Going Public with Asymmetric Information, Agency Costs and Dynamic Trading

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Going Public with Asymmetric Information, Agency Costs and Dynamic Trading

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Abstract

We study the problem of going public in the presence of moral hazard, adverse selection and multiple trading periods. In the multiperiod game managers strategically choose the level of extraction of private benefits and can develop a good reputation for expropriating low levels of private benefits. The costs of going public can be significantly reduced because of this reputation effect, and this can be an important factor in sustaining emerging stock markets that offer weak protection to minority shareholders. Also, allowing controlling managers to issue non-voting shares can increase the stock market efficiency, because the reputation effect is stronger when managers can divest more without losing control.

Introduction

This paper develops a theory of going public in the presence of moral hazard, adverse selection and dynamic trading. There is a large body of literature on the moral hazard costs of going public, that develops the idea that separation of ownership and control leads to a divergence of interests between managers and shareholders, because managers fully appropriate private benefits of control, but own only a fraction of the firm value (e.g., Berle and Means (1938), Jensen and Meckling (1976)). Another large part of the literature has studied the problems generated by selling equity when there are asymmetries of information between entrepreneur-managers and investors with respect to the value of the firm (e.g., Leland and Pyle (1977) and Rock (1986)). Most of the work in both areas is developed in a static framework, although, when going public, a firm's shares are traded in a stock market, and the implications of the existence of multiple trading periods are not explored. The contribution of this paper is to analyze what are the effects in a setup with both *moral hazard, adverse selection and multiple-trading periods* on the decision to go public and the firm market value and ownership structure.

Instead of focusing on the usual agency problem between the manager and shareholders, we emphasize the agency problem between the large shareholder and minority investors in the firm. Since the large shareholder in our model exerts full control over the management, for simplicity, we also refer to this large shareholder throughout the paper as the manager of the firm. We believe that this is the most relevant agency problem for many capital markets of the world (Shleifer and Vishny (1995)). The model developed addresses the following questions: Why do investors buy equity even when they do not get any control rights in exchange for their funds, and managers, being entrenched, are able to extract significant private benefits of control? How much should investors pay for shares, and what is the insider equity ownership and private benefits in equilibrium from the IPO and at every future trading period? Is there any role for managers' building a reputation for extracting low levels of private benefits?

We show that when stock markets are open after the company goes public and cash flow of the firm is realized across many periods, managers can implicitly commit to investors that they are not going to divert the cash flow of the firm. This reputation effect can reduce the inefficiencies caused by the moral hazard and adverse selection problems, improving the chances of going public and entrepreneurs' surplus from going public. It also increases the market value of the firm and alters the decision of insiders with respect to the amount of equity to hold in equilibria. The reputation effect serves to protect minority shareholders in the short and intermediate run even without having a formal legal framework giving them protection against expropriation. Therefore, the stock markets can start functioning even before fundamental improvements with respect to minority shareholder protection wait to take place in the future. The reputation effect is likely to be an important explanation for the empirical evidence in Singh (1995) about the heavy reliance of developing country firms on external funds and on public equity as a mechanism to finance growth opportunities, even though most of these developing countries still lack an appropriate legal framework protecting minority shareholders.

Our results have special empirical relevance for situations where managers can extract significant amounts of private benefits of control, when the reputation effect can be very significant in the equilibrium outcome. Environments where legal institutions do not offer enough protection to minority shareholders against managerial discretion, are believed to prevail in many of the world's capital markets (LaPorta, Lopez-de-Silanes, Shleifer and Vishny (1996)). Many empirical studies attempt to estimate the size of private benefits, and these studies unanimously conclude that private benefits are, on average, substantial for companies in many countries.[1] Even in the U.S., where shareholders have one of the best and most elaborate legal systems protecting their interests, there is substantial evidence documenting the discretion that managers have over investors' money, being able to waste much of it on private benefits.[2]

The main building blocks of the model can be shortly described as follows. The manager is risk-averse and initially is the sole owner of the firm. The entrepreneur-manager is motivated to sell shares of the firm in order to share the idiosyncratic risk of the firm with investors. Investors are not assured of a return on their investment, because the manager can extract private benefits from the firm, alienating minority investors (a moral hazard problem). Investors, however, having only a prior probability for the "private-benefit type" of manager, do not know how much private benefit the manager can extract (an asymmetric information problem). The model is dynamic, with both the firm's generating an uncertain future stream of cash flows over many periods and trading of shares also occurring over many periods. In the incomplete information model described above, the manager is able to build a reputation as in the models of Kreps and Wilson (1982) and Milgrom and Roberts (1982). A manager will act strategically when going public and after that, in order to credibly build a reputation for extracting low private benefits so as to sell shares at a high price. Even if investors assign a very low probability that a manager is of a good type (a high cost to extracting private benefits), she is still able to sell shares at a high price, because investors know that the manager is willing to develop a reputation for consuming low private benefits. If she started to extract high private benefits from the initial public offering (IPO), investors would discount the price of shares accordingly, and the manager's remaining shares would sell at a reduced price, a threat that is credible only if the manager has substantial equity ownership. We show that the reputation effect can play a very significant role in the equilibrium allocation through a range of numerical examples with several plausible parameter values.

In our theory, a manager holds concentrated equity ownership to provide a guarantee to investors that she is willing to build a reputation for low private benefits, which is different than Leland and Pyle's (1977) conclusion that a manager holds shares of the firm to signal to investors the information that the firm has a high market value (which is independent from the manager's action). Also, in Allen and Faulhaber (1989), Welch (1989), and Grinblatt and Hwang (1989) signalling of firm value is the explanation provided for the underpricing of new issues, because "it leaves a good taste" with investors, allowing the manager to sell future offerings at a higher price than would otherwise be the case. In our model, there are not only issues of signalling, but also moral hazard problems, since a bad type can act like a good type.

As an application of the model, we also address the following question: What are the benefits of allowing managers to concentrate control with a reduced amount of equity? Can a regulation that allows managers to dilute ownership, be efficient? We will show that in stock markets where managers derive significant private benefits of control, there is an additional benefit that has not been considered before in the literature, of issuing shares with restricted voting-rights, or more generally using various instruments to dilute ownership, such as pyramids and cross-shareholdings. We assume that managers in these markets will always want to retain majority control.[3] Since majority control can be guaranteed with a smaller equity ownership, when firms are allowed to issue restricted voting equity, managers are able to divest more equity without losing control. This gives them more room to build a reputation for extracting low levels of private benefits, because the mechanism that induces reputation is the prospect of future sales of shares. Therefore, a dual-class regime has an additional benefit over a one-share one-vote regulation, because of a more intense reputation effect. In fact, as we noted before, regulation in these stock markets often allows companies to issue two classes of equity—voting and restricted voting—and a majority of the firms in these stock markets do issue shares with differential voting rights (e.g., see Rydqvist (1992)).

Other related work in the literature is Bulow and Rogoff's (1989) model of sovereign country borrowing where sovereign countries cannot build a good reputation by making repayments and borrow primarily because of the threat of direct trade sanctions. The opposite results of our model are a consequence of our relaxing the full-information and risk neutrality specification of their model. Also, Diamond (1989, 1991) and John and Nachman (1985) developed a model of acquisition of reputation in debt markets with the same building blocks as our model (moral hazard, asymmetric information and dynamics), where reputation mitigates the conflicts of interest between borrowers and lenders. Differing from the original work of Kreps and Wilson (1982) and Milgrom and Roberts (1982), in our model, revelation does not occur at the very end of the game, and the equilibrium outcome depends on the prior belief of investors, even when the number of periods increases . Our model shares many features with existing work in the game theory literature.[4]

The remainder of the paper is organized as follows with all proofs in the appendix: Section I presents the model. Sections II and III study the costs associated with the moral hazard and adverse selection problems and analyze the static model. Section IV shows that the classic separating equilibrium no longer holds for the multiperiod game; presents an example in a 2-period game that conveys the idea of the model, and develops the general properties of the equilibrium in the multiperiod game. Section V analyzes the comparative static results and discusses the empirical and policy implications of the model. The paper ends with a summary of conclusions in Section VI.

I The Model

The problem is modeled as a stochastic dynamic game with incomplete information played by a risk-averse manager owner of a firm that generates an uncertain future stream of cash flows and investors in a competitive stock market. The action of the manager in every period is to trade shares and decide how much in private benefits to consume, and the investors' action is to bid for the offered shares. Investors have asymmetric information with respect to how much in private benefit the manager can extract (the private benefit type of the manager). Private benefits extracted during each period are assumed to be perfectly observable by investors, although they cannot be verified. It is important to note that although investors can observe current levels of private benefits, they do not know the private benefit type since a manager could be strategically extracting low levels of private benefits just to sell shares at a high price. Investors have a prior on the distribution of the manager's type.

Because the owner-manager is risk-averse, she is willing to share the risk with investors in the market. Investors, by forming a diversified portfolio, can place a higher value on the firm than the entrepreneur: investors only care about systematic or undiversifiable risk, while the entrepreneur, who has a large fraction of his wealth invested in the firm, is also concerned with the specific or idiosyncratic risk of the firm. Although there is a possibility of realization of gains from trading, the moral hazard and asymmetric information problems are an obstacle for sharing the risk and assessing the market value of shares. So, in our model the reason for going public is related not with primary equity offerings directed to finance investment and growth but to secondary equity offerings by owners who want to diversify their portfolio and to finance consumption. There is empirical evidence showing that secondary equity offerings might be a more important motivation for going public than primary equity offerings (see Pagano, Panetta and Zingales (1995), Rydvist and Hogholm (1995) and also Mayer (1990))

An important assumption of the model is that there is heterogeneity and asymmetric information with respect to private benefit types. The private benefit type of a manager is characterized as the amount of private benefits that maximizes his utility in the short-run, or in the absence of any long-run effects; it is the amount of private benefits that equates the marginal cost and the marginal benefit of an additional dollar of cash flow diverted. The cost of extracting private benefits depends both on the exogenous cost imposed by legal institutions, and on the manager's technology and/or disutility for extracting private benefits (e.g., feeling guilty for breaking the law). The motivation for asymmetric information in the model is that investors do not know the manager's technology and/or disutility for diverting cash flow. When the exogenous costs imposed by the legal system are low, not only is there more moral hazard, but there can also be more asymmetric information with respect to the manager type.[5]

In the model there are two observationally indistinguishable types of manager ($\theta = G$ and $\theta = B$), and each type can extract a constant fraction b_θ of cash flow at every period: type G manager is the good type, which for simplicity, is completely honest and never extracts private benefits ($b_G = 0$), and type B is the bad type that can extract up to a fraction b of cash flow ($b_B = b$, $0 < b \leq 1$). It is important to remark that a manager being, say of a 30% type, does not mean he will necessarily extract 30% of cash flow in private benefits in every period. It is at the heart of our model that he might want strategically to extract less in private benefits, while he still has a lot of shares, so as to convince investors that he is of a good type in order to be able to sell shares at a higher price. The 30% manager type only means that the most the manager can extract in private benefits is 30% of the cash flow. Although investors cannot distinguish the two types of managers, they have a prior probability μ that the manager is of a good type ($\Pr(\theta = G) = \mu$). Also, we assume that the manager is not able to commit not to extract private benefits by writing an enforceable contract restricting extraction of private benefits. We will assume that such enforceable contracts are too costly to write.[6]

The manager is initially the sole owner of the firm. The firm generates an uncertain cash flow for T periods $(\tilde{y}_t)_{t=1}^{T}$. Before going public at $t = 1$ the firm is wholly owned by the entrepreneur-manager. We assume, for simplicity, that the cash flow is independently distributed across time.[7] The stochastic process is assumed to be common knowledge both to the manager and investors in the market. The extensive form of the game is described below (see also Figure 1).

Insert Figure 1 here

Specification of the Extensive Form. At each period t there are two stages. At the first stage of period t, the previous history of the game is known to all players and is summarized in the history h^t. The manager moves by offering to trade a block of shares Δ_t or, equivalently, choosing a new fraction of equity ownership α_t (both are related by $\Delta_t = \alpha_{t-1} - \alpha_t$). The trading order is then auctioned in a competitive stock exchange market where many investors bid/ask for shares. Trading is realized at the market clearing price P_t (for 100% of shares), where shares are bought by investors offering prices higher than P_t or sold by investors asking prices lower than P_t. At the second stage of period t, before the random value y_t of cash flow is realized, the manager decides whether he is going to consume private benefits or not. Finally, at the end of period t cash flow is realized and the firm earns $e_t = y_t - B_t$, which is then distributed to shareholders as dividends. Investors, after observing the manager's extraction of private benefits and announced earnings, update their prior beliefs with respect to the manager's type (we assume throughout the paper, for simplicity, that all investors have the same prior beliefs about future levels of private benefits and update homogeneously their beliefs after every action of the manager). The game continues as described above at the next period, $t+1$, with the history at the next stage updated to $h^{t+1} = \{h^t, (\alpha_t, P_t, y_t, B_t)\}$ and with investors having a new belief obtained from updating posterior belief from the previous period).

The game described above is a stochastic dynamic game which is not a repeated game: the game played at period t is dependent on the state variable α_{t-1}. A description of the strategy profiles, manager's and investors' payoff function, and the equilibrium concept used throughout the paper follows.

Strategy Profile. Each type θ manager at the first stage has a pure or mixed strategy for selling shares, which is specified by the function or distribution $\Delta_t^\theta(h^t)$ or, equivalently, by $\alpha_t^\theta(h^t)$ that represents the new equity ownership of the manager. At the second stage the good type is of a commitment type and never extracts any private benefits, and the bad type has a pure or mixed strategy given by the probability distribution $B_t(h^t, \alpha_t, P_t, y_t)$. Strategy for investors is simply to quote a price on shares given by the function $P_t(h^t, \alpha_t)$. We will see that the multiperiod game might not have any equilibrium in pure strategy, so the introduction of mixed strategies might be necessary.

Specification of Manager's Payoff Structure. The manager's income at the end of period t is, $(\alpha_{t-1} - \alpha_t) P_t(h^t, \alpha_t) + \alpha_t(y_t - B_t) + B_t$, where the first term is the amount earned or spent by trading shares, the second term is the security payout, and the last term is the private benefits extracted. We assume that the risk-averse manager has negative exponential utility (for simplicity, the coefficient of risk-aversion is $a = 1$) and consumes his total wealth at the end of the game. Type θ manager objective function is to maximize the utility at every period t

$$\begin{array}{ll} \max\limits_{\{\alpha_s, B_s\}_{s=t}^{T}} & E_t\left[-\exp\left[-\left(\sum\limits_{s=t}^{T}(\alpha_{s-1} - \alpha_s) P_s(h^s, \alpha_s) + \alpha_s(\tilde{y}_s - B_s) + B_s\right)\right]\right] \\ \text{s.t.} & 0 \leq \alpha_s \leq 1 \\ & 0 \leq B_s \leq b_\theta \cdot \tilde{y}_s \end{array}$$

(manager's problem)

We emphasize that the manager is assumed not able to commit to a long term strategy, or commit today to play a predetermined strategy in the future. The short term nature of the problem or the lack of commitment is an essential feature of the model.

Investors' Payoff. Investors are risk neutral (investors diversify the idiosyncratic risk of the firm) and maximize expected return. A share of stock pays $\tilde{e}_t = \tilde{y}_t - B_t$. The return to investors at time t is $\widetilde{R}_t = \frac{e_t + P_{t+1}}{P_t}$. Investors maximize expected return

$$E\left[\widetilde{R}_t\right] = E_t\left[\frac{e_t + P_{t+1}}{P_t}\right]$$

which depends not only on the actions of the manager at time t but also on how investors will update their beliefs about the manager type. We will also assume that the market for investment in securities is competitive, and for simplicity, interest rate is equal to zero. Then, it must be the case that at every time t the expected value of $\tilde{R}_t$ is equal to 1, given manager's strategies and investors' beliefs. The competitive market condition, zero interest rate and risk-neutrality of investors imply the following restriction on prices:

$$P_t = E_t\left[\tilde{e}_t\right] + E_t\left[\tilde{P}_{t+1}\right], \text{ or}$$
$$P_t = E_t\left[\tilde{e}_t + \ldots + \tilde{e}_T\right] \qquad \text{(competive condition)}$$

Equilibrium Concept. The equilibrium concept used throughout the paper is perfect Bayesian equilibrium (PBE) with the refinement assumptions introduced in the next section. At the first stage of period t of the game, after having observed the manager trading of shares, investors update their belief about the manager being of a good type: $\Pr(\theta = G) = \rho_t(h^t, \alpha_t)$, and at the second stage, after observing the manager extraction of private benefits, investors update their beliefs again: $\Pr(\theta = G) = \mu_t(h^t, \alpha_t, P_t, y_t, B_t)$. At both stages of the game investors update beliefs using Bayes rule.

II Moral Hazard and Adverse Selection Costs

This section briefly describes the costs of going public associated with the existence of the moral hazard and adverse selection problems. These costs are in addition to the direct and indirect costs of going public, which are all borne ex-ante by the entrepreneur. The direct costs include underwriting expenses, legal expenses and registration fees, and the indirect costs are associated with the underpricing of shares at the IPO.[8] The dollar value of the direct plus the indirect costs will be referred to as a fixed cost C. Since the purpose of this paper is to model the moral hazard and adverse selection costs, we will abstract from the direct and indirect costs in most of our analysis, assuming it is zero ($C = 0$) except when otherwise explicitly noted.

Without the costs associated with the moral hazard and adverse selection problems the firm would go public with the manager achieving *the first best outcome*. The first best outcome is the outcome where the risk-averse manager diversify all the risk with risk-neutral investors. Since private benefits are assumed to depend on cash flow, it is also risky, and any outcome where the manager extracts private benefits is not the first best. The payoff to the manager, or the value for which the manager is able to sell the firm to investors in the first best, is given by $P_T = \overline{w} = E_T\left[\sum_{t=1}^{T} \tilde{y}_t\right]$ or $\overline{w} - C$ if we take into account the direct and indirect costs of going public.

In case the costs are severe enough, then the firm might not even go public and stay as a closely-held-concern. In the *outcome with no market for shares,* the manager is not able to share any of the risk with investors. In that situation the value the firm is $\underline{w}$ such that $-\exp(-\underline{w}) = -E_T\left[\exp\left(-\sum_{t=1}^{T}\tilde{y}_t\right)\right]$ or $\underline{w} = -\log\left(E_T\left[\exp\left(-\sum_{t=1}^{T}\tilde{y}_t\right)\right]\right)$, which is the value of the firm as a closely-held-concern. The reservation level of the manager, or the minimum at which he would be willing to sell the firm, is $\underline{w}$.. When the manager cannot transfer the risk to investors there is a loss of efficiency, since by Jensen's inequality, $\overline{w} > \underline{w}$.

Let σ be any Nash equilibrium where investors break even and the two types of managers maximize their payoff. Let w_G^σ and w_B^σ be the certain equivalent payoff to the good and bad manager, respectively. The costs (or amount forgone) associated with moral hazard and adverse selection is formally $\overline{w} - w_\theta^\sigma$.

Capital markets where managers can extract significant private benefits are expected to be very inefficient, in the sense that stock markets cannot accomplish one of their main roles: to share risk across owners and investors. If investors expect to receive only a fraction of the firm's cash flow, because managers can extract a fraction for themselves, then investors will not be willing to pay much for shares. In any market equilibrium, the diversification benefit of going public is greatly reduced because there is an unavoidable loss of efficiency.

Lemma 1 *In the presence of moral hazard and asymmetric information, the first best outcome cannot be implemented by any Nash equilibrium σ. The bad manager can derive rents at the expense of the good manager in equilibrium, $w_B^\sigma \geq w_G^\sigma$. The firm with manager of type θ goes public if and only if there is an equilibrium σ such that $w_\theta^\sigma - C \geq \underline{w}$.*

The intuition for this simple result is that in any equilibrium, either the risk-averse good manager or the risk-averse bad manager will be holding some of the risk. Recognizing the fact that investors break even, we conclude that the first best is never achievable, or $\mu w_G^\sigma + (1-\mu) w_B^\sigma < \overline{w}$. A bad manager derives a rent from being bad without investors knowing that he is bad. In any equilibrium with sale of shares he can always do better than the good manager that is not known to be good. In any equilibrium $w_B^\sigma \geq w_G^\sigma \geq \underline{w}$ and the last inequality comes from the fact that $\underline{w}$ is the reservation level of the managers. We will see later on examples where the bad manager gets more than the first best outcome: $w_B^\sigma \geq \overline{w}$.

This can occur when the asymmetric information parameter μ is close to 1, the good manager is willing to sell a large block of shares for the insignificantly discount $(1-\mu)\, bE\left[\sum_{t=1}^{T} \tilde{y}_t\right]$.

III The Equilibrium in the One-Period Game

The equilibrium strategies for the standard static or one-period model is developed in this section and will be used in the following sections as a benchmark to compare the equilibrium in the multiperiod game.

The static game is very similar to the Spence's job market signalling model, Spence (1974), and Leland and Pyle (1977). Here managers move in the first stage offering a fraction of shares to sell, $1-\alpha$, and then investors bid in a Bertrand fashion. Managers anticipate some pricing function $P(\alpha)$ (to simplify notation we drop the subscripts in the analysis of the static game). As in the Spence game, here perfect Bayesian equilibrium (PBE) is a very weak concept and there are many strategies and beliefs that are PBE. Many of the equilibria seem implausible because of the beliefs associated with out-of equilibrium beliefs. A number of refinements have been proposed that attempt to formalize plausible restrictions on the out-of equilibrium beliefs. For signalling games the most commonly used refinement is the intuitive criterion of Cho and Kreps (1990), which in most cases selects the Riley separating equilibrium.[9] Although, as argued by Mailath, Okuno-Fujiwara and Postlewaite (1993) the intuitive criterion refinement and the equilibrium it selects can be implausible, since in some cases the pooling equilibrium is a more natural equilibrium. They propose a belief-based refinement –undefeated equilibria– that addresses some of the problems. In our view, the Mailath, Okuno-Fujiwara and Postlewaite (1993) refinement leads to a more sensible equilibrium concept for our model. Although, we do not develop the undefeated equilibrium concept we claim that it provides a formal framework justifying the equilibrium selection based on efficiency we use next. We describe in the following paragraphs the separating and pooling equilibrium, and when each one is the most plausible equilibrium solution. The pooling equilibrium is the equilibrium selected when the probability that the manager is of a good type is above a threshold level and the separating equilibrium is selected otherwise.

For a given discounted price P, $(P < E[\tilde{y}])$, the good manager is not willing to completely diversify and will want to keep some shares. The fraction of shares the good manager is willing to supply at price P is determined by the solution of the following problem:

$$\max_{\Delta\in[0,1]} E\left[-\exp\left(-\left(\Delta \cdot P + (1-\Delta)\,\tilde{y}\right)\right)\right] \tag{1}$$

The solution is represented as the supply curve in Figure 2. Note that we can also solve for the supply function of the bad manager in a similar form. The bad manager has a lower valuation for shares and, for the same price, is willing to supply more shares. We note, also, that the single-crossing property holds: the indifference curves of the bad manager intersect the indifference curves of the good manager with a steeper slope (Figure 2).

Insert Figure 2 here

In a separating equilibrium, the good type manager sells shares at a price equal to $\overline{P} = E[\tilde{y}]$ (which is the expected value of shares if the manager is known to be good) and the bad type manager sells shares at a price $\underline{P} = (1-b)\,E[\tilde{y}]$ (which is the expected value of shares if the manager is known to be bad). The additional restriction that must be satisfied by a separating equilibrium is the incentive compatibility condition: the bad type prefers to sell all shares at $\underline{P}$ rather than to sell a few shares at $\overline{P}$. In order for this restriction to be satisfied, the maximum fraction of shares that investors can buy at $\overline{P}$ is fewer than or equal to the intersection of the indifference curve of the bad type with the horizontal line at $P = \overline{P}$ (represented by Δ_S in Figure 2). Observe that a separating equilibrium with sale of some shares always exists (provided that $b < 1$) and is independent of the value of μ. We remark that the higher the size of the moral hazard (b), the bigger the inefficiency in the separating equilibrium outcome.

It is usual to restrict investors' beliefs such that investors should infer that the manager is a good type if she sells less than Δ_S, since the bad manager would never play this strategy because he can get more by selling all shares at the discounted price $\underline{P}$. With this assumption in place, the only separating equilibrium must be the one that gives the good manager exactly the utility of selling Δ_S shares at $\overline{P}$ and the bad manager the utility of selling all shares at $\underline{P}$. This efficient separating equilibrium is also called the Riley equilibrium.

In a pooling equilibrium both managers choose to sell the same fraction of shares. The equilibrium price must then be $P_\mu = E[\tilde{y}] \cdot [\mu + (1-b)(1-\mu)]$ in order to satisfy the competitive condition, since there is a probability $1-\mu$ that the manager is bad and is always going to extract a fraction b in private benefits in the last period. At a price P_μ the good manager is willing to supply at most Δ_μ shares. Assuming that investors' beliefs are monotonic in the amount of shares retained by the manager, no point to the right of the supply curve of the good manager is chosen in equilibrium: the good manager can sell fewer shares for at least the same price, and she strictly prefers to do so. Note that if the efficient separating equilibrium is attainable then in any pooling equilibrium the good manager must get at least the utility level of selling $1-b$ shares for $E[\tilde{y}]$. Let μ^* be given by $P_{\mu^*} = P_S$, where the price P_S corresponds to the intersection of the supply curve of the good manager and her indifference curve through the separating equilibrium (see Figure 2).[10] If $\mu < \mu^*$ then there is no pooling equilibrium, and the good manager in this case is always willing to separate himself from the bad manager.

In case $\mu > \mu^*$, there can be many pooling equilibria. Any fraction of shares from Δ_μ to the point Δ'_μ represented in Figure 2 can be sustained as a pooling equilibrium for a given monotonic belief of investors. When there is a pooling equilibrium it is more efficient (strictly preferred by both the good manager and the bad manager) than the separating equilibrium. We can summarize the results in the following proposition.

Proposition 1 *The most efficient equilibrium in the static game can either be the separating equilibrium where the good manager sells $\Delta_S(b)$ shares at $E[\tilde{y}]$ and the bad manager sells 100% of shares at $(1-b)E[\tilde{y}]$ or, when $\mu \geq \mu^*(b)$, the pooling equilibrium where both managers sell Δ_μ at a price P_μ, where $\Delta_S(b)$, μ^*, Δ_μ and P_μ are given as above.*

For the static case, the decision of the firm to go public when there are fixed costs $C < \overline{w} - \underline{w}$ (potential gain from going public higher than the fixed costs) depends on the extent of the moral hazard problem (b) and asymmetry of information (μ). The decision to go public is characterized by the following result.

Corollary 1 *The firm can go public in a static game depending on the degree of moral hazard (b) and asymmetric information (μ). One of three situations can occur depending on*

the fixed costs of going public (C) :
(i) The firm does not go public if $b > \overline{b}$ *and* $\mu < \underline{\mu}(b)$;
(ii) Only the firm with the bad management goes public if $b \in \left(\underline{b}, \overline{b}\right]$ *and* $\mu < \underline{\mu}(b)$;
(iii) The firm can go public regardless of the management if either $b \leq \underline{b}$ *or* $b > \underline{b}$ *and* $\mu \geq \underline{\mu}(b)$.
The values of $\underline{b}$, $\overline{b}$ *and* $\underline{\mu}$ *are given by* $w_B^S\left(\overline{b}\right) - \underline{w} = C$ *and* $w_G^S(\underline{b}) - \underline{w} = C$ *and* $w_G^P\left(\underline{\mu}, b\right) - \underline{w} = C$, *where* w_θ^S *and* w_θ^P *are the values that the type* θ *manager get, respectively, in the efficient separating and pooling equilibria.*

The firm with a manager of type θ will go public if there can be an equilibrium such that $w_\theta^g - C \geq \underline{w}$. In proposition 1 we have seen that the most that a manager of type θ can get is either w_θ^S or w_θ^P which depends on the parameters b and μ. The three cases above correspond to the conditions under the parameters that satisfy $w_\theta^g - C \geq \underline{w}$. For example, it can be the case that when the moral hazard and adverse selection are significant enough the firm might not be able to go public (case (i) above).

We will see that even if the firm cannot go public in the static game it is still possible that it can go public when there are multiple trading periods. The addition of many trading periods can reduce the inefficiencies caused by the moral hazard and adverse selection problems and increase the gains from going public.

IV The Multi-Period Game

The first major difference from the equilibrium in the static game and the multiperiod game is that the extension of the standard separating equilibrium of the static game in general even fails to exist in the multiperiod game when the managers cannot make long-term commitments not to trade shares in the future.

A very natural and weak restriction to impose on the PBE is that once investors are convinced that the manager is of a good type or bad type for sure than the equilibrium price of shares is fixed respectively at $\overline{P}_t = E_t\left[\sum_{\tau=t}^{T} \widetilde{y}_\tau\right] = \sum_{\tau=t}^{T} \overline{y}_\tau$ or $\underline{P}_t = (1-b)\sum_{\tau=t}^{T} \overline{y}_\tau$ where $E_t\left[\widetilde{y}_\tau\right] = \overline{y}_\tau$. So, in particular, when the posterior beliefs are either $\mu_{t-1} = 0$ or 1, then the equilibrium pricing function is independent of the number of shares offered.

Assumption 1: *The share price must be* $\overline{P}_t = \sum_{\tau=t}^{T} \overline{y}_\tau$ *or* $\underline{P}_t = (1-b)\sum_{\tau=t}^{T} \overline{y}_\tau$ *if investors believe that the manager is, respectively, good or bad with probability 1.*

To simplify the notation, we define the function $w_t(.)$ as follows

$$w_t(\alpha) = -\log E\left[\exp\left(-\alpha\widetilde{y}_t\right)\right] \tag{2}$$

which represents the certainty equivalent utility of a fraction α of the $t-$period cash flow $\widetilde{y}_t$.

When assumption 1 holds and managers cannot commit to a future trading strategy, the precise conditions under which revelation of types can occur in the multiple-trading-period game are expressed by the next proposition.

Proposition 2 *(i) There exists separating equilibrium for the multiperiod game if and only if the level of moral hazard b satisfies*

$$b < \frac{\overline{y}_1 - w_1(1)}{\sum_{t=2}^{T} \overline{y}_t} \tag{3}$$

(ii) More generally, the good manager can signal her type at stage 1 of any period t of the game, when she previously owns α_{t-1} *shares, if and only there exists a solution* α_t^*

$$\alpha_t^*\overline{y}_t - \max\left\{w_t(\alpha_t^*), w_t(\alpha_t^* + b(1-\alpha_t^*)) - b\overline{P}_{t+1}\right\} = \alpha_{t-1}b\overline{P}_t - w_t(b) \tag{4}$$

with $\alpha_t^* \in [0,1)$. *The good manager can reveal her type by withholding more than* α_t^* *shares.* *(iii) There is revelation of the manager's type at period t when there is no stealing at stage 2 if and only if share ownership is less than* α_t^{**}, *the unique solution of*

$$\alpha_t^{**}b\overline{P}_t = w_t(\alpha_t^{**} + b(1-\alpha_t^{**})) - w_t(\alpha_t^{**}) \tag{5}$$

In particular, $\alpha_t^{**} < \frac{\overline{y}_t}{\overline{P}_t}$.

Condition 3 for the existence of a separating equilibrium states that the moral hazard problem must be lower than the gain from selling the first period cash flow for its expected value over the future expected value of cash flows. When there are many trading periods and/or the cash flow at period 1 is relatively small with respect to future cash flows there will be no separating equilibrium except for insignificant levels of moral hazard. The intuition for the result is that the payoff for the bad manager following separation is very high if he imitates the separation strategy, since after revelation, the manager is able to sell all the remaining shares without discount and then extract private benefits after selling. If the fraction of private benefits the bad manager can extract is large then even if investors buy no shares, it is not possible to separate the bad manager from the good manager. The bad manager would prefer not to sell any shares and wait for the next period where he could sell all shares without discount.

We note that in any equilibria there can never be separation in the first stage after the initial period. Suppose that there is an equilibrium where there is separation at the first stage of period t ($t > 1$). The bad manager in equilibria should anticipate the separation in the first stage of t, and therefore to maximize his utility, should extract private benefits in the second stage of period $t-1$, in which case separation occurs in period $t-1$, which contradicts the fact that separation can occur at period t.

The expression for revelation at the second stage (private-benefit decision node) states that separation will occur only when the manager owns few enough shares so that the gain from diverting today equates the maximum additional overpricing ($b\overline{P}_t$) of the remaining shares owned by the manager.

A Refining the equilibria concept

Since the standard separating equilibrium fails to exist in the multiperiod game one has to look for alternative equilibrium with pooling. To restrict out-of-equilibrium beliefs we will impose a Markov or stationarity assumption, and a monotonicity assumption, in addition to the conditions of the PBE and assumption 1. These two technical assumptions have been used in the bargaining literature to address the problem of multiplicity of equilibria.[11]

A perfect Bayesian equilibrium allows strategies and beliefs to be functions of the entire history h^t. We will use the Markov perfect concept, or stationarity assumption, to restrict strategies and beliefs to be only functions of payoff relevant variables, or state variables, for the players.[12] Our main state variables will be the previous manager ownership of shares and previous investors' belief about manager type (probability of being a good manager).

Hence both manager's and investors' actions and beliefs are the same for any histories h^t and $\overline{h}^t$ that have the same relevant state variable. We note that the state variables at each of the two stages of the game are not the same. We now introduce a formal definition of the stationarity property.

Assumption 2 (Stationarity): *A strategy profile σ and investors' belief are stationary. Stationarity holds if and only if for any history h^t with equity ownership and posterior beliefs α_{t-1} and μ_{t-1} : (i) the manager strategy at the first stage of period t, $\alpha_t(h^t)$, only depends on α_{t-1} and μ_{t-1} and not on the previous history; (ii) the investor's pricing strategy, $P_t(h^t, \alpha_t)$, and updating rule at the first stage of any period t, $\rho_t(h^t, \alpha_t)$, depends only on α_t, α_{t-1} and μ_{t-1}, $P_t(h^t, \alpha_t)$ (iii) the investors' updating rule at the second stage, given that $B_t = 0$, is $\mu_t(h^t, \alpha_t)$ and depends only on α_t, ρ_t.*

Therefore, for stationary strategies, we can summarize the strategies and the updating rule in the condensed form:

$$\alpha_t^\theta\left(\alpha_{t-1}, \mu_{t-1}\right), P_t\left(\alpha_t, \alpha_{t-1}, \mu_{t-1}\right), \rho_t\left(\alpha_t, \alpha_{t-1}, \mu_{t-1}\right), B_t\left(\alpha_t, \rho_t\right), \mu_t\left(\alpha_t, \rho_t\right)$$

(stationary strategies and beliefs)

It will be useful to define $V_t^\theta\left(\alpha_{t-1}, \mu_{t-1}\right)$ as the expected utility of manager of type θ in the continuation game for a given equilibrium strategy σ. Throughout the paper we also use $\underline{V}_t^\theta$ and $\underline{P}_t$ ($\overline{V}_t^\theta$ and $\overline{P}_t$) to represent the utility of a manager of type θ and price of shares when investor believe $\rho_t = 0$ ($\rho_t = 1$). The other technical assumption that is convenient to use is the monotonicity assumption.

Assumption 3 (Monotonicity): *The stationary posterior belief $\rho_t\left(\alpha_t, \alpha_{t-1}, \mu_{t-1}\right)$ is monotonic in the state variables: it is non-decreasing in α_t, μ_{t-1} and non-increasing in α_{t-1}.*

Before we move on to analyze the general properties of the multiperiod game it is useful to solve for a PBE in a 2-period game. The role played by the refinement assumptions on the PBE are highlighted and the main differences from the static game and the dynamic game are pointed out in the example.

B A 2-Period Example

We believe that the main idea of the paper can be grasped by comparing the equilibrium of the static game with the equilibrium in a 2-trading-period game. We will see how the existence of many trading periods allows for managers playing strategically to build a reputation for consuming low levels of private benefits and its effect on the price of shares and on managers' equity ownership.

Consider for simplicity the following 2-period game: the bad manager is of a type $b = 1$ and the good is of a type $b = 0$; there is a probability μ_0 that the manager is of a good type; cash flow at period t is $\widetilde{y}_t$ with $\overline{y}_t = E[\widetilde{y}_t]$ and denote as before the certainty equivalent value of a fraction α of the cash flow at period t by $w_t(\alpha)$ as in equation 2. For comparison purposes, the static game corresponding to the 2-period game is the one with cash flow $\widetilde{y} = \widetilde{y}_1 + \widetilde{y}_2$.

B.1 A Perfect Bayesian Equilibrium of the Game

We are going to construct a monotonic and stationary PBE by backwards induction. We will start by analyzing the last period of the game.

In the last period of the game... Suppose that the history at the second period is h^2, in which the manager owns a fraction α_1 of the shares, and that investors have a posterior μ_1 (obtained after investors update the prior μ_0 in the first period).

The equilibrium in the continuation game is similar to the static game with strategies and beliefs specified as follows: at a price P_2 the good manager sells $\Delta_2(\alpha_1, \mu_1)$ solution of $\max_{\Delta_2 \in [0,\alpha_1]} E[-\exp(-(\Delta_2 \cdot P_2 + (\alpha_1 - \Delta_2)\widetilde{y}_2))]$ which can be $\Delta_2 = 0$ if P_2 is small; the bad manager imitates the good manager and divert in the last period; investors' strategy is to sell shares for $\overline{y}_2$, buy at a price $P_2 = \mu_1 \overline{y}_2$, up to $\Delta_2(\alpha_1, \mu_1)$ shares and pay zero for shares if the manager tries to sell more than $\Delta_2(\alpha_1, \mu_1)$.

Investors' strategy is to buy or sell shares for their expected worth, $P_2 = E[\widetilde{e}_2]$. If the manager offers to buy shares investors would only sell for $\overline{y}_2$, since only an honest manager would be willing to buy shares in the last period, and if the manager is honest for sure, that is how much the shares are worth. In other words, the buying decision of the manager in the last period reveals her honest type and share prices appreciate immediately once this information is revealed. If the manager offers to sell a fraction Δ_2 of shares, and investors believe that both the honest and dishonest manager would be selling Δ_2 shares with probability 1, then investors buy for price $P_2 = \mu_1 \overline{y}_2$. If they believe that only the dishonest manager would be selling Δ_2 shares, then the price of shares is zero.

In the first period of the game... We start by analyzing the decision problem of the bad manager at the second stage of the first period when the cash flow is revealed and the manager decides how much in earnings to pay to the shareholders and how much in private benefits to extract.

Second Stage of the first period. Of course, the decision problem of the honest manager is not to divert any of the realized cash flow (she is of a commitment type at this stage of the game although she is active at the first stage of every period).

The decision of the bad manager depends on the strategies for the continuation game specified in the previous paragraph and how investors update their beliefs after observing no stealing at period 1. The bad manager knows that the most he can gain from not stealing is $w_1(\alpha_1) + \Delta_2(\alpha_1, 1) \cdot \overline{y}_2 + w_2(1)$ and that he can get $w_1(1) + w_2(1)$ from stealing. So if $\Delta_2(\alpha_1, 1) \cdot \overline{y}_2 \geq w_1(1) - w_1(\alpha_1)$ or the most that he can get is less than the value forgone from not diverting then he is going to divert for sure. Investors update the posterior probability of being a good type, consistent with the strategy above for the bad manager, and so $\mu_1 = 1$ when there is no stealing and the inequality above is satisfied.

On the other hand, the manager knows that if he does not divert, the posterior probability of investors will be at least equal to the prior probability μ_0. If the minimum he can get is greater than the value from stealing or if $\Delta_2(\alpha_1, \mu_0) \cdot \overline{y}_2 \geq w_1(1) - w_1(\alpha_1)$ then he should not divert. Investors will not be fooled by increasing the probability that the manager is of a good type after observing no diversion given that the manager owns α_1 shares and the prior is μ_0 and so $\mu_1(\alpha_1, \mu_0) = \mu_0$.

What happens for intermediate values α_1 when $\Delta_2(\alpha_1, \mu_0) \cdot \overline{y}_2 < w_1(1) - w_1(\alpha_1) < \Delta_2(\alpha_1, 1) \cdot \overline{y}_2$? If the strategy of the manager is not to divert for sure, then investors should not change their beliefs after observing no diversion but then the manager can do better by diverting today. Assume, on the contrary, that his strategy is to divert with probability 1. Then a Bayesian investor, after observing no diversion, will conclude that the manager is honest for sure and update the beliefs to $\mu_1 = 1$. If investors behave that way, however, the manager should not divert. We conclude that for any value of α_1 in the interval above, there is no equilibrium in pure strategies. We can show that the only equilibrium strategy for the dishonest manager is not to divert with probability $\beta_1 = \beta_1(\alpha_1, \mu_0)$ and investors update their beliefs according to $\mu_1 = \mu_1(\alpha_1, \mu_0)$, given by the following expressions

$$\begin{array}{l} \Delta_2(\alpha_1, \mu_1) \cdot \overline{y}_2 = w_1(1) - w_1(\alpha_1) \\ \beta_1 = \frac{\rho_1}{1-\rho_1} \frac{1-\mu_1}{\mu_1} \end{array} \tag{6}$$

whenever $\Delta_2(\alpha_1, \mu_0) \cdot \overline{y}_2 < w_1(1) - w_1(\alpha_1) < \Delta_2(\alpha_1, 1) \cdot \overline{y}_2$.

First stage of the first period. How much will investors pay for a fraction α_1 of shares? Let's assume for a moment that both types of managers choose α_1 with probability 1 and that investors believe that the manager is good with probability $\rho_1(\alpha_1, \mu_0) = \mu_0$. Investors should then offer $P_1(\alpha_1) = E[e_1 \mid \alpha_1] + E[e_2 \mid \alpha_1]$ for shares. It is simple to verify that given the strategies for the continuation game specified above then $E[e_1 \mid \alpha_1] = (\mu_0 + (1-\mu_0) \cdot \beta_1(\alpha_1, \mu_0))\overline{y}_1 = \frac{\mu_0}{\mu_1(\alpha_1, \mu_0)}\overline{y}_1$ where β_1 and μ_1 are, respectively, the probability that the bad manager is not going to divert cash flow and the posterior probability, both determined by equation 6. Because at the last stage the bad type steals for sure then $E[e_2 \mid \alpha_1] = \mu_0 \overline{y}_2$. So the price function is

$$P_1(\alpha_1) = (\mu_0 + (1-\mu_0) \cdot \beta_1(\alpha_1, \mu_0))\overline{y}_1 + \mu_0 \overline{y}_2 \tag{7}$$

As in the analysis of the static game, investors should be concerned with how many shares the good manager is willing to sell for the discounted price $P_1(\cdot)$. For example, with monotonic beliefs a good manager will not supply all shares for a discounted price, if she was able to sell less shares for at least the same price, which by the monotonicity assumption is always possible. Considering $V_2^G(\alpha_1, \mu_1)$ as the expected utility of the good manager in the continuation game, the maximum fraction of shares that the good manager will be willing to offer solves the maximization problem:

$$\max_{\alpha_1 \in [0,1]} E\left[\exp\left(-\left(\left(1-\alpha_1\right) P_1\left(\alpha_1\right)+\alpha_1 \tilde{y}_1\right)\right) \cdot V_2^G\left(\alpha_1, \mu_1\left(\alpha_1, \mu_0\right)\right)\right] \tag{8}$$

and let α_1^* be the solution of the problem.

We are now ready to complete the specification of our PBE strategies and beliefs for investors and managers: investors buy shares for price $P_1(\alpha_1) = \frac{\mu_0}{\mu_1(\alpha_1,\mu_0)}\overline{y}_1 + \mu_0\overline{y}_2$ and have a posterior belief of $\rho_1(\alpha_1, \mu_0) = \mu_0$ if $\alpha_1 \geq \alpha_1^*$, and otherwise for $P_1(\alpha_1) = 0$ and with posterior equal to $\rho_1(\alpha_1, \mu_0) = 0$ if $\alpha_1 < \alpha_1^*$; the optimum response for both type of manager when faced with the pricing function $P_1(\cdot)$ is to sell out $1 - \alpha_1^*$. The strategies above form a PBE, since it is easy to show that, by an extension of the single-crossing property, the bad manager will be willing to sell at least the same amount of shares as the optimum for the good manager.

B.2 Characteristics of the equilibrium

The first interesting difference from the equilibrium in the 2-period game and the static game is that the share price can be higher in the former game. The price in static game is $P_\mu = \mu_0(\overline{y}_1 + \overline{y}_2)$ since the bad manager always divert all the cash flow, and the increase in share price in the 2-period game is $\Delta P_1 = (1-\mu_0)\beta_1\overline{y}_1$. This increase occurs because there is a chance that the bad manager is not going to divert in the first period so as to mimic the behavior of the good manager, motivated by the opportunity to sell more shares in the next period. This increase in price is caused by what we call the reputation effect.

Also, observe that the good manager in her choice of divestiture in the first period, trades off risk diversification and a higher share price at the second period against a higher share price at the first period. Divesting more at the first period leads to more risk sharing and a higher price in the future. The price faced in the second period by the good manager (or price conditioned on no stealing) is $P_2 = \mu_1\overline{y}_2$ and μ_1 is increasing in the amount of shares sold at the first period (Δ_1). On the other hand, $P_1 = (\mu_0 + (1-\mu_0)\cdot\beta_1)\overline{y}_1 + \mu_0\overline{y}_2$ is decreasing in Δ_1 since the probability of no stealing β_1 is decreasing in Δ_1.

The welfare of the good manager in the 2-period game is unambiguously higher than in the static game: not only can the manager sell the same amount of shares as before for a higher price, but also she has the additional opportunity to sell more shares in the next period for a price that depends on how investors update their beliefs after observing no extraction of private benefits. In fact the manager might sell more (or less) shares than in the static game when the benefits of risk sharing and a higher share price in the future are greater (lower) than the benefits of a higher share price today. This trade off is going to be key to explain the comparative statics of the gains from going public and the dynamics of share ownership and price with respect to changes in the number of trading periods and the moral hazard and asymmetric information parameters.

The main result of the paper is that the inefficiencies caused by the moral hazard and adverse selection problems are reduced in the multi-period game because of the reputation effect. When there are additional trading periods after the company goes public and cash flow of the firm is realized across many periods, managers are able to develop a reputation for not extracting private benefits by the possibility of additional share sales. Managers can implicitly commit to investors that agency costs will be reduced.

C Properties of the multiperiod game equilibrium

The general idea used in the construction of the 2-period PBE also works for the multiperiod game. We first develop some general properties that must hold for any PBE and then establish the existence of PBE. Note that there exists a PBE even when there is no separating equilibrium for the multiperiod game.

Lemma 2 *Necessary conditions for strategy profile σ and system of beliefs (ρ_t, μ_t) to be a stationary PBE are:*

(i) the posterior probability μ_t, given that there is no stealing at stage 2 of period t, is related to the $t+1$ continuation game by

$$\mu_t(\alpha_t, \rho_t) = \begin{cases} 1 & \text{if } G_t(\alpha_t, 1) < 0 \\ \mu_t^* & \text{if } G_t(\alpha_t, \mu_t^*) = 0 \\ \rho_t & \text{if } G_t(\alpha_t, \rho_t) > 0 \end{cases} \tag{9}$$

where G_t is the additional gain from no divertion relative to diverting at period t :

$$G_t(\alpha_t, \mu_t) = V_{t+1}^B(\alpha_t, \mu_t) E\left[\exp\left(-\alpha_t \widetilde{y}_t\right)\right] - E\left[\exp\left(-\left(\alpha_t + (1-\alpha_t) b\right) \widetilde{y}_t\right)\right] \cdot \underline{V}_{t+1}^B(\alpha_t)$$

(ii) the probability of not extracting private benefits at period t, β_t, is related to the updating rule at the second stage by Bayes rule: $\beta_t = \frac{\rho_t}{1-\rho_t}\frac{1-\mu_t}{\mu_t}$;
(iii) the equilibrium pricing function is related to investors beliefs ρ_t by:

$$P_t\left(\alpha_t, \alpha_{t-1}, \mu_{t-1}\right) = \left[\delta_t \overline{y}_t + (1-\delta_t)\, \underline{y}_t\right] + \left[\delta_t P_{t+1} + (1-\delta_t)\, \underline{P}_{t+1}\right]$$

where $\delta_t = \frac{\rho_t}{\mu_t}$, or $\delta_t = \rho_t + (1-\rho_t)\beta_t$, $\underline{y}_t = (1-b)\,\overline{y}_t$ and $\underline{P}_{t+1} = \sum_{s=t+1}^{T} \underline{y}_s$ and P_{t+1} is the equilibrium price in the continuation game if there is no stealing at t.

Items (i) and (ii) state that given the strategy profile for the continuation game starting at period $t+1$ then the strategies at stage 2 of period t are determined. The decision with respect to private benefits and how investors should update their beliefs μ_t and β_t are determined by expression 9 and Bayes rule. Key in the determination of μ_t is the function G_t, which depends on $\underline{V}_{t+1}^B(\alpha_t)$ and $V_{t+1}^B(\alpha_t, \mu_t)$, respectively, the expected utility of the bad manager in the continuation game if the bad manager steals or not at period t. G_t expresses the value added to the utility of a bad manager from not diverting the cash flow at period t. So a positive (negative) G_t means that the bad manager utility from cooperating, which depends on the strategies for the continuation game, is higher (lower) than his utility from diverting and revealing himself.

Also, determined in a stationary PBE is the price response of investors for a given belief and strategies for the continuation game. The discount factor δ_t represents the probability of no stealing at period t. When the probability of the divertion is $\beta_t = 1$ then investors are sure to receive the discounted cash flow $\overline{y}_t$ without any discount, and $\delta_t = 1$. In the other extreme case where $\beta_t = 0$ investors discount the cash flow by the probability they believe the manager is of a good type, and $\delta_t = \rho_t$. The price function in item (iii) states that with probability δ_t there is no stealing in period t and investors get $\overline{y}_t$ and the firm value is P_{t+1} next period. With probability $1-\delta_t$ there is stealing and investors get $\underline{y}_t$at period t and the firm value is $\underline{P}_{t+1}$ next period.

There still remains room for a multiplicity of stationary PBE since the out-of equilibrium beliefs of investors ρ_t remains unrestricted. Once these beliefs are specified, though, the equilibrium strategies are determined. In the multi-period game there might not exist any equilibria, differently from the static game where there is always at least a separating equilibria. One needs to show first that there, in fact, always exists an equilibrium satisfying the refinement assumptions imposed.

Proposition 3 *There exists a PBE satisfying assumptions 1-3 for any multiperiod game, which can be obtained constructively by backwards induction.*

The idea of the proof is similar to the procedure used to construct the equilibria in the 2-period example above. Starting from the last period of the game with any given pair of state variables $\left(\alpha_{T-1}, \mu_{T-1}\right)$ we specify the efficient PBE equilibrium for the static continuation game as in proposition 1. Working by backwards induction and using the result of Lemma 2 the strategies of the bad manager and investors' updating rule at the second stage of period $T-1$ is uniquely determined. In order to complete the equilibrium, a set of belief ρ_{T-1} and price P_{T-1} together with strategies for the managers that are consistent with PBE and satisfies the incentive compatibility and participation constraints need to be obtained. By Lemma 2 once the beliefs and strategies for the continuation game are specified the unique price function P_{T-1} that is consistent with PBE is determined. Using the flexibility to select the belief ρ_{T-1} an equilibrium can be obtained that can be either pooling or separating. We note that, at both stages one and two, there might not be equilibrium in pure strategies and that the flexibility in the choice of the beliefs gives rise to a multiplicity of equilibria. The equilibrium that was selected in the construction of the equilibrium in Proposition 3 is the one that maximizes the utility of the good manager at every period subject to the strategies played in the continuation game.

D Comparative Statics

We use the interactive procedure developed in the proof of proposition 3 to obtain PBE solutions satisfying the refinement assumptions for a range of parameter values. Since a closed-form analytical solution for the equilibrium is not available for the game with many periods, we developed a computer program to obtain numerical solutions.[13] The relevant parameters for comparative statics purposes are the number of trading periods of the game (T), the asymmetric information parameter (μ_0) and the moral hazard parameter (b).

The game used for comparative statics is defined as $\Gamma\left(T, \mu_0, b, (\widetilde{y}_t)_{t=1}^T\right)$. The t-period cash flows $\widetilde{y}_t$ are such that $\sum_{t=1}^T \widetilde{y}_t = \widetilde{y}$, with the $\widetilde{y}_t's$ independent and identically distributed. We can compare the utilities in equilibrium for different $T-$period games since all have the same total cash flow $\widetilde{y}$. For example, the first best outcome and the outcome with no market for shares are the same for any T.

We used in our numerical results a coefficient of risk aversion $a = 1$ and a distribution $\Gamma(1,1)$ for the distributions of cash flow $\widetilde{y}$: with this specification the stochastic process $(\widetilde{y}_t)_{t=1}^T$ is such that $\widetilde{y}_t$'s are independent and have a gamma distribution $\Gamma\left(\frac{1}{T}, 1\right)$.[14] Note that the value of the firm in the first best outcome is $\overline{w} = \$1$ and the value in the case with no market for shares where the firm does not go public is $\underline{w}= \$0.69$.

Since the gain from going public and equilibrium outcome are directly dependent on both the coefficient of risk aversion and the degree of riskiness of the cash flow, it is appropriate to normalize the measures of value gain by the potential gain from going public. Therefore, value added of going public in an equilibrium σ is measured as $\frac{w^\sigma - \underline{w}}{\overline{w} - \underline{w}}$.

The numerical results are represented graphically in Figures 3 and 4. Each panel in Figure 3 represents the utilities of the good manager for different number of trading periods (ranging from 2 to 20 trading periods) and for the same level of moral hazard and asymmetric information (utilities are represented by the percentage gain from going public with respect the potential gain). The dashed line in Figure 3 represents the utilities in the static game. The dynamics of the equilibrium for the multiperiod game is represented in Figure 4. The horizontal axis in the Figure is calendar time starting from the IPO until the firm is liquidated. We used a game with 20-trading periods to illustrate the results, but games with different number of trading periods have similar paths. Each panel in Figure 4 represents both the share ownership and the expected price that investors receive given that there is no stealing until the previous period. This is the same price that a good manager gets in a pooling equilibrium. The moral hazard and asymmetric information parameter values were chosen ranging from low, medium and high values, respectively, $b = .2$, $.5$, 1 and $\mu_0 = 30\%$, 50%, 80%.

A list of our numerical results with an intuitive explanation and their empirical implications are discussed in Section V.

E Moral Hazard and Adverse Selection Unavoidable Inefficiencies

Differently from other reputation models in the literature, though, the inefficiency is not completely eliminated as the number of trading periods increase. In many dynamic game theory models in the literature with moral hazard and asymmetric information there is convergence towards efficiency when the number of trading periods increases to infinity (this result is known as the Coase conjecture).[15] The moral hazard and asymmetric information problems impose an efficiency loss in our model that does not approach zero even when the number of trading periods is increased and the cash flow is spread across many periods. So it is not surprising that in our numerical results the addition of trading periods does not lead to an increase in the gain from going public beyond a certain threshold level.

Proposition 4 *No equilibrium σ of any $T-$period version of the game with given parameters b, μ and $\tilde{y}$ converges to the first best outcome. The upper bound on the level of efficiency, $\sup_{\sigma} \{\mu w_G^{\sigma} + (1-\mu) w_B^{\sigma}\}$, is strictly lower than the first best outcome $\overline{w}$.*

In the first best outcome, the manager must be able to completely diversify the risk of the firm selling all shares to investors for the present value of expected cash flow without discount. In order for investors to break even when buying without discount, it must be the case that a bad manager has incentives not to extract private benefits, otherwise investors would face a loss. As we noted before, the only incentive that a bad manager has for not extracting private benefits is the threat of not selling additional shares for a high price. Thus, there cannot be a market equilibrium where managers diversify completely the risk with investors, since any such equilibrium violates the incentive compatibility condition for the bad manager.

V The Reputation Effect: Corporate Finance Implications

A Empirical Implications

We derive the empirical implications of the comparative statics results. The hypotheses developed are based on the numerical results obtained in Section IV.D for the multiperiod game

$\Gamma\left(T, \mu_0, b, (\tilde{y}_t)_{t=1}^{T}\right)$. We are interested in seeing how the equilibrium outcome can change with the lengthening of the number of trading periods, T, for different values of the moral hazard parameter, b, and the asymmetric information parameter, μ_0. The endogenous equilibrium variables that are of special interest, are the manager's utility from going public and the path of stock price and share ownership from the IPO until the last trading period. Our goal is to analyze how these equilibrium variables are affected by changes in the parameters of the game. The main empirical implications that come out of the model, are summarized in the following hypotheses:

H1. *The firm is more likely to go public and the value of the firm is higher, the lower is the moral hazard problem and the higher is the prior probability that the manager is good.*

H2. *The firm is more likely to go public in the multiperiod game than in the one-period game.*

H3. *The reputation effect is more intense when the moral hazard problem is more severe and there is more asymmetry of information.*

H4. *At the IPO, a block of shares is divested, and following the IPO, shares are gradually divested.*

H5. *Shares are divested faster and the revelation period occurs sooner, the lower is the moral hazard problem and the higher is the prior probability that the manager is good.*

H1 is the most straightforward result, and we can prove that it holds in general. For example, the gain from going public ($\frac{w^{\sigma}-\underline{w}}{\overline{w}-\underline{w}}$) represented in the panels of Figure 3 for $T = 20$, goes from 28% to 75% as the moral hazard parameter goes from $b = 1$ to 0.2 for $\mu = 0.5$. Also, the gain from going public goes from 35% to 75% as the asymmetric information parameter goes from $\mu = 0.3$ to 0.8 for $b = 0.5$.

Insert Figure 3 here

H2 is illustrated by Panel 3.F where the dashed line is at the zero level, indicating that in the static game, the good manager does not gain anything from going public, while in the 20-trading period game, the good manager can gain almost 30% of the total potential gain from going public. The introduction of additional trading periods enables the bad manager to develop a reputation of cooperation with minority shareholders, that leads to an increase in the likelihood of going public. It is interesting to note that the addition of a few trading periods is enough to achieve a significant gain in welfare, and that the introduction of more trading periods does not contribute much after a few periods. Also, the increase in trading periods does not necessarily lead to a monotonic increase in the good manager's utility.[16]

Note also that it can be the case that the addition of multiple periods can be harmful when both b and μ_0 are small (Panels 3.A and 3.B). This can happen because the static game has a separating equilibrium which is relatively efficient when b is small although in the multiple period game, the separating equilibrium might not exist as we have seen in Proposition 2. and for small values of μ_0 the gain from the reputation effect might not be strong enough to compensate for the loss of the separating equilibrium. Although we cannot claim that there is no PBE equilibrium in, for example, the 2-period game that dominates the one-period game equilibrium, we conjecture that this can be the case when both b and μ_0 are small.

To understand the intuition behind *H3*, note that from lemma 2, the gain in share price in the multi-period game with respect to the price in the static game is, $\Delta P_1 = \frac{b}{T}\sum_{t=1}^{T}\left(\frac{\mu_0}{\mu_t} - \mu_0\right)$, where μ_t is the equilibrium path for the posterior probabilities. For a given μ_0, a higher gain in share price can be induced by reputation when b is big at the expense of the same amount forgone in terms of risk sharing. This implies that the gap in welfare from the static and multiple period games can be larger for bigger b. The same reasoning applies to the case where b is fixed and μ_0 varies (see changes across vertical panels in Figure 3). For μ_0 close to zero, a gain in price can only be achieved at the expense of essentially no diversification of risk except at the very end of the game, so only a small increase in utility can be achieved for a given number of trading periods. At the other extreme, for μ_0 close to one, ΔP_1 is close to zero and essentially no additional gain can be obtained from reputation. The gain from reputation is more significant, though, when there is more asymmetry of information (μ_0 close to $\frac{1}{2}$).

Conditioned on there being no diversion of cash flows, there is no uncertainty with respect to manager share ownership and price, and both follow the path depicted in Figure 4 for the equilibrium strategies we consider. If the manager is of a bad type, though, there is some probability that he is going to reveal himself by extracting private benefits in the early periods after the IPO. Investors then learn that the manager is of a bad type, and the market value of the firm moves to $\underline{P}_t = (1-b)\sum_{\tau=t}^{T} \overline{y}_\tau$, or to $(1-b)$ in Figure 4. Therefore, the equilibrium path of share divestiture and price is not deterministic. Note that the path of share ownership and price of *H4* is not always, respectively, monotonic decreasing and increasing. The price of shares can be driven downwards because it can be optimum for the good manager to sell out more shares to diversify risk with investors at the cost of a reduction in the force of the reputation effect. Also, it can well be the case that in the equilibrium outcome the good manager wants to buy shares to convince investors that he is of the good type.

Insert Figure 4 here

H5 can be compared with the static game in which shares retained by the manager signal his type, and the size of the block divested is also negatively correlated with the moral hazard problem. For a given μ_0, less gain in share price can be induced by reputation when b is small, so it is better to trade off some of the reputation gain for more diversification of risk through a faster divestiture of shares. This also leads to a quicker revelation of type, because after a minimum threshold level of shares is reached (given by proposition 2), there is revelation (horizontal panels in Figure 4). The reputation effect and risk diversification trade-off also explains why shares are divested faster and revelation occurs sooner for a fixed b as μ_0 increases (vertical panels in Figure 4). As μ_0 increases, the share price gain from stimulating reputation by future sales of shares decreases, and it pays off to diversify more of the risk immediately at the IPO, leading to a bigger block being divested initially.

The hypothesis above can be tested using cross-section regression analysis. Ideally, one would like to test these theories using data where the levels of moral hazard and private benefits are significant, and where there is a large shareholder in control. Also, a successful testing of the model will depend crucially on finding good proxies for the levels of moral hazard and asymmetric information. Due to the novelty of the model presented and the fact that the empirical relevance of the results are most applicable for countries other than the U.S., it is not surprising that we could not find related tests of the model in the literature.

However, we believe that the results Singh (1995) are consistent with an extended version of our model. The main point in Singh (1995) is that developing countries rely heavily on external financing and particularly on external equity financing. Singh (1995) concluded that there are important differences with respect to the financing patterns of developing countries and the industrial countries studied by Mayer (1990). We believe that a slightly modified model in which the motivation for going public is not exclusively risk sharing but also raising equity to undertake growth opportunities, yields results consistent with Mayer (1990) and Singh (1995). A firm with higher growth opportunities is expected to be able to raise equity at a lower cost than a firm with lower growth opportunities, because it can build more reputation for reducing the moral hazard and asymmetric information costs of a primary security issue. This reinterpretation and extension of the model is likely to be particularly important in understanding the workings of emerging stock markets, where we believe that the moral hazard and asymmetric information problems are particularly intense. The significant growth opportunities encountered in most of the emerging markets (including transitional economies), are expected to reduce the costs of going public.

Another interesting extension is to consider that stock markets in developing countries have weak legal protection in the short run but in the longer run are expected to converge towards offering more protection to minority shareholders–this could easily be modeled considering a time varying level of private benefits b. Stock markets, although imperfect in the short run, can still work as an important mechanism in financing investments and diversifying risk due to reputation effects.

This effect can be important for the integration of West European countries and East European countries that are moving away from communism, both in the process of reassessing which structure of financial institutions to adopt. One important choice they face is with respect to the emphasis to place on stock markets (including equity and bond markets) as opposed to banks for providing finance. Allen (1992) argues that, for the advanced economies of Western Europe, a model favoring the importance of stock markets is recommended, while Eastern Europe should concentrate on developing bank-based financial systems.[17] The effect mentioned in the previous paragraph, leads us to opposite conclusions from the ones obtained by Allen (1992). The significantly higher growth opportunities in East European than in West European countries, implies that stock markets could work more efficiently in reducing agency costs and informational problems in Eastern Europe than in Western Europe.

B Dual-Class Shares vs One-Share One-Vote

The reputation effect also has policy implications for the debate about the use of the dual-class shares voting structure. We show that there is a potential benefit from having a dual-class structure, that has not yet been addressed in the literature. The main idea is that when managers can issue restricted voting shares, they are able to develop more reputation for not expropriating minority shareholders, which can increase the value of the firm. As we have seen in the previous sections, the mechanism that allows for the reputation effect to work, is the prospect of selling additional shares in the future for a high price; therefore, when managers are allowed to issue shares with restricted voting rights, the effect can work better since there are more shares for the managers to sell while still keeping majority control of the firm.

We use the assumption that managers of public firms want to retain majority control of the firm when the capital markets allow then to extract significant private benefits of control. There is some empirical evidence supporting this assumption.[18] To this empirical evidence, one can add the widespread occurrence of family-controlled business among public companies in many of the emerging markets of the world and less developed European stock markets (e.g., Portugal, Spain and Italy).

The regulation of the voting-structure directly affects the minimum percentage of equity needed to maintain majority control, say $\underline{\alpha}$, or equivalently, the maximum amount of shares managers are willing to divest, $1-\underline{\alpha}$. For example, if one-share one-vote were the prevailing regulatory policy , then one would expect that the manager would divest up to 50% of equity and retain majority control with the other 50% ($\underline{\alpha} = 50\%$). If issuing up to 50% of non-voting equity were allowed, then managers could divest up to 75% of equity while still retaining control, with the remaining 25% of equity corresponding to 50% of votes ($\underline{\alpha} = 25\%$). In fact, many of the world's stock markets commonly allow the issuance of up to 50% of non-voting shares.[19]

Using the framework developed in the previous sections, we can also obtain an equilibrium similar to the one in proposition 3 where managers are restricted to holding a minimum of $\underline{\alpha}$ shares.

Corollary 2 *If managers are restricted to holding* $\underline{\alpha}$ *shares, then there exists a PBE satisfying assumptions 1–3.*

The reputation effect of dual-class shares can be quantified using the same numerical procedure to compute the equilibrium, that was developed in the previous section. For example, the gain from going public for the good manager is 60%, 52% and 41%, respectively, for the cases where $\underline{\alpha}$= 0, 25% and 50%, in the 20-trading period game considered in the previous section, with parameter $b = 1$ and $\mu_0 = 80\%$. The gains are respectively, 42%, 42% and 37% for the static game with the same parameters. The reputation effect then accounts in each of the three cases for, respectively, 18%, 10% and 4% of the increase in the gains from going public in the multiple trading period game. Clearly, the improvement in efficiency that the reputation effect can bring about, diminishes as one moves from a dual-class share voting structure to a one-share one-vote structure.

VI Concluding Remarks

In this article, we develop a model of a firm that is going public in the presence of moral hazard, asymmetric information and multiple trading periods. The agency problem emphasized in the paper is not the standard agency problem between managers and shareholders, but the conflicts between a large shareholder that can extract private benefits of control, and minority investors in the firm. The large shareholder exerts control over the management, so for simplicity, we refer to this large shareholder as the manager of the firm. The motivation for the owner-manager to take the firm public, is to share the idiosyncratic risk of the firm with minority investors. However, all the participants in the market recognize that there is room for the manager to act opportunistically, expropriating minority investors. Managers face the problems of whether to take their firms public and how many shares to offer, and investors face the problem of how much to pay for shares, given the information available to them.

The equilibrium with multiple trading periods has important differences with respect to the static case. In the multiperiod game, the manager can strategically choose extraction of private benefits and can develop a reputation for expropriating low levels of private benefits. Even if investors assign a low probability that a manager is of a good type, the manager is still able to sell shares at a high price, because investors know that she has an incentive to develop a reputation for consuming low levels of private benefits. If she starts to extract high private benefits from the IPO, investors will discount the price of shares accordingly, and her remaining shares will sell at a reduced price.

We show that the reputation effect can play a very significant role in the equilibrium allocation, through a range of numerical examples with several plausible parameter values, and derive some empirical implications for the model. Because of the reputation effect, the costs of going public are significantly reduced, and the firm is more likely to go public selling at a higher price. Also, at the IPO, a block of shares is divested, and following the IPO, shares are gradually divested. Shares are divested faster and the revelation period occurs sooner, the lower is the moral hazard problem and the higher is the prior probability that the manager is good.

We believe that this model has empirical relevance in understanding the workings of stock markets in emerging economies, where despite the very weak legal institutions offering protection to minority shareholders, there is a significant amount of external financing taking place in the stock markets (Singh (1995)). Another implication of the results for emerging stock markets is that there is an additional benefit from using the dual-class voting structure, that has not yet been considered in the literature. Allowing controlling managers to issue non-voting shares can increase the efficiency of stock markets, because the reputation effect is stronger when managers can divest more without losing control.

Appendix

Proofs of Theorems

Proof of Lemma 1

Let $\widetilde{w}_G^g$ and $\widetilde{w}_B^g$ be the total income stream received in equilibrium by the good and bad managers. Since both are risk-averse then $w_G^g = u^{-1}\left(E\left[u\left(\widetilde{w}_G^g\right)\right]\right) \leq E\left[\widetilde{w}_G^g\right]$ and $w_B^g = u^{-1}\left(E\left[u\left(\widetilde{w}_B^g\right)\right]\right) \leq E\left[\widetilde{w}_B^g\right]$ with strict inequality if the either $\widetilde{w}_G^g$ or $\widetilde{w}_B^g$ are risky. In any market equilibrium the bad manager always extracts some private benefits, so that at least one inequality is strict. The total payoff to investors conditioned on the type of manager is $\widetilde{w}_I^g \mid G$ or $\widetilde{w}_I^g \mid B$ satisfying $\widetilde{w}_I^g \mid G + \widetilde{w}_G^g = \widetilde{y}_T + \ldots + \widetilde{y}_1$ and $\widetilde{w}_I^g \mid B + \widetilde{w}_B^g = \widetilde{y}_T + \ldots + \widetilde{y}_1$. Because investors in equilibria break even, and the probability of being of a good type is μ then $\mu E\left[\widetilde{w}_I^g \mid G\right] + (1-\mu) E\left[\widetilde{w}_I^g \mid B\right] = 0$. It then implies that $\mu E\left[\widetilde{w}_G^g\right] + (1-\mu) E\left[\widetilde{w}_G^g\right] = \overline{w} = E_T\left[\widetilde{y}_T + \ldots + \widetilde{y}_1\right]$ and that $\mu w_G^g + (1-\mu) w_B^g < \overline{w}$. We will see several example where $w_B^g > \overline{w}$, in Section V. ■

Proof of Corollary 1

Let $\underline{b}$, $\overline{b}$ and $\underline{\mu}$ be given as above. (i) If $b > \overline{b}$ and $\mu < \underline{\mu}(b)$ then the most that the good manager can gain in a pooling equilibrium is $w_G^P(\mu, b)$, which is less than the minimum she needs in order to be willing to go public, $w_G^P\left(\underline{\mu}, b\right)$, so there cannot be any pooling equilibrium. The bad manager cannot go public in a separating equilibrium, since his payoff is $w_B^S(b)$, which is less than the minimum payoff he would need to be willing to go public in a separating equilibrium, $w_B^S\left(\overline{b}\right)$. (ii) If $b \in \left(\underline{b}, \overline{b}\right]$ and $\mu < \underline{\mu}(b)$ then for the same reasons as above the good manager is not willing to go public in any pooling equilibrium. Also, since $b > \underline{b}$ she is not willing to go public in a separating equilibrium since the most she can get is $w_G^S(b)$, which is less than $w_G^S(\underline{b})$, the minimum payoff for which she would be willing to go public in a separating equilibrium. (iii) For the last case, observe that if $b \leq \overline{b}$ then the good manager is willing to go public regardless of μ, since she can get $w_G^S(b) > w_G^S(\underline{b})$ in a separating equilibrium. If $b > \underline{b}$ and $\mu \geq \underline{\mu}(b)$ then the good manager can go public in a pooling equilibrium because $w_G^P(\mu, b) > w_G^P\left(\underline{\mu}, b\right)$. ■

Proof of Proposition 2

We start first proving Part (ii). The condition for revelation is that it is more profitable for the bad manager to reveal himself and sell shares for a discount b, than to adopt the strategy of buying shares at the price without discounting at period t and then selling all remaining shares for the price without discounting at period $t+1$. In order to formally state the incentive compatibility condition above let us introduce the following expressions. Let $\overline{P}_t = \sum_{s=t}^{T} \overline{y}_s$ be the market value of the company without discount and $\underline{V}_{t+1}^B(\alpha)$ be the bad manager utility with revelation at stage 1 of period t, where shares are sold at a fair discounted price $(1-b)\overline{P}_{t+1}$:

$$\underline{V}_{t+1}^B(\alpha) = E\left[-\exp\left(-\left(\alpha(1-b)\overline{P}_{t+1} + b\sum_{s=t+1}^{T}\widetilde{y}_s\right)\right)\right] \tag{10}$$

Also, let $\overline{V}_{t+1}^B(\alpha)$ be the bad manager utility from selling all his shares at the price without discount:

$$\overline{V}_{t+1}^B(\alpha) = E\left[-\exp\left(-\left(\alpha\overline{P}_{t+1} + b\sum_{s=t+1}^{T}\widetilde{y}_s\right)\right)\right] \tag{11}$$

If the bad manager can convince investors that he is of a good type with probability 1 at stage 1 when he buys $(\alpha_t - \alpha_{t-1})$ shares then his utility in the continuation game is the maximum of expressions $E\left[\exp(-\alpha_t\widetilde{y}_t) \times \overline{V}_{t+1}^B(\alpha_t)\right]$ and $E\left[\exp\left(-\left(\alpha_t + b(1-\alpha_t)\right)\widetilde{y}_t\right) \times \underline{V}_{t+1}^B(\alpha_t)\right]$, which are, respectively, his utility if he decides to divert or not the cash flow at stage 2 of period t.

The incentive compatibility condition for revelation of the manager type at stage 1 of the game is that the utility of the bad manager if he reveals his type at period t, which is $\underline{V}_t^B(\alpha_{t-1})$, is bigger than or equal to what he can get by mimicking the revelation strategy:

$$\underline{V}_t^B(\alpha_{t-1}) \geq \exp\left(-(\alpha_{t-1}-\alpha_t)\overline{P}_t\right)\max\left\{\begin{array}{l} E\left[\exp(-\alpha_t\widetilde{y}_t)\times\overline{V}_{t+1}^B(\alpha_t)\right], \\ E\left[\exp(-(\alpha_t+b(1-\alpha_t))\widetilde{y}_t)\times\underline{V}_{t+1}^B(\alpha_t)\right] \end{array}\right\} \tag{12}$$

The bad manager by mimicking the revelation strategy can obtain the right hand side of expression 12. This is the maximum of the manager's payoff after trading $(\alpha_{t-1}-\alpha_t)$ shares at period t and not extracting cash flow at period t and subsequently selling α_t for $\overline{P}_{t+1}$, or trading $(\alpha_{t-1}-\alpha_t)$ shares followed by divertion of the cash flow and sellout of α_t for the discounted price. Simplifying equation 12 yields

$$\alpha_t\overline{y}_t - \max\left\{w_t(\alpha_t), w_t(\alpha_t+b(1-\alpha_t)) - b\overline{P}_{t+1}\right\} \geq \alpha_{t-1}b\overline{P}_t - w_t(b) \tag{13}$$

where $w_t(\alpha) = -\log E[\exp(-a\alpha\widetilde{y}_t)]$ is the certainty equivalent of a fraction α of the cash flow at time t.

Since the left hand side is increasing in α_t then there is a solution to equation 13 for $\alpha_t \in [0,1]$ if and only if

$$\alpha_{t-1} \leq \frac{\overline{y}_t - w_t(1) + w_t(b)}{b\overline{P}_t}$$

and let the solution be represented by $\overline{\alpha}_t^S(\alpha_{t-1})$ (for convenience let $\overline{\alpha}_t^S(\alpha_{t-1}) = 1$ when there is no solution).

Proof of Part (i). From equation 13 we can conclude that initially, when the manager owns $\alpha_0 = 1$ shares, there can be separation at the first stage of period 1 if and only

$$\overline{y}_1 - w_t(1) > b\overline{P}_1 - w_t(b)$$

or equivalently,

$$b < \frac{\overline{y}_1 - w_1(1)}{\sum_{t=2}^T \overline{y}_t} \tag{14}$$

Only when condition 14 is satisfied the manager can signal her type by withholding $\alpha_1 \in [0,1]$ shares at period 1.

Proof of Part (iii). We want to determine the maximum level of ownership $\overline{\alpha}_t^S$ that determines revelation of the type when there is no stealing in stage 2 of period t. In order words $\overline{\alpha}_t^S$ is the maximum amount of shares such that no stealing convinces investors that the manager is good for sure or $\mu_t = 1$. The expression of $\overline{\alpha}_t^S$ is given by the solution of

$$E\left[-\exp\left(-\left(\overline{\alpha}_t^S + b\left(1-\overline{\alpha}_t^S\right)\right)\tilde{y}_t\right)\right]\underline{V}_{t+1}^B\left(\overline{\alpha}_t^S\right) = E\left[-\exp\left(-\left(\overline{\alpha}_t^S\tilde{y}_t\right)\right)\right]\overline{V}_{t+1}^B\left(\overline{\alpha}_t^S\right)$$

which always exists. The manager must then be good for sure if $\alpha_t < \overline{\alpha}_t^S$ and there is no stealing at stage 2. Simplifying the above expression we get

$$\overline{\alpha}_t^S b\overline{P}_t = w_t\left(\overline{\alpha}_t^S + b\left(1-\overline{\alpha}_t^S\right)\right) - w_t\left(\overline{\alpha}_t^S\right) \tag{15}$$

which implies that

$$\overline{\alpha}_t^S < \frac{\overline{y}_t}{\overline{P}_t}$$

since $w_t\left(\overline{\alpha}_t^S + b\left(1-\overline{\alpha}_t^S\right)\right) - w_t\left(\overline{\alpha}_t^S\right) \leq w_t(b) < b\overline{y}_t$. ■

Proof of Lemma 2

Proof of Part (i). Consider the continuation game starting with state variables (α_t, ρ_t) at the stage 2 of the t-period decision node. Suppose that $G_t(\alpha_t, 1) < 0$: case in which even if investors believed that the manager were good for sure the gain from no stealing would be less than the gain from stealing. In this case the bad manager should steal for sure and investors should believe that the manager is good with probability $\mu_t = 1$ after observing no stealing.

Suppose now that $G_t(\alpha_t, \rho_t) > 0$: case in which even if investors did not updated the prior $\mu_t = \rho_t$ the gain from no stealing would be greater than the gain from stealing. The bad manager should then not steal with probability 1 and investors should believe that the manager is good with probability $\mu_t = \rho_t$ after observing no stealing.

On the other hand, if $G_t(\alpha_t, \rho_t) \leq 0$ and $G_t(\alpha_t, 1) \geq 0$ then both conditions above are not met and there is no equilibrium in pure strategy. Therefore, if there exists an equilibrium it must be a mixed strategy equilibrium in which the manager must not be diverting with probability β_t and investors update their beliefs according to μ_t^*, given implicitly by the following expressions

$$G_t(\alpha_t, \mu_t^*) = 0 \qquad (16)$$
$$\mu_t^* = \frac{\rho_t}{\rho_t + (1-\rho_t)\beta_t}$$

and the bad manager is just indifferent between stealing or not. Note that the Bayes rule condition above determines the expression for β_t as $\beta_t(\alpha_t, \rho_t) = \frac{\rho_t}{1-\rho_t}\frac{1-\mu_t}{\mu_t}$ which proves Part (ii).

Proof of Part (iii). For any equilibrium the competitive condition must hold: $P_t\left(\alpha_t, \alpha_{t-1}, \mu_{t-1}\right) = E_t\left[\tilde{e}_t \mid \alpha_t, \alpha_{t-1}, \mu_{t-1}\right] + E_t\left[\tilde{P}_{t+1} \mid \alpha_t, \alpha_{t-1}, \mu_{t-1}\right]$ where e_t is the reported earnings, $e_t = y_t - B_t$. We have

$$\begin{aligned} E\left[e_t \mid \alpha_t, \alpha_{t-1}, \mu_{t-1}\right] &= \Pr\left(\theta = \theta_G \mid \alpha_t, \alpha_{t-1}, \mu_{t-1}\right) \cdot E\left[e_t \mid \alpha_t, \alpha_{t-1}, \mu_{t-1}, \theta_G\right] \\ &+ \Pr\left(\theta = \theta_B \mid \alpha_t, \alpha_{t-1}, \mu_{t-1}\right) \cdot E\left[e_t \mid \alpha_t, \alpha_{t-1}, \mu_{t-1}, \theta_B\right] \end{aligned}$$

where $\rho_t = \Pr\left(\theta = \theta_G \mid \alpha_t, \alpha_{t-1}\mu_{t-1}\right)$ and $1 - \rho_t = \Pr\left(\theta = \theta_B \mid \alpha_t, \alpha_{t-1}, \mu_{t-1}\right)$. Also, $E_t\left[e_t \mid \alpha_t, \alpha_{t-1}, \mu_{t-1}, \theta_G\right] = \overline{y}_t$ and $E_t\left[e_t \mid \alpha_t, \alpha_{t-1}, \mu_{t-1}, \theta_B\right] = \beta_t\overline{y}_t + (1-\beta_t)\underline{y}_t$, because the bad manager does not divert with probability β_t and divert with probability $(1-\beta_t)$. Putting together the expressions above we get:

$$E\left[e_t \mid \alpha_t; \alpha_{t-1}, \mu_{t-1}\right] = \delta_t\overline{y}_t + (1-\delta_t)\underline{y}_t \qquad (17)$$

The term, $E_t\left[\tilde{P}_{t+1} \mid \alpha_t, \alpha_{t-1}, \mu_{t-1}\right] = E_t\left[\tilde{P}_{t+1} \mid \alpha_t, \rho_t\right]$, is similarly obtained conditioning on B_t :

$$\begin{aligned} E_t\left[\tilde{P}_{t+1} \mid \alpha_t, \rho_t\right] &= \Pr(B_t = 0 \mid \alpha_t, \mu_t) P_{t+1} + \Pr(B_t > 0 \mid \alpha_t, \mu_t) \underline{P}_{t+1} \qquad (18) \\ &= ((1-\rho_t)\beta_t + \rho_t) P_{t+1} + (1 - (1-\rho_t)\beta_t - \rho_t) \underline{P}_{t+1} \\ &= \delta_t P_{t+1} + (1-\delta_t)\underline{P}_{t+1} \end{aligned}$$

Putting together expressions 17 and 18 we conclude the proof of the Lemma. ■

Proof of Proposition 3

The proof is by induction. Consider the last period game that starts with a pair of shares and beliefs $\left(\alpha_{T-1}, \mu_{T-1}\right)$. Define the standard set of strategy profile for the one-period continuation game as in Proposition 1 by $\alpha_T^{\theta}\left(\alpha_{T-1}, \mu_{T-1}\right)$, $P_T\left(\alpha, \alpha_{T-1}, \mu_{T-1}\right)$, $\rho_T\left(\alpha, \alpha_{T-1}, \mu_{T-1}\right)$ with manager utilities represented by $V_T^{\theta}\left(\alpha_{T-1}, \mu_{T-1}\right)$. The static equilibrium can be either pooling or separating depending on the value of μ_{T-1} as discussed in Section III.

Suppose we are given a set of strategies and beliefs that form a PBE for the $t+1$ period continuation game. We will show how to proceed in steps in order to obtain strategies and beliefs for the t period game.

Step 1:

Let the decision strategy of the bad manager with respect to private benefits, $\beta_t\left(\alpha_t, \rho_t\right)$, and investors updating rule, $\mu_t\left(\alpha_t, \rho_t\right)$, for the continuation game starting with a history $\left(\alpha_t, \rho_t\right)$ at stage 2 of the t period game be defined as in Lemma 2 by expressions 9 and the Bayes rule.

Step 2:

We proceed to obtain a set of equilibrium strategies $\alpha_t^{\theta}\left(\alpha_{t-1}, \mu_{t-1}\right)$, $P_t\left(\alpha, \alpha_{t-1}, \mu_{t-1}\right)$, $\rho_t\left(\alpha_t, \alpha_{t-1}, \mu_{t-1}\right)$. For any given pair $\left(\alpha_t, \rho_t\right)$ the only price that is consistent with PBE according to Part (iii) of Lemma 2 is given by $P_t\left(\alpha_t, \rho_t\right) = \left[\delta_t \overline{y}_t + \left(1-\delta_t\right) \underline{y}_t\right] + \left[\delta_t P_{t+1} + \left(1-\delta_t\right) \underline{P}_{t+1}\right]$ where $\delta_t = \frac{\rho_t}{\mu_t}$. So, given any belief function $\rho_t\left(\alpha, \alpha_{t-1}, \mu_{t-1}\right)$ then the pricing function is determined by $P_t\left(\alpha_t, \alpha_{t-1}, \mu_{t-1}\right) = P_t\left(\alpha_t, \rho_t\left(\alpha_{t;}\alpha_{t-1}, \mu_{t-1}\right)\right)$.

In order to obtain a set of beliefs that is consistent with PBE start by considering temporarily a belief function $\rho_t\left(\alpha_t, \alpha_{t-1}, \mu_{t-1}\right) = \begin{cases} 1 & \text{if } \alpha_t \geq \alpha_t^*\left(\alpha_{t-1}\right) \\ \overline{\rho}_t & \text{if } \alpha_t \in \left(\alpha_t^*\left(\alpha_{t-1}\right), 0\right] \end{cases}$, where $\alpha_t^*\left(\alpha_{t-1}\right)$ is the separating level of ownership for the multiperiod game as determined by Proposition 2. The parameter $\overline{\rho}_t$ will be determined below so that the belief function is part of a PBE.

Given the belief and associated pricing function above the problem of the good manager is

$$\max_{\alpha_t \in [0,1]} E\left[\exp\left(-\left(\left(\alpha_{t-1} - \alpha_t\right) P_t\left(\alpha_t\right) + \alpha_t \widetilde{y}_t\right)\right) \cdot V_{t+1}^G\left(\alpha_t, \mu_t\left(\rho_t\left(\alpha_t\right)\right)\right)\right] \tag{19}$$

Let the solution of the good manager problem be $\alpha_t^G(\overline{\rho}_t)$ which depends on the still to be determined $\overline{\rho}_t$. Since no $\alpha_t < \alpha_t^G(\alpha_{t-1})$ is going to be chosen by the good manager in equilibrium we redefine the belief function as

$$\rho_t\left(\alpha_t, \alpha_{t-1}, \mu_{t-1}\right) = \begin{cases} 1 & \text{if } \alpha_t \geq \alpha_t^*(\alpha_{t-1}) \\ \overline{\rho}_t & \text{if } \alpha_t \in \left(\alpha_t^*(\alpha_{t-1}), \alpha_t^G(\overline{\rho}_t)\right] \\ 0 & \text{if } \alpha_t < \alpha_t^G(\overline{\rho}_t) \end{cases}$$ so that it can be part of a PBE.

The incentive compatibility solution for the bad manager is $\alpha_t^B(\overline{\rho}_t)$ solution of

$$\max_{\alpha_t \in [0,1]} \left\{ \begin{array}{l} E\left[\exp\left(-\left(\left(\alpha_{t-1} - \alpha_t\right) P_t(\alpha_t) + \alpha_t \widetilde{y}_t\right)\right) \cdot V_{t+1}^B(\alpha_t, \mu_t)\right] \\ E\left[\exp\left(-\left(\left(\alpha_{t-1} - \alpha_t\right) P_t(\alpha_t) + \alpha_t \widetilde{y}_t + b(1-\alpha_t)\widetilde{y}_t\right)\right)\right] \cdot \underline{V}_{t+1}^B(\alpha_t) \end{array} \right\} \tag{20}$$

The solution of the bad manager problem is either $\alpha_t^B(\overline{\rho}_t) = 0$ (the separating outcome), or $\alpha_t^B(\overline{\rho}_t) = \alpha_t^G(\overline{\rho}_t)$. If the bad manager tries to sell more than the optimum amount for the good manager ($\alpha_t < \alpha_t^G(\overline{\rho}_t)$) then investors will know for sure that the manager is of a bad type ($\rho_t = 0$) and so the optimum for the bad manager is either to mimic the good manager by offering $\alpha_t^B(\overline{\rho}_t) = \alpha_t^G(\overline{\rho}_t)$ or to completely divest by offering $\alpha_t^B(\overline{\rho}_t) = 0$.

Step 3:

Let $\overline{\rho}_t = \mu_{t-1}$ and consider the strategies and beliefs obtained by construction as above. If $\alpha_t^B\left(\mu_{t-1}\right) = \alpha_t^G\left(\mu_{t-1}\right)$ then we are done and the strategies and beliefs form a pooling equilibrium. We are also done if $\alpha_t^B\left(\mu_{t-1}\right) = 0$ and $\alpha_t^G\left(\mu_{t-1}\right) = \alpha_t^*(\alpha_{t-1})$ because then the strategies and beliefs form a separating equilibrium. Observe that in both cases the strategies and beliefs are consistent with the Bayes rule, are incentive compatible and satisfy the participation constraints.

However, if $\alpha_t^B\left(\mu_{t-1}\right) = 0$ and $\alpha_t^G\left(\mu_{t-1}\right) \neq \alpha_t^*(\alpha_{t-1})$ then the strategies and beliefs specified as above do not form an equilibrium. To obtain an equilibrium consider the smallest $\overline{\rho}_t$ greater than μ_{t-1} such that $\alpha_t^B(\overline{\rho}_t) = \alpha_t^G(\overline{\rho}_t)$. Such $\overline{\rho}_t$ exists by continuity and because for $\overline{\rho}_t = 1$, $\alpha_t^B(\overline{\rho}_t) = \alpha_t^G(\overline{\rho}_t) = 0$. At this new level of $\overline{\rho}_t$ the bad manager is just indifferent between either separating by selling all shares or mimicking the strategy of the good manager. Consider the following strategy for the bad manager:

$$\alpha_t^B\left(\alpha_{t-1}, \mu_{t-1}\right) = \begin{cases} \alpha_t^G(\overline{\rho}_t) & \text{with probablity } \pi \\ 0 & \text{with probablity } 1-\pi \end{cases}$$, where π is given by Bayes rule $\pi = \frac{\mu_{t-1}}{1-\mu_{t-1}} \frac{1-\overline{\rho}_t}{\overline{\rho}_t}$.

We have finally concluded the induction steps and obtained a PBE for the multiperiod game starting at period t with parameters α_{t-1} and μ_{t-1}. The PBE for the multiperiod game can be constructed proceeding with the induction steps until the first period of the multiperiod game is reached. ■

Proof of Proposition 4

Suppose by contradiction that there is a sequence of games and equilibrium whose end-of-period income is given by $\widetilde{w}_G^n$ and $\widetilde{w}_G^n$, and certain equivalent payoffs given by w_G^n and w_B^n, which approaches the payoff of the first best. This means that $\lim_{n\to\infty} \mu w_G^n + (1-\mu) w_B^n = \overline{w}$. As has been shown in proposition 1, $\mu E[\widetilde{w}_G^n] + (1-\mu) E[\widetilde{w}_B^n] = \overline{w}$ and both $w_G^n \leq E[\widetilde{w}_G^n]$ and $w_B^n \leq E[\widetilde{w}_B^n]$ with strict inequality holding whenever the total income is risky. In order for the first best be approached it then must be the case that both $\lim_{n\to\infty} w_G^n - E[\widetilde{w}_G^n] = 0$ and that $\lim_{n\to\infty} w_B^n - E[\widetilde{w}_B^n] = 0$. This implies that in the limit both the good manager and the bad manager are sharing all the risk with investors, but this cannot be true since the bad manager must then be getting private benefits that are approaching zero and at the same time holding an amount of shares that is also approaching zero, which contradicts the fact that $\widetilde{w}_G^n$ is an equilibrium and the bad manager is maximizing his utility. ■

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Notes

[1]Barclay and Holderness (1989) find for the US that the premium of block trades over the post-announcement exchange price is, on average, 4% of a firm's market value. Bergstrom and Rydqvist (1992) finds a premium of voting relative to non-voting shares of 6.5% in Sweden; Zingales (1994) finds a premium of 81% in Italy; Levy (1983) finds a premium of 45.5% for Israel; Horner (1988), 20% for Switzerland; Megginson (1990), 13.3% for the UK; Robinson and White (1990), 23.3% for Canada, and Lease, McConell, and Mikkelson (1983) find an average premium of 5.4% for the U.S.

[2]For example, oil industry firms in the mid-1980s spent significant resources in negative NPV projects (Jensen (1986)); negative returns to bidders in the announcements of acquisitions were motivated by managers' preferences for diversification and growth instead of shareholder value (Morck, Shleifer and Vishny (1990)); managers resisted takeovers to protect their private benefits rather than to serve shareholders (e.g., Jarrell and Poulsen (1987)).

[3]In the U.S., majority ownership is relatively uncommon, probably due to the superior legal protection of shareholders and to legal restrictions on high ownership and exercise of control by financial institutions. In the rest of the world, large shareholdings are the norm, as well as the use of pyramids, cross-holdings and the issuance of dual class shares. In Germany, Japan and France banks and other financial institutions often control the major industrial corporations (Franks and Mayer (1994), OECD (1995)). In most of the rest of the world, including Europe (e.g., Italy, Spain and Sweden), Latin America and East Asia, public corporations are usually majority-controlled by their founders or their offspring (Zingales (1994), Bergstrom and Rydqvist (1990)).

[4]The work of Hart and Tirole (1988) with respect to the interaction between a monopolist seller of a durable good and a buyer whose valuation for the good is his private information, has dynamics (Coasian dynamics) with similarities to ours. These same dynamics are found in many bargaining models in the literature (for references see Fudenberg and Tirole (1991)). Also, Laffont and Tirole (1988) find that in a regulatory relationship run by short-term contracts, in equilibria there may be a substantial amount of pooling in the first period of a 2-period contract.

[5]Private benefits are assumed to be inelastic with respect to ownership of equity, whereas in Jensen and Meckling (1976) elasticity of private benefits with respect to insider ownership is at the core of their theory.

[6]If the exogenous cost imposed by legal institutions on diverting cash flow is low, then it could be very costly to enforce privately a contract inhibiting the extraction of private benefits (this cost can be even higher if the judicial system is inefficient). Even if the parties decided to design such a private contract, it would be plagued by incomplete contracting problems, related to the fact that this contract would have to specify the manager's actions under the many possible states of nature affecting the firm, which would be very complex and costly to do.

[7]We could have specified the model with a general stochastic process, but for the point we want to make it is enough to use an independently distributed process.

[8]Ritter (1987) has estimated that for U.S. companies, the direct cost of going public average 21% of the realized market value of securities issued using firm commitment offers and 31% for best effort offers.

[9]For a list of Nash Equilibrium refinements for signalling games, see Fudenberg and Tirole (1991). The intuitive criterion of Cho and Kreps (1987), applied to our setting, essentially states the following: Let σ be any PBE equilibrium where the bad manager gets utility w_B. Suppose that for some Δ shares at the price $\overline{P}_1$ the utility of the bad manager is less than the equilibrium utility w_B. It then must be the case that the equilibrium outcome can be sustained by beliefs that put probability zero on Δ shares being offered by the bad manager.

[10]Analytically $\Delta_S(b)$ is the solution of:

$$E\left[\exp\left(-\left(\Delta_S \cdot E\left[\tilde{y}\right] + \left(1-\Delta_S\right)\left(1-b\right)\tilde{y} + b\tilde{y}\right)\right)\right] = E\left[\exp\left(-\left((1-b)E\left[\tilde{y}\right] + b\tilde{y}\right)\right)\right],$$

μ^* is given by

$$\max_{\Delta\in[0,1]} E\left[-\exp\left(-\left(\Delta \cdot P_{\mu^*} + \left(1-\Delta\right)\tilde{y}\right)\right)\right] = E\left[-\exp\left(-\left(\Delta_S E\left[\tilde{y}\right]\right) + \left(1-\Delta_S\right)\tilde{y}\right)\right]$$

and $\Delta_\mu = \arg\max_{\Delta\in[0,1]} E\left[-\exp\left(-\left(\Delta \cdot P_{\mu^*} + \left(1-\Delta\right)\tilde{y}\right)\right)\right]$.

[11]Ausubel and Deneckere (1989) have imposed these restrictions to narrow down the set of equilibria for bargaining games. See also Fudenberg and Tirole (1991) for other papers in the bargaining literature that use the same restrictions.

[12]Maskin and Tirole (1994) develop formally the concept of Markov perfect equilibrium

for a dynamic game with incomplete information.

[13]The program has been written in MATLAB.

[14]We have also analyzed the results using different specifications for the cash flow $\tilde{y}$ but the results are qualitative similar.

[15]For example, the Coase conjecture holds for the Hart and Tirole (1988) and for other bargaining models (see Fudenberg and Tirole (1991)).

[16]We remark that the equilibrium outcomes were computed using one of many possible PBE satisfying the refinements assumptions, and it might well be the case that the Pareto frontier of the equilibrium outcomes is monotonic with respect to the addition of trading periods. However to establish the main claims of the paper, which are based on comparisons with the static game, it is sufficient to show that there exist equilibria in the many period game where the good manager derives more utility than in the most efficient equilibrium in the static game.

[17]This conclusion is obtained in a model where there are divergence of opinions about how firms should be managed. Stock markets works relatively better when there is little consensus on how a firm should be run since it provide checks that the manager's decision are sensible. Stock markets work best in oligopolist and changing technology industries (industries important for advanced countries) and less well for basic industries where the technology is already established (industries important for developing countries).

[18]Bergstrom and Rydqvist (1990) estimate that the largest shareholder in Sweden owns 61% of the votes. Zingales (1994) estimates that for Italy the largest holds 52% of the votes. DeAngelo and DeAngelo (1985) estimated that this number for dual-class companies in the U.S. is 55%.

[19]Kristian Rydqvist (1992) describes the regulation related to dual-class shares prevailing in European stock markets and how often companies in these markets issue dual-class shares. In most of these stock markets, companies are allowed to issue restricted voting shares and very often do issue such shares: Denmark (75%), Finland (67%), Netherlands (50%), Italy (40%), Sweden (75%), and Switzerland (68%). To this list, one could also add most of the emerging stock markets (e.g., in Brazil almost 100% of the companies use dual-class shares.

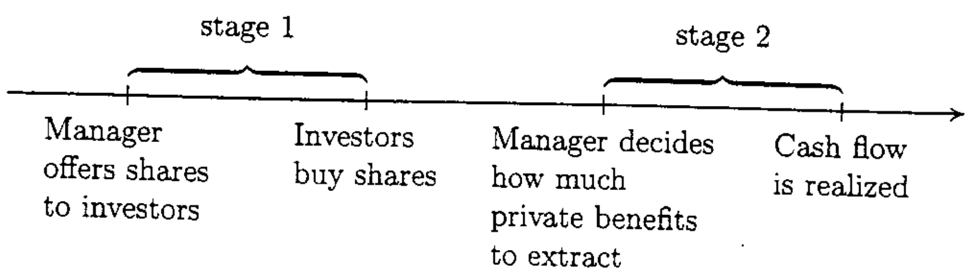

Figure 1: Timing - one period of the game

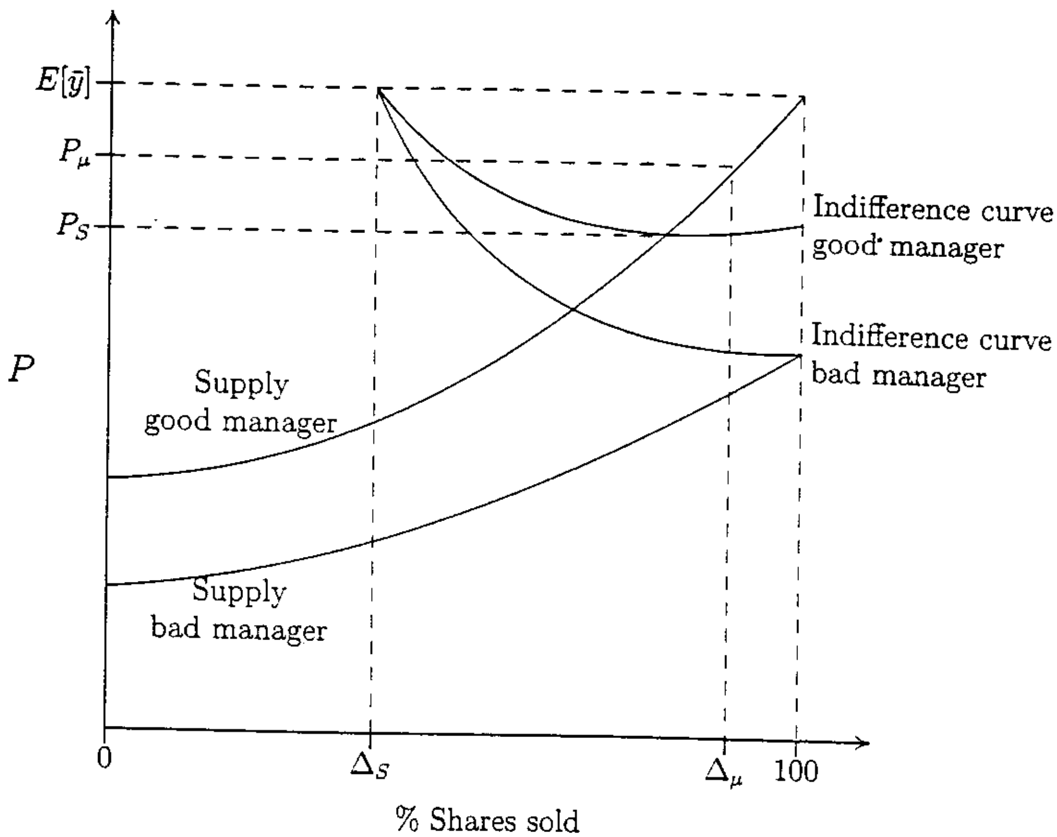

Figure 2: The static game

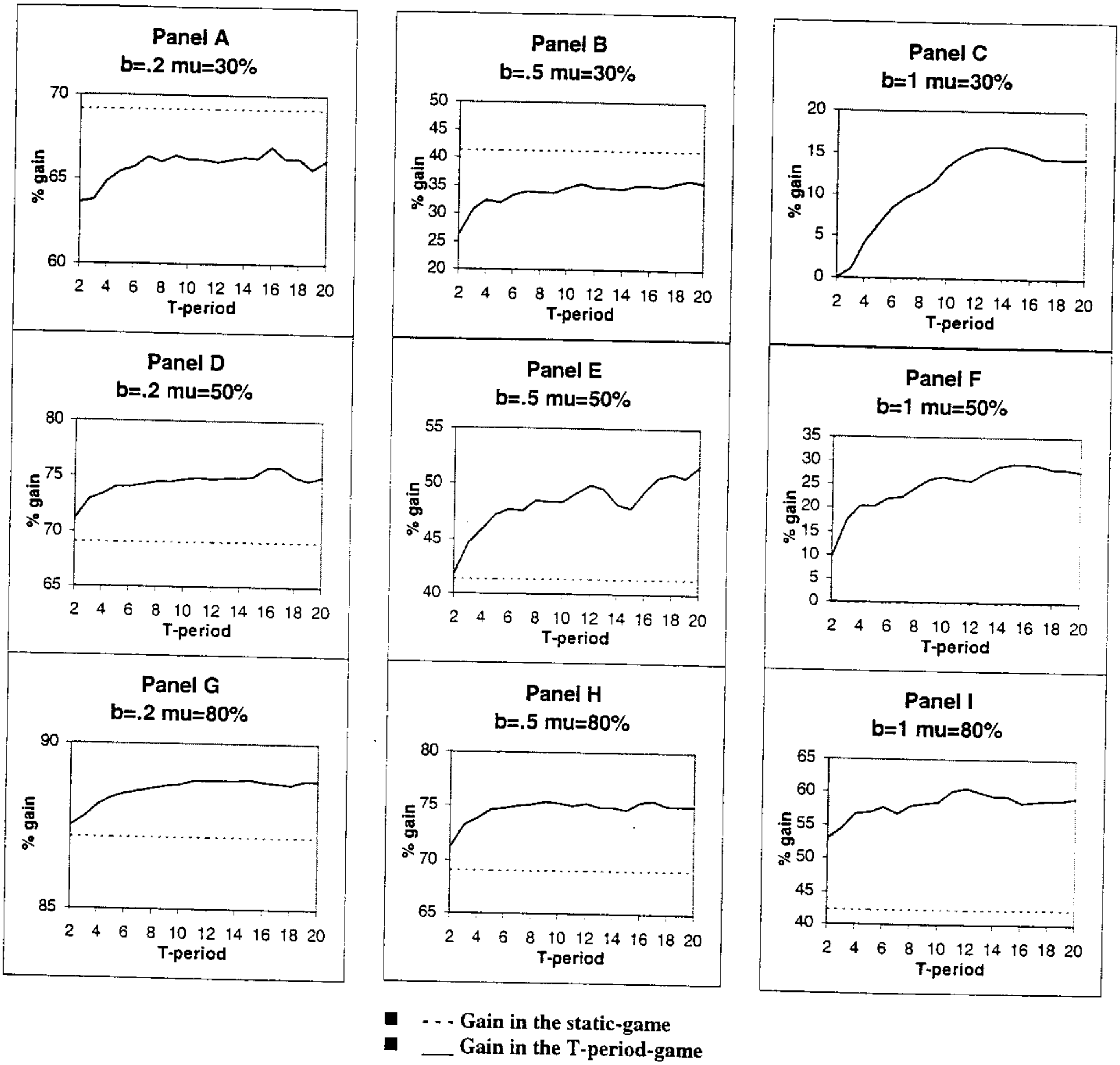

Figure 3: Gain from going public with multiple trading periods. Numerical solutions of PBE for the game with different numbers of trading periods (horizontal axis) and for different values of the moral hazard (b) and adverse selection parameters (mu). Each panel draws the equilibrium utility of the good manager for the multiple trading period game with parameters (b,mu). The good manager utility (vertical axis) is expressed as a percentage of the potential gain from going public (\$1-\$0.69). The total distribution of cash flow has gamma distribution with parameters (1,1).

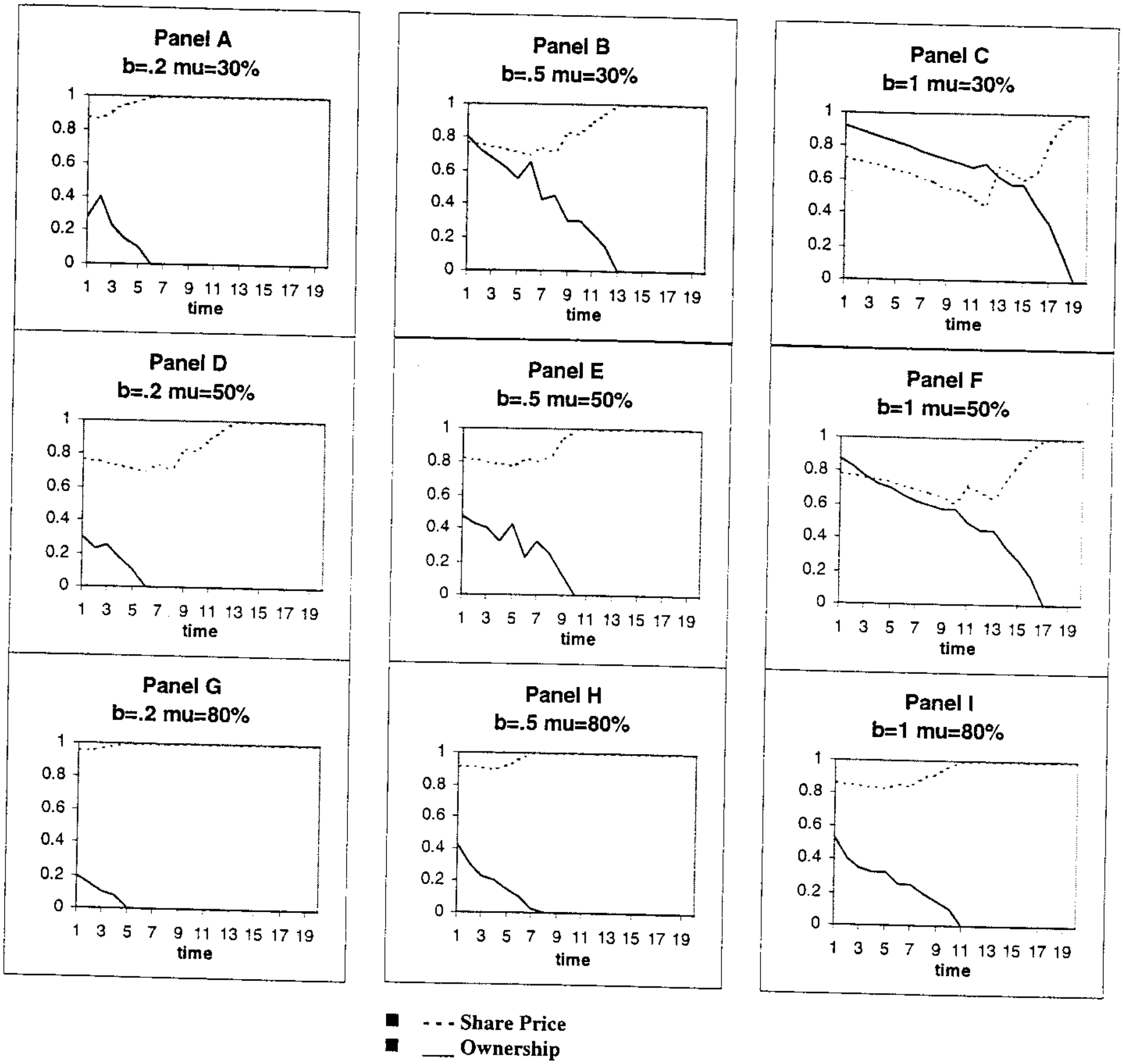

Figure 4: Dynamics of the share price and insider equity ownership. Numerical solutions of PBE for the game with 20 trading periods. The horizontal axis represents time from the IPO until the end of the game and the vertical axis represents the percentage of shares owned by the manager and the share price received by the good manager as a percentage of the price without discount. Each panel draws the equilibrium values with different parameters (b,mu). The total distribution of cash flow has gamma distribution with parameters (1,1).

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