EXECUTIVE COMPENSATION AND THE OPTIMALITY OF MANAGERIAL ENTRENCHMENT

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We solve for an optimal managerial compensation contract's wage, equity and options components, vesting dates, and control rights when firms are more complicated than standard principal-agent theory allows. Firms have assets-in-place, endure through time, and have many managers. A firm's owner can transfer some control rights to a manager, thereby entrenching her. Managerial entrenchment makes deferred compensation credible but creates a hold-up problem. Deferring some, but not all, compensation reduces a manager's incentive to free-ride on her replacement while simultaneously solving the hold-up problem. Under an optimal contract a senior manager will be entrenched, make no effort, and receive apparently performance-insensitive compensation.

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I. Introduction

When an agent must make a non-verifiable effort, compensation must be sensitive to output. This observation is the basis for the large literature on executive compensation in public corporations.[1] Kole (1994a) documents the complexity of compensation contracts in practice. One particular feature of observed contracts not predicted by a simple principal-agent paradigm is that executives are often awarded voting stock. Since votes per se do not induce effort, but do confer power, it is not clear why this "entrenchment" occurs. This paper analyzes a setting in which executive compensation contracts display many real-world features and in which, the results show, it can be optimal to award a manager both votes and claims on residual cash flows.

Rewarding an executive with voting stock is not only unexplained by the simple principal-agent paradigm, but is a puzzle in that entrenchment may exacerbate the agency problem by weakening the principal's control. A number of papers empirically document ex-post agency costs associated with entrenchment and compensation contracts that use voting equity and options on voting stock to induce effort. See for example, Mørck, Shleifer, and Vishny (1988), McConnell and Servaes (1990), Hermalin and Weisbach (1991), Chang and Mayers (1992), Eckbo and Verma (1994), Hubbard and Palia (1995), and Core (1996).[2] Corporate charters could prohibit voting shares being used to motivate managers, and could require instead that pay be linked to performance via non-voting shares. Given an ex-post cost of entrenchment, there must then be some ex-ante benefit to this contract design.

A second puzzle concerns whether executive compensation contracts employed in practice are sufficiently sensitive to firm value to induce the optimal managerial effort. On this there is some disagreement on the empirical evidence. Jensen and Murphy (1990) view their results as inconsistent with formal agency models of optimal contracting. Garen (1994), Haubrich (1994), and Haubrich and Popova (1995) argue, however, that the data are in fact consistent with formal agency models. In contrast to Jensen and Murphy's findings, Joskow and Rose (1994) and Boschen

[1] This literature is surveyed in Rosen (1990), Garvey and Swan (1994), and Gibbons (1996).
[2] For empirical work that presents an alternative view of some of this evidence see Agrawal and Knoeber (1994), Kole (1994b), and Mehran (1995).

and Smith (1995) interpret their own results as documenting both a statistically and economically significant relation between pay and performance. One difference in the empirical work concerns the length of the past performance horizon that appears to be linked to current compensation. Boschen and Smith conclude that "the cumulative response of pay to performance is roughly 10 times that of the contemporaneous response" (p. 577).[3] We show that, consistent with the Boschen and Smith conclusion, it can be important to look over a long horizon when examining the link between pay and performance. In the final period of our model there will be a manager who is entrenched, makes no effort, and earns a large fixed wage, but this observed outcome is the consequence of an optimal contract. But even looking over a longer horizon will not allow one to detect the sensitivity of pay to performance if—as the empirical literature does—one uses the unanticipated component of changes in firm value as a proxy for performance. The large fixed wage in our model's optimal contract is awarded as compensation for effort that is certain to have been undertaken in a previous period.

Our model allows us to predict when options will be used in preference to equity in an optimal contract. To date, the empirical literature has distinguished only between wages that are assumed not to induce effort and indexed compensation in general (bonuses, stock, and options). This literature proceeds by identifying firms where agency problems are arguably greater—those with high market-to-book ratios or high R&D expenditures—and looks for greater reliance on equity and/or option compensation by these firms. Clinch (1991), Smith and Watts (1992), and Gaver and Gaver (1993) documented that equity and options do appear more frequently in executive compensation contracts when a firm's "growth-opportunities" are large relative to its "assets-in-place." However Bizjak, Brickley, and Coles (1993) report the "surprising" result that the sensitivity of compensation to performance is actually lower for high-growth firms than low-growth firms. Gaver and Gaver (1995) report that the proportion of executive compensation in the form of equity and options is *not* significantly greater for high-growth firms than low-growth firms. While high

[3] Jensen and Murphy report a statistically insignificant relation between current compensation and the second and third lags of corporate performance. Joskow and Rose document a statistically and economically significant relation between compensation and each of the zero'th through second lags of performance. The Boschen and Smith conclusion is based on statistically and economically significant results for each of the zero'th through third lags of performance.

market-to-book ratios and high R&D expenditures may well be indicative of the existence of an agency problem, our model shows that, given an agency problem, the *more* valuable the other assets of the firm beyond the manager's potential contribution to firm value, the *less* likely that that manager's compensation contract will include equity and/or options. This is true irrespective of the nature of the other assets—they could be R&D projects, or plant and equipment.

Our results on an optimal managerial compensation contract's wage, equity and options components, vesting dates, and control rights flow from embedding the principal-agent problem in a firm, where a firm is defined by four characteristics.[4] First, a firm may be large in the sense that there may exist assets-in-place. This means that a manager can free-ride on the existing assets, and not be compensated only with a claim on the value added by her efforts. Second, firms are long-lived. This allows a shirking, but vested, manager to free-ride on not just the assets-in-place, but also on the efforts of potential future replacement managers. These two characteristics make inducing effort more difficult than in a setting where there is no ability to free-ride. Third, a firm may be large in the sense that there may be more than one manager. Hence the fraction of equity that can be awarded to any single manager is limited, constraining the use of effort-sensitive compensation. Finally, the notion of "control" within a firm is expanded to allow for the possibility that the managerial compensation contract can specify that some control rights, initially held by the principal, be transferred to the agent. In particular, the principal and the initial manager may wish to contract on who has the right to determine when a replacement manager is to be hired. We assume that it is feasible for a principal to cede this authority to the agent/manager (e.g., by an award of sufficient voting stock). Such a transfer "entrenches" the manager. These enrichments of the standard principal-agent model are considered in an otherwise simple setting: There is no uncertainty (but there is "nonverifiability"); the manager makes a simple zero/one effort choice (shirk or work); the principal and the agent are risk-neutral; and the form of the compensation contract is limited to a combination of wages, equity, and options.

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[4] For other theoretical analyses of the wage, equity, option, and vesting components of optimal contracts see Jackson and Lazear (1991), Eaton and Rosen (1983), and Gibbons and Murphy (1992).

This essay's basic result is as follows. The incentive of an initial manager to free-ride on a replacement's effort can be overcome by deferring compensation. This induces the initial manager to make an effort early on and stay in office in order to receive the deferred compensation. But this deferral creates an incentive for the principal to opportunistically fire the manager after she has worked but before she is fully vested. Entrenching the manager makes the promise of deferred compensation credible. However, once entrenched the manager now has an incentive to hold up the principal. We show that the free-rider and hold-up problems can be simultaneously solved by deferring some, but not all, compensation. The optimal contract incorporates the constraint on the use of equity, and includes solutions for the wage, equity, and option components, as well as their associated vesting dates. Because the firm is long-lived an entrenched manager may be compensated at several dates, and the form of the compensation can optimally differ across dates.

The paper proceeds as follows. Section II presents the model. Section III analyzes the case where the investment opportunity is short-lived in the sense that there is no opportunity to hire a replacement manager. In Section IV the issue of free-riding on a replacement manager arises because we consider a long-lived investment opportunity. We analyze the optimal contract in the absence of entrenchment. Section V reconsiders the long-lived investment opportunity, allowing the contract to entrench the manager. We show that compensation costs can be lowered when entrenchment is admitted. Extensions of the model are discussed in Section VI and our conclusions are contained in Section VII.

II. The Model

Our firm consists of a combination of existing assets-in-place and a new investment opportunity (referred to as the "project"). The owner of the firm, the principal, supplies any capital required by the firm and must design a compensation contract to attract a manager to provide the labor input required to undertake the project. The investment opportunity lasts for (up to) two periods in the sense that if effort is not expended at time 0, then the opportunity can be exploited at time 1. But delay may involve some diminution in value (potentially a complete diminution). The opportunity is short-lived if the diminution is such that it would never be optimal to hire a manager to undertake the project at time 1. Otherwise the project is said to be long-lived. If effort is not expended at time 0, then in the case of a long-lived project the question arises of whether a replacement manager will be hired at time 1. The replacement decision at time 1 is taken by the party with "managerial control rights," that is, the right to determine who is in office. Managerial control rights will have been either retained by the principal, or transferred to the manager at time 0. The sequence of events is depicted on the time line in Figure 1.

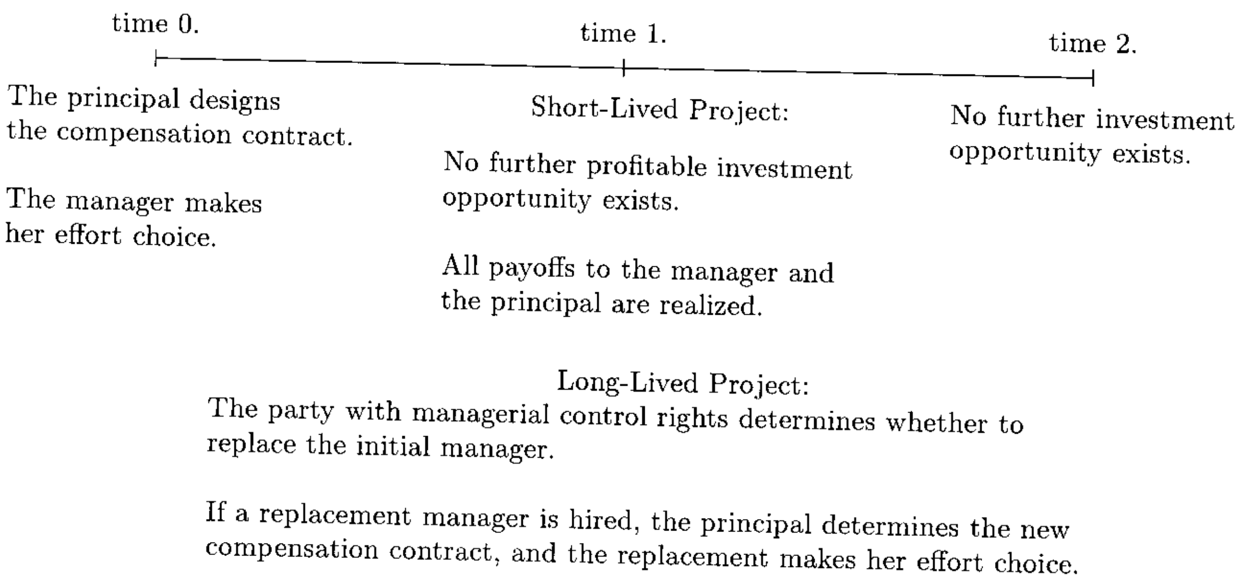

Figure 1. Sequence of Events. The project may be undertaken at time 0 or, with some possible diminution in its payoff, deferred until time 1.

The principal's and manager's decisions will reflect their utility functions, the production technology, and the contracting technology. The following assumptions provide a simple characterization of the environment in which these decisions are made. This environment reflects some essential features of contracting in a firm setting.

Preferences: Both the principal and the manager are risk-neutral. The manager's utility function is given by $C - D$, where C is lifetime consumption and D is the disutility of expending effort at any time during the manager's career.

Human Capital and the Manager's Reservation Utility: A manager's career lasts for two periods: youth and old age. A manager can make an effort only when young. A manager's reservation utility is R when young, and zero when old. Thus a manager hired for two periods has the same reservation utility as a manager hired for one period.

The Production Technology: Conditional on the manager's $\{0, 1\}$ effort choice, there is no uncertainty about the payoffs from the firm's technology. The value of the payoffs to the assets-in-place is V. Whenever $V > 0$, we assume that the project is not optimally undertaken on a stand-alone basis. The certain gross increment to firm value if effort is expended during the first period relative to effort never being expended is Δ_1. Similarly, Δ_2 is the certain gross increment to firm value if effort is expended during the second (and not the first) period, with $\Delta_2 \leq \Delta_1$. The interest rate in our world of perfect certainty is zero. At a minimum the manager must be compensated for her forgone opportunities, R, and for the disutility of effort, D. Thus, for the problem to be interesting, it must be that $\Delta_1 > R + D$.

The firm is long-lived in the sense that the principal cannot credibly commit to destroy the time 1 value of a project not undertaken at time 0. The technology is such that the manager's effort must be supplied continuously throughout a period if the project is to be undertaken. Whether a manager has worked throughout a period is not observable until the end of the period. If a manager either resigns or is fired during the period, then her project's output in that period is zero.[5]

The Transfer of Managerial Control Rights and Entrenchment: The principal's property rights to the assets-in-place and the investment opportunity give him the initial ownership of all control rights (see Grossman and Hart [1986]). "Managerial control rights" consist of the right to fire the existing manager. We assume that managerial control rights can be transferred to the manager for so long as she chooses to remain in office. The right to specify the compensation contract offered to the manager always resides with the principal. If a manager acquires managerial

[5] This implies that a manager will never be fired during a period, thereby precluding opportunistic attempts to fire a manager after she has worked but before she receives that period's compensation.

control rights, she does not acquire the right to renegotiate with herself (and award herself a higher wage, for example). Nor does she acquire the right to remain in office and hire someone else to do the work. She acquires only the right to determine when she should leave office. The transfer of managerial control rights can be accomplished through a verifiable contractual provision, or through the award of sufficient voting stock.[6] We refer to a contract that includes the transfer of managerial control rights as a "long-term contract." When the principal retains the right to fire the manager the contract is said to be a "short-term contract."

The Contracting Technology: As discussed above, a transfer of managerial control rights is assumed to be verifiable. But for the principal-agent problem to be interesting, the manager's effort choice, and anything isomorphic to it, must not be verifiable. Given our simplifying assumption of perfect certainty, firm value is isomorphic to effort. Therefore to capture an interesting agency problem in our simple firm setting it must be the case that firm value itself is not verifiable. We assume that the only feasible compensation contracts are then (i) debt-like claims (where the creditor acquires the firm in the event of default; see, e.g., Gale and Hellwig [1985]) and (ii) sharing rules that involve the verification of relative, not absolute, values (e.g., the principal must pay the same dividends per share on the stock held by the manager as on the stock retained by the principal). Given this limited set of verifiable contracts, managerial compensation will consist of debt-like wages, and long positions in stock and options.

We use the following notation to describe wages and equity shares. A wage promised at time τ to be paid at time t is denoted W_t^τ. Similarly, an equity share contracted for at time τ to vest at time t is denoted α_t^τ. The pecuniary component of a compensation contract signed at time $\tau = 0, 1$ is defined to be the specification of a wage and equity share profile $\{W_t^\tau, \alpha_t^\tau\}$ with $t = \tau + 1, \tau + 2$. (For notational simplicity we will sometimes omit the subscripts on W^τ and α^τ. It

[6] If the circumstances surrounding severance are verifiable then a manager can also be effectively entrenched via a golden parachute or severance package. If the circumstances surrounding severance are not verifiable—did she jump ship voluntarily, or was she pushed?—then promises of payments in the event of termination will not be honored by a principal retaining managerial control rights, who will always claim that the manager quit voluntarily. In practice we observe severance packages and golden parachutes in connection with a "change of control" in the ownership structure of the firm. This change of control is a verifiable event.

is then implicit that the realization date is $\tau+1$.) An executive stock option is, in fact, well-defined by our characterization of a wage, equity share profile, $\{W_t^\tau, \alpha_t^\tau\}$, and corresponds to $W_t^\tau < 0$ and $\alpha_t^\tau > 0$ for a given t. In other words, a contract signed at time τ specifying a negative wage to be paid at time t should be interpreted as involving an executive stock option such that the manager pays an exercise price of W_t^τ into the firm in exchange for the award of a fraction α_t^τ of the firm's shares.

The Constraint on Equity Compensation: We assume that $\sum_t \alpha_t^\tau$ is bounded above by $\overline{\alpha}$. This restriction may be thought of as reflecting the fact that the principal is contracting simultaneously with multiple agents. When the firm consists of n projects and n managers, then, since projects cannot be undertaken on a stand-alone basis, one cannot eliminate the principal-agent problem by selling the firm to *the* manager; at most, each manager can hold only $1/n$ of the equity. Alternatively, if votes are linked to shares in some proportion, the constraint may reflect the unwillingness of the principal to transfer so many votes that all control rights are also transferred. When, in Sections VB and VC, we assume that managerial control rights can be transferred only by granting a minimum fraction of the votes, we recognize that an $\overline{\alpha}$ constraint may preclude the transfer of managerial control rights.

Our assumptions are designed to formally embed the principal-agent problem in a simple setting that recognizes the four characteristics of firms discussed in the Introduction. In Section III we begin by focusing on a short-lived project. This setting recognizes the first two of the characteristics: firms have assets-in-place, and firms have more than one manager. When the project is short-lived the initial manager has no opportunity to free-ride on a replacement's efforts, since it will never be optimal to hire a replacement. Hence the third and fourth characteristics of firms—that they are long-lived and that some control rights may be transferred—do not arise.

III. A Short-Lived Project

An optimal contract must contain an effort-sensitive component: an executive stock option, a non-option award of equity, or an effort-sensitive wage. An effort-sensitive wage is "risky" in the sense that the promised wage can be paid in full only if the manager works. "Large" firms have assets-in-place. In this Section we show that when there is no $\overline{\alpha}$ constraint, the effort-sensitive component of an optimal executive compensation contract can depend on the size of the assets-in-place. In the presence of an $\overline{\alpha}$ constraint the form of an optimal contract depends on both the value of the assets-in-place and the bound on equity.

A. The Principal's Problem

Given a short-lived project we need only consider short-term contracts, $\{W_1^0, \alpha_1^0\}$.[7] The equity share vests at time 1; i.e., vesting is delayed until the end of the short-term contract.[8] This delay in vesting has two implications. First, the manager does not vest until after her reservation utility has declined to zero. Second, and more importantly, the manager does not vest until it is economically too late to hire a replacement to undertake the short-lived project. Hence she can not free-ride on a replacement's efforts.

At time 0 the principal seeks to determine the minimum cost compensation contract that both induces the manager to participate (the "participation constraint") and to make an effort (the "effort constraint"). Rationality on the part of the principal (the "principal's rationality constraint") dictates that the project be undertaken if this cost is less than Δ_1. The principal's contract design problem is complicated by the fact that the equity component of the compensation contract is constrained to be such that $\alpha_1^0 < \overline{\alpha}$ (the "$\overline{\alpha}$ constraint"). The principal's short-term contract design problem is then:

$$\min_{\alpha^0, W^0} \text{ Compensation}$$

$$\text{s.t.} \quad \text{Compensation if Work} - D \geq R \qquad \text{(participation constraint)}$$

$$\text{Compensation if Work} - D \geq \text{Compensation if Shirk} \qquad \text{(effort constraint)}$$

$$0 \leq \alpha^0 \leq \overline{\alpha} \qquad (\overline{\alpha} \text{ constraint})$$

[7] A long-term contract that induces a manager to both forgo her outside opportunities and expend effort can be effectively replicated by a short-term contract offering the same total wage (all to be paid at time 1) and the same total equity share (all vesting at time 1).

[8] A separate Appendix, available upon request, considers the possibility of immediate vesting at time 0. Immediate vesting is shown to be weakly dominated by delayed vesting; i.e., the cost of inducing participation and effort is weakly lower with delayed vesting.

A contract that satisfies the effort constraint and just satisfies the participation constraint will be referred to as a "least-cost contract." By definition a least-cost contract induces effort. Further, when the participation constraint is just satisfied, the manager's compensation given that she works is exactly equal to the minimum possible compensation cost of $R + D$. Thus any feasible least-cost contract must be an optimal contract. We will show that in the absence of an $\overline{\alpha}$ constraint a least-cost contract is always feasible. But, as will also be shown, the presence of an $\overline{\alpha}$ constraint can make all least-cost contracts infeasible. The cost of an optimal contract will then exceed least-cost. The $\overline{\alpha}$ constraint will be said to be binding when it rules out all least-cost contracts.

We now turn to the details of the participation and effort constraints. The manager's compensation if she works can be written in general as:

$$\max\left[0, \min[W^0, V + \Delta_1] + \alpha^0 \max[0, V + \Delta_1 - W^0]\right].$$

The functions $\min[W^0, V + \Delta_1]$ and $\max[0, V + \Delta_1 - W^0]$ reflect the firm's limited liability. When the wage is negative and the manager is compensated by an executive stock option, it is instructive to let $X := -W^0$ denote the exercise price of the warrant granted to the manager. The manager owns a warrant giving her the right to acquire the fraction α^0 of the firm if she pays X into the firm. In this case the above compensation expression simplifies to the payoff from such a warrant:

$$\max[0, \alpha(V + \Delta_1 + X) - X].$$

If the participation constraint is to be satisfied the manager's compensation must have positive value and hence the participation constraint takes the form:

$$\min[W^0, V + \Delta_1] + \alpha^0 \max[0, V + \Delta_1 - W^0] - D \geq R. \tag{1}$$

Since $\alpha^0 \geq 0$, any contract with $W^0 > \Delta_1$ will violate the principal's rationality constraint, as he will be promising more in wages than the manager adds to firm value by working. Thus we need only consider wage levels such that $W^0 < \Delta_1$. When $W^0 < \Delta_1$ the participation constraint simplifies to:

$$\alpha^0 \geq \frac{R + D - W^0}{V + \Delta_1 - W^0}. \tag{2}$$

A wage of at least $R + D$ will induce participation even absent an equity share. Conversely, absent a wage an equity share of at least $\frac{R+D}{V+\Delta_1}$ will induce participation since the value of that share will be at least $R + D$.

The effort constraint takes the form:

$$\min[W^0, V + \Delta_1] + \alpha^0 \max[0, V + \Delta_1 - W^0] - D \geq \max\left[0, \min[W^0, V] + \alpha^0 \max[0, V - W^0]\right]. \quad (3)$$

When $W^0 < \Delta_1$ the effort constraint in (3) simplifies to:

$$\alpha^0 \geq \begin{cases} \frac{D-W^0}{V+\Delta_1-W^0}, & \text{if } W^0 < -\frac{D}{\Delta_1-D}V; \\ \frac{D}{\Delta_1}, & \text{if } W^0 \in [-\frac{D}{\Delta_1-D}V, V]; \\ \frac{V+D-W^0}{V+\Delta_1-W^0}, & \text{if } W^0 \in (V, V+D]. \end{cases} \quad (4)$$

To understand the three segments of the effort constraint consider various levels of W^0. First consider "high" wages. Whenever the promised wage is in the range $(V, V + D]$, it can be paid in full only if the manager makes an effort. When the promised wage is equal to $V + D$, the actual wage received will be greater by the amount D if the manager works than if she does not work, and the effort constraint will be satisfied even for $\alpha^0 = 0$. As W^0 is reduced below $V + D$ the other performance-sensitive compensation component, α^0, must be increased if the effort constraint is to remain satisfied. When $W^0 \in [0, V]$, the promised wage can always be paid in full and her equity stake alone must motivate her to make an effort. This requires that her share of the increase in firm value if she works, $\alpha^0\Delta_1$, exceed her disutility of effort, D; i.e., $\alpha^0 \geq \frac{D}{\Delta_1}$.[9] For $W^0 \in [-\frac{\Delta_1}{\Delta_1-D}V, 0]$ the exercise price of the option is such that when the manager's equity share equals $\frac{D}{\Delta_1}$, her option always finishes in-the-money, and it finishes in-the-money by D dollars more if she works. When W^0 is very low (equivalently, the exercise price of the manager's executive stock option is very high), in particular when $W^0 < -\frac{\Delta_1}{\Delta_1-D}V$, then when the manager's equity share is equal to $\frac{D-W^0}{V+\Delta_1-W^0}$, her option finishes in-the-money by the amount D if she works and exactly at-the-money if she shirks.

Figure 2 depicts the participation and effort constraints for varying levels of V and reflects the following property of the constraints:

[9] The assumption that $\Delta_1 > R + D$ implies that $\frac{D}{\Delta_1} < 1$.

Lemma 1. *If the participation and effort constraints intersect strictly, then they do so in the horizontal section of the effort constraint.*

Proof: See Appendix A.

As V increases one moves in turn from the setting in Figure 2(a) to that in 2(b), and then to 2(c). An increase in V leads to an increase in the maximum possible wage that could be paid in full even absent any managerial effort: For $W^0 > 0$ an increase in V shifts the effort constraint horizontally to the right. An increase in V also leads to an increase in the maximum possible exercise price of an option that will finish exactly at-the-money absent any managerial effort: For $W^0 < 0$ an increase in V shifts the effort constraint horizontally to the left. An increase in V causes the participation constraint to pivot around the point $\{W^0 = R + D, \alpha^0 = 0\}$. A contract of the form $\{W^0 = R + D, \alpha^0 = 0\}$ will just satisfy the participation constraint irrespective of the value of V. The equity stake necessary to induce participation when the wage is less than $R + D$ is decreasing in V. (For example, consider the zero-wage, all-equity contract, $\{W^0 = 0, \alpha^0 = \frac{R+D}{V+\Delta_1}\}$.) Thus, the participation constraint rotates downwards as V increases.

As shown in Figure 2(a), when $V < R$, any contract that induces the manager to participate will also induce her to make an effort. Thus when $V < R$, a least-cost contract could be any of the following: an all-wage contract, an all-equity contract, a combination of wages and equity, or an executive stock option. For higher values of the assets-in-place, a contract that induces effort and participation via a risky wage becomes increasingly expensive, while a contract that uses a positive equity share and a riskless wage can induce effort and participation at least-cost irrespective of V.

Figures 2(b) and 2(c) are both situations where $V > R$ and all least-cost contracts contain an equity share. The more valuable the assets-in-place, the more valuable is any such equity share. Whether a least-cost contract involves a positive wage in addition to the equity share depends on the value of the manager's equity claim on the assets-in-place. Consider the zero-wage, all-equity contract that just satisfies the effort constraint, $\{W^0 = 0, \alpha^0 = \frac{D}{\Delta_1}\}$. The value of the manager's claim given that she works is:

$$\alpha^0(V + \Delta_1) = \frac{D}{\Delta_1}(V + \Delta_1) = \frac{D}{\Delta_1}V + D.$$

Whether this contract induces participation depends on whether the fraction of the assets-in-place that the manager receives, $\frac{D}{\Delta_1}V$, compensates her for forgoing her outside opportunities. When $\frac{D}{\Delta_1}V < R$ a positive wage must be added to induce participation. But when $\frac{D}{\Delta_1}V > R$ the manager is overcompensated for having forgone R. This zero-wage, all-equity contract cannot then be an optimal contract. Charging the manager a "strike price" to receive her equity share can reduce the cost of the contract without affecting the manager's incentive to work.

B. The Design and Cost of the Optimal Contract Given a Short-Lived Project

From Figures 2(a), (b), and (c) we see that, absent an $\overline{\alpha}$ constraint, least-cost contracts always exist. Inspection of Figures 2(b) and 2(c) reveals that when $V > R$, all least-cost contracts involve $\alpha^0 \geq \frac{D}{\Delta_1}$. But when the $\overline{\alpha}$ constraint precludes awarding such a high equity share, the cost of an optimal contract will exceed least-cost. Thus we have shown:

Lemma 2. *The $\overline{\alpha}$ constraint will be binding if $V > R$ and $\overline{\alpha} < \frac{D}{\Delta_1}$.*

Proposition 1 formalizes the above discussion of the design of an optimal contract given a short-lived project in terms of the necessary and sufficient conditions for the contract to include executive stock options, a non-option equity award, and effort-sensitive wages.

Proposition 1. (The Form of the Optimal Contract for a Short-Lived Project)

1. Executive Stock Options: $\{W^0 < 0, \alpha^0 > 0\}$

If an optimal contract for a short-lived project takes the form of an executive stock option, then either (i) $V \leq \frac{\Delta_1}{D}R$ and $\overline{\alpha} > \frac{R+D}{V+\Delta_1}$, or (ii) $V > \frac{\Delta_1}{D}R$ and $\overline{\alpha} \geq \frac{D}{\Delta_1}$. The optimal contract for a short-lived project must be an executive stock option if $V > \frac{\Delta_1}{D}R$ and $\overline{\alpha} > \frac{D}{\Delta_1}$.

2. A Non-Option Equity Award: $\{W^0 \geq 0, \alpha^0 > 0\}$

If a non-option equity award is included in an optimal contract for a short-lived project, then either (i) $V \leq \frac{\Delta_1}{D}R$, or (ii) $V > \frac{\Delta_1}{D}R$ and $\overline{\alpha} < \frac{D}{\Delta_1}$. The optimal contract for a short-lived project must contain a non-option equity award if $R < V \leq \frac{\Delta_1}{D}R$ and $V\frac{D}{\Delta_1} \leq \overline{\alpha} \leq \frac{R+D}{V+\Delta_1}$.

3. An Effort-Sensitive Wage: $\{W^0 > V, \alpha^0 \geq 0\}$

If an effort-sensitive wage is included in an optimal contract for a short-lived project, then either (i) $V > R$ and $\overline{\alpha} < \frac{D}{\Delta_1}$, or (ii) $V \leq R$. The optimal contract for a short-lived project must contain an effort-sensitive wage if either (i) $V > R$ and $\overline{\alpha} < \frac{D}{\Delta_1}$, or (ii) $V \leq R$ and $\overline{\alpha} < \frac{R+D-V}{\Delta_1}$.

Proof: See Appendix A.

When assets-in-place are small an optimal contract need not involve any equity. The promised wage component of an optimal contract can be honored only if the manager works, and hence the wage received is sensitive to the manager's effort. At higher levels of assets-in-place, optimal contracts must involve an equity component; there may be wages plus equity, or a zero-wage, all-equity contract, or executive stock options. When the value of the assets-in-place becomes sufficiently high, optimal contracts must take the form of stock options.[10] "Large" firms have more than one manager and hence an $\overline{\alpha}$ constraint. When the $\overline{\alpha}$ constraint proves binding the manager must be rewarded, at least in part, with a promised wage component that is "risky." Since only positive wages can be "risky," executive stock options are ruled out.

Having characterized the set of optimal contracts we turn next to their cost. Let $C^{\tau}(\mathcal{S}, ST)$ denote the cost of an optimal short-term contract (ST) signed at time τ when the project is short-lived $(\mathcal{S})$. Proposition 2 sets out the optimal contract cost $C^0(\mathcal{S}, ST)$.[11]

Proposition 2. (The Cost of the Optimal Time 0 Contract for a Short-Lived Project) *The cost of an optimal contract is:*

$$C^0(\mathcal{S}, ST) = \begin{cases} V + D; & \text{if } V > R \text{ and } \overline{\alpha} < \frac{D}{\Delta_1} \quad (\overline{\alpha} \textit{ constraint binding}), \\ R + D; & \textit{otherwise.} \end{cases} \tag{5}$$

Proof: Lemma 2 provides the condition under which the $\overline{\alpha}$ constraint will be binding. When the $\overline{\alpha}$ constraint is binding, inspection of Figures 2(b) and 2(c) reveals, optimal contracts are those contracts involving an effort-sensitive wage that just satisfy the effort constraint in (4), while also satisfying the $\overline{\alpha}$ constraint. When α^0 is so determined, the cost of a compensation

[10] The empirical literature discussed in the Introduction considers "assets-in-place" to be those assets the optimal management of which involves a verifiable effort choice, and considers "growth opportunities" to be those assets whose optimal management involves an agency problem. Expenditures on R&D and market-to-book ratios are used as empirical proxies for "growth opportunities." But "assets-in-place" in our model refers to all the other assets of the firm beyond the particular project to be managed. These other assets may in fact be other projects; i.e., they may take a form that the empirical literature would label "growth opportunities."

[11] If a long-lived project is not undertaken at time 0, it becomes, by default, a short-lived project at time 1, and $C^1(\mathcal{S}, ST)$ is the cost of then hiring a manager under an optimal contract.

contract is:

$$W^0 + \alpha^0(V + \Delta_1 - W^0) = W^0 + \frac{V + D - W^0}{V + \Delta_1 - W^0}(V + \Delta_1 - W^0) = V + D. \quad \blacksquare$$

The cost of an optimal contract for a short-lived project is depicted in Figure 3. Figure 3(a) depicts the case where $\overline{\alpha} < \frac{D}{\Delta_1}$, the situation where effort cannot be induced through the equity component alone. At least part of the effort must be induced through an effort-sensitive wage. When $V > R$, any contract with $\alpha^0 < \overline{\alpha}$ that just satisfies the participation constraint will not induce effort; the assets-in-place available to back the promised wage component of such a contract are so large that any difference in the wage actually received if the manager does make an effort does not adequately compensate her for the disutility of doing so. To induce her to work requires a higher promised wage. The set of optimal contracts is equivalent to the all-wage, zero-equity contract that promises the manager all the assets-in-place plus an additional amount equal to D. She can receive the additional amount only if she works. The resulting contract is more expensive than least-cost.[12]

Figure 3(b) depicts the case where $\overline{\alpha} \geq \frac{D}{\Delta_1}$, the situation where a least-cost contract can always be achieved irrespective of V. For example, an equity share of $\frac{D}{\Delta_1}$ and a wage of zero would exactly compensate the manager for the disutility of effort. Adjusting the wage, either positively or negatively as required, can then induce least-cost participation.

C. *Discussion of Results for a Short-Lived Project*

In the absence of an $\overline{\alpha}$ constraint the principal will always find it optimal to hire a manager to undertake the project (by assumption the value of the project exceeds the least-cost compensation cost). But when the $\overline{\alpha}$ constraint is binding and $C^0(S, ST) = V + D > \Delta_1$, the principal will forgo the project. Thus when assets-in-place are sufficiently large, the presence of multiple

[12] This suggests that potential managers would bid for the position of manager under this contract, effectively lowering its cost to least-cost. Potential managers may be cash-constrained, and borrowing against future compensation creates a new agency problem between the lender and the manager. Alternately, the compensation in excess of least-cost might be viewed as a franchise fee that is paid to the principal for the right to manage one of the firm's outlets, and the conditions under which least-cost cannot be achieved may be interpreted as conditions under which we should expect to observe franchise operations.

managers can make the $\overline{\alpha}$ constraint so tight that it becomes binding, and a socially valuable project—a project with $\Delta_1 > R + D$—will not be undertaken. Minimizing this social loss provides a natural theory for the existence of debt. The loss could be avoided if the assets-in-place could be separated from the investment opportunity. But by assumption this is technologically impossible. However, claims to the assets-in-place can be effectively separated from claims on the investment opportunity if the firm's cash flows accrue through time in such a way that they can be used to pay off a debt claim that matures prior to the promised wage payment. The earlier maturity of the debt can guarantee that the manager's promised compensation is then backed only by the increment in firm value if she works. Appendix B provides the details of the argument that in this case debt financing can make least-cost compensation contracts feasible. Note that this theory of debt reflects internal firm incentive problems rather than conflicts with outside investors. Our failure to further examine such a role for debt in the remainder of this paper reflects only a desire to develop a parsimonious model of the principal-agent problem in a firm setting. To continue that model we now wish to recognize that firms are long-lived by considering optimal contracting when the project is long-lived.

IV. A Long-Lived Project and a Short-Term Contract

When the project is long-lived, the possibility of the long-lived firm hiring a replacement manager arises. In this Section we consider a long-lived project with an initial manager hired under a short-term contract. In the following Section we consider using a long-term contract to hire the initial manager of that same long-lived project. Thus we delay, until Section V, the addition of the final characteristic of a principal-agent problem in a firm setting, namely the ability to entrench the agent by transferring some control rights to her.

When the executive compensation contract does not involve a transfer of managerial control rights, the principal is free to fire the manager after one period. Consequently an optimal short-term contract for the initial manager will not involve wages promised at time 2, or stock or options that do not vest until time 2; i.e., our initial manager will be fully vested at time 1. When the initial manager is fully vested at time 1 and, as here, the project is long-lived, she has an incentive to shirk and free-ride on her replacement. By definition, a long-lived project potentially remains valuable even if the initial manager shirks. In order to determine the optimal short-term contract with a manager at time 0, we must proceed recursively and first determine the compensation contract for a potential replacement manager hired at time 1. In determining the optimal contract for such a replacement manager, we must be explicit as to the priority of the claims held by the initial and replacement managers.

Seniority of Claims Given Replacement: For tractability we assume that potential replacement managers are offered a wage and an equity package that is senior to any other claim on the firm. This assumption makes the cost of compensating a replacement manager independent of the design of the initial manager's compensation package. We also assume that the claims of an initial shirking manager maintain their original priority relative to the stockholders. If the initial manager shirks, she retains her equity claim, and any promised wage is replaced by a debt claim to that amount with the debt maturing at time 2. In compensating the replacement manager there may again be a constraint on using equity—i.e., a constraint that $\alpha^1 < \overline{\alpha}$. For simplicity only, we take the upper bound on the equity claim available to offer a manager at time 1 to be the same as that available to offer a manager at time 0. In the event that the $\overline{\alpha}$ constraint was binding at time 0, the question then arises of how the replacement manager could be compensated with any equity. The answer is that all shareholders would be willing to have the firm issue a sufficient amount of new equity (and hence dilute their own holdings) when doing so can enable the replacement manager to be hired at a cost less than Δ_2.[13]

A. Hiring a Replacement Manager at Time 1

At time 1 the project will be short-lived. Therefore the analysis of a replacement manager's optimal compensation package is the same as the Section III analysis of optimal contracting given a short-lived project, except that Δ_2 replaces Δ_1 throughout. By analogy with Proposition 2, the

[13] Shareholders' willingness to dilute their holdings at time 0 in order to reduce the compensation of any particular manager will be tempered by the adverse incentive effects on other managers who are also compensated with equity and hence whose holdings are also diluted. The $\overline{\alpha}$ constraint should be interpreted as the maximum fraction of equity available to compensate a particular manager taking into account the optimal amount of such dilution.

cost of an optimal contract for the replacement manager, $C^1(\mathcal{S}, ST)$ is:

$$C^1(\mathcal{S}, ST) = \begin{cases} V + D; & \text{if } V > R \text{ and } \overline{\alpha} < \frac{D}{\Delta_2} \qquad (\overline{\alpha} \textit{ constraint binding}), \\ R + D; & \textit{otherwise.} \end{cases} \tag{6}$$

A replacement manager will be hired whenever $\Delta_2 > C^1(\mathcal{S}, ST)$.

B. *A Characterization of the Time 0 Problem*

Since the opportunity to free-ride on a replacement manager's effort exists only when $\Delta_2 > C^1(SL)$, it is convenient to define the value of the option to replace a shirking manager as $\mathcal{O} := \max[0, \Delta_2 - C^1_{\mathcal{S}}]$. From the point of view of the initial manager the assets-in-place are not V; they are $V + \mathcal{O}$. From the point of view of the initial manager the increment that she adds to firm value by working is less than the full amount Δ_1; she views her marginal contribution to firm value if *she* bears the disutility of effort as only $\Delta_1 - \mathcal{O}$.

If the initial manager works, no replacement need be hired. Hence the participation constraint is the same as that for a short-lived project. If the initial manager shirks, she has a claim not only on V but on $\mathcal{O}$ as well. The effort constraint takes the form:

$$\alpha^0 \geq \begin{cases} \frac{D - W^0}{V + \Delta_1 - W^0}, & \text{if } W^0 < -\frac{D}{\Delta_1 - \mathcal{O} - D}(V + \mathcal{O}); \\ \frac{D}{\Delta_1 - \mathcal{O}}, & \text{if } W^0 \in [-\frac{D}{\Delta_1 - \mathcal{O} - D}(V + \mathcal{O}), V + \mathcal{O}]; \\ \frac{V + \mathcal{O} + D - W^0}{V + \Delta_1 - W^0}, & \text{if } W^0 \in (V + \mathcal{O}, V + \mathcal{O} + D]. \end{cases} \tag{7}$$

Note that replacing V in (4) with $V + \mathcal{O}$, and replacing Δ_1 in (4) with $\Delta_1 - \mathcal{O}$, gives the effort constraint for a long-lived project in (7). As $C^1(\mathcal{S}, ST)$ increases, $\mathcal{O}$ decreases and the effort constraint converges towards that of a short-lived project. An increase in the cost of hiring a replacement reduces the incentive for the initial manager to free-ride. When $\mathcal{O}$ is zero there is no possibility of free-riding, and the effort constraint in (7) becomes identical to the effort constraint in (4), the constraint for a short-lived project.

The effort and participation constraints are as portrayed in Figure 4. It is instructive to compare Figure 4 to Figure 2. Whatever the technological length of the project, the participation constraint remains the same. However, the effort constraint is affected by the existence of the option to replace a shirking manager. The presence of $\mathcal{O}$ moves the effort constraint up and to the right for positive values of W^0 and up and to the left for negative values of W^0. The replacement option associated with a long-lived project can affect both the terms of an optimal short-term contract entered into at time 0 and the cost of that contract. To illustrate these effects consider the scenario depicted in Figure 5, which shows the effort constraints facing the principals of two different firms. The two firms share the same exogenous parameter values V, D, R, and $\overline{\alpha}$, but they face different investment opportunities. Although the value of Δ_1 is the same for each firm, the value of Δ_2 differs. For one firm the investment opportunity is short-lived; if delayed, the diminution in project value is so large that it is no longer profitable to undertake the project. For the other firm the opportunity is long-lived, and, in fact, $\mathcal{O} > 0$. For the firm with the short-lived opportunity, the parameter values are such that the effort constraint takes the form depicted in Figure 2(a), and a least-cost contract can be achieved. But for the firm with the long-lived opportunity, the effort constraint is as depicted in Figure 4(c), the $\overline{\alpha}$ constraint is binding at time 0, and a least-cost contract cannot be achieved. Even if the $\overline{\alpha}$ constraint were weaker and were not binding on the firm with the long-lived opportunity, then although both firms could achieve least-cost, the set of optimal contracts for the firm facing the long-lived project would all take the form of executive stock options. The set of optimal contracts for the firm with the short-lived project would include executive stock options, but would not be limited to them.

C. *The Design and Cost of the Optimal Short-Term Contract Given a Long-Lived Project*

When the replacement option has value, it may be harder to motivate the initial manager with a wage—the potential sensitivity of the realized wage to effort is reduced by $\mathcal{O}$ relative to the situation where the project is short-lived. It may also be harder to motivate the initial manager with an equity claim—the increase in the value of the firm if she works is reduced by $\mathcal{O}$ relative to the situation where the project is short-lived. Replacing V by $V + \mathcal{O}$, and Δ_1 by $\Delta_1 - \mathcal{O}$, in Propositions 1 and 2 gives the form and cost of an optimal short-term contract for a long-lived project. Since the manager of a long-lived ($\mathcal{L}$) project could have been hired under either a short-term (ST) or a long-term (LT) contract, we use the notation $C^0(\mathcal{L}, ST)$ and $C^0(\mathcal{L}, LT)$ to denote the respective costs of optimal short-term contracts and optimal long-term contracts signed at time 0 when the project is long-lived.

The cost of the optimal short-term contract for a long-lived project is:

$$C^0(\mathcal{L}, ST) = \begin{cases} V + \mathcal{O} + D; & \text{if } V + \mathcal{O} > R \text{ and } \overline{\alpha} < \frac{D}{\Delta_1 - \mathcal{O}} \quad (\overline{\alpha} \textit{ constraint binding}), \\ R + D; & \textit{otherwise.} \end{cases} \tag{8}$$

Propositions 3 and 4 compare the form and cost of an optimal short-term contract for otherwise equivalent short- versus long-lived projects. Projects are otherwise equivalent if they differ in their Δ_2 values, but share common values of V, Δ_1, R, D, and $\overline{\alpha}$.

Proposition 3. *Consider a long-lived project and a short-term contract. For any given W^0, the equity fraction, α^0, required to induce effort from the initial manager is at least as great as that necessary for an otherwise equivalent short-lived project. When the replacement option has value, $\mathcal{O} > 0$, that equity fraction is strictly greater.*

Proof: Compare the effort constraint in (7) to the corresponding constraint in (4). ▮

Proposition 4. *The cost of an optimal time 0 short-term contract for a long-lived project, $C(\mathcal{L}, ST)$, is always at least as high as the cost of an optimal short-term contract for an otherwise equivalent short-lived project, $C^0(\mathcal{S}, ST)$.*

Proof: Compare expressions (8) and (5). ▮

The value of the replacement option, $\mathcal{O}$, is a determinant of the cost of an optimal short-term contract, $C^0(\mathcal{L}, ST)$. The replacement option can be characterized in one of three ways. If a replacement can be hired at least-cost, $\mathcal{O} = \Delta_2 - C^1(\mathcal{S}, ST) = \Delta_2 - (R + D)$, which by the definition of a long-lived project exceeds zero. If a replacement will be hired but at greater than least-cost, then $\mathcal{O} = \Delta_2 - (V + D)$. Finally, if it is prohibitively expensive to hire a replacement, $\mathcal{O} = 0$. What remains is to determine $\mathcal{O}$ and hence $C^0(\mathcal{L}, ST)$ purely in terms of the exogenous parameters V, Δ_1, Δ_2, R, D and $\overline{\alpha}$.

$C^0(\mathcal{L}, ST)$ is depicted in Figure 6 as a function of the $\overline{\alpha}$ constraint and the value of the assets-in-place, V.[14] Comparing Figures 6 and 3 we see that when the replacement option has value, the cost of compensating the initial manager strictly increases for values of V corresponding to the shaded regions of Figure 6. The shaded regions correspond to the increase in cost.

[14] Figure 6 is drawn to reflect the situation when $2R + D - \Delta_2 > 0$. Nothing of substance is altered when $2R + D - \Delta_2 < 0$. The $C(ST, LL)$ axis is simply shifted to a point between $2R + D - \Delta_2$ and R.

Moving from Figure 6(a) through to Figure 6(d) corresponds to successively loosening the $\overline{\alpha}$ constraint. Loosening the $\overline{\alpha}$ constraint can affect the compensation of both a replacement manager and the initial manager; moreover, these changes in compensation cost are linked as follows. If loosening the $\overline{\alpha}$ constraint lowers the cost of hiring a replacement, it increases the incentive for the initial manager to free-ride, in effect shifting upward the initial manager's effort constraint. Whether this leads to an increase in the cost of compensating the initial manager depends on whether the $\overline{\alpha}$ constraint has moved up even more. Compare Figures 6(b) and 6(c). For the parameter values of Figure 6(ci), loosening the $\overline{\alpha}$ constraint can lead to an increase in $C^0(\mathcal{L}, ST)$, while for the parameter values of Figure 6(cii), loosening the $\overline{\alpha}$ constraint can lead to an decrease in $C^0(\mathcal{L}, ST)$. A detailed discussion of the various possibilities reflected in Figure 6 is contained in a separate Appendix available upon request.

D. *Summary of Results for a Long-Lived Project and a Short-Term Contract*

In this Section we have recognized that firms are not only large, they are also long-lived. Given a long-lived firm, a replacement manager may be hired when a long-lived project is not undertaken by the initial manager. One might think that the managerial control right to hire a replacement would be of some value to the principal. However, the principal is always at least as well off, and potentially better off, if he can commit to never exercising this option. The reason is that the replacement option introduces the possibility of free-riding by the initial manager. From her point of view $\mathcal{O}$ is part of the assets-in-place, since the principal is unable to separate the time 0 investment opportunity from the delayed investment opportunity. The existence of a valuable replacement option alters the form of the initial manager's effort constraint because the payoff from shirking is a claim on both V and $\mathcal{O}$. To induce effort by the initial manager can then require that she receive even greater compensation for working than would be necessary in the absence of an option to replace a shirking manager.

Interestingly, if the $\overline{\alpha}$ constraint is binding on a replacement manager, it can reduce the cost of hiring an initial manager. Increasing the cost of hiring a replacement reduces the value of the replacement option and may make hiring the replacement prohibitively expensive, in which case the replacement option has no value. When the replacement option does have value, it may still be possible to make long-lived projects economically short-lived by designing a compensation package for the initial manager such that her *claim* on the replacement option is valueless. Designing such a contract is the subject of Section V, where we introduce our final characteristic of the principal agent problem in a firm setting: the ability to transfer managerial control rights.

V. A Long-Lived Project and a Long-Term Contract

Long-term contracting via the transfer of managerial control rights has two distinct roles in precluding free-riding. The first is that by entrenching the manager, the principal commits not to exercise the replacement option; i.e., the manager will not be fired if she shirks. But entrenching the manager does not preclude her from voluntarily resigning, thereby exercising the replacement option. To preclude free-riding, the principal must not only entrench the initial manager, the principal must design the compensation contract so as to impose a penalty on her if she voluntarily resigns. Entrenchment plays its second role here. By deferring part of the initial manager's compensation until time 2, the principal makes it costly to her to voluntarily depart at time 1. But the promise of deferred compensation at time 2 is credible only if the initial manager cannot be fired after she has done the work but before she is fully vested. Thus the initial manager must vest in entrenchment by time 1. While deferring compensation can solve the free-rider problem, deferring too much of the compensation can introduce a new problem: the hold-up problem. When all compensation is deferred until time 2, a manager who shirks may be able to credibly threaten to remain in office and continue to shirk unless she receives a bribe equal to the full value of the replacement option.

In subsections A and B the hold-up problem is temporarily put aside. Subsection A analyzes long-term contracts maintaining the assumption that managerial control rights can be transferred via a verifiable contract provision. Subsection B then considers the more realistic setting where managerial control can be transferred only via the award of a minimum number of voting shares at time 1. Subsection C then recognizes the hold-up problem and determines the optimal long-term contract when entrenchment requires a (possibly zero) transfer of votes.

A. A Contractual Transfer of Managerial Control Rights

Under a short-term contract the initial manager must, by default, vest at time 1. Under a long-term contract there can be credible promises of wages at both times 1 and 2, and awards of equity and executive stock options can vest at both times 1 and 2.

Proposition 5. *When the project is long-lived, the set of long-term contracts that overcome the free-rider problem includes* $\{\alpha_1^0 = W_1^0 = 0, \alpha_2^0 = \alpha^{0^*}, W_2^0 = W^{0^*}\}$, *where* α^{0^*} *and* W^{0^*} *are the equity and wage components of any one of the set of optimal contracts for a short-lived project having the same exogenous parameters* V, Δ_1, R, D, *and* $\overline{\alpha}$.

Proof: See Appendix A.

Proposition 5 illustrates the extreme case where all rights to compensation vest at time 2. More generally, so long as the value of all claims vesting at times 1 and 2 on $V + \Delta_1$ exceeds the value of claims vesting at time 1 on the lesser quantity $V + \mathcal{O}$ by at least D, the entrenched manager will not be tempted to free-ride on her replacement's effort by voluntarily resigning at time 1.

B. The Transfer of Managerial Control Rights via an Award of Voting Stock

The ability to contractually transfer managerial control rights through a verifiable contract provision may be limited.[15] Therefore in this subsection we consider transferring managerial control by awarding voting stock and assume that managerial control rights can be transferred if at least the fraction $\underline{\alpha}$ of the equity is awarded to the manager prior to the possible exercise of the replacement option at time 1. ($\underline{\alpha} = 0$ corresponds to the case when managerial control rights can be contractually transferred.) Corresponding to the empirical evidence, the fraction of votes necessary to transfer managerial control rights may be small, certainly less than the fraction necessary to acquire all control rights.[16] Whatever the fraction of votes necessary to transfer managerial control rights, the important consideration for our analysis is whether $\underline{\alpha}$ is greater than, or less

[15] Still, 37 of the 147 employment agreements of Fortune 500 senior executives for the year 1980 studied in Kole (1994a) contain a provision guaranteeing employment. In seven cases the guarantee lasts more than five years.

[16] Dennis, Dennis, and Sarin (1994) document that managers can become entrenched with ownership stakes as low as 1%. See also Mikkelson and Partch (1994).

than, $\overline{\alpha}$. We consider these two possibilities in turn.[17]

Proposition 6. *When the project is long-lived and a transfer of managerial control rights is feasible (i.e., $\overline{\alpha} > \underline{\alpha}$), the set of long-term contracts that overcome the free-rider problem includes* $\{W_1^0 = -\frac{\underline{\alpha}}{1-\underline{\alpha}}(V + \Delta_1), \alpha_1^0 = \underline{\alpha}, W_2^0 = W^{0^*} - W_1^0, \alpha_2^0 = \alpha^{0^*} - \underline{\alpha}\}$*, where* α^{0^*} *and* W^{0^*} *are any equity and wage pair that (i) is a member of the set of optimal contracts for an otherwise identical short-lived project, and (ii) satisfies* $\alpha^{0^*} \geq \underline{\alpha}$.

Proof: Given the equity component of the candidate optimal contract that vests at time 1 and entrenches her, $\underline{\alpha}$, the wage component of the candidate optimal contract due at time 1, W_1^0, is chosen to satisfy

$$W_1^0 + \underline{\alpha}(V + \Delta_1 - W_1^0) = 0;$$

i.e., $W_1^0 < 0$, and the manager becomes entrenched via an executive stock option. If the manager works and then leaves the firm at time 1, her option finishes "at-the-money." The option will be "out-of-the-money" if the manager shirks and then leaves the firm at time 1. Having achieved entrenchment without giving the manager a valuable claim on the replacement option, we are back in the setting of Proposition 5. The remainder of the contract therefore consists of the following: $\alpha_2^0 = \alpha^{0^*} - \underline{\alpha}$ and $W_2^0 = W^{0^*} - W_1^0$. ▮

Proposition 5 is simply the special case of Proposition 6 with $\underline{\alpha} = 0$. Notice that since the contract component of Proposition 6 that vests at time 1 is an executive stock option, the optimal time 2 wage component of Proposition 6 exceeds the optimal W^{0^*} of Proposition 5 by exactly the amount of the strike price. In other words, the optimal contract can have the manager buy votes/entrenchment at time 1 by exercising a stock option, with the purchase/exercise price returned at time 2.[18] The option contract of Proposition 6 is designed to illustrate the extreme case

[17] If residual cash flow rights and votes are bundled as one share-one vote, the upper bound on the award of cash flow claims $\overline{\alpha}$ is also an upper bound on the award of votes. Even when cash flow rights and votes attach separately to non-voting and voting stock, the existence of multiple managers will still constrain the maximum fraction of votes that can be awarded to any one manager. For simplicity, we assume that this bound is also equal to $\overline{\alpha}$.

[18] Huddart (1994) and Huddart and Lang (1995) empirically document "early exercise" of executive stock options. They attribute "early exercise" to executives' risk aversion and the inalienable nature of the option. For our manager the payoff from exercising is not just the shares received immediately, but also entrenchment and hence the future receipt of the compensation

where there is a zero payoff to a manager who resigns in order to free-ride. In general (as will be seen in the proof of Proposition 9) an optimal contract can involve a positive payoff to free-riding. What is important is that under an optimal contract the payoff to working must exceed the payoff to free-riding by at least D.

When entrenchment is not feasible, $\overline{\alpha} < \underline{\alpha}$, the principal endowed with a long-lived project faces a choice between using a short-term contract to induce effort at time 0, or delaying the project until time 1. If delayed until time 1 the project will have become short-lived—there will no longer be a replacement option on which a manager could free-ride. Motivating a manager to work at time 1 is then potentially cheaper than motivating her to work at time 0. Any saving in contracting cost must be balanced against the diminution in project value if it is delayed until time 1. When entrenchment is not feasible, delaying the project until time 1 will dominate undertaking the project at time 0 provided the saving in compensation cost, $C^0(\mathcal{L}, ST) - C^1(\mathcal{S}, ST)$, exceeds the diminution in project value, $\Delta_1 - \Delta_2$.

C. *Entrenchment and the Hold-Up Problem*

When entrenchment is feasible, deferring compensation can overcome the free-rider problem. But in doing so it may create a new problem. Once entrenched, the manager has the ability to hold up the principal. Suppose the manager were to shirk during the first period, and then threaten to stay in office unless she received a bribe equal to the full value of the replacement option. When all compensation is deferred until time 2 in order to preclude free-riding (as in Proposition 5), the manager's hold-up threat will be credible. She has no incentive to resign voluntarily without a bribe.[19] In this subsection we will show that there exist long-term contracts that are not vulnerable

that will not vest until time 2. Desired entrenchment provides a possible additional reason for the "early exercise" of employee stock options.

[19] Although her threat may be credible at time 1, it may still be optimal for her to work during the first period. As one example, suppose that absent this hold-up threat the manager could be hired under a least-cost, long-term contract that (i) deferred all compensation until time 2 and (ii) took the form of an executive stock option that would be valueless if she did not work. If she works she receives compensation worth $R + D$ and bears the disutility of effort. Net, she receives R. If she shirks and can extract the maximum bribe of $\mathcal{O}$, she has $\mathcal{O}$ plus her executive stock option on the assets-in-place. But this option is valueless. Thus whenever $\mathcal{O} < R$ she is better off working during the first period, even recognizing the potential for extortion at time 1.

to this hold-up threat and that can achieve the same low cost that would be achieved if the project were short-lived. Such contracts involve deferring some, but not all, compensation.

The threat to hold up the principal is not credible if the manager's payoff from staying in office,

$$\min[W_1^0 + W_2^0, V] + (\alpha_1^0 + \alpha_2^0)\max[0, V - (W_1^0 + W_2^0)], \tag{9}$$

is dominated by her payoff from resigning and free-riding,

$$\min[W_1^0, V + \mathcal{O}] + \alpha_1^0 \max[0, V + \mathcal{O} - W_1^0]. \tag{10}$$

Thus in order for the hold-up problem to be avoided, sufficient compensation must vest at time 1, and that compensation must be sufficiently sensitive to firm value that a manager who did shirk during the first period would then find it optimal to voluntarily resign and free-ride with a time 1 claim on $V + \mathcal{O}$, rather than stay in office merely to vest in the remaining deferred compensation component. If she did stay in office her increased claim would be backed by the lesser amount V.

But free-riding cannot be made too attractive if the manager is to be motivated to work. Although a sufficient portion of the compensation package must vest at time 1 if the hold-up problem is to be avoided, it will generally be the case that the package cannot fully vest at time 1 if the free-rider problem is to be avoided. If the manager shirks and resigns in order to free-ride, she forfeits the non-vested components W_2^0 and α_2^0. Her remaining claim is backed only by $V + \mathcal{O}$. If she works, she not only has a claim on the increased amount $V + \Delta_1$, she will become fully vested. To preclude free-riding the payoff in (10) must be dominated by the payoff from working:

$$\min[W_1^0 + W_2^0, V + \Delta_1] + (\alpha_1^0 + \alpha_2^0)\max[0, V + \Delta_1 - (W_1^0 + W_2^0)] - D. \tag{11}$$

Proposition 7. (The Optimal Long-Term Contract for a Long-Lived Project) *Consider a long-lived project. Suppose that a transfer of managerial control rights is feasible, $\underline{\alpha} \leq \overline{\alpha}$, and that least-cost compensation cannot be achieved with a short-term contract. There always exists a long-term contract with the properties that (i) the manager prefers to work rather than free-ride, (ii) the manager's threat to hold up the principal is not credible, and (iii) the contract's cost is the same as that of an optimal contract for an otherwise equivalent short-lived project.*

Proof: See Appendix A.

The total of the equity components and the total of the wage components of the optimal compensation package are chosen to match the equity and wage components of what would have been an optimal contract had the project been short-lived. Thus when the manager works, the cost is the same as that of a short-lived project. The equity component has $\underline{\alpha}$ vest at time 1 either directly or via an option. All else equal, the greater the value of the replacement option, the greater the incentive to free-ride. An optimal contract balances any increased incentive to free-ride as $\mathcal{O}$ increases by deferring more of the wage component until time 2. The increased deferred compensation does not make the hold-up threat any more credible, since that threat involves the total wage component. As the time 2 wage increases, the contract reduces the time 1 wage so as to hold the total wage constant. Even if the time 1 wage becomes negative, this time 1 option must be exercised if the manager is to become entrenched.

Part (iii) of Proposition 7 states that the cost of an optimal long-term contract for a long-lived project, $C^0(\mathcal{L}, LT)$, is the same as the cost of the optimal contract for an otherwise equivalent short-lived project, $C^0(\mathcal{S}, ST)$. Recall that the shaded regions in Figure 6 show the difference between $C^0(\mathcal{L}, ST)$ and $C^0(\mathcal{S}, ST)$. From Proposition 7 these shaded regions must also show the difference between $C^0(\mathcal{L}, ST)$ and $C^0(\mathcal{L}, LT)$, and hence the additional cost associated with short-term rather than long-term contracting when the project is long-lived.

Proposition 8. (The Optimality of Entrenchment) *An optimal long-term contract (weakly) dominates an optimal short-term contract; i.e.,* $C^0(\mathcal{L}, LT) \leq C^0(\mathcal{L}, ST)$.

Proof: For given values of V, Δ_1, R, D, and $\overline{\alpha}$, $C^0(\mathcal{L}, LT) = C^0(\mathcal{S}, ST)$ (Proposition 8). For the same given values of V, Δ_1, R, D, and $\overline{\alpha}$, $C^0(\mathcal{S}, ST) \leq C^0(\mathcal{L}, ST)$ (Proposition 4). ∎

Figure 3 can now be thought of as depicting the cost of an optimal contract whenever the initial manager's claim on the replacement option has no value; i.e., Figure 3 depicts both $C^0(\mathcal{S}, ST)$ and $C^0(\mathcal{L}, LT)$. The initial manager's claim on the replacement option will be valueless either if the replacement option itself has no value (as with a short-lived project), or if, although the project is long-lived and the option has value, the manager's claim thereon under a long-term contract does not vest until it is too late for her to exercise the replacement option. In other words, the optimal long-term contract dominates the optimal short-term contract when the project is long-lived, because under the optimal long-term contract the technologically long-lived project becomes economically short-lived; i.e., the initial manager is never replaced.

D. *Summary of Results for a Long-Lived Project and a Long-Term Contract*

When the project is long-lived, and entrenchment is feasible ($\underline{\alpha} \leq \overline{\alpha}$), and the $\overline{\alpha}$ constraint is binding on all short-term contracts, then a long-term contract dominates a short-term contract: it is (weakly) less costly. A long-lived project introduces the possibility of free-riding. To preclude free-riding managerial control rights must be transferred. Absent the transfer, the principal would fire both a shirking manager and a manager who has worked but not yet vested in any deferred compensation component. The transfer makes the promise of deferred compensation credible and hence the loss of deferred compensation costly to a manager who voluntarily resigns in order to free-ride.[20] Deferring too much compensation can introduce its own problem however: the hold-up problem. An optimal long-term contract will defer sufficient compensation to preclude free-riding, while awarding sufficient appropriately designed compensation at time 1 that any hold-up threat is not credible.

VI. Extensions

Our model assumes that only young managers can work. There are several ways in which one might consider weakening this assumption. First, a young manager may develop firm-specific human capital that allows her to make an effort when old simply by virtue of having been in office when young. This means that she could possibly replace herself if she shirked when young. If she has shirked, rehiring her when she is old is potentially cheaper than hiring a new young manager since, unlike her young rivals, she has zero reservation utility.[21] Whether this would actually result

[20] In Berkovitch and Israel's (1996) model, managers are not entrenched. Whether a manager of initially unknown quality is fired, and a replacement hired, after new information arrives depends on both the manager's effort and the incentives of the party with managerial control rights. In their model managerial control rights are sometimes allocated to bondholders rather than stockholders in order to induce the optimal managerial effort.

[21] It may also appear to be cheaper to rehire her because she already owns a claim on the firm and that claim provides an incentive to bear the disutility of effort. To eliminate this

in a lower replacement cost depends upon the relative bargaining power of the principal and the initial manager.

A second alternative assumption is that both young and old managers can work, in which case the reservation utility of a manager is R per period. In this case rehiring the initial manager is no cheaper than hiring any other manager. However, a firm with a long-lived project now faces an initial choice between hiring a young or an old manager. If the firm wants to hire a manager under a long-term contract, that manager must be a young manager—that is, the manager must be able to outlive the replacement option in order to vest in deferred compensation. A young manager's reservation utility is $2R$ if hired under a long-term contract, and R if hired under a short-term contract. Since an old manager cannot commit to outlive the replacement option, she will not value deferred compensation as highly as a young manager. An old manager with a reservation utility of R can be hired only under a short-term contract, and hence will be tempted to free-ride on a replacement's effort. Whether the firm would then prefer to hire a young manager under a long-term contract or an old manager under a short-term contract depends on both their relative reservation utilities and the value of the replacement option.[22] Apparent "age discrimination" may arise in that, for some parameter values, old managers will be hired only to manage short-lived projects.

We are currently pursuing two extensions of this model. The first extension concerns a multiperiod model where an entrenched manager has a choice of reinvesting or distributing the net cash flows realized through time. In this case ex-post costs of entrenchment can arise. The

incentive and increase her bargaining power, however, she can sell all but an ϵ fraction of her existing claim, including the right to the existing future promised compensation she will receive if she vests therein. The ϵ fraction retained means that she will be rehired, and hence she can sell both her existing vested and her existing, as yet, non-vested compensation. She retains the new compensation package that provides her the incentive to work.

[22] Suppose that the long-lived project involves a valuable replacement option. Suppose in addition that a new short-lived project will arrive at time 1; i.e., the firm has a series of two projects, a long-lived project at time 0 followed by a new short-lived project at time 1. In this case the firm will strictly prefer to hire a young manager at time 0 under a long-term contract. The young manager will manage both projects at an opportunity cost of $2R$. Hiring a series of two old managers will involve the same total opportunity cost, but the old manager who oversees the initial long-term project will be tempted to free-ride.

manager faces a trade-off between adopting a suboptimal investment policy in order to maintain control and adopting the policy that would maximize the value of the equity, and hence her own share claims. The agency cost of a suboptimal investment policy will be bounded whenever the manager prefers to give up control of the firm rather than pursue an investment policy that causes too large a loss on her equity position. In other words, a policy of initially entrenching the manager with votes and compensating her with claims on residual cash flows, as outlined in this paper, is "self-correcting" over time in the sense that the manager has a built-in incentive not to deviate too far from first-best.

Our embedding of the agency problem within a firm setting suggests the second extension: constructing a theory of the firm. In this paper we have taken the $\overline{\alpha}$ constraint as exogenous. The constraint can be interpreted as an inverse measure of the number of projects within the firm that each require a manager. Clearly it would be desirable to endogenize $\overline{\alpha}$.[23] This raises the question of how many projects should optimally be combined within one firm. There may be a technological benefit of increasing the number of projects due to economies of scale. However, there can be diseconomies due to rising agency costs as the number of managers in the firm increases. The marginal agency cost of hiring an additional manager is increasing because of the additional cost of inducing effort—the increase in firm size due to adding the additional project gives all managers an increased opportunity to free-ride.

VII. Conclusions

We have enriched the principal-agent problem by embedding it in a long-lived firm with assets-in-place and multiple managers. In all other respects we made simplifying assumptions—risk neutrality, no uncertainty, and a zero/one effort choice. The firm setting produces a rich set of outcomes. Wages, equity, and option components of an optimal contract play different roles

[23] It is tempting to interpret the participation and effort constraints in (2) and (4) as applicable in a multi-agent setting by considering a principal with n projects, each requiring a manager. The natural upper bound on α is then $\overline{\alpha} = 1/n$. From the point of view of any one manager, the value of the assets-in-place is the value of the firm conditional on the effort choice of the other $n-1$ managers. Interpreting the constraints in this manner, however, assumes that only the nth manager's contract includes a wage component.

at different dates in motivating the manager, entrenching her, and overcoming hold-up problems at the minimum possible cost. Entrenchment can be part of an optimal contract. A researcher studying a manager in the second period of a long-term contract (i.e., "late" in her career) will observe an entrenched manager making no effort and receiving compensation not apparently linked to performance. Entrenchment is desirable when it is necessary to make credible the promise of deferred compensation, and it is the loss of deferred compensation that penalizes a manager who resigns in order to free-ride on a replacement's effort.

A manager must vest in sufficient voting equity to be entrenched before completing the task. Otherwise she will be opportunistically fired. But she cannot fully vest in all her compensation prior to the date at which it is no longer profitable for the principal to hire a manager to undertake the project. Otherwise she will have an incentive to opportunistically resign in order to free-ride. Although she cannot become fully vested, enough compensation must vest while it is still profitable to hire a replacement in order to make a hold-up threat by the manager incredible. Note that the optimal time until full vesting could be either shorter or longer than the time necessary to actually complete the project. It should be clear that the performance sensitivity of an optimal contract is more complicated than any simple period-by-period correlation. The timing of the link between a manager's pay and her performance depends on the vesting dates of an optimal contract. These dates depend on both how fast the project's payoffs decay through time and on the future agency problems inherent in hiring a replacement manager.

Appendix A

Proof of Lemma 1

Recall that the participation constraint is:

$$\alpha^0 \geq \frac{R + D - W^0}{V + \Delta_1 - W^0}. \tag{A1}$$

The upward sloping sections of the effort constraint correspond to wage levels such that W^0 is either less than $-\frac{D}{\Delta_1 - D}V$ or greater than V. For $W^0 < -\frac{D}{\Delta_1 - D}V$, the effort constraint is:

$$\alpha^0 \geq \frac{D - W^0}{V + \Delta_1 - W^0}. \tag{A2}$$

The righthand side of $(A1)$ always exceeds that of $(A2)$, and hence the two constraints cannot intersect at a wage level less than $-\frac{D}{\Delta_1 - D}V$. For $W^0 \in (V, V + D)$ the effort constraint is:

$$\alpha^0 \geq \frac{V + D - W^0}{V + \Delta_1 - W^0}. \tag{A3}$$

The righthand side of $(A3)$ exceeds (is less than) that of $(A1)$ if $R > V$ (if $R < V$). When $R = V$ the participation and effort constraints overlap exactly for all $W^0 \in (V, V + D)$. Hence the two constraints cannot have a strict intersection at a wage level in the range $(V, V + D)$. ∎

Proof of Proposition 1

With one exception Proposition 1 follows by direct inspection of Figure 2. The exception concerns the second of the two sets of sufficient conditions for an effort-sensitive wage to be part of an optimal contract: An optimal contract must contain an effort-sensitive wage whenever $V \leq R$ and $\overline{\alpha} < \frac{R+D-V}{\Delta_1}$. Suppose otherwise; i.e., suppose that $V \leq R$ and $\overline{\alpha} < \frac{R+D-V}{\Delta_1}$, yet the wage component of a candidate optimal contract is $W^0 \leq V$. Such a wage is not effort-sensitive. To be optimal the contract must satisfy the participation constraint. The α^0 component of a contract that just satisfies the participation constraint is decreasing in W^0. Substitution of even the largest possible value for W^0 that is not effort-sensitive (i.e., substituting $W^0 = V$) into the participation constraint in (2) reveals that the α^0 component of this candidate optimal contract must be at least $\alpha^0 \geq \frac{R+D-V}{\Delta_1}$. But any such value for α^0 violates the $\overline{\alpha}$ constraint, and the candidate optimal contract is infeasible. ∎

Proof of Proposition 5

When the manager's compensation does not vest until time 2, it is never in her interest to sign, shirk, and resign at time 1—she forgoes the R of her youth and gains nothing. The remaining possibilities are: (i) sign the contract at time 0, and work during the first period; (ii) sign, and shirk during both periods; (iii) sign, shirk during the first period, and then work; and (iv) don't sign. Strategy (i) dominates strategy (iii), since if she works prior to time 1 she has a claim on $V + \Delta_1$. If she works after time 1, her claim is backed only by the lesser amount $V + \Delta_2$. Either way she bears the disutility of effort. Under the contract strategy (i) also dominates strategy (ii), since this dominance is guaranteed by the satisfaction of the effort constraint in (3) associated with an otherwise equivalent short-lived project. Finally, under the contract strategy (i) also dominates strategy (iv), since this dominance is guaranteed by the satisfaction of the participation constraint in (1) associated with an otherwise equivalent short-lived project. ∎

Proof of Proposition 9

The Proposition 9 condition that a short-term contract is not least-cost implies that both $V + \mathcal{O} > R$ and $\frac{D}{\Delta_1 - \mathcal{O}} > \overline{\alpha}$ are satisfied; i.e., the $\overline{\alpha}$ constraint is binding under a short-term contract. By assumption a transfer of managerial control rights is feasible; i.e., $\underline{\alpha} \leq \overline{\alpha}$.

Consider an otherwise equivalent short-lived project. The parameter values will be such that the associated compensation cost is either least-cost (Case A) or greater than least-cost (Case B). For each of these two possible sets of parameter values we will demonstrate for the actual long-lived project of interest that there exists a long-term contract with properties (i), (ii), and (iii) of Proposition 9. In Case A the parameter values are such that at least one of the following two inequalities must be satisfied: $V \leq R$ and/or $\frac{D}{\Delta_1} \leq \overline{\alpha}$. Consider the following long-term contract for the long-lived project:

$$\alpha_1^0 = \underline{\alpha} \tag{A4a}$$

$$\alpha_2^0 = \overline{\alpha} - \underline{\alpha} \tag{A4b}$$

$$W_1^0 = \frac{R - \underline{\alpha}(V + \mathcal{O})}{1 - \underline{\alpha}} \tag{A4c}$$

$$W_2^0 = \frac{R + D - \overline{\alpha}(V + \Delta_1)}{1 - \overline{\alpha}} - W_1^0. \qquad (A4d)$$

The payoff to the manager from working, becoming entrenched at time 1 (potentially through the exercise of an option), and exercising any time 2 option is:

$$\min[W_1^0 + W_2^0, V + \Delta_1] + (\alpha_1^0 + \alpha_2^0)\max[0, V + \Delta_1 - (W_1^0 + W_2^0)] - D.$$

Because $R + D < \Delta_1 \leq V + \Delta_1$, $(A4d)$ implies $W_1^0 + W_2^0 < V + \Delta_1$ and hence the payoff is:

$$\begin{aligned} & W_1^0 + W_2^0 + (\alpha_1^0 + \alpha_2^0)[V + \Delta_1 - (W_1^0 + W_2^0)] - D \\ &= (W_1^0 + W_2^0)(1 - \overline{\alpha}) + \overline{\alpha}(V + \Delta_1) - D \\ &= \frac{R + D - \overline{\alpha}(V + \Delta_1)}{1 - \overline{\alpha}}(1 - \overline{\alpha}) + \overline{\alpha}(V + \Delta_1) - D = R. \end{aligned}$$

The payoff to the manager from free-riding (i.e., shirking and then voluntarily resigning at time 1) is:

$$\max\left[0, \min[W_1^0, V + \mathcal{O}] + \alpha_1^0 \max[0, V + \mathcal{O} - W_1^0]\right].$$

Because of the Proposition 9 condition that $V + \mathcal{O} > R$, $(A4c)$ implies $W_1^0 < V + \mathcal{O}$ and hence the payoff is:

$$\max\left[0, W_1^0 + \alpha_1^0(V + \mathcal{O} - W_1^0)\right] = \max[0, \frac{R - \underline{\alpha}(V + \mathcal{O})}{1 - \underline{\alpha}}(1 - \underline{\alpha}) + \underline{\alpha}(V + \mathcal{O})] = R.$$

Therefore the manager will not strictly prefer free-riding to working, and property (i) of the Proposition is satisfied.

The payoff to a manager who shirks, does not resign at time 1, and exercises her options at both times 1 and 2 is:

$$\min[W_1^0 + W_2^0, V] + (\alpha_1^0 + \alpha_2^0)\max[0, V - (W_1^0 + W_2^0)].$$

Recall that in Case A, $V \leq R$ and/or $\frac{D}{\Delta_1} \leq \overline{\alpha}$. Suppose $V \leq R$. Then since the manager's payoff can be no more valuable than the assets backing it (due to limited liability), her payoff must also be less than or equal to R. Alternately, suppose $V > R$. Then it must be that $\frac{D}{\Delta_1} \leq \overline{\alpha}$. Applying these two inequalities to $(A4d)$ yields

$$W_1^0 + W_2^0 \leq \frac{R - \overline{\alpha}V}{1 - \overline{\alpha}} \leq V.$$

Hence the manager's payoff becomes:

$$W_1^0 + W_2^0 + (\alpha_1^0 + \alpha_2^0)(V - (W_1^0 + W_2^0)) = R + D - \overline{\alpha}\Delta_1 \leq R.$$

Therefore the threat to use this strategy to hold up the principal is not credible, since resigning and free-riding would give R. An alternate strategy open to a manager attempting to hold up the principal would be not to exercise any time 2 option. But then her payoff would be

$$\min[W_1^0, V] + \alpha_1^0 \max[0, V - W_1^0],$$

which is clearly less than the payoff from free-riding. Thus property (ii) is satisfied.

Working in the first period dominates initially shirking and then working in the second period. Whenever she works she bears the disutility of effort, while the assets backing her claim are larger if she works in the first rather than second period ($V + \Delta_1$ exceeds $V + \Delta_2$). Since under this long-term contract working has the same payoff as free-riding, and free-riding dominates the hold-up threat point of never working, we have established that working dominates the payoffs from each of the alternatives available to a manager who signs the contract and then shirks during the first period; i.e., the effort constraint is satisfied.

It remains to be demonstrated that property (iii) is satisfied. We have shown above that the total compensation of a manager who both works and exercises any time 2 option is $R + D$. In fact she will always want to exercise any time 2 option if she works. To see this consider her compensation if she does not exercise any time 2 option:

$$\min[W_1^0, V + \Delta_1] + \alpha_1^0 \max[0, V + \Delta_1 - W_1^0] = R + \underline{\alpha}(\Delta_1 - \mathcal{O}) \leq R + D.$$

This last inequality follows from the Proposition 9 conditions that entrenchment is feasible ($\underline{\alpha} \leq \overline{\alpha}$) and that least-cost cannot achieved with a short-term contract (and hence $\overline{\alpha}$ must be less than $\frac{D}{\Delta_1 - \mathcal{O}}$).

In Case B the parameter values are such that both the following two inequalities must be satisfied: $V > R$ and $\frac{D}{\Delta_1} > \overline{\alpha}$. Consider the following long-term contract:

$$\alpha_1^0 = \underline{\alpha} \tag{A5a}$$

$$\alpha_2^0 = \overline{\alpha} - \underline{\alpha} \tag{A5b}$$

$$W_1^0 = \frac{V - \underline{\alpha}(V + \mathcal{O})}{1 - \underline{\alpha}} \tag{A5c}$$

$$W_2^0 = \frac{V + D - \overline{\alpha}(V + \Delta_1)}{1 - \overline{\alpha}} - W_1^0. \tag{A5d}$$

The payoff to the manager from working, becoming entrenched at time 1 (potentially through the exercise of an option), and exercising any time 2 option is:

$$\min[W_1^0 + W_2^0, V + \Delta_1] + (\alpha_1^0 + \alpha_2^0)\max[0, V + \Delta_1 - (W_1^0 + W_2^0)] - D.$$

Because $D < \Delta_1$ and hence $V + D < V + \Delta_1$, $(A5d)$ implies $W_1^0 + W_2^0 < V + \Delta_1$, and hence the payoff is:

$$\begin{aligned} & W_1^0 + W_2^0 + (\alpha_1^0 + \alpha_2^0)[V + \Delta_1 - (W_1^0 + W_2^0)] - D \\ &= (W_1^0 + W_2^0)(1 - \overline{\alpha}) + \overline{\alpha}(V + \Delta_1) - D \\ &= \frac{V + D - \overline{\alpha}(V + \Delta_1)}{1 - \overline{\alpha}}(1 - \overline{\alpha}) + \overline{\alpha}(V + \Delta_1) - D = V. \end{aligned}$$

The payoff to the manager from free-riding is:

$$\max\left[0, \min[W_1^0, V + \mathcal{O}] + \alpha_1^0 \max[0, V + \mathcal{O} - W_1^0]\right].$$

Because $V < V + \mathcal{O}$, $(A5c)$ implies $W_1^0 < V + \mathcal{O}$ and hence the payoff is:

$$\max[0, W_1^0 + \alpha_1^0(V + \mathcal{O} - W_1^0)] = V.$$

Therefore the manager will not strictly prefer free-riding to working, and property (i) of the Proposition is satisfied. Given limited liability, the payoff to a manager who shirks and does not resign at time 1 is never greater than V. But this is less than the payoff from free-riding. Thus property (ii) of the Proposition is satisfied. Again, the effort constraint is satisfied.

Finally it remains to be demonstrated that property (iii) is satisfied. We have shown that if a manager who works does exercise any time 2 option her total compensation is $V + D$. If she does not exercise any time 2 option, her total compensation is:

$$\min[W_1^0, V + \Delta_1] + \alpha_1^0 \max[0, V + \Delta_1 - W_1^0] = V + \underline{\alpha}(\Delta_1 - \mathcal{O}) \leq V + D.$$

Again, this last inequality follows from the Proposition 9 conditions; i.e., $\underline{\alpha} \leq \overline{\alpha} < \frac{D}{\Delta_1 - \mathcal{O}}$. ∎

Appendix B A theory of debt based on agency costs

Consider a short-lived project with a binding $\overline{\alpha}$ constraint. The $\overline{\alpha}$ constraint will be binding if $\overline{\alpha} < \frac{D}{\Delta_1}$ and the assets-in-place technologically bundled with the investment project are worth more than R. But if that same project could be implemented on a stand-alone basis (i.e., if instead $V = 0$), a manager could be hired at least-cost. We wish to consider the implications of issuing debt maturing prior to the wage payment in order to effectively separate the cash flows to the assets-in-place from the cash flows to the incremental project. For expositional purposes we model this as if the debt matured at the end of the period, but was senior to the wage claim. In order to actually be able to repay debt maturing prior to the wages, it must be that sufficient cash flows arrive during the period.[B1]

Proposition B1. *Suppose the $\overline{\alpha}$ constraint is binding on an unlevered firm. An otherwise equivalent levered firm could hire a manager at least-cost provided the face value of its debt, B, satisfied $B \in [V - R, V - R + \Delta_1 - D]$.*

Proof: Given an outstanding debt issue (senior to the wage), the participation and effort constraints take the respective forms:

$$\min\left[W^0, \max[0, V - B + \Delta_1]\right] + \alpha^0 \max[0, V - B + \Delta_1 - W^0] - D \geq R \tag{B1}$$

and

$$\begin{aligned}&\min\left[W^0, \max[0, V - B + \Delta_1]\right] + \alpha^0 \max[0, V - B + \Delta_1 - W^0] - D \geq \\ &\max\left[0, \min\left[W^0, \max[0, V - B]\right] + \alpha^0 \max[0, V - B - W^0]\right].\end{aligned} \tag{B2}$$

For values of $B \in [V - R, V - R + \Delta_1 - D]$,

$$V - B < R, \tag{B3a}$$

and

$$V - B + \Delta_1 > R + D > 0. \tag{B3b}$$

[B1] For consistency with our production technology assumption that it is not possible to observe whether the manager works throughout a period until the end of the period, the incremental cash flow due to making a continuous effort must arrive at the end of the period.

Given $(B3b)$ the participation and effort constraints simplify to:

$$\min[W^0, V - B + \Delta_1] + \alpha^0 \max[0, V - B + \Delta_1 - W^0] - D \geq R. \qquad (B1')$$

and

$$\min[W^0, V - B + \Delta_1] + \alpha^0 \max[0, V - B + \Delta_1 - W^0] - D \geq$$
$$\max\left[0, \min[W^0, \max[0, V - B]] + \alpha^0 \max[0, V - B - W^0]\right]. \qquad (B2')$$

Now consider a wage satisfying $W^0 \in [V - B, V - B + D]$. Since $D < \Delta_1$, wages in this range will also satisfy the inequality $W^0 < V - B + \Delta_1$. For such wage levels the participation and effort constraints further simplify to:

$$W^0 + \alpha^0(V - B + \Delta_1 - W^0)D \geq R. \qquad (B1'')$$

and

$$W^0 + \alpha^0(V - B + \Delta_1 - W^0) - D \geq \max[0, V - B]. \qquad (B2'')$$

Given $(B3a)$, it follows that for all α^0 values such that the participation constraint $(B1'')$ is just satisfied when $W^0 \in [V - B, V - B + D]$, it is the case that the effort constraint $(B2'')$ is also satisfied. Hence all such $\{W^0, \alpha^0\}$ combinations are least-cost contracts. This set contains the pair $\{W^0 = D + R, \alpha^0 = 0\}$, which clearly satisfies any $\overline{\alpha}$ constraint. ∎

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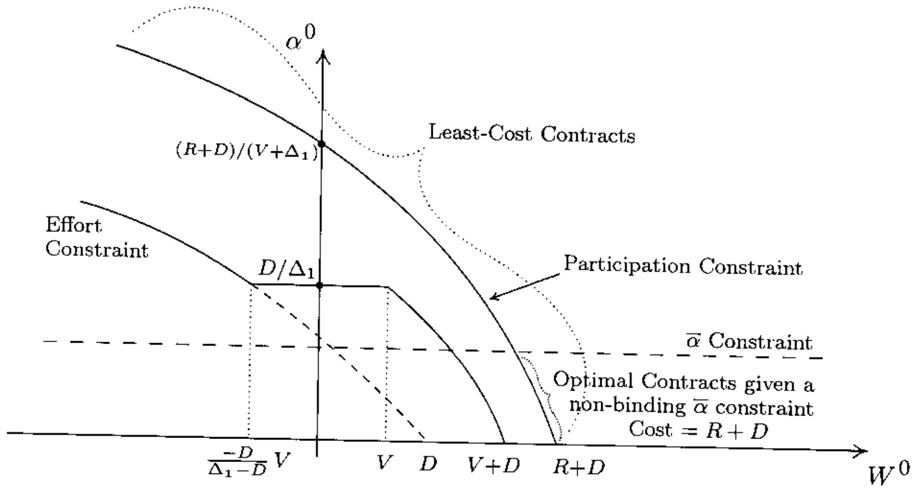

Figure 2(a) $V < R$

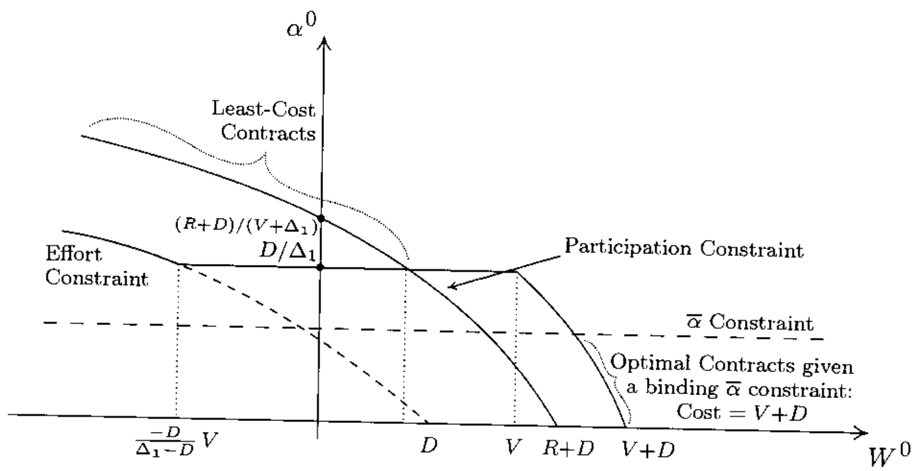

Figure 2(b) $R < V < \frac{\Delta_1}{D} R$

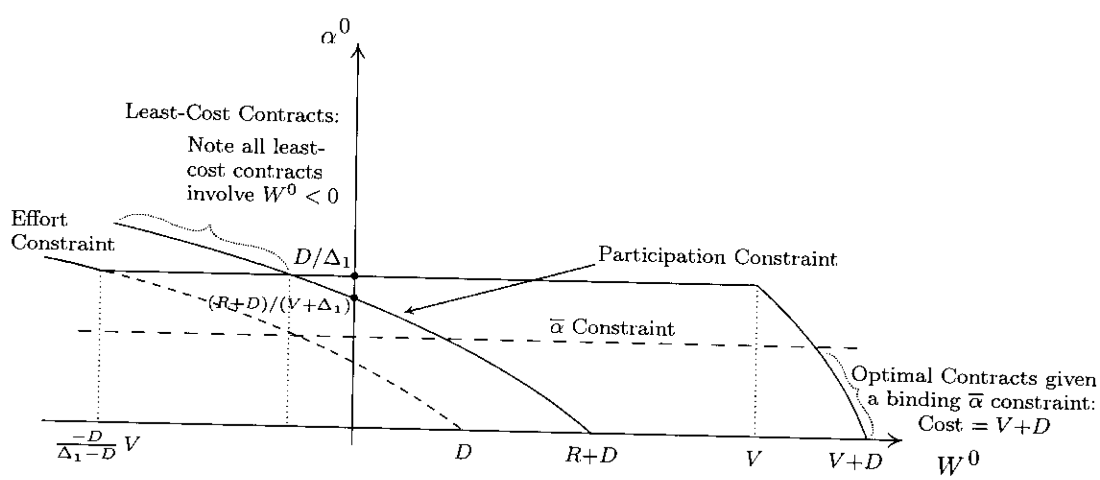

Figure 2(c) $\frac{\Delta_1}{D} R < V$

Figure 2. Time 0 compensation contracts, $\{\alpha^0, W^0\}$, given a short-lived project and vesting at time 1. α^0 is the manager's equity share. W^0 is the wage. A negative wage corresponds to an executive stock option.

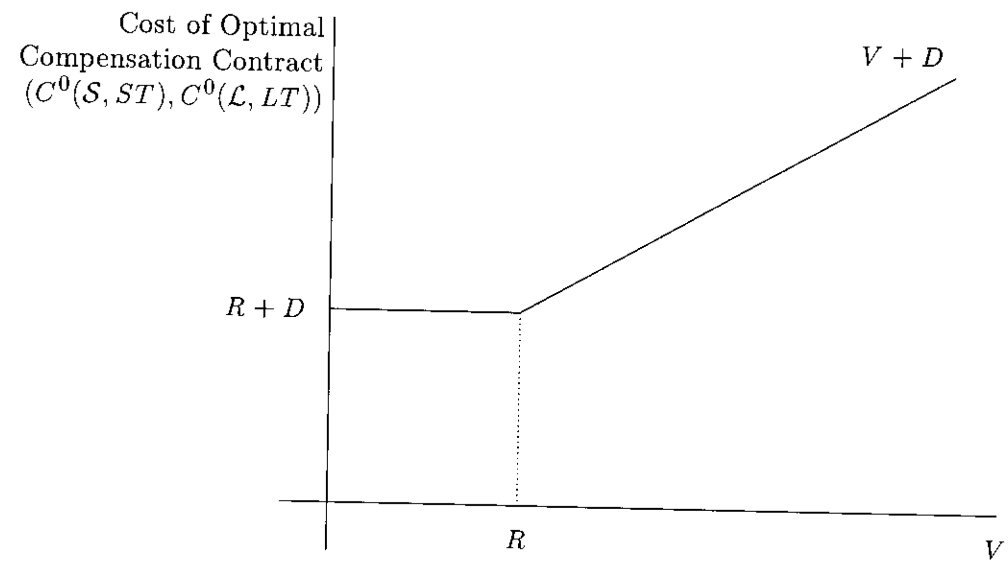

Figure 3a. $\overline{\alpha} < \frac{D}{\Delta_1}$

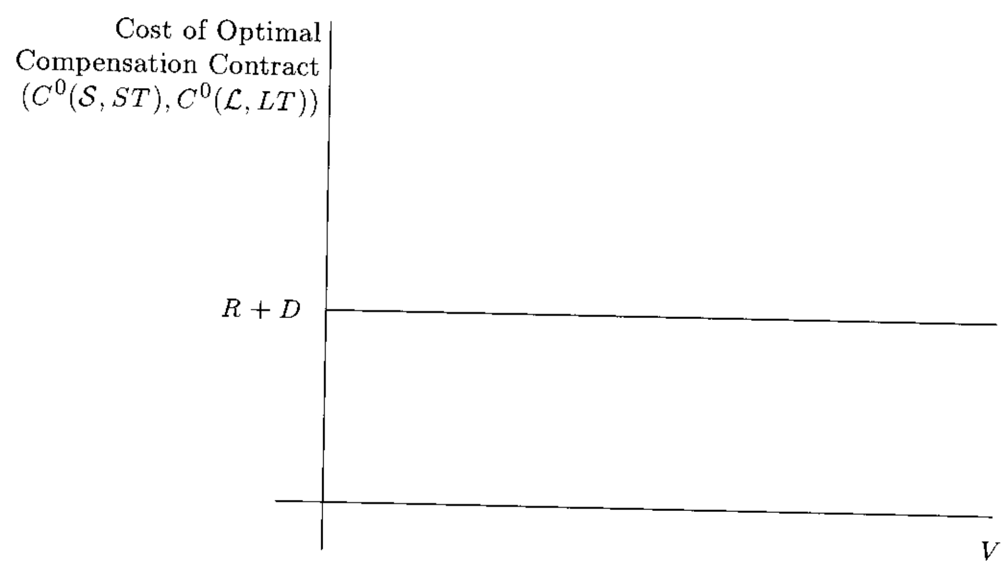

Figure 3b. $\overline{\alpha} > \frac{D}{\Delta_1}$

Figure 3. Cost of optimal time 0 compensation contracts, when the initial manager will never be replaced. The initial manager will never be replaced either if the project is short-lived, in which case we denote the cost by $C^0(\mathcal{S}, ST)$, or, if, although the project is long-lived, the initial manager is hired under an optimal long-term contract, in which case we denote the cost by $C^0(\mathcal{L}, LT)$.

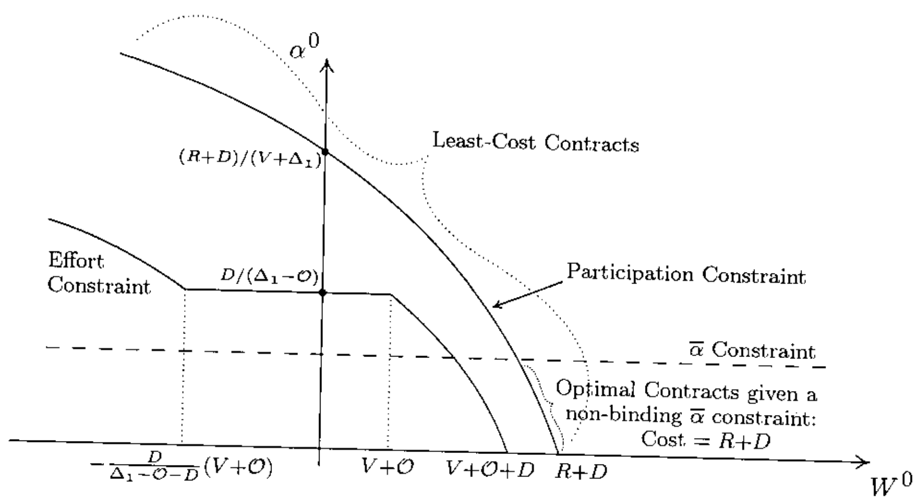

Figure 4(a) $V+\mathcal{O}<R$

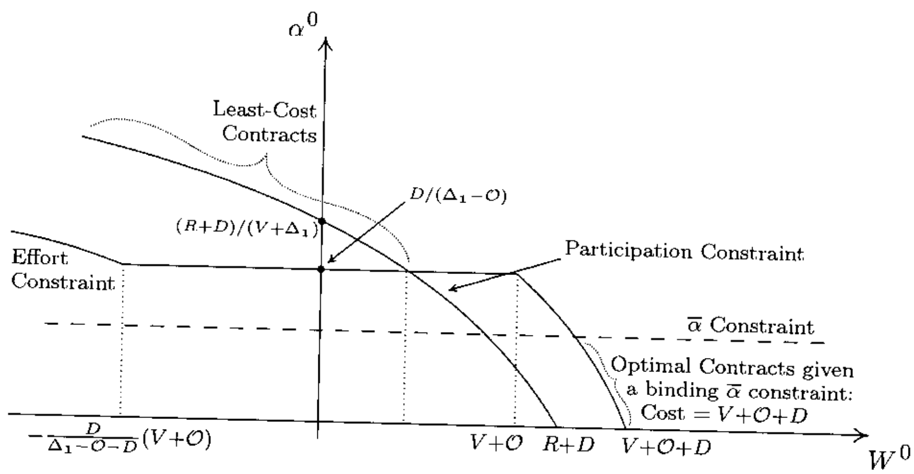

Figure 4(b) $R<V+\mathcal{O}<\frac{\Delta_1-\mathcal{O}}{D}R$

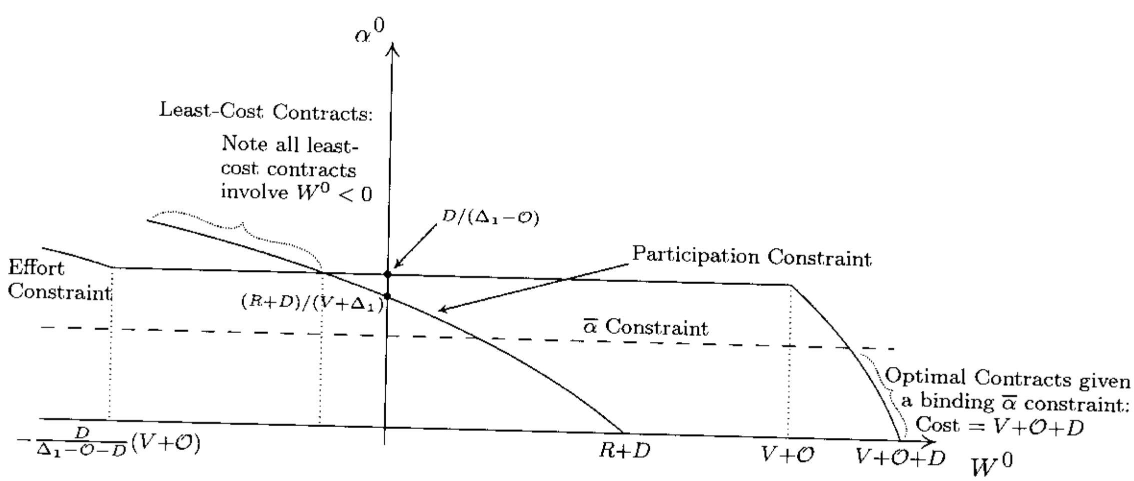

Figure 4(c) $\frac{\Delta_1-\mathcal{O}}{D}R<V+\mathcal{O}$

Figure 4. Time 0 short-term compensation contracts, $\{\alpha^0, W^0\}$, given a long-lived project and vesting at time 1. α^0 is the equity share promised the initial manager hired at time 0. W^0 is the wage promised the manager hired at time 0 with the wage due at time 1. $\mathcal{O} := \max[0, \Delta_2 - C^1(\mathcal{S}, ST)]$ is the value of the replacement option, and $C^1(\mathcal{S}, ST)$ is the cost of hiring a replacement manager at time 1.

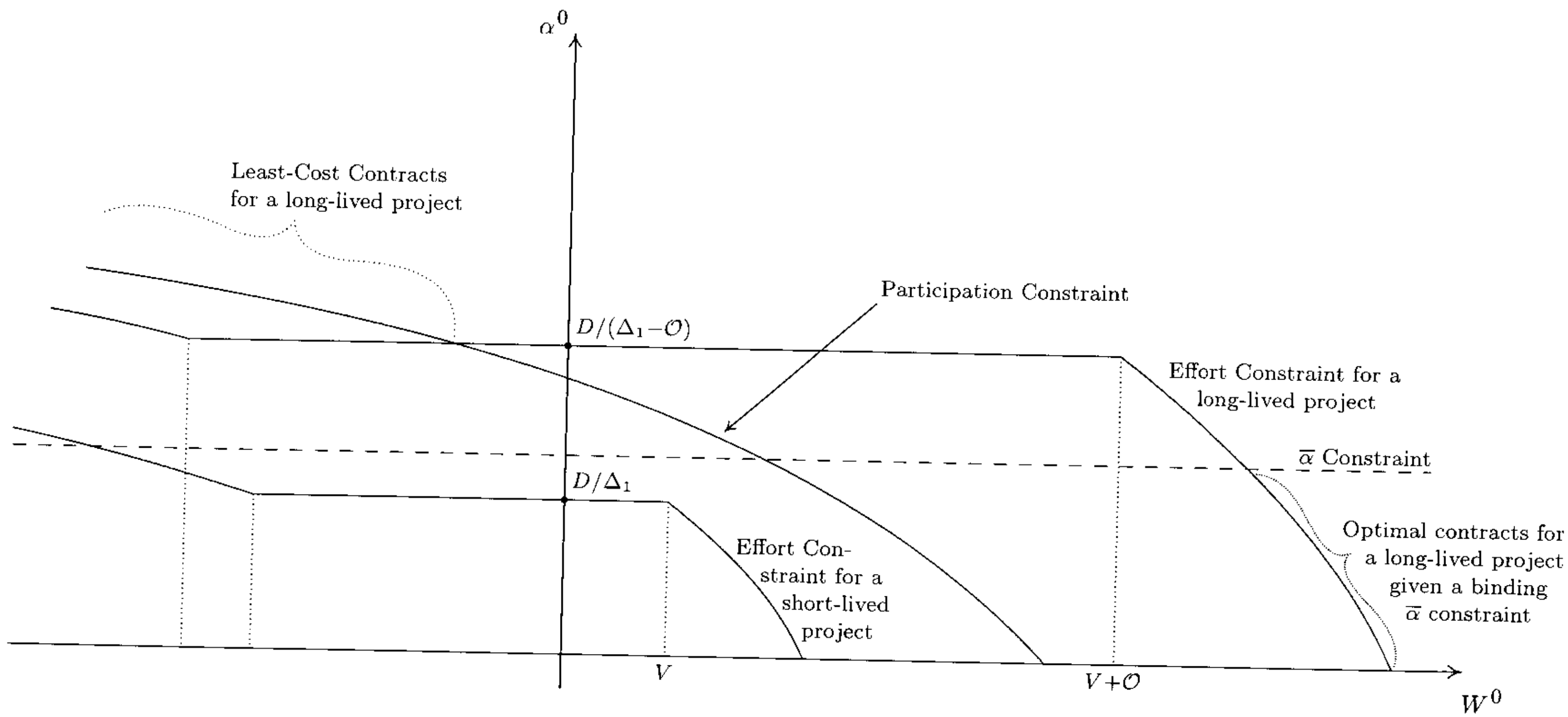

Figure 5. Illustration of the difference in the effort constraints when a manager is hired under a short-term contract to manage a short- versus long-lived project. For the long-lived project $\mathcal{O} > 0$. The $\overline{\alpha}$ constraint is binding (not binding) at time 0 if the project is long-lived (short-lived). $\mathcal{O} := \max[0, \Delta_2 - C^1(\mathcal{S}, ST)]$ is the value of the replacement option, and $C^1(\mathcal{S}, ST)$ is the cost of hiring the replacement manager at time 1.

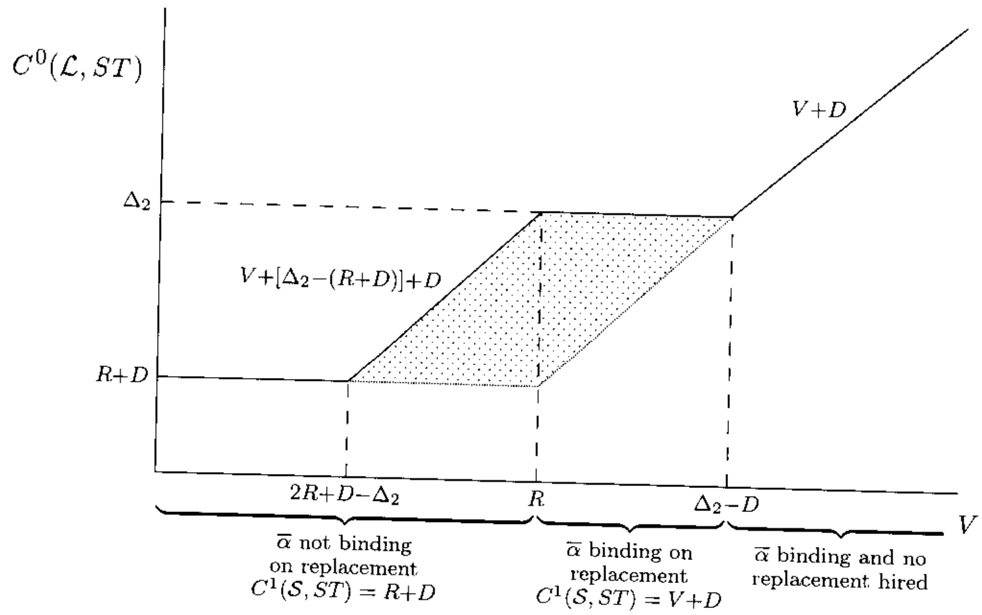

Figure 6(a). $\overline{\alpha}$ such that $\overline{\alpha} < \frac{D}{\Delta_1} < \min\left[\frac{D}{\Delta_2}, \frac{D}{\Delta_1-[\Delta_2-(R+D)]}\right]$.

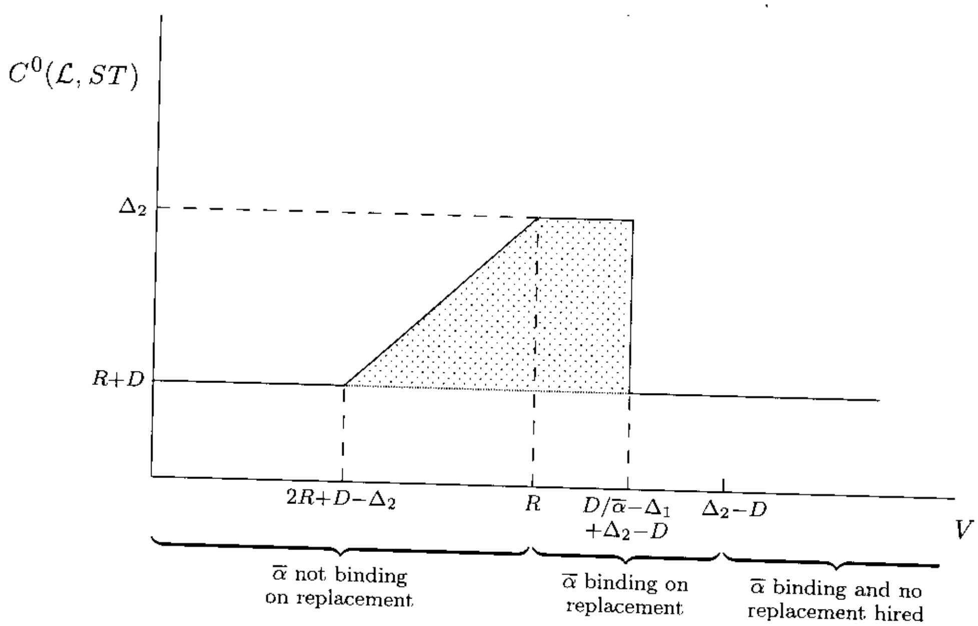

Figure 6(b). $\overline{\alpha}$ such that $\frac{D}{\Delta_1} < \overline{\alpha} < \min\left[\frac{D}{\Delta_2}, \frac{D}{\Delta_1-[\Delta_2-(R+D)]}\right]$.

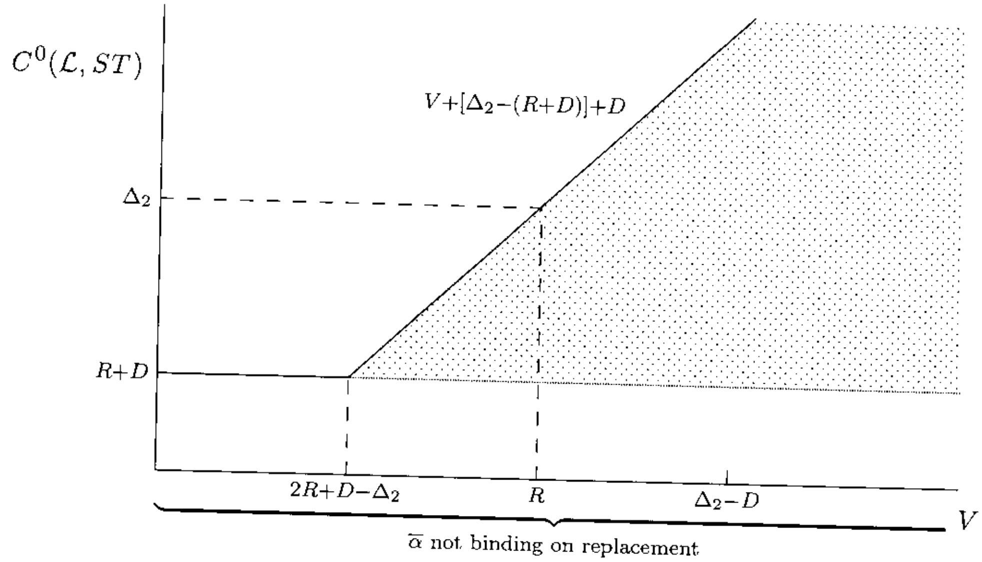

Figure 6(ci). $\overline{\alpha}$ such that $\frac{D}{\Delta_1} < \frac{D}{\Delta_2} < \overline{\alpha} < \frac{D}{\Delta_1-[\Delta_2-(R+D)]}$.

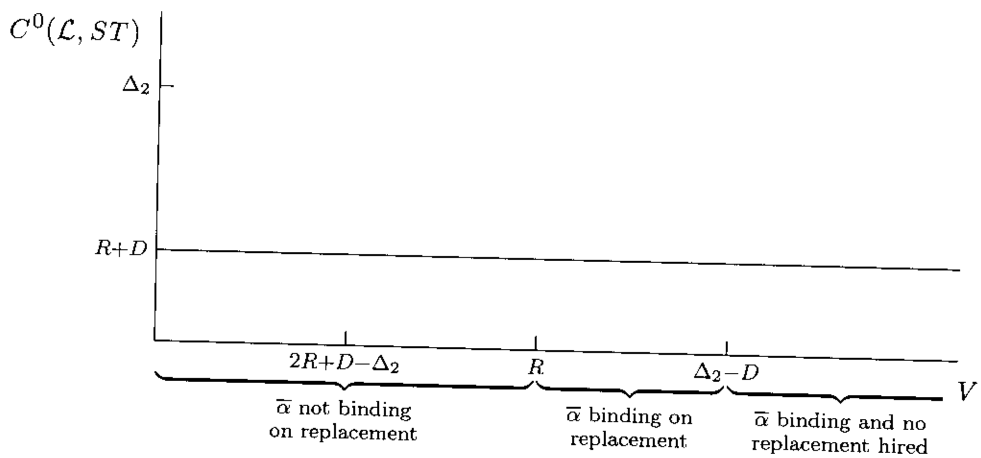

Figure 6(cii). $\overline{\alpha}$ such that $\frac{D}{\Delta_1} < \frac{D}{\Delta_1-[\Delta_2-(R+D)]} < \overline{\alpha} < \frac{D}{\Delta_2}$.

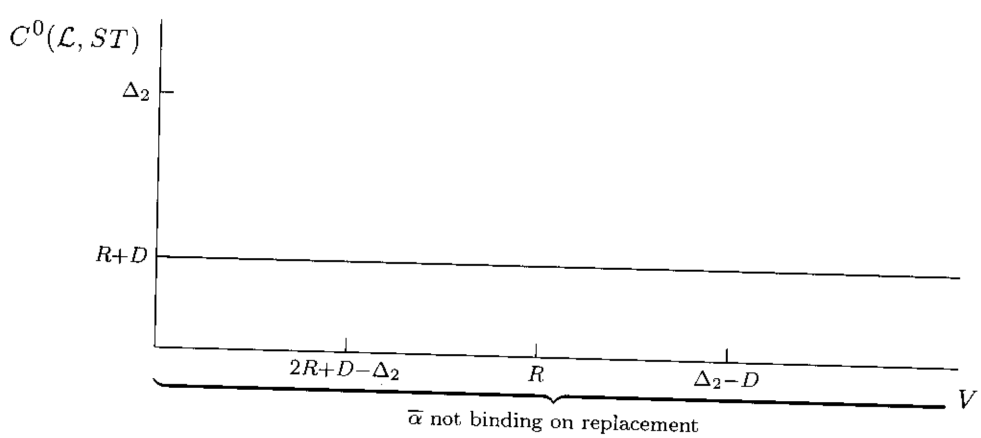

Figure 6(d). $\overline{\alpha}$ such that $\frac{D}{\Delta_1} < \max\left[\frac{D}{\Delta_2}, \frac{D}{\Delta_1-[\Delta_2-(R+D)]}\right] < \overline{\alpha}$.

Figure 6. Cost of an optimal time 0 short-term contract given a long-lived project, $C^0(\mathcal{L}, ST)$.