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Going Public with Asymmetric Information,
Agency Costs, and Dynamic Trading

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Going Public with Asymmetric Information, Agency Costs, and Dynamic Trading

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Abstract

We study the problem of a firm that is controlled by a large shareholder and is going public in the presence of agency problems, asymmetric information, and trading of shares over time. In this multiperiod signalling game, a shareholder-manager can develop a reputation for expropriating low levels of private benefits, and this effect causes a significant increase in the stock price and in the likelihood that the firm will go public. Also, this reputation effect is unrelated to the firm's needs to raise external financing in the future. Moreover, insiders divest shares gradually over time, at a rate that is negatively related to the degree of moral hazard. We argue that this model of outside equity is empirically relevant and can be important in understanding the workings of emerging stock markets.

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Introduction

There is a large body of literature on the moral hazard costs associated with the separation of ownership and control. These papers study the conflict of interest between managers and shareholders that results because managers bear the full cost of their effort and do not fully appropriate the benefits (e.g., Berle and Means (1932), Jensen and Meckling (1976)). Another large part of the literature has studied signalling problems that exist when informed insiders are selling claims to uninformed investors (e.g., Leland and Pyle (1977)). Most of the work in both areas is developed in a static framework. The contribution of this paper is to show that a model in which a firm goes public in the presence of agency problems, asymmetric information, and trading of shares in the stock market over time has several novel properties and empirical implications not found in static models.

Instead of focusing on the usual agency problem between the manager and shareholders, we emphasize the agency problem between a large shareholder that exerts full control over the management (owner-manager) and minority shareholders.[1] The risk-averse owner-manager is initially the sole owner of the firm and is motivated to sell equity claims in order to share the idiosyncratic risk of the firm with investors. The value of shares to investors, though, depends on the manager's actions throughout the life of the firm, such as how much effort he exerts or how much he diverts from the firm's cash flow. We refer to effort or diversion of cash flow as private benefits from control (Grossman and Hart (1988)). The manager's actions depend on his private and unobservable information about his costs of extracting private benefits–a manager can be of a high type or a low type, depending on whether his costs of extraction of private benefits are high or low, respectively. Finally, the firm generates an uncertain stream of cash flows, and trading of shares occurs over time.

How much should investors pay for shares, and what is the path of insider equity ownership and private benefits in equilibrium? Is there any role for managers' building a reputation for extracting low levels of private benefits? We show that when stock markets are open after the company goes public and cash flow of the firm is realized across many periods, managers can implicitly commit to investors that they are going to extract low amounts of private

[1] We believe that this is the most relevant agency problem for many capital markets of the world. See, for example, Shleifer and Vishny (1997).

benefits or exert high levels of effort. This is credible because investors know that if a low type manager started to extract high levels of private benefits just after going public, when she still owned a substantial amount of shares, investors would discount the stock price accordingly, and the insider's remaining shares would sell at a reduced price. This reputation effect, similarly to Kreps and Wilson (1982) and Milgrom and Roberts (1982), can reduce the inefficiencies caused by the moral hazard and adverse selection problems, improving the chances of going public and increasing entrepreneurs' surplus when going public. It also increases the stock price and alters the decision of insiders with respect to the amount of equity to hold in equilibria.

Our model proposes a new theoretical explanation for the existence of outside equity. Most of the models in the finance literature in which managers are allowed to divert cash flow and cash flow is nonverifiable (or costly to verify) are incompatible with outside equity, and only debt financing works (see Harris and Raviv (1992) for a survey of models). Two exceptions are Fluck (1998) and Myers (1996), which develop a theory of outside equity that is compatible with the two features mentioned above. This is similar to our model, although the mechanism developed here is quite different. Fluck's (1998) theory of outside equity is based on the control rights and the maturity design of equity, and depends on the ability of equity-holders to dismiss management, while in this paper managers are unambiguously in charge and cannot be disciplined by a threat of dismissal. Also, in Myers (1996) equity investors own the firm and have a "primitive right" to withdraw their share of assets at any time, giving investors some control rights that indirectly prevent insiders from capturing the cash flow of the firm. The assumptions imposed here are thus more unfavorable for the existence of outside equity; the motivation for existence of outside equity here is diversification of risk with outside investors, and the mechanism that sustains equity–despite the lack of any controlling mechanism by investors–is the multiperiod nature of the realization of cash flow and trading of shares, which allow managers to implicitly commit not to expropriate investors.

This model has several new empirical implications for corporate finance. The model predicts that the insider ownership time-series and cross-sectional variation is related to the degree of moral hazard. At the initial public offering (IPO), the size of the block of shares divested is negatively related to the degree of moral hazard. Likewise, following the IPO, shares are gradually divested over time at a rate that is negatively related to the degree of moral hazard. Unlike an extension of Jensen and Meckling (1976) to a multiperiod setting, in our model, there would not be an almost instantaneous divestiture of shares.

Our results have special empirical relevance for situations where managers can extract significant amounts of private benefits of control. Environments where legal institutions do not offer enough protection to minority shareholders against managerial discretion are believed to prevail in many of the world's capital markets (La Porta, Lopez-de-Silanes, Shleifer, and Vishny (1996) and Shleifer and Vishny (1997)). Why do investors buy equity even when they do not get any control rights in exchange for their funds, and managers, being entrenched, are able to extract significant private benefits of control? Through a series of simulations with several parameter values, we show that the reputation effect is economically significant whenever the moral hazard problem is significant. For example, when managers can divert an expected amount of 50% of the cash flow, the value of the stock price with the reputation effect is 30% higher than the price that would be predicted based only on the actual level of protection of minority shareholders. Furthermore, this increase in stock price is positively correlated with the degree of moral hazard problem (coefficient of 0.95 in our simulations). The reputation effect serves to protect minority shareholders in the short and intermediate run even in the absence of a formal legal framework giving them protection against expropriation. Therefore, the stock markets can start functioning even before fundamental improvements are made to protect minority shareholders.

The reputation effect is likely to be an important explanation for the empirical evidence in Singh (1995) that firms in developing markets, where there are significant growth opportunities, rely more heavily on external financing than firms in developed countries, and also more intensively use equity rather than debt financing for their external financing needs. To be sure, firms with higher growth rates are more likely not to generate internally funds for their investment needs, having to more often raise external funds. But why would they more intensively use equity rather than debt financing? Could this finding be related to firms' ability to build more reputation in the presence of more growth opportunities? Surprisingly, contrary to this intuition, our results show that the amount of reputation that owners can build is largely independent of firms' growth rates and investment opportunities if firms have the ability to finance growth prospects by issuing both equity and debt. Intuitively, this result holds because the reputation-building mechanism comes from the owner-managers' dependency on the unique feature of the stock market–its ability to diversify risks–and no reputation can be built from the stock market's role as a source of funding while alternative funding channels are available. We conclude by attributing the relative advantage of equity with respect to debt financing to the unique risk-diversification benefits of equity, which are intensified in developing countries because of the joint presence of the reputation effect, a more concentrated ownership structure, and a more volatile cash flow.

Finally, the model suggests an additional benefit that has not been considered before in the literature on allowing companies to use dual-class shares and pyramidal structures.[2] In markets with low protection to minority shareholders, it is natural that managers will want to retain majority control regardless of whether or not they are allowed to use dual-class shares and pyramidal structures. However, when firms are allowed to use the above-mentioned structures, managers are able to divest more equity without losing control. This gives them more room to build a reputation for extracting low levels of private benefits, because the mechanism that induces reputation is the prospect of future sales of shares. Therefore, regimes that facilitate the dilution of ownership indirectly benefit from a more intense reputation effect, in addition to directly allowing owners to increase diversification of risks.

There is an extensive finance literature related to reputation building. Diamond (1989, 1991) and John and Nachman (1985) developed a model of acquisition of reputation in debt markets with the same building blocks as our model (moral hazard, asymmetric information and dynamics), where reputation mitigates the conflicts of interest between borrowers and lenders in taking projects with excessive risk. In Diamond (1989), if there is initially substantial adverse selection, the reputation effect will be weak and does not induce good behavior in borrowers with a short track record. A longer track record reduces the adverse selection problem and eliminates the conflict of interest of borrowers. Alternatively, if there

[2] We will argue later on that the costs associated with structures that increase the separation of ownership and control are not so significant for markets with low protection to minority shareholders.

is not substantial initial adverse selection, reputation can begin to work immediately. The equilibrium in our game has similar characteristics, although, our model differs in two main ways from those of Diamond (1989, 1991) and John and Nachman (1985). First, their modles are applied to explain properties of debt markets as opposed to equity markets, and second, their models are based on repeated games (following Kreps and Wilson (1982) and Milgrom and Roberts (1982)), while our equity model is not a repeated game, because of its dependence on a state variable (ownership).[3]

There is also an extensive finance literature on going public and signalling problems. Leland and Pyle (1977) concluded that a manager holds shares of the firm to signal to investors the information that the firm has a high market value (which is independent from the manager's action). In this work, a manager holds concentrated equity ownership to provide a guarantee to investors that she is willing to build a reputation for consuming low levels of private benefits. Another, well-known signalling model is John and Williams (1985), in which dividend payments is used as a signal. The main difference between these signalling models and our model is that we address a multiperiod signalling problem. Also, the focus in our model is not on questions related to underpricing of new issues, such as in Allen and Faulhaber (1989), Welch (1989), Grinblatt and Hwang (1989), Stoughton and Zechner (1998), and Chemmanur (1993). Importantly, in our model the motivation to go public is diversification of risks, unlike Zingales (1995), where the motivation to go public is to extract more surplus from potential buyers.[4]

The remainder of the paper is organized as follows: Section I presents the model. Section II solves the complete information version of the game and develops a refinement for the perfect Bayesian equilibrium concept for signalling games. Section III solves for the multiperiod signalling equilibrium of the game. Section IV develops the main results of the paper. Section V discusses the empirical implications and relevance of the model, and Section VI concludes. The proofs of all propositions are in the appendix.

[3]See also Chemmanur and Fulghieri (1994) for a model where investment banks can develop a reputation for production of information.

[4]See also Mello and Parsons (1998) for a model of going public that also takes into account the final ownership structure of the firm.

I The Model

The problem is modeled as a stochastic dynamic game with incomplete information and a finite number of periods (T periods). The players in the game are a risk-averse owner-manager and investors in a competitive stock market. The manager chooses in every period how many shares to trade and decides how much effort to exert; the more effort he exerts, the higher are the expected cash flows of the firm. The costs of exerting effort are incurred by the manager, and investors do not know the manager's cost function (his type); he can be of a high type or low type if his costs of undertaking effort are low or high, respectively. The level of effort exerted by the manager during every period is assumed to be perfectly observable by investors, with a lag, although it cannot be verified. An alternative interpretation of effort levels in the model, following Grossman and Hart (1988), is that the manager can divert for himself a fraction of the cash flow or extract private benefits of control; in this interpretation investors know neither how much the manager can divert nor the cost of diverting cash flow.

Initially, at period $t = 0$, the manager is the sole owner of a firm that produces a risky stream of cash flows for a total of T-periods, $(y_t)_{t=1}^{T}$. The firm's cash flows in any period depend both on independent random shocks η_t and on the manager's effort level choices. In every period, $t = 1, ..., T$, the manager is allowed to trade shares with investors. The owner-manager is risk-averse and thus there exists a possibility of gains from trading when the owner-manager can share the idiosyncratic risk with investors. We denote the manager equity ownership choice in period t as α_t (percentage of the total number of shares) and, since he is initially the sole owner, $\alpha_0 = 1$. Managers also choose an effort level e_t in every period; they choose effort before observing the realization of the random shock, and their choices are perfectly observed by outside investors in the following period (after effort has influenced the publicly observed cash flows). The problem is formally equivalent to one in which the manager chooses simultaneously at every period a multidimensional signal (α_t, e_t), which is observed by investors, where the effort choice e_t influences the $t-1$ period cash flow. The production function is for convenience defined as $y_t = y_t(e_{t+1}, \eta_t)$ and the effort variable has been normalized so that the expected cash flow is equal to the level of effort: $E[y_t(e_{t+1}, \eta_t)] = e_{t+1}$.

In the model there are two observationally indistinguishable types of manager–a high type, $\theta = \overline{\theta}$, and a low type, $\theta = \underline{\theta}$, with different costs of exerting effort. Although the manager knows his own type, investors know only the prior probability distribution of types, $\mu_0 = P(\theta = \overline{\theta})$. The cost of exerting effort is, in monetary equivalent terms, equal to $c_{t-1}\left(e_t, \eta_{t-1}, \theta\right)$ and depends on the unobservable types.

Equivalently, instead of managers having different and unobservable costs of exerting effort, managers could be different with respect to the amount of private benefits they get from diverting cash flows from the firm. A simple diversion model that will be used in examples is specified as follows. A manager of type θ can extract up to a fraction b_θ of cash flow at every period: type $\overline{\theta}$ is the high type that is completely honest and never extracts private benefits ($b_{\overline{\theta}} = 0$), and type $\underline{\theta}$ is the low type that can extract up to a fraction b of cash flow ($b_{\underline{\theta}} = b$, $0 < b \leq 1$). So, if the cash flow in period t is y_t the low type could extract in private benefits by_t and the minority shareholders receive a verifiable cash flow of only $(1-b)\,y_t$.

The extensive form of the game is as follows. At each period t the previous history of the game is known by all players. The manager moves by choosing simultaneously a new fraction of equity ownership and effort level (α_t, e_t), which are the manager's t-period signals. After observing the manager's signals, investors update their prior beliefs about the manager type and price shares in the market accordingly. Investors price shares competitively–Bertrand pricing competition–and trading is realized at the market clearing price P_t (price per share multiplied by the number of shares). The game continues as described above at the next period until a final period is reached.[5]

We emphasize that the manager is assumed not to be able to commit to a long-term strategy, or commit today to play a predetermined strategy in the future. The short-term nature of the problem or the lack of commitment is an essential feature of the model.[6]

The payoffs for managers and investors are specified as follows, for any given history of

[5] In the initial period $t = 1$ there is no choice of effort, $e_1 = \emptyset$ (because the firm generates no cash flow in period $t = 0$), and in the last period $T + 1$ there is no trading, $\alpha_{T+1} = \alpha_T$, but the manager chooses an effort level e_{T+1} (which determines the T–period cash flow).

[6] We depart from the principal-agent literature by assuming that no long-term incentive contracts can be written and/or enforced. This can be motivated by the lack of verifiability of variables, the lack of ability of courts to enforce contracts, or the ability of managers with control to manipulate variables or to change contracts during their life.

moves $h^{T+1} = \{(\alpha_t, e_t, P_t)\}_{t=1}^{T+1}$. The manager's utility depends on his net worth at the end of the game, equal to $w_T = \sum_{t=1}^{T+1} (\alpha_{t-1} - \alpha_t) P_t + \alpha_{t-1} y_{t-1} (e_t, \eta_{t-1}) - c_{t-1} (e_t, \eta_{t-1}, \theta)$ where the first term corresponds to revenues from trading shares, the second term to dividends received, and the last term to his costs of exerting effort (define $c_0 \equiv 0$ and $y_0 \equiv 0$).

The risk-averse manager has a von Neumann-Morgenstern utility function that is negative exponential in his total wealth.[7] The manager's utility is then $E[-\exp(-w_T)]$ and his objective is to maximize his utility (for simplicity, the coefficient of risk-aversion is made equal to 1). Expected values are taken with respect to the distribution of the exogenous random shocks to the cash flow. A nice property of the negative exponential utility function is that the manager's utility at time t is independent of his current wealth, and depends only on the incremental income from the decisions from period t to the future: $w_{t,T} = \sum_{\tau=t}^{T} (\alpha_{\tau-1} - \alpha_\tau) P_\tau + \alpha_{\tau-1} y_{\tau-1} (e_\tau, \eta_{\tau-1}) - c_{\tau-1} (e_\tau, \eta_{\tau-1}, \theta)$. Thus, the problem of the manager of type θ at the start of every period t is to choose strategies $\{\alpha_\tau, e_\tau\}_{\tau=t}^{T+1}$ that maximize the objective function $E[-\exp[-w_{t,T}]]$. Another convenient property of the exponential utility and the independence of the random shocks allows for a further simplification of the manager's objective function (see Lemma 1 in the appendix). Managers' problem can be re-written in a simplified manner as

$$\max_{\{\alpha_\tau, e_\tau\}_{\tau=t}^{T+1}} \sum_{\tau=t}^{T+1} (\alpha_{\tau-1} - \alpha_\tau) P_\tau + v_{\tau-1} (\alpha_{\tau-1}, e_\tau, \theta) \tag{1}$$

where the function $v_t(\alpha_t, e_{t+1}, \theta) = -\log E[\exp - [\alpha_t y_t (e_{t+1}, \eta_t) - c_t (e_{t+1}, \eta_t, \theta)]]$. Throughout the paper, we will be considering the reformulated utility function with the following regularity conditions imposed on $v_t(\alpha_t, e_{t+1}, \theta)$:

Assumption 1 *For all t, the function $v(\alpha, e, \theta)$ is twice differentiable and satisfies the following conditions:*

[7] To avoid complications associated with savings and consumption decisions in an intertemporal framework, the manager is assumed to consume his total wealth at the end of the game.

(A1) Regularity conditions: $v_\alpha \geq 0$, $v(\alpha, e, \theta)$ *is concave in* (α, e), $v_{\alpha e} \geq 0$ *and* $v_\alpha \leq e$;
(A2) Single-crossing property (SC-P): $v_\alpha\left(\alpha, e, \overline{\theta}\right) \geq v_\alpha\left(\alpha, e, \underline{\theta}\right)$ *and* $v_e\left(\alpha, e, \overline{\theta}\right) \geq v_e\left(\alpha, e, \underline{\theta}\right)$.

The regularity conditions imposed essentially state that the marginal value of shares decreases with ownership, the marginal value of effort decreases with effort, and diversification of risk is efficient. The second condition is the familiar single-crossing property of the static multidimensional signalling game.[8]

Investors are risk-neutral (investors diversify the idiosyncratic risk of the firm), maximize expected return, and the required rate of return is equal to zero. A share of stock pays at period t a dividend equal to the expected cash flow y_t. Investors' returns thus depend not only on the actions of the manager at time t but also on his future actions. Stock prices are then, in a competitive investment market, equal to the sum of future expected cash flows:

$$P_t = E_t\left[y_t + \ldots + y_T\right] \tag{2}$$

Finally, the equilibrium concept used throughout the paper is perfect Bayesian equilibrium (PBE), with the refinement concept introduced in the following section.

II Preliminaries and Refinement Concepts

A The Complete Information Game

The dynamic game proposed in the previous section is particularly simple to solve when there is complete information about the manager type. This is the dynamic version of the Jensen and Meckling (1976) agency problem, where there is a known moral hazard problem and the manager is assumed to be unable to commit not to shirk or divert cash flow from the firm after having reduced his ownership stake.

The complete information game with no commitment has a unique subgame perfect equilibrium that can be easily obtained by backwards induction as follows. In the last decision period, $t = T + 1$, there is no trading, so $\alpha_{T+1} = \alpha_T$ and the manager of type θ chooses a level of effort e_{T+1} that solves the problem $\max_{e_{T+1}} v_T\left(\alpha_T, e_{T+1}, \theta\right)$. Let $e_{T+1}\left(\alpha_T, \theta\right)$

[8]This is the same condition used by Cho and Sobel (1990).

be the solution of the problem and let $V_T(\alpha_T, \theta)$ be the value of the maximization problem. The equilibrium price of shares in period $t = T$ is then given by $P_T(\alpha_T, \theta) = e_{T+1}(\alpha_T, \theta)$ because investors anticipate the manager's choice of effort in the next period.

Proceeding recursively, we have that the choice of effort and ownership by the manager at any period $t = 1, ..., T$ is given by $\{e_t(\alpha_{t-1}, \theta), \alpha_t(\alpha_{t-1}, \theta)\}$, the solution of *the complete information problem*:

$$V_{t-1}(\alpha_{t-1}, \theta) = \max_{\{\alpha_t, e_t\}} (\alpha_{t-1} - \alpha_t) P_t(\alpha_t, \theta) + v_{t-1}(\alpha_{t-1}, e_t, \theta) + V_t(\alpha_t, \theta) \tag{3}$$

where the price of shares at period t is defined recursively as $P_t(\alpha_t, \theta) = e_{t+1}(\alpha_t, \theta) + P_{t+1}(\alpha_{t+1}(\alpha_t, \theta), \theta)$, and $V_{t-1}(\alpha_{t-1}, \theta)$ is the equilibrium utility of the manager who owns α_{t-1} shares in the subgame starting in period t (value of the period t maximization problem).

We have just established the following result.

Proposition 1 *There is a unique subgame perfect equilibrium of the game with complete information and no commitment. The unique equilibrium strategies are as follows: in any period t where the manager owns α_{t-1} shares, the equilibrium choices are $e_t(\alpha_{t-1}, \theta)$ and $\alpha_t(\alpha_{t-1}, \theta)$ and investors responds with a price $P_t(\alpha_t, \theta)$ given by the solution of the complete information problem (3).*

It is also of interest to develop *the first best outcome* that owner-managers could achieve if they were able to commit to an efficient level of effort for the future or write enforceable contracts based on observable variables that could give them incentives to exert an optimal level of effort. Managers would then commit to a path of ownership and effort $\{\alpha_t, e_t\}_{t=1}^{T+1}$, with a share price equal to $P_t = \sum_{\tau=t}^{T} e_{\tau+1}$, which maximizes $\displaystyle\sum_{t=1}^{T+1} (\alpha_{t-1} - \alpha_t) P_t + v_{t-1}(\alpha_{t-1}, e_t, \theta)$. The conditions in Assumption 1 easily imply that the value of the firm for the manager in the first best–defined as $\overline{V}_0(\theta)$–is such that managers share all the risk with investors, $\alpha_t^* = 0$, and choose an efficient level of effort $e_{t+1}^*(\theta)$, solution of $\max e_{t+1} + v_t(0, e_{t+1}, \theta)$.

At the other extreme, firms do not go public and remain as closely held concerns. In the *outcome with no market for shares,* the closely held concern value is equal to $\underline{V}_0(\theta) = \max_{\{e_t\}_{t=1}^{T+1}} \sum_{t=1}^{T+1} v_{t-1}(1, e_t, \theta)$.

We will be comparing the equilibrium in the three extreme cases developed above with the equilibrium when there is incomplete information about the manager type. Obviously, in the incomplete information game the first best outcome cannot be implemented by any Nash equilibrium, and we will also see that the outcome of the low type (high type) manager with incomplete information is better than (worse than) the outcome in the complete information game. In other words, the low type derives rents at the expense of the high type in the incomplete information equilibrium. A profit-maximizer high-type manager then might be driven out of the public markets and not go public because of a "lemons problem." When going public a high type will certainly try to send signals to investors that he is of a high type, through his trading of shares and other observable variables such as his choice of effort, in order to fetch a higher stock price. Obviously, investors know that low types also might have incentives to be mimicking the high types. Our goal is to determine the equilibrium of this multiperiod signalling game.

B The Refinement Concept

It is well known that signalling games such as the one we are analyzing have a multiplicity of perfect Bayesian equilibria (PBE). We will develop in this sub-section a natural refinement concept that will yield a unique equilibrium outcome for our multiperiod signalling problem.

There are many PBE for signalling games because this equilibrium concept does not impose any restrictions on the out-of-equilibrium beliefs. A number of refinements have been proposed that attempt to formalize plausible restrictions on the out-of-equilibrium beliefs. For signalling games the most commonly used refinement is the intuitive criterion of Cho and Kreps (1987), which selects a separating equilibrium for our signalling game.

Mailath, Okuno-Fujiwara, and Postlewaite (1993) criticize the logic of the intuitive criterion and introduce the concept of undefeated equilibrium along with the concept of *lexicographically maximum (lex max)* perfect Bayesian equilibrium. They show that an equilibrium that is lex max is also undefeated and argue that these refinements are more appropriate for signalling games such as the ones we are studying. We will be using through this paper the lex max PBE equilibrium concept. Even though a comprehensive discussion of equilibrium refinements is beyond the scope of this paper, we will briefly present an intuitive and formal definition of the lex max concept.

Intuitively, an equilibrium σ satisfies the *lex max property* if it maximizes the utility for the high-type manager among a set of PBE and if, conditional on maximizing the utility for the high type, it also maximizes the utility for the low type. We extend this intuitive notion to a multiperiod setting, requiring that the lex max property be satisfied for all continuation games. We refer to any subgame starting from a history of past moves h^t with prior distribution μ over types as the continuation game $G(h^t, \mu)$.

More formally, we say that a PBE (σ, μ) is a lex max equilibrium of the multiperiod game if it satisfies the following two conditions for all continuation games. First–condition LM1–is that for any history h^t the restriction of (σ, μ) to the history h^t, $(\sigma, \mu)_{|h^t}$, satisfies the lex max property for the continuation game $G(h^t, \mu(.|h^t))$. Second–condition LM2–is that the maximization in the lex max property is not over the set of all PBE of the continuation game, but is restricted to a subset that is consistent with the lex max concept in all continuation games starting at $t+1$. The motivation for this restriction is that if lex max property is the concept that players use to choose an equilibrium, then the only candidates for equilibrium are the PBE strategies that also satisfy the lex max property when restricted to its subgames. These ideas are formalized below.

Assumption 2 *(σ, μ) is a lex max PBE, if it is a PBE and for any history h^t the restriction to the continuation game, $(\sigma, \mu)_{|h^t}$, satisfies the lex max property. The set of equilibria that satisfies the lex max property for any continuation game is defined recursively as follows: $(\sigma, \mu)_{|h^t} \in LM(h^t, \mu(.|h^t))$ if and only if $(\sigma, \mu)_{|h^t}$ is a PBE of the continuation game $G(h^t, \mu(.|h^t))$ and there exists no $(\sigma', \mu')_{|h^t}$, PBE of $G(h^t, \mu(.|h^t))$, such that:*
(LM1) $\exists\, \theta$ such that $u(\sigma'|h^t, \theta) > u(\sigma|h^t, \theta)$ and for $\theta' > \theta$, $u(\sigma'|h^t, \theta') \geq u(\sigma|h^t, \theta')$; and
(LM2) $\forall h^{t+1}$ consistent with h^t with $t+1 \leq T$: $(\sigma', \mu')_{|h^{t+1}} \in LM(h^{t+1}, \mu'(.|h^{t+1}))$.

C The Static Signalling Game

The equilibrium in the static game is developed in this section as a way to illustrate the equilibrium that is selected by the refinement Assumption 2 in a simple setting. This one-shot game also serve as a benchmark to compare the multiperiod signalling game with a signalling game where managers cannot develop any reputation.

The static game is similar to Spence's (1974) job market signalling model and to the equity offering model of Leland and Pyle (1977). There are many PBE in the one-period signalling game, and the intuitive criterion of Cho and Kreps (1987) selects a unique PBE equilibrium: *the Riley separating equilibrium*. This is the equilibrium in which the low type chooses to reveal his type and maximizes his complete information utility. This corresponds to the low type offering a fraction $\alpha(\underline{\theta}) = \alpha_1(1, \underline{\theta})$ of shares at a price equal to $P(\alpha(\underline{\theta}), \underline{\theta})$ and yields a total utility of $V_0(1, \underline{\theta})$ to him (see Section A). The high type, on the other hand, chooses to offer α shares, which maximizes his utility $(1-\alpha) P\left(\alpha, \overline{\theta}\right) + V_1\left(\alpha, \overline{\theta}\right)$ subject to the incentive compatibility constraint (IC), $(1-\alpha) P\left(\alpha, \overline{\theta}\right) + V_1(\alpha, \underline{\theta}) \leq V_0(1, \underline{\theta})$.

We will see in Proposition 2 below that there is a unique PBE equilibrium outcome that satisfies the refinement Assumption 2 for the static signalling game. This equilibrium can be either the Riley separating equilibrium or a pooling equilibrium depending, respectively, on whether the proportion of high-type managers in the economy is small or large. This is in contrast with the intuitive criterion that always selects the Riley equilibrium, regardless of the proportion of high types. As argued by Mailath et al. (1993) and Fudenberg and Tirole (1991), choosing a separating equilibrium when there is only a very small probability of low-types in the economy does not make much economic sense because a small chance of existence of low types causes a large distortions in allocations. A pooling equilibrium seems to be a more sensible equilibrium selection for such priors because it can support an equilibrium allocation with small distortions.

Another advantage of the equilibrium concept that we are using is that we can naturally associate the equilibrium outcome with a constrained maximization problem. Assume that (σ, μ) is a PBE in which the high type chooses α and the response of investors to this choice is to update their beliefs to μ and offer a price P for shares. Necessary conditions for the triple $\{\alpha, \mu, P\}$ to be on-the-equilibrium path of a PBE are: (1) Bayes rule restricts beliefs to the interval $1 \geq \mu \geq \mu_0$; (2) the response of investors is consistent with beliefs so that $P = P\left(\alpha, \overline{\theta}\right)\mu + P(\alpha, \underline{\theta})(1-\mu)$; and (3) if the equilibrium is not pooling, $\mu > \mu_0$, then the incentive compatibility condition must be satisfied, $(1-\alpha) P + V_1(\alpha, \underline{\theta}) \leq V_0(1, \underline{\theta})$.

The problem of finding an equilibrium that satisfies Assumption 2 can be mapped into the more tractable problem of solving the constrained maximization:

$$\begin{array}{lll} \max\limits_{\{\alpha,\mu,P\}} & (1-\alpha)\,P + V_1\left(\alpha,\overline{\theta}\right) & \\ s.t. & 1 \geq \mu \geq \mu_0 & \text{(B)} \\ & P = P\left(\alpha,\overline{\theta}\right)\mu + P\left(\alpha,\underline{\theta}\right)(1-\mu) & \text{(CC)} \\ & \left[(1-\alpha)\,P + V_1\left(\alpha,\underline{\theta}\right) - V_0\left(1,\underline{\theta}\right)\right].\left(\mu-\mu_0\right) \leq 0 & \text{(IC)} \end{array} \tag{4}$$

where the solution $\{\alpha^*, \mu^*, P^*\}$ of the problem provides the equilibrium path and the value of the problem, $V_0\left(1,\mu_0,\overline{\theta}\right)$, is the equilibrium outcome for the high type, and the equilibrium outcome for the low type is $V_0\left(1,\mu_0,\overline{\theta}\right) = \max\left\{(1-\alpha^*)\,P^* + V_1\left(\alpha^*,\underline{\theta}\right), V_0\left(1,\underline{\theta}\right)\right\}$.

We prove in the next proposition that the solution of problem (4) is associated with the equilibrium of the static signalling game that satisfy Assumption 2.

Proposition 2 *There is a unique PBE outcome for the static game satisfying refinement Assumption 2 that is associated with the solution of program (4). There exists a prior probability* $\overline{\mu}_0$ *such that this equilibrium outcome can be either the Riley separating equilibrium if* $\mu_0 < \overline{\mu}_0$ *or a pooling equilibrium for* $\mu_0 \geq \overline{\mu}_0$. *The equilibrium payoffs are constant in the prior probability* μ_0 *if* $\mu_0 < \overline{\mu}_0$ *and are monotonically increasing and continuous in* μ_0 *if* $\mu_0 \geq \overline{\mu}_0$.

So the equilibrium outcome is a pooling equilibrium that changes continually and monotonically with the proportion of types if the probability of being a high type is above a certain threshold $\overline{\mu}_0$. The equilibrium outcome has a discontinuity only at the point where the prior belief is equal to $\overline{\mu}_0$, when there is a transition from the separating equilibrium to the pooling equilibrium. See Figure 1 for a graphical description of the solution of the static equilibrium.

Insert Figure 1 here

III The Multiperiod Signalling Game

This section characterizes the equilibrium of the multiperiod signalling game. This characterization is then used in the next section to derive the corporate finance implications of the model.

The first interesting difference between the equilibrium in the static game and in the multiperiod game is that the extension of the standard separating equilibrium of the static game in general fails even to exist in the multiperiod game when the managers cannot make long-term commitments not to trade shares in the future.

The intuition for this result is that the payoff for the low-type manager following separation at an IPO is very high if he imitates the separation strategy of a high-type manager. This is the case because if the manager plays the separating strategy of a high type at the IPO the market would assign probability one that the manager is of the high type. In the next trading period, though, a high-type manager will be willing to diversify the risk and divest all shares, and investors are willing to pay a fair price (with no agency discount) for all shares because they now believe that the manager is definitely of the high type. However, a low-type manager can potentially imitate the strategy of the high type and therefore sell all shares at a price with no discount and, naturally, extract as much private benefits as possible after selling. If the fraction of private benefits the low type can extract is relatively large, then, even if investors buy no shares in the first period this is not costly enough to separate the low type from the high type. The low type would prefer not to sell any shares and to wait for the next period when he could sell all shares without discount. It then follows that it can well be the case that no separating equilibrium can exist in the multiperiod game.

Consider for concreteness the diversion model where the high-type manager cannot divert the cash flows and the low type can divert up to a fraction b of the cash flow. In the separating equilibrium, the low type reveals himself in the first period, selling all the shares with a discount of b, and thereafter extracts a fraction b of the cash flow, so that the utility of the low type is $V_0\left(1,\underline{\theta}\right)=\sum_{t=1}^{T}\left(1-b\right)\overline{y}_t+v_t\left(b\right),$ where $v_t\left(\alpha\right)=-\log E\left[\exp -\alpha y_t\right]$ and $\overline{y}_t=E\left[\widetilde{y}_t\right].$ The high type maintains a high ownership stake in the first period, and after revealing himself in the first period then divests all shares at a price with no discount equal to $\sum_{t=2}^{T}\overline{y}_t$. By imitating the high type the low type can get $\left(1-\alpha_1\right)P_1\left(\alpha_1,\overline{\theta}\right)+V_1\left(\alpha_1,1,\underline{\theta}\right),$ where the price of share in the first period is $P_1\left(\alpha_1,\overline{\theta}\right)=\sum_{t=1}^{T}\overline{y}_t$, and low-type utility after the first period when investors believe that the manager is definitely of the high type is $V_1\left(\alpha_1,1,\underline{\theta}\right)=\alpha_1\sum_{t=2}^{T}\overline{y}_t+v_1\left(\alpha_1\right)+\sum_{t=2}^{T}v_t\left(b\right).$ This is true because the low type can sell all remaining α_1 shares at the price with no discount in period 2, get the value of the risky cash flow of period 1 with no stealing, $v_1(\alpha_1)$, and then divert a fraction b of the cash flows from period 2 on, after having sold all shares.

The incentive compatibility condition that must be satisfied for the existence of a separating equilibrium is

$$(1-\alpha_1) P_1\left(\alpha_1, \overline{\theta}\right) + V_1\left(\alpha_1, 1, \underline{\theta}\right) \leq V_0\left(1, \underline{\theta}\right) \tag{5}$$

which is equivalent to $\sum_{t=1}^{T} b\overline{y}_t \leq \alpha_1 \overline{y}_1 - v_1(\alpha_1) + v_1(b)$. A simple inspection of this expression indicates that whenever the moral hazard parameter (b) is large enough there is no separating equilibrium, because the LHS is larger than the RHS for all values of α_1. The above results are summarized in the following proposition.

Proposition 3 *There exists a separating PBE in the multiperiod signalling game if and only if the incentive compatibility condition (5) holds for some $\alpha_1 \in [0,1]$. Furthermore, if the level of moral hazard is large then the multiperiod game has no separating equilibria.*

If the standard separating equilibrium does not in general even exist, what is then the equilibrium of the game? In the remainder of this section we prove the existence and uniqueness of the equilibrium for the multiperiod game. We also show how to reduce the problem of obtaining equilibria of the game to the simpler problem of solving a dynamic program.

We start this analysis by assuming that we are given equilibrium strategies for all continuation games starting in period $t+1$ where the manager owns α_t shares and the market believes that the manager's type is given by probability μ_t. Proceeding recursively, we can then obtain what is the equilibrium at time $t=0$, when the manager owns all shares and prior beliefs are μ_0. Suppose that the equilibrium value in each such continuation game is equal to $V_t(\alpha_t, \mu_t, \theta)$ for a manager of type θ and the value of shares to investors is $P_t(\alpha_t, \mu_t)$. What is the equilibrium of the $t-$period signalling game that satisfies refinement Assumption 2 given any initial ownership stake α_{t-1} and prior beliefs μ_{t-1}? In other words, what strategy profile and system of beliefs (σ_t, μ_t) for the period t play is both PBE and maximizes the utility of the high type?

PBE imposes the following several restrictions on the strategies that can be equilibria.

Say that the equilibrium strategy for the high type is to play (α_t^*, e_t^*).[9] The low-type manager can either pool with the high type and play (α_t^*, e_t^*) or reveal himself, choosing any other action. Of course, if the low type decides not to pool with the high type he should then play according to his complete information optimum (see proposition 1), which corresponds to $(\underline{\alpha}_t, \underline{e}_t) = (\alpha_t(\alpha_{t-1}, \underline{\theta}), e_t(\alpha_{t-1}, \underline{\theta}))$ and yields utility for the low type equal to $V_{t-1}(\alpha_{t-1}, \underline{\theta})$. So we can restrict our attention to strategies for the low type where he plays (α_t^*, e_t^*) with probability β_t and $(\underline{\alpha}_t, \underline{e}_t)$ with probability $1-\beta_t$. We will see that it is necessary to consider mixed strategies for the low type, since there might not exist any equilibrium in which all players use pure strategies. Investors should respond to the signals (α_t^*, e_t^*) and $(\underline{\alpha}_t, \underline{e}_t)$ that are played in equilibrium consistently with Bayes' rule: first, when (α_t^*, e_t^*) is played they update their beliefs to $\mu_t^* = \frac{\mu_{t-1}}{\mu_{t-1}+(1-\mu_{t-1})\beta_t}$, with $\mu_t^* \in [\mu_{t-1}, 1]$ (condition B), and price shares consistently at $P_t^* = P_t(\alpha_t^*, \mu_t^*)$, the equilibrium value of shares given by the equilibrium strategies in the continuation game (condition CC); second, when $(\underline{\alpha}_t, \underline{e}_t)$ is played they are now sure that the manager is of the low type, so $\mu_t = 0$, and they price shares at $\underline{P}_t = P_t(\underline{\alpha}_t, \underline{\theta})$ given by the equilibrium in the complete information game with type $\underline{\theta}$.

Furthermore, the Nash condition for the low type requires that given the response of investors, the low type should play (α_t^*, e_t^*) for sure (pool) whenever the utility of the low type from pooling is greater than the utility from separation, which then implies that $\mu_t^* = \mu_{t-1}$ (condition IC_1). In addition, whenever the utility from pooling is lower than the utility from separation, the low type should play $(\underline{\alpha}_t, \underline{e}_t)$ with probability 1, which then implies that $\mu_t^* = 1$ (condition IC_2).

We will show that the equilibrium that is PBE and satisfies Assumption 2, and thus maximizes the utility of the high-type manager, can be associated with the solution of a dynamic programming problem. Let us define the utility function for the t-period signalling problem as

$$u_t(\alpha_t, e_t, P_t, \mu_t, \theta) = (\alpha_{t-1} - \alpha_t) P_t + v_{t-1}(\alpha_{t-1}, e_t, \theta) + V_t(\alpha_t, \mu_t, \theta) \tag{6}$$

[9] Without any loss of generality, we will restrict our attention to strategies where the high type plays only pure strategies (see Mailath (1992)).

where $V_t(\alpha_t, \mu_t, \theta)$ and $P_t(\alpha_t, \mu_t)$ are the equilibrium values of the $t+1-$period continuation game for managers and investors. The discussion above suggests that we consider the following dynamic program:

$$\begin{array}{lll} \max\limits_{\{\alpha_t, e_t, \mu_t, P_t\}} & u_t\left(\alpha_t, e_t, P_t, \mu_t, \overline{\theta}\right) & \\ s.t. & 1 \geq \mu_t \geq \mu_{t-1} & \text{(B)} \\ & P_t = P_t(\alpha_t, \mu_t) & \text{(CC)} \\ & \left(u_t(\alpha_t, e_t, P_t, \mu_t, \underline{\theta}) - V_{t-1}(\alpha_{t-1}, \underline{\theta})\right).\left(\mu_t - \mu_{t-1}\right) \leq 0 & (\text{IC}_1) \\ & \left(u_t(\alpha_t, e_t, P_t, \mu_t, \underline{\theta}) - V_{t-1}(\alpha_{t-1}, \underline{\theta})\right).(1-\mu_t) \geq 0 & (\text{IC}_2) \end{array} \quad (7)$$

where α_t^*, e_t^*, μ_t^* and P_t^* are the solutions of the problem and $V_t\left(\alpha_t, \mu_t, \overline{\theta}\right)$ is the value of the program.

In order to complete the specification of the dynamic program we need to provide the expression for the terminal period $V_T(\alpha_T, \mu_T, \theta)$ and $P_T(\alpha_T, \mu_T)$, as well as show how to proceed recursively to obtain $V_{t-1}\left(\alpha_{t-1}, \mu_{t-1}, \underline{\theta}\right)$ and $P_{t-1}\left(\alpha_{t-1}, \mu_{t-1}\right)$.

First, the terminal period, $t = T+1$, is exactly like the terminal period of the complete information game analyzed in section II, and thus the expressions for $V_T(\alpha_T, \mu_T, \theta)$ and $P_T(\alpha_T, \mu_T)$ can be similarly obtained. Second, low-type payoff is simply $V_{t-1}\left(\alpha_{t-1}, \mu_{t-1}, \underline{\theta}\right) = \max\left\{(\alpha_{t-1} - \alpha_t^*) P_t^* + v_{t-1}(\alpha_{t-1}, e_t^*, \underline{\theta}) + V_t(\alpha_t^*, \mu_t^*, \underline{\theta}), V_{t-1}(\alpha_{t-1}, \underline{\theta})\right\}$ because he can either pool or separate. Finally, the stock price is given by the competitive equilibrium condition and Bayes' rule as follows:[10]

$$P_{t-1}\left(\alpha_{t-1}, \mu_{t-1}\right) = \frac{\mu_{t-1}}{\mu_t^*}\left[e_t^* + P_t^*\right] + \left(1 - \frac{\mu_{t-1}}{\mu_t^*}\right) P_{t-1}(\alpha_{t-1}, \underline{\theta}) \quad (8)$$

The next proposition shows that there exists a solution to program (7) and shows how to relate this solution to an equilibrium of the game.

Proposition 4 *The multiperiod game has a unique PBE equilibrium outcome that satisfies Assumption 2. Furthermore, the equilibrium path and the payoffs for managers and investors are associated with the solution of the dynamic programming Problem (7).*

[10]Note that $P_{t-1}\left(\alpha_{t-1}, \mu_{t-1}\right) = E\left[e_t + P_t\right] = \left[e_t^* + P_t^*\right] P\left(e_t = e_t^*\right) + \left[\underline{e}_t + \underline{P}_t\right] P\left(e_t = \underline{e}_t\right)$, and Bayes' rule implies that, $P\left(e_t = e_t^*\right) = \frac{\mu_{t-1}}{\mu_t^*}$, which yields the recursion expression for the stock price.

The equilibrium of the multiperiod signalling game has several novel characteristics not present in the static game. For ease of exposition, it is useful to concentrate on the diversion model in which the low type can divert a fraction b of the cash flow.

Note first that the equilibrium path for the high-type manager is deterministic and is given by his ownership stake α_t^* and the stock price P_t^*, the solution of Problem (7). However, the equilibrium path for the low-type manager is stochastic because with some probability he mimics the high type or otherwise reveals himself to be expropriating shareholders. In the first time where the low type behaves badly, investors learn his low type and the stock price adjusts to $\underline{P}_t = \sum_{\tau=t}^{T} \underline{y}_\tau$, where $\underline{y}_t = (1-b)\overline{y}_t$, and he divests all his shares.

The rate at which the market learns the information about the manager is given by the variable μ_t^* which is the probability that the manager is of a high type given that he has been behaving well. Naturally, the path of μ_t^* is non-decreasing. Moreover, $\mu_{t+1}^* = \mu_t^*$ means that in equilibrium there is no diversion of the t-period cash flow (pooling); $1 > \mu_{t+1}^* > \mu_t^*$ means that there is some probability of diversion of the t-period cash flow (semi-pooling); and $\mu_{t+1}^* = 1$ means that the low types definitely divert the t-period cash flow (separation). Of course, at the last period of the game, the low types always divert, and thus $\mu_{T+1}^* = 1$.

Furthermore, the stock price P_t^* and the probability μ_t^* are related in an economically meaningful way by an expression derived from equation (8). For example, the stock price at the IPO is given by:

$$P_1^* = \sum_{t=1}^{T} \left[\frac{\mu_1^*}{\mu_{t+1}^*} \overline{y}_t + \left(1 - \frac{\mu_1^*}{\mu_{t+1}^*} \right) \underline{y}_t \right] \tag{9}$$

where the term $\frac{\mu_1^*}{\mu_{t+1}^*}$ is the unconditional probability of no diversion of the t-period cash flow, $\overline{y}_t$ is the expected cash flow given that there is no diversion, and $\underline{y}_t$ is the expected cash flow with diversion.

We will show that the multiperiod equilibrium is such that there is a positive probability that the low type will behave well in the beginning of the game followed by expropriation toward the end of the game. These and other novel properties of the multiperiod equilibrium are discussed in more detail in the following section.

IV Corporate Finance Results

This section develops the main results of the paper. We show that managers can develop a reputation for not expropriating minority shareholders or exerting high levels of effort. This reputation effect causes a significant increase in the stock price and reduces the inefficiencies associated with agency problems facilitating firms to go public. Likewise, our results suggest that the rate of growth of the firm does not have a significant effect on its reputation-building ability. Furthermore, we show that there is a negative relationship between the degree of moral hazard and both the size of the block that is divested at the IPO and the rate of divesture of shares over time after the IPO.

Our goal is to analyze the equilibrium outcome of the multiperiod game and develop the comparative statics analysis of the equilibrium. The variables of interest to us are the path of insider ownership α_t^*, the share price P_t^*, and investors' information about the manager μ_t^*, all given by the results of Proposition 4. Also, we use the equilibrium payoffs of owner-managers to determine how likely the firm is to go public.

Since a close form solution for the multiperiod equilibrium is not obtainable, we use numeric simulations to derive our results. The simulations were performed using the diversion model in which managers can either be of a high type that does not divert cash flows or a low type that can divert a fraction b of the cash flow. The parameters in the comparative statics analysis are the initial probability $1-\mu_0$ that the manager is of a low type (the asymmetry of information parameter) and the moral hazard parameter b. As a proxy for the degree of moral hazard we use $b(1-\mu_0)$, which is equal to the expected fraction of the cash flow that can be diverted by the manager.[11]

We will formulate and then explain each of the main results in the remainder of the section.

Result 1 *At the IPO, a block of shares is divested, and following the IPO, shares are gradually divested. The size of the block sold in the IPO and the rate at which shares are divested over time are negatively related to the degree of moral hazard.*

[11] We developed a computer program to obtain the numerical results. The program was written in MATLAB and is available upon request.

Our first result extends to a multiperiod game the static game result that the amount of shares retained by the manager to signal his type is negatively correlated with the degree of moral hazard.

The main trade-off faced by the high-type manager in his choice of divestiture is, intuitively, one of selling shares at a low price today in order to be able to sell shares at a higher price in the future. More specifically, when he diverts more shares in the current period the stock price in the current period is lower, but there is more diversification of risk and the stock price in future periods is higher because investors update their beliefs that the manager is of a high type by a larger amount when the manager owns fewer shares and there is no diversion of cash flows. Therefore, he trades off more diversification of risk and a higher share price in the future against a higher share price in the current period in his choice of divestiture of shares.

Figure 2 depicts the equilibrium path of insider ownership, α_t^*, and the path for the probability of being a high type, μ_t^*, for a game with 20 periods. The figure illustrates the content of our first comparative statics result. The horizontal axis in Figure 2 is calendar time starting from the IPO until the firm is liquidated.

Insert Figure 2 here

The intuition for the relationship between the rate of divestiture of shares and the degree of moral hazard is as follows. Note first that the degree of moral hazard increases with higher values of b and lower values of μ_0. Besides, for any given μ_0, a smaller value of b is associated with a smaller gain in the stock price that can be induced by retention of shares, and therefore it is optimum to diversify more risk early on through a faster divestiture of shares. This also leads to a quicker revelation of information about the manager. Analogously, as μ_0 increases, there is less room to increase the share price by making investors believe that the manager is more likely to be of the high type, and thus it pays off to diversify more of the risk immediately at the IPO, leading to a bigger block being divested initially.

Result 2 *Managers can develop a reputation for behaving well, extracting low levels of private benefits or exerting high levels of effort. In equilibrium, the stock price is significantly* *higher than it would otherwise be without this reputation effect. Furthermore, the increase in the stock price associated with the reputation effect is positively correlated with the degree of moral hazard.*

Managers in our model are able to build a reputation in a way similar to that in the models of Kreps and Wilson (1982) and Milgrom and Roberts (1982).[12] A manager will act strategically, extracting low levels of private benefits in order to sell shares at a high price. A low-type manager can credibly commit not to divert the cash flows because if he started to extract high levels of private benefits from the IPO, investors would discount the price of shares accordingly, and the manager's remaining shares would sell at a reduced price. Unlike in Leland and Pyle (1977), where a manager holds a concentrated block of shares to signal to investors his private information that the firm has a high market value, in this paper a manager holds concentrated equity ownership to provide a guarantee to investors that he will exert high effort levels or extract low levels of private benefits in the future.

The stock price at the IPO with the reputation effect is given by expression (9), $P_1^* = \sum_{t=1}^{T}\left[\frac{\mu_1^*}{\mu_{t+1}^*} + \left(1 - \frac{\mu_1^*}{\mu_{t+1}^*}\right)(1-b)\right]\overline{y}_t$, where $\frac{\mu_1^*}{\mu_{t+1}^*}$ is the unconditional probability that there is no diversion of the t-period cash flow. If the manager could not establish any reputation, and therefore expropriated shareholders as much as possible in every period, the stock price would be equal to $P_1 = \sum_{t=1}^{T}\left[\mu_0 + (1-\mu_0)(1-b)\right]\overline{y}_t$. Note that $P_1^* \geq P_1$ because $\mu_0 \leq \mu_1^* \leq \mu_t^* \leq \mu_{t+1}^*$. The gain from the reputation effect is $P_1^* - P_1$. At one extreme, when there is separation early on, the gain is not very significant, and at the other extreme, when there is pooling or semi-pooling for several periods, the gain is very significant. The average increase in stock price for the 20-period game with expected cash flow equal to $\sum_{t=1}^{20}\overline{y}_t = \$100$ is shown in Figure 3 for several different parameter values.

Insert Figure 3 here

[12] There are, though, several differences between our model and Kreps and Wilson (1982) and Milgrom and Roberts (1982). In this paper a long-run player (the manager) faces many long-lived opponents (investors), as in Fudenberg and Kreps (1987). Also, the game studied is not a repeated game; the game played at any period is dependent on the insider ownership variable as a state variable.

The figure shows both that the increase in the stock price associated with the reputation effect is very significant and that this increase is positively related to the degree of moral hazard. For example, when $b = 1$ and $\mu_0 = 30\%$, the degree of moral hazard is very severe ($b(1 - \mu_0) = 0.7$), and the stock price goes from \$30 without any reputation effect to \$57 with the reputation effect, an increase of \$27 or 90%. Also, when $b = 1$ and $\mu_0 = 50\%$, the degree of moral hazard is 0.5 and the stock price increases from \$50 to \$66, an increase of \$16 or 32%. For moderate values of the moral hazard problem, gains over 5% can be easily obtained due to the reputation effect. For low values of moral hazard the gains from reputation are not very significant. There is also a strong correlation (0.95) between the dollar gains in stock price attributed to the reputation effect and the degree of moral hazard.

Another interesting result is that *the firm is more likely to go public in the multiperiod setting.* The idea is that the "lemons problem" that can drive good firms out the stock market is ameliorated because owner-managers are able to gradually sell shares and build a reputation for behaving well. For example, if $b = 1$, $\mu_0 = 50\%$ and the cash flow is realized over 20 periods, then a low-type manager gets a total value of \$84 when going public and the high type gets \$75. In a static game the "lemons problem" is so severe that the firm would not go public and the closely held concern value would be only \$69. Likewise, of course, the firm is more likely to go public and is worth more for owner-managers the lower the moral hazard problem and the higher the prior probability that the manager is of a high type.

So far this paper has emphasized the role of secondary equity offerings and the need to diversify the idiosyncratic risk of the firm as a mechanism that allow the manager to build a reputation for good behavior. Is this reputation effect even more intense for a growth company with large financing requirements, which is likely to revisit the primary capital markets more frequently? In other words, is a firm with more growth opportunities able to raise equity financing at a lower cost because it can build more reputation for reducing the moral hazard problem?[13] According to the evidence in Singh (1995), firms in developing markets, where there are significant growth opportunities, rely more heavily on external equity financing than firms in developed markets. Could this finding be related to firms' ability to build more reputation in the presence of more growth opportunities?

[13]I thank an anonymous referee for suggesting this relationship.

Result 3 *The reputation effect is not significantly dependent on the firm's growth opportunities as long as the firm can raise debt financing or the owner-manager is not credit constrained.*

Contrary to the intuition, our results indicate that firms' growth opportunities are largely unrelated to a lower cost of equity financing as long as firms have the ability to finance growth prospects by issuing riskless debt. We believe that, since firms invariably have the option of tapping the debt markets, any claim that managers should behave better with minority shareholders because of the need to revisit the capital markets should be robust to the availability of debt financing. We will develop Result 3 by first showing that the reputation effect is not dependent on the financing needs of the firm, and then showing that it is unrelated to the rate of growth of cash flows.

First, let us see why the reputation effect is unrelated to the firm's future financing needs. Intuitively, this result holds because the reputation-building mechanism comes from the owner-managers' dependence on the unique feature of the equity markets: the ability to diversify idiosyncratic risks. No reputation can be developed for treating equityholders well based on the firm's future needs for additional funds, as long as other forms of financing besides equity are available. The owner-managers' decisions with respect to ownership, effort, and investments are neutral with respect to the firm's external financing needs because the firm is able to obtain external financing raising fairly priced and riskless debt without changing the insider ownership stake α_t (no equity issuance) and the manager's incentive for exerting effort. More specifically, let the total cash flows under any investment choice be $\widetilde{y}_t$, which is financed issuing riskless debt D_t, where $\sum D_t = 0$ (interest rate has been set equal to zero).[14] The cash flow to equity holders is equal to $\widetilde{y}_t - D_t$ and therefore the manager's choice of divesture and effort that maximizes problem (1) is not dependent on the level of riskless debt in the firm's capital structure. Likewise, we obtain the same neutrality result with respect to the financing needs of the firm if the owner-manager is wealthy or not credit constrained.[15]

[14] The assumption that the firm is able to finance its external capital requirements issuing riskless debt is less stringent than it might seem. For example, allowing for efficient renegotiations between the firm and creditors the set of investment opportunities that can be financed with riskless debt can be quite large.

[15] By the same reasoning as above, the owner-manager's choices of effort and divesture are unaffected, as

We are still left with the possibility that the growth rate of cash flows per se can influence the manager's ability to develop a reputation for treating minority shareholders well. In order to explore the potential for a relationship between the growth rate of cash flows and the reputation effect we conducted some numeric analysis. Say that the real rate of growth of cash flow is equal to g, so that $E[\widetilde{y}_t] = (1+g)\,E[\widetilde{y}_{t-1}]$ and $\sum_{t=1}^{T} \widetilde{y}_t = \widetilde{y}$.[16] Is there any relation between g and the stock price and the value of the firm for entrepreneurs? Our quantitative results suggest that there is no significant relation between the relevant variables and the growth rate.

For example, with the moral hazard and asymmetric information parameters equal to $(b, \mu_0) = (1, 50\%)$ and total cash flow equal to $\sum_{t=1}^{20} \overline{y}_t = \$100$, the stock price at the IPO is \$67, \$66, and \$63, respectively, for growth rates equal to -20%, 0%, and 20%. Also, the average value from going public for the owners, which includes their expected extraction of private benefits, is \$80, \$83, and \$82, respectively. Therefore, the growth rate does not significantly influence the reputation effect, for values of g in a 20% range of zero. In spite of that, the growth rate does have an impact on the insider ownership variable. Insiders in mature firms, with a real growth rate of -20%, hold only 22% of their firm's equity 10 periods after going public, while insiders in firms with real growth rates equal to zero and 20% hold, respectively, 60% and 68% of their firm's equity 10 periods after going public. Nevertheless, as we allow for extreme values of growth, e.g., growth rates close to $+100\%$ and -100%, the outcome converges to the outcome of the static game–all the cash flow comes, respectively, in the last period and first period of the game. As we have argued before, in static games managers are not able to develop any reputation and the stock price is much lower; therefore, extreme values of growth rates can have an adverse effect on the value of the firm.

We explore the empirical implications of our results as well as the empirical relevance of the model in the next section.

long as the firm can finance investments issuing equity (e.g., using rights offering) in which the manager has enough funds to maintain the same fractional ownership stake he had before the issuance.

[16] In our previous simulations the real growth rate was set equal to zero.

V Empirical Relevance and Implications

This model provides several new empirical implications for the dynamics of insider ownership. The model predicts that insiders divest shares gradually. There is an initial public offering (IPO) where a block of new shares (primary offering) and/or existing owners' shares (secondary offering) are sold to the public followed by gradual secondary and/or primary offerings over time. Furthermore, insider ownership time series is related to the degree of agency problems: the size of the block divested at the IPO and the rate at which shares are divested are negatively related to the degree of agency problems expected by investors. Alternatively, the time it takes to reach a certain insider ownership level is positively related to agency costs. Those predictions do not follow from Jensen and Meckling (1976) extended to a multiperiod model–see the multiperiod complete information model of section II. If one allowed managers to trade their shares continuously there would be almost instantaneous convergence to a lower level of ownership.[17] This complete information model would predict that insiders divest a large block of their shares soon after the IPO and then maintain essentially constant their ownership after the IPO. Our incomplete information model, on the other hand, predicts a time series with gradual divestiture of shares, with a cross-sectional variation that depends on how severe the agency problem is.

Rydqvist and Hogholm (1995) provide evidence that is consistent with the predictions of our model. They show evidence of insider ownership for corporations that went public in Sweden from 1970 to 1991. Two years before going public the mean ownership concentration is 90%. Immediately after the IPO the average ownership retention is reduced to 57%, and five years after the IPO the ownership is reduced to 36%. It would be interesting to test the time series and the cross-section patterns of insider ownership for other markets–e.g., we did not find any evidence on the cross-sectional empirical implications of the model.

It is important to highlight that the model is consistent with the widely documented evidence that equity markets are not a relevant source of financing for corporations. Mayer (1990) has shown that internally generated funds are the major source of financing for industrial countries over the 1970-1985 period and that equity financing represents only a

[17] This is similar to Coase's analysis of pricing by a durable-good monopolist. When offers by the monopolist take place very quickly, the price converges to the competitive prices almost instantaneously.

negligible amount. In our view the main role of the equity markets is not to provide financing to firms but to provide owners with the ability to diversify idiosyncratic risks and to provide them with liquidity–see Rydqvist and Hogholm (1995). Even though, for example, the U.S. stock market represented only 0.8% of the total financing during 1970-1985, it performed a very significant role as a divestiture mechanism; Demsetz and Lehn (1985) have shown that the largest 500 firms in the United States in 1980 had the five largest shareholders owning on average only 25% of shares. For this reason, our model is based not on primary offerings but on secondary offerings of equity; the motivation to go public is not financing but diversification of risks; and the implications of the model are not with respect to the dynamics of seasoned equity offerings but for the dynamics of insider ownership.

Our results have special empirical relevance for countries where legal institutions do not offer enough protection to minority shareholders against managerial discretion. Many empirical studies attempt to estimate the size of private benefits, and these studies unanimously conclude that private benefits are, on average, substantial for companies in many countries (see La Porta et al. (1998)).[18] Even in the United States, where shareholders have one of the best and most elaborate legal systems protecting their interests, there is substantial evidence documenting the discretion that managers have over investors' money, being able to waste much of it on private benefits.[19] This paper explains why firms can go public even when investors do not get any control rights in exchange for their funds, and managers have the ability to extract significant private benefits of control. Our results show that these firms can go public because of the multiperiod nature of share trading and the generation of cash

[18] Barclay and Holderness (1989) find for the U.S. that the premium of block trades over the post-announcement exchange price is, on average, 4% of a firm's market value. Bergstrom and Rydqvist (1992) find a premium of voting relative to non-voting shares of 6.5% in Sweden; Zingales (1994) finds a premium of 81% in Italy; Levy (1983) finds a premium of 45.5% for Israel; Horner (1988), 20% for Switzerland; Megginson (1990), 13.3% for the U.K.; Robinson and White (1990), 23.3% for Canada; and Lease, McConell, and Mikkelson (1983), 5.4% for the U.S. See also Barclay, Holderness, and Pontiff (1993) for private benefits of control in closed-end mutual funds and DeAngelo and DeAngelo (1985) for a study of the concentration of managerial ownership and control in dual-class shares corporations in the U.S.

[19] For example, oil industry firms in the mid-1980s spent significant resources in negative NPV projects (Jensen (1986)); negative returns to bidders in the announcements of acquisitions were motivated by managers' preferences for diversification and growth instead of shareholder value (Morck, Shleifer, and Vishny (1990)); managers resisted takeovers to protect their private benefits rather than to serve shareholders (e.g., Jarrell and Poulsen (1987)).

flows. It is precisely when the agency problem is more severe that managers can implicitly commit to investors that they are not going to divert the cash flow of the firm. The chances of going public and the value that entrepreneurs can gain from going public are increased in the multiperiod setting, and the average value of firms that go public is enhanced by a reputation effect that implicitly protects minority shareholders against expropriation.

Our quantitative results show that, for example, when managers can divert an expected amount of 50% of the cash flow, the value of the stock price is more than 30% higher, because of the reputation effect, than the price that would be predicted in the absence of this effect, based only on the level of protection of minority shareholders. So, in equilibrium managers are expected to divert much less than what they can. In addition, this increase in stock price is positively correlated with the degree of moral hazard problem (coefficient of 0.95 in our simulations).

Singh (1995) concluded that there are important differences between the financing patterns of developing countries and those of the industrial countries studied by Mayer (1990). Singh provided evidence that developing countries rely more heavily on external financing than developed countries, and also use equity more intensively than debt financing for their external financing needs. This evidence is especially puzzling given the results of La Porta et al. (1998) that investors in these countries are the ones with lower levels of protection. Our model of equity is consistent with this evidence and points out to variables that are relevant to explaining it.

Several independent variables are candidates for explaining the above-mentioned disparities in the use of external equity financing and we believe that the most important ones are the level of protection of minority shareholders, growth opportunities, volatility of cash flows, and levels of concentration of insider ownership. Our model shows why firms in countries with low levels of protection of minority shareholders are able to obtain equity financing and investors are willing to buy shares: entrepreneur-owners can develop a reputation for treating minority shareholders well. Firms in developing countries are also expected to have higher growth opportunities and therefore are more likely not to be able to generate internally funds for their investment needs, having to revisit the primary capital markets more often. While this simple fact alone could explain part of the evidence in Singh that firms in developing countries are more likely to use external financing, it still does not explain the more intense use of equity rather than debt financing. Intuitively, the commitment to revisit the capital markets could further enhance the reputation effect and lead to additional reductions of the cost of equity financing, therefore also stimulating a more heavy use of equity by firms. Contrary to this intuition, we have shown that the growth rate of firms does not have any additional significant influence on the reputation effect whenever debt financing remains available for firms to tap.

A lower level of protection and higher growth rate imply that firms in equilibrium should have a more concentrated ownership structure. In fact, La Porta, Lopez-de-Silanes, and Shleifer (1998) document that among the world's largest publicly traded firms, 73% of the firms in countries with low protection–mostly developing countries–have a large shareholder (stake above 20%), while only 52% of the firms in countries with more protection have a large shareholder. Obviously, however, firms with a more concentrated ownership and a more volatile cash flow find equity financing relatively more desirable than debt financing because risk-averse entrepreneurs' marginal gains from diversification are higher. For example, entrepreneur-owners of firms with concentrated ownership are only willing to undertake some risky projects if they can issue risky claims to outside investors, while diversified firms in the United States and many other developed countries receive insignificant marginal diversification gains using equity financing and prefer to finance at lower costs, issuing fixed-income claims.

In summary, Singh's findings that firms in developing countries are willing to issue more equity for financing than developed country firms are not surprising: the reputation-building effect reduces the importance of the absence of protection to shareholders; high growth rates imply that internally generated funds are insufficient to finance investment opportunities; and a high concentration of ownership coupled with high volatilities of cash flows makes equity financing a relatively more attractive source of financing than debt.

An important assumption in our application of the model to equity markets of low-protection countries is that managers are unambiguously in charge and cannot be disciplined by takeovers, or by dismissal by the board of directors or other large blockholders. The empirical evidence that is starting to mount in markets around the world seems to be consistent with this assumption (Franks and Mayer (1990), La Porta et al. (1998)). In fact, one can argue that in those markets the only equilibrium is for firms to have a controlling shareholder. Any firm widely held without a controlling shareholder, even if well managed, would likely be the target of a raider who would buy enough shares to form a controlling block. This blockholder would find it more profitable to extract private benefits for himself, instead of acting as a monitor that increases the value of the firm for minority shareholders. For the same reason, other large blockholders in these environments, such as banks and institutional investors, are not likely to act as monitors for minority shareholders; it is much more profitable for them to get "side payments" in exchange for their compliance than to spend resources opposing management. In addition, La Porta et al. (1998) provide evidence that controlling shareholders are often alone and there are no other large blocks in the hands of banks and other financial institution.[20]

The assumption that the managers cannot be disciplined by outsiders seems to be at odds with the documented evidence on large voting premiums, e.g., Zingales (1994) and Levy (1983); a large voting premium suggests that managers might be ousted by outsiders that act as a disciplinary agent for minority shareholders.[21] Several plausible explanations can reconcile a large voting premium with a lack of outside discipline of managers. First, in many countries where there are dual-class shares, regulation forces a bidder for control to make the same offer per share to all target shareholders, with the exception that he may differentiate the bid between classes of shares (Rydqvist (1992)). A marginal voting share might trade at a premium not because of the possibility of a disciplining takeover but because of a negotiated sale of control (Bebchuk (1994)) in which marginal voting shares will receive the higher price paid for the controlling block–Italy, Canada, Sweden and Brazil are some countries with this type of regulation.[22] In addition, a premium can exist when there is more than one shareholder fighting for control and the marginal voting share can be pivotal; none of the potential controllers, though, are expected to monitor and increase value for minority

[20] In Germany, Japan, and France, banks and other financial institutions often control the major industrial corporations (Franks and Mayer (1994)). In most of the rest of the world, public corporations are often controlled by their founders or the founders' offspring (La Porta et al. (1998) and Zingales (1994)).

[21] I thank an anonymous referee for bringing out this point.

[22] See Zingales (1994, p. 140) for Italy, Smith and Amoaku-Adu (1995, p. 226) for Canada, and Rydqvist (1992) for Sweden.

shareholders after gaining control, but rather to extract private benefits for themselves (see Gomes and Novaes (1998) for a model with multiple controlling shareholders).

The existing literature suggests many disadvantages to using structures that increase the separation between ownership and control, e.g., Grossman and Hart (1988). The widespread use of these structures, e.g., La Porta et al. (1998) and Rydqvist (1992), is puzzling because the initial owners bear ex-ante the costs of ex-post inefficiencies, and it seems reasonable that owners could commit ex-ante not to use pyramidal and dual-class shares structures. This suggests that, in many cases, the benefits of using those structures are lower than the costs.

Finally, the model provides a new explanation for the benefits of using pyramidal and dual-class shares structures. The main idea is that when managers can issue restricted voting shares or build pyramids, they are able to increase their reputation for not expropriating minority shareholders, which can increase the value of the firm. As we have seen in the previous sections, the mechanism that allows the reputation effect to work is the prospect of selling additional shares in the future for a high price; therefore, when managers are allowed to issue shares with restricted voting rights, or develop a pyramidal structure, the effect can work better since there are more shares for managers to sell while still keeping majority control of the firm. In addition, the costs of using those structures that increase the separation of ownership and control are not so significant because managers in environments with low protection will want to keep control whether dual-class shares and pyramids are allowed or not. As we argued above, keeping control seems to be the only possible equilibrium in many countries where, for example, negotiated transfer of control replaces disciplining mechanisms like takeovers, consistent with the empirical evidence. Furthermore, the ability to divest more shares without losing control can increase the free float (percentage that the owners sell to the public) and thus improve the amount of risks that owners can diversify.

VI Concluding Remarks

In this article, we develop a model of a firm that is going public in the presence of moral hazard, asymmetric information, and multiple trading periods. The agency problem emphasized in the paper is not the standard agency problem between managers and shareholders, but the conflicts between a large shareholder that exerts control over the management and can extract private benefits of control, and minority shareholders in the firm. The motivation for the owner-manager to take the firm public is to share the idiosyncratic risk of the firm with minority shareholders; however, all the participants in the market recognize that there is room for the manager to act opportunistically, expropriating minority investors. Managers face the problem of whether to take their firms public and how many shares to offer, and investors face the problem of how much to pay for shares, given all the information available to them.

The equilibrium with multiple trading periods has important differences from the static case. In the multiperiod game, the manager can strategically choose extraction of private benefits and can develop a reputation for expropriating low levels of private benefits. Even if investors assign a low probability that a manager is of a high type, the manager is still able to sell shares at a high price, because investors know that she has an incentive to develop a reputation for consuming low levels of private benefits. If she starts to extract high private benefits from the IPO, investors will discount the price of shares accordingly, and her remaining shares will sell at a reduced price. We showed that the reputation effect can play a very significant role in the equilibrium allocation, through a range of numerical simulations of the model.

The model provided several novel empirical implications. First, the model predicts that the insider ownership time-series and cross-section variation is related to the degree of moral hazard. At the IPO, the size of the block of shares divested is negatively related to the degree of moral hazard problem, and following the IPO, shares are gradually divested at a rate that is also negatively related to the degree of moral hazard. Second, we believe that this model has empirical relevance in explaining the existence of stock markets in emerging economies, despite the very weak legal institutions offering protection to minority shareholders. Another implication of the results for emerging stock markets is that there are additional benefits from using the dual-class voting structure and pyramidal structures that have not yet been considered in the literature. Allowing controlling managers to increase the separation of ownership and control does not increase inefficiencies significantly, but can increase the efficiency of stock markets, because the reputation effect is stronger when managers can divest more without losing control.

The use of multiperiod signalling models can have promising applications in other areas of finance. For example, the model can be extended to allow the manager to use additional signals, such as the level of debt or dividend policy. Most other studies of asymmetric information problems in corporate finance have been conducted in a static setting. It is an interesting issue for future research to determine what new insights arise from the extension of static models to a more realistic multiperiod signalling framework. The results of this paper suggest that multiperiod signalling games have several economic properties that do not exist in static games.

Appendix

Proofs of Theorems

Lemma 1 *The independence of the random shocks implies that*

$$-\log E\left[\exp\left[-w_{t,T}\right]\right]=\sum_{\tau=t}^{T+1}\left(\alpha_{\tau-1}-\alpha_{\tau}\right)P_{\tau}+v_{\tau-1}\left(\alpha_{\tau-1},e_{\tau},\theta\right)$$

where $v_{\tau}\left(\alpha_{\tau},e_{\tau+1},\theta\right)=-\log E\left[\exp-\left[\alpha_{\tau}y_{\tau}\left(e_{\tau+1},\eta_{\tau}\right)-c_{\tau}\left(e_{\tau+1},\eta_{\tau},\theta\right)\right]\right]$.

Proof of Lemma 1:

Expected values are taken with respect to the distribution of random shocks to the cash flows (conditional and unconditional expectations are identical because of the independence of random shocks). The proof uses the fact that the choices of signals in any period τ, α_{τ} and e_{τ}, only depend on the previous history of actions $h^{\tau-1}$ until period $\tau-1$, but do not depend on realizations of random shocks to the cash flows. We will see that this true because past realizations of random shocks are irrelevant for future payoffs (see section III.A).

First we have that,

$$E\left[\exp\left[-w_{t,T}\right]\right]=E\left[\prod_{\tau=t}^{T}\left[\exp-\left[\left(\alpha_{\tau-1}-\alpha_{\tau}\right)P_{\tau}+\alpha_{\tau-1}y_{\tau-1}\left(e_{\tau},\eta_{\tau-1}\right)-c_{\tau-1}\left(e_{\tau},\eta_{\tau-1},\theta\right)\right]\right]\right]$$

which is equal to $\prod_{\tau=t}^{T}E\left[\exp-\left[\left(\alpha_{\tau-1}-\alpha_{\tau}\right)P_{\tau}+\alpha_{\tau-1}y_{\tau-1}\left(e_{\tau},\eta_{\tau-1}\right)-c_{\tau-1}\left(e_{\tau},\eta_{\tau-1},\theta\right)\right]\right]$, because of the independence of the η_t's. The terms $\left(\alpha_{\tau-1}-\alpha_{\tau}\right)P_{\tau}$ can be taken out of the expectation operator because they do not depend on the random shock. This implies that $-\log E\left[\exp\left[-w_{t,T}\right]\right]=\sum_{\tau=t}^{T+1}\left(\alpha_{\tau-1}-\alpha_{\tau}\right)P_{\tau}+v_{\tau-1}\left(\alpha_{\tau-1},e_{\tau},\theta\right)$, where $v_{t}\left(\alpha_{t},e_{t+1},\theta\right)=-\log E\left[\exp-\left[\alpha_{t}y_{t}\left(e_{t+1},\eta_{t}\right)-c_{t}\left(e_{t+1},\eta_{t},\theta\right)\right]\right]$. ■

Proof of Proposition 2

Define the utility function for the manager as $u(\alpha, P, \theta) = (1-\alpha) P + V_1(\alpha, \theta)$ and consider the strategy profile σ and system of beliefs μ defined by the statement of the proposition.

First, (σ, μ) is a PBE. (1) The strategy prescribed for the high type is a Nash equilibrium: suppose $\exists \alpha \neq \alpha^*$ such that $u\left(\alpha, P(\alpha, \underline{\theta}), \overline{\theta}\right) > u\left(\alpha^*, P^*, \overline{\theta}\right)$. But $u\left(\alpha, P(\alpha, \mu_0), \overline{\theta}\right) > u\left(\alpha, P(\alpha, \underline{\theta}), \overline{\theta}\right)$ and the triple α, μ_0 and $P = P(\alpha, \mu_0)$ satisfy all the constraints of problem 4. This is in contradiction with α^*, μ^* and P^* being a solution of the problem. (2) The strategy prescribed for the low type is a Nash equilibrium: say that $\mu^* = 1$ and thus $u\left(\alpha^*, P^*, \overline{\theta}\right) \leq V_0(1, \underline{\theta})$, so that playing $\alpha(\underline{\theta})$ with probability 1 is the optimal response for the low type; say that $\mu^* = \mu_0$. Then it must be the case that $u(\alpha^*, P^*, \underline{\theta}) \geq V_0(1, \underline{\theta})$; otherwise one could choose $\mu > \mu_0$ that satisfy all the constraints and increase the value of problem 4. Note that μ^* is equal to either μ_0 or 1 because if $\mu^* \in (\mu_0, 1)$ then by the single-crossing property there would exist a pair $\mu > \mu^*$ and $\alpha > \alpha^*$ that would satisfy $u(\alpha^*, P^*, \underline{\theta}) = u(\alpha, P(\alpha, \mu), \underline{\theta})$ with $u\left(\alpha^*, P^*, \overline{\theta}\right) < u\left(\alpha, P(\alpha, \mu), \overline{\theta}\right)$. The competitive equilibrium condition and Bayes rule are automatically satisfied by (σ, μ).

Second, (σ, μ) is a lex max PBE. Assume that there is PBE equilibrium σ' that lexicographically dominates σ. Consider any action α' that is played by the high type such that the associated on-the-equilibrium path beliefs are $\mu' \geq \mu_0$ and prices are equal to $P' = P(\alpha', \mu')$. Such triple must satisfy all the constraints of problem 4 and thus $u\left(\sigma, \overline{\theta}\right) \geq u\left(\sigma', \overline{\theta}\right)$. Moreover, the equilibrium σ is such that there exists no σ' such that $u(\sigma', \underline{\theta}) > u(\sigma, \underline{\theta})$ and $u\left(\sigma', \overline{\theta}\right) \geq u\left(\sigma, \overline{\theta}\right)$, which is in contradiction with σ' lexicographically dominating σ. The uniqueness of the equilibrium outcome is obvious–note, though, that there can be several strategies implementing such outcome.

Third, consider for any μ the value function $f(\mu) = \max_\alpha u\left(\alpha, P(\alpha, \mu), \overline{\theta}\right)$. The function f is the most utility that can be obtained in any pooling equilibrium by the high type and is an increasing and continuous function. Let $\overline{\mu}_0 \in (0, 1]$ be the value μ (which must always exist) such that $f(\mu)$ is equal to the value of the high type in the Riley separating equilibrium.

Finally, using Theorem 1 of Mailath, Okuno-Fujiwara and Postlewaite (1993), which states that any lex max equilibrium is undefeated, we conclude the proof of the proposition. We refer to their work for a proof of this result. ■

Proof of Proposition 3

In a separating equilibrium there must be a level of ownership α_1 in the first period such that the incentive compatibility condition $(1-\alpha_1) P_1 \left(\alpha_1, \overline{\theta}\right) + V_1 (\alpha_1, 1, \underline{\theta}) \leq V_0 (1, \underline{\theta})$ is satisfied. We first show how to obtain, in general, the expression for $V_1 (\alpha_1, 1, \underline{\theta})$ that appears in the IC condition.

We start by determining the value of $V_{t-1} (\alpha_{t-1}, 1, \underline{\theta})$. The concept of PBE and its refinements impose very weak restrictions on the equilibrium outcome for games with incomplete information with degenerate priors, where the uninformed player puts all the weight on one of the types. Although the concept of PBE does not provide a unique solution to the problem it seems natural to consider that the equilibrium outcome is similar to the case where there is no incomplete information and the concept of subgame perfection pins down a unique equilibrium outcome. The complete information case where the manager is of type $\overline{\theta}$ was solved in section II; the manager strategy is to play $\alpha_t \left(\alpha_{t-1}, \overline{\theta}\right)$ and $e_t \left(\alpha_{t-1}, \overline{\theta}\right)$ and the investor response is $P_t \left(\alpha_t, \overline{\theta}\right)$. Playing $(\alpha_t, e_t) \neq \left(\alpha_t \left(\alpha_{t-1}, \overline{\theta}\right), e_t \left(\alpha_{t-1}, \overline{\theta}\right)\right)$ in period t is a dominated strategy of the high-type manager and thus it is reasonable that investors' should believe that such a play must come from a low-type manager. Thus by equilibrium dominance, investors out-of-equilibrium belief must be given by $\mu \left(\overline{\theta} | \alpha_t, e_t\right) = 1$ if $(\alpha_t, e_t) = \left(\alpha_t \left(\alpha_{t-1}, \overline{\theta}\right), e_t \left(\alpha_{t-1}, \overline{\theta}\right)\right)$ and $\mu \left(\overline{\theta} | \alpha_t, e_t\right) = 0$ otherwise.

Given the above considerations restricting the out-of-equilibrium beliefs we can obtain the expression for $V_{t-1} (\alpha_{t-1}, 1, \underline{\theta})$ recursively as equal to:

$$\begin{array}{ll} \max\limits_{\alpha_t, e_t} & (\alpha_{t-1} - \alpha_t) P_t (\alpha_t, \mu (\alpha_t, e_t)) + v_{t-1} (\alpha_{t-1}, e_t, \theta) + V_t (\alpha_t, \mu (\alpha_t, e_t), \underline{\theta}) \\ \text{s.}t. & \mu (\alpha_t, e_t) = \left\{ \begin{array}{ll} 1 & \text{if } (\alpha_t, e_t) = \left(\alpha_t \left(\alpha_{t-1}, \overline{\theta}\right), e_t \left(\alpha_{t-1}, \overline{\theta}\right)\right) \\ 0 & \text{otherwise} \end{array} \right. \end{array} \tag{10}$$

where $V_T (\alpha_T, 1, \underline{\theta}) = \max_{e_{T+1}} v_T (\alpha_T, e_{T+1}, \underline{\theta})$ and $V_t (\alpha_t, 0, \underline{\theta})$ is the value for the low type in the complete information case. Let the solution of the above problem be $e_t (\alpha_{t-1}, 1, \underline{\theta})$ and $\alpha_t (\alpha_{t-1}, 1, \underline{\theta})$.

The second part of the proposition follows from the analysis of the example proposed before the statement of the proposition. The IC condition for the example is $\sum_{t=1}^{T} b\overline{y}_t \leq \alpha_1\overline{y}_1 - v_1(\alpha_1) + v_1(b)$ and it is straightforward that the condition is not satisfied for any α_1 for large values of b.

■

Proof of Proposition 4

We first state and prove a Lemma that guarantees that there exists a well-behaved solution to the dynamic programming problem. We apply the Theorem of the Maximum in order to prove the result; see Lucas and Stokey (1989).

Lemma 2 *The dynamic programming problem 7 has a value* $V_{t-1}\left(\alpha_{t-1}, \mu_{t-1}, \overline{\theta}\right)$ *that is continuous in the state variables* $\left(\alpha_{t-1}, \mu_{t-1}\right)$, *and also a non-empty solution correspondence* $\left(\alpha_t, e_t, \mu_t, P_t\right) \in G_t\left(\alpha_{t-1}, \mu_{t-1}\right)$ *that is upper-hemi-continuous in the state variables.*

Proof of Lemma 2:

We first restate the problem in formal terms and then prove that the conditions of the Theorem apply to the problem.

The maximization problem at any period $t = 1, ..., T$ is equivalent to:

$$V_{t-1}\left(x, \overline{\theta}\right) = \max_{y \in \Gamma_t(x)} F_t(x, y)$$

where the beginning-of-period state variable is $x = \left(\alpha_{t-1}, \mu_{t-1}\right) \in X$, $X = A \times [0, 1]$ and the maximization is over $y = (\alpha_t, e_t, \mu_t, P_t, V_t) \in Y$, $Y = A \times E \times [0, 1] \times \Re_+^2$. The value of the maximum of the t-period problem is denoted as $V_{t-1}\left(x, \overline{\theta}\right)$, and the solution correspondence by $H_{t-1}(x) : X \to Y$.

The maximand of the t-period problem, $F_t(x, y)$, is defined as a function of the value for the future periods $V_t\left(x, \overline{\theta}\right)$ as $F_t(x, y) = (\alpha_{t-1} - \alpha_t) P_t + v_{t-1}\left(\alpha_{t-1}, e_t, \overline{\theta}\right) + V_t\left(\alpha_t, \mu_t, \overline{\theta}\right)$.

The set of feasible values is given by the correspondence $\Gamma_t : X \to Y$ defined based on the solution correspondence H_t as follows: for any $x = \left(\alpha_{t-1}, \mu_{t-1}\right) \in X$ and $y = (\alpha_t, e_t, \mu_t, P_t, V_t) \in Y$, $y \in \Gamma_t(x)$ if and only if:

$\exists z = \left(\alpha_{t+1}, e_{t+1}, \mu_{t+1}, P_{t+1}, V_{t+1}\right) \in H_t\left(\alpha_t, \mu_t\right)$ such that
(1) $P_t = \frac{\mu_t}{\mu_{t+1}}\left[e_{t+1} + P_{t+1}\right] + \left(1 - \frac{\mu_t}{\mu_{t+1}}\right)\left[P_t\left(\alpha_t, \underline{\theta}\right)\right]$;
(2) $V_t = \max\left\{\left(\alpha_t - \alpha_{t+1}\right) P_{t+1} + v_t\left(\alpha_t, e_{t+1}, \underline{\theta}\right) + V_{t+1}, V_t\left(\alpha_t, \underline{\theta}\right)\right\}$;
(3) $\mu_t \in \left[\mu_{t-1}, 1\right]$;
(4) $\left(\left(\alpha_{t-1} - \alpha_t\right) P_t + v_{t-1}\left(\alpha_{t-1}, e_t, \underline{\theta}\right) + V_t - V_{t-1}\left(\alpha_{t-1}, \underline{\theta}\right)\right) . \left(1 - \mu_t\right) \geq 0$;
(5) $\left(\left(\alpha_{t-1} - \alpha_t\right) P_t + v_{t-1}\left(\alpha_{t-1}, e_t, \underline{\theta}\right) + V_t - V_{t-1}\left(\alpha_{t-1}, \underline{\theta}\right)\right) . \left(\mu_t - \mu_{t-1}\right) \leq 0$.

At the terminal period the initial conditions are defined for any $x = \left(\alpha_T, \mu_T\right)$ as $V_T\left(x, \theta\right) = \max_{e_{T+1}} v_T\left(\alpha_T, e_{T+1}, \theta\right)$ and $H_T\left(x\right) = \left\{\left(\alpha_T, e_{T+1}\left(\alpha_T, \overline{\theta}\right), 1, 0, 0\right)\right\}$, a set with a unique element, $\left(\alpha_{T+1}, e_{T+1}, \mu_{T+1}, P_{T+1}, V_{T+1}\right) = \left(\alpha_T, e_{T+1}\left(\alpha_T, \overline{\theta}\right), 1, 0, 0\right)$. Note that both $V_T\left(x, \theta\right)$ and $H_T\left(x\right)$ are continuous in X and compact-valued.

Note that the correspondence $G_t\left(x\right) = \pi\left(H_t\left(x\right)\right)$, where $\pi\left(y\right) = \left(\alpha_t, e_t, \mu_t, P_t\right)$ for any $y = \left(\alpha_t, e_t, \mu_t, P_t, V_t\right) \in Y$, is a projection. Furthermore, $G_t\left(x\right)$ is upper-hemi-continuous (u.h.c.) if $H_t\left(x\right)$ is u.h.c.

The Theorem of the Maximum states that the value function $V_{t-1}\left(x, \overline{\theta}\right)$ is continuous and the solution correspondence $H_{t-1}\left(x\right)$ is u.h.c. and compact-valued as long as the maximand $F_t\left(x, y\right)$ is continuous and the correspondence $\Gamma_t : X \rightarrow Y$ is continuous–u.h.c and lower-hemi-continuous (l.h.c)–and compact-valued. By recursion, the function $F_t\left(x, y\right)$ is continuous. We next show that the correspondence $\Gamma_t : X \rightarrow Y$ is compact-valued and continuous if $H_t\left(x\right)$ is u.h.c. We will use several results about u.h.c and l.h.c correspondences (see Lucas and Stokey (1989, pp. 55-65).

Define $\Psi_1\left(y, z\right) = \frac{\mu_t}{\mu_{t+1}}\left[e_{t+1} + P_{t+1}\right] + \left(1 - \frac{\mu_t}{\mu_{t+1}}\right)\left[P_t\left(\alpha_t, \underline{\theta}\right)\right] - P_t$ for any $y = \left(\alpha_t, e_t, \mu_t, P_t, V_t\right) \in Y$ and $z = \left(\alpha_{t+1}, e_{t+1}, \mu_{t+1}, P_{t+1}, V_{t+1}\right)$ and $\Psi_2\left(y, z\right)$ analogously using restriction (2). Both $\Psi_1\left(y, z\right)$ and $\Psi_2\left(y, z\right)$ are continuous functions. Let $\Phi_1 : Y \rightarrow \Re$ be the correspondence $\Phi_1\left(y\right) = \Psi_1\left(y, H_t\left(\pi_1\left(y\right)\right)\right)$ where $\pi_1\left(y\right) = \left(\alpha_t, \mu_t\right)$ is a projection of y. Φ_1 is u.h.c. because $H_t\left(.\right)$ is u.h.c. and $\Psi_1\left(y, z\right)$ is continuous. Let $\Phi_2 : Y \rightarrow \Re$ be defined analogously. Restrictions (1) and (2) are equivalent to $\Phi_1^{-1}\left(0\right)$ and $\Phi_2^{-1}\left(0\right)$, respectively. Both $\Phi_1^{-1}\left(0\right)$ and $\Phi_2^{-1}\left(0\right)$ are closed sets. This is true because any u.h.c. correspondence is such that the inverse of a closed set is also a closed set.

Also, let $\Psi_4\left(x, y\right) = \left(\left(\alpha_{t-1} - \alpha_t\right) P_t + v_{t-1}\left(\alpha_{t-1}, e_t, \underline{\theta}\right) + V_t - V_{t-1}\left(\alpha_{t-1}, \underline{\theta}\right)\right) . \left(1 - \mu_t\right)$ for any $y = \left(\alpha_t, e_t, \mu_t, P_t, V_t\right) \in Y$ and $x = \left(\alpha_{t-1}, \mu_{t-1}\right) \in X$. Similarly, define $\Psi_5\left(x, y\right)$

using restriction (5). Of course, both $\Psi_4(x,y)$ and $\Psi_5(x,y)$ are continuous functions. Restrictions (4) and (5) are equivalent to $\Phi_4(x) = \left\{y \in Y : (x,y) \in \Psi_4^{-1}([0,+\infty])\right\}$ and $\Phi_5(x) = \left\{y \in Y : (x,y) \in \Psi_5^{-1}([0,+\infty])\right\}$ which are both closed sets. Let also $\Phi_3(x) = \left\{y \in Y : \mu_t \in \left[\mu_{t-1}, 1\right]\right\}$ for any $x = \left(\alpha_{t-1}, \mu_{t-1}\right)$. We have just shown that $\Gamma_t(x) = \Phi_1^{-1}(0) \cap \Phi_2^{-1}(0) \cap \Phi_3(x) \cap \Phi_4(x) \cap \Phi_5(x)$, where $\Phi_3(x)$, $\Phi_4(x)$, and $\Phi_5(x)$ all continuous functions of x. This implies that $\Gamma_t(x)$ is compact-valued (closed and bounded): $\Gamma_t(x)$ is obviously bounded, and is an intersection of closed sets, thus also a closed set. We have also shown that $\Gamma_t(x)$ is continuous in x. ■

We continue the proof of proposition showing how to associate solutions of the dynamic programming problem to equilibrium of the multiperiod game.

We first construct the PBE (σ, μ) that is a candidate for lex max. Let for any $\left(\alpha_{t-1}, \mu_{t-1}\right)$, $(\alpha_t^*, e_t^*, \mu_t^*, P_t^*, V_t^*) \in G\left(\alpha_{t-1}, \mu_{t-1}\right)$ be a solution of the dynamic program that maximizes $V_{t-1}\left(\alpha_{t-1}, \mu_{t-1}, \underline{\theta}\right) = \max\left\{(\alpha_{t-1} - \alpha_t^*) P_t^* + v_{t-1}(\alpha_{t-1}, e_t^*, \underline{\theta}) + V_t(\alpha_t^*, \mu_t^*, \underline{\theta})\right\}$. For any history h^{t-1} and prior μ_{t-1} define a strategy and system of beliefs (σ, μ) as follows: proceed forward to define the strategies for the high type as $\alpha\left(h^{t-1}, \overline{\theta}\right) = \alpha_t^*$ and equivalently for effort. Define the rule to update beliefs as $\mu_t\left(\overline{\theta}|h^{t-1}, \alpha_t, e_t\right) = \mu_t^*$ if $(\alpha_t, e_t) = (\alpha_t^*, e_t^*)$ and equal to 0 otherwise, and a consistent response $P_t(h^{t-1}, \alpha_t, e_t) = P_t^*$ if $(\alpha_t, e_t) = (\alpha_t^*, e_t^*)$ and equal to $P_t(\alpha_t, \underline{\theta})$ otherwise. Define the strategy for the low type as $\alpha(h^{t-1}, \underline{\theta})$ and $e(h^{t-1}, \underline{\theta})$ where he plays (α_t^*, e_t^*) with probability $\beta_t = \frac{\mu_{t-1}}{1-\mu_{t-1}} \frac{1-\mu_t^*}{\mu_t^*}$ and $(\underline{\alpha}_t, \underline{e}_t) = (\alpha_t(\alpha_{t-1}, \underline{\theta}), e_t(\alpha_{t-1}, \underline{\theta}))$ with probability $1-\beta_t$. The strategy profile and system of beliefs (σ, μ) are well-defined and the players' payoff in any continuation game starting with state variables (α_t, μ_t) is given by $V_t(\alpha_t, \mu_t, \theta)$ and $P_t(\alpha_t, \mu_t)$, solution of the dynamic programming problem.

Note that the strategy profile and system of beliefs (σ, μ) is Markovian with respect to the state variables (α_t, μ_t). A strategy profile σ and system of beliefs is Markovian with respect to the state variables (α_t, μ_t) if and only if for any two histories h^t and $\overline{h}^t$ such that the posterior beliefs are the same, $\mu_t = \mu(.|h^t) = \mu\left(.|\overline{h}^t\right)$, and also the equity ownership are the same, $\alpha_t = \alpha_t(h^t) = \alpha_t\left(\overline{h}^t\right)$, then the strategies and beliefs in the continuation game are equal, $\sigma_{|h^t} \equiv \sigma_{|\overline{h}^t}$ and $\mu_{|h^t} \equiv \mu_{|\overline{h}^t}$.

It is straightforward to verify that (σ, μ) as defined above is a PBE equilibrium. This is the case because $(\alpha_t^*, e_t^*, \mu_t^*, P_t^*, V_t^*) \in G_t\left(\alpha_{t-1}, \mu_{t-1}\right)$ is a solution of the dynamic programming problem with restrictions that guarantee that the Nash equilibrium condition (P1), the competitive condition (P2) and Bayes rule for updating beliefs (P3) as in the definition of PBE are satisfied.

It remains to show that $(\sigma, \mu) \in LM\left(h^0, \mu_0\right)$ and that there is a unique equilibrium outcome in $LM\left(h^0, \mu_0\right)$. We now proceed inductively to establish that $(\sigma, \mu)_{|h^t} \in LM\left(h^t, \mu_t\right)$ and all elements in $LM\left(h^t, \mu_t\right)$ produce the same equilibrium outcome, for any period t and history h^t and $\mu_t = \mu\left(.|h^t\right)$. First note that at the terminal period $T+1$ there is a unique $(\sigma, \mu)_{|h^T} \in LM\left(h^T, \mu_T\right)$. Assume as the induction hypothesis that $(\sigma, \mu)_{|h^{t+1}} \in LM\left(h^{t+1}, \mu_{t+1}\right)$ and all elements in $LM\left(h^{t+1}, \mu_{t+1}\right)$ produce the same equilibrium outcome, for any period $t+1$. We want to prove that the induction hypothesis also holds for period t. Assume by contradiction that there exists a $(\sigma', \mu')_{|h^t} \in PBE\left(h^t, \mu_t\right)$ with $(\sigma', \mu')_{|h^t} \neq (\sigma, \mu)_{|h^t}$ that lexicographically dominates $(\sigma, \mu)_{|h^t}$. It then must be the case that $\exists\ \theta$ such that $u\left(\sigma'|h^t, \theta\right) > u\left(\sigma|h^t, \theta\right)$ and for $\theta' > \theta$, $u\left(\sigma'|h^t, \theta'\right) \geq u\left(\sigma|h^t, \theta'\right)$; and $(\sigma', \mu')_{|h^{t+1}} \in LM\left(h^{t+1}, \mu'\left(.|h^{t+1}\right)\right)$. By the induction hypothesis then $u\left(\sigma'|h^{t+1}, \theta\right) = V_t\left(\alpha_t, \mu_t', \theta\right)$ and $P_t\left(\alpha_t, \mu_t'\right)$. But the existence of θ such that $u\left(\sigma'|h^t, \theta\right) > u\left(\sigma|h^t, \theta\right)$ and for $\theta' > \theta$, $u\left(\sigma'|h^t, \theta'\right) \geq u\left(\sigma|h^t, \theta'\right)$ is in contradiction with the fact that $u\left(\sigma|h^t, \theta\right) = V_t\left(\alpha_t, \mu_t, \theta\right)$ is the value of dynamic program at period t. $\blacksquare$

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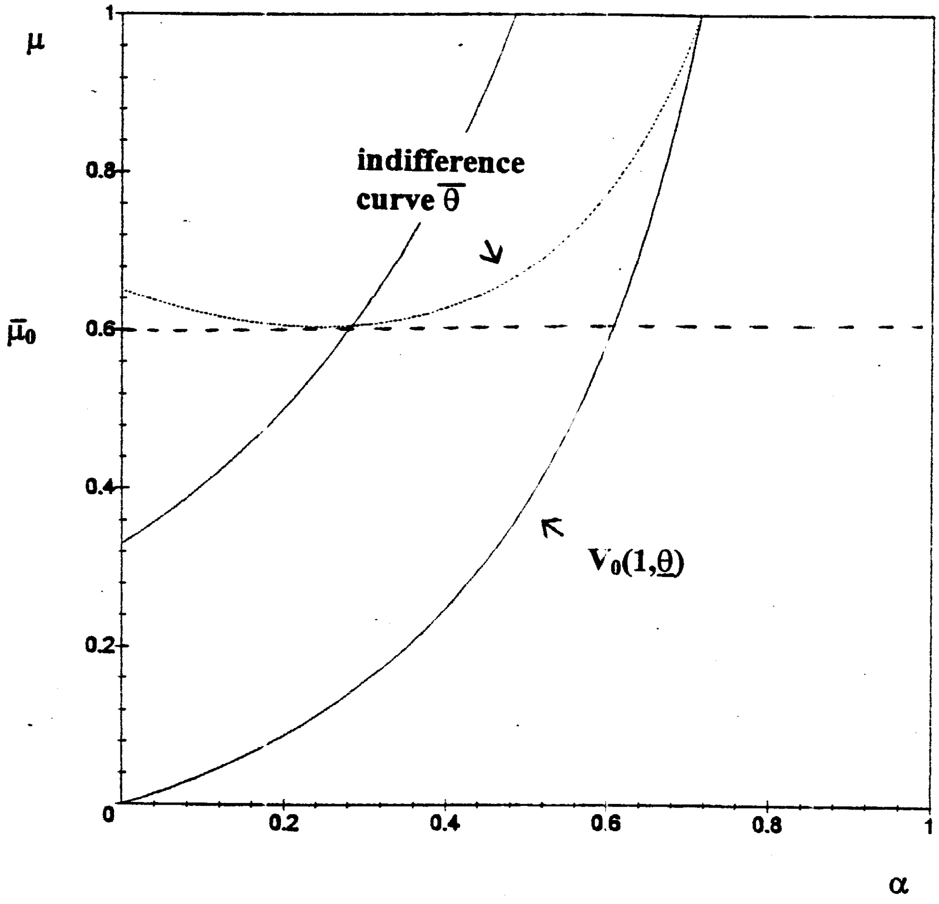

Figure 1: The static signalling game

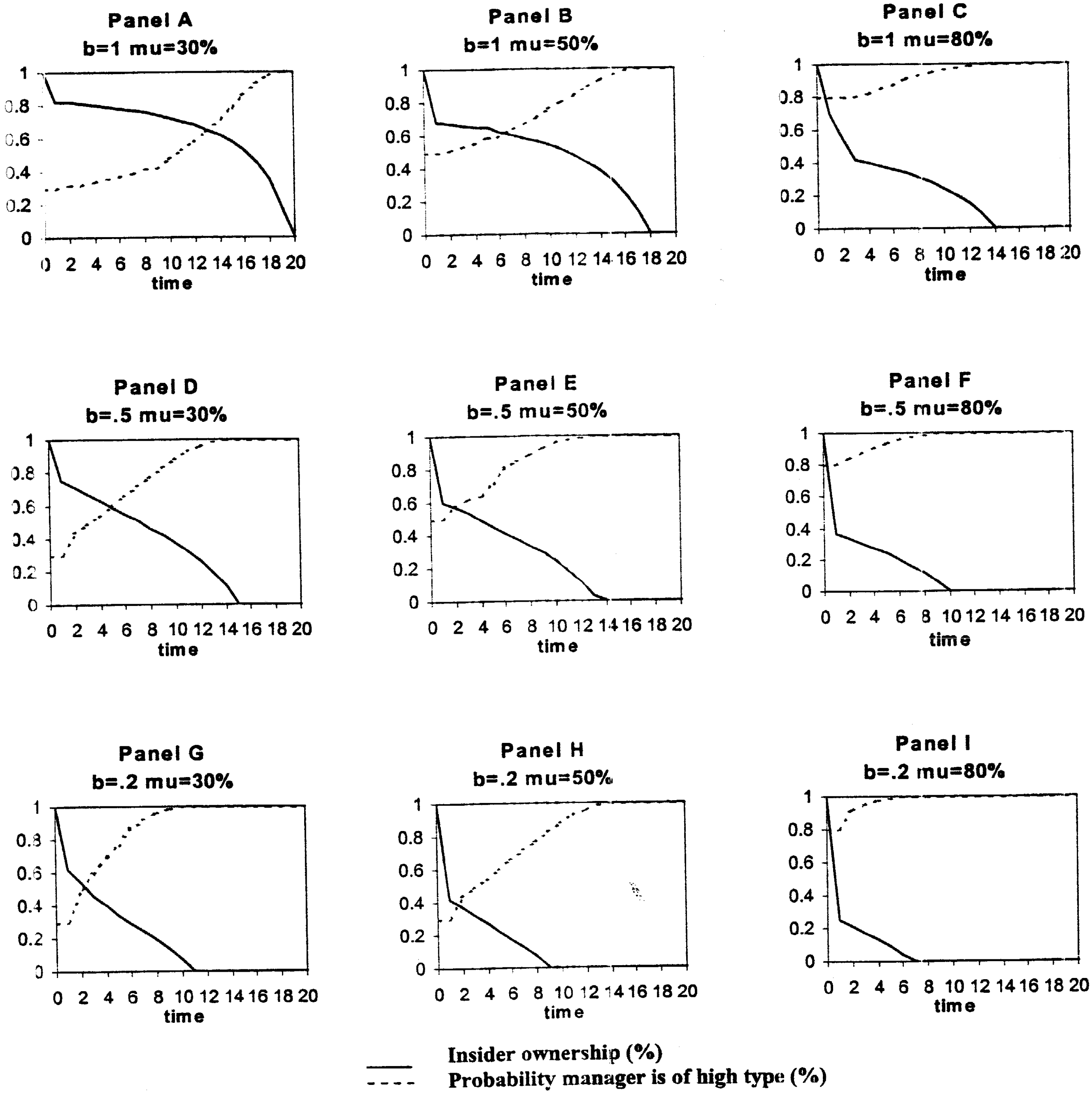

Figure 2: Dynamics of insider ownership and revelation of information.
Equilibrium path for the 20-period game for different parameters (b,μ_0). The horizontal axis represents time: t=0 is the period before IPO, t=1 is the moment of the IPO, and t=20 is the last period. The vertical axis represents the percentage of shares owned by the manager and the probability that the manager is of a high type conditional on no diversion of cash flow. Cash flows are independently and identically distributed with gamma distribution with parameters (1,1).

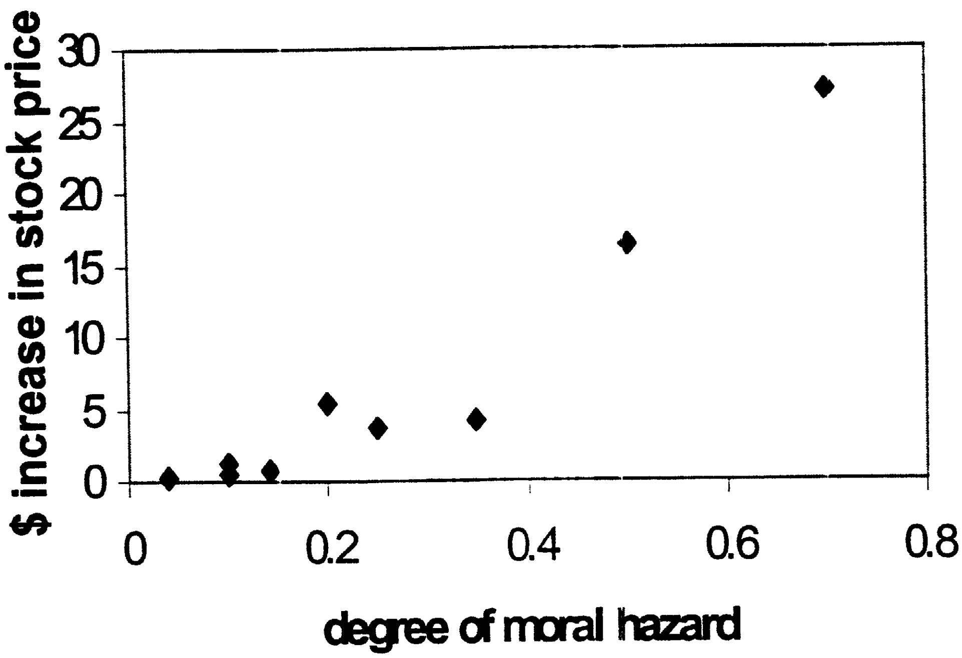

Figure 3: The reputation effect. Increase in stock price at the IPO, $P_1^* - P_1$, for the 20-period game associated with the reputation effect for several moral hazard (0.2, 0.5 and 1) and asymmetric information parameters (30%, 50% and 80%). The horizontal axis is the proxy for the degree of moral hazard, $b(1-\mu_0)$. The sum of expected cash flows with no diversion is equal to \$100, and cash flows are independent and identically distributed with gamma distribution.