Costs of Equity from Factor-Based Models

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Abstract

Equity costs of capital for individual firms are estimated using several models that relate expected returns to betas on one or more pervasive factors. A Bayesian approach incorporates prior uncertainty about an asset's mispricing as well as uncertainty about betas and factor means. Substantial prior uncertainty about mispricing results in an estimated cost of equity close to that obtained with mispricing ruled out. Uncertainty about which pricing model to use appears to be less important, on average, than within-model parameter uncertainty. In the absence of mispricing uncertainty, uncertainty about factor means is generally the most important source of overall uncertainty about a firm's cost of equity, although uncertainty about betas is nearly as important.

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1. Introduction

The expected rate of return on a firm's stock, the "cost of equity," is regarded as a key ingredient for making decisions affecting the firm. For example, the cost of equity affects which projects are undertaken by a firm and how prices of public-utility services are regulated.

One approach to estimating the cost of equity uses a standard asset-pricing model, in which the cost of equity hinges on sensitivities of the firm's stock return to market-wide factors.[1] If r_t denotes the stock's return in excess of a riskless rate and f_t denotes a $K \times 1$ vector of the factors, all realized in period t, then the stock's sensitivities, or "betas," are the slope coefficients in the regression,

$$r_t = \alpha + \beta' f_t + \epsilon_t, \tag{1}$$

where ϵ_t is the mean-zero regression disturbance. When the factors appropriate to the given model are constructed as excess portfolio returns or payoffs on zero-investment positions, as will be the case in the models analyzed below, then the pricing model implies $\alpha = 0$.[2] That is, the pricing model implies that the firm's cost of equity, μ, is given by

$$\mu = \beta' \lambda, \tag{2}$$

where λ is the vector of expected values of the factors.

The elements of β and λ must be estimated, so the true cost of equity is uncertain. Moreover, even if β and λ were known for certain, the pricing model might not deliver the precise cost of equity for every stock. That is, the model might misprice the given stock in question, so that the cost of equity is actually

$$\mu = \alpha + \beta' \lambda, \tag{3}$$

where $\alpha \neq 0$. This "mispricing" uncertainty about α contributes further to the uncertainty about the cost of equity. Finally, if the decision maker has any doubts about which pricing model to use, then the uncertainty about μ also includes that "model" uncertainty. This study attempts to quantify these various sources of uncertainty and gauge the relative importance of each in estimating a firm's cost of equity.

We estimate the cost of equity using a Bayesian approach. In this setting, the decision maker does not know the true cost of equity but instead uses the conditional expectation

[1] Such models include, for example, the Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965), the intertemporal CAPM of Merton (1973), and the Arbitrage Pricing Theory of Ross (1976).

[2] See Huberman, Kandel, and Stambaugh (1987) for a deeper discussion of this point.

$E\{r_t|\Phi\}$, where Φ denotes the information available at the time of the decision. We assume that (excess) returns have constant mean μ, so the decision maker's estimated cost of equity is then simply the posterior mean of μ given Φ, and the decision maker's uncertainty about the cost of equity is reflected in the posterior variance of μ. One feature of this Bayesian approach is that we can explore the effects of prior mispricing uncertainty on the estimated cost of equity and its posterior uncertainty. We find, for example, that mispricing uncertainty that seems important in economic terms, say with an annual standard deviation for α of 5%, does not impact greatly the estimated cost of equity. That is, the posterior mean of μ in that case is generally close to the posterior mean obtained when mispricing is ruled out, even when the sample least-squares estimate of α departs substantially from zero. In this sense, a pricing model that might be viewed by the decision maker as being only mediocre in its ability to price stocks accurately is still relied upon fairly heavily in estimating the cost of equity.

This study investigates factor-based models with a focus on the estimates they produce rather than on their asset-pricing abilities versus each other or versus non-factor-based approaches. Even though the latter issues continue to invite debate in the academic literature, we suggest that these factor-based models have received sufficient interest to merit investigating their potential use by decision makers. Three pricing models are used to illustrate our approach. The first is the CAPM, where the single factor is specified to be the excess return on a market index portfolio. The second model, proposed by Fama and French (1993), contains that market factor plus two additional factors: the difference in returns between small and large firms and the difference in returns between firms with high and low ratios of book value to market value. The third model also has three factors, but, instead of prespecifying them, we extract them from returns on a large cross-section of stocks using the asymptotic principal components method of Connor and Korajczyk (1986). Uncertainty about which of these three models to use can contribute nontrivially to a decision maker's overall uncertainty about the cost of equity, but this source of uncertainty is typically less important than the parameter uncertainty within any given model. For example, when each model is assigned an equal probability of being the "correct" one, we obtain an overall posterior standard deviation for the cost of equity of 5% per year or more, depending on the prior uncertainty about α, but that value is typically no more than 0.75% above the posterior standard deviation of μ obtained within any single model. We also find a close relationship between the cost of equity estimates produced by the CAPM and the Connor-Korajczyk model, particularly for utilities.

Uncertainty about β contributes substantially to the overall uncertainty about the cost of equity for an individual firm, but somewhat more important is the uncertainty about λ, the vector of factor means. In a recent article, Fama and French (1997) estimate costs of equity for industries using both the CAPM as well as the Fama-French (1993) three-factor model. Based on frequentist standard errors, they conclude that by far the largest source of imprecision in industry costs of equity arises from estimation of λ. Although uncertainty about β, not surprisingly, is more important for individual firms than for industry portfolios, our conclusion regarding the importance of uncertainty about λ is otherwise similar to theirs. In all three of the models we use, the histories of the factors are available beginning in July 1963, but the factors are correlated with other series whose histories begin earlier. As a result, the longer-history series contain additional information about λ, as discussed by Stambaugh (1997). We find that, in the absence of uncertainty about mispricing, uncertainty about λ remains the most important source of uncertainty about a firm's cost of equity, even after incorporating information about λ that is contained in series whose histories begin in 1926.

In keeping with the spirit of a factor-based approach, much of our analysis assumes that the information set used by the decision maker consists of histories of factors and stock returns. That is, the decision maker does not make use of firm-specific characteristics, except perhaps in constructing the factors (as in, for example, the Fama-French model). Previous studies have recommended the use of firm-specific characteristics in estimating the cost of equity (e.g., Elton, Gruber, and Mei, 1994, or Schink and Bower, 1994), and the usefulness of various firm-specific characteristics in explaining expected returns has been argued recently by Daniel and Titman (1997). Another feature of the Bayesian approach is that it allows the decision maker to introduce additional prior information about the firm whose cost of equity is to be estimated, and our methodology allows the decision maker to either ignore or incorporate such prior information. In specifying the prior, the firm can be regarded as a random draw either from the whole cross-section of stocks, when firm-specific characteristics are ignored, or from a group of firms with similar characteristics, when the firm's characteristics are incorporated. As a simple illustration of the latter case, we incorporate a firm's industry classification as additional prior information and estimate costs of equity for utilities, an industry in which a firm's estimated cost of equity has clear practical relevance.

The remainder of the paper is organized as follows. The methodology is developed in Section 2, wherein we present the general form of the priors used in our Bayesian approach and explain how we analyze the resulting posterior distributions of μ and its components. Section 3 begins with a description of the empirical Bayes procedure used to obtain parameters in the prior distributions and then presents our empirical findings regarding the estimation of the cost of equity for individual stocks. The empirical results include a detailed analysis for one stock as well as analyses based on two cross-sections: one of 1,994 stocks and the other of 124 utility stocks. Section 4 reviews the conclusions.

2. Methodology

2.1. Stochastic Setting

Let r denote the $T \times 1$ vector of returns on the stock of the firm whose cost of equity is to be estimated. In many cases, the stock's return history, or at least the portion of that history used in the estimation, may be shorter than the history of the factors. It is assumed that there are S observations of the factors, with $S \geq T$. Let $F^{(T)}$ denote the $T \times K$ matrix containing the T observations of the factors corresponding to the same periods as the returns in r. The regression disturbance ϵ_t in (1) is assumed to be, in each period t, an independent realization from a normal distribution with zero mean and variance σ^2, so the most recent T observations of the returns and the factors obey the regression relation

$$r = Xb + \epsilon, \qquad \epsilon \sim N(0, \sigma^2 I_T), \tag{4}$$

where $b' \equiv [\alpha \ \beta']$, $X = [\iota_T \ F^{(T)}]$, ϵ contains the T regression disturbances, ι_T is a T-vector of ones, I_T is a $T \times T$ identity matrix, and the notation "$\sim$" is read "is distributed as."

In addition to the S observations of the K factors, there exist L observations of K_L variables that are correlated with the factors. If $L > S$, then, as shown by Stambaugh (1997), the longer histories of these additional variables contain information about λ, the $K \times 1$ vector of factor means, beyond that contained in the factor histories alone. Let y_t denote the $K_L \times 1$ vector containing the observations of the additional variables in period t, and let $Y^{(L)}$ denote the $L \times K_L$ matrix containing all L observations of y_t. For each of the S periods over which both f_t and y_t are observed, define the "augmented" set of factors $f_t^{a'} = [f_t' \ y_t']$, and assume that

$$f_t^a \sim N(\theta, \ G), \tag{5}$$

where the realizations are independent across t, and $\theta' = [\lambda' \ \theta_2']$. For the $L - S$ periods in which only y_t is observed, it is also assumed that

$$y_t \sim N(\theta_2, \ G_{22}), \tag{6}$$

again with independent realizations across t, where G_{22} is the corresponding submatrix of G. That is, the marginal distribution of y_t is given by (6) for all L periods. Finally, it is assumed that f_t^a is independent of ϵ for all t.

Given the above assumptions, it follows that the likelihood function for the parameters $(b,\ \sigma,\ \theta,\ G)$ can be factored as

$$p(r, F^{(S)}, Y^{(L)}|b, \sigma, \theta, G) = p(r|F^{(T)}, b, \sigma)p(F^{(S)}, Y^{(L)}|\theta, G), \tag{7}$$

where the likelihood function for the regression parameters is

$$p(r|F^{(T)}, b, \sigma) \propto \frac{1}{\sigma^T} \exp\left\{\frac{1}{2\sigma^2}(r - Xb)'(r - Xb)\right\}, \tag{8}$$

and the likelihood function for the moments of the factors and additional variables is

$$\begin{aligned} p(F^{(S)}, Y^{(L)}|\theta, G) \quad \propto \quad & |G_{22}|^{-\frac{L-S}{2}} \exp\left\{-\frac{1}{2}\sum_{t=1}^{L-S}(y_t - \theta_2)'(G_{22})^{-1}(y_t - \theta_2)\right\} \\ & \times |G|^{-\frac{S}{2}} \exp\left\{-\frac{1}{2}\sum_{t=L-S+1}^{L}(f_t^a - \theta)'(G)^{-1}(f_t^a - \theta)\right\}. \end{aligned} \tag{9}$$

2.2. Priors

We propose a normal-inverted-gamma prior on the regression parameters b and σ:

$$b|\sigma \sim N(\bar{b}, \Psi(\sigma)) \tag{10}$$

$$\sigma^2 \sim \frac{\nu s_0^2}{\chi_\nu^2}, \tag{11}$$

where

$$\Psi(\sigma) = \begin{bmatrix} \left(\frac{\sigma^2}{E(\sigma^2)}\right)\sigma_\alpha^2 & \left(\frac{\sigma}{E(\sigma)}\right)\sigma_\alpha(\rho_{\alpha\beta}\sigma_\beta)' \\ \left(\frac{\sigma}{E(\sigma)}\right)\sigma_\alpha(\rho_{\alpha\beta}\sigma_\beta) & V_\beta \end{bmatrix}. \tag{12}$$

In the above, σ_β is a $K \times 1$ vector containing the square roots of the diagonal elements of V_β, and $\rho_{\alpha\beta}$ is a $K \times K$ diagonal matrix with the simple correlations between α and the individual β's on the main diagonal. Since $\bar{b}$ does not depend on σ, the marginal prior covariance matrix of b equals

$$\begin{aligned} V_b \equiv \text{cov}(b, b') &= E\{\Psi(\sigma)\} \\ &= \begin{bmatrix} \sigma_\alpha^2 & \sigma_\alpha(\rho_{\alpha\beta}\sigma_\beta)' \\ \sigma_\alpha(\rho_{\alpha\beta}\sigma_\beta) & V_\beta \end{bmatrix}, \end{aligned} \tag{13}$$

and it is assumed that V_b is positive definite. In order to have $\Psi(\sigma)$ be positive definite, we also require

$$\sigma'_\beta \rho'_{\alpha\beta} V_\beta^{-1} \rho_{\alpha\beta} \sigma_\beta < \frac{[E(\sigma)]^2}{E(\sigma^2)} = \frac{\nu - 2}{2} \left(\frac{\Gamma[(\nu-1)/2]}{\Gamma(\nu/2)} \right)^2, \tag{14}$$

where the equality of the second and third expressions follows from the properties of the inverted gamma distribution for σ,[3]

$$E(\sigma^2) = \frac{\nu s_0^2}{\nu - 2} \tag{15}$$

and

$$E(\sigma) = \frac{\Gamma[(\nu-1)/2]}{\Gamma(\nu/2)} (\frac{\nu s_0^2}{2})^{1/2}. \tag{16}$$

In specifying the parameters for the above priors, we use an empirical Bayes procedure that relies on data for a cross-section of individual stocks. The effects of "mispricing" uncertainty are investigated by entertaining a wide range of values for σ_α^2. Details of that approach are provided in section 3.

Observe in (12) that the conditional prior variance of α is proportional to σ^2, the variance of ϵ_t. This feature of our prior recognizes that a high value of $|\alpha|$ accompanied by a low value of σ^2 implies a high Sharpe ratio for some combination of the asset, the factor-mimicking positions, and cash (earning the riskless rate).[4] In particular, $(\alpha/\sigma)^2$ is the difference between the maximum squared Sharpe ratio for such a combination and the maximum squared Sharpe ratio for combinations of only the factor-mimicking positions and cash. Following MacKinlay (1995), a prior positive association between α and σ is imposed to reduce the probability of high Sharpe ratios as compared to priors that treat those parameters as independent. In contrast, we do assume independence between β and σ in the absence of a compelling a priori argument to the contrary.

The structure of the covariance matrix for b, $\Psi(\sigma)$ in (12), produces a prior that is essentially a hybrid of two more standard alternative priors for the regression model. In one alternative, the normal density for b and the inverted-gamma density for σ^2 are independent, so that no part of the covariance matrix for b involves σ^2 (e.g., Chib and Greenberg (1996)). As explained above, this prior would make α independent of σ^2 and hence allow for high Sharpe ratios. In the other alternative, the well known natural-conjugate prior, the marginal prior for σ^2 is still inverted gamma, but the entire covariance matrix of b is proportional to σ^2 (e.g., Zellner (1971, chapter 3)). As a result, in the formula for the posterior mean of β,

[3] See Zellner (1971, p.372).

[4] A portfolio's Sharpe ratio is its expected excess return divided by its standard deviation of return.

the relative weights on the sample estimate and the prior mean do not depend on sample information about σ. That is, $\hat{\beta}$ is given no more weight when the sample residual variance is small than when it is large. Vasicek (1973) argues that the natural conjugate prior is inappropriate when the prior parameters are estimated from a cross-section of stocks.

Finally, we assume that the regression parameters are independent of the moments of f_t^a:

$$p(b, \sigma, \theta, G) = p(b, \sigma)p(\theta, G). \tag{17}$$

The prior density for θ and G is specified as

$$p(\theta, G) \propto |G|^{-\frac{K+K_L+1}{2}}, \tag{18}$$

which is the standard diffuse prior used to represent "noninformative" beliefs about the parameters of a multivariate normal distribution (e.g., Box and Tiao (1973)).

2.3. Posteriors

The posterior density for the parameters is proportional to the product of the prior density and the likelihood function. Given the factorizations of the likelihood function in (7) and the prior density in (17), the posterior density can also be factored as the posterior for b and σ multiplied by the posterior for θ and G. We analyze these two posteriors separately and then explain how we combine the posterior moments for b and λ to obtain posterior moments for the cost of equity.

2.3.1. Regression Parameters

The joint prior density $p(b, \sigma)$ is equal to the product $p(b|\sigma)p(\sigma)$, where the normal prior density for b given σ in (10) can be written as[5]

$$\begin{aligned} p(b|\sigma) &\propto |\Psi(\sigma)|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(b-\bar{b})'\Psi(\sigma)^{-1}(b-\bar{b})\right\} \\ &\propto \frac{1}{\sigma} \exp\left\{-\frac{1}{2\sigma^2}(b-\bar{b})'(\frac{1}{\sigma^2}\Psi(\sigma))^{-1}(b-\bar{b})\right\}, \end{aligned} \tag{19}$$

[5]The second line in (19) follows from

$$\begin{aligned} |\Psi(\sigma)| &= \sigma_\alpha^2 \left| \left(\frac{\sigma^2}{E(\sigma^2)}\right) - \left(\frac{\sigma}{E(\sigma)}\right)^2 \sigma_\beta' \rho_{\alpha\beta}' V_\beta^{-1} \rho_{\alpha\beta}\sigma_\beta \right| \cdot |V_\beta| \\ &\propto \sigma^2. \end{aligned}$$

and the marginal inverted-gamma prior density for σ in (11) can be written as

$$p(\sigma) \propto \frac{1}{\sigma^{\nu+1}} \exp\{-\frac{\nu s_0^2}{2\sigma^2}\}. \tag{20}$$

Multiplying the prior densities in (19) and (20) and the likelihood in (8) gives the joint posterior for b and σ, which can be written as

$$\begin{aligned} p(b, \sigma | r, F^{(T)}) &\propto \frac{1}{\sigma^{\nu+T+2}} \exp\left\{-\frac{1}{2\sigma^2}\left[\nu s_0^2 + T\hat{\sigma}^2 + (b-\bar{b})'(\frac{1}{\sigma^2}\Psi(\sigma))^{-1}(b-\bar{b}) \right.\right. \\ &\qquad \left.\left. + (b-\hat{b})'X'X(b-\hat{b})\right]\right\}, \end{aligned} \tag{21}$$

where $\hat{b} = (X'X)^{-1}X'r$ and $T\hat{\sigma}^2 = (r - X\hat{b})'(r - X\hat{b})$. We compute moments of this joint posterior using Gibbs sampling, a Markov-chain Monte Carlo procedure introduced by Geman and Geman (1984). (For an introduction to the Gibbs sampler, see Casella and George (1992).) We implement the Gibbs Markov chain by initializing b at its prior mean and then making draws in turn from the conditional posterior densities $p(\sigma|b, r, F^{(T)})$ and $p(b|\sigma, r, F^{(T)})$. After a number of draws, the effect of the startup value for b disappears and the draws are then made from the joint posterior density $p(b, \sigma|r, F^{(T)})$. The posterior moments of the parameters are taken as the sample estimates over a large number of draws. We simulate a Gibbs chain of 50,500 draws, discard the first 500 draws, and estimate the posterior moments of b over the remaining 50,000 draws of b. The number of draws is chosen such that, across repeated independent runs of the Gibbs sampler, differences in the first and second moments of b are small enough for us to report at least two decimal places in our results.

In order to implement the Gibbs sampling described above, we must make draws from the conditional posteriors of b and σ. From (21), we see that the conditional posterior for b given σ can be written as

$$\begin{aligned} p(b|\sigma, r, F^{(T)}) &\propto \exp\left\{-\frac{1}{2}\left[(b-\bar{b})'\Psi(\sigma)^{-1}(b-\bar{b}) + (b-\hat{b})'(\frac{1}{\sigma^2}X'X)(b-\hat{b})\right]\right\} \\ &\propto \exp\left\{-\frac{1}{2}(b-\tilde{b}_\sigma)'M(b-\tilde{b}_\sigma)\right\}, \end{aligned} \tag{22}$$

where

$$M = \Psi(\sigma)^{-1} + \frac{1}{\sigma^2}X'X \tag{23}$$

and

$$\tilde{b}_\sigma = M^{-1}\left[\Psi(\sigma)^{-1}\bar{b} + \frac{1}{\sigma^2}X'X\hat{b}\right]. \tag{24}$$

Hence, the conditional posterior distribution for b given σ is normal with mean $\tilde{b}_\sigma$ and covariance matrix M^{-1}. Note that $\tilde{b}_\sigma$ is a (matrix) weighted average of the prior mean $\bar{b}$

and the sample estimate $\hat{b}$, where the weights are the precisions of $\bar{b}$ and $\hat{b}$ conditional on σ. This weighting can be interpreted as shrinking the sample estimate $\hat{b}$ toward its prior mean $\bar{b}$, where the degree of shrinkage depends on the relative reliability of the sample estimate.

The joint posterior in (21) implies that the conditional posterior for σ given b is

$$p(\sigma|b,r,F^{(T)}) \propto \frac{1}{\sigma^{\nu+T+2}} \exp\left\{-\frac{1}{2\sigma^2}\left[\nu s_0^2 + T\hat{\sigma}^2 + (b-\bar{b})'(\frac{1}{\sigma^2}\Psi(\sigma))^{-1}(b-\bar{b}) + (b-\hat{b})'X'X(b-\hat{b})\right]\right\}, \tag{25}$$

which does not correspond to a well known density. We use a two-pass grid method with a piecewise linear approximation to the cumulative distribution function (cdf) to make draws from the density in (25). Our approach relies on the "griddy Gibbs" sampler discussed by Tanner (1993). In the first pass, we use an equally spaced grid, centered around the density's (unique, strictly positive) mode, to fit a piecewise linear cdf $P_1(\sigma|b,r,F^{(T)})$ corresponding to $p(\sigma|b,r,F^{(T)})$. The second-pass grid is created by taking $P_1^{-1}(\frac{j}{J}|b,r,F^{(T)})$, for $j=1,\ldots,J$, where J determines the fineness of the grid. This second grid increases the accuracy of the procedure by putting more grid points in the regions of greater mass. Through this finer grid we then refit the piecewise linear cdf, denoted as $P_2(\sigma|b,r,F^{(T)})$, and use the inverse cdf method to draw σ. That is, we generate a $U(0,1)$ variate u and take σ as $P_2^{-1}(u|b,r,F^{(T)})$.

We find that the first and second posterior moments of b, computed using Gibbs sampling, are approximated well by the moments of $p(b|\sigma,r,F^{(T)})$ evaluated at a reasonable estimate of σ (using (23) and (24)). An estimate of σ for this purpose is computed in two steps. Using (24), the posterior mean of b conditioned on $\sigma=\hat{\sigma}$ is computed, and its value is denoted as b^*. The final estimate of σ is computed as the posterior mean of σ conditioned on $b=b^*$, using the conditional posterior density for σ that arises when σ and b are made independent in the normal-inverted-gamma prior.[6] In the empirical analysis presented in Section 3, we report results for one stock based on Gibbs sampling, but we use the approximation described above to compute posterior moments for a large number of stocks, since performing a Gibbs sampling for every stock would be computationally prohibitive.

2.3.2. Factor Means

Define the first and second sample moments of y_t,

$$\hat{\theta}_2 = \frac{1}{L}Y^{(L)\prime}\iota_L \tag{26}$$

[6]The prior for σ is the same as in (11), and the prior for b is specified as normal with mean $\bar{b}$ and covariance matrix V_b. The conditional posterior for σ given b is inverted gamma in that case.

and

$$\hat{G}_{22} = \frac{1}{L}(Y^{(L)} - \iota_L \hat{\theta}_2')'(Y^{(L)} - \iota_L \hat{\theta}_2'). \tag{27}$$

Let $Y^{(S)}$ denote the $S \times K_L$ matrix containing the S observations of y_t corresponding to the same S periods as those in $F^{(S)}$, and define $Z = [\iota_S \; Y^{(S)}]$. The least-squares coefficient matrix in a multivariate regression of $F^{(S)}$ on $Y^{(S)}$ is

$$\hat{H} = \begin{bmatrix} \hat{h}_1' \\ \hat{H}_2' \end{bmatrix} = (Z'Z)^{-1}Z'F^{(S)}, \tag{28}$$

where $\hat{h}$ is $K \times 1$ and $\hat{H}_2$ is $K \times K_L$, and the sample covariance matrix of the residuals is

$$\hat{\Sigma} = \frac{1}{S}(F^{(S)} - Z\hat{H})'(F^{(S)} - Z\hat{H}). \tag{29}$$

The sample statistics in (26) through (29) prove useful in computing the posterior first and second moments of λ, which are derived in the Appendix. The posterior mean of λ is

$$\tilde{\lambda} = \hat{h}_1 + \hat{H}_2\hat{\theta}_2, \tag{30}$$

and the posterior covariance matrix of λ is

$$\begin{aligned} \tilde{V}_\lambda \;=\; & \left(\frac{S}{S-K-2}\right) \mathrm{tr}\left\{(Z'Z)^{-1}\begin{bmatrix} 1 & \hat{\theta}_2' \\ \hat{\theta}_2 & \left(\frac{1}{L-K-2K_L-1}\right)\hat{G}_{22} + \hat{\theta}_2\hat{\theta}_2' \end{bmatrix}\right\} \cdot \hat{\Sigma} \\ & + \left(\frac{1}{L-K-2K_L-1}\right)\hat{H}_2\hat{G}_{22}\hat{H}_2'. \end{aligned} \tag{31}$$

Note that when $S = L$, $\tilde{\lambda}$ in (30) simplifies to the vector of sample means of the factors over the S periods. That is, the more common estimate of the factor premia arises as a special case of our estimate when no longer-history asset returns are included in the estimation.

2.3.3. Cost of Equity

Recall that the cost of equity (as an excess return) is given by

$$\mu = \alpha + \lambda'\beta = [1 \; \lambda']b. \tag{32}$$

Once we have obtained the posterior first and second moments of λ and b, it is straightforward to compute the first and second moments of μ, since the posterior distributions of those parameters are independent. As noted at the outset, the decision maker's estimate of the cost of equity is the posterior mean of μ, which is simply

$$\mathrm{E}\{\mu | r, F^{(S)}, Y^{(L)}\} = \tilde{\alpha} + \tilde{\lambda}'\tilde{\beta}, \tag{33}$$

where $\tilde{\alpha}$ and $\tilde{\beta}$ denote posterior means of α and β. The posterior variance of μ is easily verified to be

$$\text{var}\{\mu|r, F^{(S)}, Y^{(L)}\} = \text{tr}\left(\tilde{V}_b \begin{bmatrix} 1 & \tilde{\lambda}' \\ \tilde{\lambda} & \tilde{V}_\lambda + \tilde{\lambda}\tilde{\lambda}' \end{bmatrix}\right) + \tilde{\beta}'\tilde{V}_\lambda\tilde{\beta}, \tag{34}$$

where $\tilde{V}_b$ and $\tilde{V}_\beta$ denote the posterior covariance matrices of b and β.

In the empirical results presented in the next section, we compute the posterior variance of μ and its components, α and $\beta'\lambda$. For the latter quantity, we report the unconditional variance as well as variances that condition on either β or λ set equal to their posterior means. The conditional variances provide additional insight into the sources of uncertainty about the cost of equity. These variances of $\beta'\lambda$ are computed as

$$\text{var}\{\beta'\lambda|r, F^{(S)}, Y^{(L)}\} = \text{tr}\left(\tilde{V}_\beta\left[\tilde{V}_\lambda + \tilde{\lambda}'\tilde{\lambda}\right]\right) + \tilde{\beta}'\tilde{V}_\lambda\tilde{\beta}, \tag{35}$$

$$\text{var}\{\beta'\lambda|\lambda = \tilde{\lambda}, r, F^{(S)}, Y^{(L)}\} = \tilde{\lambda}'\tilde{V}_\beta\tilde{\lambda}, \tag{36}$$

and

$$\text{var}\{\beta'\lambda|\beta = \tilde{\beta}, r, F^{(S)}, Y^{(L)}\} = \tilde{\beta}'\tilde{V}_\lambda\tilde{\beta}. \tag{37}$$

3. Empirical Analysis

3.1. Prior Parameters

In order to construct the prior distribution for the regression parameters in (10) and (11), we specify the elements in $\bar{b}$ and V_b and the scalar quantities s_0 and ν. (Note from (12) through (16) that V_b, s_0^2 and ν determine the conditional covariance matrix $\Psi(\sigma)$.) The prior values are chosen with the objective that the prior mean of b for any given stock be the mean of b in a large cross-section of stocks and that the prior unconditional covariance matrix of b for that stock, V_b, be the covariance matrix of b in the cross-section. Similarly, the prior mean and variance of σ^2 for the stock, determined by s_0 and ν, correspond to moments of σ^2 in the cross-section. In essence, the stock to be analyzed is viewed as a random draw from the universe of all stocks. Although this approach strikes us as a reasonable starting point, at least for our exploratory study, it is only one of many methods that might be used to specify the prior. In a statistical sense, the normal-inverted-gamma prior in (10) and (11) is generally characterized as "informative" as opposed to diffuse (non-informative), but our approach to specifying that prior does not rely on specific knowledge about the firm. In an economic sense, therefore, our prior is rather uninformative. One could instead, for example, use knowledge about the firm's industry and base the prior values on a cross-section of firms within only that industry. Such an approach is illustrated later in this section for the utilities industry.

The cross-sectional moments of b and σ^2 are not directly observable. We take an empirical Bayes approach and estimate those moments using values of $\hat{b}$ and $\hat{\sigma}^2$ computed for a large cross-section.[7] Fama and French (1997) apply a similar methodology, following Blattberg and George (1991), in computing shrinkage estimates of β for industry portfolios. For each stock in the CRSP monthly NYSE-AMEX file with at least 24 months of data in the period from July 1963 through December 1995, we compute $\hat{b}$ and $\hat{\sigma}^2$ using all of that stock's available data during that period. The stock returns are in excess of the return on a one-month Treasury bill (from CRSP's SBBI file). For the CAPM and the Fama-French (FF) three-factor model, the factor data begin in July 1963 and consist of monthly realizations of the three FF factors: (i) the excess return on the value-weighted portfolio of NYSE, AMEX, and NASDAQ stocks, (ii) the difference in returns between a small-stock portfolio and a large-stock portfolio, and (iii) the difference in returns between a portfolio of high book-to-market (B/M) stocks and a portfolio of low B/M stocks.[8] Only the first of these factors is used in the CAPM. To construct the three factors for the Connor-Korajczyk (CK) model, we take all stocks with at least one year of data on the NYSE-AMEX monthly CRSP file for the 7/63–12/95 period and then extract one set of factors for that entire period using the method in Connor and Korajczyk (1988) that allows for missing observations.[9]

The statistics $\hat{b}$ and $\hat{\sigma}^2$, computed for each stock, are used to construct the prior parameters b, V_b, s_0^2, and ν. The prior mean of b, $\bar{b}$, is set equal to the cross-sectional average of the $\hat{b}$'s, except that the first element, $\bar{\alpha}$, is set to zero. That is, the prior mean of α corresponds to an exact version of the factor-based pricing model. The prior covariance matrix of b, V_b, is constructed as follows. First, we compute the matrix

$$\hat{V}_b = \Xi(\hat{b}) - \overline{\hat{\sigma}_i^2 (X'X)_i^{-1}}, \tag{38}$$

where $\Xi(\hat{b})$ is the sample cross-sectional covariance matrix of the $\hat{b}$'s. The second term in (38) is the average across stocks of the usual estimate for the sampling variance of $\hat{b}$, where $\hat{\sigma}_i^2$ and $(X'X)_i$ are based on the observations available for stock i. As noted by Fama and French

[7]Vasicek (1973) first proposed using a cross-section of stocks to obtain the parameters of the prior distribution for the market beta. See Berger (1985) for a general discussion of empirical Bayes methods.

[8]We thank Ken French for providing these data.

[9]The factors are the first three eigenvectors of the $T \times T$ matrix ($T = 390$) whose (s,t) element is $(1/N_{s,t}) \sum_{i=1}^{N_{s,t}} r_{i,s} r_{i,t}$, where $r_{i,t}$ is the excess return on stock i in month t and $N_{s,t}$ denotes the number of stocks that have returns in months s and t.

(1997), under standard assumptions, $\hat{V}_b$ is an estimate of the cross-sectional covariance of the b's. For all three models, it happens that $\hat{V}_b$ is positive definite (not guaranteed in general). To construct the matrix V_b, as represented in (13), V_β is set equal to the corresponding submatrix of $\hat{V}_b$, and $\rho_{\alpha\beta}$ is taken from the correlation matrix associated with $\hat{V}_b$. Rather than set σ_α^2 equal to the (1,1) element of $\hat{V}_b$, however, we instead let it take a wide range of values, ranging from zero to infinity.[10] Each value of σ_α^2 is then combined with the fixed values of V_β and $\rho_{\alpha\beta}$, using (13), to form the matrix V_b used in the prior. The inverted gamma density for σ implies[11]

$$\nu = 4 + \frac{2(\mathrm{E}\{\sigma^2\})^2}{\mathrm{var}\{\sigma^2\}}. \tag{39}$$

We substitute the cross-sectional mean and variance of the $\hat{\sigma}^2$'s for the corresponding moments in (39) and then set the value of ν in the prior to be the next largest integer of the resulting value on the right-hand side. Finally, given that value of ν, the value of s_0^2 used in the prior is obtained from (15), where the cross-sectional average of the $\hat{\sigma}^2$'s is substituted for $\mathrm{E}\{\sigma^2\}$. The estimates of $\bar{b}$, $\hat{V}_b$, ν and s_0^2 are reported in Table 1.

3.2. Posterior Moments

3.2.1. An Individual Stock

We first compute, for a specific stock, the moments of the posterior distribution for the cost of equity and its various components. If a "typical" stock were chosen for this exercise, then presumably that stock's estimated regression coefficient vector $\hat{b}$ would be close to the cross-sectional average $\bar{b}$, the prior mean for b. The shrinkage effects in (24) would therefore be minimal, and such a stock might offer a less interesting illustration of the methodology. Instead, the stock used in this analysis is selected to be, loosely speaking, "typically atypical." For each stock on the NYSE and AMEX having at least 60 months of data continuing through December 1995, we compute the regression statistics $\hat{b}$ and $\hat{\sigma}$, where each stock's available monthly history back through July 1963 in used in the estimation. For a given sample statistic, say $\hat{\beta}_1$, we compute for each stock the absolute deviation of $\hat{\beta}_1$ from the cross-sectional average of $\hat{\beta}_1$. The 1,994 stocks are then sorted by these absolute deviations, and the set of $2p + 1$ stocks that centers on the median value for that statistic (p stocks on each side of the median) is identified. This sorting is performed separately for

[10]Technically, the priors and posteriors given in our formulas are defined only for finite positive values of σ_α^2, so the results reported for "zero" and "infinity" are actually computed by setting σ_α to very small and very large values.

[11]This follows directly from the moments given by Zellner (1971, p. 372).

each sample statistic, and the value of p is increased until the intersection of these sets across statistics contains one stock. In order to use the same stock to illustrate all three pricing models, this procedure is conducted over the combined set of regression statistics for the three models (13 statistics in total), and the stock thus selected is KN Energy, Incorporated. The final value of p is 332, which implies that each of KN Energy's statistics lies roughly within the middle third in terms of absolute deviation from the cross-sectional average.

As explained in the previous section, given the form of the likelihood and the assumed prior independence between the regression parameters (b and σ) and the factor means (λ), the posterior moments of the regression parameters depend only on the data used in the regression model. The monthly history of KN Energy begins in December 1970, so in this case, the regression-model data consist of monthly returns on the stock and the factors for the 301 months in the period from December 1970 through December 1995. For KN Energy, Gibbs sampling is used to compute the posterior means and standard deviations of the regression parameters, as described in the previous section. Table 2 reports, in Part A, the posterior means and standard deviations of the CAPM α and β. These posterior moments are reported for seven values of σ_α, the prior standard deviation of α. As σ_α increases from zero to infinity, the posterior mean of α moves from zero to 6.63%; the latter value is close (but not equal) to the least-squares regression estimate of 6.68%. (All values are annualized.) Observe that the posterior mean of α moves away from zero rather slowly. For example, the posterior mean of α is only 66 basis points (bp) above zero at $\sigma_\alpha = 3\%$ and only 155 bp above zero at $\sigma_\alpha = 5\%$. As discussed earlier, the shrinkage applied to β depends on both $\hat{\alpha}$ and σ_α as well as the regression sample size, T. In this case, given $T = 301$, the posterior mean of KN Energy's β is close to the least-squares estimate of 0.76 and moves only slightly, from 0.78 to 0.77, as σ_α goes from zero to infinity. For smaller values of T, the posterior mean of β is shrunk more toward the prior mean.

The cost of equity has α as one of its components. Part B of Table 2 reports posterior moments for the other component, $\beta'\lambda$, and the overall cost of equity in excess of the riskless rate, μ. Recall that information about λ is contained not only in the available histories of returns on the factors but also in the longer histories of other series that are correlated with the factors. For example, the Fama-French market factor has been constructed back through July 1963, but it is highly correlated with the value-weighted NYSE index, which CRSP supplies beginning in January 1926. The first panel in Part B reports posterior moments based on the longer period from January 1926 through December 1995, whereas the second panel reports moments based on the shorter period beginning in July 1963. The posterior mean of λ, $\tilde{\lambda}$, is 8.11% based on the longer period but only 5.52% based on the shorter period. This difference reflects the fact that the average return on the value-weighted NYSE portfolio is higher over the 1926–95 period than during the shorter 1963–95 period. Given the high positive correlation between the NYSE index and the Fama-French NYSE-AMEX-NASDAQ index, the posterior mean of the latter index is adjusted upward. (See Stambaugh (1997).) This adjustment produces a cost of equity for KN Energy that is above the shorter-period estimate by about 2%. For the overall period, the posterior mean of μ is about 6.4% based on a strict CAPM ($\sigma_\alpha = 0$) and, given the behavior of the posterior mean of α discussed above, the posterior mean of μ remains between 7% and 8% for values of σ_α up to 5%. That is, prior uncertainty about KN Energy's CAPM mispricing (α) that seems substantial in economic terms still results in a posterior mean fairly close to the CAPM value. As will be demonstrated below, this observation generalizes across stocks and across the three pricing models considered.

The results in the second panel in Part B of Table 2 ignore the longer-history asset returns in the estimation of the factor premia. As was pointed out earlier, in such a case $\tilde{\lambda}$ is simply equal to $\bar{\lambda}$, the vector of sample averages of the factors. Also, due to the relatively large T for KN Energy, the posterior mean of β is very close to $\hat{\beta} = 0.76$. The posterior mean of α ranges from 0% to 6.61%, and the latter value is close to $\hat{\alpha} = 6.68\%$. As a result, in the extreme cases when σ_α equals zero and infinity, our estimates of the cost of equity are close to alternative textbook-recommended estimates (e.g., Benninga and Sarig, 1997). For $\sigma_\alpha = 0$, our estimate of 4.33% is close to the simpler CAPM-based estimate $\hat{\beta}\bar{\lambda} = 4.20\%$. For $\sigma_\alpha = \infty$, our estimate of 10.86% is close to the sample mean of 11.52% for the excess returns on KN Energy's stock, and the corresponding standard deviation of 5.21% is close to the frequentist estimate of 5.34% for the standard error of the sample mean. The close correspondence between our extreme estimates and the two alternative estimates is also observed for the two multifactor models.

Posterior standard deviations of μ, α, and $\beta'\lambda$, also reported in Table 2, summarize the uncertainty about KN Energy's cost of equity and its components. The values reported for $\beta'\lambda$ include both the unconditional standard deviation as well as standard deviations that condition on either β or λ set equal to their posterior means, $\tilde{\beta}$ and $\tilde{\lambda}$. (The calculations rely on equations (34) through (37) in the previous section.) Based on the 1926–95 period, the posterior standard deviation of KN Energy's (annualized) cost of equity ranges from 1.97%, in the case of a dogmatic belief in the CAPM ($\sigma_\alpha = 0$), to 5.12%, in the case of a diffuse prior about deviations from the model ($\sigma_\alpha = \infty$). The first value is essentially the posterior standard deviation of $\beta'\lambda$, which is largely unaffected by σ_α. Further discussion of posterior standard deviations is deferred to the later analysis of cross-sectional averages.

Tables 3 and 4 report posterior moments for the components of KN Energy's cost of equity under the three-factor Fama-French (FF) model and the three-factor Connor-Korajczyk (CK) model. In general, the observations made above for the CAPM apply to these cases as well. In particular, KN Energy's $\hat{\alpha}$ is 6.5% in the FF model and 4.8% in the CK model, but, even with σ_α as large as 5%, the posterior means for α are at most 1.5% above zero. Also, the information about λ contained in the longer histories of the additional assets has a substantial effect on the estimated cost of equity. In these three-factor models, the available histories of the factors, which begin in July 1963, are augmented by three return series whose histories begin in January 1926: the value-weighted NYSE, the equally weighted NYSE, and the Ibbotson small-stock portfolio (all obtained from CRSP). For both of the three-factor models, the cost of equity for KN Energy based on the longer 1926–95 period is, as observed previously for the CAPM, about 2% higher than the cost of equity based on the shorter 1963–95 period. For the longer period, the CAPM and the FF model produce similar estimates for KN Energy's cost of equity, whereas the CK model produces values one or two percent higher. Differences in expected costs of equity from the three models are analyzed later using a cross-section of stocks.

3.2.2. Cross-Sectional Results

For each stock on the NYSE and AMEX having at least 60 months of data continuing through December 1995, we compute the same posterior moments reported for KN Energy in Tables 2–4 using a stock's available monthly history back through July 1963. Each value in Tables 5–7 is the arithmetic average across the 1,994 stocks of the corresponding value reported in Tables 2–4. Computing the posterior moments for each of these stocks using Gibbs sampling would be computationally prohibitive. In constructing Tables 5–7, we instead use the approximations to the first and second posterior moments of b discussed in the previous section. The approximations appear to work well. For example, when the values in Tables 2–4 are recomputed using the approximations, none of the posterior means change by more than 3 basis points (bp), and none of the standard deviations change by more than 5 bp. (When σ_α is 10% or less, none of the means and standard deviations change by more than 2bp.)

Unless stated otherwise, our discussion will center on results obtained for all three models using the longer 1926–95 period. The FF and CK models yield posterior means of μ for the typical (average) stock in the range of 11 to 12 percent, roughly 3 percent higher than the corresponding mean under the CAPM. For the FF model, this difference relative to the

CAPM is due largely to the second and third factors, since the average posterior means of the market betas are similar for the two models (1.00 versus 0.98). The average posterior means of the betas on SMB_t and HML_t are 0.68 and 0.32, which indicates that the average firm in the cross-section is tilted toward smaller capitalization and higher book-to-market. When combined with the posterior means for SMB_t and HML_t of 3.6% and 5.3%, those betas account for the bulk of the difference between the CAPM and FF costs of equity for the average firm. The difference between the CAPM and the CK model is more difficult to describe, given that the factors are less easily identified, but one might reasonably conjecture that the CK factors similarly capture additional priced components of returns on small-capitalization and high book-to-market stocks.[12]

The average posterior standard deviations in Tables 5–7 reveal various aspects of uncertainty about the cost of equity for a typical individual stock. An exact version of a pricing model, where $\alpha = 0$, implies a cost of equity equal to $\beta'\lambda$, and that quantity's average posterior standard deviation is largely unaffected by the prior uncertainty about α, as is evident from Tables 5–7. The average posterior standard deviation of $\beta'\lambda$ is about 2.9% for the CAPM, 4.3% for the FF model, and 4.5% for the CK model. These values reflect the uncertainty in both β and λ. For the typical stock, we see that uncertainty about β alone contributes substantially to the overall uncertainty about the cost of equity for an individual stock. Specifically, the average conditional standard deviation of $\beta'\lambda$ given $\lambda = \tilde{\lambda}$ is about 1.4% for the CAPM, 2.7% for the FF model, and 2.6% for the CK model. On average, uncertainty about β is less important than uncertainty about λ, but not dramatically so: the average conditional standard deviation of $\beta'\lambda$ given $\beta = \tilde{\beta}$ is about 2.3% for the CAPM, 3.1% for the FF model, and 3.2% for the CK model. Note also from these conditional standard deviations that the higher unconditional posterior standard deviations of $\beta'\lambda$ in the three-factor models, as compared to the CAPM, reflect additional uncertainty about both β and λ.

In all three models, the posterior means of λ are affected substantially by augmenting the factor histories, which begin in July 1963, with the longer histories of additional series that begin in 1926. These effects on posterior means indicate an important reliance on the information in the longer histories of the additional variables, but the posterior standard deviations of $\beta'\lambda$ for the longer period are generally of about the same magnitude, or even slightly larger, than the posterior standard deviations for the shorter period. This outcome might seem puzzling, but the comparison of posterior standard deviations does not really

[12]Brennan, Chordia, and Subrahmanyam (1996) conclude that Connor-Korajczyk factor sensitivities can at least partially account for cross-sectional differences in expected returns related to size and book-to-market.

provide a sensible measure of the additional information provided by the longer histories. The reason is that the longer histories can also provide additional information about uncertainty. In particular, if the sample volatility of the long-history series is higher prior to 1963 than after, then posterior beliefs about the factors' variances will center on higher values when based on the overall period. This increase in posterior variance of the factors, ceteris paribus, raises the posterior variance of λ, the vector of factor means. In effect, more information can reveal greater uncertainty than otherwise perceived. That effect then works in opposition to the more obvious one (also present): longer histories provides more information about factor means and, ceteris paribus, lower their posterior variances.

When σ_α is very large, then the posterior standard deviation of α is fairly close to the usual frequentist standard error for the estimated regression intercept. In that case, not surprisingly, the posterior uncertainty about α dominates the posterior uncertainty about the cost of equity. At lower values of σ_α, the posterior standard deviation of α is typically about 1/2 to 3/4 of σ_α. For example, when $\sigma_\alpha = 5\%$, the posterior standard deviation of α is just over 3% in all three models. The difference between the posterior standard deviation of μ and the posterior standard deviation of $\beta'\lambda$ arises due to uncertainty about α. In general, for values of σ_α between 3% and 5%, it seems that uncertainty about α is of roughly similar importance to uncertainty about β and λ in explaining the overall posterior uncertainty about a typical firm's cost of equity.

Recall that, for each of the three models, the estimated cost of equity for KN Energy, i.e., the posterior mean of μ, is not very sensitive to the presence of economically plausible "pricing uncertainty," represented by σ_α. As the results in Tables 2–4 demonstrate, for values of σ_α up to 5%, the posterior mean of KN Energy's α remains within 150 basis points of its prior mean of zero, even though the least-squares estimate $\hat{\alpha}$, based on over 25 years (301 months) of data, ranges between 4.8% and 6.7% for the three models. For the other firms in our cross-section, the degree to which the cost of equity is sensitive to σ_α cannot be discerned from the cross-sectional averages reported in Tables 5–7. In order to explore this issue, we plot in Figures 1 through 3, for the three pricing models, each stock's posterior mean of μ obtained with $\sigma_\alpha = 0$ versus the stock's posterior mean of μ obtained with a non-zero value of σ_α. The latter value of σ_α is, in different plots, 3%, 5%, 10%, and ∞. A stock's vertical deviation from a 45-degree line is approximately $\tilde{\alpha}$, the posterior mean of α for that stock, since the values plotted are $\tilde{\beta}'\tilde{\lambda}$ (horizontal axis) versus $\tilde{\alpha} + \tilde{\beta}'\tilde{\lambda}$ (vertical axis), and $\tilde{\beta}$ is essentially unaffected by σ_α. In all three figures, the upper-left plot reveals that, across the 1,994 stocks in the cross-section, estimated costs of equity obtained with $\sigma_\alpha = 3\%$ are generally quite close to those obtained with $\sigma_\alpha = 0$. The scatter of points becomes more disperse as the nonzero value of σ_α increases, but not very quickly. Even for $\sigma_\alpha = 10\%$, the estimated costs of equity from all three models display a clear association with those obtained using an exact pricing relation.

Note that the elements of b are assumed to be constant during the T periods for which the stock's historical returns are used in (4) and (8). In the empirical analysis reported above, we take T to be the stock's entire history, at least back through July 1963. Thus, we essentially use "long-run" betas and ignore potential fluctuations in individual-stock betas over time. Several alternative approaches could be pursued. For example, T might be restricted to at most 60 months, as is consistent with common practice. We have redone the calculations for that case and find similar results, except that, not surprisingly, the estimated cost of equity is then affected even less by $\hat{\alpha}$. In other words, for any economically reasonable prior uncertainty about mispricing, the estimated cost of equity is very close to the estimate produced by zero prior uncertainty. Also, the uncertainty associated with b rises somewhat for most stocks. Although we could have just as easily reported those results, we find the longer-period results, especially those involving α, to be more interesting. Another approach that might be a fruitful direction for research would be to reformulate the Bayesian model to allow changes in b. In a frequentist setting, for example, Shanken (1990) specifies b to be a linear function of observable state variables. Fama and French (1997) implement such a procedure by letting an industry's betas depend on its size and book-to-market ratio.[13]

3.2.3. An Industry-Specific Approach: Utilities

As noted earlier, the prior constructed by using the entire cross-section of stocks can be viewed as uninformative compared to a prior that makes use of the firm's industrial classification. For example, if a public utility's cost of equity is to be estimated, the prior parameters can be obtained from a cross-section of utilities rather than the cross-section of all stocks. We construct such a prior using the cross-section of utility firms (SIC codes between 4900 and 4999) with at least 48 months of data in the period from July 1963 through December 1995. The same approach described earlier for the entire cross-section is applied here, except that the off-diagonal elements of $\hat{V}_b$ are set to zero in order to obtain a positive-definite covariance matrix. In the same manner as discussed previously, posterior moments are then computed

[13]Fama and French (1997) find support for such a specification, although they do not find its merits over the simpler procedure to be clear cut. Moreover, they also suggest (p. 170) that, because variables such as size and book-to-market may be somewhat under management's control, "firms might be better off using full-period constant-slope [costs of equity] for capital budgeting." Schink and Bower (1994), for example, use full-period betas in estimating the cost of equity for individual public utilities.

for the 124 utilities having at least 60 months of data continuing through December 1995.

In the interest of space, we present only a brief summary of the results corresponding to those reported for the large cross-section in tables 5 through 7. Compared to those previous values, the average posterior means of μ for the utilities are smaller, ranging roughly from 5 to 8 percent. As before, the CAPM estimates are on average the smallest, and the FF estimates are the largest. The posterior standard deviations of μ are also smaller than their counterparts in the whole cross-section, by a factor of roughly two. This higher precision of the estimated cost of equity for utilities is due both to lower average betas and to lower posterior standard deviations of the betas. For example, the average posterior mean of the CAPM betas for utilities is only 0.59, which is less than average of 1.00 for the whole cross-section, and the average posterior standard deviation of the CAPM betas is only 0.07, which is less than the corresponding value of 0.17 for the whole cross-section. The uncertainty about λ is more important than the uncertainty about β and, not surprisingly, this effect is more pronounced than in the whole cross-section. Again taking the CAPM as an example, the average conditional standard deviation of $\beta'\lambda$ given $\lambda = \tilde{\lambda}$ is about 0.6%, whereas the average conditional standard deviation of $\beta'\lambda$ given $\beta = \tilde{\beta}$ is about 1.4%.

Figure 4 displays six plots corresponding to those displayed in Figures 1–3, where the non-zero values of σ_α are set equal to 3% and 5% (results for $\sigma_\alpha = 10\%$ and $\sigma_\alpha = \infty$ are not shown). That is, for all three models, each utility's cost of equity estimated with $\sigma_\alpha = 0$ is plotted against its cost of equity estimated with $\sigma_\alpha = 3\%$ or $\sigma_\alpha = 5\%$. As before, the plots exhibit clear positive associations, with deviations from a 45-degree line of roughly the same magnitude as observed previously in the larger cross-section.

3.3. Model Uncertainty

Recall from Tables 2 through 4 that the cost of equity for KN Energy obtained using the CAPM or the Fama-French model is roughly 2 percent less than the cost of equity obtained from the three-factor Connor-Korajczyk model. In their analysis of industries, Fama and French (1997) find that the CAPM produces estimated costs of equity that can differ from those produced by the FF model by 2% or more for some industries. Such differences across models essentially produce additional uncertainty about the cost of equity for a decision maker who remains uncertain about which model to use. As a first step in exploring the potential importance of differences across models in costs of equity for individual firms, we simply plot the cost of equity (posterior mean of μ) obtained using one model versus that obtained using another model. Figure 5 plots, for the previously analyzed cross-section of 1,994 stocks, the costs of equity from the CAPM versus those from the FF model. Figure 6 plots the CAPM costs of equity versus the CK costs of equity, and Figure 7 plots the FF values versus the CK values. Each figure contains four plots, produced with σ_α equal to zero, five percent, ten percent, and infinity. In general, the plots reveal positive correlation between costs of equity produced using different models, although the degree of correlation depends on σ_α as well as the pair of models being compared. The plots in Figure 6, for the CAPM versus the CK model, exhibit the highest correlation, but even those plots exhibit more dispersion than any of the top two plots in Figures 1 through 3. That is, the disagreement in costs of equity across models appears to be greater than the disagreement within a given model produced by changing the degree of prior pricing uncertainty (σ_α) from zero to five percent.

The disagreements among models can be quantified further by assigning subjective probabilities to each model and then computing the variance of a given stock's μ associated with model uncertainty. For each model, the prior and posterior distributions of the parameters in the model are conditioned on that model's being the correct one. If there are Q models under consideration, $q = 1, \ldots, Q$, let $\tilde{\mu}_{[q]}$ denote the posterior mean of μ obtained under model q, and let π_q denotes the decision maker's posterior probability that model q is the correct model. Then, taking the expectation across models, the decision maker ultimately estimates the cost of equity to be

$$\mu^* = \sum_{q=1}^{Q} \pi_q \tilde{\mu}_{[q]}. \tag{40}$$

For example, the New York State Public Service Commission has endorsed the use of equal weights across three different models to estimate the cost of equity for public utilities under its supervision. The three models used by the Commission are the CAPM (more precisely, an average of four CAPM-based estimates) and two non-factor-based models—the "Discounted Cash Flows" model and the "Comparable Earnings" model. The commission is also evaluating the usefulness of multifactor models in estimating costs of equity for public utilities. (See DiValentino, 1994.)

Let $\tilde{v}_{\mu[q]}$ denote the posterior variance of μ obtained under model q. When estimating the cost of equity, the decision maker is left with overall uncertainty given by the unconditional variance across models:

$$v_\mu^* = \sum_{q=1}^{Q} \pi_q \tilde{v}_{\mu[q]} \ + \ \sum_{q=1}^{Q} \pi_q (\tilde{\mu}_{[q]} - \mu^*)^2. \tag{41}$$

The first term on the right-hand side of (41), the expected value across models of the posterior variance of μ, is essentially the average within-model uncertainty about the cost of equity. This component of the overall uncertainty was analyzed in the previous subsection. The second term on the right-hand side of (41), the variance across models of the posterior mean of μ, might be termed "model" uncertainty, or the component of the overall variance of μ attributable to uncertainty about which model to use.

Calculation of model probabilities (π_q's) is beyond the intended scope of this study. As noted at the outset, we focus more on issues related to using various factor-based models for cost-of-equity estimation rather than on issues related to testing such models or evaluating their relative merits. In order to illustrate the calculation of model uncertainty, we consider various sets of candidate models and, for each set, the π_q's are made equal across models.[14] As one model is assigned increasingly higher probability relative to others, model uncertainty generally decreases. In that sense, specifying equal probabilities across models is likely to overstate the model uncertainty, but we suggest that such an exercise nevertheless reveals the potential importance of such uncertainty relative to the components of within-model parameter uncertainty discussed previously.

Table 8 reports the model uncertainty about μ as well as the amount of overall uncertainty, which includes the within-model parameter uncertainty. Calculations are reported for the various two-way subsets of the models as well as for the set of all three models. The results are based on the longer 1926–95 period and are computed for the same alternative values of the prior within-model pricing uncertainty, σ_α, as in the previous analysis. All values are reported as annualized percentage standard deviations. Also shown, for comparison, are (square roots of) the expected values across the three models of the posterior variances of μ, α, and $\beta'\lambda$. Part A of Table 8 displays results for KN Energy, the individual stock examined previously. Recall from Tables 2 and 3 that the CAPM produces posterior means for KN Energy's cost of equity that are reasonably close to those from the FF model across the different values of σ_α. That observation is consistent with the result in Table 8 that, when considering only the CAPM and FF models, the model uncertainty about μ is 0.26% for $\sigma_\alpha = 0$ and only 0.02% for large σ_α. Similarly, given that the CK model produces higher posterior means than the other two models for KN Energy's cost of equity (cf. Table 4), the model uncertainty for the three sets of models that include CK is higher—roughly 1% for $\sigma_\alpha = 0$ and declining to 0.25% as σ_α becomes infinite. In general, however, the uncertainty about KN Energy's cost of equity arising from model uncertainty is less than that arising from uncertainty about parameters within a given model. Consider, for example,

[14] Bower, Bower, and Logue (1984) suggest putting more weight on the estimates obtained from multifactor models than on the CAPM estimates.

the case where a decision maker entertains all three models. At $\sigma_\alpha = 0$, KN Energy's model uncertainty is highest and within-model parameter uncertainty is lowest, but even then the first component (1.01%) is about half of the latter (2.18%) in terms of standard deviation.

Part B of the Table 8 reports the averages across the 1,994 stocks of each value in part A. For the typical stock, both model uncertainty and overall uncertainty are higher than for KN Energy. Otherwise, the conclusions are similar. In particular, the average model uncertainty is less than the average within-model parameter uncertainty. As was the case with KN Energy, as σ_α grows large the average model uncertainty decreases and the average within-model parameter uncertainty increases. Again in the example where a decision maker entertains all three models and $\sigma_\alpha = 0$, the average model uncertainty is 2.5%, whereas the average within-model uncertainty is 4.0%. The average overall uncertainty about μ in that case is 4.74%, only about 0.75% higher than the average uncertainty produced by within-model parameter uncertainty alone. In general, for the three beta-pricing models entertained, although model uncertainty is nontrivial, it appears to be less important than within-model parameter uncertainty in estimating costs of equity for individual firms. Moreover, when entertaining several different models, the decision maker is likely not to have a dogmatic belief in any single model. Therefore, the case in which σ_α is very small, which produces the highest relative importance of the model uncertainty, may be less plausible. Also recall that, because we specify equal probabilities across models, the effects of model uncertainty might even be overstated. Both of these observations tend to strengthen our findings of the relative unimportance of model uncertainty versus within-model uncertainty.

We also conduct a similar analysis the utilities industry. Figure 8 displays, for σ_α set to 3% and 5%, the plots corresponding to those in Figures 5–7. That is, each utility's cost-of-equity estimates obtained from two different models are plotted against each other. The associations between the estimates obtained from different models appear to be stronger than those observed in Figures 5–7 for the whole cross-section of stocks. All three models typically produce rather similar estimates, and the fit between the estimates from the CAPM and the three-factor CK model is especially close. Note that, contrary to the observation for the whole cross-section, the across-model plots in Figure 8 are less disperse than the within-model plots in Figure 4. In other words, the disagreements in utilities' costs of equity across models appear to be smaller than the disagreements within a given model produced by changing the degree of prior mispricing uncertainty (σ_α) from zero to five percent.

Table 9 is the equivalent of part B of Table 8, except that it is constructed for the utilities industry rather than for the whole cross-section. On average across the 124 utility stocks, both model uncertainty and overall uncertainty about the cost of equity are much smaller for utilities than for the whole cross-section. In particular, the model uncertainty for the pair of CAPM and CK is quite low, around 0.5%, confirming our finding of a close correspondence between the cost-of-equity estimates produced by the two models. Despite some differences in magnitude, the relative proportions of model uncertainty and overall uncertainty are similar to those observed in Table 8. Therefore, in the utilities industry, uncertainty about which model to use is again less important than within-model uncertainty.

4. Conclusions

Costs of equity capital implied by factor-based pricing models can be estimated in a Bayesian setting. Such an approach reveals that, after making use of available returns data, the decision maker's uncertainty about the cost of equity (μ) remains high. The posterior standard deviation of μ is typically at least 3% per year in a one-factor model and 4% per year in a three-factor model, even if the possibility that the model might misprice the stock is completely ruled out. For utilities, this standard deviation is smaller, but typically at least 2% per year. Uncertainty about that potential pricing error (α) increases the uncertainty about μ, but the posterior mean of μ—the decision maker's estimated cost of equity—is not affected greatly by uncertainty about α that is substantial in economic terms. A decision maker's uncertainty about which factor-based model to use contributes nontrivially to the overall uncertainty about μ, but, on average, model uncertainty is rather less important than the within-model parameter uncertainty.

In the absence of uncertainty about α, uncertainty about factor means makes the largest contribution to overall uncertainty about the cost of equity, but, for individual stocks, uncertainty about betas is nearly as important. The uncertainty about betas is relatively even less important in the utilities industry. The importance of the uncertainty about factor means remains even after incorporating the additional information in series whose histories are longer than those of the factors. That additional information does, however, produce posterior means for the factors, and thus for μ, that differ from those based on the factor histories alone.

Appendix

This appendix extends results in Stambaugh (1997) and derives the posterior mean and variance-covariance matrix of λ in (30) and (31) when the likelihood function is given by (9) and the prior is given by (18). Recall that λ contains the first K elements of θ. Let Φ denote the data set consisting of $F^{(S)}$ and $Y^{(L)}$, the sample information about the moments of f_t^a. Define the population counterparts to the quantities in (28) and (29),

$$H_2 = G_{12}G_{22}^{-1}, \tag{A.1}$$

$$h_1 = \lambda - H_2\theta_2, \tag{A.2}$$

and

$$\Sigma = G_{11} - H_2 G_{22} H_2', \tag{A.3}$$

where G_{11}, G_{12}, and G_{22} are the submatrices of G in (5) that correspond to the partitioning of $f_t^{a'} = [f_t' \; y_t']$, and let

$$H = \begin{bmatrix} h_1' \\ H_2' \end{bmatrix}. \tag{A.4}$$

It is shown in Stambaugh (1997) that

$$p(H, \Sigma, \theta_2, G_{22}|\Phi) = p(H, \Sigma|\Phi)p(\theta_2, G_{22}|\Phi), \tag{A.5}$$

where

$$p(H, \Sigma|\Phi) \propto |\Sigma|^{-\frac{S+K+K_L+1}{2}} \exp\left\{-\frac{1}{2}\mathrm{tr}\,[S\hat{\Sigma} + (H-\hat{H})'Z'Z(H-\hat{H})]\Sigma^{-1}\right\}, \tag{A.6}$$

and

$$p(\theta_2, G_{22}|\Phi) \propto |G_{22}|^{-\frac{L-K+K_L+1}{2}} \exp\left\{-\frac{1}{2}L \cdot \mathrm{tr}\,[\hat{G}_{22} + (\theta_2 - \hat{\theta}_2)(\theta_2 - \hat{\theta}_2)']G_{22}^{-1}\right\}. \tag{A.7}$$

From (A.7), the conditional posterior of θ_2 given G_{22} is

$$p(\theta_2|G_{22}, \Phi) \propto |G_{22}|^{-\frac{K_L}{2}} \exp\left\{-\frac{1}{2}L(\theta_2 - \hat{\theta}_2)'G_{22}^{-1}(\theta_2 - \hat{\theta}_2)\right\},) \tag{A.8}$$

which is a multivariate normal density with

$$\mathrm{E}\{\theta_2|G_{22}, \Phi\} = \mathrm{E}\{\theta_2|\Phi\} = \hat{\theta}_2 \tag{A.9}$$

and

$$\mathrm{cov}\{\theta_2, \theta_2'|G_{22}, \Phi\} = \frac{1}{L}G_{22}. \tag{A.10}$$

From (A.7) and (A.8), the marginal posterior density of G_{22} is

$$p(G_{22}|\Phi) \propto |G_{22}|^{-\frac{L-K+1}{2}} \exp\left\{-\frac{1}{2}L \cdot \mathrm{tr}\, \hat{G}_{22}G_{22}^{-1}\right\}, \tag{A.11}$$

which is an inverted Wishart density with

$$\mathrm{E}\{G_{22}|\Phi\} = \frac{L}{L-K-2K_L-1}\hat{G}_{22}, \tag{A.12}$$

where (A.12) follows from properties of the inverted Wishart distribution. (See, for example, Anderson (1984, pp. 268–270).) Therefore, since the conditional mean in (A.9) does not involve G_{22}, the unconditional posterior covariance matrix of θ_2 is the expectation of (A.10), which, using (A.12), is

$$\mathrm{cov}\{\theta_2, \theta_2'|\Phi\} = \frac{1}{L-K-2K_L-1}\hat{G}_{22}. \tag{A.13}$$

Next rewrite equation (A.2) as

$$\lambda = Dc, \tag{A.14}$$

where

$$D = I_K \otimes [1\ \theta_2'], \tag{A.15}$$

$$c = \mathrm{vec}\{H\}, \tag{A.16}$$

and "vec$\{H\}$" denotes the $K \times (K_L+1)$ column vector formed by stacking the successive columns of H. Similarly, define

$$\hat{c} = \mathrm{vec}\{\hat{H}\}. \tag{A.17}$$

From (A.6) and the analysis of the multivariate regression model in Zellner (1971, p. 227), the conditional posterior density of c given Σ can be written as

$$p(c|\Sigma,\Phi) \propto |\Sigma|^{-\frac{K_L+1}{2}} \exp\left\{-\frac{1}{2}(c-\hat{c})'(\Sigma^{-1}\otimes Z'Z)(c-\hat{c})\right\}, \tag{A.18}$$

which is a multivariate normal density with

$$\mathrm{E}\{c|\Sigma,\Phi\} = \hat{c} \tag{A.19}$$

and

$$\mathrm{cov}\{c, c'|\Sigma,\Phi\} = \mathrm{cov}\{c,c'|\Phi\} = \Sigma \otimes (Z'Z)^{-1}. \tag{A.20}$$

Because c and θ_2 are independent (cf. (A.5)), it follows immediately from (A.9) and (A.14) through (A.17) that

$$\mathrm{E}\{\lambda|\theta_2,\Phi\} = \hat{h}_1 + \hat{H}_2\theta_2, \tag{A.21}$$

and the unconditional posterior mean of λ, $\tilde{\lambda}$, is given by (30).

From (A.6) and (A.18), and again relying on the analysis in Zellner (1971, p. 227), the marginal posterior density of Σ is given by

$$p(\Sigma|\Phi) \propto |\Sigma|^{-\frac{S+K}{2}} \exp\left\{-\frac{1}{2}S \cdot \mathrm{tr}\,\hat{\Sigma}\Sigma^{-1}\right\}, \tag{A.22}$$

and, using the same property of the inverted Wishart distribution as in (A.12), the unconditional posterior mean of Σ is

$$\mathrm{E}\{\Sigma|\Phi\} = \frac{S}{S-K-2}\hat{\Sigma}. \tag{A.23}$$

Given (A.19), the unconditional posterior covariance matrix of c is the expectation of the conditional covariance matrix in (A.20), which, using (A.23), is equal to

$$\mathrm{cov}\{c, c'|\Phi\} = \frac{S}{S-K-2}\hat{\Sigma} \otimes (Z'Z)^{-1}.) \tag{A.24}$$

Combining (A.14) and (A.24) gives

$$\begin{aligned} \mathrm{cov}\{\lambda, \lambda'|\theta_2, \Phi\} &= \frac{S}{S-K_L(K-1)-2K-1} D(\hat{\Sigma} \otimes (Z'Z)^{-1})D' \\ &= \frac{S}{S-K-2}\left([1\ \lambda']\,(Z'Z)^{-1}\begin{bmatrix} 1 \\ \lambda \end{bmatrix}\right)\hat{\Sigma}, \end{aligned} \tag{A.25}$$

and taking the unconditional expectation of (A.25), using (A.9) and (A.12), gives

$$\begin{aligned} &\mathrm{E}\left\{\mathrm{cov}\{\lambda, \lambda'|\theta_2, \Phi\}|\Phi\right\} = \\ &\left(\frac{S}{S-K-2}\right) \mathrm{tr}\left\{(Z'Z)^{-1}\begin{bmatrix} 1 & \hat{\theta}_2' \\ \hat{\theta}_2 & \left(\frac{1}{L-K-2K_L-1}\right)\hat{G}_{22} + \hat{\theta}_2\hat{\theta}_2' \end{bmatrix}\right\} \cdot \hat{\Sigma}. \end{aligned} \tag{A.26}$$

Also, from (A.21) and (A.13),

$$\begin{aligned} \mathrm{cov}\left\{\mathrm{E}(\lambda|\theta_2, \Phi), \mathrm{E}(\lambda'|\theta_2, \Phi)|\Phi\right\} &= \mathrm{cov}\left\{\hat{H}_2\theta_2, \theta_2'\hat{H}_2'|\Phi\right\} \\ &= \hat{H}_2 \mathrm{cov}\{\theta_2, \theta_2'|\Phi\}\hat{H}_2' \\ &= \frac{1}{L-K-2K_L-1}\hat{H}_2\hat{G}_{22}\hat{H}_2'. \end{aligned} \tag{A.27}$$

By the variance decomposition rule, the sum of the matrices in (A.26) and (A.27) gives $\tilde{V}_\lambda$, the unconditional variance-covariance matrix of λ, and that result is displayed in (31).

Table 1

Cross-sectional Estimates of Prior Parameters

For every stock with at least 24 months of data in the period July 1963 - December 1995, the statistics $\hat{b}$ and $\hat{\sigma}^2$ are computed using monthly data to estimate the appropriate factor-model regressions. The estimate of the prior mean of b, $\bar{b}$, is computed as the cross-sectional average of the $\hat{b}$'s, except that its first element is set to zero. The matrix $\hat{V}_b$ is computed as the cross-sectional covariance matrix of the $\hat{b}$'s minus the cross-sectional average of the time-series sampling variances of the $\hat{b}$'s. The estimate of the prior covariance matrix of b, V_b, is computed from $\hat{V}_b$ by varying σ_α and preserving the correlation structure of $\hat{V}_b$. The estimates of ν and s_0^2 are computed from the cross-sectional sample moments of the $\hat{\sigma}^2$'s using the properties of the inverted gamma density.

	Prior parameter estimates						
Model	$\bar{b}$	$\hat{V}_b$				ν	s_0^2
CAPM	0.0005	0.0003	-0.0008			5	0.0101
	1.1217		0.3844				
3-factor FF	-0.0026	0.0003	-0.0020	-0.0023	-0.0047	5	0.0089
	1.0056		0.3717	0.0611	0.1632		
	0.9671			1.1616	0.3452		
	0.3820				1.0241		
3-factor CK	-0.0007	0.0002	-0.0029	0.0014	-0.0001	5	0.0086
	1.0513		0.9596	0.0427	-0.3175		
	0.0170			1.3080	-0.1051		
	0.0557				0.4083		

Table 2

Posterior Means and Standard Deviations for the Components of KN Energy's Expected Excess Return from the CAPM

The expected excess return on the stock, μ, is given by $\mu = \alpha + \beta\lambda$, where λ is the expected excess return on the value-weighted market portfolio of NYSE, AMEX, and NASDAQ stocks ($R_{M,t}$), and α and β are parameters in the regression of the stock's monthly excess return (R_t) on the market excess return:

$$R_t = \alpha + \beta R_{M,t} + \epsilon_t.$$

The moments for the parameters of the regression model, reported in part A., are based on monthly excess returns for the period 12/1970–12/1995 (301 months). The ordinary least-squares estimates are $\hat{\alpha} = 6.68\%$ (annualized) and $\hat{\beta} = 0.76$. The moments for the quantities involving λ, reported in part B., are based on monthly excess returns for the periods indicated and, for the longer period, make use of the additional information in the history of returns on the value-weighted NYSE portfolio. Also reported for each period is $\hat{\mu} = \hat{\alpha} + \hat{\beta}\tilde{\lambda}$, which is the posterior mean of μ obtained with diffuse priors on all parameters, where $\tilde{\lambda}$ denotes the posterior mean of λ. Except for the moments of β, all posterior means and standard deviations are reported as annualized percentage values.

	Prior Standard Deviation of α (σ_α)						
	0	1%	3%	5%	10%	30%	∞
Part A. Regression parameters							
				Means			
α	0.00	0.08	0.66	1.55	3.64	6.07	6.61
β_1	0.78	0.78	0.78	0.78	0.78	0.77	0.77
				Standard deviations			
α	0.00	0.53	1.52	2.33	3.58	4.61	4.82
β_1	0.09	0.09	0.09	0.09	0.09	0.09	0.09
Part B. Components involving the expected market return							
		1/1926–12/1995; $\hat{\mu} = 12.86$, $\tilde{\lambda} = 8.11$					
				Means			
μ	6.36	6.43	7.00	7.88	9.93	12.32	12.86
$\beta'\lambda$	6.36	6.35	6.34	6.33	6.29	6.25	6.24
				Standard Deviations			
μ	1.97	2.04	2.47	3.02	4.02	4.93	5.12
$\beta'\lambda$	1.97	1.97	1.96	1.96	1.95	1.94	1.94
$\beta'\lambda \mid \lambda = \tilde{\lambda}$	0.72	0.72	0.72	0.72	0.72	0.72	0.72
$\beta'\lambda \mid \beta = \tilde{\beta}$	1.82	1.82	1.82	1.81	1.80	1.79	1.79
		7/1963–12/1995; $\hat{\mu} = 10.89$, $\tilde{\lambda} = 5.52$					
				Means			
μ	4.33	4.41	4.98	5.86	7.92	10.32	10.86
$\beta'\lambda$	4.33	4.33	4.32	4.31	4.28	4.26	4.25
				Standard Deviations			
μ	2.13	2.20	2.61	3.14	4.12	5.02	5.21
$\beta'\lambda$	2.13	2.13	2.13	2.13	2.12	2.10	2.10
$\beta'\lambda \mid \lambda = \tilde{\lambda}$	0.49	0.49	0.49	0.49	0.49	0.49	0.49
$\beta'\lambda \mid \beta = \tilde{\beta}$	2.07	2.06	2.06	2.06	2.04	2.03	2.03

Table 3

Posterior Means and Standard Deviations for the Components of KN Energy's Expected Excess Return from the Three-Factor Fama-French Model

The expected excess return on the stock, μ, is given by $\mu = \alpha + \beta'\lambda$, where λ is the vector of expected values of the three Fama-French factors, and α and $\beta = [\beta_1\ \beta_2\ \beta_3]'$ are parameters in the regression of the stock's monthly excess return (R_t) on the factors:

$$R_t = \alpha + \beta_1 R_{M,t} + \beta_2 SMB_t + \beta_3 HML_t + \epsilon_t.$$

The moments for the parameters of the regression model, reported in part A., are based on monthly data for the period 12/1970–12/1995 (301 months). The ordinary least-squares estimates are $\hat{\alpha} = 6.53\%$ (annualized) and $\hat{\beta} = [0.76\ 0.04\ 0.02]'$. The moments for the quantities involving λ, reported in part B., are based on monthly data for the periods indicated and, for the longer period, make use of the additional information in the history of returns on the value-weighted and equally weighted NYSE portfolios and the Ibbotson small-stock portfolio. Also reported for each period is $\hat{\mu} = \hat{\alpha} + \hat{\beta}'\tilde{\lambda}$, which is the posterior mean of μ obtained with diffuse priors on all parameters, where $\tilde{\lambda}$ denotes the posterior mean of λ. Except for the moments of β, all posterior means and standard deviations are reported as annualized percentage values.

	Prior Standard Deviation of α (σ_α)						
	0	1%	3%	5%	10%	30%	∞
Part A. Regression parameters							
	Means						
α	0.00	0.09	0.65	1.52	3.57	5.95	6.48
β_1	0.78	0.78	0.78	0.78	0.77	0.76	0.76
β_2	0.06	0.06	0.06	0.06	0.06	0.06	0.06
β_3	0.06	0.06	0.06	0.05	0.04	0.02	0.02
	Standard deviations						
α	0.00	0.54	1.55	2.39	3.68	4.77	4.99
β_1	0.10	0.10	0.10	0.10	0.10	0.10	0.10
β_2	0.15	0.15	0.15	0.15	0.15	0.15	0.15
β_3	0.16	0.16	0.16	0.16	0.16	0.16	0.16
Part B. Components involving the expected factors							
	1/1926–12/1995; $\hat{\mu} = 12.88$, $\tilde{\lambda} = [8.05\ 3.63\ 5.32]'$						
	Means						
μ	6.88	6.96	7.48	8.29	10.20	12.42	12.91
$\beta'\lambda$	6.88	6.87	6.83	6.77	6.63	6.47	6.44
	Standard Deviations						
μ	2.35	2.41	2.76	3.23	4.14	5.01	5.19
$\beta'\lambda$	2.35	2.35	2.35	2.35	2.35	2.35	2.34
$\beta'\lambda\ \vert\lambda = \tilde{\lambda}$	1.38	1.38	1.38	1.39	1.40	1.42	1.42
$\beta'\lambda\ \vert\beta = \tilde{\beta}$	1.87	1.87	1.86	1.86	1.84	1.83	1.83
	7/1963–12/1995; $\hat{\mu} = 10.93$, $\tilde{\lambda} = [5.52\ 3.01\ 5.05]'$						
	Means						
μ	4.84	4.92	5.45	6.27	8.20	10.45	10.95
$\beta'\lambda$	4.84	4.83	4.80	4.75	4.63	4.50	4.47
	Standard Deviations						
μ	2.41	2.46	2.81	3.29	4.21	5.09	5.27
$\beta'\lambda$	2.41	2.41	2.41	2.41	2.40	2.40	2.39
$\beta'\lambda\ \vert\lambda = \tilde{\lambda}$	1.15	1.15	1.16	1.16	1.17	1.18	1.19
$\beta'\lambda\ \vert\beta = \tilde{\beta}$	2.08	2.08	2.07	2.07	2.06	2.04	2.04

Table 4

Posterior Means and Standard Deviations for the Components of KN Energy's Expected Excess Return from the Three-Factor Connor-Korajczyk Model

The expected excess return on the stock, μ, is given by $\mu = \alpha + \beta'\lambda$, where λ is the vector of expected values of the three Connor-Korajczyk factors, and α and $\beta = [\beta_1\ \beta_2\ \beta_3]'$ are parameters in the regression of the stock's monthly excess return (R_t) on the factors:

$$R_t = \alpha + \beta_1 F_{1,t} + \beta_2 F_{2,t} + \beta_3 F_{3,t} + \epsilon_t.$$

The moments for the parameters of the regression model, reported in part A., are based on monthly data for the period 12/1970–12/1995 (301 months). The ordinary least-squares estimates are $\hat{\alpha} = 4.84\%$ (annualized) and $\hat{\beta} = [0.58\ \ -0.05\ 0.37]'$. The moments for the quantities involving λ, reported in part B., are based on monthly data for the periods indicated and, for the longer period, make use of the additional information in the history of returns on the value-weighted and equally weighted NYSE portfolios and the Ibbotson small-stock portfolio. Also reported for each period is $\hat{\mu} = \hat{\alpha} + \hat{\beta}'\tilde{\lambda}$, which is the posterior mean of μ obtained with diffuse priors on all parameters, where $\tilde{\lambda}$ denotes the posterior mean of λ. Except for the moments of β, all posterior means and standard deviations are reported as annualized percentage values.

	Prior Standard Deviation of α (σ_α)						
	0	1%	3%	5%	10%	30%	∞
Part A. Regression parameters							
				Means			
α	0.00	0.07	0.53	1.22	2.78	4.48	4.84
β_1	0.59	0.59	0.59	0.59	0.58	0.58	0.58
β_2	-0.06	-0.06	-0.05	-0.05	-0.05	-0.05	-0.05
β_3	0.37	0.37	0.37	0.37	0.37	0.37	0.37
				Standard deviations			
α	0.00	0.56	1.60	2.44	3.69	4.69	4.88
β_1	0.08	0.08	0.08	0.08	0.08	0.08	0.08
β_2	0.07	0.07	0.07	0.07	0.07	0.07	0.07
β_3	0.07	0.07	0.07	0.07	0.07	0.07	0.07
Part B. Components involving the expected factors							
	1/1926–12/1995; $\hat{\mu} = 13.39$, $\tilde{\lambda} = [10.85\ \ -2.22\ 5.94]'$						
				Means			
μ	8.72	8.78	9.23	9.90	11.40	13.04	13.40
$\beta'\lambda$	8.72	8.72	8.70	8.68	8.63	8.57	8.56
				Standard Deviations			
μ	2.21	2.27	2.69	3.22	4.18	5.03	5.19
$\beta'\lambda$	2.21	2.21	2.21	2.20	2.20	2.19	2.19
$\beta'\lambda\ \|\lambda = \tilde{\lambda}$	0.99	0.99	0.99	1.00	1.00	1.00	1.01
$\beta'\lambda\ \|\beta = \tilde{\beta}$	1.94	1.94	1.93	1.93	1.92	1.91	1.90
	7/1963–12/1995; $\hat{\mu} = 11.49$, $\tilde{\lambda} = [7.65\ \ -3.17\ 5.66]'$						
				Means			
μ	6.79	6.85	7.30	7.97	9.49	11.14	11.50
$\beta'\lambda$	6.79	6.79	6.77	6.75	6.71	6.67	6.66
				Standard Deviations			
μ	2.30	2.37	2.77	3.30	4.25	5.09	5.26
$\beta'\lambda$	2.30	2.30	2.30	2.30	2.29	2.27	2.27
$\beta'\lambda\ \|\lambda = \tilde{\lambda}$	0.79	0.79	0.79	0.79	0.80	0.80	0.80
$\beta'\lambda\ \|\beta = \tilde{\beta}$	2.13	2.13	2.12	2.12	2.10	2.09	2.09

Table 5

Averages Across 1,994 Stocks of the Values Shown in Table 2

The means and standard deviations reported for the individual stock in Table 2 are computed for each of the 1,994 stocks that have data through December 1995 and have at least 60 months of historical returns, and the averages of those values across all stocks are reported below. The expected excess return on the stock, μ, is given by $\mu = \alpha + \beta\lambda$, where λ is the expected excess return on the value-weighted market portfolio of NYSE, AMEX, and NASDAQ stocks ($R_{M,t}$), and α and β are parameters in the regression of the stock's monthly excess return (R_t) on the market excess return:

$$R_t = \alpha + \beta R_{M,t} + \epsilon_t.$$

The moments for the parameters of the regression model, reported in part A., are based on each stock's available history of monthly returns, back through July 1963. The moments for the quantities involving λ, reported in part B., are based on monthly excess returns for the periods indicated and, for the longer period, make use of the additional information in the history of returns on the value-weighted NYSE portfolio. Also reported for each period is $\tilde{\lambda}$, the posterior mean of λ. Except for the moments of β, all posterior means and standard deviations are reported as annualized percentage values.

	Prior Standard Deviation of α (σ_α)						
	0	1%	3%	5%	10%	30%	∞
Part A. Regression parameters							
				Means			
α	0.00	0.02	0.17	0.40	0.90	1.45	1.56
β_1	1.00	1.00	1.00	1.00	1.00	1.00	1.00
				Standard deviations			
α	0.00	0.74	2.16	3.39	5.61	8.43	9.31
β_1	0.17	0.17	0.17	0.17	0.17	0.17	0.17
Part B. Components involving the expected market return							
				1/1926–12/1995; $\tilde{\lambda} = 8.11$			
				Means			
μ	8.14	8.17	8.32	8.54	9.03	9.57	9.68
$\beta'\lambda$	8.14	8.14	8.14	8.14	8.13	8.12	8.12
				Standard Deviations			
μ	2.85	2.95	3.58	4.42	6.23	8.77	9.60
$\beta'\lambda$	2.85	2.85	2.85	2.85	2.85	2.85	2.86
$\beta'\lambda \mid \lambda = \tilde{\lambda}$	1.40	1.40	1.40	1.40	1.41	1.41	1.42
$\beta'\lambda \mid \beta = \tilde{\beta}$	2.33	2.33	2.33	2.33	2.33	2.33	2.33
				7/1963–12/1995; $\tilde{\lambda} = 5.52$			
				Means			
μ	5.54	5.57	5.72	5.94	6.44	6.97	7.09
$\beta'\lambda$	5.54	5.54	5.54	5.54	5.53	5.53	5.53
				Standard Deviations			
μ	2.92	3.02	3.66	4.50	6.32	8.88	9.72
$\beta'\lambda$	2.92	2.92	2.92	2.92	2.92	2.91	2.92
$\beta'\lambda \mid \lambda = \tilde{\lambda}$	0.95	0.95	0.96	0.96	0.96	0.96	0.96
$\beta'\lambda \mid \beta = \tilde{\beta}$	2.65	2.65	2.65	2.64	2.64	2.64	2.64

Table 6

Averages Across 1,994 Stocks of the Values Shown in Table 3

The means and standard deviations reported for the individual stock in Table 3 are computed for each of the 1,994 stocks that have data through December 1995 and have at least 60 months of historical returns, and the averages of those values across all stocks are reported below. The expected excess return on the stock, μ, is given by $\mu = \alpha + \beta'\lambda$, where λ is the vector of expected values of the three Fama-French factors, and α and $\beta = [\beta_1\ \beta_2\ \beta_3]'$ are parameters in the regression of the stock's monthly excess return (R_t) on the factors:

$$R_t = \alpha + \beta_1 R_{M,t} + \beta_2 SMB_t + \beta_3 HML_t + \epsilon_t.$$

The moments for the parameters of the regression model, reported in part A., are based on each stock's available history of monthly returns, back through July 1963. The moments for the quantities involving λ, reported in part B., are based on monthly data for the periods indicated and, for the longer period, make use of the additional information in the history of returns on the value-weighted and equally weighted NYSE portfolios and the Ibbotson small-stock portfolio. Also reported for each period is $\tilde{\lambda}$, the posterior mean of λ. Except for the moments of β, all posterior means and standard deviations are reported as annualized percentage values.

	Prior Standard Deviation of α (σ_α)						
	0	1%	3%	5%	10%	30%	∞
Part A. Regression parameters							
				Means			
α	0.00	0.00	0.03	0.07	0.17	0.30	0.32
β_1	0.98	0.98	0.98	0.98	0.98	0.98	0.98
β_2	0.68	0.68	0.68	0.68	0.68	0.68	0.68
β_3	0.32	0.32	0.32	0.32	0.32	0.32	0.32
				Standard deviations			
α	0.00	0.72	2.10	3.30	5.47	8.23	9.08
β_1	0.18	0.18	0.18	0.18	0.18	0.18	0.18
β_2	0.27	0.27	0.27	0.27	0.27	0.27	0.27
β_3	0.30	0.30	0.30	0.30	0.30	0.30	0.30
Part B. Components involving the expected factors							
		1/1926–12/1995; $\tilde{\lambda} = [8.05\ 3.63\ 5.32]'$					
				Means			
μ	12.03	12.03	12.06	12.10	12.19	12.31	12.34
$\beta'\lambda$	12.03	12.03	12.03	12.02	12.02	12.01	12.01
				Standard Deviations			
μ	4.31	4.36	4.74	5.32	6.72	8.90	9.64
$\beta'\lambda$	4.31	4.31	4.32	4.32	4.33	4.34	4.35
$\beta'\lambda \mid \lambda = \tilde{\lambda}$	2.71	2.71	2.71	2.72	2.73	2.75	2.76
$\beta'\lambda \mid \beta = \tilde{\beta}$	3.10	3.10	3.10	3.10	3.10	3.10	3.10
		7/1963–12/1995; $\tilde{\lambda} = [5.52\ 3.01\ 5.05]'$					
				Means			
μ	9.05	9.05	9.08	9.12	9.21	9.33	9.36
$\beta'\lambda$	9.05	9.05	9.05	9.04	9.04	9.04	9.04
				Standard Deviations			
μ	4.14	4.19	4.60	5.21	6.67	8.92	9.68
$\beta'\lambda$	4.14	4.14	4.14	4.14	4.15	4.16	4.16
$\beta'\lambda \mid \lambda = \tilde{\lambda}$	2.26	2.27	2.27	2.27	2.28	2.30	2.30
$\beta'\lambda \mid \beta = \tilde{\beta}$	3.23	3.23	3.23	3.23	3.23	3.23	3.23

Table 7

Averages Across 1,994 Stocks of the Values Shown in Table 4

The means and standard deviations reported for the individual stock in Table 4 are computed for each of the 1,994 stocks that have data through December 1995 and have at least 60 months of historical returns, and the averages of those values across all stocks are reported below. The expected excess return on the stock, μ, is given by $\mu = \alpha + \beta'\lambda$, where λ is the vector of expected values of the three Connor-Korajczyk factors, and α and $\beta = [\beta_1\ \beta_2\ \beta_3]'$ are parameters in the regression of the stock's monthly excess return (R_t) on the factors:

$$R_t = \alpha + \beta_1 F_{1,t} + \beta_2 F_{2,t} + \beta_3 F_{3,t} + \epsilon_t.$$

The moments for the parameters of the regression model, reported in part A., are based on each stock's available history of monthly returns, back through July 1963. The moments for the quantities involving λ, reported in part B., are based on monthly data for the periods indicated and, for the longer period, make use of the additional information in the history of returns on the value-weighted and equally weighted NYSE portfolios and the Ibbotson small-stock portfolio. Also reported for each period is $\tilde{\lambda}$, the posterior mean of λ. Except for the moments of β, all posterior means and standard deviations are reported as annualized percentage values.

	Prior Standard Deviation of α (σ_α)						
	0	1%	3%	5%	10%	30%	∞
Part A. Regression parameters							
				Means			
α	0.00	0.01	0.07	0.18	0.46	0.93	1.09
β_1	0.97	0.97	0.97	0.97	0.97	0.97	0.97
β_2	0.02	0.02	0.02	0.02	0.02	0.02	0.02
β_3	0.14	0.14	0.14	0.14	0.14	0.14	0.14
				Standard deviations			
α	0.00	0.75	2.17	3.39	5.55	8.18	8.96
β_1	0.26	0.26	0.26	0.26	0.26	0.26	0.26
β_2	0.13	0.13	0.13	0.13	0.13	0.13	0.13
β_3	0.17	0.17	0.17	0.17	0.17	0.17	0.17
Part B. Components involving the expected factors							
		1/1926–12/1995; $\tilde{\lambda} = [10.85\ \ -2.22\ 5.94]'$					
				Means			
μ	11.38	11.39	11.45	11.55	11.83	12.27	12.42
$\beta'\lambda$	11.38	11.38	11.38	11.37	11.36	11.34	11.33
				Standard Deviations			
μ	4.46	4.52	4.94	5.55	6.96	9.03	9.70
$\beta'\lambda$	4.46	4.46	4.46	4.46	4.47	4.47	4.47
$\beta'\lambda\ \lvert\lambda = \tilde{\lambda}$	2.59	2.59	2.59	2.60	2.60	2.61	2.62
$\beta'\lambda\ \lvert\beta = \tilde{\beta}$	3.17	3.17	3.17	3.17	3.16	3.16	3.16
		7/1963–12/1995; $\tilde{\lambda} = [7.65\ \ -3.17\ 5.66]'$					
				Means			
μ	8.21	8.22	8.28	8.38	8.66	9.11	9.26
$\beta'\lambda$	8.21	8.21	8.21	8.20	8.20	8.18	8.17
				Standard Deviations			
μ	4.10	4.17	4.63	5.29	6.79	8.96	9.65
$\beta'\lambda$	4.10	4.10	4.10	4.10	4.10	4.11	4.11
$\beta'\lambda\ \lvert\lambda = \tilde{\lambda}$	1.90	1.90	1.91	1.91	1.91	1.92	1.92
$\beta'\lambda\ \lvert\beta = \tilde{\beta}$	3.26	3.26	3.26	3.26	3.26	3.25	3.25

Table 8

Model Uncertainty and Overall Uncertainty About the Cost of Equity

The table reports the uncertainty about a firm's cost of equity (μ) that arises from entertaining multiple models, the overall uncertainty about μ that incorporates both model uncertainty and within-model parameter uncertainty, and the average within-model parameter uncertainty. The three beta-pricing models are the CAPM, the three-factor Fama-French model (FF), and the three-factor Connor-Korajczyk model (CK). For any given subset of models entertained, each model is assigned equal probability. All values are reported as annualized percentage standard deviations.

	Prior Standard Deviation of α (σ_α)						
	0	1%	3%	5%	10%	30%	∞
Part A. Results for KN Energy							
Model uncertainty about μ when the set of models entertained is							
CAPM and FF	0.26	0.26	0.24	0.20	0.12	0.04	0.02
CAPM and CK	1.18	1.17	1.11	1.00	0.73	0.35	0.26
FF and CK	0.92	0.91	0.87	0.80	0.60	0.31	0.24
CAPM, FF, and CK	1.01	1.01	0.95	0.87	0.63	0.31	0.24
Overall uncertainty about μ when the set of models entertained is							
CAPM and FF	2.18	2.24	2.63	3.14	4.09	4.99	5.17
CAPM and CK	2.40	2.46	2.81	3.28	4.17	5.01	5.18
FF and CK	2.46	2.51	2.86	3.32	4.20	5.02	5.19
CAPM, FF, and CK	2.41	2.46	2.81	3.28	4.17	5.01	5.18
Average across the three models of the within-model uncertainty of[a]							
μ	2.18	2.24	2.64	3.16	4.12	5.00	5.17
α	0.00	0.54	1.56	2.39	3.65	4.70	4.90
$\beta'\lambda$	2.18	2.18	2.18	2.18	2.17	2.16	2.16
Part B. Average Across 1,994 Stocks of the Values in Part A							
Model uncertainty about μ when the set of models entertained is							
CAPM and FF	2.20	2.18	2.10	1.99	1.74	1.48	1.43
CAPM and CK	1.79	1.78	1.72	1.65	1.50	1.43	1.46
FF and CK	1.79	1.78	1.70	1.59	1.33	1.02	0.98
CAPM, FF, and CK	2.47	2.45	2.36	2.24	1.96	1.68	1.66
Overall uncertainty about μ when the set of models entertained is							
CAPM and FF	4.45	4.49	4.86	5.42	6.81	9.03	9.80
CAPM and CK	4.23	4.29	4.72	5.35	6.84	9.08	9.82
FF and CK	4.90	4.94	5.27	5.79	7.06	9.08	9.77
CAPM, FF, and CK	4.74	4.78	5.13	5.66	6.99	9.12	9.85
Average across the three models of the within-model uncertainty of[a]							
μ	3.96	4.02	4.47	5.13	6.65	8.90	9.65
α	0.00	0.74	2.14	3.37	5.54	8.28	9.12
$\beta'\lambda$	3.96	3.96	3.96	3.96	3.97	3.97	3.98

[a] The posterior variances are averaged across models before taking the square root.

Table 9

Model Uncertainty and Overall Uncertainty About the Cost of Equity for Utilities

The table reports the uncertainty about a firm's cost of equity (μ) that arises from entertaining multiple models, the overall uncertainty about μ that incorporates both model uncertainty and within-model parameter uncertainty, and the average within-model parameter uncertainty. The three beta-pricing models are the CAPM, the three-factor Fama-French model (FF), and the three-factor Connor-Korajczyk model (CK). For any given subset of models entertained, each model is assigned equal probability. All values are reported as annualized percentage standard deviations and are averaged across 124 utilities.

	Prior Standard Deviation of α (σ_α)						
	0	1%	3%	5%	10%	30%	∞
Model uncertainty about μ when the set of models entertained is							
CAPM and FF	1.54	1.47	1.09	0.77	0.44	0.27	0.25
CAPM and CK	0.67	0.64	0.47	0.33	0.23	0.26	0.27
FF and CK	0.99	0.95	0.71	0.51	0.35	0.32	0.32
CAPM, FF, and CK	1.35	1.29	0.96	0.70	0.45	0.37	0.36
Overall uncertainty about μ when the set of models entertained is							
CAPM and FF	2.36	2.45	2.91	3.34	3.87	4.21	4.27
CAPM and CK	1.74	1.91	2.67	3.24	3.86	4.23	4.29
FF and CK	2.11	2.24	2.83	3.32	3.89	4.24	4.30
CAPM, FF, and CK	2.21	2.32	2.85	3.32	3.88	4.23	4.29
Average across the three models of the within-model uncertainty of[a]							
μ	1.70	1.88	2.67	3.24	3.85	4.21	4.27
α	0.00	0.83	2.10	2.82	3.53	3.94	4.00
$\beta'\lambda$	1.70	1.70	1.70	1.70	1.70	1.70	1.70

[a]The posterior variances are averaged across models before taking the square root.

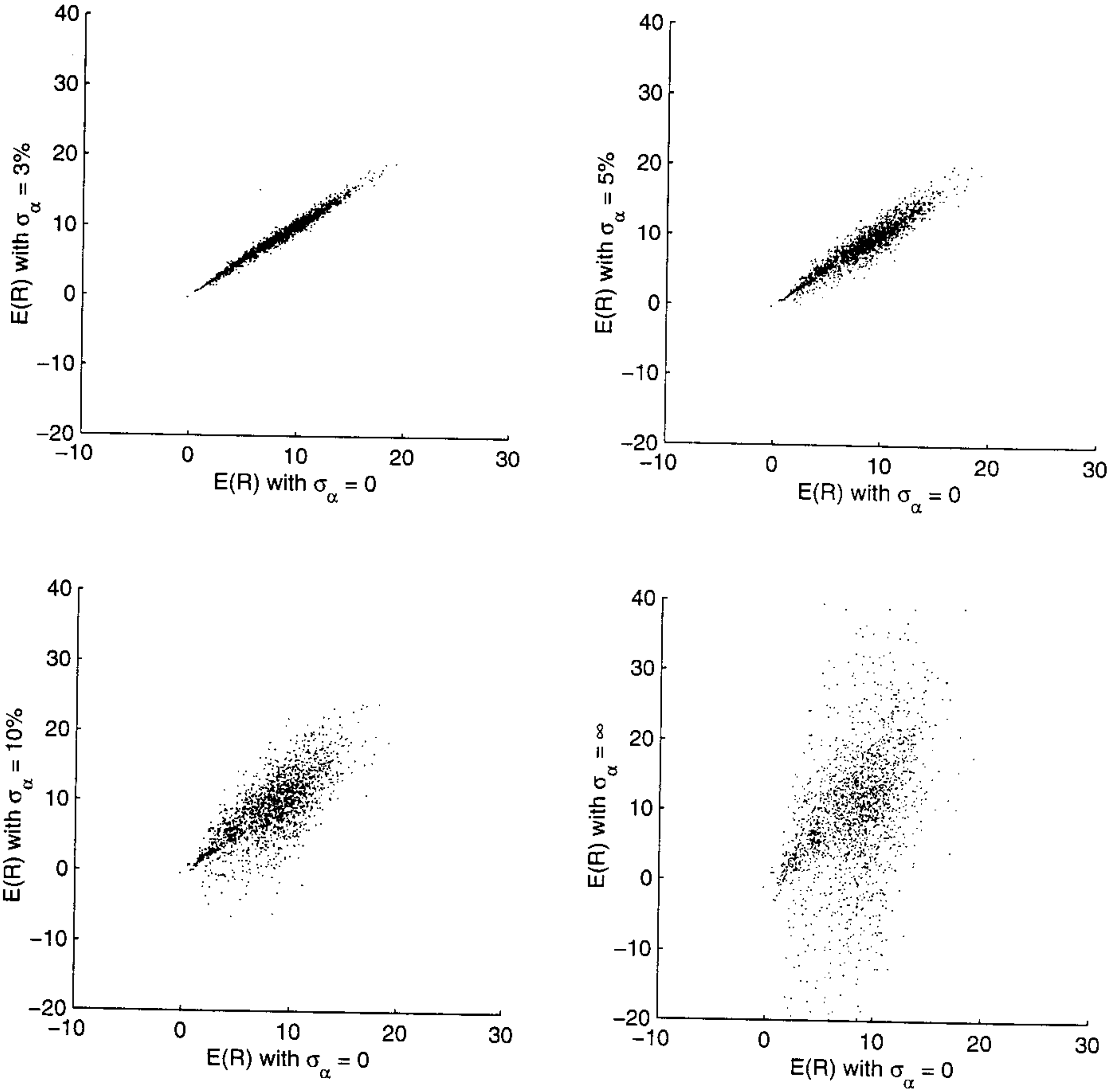

Figure 1. Costs of equity (E(R)) from the CAPM for various degrees of prior uncertainty about mispricing. The prior mispricing uncertainty, σ_α, is the annualized prior standard deviation of α.

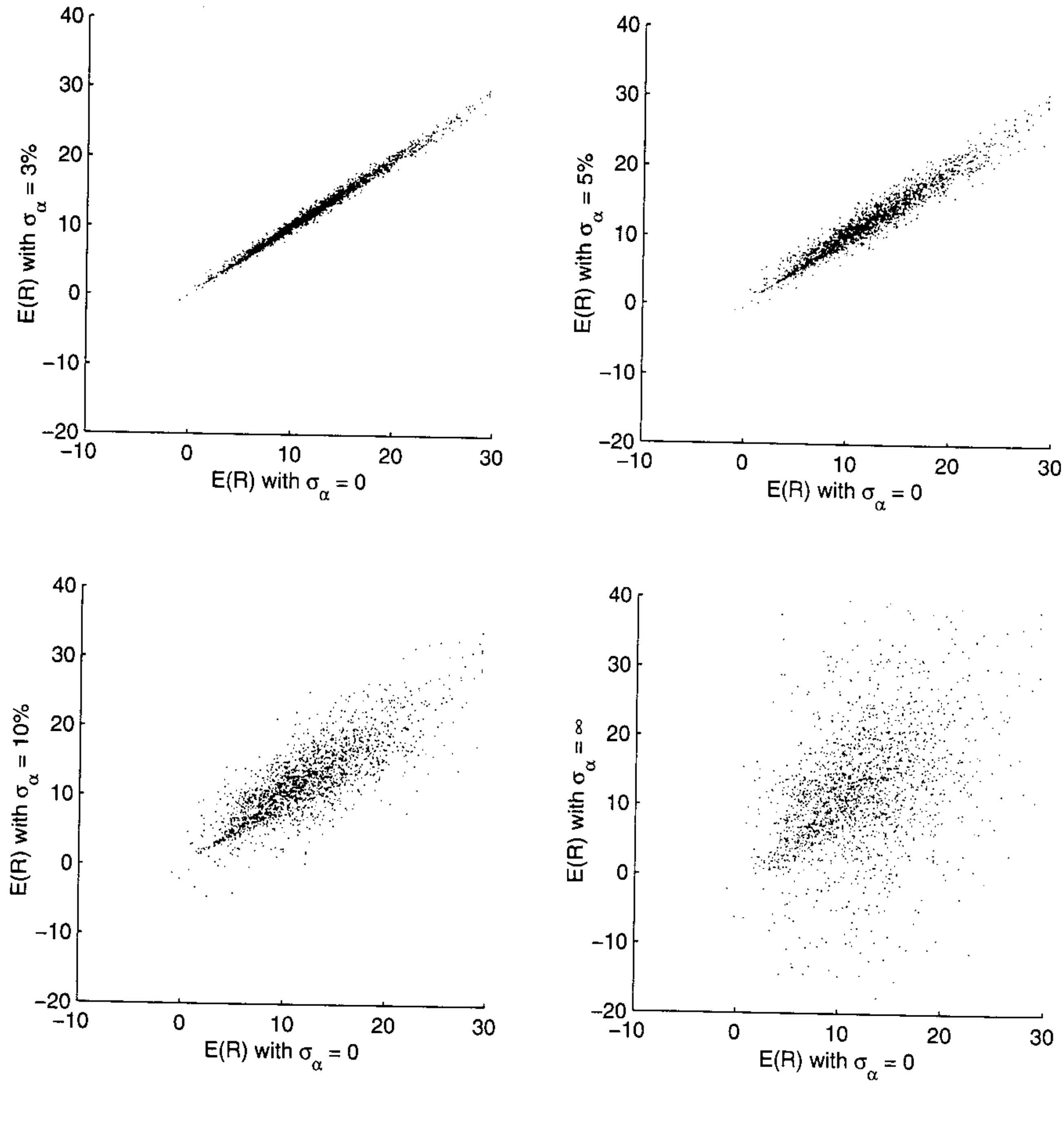

Figure 2. Costs of equity (E(R)) from the three-factor Fama-French model for various degrees of prior uncertainty about mispricing. The prior mispricing uncertainty, σ_α, is the annualized prior standard deviation of α.

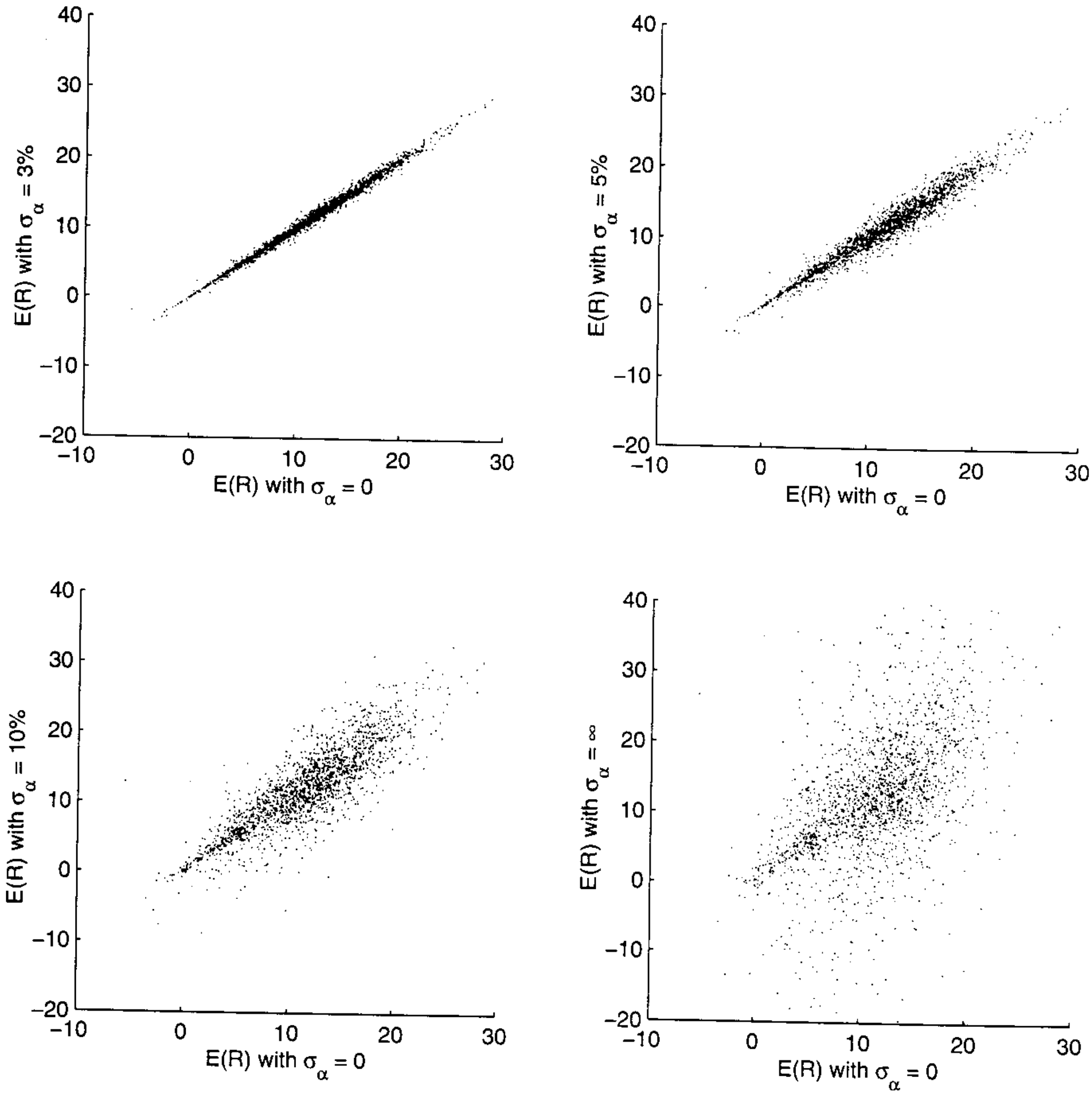

Figure 3. Costs of equity (E(R)) from the three-factor Connor-Korajczyk model for various degrees of prior uncertainty about mispricing. The prior mispricing uncertainty, σ_α, is the annualized prior standard deviation of α.

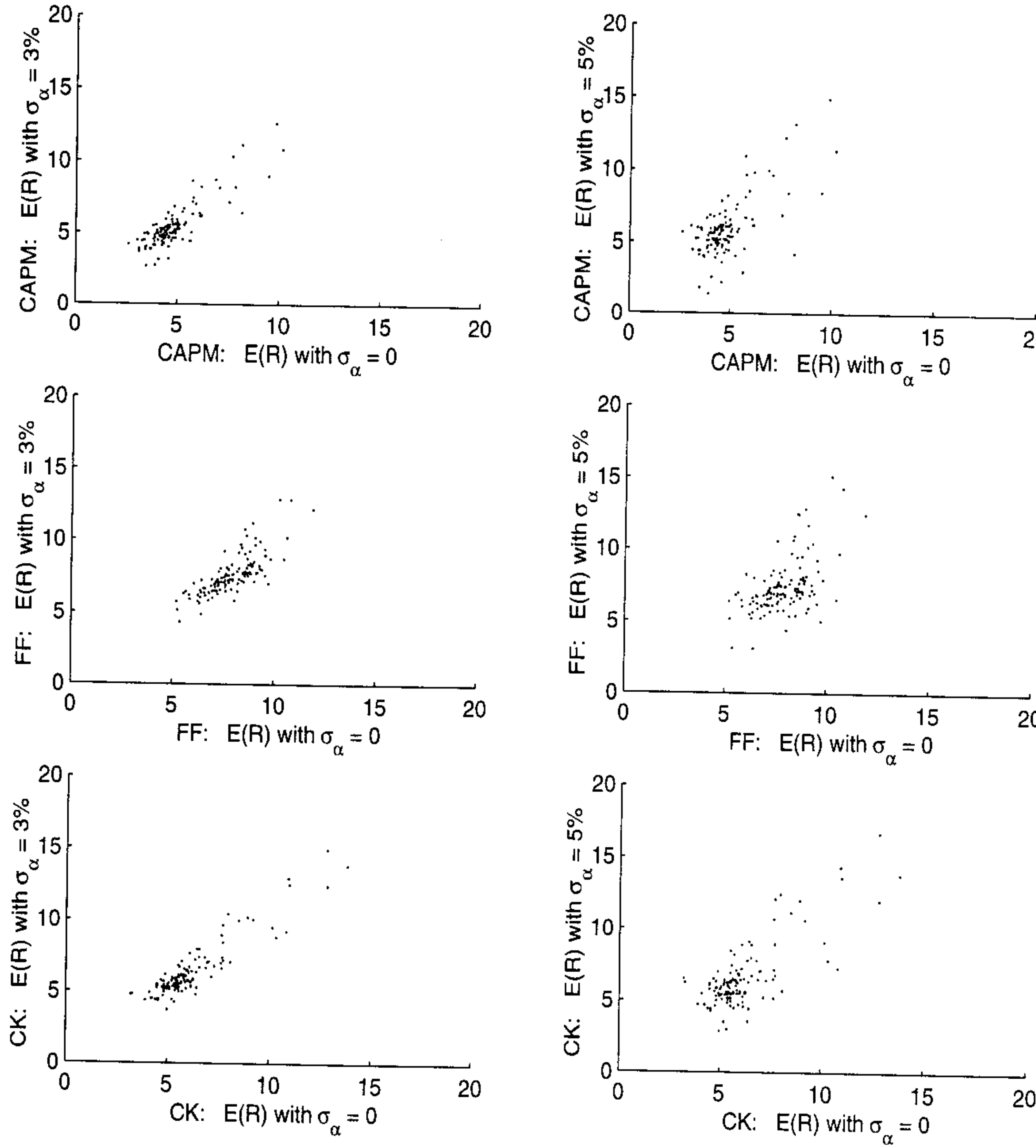

Figure 4. Utility costs of equity (E(R)) for different degrees of prior uncertainty about mispricing. Costs of equity are computed using the CAPM, the three-factor Fama-French model (FF), and the three-factor Connor-Korajczyk model (CK). The prior mispricing uncertainty, σ_α, is the annualized prior standard deviation of α.

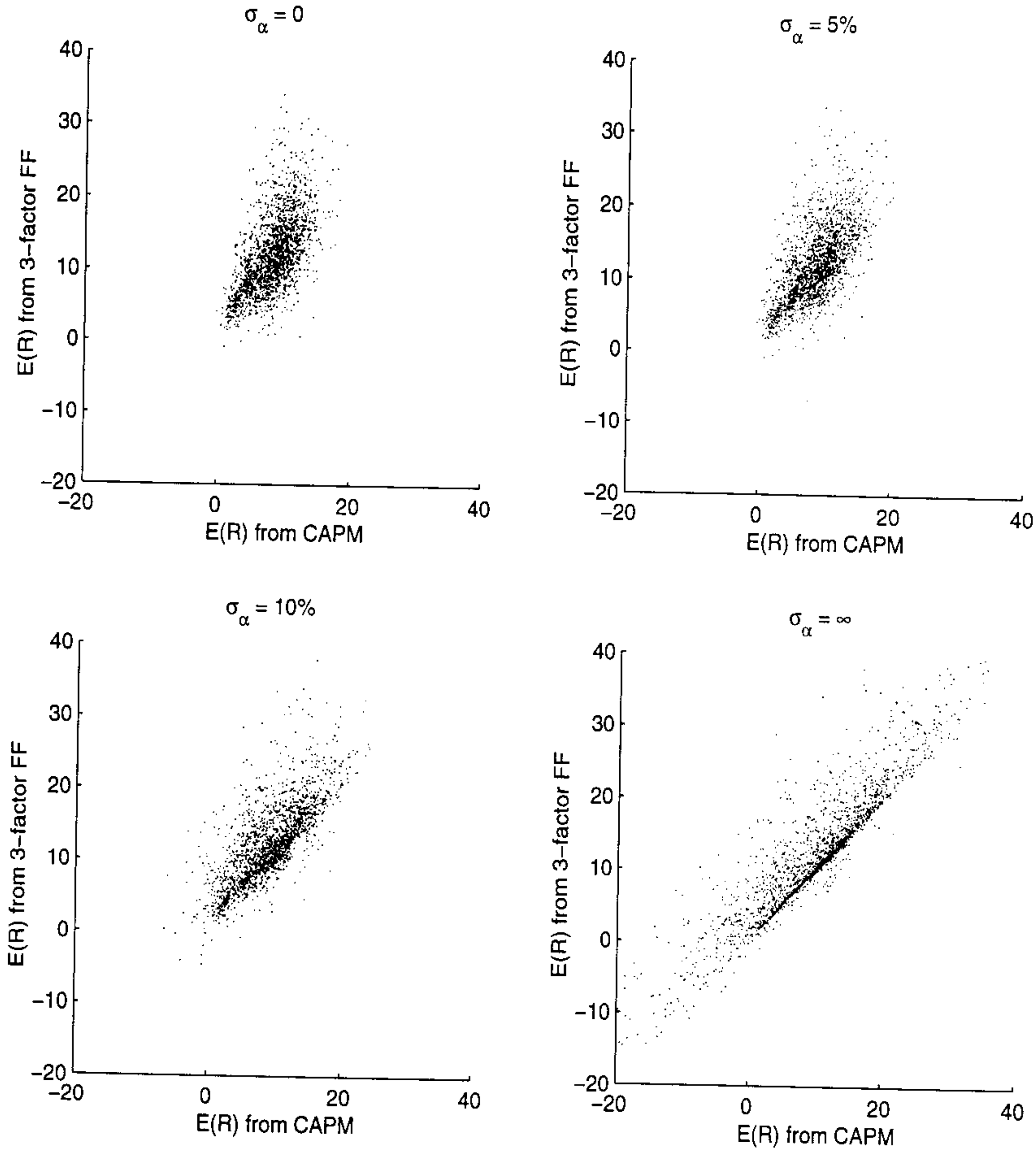

Figure 5. Costs of equity (E(R)) from the CAPM versus costs of equity from the three-factor Fama-French (FF) model for various degrees of prior uncertainty about mispricing. The prior mispricing uncertainty, σ_α, is the annualized prior standard deviation of α.

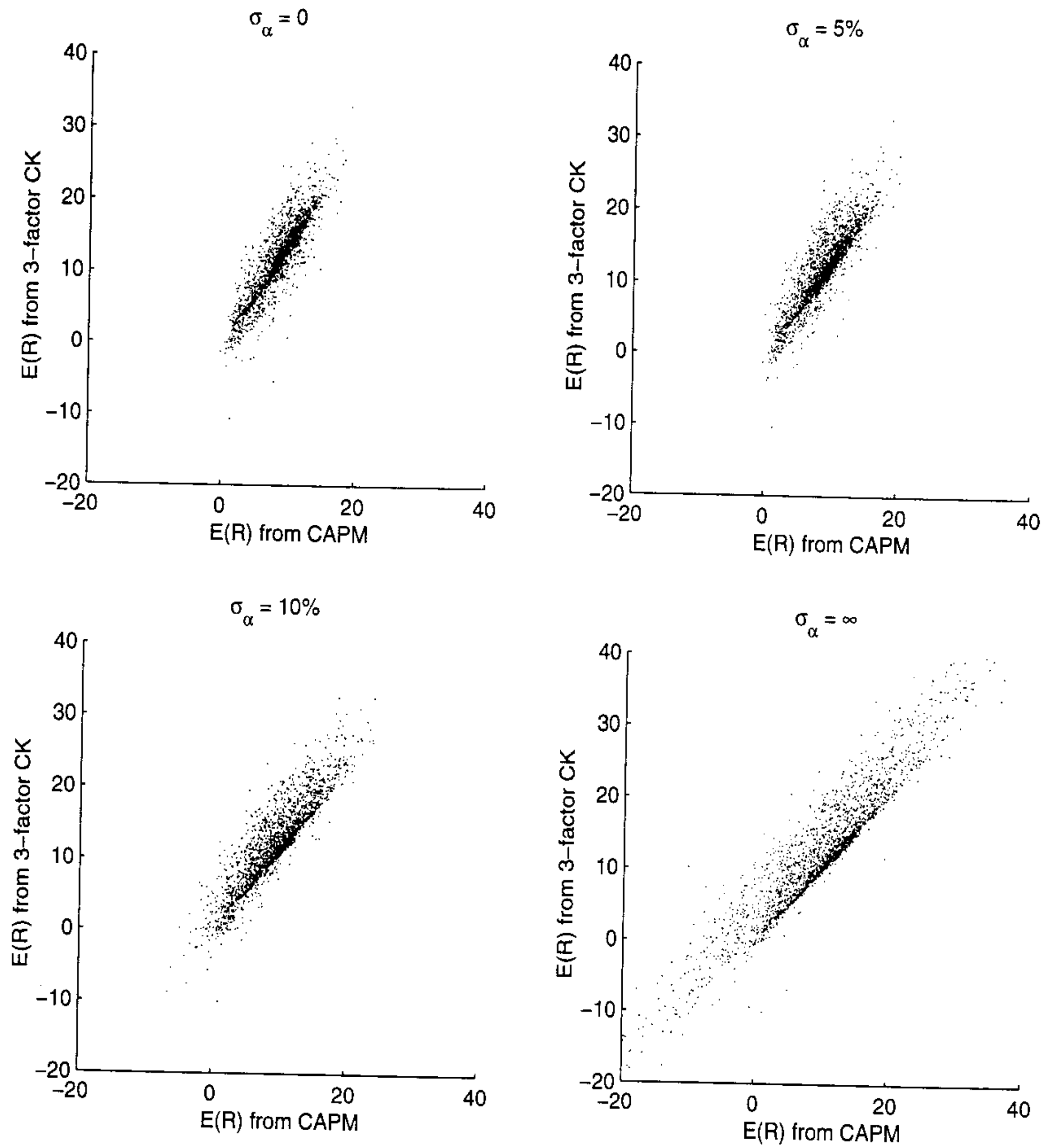

Figure 6. Costs of equity (E(R)) from the CAPM versus costs of equity from the three-factor Connor-Korajczyk (CK) model for various degrees of prior uncertainty about mispricing. The prior mispricing uncertainty, σ_α, is the annualized prior standard deviation of α.

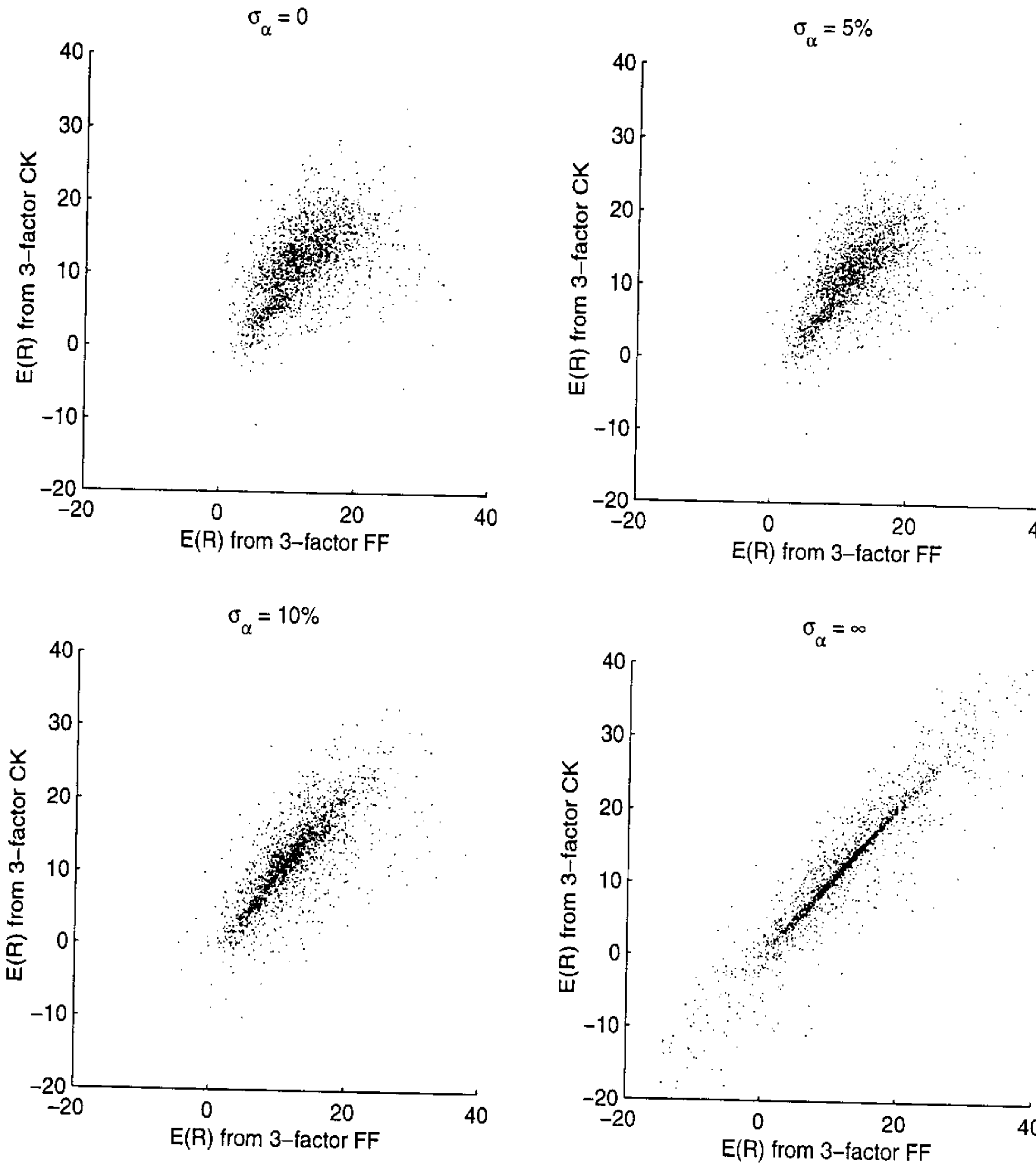

Figure 7. Costs of equity (E(R)) from the three-factor Fama-French (FF) model versus costs of equity from the three-factor Connor-Korajczyk (CK) model for various degrees of prior uncertainty about mispricing. The prior mispricing uncertainty, σ_α, is the annualized prior standard deviation of α.

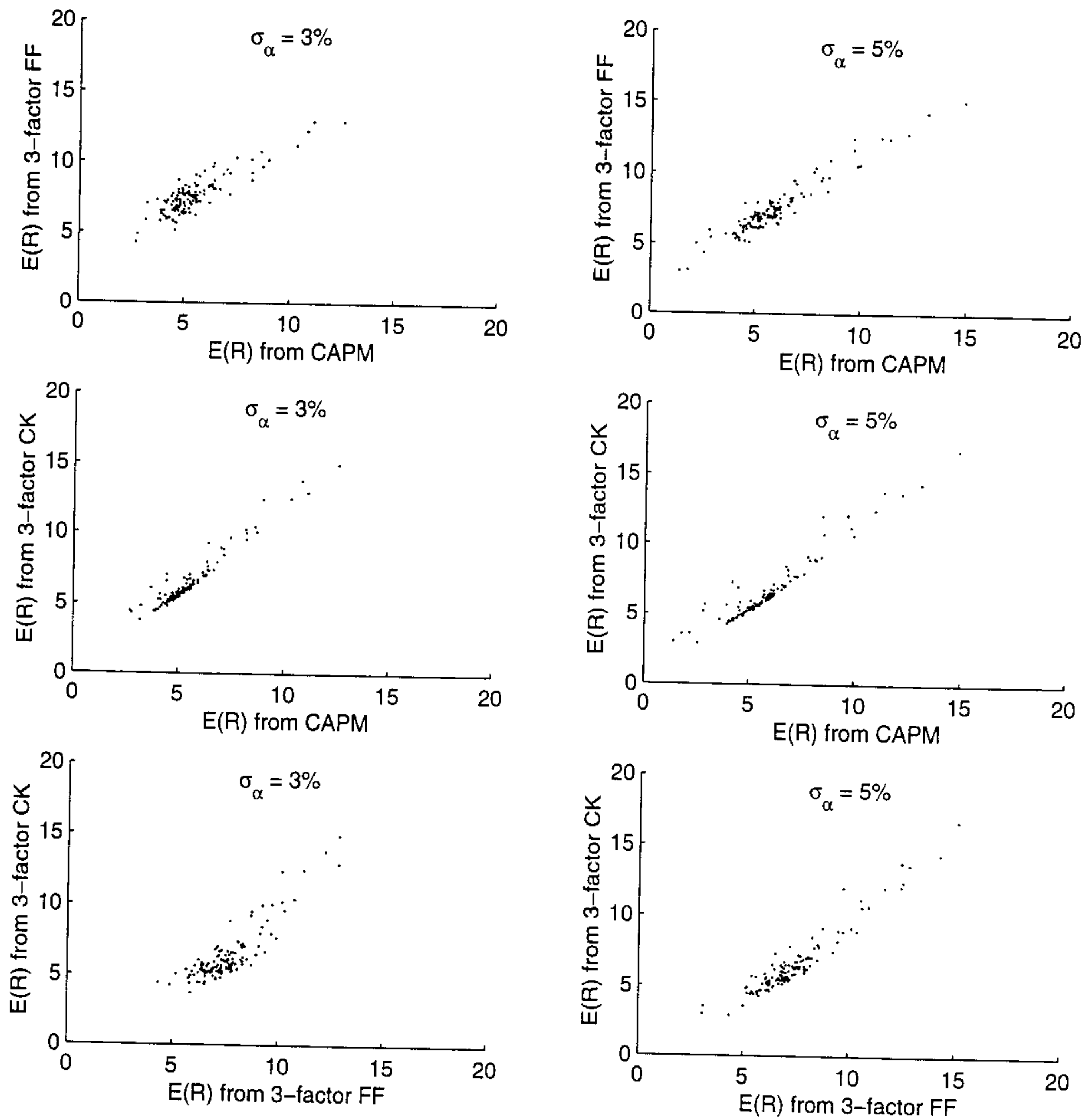

Figure 8. Utility costs of equity (E(R)) from one model versus another for different degrees of prior uncertainty about mispricing. Costs of equity are computed using the CAPM, the three-factor Fama-French model (FF), and the three-factor Connor-Korajczyk model (CK). The prior mispricing uncertainty, σ_α, is the annualized prior standard deviation of α.

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