Free Cash Flow, Optimal Contracting, and Takeovers

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Abstract

A commonly held view in the financial and economic literature is that "free cash flow is bad" in the sense that, given the opportunity, shareholders would always choose to minimize its existence. This view of the world has motivated economists such as Jensen (1988, 1993) to conclude that takeovers, to the extent that they are driven by an overinvestment problem, are beneficial because they both facilitate ex post divestiture and also pose an ex ante threat on managers who overinvest. In this paper we challenge these widely-held beliefs and show that not only might shareholders optimally choose to allow for the existence of free cash flow in the future, but also that the existence of takeovers may actually exacerbate the problem of overinvestment rather than help resolve it.

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1 Introduction

A commonly held view in the financial and economic literature is that "free cash flow is bad". When asked to elaborate most will say that given the opportunity shareholders will always choose to minimize the existence of free cash flow.[2] This view of the world has motivated economists such as Jensen (1988, 1993) to conclude that takeovers, to the extent that they are driven by an overinvestment problem, are beneficial because they both facilitate ex post divestiture and also pose an ex ante threat on managers who overinvest. Thus takeovers always help alleviate the problem of free cash flow. In this paper we challenge these widely-held beliefs and show that not only might shareholders optimally choose to allow for the existence of free cash flow in the future, but also that the existence of takeovers may actually exacerbate the problem of overinvestment rather than help resolve it.

Interestingly, we obtain these results in an economy very similar in spirit to that of Jensen's (1988, 1993). In his economy industries face endogenous shocks which may result in overcapacity. Ideally one would see some of the companies in the industry reduce productive capacity or exit the industry completely. Due to agency problems, however, managers may be reluctant to do so. This leads to a free cash flow problem which internal control systems, such as the board of directors, are unable to resolve. Hence the only available mechanism to force divestiture of these inefficient investments is a takeover.

The difference in our analysis is that we endogenize into the model the contracting stage between shareholders and the manager. We believe that to fully understand how agency costs, incomplete contracting, and takeovers affect the economy in equilibrium, it is not enough to look at these factors independently of shareholders' optimal responses to them. In taking this into account our paper sheds new light on scenarios in which one observes the existence of free cash flow. Specifically, we show that it is possible that shareholders will actually choose an equilibrium in which overinvestment may arise in the future rather than offering the manager a contract that eliminates all future overinvestment. This will be the case when the latter "incentivising" contract, while feasible, is ex ante too costly for

[2] In this paper we define free cash flow to be cash left over at the discretion of the manager after investing in all positive NPV projects.

the shareholders.

Although takeovers do serve to reverse overinvestment, we find that they have an additional effect through the optimal contracting of management and shareholders that may worsen the overinvestment problem. The intuition here is very clear. Since shareholders know that in the future takeovers will force divestiture of bad investments in some states of nature, they have less of an incentive to offer the manager an aligning contract today. Thus as the cost of engaging in a takeover decreases, we will actually observe more free cash flow in equilibrium. Furthermore, it is the case that free cash flow may be more prevalent in a world without takeovers than in a world with them.

Our model is driven by the combination of an agency problem and an incomplete contracting environment. The agency problem stems from the manager's preference for empire-building, or "utility from control". This preference is detrimental to shareholders because it causes the manager to invest in projects that do not have a positive net present value. This potential problem can not be resolved through contracting because of the assumed incompleteness. Incomplete contracting exists because the type of project in which the manager invests is random and can not be verified. This forces contracts to be contingent only on observables such as firm value, and hence they can not directly dictate the managers investment decision.

The agency and incomplete contracting would not be so problematic were internal control a viable way to discipline an errant manager. As pointed out by Jensen (1993, p. 854), "it appears that the infrequency with which large corporate organizations restructure or redirect themselves solely on the basis of the internal control mechanisms in the absence of crises in the product, factor, or capital markets or the regulatory sector is strong testimony to the inadequacy of these control mechanisms". We therefore ignore the possibility that internal control mechanisms such as the board of directors can exert any real control on the manager. In the absence of internal control, the only disciplinary device available is the takeover.

It is worth noting that unlike the usual paradigm for explaining the benefits of a takeover, our model gives the raider no special skills or superior information (i.e. no synergistic gains). Rather, it is the timing at which the raider enters that create the social gains. While the initial contract with the manager must be signed before the investment opportunity set is realized, the raider need not act until these investment opportunities become known. Hence, by taking over he can liquidate bad investments made by the previous manager and thereby create value. This idea was first argued informally by Grossman and Hart (1980) and later extended more formally by Scharfstein (1988). Both claim that contracts tend to become outdated due to changing environments, and this in turn may lead to opportunistic behavior by managers that can be rectified through the takeover mechanism.

The closest paper to our own is that of Scharfstein (1988), who illustrates the role that takeovers play in disciplining self-serving behavior by managers. Somewhat different from our model, Scharfstein's shareholders suffer from an agency problem with the manager because they do not observe and cannot contract on either the state of nature or managerial effort. Raiders, however, are better informed regarding the state of nature. Thus if firm value is low due to managerial shirking, then the raider can take over the firm.

Perhaps the most important difference between the model we present here and that of Scharfstein is that while our agency problem is one of utility from control that leads to overinvestment, Scharfstein's agency problem is one of shirking, which leads to what one might interpret as underinvestment. While our paper is consistent with a world in which takeovers are followed by a downsizing of the firm, his model is consistent with a world in which after a takeover one observes an expansion of the firms activities. Scharfstein's model is therefore incapable of addressing the problem of free cash flow which is the heart of our paper.

Our main modeling features find strong support in the empirical literature. Bhide (1989), Ravenscraft and Scherer (1989), and Bhagat, Shleifer, and Vishny (1990) all find that following a hostile takeover firms tend to engage in divestiture of marginal projects. This is consistent with our model, in which takeovers serve as a mechanism for efficient downsizing. As suggested by papers by Murphy (1986), Chang (1993), and Lippert and Williams (1994), who find support for the existence and importance of compensation contracts in effecting managerial behavior, we allow for incentive contracts in the form of stocks and call options as a way to influence managerial actions and decisions. Weston, Chung, and Hong (1990) and Bradley, Desai, and Kim (1988) find evidence that shareholders of a raided firm typically get all the benefits from the takeover while raiders on average break even. We model this by having the raider bid for the shares at the post takeover value net of the raiding costs. Finally, as noted by Stulz, Wong, and Song (1990), among others, the higher is the fraction of the firm held by the manager the more costly it is for a raider to take over the firm. We capture this by introducing a maximum fraction of ownership by the manager above which takeovers are not feasible.

Our analysis provides some interesting empirical predictions. First, in industries where the future investment opportunity set is more uncertain (in an appropriate sense) our analysis predicts higher levels of overinvestment. Second, in companies where the incumbent needs a lower fraction of the equity to effectively control the firm (i.e. make a takeover infeasible), we would expect free cash flow to be more prevalent. Third, if an industry tends to have a free cash flow problem, then that industry will attract managers with a high utility from control, whereas industries in which overinvestment is not a major problem will tend to attract managers who get less utility from control. This will tend to exacerbate the free cash flow problem within an industry that is already suffering from overinvestment.

The rest of the paper will proceed as follows: In section 2 we present the formal description of the model. In section 3 we solve the model. Section 4 illustrates some comparative statics. In section 5 we compare our results to the case of an economy in which there are no raiders. Section 6 concludes. For the sake of making the paper easier to read we have moved all formal proofs and mathematical definitions to the appendix.

2 The model

In the model we present, the manager of a firm, given her compensation contract, decides whether or not to invest the firm's capital in a project. Depending on its type, this project may or may not be a desirable undertaking from the perspective of the shareholders. Given a compensation contract that pays the manager a flat wage, the manager's investment choice will generally not coincide with the interests of the shareholders. An incentive-giving compensation contract has the potential of aligning the interests of shareholders and management, so that projects are only undertaken when it is optimal from the point of view of the shareholders (we will say in this case that the manager is "aligned".) In some cases, however, the cost to the shareholders of writing such a contract with the manager is prohibitively high. In these cases, the optimal contract allows for a free cash flow problem to arise in some states, in part because this problem is ameliorated by the possibility of a takeover, which serves both to discipline the manager and to reallocate capital more efficiently.

The formal description of the model now follows.

2.1 The agents

There are three types of agents in our economy. The first type, the shareholders, are homogeneous and risk neutral. They seek to maximize the expected value of their holdings in the firm. To operate the firm, shareholders contract with a manager, who is the second agent type.

Hiring a manager is not optional for shareholders. We assume that no investment opportunity will become available to the firm absent a manager. Even if the firm is engaging in no physical production, we assume that a manager is required to perform "caretaker" functions.

Managers, who are homogeneous, differ from the shareholders in that in addition to utility from monetary compensation, they also receive utility from "control". Utility from control may be interpreted as the manager's preference for managing a firm engaging in real production. This should be distinguished from managing a firm with a high market value, which may be derived to a great extent from the firm's portfolio of financial assets. The benefits of managing a firm that is engaged in higher levels of production could include more perquisites or greater satiation of hubris.

For tractability we assume that the manager's utility function takes the form $C + \lambda I$, where C is the manager's monetary compensation and λI represents utility from control. I is an indicator function representing whether or not the manager has invested in a project ($I = 1$) or declined to invest ($I = 0$). We assume that the manager gets positive utility from investing, so λ is strictly positive.[3]

[3]If $\lambda = 0$, then in the model we develop the manager would get no utility from control, eliminating the

The market for managers is assumed to be competitive. Since all managers are identical, each firm can induce a manager to manage the firm as long as the contract offered at the initial period gives her an expected utility of at least some reservation level u. As we will see later, there may be situations in which it is optimal for the shareholders to offer a manager a contract that gives her a higher level of expected utility.[4]

The last type of agent is the raider. We do not specify an objective function for the raider because we assume that competition among the identical raiders causes them to participate in a takeover as long as they earn their opportunity cost. More on the raider's role will be described in section 2.3.

2.2 The firm's investment opportunity set

The firm starts out at period zero with an exogenously given amount of capital which without loss of generality we set equal to $1.[5] At period one there is a single project available to the firm and the manager decides whether or not to undertake it. The project requires an investment of $1 in period one and pays off some amount in a later period. Once the project pays off the firm is liquidated and the liquidation value is distributed to the claimants. If the manager opts not to invest in the project, then she must immediately pay out the unused capital as a dividend to shareholders, retaining just enough to cover her own compensation. We assume that the time value of money is zero, so discounting is not required.

The type of the project is random, either "good" or "bad" with equal probability. The good project has a certain gross return of $R > 1$ while the bad project, for simplicity, has a certain gross return of 0.

Once the manager chooses to invest in the project, she is unable to reverse her decision in order to avert a takeover attempt. There are several reasons for this. First, in our model the manager does not know whether a takeover will be successful or not until the takeover

conflict of interest between management and the shareholders.

[4]We do not assume that the reservation utility level is constant over time, so managers that are hired at different times may potentially be willing to accept different contracts.

[5]The firm is assumed to exist with the capital already in the firm. We make no claim that the amount of capital available to the firm is optimal.

actually occurs and it is too late to reverse her decision. Secondly, even if the manager could reverse a project, the raider, as an outsider, may be able to reverse a project more efficiently than the incumbent manager, so that shareholders would prefer to tender their shares to the raider.[6] An additional argument in support of this assumption is given by Boot(1992). In his model he is able to show that managers may be reluctant to divest because the negative reputational effect of doing so may outweigh a takeover's reputation effect. Divestiture therefore more adversely affects the manager's future salary.

2.3 The market for corporate control

When a manager acts against the interests of the shareholders by investing in a project that is not value maximizing, and the effect of this action is sufficiently detrimental, then it may be optimal for the shareholders to tender their shares to a raider. We assume that there exists a competitive market for firm control. The participants of this market, the "raiders", will initiate a takeover if the gains from takeover are large enough to cover their costs. All gains in excess of the cost accrue to the shareholders.

The cost of takeover is random and assumes one of two values with equal probability. The low cost is C_L and the high cost is C_H. This cost includes, among other things, the cost of hiring a replacement manager, fees for lawyers and investment bankers, and a salary/opportunity cost of the raider. The distribution of the takeover cost is common knowledge as of period zero. However, the actual realization of this cost is not observed by the managers and shareholders until the takeover bid is made, by which time the manager's investment decision is fixed.

Takeovers are beneficial to shareholders because by paying the cost of a takeover the raider is able to reverse the manager's investment decision. In the cases in which the manager chooses to invest in the negative NPV project, forcing divestiture has an obvious benefit.

We assume that C_L is low enough to make takeovers optimal in some states, but that

[6]The raider may have greater expertise or leverage in reversing projects. Jensen's (1988) example of this is the takeover of TWA by Carl Icahn, who was able to force concessions from the pilots' union, a feat which previous management had been unable or unwilling to do.

C_H is too high for takeovers to be beneficial. Specifically, takeovers will be optimal only when the manager chooses to invest following the realization of the bad project (which has a negative net present value) and the low cost, C_L, is subsequently realized.

Since each shareholder knows the post takeover value of the firm, and since raiders operate in a competitive market, each tenders his shares at their post takeover value net of the takeover cost.

As in Grossman and Hart (1980), the raider has the ability to dilute the value of shares after he takes over which eliminates free rider problems and causes all shareholders to tender their shares to the raider. Furthermore, the competitive market for firm control ensures that there will always be a raider willing to agree to purchase all shares that are tendered. Thus in every takeover all shares are acquired by the raider.[7]

Takeovers also benefit shareholders because they serve to discipline the manager. When a takeover occurs the manager loses her utility from control. This is natural since takeovers exist in our model only to disinvest the firm of the project giving the manager this utility. This may be thought of as the manager being fired, although the interpretation is not crucial.[8]

Although the manager's job is at risk, her compensation contract is legally protected and must be honored by the raider. Raiding will therefore not occur in order to avoid payment of the manager's compensation.

2.4 Contracting

Shareholders contract with a manager in a way that maximizes the value of their holdings in the firm. We allow the manager's compensation as specified by this contract to depend only on the liquidation value of the firm. In particular, we consider compensation contracts that

[7] An example of an offer that would induce all shareholders to tender their shares would be one in which the raider agreed to spend an amount equal to the post-takeover value of the firm minus the takeover cost on however many shares were tendered. Under this offer, the value of tendering will be higher than the value of not tendering unless all shares are tendered, in which case the values will be equal.

[8] The manager could be retained following a takeover, but because takeovers force project liquidation the manager will not receive any utility from control anyway. We assume that the manager is fired as an expositional simplification.

are positive combinations of salary, equity in the firm, and a call option which we believe are representative of the main contracting features observed in reality.[9] We specifically do not allow the contract to depend on any unrealized "states of nature" such as project type. Furthermore, the contract may not depend on whether or not the manager decides to invest.[10]

An additional feature of the contract is that if the project type is known at the time of the contracting, then the shareholders may specify whether the manager may invest or not. Thus the shareholders of a firm with uniformly poor investment opportunities can instruct their manager to forgo investment regardless of the state of nature and pay out a liquidation dividend.

As we have seen thus far, there are two dimensions of uncertainty in this model - the project type and the cost of takeover. At the time the contract is written, neither uncertainty has been resolved. Furthermore, although these states of nature are observable once they are realized, they are not contractable. So a contract of the form "invest only if the good project is realized" is not admissible since project quality, although observable, is not verifiable.

In constructing optimal incentive contracts the shareholders will never choose to offer a nonzero base salary. This is so because a promised salary does not offer the manager any incentive to invest optimally. Shareholders would be better off giving her equity or a call option on equity. The risk neutrality of the manager and the shareholders means that the substitution would be one-for-one in expected value, so all in all the shareholders would prefer to give an additional dollar in stock rather then in salary. We therefore focus on the ingredients of the contract which have the potential of giving the manager the right incentives to act optimally.[11] Denoting by lv the liquidation value of the firm, including any

[9] In the formal proofs we allow for any positive combination of stocks and call options. The optimization shows, however, that by using just one call option or simple equity the shareholders are able to achieve their best contract.

[10] If we reinterpret no investment as investing in a safe or "passive" project that yields no utility from control, rather than an "active" project which does provide utility from control, it is natural to imagine that an investment's active or passive nature would not be contractable.

[11] It is possible to imagine situations in which it would be optimal to give the risk-neutral manager a large

early dividends paid, the manager's compensation must be of the form $\alpha\, lv + \beta\,(lv - K)^+$ for some non-negative constants α, β, and K.[12] The fractional share of equity given to the manager is denoted by α, while β and K represent the fractional share and strike price of the manager's call option. The triplet (α, β, K) therefore exactly characterizes the manager's compensation contract. For sake of consistency we require that $0 \leq \alpha\, lv + \beta\,(lv - K)^+ \leq lv$ for every possible outcome of lv and that the manager be "gross long" in every security (i.e. $\alpha, \beta \geq 0$).

Keeping this in mind, we impose one additional constraint on the allowable contract. If $\alpha + \beta \geq \frac{1}{2}$, then the manager would have a potential claim on the majority of the firm's shares, and it would be difficult to justify how a takeover attempt could ever be successful. Even if $\alpha + \beta$ was less than a half but still large, we might think that the manager was a large enough shareholder to block takeovers. Let β^* represent the maximum value of $\alpha + \beta$ for which a takeover may still occur. Clearly β^* will be less than $\frac{1}{2}$. In fact, we define β^* so that it will be far enough below $\frac{1}{2}$ so as to insure that the manager cannot acquire a controlling interest in the firm by purchasing shares with his own wealth. Contracts will be restricted to have $\alpha + \beta \leq \beta^*$.[13]

2.5 Sequence of events

To summarize, the sequence of events of the model is as follows:

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equity share in the firm (or call option on that equity) in exchange for a negative "salary". Owing to the apparent lack of such contracts in real life and because we presume limited liability of the *manager*, we restrict the manager's salary to be non-negative.

[12] $(x)^+ \equiv max\,(x, 0)$

[13] Our model does not address cases in which the shareholders might wish to give the manager a controlling interest in the firm, even though such contracts may be optimal for the shareholders.

Period 0

Shareholders contract with the manager.

Period 1

Investment opportunity revealed.

Manager chooses whether to invest or pay out a liquidating dividend.

Period 2

Cost of takeover is revealed.

Raider decides whether to contract with new manager and take over.

Project may be reversed.

Period 3

Returns are realized and the firm is liquidated.

An illustration of the model in a state/decision tree appears in Figure 1. The ordering of the nodes and branches follows from the description of the sequence of events. The first split shows the realization of the project type. Following the realization of the project type, the manager then chooses whether or not to invest. The realization of the cost of takeover, either C_L or C_H, makes up the last set of branches. Those end nodes which result in a takeover are labeled "T-O".

Next to each end node are three expressions. The third column provides the liquidation value of the firm. The liquidation value lv is R if the good project is undertaken, 1 if no project is undertaken, $1 - C_L$ if the bad project is chosen but reversed by a takeover, or 0 if the bad project is chosen and no takeover occurs to reverse it. Note that the end nodes corresponding to "no investment when the good project is realized" are irrelevant, since the choice of no investment in this situation is suboptimal for both the manager and the shareholders.

In the first column we show the manager's utility corresponding to this liquidation value and whether or not a project is carried out to completion. Given the structure of the allowable contracts and the form of the manager's utility function, this utility takes a value of the form $\alpha\, lv + \beta\,(lv - K)^+ + \lambda I$, where I is the indicator function for whether or not a project is undertaken.

The second column shows the residual value to shareholders, which is equal to the firm liquidation value minus the manager's compensation.

3 Solving the model

We solve the model starting from the end of the decision tree and then going back. First, we determine the manager's optimal actions given her compensation contract (α, β, K), the investment opportunity set and the distribution of the takeover cost. Then we solve for the optimal contract shareholders' choose to offer to the manager when taking into account the investment decision which the contract induces the manager to make.

Before we begin, recall that a contract may specify that the manager must forgo investment regardless of project type. Since our main objective is to look at scenarios in which it is profitable for the shareholders to hire a manager to actually *manage* the firm, we wish, for expositional purposes, to exclude the cases in which the optimal decision by the shareholders is to liquidate in period 0. Assumption 4 and Proposition 4 at the end of this section will provide sufficient conditions to ensure that this is indeed the case.

Although a contract which prohibits investment will, under this condition, never be offered to the original manager, this feature of the contract will become crucial when a takeover occurs and a new manager is hired. This is because a takeover only occurs when the bad project is realized, when the raider wants to insure that the manager he hires to liquidate the previous manager's investment in the bad project will in fact do so.

The following assumption will determine the states for which a takeover may be profitable.

Assumption 1 $0 \leq C_L < 1 < C_H$

Takeovers occur in response to the decision of the manager to invest even though the bad project is realized. Reversal of this project yields a gross firm value of \$1. Since the value of the firm in this state will be zero if no takeover occurs Assumption 1 then simply implies that the benefit net of costs will be positive to all of the firms' claimants in the low cost state and negative in the high cost state. Thus, takeovers will never occur in the high cost state but will occur in the low cost state if the bad project is undertaken.

3.1 The manager's decision

The managers investment decision is made after the investment opportunity set is realized but before the cost of takeover becomes known. When the good project is realized the manager, regardless of her compensation contract, always chooses to invest.

This can be verified from looking at Figure 1 or simply by noting that the manager has utility for both compensation and undertaking projects. When the good project is realized then firm value, and therefore managerial compensation, is higher when the manager chooses to invest.

Thus in what follows we will focus on the interesting case where the bad project is realized, shown in the lower half of the state/decision tree in Figure 1. We now construct the conditions on (α, β, K) under which the manager chooses not to invest in the bad project. As α and β increase and as K decreases, the manager holds a more significant stake in the firm, causing her incentives to become more aligned with those of the shareholders.[14]

Specifically, the manager will be aligned when, conditional on the realization of bad project, her expected utility from investing is lower than her expected utility from not investing. Should the manager invest when the bad project is realized, she faces a 50% chance of the low takeover cost being realized, causing a takeover to occur. Her expected utility in this case is then $\frac{1}{2}\lambda + \frac{1}{2}\left[\alpha(1-C_L) + \beta(1-C_L-K)^+\right]$.

If the manager decides to forgo the bad project, the firm is not taken over and the capital of $1 is paid out as a dividend. Her expected utility in this case is the certain amount $\alpha + \beta(1-K)^+$.

Thus the manager is aligned if

$$\alpha + \beta(1-K)^+ \geq \frac{1}{2}\lambda + \frac{1}{2}\left[\alpha(1-C_L) + \beta(1-C_L-K)^+\right]. \qquad (1)$$

Should the manager forgo investment when the bad project is realized, her expected utility is given by

$$U(\alpha, \beta, K, I=0) = \frac{1}{2}\lambda + \frac{1}{2}(\alpha R + \beta(R-K)^+) + \frac{1}{2}(\alpha + \beta(1-K)^+). \qquad (2)$$

[14] Note that there is no guarantee that these contracts will have $\alpha + \beta$ less than 1.

If, however, the manager invests in the bad project, then her period 0 expected utility is

$$U(\alpha, \beta, K, I = 1) = \frac{3}{4}\lambda + \frac{1}{2}(\alpha R + \beta (R - K)^{+}) + \frac{1}{4}(\alpha(1 - C_L) + \beta (1 - K - C_L)^{+}) \quad (3)$$

To optimize, the manager chooses the maximum of these two expressions, and the result,

$$U^{*}(\alpha, \beta, K) = max\,\{U(\alpha, \beta, K, I = 0)\,,\; U(\alpha, \beta, K, I = 1)\},$$

is the manager's value function. As anticipated, $U^{*}(\alpha, \beta, K)$ is increasing in α, and β and decreasing in K.

3.2 Choosing the optimal contract

To derive the optimal contract the shareholders maximize the expected value of their holdings subject to the manager's participation constraint and the feasibility of the contract. Denoting by $V(\alpha, \beta, K)$ the expected end-period value of the part of the firm retained by the shareholders, this problem can be written as:

$$\begin{aligned} &max_{(\alpha,\beta,K)}\; V(\alpha, \beta, K) \\ &s.t:\;\; U^{*}(\alpha, \beta, K) \geq u. \\ &\alpha + \beta \leq \beta^{*} \end{aligned} \quad (4)$$

To solve this problem it is best to look separately at the two classes of contracts which may turn out to be optimal. The first class is what we call incentive-aligned contracts. These contracts align the manager, so that the manager invests if and only if the good project is realized. The other contract class is non-incentive-aligned contracts. Under a contract in this class, the manager will invest regardless of project type.

3.2.1 Best incentive-aligned contract

At this stage we ignore the manager's participation constraint and determine the best contract out of all the incentive-aligned contracts. Because we are ignoring the participation constraint, this contract could offer an expected utility below u, making it unacceptable to the manager. Remember that even if the reservation utility level is met by this contract, the contract will not always be optimal since it's cost could be very high relative to the non-incentive-aligned one. We will later show the conditions under which this incentive-aligned contract will be chosen by the shareholders.

Recall the general formulation of the contracts available to the shareholders: $\alpha\, lv + \beta\,(lv-K)^+$ for some constants α, β, and K. Equation (1) states that the manager is aligned if and only if

$$\frac{1}{2}\lambda+\frac{1}{2}\alpha(1-C_L)+\frac{1}{2}\beta\,(1-C_L-K)^+\leq\alpha+\beta\,(1-K)^+.$$

Given that the shareholders are aligning the manager, the problem of maximizing the shareholders expected payoff is equivalent to the problem of minimizing the expected cost of the compensation contract that is signed with the manager.

Since an incentive-aligned contract is one which causes the manager never to invest in the bad project, the takeover cost becomes an irrelevant source of uncertainty. Hence the shareholders' problem is equivalent to minimizing the expected payment to the manager,

$$\frac{1}{2}\left[\alpha+\beta\,(1-K)^+\right]+\frac{1}{2}\left[\alpha R+\beta\,(R-K)^+\right], \tag{5}$$

subject to the incentive constraint (1) and non-negativity constraints on α, β, and K.

Proposition 1 *Assuming that $\beta^*\geq\frac{\lambda}{2C_L}$, and ignoring the manager's participation constraint, the incentive-aligned contract that is optimal for shareholders is given by*

$$(\alpha,\beta,K)=\begin{cases}(0,\frac{\lambda}{2C_L},1-C_L) & if\ \ R<1+2C_L;\\ (\frac{\lambda}{1+C_L},0,0) & if\ \ R\geq 1+2C_L.\end{cases}$$

Shareholders either use a pure equity contract (when $R\geq 1+2C_L$) or a single call option (when $R<1+2C_L$).[15] Note that the equity contract is equivalent to a call option with strike price equal to zero, or

$$(\frac{\lambda}{1+C_L},0,0)\equiv(0,\frac{\lambda}{1+C_L},0).$$

[15]The solution for $R=1+2C_L$ is not unique. In this case for any $K\leq 1-C_L$ we can find a β such that $(0,\beta,K)$ is optimal.

For the sake of reduced notation and without loss of generality we take $\alpha = 0$.

The intuition for why the level of R is important follows from the interpretation of equity as a call option with a zero strike price. In order to maintain an incentive-aligned contract while raising the strike price above zero, the call option the shareholders offer the manager must be for a greater "share" of the firm (β). Giving the manager a greater share of the firm will be costlier when there is a potential for the firm's shares to become very valuable, or when R is high. In this case, the shareholders will chose equity because they can align the manager with a smaller β, thereby retaining more of their claims on the firm's potential upside profits.

Proposition 2 *Assuming that $\frac{\lambda}{1+C_L} \leq \beta^* \leq \frac{\lambda}{2C_L}$, and ignoring the managers participation constraint, the incentive-aligned contract that is optimal for shareholders is given by*

$$(\alpha, \beta, K) = \begin{cases} (0,\ \beta^*, 1 + C_L - \frac{\lambda}{\beta^*}) & if \;\; R < 1 + 2C_L; \\ (0,\ \frac{\lambda}{1+C_L}, 0) & if \;\; R \geq 1 + 2C_L. \end{cases}$$

Proof: Similar to the proof of Proposition 1, but also accounting for the β^* constraint. □

Again, α is set equal to zero without loss of generality, even though the contract with $R \geq 1 + 2C_L$ is essentially pure equity. Because α is redundant, it will be eliminated from the remainder of the paper.

Corollary 1 *If $\beta^* < \frac{\lambda}{1+C_L}$ then the shareholders are not able to align the managers incentives.*

The corollary follows from the fact that between all contracts which induce the manager to forgo the bad project, a pure equity contract is the one that gives him the smallest stake in the company. This contract has $\beta = \frac{\lambda}{1+C_L}$ and $K = 0$. Thus if this contract is infeasible then there is no other feasible contract which will align the manager.

3.2.2 Non-incentive-aligned contracts

There may be cases in which contracts which align the manager are suboptimal because they are too costly. When considering contracts which do not align the manager (i.e., the manager chooses to invest in the bad project if it is realized), the shareholders need only consider contracts which offer the manager exactly her reservation utility. This is because when not aligning the manager the shareholders will always offer the manager the least expensive contract that will still induce her to take the job.

For the moment we shall ignore the constraint $\alpha+\beta \leq \beta^*$ and derive the firm' maximum value under a contract that is not incentive-aligning. (Assume that there exists a contract of this form that meets the manager's participation constraint.) Since the non-aligning contract exactly provides the manager's expected utility, the manager's expected compensation under such a contract must equal her reservation expected utility level minus her expected utility from control. In other words we look at all the contracts (α, β, K) which satisfy

$$\frac{1}{2}(\alpha R + \beta (R-K)^+) + \frac{1}{4}(\alpha(1-C_L) + \beta(1-K-C_L)^+) = u - \frac{3}{4}\lambda.$$

Given the above we have the following claim:

Claim 1 *The time zero value of the shareholders claim on the firm when offering the manager a non-incentive-aligning contract is given by*

$$V_1^{OPT} = \frac{1}{4}(2R + 1 - C_L) - (u - \frac{3}{4}\lambda) \tag{6}$$

where the subscript 1 denotes the managers decision to invest in the bad project if it is realized.

3.2.3 The optimal contract

After introducing both the incentive-aligned and non-incentive-aligned contracts a few words regarding the managers participation constraint are in order. Recall that a firm can contract with a manager as long as the proposed contract offers her an exogenously specified level of expected utility u.

Notice that if $u < U^*(0,0,0)$ then the manager is actually willing to pay for the right to manage and does not require any compensation. By equations (2) and (3), the following assumption is sufficient to eliminate such possibilities.

Assumption 2 $u \geq \frac{3}{4}\lambda$.

The period zero expected value of the shareholders' claim when holding fixed the manager decision (in the bad state) to invest in zero or one projects, respectively is given by:

$$V_0(\alpha,\beta,K) \equiv \frac{1}{2}(R-\alpha R-\beta(R-K)^+)+\frac{1}{2}(1-\alpha-\beta(1-K)^+) \tag{7}$$

$$V_1(\alpha,\beta,K) \equiv \frac{1}{2}(R-\alpha R-\beta(R-K)^+)+\frac{1}{4}(1-\alpha(1-C_L)-C_L-\beta(1-C_L-K)^+) \tag{8}$$

It can be directly verified that both V_0 and V_1 are decreasing in α and β and increasing in K.

The following assumption guarantees that there exists a feasible contract which solves the shareholders' maximization problem.

Assumption 3 $\frac{4u-3\lambda}{2R+1-C_L} \leq \beta^*$.

Proposition 3 *If assumption 3 holds then there exists a feasible contract which solves the shareholders' maximization problem.*

We now present our main result. This lemma provides conditions on the exogenous parameters of the model which determine the existence of a free cash flow problem.

Lemma 1 *If* $\beta^* \geq \frac{\lambda}{2C_L}$ *then a free cash flow problem exists if and only if*

$$\begin{cases} u < \lambda\frac{3C_L+5+2R}{4C_L+4} - \frac{1+C_L}{4} & if \;\; R > 1+2C_L; \\ u < \lambda\frac{5C_L+R-1}{4C_L} - \frac{1+C_L}{4} & if \;\; R \leq 1+2C_L. \end{cases}$$

The idea behind the derivation of this lemma is straightforward. In deciding which contract to offer the manager, shareholders must consider two alternatives: aligning or not aligning the manager. Regardless of which alternative they choose, shareholders will offer the least expensive contract which achieves the desired action by the manager. Shareholders optimize by choosing the alternative which maximizes the value of their holdings. The conditions in the above lemma are simply the result of a comparison of this value under the two alternatives.

Lemma 2 *If* $\frac{\lambda}{1+C_L} \leq \beta^* \leq \frac{\lambda}{2C_L}$, *then a free cash flow problem exists if and only if*

$$\begin{cases} u < \lambda\frac{3C_L+5+2R}{4C_L+4} - \frac{1+C_L}{4}, & R > 1+2C_L; \\ u < \frac{7}{4}\lambda - \frac{1+C_L}{4} - \frac{\beta^*}{2}(2C_L+1-R), & R \leq 1+2C_L. \end{cases}$$

Lemma 3 *If $\beta^* \leq \frac{\lambda}{1+C_L}$ then free cash flow always exists.*

To close the model we need to ensure that the firm value to shareholders obtained under the optimal contract is higher than what they would receive by liquidating the firm at the initial date. The following provides a sufficient condition to ensure that the firm has a higher expected value as a going concern.

Assumption 4 $R + \frac{3}{2}\lambda - \frac{1}{2}(3 + C_L) > 0$

Proposition 4 *As long as assumption 4 holds, the firm's value is greater than its value under liquidation.*

4 Comparative statics

We now use the results obtained in previous sections to give some insight about when a free cash flow problem is likely to arise and when this problem will be resolved by a takeover.

The following results are direct consequences of Lemmas 1 through 3. For each comparative static we vary one parameter while holding all the others constant. Unless stated otherwise, we are assuming that all assumptions continue to hold even as we vary the parameters.

- increase in u: The higher the reservation utility of the prospective manager the less likely it is that a free cash flow problem will exist. As the manager's reservation utility increases, the shareholders are forced to offer her a more lucrative compensation contract. Since the contracts offered are always either equity or a call option, the more expensive contract gives the manager a greater stake in the firm, reducing the conflict of interest between shareholders and management and potentially eliminating the free cash flow problem. This, however, does *not* mean that shareholders are better off when u is higher since the more lucrative contract had been feasible for the lower reservation utility level as well. By constraining the shareholders to offer a more lucrative contract, we may be forcing them to eliminate a free cash flow problem but at a very high expense.

- increase in λ: As λ increases the probability of having a free cash flow problem increases. Since λ represents utility from control, higher λ effectively means that *ex ante* the manager's and shareholders' objectives are less aligned. In order to align the incentives of a manager who likes control more, the shareholders will be forced to give her a bigger stake in the firm. For a high enough λ, it will not be worthwhile for the shareholders to try to align the manager, and they will prefer to allow that in some states of the world the manager's decision will not be firm value-maximizing. In those cases a free cash flow problem will arise, and if the takeover costs are low enough the firm will be taken over and the manager's decision reversed. Note that conditional on not aligning the manager the shareholders actually prefer a manager with a higher λ since such a manager will require less compensation. On the other hand, conditional on aligning the manager, the shareholders prefer a manager with a lower λ. One could conjecture that if an industry tends to have a free cash flow problem, then that industry will attract managers with a high utility from control, whereas industries in which overinvestment is not a major problem will tend to attract managers who get less utility from control. This will tend to exacerbate the free cash flow problem within an industry that is already suffering from overinvestment.

- increase in R: The higher R is the more likely the free cash flow problem becomes. If R is very high then the potential payoff, if a good project is realized, is so high that the shareholders are not willing to part with a large enough fraction of the firm to align the manager.

  Intuitively, we can explain this as follows. Suppose that, given R, shareholders came up with an optimal contract that was incentive-aligning, but then found out that R was actually higher than they had thought. According to equation 1, the contract that they were considering would still be incentive-aligning. However it would be more expensive than they initially figured since it is a claim on a potentially more valuable asset. If R is high enough this increased cost will make the incentive-aligned contract suboptimal.

  We believe that R being high can be interpreted as the case in which *ex ante* projects that are potentially available to the firm vary a lot in their quality. Although not directly implied by our model due to it's simple structure, one might conjecture that in an industry in which there is high variability in the quality of the future projects available to each firm, one can expect the free cash flow problem to be more prevalent and for takeovers to occur more frequently to alleviate this problem.

- decrease in β^*: As the fraction of the firm shares that is required in order to control the firm decreases it is more likely that a free cash flow problem will arise. In order to insure that shareholders do not give the manager a controlling interest in the firm, we require that $\beta \leq \beta^*$. It could be, however, that the amount of shares or call options that is required to align the manager violates this constraint, making it impossible to align the manager. In such a scenario the shareholders will have to settle for a contract that has a potential of generating a free cash flow problem. This result implies that the free cash flow problem will be more prevalent in firms were one needs a relatively small fraction of the shares in order to effectively control the firm.

  In a firm whose shareholders are dispersed, a relatively small block of shares may be all that is required to control the firm. Shareholders, realizing that agency problems will be compounded when the manager is entrenched, do not offer the manager too big a stake in the firm. The manager, in this situation, is inclined towards overinvestment since her stake in the firm is not large enough to be incentive-aligning. Takeovers will arise more frequently to eliminate the resulting free cash flow problem.

- increase in C_L: When the takeover cost rises, but not by enough to make takeovers unprofitable, one can expect to observe fewer free cash flow problems. As long as C_L remains below 1, a takeover will still be beneficial for the shareholders when the manager invests in the bad project and the low takeover cost is realized.

  The cost of takeover is borne by shareholders and management, so higher costs provide a greater incentive for the shareholders to try to avoid takeovers. They do so by giving the manager an incentive-aligning contract. In addition, since the manager is compensated with equity and call options, part of the takeover costs are borne by her as well. The motivation of avoiding these costs makes the manager more likely to forgo investing in the bad project.

The conclusion is that a "tax" on takeovers actually reduces the problem of free cash flow. While a straightfoward consequence of our model, this result runs against the spirit of the previous literature, which has argued that takeovers should be made as easy as possible to implement in order to combat the free cash flow problem.

The weakness of this conclusion is that our model thus far does not deal explicitly with situations in which the takeover cost rises enough to make takeovers unprofitable. In other words, if we go from an economy in which takeovers are (sometimes) cheap to one in which they are (always) prohibitively expensive, do we increase or decrease the extent of the free cash flow problem? This is the subject of the next section.

5 The no-raid economy

A commonly held belief is that takeovers always help to alleviate the free cash flow problem. The argument is, loosely, that existence of the takeover threat poses an ex ante threat on the manager who wants to overinvest, and that, in addition, takeovers facilitate ex post divestiture in cases in which a free cash flow problem arises. In this section we show that although takeovers serve to reverse overinvestment, the existence of potential takeovers may actually worsen the overinvestment problem.

We compare the model presented in this paper to a simplified version in which takeovers do not exist. Figure 2 illustrates the no-takeover economy.

The following result demonstrates that free cash flow can actually be more prevalent in an economy in which takeovers occur, as opposed to one in which takeovers are "outlawed".[16]

Lemma 4 *If $\lambda \geq \frac{2u}{R+2}$, $\beta^* > \max(\lambda, \frac{\lambda}{2C_L})$ and $R < 1 + 2C_L$ Then $\exists \bar{C_L} > 0$ s.t. $\forall 0 < C_L \leq \bar{C_L}$ there exists a free cash flow problem in the economy where takeovers occur, but there does not exist a free cash flow problem in the economy in which they are "outlawed".*[17]

[16]The claim shows that one may observe more free cash flow incidences in economy in which takeovers exist as opposed to an economy in which they do not. We make no comparison of shareholder welfare under the two regimes.

[17]If $R \geq 1 + 2C_L$ and the rest of the conditions hold than the free cash flow problem becomes more

The conditions of the lemma are quite intuitive. The inequality $\lambda \geq \frac{2u}{R+2}$ insures that the manager's participation constraint is non-binding. The condition $\beta^* > \lambda$ guarantees that the contracts of interest are feasible.

Although it is the case that outlawing takeovers, under some conditions, may worsen the overinvestment problem, this lemma shows that there are cases in which the existence of takeovers exacerbates the free cash flow problem. The reason is simply that a ban on takeovers may induce shareholders to give incentives to managers through the contracts they offer them, since they realize that there is no white knight raider to rescue them should their manager start to overinvest.

6 Conclusion

The purpose of this paper has been to demonstrate that common and intuitive hypotheses about free cash flow and its relation to takeovers may fail to hold in an equilibrium where managers, shareholders, and raiders behave strategically. Two such hypotheses are implicit in Jensen (1988, 1993). The first is that the shareholders' optimal policy is to minimize free cash flow. The second is that low-cost takeovers, both by posing an ex ante threat to managers who overinvest and by the ex post facilitation of divestiture, reduce the prevalence of free cash flow. In our model, both statements are in general untrue.

The key to our analysis of free cash flow is the observation that shareholders' optimal contracting with managers will reflect both the cost of preventing a free cash flow problem and the potential benefit of an outside takeover. In part because of this takeover benefit, shareholders may optimally choose to offer a contract which they know will result in a free cash flow problem in some states of nature. Furthermore, by forcing divestiture of bad projects, the presence of takeovers reduces the incentive of shareholders to align the manager. The effect may be to increase the severity of free cash flow relative to the corresponding "no-raid" economy.

Our model, in addition, has a number of empirical implications. The first is that managers of firms that get taken over will generally have had a smaller part of their salary in

prevalent in the economy in which takeovers are "outlawed".

the form of stocks and call options, since it was these managers who were not provided with enough incentive to forgo overinvesting. Second, takeovers should more likely occur in firms in which the number of shares required to effectively control the firm is low, i.e., in firms where shareholdings are relatively dispersed. Because contracting options are more limited for these firms, takeover-inducing mismanagement is harder to avoid. Third, we conjecture that one is more likely to observe takeovers in industries which have relatively high uncertainty regarding the quality of potential projects that will be available in the future.

Lastly, in industries in which the cost of takeover is very low, free cash flow problems will be more common, and these problems will be resolved more frequently through the market for corporate control. When takeover costs are moderate, shareholders will attempt to avoid these costs by aligning the manager, and the free cash flow problem will be less severe. As the cost of takeover becomes extremely or prohibitively high, the free cash flow problem may be severe or may be resolved completely, depending on whether the cost of alignment is high or low.

A Appendix: Proofs

The appendix contains three parts. In the first we introduce some definitions that are used throughout the proofs. In the second we present some auxiliary claims and propositions. In the third we prove the different propositions which appear throughout the paper.

A.1 Definitions

The following definitions are used throughout the appendix.

Definition 1

$$\begin{aligned} S &\equiv \{(\alpha,\beta,K) : \; U(\alpha,\beta,K,I=0) \geq U(\alpha,\beta,K,I=1)\} \\ \partial S &\equiv \{(\alpha,\beta,K) : \; U(\alpha,\beta,K,I=0) = U(\alpha,\beta,K,I=1)\} \\ S^C &\equiv \{(\alpha,\beta,K) : \; U(\alpha,\beta,K,I=0) < U(\alpha,\beta,K,I=1)\} \end{aligned}$$

S is the set of all (α, β, K) for which, given the realization of a bad project, the manager will choose not to invest. ∂S is the set of all $(\alpha, \beta,\ K)$ for which, given a realization of the bad project, the manager is indifferent between investing in it and forgoing the investment. We assume that when the manager is indifferent between actions she chooses not to invest, since, as we will show later, this is preferred by shareholders.

Definition 2

$$\begin{aligned} A &\equiv \{(\alpha,\beta,K) : U^*(\alpha,\beta,K) \geq u\} \\ \partial A &\equiv \{(\alpha,\beta,K) : U^*(\alpha,\beta,K) = u\} \\ \partial A_1 &\equiv \{(\alpha,\beta,K) \in \partial A \bigcap S^C\} \\ \partial A_0 &\equiv \{(\alpha,\beta,K) \in \partial A \bigcap S\} \end{aligned}$$

Clearly A is the set of all contracts which give the manager an expected utility of at least u. ∂A is the indifference curve of the manager corresponding to the reservation expected utility level u. For reasons which will become apparent shortly it is useful to divide ∂A into two sets, ∂A_1 and ∂A_0. ∂A_1 is the set of all contracts in ∂A which induce the manager to invest in the bad project. ∂A_0 is the set of all contracts in ∂A for which the manager does not invest in the bad project. Since ∂A_0 and ∂A_1 are subsets of ∂A, the manager's expected utility on each is equal to u.

A.2 Auxiliary claims and propositions

Claim 2 $\{(0,\ \beta,\ K) : (0,\ \beta,\ K) \in \partial A_1\} \neq \emptyset$. *Furthermore, if* $\{(0,\ \beta,\ K) : (0,\ \beta,\ K) \in \partial A_1$ *and* $\beta \leq \beta^*\} = \emptyset$ *then* $\partial A \bigcap \partial S \neq \emptyset$.

Proof: We shall look at the region $1 - C_L \leq K < 1$. First note that $\forall K < 1$, $\exists\ \beta > 0$ s.t. $(0, \beta, K) \in \partial A$. By contradiction assume the opposite. Thus by (2), $\forall\ (0,\ \beta_{\partial A_0}, K) \in \partial A_0 \ \Rightarrow\ \beta_{\partial A_0}(K) = \frac{2u-\lambda}{R+1-2K}$. Furthermore by (2) and (3), $\forall\ (0,\ \beta_{\partial S}, K) \in \partial S \ \Rightarrow\ \beta_{\partial S}(K) = \frac{\frac{1}{2}\lambda}{1-K}$. Thus with a little abuse of notation we have that $lim_{K\uparrow 1}\beta_{\partial A_0} < \infty$, and $lim_{K\uparrow 1}\beta_{\partial S} = \infty$. Thus for K close to 1 there exists corresponding values of β such that $(\beta, K) \in \partial A$ and $(\beta, K) \in S$, implying that $(\beta, K) \in \partial A_1$, so that $\partial A_1 \neq \emptyset$.

The second part of the proof is obtained directly by using the first part in conjunction with observing that both $\beta_{\partial A_0}$ and $\beta_{\partial S}$ are continuous. □

Proposition 5 *Given u,* $\forall(\alpha, \beta, K) \in \partial A_1$ V_1 *is a constant and* $\forall(\alpha, \beta, K) \in \partial A_0$ V_0 *is a constant.*

Proof: Directly obtained by using the definition of V_0 and V_1 in conjunction with the condition that the contract is on the managers indifference curve (∂A_0 and ∂A_1, respectively). □

The intuition behind this proposition is straightforward. Taking the manager's decision in the bad state as fixed, the expected value of the manager's compensation is constant along any of the manager's indifference curves. Thus the expected payout to shareholders must also be the same.

We use Proposition 5 throughout the proof of the following proposition, which presents a simple sufficient condition for *no* free cash flow problem. In proposition 1 we show that the optimal contract may always be written with $\alpha = 0$.[18] Thus, from here on we omit α as an argument from the functions in which it previously appeared.

Proposition 6 *If* $\partial A \bigcap \partial S \neq \emptyset$ *then there exists no free cash flow problem. Furthermore for any contract* $(\hat{\beta}, \hat{K}) \in \partial A_0$, $V_0(\hat{\beta}, \hat{K}) \geq V_1^{OPT}$.

Proof: Let $(\beta_{int}, K_{int}) \in \partial A \bigcap \partial S$. By definition $\forall(\beta,\ K) \in \partial S$, $U(\beta, K, I = 0) = U(\beta, K, I = 1)$ and therefore we can assume that for such (β, K) the manager decides not to invest in the bad project. Since V_1 is decreasing in α, and β and nonincreasing in K, and since U^* is increasing in α, and β and nondecreasing in K we obtain that V_1^{OPT} will be achieved on ∂A_1 and since by Proposition 5 V_1^{OPT} is a constant on ∂A_1 and $(\beta_{int}, K_{int}) \in \partial A_1$ then $V_1(\beta_{int}, K_{int}) = V_1^{OPT}$. Now since $\forall\beta,\ K \in R_+^2$, $V_0(\beta,\ K) \geq V_1(\beta,\ K)$ we obtain that $V_0(\beta_{int}, K_{int}) \geq V_1(\beta_{int}, K_{int}) = V_1^{OPT}$. Hence the shareholders, in this case, never offer the manager a contract which induces him to invest in the bad project and there is no free cash flow problem. Also by Proposition 5 V_0 is constant along ∂A_0, and the second part of the proposition follows. □

[18]The proof of proposition 1 is self contained.

The above proposition simply states that if there is an incentive-aligning contract which provides the manager with only her reservation utility, then the shareholders will choose to offer the manager a contract of this type, and thus there will not be a free cash flow problem. Furthermore in such a situation any incentive-aligning contract that gives her her reservation utility will be preferred by the shareholders over any feasible (i.e., that gives her at least her reservation utility) non-incentive contract.

A.3 Proofs

A.3.1 Proof of Proposition 1

Proof: Since we are restricting the set of contracts to be any positive combination of stocks and call options it is easily verified that the payoffs to the manager from these contracts are non-negative convex and increasing functions of the liquidation value of the company.

If we write the manager's compensation as some function $f : R_+ \rightarrow R_+$ which depends only on the liquidation value of the firm, then the "incentive" condition (1) is equivalent to

$$f(1) \geq \frac{1}{2}(\lambda + f(0)) + \frac{1}{2}f(1 - C_L).$$

Given the above definition we can write the shareholders problem as[19]

$$min_f \ \frac{1}{2}f(1) + \frac{1}{2}f(R)$$

$$s.t \begin{cases} f \geq 0, \text{ convex, increasing, and} \\ f(1) \geq \frac{1}{2}(\lambda + f(0)) + \frac{1}{2}f(1 - C_L). \end{cases}$$

First notice that $f(0) = 0$ since stocks and calls are valueless when the liquidation value is 0. Secondly we must have for the optimum that the incentive constraint be binding, or

$$f(1) = \frac{1}{2}(\lambda + f(0)) + \frac{1}{2}f(1 - C_L) = \frac{1}{2}\lambda + \frac{1}{2}f(1 - C_L).$$

If the constraint were not binding then we could find some $\epsilon > 0$ such that $g(x) \equiv f(x)(1-\epsilon)$ will be feasible and will, in addition, be strictly cheaper than f.

[19] If the good project is realized the liquidation value is R. If the bad project is realized and the manager decides not to invest in it then the liquidation value is 1

Furthermore it is the case that for the optimal f we get $f(R) = f(1) + f\prime(1)(R-1)$. Since f is convex $f(R) \geq f(1) + f\prime(1)(R-1)$, where the derivative is taken as the limit from below. At the optimum f this must hold as an equality or else we can choose g such that $g(x) = f(x)$ for $0 \leq x \leq 1$ and $g(x) = f(1) + f\prime(1)(x-1)$ for $1 < x \leq R$. Clearly g is feasible and convex, and it is cheaper than f, since $g(R) < f(R)$.

Given any value for $f(1-C_L)$, we can compute a value for $f(1)$. If, in addition, we knew $f\prime(1)$, then we could determine $f(R)$. In order to make $f(R)$ as small as possible, we will make $f\prime(1)$ as small as possible. Since f must be convex, this is achieved by making $f(x)$ a straight line from $1-C_L$ to R. Thus $f\prime(1) = \frac{f(1)-f(1-C_L)}{C_L}$.

We can now write $f(1)$ and $f(R)$ as functions of $f(1-C_L)$ which incorporate the incentive constraint and which insure the convexity, non-decreasingness, and non-negativity of the function $f(x)$ for $x \geq 1-C_L$. We have also shown that it is always the case that $f(0) = 0$. Thus $f(1-C_L)$ is the only free parameter left for the shareholders to minimize over. The only constraints that remain to be imposed are that $f(x)$ be non-decreasing and convex for $0 \leq x \leq 1-C_L$. A necessary condition for convexity is that the value $f(1-C_L)$ lie on or below the line connecting $(0, f(0))$ and $(1, f(1))$. Since $f(0) = 0$ and $f(1) = \frac{1}{2}\lambda + \frac{1}{2}f(1-C_L)$ this is equivalent to $f(1-C_L) \leq \frac{\lambda(1-C_L)}{1+C_L}$. For non-decreasingness we require that $f(1-C_L) \geq 0$. Letting $w = f(1-C_L)$, we rewrite the shareholders' optimization problem as $min_w\ F(w)$, where $F(w) \equiv f(1) + f(R) = w + \lambda + \frac{\lambda - w}{2C_L}(R-1)$, subject to $0 \leq w \leq \frac{\lambda(1-C_L)}{1+C_L}$. By differentiating we obtain that $F\prime(w) \geq 0 \Leftrightarrow R \leq 1 + 2C_L$.

The result is always a corner solution:

$$w^* = \begin{cases} 0 & R < 1 + 2C_L \\ \frac{\lambda(1-C_L)}{1+C_L} & R \geq 1 + 2C_L. \end{cases}$$

This value of w^* is sufficient to characterize the function f. On inspection of the values $f(0), f(1-C_L), f(1)$, and $f(R)$, we see that for $R < 1 + 2C_L$ the function f represents the payoffs of a call option with $\beta = \frac{\lambda}{2C_L}$ and $K = 1 - C_L$, and for $R > 1 + 2C_L$ it represents the payoffs of a pure equity contract with $\beta = \frac{\lambda}{1+C_L}$ (and $K = 0$). Thus the proposition is proved. □

A.3.2 Proof of Proposition 3

Proof: By plugging a contract of the form $(0, \hat{\beta}, 0)$ into (3) and setting the LHS of (3) equal to u one obtains that $\hat{\beta} = \frac{4u-3\lambda}{2R+1-C_L}$. This contract belongs to either ∂A_1, or to S or to both. Assumption 3 guarantees that it is feasible to offer the manager such a contract. □

A.3.3 Proof of Claim 1

Proof: Use (8) and Proposition 5 to obtain that the shareholders best expected payoff under a contract which does **not** induce the manager to forgo the bad project is

$$V_1^{OPT} = \frac{1}{4}(2R + 1 - C_L) - (u - \frac{3}{4}\lambda) \tag{9}$$

□

Although, it may be the case that when we take into account the constraint $\alpha + \beta \leq \beta^*$ there will be no feasible non-incentive aligning contract, we know (by using Claim 2) that even though V_1^{OPT} is not feasible it is still well-defined.

A.3.4 Proof of Lemma 1

Proof: Denote by V_0^{CON} the optimal value of V_0 subject to the participation constraint and by V_0^{UNCON} the optimal value of V_0 when we disregard the constraint.

We shall look at two different cases:

1. If $\{(0, \beta, K) : (0, \beta, K) \in \partial A_1 \text{ and } \beta \leq \beta^*\} = \emptyset$, then by Claim 2 we obtain that $\partial A \bigcap \partial S \neq \emptyset$. Therefore, by Proposition 6 there does not exist a free cash flow problem and $V_0^{CON} \geq V_1^{OPT}$. Proposition 3 in conjunction with the second part of Proposition 6 guarantees that V_0^{CON} is feasible. Thus in this case clearly $V_0^{UNCON} \geq V_1^{OPT}$.

2. If $\{(0, \beta, K) : (0, \beta, K) \in \partial A_1 and \beta \leq \beta^*\} \neq \emptyset$, then Proposition 3 guarantees that both V_1^{OPT} and V_0^{CON} are feasible. Thus, a free cash flow problem exists if and only if

$$V_1^{OPT} - V_0^{CON} > 0.$$

Now we claim that

$$V_1^{OPT} - V_0^{CON} > 0 \Leftrightarrow V_1^{OPT} - V_0^{UNCON} > 0.$$

"$\Longleftarrow$" This direction is trivial.

"$\Longrightarrow$" If $V_1^{OPT} - V_0^{CON} > 0$ then $\partial A \bigcap \partial S = \emptyset$ by Proposition 6 which showed that $\partial A \bigcap \partial S \neq \emptyset \Rightarrow V_1^{OPT} - V_0^{CON} \leq 0$. Hence $\forall(\beta, K) \in \partial S$ the participation constraint is slack and therefore $V_0^{CON} = V_0^{UNCON}$, which verifies our claim.

From (6), $V_1^{OPT} = \frac{1}{4}(2R + 1 - C_L) + \frac{3}{4}\lambda - u$. From Proposition 1 we have

$$V_0^{UNCON} = \begin{cases} \frac{1}{2}(R+1) - \frac{1}{2}\lambda - \frac{\lambda}{4C_L}(R-1), & R < 1 + 2C_L; \\ \frac{1}{2}(R+1) - \frac{\lambda}{2(1+C_L)}(R+1), & R \geq 1 + 2C_L. \end{cases}$$

By calculating the difference between the two we obtain the conditions specified in the proposition. □

The Proof of Lemma 2 is similar.

A.3.5 Proof of Lemma 3

Proof: This is a direct consequence of Corollary 1 in conjunction with the fact that since there is no feasible incentive-aligning contract in this case we know by Proposition 3 that V_1^{OPT} is feasible. □

A.3.6 Proof of Proposition 4

Proof: From the proof of Lemma 1 - Lemma 3 one obtains that the value to the shareholders of keeping the firm alive is at least V_1^{OPT}. On the other by liquidating at date 0 the shareholders receive $1 - u$. The proposition follows by comparing these two numbers and using Assumption 4. □

A.3.7 Sketch of the proof of Lemma 4

The proof is obtained in two steps. In the first we find, under the conditions of the lemma, a necessary and sufficient condition under which there exists a free cash flow problem in the no-raid economy. In the second we compare this condition to the one obtained in Lemma 1 and thus find conditions on C_L in which a free cash flow problem exists in the economy where takeovers occur, but does not exist in the one in which they are "outlawed".

Proof: In the economy in which free cash flow is "outlawed": If the shareholders decide to give the manager a non incentive aligning contract they will pay her $u - \lambda$ and thus the shareholders payoff will be $\frac{1}{2}R - (u - \lambda)$. The best incentive aligning contract is to give the manager $\alpha = \lambda$ shares. This result is obtained as follows. First, by ignoring the managers participation constraint and the β^* constraint one can use a similar, but simpler, argument to the one used in the proof of Proposition 1 to obtain that the best incentive aligning contract is obtained by setting $\alpha = \lambda$ and $\beta = 0$. Second, the condition $\beta^* > \lambda$ guarantees that the β^* constraint is non binding. Third, the condition $\lambda \geq \frac{2u}{R+2}$ guarantees that the managers participation constrain is non binding. Thus, the shareholders payoff under the best incentive aligning contract is of the form $\frac{1}{2}(R - \lambda R) + \frac{1}{2}(1 - \lambda)$. Comparing the shareholders payoffs under an incentive aligning contract and a non incentive aligning contract yields the following necessary and sufficient condition for the existence of a free cash flow problem: $\lambda > \frac{2u+1}{R+3}$.

Using Lemma 1 ,since the conditions of the claim state that $\beta^* \geq \frac{\lambda}{2C_L}$, and restricting to the case in which $R \leq 1 + 2C_L$ the following necessary and sufficient condition for the existence of a free cash flow problem, in an economy in which takeovers are not "outlawed", is obtained: $\lambda \geq \frac{(4u+1+C_L)C_L}{5C_L+R+1}$. Note that the right hand side of this expression is a continuous and increasing function of C_L.

The claim follows by comparing the condition obtained for the economy in which takeovers are "outlawed" to the one in which they are not "outlawed" for a value of C_L close enough to zero. □

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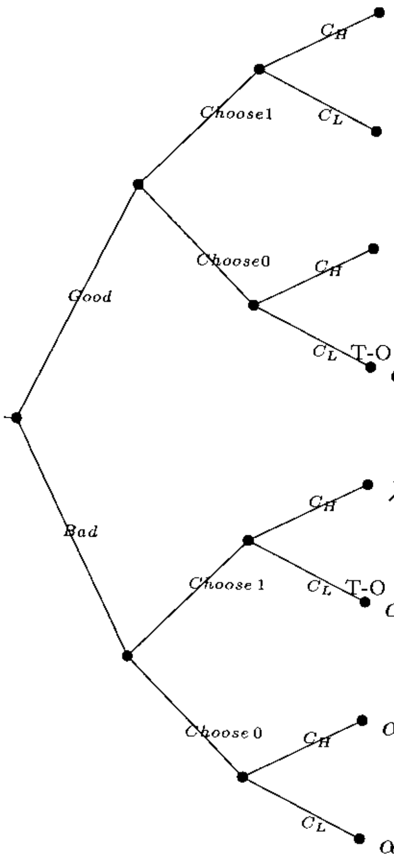
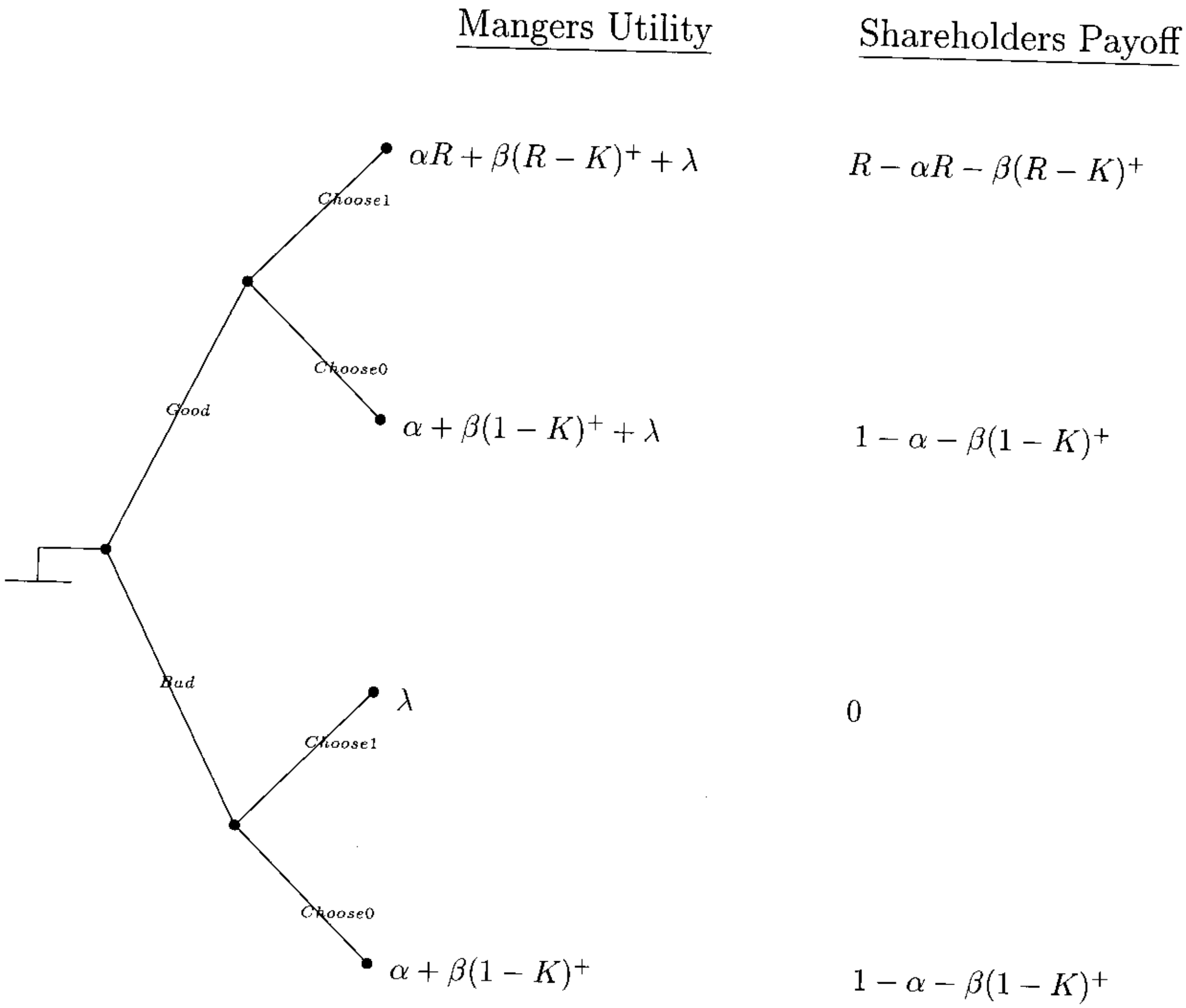
Figure 1



Mangers Utility	Shareholders Payoff	Firm Value
$\alpha R + \beta(R-K)^+ + \lambda$	$R - \alpha R - \beta(R-K)^+$	R
$\alpha R + \beta(R-K)^+ + \lambda$	$R - \alpha R - \beta(R-K)^+$	R
$\alpha + \beta(1-K)^+$	$1 - \alpha - \beta(1-K)^+$	1
$\alpha(R-C_L) + \beta(R-C_L-K)^+$	$R - C_L - \alpha(R-C_L) - \beta(R-C_L-K)^+$	$R - C_L$
λ	0	0
$\alpha(1-C_L) + \beta(1-C_L-K)^+$	$1 - C_L - \alpha(1-C_L) - \beta(1-C_L-K)^+$	$1 - C_L$
$\alpha + \beta(1-K)^+$	$1 - \alpha - \beta(1-K)^+$	1
$\alpha + \beta(1-K)^+$	$1 - \alpha - \beta(1-K)^+$	1

Figure 2


Mangers Utility
Shareholders Payoff
Good
Choose1
$\alpha R + \beta(R-K)^+ + \lambda$
$R - \alpha R - \beta(R-K)^+$
Choose0
$\alpha + \beta(1-K)^+ + \lambda$
$1 - \alpha - \beta(1-K)^+$
Bad
Choose1
λ
0
Choose0
$\alpha + \beta(1-K)^+$
$1 - \alpha - \beta(1-K)^+$

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