ and $\underset{x_3+\cdots+x_n \geq \overline{x}_1-\underline{x}_2}{\int\cdots\int} (x_3+\cdots+x_n-(\overline{x}_1-\underline{x}_2))dx_2 \prod_{k=3}^n f_k(x_k)dx_k$ have exactly the same form. So we may use this recursive property to get:

$$\underset{x_3+\cdots+x_n \geq \overline{x}_1-\underline{x}_2}{\int\cdots\int} (x_3+\cdots+x_n-(\overline{x}_1-\underline{x}_2))dx_2 \prod_{k=3}^n f_k(x_k)dx_k$$
$$= - \underset{x_3+\cdots+x_n \geq \overline{x}_1-\underline{x}_2}{\int\cdots\int} (F_3(x_2)-1)dx_3 \prod_{k=4}^n f_k(x_k)dx_k$$
$$+ \underset{x_4+\cdots+x_n \geq \overline{x}_1-\underline{x}_2-\underline{x}_3}{\int\cdots\int} (x_4+\cdots+x_n-(\overline{x}_1-\underline{x}_2-\underline{x}_3))dx_3 \prod_{k=4}^n f_k(x_k)dx_k.$$

Hence,

$$\underset{x_2+\cdots+x_n \geq \overline{x}_1}{\int\cdots\int} (x_2+\cdots+x_n-\overline{x}_1) \prod_{k=2}^n f_k(x_k)dx_k$$
$$= - \underset{x_2+\cdots+x_n \geq \overline{x}_1}{\int\cdots\int} (F_2(x_2)-1)dx_2 \prod_{k=3}^n f_k(x_k)dx_k$$

$- \int\cdots\int_{x_3+\cdots+x_n \geq \overline{x}_1-\underline{x}_2} (F_3(x_2)-1)dx_3 \prod_{k=4}^n f_k(x_k)dx_k$
$+ \int\cdots\int_{x_4+\cdots+x_n \geq \overline{x}_1-\underline{x}_2-\underline{x}_3} (x_4+\cdots+x_n-(\overline{x}_1-\underline{x}_2-\underline{x}_3))dx_3 \prod_{k=4}^n f_k(x_k)dx_k$
$= - \int\cdots\int_{x_2+\cdots+x_n \geq \overline{x}_1} (F_2(x_2)-1)dx_2 \prod_{k=3}^n f_k(x_k)dx_k$
$- \int\cdots\int_{x_3+\cdots+x_n \geq \overline{x}_1-\underline{x}_2} (F_3(x_3)-1)dx_3 \prod_{k=4}^n f_k(x_k)dx_k$
$- \int\cdots\int_{x_4+\cdots+x_n \geq \overline{x}_1-\underline{x}_2-\underline{x}_3} (F_4(x_4)-1)dx_4 \prod_{k=5}^n f_k(x_k)dx_k$
$+ \int\cdots\int_{x_5+\cdots+x_n \geq \overline{x}_1-\underline{x}_2-\underline{x}_3-\underline{x}_4} (x_5+\cdots+x_n-(\overline{x}_1-\underline{x}_2-\underline{x}_3-\underline{x}_4))dx_4 \prod_{k=5}^n f_k(x_k)dx_k$
$= - \int\cdots\int_{x_2+\cdots+x_n \geq \overline{x}_1} (F_2(x_2)-1)dx_2 \prod_{k=3}^n f_k(x_k)dx_k$
$- \int\cdots\int_{x_3+\cdots+x_n \geq \overline{x}_1-\underline{x}_2} (F_3(x_3)-1)dx_3 \prod_{k=4}^n f_k(x_k)dx_k - \cdots$
$- \int\int_{x_{n-1}+x_n \geq \overline{x}_1-\underline{x}_2-\cdots-\underline{x}_{n-2}} (F_{n-1}(x_{n-1})-1)dx_{n-1} f_n(x_n)dx_n$
$+ \int_{x_n \geq \overline{x}_1-\underline{x}_2-\cdots-\underline{x}_{n-2}-\underline{x}_{n-1}} (x_n-(\overline{x}_1-\underline{x}_2-\cdots-\underline{x}_{n-2}-\underline{x}_{n-1}))f_n(x_n)dx_n.$

Let Ω be the set $\{(x_2,\cdots x_n)|\underline{x}_2 \leq x_2 \leq \overline{x}_2, \cdots, \underline{x}_{i-1} \leq x_{i-1} \leq \overline{x}_{i-1}, \cdots, \underline{x}_n \leq x_n \leq \overline{x}_n\}$.

For any i satisfying $2 \leq i \leq n-1$, we have

$\{(x_2,\cdots x_n) \in \Omega|\underline{x}_2 \leq x_2 \leq \overline{x}_2, \cdots, \underline{x}_{n-1} \leq x_{i-1} \leq \overline{x}_{i-1}, x_i+\cdots+x_n \geq \overline{x}_1-\underline{x}_2-\cdots-\underline{x}_{n-1}\}$

$\subset \{(x_2,\cdots x_n) \in \Omega|x_2+\cdots+x_n \geq \overline{x}_1\}$. Using the fact that $F_{i-1}(x_{i-1})-1 \leq 0$, we have the following inequality:

$\int\cdots\int_{x_i+\cdots+x_n \geq \overline{x}_1-\underline{x}_2-\cdots-\underline{x}_{i-1}} \int_{\underline{x}_{i-1}}^{\overline{x}_{i-1}} \cdots \int_{\underline{x}_2}^{\overline{x}_2} (F_i(x_i)-1) \prod_{k=2,k\neq i}^n f_k(x_k)dx_k$
$= \int\cdots\int_{x_i+\cdots+x_n \geq \overline{x}_1-\underline{x}_2-\cdots-\underline{x}_{i-1}} \int_{\underline{x}_{i-1}}^{\overline{x}_{i-1}} \cdots \int_{\underline{x}_2}^{\overline{x}_2} (F_i(x_i)-1)F_1(x_2+\cdots+x_n) \prod_{k=2,k\neq i}^n f_k(x_k)dx_k$
$\geq \int\cdots\int_{x_2+\cdots+x_n \geq \overline{x}_1} (F_i(x_i)-1)F_1(x_2+\cdots+x_n) \prod_{k=2,k\neq i}^n f_k(x_k)dx_k.$

But

$\int\cdots\int_{x_i+\cdots+x_n \geq \overline{x}_1-\underline{x}_2-\cdots-\underline{x}_{i-1}} \int_{\underline{x}_{i-1}}^{\overline{x}_{i-1}} \cdots \int_{\underline{x}_2}^{\overline{x}_2} (F_i(x_i)-1) \prod_{k=2,k\neq i}^n f_k(x_k)dx_k$
$= \int\cdots\int_{x_i+\cdots+x_n \geq \overline{x}_1-\underline{x}_2-\cdots-\underline{x}_{i-1}} (F_i(x_i)-1) \prod_{k=i+1}^n f_k(x_k)dx_k \int_{\underline{x}_{i-1}}^{\overline{x}_{i-1}} \cdots \int_{\underline{x}_2}^{\overline{x}_2} \prod_{k=1}^{i-1} f_k(x_k)dx_k$
$= \int\cdots\int_{x_i+\cdots+x_n \geq \overline{x}_1-\underline{x}_2-\cdots-\underline{x}_{i-1}} (F_i(x_i)-1) \prod_{k=i+1}^n f_k(x_k)dx_k.$

Hence, we get the following equation:

$$- \int\cdots\int_{x_i+\cdots+x_n \geq \overline{x}_1-\underline{x}_2-\cdots-\underline{x}_{i-1}} (F_i(x_i)-1) \prod_{k=i+1}^n f_k(x_k)dx_k \leq \tag{25}$$

$$- \int \cdots \int_{x_2+\cdots+x_n \geq \overline{x}_1} (F_i(x_i) - 1)F_1(x_2 + \cdots + x_n) \prod_{k=2,k\neq i}^{n} f_k(x_k)dx_k$$

To estimate $\int_{x_n \geq \overline{x}_1 - \underline{x}_2 - \cdots - \underline{x}_{n-2} - \underline{x}_{n-1}} (x_n - (\overline{x}_1 - \underline{x}_2 - \cdots - \underline{x}_{n-2} - \underline{x}_{n-1}))f_n(x_n)dx_n$, we use the fact that $\underline{x}_2 + \cdots + \underline{x}_n \leq \overline{x}_1$ which is equivalent to $\sum \underline{y}_i < 0$. We then get the following :

$\{(x_2, \cdots x_n) \in \Omega | x_n \geq \overline{x}_1 - \underline{x}_2 - \cdots - \underline{x}_{n-2} - \underline{x}_{n-1}\} \subset \{(x_2, \cdots x_n) \in \Omega | x_2 + \cdots + x_n \geq \overline{x}_1\}$.

Therefore,

$\int_{x_n \geq \overline{x}_1 - \underline{x}_2 - \cdots - \underline{x}_{n-2} - \underline{x}_{n-1}} (x_n - (\overline{x}_1 - \underline{x}_2 - \cdots - \underline{x}_{n-2} - \underline{x}_{n-1}))f_n(x_n)dx_n$
$= \int_{x_n \geq \overline{x}_1 - \underline{x}_2 - \cdots - \underline{x}_{n-2} - \underline{x}_{n-1}} (x_n - (\overline{x}_1 - \underline{x}_2 - \cdots - \underline{x}_{n-2} - \underline{x}_{n-1}))dF_n(x_n)$
$= (x_n - (\overline{x}_1 - \underline{x}_2 - \cdots - \underline{x}_{n-2} - \underline{x}_{n-1}))F_n(x_n)|_{\overline{x}_1 - \underline{x}_2 - \cdots - \underline{x}_{n-2} - \underline{x}_{n-1}}^{\overline{x}_n}$
$- \int_{x_n \geq \overline{x}_1 - \underline{x}_2 - \cdots - \underline{x}_{n-2} - \underline{x}_{n-1}} F_n(x_n)dx_n$
$= (\overline{x}_n - (\overline{x}_1 - \underline{x}_2 - \cdots - \underline{x}_{n-2} - \underline{x}_{n-1}) - \int_{x_n \geq \overline{x}_1 - \underline{x}_2 - \cdots - \underline{x}_{n-2} - \underline{x}_{n-1}} F_n(x_n)dx_n$
$= - \int_{x_n \geq \overline{x}_1 - \underline{x}_2 - \cdots - \underline{x}_{n-2} - \underline{x}_{n-1}} (F_n(x_n) - 1)dx_n$
$= - \int_{x_n \geq \overline{x}_1 - \underline{x}_2 - \cdots - \underline{x}_{n-2} - \underline{x}_{n-1}} (F_n(x_n) - 1)dx_n \int_{\underline{x}_{n-1}}^{\overline{x}_{n-1}} \cdots \int_{\underline{x}_2}^{\overline{x}_2} \prod_{k=2}^{n-1} f_k(x_k)dx_k$
$\leq - \int \cdots \int_{x_2+\cdots+x_n \geq \overline{x}_1} (F_n(x_n) - 1) \prod_{k=2}^{n-1} f_k(x_k)dx_k$.

Putting together the above inequality and inequality 25 we finally get,

$\int \cdots \int_{x_2+\cdots+x_n \geq \overline{x}_1} (x_2 + \cdots + x_n - \overline{x}_1) \prod_{k=2}^{n} f_k(x_k)dx_k$
$\leq - \sum_{i=2}^{n} \int \cdots \int_{x_2+\cdots+x_n \geq \overline{x}_1} [F_i(x_i) - 1] \prod_{k=2,k\neq i}^{n} f_k(x_k)dx_k$
$= - \sum_{i=2}^{n} \int \cdots \int_{x_2+\cdots+x_n \geq \overline{x}_1} [F_i(x_i) - 1]F_1(x_2 + \cdots + x_n) \prod_{k=2,k\neq i}^{n} f_k(x_k)dx_k$.

Therefore, we have

$I = \sum_{i=2}^{n} \int_{\underline{x}_n}^{\overline{x}_n} \int_{\underline{x}_{n-1}}^{\overline{x}_{n-1}} \cdots \int_{\underline{x}_2}^{\overline{x}_2} [F_i(x_i) - 1]F_1(x_2 + \cdots + x_n) \prod_{k=2,k\neq i}^{n} f_k(x_k)dx_k$
$+ \int \cdots \int_{x_2+\cdots+x_n \geq \overline{x}_1} (x_2 + \cdots + x_n - \overline{x}_1) \prod_{k=2}^{n} f_k(x_k)dx_k$
$\leq \sum_{i=2}^{n} \int_{\underline{x}_n}^{\overline{x}_n} \int_{\underline{x}_{n-1}}^{\overline{x}_{n-1}} \cdots \int_{\underline{x}_2}^{\overline{x}_2} [F_i(x_i) - 1]F_1(x_2 + \cdots + x_n) \prod_{k=2,k\neq i}^{n} f_k(x_k)dx_k$
$- \sum_{i=2}^{n} \int \cdots \int_{x_2+\cdots+x_n \geq \overline{x}_1} [F_i(x_i) - 1]F_1(x_2 + \cdots + x_n) \prod_{k=2,k\neq i}^{n} f_k(x_k)dx_k$
$= \sum_{i=2}^{n} \int \cdots \int_{x_2+\cdots+x_n < \overline{x}_1} [F_i(x_i) - 1]F_1(x_2 + \cdots + x_n) \prod_{k=2,k\neq i}^{n} f_k(x_k)dx_k < 0$

because $\underline{x}_2 + \cdots + \underline{x}_n < \overline{x}_1$. Inequality 23 then follows.

□

Table 1
Close Corporations in the U.S.

	Full Sample	Manager-Owned Firms	Non-Manager-Owned Firms
Observations	310	223	87
Number of firms	64,708	46,990	17,718
Average number of employees	98.0	98.7	95.9
Average asset value ($ million)	7.5	6.9	9.2
Average annual sales ($ million)	23.3	22.4	25.6
Median (Average) number of shareholders	3.0 (74.4)	3.0 (7.9)	10.0 (250.7)
Number of firms with at least one large shareholder -- %	86.9	93.7	69.0
Number of firms with multiple large shareholders -- %	57.2	69.4	24.9
Probability of having multiple large shareholders conditioned on having at least one large shareholder -- %	65.8	74.0	36.1
Number of firms with minority shareholders -- %	67.7	60.6	86.7
Probability of having multiple large shareholders conditioned on having minority shareholders and at least one large shareholder -- %	45.6	55.2	20.7

The NSSBF database includes 4,637 firms with less than 500 employees that answered the 1992 National Survey of Small Business Finances. Out of these 4,637 firms we extracted the close corporations with more than one shareholder and annual sales above $10 million, ending up with our full sample of 310 firms. Manager-Owned Firms *excludes* firms where the manager *is not* a shareholder, while Non-Manager-Owned Firms *includes only* firms where the manager *is not* a shareholder. Observations give the number of firms in each sub-sample. Number of firms gives the NSSBF estimate of the number of firms that each sub-sample would have if all firms had answered the survey. For the estimated number of firms of each sub-sample, Average number of employees, Average asset value ($ million), and Average annual sales ($ million) give, respectively, the average number of employees, and the average book value of assets and annual sales in millions of dollars. Median (Average) number of shareholders is the median (average) number of shareholders of the estimated number of firms in each sub-sample. Number of firms (condition)-% is the fraction of the estimated number of firms satisfying the (condition) over the estimated number of firm in each sub-sample in percentage terms. A firm is said to have multiple large shareholders if two shareholders have at least 10% of the shares and one of them has more than 20%. A firm has one large shareholder if at least one shareholder has more than 20% of the shares while the other shareholders have less than 10% of the shares. A firm has minority shareholders if there are no large shareholders or if there is a shareholder with less than half of the shares of a large shareholder.

Table 2
Public Companies with Shareholders' Agreements in Italy

	Number of Companies	Median number of shareholders in the agreement	Median percentage of shares in the agreement
Buy-out Agreements	52 (17.16%)	4.5	50.6%
Voting Agreements	39 (12.87%)	4.0	51.0%
Control Agreements	20 (6.6%)	6.0	50.4%
Any Agreement	58 (19.14%)	4.0	50.8%

This table shows the fraction of the 303 Italian public firms with shareholders' agreements, as reported by the "Ownership Structure of Companies Listed on Stock Exchanges in Italy." CONSOB, December 1996. *Buy-out agreements* impose restrictions on the transferability of shares. *Voting (or pooling) agreements* force the participants of the agreement to vote with the group. *Control agreements* set corporate policies that otherwise would be decided by the board of directors. *Any agreement* refers to firms that have any of the previously mentioned shareholders' agreements. Numbers in parenthesis show the percentage of the 303 public firms that have the agreements.

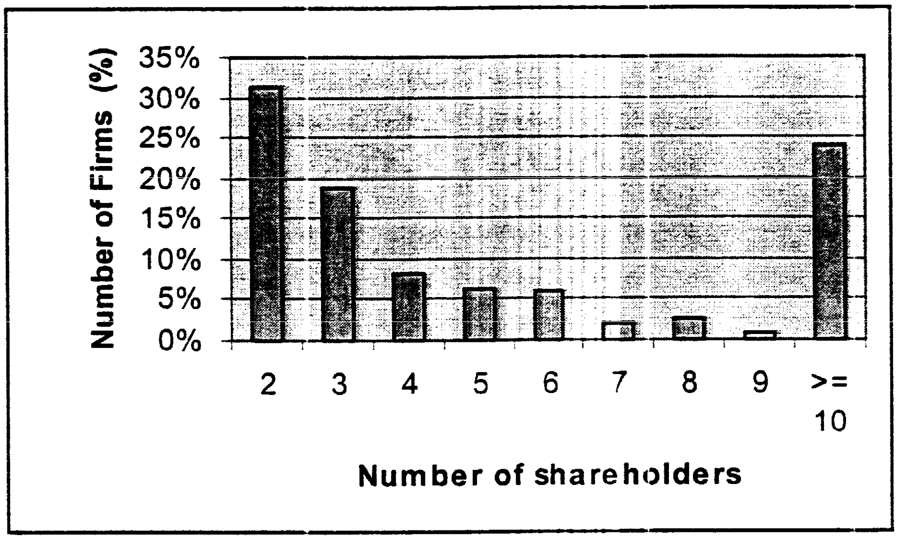

Figure 2
Distribution of the number of shareholders in close corporations