Risk Arbitrage in Takeovers*

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Abstract. The paper studies the role of risk arbitrage in takeover contests. We show that arbitrageurs have an incentive to accumulate non-trivial stakes in a company target of a takeover. For each arbitrageur, the knowledge of his own presence (and that he will tender a positive fraction of his shares) is an informational advantage which guarantees that there is a scope for trade with the other shareholders. In equilibrium, the number of arbitrageurs buying shares and the number of shares they buy are determined endogenously. The paper also presents a range of empirical implications, including the relationship between trading volume, takeover premium, bidder's toehold, liquidity of the shares and the probability that the takeover will succeed.

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1. Introduction

It is well known that risk arbitrageurs play an important role in the market for corporate control. After a tender offer, the trading volume increases dramatically in large part because of risk arbitrageurs activity.[1] They take long positions in the target stock, in the hope that the takeover will go through. They are also usually hedged by taking short positions in the acquirer's stock.

Risk arbitrage used to be a very inconspicuous activity, but in the mid-70s the emergence of Ivan Boesky and the increasing volume of corporate takeover deals contributed to make it more visible.[2] Attracted by the high rewards, many firms started new arbitrage departments and more people became involved in this activity. As a consequence of the large volume of new arbitrage capital, in more recent years spreads narrowed and the share price, after a takeover announcement, rises much more rapidly. This clearly reduced profits margins. However, despite the increasing competition, risk arbitrageurs still make profits and they are perceived as a crucial element in determining the success of a takeover. They are typically perceived as favoring the acquirer since they are more likely to tender.[3] They have been helped by the fact that the arbitrage community has often come to control, in total, 30 to 40 per cent of the stock and therefore they have become the single most important element in making many deals happening.

In this paper we study why arbitrageurs have an incentive to take part in takeover contests. In other words, what is the source of their advantage and why the competion of an increasing number of arbitrageurs does not erode it. We start by abstracting from differences in attitude towards risk and focus on the explanation most commonly

[1]Numerous case studies reveal that the increased trading volume is largely due to arbitrage activities. See Harvard Business School case 9-282-065: *Note on Hostile Takeover Bid Defense Strategies.* For specific examples see Harvard Business School (HBS) case 9-285-053: *Gulf Oil Corp—Takeover,* HBS case 9-285-018: *The Diamond Shamrock Tender for Natomas (A)* and D. Commons: *Tender Offer.* On the other hand, it is common knowledge among financial arbitragers that a takeover bid represents one of the best opportunities for them to operate, see Ivan Boesky, *Merger Mania — Arbitrage: Wall Street's Best Kept Money-making Secret.*

[2]See Welles (1981).

[3]See Grinblatt and Titman (1998).

given: difference in information. It is often argued that arbitrageurs have better information about the chance of a successful takeover and purchase shares as long as their forecast of the "correct" security price exceeds the current market price.

We argue that it is not necessary to assume that risk arbitrageurs have specific knowledge on the takeover fight. Instead, the information advantage can arise endogenously from the choice of a risk arbitrageur to enter the contest. The intuition is quite simple: if the presence of risk arbitrageurs increases the probability of a takeover, then the fact that one risk arbitrageur bought shares is per se relevant for the value of these shares. Therefore the risk arbitrageur has an informational advantage: he knows he is in. After all, risk arbitrageurs are often quoted saying that a crucial part of their activities is trying to predict what other arbitrageurs will do.[4]

We model the decision of risk arbitrageurs to enter the contest and the way they accumulate shares. The number of arbitrageurs who choose to take positions, the number of shares they buy and the price they pay are determined endogenously in equilibrium. In our model, we start from a company with diffuse ownership, with no large shareholders who can facilitate the takeover. After a bidder has made a tender offer, arbitrageurs decide whether to buy shares. If they succeed in accumulating non-trivial stakes, they become temporary large shareholders. Unlike small shareholders, they tend to sell their shares to the bidder and therefore facilitate the takeover. For this to happen, however, it is necessary to show how they can be successful in accumulating these positions without driving the price up so much that they end up losing money. In other words, we ask ourselves how arbitrageurs can afford to pay a price which is high enough to persuade small shareholders to give up their shares.

The value of the shares depends on the probability that the takeover will take place, and therefore it should be higher the larger is the number of risk arbitrageurs in the market (since they are more likely to tender). Both small shareholders and risk arbitrageurs do not know how many arbitrageurs have entered the contest and update their beliefs looking at the trading volume. However, a risk arbitrageur always has

[4] In general, risk arbitrageurs talk to a small subset of other arbitrageurs about whether or not they are involved in a specific deal. In the conclusions, we briefly discuss this possibility.

an informational advantage on the small shareholders: he knows that at least he is buying shares. This informational advantage guarantees that he is willing to pay a price which is high enough to persuade the small shareholders to sell their shares.

As the trading volume increases, small shareholders think it is more likely that some arbitrageurs are buying shares. Consequently, the share price increases. We show that there exists an equilibrium in which risk arbitrageurs buy shares and earn positive expected profits.

As long as the expected profits are strictly positive, more arbitrageurs will choose to buy shares. If too many arbitrageurs are buying shares, however, the price will rise too much and the profits will be negative. We show that there exists a symmetric equilibrium where each arbitrageur randomizes between entering or not. In equilibrium, even if more than one arbitrageur has entered, there are cases in which they do not compete away the entire rent.

While the press has often depicted risk arbitrageurs in an unfavorable way, this paper shows that they can actually increase welfare, facilitating takeovers which increase the value of a company. Moreover, small shareholders, who sell their shares to risk arbitrageurs, do not lose money but they actually appropriate the ex ante surplus, which would be lost if the takeover did not succeed.

In a similar spirit, Kyle and Vila (1991) studies a case in which the bidder buys shares before announcing the takeover. Because of noise trading, the bidder succeeds in hiding at least partially his presence. In our paper, since we focus on post-announcement trading, risk arbitrageurs do not have any initial private information: the informational advantage arises endogenously when they start buying. Moreover, since there is more than one arbitrageur with the same informational advantage, we have to control that they do not compete away their rent.

Larcker and Lys (1987) offers a careful empirical study of risk arbitrageurs in takeovers. Their hypothesis is that risk arbitrageurs are better informed than the market about the takeover success rate. They find that firms purchased by arbitrageurs have an actual success rate higher than the average probability of success implied by market prices. As a result, they can generate substantive positive returns on their portfolio positions. This is compatible with the results of our model, although we argue that the explanation could be different: the risk arbitrageurs may not know ex ante which takeover attempt are more likely to be successful, but it is their presence which increases the probability of success. Larcker and Lys (1987) also shows that the amount of shares arbitrageurs buy is not significantly correlated with their return rate, which is consistent with our result that the limit to the number of shares bought come from the need of risk arbitrageurs to hide their presence.

Focusing on the role of risk arbitrageurs allows us to explain certain empirical patterns during takeover activity and derive testable implications. A widely observed phenomenon is that, after the takeover announcement, both the stock price and the transaction volume of the target rise tremendously relative to their pre-announcement levels.[5] We find a positive relationship between trading volume, takeover premium and the probability that the takeover is successful. We also find that the more liquid is the target stock, the better risk arbitrageurs can hide their trade and, as a consequence, the higher is the probability of success of the takeover (or the lower is the required takeover premium). Finally, we look at the bidder's choice of a toehold and we show that an increase of the toehold may discourage the risk arbitrageurs from entering and, as a result, reduce the probability of success of a takeover.

Note that an implication of this paper is that the well known free-riding problem of Grossman and Hart (1980) is mitigated.[6] The reason is that risk arbitrageurs have the role of large shareholders, as in Shleifer and Vishny (1986) and Hirshleifer and Titman (1990). The contribution of this paper is to show that, even if at the time of the tender offer there are no large shareholders, there exist equilibria where arbitrageurs enter and buy shares, becoming in this way large shareholders.

[5]Based on a sample of mergers before the 1980, Jensen and Ruback (1983) found in a comprehensive survey that the average jump in share price of the target firm ranges from 17% to 35%. For the 1980's merger wave, Bradley, Desai and Kim (1988) found similar results.

[6]Several other papers have shown how this problem can be mitigated because of various reasons. See, among others, Bagnoli and Lipman (1988), Giammarino and Heinkel (1986), Harrington and Prokop (1993), Bebchuk (1989) and Yilmaz (1997). Hirshleifer (1995) gives an overview of the subject.

We assume that small shareholders take the probability of a takeover for given in order to simplify the analysis. In fact, in this way the behavior of small shareholders is straightforward and we can focus on risk arbitrageurs. However, the intuition remains the same also with more strategic players (as long as the probability of the takeover, at the announcement of the tender offer, is less than one): an individual who owns a larger stake will suffer less of the free-riding problem and tender a larger proportion of his shares, thereby facilitating the takeover. The rest of the analysis would then be as above.

In most of the paper, we abstract from the issue of difference in the attitude towards risk and look only at the information advantage. In Section 6 we show that if risk arbitrageurs are also less risk averse our result is stronger. Moreover, we consider the possibility that the only advantage of the risk arbitrageurs were their lower risk aversion and show that in this case their demand for shares would drive the price up to the point where their profits are equal to zero.

Finally, we focus on the case in which risk arbitrageurs buy shares only after the takeover announcement. In the data set of Larcker and Lys (1987), only in three cases the transaction date was prior to the first tender offer. In general, risk arbitrageurs do not attempt to forecast acquisition candidates, but rather to resolve the uncertainty surrounding an announced proposal. The model can be extended to the case in which risk arbitrageurs can take positions also prior to the takeover announcement, speculating on the probability that the announcement will indeed happen. As long as their presence increases the chances of success of a (possible) takeover, risk arbitrageurs have an informational advantage and the same result holds. This would imply that the run-ups in share prices of target firms before takeover announcements may be at least in part due not to insider traders but to people who by the simple fact that they are buying shares are increasing the probability of a successful takeover.

Following the introduction, the model is described in detail. Section 3 studies the tendering strategies of risk arbitrageurs, once they have taken position in the target shares. Section 4 studies the choice of risk arbitrageurs to buy shares and Section

5 their decision to enter the contest. Section 6 considers several empirical implications and extensions. Finally, the conclusions summarize the results and discuss the implications.

2. The Model

In order to focus on the role of arbitrageurs, we assume that at the beginning small shareholders control 100% of the outstanding shares (as in Grossman and Hart (1980)). The model does not consider situations where there are large shareholders, but they could be easily incorporated into the model. The crucial feature is that small shareholders free ride at least partially, so that when the tender offer is made the probability of a successful takeover is less than 1.

At time 0, a bidder announces a cash tender offer of P_T for all shares. If more than 50% shares are tendered, the bidder purchases them all at the price P_T, otherwise all tendered shares are returned.

We assume that P_0 is the initial share price and both P_0 and P_T are observable to all. So is the value improvement per share that the bidder can bring to the firm, ΔP. Naturally, we assume that

$$P_0 + \Delta P \geq P_T \geq P_0.$$

The bidding price is between the *status quo* share price and the potential improved value of the share. In addition, we assume that if the takeover bid proves to be a failure, the stock price goes back to P_0.[7]

At time 1, arbitrageurs decide whether to enter and speculate. At time 2, stock trading takes place — arbitrageurs take positions, hiding among small investors. Finally, at time 3, all shareholders decide how many shares to tender and the outcome of the takeover is determined.

[7] The implied assumption is that the occurrence of this takeover bid does not change the probability of new takeover bids and their success. If the stock price falls to a different value than P_0, a similar analysis can still be performed.

Let us now look at the players.

The Small Shareholders

A small shareholder controls so few shares that he believes that his decision to tender his shares will have no effect on either the trading price or the outcome of the takeover. For the moment, we assume that the small shareholders are risk neutral. In Section 6 we assume that the small shareholders are risk averse and show that our result is actually strengthened.

The Arbitrageurs

There are N potential arbitrageurs, who can choose to take a position. The arbitrageurs are assumed to be risk neutral. After the takeover announcement, each of them has to decide whether to arbitrage or not in the stock of the target firm. If an arbitrageur A_i decides to arbitrage (i.e. decides to "enter the contest"), he must bear a cost c. Such cost can be interpreted as the cost of collecting information or as the opportunity cost of other investment opportunities, given that risk arbitrageurs, as argued in Shleifer and Vishny (1997), do not have unlimited financial resources. If he decides to enter, the arbitrageur buys a portion δ_i of the total outstanding shares of the firm, where δ_i is endogenously determined. Legally, there is an upper limit $\bar{\delta}$ (which in US is 5%) so that if $\delta_i > \bar{\delta}$, the risk arbitrageur has to declare the amount of shares he owns to comply with Section 13D of the Security Exchange Act.

Let us call n the number of arbitrageurs who enter. In equilibrium, the entry decision of each arbitrageur is endogenized. For the moment, let us call $G(n)$ the distribution of n (with density $g(n)$), which will be endogenously derived in equilibrium. For technical tractability, n will be treated as a real (continuous) number and, likewise, $g(n)$ as a continuous function.

After the trading session is closed, each arbitrageur who purchased shares makes a decision regarding the portion of shares to be tendered. For A_i, let us define the portion as $\gamma_i \in [0, 1]$.

Noise Traders and the Total Trade Volume

Shares can be bought also by small investors, who exist due to external reasons (such as diversification of their investment portfolio)[8]. Since in this paper we focus on symmetric equilibria, we need to assume the presence of noise traders in order to guarantee that the equilibrium in the trading game is not perfectly revealing. However, if we looked instead at the asymmetric equilibria, the presence of noise traders would not be necessary, since the uncertainty about the number of arbitrageurs n would be enough to guarantee that the equilibrium is not fully revealing.

The trading volume from noise traders, ω, is random and independent of both the share price and the demand of arbitrageurs. By definition, the volume ω is non-negative and it is common knowledge that it is distributed uniformly on the interval $[0, 1]$. [9]

Let y be the total trading volume of the shares of the firm, then

$$y = \omega + \sum_{i=1}^{n} \delta_i.$$

The number of arbitrageurs n who entered is unknown to both risk arbitrageurs and small shareholders, but everybody knows that n is distributed according to $G(n)$ (which will be determined in equilibrium) and can observe the trading volume y.

We use the concept of Perfect Bayesian equilibrium and focus on the symmetric equilibria, where each arbitrageur buys and tenders the same proportions, δ and γ, of shares. To determine the equilibria, we solve the game backwards. We start from the tendering game: after n arbitrageurs entered and bought shares, we determine their optimal tendering strategy, given their beliefs on how many other arbitrageurs are around. Then, given their tendering strategy, we look at the trading game. We find the rational expectations equilibria and whether there exists an equilibrium where

[8]They can also be program or package traders whose decision to trade is based on information uncorrelated to the takeover process.

[9]We are assuming that noise traders and risk arbitrageurs cannot short sell the shares of the target firm. In reality, risk arbitrageurs usually short sell the bidder's shares in order to hedge. Since risk arbitrageurs are risk neutral in this paper, they have no reason to hedge. In the conclusions, we discuss the possibility to let risk arbitrageurs short sell the target's shares.

the risk arbitrageurs buy shares. Finally, we look at the choice of the arbitrageurs whether to enter or not.

3. The Tendering Game

The tendering game is played among the arbitrageurs. Small shareholders stay out of the picture, since they take the probability of a takeover for given and therefore, by the Grossman-Hart (1980) argument, they never tender their shares.

At the beginning of period 3, n arbitrageurs have entered. Each arbitrageur A_i bought δ_i shares and observed the transaction volume y, but does not know exactly how many other arbitrageurs entered. From y he updates his belief about n. Then, the arbitrageur chooses how many shares to tender, given y and given the strategies of the other risk arbitrageurs.[10]

We first look at the updating process of the arbitrageur, after observing y, and derive his *posterior* probability that the takeover is successful. Then the decision problem of the arbitrageur is analyzed. Subsequently, the existence and properties of the equilibrium are established.

3.1. Posterior Probability of Success of the Takeover. We want to compute the probability τ_i^a that the takeover will succeed for an arbitrageur A_i who bought δ_i shares and plans to tender γ_i shares, given that all other arbitrageurs tender a portion γ of their δ shares.[11]

Since small shareholders do not tender, when $y < 0.5$, ${\tau^a}_i = 0$. If $y \geq 0.5$ and no arbitrageur has declared to have $\bar{\delta}$ or more shares[12],

[10]We rule out the possibility of collusion between arbitrageurs and acquirer. If the acquirer colludes with a few arbitrageurs, he becomes a large shareholder in the sense of Shleifer and Vishny (1986), although this can be illegal. The rest of the arbitrageurs still have to play a tendering game. Thus, the takeover can be successful with less than 50% shares tendered. In this sense, the model can be expanded to cover collusion.

[11]Equivalently, one can assume that A_i believes that A_j tenders γ_j. This seemingly more general assumption does not change the following derivation at all, since all that matters is the sum of the shares tendered by other arbitrageurs. In other words, γ in the formal assumption can be regarded as the average portion of shares tendered.

[12]If other arbitrageurs have filed 13D, A_i will take that into account in computing τ_i^a.

$$\tau^a{}_i = \tau^a{}_i(y,\gamma,\gamma_i,\delta) = Prob\ [\ \gamma(n-1)\delta + \gamma_i\delta_i \geq 0.5\ \ \mid y-\delta_i\ ]$$
$$= Prob\ [\ n-1 \geq \frac{0.5}{\delta\gamma} - \frac{\gamma_i\delta_i}{\gamma\delta}\ \ \mid y-\delta_i\ ]$$

where $y-\delta_i$ is his observation of the total transaction volume excluding his own. Clearly, A_i has to compute the conditional probability distribution of the number of arbitrageurs other than himself. Let $d\ (.)$ denote the density, then

$$d\ (n-1=s\ \ \mid y-\delta_i\ ) = \frac{d\ (n-1=s, \omega+(n-1)\delta = y-\delta_i\ )}{d\ (\omega+(n-1)\delta = y-\delta_i\ )}$$

$$= \frac{d\ (n-1=s, \omega = y-s\delta-\delta_i\ )}{d\ [\omega+(n-1)\delta = y-\delta_i\ ]}.$$

Under the assumption that, *ex ante,* n and ω are independent and that the coming of arbitrageurs is mutually independent (which will be shown to be true in equilibrium), it follows

$$d\ (n-1=s, \omega = y-s\delta-\delta_i) = g(s+1)f(y-s\delta-\delta_i)$$

where $f(\cdot)$ is the density of the noise traders distribution and

$$Prob\ [\omega+(n-1)\delta = y-\delta_i\ ] = \int_0^{\frac{y-\delta_i}{\delta}} g(t+1)f[y-t\delta-\delta_i]dt$$

Therefore, we have

$$Prob\ [\ n-1 \geq \frac{0.5}{\delta\gamma} - \frac{\gamma_i\delta_i}{\gamma\delta},\ \omega+(n-1)\delta = y-\delta_i\ ] = \int_{\frac{0.5}{\delta\gamma}-\frac{\gamma_i\delta_i}{\gamma\delta}}^{\frac{y-\delta_i}{\delta}} g(s+1)f[y-\delta_i-s\delta]ds \quad (1)$$

and consequently, given that noise traders are uniformly distributed,

$$\tau^a{}_i = \frac{\int_{\frac{0.5}{\delta\gamma}-\frac{\gamma_i\delta_i}{\gamma\delta}}^{\frac{y-\delta_i}{\delta}} g(s+1)ds}{\int_0^{\frac{y-\delta_i}{\delta}} g(t+1)dt} \quad (2)$$

Notice that if $\delta_i = \delta$ and $\gamma_i = \gamma$ (i.e. in a symmetric equilibrium),

$$\tau^a = \frac{\int_{\frac{0.5}{\delta\gamma}}^{\frac{y}{\delta}} g(s)ds}{\int_1^{\frac{y}{\delta}} g(t)dt} \tag{3}$$

3.2. An arbitrageur's Tendering Decision. If the takeover is successful, the average payoff per share for arbitrageur A_i is $P_T\gamma_i + (P_0 + \Delta P)(1 - \gamma_i)$; if the takeover fails, the price of his shares returns to P_0. Clearly, his problem is

$$MAX_{\gamma_i}\ \delta_i\ \{[P_T\gamma_i + (P_0 + \Delta P)(1 - \gamma_i)]\ \tau^a{}_i + P_0(1 - \tau^a{}_i)\} =$$

$$= \delta_i\ \{[\Delta P - \gamma_i(P_0 + \Delta P - P_T)]\ \tau^a{}_i + P_0\}.$$

Define

$$P_{\tau_i} \equiv [\Delta P - \gamma_i(P_0 + \Delta P - P_T)]\tau^a{}_i(\gamma_i). \tag{4}$$

Then A_i is actually maximizing P_{τ_i}. We can now characterize the reaction function of arbitrageur A_i holding a fraction δ_i of shares.

PROPOSITION 1: *For an arbitrageur who has bought δ_i shares, if $\gamma > \frac{0.5-\delta_i}{y-\delta_i}$, the reaction function $\gamma_i = \gamma_i(y, \gamma, \delta, \delta_i)$ is unique and non-increasing in y. Furthermore, there exist a $\underline{y}_i \geq 0.5$ and a $\bar{y}_i \leq 1$ such that*

$$\gamma_i = 1, \quad \forall\ y \leq \underline{y_i};$$

$$0 < \gamma_i < 1, \quad \textit{when}\ \underline{y_i} < y < \overline{y_i};$$

$$\gamma_i = 0, \quad \forall\ y \geq \overline{y_i}.$$

Proof: See Appendix I.

The condition $\gamma > \frac{0.5-\delta_i}{y-\delta_i}$ is equivalent to $\gamma(y - \delta_i) + \delta_i > 0.5$, which guarantees that there is a non-zero chance of takeover success. In fact, if the inequality is not satisfied it means that even when all trading volume $y - \delta_i$ is from arbitrageurs and i

tenders all its shares it is not enough for the takeover to succeed. Then i is indifferent to any choice of γ_i, because the takeover is doomed to fail. When the inequality is satisfied, the ith arbitrageur's tendering has non-zero marginal contribution to the success.

The characterization of the reaction function is intuitive. Given the portion of shares tendered by other arbitrageurs, when $y - \delta_i$ increases A_i will infer that there are more arbitrageurs and therefore will tender fewer shares in order to free ride.

It will be useful later to know how the total number of shares tendered changes with δ_i, so we give it in the following corollary.

COROLLARY 1: *If* $\gamma > \frac{0.5-\delta_i}{y-\delta_i}$, $\frac{\partial(\gamma_i\delta_i)}{\partial\delta_i} > 0$

Proof: See Appendix II.

In other words, if a risk arbitrageur holds more shares, the tendered fraction may decrease or increase, but the absolute number of shares tendered increases.

Since we are going to focus on the symmetric equilibrium, here we want to show the existence of a symmetric equilibrium in this last stage of the game, when $\delta_i = \delta$ for any i.

PROPOSITION 2: *Define* $\underline{\gamma} \equiv \frac{.5-\delta}{y-\delta}$. *When all arbitrageurs hold the same fraction of shares* δ, *for any given* y *there exists a symmetric Bayesian equilibrium* $\gamma = \gamma(y)$, *where* $\gamma(y)$ *is non-increasing in* y.

Furthermore, there exists a $\underline{y} > 0.5$ *such that*

$$\gamma = 1, \quad \forall\ y \leq \underline{y}$$

$$0 < \underline{\gamma} < \gamma < 1, \quad \textit{when } y > \underline{y}$$

Proof: See Appendix III.

There exist other symmetric Bayesian equilibria, where $\gamma_i = \gamma < \underline{\gamma}$. This is easy to see, since if all other arbitrageurs tender $\gamma < \frac{0.5-\delta}{y-\delta}$, then even if arbitrageur i tenders all of his shares, the takeover fails. Therefore, arbitrageur i might as well tender the same proportion γ. In these cases $\tau_i^a = 0$. We however focus on the more interesting symmetric equilibria where $\gamma \geq \underline{\gamma}$.

In Figure 1, we have simulated how the equilibrium γ changes as a function of y for some realistic values. We used the actual values of the Diamond Shamrock tender for Natomas (HBS 9-285-018), where we know arbitrageurs played an important role. The company has an initial value (P_0) of \$924.6 millions and the tender offer price is $P_T =$ \$1.4 billions. Moreover, we assume that the increase in value following the takeover (ΔP) is \$600 millions. We also set $\bar{\delta} = 5\%$ and assume that the number of potential risk arbitrageurs N is 60 (notice that at least 10 arbitrageurs must enter for the takeover to succeed).

4. The accumulation of shares

Given the symmetric equilibrium of the last subgame, in which risk arbitrageurs tender a positive fraction of their shares, we now look at how risk arbitrageurs buy shares. The players in this stage are the small shareholders—who own all the shares at the beginning of the game and may choose to sell them—the risk arbitrageurs and the noise traders, who buy shares. We want to show that although the price increases when arbitrageurs buy shares, they do succeed in hiding, at least partially, their presence, so that it is profitable for them to buy shares.

We use the concept of rational expectation equilibrium. For each realization of the random variables, n and ω, we characterize an equilibrium where each arbitrageur buys δ shares, the total trading volume is $y = n\delta + \omega$ and the share price is P. The equilibrium is such that (1) given the volume y, risk arbitrageurs and small shareholders are maximizing their utility; (2) the beliefs τ and τ_i^a are consistent with the players' strategies.

4.1. The Post-announcement Share Price and Updated Beliefs. The price is determined as follows. Since we are assuming that the small shareholders own 100% of the shares, if the total demand of shares (by risk arbitrageurs and noise traders) is less than 100%, then the market price P equals the reservation price of the small shareholders, so that each small shareholder is indifferent between selling the share and holding it (and waiting to see if the takeover takes place):

$$P = \tau(P_0 + \Delta P) + (1 - \tau)P_0 = P_0 + \tau\Delta P^{13} \tag{5}$$

where τ is the probability of success of the takeover bid, *as perceived by small shareholders.*

The reservation price V^a of a risk arbitrageur is given by

$$V^a = \tau_i^a[\gamma P_T + (1 - \gamma)(P_0 + \Delta P)] + (1 - \tau_i^a)P_0 \tag{6}$$

where τ_i^a is the success rate *calculated by the arbitrageur* A_i, who has bought δ_i shares. Note that if $\tau_i^a = \tau$ then $V^a < P$ and there is no trade. However, if τ_i^a is sufficiently larger than τ, then there is room for trade between arbitrageurs and small shareholders.

If instead the demand is above 100%, then P in (5) does not clear the market and the competition between risk arbitrageurs in order to obtain the shares will drive the price up to their reservation price.

Both τ and τ_i^a are endogenous and depend on the transaction volume y, which conveys new information about the number of arbitrageurs and their positions. In Section 3 we computed τ_i^a in (2) and τ^a for the symmetric case in (3). The updating of the small shareholders is different, since they may think that perhaps all of y is from noise traders. If we repeat the calculation we did for τ_i^a for the case of the small shareholders, the *posterior* probability of success of the takeover (conditional on y)

[13]Notice that this is exactly the expression used in Larcker and Lys (1987), in order to estimate the market-determined probability of success, i.e. τ.

is[14]

$$\tau = \frac{\int_{\frac{0.5}{\delta\gamma}}^{\frac{y}{\delta}} g(s)ds}{\int_0^{\frac{y}{\delta}} g(t)dt}. \tag{7}$$

Notice that, even if in general we cannot compare τ and τ_i^a, in the symmetric case it is easy to see that $\tau^a > \tau$. However, if one arbitrageur A_i declares $\bar{\delta}$, then $\tau = \tau_i^a$.[15]

Therefore, the probability of success of takeover as assessed by the risk arbitrageurs can be higher than the probability assessed by the small shareholders. If the difference is sufficiently high, $V^a(y) > P(y)$ and the arbitrageurs are willing to buy shares at the price P, which makes the small shareholders indifferent.[16]

4.2. The equilibrium. Since both τ and τ_i^a depend on y, in equilibrium the beliefs will have to be consistent with the strategies. To derive the equilibrium, we proceed in the following way: for given beliefs τ and τ^a, we derive the optimal choice of δ_i. Given this strategy, we then find the beliefs which are consistent in equilibrium.

In what follows, we start by proving that, given the beliefs, the optimal choice for a risk arbitrageur is always either to buy no shares at all, or to buy up to $\bar{\delta}$ shares. There always exists an equilibrium where arbitrageurs buy no shares at all and the takeover will never succeed (since one arbitrageur alone, by deviating and buying $\bar{\delta}$, cannot make the takeover succeed). We want to find out whether there exist an equilibrium where arbitrageurs buy $\bar{\delta}$ shares and the takeover has a positive

[14]Assuming no arbitrageur filed 13D.

[15]Even if the arbitrageur i is planning to buy more than $\bar{\delta}$ shares, the small shareholders are perfectly able to compute his optimal δ_i, once they know he is buying shares, and therefore there is no asymmetry of information.

[16]It should be pointed out that the model only catches one aspect of the informational advantage of the arbitrageurs over small shareholders. It is not difficult to extend the model to explicitly analyze major informational advantage of the arbitrageurs. One example is to model the risk that the takeover will not go through even though the tender offer per se is successful. This risk stems from legal fights (anti-trust suits are often involved) or the bidder's failing to secure financing for the takeover. Let p be this failure rate. The arbitrageurs have much better estimation of p than the small shareholders. Due to the existence of noise traders, the arbitrageurs knowledge of p is only partially reflected through their purchasing of shares and therefore the share price should be low enough for the arbitrageurs to make non-negative profits.

probability to be successful. In the next proposition we characterize all the symmetric equilibria. The intuition is immediately after.

PROPOSITION 3: *Given that n arbitrageurs entered the contest and the noise trade is ω, then*

a) *If $n\bar{\delta}+\omega < 0.5$, there is a unique equilibrium where risk arbitrageurs buy no shares and the trading volume is $y = \omega < 0.5$. The takeover fails.*

b) *If $0.5 \leq n\bar{\delta}+\omega \leq 1$, then in equilibrium risk arbitrageurs buy either $\bar{\delta}$ or 0 shares. If P_T is not too low, then at least for $y \leq \underline{y}$ and for values larger but close to $\underline{y}$ there exists an equilibrium where the risk arbitrageurs buy $\bar{\delta}$ shares and the trading volume is $y = n\bar{\delta}+\omega$. The proportion of shares γ tendered by each risk arbitrageur is equal to 1 for $y \leq \underline{y}$ and then decreases with y. The probability that the takeover is successful is strictly positive.*

c) *If $n\bar{\delta} + \omega > 1$ risk arbitrageurs buy no shares and the takeover fails.*

PROOF: See Appendix IV.

The intuition of the proposition is the following. If the trading volume is less than 50%, everybody knows that the takeover is going to fail and the share price is P_0. In equilibrium risk arbitrageurs are indifferent between buying and not buying shares at the price P_0. Since they are usually not interested in long term positions, we assume that in equilibrium they buy no shares at all. The expectation that the takeover will fail is therefore correct. This gives point (a) of the proposition.

If the trading volume is larger than 50%, the takeover could be successful. We have therefore to check that in equilibrium risk arbitrageurs are indeed buying shares. To find the equilibrium we first show that a risk arbitrageur always wants to buy either no shares at all or as many shares as possible without revealing his presence ($\bar{\delta}$). The intuition is quite simple: P_{τ_i} is the expected benefit from holding one share, while $\tau \Delta P$ is the actual cost (both in excess of P_0); since risk arbitrageurs are risk neutral, they will buy shares if and only if the expected benefit is higher than the cost. Moreover, a risk arbitrageur will never want to buy more than $\bar{\delta}$ shares: if he does, he will have to declare his transaction, $\tau = \tau_i^a$ and his expected profits become negative.[17] Therefore the arbitrageur will never buy more than $\bar{\delta}$ shares.[18]

Once we have restricted the choice to 0 or $\bar{\delta}$ shares, we can find out whether for some values of n and ω there exists an equilibrium where arbitrageurs buy $\bar{\delta}$ shares. If, when buying $\bar{\delta}$ shares, risk arbitrageurs obtain positive expected profits, this is an equilibrium. If instead the expected profits are negative, the risk arbitrageurs buy no shares at all.

When $0.5 \leq n\bar{\delta} + \omega \leq \underline{y}$, $\gamma = 1$ from Proposition 2 and we show in Appendix IV that profits are either negative or positive depending on P_T. For P_T sufficiently high (but bounded away from $P_0 + \Delta P$) the profits are positive and increasing and is therefore an equilibrium to buy $\bar{\delta}$.

As y increases, two effects happen. First of all, more arbitrageurs are likely to be in position and this promises a greater chance of success of the takeover. On the other hand, also the price at which arbitrageurs can buy shares increases. Moreover, γ decreases with y: each arbitrageur feels less pivotal for the success of the takeover and reacts by tendering less shares. In general, the net effect of an increase in y on the probability of success of the takeover can be either positive or negative. In the Appendix we show that the expected (interim) profits by buying $\bar{\delta}$ shares are in

[17] Holderness and Sheehan (1985) study the price reaction to announcements that some "corporate raiders" had acquired stock in a specific firm. They show that there are positive abnormal returns at the announcement that the investors bought stock in a firm which was target of a reorganization. Unfortunately, they do not distinguish between the case in which the investor was the acquirer and the case in which a third party was the acquirer (although both cases are in the sample).

[18] In the reality, there is a delay between when $\bar{\delta}$ shares are bought and when 13D is filed. Consequently, risk arbitrageurs—although in general try to avoid it—may buy more than 5%. In this event, Larcker and Lys (1987) show that arbitrageurs buy most of the shares at one date, after which they reveal themselves. This cannot happen in this model, where all trade is happening in a one-shot period. However, one may see the limit $\bar{\delta}$ as the amount of shares an arbitrageurs can buy in one day, disguising themselves among small trades. The 5% limit guarantees that after that day they will have to reveal themselves and therefore they will not buy any more shares. The fact that the amount of shares bought is not related with expected returns seems to confirm that there is an exogenous limit to the amount of shares an arbitrageur can buy without revealing his presence. The present model is therefore trying to capture these features without explicitly modelling trade over time.

general non-monotonic and could become negative. In Figure 2 we show one possible configuration of the (interim) expected profits of risk arbitrageurs if they bought $\bar{\delta}$ shares: if $y < 0.5$ the profits are 0. In the interval between 0.5 and $\underline{y}$ profits are positive and increasing (so this would be a case where P_T is sufficiently high). As y increases, profits first increase, but then begin to decrease and eventually become negative.[19] All the interval where profits are positive corresponds to values of n and ω for which there exists an equilibrium where risk arbitrageurs buy $\bar{\delta}$ shares. In Appendix IV we also show that if P_T is sufficiently high expected profits are always positive for the entire range $0.5 < y < 1$. This concludes case (b) of Proposition 3.

Finally, if $n\bar{\delta} + \omega > 1$, when risk arbitrageurs demand $\bar{\delta}$ shares, the demand exceeds supply and the price rises up to their reservation price. In equilibrium, risk arbitrageurs buy no shares and the takeover fails. This is case (c) of Proposition 3.

We have therefore shown that if P_T is not too low there exist trading volumes at which it is an equilibrium for the arbitrageurs to buy shares.[20]

5. The decision to enter

To complete the equilibrium, we still have to endogenize arbitrageurs' decision to enter. In the previous stage arbitrageurs always had the option to buy no shares. As a result, the expected profits of an arbitrageur who has chosen to enter are always non-negative. However, when the announcement of the takeover bid is made, some

[19]Because of the two effects we just described, we cannot exclude that profits might become positive again.

[20]Since arbitrageurs can only buy shares without being recognized up to 5%, we have considered this limit small enough for the risk arbitrageurs to be price takers and have therefore used the concept of noisy rational expectation equilibria. Alternatively, if one thinks arbitrageurs are not price takers, one could modify the trading game as in Kyle (1989), where risk arbitrageurs submit demand functions and therefore take into account the effect that their demand has on the price. As a result, arbitrageurs may buy less than $\bar{\delta}$ shares. Notice, however, that the incentive to buy less shares is lower than in Kyle (1984, 1989), since here, as δ_i decreases, also the informational advantage of the risk arbitrageur decreases. Finally, it is also possible to consider a model as in Kyle (1985) and Giammarino, Heinkel and Hollifield (1994), where risk arbitrageurs demand an amount δ_i independent from the volume and the market makers set the price at the reservation value of the small shareholders based upon total trading volume. In this last case, for some level of volume arbitrageurs' profits are negative, so it is not necessary to introduce a cost of entry in the next section.

of the arbitrageurs are engaged in other operations and their financial resources—including their bounded debt capacity—are tied up. Shleifer and Vishny (1997) show that arbitrageurs do not have unlimited capital they can invest and this is crucial in determining their strategy. An equivalent situation is depicted here, where they have to free some resources and this may be costly for them. This cost may also be interpreted as a lost opportunity to invest in a different deal, whose expected profits are strictly positive. Moreover, even before they start buying, arbitrageurs must collect some information, which is costly. We therefore assume that arbitrageurs decide whether to enter or not, where entry has a cost $c > 0$, which can be arbitrarily small.[21]

We focus on the symmetric equilibria. It is clear that the case in which no arbitrageur enters is always an equilibrium. In fact, one arbitrageur alone, who deviates and enters, can at most buy $\bar{\delta}$ shares without revealing his presence, and that is not enough for the takeover to succeed. We want to show, however, that there exists another equilibrium where the takeover can succeed. First of all, there cannot exist an equilibrium where all arbitrageurs enter with probability 1. In this case, the small shareholders would know that there are exactly N arbitrageurs buying with probability one. Therefore the arbitrageurs would have no information advantage at all and there would be no room for trade. The only other possibility for a symmetric equilibrium is therefore a mixed strategy equilibrium where each arbitrageur enters with a probability p which makes him indifferent between entering and not entering (i.e. such that the expected profits from entering are equal to c).

Given p, the probability that exactly n out of the N potential arbitrageurs entered is

$$g(n) = \left(\begin{array}{c} N \\ n \end{array} \right) p^n (1-p)^{N-n} \tag{8}$$

Each arbitrageur is able to forecast what is the equilibrium corresponding to each

[21] Alternatively, one could imagine that the trading game were not a one-shot but a dynamic game, with the arbitrageurs beginning to buy some shares and observing the volume over time. Then, when the arbitrageurs see that the volume is too high and decide to stop buying shares, they have already bought some at a price which is too high and therefore they realize a loss.

realization of ω and n. If the equilibrium implies they buy 0 shares, then entry implies a loss equal to c. The ex-ante expected profits can therefore be written as

$$\Pi(p, N, c) \equiv E_{n,\omega}\left[\pi(n, \omega)\right] \tag{9}$$

where $\pi(n, \omega)$ are the ex post profits for each realization of n and ω.

PROPOSITION 4: *If the cost c is not too high and N not too low, there always exists a symmetric equilibrium where each arbitrageur enters with probability p such that $0 < p < 1$ and the ex-ante expected profits in (9) are equal to 0. This is also an equilibrium of the general game.*

PROOF: See Appendix V.

The condition that c is not too high guarantees that ex-ante profits are not always negative and the condition on N guarantees that there are potentially enough risk arbitrageurs for the takeover to be successful.

We have therefore characterized the symmetric equilibrium of the entire game. In equilibrium, each of the N arbitrageurs randomizes between entering and not entering the contest with a probability p, where p is endogenously determined so that each risk arbitrageur is indifferent between entering and not entering. Out of the N potential arbitrageurs, n will enter the contest and invest in shares of the target company. Depending on the realization of n and of ω, arbitrageurs either buy no shares (and therefore bear a loss c) or buy $\bar{\delta}$ shares. In the first case the takeover will be for sure unsuccessful, while in the second case the risk arbitrageurs will tender a fraction γ of their shares and the takeover has a positive probability to be successful.

Note that, if we call S the ex-ante probability of success of the takeover, in equilibrium the ex-ante expected profits of the small shareholders is $S\Delta P + P_0$. The reason is that each of them is indifferent between selling and holding and, if they hold, they get $\Delta P + P_0$ if the takeover is successful and P_0 otherwise.

6. Extensions and Empirical implications

Now that we have characterized the equilibrium, we can derive several empirical implications. Before doing that, however, we have to make two assumptions. First of all, in Appendix IV, we show that, in general, τ^a and τ are a non-monotonic function of y. This makes it difficult to give empirical predictions. However, if we assume that risk arbitrageurs tender all their shares for any volume observed ($\gamma = 1$ for any y), then both τ and τ^a increase when y increases. Since in the reality risk arbitrageurs very often liquidate their entire position, this seems quite a realistic assumption. Therefore, in this section we will often assume that risk arbitrageurs liquidate their entire position in order to simplify the analysis and have clear cut empirical implications.

Second, in Appendix IV we showed that in general there may be two equilibria where arbitrageurs randomize between entering and not entering with two different p^*. From the point of view of characterizing the equilibrium, they are the same, therefore this was not an issue until now. However, in this section we will in some cases look at how the p^* of equilibrium changes as some parameters change. In this cases we choose to look only the equilibrium with the highest p^*, since it is more stable.

6.1. Volume and Price. If we look at the stage in which arbitrageurs buy shares and assume that $\gamma = 1$ for all y, then it is easy to see that the probability of success of the takeover increases with y and is equal to zero if $y < 50\%$.[22] Moreover, the higher is the volume the higher is the price of the shares in the market $(P_0 + \Delta P \tau)$.

We can also analyze the relationship between the number of arbitrageurs taking position (n) and the success rate of the takeover. First of all, for a given ω, the higher is n the higher is the trading volume. Therefore as n increases, also the expected volume increases and the takeover is more likely to be successful.

[22] This is due to the fact that we assumed that initially there are no large shareholders. If we relax this assumption, the takeover may succeed also when the trading volume is low.

6.2. Shares Liquidity. We assumed that noise trade volume was distributed uniformly on $[0, 1]$. In the reality, some companies' shares are traded more frequently than other ones. When shares are traded more often, risk arbitrageurs can hide their trade more easily. We can then look at the implications of a different level of liquidity.

Let us assume noise trading volume is distributed uniformly on the interval $[0, \bar{\omega}]$, with $0.5 < \bar{\omega} \leq 1$ and let us see how the equilibrium changes as $\bar{\omega}$ decreases. Using (1) and the fact that $f(y - s\delta) = \frac{1}{\bar{\omega}}$ if $y - s\delta \leq \bar{\omega}$ and 0 otherwise, we obtain

$$\tau = \frac{\int_{\frac{0.5}{\delta}}^{\frac{y}{\delta}} g(s)ds}{\int_{\frac{y-\bar{\omega}}{\delta}}^{\frac{y}{\delta}} g(t)dt}. \tag{10}$$

Moreover, if $y \geq \bar{\omega} + \delta$, τ^a is equal to τ while if $y < \bar{\omega} + \delta$

$$\tau^a = \frac{\int_{\frac{0.5}{\delta}}^{\frac{y}{\delta}} g(s)ds}{\int_{1}^{\frac{y}{\delta}} g(t)dt}. \tag{11}$$

The intuition is the following. If $y > \bar{\omega} + \delta$, everybody knows that there is at least one risk arbitrageur, therefore the arbitrageur's advantage disappears. If instead $y < \bar{\omega} + \delta$, the risk arbitrageur still has an advantage.[23] As the liquidity decreases, the risk arbitrageur can hide less well: in fact, the difference $\tau^a - \tau$ decreases as $\bar{\omega}$ decreases.

As a result, when the liquidity of the target stock decreases, the (interim) expected profits of the risk arbitrageurs decrease. However, we know that ex ante profits in equilibrium must be equal to 0, therefore the equilibrium p^* must change. In the following proposition we summarize the change of equilibrium in terms of the ex-ante probability of a takeover.

PROPOSITION 5: *If the difference in liquidity is sufficiently high, the probability that the takeover will be successful is higher the higher is the liquidity of the target shares.*

[23] Note that for $y > \bar{\omega}$ it is already clear that at least one risk arbitrageur is buying shares. However, the risk arbitrageur knows that he is in and this still constitutes an advantage.

PROOF: See Appendix VI.

We should therefore expect on average takeovers to be more successful the more liquid the target shares are. Alternatively, using what will be shown in the next section, we can expect the average takeover premium of successful takeover to be higher when the company shares are less liquid: in other words, the bidder must pay more to take over a less liquid target, because he can rely less on the help of risk arbitrageur.

6.3. The takeover premium. *Remark 1.* The higher is the takeover premium, the more shares are tendered by the risk arbitrageurs. In fact, when the takeover premium increases, the trade-off is less acute and the risk arbitrageurs tender more shares.

Proof: by implicit function theorem, $\frac{d\gamma}{dP_T} = \frac{\frac{d\phi}{dP_T}}{1+\frac{\partial q}{\partial \gamma}} > 0$, where ϕ is defined in (A1). Moreover, the definition of $\underline{y}$ is

$$\phi - 1 - \frac{\int_{\frac{0.5}{\delta}}^{\frac{\underline{y}}{\delta}} g(s)ds}{g(\frac{0.5}{\delta})} = 0$$

since the last term is increasing in $\underline{y}$, it is clear that an increase in ϕ caused by an increase in P_T does not decrease $\underline{y}$. Therefore the range of y for which $\gamma = 1$ becomes larger.

Remark 2. For any given y, the higher the takeover premium, the higher the probability of success of the takeover and the trading price of the shares P.

Proof: this is easily checked by seeing that $\frac{d\tau}{dP_T} = \frac{\frac{0.5}{\delta\gamma^2} g(\frac{0.5}{\delta\gamma})}{\int_0^{\frac{y}{\delta}} g(s)ds} \frac{d\gamma}{dP_T} > 0$

Remark 3. Assume risk arbitrageurs always liquidate their entire position ($\gamma = 1$). Then, the (interim) expected profits of the risk arbitrageurs increase with P_T.

Proof: the effect of P_T on the risk arbitrageurs' profits for any given y is positive for two reasons. First of all, we showed in Appendix IV that there are situations in which the equilibrium will imply that risk arbitrageurs buy no shares at all (and therefore have 0 profits) if P_T is low and instead they buy shares and earn positive profits if P_T is sufficiently high. Moreover, $\frac{d\pi}{dP_T} = \tau^a > 0$.

As a consequence, we have also the following remark:

Remark 4 If the risk arbitrageurs liquidate their entire position, the expected number of risk arbitrageurs is higher the higher is P_T.

Proof: Starting from an equilibrium, when P_T increases ex-ante expected profits become positive. Since in equilibrium ex-ante expected profits must be equal to zero, the risk arbitrageurs must randomize between entering or not with a higher probability p. As a consequence, the expected number of risk arbitrageurs buying shares is higher.

6.4. The bidder's toehold. Until now we assumed that the bidder owned no shares at all. It has been argued that if the bidder does own shares, then the takeover has a higher probability to succeed.[24] However, most of the times bidders do not try to buy shares in the market before the tender offer or they buy less than they could.[25] Chowdhry and Jegadeesh (1994) and Bris (1997) show that when there is asymmetry of information, the bidder may have an incentive not to buy shares before the announcement since this would convey information about the improvement in the value of the firm, following the takeover, and would therefore increase the takeover premium. The framework of this paper can be used to find an alternative and complementary explanation.[26]

The presence of arbitrageurs increases the probability that the takeover is successful and therefore the bidder would like to encourage risk arbitrageurs to enter. If everybody knows that the bidder owns some shares (since the toehold must be declared at the takeover announcement), then the share price increases and the expected (interim) profits may decrease. As a result, fewer arbitrageurs enter the contest. The overall effect on the probability of success of the takeover may be negative.

[24] See, for example, Holmstrom and Nalebuff (1992), Burkart (1995), Bulow, Huang and Klemperer (1996) and Singh (1998).

[25] See Jarrell and Poulsen (1989) and Hirshleifer (1995).

[26] Qian (1997) provides an alternative explanation based on executive compensation and managers' risk aversion.

The bidder chooses how many shares α to buy before making the tender offer. Let us assume for simplicity that he will be able to buy these shares at the price P_0. At the announcement of the tender offer, the bidder discloses α. The bidder chooses α in order to maximize

$$\mathrm{MAX}_{\alpha}(P_0 + \Delta P - P_T)E\{\delta n \mid \alpha\} + \alpha \Delta P \Pr\{\delta n > 0.5 - \alpha \mid \alpha\} \tag{12}$$

where $E\{\delta n \mid P_T\}$ is the expected number of tendered shares times the probability that it is greater than $50\% - \alpha$. The maximization in (12) can be rewritten as

$$\mathrm{MAX}_{\alpha}(P_0+\Delta P-P_T)\bar{\delta}\int_{(0.5-\alpha)}^{\bar{y}}\int_{\frac{(0.5-\alpha)}{\delta}}^{\frac{y}{\delta}} sg(s \mid \alpha)dsdy+\Delta P\alpha\int_{(0.5-\alpha)}^{\bar{y}}\int_{\frac{(0.5-\alpha)}{\delta}}^{\frac{y}{\delta}} g(s \mid \alpha)dsdy$$

When α increases, it is easier for the bidder to reach the majority (in other words, $.5 - \alpha$ decreases). However, there is also an indirect effect. In fact, both τ^a and τ change to take α into account. The effect on profits is ambiguous and depends on the parameters.[27]

Although we cannot give a unique answer in this general model, what is important is that arbitrageurs' interim profits may decrease as a result of an increase in α. The equilibrium p^* should then decrease, implying that an increase in α may discourage arbitrageurs from entering (i.e. $G(s \mid \alpha)$ changes in the sense of first order stochastic dominance).

6.5. Differences in risk preferences. Risk arbitrageurs may have an advantage since, being better diversified than small shareholders, they are less risk averse. To take into account also this aspect, we can modify the previous set-up by assuming that risk arbitrageurs are still risk neutral, but small shareholders are risk averse. Then the

[27] There is an additional effect that for simplicity is not included in (12): if the bidder has a toehold, the shares which can be traded are only $1 - \alpha$: this is equivalent to a lower liquidity, which decreases the arbitrageurs profits, as we showed in Section 6.2.

reservation price of the small shareholders is

$$P < P_0 + \tau \Delta P.$$

Let us first assume that the difference in risk preferences is the only advantage that risk arbitrageurs have (i.e. $\tau_i^a = \tau$), then the reservation price is

$$P < V^a = \tau \left[\gamma P_T + (1 - \gamma)(P_0 + \Delta P) \right] < P_0 + \tau \Delta P.$$

Each risk arbitrageur has no incentive not to reveal himself, therefore he will demand as many shares as possible and this will raise the price up to V^a.[28] The expected profit of a risk arbitrageur will then always be equal to $-c$ and he will never enter.

If instead we assume that risk arbitrageurs have an informational advantage (i.e. they know their own presence) and introduce risk aversion of the small shareholders, the same results go through but the expected (interim) profits of risk arbitrageurs are higher. This implies that the equilibrium p will be higher and therefore on average there will be more risk arbitrageurs entering the contest.

The conclusion is therefore that differences in risk preferences are important in determining the expected (interim) profits of the risk arbitrageurs, and therefore how many of them will take positions, but taken alone they cannot explain why the risk arbitrageurs' demand is not so high to raise the price up to V^a. The asymmetry of information of our model guarantees that the risk arbitrageurs will not want to buy too many shares, in order not to reveal themselves.

7. Conclusions

We have provided an explanation of why arbitrageurs have an incentive to enter the market of corporate control and why in so doing they do not drive the price up until the returns are 0 (which would discourage them from bearing the cost of entry).

[28] Assuming that risk arbitrageurs are still price takers.

We have also characterized the equilibrium in which arbitrageurs enter, buy shares and tender a fraction of them. Such characterization has allowed us to derive some relationships that link the trading volume and the number of arbitrageurs buying shares to the success rate of the takeover and the market price. Moreover, the role of the arbitrageurs in determining the success of the takeover influences the acquirer's choice of the takeover premium and of the initial toehold.

The model can be extended to take into account other characteristics. One possibility would be to allow noise traders and risk arbitrageurs to short sales. Risk arbitrageurs may have an incentive to short sell if their assessment of the probability that the takeover will be successful is lower than the assessment of the small shareholders. When short sales are allowed, cases where we found that arbitrageurs do buy shares remain unchaged as equilibrum choice. However, in the cases where we found it was optimal for arbitrageurs not to buy shares, it may become optimal to short sell shares. Therefore, though our result that arbitrageurs do buy shares in equilibrium would still hold, the introduction of short sales allows for a richer and more complex behavior.

Another interesting extension would be to allow risk arbitrageurs to communicate between themselves. Usually risk arbitrageurs belong to small "clubs" and they talk only to arbitrageurs in the same club. In this case they will be informed not only of their own presence, but also of the presence of everybody in the same club. We could model this behavior by assuming that risk arbitrageurs in the same club commit, before randomizing, to inform each other if they choose to enter the contest. In general, it will be incentive compatible for a risk arbitrageur who entered to tell the truth. If the agreement is among two people only, their ex-ante expected profits increase, so that they will choose to enter more often. However, as the number of members of each club increases, the expected number of arbitrageurs entering increases, until it starts having an adverse effect. As a result, there is an optimal size of the club. An exhaustive treatment of this aspect is however beyond the scope of this paper.

Finally, this paper allows to study the role of the legal limit to the number of shares an arbitrageur can buy without disclosing his presence (such as the 5% limit according to Section 13D of the Security Exchange Act). This limit is usually interpreted as an obstacle to takeovers, while we show that it can actually favor takeovers. In fact, it reduces competition among risk arbitrageurs. It would therefore be interesting to study how different limits across countries influence the takeover activity.

APPENDIX

Appendix I. Proof of Proposition 1

Let us proceed with the maximization of P_{τ_i} with respect to γ_i. From (4) the first order condition is

$$\frac{\partial P_{\tau_i}}{\partial \gamma_i} = -(P_0 + \Delta P - P_T){\tau^a}_i + [\Delta P - \gamma_i(P_0 + \Delta P - P_T)]\frac{\partial {\tau^a}_i}{\partial \gamma_i}$$

$$= (P_0 + \Delta P - P_T)\ [\frac{\Delta P}{P_0 + \Delta P - P_T} - \gamma_i - \frac{{\tau^a}_i}{\frac{\partial {\tau^a}_i}{\partial \gamma_i}}\ ]\ \frac{\partial {\tau^a}_i}{\partial \gamma_i}.$$

Define

$$\phi \equiv \frac{\Delta P}{P_0 + \Delta P - P_T} > 1 \tag{A1}$$

$$q_i = q_i(y, \gamma, \gamma_i, \delta, \delta_i) \equiv \frac{{\tau^a}_i}{\frac{\partial {\tau^a}_i}{\partial \gamma_i}}$$

and

$$\Psi_i \equiv \phi - \gamma_i - q_i(y, \gamma, \gamma_i, \delta, \delta_i). \tag{A2}$$

From (2), the partial derivative of ${\tau^a}_i$ with respect to γ_i is:

$$\frac{\partial {\tau^a}_i}{\partial \gamma_i} = \frac{\delta_i}{\gamma\delta}\frac{g(\frac{0.5}{\delta\gamma} - \frac{\gamma_i\delta_i}{\gamma\delta} + 1)}{\int_0^{\frac{y-\delta_i}{\delta}} g(t+1)dt} \geq 0$$

Notice that with the assumption that $\gamma > \frac{0.5-\delta_i}{y-\delta}$ and $g(\cdot) > 0$, there is a chance for i to make the takeover successful. Mathematically, with this assumption $\frac{\partial \tau^a_i}{\partial \gamma_i}$ is positive and

$$q_i(y, \gamma, \gamma_i, \delta, \delta_i) = \frac{\gamma\delta}{\delta_i}\frac{\int_{\frac{0.5-\gamma_i\delta_i}{\delta\gamma}}^{\frac{y-\delta_i}{\delta}} g(s+1)ds}{g(\frac{0.5-\gamma_i\delta_i}{\gamma\delta} + 1)}. \tag{A3}$$

The reaction function $\gamma_i = \gamma_i(\gamma)$ is such that:

$$\Psi_i(y, \gamma, 0, \delta, \delta_i) \leq 0 \quad if\ \ \gamma_i(\gamma) = 0;$$

$$\Psi_i(y,\gamma,\gamma_i,\delta,\delta_i) = 0 \quad if\ 0 < \gamma_i(\gamma) < 1;$$

$$\Psi_i(y,\gamma,1,\delta,\delta_i) \geq 0 \quad if\ \gamma_i(\gamma) = 1.$$

Let us focus on the case in which $\Psi_i(y,\gamma,\gamma_i,\delta,\delta_i) = 0$. To check the second order condition, notice that $\frac{\partial \Psi_i}{\partial \gamma_i} = -1 - \frac{\partial q_i}{\partial \gamma_i} < 0$ iff $\frac{\partial q_i}{\partial \gamma_i} > -1$. This condition can be rewritten as

$$g'(\frac{0.5}{\delta\gamma} - \frac{\gamma_i\delta_i}{\gamma\delta} + 1)\int_{\frac{0.5}{\delta\gamma}-\frac{\gamma_i\delta_i}{\gamma\delta}}^{\frac{y-\delta_i}{\delta}} g(s+1)ds > -2[g(\frac{0.5}{\delta\gamma} - \frac{\gamma_i\delta_i}{\gamma\delta} + 1)]^2 \tag{A4}$$

which is satisfied if $G(n)$ has a monotone increasing hazard rate. For the moment, we assume that $G(n)$ has a monotone increasing hazard rate. When we endogenize $G(n)$, we will check that this is indeed true. If (A4) is satisfied, the existence and uniqueness of $\gamma_i(\gamma)$ is guaranteed.

As for the non-increasingness of γ_i in y, notice that if $\delta_i \neq 0$

$$\frac{\partial q_i}{\partial y} = \frac{\gamma}{\delta_i}\frac{g(\frac{y-\delta_i}{\delta}+1)}{g(\frac{0.5}{\delta\gamma} - \frac{\gamma_i\delta_i}{\gamma\delta}+1)} > 0,$$

which, by implicit function theorem, implies $\frac{d\gamma_i}{dy} < 0$.

If we define $\underline{y}_i \equiv max\ \{\ 0.5,\ \ y_1 : \Psi_i(y_1,\gamma,1,\delta,\delta_i) = 0\ \}$ and $\overline{y_i} \equiv min\ \{1, y_2 : \Psi_i(y_2,\gamma,0,\delta,\delta_i) = 0\ \}$, then if $y \leq \underline{y_i}$, $\forall \gamma_i < 1$,

$$\Psi_i(y,\gamma,\gamma_i,\delta,\delta_i) > \Psi_i(\underline{y},\gamma,1,\delta,\delta_i) = 0.$$

Recall that Ψ_i shares its sign with $\frac{\partial P_{\tau_i}}{\partial \gamma_i}$. Therefore, the best reaction to any γ is $\gamma_i = 1$ when $y \leq \underline{y}$.

The proof for the other cases are similar, and hence omitted here.

¶Q.E.D.

Appendix II. Proof of Corollary 1

$$\frac{\partial(\gamma_i\delta_i)}{\partial\delta_i} = \gamma_i\left[1+\frac{\partial\gamma_i}{\partial\delta_i}\frac{\delta_i}{\gamma_i}\right]$$

therefore $\frac{\partial\gamma_i\delta_i}{\partial\delta_i}$ is positive if the elasticity of γ_i with respect to δ_i is larger than -1. By implicit function theorem

$$\frac{d\gamma_i}{d\delta_i} = -\frac{\frac{dq_i}{d\delta_i}}{1+\frac{dq_i}{d\gamma_i}}$$

where q_i is given in (A3). It is possible to compute that

$$\frac{\partial\gamma_i}{\partial\delta_i}\frac{\delta_i}{\gamma_i} = -\frac{[g(a+1)]^2+g'(a+1)\int_a^b g(s+1)ds}{2[g(a+1)]^2+g'(a+1)\int_a^b g(s+1)ds}+\frac{\gamma\delta}{\gamma_i\delta_i}\frac{g(a+1)\int_a^b g(s+1)ds}{2\,[g(a+1)]^2+g'(a+1)\int_a^b g(s+1)ds}$$

where the first term is strictly less than 1 in absolute value and the second term is strictly positive since the second order condition imply that $2[g(a+1)]^2+g'(a+1)\int_a^b g(s+1)ds>0$ therefore, the elasticity is larger than -1.

¶Q.E.D.

Appendix III. Proof of Proposition 2

At a symmetric Bayesian equilibrium, it must be true that $\gamma_i(\gamma)=\gamma$. Define

$$\Psi(y,\gamma,\delta)\equiv\Psi_i(y,\gamma,\gamma,\delta,\delta)=\phi-\gamma-q(y,\gamma,\delta) \tag{A5}$$

where $q(y,\gamma,\delta)$ is defined as

$$q(y,\gamma,\delta)=\gamma\frac{\int_{\frac{0.5}{\delta\gamma}}^{\frac{y}{\delta}} g(s)ds}{g(\frac{0.5}{\delta\gamma})}. \tag{A6}$$

and $\frac{\partial q}{\partial y}>0$. If we define $\underline{y}\equiv Max\{y_1:\Psi(y_1,1,\delta)\geq 0\}$, notice that from the equation:

$$\phi-1-\frac{\int_{\frac{0.5}{\delta}}^{\frac{y_1}{\delta}} g(t)dt}{g(\frac{0.5}{\delta})}\geq 0$$

it is clear that $y_1 > 0.5$ (since $\phi > 1$) and so $\underline{y} > 0.5$. By the definition given by (A5)

$$\Psi(y, \gamma, \delta) = \phi - \gamma - q(y, \gamma, \delta)$$

where $\frac{\partial \Psi}{\partial y} < 0$ and $\frac{\partial \Psi}{\partial \gamma} < 0$.

Moreover, since $\phi > \gamma$, (A5) implies that $\frac{y}{\delta} > \frac{0.5}{\delta\gamma}$, which implies that the minimum gamma $\underline{\gamma} > 0$. When γ goes from $\underline{\gamma}$ to 1, Ψ decreases continuously. If $\Psi(y, 1, \delta) < 0$, then there is a unique γ such that $\Psi(y, \gamma, \delta) = 0$. If $\Psi(y, 1, \delta) \geq 0$, then $\gamma = 1$ is the equilibrium. Thus, we have the existence and uniqueness of the equilibrium.

As for the non-increasingness of the equilibrium, by implicit function theorem,

$$\frac{d\gamma}{dy} = \frac{-q'_y}{1 + q'_\gamma} < 0.$$

¶Q.E.D.

Appendix IV. Proof of Proposition 3

The arbitrageur chooses δ_i in order to maximize

$$MAX_{\delta_i} \delta_i \{[\Delta P - \gamma_i (P_0 + \Delta P - P_T)] \ \tau^a{}_i + P_0 - P_1\} \tag{A7}$$

If the total volume $y \leq 1$, the objective function becomes

$$MAX_{\delta_i} \delta_i \left[P_{\tau_i} - \Delta P \tau\right] \tag{A8}$$

the first order conditions are

$$P_{\tau_i} - \Delta P \tau + \delta_i \frac{\partial P_{\tau_i}}{\partial \delta_i} \tag{A9}$$

where by envelope theorem

$$\frac{dP_{\tau_i}}{d\delta_i} = \frac{\partial \tau_i^a}{\partial \delta_i} [\Delta P - \gamma_i (P_0 + \Delta P - P_T)] > 0$$

If $\delta_i = \delta \leq \bar{\delta}$ (i.e. the risk arbitrageur A_i buys as many shares as the others), then

$\tau_i^a = \tau^a > \tau$. Moreover, $P_{\tau_i} - \Delta P \tau$ is increasing in δ_i. Therefore, given τ_i^a and τ, define $\hat{\delta}_i$ such that $P_{\tau_i} = \Delta P \tau$ (if such $\hat{\delta}_i > 0$ exists). Then for all $\delta_i < \hat{\delta}_i$ the objective function is negative and the optimum is $\delta_i = 0$. For all $\delta_i \geq \hat{\delta}_i$, the first order conditions are strictly positive. Therefore, the solution is always a corner solution: the arbitrageur wants to buy either no shares at all or as many shares as possible.

Assume now the risk arbitrageur buys more than $\bar{\delta}$ shares. He should then declare his transaction and the price will become $P_0 + \Delta P \tau_i^a$. Therefore the arbitrageur will buy either $\bar{\delta}$ or 0 shares.

Let us check if it is ever an equilibrium to buy $\bar{\delta}$ shares. For each n and ω, if the arbitrageurs buy $\bar{\delta}$ shares, the volume is $y = n\bar{\delta} + \omega$ and τ and τ^a depend on such a y. We have therefore to see if the profits from buying $\bar{\delta}$ shares are positive or negative. First of all, if $y < 0.5$, then $\tau_i^a = \tau = 0$. The share price is P_0 and the risk arbitrageurs are indifferent between buying and not buying shares. This gives us case (a) of Proposition 3.

Let us now consider the case with $1 \geq n\bar{\delta} + \omega \geq .5$. If the risk arbitrageur bought $\bar{\delta}$ shares,

$$\tau^a(y) = Pr\{n > \frac{.5}{\gamma\bar{\delta}} \mid y - \bar{\delta}\} = \frac{\int_{\frac{0.5}{\bar{\delta}\gamma}}^{\frac{y}{\bar{\delta}}} g(s)ds}{\int_1^{\frac{y}{\bar{\delta}}} g(t)dt} \tag{A10}$$

Therefore, the expected (interim) profits of arbitrageur A_i are

$$\pi(y) \equiv \bar{\delta}\,[P_{\tau_i} - \Delta P \tau] \tag{A11}$$

Moreover

$$\frac{d\pi}{dy} = \frac{\partial \pi}{\partial y} + \frac{\partial \pi}{\partial \gamma}\frac{d\gamma}{dy}$$

Let us consider the two parts separately. By envelope theorem,

$$\frac{\partial \pi}{\partial \gamma} = \frac{d\tau^a}{d\gamma_{-i}}[\Delta P - \gamma_i(P_0 + \Delta P - P_T)] - \frac{d\tau}{d\gamma}\Delta P \tag{A12}$$

and

$$\frac{\partial \pi}{\partial y} = \frac{\partial \tau^a}{\partial y}[\Delta P - \gamma_i(P_0 + \Delta P - P_T)] - \frac{\partial \tau}{\partial y}\Delta P \tag{A13}$$

where $\frac{\partial \tau^a}{\partial y} > 0$ and $\frac{\partial \tau}{\partial y} > 0$. However, $\frac{d\tau^a}{dy}$ and $\frac{d\tau}{dy}$ can be both positive or negative and the profits can decrease or increase as y increase. Let us first look at the interval $0.5 \leq n\bar{\delta}+\omega \leq \underline{y}$. We know that in that interval $\pi(0.5, \bar{\delta}) = 0$ and $\gamma = 1$. Therefore,

$$\frac{d\pi}{dy} = (P_T - P_0)\frac{\partial \tau^a}{\partial y} - \Delta P \frac{\partial \tau}{\partial y}.$$

$\frac{\partial \tau^a}{\partial y} > \frac{\partial \tau}{\partial y}$ if and only if $\left[G(\frac{y}{\bar{\delta}})\right]^2 < G(\frac{05}{\bar{\delta}})\left[2 - G(1)\right]$. It is easy to see that this is always satisfied if y is close to 0.5. Therefore, if P_T is sufficiently high, profits are positive and increasing: in this case it is an equilibrium to buy $\bar{\delta}$ shares for any values of y in $[0.5, \underline{y}]$. Moreover, by continuity, it still is an equilibrium to buy $\bar{\delta}$ shares for at least a range of y larger but close to $\underline{y}$. If instead P_T is low, profits are negative and decreasing.

For higher level of volumes, however, the profits can be a non-monotonic function of y. However, profits equal to 0 imply that $\gamma(y)G(\frac{y}{\bar{\delta}}) = \phi G(1)$. The LHS is always less than 1 and as $P_T \to P_0 + \Delta P$, $\phi \to \infty$. Therefore, if $G(1)$ is not infinitesimal (which we can check when we endogenize $G(n)$), and P_T is sufficiently high, the profits are always positive. Therefore, if P_T is not too low there exists values of n and ω (and therefore y) for which it is an equilibrium to buy $\bar{\delta}$ shares.

The last thing we want to show is that when it is not an equilibrium to buy $\bar{\delta}$ shares, the only symmetric pure strategy equilibrium is the one in which risk arbitrageurs buy 0 shares. Let us assume that each risk arbitrageur buys a quantity $\delta_0 \leq \bar{\delta}$ (while $\frac{\partial \pi}{\partial \delta_i} > 0$, the sign of $\frac{\partial \pi}{\partial \delta}$ is ambiguous, so it could be that if the reduction is sufficiently large the profits become non negative). However, then for given beliefs $\tau(\delta_0)$ a single risk arbitrageurs always has an incentive to deviate and increase his number of shares up to $\bar{\delta}$. Therefore δ_0 cannot be an equilibrium. This gives us case (b) of Proposition 3.

To see case (c) notice that if $n\bar{\delta} + \omega > 1$ the price should increase up to $P_0 + \tau^a \Delta P$. However, at that price the risk arbitrageurs are indifferent between buying and not buying shares.

¶Q.E.D.

Appendix V. Proof of Proposition 4

For each (n, ω), we can compute the ex-post profits $\pi(n, \omega)$. Define two sets: $Y^+ \equiv \{(n, \omega) : \pi(y = n\bar{\delta} + \omega) > 0\}$ and $Y^- \equiv \{(n, \omega) : \pi(y = n\bar{\delta} + \omega) \leq 0\}$. In other words, Y^+

is the set of all the (n, ω) such that in equilibrium arbitrageurs buy $\bar{\delta}$ shares and Y^- is its complement. If $(n, \omega) \in Y^-$, then $\pi(n, \omega) = -c$. If instead $(n, \omega) \in Y^+$ and $n\bar{\delta} \geq .5$ then $\pi(n, \omega) = \Delta P(1-\tau) - \gamma(n\bar{\delta} + \omega)(\Delta P + P_0 - P_T) - c$. Finally, if $(n, \omega) \in Y^+$ and $n\bar{\delta} < .5$ then $\pi(n, \omega) = -\Delta P\tau - c$.

The ex-ante expected profits are given by

$$\Pi(p, N, c) \equiv E_{n,\omega}\left[\pi(n, \omega)\right] = E_n\left[E_\omega\left[\pi(n, \omega)\right]\right] \tag{A14}$$

Let us define a distribution of n, $\hat{g}(n)$, such that $\hat{g}(n+1) = g(n)$, where $g(n)$ is the binomial defined in (8). Suppose $N-1$ arbitrageurs randomize their entry decision with probability p and let us consider the entry decision of the N-th arbitrageur. If he decides not to enter, the expected payoff is 0. Suppose he decides to enter with probability 1 and let us analyze his expected payoff. Define $\bar{\pi}(n) \equiv E_\omega\left[\pi(n, \omega) \mid n\right]$ then his expected payoff is

$$E_n\left[\bar{\pi}(n)\right] = \sum_{n=0}^{N}\left[\bar{\pi}(n)\hat{g}(n)\right],$$

where $\hat{g}(n)$ is the distribution of the number of arbitrageurs in the game taking into account the decision of the last arbitrageur.

It is easy to see that if all other arbitrageurs randomize with probability $p = 0$, then $E_n\left[\bar{\pi}(n)\right] < 0$, since one arbitrageur alone is not enough for the takeover to succeed. On the other hand, we know that if P_T is not too low, there exists some y for which $\pi(y) > 0$. Since $E_n\left[\bar{\pi}(n)\right]$ is continuous in p, as p increases the ex-ante expected profits increase. Therefore, if c is small enough and N is large enough, there exists a p such that $E_n\left[\bar{\pi}(n)\right] > 0$. By continuity, there exists at least one p such that $E_n\left[\bar{\pi}(n)\right] = 0$. When the other arbitrageurs randomize with this probability p, the last arbitrageur is indifferent between entering and not and then he might as well randomize between the two with probability p. Such a p is therefore an equilibrium of this game. As N increases, the profits can become negative again and in that case there is another equilibrium p at which ex ante profits are equal to 0.

Finally, notice that we still have to check condition (A4) which guarantees that the second order conditions in the tendering game are satisfied. We showed in Appendix I that this condition is automatically satisfied if $G(n)$ has a monotonic increasing hazard rate. We just showed that $G(n)$ is a binomial, which, for n continuous, is always approximated by a normal, which has a monotonic increasing hazard rate. The condition is therefore satisfied and p is an equilibrium of the entire game.

¶Q.E.D.

Appendix VI. Proof of Proposition 5

From the text, we know that when $\bar{\omega}$ decreases the interval in which the profits could be positive $[0.5, \bar{\omega} + \bar{\delta}]$ shrinks. Moreover, in such interval, τ^a is not changing, while τ is increasing. As a result, ex-ante expected profits decrease and become negative. In equilibrium, therefore, p^* should decrease. When p^* decreases, $E(n)$ decreases, which reduces the ex-ante probability of success of the takeover. However, it could happen that $E(\delta)$ increases (risk arbitrageurs buy shares more often) which has the opposite effect. We could not determine the overall effect in general. However, if the reduction in $\bar{\omega}$ is sufficiently high, it is easy to see that the interval reduction is dominating and $E(\delta)$ is also decreasing. As a result, the ex-ante probability of success of the takeover is lower.

¶Q.E.D.

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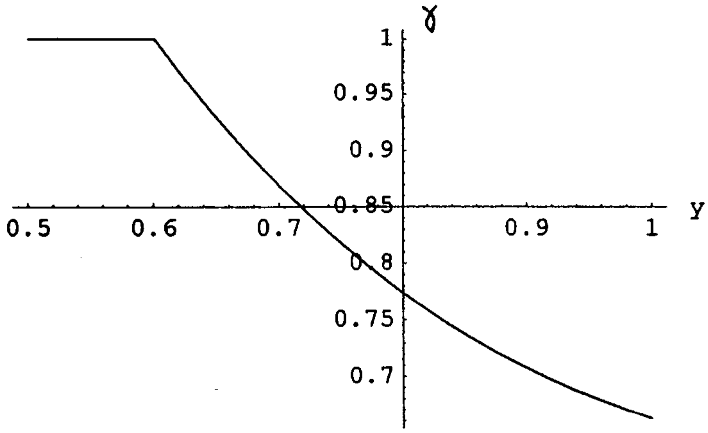

Figure 1: The Equilibrium Tendering Strategy

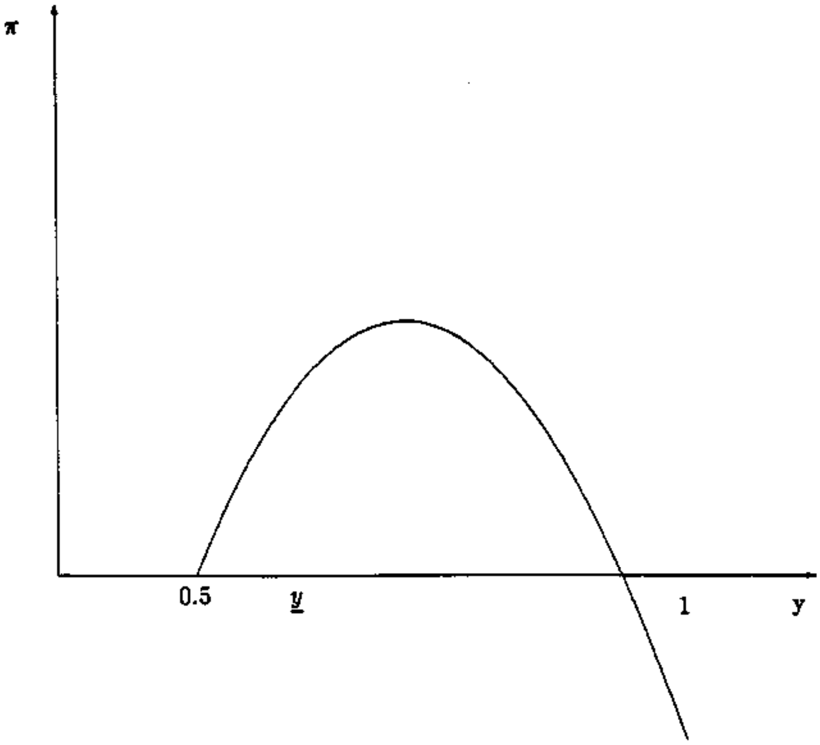

Figure 2: Expected Interim Profits