Executive Compensation & the Boundary of the Firm: The Case of Short-Lived Projects

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We consider the problem of moral hazard in the team of managers employed in a firm when the principal/firm owner can play an active role in determining team output. Unless the principal's compensation is non-decreasing in firm value there is an additional moral hazard problem since the principal will have an incentive to reduce output. In this context we determine team, and hence firm, size and solve for the form of optimal managerial compensation contracts. In particular we determine conditions under which optimal contracts will have option-like features.

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The theory of the firm examines why projects are combined and the limits to such integration. In their survey of the theory of the firm Holmstrom and Tirole (1989, p.66) note that "[w]hile it is relatively easy to envision reasons for integration, it is substantially harder to articulate costs of increased size." We examine a setting where adding a project to the firm requires hiring an additional manager. We show how increasing agency problems provide a natural boundary on firm size.[1] Determining optimal firm size requires simultaneously solving for optimal managerial compensation contracts. In doing so, we establish conditions under which compensation will optimally involve options as distinct from equity-like compensation.[2]

We formally embed the principal-agent problem in a firm, where a firm is defined by three characteristics. First, a firm may have assets-in-place, in which case a manager can free-ride on existing assets. Second, a firm may have more than one manager, in which case a manager can free-ride on her colleagues' efforts. These first two characteristics can mean that inducing effort in a firm setting is more difficult than in a setting where a manager is compensated only with claims on the value added by her own efforts. Third, the principal is an active participant in the firm.

This third characteristic is the most important for our results and distinguishes the model of this paper from that of Holmstrom (1982). Holmstrom focuses on the second characteristic noted above, namely the problem of contracting with multiple managers. For any given number of managers, first best effort levels are achievable if managers are promised a bonus in the event that firm value exceeds a level consistent with them all working. This outcome is the result of introducing a "principal" whose sole purpose is to be the residual claimant on any excess of firm value over and above the bonuses earned—unless all managers work, each will receive nothing and any output will go to the principal. The principal is a device allowing the managers to commit to not share output in the event that they do not all work, and the forfeiture of output to the

[1] For an quite different analysis of the relation between agency problems and organizational structure see Hermalin (1996).

[2] Thus we go part way in addressing Kole's (1997, p. 79) observation that "variation in compensation contracting challenges theorists to incorporate the richness of management contracts into models of incentive pay."

principal is the enforceable group penalty which implements first-best.

Holmstrom (1982, p. 328) observes that "it is important that the principal not provide any (unobservable) productive inputs or else a free-rider problem remains." In contrast to Holmstrom's principal, the principal in our paper is active and can provide non-verifiable productive inputs; in particular, negative inputs. The principal will have an incentive to reduce output whenever doing so would increase the value of his residual claim.

When the principal is purely an inactive budget-breaker as in Holmstrom's model, the issue of firm size never arises. When the principal is active, potential managers will be loath to accept contracts that give the principal the incentive to reduce output. The principal will not have this incentive if managerial contracts are such that the principal's payoff is non-decreasing in firm value. As will be seen, managerial contracting in the face of a potential moral hazard problem on the part of the principal constrains the number of managers who can be motivated to work, and hence underlies our results on firm size.

Our setting is otherwise quite simple: There is nonverifiability of a simple zero/one (shirk or work) effort choice on the part of each manager; there is no uncertainty concerning output given the effort choices; and the principal and managers are risk-neutral. The paper's title refers to "short-lived projects." The projects considered in this paper are short-lived in the sense that they can only be undertaken in the current period. The ability to defer long-lived projects and the resultant possibility that a manager may attempt to free-ride on the firm's future managers is considered in a companion paper: "Executive Compensation and the Boundary of the Firm: The Case of Long-Lived Projects."

The paper proceeds as follows. Section 1 presents the model. Section 2 analyzes a restricted version of the problem in which the number of managers is fixed, all managers are compensated with identical contracts involving wages, equity shares and options on equity shares, and the principal and managers possess limited liability. Section 3 solves the general problem of minimizing the cost of hiring and motivating a fixed number of managers. The optimal number of managers to hire, and hence optimal size, is the subject of Section 4. Section 5 analyzes the uniqueness of the equilibrium and highlights the link between managerial limited liability and uniqueness. Section 6 extends our results on firm size to the case of stochastic output and managerial risk aversion. Our conclusions are contained in Section 7.

1. The Model

Our firm consists of a combination of assets-in-place and m new investment opportunities. Each such project requires the labor services of a manager. The owner of the firm, the principal, supplies any capital required by the firm, chooses the number of projects, $n \leq m$, to actually be undertaken and must design compensation contracts to attract n managers, one for each project. Managers' effort choices are not verifiable. Firm size is measured by n, the number of projects undertaken.

1.1 The assumptions

Utility Functions and Opportunity Costs: The principal and the managers are risk-neutral. Each manager i's utility function is given by $C - e_i D$, where C is her lifetime consumption, e_i is her zero/one effort choice and D is the disutility of expending effort. A manager's reservation utility is R. At a minimum the manager must be compensated for her forgone opportunities, R, and for the disutility of effort, D. If a manager can be induced to work at a cost of $R + D$, then we say that her compensation is "least-cost."

The Production Technology: Conditional on knowing the zero/one effort choice of a given manager there is no uncertainty about her project's payoff. For simplicity, we assume that all projects have identical potential payoffs. If a manager doesn't make an effort, the associated project's payoff is zero. If effort is expended each project's payoff is Δ. The projects are each potentially positive NPV investments in that the payoff is greater than the minimum possible cost of hiring and motivating a manager; i.e., $\Delta > R + D$. The liquidation value of the firm's assets-in-place at the end of the period is $\mathcal{A} \geq 0$. The interest rate is zero.

The Contracting Technology: Since the managers' effort choices are not verifiable, contracts are limited to functions of firm value. Firm value itself is verifiable. Non-contractibility of effort implies that the cash flows from the combination of the assets-in-place and the projects undertaken can not be verifiably attributed to particular projects. Firm value depends on the total effort expended, $\sum_{j=1}^{n} e_j$, and is independent of the identity of the particular managers making an effort.[3] Managers may or may not have limited liability and we consider both possibilities.

An Active Principal: Our principal is assumed active in the sense that he can non-verifiably reduce output. In the extreme, such a principal could destroy output. More generally, an active principal has an incentive to secretly renegotiate with some subset of the managers whenever bribing this subset not to work would increase the value of his claim. The principal will not have an incentive to reduce output, so long as managerial compensation contracts are designed so that the residual payoff to the principal is non-decreasing in firm value.

1.2 The principal's problem for fixed n

The principal must design a set of contracts which will be offered to the potential managers. Each manager who agrees to participate then makes an effort choice given her conjecture concerning the effort choices of the other managers. We solve this problem as a mechanism design problem in which the principal optimally chooses contracts when the managers' effort choices and conjectures constitute a Nash equilibrium. The analysis proceeds by first taking the number of projects, n, as given, and then (in Section 4) determines the optimal firm size by finding the optimal number of projects to undertake, n^*.

The gross firm value associated with undertaking n projects is simply $\mathcal{A}+n\Delta$. The principal seeks to minimize the total compensation cost of inducing n managers to participate and make an effort. Let Ω_i denote the terms of manager i's contract. Let $C(\Omega_i | \sum_{j=1}^{n} e_j = k)$ be the compensation received by manager i conditional on firm value being $\mathcal{A} + k\Delta$; i.e., conditional on

[3] It is precisely the fact that firm value is independent of the identity of the particular managers making an effort which explains why the value of the firm can be verifiable, yet there can still be an agency problem. As will be seen, our results on firm size apply so long as aggregate firm value is observable. Verifiability of firm value is only relevant for the form of optimal managerial contracts.

a total of k of the managers making an effort. Recall that since an individual manager's effort is not verifiable, compensation contracts must be functions of firm value and firm value depends on the total effort expended, $\sum_{j=1}^{n} e_j$. For n fixed, the principal's problem is:

$$\min_{\{\Omega_i\},\ i=1,\ldots,n} \quad \sum_{i=1}^{n} C\left(\Omega_i \middle| \sum_{j=1}^{n} e_j = n\right) \qquad \text{(Problem PI)}$$

$$s.t. \qquad C\left(\Omega_i \middle| \sum_{j=1}^{n} e_j = n\right) - D \geq R \ \forall\ i = 1, \ldots, n. \qquad (1)$$

$$C\left(\Omega_i \middle| \sum_{j=1}^{n} e_j = n\right) - D \geq C\left(\Omega_i \middle| \sum_{j=1}^{n} e_j = n-1\right) \forall\ i = 1, \ldots, n. \qquad (2)$$

$$\mathcal{A} + n\Delta - \sum_{i=1}^{n} C\left(\Omega_i \middle| \sum_{j=1}^{n} e_j = n\right) \geq \mathcal{A} + (k-1)\Delta - \sum_{i=1}^{n} C\left(\Omega_i \middle| \sum_{j=1}^{n} e_j = k-1\right) \forall\ k = 1, \ldots, n. \quad (3)$$

Constraint (1) is the participation constraint. Constraint (2) is the managerial effort constraint. Under an optimal contract the managers' effort choices are assumed to constitute a Nash equilibrium in which each manager works believing that the other $n-1$ managers will also work. Constraint (3) is the principal's incentive compatibility constraint guaranteeing that the principal does not have an incentive to reduce output. Constraint (3) states that managerial contracts must be designed such that the residual payoff to the principal is never higher than it is when all n managers work. Imagine solving (PI) subject only to constraints (1) and (2). One solution to this reduced problem would be to pay each manager a fixed amount, $F \leq R$, and a bonus of $D+R-F$ to be received in the event that firm value is $\mathcal{A} + n\Delta$ or more. Such a contract satisfies both constraints (1) and (2) at least-cost.

But given an active principal, such a contract will not succeed in inducing n managers to sign and work. If an active principal were to secretly renegotiate with any one manager, paying her a bribe of $R-F$ not to work, then the principal would not have to pay any bonuses. Although output would be reduced by Δ, total managerial compensation would be reduced by $n(D+R-F)-(R-F)$. Whenever $n > \frac{\Delta+R-F}{D+R-F}$, the principal would be better off by having reduced output. But all the other managers will recognize that someone else will have been bribed not to work, and hence they too would find it optimal not to work. Recognizing that, in the end, no-one was going to work, no manager would agree to participate. By designing managerial contracts such that the residual payoff to the principal is always non-decreasing in firm value, the principal can guarantee that he will not be tempted by such a moral hazard.

2. The Restricted Problem for Fixed n

For fixed n we first solve the principal's problem, (PI), by adding two constraints on contract form and return in Section 3 to the more general problem of Subsection 1.2. The two additional constraints are that: i) Managerial contracts take the form of identical wage/equity/option contracts for each manager; and ii) All parties enjoy limited liability. We consider only wage/equity/option contracts for two reasons. First, these are the contracts observed in practice and we would like to determine conditions under which each is used.[4] Second, the model and the results are easier to understand when explicated in terms of these familiar contract forms. Given identical contracts for each manager, each project-manager combination is the same. In solving for an optimal contract we focus on a representative combination. Henceforth whenever we refer to "the manager" we mean this representative manager.

These two constraints aid the exposition. But, as we will show, they do not affect either the cost of an optimal managerial compensation contract or the number of managers who can be hired and motivated to work. Managerial limited liability can though affect the uniqueness of the equilibrium.

A wage/equity/options contract is defined by the specification of a wage-equity share pair, $\{W, \alpha\}$, where W denotes a promised wage and α denotes an equity share. The equity vests at the end of the contract; i.e., after the manager's reservation utility has declined to zero.[5] The fraction

[4] Jackson and Lazear (1991) also analyze the wage, equity and option components of optimal contracts. Their focus is on risk-aversion and output uncertainty rather than, as here, on multiple managers and the principal's moral hazard.

[5] Immediate vesting is weakly dominated by delayed vesting; i.e., the cost of inducing participation and effort is weakly lower when a manager is unable to sign, resign immediately, regain

α measures the holder's claim on the fully diluted number of shares outstanding after any exercise of options. An executive stock option corresponds to $W < 0$ and $\alpha > 0$; i.e., a contract specifying a negative wage should be interpreted as an executive stock option where the manager pays an exercise price W into the firm in exchange for the award of a fraction α of the equity.

The principal's restricted problem is:

$$\min_{\alpha,W} C\left(\alpha, W \;\middle|\; \sum_{j=1}^{n} e_j = n\right)$$

$$s.t. \quad C\left(\alpha, W \middle| \sum_{j=1}^{n} e_j = n\right) - D \geq R \qquad (\textit{participation constraint}) \quad (i)$$

$$C\left(\alpha, W \middle| \sum_{j=1}^{n} e_j = n\right) - D \geq C\left(\alpha, W \middle| \sum_{j=1}^{n} e_j = n-1\right) \qquad (\textit{effort constraint}) \quad (ii)$$

$$\alpha \leq 1/n \qquad (\textit{no principal moral hazard}) \quad (iii)$$

For $W < 0$, (*managers' limited liability*)

$$C\left(\alpha, W \middle| \sum_{j=1}^{n} e_j = n\right) := \max\left[0, W + \alpha\left[\mathcal{A} + n\Delta - nW\right]\right]. \qquad (iv)$$

For $W \geq 0$, (*principal's limited liability*)

$$C\left(\alpha, W \middle| \sum_{j=1}^{n} e_j = n\right) := \min\left[W, \frac{\mathcal{A} + n\Delta}{n}\right] + \alpha \max\left[0, \mathcal{A} + n\Delta - nW\right]. \qquad (v)$$

The fact that the principal's payoff must be nondecreasing in firm value rules out short-selling by the principal. Since the principal can not go short, the fraction of the equity that can vest with any individual manager is at most one minus the aggregate of the fractions that vest with other managers. When the firm consists of n projects and n managers, symmetric wage/equity/option compensation contracts imply that each manager can hold at most only $1/n$ of the fully-diluted equity (constraint (iii) above).[6]

her outside opportunities and leave with a valuable equity or option claim on the firm.

[6] Alternatively, if votes are linked to shares in some proportion, the constraint may reflect the

The managers' limited liability means that whenever $W < 0$ the pair should be interpreted as the terms of an option contract (constraint (iv) above).[7] The reader may notice that we have not restricted α to be non-negative. We will show in Proposition 2 that when a solution to the restricted problem exists, the solution satisfying $\alpha \geq 0$ can not be improved upon since it achieves least-cost. Hence managerial limited liability will be satisfied since optimal managerial contracts will either take the form of stock options or a combination of a non-negative promised wage and a non-negative equity share. The principal's limited liability means that the firm's equity is analogous to an option on the firm's assets (constraint (v) above). For example, if the aggregate promised wage exceeds aggregate firm value, then the firm will be bankrupt and each manager/debtholder will receive 1/nth of the firm in partial payment of her wage claim. Hence the wage received component of (v) is $\min[W, \frac{\mathcal{A}+n\Delta}{n}]$.

2.1 The participation and effort constraints of the restricted problem

To analyze the effort and participation constraints we need an expression for the represen-

unwillingness of the principal to transfer so many votes that all control rights are also transferred. With this interpretation a constraint restricting α to be strictly less than one would still exist with but one manager. For an analysis of this problem, see Gorton and Grundy (1997).

[7] It is instructive to let $X := -W$ denote the exercise price of the warrant granted to the manager. She owns a warrant giving her the right to acquire the fraction α of the firm if she pays X into the firm. The compensation expression in (iv) then simplifies to the payoff from such a warrant:

$$\max[0, \alpha(\mathcal{A} + n\Delta + nX) - X].$$

Note that the manager need not literally pay money into the firm. A warrant that satisfies condition (iii) that $\alpha \leq 1/n$ can take the form of a stock appreciation right. To see this, let $Y := \frac{1}{1-\alpha n}X$. Since $\alpha \leq 1/n$, $Y \geq 0$. Let $f := \frac{\alpha}{1-\alpha n}$ and normalize the number of shares outstanding to one. Suppose each manager were compensated with a stock appreciation right giving her the right to a payment equal to any excess of the value of f shares over and above Y. Conditional on k managers working, each manager would receive compensation, C, satisfying

$$C = \max[0, f(\mathcal{A} + k\Delta - nC) - Y],$$

and the value of the shares held by the principal would be $\mathcal{A} + k\Delta - nC$. Thus

$$\begin{aligned} C &= \max[0, \frac{f}{1+fn}(\mathcal{A} + k\Delta) - \frac{1}{1+fn}Y] \\ &= \max[0, \alpha(\mathcal{A} + k\Delta + nX) - X]. \end{aligned}$$

tative manager's compensation cost. Conditional on all other managers working, the representative manager's compensation if she too works can be written (by combining constraints (*iv*) and (*v*)) as:

$$C\left(\alpha, W \middle| \sum_{j=1}^{n} e_j = n\right) := \max\left[0, \min\left[W, \frac{\mathcal{A}+n\Delta}{n}\right] + \alpha \max\left[0, \mathcal{A}+n\Delta - nW\right]\right].$$

If the participation constraint is to be satisfied, the manager's compensation when she works must be positive. Hence the participation constraint takes the form:

$$\min\left[W, \frac{\mathcal{A}+n\Delta}{n}\right] + \alpha \max[0, \mathcal{A}+n\Delta - nW] - D \geq R.$$

For $W \geq \frac{\mathcal{A}}{n} + \Delta$, the participation constraint simplifies to:

$$\frac{\mathcal{A}}{n} + \Delta - D \geq R,$$

which, given $\Delta > R+D$, is always satisfied. For $W < \frac{\mathcal{A}}{n}+\Delta$, the participation constraint simplifies to:

$$\alpha \geq \frac{R+D-W}{\mathcal{A}+n\Delta-nW}.$$

A wage of at least $R+D$ will induce participation even absent an equity share. Conversely, absent a wage, an equity share of at least $\frac{R+D}{\mathcal{A}+n\Delta}$ will induce participation since the value of that share will be at least $R+D$.

If the effort constraint is to be satisfied, the manager's compensation when she works must be positive. Hence the effort constraint takes the form:

$$\begin{aligned} &\min\left[W, \frac{\mathcal{A}+n\Delta}{n}\right] + \alpha \max\left[0, \mathcal{A}+n\Delta - nW\right] - D \\ &\geq \max\left[0,\ \min\left[W, \frac{\mathcal{A}+(n-1)\Delta}{n}\right] + \alpha \max\left[0, \mathcal{A}+(n-1)\Delta - nW\right]\right]. \end{aligned} \tag{4}$$

The set of $\{W, \alpha\}$ pairs satisfying the effort constraint depends crucially on the number of projects to be simultaneously undertaken. The form of the effort constraint depends on whether n is "large" or "small", where large and small are defined as follows:

Definition: The number of projects to be undertaken is said to be *large* if $n > \frac{\Delta}{D}$. Otherwise the number is said to be *small*.

Notice that this definition involves a comparison between the aggregate number of projects to be undertaken, n, and managerial productivity as measured by the output-input ratio, $\frac{\Delta}{D}$. When $W \geq \frac{\mathcal{A}+n\Delta}{n}$ the effort constraint in (4) simplifies to $n \leq \frac{\Delta}{D}$. Thus when n is large, the effort constraint can not be satisfied by contracts involving $W \geq \frac{\mathcal{A}+n\Delta}{n}$.

Lemma 1. *When the number of projects is large the effort constraint takes the form:*

$$\alpha \geq \begin{cases} \frac{D}{\Delta}, & \text{if } W \leq \frac{\mathcal{A}+(n-1)\Delta}{n}; \\ \frac{\frac{1}{n}(\mathcal{A}+n\Delta-nW-(\Delta-nD))}{\mathcal{A}+n\Delta-nW}, & \text{if } \frac{\mathcal{A}+(n-1)\Delta}{n} < W < \frac{\mathcal{A}+n\Delta}{n}. \end{cases} \tag{5}$$

Proof: See Appendix A.

The effort constraint given large n is depicted in Figure 1. The intuition underlying Figure 1 is straightforward. Suppose $W \leq \frac{\mathcal{A}+(n-1)\Delta}{n}$. Any such promised wage can always be paid in full so long as $n-1$ managers work. Any such option will always finish in-the-money and be exercised. Thus the realized wage is not effort sensitive. Thus if the manager is to be induced to work when $W \leq \frac{\mathcal{A}+(n-1)\Delta}{n}$, her equity share alone must provide the appropriate incentives. An equity share of $\frac{D}{\Delta}$ of the increment to equity value if she works will give her $\frac{D}{\Delta} \times \Delta$, and thereby compensates her for the disutility of effort, D.

When $\frac{\mathcal{A}+(n-1)\Delta}{n} < W < \frac{\mathcal{A}+n\Delta}{n}$, the realized wage is effort sensitive. As depicted in Figure 1, the effort constraint is upward-sloping for W in this range. To understand this, compare two contracts. One contract offers the highest possible wage that is not effort sensitive, i.e., $W = \frac{\mathcal{A}+(n-1)\Delta}{n}$, and an equity share of $\alpha = \frac{D}{\Delta}$. Clearly this contract satisfies the effort constraint. The second contract offers a promised wage that is \$1 greater than that of the first contract and the same equity share. Provided that she works, our representative manager's realized wage will be \$1 more under the second contract than the first. In fact, every manager will receive an additional dollar in wages if she works. Thus conditional on working, her equity share is less valuable under the second contract than under the first by an amount equal to her share of the increased wage bill; i.e., by $\alpha \times n \times \$1$. When n is large, this decline in value of her equity share under the second contract relative to the first actually exceeds the \$1 increase in her wage: $\alpha \times n \times \$1 = \frac{D}{\Delta} \times n \times \$1 > \$1$

for n large. Conditional on not working, she receives $\frac{\mathcal{A}+(n-1)\Delta}{n}$ under either contract. Therefore, to induce effort when her wage is increased by \$1 requires that her equity share also be increased.

It follows immediately from Figure 1 that:

Proposition 1. *When compensation contracts are restricted to identical wage/equity/option contracts with limited liability and the principal is active, it is impossible to motivate a large number of managers to work.*

When n is large, satisfaction of the effort constraint requires that each manager receive an equity share of at least $\frac{D}{\Delta}$ or more. The maximum equity share that can be awarded to each manager without creating an incentive for the principal to engage in moral hazard is $1/n$. A large value for n means that this maximum is less than $\frac{D}{\Delta}$.

Now consider the effort constraint when the number of projects is small.

Lemma 2. *When the number of projects is small, the effort constraint takes the form:*

$$\alpha \geq \begin{cases} \frac{D-W}{\mathcal{A}+n\Delta-nW}, & \text{if } W < -\frac{D(\mathcal{A}+(n-1)\Delta)}{\Delta-nD}; \\ \frac{D}{\Delta}, & \text{if } W \in [-\frac{D(\mathcal{A}+(n-1)\Delta)}{\Delta-nD}, \frac{\mathcal{A}+(n-1)\Delta}{n}]; \\ \frac{\frac{1}{n}(\mathcal{A}+n\Delta-nW)-(\Delta-nD)}{\mathcal{A}+n\Delta-nW}, & \text{if } W \in (\frac{\mathcal{A}+(n-1)\Delta}{n}, \frac{\mathcal{A}+n\Delta}{n}); \\ -\infty, & \text{if } W \geq \frac{\mathcal{A}+n\Delta}{n}; \end{cases} \tag{6}$$

Proof: See Appendix A.

Promised wages that in aggregate exceed total output plus the value of any assets-in-place (i.e., values of $W \geq \frac{\mathcal{A}+n\Delta}{n}$) effectively give each manager ownership of $1/n$ of the firm. This is so because such promised wages will always bankrupt the firm. By working, each manager adds Δ to the firm. Of this increase in firm value, she will receive $\frac{1}{n} \times \Delta$, which, since n is small, exceeds the disutility of her efforts.[8] The relevant region of the effort constraint for small n is depicted in Figure 2.

To understand the three segments depicted in Figure 2, consider the various corresponding levels of W. Consider first the segment with promised wages in the range $(\frac{\mathcal{A}+(n-1)\Delta}{n}, \frac{\mathcal{A}+n\Delta}{n}]$. Such

[8] Not surprisingly, satisfying the effort constraint by promising everything to the managers is not the minimum cost way of hiring and motivating a small n number of managers.

high promised wages can only be paid in full if the manager works. When the promised wage is equal to $\frac{\mathcal{A}+(n-1)\Delta}{n} + D$, the difference between the wage received when the manager works versus that received when she shirks is exactly D. Hence the effort constraint is satisfied for $\alpha = 0$. As W is reduced below $\frac{\mathcal{A}+(n-1)\Delta}{n} + D$ the other performance-sensitive compensation component, α, must be increased if the effort constraint is to remain satisfied. Consider now the horizontal segment. Divide it into positive and negative wage regions. When $W \in [0, \frac{\mathcal{A}+(n-1)\Delta}{n}]$, the promised wage can always be paid in full and her equity stake alone must motivate her to make an effort. This requires that her share of the increase in firm value if she works, $\alpha\Delta$, exceeds her disutility of effort, D; i.e., $\alpha \geq \frac{D}{\Delta}$. When $W \in [-\frac{D(\mathcal{A}+(n-1)\Delta)}{\Delta-nD}, 0]$ the exercise price of the option is such that when the manager's equity share equals $\frac{D}{\Delta}$, her option finishes in-the-money whether she works or not, but it finishes in-the-money by D dollars more if she works than if she shirks. Finally in the third segment W is very low; i.e., the exercise price of the manager's executive stock option is very high. When $W < -\frac{D(\mathcal{A}+(n-1)\Delta)}{\Delta-nD}$ and the manager's equity share is equal to $\frac{D-W}{\mathcal{A}+n\Delta-nW}$, then her option finishes in-the-money by the amount D if she works and out-of-the-money if she shirks.

For a fixed small n, Figure 3 depicts the participation and effort constraints for varying levels of $\mathcal{A}$, and reflects the following property of the constraints:

Lemma 3. *Suppose n is small. If the participation and effort constraints intersect strictly, then they do so in the horizontal section of the effort constraint.*

Proof: See Appendix A.

Which panel of Figure 3 is relevant depends on the size of $\mathcal{A} + n\Delta$. Note that $\mathcal{A} + n\Delta$ is increasing in $\mathcal{A}$, but not necessarily in n. Since we are considering only a candidate fixed n, differences across panels reflect only differences in $\mathcal{A}$. As $\mathcal{A}$ increases, one moves from the setting in Figure 3(a) to that in 3(b), and finally to 3(c). An increase in $\mathcal{A}$ leads to an increase in the maximum possible wage that could be paid in full even absent any managerial effort: For $W > 0$ an increase in $\mathcal{A}$ shifts the effort constraint horizontally to the right. An increase in $\mathcal{A}$ also leads to an increase in the maximum possible exercise price of an option that will finish exactly at-the-money absent any managerial effort: For $W < 0$ an increase in $\mathcal{A}$ shifts the effort constraint horizontally to the left. An increase in $\mathcal{A}$ causes the participation constraint to pivot around the point $\{W = R + D, \alpha = 0\}$. A contract of the form $\{W = R + D, \alpha = 0\}$ will just satisfy the participation constraint irrespective of the value of $\mathcal{A}$. The equity stake necessary to induce participation when the wage is less than $R + D$ is decreasing in $\mathcal{A}$. For example, consider the zero-wage, all-equity contract: $\{W = 0, \alpha = \frac{R+D}{\mathcal{A}+n\Delta}\}$. Thus, the participation constraint rotates downwards as $\mathcal{A}$ increases.

As shown in Figure 3(a), when $\mathcal{A} + n\Delta \leq nR + \Delta$, any contract that induces the manager to participate will also induce her to make an effort at least-cost. The form of an optimal contract could be any of the following: an all-wage contract, an all-equity contract, a combination of wages and equity, or an executive stock option. Figures 3(b) and 3(c) are both situations where $\mathcal{A} + n\Delta > nR + \Delta$ and an all-wage contract is never optimal. To understand the difference in optimal contract form between panels (b) and (c), consider the zero-wage, all-equity contract that just satisfies the effort constraint: $\{W = 0, \alpha = \frac{D}{\Delta}\}$. The value of the manager's claim when she works is:

$$\alpha(\mathcal{A} + n\Delta) = \alpha(\mathcal{A} + (n-1)\Delta + \Delta) = \frac{D}{\Delta}(\mathcal{A} + (n-1)\Delta) + D.$$

Whether this contract will induce participation depends on whether $\frac{D}{\Delta}(\mathcal{A}+(n-1)\Delta)$ compensates her for forgoing her outside opportunities. When $\frac{D}{\Delta}(\mathcal{A} + (n-1)\Delta) < R$, a positive wage must be added to induce participation. But, when $\frac{D}{\Delta}(\mathcal{A}+(n-1)\Delta) > R$, the manager is overcompensated for having forgone R and this zero-wage, all-equity contract is not an optimal contract. Charging the manager a "strike price" to receive her equity share can reduce the cost of the contract without affecting the manager's incentive to work or eliminating her desire to participate.

2.2 Optimal contracts in the restricted problem for fixed small n

Let C^* denote the cost of an optimal contract. Proposition 2 follows immediately from inspection of Figures 3(a), (b), and (c).

Proposition 2. (The Cost of An Optimal Contract given n Small)

When n is small the solution to the principal's problem is least-cost; i.e., $C^* = R + D$.

We now turn to the form of an optimal contract when managers are rewarded with identical wage/equity/options packages and managers enjoy limited liability. Inspection of Figures 3(a), (b), and (c) also reveals that least-cost can be achieved without ever having to offer the managers a contract with $\alpha < 0$. Proposition 3 formalizes the discussion of Figure 3 in terms of necessary and/or sufficient conditions for the contract to include effort-sensitive wages, a non-option equity award, and executive stock options.

Proposition 3. (The Form of Optimal Identical Wage/Equity/Options Contract with Limited Liability Given Small n)

For n small, optimal contracts that take the form of identical wage/equity/options packages with limited liability have the following properties:

1. *An Effort-Sensitive Wage, $\{W > \frac{\mathcal{A}+(n-1)\Delta}{n}, \alpha \geq 0\}$, can be part of an optimal contract if $\mathcal{A} + n\Delta \leq nR + \Delta$.*
2. *A Non-Option Equity Award, $\{W \geq 0, \alpha > 0\}$, can be part of an optimal contract if $\mathcal{A} + n\Delta \leq \frac{R+D}{D}\Delta$.*
3. *Executive Stock Options, $\{W < 0, \alpha > 0\}$, can always be part of an optimal contract. The optimal contract must be an executive stock option if $\mathcal{A} + n\Delta > \frac{R+D}{D}\Delta$.*

The proof follows by inspection of Figure 3. Compensation under an optimal contract must be sensitive to the effort choice. When the value of the firm as measured by $\mathcal{A} + n\Delta$ is sufficiently low, an optimal contract need not involve any equity because the promised wage component can be honored only if the manager works—the wage received is sensitive to the manager's effort. At higher levels of firm value, it is not optimal to promise a wage that is so large as to be sensitive to the manager's effort choice. Optimal contracts will then involve an equity component: there may be wages plus equity; or a zero-wage, all-equity contract; or executive stock options. As mentioned above, when firm value becomes sufficiently high, an equity share large enough to induce effort overpays the manager. Equity gives her a claim not only on what she adds, but also on the efforts of her colleagues and on the assets-in-place. The solution is then to 'sell' the manager her effort-inducing equity share in return for a strike price; i.e., optimal contracts take the form of stock options.[9]

Real world firms are not 100% owned by managers. The same is true here in the following sense. The principal owns all the outstanding equity until the end of the period at which point the managers vest in a maximum of 100% of the equity. In this article the project ends after one period. If single period projects are simply repeated then the principal in a given period is the prior principal and the set of managers who received equity in the prior period.

3. The General Problem for Fixed n

We solve the principal's general problem for fixed n by showing that the imposition of the additional constraints (of identical wage/equity/options contracts and limited liability) in the restricted problem does not affect either the feasibility or cost of hiring n managers. We have shown that for small n, least-cost compensation can be achieved in the restricted problem. Hence, for small n, the solution given identical wage/equity/options contracts with limited liability can not be improved upon. We now show that for large n the general problem lacks a solution.

Theorem 1. (The Bound on Firm Size) *Given an active principal and large n, there does not exist any contract that can induce effort from the managers.*

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[9] Other rationales for the use of options in executive compensation are quite distinct from this fundamental result that links the use of options to firm value even given risk-neutrality and no uncertainty about project payoffs conditional on the effort choice. In the now standard principal-agent setting with a risk-averse agent, Ross (1974) established the link between the convexity versus concavity of an optimal fee schedule and the specifics of the trade-off between risk-sharing and incentives. Lambert (1986) and Hirshleifer and Suh (1992) examine settings in which the compensation scheme affects both the agent's choice between risky projects and the effort expended. Managerial effort can either generate information about the distribution of returns on the future projects from which the manager will select (as in Lambert) or increase the payoff from the particular project that she has previously selected (as in Hirshleifer and Suh). These authors establish that given a risk-reward tradeoff between the potential projects, optimal compensation schemes can involve managerial stock options. Hagerty, Ofer and Siegel (1990) model and empirically examine the link between option-like features of compensation schemes and the choice between projects with immediate versus future payoffs. Smith and Watts (1992) empirically examine the relation between measures of a firm's investment opportunity set and its use of option-like compensation schemes.

Proof: Summing the effort constraint in (2) over all $i = 1, \ldots, n$ gives

$$\sum_{i=1}^{n} C\left(\Omega_i \left| \sum_{j=1}^{n} e_j = n \right.\right) - \sum_{i=1}^{n} C\left(\Omega_i \left| \sum_{j=1}^{n} e_j = n-1 \right.\right) \geq nD. \tag{7}$$

For $k = n$ in (3) we have that the left-hand-side of (7) is less than Δ, implying $\Delta \geq nD$. ▮

Despite the fact that the technology exhibits constant returns to scale (each additional manager hired and motivated forgoes outside opportunities worth R, bears disututility of effort D, and produces Δ), Theorem 1 shows that there is a limit to the size of the firm: A firm can not be large in the sense that n can not exceed $\frac{\Delta}{D}$. At root this boundary on firm size is due to the non-verifiability of effort and the potential for moral hazard on the part of the principal. The non-verifiability of effort limits managers' contracts to claims on firm value. For any given n, motivating each manager to make an effort in the Nash equilibrium requires that *each* manager's compensation increase by at least D when firm value increases by the last Δ increment. Each manager conjectures that the others work and that it is her effort that adds the last Δ to firm value. Thus, in *aggregate*, at least nD in additional managerial compensation must be paid when firm value increases from $\mathcal{A} + (n-1)\Delta$ to $\mathcal{A} + n\Delta$. But this aggregate increase in the managers' compensation can not be greater than Δ, otherwise the residual payoff to the principal will decrease when firm value increases.

The bound on firm size was established considering only pure strategies. Each manager works and conjectures that each other manager will choose to work. In a mixed-strategy equilibrium, each manager works with some probability π and conjectures that the effort choices of the other managers are independent and that each will work with probability π. Does there exist a contract associated with a mixed-strategy equilibrium which allows the principal to hire a large number of managers and with some probability have a large number of projects completed? Appendix B establishes that the answer is no. The same bound on firm size, $n \leq \frac{\Delta}{D}$, applies when managers are hired under a contract consistent with mixed strategies in managerial effort choices.

Part 3 of Proposition 3 states that when optimal contracts take the form of identical wage/equity/options packages with limited liability and $\mathcal{A} + n\Delta > \frac{R+D}{D}\Delta$, then the optimal contract must be an executive stock option. We now show that this is a special case of a more general result: Whenever managers have limited liability and $\mathcal{A} + n\Delta > \frac{R+D}{D}\Delta$, then an optimal contract must have option-like features; i.e., over some range the manager's payoff must be convex in firm value.[10]

Theorem 2. (Option-like Feature of Optimal Contracts Given Limited Liability) *Suppose that n is small, that managers have limited liability and that the principal is active. An optimal contract can not be everywhere concave in firm value if*

$$\mathcal{A} + n\Delta > \frac{R+D}{D}\Delta.$$

Proof: If the contractual payoff is everywhere concave in firm value and managers have limited liability then

$$\begin{aligned} C\left(\Omega_i \middle| \sum_{j=1}^{n} e_j = n\right) &\geq (\mathcal{A} + n\Delta) \times \frac{C\left(\Omega_i \middle| \sum_{j=1}^{n} e_j = n\right) - C\left(\Omega_i \middle| \sum_{j=1}^{n} e_j = n-1\right)}{\Delta} \\ &\geq (\mathcal{A} + n\Delta) \times \frac{D}{\Delta}. \end{aligned} \tag{8}$$

where the second inequality follows from the satisfaction of the effort constraint (2). Combining the participation constraint in (1) with the inequality in (8) gives:

$$R + D \geq C\left(\Omega_i \middle| \sum_{j=1}^{n} e_j = n\right) \geq \frac{D}{\Delta}\left(\mathcal{A} + n\Delta\right),$$

which implies

$$\mathcal{A} + n\Delta \leq \frac{R+D}{D}\Delta. \quad \blacksquare$$

This result is presented graphically in Figure 4. In Figure 4A, $\mathcal{A}'$ and n' are such that $\mathcal{A}' + n'\Delta \leq \frac{R+D}{D}\Delta$ and each manager's compensation can be concave in firm value without violating managerial limited liability. In Figure 4B, $\mathcal{A}'' > \mathcal{A}'$. In Figure 4C, $n'' > n'$. For the parameter values of Figures 4B and 4C, a contract that is everywhere concave in firm value will violate managerial limited liability.

[10] Kim (1997) examines the issue of contract form and the optimality of a bonus contract in a setting with a principal and a single agent who makes a continuous effort choice and enjoys limited liability.

To this point we have examined optimal contracts taking n to be fixed. But the number of projects to be undertaken from the investment opportunity set (of size m) is itself a choice variable of the principal. We now turn to the determination of optimal firm size.

4. Optimal Firm Size

For n small the value of the principal's claim on the firm is $\mathcal{A}+n(\Delta-C^*)$ where (as shown in Proposition 2) $C^*=R+D$.

Theorem 3. (Optimal Firm Size) *The optimal firm size, n^*, is the largest number of projects less than or equal to m that satisfies the constraint $n \leq \frac{\Delta}{D}$.*

Proof: So long as it is feasible to hire and motivate another manager it is optimal to do so, since (from Proposition 2) whenever it is feasible, each manager can be hired at least-cost, and, by assumption, $R+D<\Delta$. Theorem 1 establishes that it is feasible to hire and motivate n managers so long as n is small. Therefore the optimal n solves:

$$\begin{aligned} \max_{n} \quad & \mathcal{A}+n(\Delta-(R+D)) \\ s.t. \quad & n \leq \min\left[m, \mathrm{int}\left(\frac{\Delta}{D}\right)\right]. \quad \blacksquare \end{aligned}$$

Optimal firm size is determined by managerial productivity which is precisely defined by $\frac{\Delta}{D}$. As productivity increases, the increment to firm value from a given manager working can support a larger number of managers making an effort.

5. Uniqueness

Each manager contemplating her participation at the firm will have a conjecture about the effort choices of the other managers hired. Problem (PI) has been solved when managers conjecture that all other managers work. Given this conjecture, the contract induces effort choices consistent with the conjecture. In this section we consider the question of whether an executive compensation contract that is a solution to (PI) can also be associated with other equilibria.

5.1 Existence of optimal contracts associated with unique equilibria

For fixed small n, different contracts can solve (PI). Not all solutions to (PI) are necessarily associated with a unique equilibrium in which n managers sign and work. Some contracts solving (PI) can be associated with an alternate equilibrium in which managers do not sign because they rationally conjecture that any managers who do sign will not work. For example, consider a solution to (PI) in which managers are compensated with options. Suppose the exercise price of these options is such that they will finish out-of-the-money if only one manager works. If each manager conjectures that every other manager who does sign will not work, then that manager will not find it optimal to bear the disutility of effort—her options will finish out-of-the-money whether she works or shirks. But given such a conjecture, each manager will find it optimal not to sign in the first place.

The principal however is in a position to select a contract from the set of contracts solving (PI). By doing so he selects the set of possible equilibria, and he will choose the set that maximizes his utility. In the absence of any additional constraints on (PI) we show that the principal can always design a contract that implements a unique equilibrium in which n managers sign and work at least-cost. The principal will optimally select such a contract. In the following subsection, we will re-examine this issue when the principal is constrained to select contracts that reflect limited liability on the part of managers.

Theorem 4. (Uniqueness) *Within the set of contracts that solve (PI) for a fixed small n, there always exists a contract such that the equilibrium is unique and n managers will sign and work.*

Proof: The proof is by construction. Each manager i who signs finds working to be a dominant strategy if:

$$C\left(\Omega_i \middle| \sum_{j=1}^{n} e_j = k\right) - D \geq C\left(\Omega_i \middle| \sum_{j=1}^{n} e_j = k-1\right) \;\; \forall\; k = 1, \ldots, n. \tag{9}$$

Consider a contract where each manager receives the fraction $\frac{D}{\Delta}$ of the end of period firm value and pays an amount to (or receives an amount from) the principal. This contract guarantees satisfaction of the inequality in (9); i.e., every manager who signs will prefer to work. Given a transfer between the principal and each agent of the difference

$$\frac{D}{\Delta}(\mathcal{A} + n\Delta) - (R + D),$$

n managers will sign.[11] ∎

The particular contract constructed in the proof of Theorem 4 gave each manager the fraction $\frac{D}{\Delta}$ of firm value coupled with a transfer payment. More generally, the fraction could depend on the realized firm value.

Corollary 1: (Concavity and Uniqueness) *Whenever under a contract that solves (PI) each manager's payoff is everywhere concave in firm value for all values of the firm exceeding the liquidation value of the assets-in-place and n is small, then the Nash equilibrium in which all managers work is the unique equilibrium under the contract.*

Proof: Any contract that is a solution to (PI) must satisfy the effort constraint. Given such a contract, if each manager i's payoff is everywhere concave in firm value for all firm values exceeding $\mathcal{A}$, then the inequality in (9) must be satisfied. Hence working is a dominant strategy irrespective of one's beliefs about the actions of others. ∎

We now turn to the question of the uniqueness of the equilibrium under contracts that solve (PI) when those contracts are constrained by limited liability on the part of managers.[12]

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[11] Suppose there is some possibility that less than $n-1$ other managers sign, say, because they happen to resolve their indifference by not signing. Given a transfer of size $\frac{D}{\Delta}(\mathcal{A}+n\Delta)-(R+D)$, every potential manager will *strictly* prefer not to sign. Alternately suppose that there is some chance that a manager will suffer a heart attack after signing but before working. Again, given a transfer of size $\frac{D}{\Delta}(\mathcal{A}+n\Delta)-(R+D)$, every potential manager will *strictly* prefer not to sign. This concern can be overcome by including in the contractual terms, Ω_i, the number of managers who sign and survive. Let s denote this number. A transfer of

$$\frac{D}{\Delta}(\mathcal{A} + s\Delta) - (R + D)$$

will ensure that each manager is indifferent to signing irrespective of the participation decisions and survival of others.

[12] The link between managerial limited liability and possibility of multiple equilibria is quite distinct from the issues of implementation of first-best effort levels given limited liability and a single agent that are examined in Sappington (1983), Kim (1990) and Innes (1990).

5.2 Uniqueness when managers have limited liability

Whenever the contract considered in the uniqueness proof of Theorem 4 involves a transfer *from* the manager to the principal, it will be ruled out if compensation contracts are constrained to reflect managerial limited liability.

Theorem 5. (Uniqueness when Managers have Limited Liability) *For fixed small n, there exists a contract that solves (PI), satisfies limited liability, and implements a unique equilibrium in which all n managers sign and work if and only if $n < \frac{R+D}{D}$.*

Proof: For working to be a dominant strategy irrespective of the number of other managers who work, condition (9) must be satisfied. Condition (9) implies

$$C\left(\Omega_i \middle| \sum_{j=1}^{n} e_j = n\right) - C\left(\Omega_i \middle| \sum_{j=1}^{n} e_j = 0\right) \geq nD. \tag{10}$$

Recall from Proposition 2 that the solution to (PI) satisfies

$$C\left(\Omega_i \middle| \sum_{j=1}^{n} e_j = n\right) = R + D. \tag{11}$$

Given limited liability

$$C\left(\Omega_i \middle| \sum_{j=1}^{n} e_j = 0\right) \geq 0. \tag{12}$$

Relations (10), (11) and (12) imply

$$n < \frac{R+D}{D}. \quad \blacksquare$$

Note that when $n < \frac{R+D}{D}$ a contract of the form considered in the proof of Theorem 4 never involves a transfer from the manager to the principal. It follows immediately from Theorem 5 that when $m \geq \frac{R+D}{D}$, the principal is active and managers enjoy limited liability, then it is impossible to design a least-cost contract such that n^* projects are certain to be undertaken. Faced with $m \geq \frac{R+D}{D}$ and managerial limited liability, the principal has three choices:

i) Attempt to undertake n^* projects by offering a contract that solves (PI) and is associated with multiple equilibria (one of which is that no managers will sign);

ii) Undertake only $\frac{R+D}{D} < n^*$ projects using a contract that ensures a unique equilibrium with least-cost managerial compensation; or

iii) Undertake more than $\frac{R+D}{D}$ projects, but pay the managers sufficiently more than least-cost that a unique equilibrium in which all managers work can be ensured.

The optimal choice among these three possibilities remains an open question. Choice (i) is preferred by the principal if all managers always conjecture that all other managers work. Such a set of conjectures makes for a more valuable firm. See Kreps (1990) for a general analysis of the economics of "corporate culture".

Contracts consistent with possibilities i), ii) and iii) are depicted in Figures 5B, 5C and 5D respectively. Figure 5 illustrates a setting where $m > \frac{\Delta}{D} = n^*$. Figure 5A depicts the contract used in the proof of Theorem 4. This contract involves a transfer *from* the manager to the principal, and hence violates limited liability. Figure 5B depicts a limited-liability contract associated with two equilibria: In one, n^* managers sign and work at least-cost; in the other, no one signs. Figure 5C depicts limited-liability contractual payoffs consistent with hiring the maximum possible number of managers that it is possible to hire at least-cost and still ensure a unique equilibrium in which all managers work. That maximum number is $\frac{R+D}{D}$. Figure 5D depicts limited-liability contractual payoffs consistent with hiring more than $\frac{R+D}{D}$ managers at greater than least-cost and being assured that all managers will work. Figure 5D makes clear that if n^* managers were to be hired under a limited-liability contract associated with a unique equilibrium in which all managers worked, then each manager would have to receive compensation of Δ! The principal would clearly be better off with, say, the smaller number of managers hired under the contract depicted in Figure 5C. Suppose the principal wished to determine optimal firm size conditional on his offering only limited-liability contracts associated with unique equilibria in which all managers work. The principal would then solve:

$$\begin{aligned} \max_n \quad & \mathcal{A} + n(\Delta - \max[R + D, nD]) \\ s.t. \quad & n \leq \min\left[m, \operatorname{int}\left(\frac{\Delta}{D}\right)\right], \end{aligned}$$

and would choose to hire $\min\left[m, \max\left[\operatorname{int}\left(\frac{R+D}{D}\right), \operatorname{int}\left(\frac{\Delta}{2D}\right)\right]\right]$ managers.

6. Verifiability, Uncertainty and Risk Aversion

Our bound on firm size reflects the lack of verifiability of effort and an active principal. In deriving this bound we assumed that firm value was verifiable, that output conditional on the effort choice was certain, and that managers were risk-neutral. In this section we show that our basic result on firm size is unchanged when each of these assumptions is relaxed.

Even though effort is not verifiable, we have assumed that aggregate firm value is. What if, say because the firms' stock is not traded, firm value can not be verified? Recall that our results on firm size were unchanged when managerial contracts were restricted to wage/equity/options contracts. Wages are debt-like claims where the managers will acquire the firm in the event of a default. Default is verifiable, and the contract can be enforced even when firm value itself is not—see, e.g., Gale and Hellwig (1985). Equity and options on equity require only that relative, not absolute, values be verifiable. In particular, so long as the principal must pay the same dividends per share on the stock held by the managers as on the stock retained by the principal, the implementation of wage/equity/options contracts requires only that firm value be observable. The managers do need to observe firm value in order to make their option exercise decisions.

Now suppose that, conditional on the effort choice, output is not perfectly certain. Given stochastic output, the effort constraint in a Nash equilibrium in pure strategies takes the form:

$$E\left\{\widetilde{C}\left(\Omega_i\middle|\sum_{j=1}^{n} e_j = n\right)\right\} - D \geq E\left\{\widetilde{C}\left(\Omega_i\middle|\sum_{j=1}^{n} e_j = n-1\right)\right\} \quad \forall\ i = 1,\ldots,n,$$

where the tilde denotes that output, and hence the managers' compensation, is uncertain given the managers' effort choices. Summing over all i gives:

$$\sum_{i=1}^{n} E\left\{\widetilde{C}\left(\Omega_i\middle|\sum_{j=1}^{n} e_j = n\right) - \widetilde{C}\left(\Omega_i\middle|\sum_{j=1}^{n} e_j = n-1\right)\right\} \geq nD. \tag{13}$$

From the inequality constraint that the principal's payoff must be non-decreasing in firm value, expression (3), we have:

$$\widetilde{\Delta} \geq \sum_{i=1}^{n}\left[\widetilde{C}\left(\Omega_i\middle|\sum_{j=1}^{n} e_j = k\right) - \widetilde{C}\left(\Omega_i\middle|\sum_{j=1}^{n} e_j = k-1\right)\right]. \tag{14}$$

Taking expectations in (14) and combining the result with (14) gives:

$$n \leq \frac{E\{\widetilde{\Delta}\}}{D}.$$

Firm size is bounded by the managers' *expected* productivity.

Finally, suppose that managers are risk averse? For simplicity only, assume that output conditional on the managers' effort choices is perfectly certain and that each manager's utility function is given by $\mathcal{U}(C) - eD$, with $\mathcal{U}_1 > 0$ and $\mathcal{U}_{11} < 0$. The effort constraint now takes the form:

$$\mathcal{U}\left(C\left(\Omega_i \middle| \sum_{j=1}^{n} e_j = n\right)\right) - D \geq \mathcal{U}\left(C\left(\Omega_i \middle| \sum_{j=1}^{n} e_j = n-1\right)\right) \quad \forall\, i = 1, \ldots, n.$$

Without loss of generality we can assume that all managers are hired under contracts with identical terms, Ω. It follows from the mean-value theorem that there exists a value, U_1, such that

$$\mathcal{U}_1\left(C\left(\Omega \middle| \sum_{j=1}^{n} e_j = n-1\right)\right) < \mathrm{U}_1 < \mathcal{U}_1\left(C\left(\Omega \middle| \sum_{j=1}^{n} e_j = n\right)\right)$$

and

$$\begin{aligned}\mathcal{U}\left(C\left(\Omega \middle| \sum_{j=1}^{n} e_j = n\right)\right) &= \mathcal{U}\left(C\left(\Omega \middle| \sum_{j=1}^{n} e_j = n-1\right)\right) \\ &\quad + \mathrm{U}_1\left[C\left(\Omega \middle| \sum_{j=1}^{n} e_j = n\right) - C\left(\Omega \middle| \sum_{j=1}^{n} e_j = n-1\right)\right].\end{aligned}$$

Thus

$$\mathrm{U}_1\left[C\left(\Omega_i \middle| \sum_{j=1}^{n} e_j = n\right) - C\left(\Omega_i \middle| \sum_{j=1}^{n} e_j = n-1\right)\right] \geq D.$$

Summing over all i gives

$$\mathrm{U}_1 \sum_{i=1}^{n}\left(C\left(\Omega \middle| \sum_{j=1}^{n} e_j = n\right) - C\left(\Omega \middle| \sum_{j=1}^{n} e_j = n-1\right)\right) \geq nD. \tag{15}$$

Combining (15) with the no principal moral hazard constraint in (3) gives:

$$n < \frac{\mathrm{U}_1 \Delta}{D}.$$

Firm size is bounded by a measure of the marginal utility each manager would derive from consuming her output relative to the disutility of creating it. (When managers are risk neutral, the disutility of effort can be appropriately defined in units of marginal utility of consumption such that $U_1 = 1$.)

7. Conclusions

The classic Berle and Means (1932) agency problem arises because a growing but cash constrained firm requires more owners to provide capital inputs. We consider the agency problem that arises when an entrepreneur has plenty of cash to acquire any capital inputs necessary for the firm's growth, but needs more managerial labor inputs. Even if the n managers were to purchase the firm from the principal, an agency problem would still exist within the management team. Holmstrom (1982) shows that employing a principal to act purely as a budget-breaker can lead each manager to exert the first-best level of effort. But when the principal can renegotiate with individual managers to reduce output, the introduction of an active principal merely shifts the moral hazard problem from the team to the principal. We have shown that given an active principal, firm size is limited in the same way that team size is limited in the absence of a principal.[13] Neither a firm with an active principal nor a partnership can be larger then a natural measure of managerial productivity. That measure is the ratio of a manager's marginal utility from the consumption of her output to her disutility of effort.

Our focus on the limits to integration means that we take as given the benefits of combining projects within a firm. The cost of an agency-related limit to firm size depends on the nature of these benefits. Suppose, as in Grossman and Hart (1986), the optimal allocation of residual ownership rights precludes combining the assets-in-place with managerial effort by contracts across firms. Consider a principal with property rights to a set of $m > n^*$ projects. The cost to society of a limit on firm size is the cost of duplicating the necessary assets-in-place in a second firm. The cost to the principal may be higher still if the principal can not replicate the necessary assets-in-place at

[13] For a detailed analysis of the efficiency of partnerships see Legros and Matthews (1993).

the same cost as a potential purchaser of those rights can. A corporate spin-off can be interpreted as the optimal sale of the excess $m - n^*$ projects.

In a companion paper we re-examine the issue of optimal contracting and firm size when projects are long-lived in the sense that their implementation can be delayed for some time. Consider a long-lived project with a manager who is vested in equity and options. Her view of what she adds to the firm by bearing the disutility of effort is not Δ. She recognizes that, if she shirks and the project is delayed, then a replacement manager can be hired. Her vested equity and/or options allow her to free-ride on her replacement's efforts. She reasons that what she adds to the firm by working is only any excess of Δ over and above what a replacement would add. A vested manager's perception that her own productivity is, in effect, lower when projects can be delayed can then make for an even tighter bound on firm size. We show that when projects are long-lived, an optimal contract is such that the manager is not immediately fully vested. Optimal contracts involve both entrenchment and the deferral of compensation. Entrenchment makes the promise of deferred compensation credible. Deferral removes the possibility of free-riding on one's replacement. While resigning does allow a replacement manager to be hired, it also means the forfeiture of any nonvested claims on output. A manager hired under an optimal contract will then view her productivity correctly; i.e., as $\frac{\Delta}{D}$. Thus, under an optimal contract, the firm is able to hire and motivate $\frac{\Delta}{D}$ managers even when projects are long-lived.

We began by noting the potential for moral hazard on the part of the principal when managers are hired under bonus contracts. This does not mean that the observed use of bonus contracts in practice is anomalous. When output is lumpy, a bonus contract need not imply that the principal has an incentive to reduce output. Consider again a contract that pays each manager a fixed amount, $F \leq R$, and a bonus of $D + R - F$ in the event that firm value is $\mathcal{A} + n\Delta$ or more. Paying one manager a bribe of $R - F$ not to work will only make the principal better off if the aggregate saving in bonus payments less the cost of the bribe, $n(D + R - F) + (R - F)$, exceeds the forgone output, Δ. A bonus contract can be used to motivate managers without creating an incentive for moral hazard on the principal's part whenever the number of managers satisfies:

$$n \leq \frac{\Delta + R - F}{D + R - F}. \tag{16}$$

How large could such a team be? Setting $F = R$ in (16) gives a maximum team size of, lo and behold, $\frac{\Delta}{D}$.

Appendix A

Proof of Lemma 1:

When $W = \frac{\mathcal{A}+(n-1)\Delta}{n}$ and $\alpha = \frac{D}{\Delta}$, the effort constraint is just satisfied. Our representative manager receives the promised wage whether she works or not, and effort must be induced by the equity share. The equity share $\frac{D}{\Delta}$ of the increment to equity value if she also works, Δ, just compensates her for her disutility of effort. The same is true for all $W \in \left[0, \frac{\mathcal{A}+(n-1)\Delta}{n}\right)$. The intuition for the options case, $W < 0$ is as follows. From the point of view of our representative manager an equity share in exchange for the strike price is a claim on two components of firm value. The first component consists of the assets-in-place plus the Δ-sized increments to value added by each manager who works. The second component consists of the cash paid in by each manager exercising her options. When n is large (i.e., $n > \frac{\Delta}{D}$) and $\alpha \geq \frac{D}{\Delta}$ then $\alpha \times n > 1$. In this case, the value of the manager's share of the second component is sufficient to induce exercise: The manager pays W to receive the share α of the $n \times W$ paid into the firm. The manager also has a claim on the assets-in-place plus the projects' payoffs. It is the effect on the value of this claim that determines the representative manager's effort choice. A claim on $\frac{D}{\Delta}$ of the addition to equity value if she works will compensate her for the disutility of effort.

Finally consider values of W greater than $\frac{\mathcal{A}+(n-1)\Delta}{n}$. For such high promised wages, the realized wage will be effort-sensitive. So long as $W < \frac{\mathcal{A}}{n} + \Delta$ the promised wage can be paid in full provided the manager works: The firm's total wage bill is nW and the assets when all managers work are $\mathcal{A} + n\Delta$. If she does not work, she receives $\frac{\mathcal{A}+(n-1)\Delta}{n}$. As depicted in Figure 1 the effort constraint in this range is upward-sloping. For large n a manager who works will find that an increase in her wage will lead to a decline in the value of her equity share that exceeds the wage increase. Her compensation if she shirks will still be $\frac{\mathcal{A}+(n-1)\Delta}{n}$. Therefore, if the effort constraint is to be satisfied, her equity share must be increased along with the increase in wages. ∎

Proof of Lemma 2:

The effort constraint for the case $W \geq 0$ is straightforward and is discussed in the main text. The case $W < 0$ (i.e., an options package) is more involved. For an options package the manager's payoff should she shirk is the maximum of the payoff to exercising the option and the payoff to not exercising. She will work if working and exercising dominates the payoff to shirking. This could be satisfied in either of two ways. First working and exercising could dominate shirking and optimally not exercising:

$$W + \alpha[\mathcal{A} + n\Delta - nW] - D \geq 0 \geq W + \alpha[\mathcal{A} + (n-1)\Delta - nW]. \tag{A1}$$

Alternately, working and exercising could dominate shirking and optimally exercising:

$$W + \alpha[\mathcal{A} + n\Delta - nW] - D \geq W + \alpha[\mathcal{A} + (n-1)\Delta - nW] \geq 0. \tag{A2}$$

For a given W we first determine the set of α values that will both induce effort and make it suboptimal to exercise the option should the manager shirk; i.e., that satisfy both inequalities of $(A1)$. We then determine the set of α values that will both induce effort and make it optimal to exercise the option should the manager shirk; i.e., that satisfy both inequalities of $(A2)$. For the given W value, the effort constraint is determined by the smallest value of α that is contained in either of these two sets.

For $W \in [-\frac{D(\mathcal{A}+(n-1)\Delta)}{\Delta - nD}, 0]$ and n small, the set of α values satisfying $(A1)$ is the null set, while $(A2)$ is satisfied for all $\alpha \geq \frac{D}{\Delta}$. Hence we have the flat portion of the effort constraint applicable when compensation takes the form of an option. For $W < -\frac{D(\mathcal{A}+(n-1)\Delta)}{\Delta - nD}$ and n small, the set of α values satisfying $(A1)$ consists of all $\alpha \in [\frac{D-W}{\mathcal{A}+n\Delta-nW}, \frac{-W}{\mathcal{A}+(n-1)\Delta-nW}]$. Note that $\frac{D-W}{\mathcal{A}+n\Delta-nW} < \frac{-W}{\mathcal{A}+(n-1)\Delta-nW}$. The set of α values satisfying $(A2)$ consists of all $\alpha \geq \frac{-W}{\mathcal{A}+(n-1)\Delta-nW}$. Hence the smallest α that will induce effort is $\alpha = \frac{D-W}{\mathcal{A}+n\Delta-nW}$, and we have the negatively sloped portion of the effort constraint applicable when compensation takes the form of an option. ∎

Proof of Lemma 3:

Recall that the participation constraint is:

$$\alpha \geq \frac{R+D-W}{\mathcal{A}+n\Delta-W}. \tag{A3}$$

The sloping sections of the effort constraint in Figure 2 correspond to wage levels such that W is either less than $-\frac{D(\mathcal{A}+(n-1)\Delta)}{\Delta-nD}$ or greater than $\frac{\mathcal{A}+(n-1)\Delta}{n}$.

For $W < -\frac{D(\mathcal{A}+(n-1)\Delta)}{\Delta-nD}$, the effort constraint is:

$$\alpha \geq \frac{D-W}{\mathcal{A}+n\Delta-nW}. \tag{A4}$$

The righthand side of $(A3)$ always exceeds that of $(A4)$, and hence the two constraints cannot intersect at a wage level less than $-\frac{D(\mathcal{A}+(n-1)\Delta)}{\Delta-nD}$.

For $W \in (\frac{\mathcal{A}+(n-1)\Delta}{n}, \frac{\mathcal{A}+(n-1)\Delta}{n}+D)$ the effort constraint is:

$$\alpha \geq \frac{\frac{1}{n}(\mathcal{A}+n\Delta-nW)-(\Delta-nD)}{\mathcal{A}+n\Delta-nW}. \tag{A5}$$

The righthand side of $(A5)$ exceeds (is less than) that of $(A3)$ if $\frac{R+D}{D}\Delta > \mathcal{A}+n\Delta$ (if $\frac{R+D}{D}\Delta < \mathcal{A}+n\Delta$). When $\frac{R+D}{D}\Delta = \mathcal{A}+n\Delta$, the participation and effort constraints overlap exactly for all $W \in (\frac{\mathcal{A}+(n-1)\Delta}{n}, \frac{\mathcal{A}+(n-1)\Delta}{n}+D)$. Hence the two constraints cannot have a strict intersection at a wage level in the range $(\frac{\mathcal{A}+(n-1)\Delta}{n}, \frac{\mathcal{A}+(n-1)\Delta}{n}+D)$. ∎

Appendix B

The Bound on Firm Size Given a Mixed Strategy Equilibrium

Consider a contract consistent with a mixed strategy equilibrium in which each of n managers works with probability π. For each manager to be just indifferent between working and shirking, it must be that for all i:

$$\sum_{k=1}^{n-1} \binom{n-1}{k} \pi^k (1-\pi)^{n-1-k} C\left(\Omega_i \middle| \sum_{j=1}^{n} e_j = k+1\right) - D = \sum_{k=1}^{n-1} \binom{n-1}{k} \pi^k (1-\pi)^{n-1-k} C\left(\Omega_i \middle| \sum_{j=1}^{n} e_j = k\right). \qquad (B1)$$

Summing $(B1)$ over all i gives:

$$\sum_{k=1}^{n-1} \binom{n-1}{k} \pi^k (1-\pi)^{n-1-k} \left[C\left(\Omega_i \middle| \sum_{j=1}^{n} e_j = k+1\right) - C\left(\Omega_i \middle| \sum_{j=1}^{n} e_j = k\right)\right] = nD. \qquad (B2)$$

Ruling out principal moral hazard given an equilibrium in pure strategies really requires only that inequality (3) be satisfied. Ruling out principal moral hazard given an equilibrium in mixed strategies requires a stronger restriction that the principal's payoff be everywhere non-decreasing in firm value.

$$\mathcal{A} + k\Delta - \sum_{i=1}^{n} C\left(\Omega_i \middle| \sum_{j=1}^{n} e_j = k\right) \geq \mathcal{A} + (k-1)\Delta - \sum_{i=1}^{n} C\left(\Omega_i \middle| \sum_{j=1}^{n} e_j = k-1\right) \ \forall\, k = 1, \ldots, n. \qquad (B3)$$

Multiplying through inequality $(B3)$ by $\binom{n-1}{k} \pi^k (1-\pi)^{n-1-k}$, and then summing over all $k = 1, \ldots n-1$ gives:

$$\Delta \geq \sum_{k=1}^{n-1} \binom{n-1}{k} \pi^k (1-\pi)^{n-1-k} \left[C\left(\Omega_i \middle| \sum_{j=1}^{n} e_j = k+1\right) - C\left(\Omega_i \middle| \sum_{j=1}^{n} e_j = k\right)\right]. \qquad (B4)$$

Together the inequalities $(B2)$ and $(B4)$ imply that ruling out principal moral hazard in a mixed strategy equilibrium requires that

$$n \leq \frac{\Delta}{D}.$$

Thus choosing a contract consistent with a mixed strategy equilibrium does not allow the principal to hire a large number of managers.

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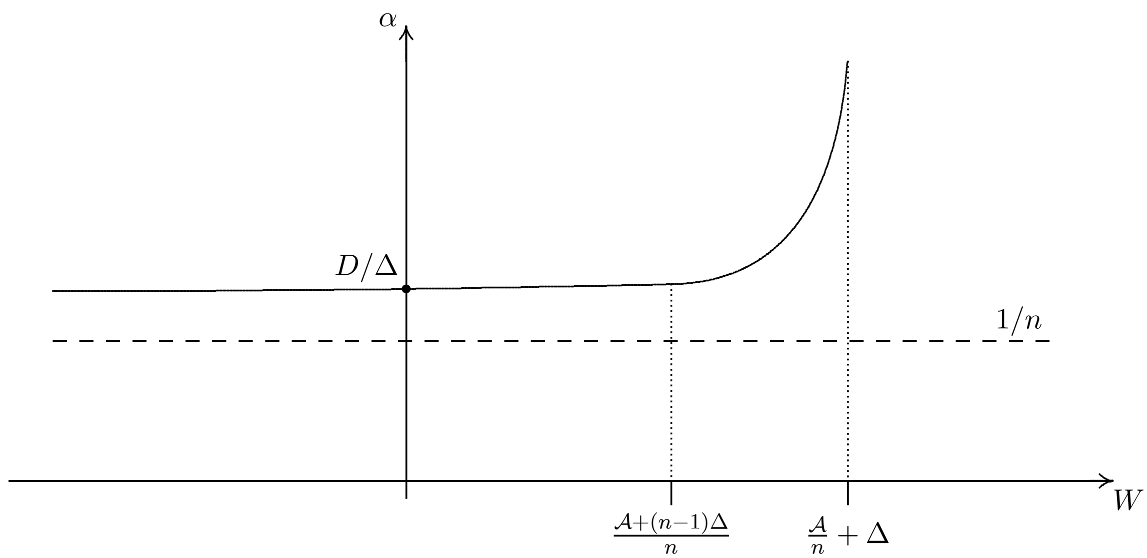

Figure 1. The effort constraint, given short-lived projects and large n. α is the manager's equity share. W is the wage. A negative wage corresponds to an executive stock option.

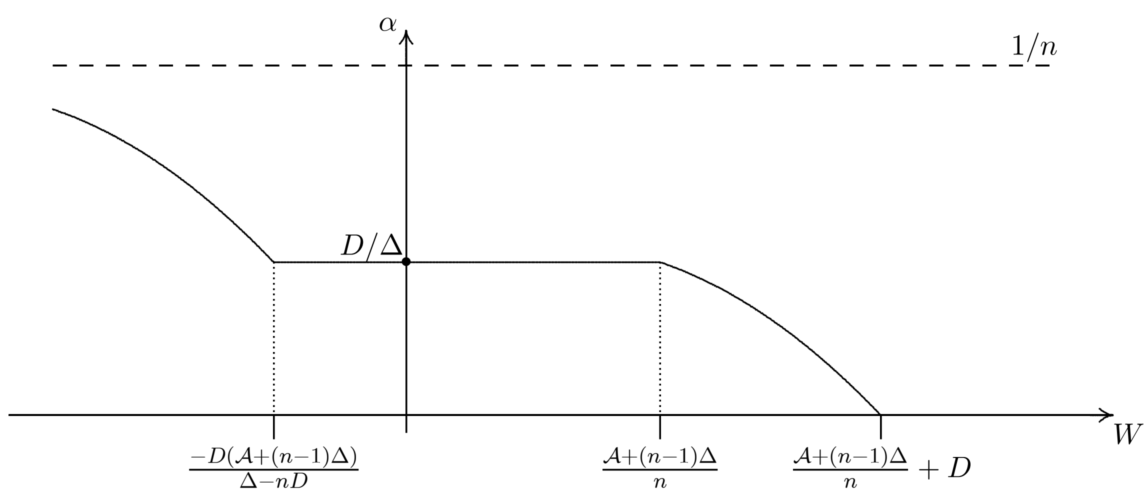

Figure 2. The effort constraint, given short-lived projects and small n. α is the manager's equity share. W is the wage. A negative wage corresponds to an executive stock option.

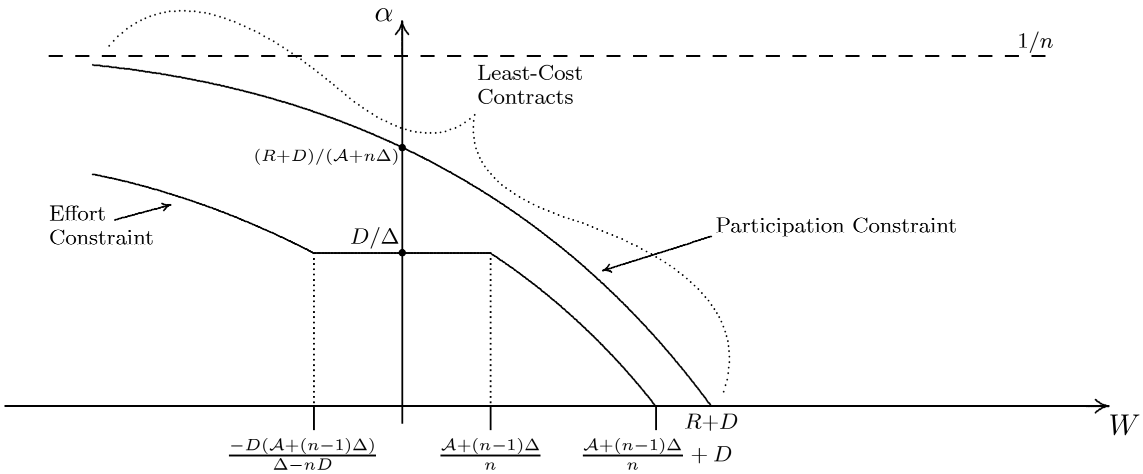

Figure 3(a) $\mathcal{A}+n\Delta < nR+\Delta$

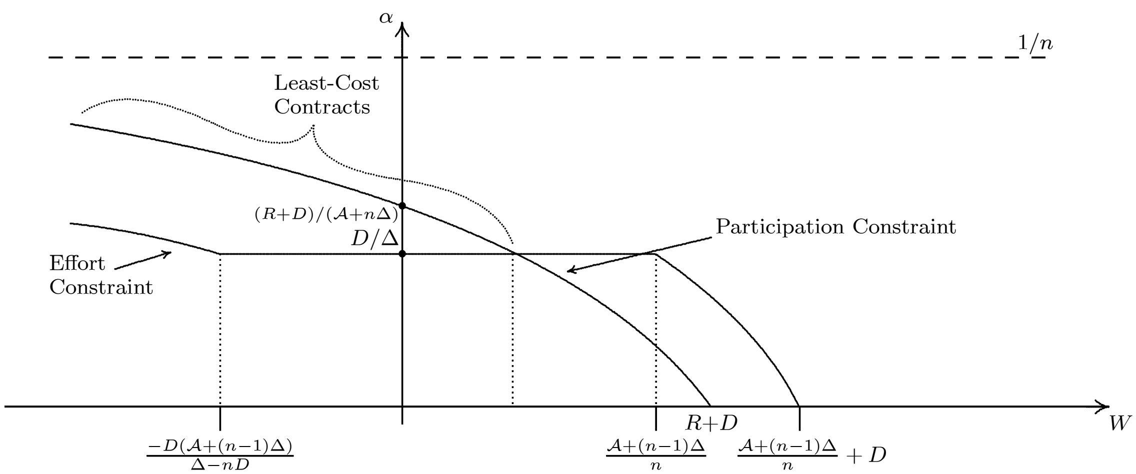

Figure 3(b) $nR+\Delta < \mathcal{A}+n\Delta < \frac{R+D}{D}\Delta$

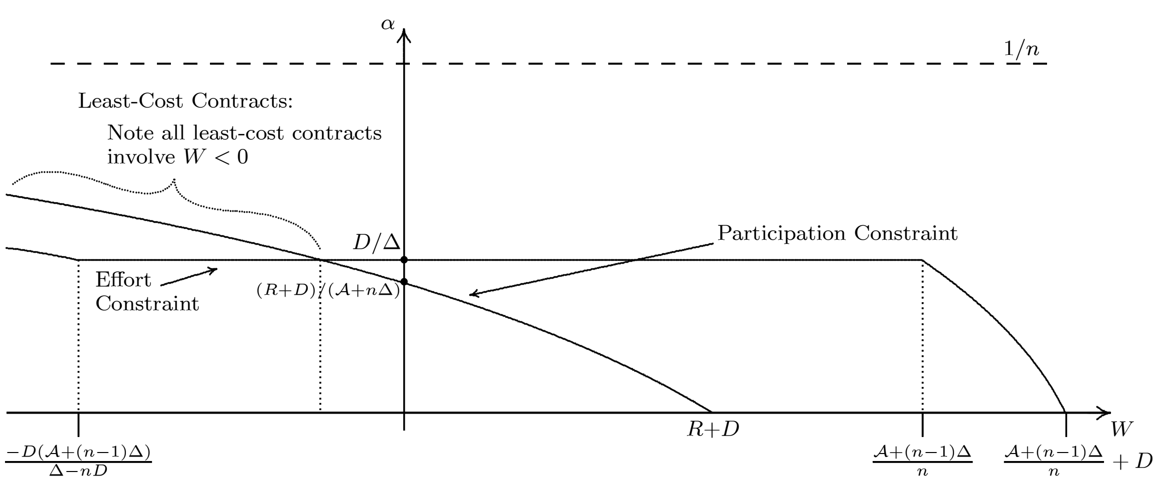

Figure 3(c) $\frac{R+D}{D}\Delta < \mathcal{A}+n\Delta$

Figure 3. Time 0 compensation contracts, $\{\alpha, W\}$, given short-lived projects and small n. α is the manager's equity share. W is the wage. A negative wage corresponds to an executive stock option.

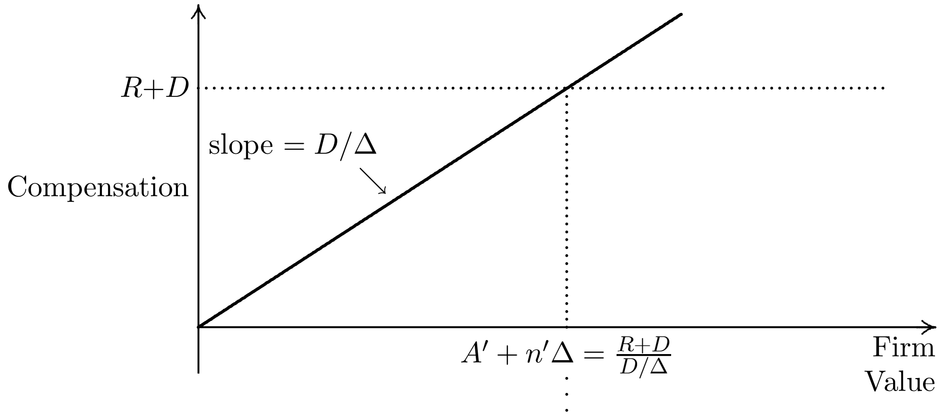

Figure 4A. The liquidation value of the assets-in-place, A', and the number of managers/projects, n', happen to be such that the payoff to a limited liability, least cost contract is everywhere weakly concave in firm value. A' and n' satisfy: $A' + n'\Delta = \frac{R+D}{D/\Delta}$.

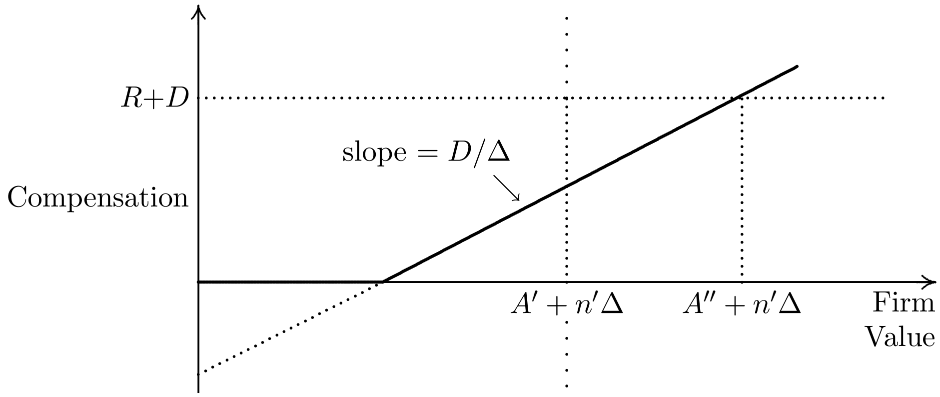

Figure 4B. The liquidation value of the assets-in-place, A'', is greater than the value of A' in Figure 4A. The payoff to a limited liability, least cost contract can not then be everywhere concave in firm value.

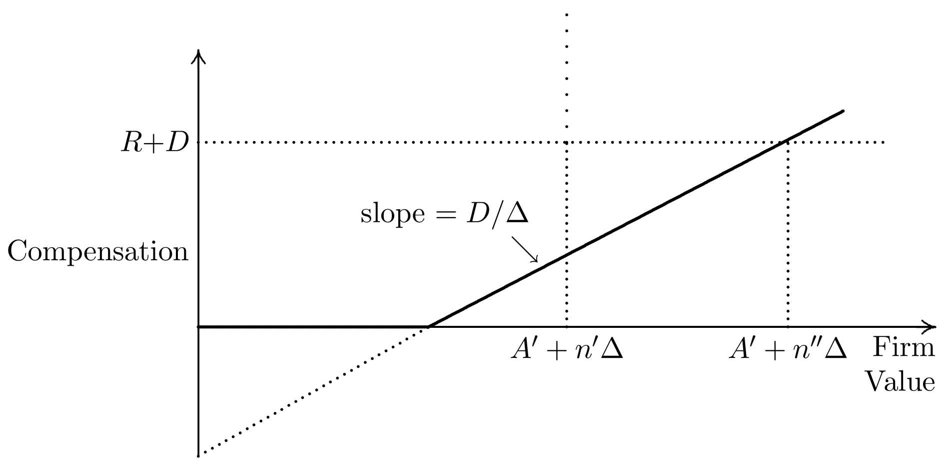

Figure 4C. The number of managers/projects, n'', is greater than the number n' in Figure 4A. The payoff to a limited liability, least cost contract can not then be everywhere concave in firm value.

Figure 4. Illustration of the link between the form of the payoff to a limited liability, least cost contract and the liquidation value of the assets-in-place, A, and the number of managers/projects, n. Whenever A and n are such that $A + n\Delta > (R + D)/(D/\Delta)$, then the payoff must be convex in firm value.

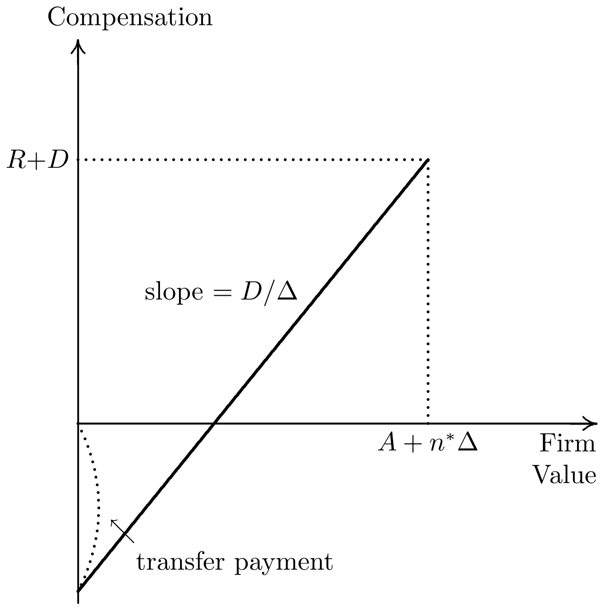

Figure 5A. Least-Cost Contract. Unique Equilibrium. Implements n^* projects. Violates Limited Liability.

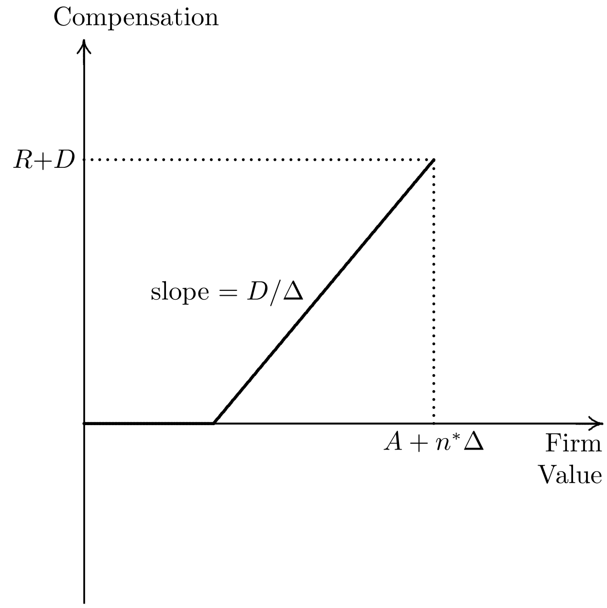

Figure 5B. Least-Cost Contract. Multiple Equilibria: One equilibrium implements n^* projects; in the other, no one signs. Managers have Limited Liability.

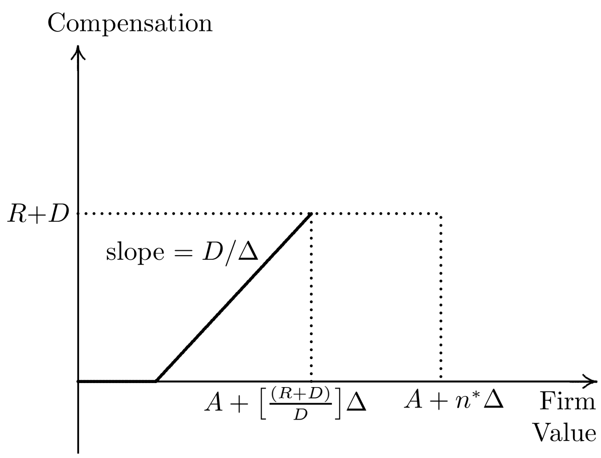

Figure 5C. Least-Cost Contract. Unique Equilibrium. Implements only $(R{+}D)/D$ projects. Managers have Limited Liability.

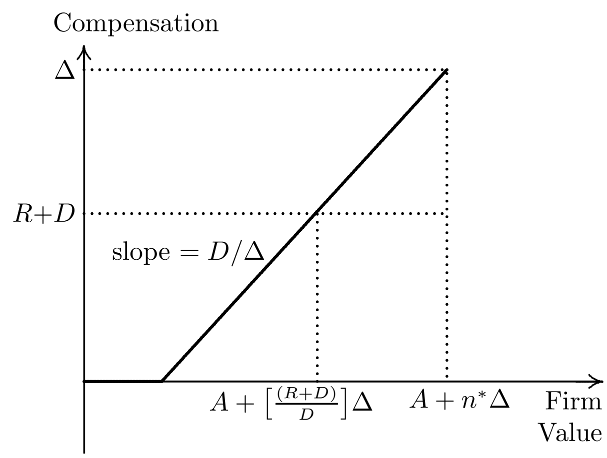

Figure 5D. Greater than Least-Cost Contract. Unique Equilibrium. Implements n^* projects. Managers have Limited Liability.

Figure 5. Contractual Payoff Functions and their Associated Properties. The maximum possible number of managers that can be motivated to work given a Nash equilibrium in which each manager conjectures that all other managers will work is n^* (Figures 5A, 5B and 5D). The maximum number of managers that can be hired at least-cost without violating limited liability such that the equilibrium is unique and all managers work is $(R{+}D)/D$ (Figure 5C). The various contractual payoffs depict settings with unique equilibrium in which all managers work (Figures 5A, 5C and 5D); a setting with multiple equilibria (Figure 5B); settings with limited liability (Figures 5B, 5C and 5D); and a setting which violates limited liability (Figure 5A).