# Ex-Ante Real Rates and Inflation Risk Premiums:
# A Consumption-Based Approach

by

**Ayelet Balsam*, Shmuel Kandel**, and Ori Levy***

* Economics Department, Tel-Aviv University.

** Recanati School of Business, Tel-Aviv University, and the Wharton School, the University of Pennsylvania.

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## Abstract

This paper sets out to quantify, with the use of a consumption-based CAPM, the risk premiums inherent in the Israeli market for index-linked and non-index-linked bonds. In contrast to what has appeared in the macroeconomics literature, this study quantifies the size and dynamics of two such premiums: one is related to the inflation uncertainty in a nominal risk-free bond, and the other is related to the inflation uncertainty in an index-linked bond, caused by the indexation lag. This enables an approximation of the size and time-variation of the real ex-ante risk-free rate of return, and an evaluation of the accuracy of the method used by the Bank of Israel to measure inflation expectations. It is shown that the inflation risk premium term and the indexation-lag risk premium term depend heavily and positively on the degree of relative risk aversion, and that the latter is inconsequential. As a result, we claim that the bias caused due to overlooking both these risk premiums in the computation of inflation expectations depends on assumptions regarding the degree of relative risk aversion.



Keywords: Consumption-based CAPM, capital-market-based inflation expectations, inflation risk premium, indexation-lag risk premium, real ex-ante risk-free rate of return.

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## 1. Introduction

The identification of a real ex-ante risk-free rate of return has been a challenge in financial economics literature, since it is an important ingredient in numerous theoretical settings. However, these efforts have been somewhat unsuccessful. The main hurdle is the fact that the bond market does not contain any financial asset that provides full insurance against inflation uncertainty. In most economies, there is no asset that is able to hedge against even part of this price risk, consequently guaranteeing only a fixed risk-free nominal return. In several bond markets, for instance the Israeli market, there is an additional asset that is index-linked, the yield on which appears to represent the real ex-ante risk-free rate of return. However, it turns out that the latter is not entirely risk-free (in real purchasing power terms), since there is an indexation lag, which makes it impossible to eliminate price uncertainty in the period prior to maturity.

Evans and Wachtel (1992) and Chan (1994) consider an empirical setting where there are only two assets, a nominal risk-free bond and a hypothetical real risk-free bond, and quantify the premium needed to be paid on the former due to price instability in equilibrium. They refer to this variable as the "inflation risk premium". These papers rely on a parameterized consumption-based CAPM setting, which makes it possible to derive the fictional asset that guarantees one consumption unit one period ahead.

The existence of an index-linked bond market permits the adoption of an alternative route. Concretely, Kandel, Ofer and Sarig (1996) show how a proxy for the real ex-ante risk-free rate of return can be extracted, non-parametrically, from data on Israeli market prices of index-linked and non-index-linked bonds (see also Barr and Campbell [1997]). This estimate is the exact real rate under the assumption that all core variables are serially independent. Evans (1998), in the index-linked UK bond market, provides further insight with regard to the quantification of the real risk-free rate of return from asset prices data. In particular, he derives the theoretical structure of a second risk premium, which is the general equilibrium outcome of the indexation lag inherent in the index-linked bond. Furthermore, he shows that the annual “indexation-lag risk premium” is significant, hovering around 1.5 percentage points, emphasizing the importance of incorporating such a premium to attain a proxy for the real ex-ante risk-free rate of return. It is relevant to note that the UK bond market, which serves as Evans’ laboratory, is equipped with index-linked bonds with a non-trivial eight-month indexation lag, in stark contrast with the Israeli bond market where the average lag is one month.[1] This paper presents evidence that, despite the different inflation environments, such a difference in the compensation scheme can have important consequences for the equilibrium risk structure (and as a by-product, for the government’s cost of credit).

In Israel’s economy, as in many others, inflation expectations are one of the most important variables that enter into the reaction function of monetary policy makers in the Bank of Israel. In extracting capital-market-based inflation expectations, the core assumption in the calculation process is that the two premiums are constant and non-existent. As a by-product, such a practice can, under certain circumstances, lead to a non-trivial bias in this monetary indicator. Hence, on both accounts, the quantification of the size and dynamics of these two premiums seems important.

Consequently, in this paper, the following question is tackled: how significant are the inflation risk premium and the indexation-lag risk premium in the Israeli bond market? In particular, we use a consumption-based CAPM framework, in the spirit of Lucas (1978), in an environment in which there exist three financial assets: a nominal risk-free bond, an *index-linked bond*, and a hypothetical real risk-free bond. It is well known that the consumption-based CAPM that is applied to our environment performs poorly in the US economy. Specifically, it is unable to capture the co-movement of consumption and asset returns, for example, equities and bonds, adequately, as was made clear in Hansen and Singleton (1982, 1983). This is due primarily to its inability to generate sufficient variability in the stochastic pricing kernel.[2] In a recent paper, Levy (1997) performs a

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[1] The US Treasury started issuing index-linked bonds at the beginning of 1997. There, too, the indexation lag is significant and has a duration of six months. Campbell and Shiller (1996) give an exhaustive list of pros and cons for index-linked debt.

[2] See, for example, Hansen and Jagannathan (1991), Cochrane and Hansen (1992).

similar exercise for the Israel's volatile economy. He does not corroborate the latter finding. In essence, a numerical simulation, a parametric exercise and a non-parametric exercise all point to the conclusion that the model is capable of tracking the co-variation of consumption, equities and bonds successfully. Consequently, we are more confident making use of this theoretical framework in the current paper. The approach herein offers a complementary one to the non-parametric taken by Evans; in the current approach the size and dynamic behavior of the different premiums are characterized, as are the real ex-ante risk-free rate of return and the inflation expectations bias, and their interaction with the degree of relative risk aversion of the representative agent is demonstrated.

We derive a positive mapping between the inflation risk premium and the coefficient of relative risk aversion. In particular, when the latter coefficient is calibrated to possess a value of one (ten), the unconditional mean of the premium turns out to be rather low (high). Moreover, the fact that the whole time-series of the premium is estimated helps to show how it is related to significant occurrences in the Israeli economy. For example, in the hyperinflation period in the mid-eighties, the average premium's size turns out to be substantial. Also, throughout the 1990s, despite the Bank of Israel's inflation target policy, its average seems to be sub-period independent, a finding that could be attributed to the persistent *instability* of the inflation rate, despite the decline in its *level*. Second, we find that in contrast with Evans' results for the index-linked UK bond market, the unconditional mean of the indexation-lag risk premium is small, and dependent on the degree of relative risk aversion. Consequently, it seems that the real ex-ante risk-free rate of return computed by Kandel, Ofer and Sarig (1996) very closely approximates the real risk-free rate, thus, corroborating their serial independence assumption. Third, an upward bias was found in the calculation of inflation expectations made by the Bank of Israel, that is positively related to the intensity of risk aversion of the representative agent in the economy, and is non-trivial when the relative risk aversion coefficient attains relatively high levels.

The paper proceeds as follows. In section two, a structural model is proposed with which the inflation risk premium and the indexation-lag risk premium are quantified. Also reported are the size and time-variation of their combination (i.e., the inflation expectations bias of the Bank of Israel), as well as the real ex-ante risk-free rate series. Section three presents empirical results. Section four contains some concluding comments and ideas for future research.

## 2. The Theoretical Framework

This section investigates the theoretical relationship between three financial assets: a nominal risk-free bond, a real risk-free (hypothetical) bond, and an index-linked bond. Such links will allow us to derive structural closed-form expressions for the two risk premiums, i.e., the inflation and indexation-lag risk premiums. We use a standard theoretical structure, such as in Lucas (1978), where we assume the existence of a representative consumer, who maximizes expected utility over an infinite horizon. The utility metric is taken to be time and state separable, with a unique argument, $C_t$, representing per-capita real consumption of non-durables and services at time period t. Each period, the consumer chooses stochastic consumption and investment plans, so as to maximize his utility metric and satisfy his standard budget constraint. The kernel with which the equilibrium risk structure is quantified emanates from the consumption-based CAPM structure, and is a function of consumption growth and preferences' parameters.

We start, in subsection 2.1., with some definitions and notations that will serve throughout the paper. We then continue with setting the formal environment for pricing financial assets. The analysis draws heavily on Evans (1998). Next, in subsection 2.2.1., we show how one can identify the size and variation over time of the first risk component, the inflation risk premium. Finally, in subsection 2.2.2., we employ a rather similar methodology that permits the identification of the conditional second risk component, the indexation-lag risk premium.

## 2.1. Definitions, Notations and the Pricing of Financial Assets

Let $Q_t(h)$ be the nominal price of a nominal discount bond, which pays NIS 1,[3] h months ahead. Consequently, the yield on such an asset, $Y_t(h)$, is defined as,

$$Y_t(h) \equiv Q_t(h)^{-\frac{1}{h}}. \tag{1}$$

Similarly, we can define the real rate of return on a hypothetical, real discount bond. Specifically, $Q^*_t(h)$ is taken to represent the nominal price at time t of such an asset, that pays one unit of consumption h months ahead, or alternatively, pays NIS $\frac{p_{t+h}}{p_t}$ at t+h. Therefore, the real yield, $Y^*_t(h)$ is defined as,

$$Y^*_t(h) \equiv Q^*_t(h)^{-\frac{1}{h}}. \tag{2}$$

It is important to note that the holder of the latter asset is entitled to full compensation for the price change throughout the period. Such hypothetical terms contrast with those promised on index-linked bonds traded in the Israeli and other bond markets. This is due to the lag that characterizes the publication of the CPI (Consumer Price Index), which results in an inescapable incomplete compensation (in Israel, the lag is usually two weeks). Hence, let $Q_t^+(h)$ be the nominal price at time t of an index-linked bond which pays NIS $P_{t+\tau} / P_t$ at time period t+h, where $\tau$ represents the period on which the holder is compensated, and $\ell$ stands for the remaining time period for which the bond does not incorporate a hedge against inflation uncertainty, i.e., $\ell \equiv h - \tau$. Therefore, the spot yield, $Y^+{}_t(h)$ is defined as,

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[3] One New Israeli Shekel, in 1998, is about US$ 0.27.

$$Y^{+}{}_{t}(h) \equiv Q_t{}^{+}(h)^{-\frac{1}{h}}. \qquad (3)$$

Evans (1998) presents the general equilibrium prices of the nominal, real and index-linked bonds, shown in equations (4), (5) and (6) below,

$$Q_t(h) = E_t\left[\prod_{i=1}^{h} M_{t+i}\right] \qquad (4)$$

$$Q^{*}{}_{t}(h) = E_t\left[\prod_{i=1}^{h} M^{*}{}_{t+i}\right] \qquad (5)$$

$$Q^{+}{}_{t}(h) = E_t\left[\prod_{i=1}^{h-\ell} M^{*}{}_{t+i} Q_{t+\tau}(\ell)\right], \qquad (6)$$

where $E_t$ denotes the mathematical expectation conditional on all information known at time period t,[4] $M_{t+1}$ is the one month ahead nominal stochastic pricing kernel and $M^{*}{}_{t+1} \equiv M_{t+1} \cdot (P_{t+1} / P_t)$ is the one month ahead real kernel.

In order to be able to quantify the different risk premiums, we need to describe how bonds, including the hypothetical real-risk-free bond, are priced. The use of the fictional real-risk-free asset, which extends the portfolio universe choice, enables the theoretical identification of the two premiums, since it serves as a real-risk-free benchmark asset against which the two premiums are defined. Thus, equations (4), (5) and (6) will now be used to generate, in a consumption-based framework, the risk structure's components.

[4] In particular, we assume throughout, in accordance with the common practice in the asset pricing econometrics literature, that the time t price level is part of the representative agent's information set. Abandoning this supposition results in an intractable optimization problem, since it causes investors to face stochastic opportunity sets when they are constructing their utility-maximizing infinite horizon consumption-saving plans.

## 2.2. Deriving the Inflation and Indexation-Lag Risk Premiums Intrinsic in the Bond Market in a Consumption-Based CAPM Framework

The analysis below is comprised of two parts. Subsection 2.2.1. deals with the derivation of the inflation risk premium, which, later in the study, will permit the identification of the bias inherent in the Bank of Israel's capital-market-based inflation expectations estimate. In subsection 2.2.2. we derive the indexation-lag risk premium, which, later in the study, will permit the identification of the real ex-ante risk-free rate of return.

### 2.2.1. The Inflation Risk Premium

Equations (7) and (8) are two first order necessary conditions (i.e., Euler equations) of the representative agent's optimization problem (see equations (5) and (4) above),

$$Q^*_t(3) = E_t\left(M^*_{t+3}\right) \tag{7}$$

$$Q_t(3) = E_t\left[M^*_{t+3} \cdot \left(P_t / P_{t+3}\right)\right]. \tag{8}$$

By decomposing equation (8), one gets the following equation,

$$Q_t(3) = E_t\left(M^*_{t+3}\right) \cdot E_t\left(\frac{P_t}{P_{t+3}}\right) + \mathrm{cov}_t\left(M^*_{t+3}, \frac{P_t}{P_{t+3}}\right). \tag{9}$$

We use throughout the power utility function, specified as follows,

$$U\left(C_t\right) = C_t^{1-\theta} / (1-\theta), \tag{10}$$

where $\theta>0$ is the (Arrow [1970]) constant relative risk aversion (CRRA) coefficient. Thus, we get that the real kernel, which in the present model is equivalent to the discounted ratio of marginal utilities, is equal to,

$$M^*_{t+3} = \beta\left(C_{t+3} / C_t\right)^{-\theta}, \tag{11}$$

where $\beta$ is the time preference coefficient.[5] Inserting the latter expression in (11) into equation (9) gives the following,

$$Q_t(3) = E_t\left(\beta\left(\frac{C_{t+3}}{C_t}\right)^{-\theta}\right)\cdot E_t\left(\frac{P_t}{P_{t+3}}\right)\cdot\left(\psi_t\right), \tag{12}$$

where, $\psi_t \equiv 1 + \dfrac{\operatorname{cov}_t\left(\beta\left(C_{t+3} / C_t\right)^{-\theta}, P_t / P_{t+3}\right)}{E_t\left[\beta\left(C_{t+3} / C_t\right)^{-\theta}\right]\cdot E_t\left[P_t / P_{t+3}\right]}$.

We have derived in equation (12) above the conditional Fisher equation (in the prices' domain), in which the risk premium attached to the nominal risk-free bond is captured by $\psi_t$, a function of the conditional covariance between the real kernel and the purchasing power of money.

The underlying intuition for this conditional risk premium is as follows: suppose that there is an unexpected positive price shock, so that there is an increase in the rate of inflation. If such a scenario is accompanied by a higher future marginal utility of consumption, the concavity of the utility function requires that future consumption be lower. If this is the case, i.e., if the conditional covariance is negative, adverse shocks in consumption growth are accompanied by adverse shocks in the financial asset's real ex-post yield, thus

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[5] See, for example, Hansen and Singleton (1982).

augmenting the volatility of the consumption path. Consequently, the nominal bond is to be considered risky and in equilibrium will be traded at a discount.[6]

The Israeli bond market offers the financial investor an additional asset that is index-linked, serving, among other things, as an insurance against price instability.[7] Huberman and Schwert (1985) and Kandel, Ofer and Sarig (1996) have used this additional price information to test rationality of inflation expectations and to construct the real ex-ante risk-free rate of return, while Yariv (1995) and Kandel, Ofer and Sarig (1991, 1993) have used it to derive capital-market-based inflation expectations. These authors have used different simplifying assumptions with respect to the inflation risk structure in the Israeli bond market. However, since the CPI is announced with a lag, the insurance is not complete, causing a risk averse individual to require a compensation in equilibrium. Therefore, in the spirit of Evans (1998), we wish to study the time-series' properties of this indexation-lag risk premium in order to accurately assess capital-market-based inflation expectations and the real ex-ante risk-free rate of return. The next subsection derives the specific structure of this second risk component.

### 2.2.2. The Indexation-Lag Risk Premium

In this subsection, the representative consumer is faced with an extended empirical choice universe of financial assets in which he or she is allowed to trade in a third bond that is index-linked. The latter bond promises to pay a nominal payoff of NIS $P_{t+\tau} / P_t$ at maturity in time period t+h, where $\tau$ represents the period in which the consumer is fully hedged against price uncertainty, and $\ell$ is the remaining period (i.e., $\ell \equiv h - \tau$) in which the consumer is not compensated for price variation, due to the indexation lag.

Consistent with a representative indexation lag for the index-linked bonds traded in the Israeli bond market, we choose the bond’s duration to be of a length of four months (i.e., h=4), and the indexed period to be of a length of three months (i.e., $\tau$=3). This is done

[6] The conditional risk premium term in (12) is independent of any specific distribution that potentially characterizes the real kernel, and is taken in the literature to represent the pure inflation risk, capturing the fundamental risk in a non-index-linked asset.

since we wish to generate a premium against an asset which promises a real risk-free quarterly return, and hence are obliged to produce an index-linked asset for which the fully hedged period is of a duration of a quarter.

For the derivation of the indexation-lag risk premium, we make use of the necessary equilibrium parity, stated above in equation (6). Clearly, the conditional expectation term can be broken down into the product of the conditional expectations of the real kernel and the price of a one-month nominal risk-free bond purchased three months hence, and a conditional covariance term,

$$Q_t^{+}(4) = E_t\left(\prod_{i=1}^{3} M_{t+i}^{*}\right) \cdot E_t\left(Q_{t+3}(1)\right) \cdot \gamma_t, \tag{13}$$

where, $\gamma_t \equiv 1 + \dfrac{\operatorname{cov}_t\left(Q_{t+3}(1), \prod_{i=1}^{3} M_{t+i}^{*}\right)}{E_t\left[Q_{t+3}(1)\right] \cdot E_t\left[\prod_{i=1}^{3} M_{t+i}^{*}\right]}$.

After some supplementary manipulations of equation (13), we get the following dependence between nominal, index-linked and real measures, coupled with an uncertainty premium term,

$$\left[Y_t^{+}(4)\right]^4 = \left[Y_t^{*}(3)\right]^3 \cdot \left[E_t\left(Q_{t+3}(1)\right)\right]^{-1} \cdot \left[\gamma_t\right]^{-1}. \tag{14}$$

The intuition for the risk premium component $\gamma$ identified in equations (13) and (14) is as follows: first, note that the nominal compensation for the price increase throughout the quarter is received by a holder of the index-linked bond with a lag of one month in comparison with a holder of the hypothetical real risk-free bond. As such, and in order to compute the three-month real yield, one needs to deflate the delayed nominal payoff by a

[7] Other capital markets where indexed bonds exist are described in Campbell and Shiller (1996).

discount factor, in the shape of the one-month nominal rate, three months ahead. Now, suppose that there is an unexpected decrease in the future price of the nominal asset, and as such a *decrease* in the quarterly real rate of return paid to an index-linked bond holder. If the decline in the real yield on the index-linked asset is accompanied by a relative increase in future marginal utility (i.e., the agent raises her evaluation of every future consumption unit), rendering the conditional covariance term negative, then the index-linked asset will be considered risky, since it does not possess a hedging property, and will pay a higher return, or alternatively, will be traded in equilibrium at a discount.

On the right hand side of equation (14) appears $\left[E_t\left(Q_{t+3}(1)\right)\right]^{-1}$, the inverse of the price of a one-month nominal asset, purchased at the beginning of the non-indexed period of the index-linked asset, as perceived at time period t (i.e., the one-month expected nominal return in the course of the non-indexed period). However, since the formation process of conditional expectations is not known to the econometrician, an approximation is made with the use of the forward nominal rate (see, for example, Kandel, Ofer and Sarig [1996]). This forward rate is computed by dividing the period t nominal price of a three-month nominal bond, $Q_t(3)$, by the period t nominal price of a four-month nominal bond, $Q_t(4)$. Using the nominal Euler equation (4), the factorization property of the conditional expectation of the product of two random variables, and the law of iterated expectations, we get that the mapping between the forward nominal yield and the expected future spot yield can be described by,

$$\frac{Q_t(3)}{Q_t(4)} = \frac{E_t\left[\prod_{i=1}^{3} M_{t+i}\right]}{E_t\left[\prod_{i=1}^{4} M_{t+i}\right]} = \frac{E_t\left[\prod_{i=1}^{3} M_{t+i}\right]}{E_t\left[\prod_{i=1}^{3} M_{t+i}\right] \cdot E_t\left[Q_{t+3}(1)\right] \cdot \eta_t} = \qquad (15)$$

$$\frac{1}{E_t\left[Q_{t+3}(1)\right] \cdot \eta_t},$$

$$\text{where, } \eta_t = 1 + \frac{\operatorname{cov}_t\left(\prod_{i=1}^{3} M_{t+i}, Q_{t+3}(1)\right)}{E_t\left[\prod_{i=1}^{3} M_{t+i}\right] \cdot E_t\left[Q_{t+3}(1)\right]}.$$

The intuition for the residual term in equation (15) proceeds as follows:[8] consider a representative agent in the economy, who maximizes nominal (not real) wealth. Imagine that she faces a choice problem between two alternatives. The first involves buying a discount nominal bond at time period t, with a nominal price of NIS $Q_t(3)$, which promises to pay NIS 1 at maturity at t+3, and hence guaranteeing a certain quarterly nominal yield. The second involves buying a discount nominal bond at time period t, with a nominal price of NIS $Q_t(4)$, which promises to pay NIS 1 at maturity at t+4, its three-month nominal yield depending on the inverse of the three months ahead nominal price of a one-month discount nominal bond, $Q_{t+3}(1)$. Consequently, if it happens that whenever the nominal kernel is high (i.e., the future marginal utility is relatively high, thus making any additional NIS more valuable), the future price of the one-month nominal discount bond decreases, a risk averse individual would be deterred from the uncertain alternative, since it is not structured to serve as a smoothing mechanism. It can easily be shown that if the conditional covariance term is negative, then the forward rate contains a risk premium in addition to the conditional expectations for the future spot yield.

If we insert (15) into (14), we get an equation that summarizes the relationship between the four-month yield accrued to a holder of an index-linked asset, the quarterly risk-free (in real terms) yield, the forward nominal yield during the risky period of the index-linked asset as it is priced today, and what we designate as the equilibrium indexation-lag risk premium (comprised of two elements, where its nominator and denominator are described in equations (15) and (13) respectively),

[8] Under the expectations hypothesis, forward rates exactly equal market expectations of future spot rates. Backus, Gregory and Zin (1989) derive, in a C-CAPM framework, the mapping between forward rates, forecasts of future spot rates, and a time-varying premium, similar to the one depicted in equation (15) above.

$$\left[Y_t^{+}(4)\right]^4 = \left[Y_t^{*}(3)\right]^3 \cdot \frac{Q_t(3)}{Q_t(4)} \cdot \chi_t , \tag{16}$$

where, $\chi_t \equiv \dfrac{\eta_t}{\gamma_t}$.

Kandel, Ofer and Sarig (1996) have used an equation similar to (16) in order to derive, non-parametrically, the real ex-ante risk-free rate of return. They maintain the premise of serial independence of all variables. Consequently, the indexation-lag risk premium in their case equals zero, by definition.

Evans (1998), on the other hand, does allow for the existence of an indexation-lag risk premium. However, due to his assumption of joint conditional lognormal distribution of the nominal kernel and the rate of price increase, the premium turns out to be independent of the kernels, thereby inhibiting him from relating it to the degree of relative risk aversion in the economy.

In addition, the two premiums allow us to evaluate the bias in the Bank of Israel's capital-market-based inflation expectations estimate. Specifically, the Bank of Israel derives quarterly inflation expectations, $E_{t,BoI}\left[P_{t+3} / P_t\right]$, by dividing the quarterly nominal yield by the quarterly real ex-ante yield on the indexed bond,

$$E_{t,BoI}\left[P_{t+3} / P_t\right] = \frac{\left[Y_t(3)\right]^3}{\left\{\left[Y_t^{+}(4)\right]^4 / \dfrac{Q_t(3)}{Q_t(4)}\right\}} . \tag{17}$$

Since both risk premiums are ignored, $E_{t,BoI}\left[P_{t+3} / P_t\right]$ is potentially biased. In particular, using equations (12) and (16), it turns out that this bias, $B_t$, is of an order of magnitude which depends on the product of the inflation risk premium and the indexation-lag risk premium,[9]

[9] Note that the following holds: $\left[E_t\left[P_t / P_{t+3}\right]\right]^{-1} \approx E_t\left[P_{t+3} / P_t\right]$.

$$B_t = (\psi_t) \cdot (\chi_t). \quad (18)$$

Equation (18) demonstrates that this bias holds even if we eliminate the existence of the inflation risk premium.

## 3. Empirical Results

This section turns to the time-dependent quantification of the pure inflation risk premium and the indexation-lag risk premium inherent in the Israeli index-linked and non-index-linked bond markets, i.e.,

$$\psi_t = 1 + \frac{\mathrm{cov}_t\left(M^*_{t+3}, P_t / P_{t+3}\right)}{E_t\left[M^*_{t+3}\right] \cdot E_t\left[P_t / P_{t+3}\right]}.$$

$$\chi_t = \left[1 + \frac{\mathrm{cov}_t\left(\prod_{i=1}^{3} M_{t+i}, Q_{t+3}(1)\right)}{E_t\left[\prod_{i=1}^{3} M_{t+i}\right] \cdot E_t\left[Q_{t+3}(1)\right]}\right] \Big/ \left[1 + \frac{\mathrm{cov}_t\left(\prod_{i=1}^{3} M^*_{t+i}, Q_{t+3}(1)\right)}{E_t\left[\prod_{i=1}^{3} M^*_{t+i}\right] \cdot E_t\left[Q_{t+3}(1)\right]}\right].$$

First, we define a permissible domain for the preference parameters. In particular, we calibrate the time preference coefficient throughout such that $\beta=1$. In addition, we allow the average individual's risk preferences to vary, and let the coefficient of relative risk aversion $\theta$ be equal to 1 (near risk neutrality), 5 and 10. The degree of the representative agent's aversion to risk in the Israeli economy was estimated in several studies using an instrumental variables approach (the generalized method of moments), and was found to be relatively low (see, for example, Bufman and Leiderman (1990), Bental and Eckstein (1997) and Levy (1997)). On the other hand, Kandel and Stambaugh (1991) have used for calibrated US data a CRRA of 29. Thus, the upper limit we have enforced on its value in the calibration exercise seems innocuous.

Second, we use information on the real and nominal kernels, CPI change and the future spot price of a one-month (i.e., the non-indexed period) nominal bond, and characterize the conditional expectations and co-variations terms, in a way similar to Chan (1994). In order to do that, we uphold the rational expectations hypothesis throughout, i.e., the claim that the true realization of any random variable deviates from its expected value conditional on all available information as of time period t, by a stochastic (zero-mean) error term. Consequently, the following four equations emerge, representing the innovations' evolution,

$$u_x(t+3) = x_{t+3} - E_t x_{t+3}, \tag{19}$$

where, $x = M^*_{t+3}, M_{t+3}, (P_t / P_{t+3}), Q_{t+3}(1)$.

Utilizing elementary covariance properties along with the characteristics of the random errors mentioned above, the conditional risk premiums are written as follows,

$$\psi_t = 1 + \left[E_t\left(u_{M*}(t+3) \cdot u_p(t+3)\right)\right] / \left[E_t\left[M^*_{t+3}\right] \cdot E_t\left[P_t / P_{t+3}\right]\right] \tag{20}$$

$$\chi_t = \left[1 + \frac{E_t\left(u_M(t+3) \cdot u_Q(t+3)\right)}{E_t\left[\prod_{i=1}^{3} M_{t+i}\right] \cdot E_t\left[Q_{t+3}(1)\right]}\right] / \left[1 + \frac{E_t\left(u_{M*}(t+3) \cdot u_Q(t+3)\right)}{E_t\left[\prod_{i=1}^{3} M^*_{t+i}\right] \cdot E_t\left[Q_{t+3}(1)\right]}\right]. \tag{21}$$

We estimate the premiums in equations (20) and (21) by employing a two-stage procedure. First, we compute the respective innovations for the real (nominal) kernel $M^*_{t+3}$ ($M_{t+3}$), the inverse of the inflation rate $(P_t / P_{t+3})$ and the nominal bond price $Q_{t+3}(1)$. We assume that all follow an unrestricted first order auto-regressive process.[10]

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[10] Unless conditional heteroskedasticity is detected, in which case the process is modeled as in Bollerslev (1986). Note that the real and nominal kernels, which are functions of consumption growth, are used throughout in this paper. This slightly differs from (e.g.) Mehra and Prescott (1985) and Kandel and Stambaugh (1996) who calibrate the

Second, in order to generate the conditional co-variation of any pair of random variables, we take the innovations' product to follow an unrestricted auto-regressive process, specified as follows,

$$u_{M*}(t+3)u_p(t+3) = \sum_{t=1}^{n} L^{t-1}\alpha_t u_{M*}(t)u_p(t) + \varepsilon_{rp}(t+3) \qquad (22)$$

$$u_{M*}(t+3)u_Q(t+3) = \sum_{t=1}^{n} L^{t-1}\alpha_t u_{M*}(t)u_Q(t) + \nu_{rp}(t+3) \qquad (23)$$

$$u_M(t+3)u_Q(t+3) = \sum_{t=1}^{n} L^{t-1}\alpha_t u_M(t)u_Q(t) + \mu_{rp}(t+3), \qquad (24)$$

where $L$ is the lag operator. Since the error terms on the right hand side of equations (22), (23) and (24), $\varepsilon_{rp}$, $\nu_{rp}$ and $\mu_{rp}$ are assumed to be white noises, we get that the pure inflation risk premium and the indexation-lag risk premium terms for any time period t have to satisfy the following,

$$\psi_t = 1 + \left[\sum_{t=1}^{n} L^{t-1}\alpha_t u_{M*}(t)u_p(t)\right] / \left[E_t\left[M^*_{t+3}\right] \cdot E_t\left[P_t / P_{t+3}\right]\right] \qquad (25)$$

$$\chi_t = \left[1 + \frac{\sum_{t=1}^{n} L^{t-1}\alpha_t u_M(t)u_Q(t)}{E_t\left[\prod_{i=1}^{3} M_{t+i}\right] \cdot E_t\left[Q_{t+3}(1)\right]}\right] / \left[1 + \frac{\sum_{t=1}^{n} L^{t-1}\alpha_t u_{M*}(t)u_Q(t)}{E_t\left[\prod_{i=1}^{3} M^*_{t+i}\right] \cdot E_t\left[Q_{t+3}(1)\right]}\right]. \qquad (26)$$

The real (nominal) kernel (see equation (11) above) is generated with the use of consumption data, for which we take the seasonally adjusted, non-durables and services series, in fixed prices of 1995 (current prices). The price measure is the consumption price

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consumption growth process itself. In addition, it should be emphasized that our choice of modeling is somewhat arbitrary and that many other parameterizations for the dynamics of the different mean-reverting processes are possible (e.g., allowing for regime shift in the regression's coefficients).

deflator, derived from the nominal and real consumption series.[11] The data are quarterly, and are divided by the population data, to give the per-capita consumption series, suitable for our representative agent framework. All data are taken from the Bank of Israel series. The sample extends from the first quarter of 1964 until the fourth quarter of 1997 (a total of 136 observations). The sample of nominal bond prices extends from the first quarter of 1988 until the fourth quarter of 1997 (a total of 40 observations). Extracted information is from the Tel-Aviv Stock Exchange database. The generating processes for the respective kernels are adjusted accordingly.

Table 1 presents summary statistics for the quarterly real and nominal per-capita consumption growth, the quarterly rate of inflation and the nominal monthly yield on non-indexed bonds for the period 1988:01 through 1997:04 in Israel.

[Table 1, here]

We checked that the vector of stochastic processes used in the course of our study was stationary, employing two prevalent tests, the Dickey-Fuller test and the Phillips-Peron test, and using the standard 5% significance level. We found no evidence of a unit root type behavior in either of the processes.

In Figure 1, under diverse scenarios for the degree of relative risk aversion, we confront the quarterly actual and fitted values for the real kernel and the purchasing power of money. As can be seen, the fit of both our stochastic processes seems to be satisfactory.[12]

[Figure 1, here]

In Figure 2 and in Table 2 we depict the evolution of the inflation risk premium over time and its unconditional mean respectively for different values of the CRRA.

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[11] We have checked the robustness of our results with the use of the consumer price index as our price measure. The results remain practically unchanged.

[12] Since the indexation-lag risk premium will be calculated, due to data availability, from the first quarter of 1988, all Figures and Tables henceforth are adjusted accordingly.

[Figure 2, here]

[Table 2, here]

We find that the average premium (given on a quarterly basis), if the representative individual is only mildly averse to risk (i.e., $\theta$=1), is approximately 0.04%! Furthermore, as can be seen in Table 2 above, the premium depends critically on the individual's assumed relative risk aversion. In particular, if she has a relatively strong risk aversion (i.e., $\theta$=10), then the inflation risk premium is approximately 0.62%, seventeen times greater. The latter can no longer be treated as negligible.

Moreover, along with the strong positive dependence on the coefficient of relative risk aversion, the inflation risk premium does not seem to be constant over time. In particular, it turns out (see Figure 2b) that it was a great deal more significant in the course of the hyperinflation era in the mid-eighties. Specifically, during the period starting in 1983:04 and ending in Israel in 1985:03 (8 observations),[13] the quarterly average premium is approximately 2.2% (for $\theta$=5)! Furthermore, the average of the inflation risk premium seems to be sub-period independent in the course of the 1990s, despite the inflation target policy the Bank of Israel has been conducting for the last few years, a fact which can be related to the persistent instability of the inflation rate, despite the decline in its level.

In Figure 3, we depict the actual and fitted values for the nominal kernel and the future spot nominal price of a one-month non-indexed bond. We find, as in all previous cases, that the proposed calibration of the stochastic processes seems to track the data.

[Figure 3, here]

Figure 4 depicts the time variation of the indexation-lag risk premium for three different degrees of relative risk aversion. Table 2 shows an equivalent mapping for its unconditional mean.

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[13] A period identified by Liviatan and Melnick (1998) as the hyperinflation step.

[Figure 4, here]

Our calibration exercise yielded a few interesting results: first, it turns out (see Table 2) that the nominal term's premium, discussed in equation (15) above, is on average smaller than one, i.e., the equilibrium one-month forward nominal rate of return (as observed on t) is bigger than the expectations with respect to the future spot nominal rate of return in the course of the non-indexed period. In addition, and rather intuitively so, its absolute value increases with the degree of relative risk aversion.[14]

Second, it turns out from Table 2 that the average of the indexation-lag risk premium is inconsequential.[15] This result contrasts with the results obtained by Evans (1998) who found a sizable 1.5 % (annual basis) risk premium in the UK bond market. The UK market is equipped with index-linked bonds with a non-trivial eight-month indexation lag, in contrast with the Israeli bond market, where the average lag is one month. It turns out that, despite the different inflation environments, this may be consequential regarding the compensation required by a risk averse individual.

Moreover, the premium seems to be rather dependent on the way the Israeli hypothetical representative agent evaluates risk. Evans, using the assumption that the nominal kernel and the rate of price increase are jointly lognormal, derives a relation similar to equation (16) above. However, this distribution assumption has some implications for the theoretical risk structure. In particular, he identifies the risk premium attached to an index-linked asset as a conditional covariance term between the inflation rate during the indexed period and the price of a $\ell$-month nominal bond as it is perceived at the start of the non-indexed period $t+\tau$. He then uses a bi-variate unconditional VAR structure in order to track the (time-invariant in his case) risk premium. Evidently, the entire exercise is non-parametric in the sense that the premium is not a function of either nominal or real kernels,

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[14] It is commonly observed in the data that the forward rate is the sum of the expected future spot rate and a positive risk premium. In contrast to the above-mentioned results, Backus et al. (1989), in a monetary version of the Mehra-Prescott (1985) environment, are unable to generate the correct sign and magnitude of the premium. This failure, taken in conjunction with the model's success in the Israeli economy, demonstrates, as in the case of the equity premium, the contrast between the model's performance in a smooth (i.e., US) and volatile (i.e., Israel's) economy.

[15] Since the CPI is announced with a lag of two weeks, investors are unaware at the day of the bond's purchase of last month's index, thus, a premium for past inflation risk is required. Kandel, Ofer and Sarig (1993) are unable to reject the hypothesis that this risk premium equals zero in the Israeli indexed bond market.

and in particular, does not necessitate the use of consumption data. Furthermore, such exercise is unable to perform a sensitivity analysis of the size of the premium with respect to the degree of relative risk aversion. Our calibration results indicate that such neglect may be consequential.

In addition, we have undertaken the task of reconstructing Evans' theoretical structure (using our empirical methodology) in order to give further, non-parametric support to the above hypothesis, regarding the insignificance of the indexation-lag risk premium in the Israeli index-linked bond market. We found that the indexation-lag risk premium's size is very small, equaling approximately *0.0003%* (on a quarterly basis). It looks like this non-parametric result strengthens the conclusions we have derived previously, given a parametric framework, a representative consumer with relatively mild aversion toward uncertainty, and pre-determined calibration structure for the various mean-reverting stochastic processes.

Moreover, the finding that the methodology proposed recently by Kandel, Ofer and Sarig (1996) for quantifying the ex-ante real risk-free rate of return exclusively from information on index-linked and non-index-linked bonds is to a reasonable approximation not biased, derives immediately from the discussion above.

Finally, in accordance with equation (18), we generate the evolution over time and the average size of the bias of the Bank of Israel's capital-market-based inflation expectations estimate (conditional on $\theta$), depicted respectively in Figure 5 and Table 2.

[Figure 5, here]

We find that the bias (on a quarterly basis) is negative over the entire permissible range of the relative risk aversion coefficient (i.e., the Bank of Israel's methodology overestimates, on average, the genuine value of inflation expectations). Furthermore, the average bias seems to depend critically on the relative risk aversion coefficient, being immaterial for minor risk aversion, and gathering momentum as the coefficient is increased. Consequently, as we have concluded in the case of the magnitude of the inflation risk premium's average, a thorough understanding of the way risky prospects are evaluated in the economy is required prior to embracing any particular belief as to the degree of bias in the inflation expectations estimate.

- **Calculating the Real Ex-Ante Risk-Free Rate of Return**

Building on the work of Kandel, Ofer and Sarig (1996), the real ex-ante risk-free rate of return is derived (on a quarterly basis) directly from equation (16) above, as follows,

$$Y_t^*(3)^3 = \frac{\left[(F + C) \cdot \left(\hat{I} / I^{Base}\right)\right] / P}{\left(Q_t(3) / Q_t(4)\right) \cdot \chi_t}, \qquad (27)$$

where P is the price of a bond with four months left to maturity, F is the face value of the bond (100), C is the pre-tax coupon, $I^{Base}$ is the known index when the bond is issued, and $\hat{I}$ constitutes the previous month's CPI forecast, to be published on the fifteenth of the following month. We have direct information on the following variables, from characteristics of index-linked bond: P, F, C, and $I^{Base}$. In order to complete the calculation, we need to quantify $\hat{I}$. We extract these implied inflation expectations directly from bond market information, as in Kandel, Ofer and Sarig (1993, 1996). Price information comes from the Tel-Aviv Stock Exchange database. As above, data availability limits the bond prices sample to the period from the first quarter of 1988 until the fourth quarter of 1997 (a total of 40 observations). Data are available for all quarters in the sample period except for the first and second quarters of 1995.[16]

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[16] In the first few observations where the Treasury has issued an 80 percent index-linked debt instead of a 100 percent linked one, we calculate the corresponding yields along the lines put forward by Kandel, Ofer and Sarig (1996). When implied inflation expectations could not be extracted directly (first quarter of 1988, first and second quarters of 1995), we used instead the actual CPI. In order to preserve continuity, the quarterly yields for the fourth quarter of 1994 and the first quarter of 1995 were approximated with the use of information on an indexed bond with ten months to maturity, coupled with price information on non-indexed bonds with nine and ten months to maturity.

Table 1 presents summary statistics for the quarterly real rate of return on an index-linked bond and on the quarterly nominal rate of return on a non-index-linked bond for the period 1988:01 through 1997:04 in Israel.

The quarterly real interest rate is depicted in Figure 6 and its unconditional mean is provided in Table 2, under three different scenarios for the degree of relative risk aversion.

[Figure 6, here]

## • Matching the First Moment

Here, we confront the model's predictions with the data and check for which values of the CRRA the equilibrium parities for the yields on non-index-linked and index-linked bonds, summarized in equations (12) and (16) above, are satisfied. Both parities are written slightly differently in equations (28) and (29) below,

$$Y_t(3) = Y^*_{t,p}(3,\theta) \cdot E_{t,p}\left(P_{t+3} / P_t\right) \cdot \left(1 / \psi_{t,p}(\theta)\right) \tag{28}$$

$$\frac{Y_t^+(4)}{Q_t(3) / Q_t(4)} = Y^*_{t,p}(3,\theta) \cdot \left(\eta_{t,p}(\theta) / \gamma_{t,p}(\theta)\right). \tag{29}$$

For every period and for a given value for the CRRA, our model generates estimates for the (hypothetical) quarterly real rate of return, $Y^*_{t,p}(3,\theta)$, the inflation risk premium, $\left(1/\psi_{t,p}(\theta)\right)$, and the indexation-lag risk premium, $\left(\eta_{t,p}(\theta)/\gamma_{t,p}(\theta)\right)$. Alongside, for every period our model generates a forecast for the quarterly rate of inflation. Having done that, we compute for every period (given θ) the model's predictions for the quarterly yields on indexed and non-indexed bonds and average over the entire sample (i.e., 1988:01-1997:02). The procedure is repeated until a value for the CRRA, for which the sampled means are perfectly mimicked, is found.

We find that the model is able to generate the unconditional mean of both yields for relatively low values of the relative risk aversion coefficient. In particular, the actual (3.33%) and predicted quarterly averages of the nominal rate of return on the non-indexed bond (i.e., equation (28)) are identical for $\theta$ hovering around *0.15*. Similarly, the actual (0.206%) and predicted quarterly averages of the real rate of return on the indexed bond (i.e., equation (29)) are identical for $\theta$ hovering around *0.3*. We do not find these results surprising given the C-CAPM good statistical fit and the mild aversion to risk estimated in the Israeli economy, discussed above.

## 4. Conclusions

The understanding of the time-series properties of the real ex-ante risk-free rate of return and of the inflation expectations is of interest to theoreticians and practitioners alike. This paper shows that for a complete comprehension of the size and dynamic behavior of these two variables, two omitted risk components need to be thoroughly understood: the premium attached to the nominal risk-free bond, the inflation risk premium, and the premium attached to the index-linked bond, the indexation-lag risk premium. In order to accomplish such a task, we have proposed a consumption-based capital asset pricing model framework, which has the merit of being able to depict the dynamic properties of the yield on a hypothetical risk-free (in real terms) asset, against which both premiums are defined. The general equilibrium compensation for bearing the different price uncertainties has translated in our consumption-based, three-bond environment into a set of conditional mean and covariance terms that were then calibrated with the use of prevalent stochastic processes.

We arrived at three main conclusions: first, we found that the average inflation risk premium has been sub-period independent in the course of the 1990s and its unconditional mean being positively related to the degree of relative risk aversion in the economy. Second, we found that the unconditional mean of the indexation-lag risk premium is tiny yet dependent on the degree of aversion the representative agent exhibits toward price instability. Third, we found that the general practice of explicitly presuming a null bias in capital-market-based inflation expectations used, for example, by the Bank of Israel is approximately accurate in the case of a preferences structure characterized by a relatively mild aversion to risk. However, such a supposition seems to be invalidated the more the individual is willing to sacrifice in order to avoid risk.

We suggest several avenues for future research: first, tastes in this paper were characterized by a time and state separable utility function. However, such a choice for the preferences' representation is not unique, since it is well known by now that more elaborate parameterization are feasible (see, for example, Constantinides [1990], Epstein and Zin [1989]). Since such a pick is not without consequences for the real stochastic pricing kernel and hence for the real risk-free yield, future work should check the robustness of our results to other specifications. Second, having computed the vector of observations for both premiums, it seems necessary to try to track the different macroeconomic fundamentals that determine the time-series properties of the inflation risk premium and the indexation-lag risk premium, probably with a regime-switching mechanism (Evans and Lewis (1995) analyze a case of switching inflation regimes). Such a structural shape would eventually be capable of providing a forecast for these additional compensations the government must pay when it uses these particular index-linked and non-index-linked instruments to raise credit from the public. Third, the calculation of the indexation-lag risk premium provides the means to approximate the real ex-ante risk-free rate of return. This extracted yield can then be applied in trying to solve numerous open questions in the asset pricing literature, such as ones related to the real term structure of interest rates. Fourth, the Hansen-Jagannathan (1991) non-parametric technique of constructing a lower envelope for the set of stochastic discount factors can be extended to a more elaborate information environment, one where there is no uncertainty as to what should be the true value of the mean of the real kernel. In addition, being aware of the often-heard criticism of the use of a consumption-based methodology, we consider it worthwhile to augment our parametric approach with a non-parametric one, which does not rely on any structural assumption.

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**Table 1**

**Summary Statistics**

| Variable | Mean | SD | ρ | Minimum | Maximum |
|---|---|---|---|---|---|
| Real CG | 0.6% | 2.1% | -0.35 | -3.8% | 4.5% |
| Nominal CG | 3.6% | 2.7% | -0.20 | -3.2% | 10.0% |
| Inflation | 3.0% | 1.8% | 0.15 | -3.1% | 7.9% |
| Nominal | 1.0% | 0.1% | 0.40 | 0.7% | 1.6% |
| Indexed | 0.261% | 1.3% | 0.42 | -2.98% | 2.81% |
| Non-Indexed | 3.3% | 0.6% | 0.49 | 2.0% | 5.0% |

Summary statistics for quarterly real per-capita consumption growth (Real CG), quarterly nominal per-capita consumption growth (Nominal CG), rate of inflation (Inflation), the nominal monthly yield on non-indexed bonds (Nominal), the quarterly real rate of return on an index-linked bond (Indexed), and the quarterly nominal rate of return on a non-index-linked bond (Non-Indexed) for the period 1988:01-1997:04 in Israel. The price measure is the consumption price deflator, derived from the nominal and real consumption series. The quarterly real rate of return on an index-linked bond is calculated from prices of indexed and nominal bonds simultaneously observed at the beginning of each quarter, as in Kandel, Ofer and Sarig (1996). The quarterly nominal rate of return on a non-index-linked bond is the yield to maturity of a three-month nominal bond. All rates are expressed in terms of percent per quarter. SD denotes the standard deviations of the variables and ρ their first order serial correlation.

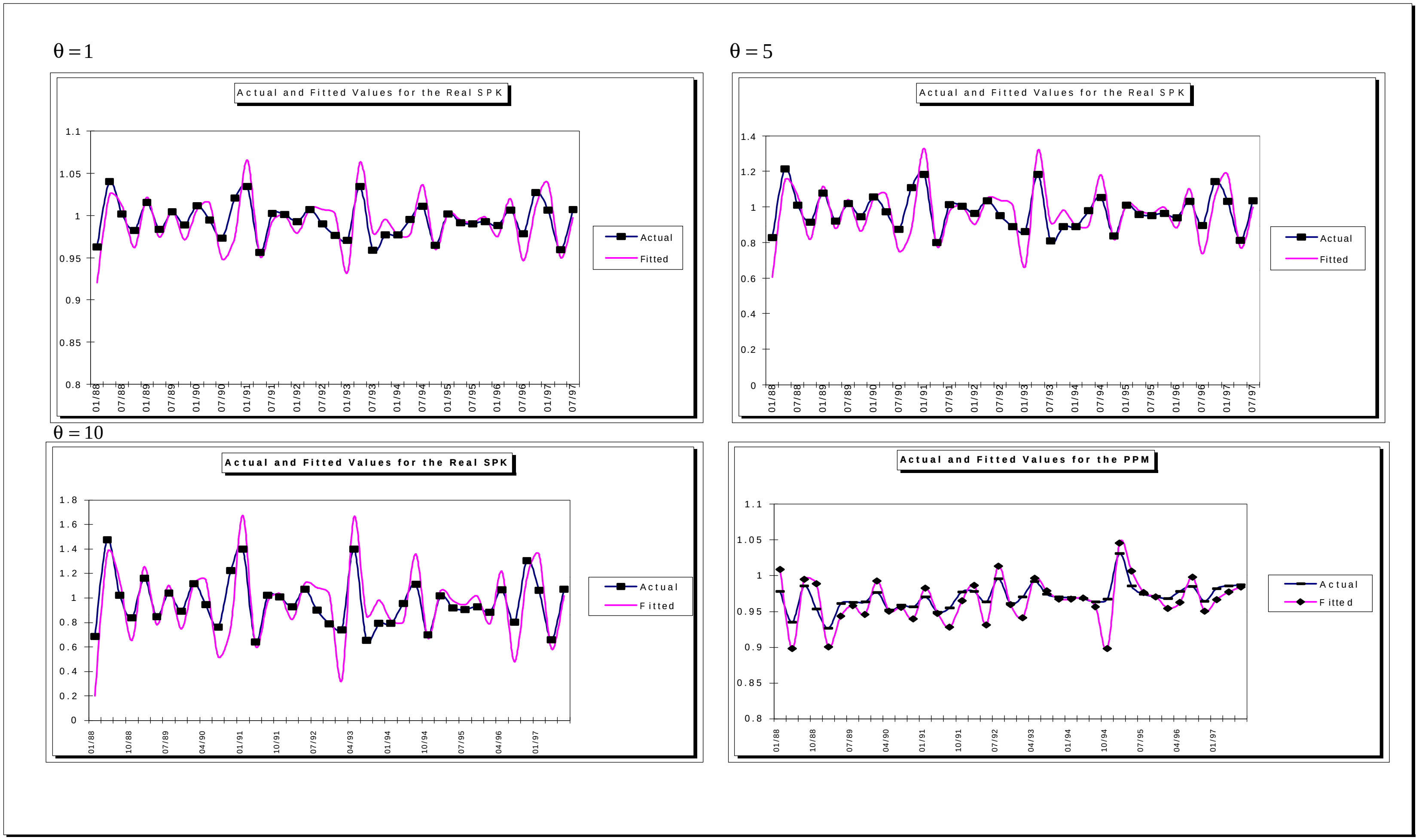
**Table 2**

**Risk Aversion and the Unconditional Mean of the Pure Inflation Risk Premium, Term Premium, Indexation-Lag Risk Premium, Inflation Expectations Bias, and the Real Ex-Ante Risk-Free Rate of Return, 1988:01-1997:02**

| $\theta$ | IRP | TP | ILRP | Bias | Real |
|---|---|---|---|---|---|
| 1 | 0.0366% | 0.0007% | 0.0001% | -0.0365% | 0.2059% |
| 5 | 0.1973% | 0.003% | 0.0004% | -0.1969% | 0.2056% |
| 10 | 0.6165% | 0.007% | 0.002% | - 0.6146% | 0.2039% |

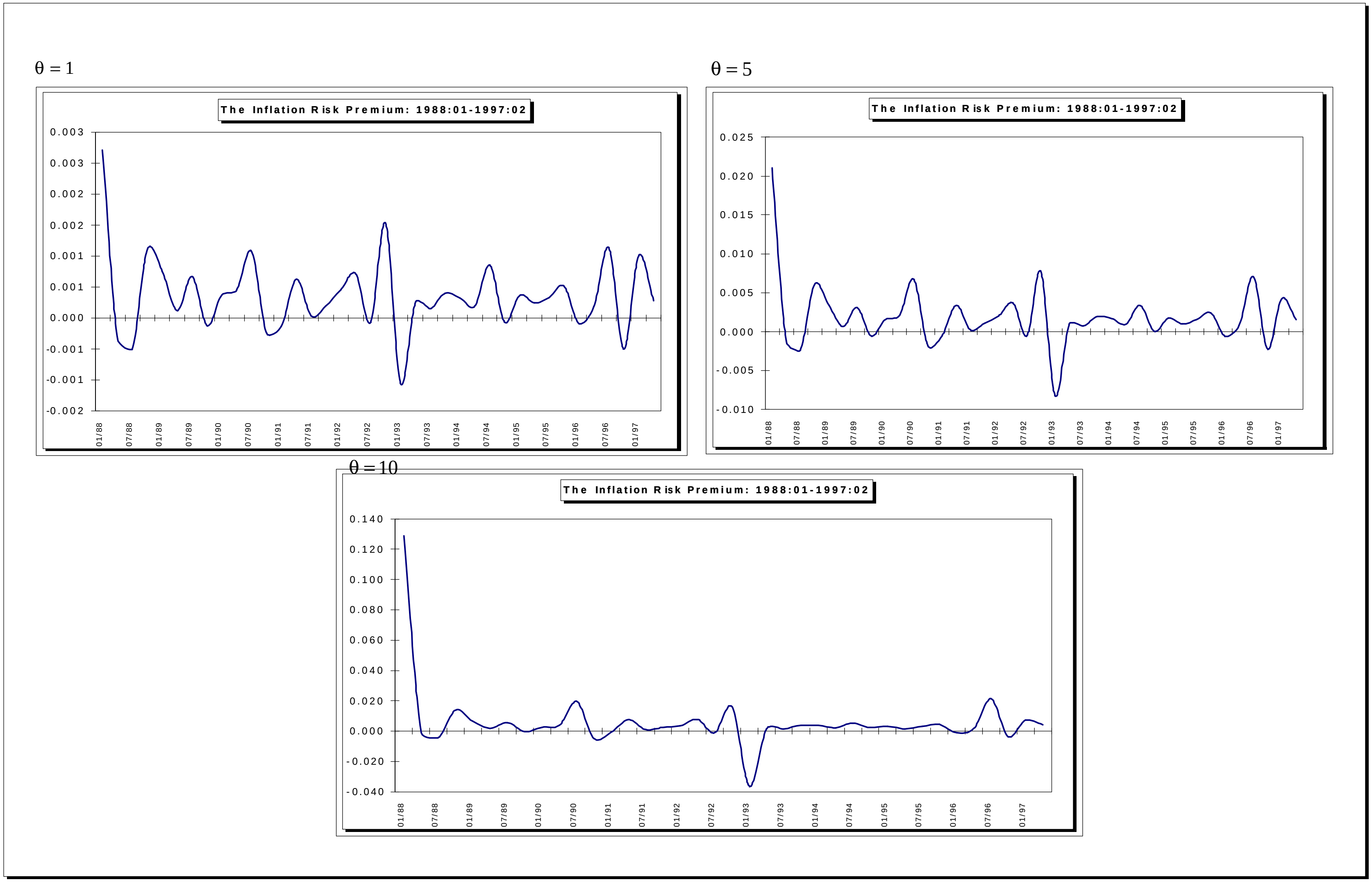
"IRP" is the unconditional mean of the pure inflation risk premium. "TP" is the unconditional mean of the term premium. The reported values are defined on the yields' domain, i.e., they are the inverse of the premiums identified in equation (12) and (15) respectively. "ILRP" is the unconditional mean of the indexation-lag risk premium, defined in equation (16). "Bias" is the unconditional mean of the bias in the Bank of Israel's capital-market-based inflation expectations estimate, defined in equation (18). A negative value means that the Bank of Israel's methodology overestimates, on average, the genuine value of inflation expectations. "Real" is the unconditional mean of the real ex-ante risk-free rate of return, defined in equation (27). $\theta$ is the coefficient of relative risk aversion. The reported values are expressed in terms of percent per quarter.

Figure 1




"Real SPK" is the real stochastic pricing kernel, which equals $\beta\left(\frac{c_{t+3}}{c_t}\right)^{-\theta}$. c is the real per-capita consumption. θ is the coefficient of relative risk aversion. β is the time preference coefficient. We let β=1 throughout. "PPM" is the inverse of the inflation rate. "Actual" are the sampled observations whereas "Fitted" are the values predicted by a first order auto-regressive equation. By subtracting the latter from the former we generate the respective innovations, as in equation (19). The quarterly sample begins in the first quarter of 1964 and ends in the fourth quarter of 1997. Since the indexation-lag risk premium will be calculated, due to data availability, from the first quarter of 1988, the Figures are adjusted accordingly.

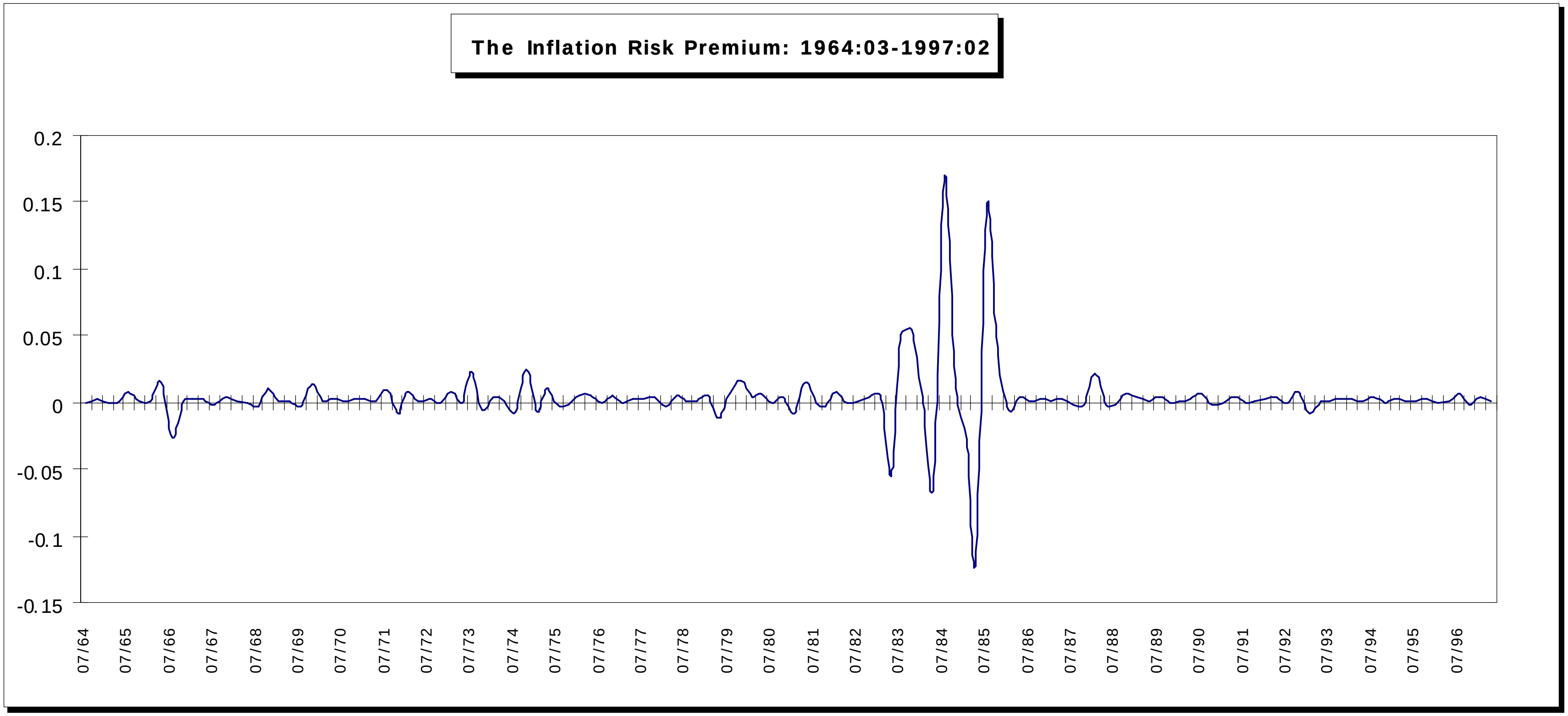
Figure 2a




The "Inflation Risk Premium" is generated in accordance with equation (25). The innovations' product in equation (22) is taken to follow a first order auto-regressive process. θ is the coefficient of relative risk aversion. The reported estimates are on a quarterly basis, starting in the first quarter of 1988 and ending in the second quarter of 1997, and represent the net (in real terms) additional yield on a non-indexed bond (i.e., the inverse of the premium identified in equation (12) minus one).

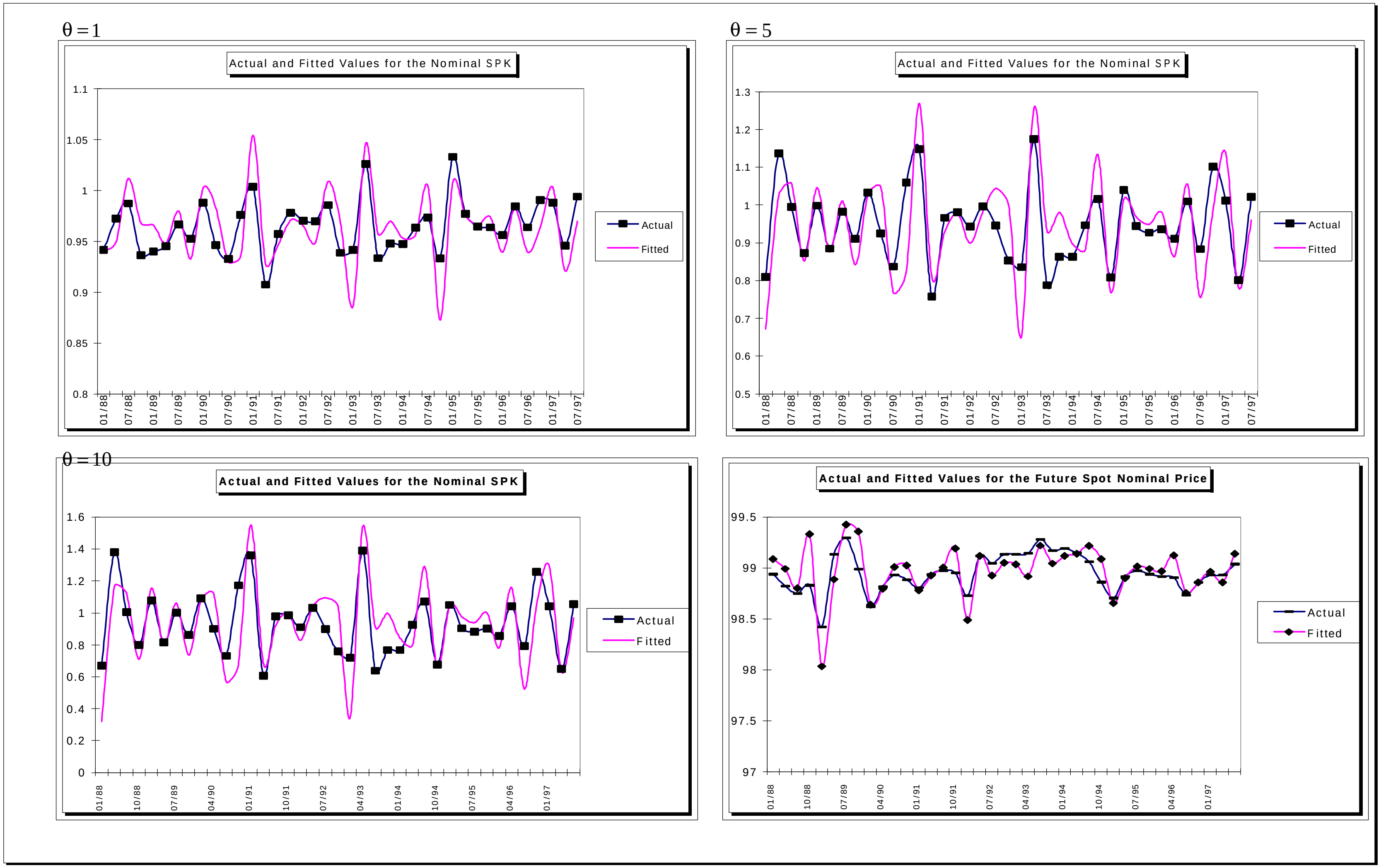
Figure 2b

θ = 5



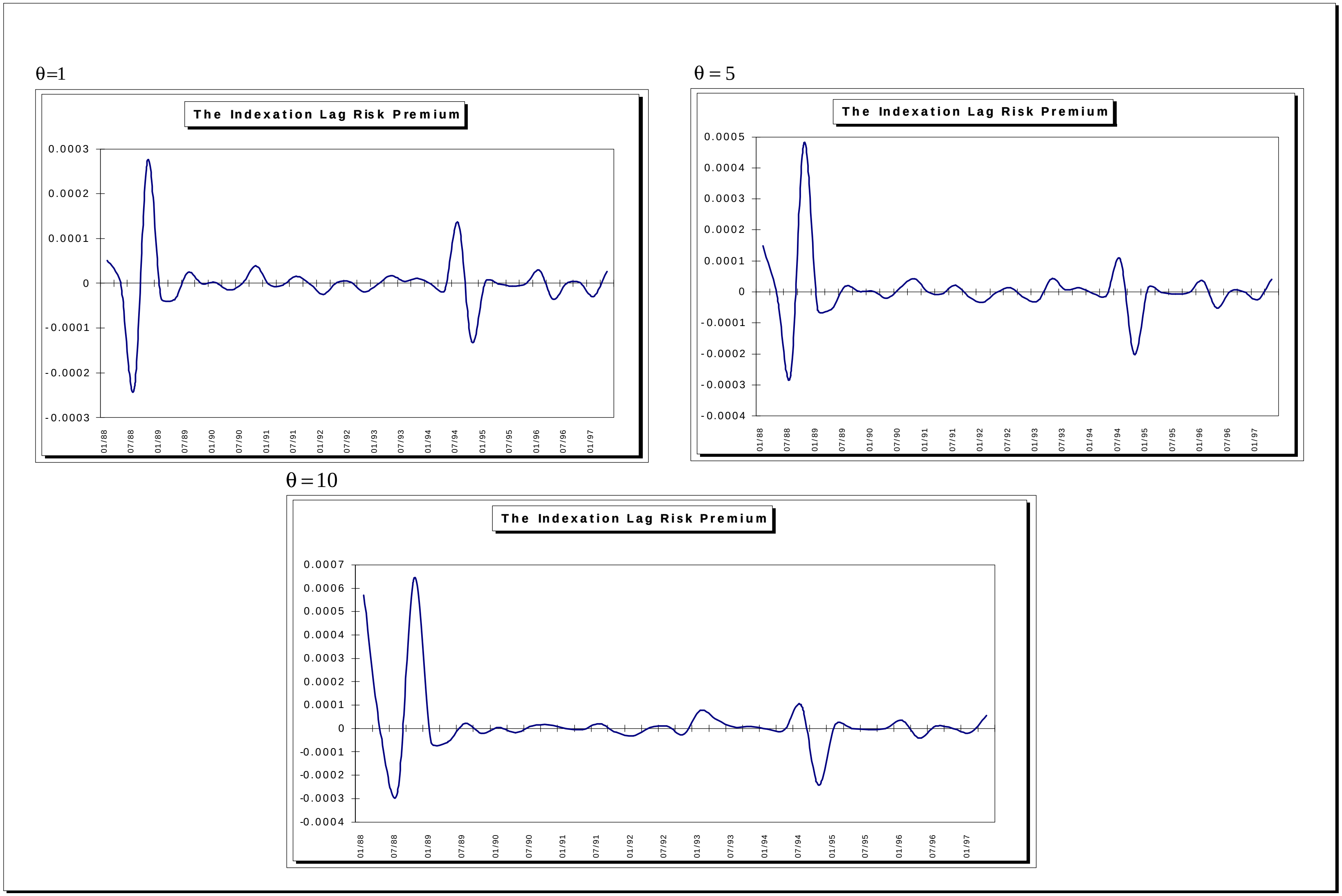

The "Inflation Risk Premium" is generated in accordance with equation (25). The innovations' product in equation (22) is taken to follow a first order auto-regressive process. θ is the coefficient of relative risk aversion. The reported estimates are on a quarterly basis, starting in the third quarter of 1964 and ending in the second quarter of 1997, and represent the net (in real terms) additional yield on a non-indexed bond (i.e., the inverse of the premium identified in equation (12) minus one).

Figure 3



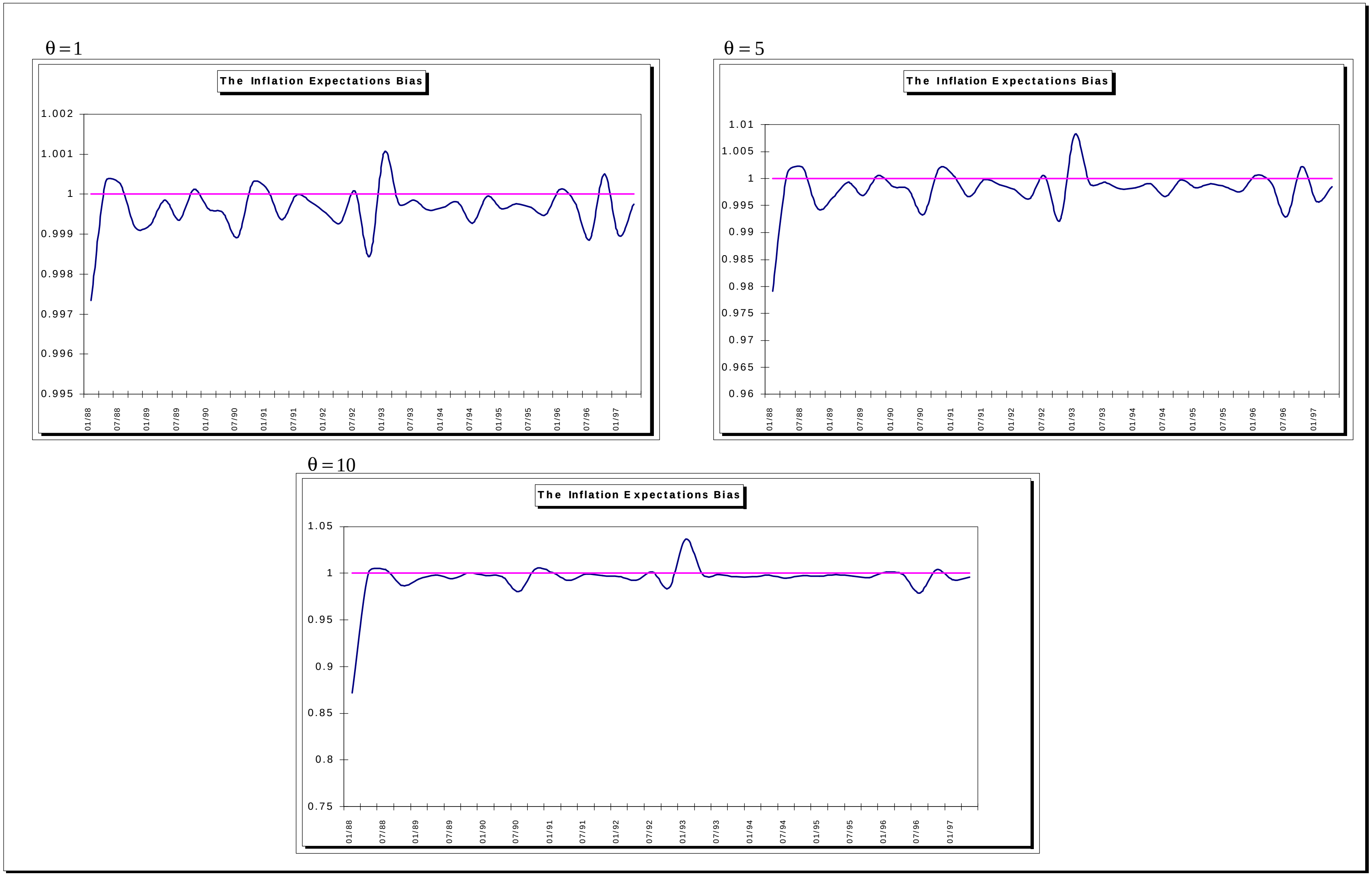

"Nominal SPK" is the nominal stochastic pricing kernel, which equals $\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\theta}$. c is the nominal per-capita consumption. θ is the coefficient of relative risk aversion. β is the time preference coefficient. We let β=1 throughout. "Future Spot Nominal Price" is the nominal price, three-months ahead of a one-period non-indexed bond. "Actual" are the sampled observations whereas "Fitted" are the values predicted by a first order auto-regressive equation. By subtracting the latter from the former we generate the respective innovations, as in equation (19). The quarterly sample begins in the first quarter of 1988 and ends in the fourth quarter of 1997.

Figure 4



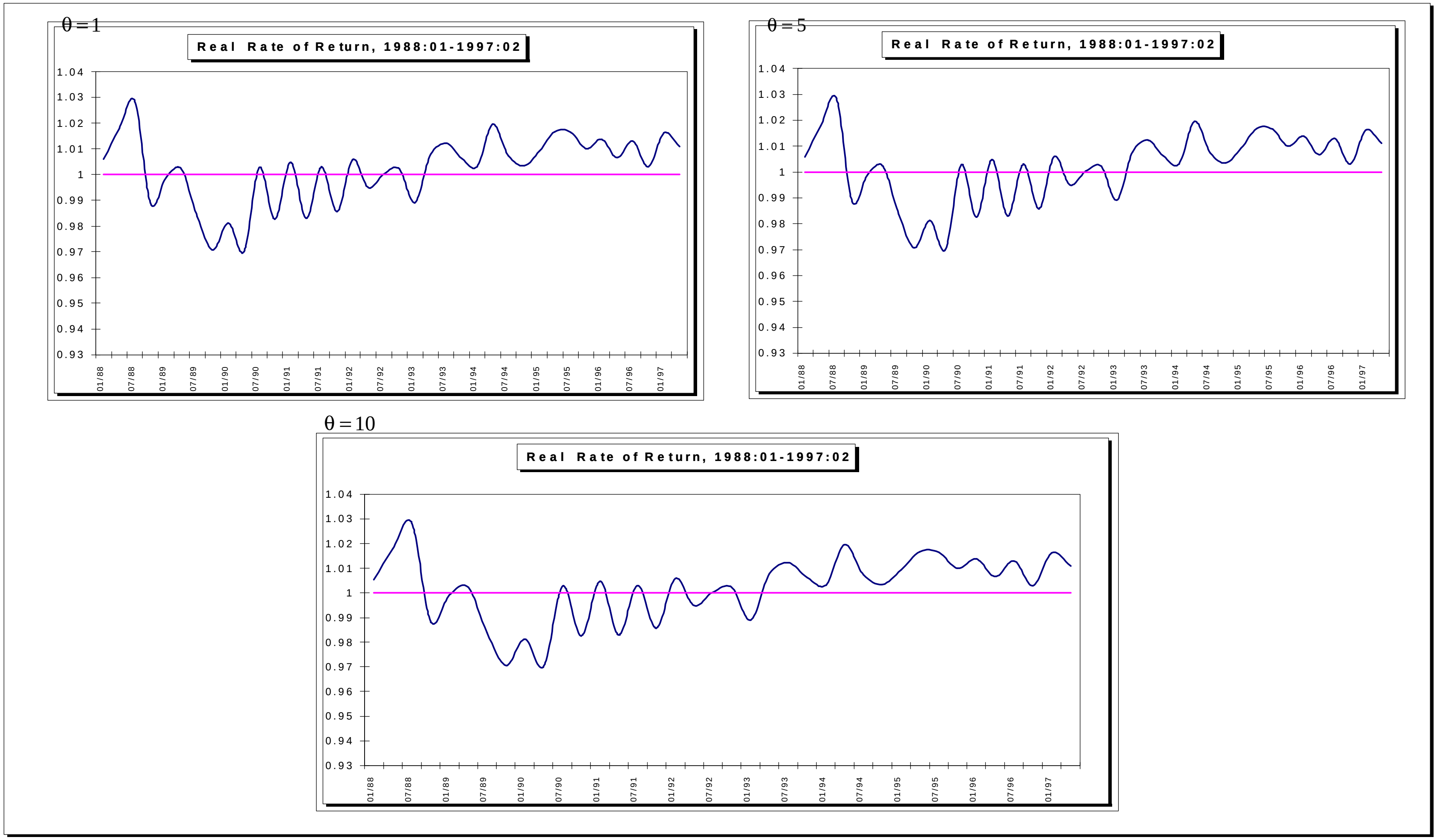

The "Indexation-Lag Risk Premium" is generated in accordance with equation (26). The innovations' products in equations (23) and (24) are taken to follow a first order auto-regressive process. θ is the coefficient of relative risk aversion. The reported estimates are on a quarterly basis, starting in the first quarter of 1988 and ending in the second quarter of 1997, and represent the net (in real terms) additional yield on an indexed bond (i.e., the premium identified in equation (16) minus one).

Figure 5




The "Inflation Expectations Bias", which measures the bias in the Bank of Israel's capital-market-based inflation expectations estimate, is generated, in accordance with equation (18), as the product of the calibrated inflation risk premium $\Psi$ and the indexation-lag risk premium $\chi$. $\theta$ is the coefficient of relative risk aversion. A value smaller than one means that the Bank of Israel's methodology overestimates the genuine value of inflation expectations. The reported estimates are on a quarterly basis, starting in the first quarter of 1988 and ending in the second quarter of 1997.

Figure 6




"Real Rate of Return" is the gross real ex-ante risk-free rate of return, generated in accordance with equation (27). θ is the coefficient of relative risk aversion. The reported estimates are on a quarterly basis, starting in the first quarter of 1988 and ending in the second quarter of 1997.