FINANCIAL MARKETS, INTERMEDIARIES, AND INTERTEMPORAL SMOOTHING

by

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Abstract

The returns of assets that are traded on financial markets are more volatile than the returns offered by intermediaries such as banks and insurance companies. This suggests that individual investors are exposed to more risk in countries which rely heavily on financial markets. In the absence of a complete set of Arrow-Debreu securities, there may be a role for institutions that can smooth asset returns over time. In this paper, we consider one such mechanism. We present an example of an overlapping generations economy in which the incompleteness of financial markets leads to underinvestment in reserves. There exist allocations where by building up large reserves it is possible to smooth asset returns and eliminate nondiversifiable risk. This allows an ex ante Pareto improvement. We then argue that a long-lived intermediary may be able to implement this type of smoothing. However, the position of the intermediary is fragile; competition from financial markets can cause the intertemporal smoothing mechanism to unravel, in which case the intermediary will do no better than the market.

1 Introduction

In the early nineteen-seventies most industrialized countries were adversely affected by a sharp rise in oil prices. This "oil shock" had a dramatic effect on the value of US firms. As illustrated in Figure 1, the real value of shares listed on the New York Stock Exchange fell by almost half compared to their value at the peak in 1972. This collapse in share prices had a severe negative impact on the wealth of any investor whose portfolio contained a significant amount of stocks. Any investor who was forced to liquidate stocks after market prices fell would have suffered from lower consumption over the remainder of his life. Retirees in particular might have been affected in this way.

In Germany, where savings are mostly placed with intermediaries such as banks and insurance companies and assets are not marked to the market, the effect was rather different. Since their claims on intermediaries were fixed in nominal terms, these individuals did not suffer a fall in wealth like their counterparts in the US and would not have been forced to reduce their consumption. Somehow the German financial system was able to smooth the oil-price shock rather than passing it on to investors.

In the nineteen-eighties, the situation was reversed. The economies of most industrialized countries performed relatively well. In the US, the stock market boomed, as shown in Figure 1. Investors who held stocks were able to achieve higher than expected returns and could use these returns to finance a higher level of consumption. The dissaving generation in Germany did less well, by comparison. Since their savings were placed with intermediaries, such as banks, on which they held fixed claims, there was no windfall gain for them.

The effect of the "oil shock" on the US market is an example of what is usually considered a non-diversifiable risk. The shock causes highly correlated changes in most asset values, so investors cannot avoid the risk by holding a diversified portfolio. Nonetheless, these episodes illustrate that the risks borne by individuals in two countries may be very different, even though the countries are subjected to similar shocks. This raises the interesting question of whether and how different financial systems can cope with this sort of risk.

Traditional financial theory has little to say about hedging non-diversifiable risks. It assumes that the set of assets is given and focuses on the efficient sharing of these risks through exchange. For example, the standard diver-

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sification argument requires individuals to exchange assets so that each individual holds a relatively small amount of any one risk. Risks will also be traded so that more risk averse people bear less of the risk than people who are less risk averse. These strategies do not eliminate macroeconomic shocks which affect all assets in a similar way. We call this kind of risk sharing *cross-sectional risk sharing*, because it is achieved through exchanges of risk among individuals at a given point in time.

Departing from the traditional approach, this paper focuses on the *intertemporal smoothing* of risk. Risks which cannot be diversified at a given point in time can nevertheless be averaged over time in a way that reduces their impact on individual welfare. One hedging strategy for non-diversifiable risks is *intergenerational risk sharing*, which spreads the risks associated with a given stock of assets across generations with heterogeneous experiences. Another strategy involves *asset accumulation* in order to reduce fluctuations in consumption over time. Both of these strategies are examples of the intertemporal smoothing of asset returns.

In standard financial models with fixed asset supplies and a single period, it is usually argued that somebody must bear the non-diversifiable risk. Such models implicitly overlook possibilities for intertemporal smoothing. At the other extreme, in an ideal, Arrow-Debreu world, cross-sectional risk sharing and intertemporal smoothing are undertaken automatically because markets are complete and there is complete participation in those markets. Neither the standard financial models, which assume a fixed set of assets, nor the idealized Arrow-Debreu model, which does not explicitly deal with institutions, provide much insight into the relationship between the structure of a country's financial system and the stock of assets accumulated. In particular, they don't tell us how a country's reliance on financial markets or intermediaries affects its ability to smooth asset returns by changing its dynamic accumulation path.

The purpose of this paper is to consider the consequences of intertemporal smoothing for welfare and for positive issues such as asset pricing in a model with incomplete markets. In practice, markets may not be complete in the Arrow-Debreu sense for a wide variety of reasons, including moral hazard, adverse selection, transaction costs and incomplete participation. For simplicity we consider an economy with an overlapping generations structure, which results in incomplete participation. This is a tractable paradigm for the analysis of intertemporal smoothing and captures many of the features common to a wide range of models of market incompleteness.

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Our analysis is related to a number of strands of the literature. First, Scheinkman (1980), McCallum (1987) and others have showed that incorporating a long-lived asset rules out the possibility of over-accumulation. These papers are not concerned with risk. In contrast, our paper analyzes how the risk arising from the dividend stream of long-lived assets is not eliminated by financial markets but can be eliminated by an intermediary. Secondly, Qi (1994) extends the Diamond-Dybvig model to an overlapping generations model. In his model there is no aggregate risk and no role for intertemporal smoothing. Fulghieri and Rovelli (1994) and Bhattacharya and Padilla (1996) also compare the performance of markets and intermediaries in achieving an efficient intertemporal allocation of resources in an overlapping generations model. There is again no aggregate uncertainty in their models and they do not consider intertemporal smoothing. Thirdly, Gordon and Varian (1988) consider how governments can implement policies such as social security to allow intergenerational risk sharing in the context of a model with a single asset. They do not consider market allocations and asset pricing or the role of intermediation, which are the focus of the present paper.

In Section 2 we describe a standard overlapping generations (OLG) model with two assets, a risky asset in fixed supply and a safe asset that can be accumulated over time. In Section 3, we show that under certain conditions the safe asset is never held in the market equilibrium; in fact, it is dominated by the risky asset. Then we show, in Section 4, that intertemporal smoothing can lead to a higher level of average expected utility than is possible in the market equilibrium. Section 5 shows that the market equilibrium is in fact *ex ante* Pareto inefficient: there exist allocations with intertemporal smoothing which make all generations better off *ex ante* compared to the market equilibrium. We suggest that intertemporal smoothing could be implemented by a long-lived intermediary. In Section 6 we show that this mechanism is fragile and that competition from financial markets can lead to disintermediation, which causes the smoothing mechanism to unravel. Section 7 contains concluding remarks. Formal proofs are contained in the appendix.

2   The Model

As a vehicle for the analysis of intertemporal risk smoothing, we use a standard, infinite-horizon, OLG model. Time is divided into a countable number of dates $t = 1, 2, ...$ and a new generation is born at each date t. Each gen-

eration consists of an equal number of identical agents, so there is no loss of generality in treating each generation as if it consists of a single representative agent. There is an initial old generation which lives for one period; each subsequent generation lives for two periods.

There is a single good available for consumption in each period and an agent born at date t has an endowment of e units of the good when young and nothing when old.

There are two types of assets, a safe asset and a risky asset, which are held to provide for future consumption. The supply of the risky asset is normalized to unity, and is initially owned by the old generation. The risky asset lasts forever and pays a dividend of y_t units of the consumption good at each date t. The only exogenous uncertainty in this economy comes from the stochastic process y_t. We assume that y_t is i.i.d. and non-negative, with a positive and finite expectation and variance.

The safe asset is represented by a storage technology, which converts one unit of the consumption good at date t into a unit of consumption at date $t+1$. None of the safe asset is owned by the initial old generation so $s_0 = 0$.

Agents choose their investments to maximize their von Neumann Morgenstern expected utility. Their risk preferences are represented by the additive utility function

$$U(c_1, c_2) = u(c_1) + v(c_2),$$

where c_i is the agent's consumption in the $i - th$ period of life. The functions $u(\cdot)$ and $v(\cdot)$ satisfy the usual properties: both are twice continuously differentiable, increasing and strictly concave.

The special features of this model are chosen for the sake of simplicity. In particular, the OLG structure is a metaphor for all the other sources of market incompleteness that may arise in practice. Extensions are discussed below.

3   Market Equilibrium

Let $x_t \geq 0$ denote the amount of the risky asset and $s_t \geq 0$ the amount of the safe asset held by the young generation at date t. For simplicity, we do not allow short sales, but nothing is changed in equilibrium if short sales are allowed, as we explain at the end of the section. The agent's first-period budget constraint restricts the sum of his first-period consumption and the

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value of his portfolio to be equal to his first-period endowment:

$$c_{1t} + s_t + p_t x_t = e,$$

where p_t is the price of the risky asset at date t. The second-period budget constraint restricts his second-period consumption to be equal to the portfolio's liquidation value *plus* the dividend on the risky asset:

$$c_{2t+1} = s_t + p_{t+1} x_t + y_{t+1} x_t.$$

At every date the agents know the present and past values of asset returns. In other words, the (common) information set is $y^t \equiv \{y_1, ..., y_t\}$. Since an agent's decision can at most depend on the information available to him, his choice of (x_t, s_t) is a function of y^t, i.e., it is adapted to the stochastic process $\{y_t\}$. Asset prices and consumption satisfy the same condition since they are functions of the agents' portfolio decisions. At each date the representative agent chooses his portfolio to maximize his expected utility, conditional on the available information and subject to the period budget constraints.

An *equilibrium* consists of a sequence of portfolios $\{(s_t, x_t)\}$ and prices $\{p_t\}$, adapted to the stochastic process $\{y_t\}$, and satisfying the following conditions. First, at each date t, the portfolio $(s_t, x_t) \geq 0$ chosen by the representative young agent solves the problem:

$$\begin{aligned} \max \quad & E_t[u(c_{1t}) + v(c_{2t+1})] \\ \text{s.t.} \quad & c_{1t} + p_t + s_t = e \\ & c_{2t+1} = s_t + (y_{t+1} + p_{t+1})x_t. \end{aligned}$$

Secondly, the market for the risky asset must clear, that is, $x_t = 1$ for every date t. In what follows, we focus our attention on *Markov* equilibria, that is, equilibria with the property that the endogenous variables (p_t, s_t, x_t) are functions of the contemporaneous shock y_t. A Markov equilibrium is said to be *stationary* if this functional relationship is time-invariant:

$$(p_t, s_t, x_t) = f(y_t), \forall t.$$

Although the safe asset would seem to be a useful hedge against the uncertainty generated by the risky asset, it turns out that this is not the case. Since the returns to the risky asset are assumed to be i.i.d., the representative young agent in a stationary Markov equilibrium solves the same decision problem at each date, regardless of the state in which he is born, and since

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the old supply the risky asset inelastically, the equilibrium price is constant and non-stochastic. The net return to holding the risky asset in such an equilibrium is $r_t = y_{t+1} + p_{t+1} - p_t = y_{t+1}$ and, since y_{t+1} is non-negative and sometimes positive, the safe asset is clearly dominated and will never be held in equilibrium.

Proposition 1 *There exists a stationary, Markov equilibrium $\{(p_t, s_t, x_t)\}$ in which the price of the risky asset is a constant $p_t = p$ and the demand for the safe asset is $s_t = 0$ at every date t if $\sup u'(\cdot) > \inf v'(\cdot)$.*

The assumption that $\sup u'(\cdot) > \inf v'(\cdot)$ is needed to ensure that there exists a positive rate of return at which the representative young agent wants to transfer wealth from the first to the second period of his life. Otherwise, there is no (constant) asset price at which the young agent is willing to hold the risky asset and a stationary equilibrium cannot exist.

To illustrate the operation of the market equilibrium, consider the following example:

$$
\begin{aligned}
U(c_1, c_2) &= \ln(c_1) + \ln(c_2) \\
e &= 1 \\
y_t &= \begin{cases} 0 \text{ w.pr. } 1/2 \\ 1 \text{ w.pr. } 1/2 \end{cases}
\end{aligned}
$$

For this case it can be shown that the stationary equilibrium price is $p_t = 0.5$ and the allocation of consumption in equilibrium is

y_t	c_{1t}	c_{2t}
0	0.5	0.5
1	0.5	1.5

The levels of expected utility attained are $E[v(c_{21})] = -0.14$ for the initial generation and $E[U(c_{1t}, c_{2t+1})] = -0.84$ for each subsequent generation. The long-run average expected utility is therefore also -0.84.

In this example the risky asset is very attractive. It can be bought in youth for 0.5 and sold for the same amount in old age; it also pays non-negative dividends which are 1 half of the time. Since the risky asset is so attractive, investors sacrifice consumption in youth in order to be able to consume in old age. They only consume 0.5 in youth but in old age consume 0.5 or 1.5 with equal probability, or 1 on average.

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It has been assumed so far that no short sales are allowed. This assumption can be dropped at no cost, however, since the equilibrium allocations would be exactly the same if short sales were allowed. The existence of a representative agent and the fact that net asset holdings must be non-negative in equilibrium, together ensure that no short sales can actually take place in equilibrium, even if they were allowed. Furthermore, market clearing requires $x_t = 1$, so the short-sale constraint is never binding for the risky asset. The short-sale constraint $s_t \geq 0$ for the safe asset may be binding in equilibrium. If it is, then we need to introduce a price $q_t < 1$ for claims to the safe asset in order to clear the market at zero net supply.

The model can also be extended to allow for (random) endowments (e_{1t}, e_{2t}) in both periods of an agent's life. In this case, the safe asset may sometimes be used in equilibrium. However, it is always true that $s_t = 0$ with positive probability in a stationary Markov equilibrium. Furthermore, by restricting the distribution of e_{1t}, we can ensure that the conclusion of Proposition 1 continues to hold. In any case, financial markets do not eliminate the risk created by random fluctuations in endowments and asset returns. Similarly, if the return on the safe asset is positive or it is possible that $y_t < 0$ the yield on the risky asset will no longer be uniformly higher than the yield on the safe asset and some of the safe asset may be held in equilibrium. Again, however, financial markets will not eliminate risk. In the next section, we shall see that there exist feasible allocations in which risk is almost entirely eliminated in the long run.

4   Intertemporal Smoothing

In a stationary, Markov equilibrium, the safe asset is not used to hedge against the uncertainty of the risky asset's return. However, in an infinite-horizon economy, almost all of the risk can be eliminated through a program of accumulating buffer stocks of the risk-free asset. This is simply an application of a well known theorem of Schechtman (1976). Schechtman considered the problem of an individual who has a risky income ω_t and wants to maximize the expected value of his long-run average utility:

$$E\{\lim_{T \to \infty} T^{-1} \sum_1^T u(c_t)\}.$$

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The individual cannot borrow, but is able to self-insure by investing in a safe asset (storage technology). Consider the following policy: at each date t, the individual, who has accumulated savings s_{t-1}, consumes $E\omega_t$ if this is feasible and $\omega_t + s_{t-1}$ otherwise. Then the individual's savings at date t will be

$$s_t = \max\{\omega_t + s_{t-1} - E\omega_t, 0\}.$$

Let $M_T \equiv \#\{t \leq T | s_t = 0\}$ be the (random) number of periods that this process spends at the boundary in the first T periods. The renewal theorem tells us that if the random variables $\{\omega_t\}$ are i.i.d. then, with probability 1, M_T/T converges to 0 as T approaches infinity. Since the individual's consumption is less than $E\omega_t$ only when $s_t = 0$, this implies that the individual's consumption is equal to $E\omega_t$ for all but a negligible fraction of the time, and his long run average utility will converge to $u(E\omega_t)$ almost surely.

The same policy works in the present framework. Suppose that a planner wants to maximize the long-run average of the expected utilities of the different generations. To this end, the planner accumulates part of the economy's total endowment using the storage technology. Let s_t denote the accumulated savings at the end of date t and let $\omega_t \equiv e + y_t$ denote the total endowment of the economy at date t. Then by following the policy of setting

$$s_t = \max\{\omega_t + s_{t-1} - E\omega_t, 0\}$$

we can provide the two generations at each date with a total consumption equal to $\bar{\omega} \equiv E\omega_t$ in almost every period, with probability one. The planner will divide the consumption between the two generations in a way that maximizes the typical generation's utility. If we let

$$(c_1(w), c_2(w)) \equiv \arg \max_{c_1 + c_2 = w} u(c_1) + v(c_2),$$

and put $U^*(w) \equiv u(c_1(w)) + v(c_2(w))$, then we have shown that the planner can achieve

$$E[\lim_{T \to \infty} T^{-1} \sum_1^T u(c_1(\min\{\bar{\omega}, \omega_t + s_{t-1}\})) + v(c_2(\min\{\bar{\omega}, \omega_{t+1} + s_t\}))] = U^*(\bar{\omega}).$$

Proposition 2 *There exists a feasible policy $\{s_t\}$ that ensures with probability one that all but a negligible fraction of generations are able to achieve the expected utility level $U^*(\bar{\omega})$.*

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The utility level $U^*(\bar{\omega})$ is at least as great as the level achieved in the market equilibrium on average. In fact, this must be true for any feasible allocation in which the long-run average consumption levels of the old and young are well defined. Let $\{(c_{1t}, c_{2t})\}$ be a feasible consumption process and suppose that

$$\bar{c}_i \equiv \lim_{T \to \infty} T^{-1} \sum_1^T c_{it}$$

is well defined. Then, by concavity,

$$E[\lim_{T \to \infty} T^{-1} \sum_1^T \{u(c_{1t}) + v(c_{2t+1})\}]$$
$$\leq \quad E[u(\lim_{T \to \infty} T^{-1} \sum_1^T c_{1t}) + v(\lim_{T \to \infty} T^{-1} \sum_1^T c_{2t+1})\}]$$
$$= \quad E[u(\bar{c}_1) + v(\bar{c}_2)]$$
$$\leq \quad u(E\bar{c}_1) + v(E\bar{c}_2).$$

Now, we have assumed that $\{(c_{1t}, c_{2t})\}$ is feasible so, with probability one,

$$T^{-1} \sum_1^T (c_{1t} + c_{2t}) \leq T^{-1} \sum_1^T \omega_t \to \bar{\omega}.$$

From this it follows that $E\bar{c}_1 + E\bar{c}_2 \leq \bar{\omega}$, which in turn implies that $u(E\bar{c}_1) + v(E\bar{c}_2) \leq U^*(\bar{\omega})$, and from this we conclude that

$$E[\lim_{T \to \infty} T^{-1} \sum_1^T \{u(c_{1t}) + v(c_{2t+1})\}] \leq U^*(\bar{\omega}).$$

The inequality will be strict when $\{(c_{1t}, c_{2t})\}$ corresponds to the market allocation, since the agents' risk preferences are strictly concave in old age and the variance of y_t is strictly positive. This result can be summarized as follows.

Proposition 3 *For any feasible allocation $\{(c_{1t}, c_{2t}, s_t)\}$ for which long-run average consumptions are well defined,*

$$E[\lim_{T \to \infty} T^{-1} \sum_1^T \{u(c_{1t}) + v(c_{2t+1})\}] \leq U^*(\bar{\omega})$$

and the inequality is strict if $\{(c_{1t}, c_{2t}, s_t)\}$ is the market equilibrium allocation.

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In the example from the previous section, $\bar{\omega} = 1.5$ and the additive logarithmic utility function implies that the long-run average expected utility is maximized by setting $c_1(w) = c_2(w) = \bar{\omega}/2 = 0.75$. In this case, $U^*(\bar{\omega}) = -0.58$, which compares favorably with the long-run average expected utility in the market equilibrium $E[U(c_{1t}, c_{2t+1})] = -0.84$.

Propositions 2 and 3 extend immediately to the case where there are both random endowments (e_{1t}, e_{2t+1}) and random asset returns y_t, as long as we assume that the aggregate endowments $\omega_t \equiv e_{1t} + e_{2t} + y_t$ are i.i.d.

5 Ex Ante Efficiency and the Genesis of Intertemporal Smoothing

In the preceding section, we saw that a long-lived agent, or a planner who maximized the long-run average of expected utility, might behave very differently from the successive generations in the OLG model, who maximized their own expected utility over a two-period horizon. The former would have an incentive to accumulate large stocks of the safe asset in order to provide insurance against rate of return risk, whereas the latter have no incentive to hold the safe asset at all in a stationary, Markov equilibrium.

This raises the question of whether there is some sort of market failure, some form of inefficiency, in the equilibrium described in Proposition 1. Before we can answer this question, we have to be more precise about how we define the welfare of an individual agent. There are two salient definitions. The first identifies the individual's welfare with his expected utility $E[U(c_{1t}, c_{2t+1})|y^t]$, *conditional on the information that is available when he is born.* In effect, it treats the "same" individual born at two different information sets as two different individuals. The second definition identifies the individual's welfare with his *unconditional* expected utility $E[U(c_{1t}, c_{2t+1})]$, implicitly assuming that there is only one individual born at any date, regardless of the information available at that date.

Correspondingly, there are two notions of Pareto efficiency, *ex ante* and *ex post*, depending on whether or not we take into account the state in which an agent is born. A feasible allocation is *ex post efficient* if it is impossible to increase the *ex post* expected utility $E[U(c_{1t}, c_{2t+1})|y^t]$ of some generations without reducing the *ex post* expected utility of other generations. On the other hand, a feasible allocation is *ex ante efficient* if it is impossible

to increase the *ex ante* expected utility $E[U(c_{1t}, c_{2t+1})]$ of some generations without reducing the *ex ante* expected utility of other generations. We consider efficiency initially using the *ex ante* notion and then using the *ex post* notion.

It is easy to see that a market equilibrium allocation will not be *ex ante* efficient in general, because agents are not allowed to trade before they are born. Hence, all trades are undertaken by an agent after the state in which he is born has been revealed. In other words, the birth state y^t is a "pre-existing condition", against which an agent cannot insure. However, a planner could provide such insurance by making appropriate transfers between the old and the young at each date. Thus, even without making use of the storage technology, the planner could achieve a Pareto improvement from the *ex ante* point of view. However, the expected utility of the typical generation will be even higher if intergenerational smoothing is carried out, accumulating reserves of the safe asset and using them to smooth fluctuations in consumption. Intergenerational risk sharing by means of transfers between the old and young at each date does not remove the aggregate uncertainty caused by the randomness of the aggregate endowment. Intertemporal smoothing eliminates this uncertainty, at no cost in terms of long-run average consumption.

Although it is easy to see that intertemporal smoothing can increase long-run average expected utility, some care must be taken about the way in which intertemporal smoothing is introduced in order to ensure that *each* generation is better off *ex ante* compared to the equilibrium allocation. Consider the policy described in Section 4. Under that policy, $c_{1t} + c_{2t} = \bar{\omega}$ when $\omega_t + s_t \geq \bar{\omega}$ and $c_{1t} + c_{2t} = \omega_t + s_t$ otherwise. In the market equilibrium, $c_{1t} + c_{2t} = e + y_t$ in all periods. Since $s_0 = 0$, it follows that if the intertemporal smoothing scheme were implemented at the first date, either the initial old generation or the initial young generation or both would be worse off in an *ex ante* sense compared to the market allocation. A similar argument applies in subsequent periods. In order to achieve an *ex ante* Pareto improvement, intertemporal smoothing has to be introduced in two stages. The first stage achieves an increase in expected utility by means of intergenerational risk sharing (transfers) which allows the planner to accumulate some of the endowment in the form of reserves of the safe asset, without making any generation worse off. Once reserves are sufficiently large, it is possible to switch to a policy of intertemporal smoothing and make every generation better off *ex ante* than they would be with intergenerational risk sharing alone.

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To see how the first stage is implemented, consider some necessary conditions for *ex ante* efficiency. If the equilibrium is *ex ante* efficient, it must be impossible to make both generations at date t better off by reallocating consumption at that date. That means that the equilibrium consumption allocation (c_{1t}, c_{2t}) must solve the maximization problem

$$\max \quad E[\lambda u(c_{1t}) + (1 - \lambda)v(c_{2t})]$$
$$\text{s.t.} \quad c_{1t} + c_{2t} = \omega_t \equiv e + y_t,$$

for some constant $0 \leq \lambda \leq 1$. A necessary condition for this to be true is that

$$u'(c_{1t})/v'(c_{2t}) = \text{constant}$$

with probability one. In the market equilibrium $c_{1t} = e - p_t$ is non-stochastic, whereas $c_{2t} = p_t + y_t$ is stochastic, so the necessary condition for *ex ante* Pareto efficiency cannot be satisfied and the market equilibrium is *ex ante* inefficient. In fact, it is possible to find an *ex ante* Pareto-preferred allocation by making stationary transfers contingent on the contemporaneous asset returns. Let $\tau(y_t)$ be the transfer from young to old at period t when the asset return is y_t and define the new consumption allocation by putting $\hat{c}_{1t} = c_{1t} - \tau(y_t)$ and $\hat{c}_{2t} = c_{2t} + \tau(y_t)$. For an appropriate specification of the transfer function $\tau(\cdot)$,

$$E[v(\hat{c}_{21})] > E[v(c_{21})]$$

and

$$E[u(\hat{c}_{1t}) + v(\hat{c}_{2t+1})] > E[u(c_{1t}) + v(c_{2t+1})]$$

for every date t. By continuity, the same will be true if we reduce the consumption of the young at each date by a constant amount $\eta > 0$ and add this amount to the stock of the safe asset, so that by period t we have accumulated $s_t = \eta t$. Hence by using intergenerational transfers an arbitrarily large level of reserves can be built in preparation for switching to the intertemporal smoothing program.

Let S denote the target level of reserves accumulated in the first stage and let T denote the end of the first stage, that is, choose T so that $s_T = S$. To show that a Pareto-improving scheme with intertemporal smoothing can be implemented, the following lemma is needed.

Lemma 1 For any $\epsilon > 0$ there is a level of initial reserves S sufficiently large that, starting the intertemporal smoothing plan at date T the probability of $s_t = 0$ at any $t \geq T$ is less than ϵ.

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In the short run, this is obvious because it will take some time to run down the reserves to 0. In the longer run, it is not so obvious that the probability of running out of reserves is uniformly small at all future dates. The lemma follows from the fact that reserves follow a "random walk" when $s_t > 0$ and are expected to increase when $s_t > 0$. This means that reserves are expected to increase on average without limit under the intertemporal smoothing policy. Even though the event $s_t = 0$ will occur infinitely often, the probability that it happens at any fixed date t is becoming vanishingly small as t approaches infinity.

The fact that the probability of the event $s_t = 0$ is bounded by ϵ for each future generation means that the ex ante expected utility of any generation living after date T is at least $(1 - \epsilon)U^*(\bar{\omega})$, which is greater than the equilibrium *ex ante* expected utility $E[U(c_{1t}, c_{2t+1})]$ for ϵ sufficiently small. Thus, generations $t \geq T$ will prefer the intertemporal smoothing plan *ex ante* to their market allocation. There is a problem with the generation born at date $T - 1$ since this generation does not get the full benefit of intergenerational risk sharing but has its second-period consumption reduced on average. To compensate this generation, we make a one-time transfer out of the reserves. With this adjustment, every generation is *ex ante* better off.

Furthermore, since the first stage with intergenerational sharing is finite in length, the analysis in Section 4 shows that the long-run average expected utility will converge to $U^*(\bar{\omega})$. All this can be summarized in the following result.

Proposition 4 *The market equilibrium allocation is ex ante Pareto inefficient. There exists an attainable allocation with intertemporal smoothing which provides every generation with higher ex ante expected utility and achieves the long-run average expected utility $U^*(\bar{\omega})$.*

The existence of an allocation that improves *ex ante* welfare for all generations compared to the market equilibrium can be illustrated in the context of the numerical example used above. In the initial stage, the market allocation is altered by intergenerational transfers:

$$\tau(y_t) = \begin{cases} 0.1125 & \text{if } y_t = 0 \\ -0.275 & \text{if } y_t = 1 \end{cases}$$

When $y_t = 0$, the old receive a transfer of 0.1125 from the young and when $y_t = 1$ the young receive a transfer of 0.275 from the old. These transfers

ensure the expected utility of the initial generation is slightly greater than the market equilibrium level of -0.84. They also allow an addition to reserves of 0.028 to be extracted from each generation except the initial one, while still leaving them slightly better off than the market allocation.

To see how the second stage operates consider the effect on the *ex ante* expected utilities of the generations around date T if intertemporal smoothing were implemented at date T and there were no reserves at that date. We assume that the generation born at date $T - 1$ simply receives the market allocation when it is young.

Gen. born at	c_{1t}	c_{2t+1}	$E[U(c_{1t}, c_{2t+1})]$
$T - 1$	0.5	0.5 w.pr. 0.5 0.75 w.pr. 0.5	-1.18
T	0.5 w.pr. 0.5 0.75 w.pr. 0.5	0.5 w.pr. 0.25 0.75 w.pr. 0.75	-0.88
$T + 1$	0.5 w.pr. 0.25 0.75 w.pr. 0.75	0.5 w.pr. 0.25 0.75 w.pr. 0.75	-0.78

The generation born at date $T+1$ is clearly better off than in the equilibrium, but generations $T - 1$ and T are worse off. If there were positive reserves at date T, $s_t > 0$, the generations born after T would be even better off. For the generation born at $T - 1$, a transfer of 0.21 in their youth at date $T - 1$ would be sufficient to make them better off than in the market equilibrium. For the generation born at date T, a transfer of 0.03 would be sufficient. Hence the total reserves at the time of the transition must be at least 0.24. Remembering that the initial generation and generation $T - 1$ do not help build reserves this implies the initial stage must last at least six periods so $T = 6$.

Comparing the paths of utility in this *ex ante* Pareto-superior allocation with that in the market equilibrium, it can be seen that the first six generations in this example have slightly higher utility, but all subsequent generations are significantly better off. The long-run average expected utility is -0.58, compared to -0.84 in the market equilibrium. Note also that the average expected utility is also significantly greater than can be achieved with intergenerational transfers alone. The consumption allocation that maximizes long-run average expected utility through intergenerational transfers with no accumulation of reserves is

$$c_{it} = \begin{cases} 0.5 & \text{w. pr. } 0.5 \\ 1.0 & \text{w. pr. } 0.5 \end{cases}$$

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for $i = 1, 2$, which gives expected utility of $E[U(c_{1t}, c_{2t+1})] = -0.69$.

At the start of the section it was pointed that there exist two notions of efficiency depending on whether we take into account the state in which an agent is born. *Ex ante* efficiency, which Proposition 5 focused on, takes the expectation of utility across all possible states. An alternative view is that of *ex post* efficiency where an individual's welfare is conditional on the information available when he is born. The discussion of Lemma 1 provides insights into *ex post* expected utility in the model when intertemporal smoothing is adopted. Aggregate consumption is $\bar{\omega}$ if and only if $s_t > 0$ and $\Pr[s_t = 0] \to 0$ as t approaches infinity. Hence, *ex post* expected utility will converge to $U^*(\bar{\omega})$ in probability as t approaches infinity. In other words, except when reserves are low the *ex post* expected utility of each generation will be $U^*(\bar{\omega})$ which is higher than in the market equilibrium. As t becomes large the probability of reserves being low and a generation being worse off *ex post* than in the market equilibrium becomes vanishingly small. In fact, in the long run all but a negligible fraction of generations can be made better off *ex post*.

We have so far studied the existence of allocations which allow the introduction of intertemporal smoothing *and* an *ex ante* Pareto improvement over the market allocation, without specifying the institutional framework that implements them. The existence of such allocations suggests a story of how intertemporal smoothing by intermediaries might come into existence. Given the opportunity to make individuals better off, some institution will try to exploit that opportunity and capture part of the surplus. One possibility is that a long-lived intermediary is set up to provide insurance against uncertain returns by averaging high and low returns over time. Such an intermediary could hold all the assets and offer a deposit contract to each generation. Initially, the intermediary offers intergenerational insurance. Later on, after accumulating large reserves, the intermediary can offer almost all generations a constant return on deposits, independently of the actual returns.

Some degree of market power will be required to ensure that individuals participate in this scheme, as we shall see in the next section. This market power may arise naturally or it may be the result of government intervention. For example, the government may give the intermediary an exclusive license in order to achieve an *ex ante* Pareto improvement.

Of course, we do not suggest that intertemporal smoothing would always occur in this way, merely that the introduction of intertemporal smoothing is consistent with market incentives.

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6 Competition between Intermediaries and Financial Markets

A commonly heard argument is that financial markets are desirable because of the risk-sharing opportunities they provide. It is well known that this is correct as far as cross-sectional risk sharing opportunities are concerned, but the results of the preceding sections suggest that this argument ignores the possibilities for intertemporal risk smoothing. We have shown in the context of a simple OLG model that an intermediated financial system can make every generation better off than it would be with financial markets alone. Note that, in this interpretation, financial markets and intermediaries are not simply veils thrown over a fixed set of assets. They actually determine, in conjunction with other factors, the set of assets accumulated by the agents in the economy. By adopting one or another set of institutions, the economy is placed on a different trajectory, with important implications for risk smoothing.

A natural question that arises is whether it is possible to combine the cross-sectional risk sharing advantages of financial markets with the intertemporal risk smoothing advantages of an intermediated system. There is a significant obstacle in the path of trying to combine the two types of systems. Risk sharing of the kind discussed in the last few sections implies some form of arbitrage opportunity. Taking advantage of arbitrage opportunities is rational for the individual, but it undermines the insurance offered by the intermediary. For this reason, an open financial system may not be able to provide intertemporal risk smoothing, although it provides a tremendous variety of financial instruments.

One way to illustrate the effect of competition from financial markets is to consider the effect of opening up a relatively small, closed, and intermediated financial system to global financial markets. Initially, the small country's financial system is monopolized by a cartel of banks that engage in intertemporal smoothing without the threat of competition. After opening the small country's financial system, the banks now face the constraint that individuals can opt out and invest in global markets instead. The assumption that the country is small relative to the rest of the world implies that prices in the global market are not affected by the financial system of the small country or its investors' decision to participate in the risk sharing mechanism provided by the intermediary.

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Let $\{(p_t, s_t, x_t)\}$ be the equilibrium in the global market and let $\{(c'_{1t}, c'_{2t}, s'_t)\}$ be the optimal allocation implemented in the small country. The global equilibrium represents a benchmark for the welfare of investors in the absence of a long-lived intermediary, as well as an outside option for the individuals when the intermediary is in operation. We assume that all investors in the small country make use of the intermediary. Since the intermediary can always replicate the investment opportunities available through the market, there is no loss of generality in this assumption.

Disintermediation can take several forms, depending on whether investors are able to make side trades while taking advantage of the intermediary. We assume that the intermediary can enforce exclusivity, which means that an agent who wants to trade in the market is unable to make use of the intermediary at all. This assumption makes disintermediation less attractive and hence produces a weaker constraint on the intermediary's problem of designing a risk smoothing scheme. We can show that even this weak constraint on the intermediary is sufficient to rule out any welfare improvement from intertemporal risk smoothing. Alternative (stronger) specifications of the disintermediation constraint would only strengthen this result.

The *disintermediation constraint* (DC), which ensures that people do not abandon the small country's risk sharing mechanism once they have access to global markets, can be stated as follows: for any history $y^t = (y_1, ..., y_t)$ the allocation $\{(c'_{1t}, c'_{2t}, s'_t)\}$ satisfies

$$E[u(c'_{1t}) + v(c'_{2t})|y^t] \geq \max_{(x,s) \geq 0} E[u(e - p_t) + v(x(y_t + s))|y^t].$$

The expression on the right is the maximum expected utility an agent born at date t could obtain from trading on the open market. The expression on the left is the expected utility offered by the risk-sharing mechanism. The crucial point is that both expressions are conditioned on all the information available at date t. An agent makes his decision whether to join the risk-sharing mechanism after he has observed y^t.

The possibility of disintermediation implies that an intermediated financial system in a small open country does not allow any improvement in expected utility over that obtained by investors in global financial markets. To prove this result, we need two additional assumptions. The first rules out the possibility of a welfare-increasing Ponzi scheme: we assume there exists a constant K such that if $U(c_{1t}, c_{2t+1}) \geq U(e, 0)$ then $c_{1t} \geq -K$ and $c_{2t+1} \geq -K$ with probability one. Since the utility level $U(e, 0)$ is always at-

tainable, an agent's expected utility must be at least this high in equilibrium and that means his consumption will be bounded below with probability one. The second assumption is purely technical: we assume that the random asset return y_t assumes a finite number of values. Under these assumptions we can show that the equilibrium allocation is *ex post* Pareto-efficient and so there is no feasible allocation that makes any generation better off without making some generation worse off *ex post*. The DC requires each generation to be at least as well off *ex post* as it was under the equilibrium allocation and hence no better off.

Proposition 5 *If the allocation $\{(c'_{1t}, c'_{2t}, s'_t)\}$ is feasible and satisfies the disintermediation constraint, then each agent is ex post no better off under $\{(c'_{1t}, c'_{2t}, s'_t)\}$ than he would be in the market equilibrium $\{(p_t, s_t, x_t)\}$.*

To understand Proposition 5, it is helpful to think about the policy described in Proposition 2. That policy provides the two generations at each date with a total consumption equal to the lesser of $\bar{\omega}$ and the sum of the actual return and the reserves held by the intermediary, so that the total amount consumed each period is:

$$c'_{1t} + c'_{2t} = \min\{\bar{\omega}, y_t + s'_{t-1} + e\}$$

If the reserves held by the intermediary endowment are very low (close to zero), the expected utility of an agent must be lower than in the market equilibrium. Compared to the equilibrium allocation, he loses the high returns from the risky asset when $y_t > \bar{\omega}$ and still suffers the probability of loss when $y_t < \bar{\omega}$. So any generation will only be better off if it inherits a large reserve from the previous generation. This will be true most of the time, but occasionally a generation will be born when reserves are low and that generation will be worse off *ex post*. If that generation can opt out of the risk-sharing mechanism, the whole scheme will break down, leaving us in the situation described by Proposition 5.

To see this in the context of the numerical example, suppose reserves are at zero at date T^*. If intermediation is initiated or continued and intertemporal smoothing were implemented the allocation of consumption would be as in the table in the previous section with $T = T^*$. The young generation (born at T^*) would obtain $E[U(c_{1T^*}, c_{2T^*+1})] = -0.88$. However, with markets they would obtain the usual market allocation which gives $E[U(c_{1T^*}, c_{2T^*+1})] = -0.84$. Hence when there are no reserves the young

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generation will prefer the competitive market allocation and will defect if given the opportunity. This is why some monopoly power is important in establishing and maintaining intermediation as discussed in the previous section. Building up the reserves necessary to start intertemporal smoothing requires intergenerational transfers initially. Access to competitive financial markets ensures this type of insurance will not be feasible. Any allocation of consumption offered by an intermediary must match the market and give the young generation $E[U(c_{1T^*}, c_{2T^*+1})] = -0.84$. This means an intermediary cannot improve on the market.

Proposition 4 implies that *ex ante* expected utility will be higher for all generations in an intermediated economy than in an economy with financial markets only. Incomplete financial markets do not allow intertemporal smoothing, while intermediaries in principle can, provided investors do not have ready access to financial markets. This suggests that economies which are intermediary-based may be worse off by allowing access to financial markets. As discussed below this result may have important policy implications for the European Union and other regions considering liberalizing access to global financial markets.

7 Concluding Remarks

Our formal analysis has focused on a simple, overlapping-generations model. This benchmark is meant to illustrate the absence of intertemporal smoothing that can result from incomplete markets and to show how an intermediated financial system can eliminate the resulting inefficiencies. It is important to stress that the overlapping-generations structure is chosen because of its tractability. We believe that there are many other types of incompleteness that lead to the absence of intertemporal smoothing.

In our model, investors have a short time horizon and this means that they do not self-insure. Individuals live more than two "periods", but whether self-insurance can realistically be achieved in a single individual's lifetime is questionable. In the first place, the number of independent shocks may be small. We can think of the Great Depression as being one shock and the boom of the 1950's and 1960's as another. With this interpretation, the number of periods each generation lives through is small. In addition, there are life-cycle considerations which may prevent households from self-insuring. For example, the desire to purchase a house and provide an education for their

children means that many households do not start saving for retirement until fairly late in life. For both these reasons, the possibilities for self-insurance may be limited.

Finally, note that incomplete market participation will not be a problem when agents have a bequest motive that causes successive generations to act like a single infinitely-lived individual. There is some evidence that in the general population bequest motives and risk sharing within extended families are limited (see Altonji, Hayashi and Kotlikoff (1992; 1996) and the references cited therein). The issue here is whether the wealthy, who own most of the capital, have a sufficient bequest motive for intertemporal smoothing not to be a problem. Altonji, Hayashi and Kotlikoff (1992) point out that wealthy individuals are underrepresented in the data sets most commonly studied in this area and we are unaware of any evidence regarding this group specifically.

In the introduction, we used the comparison of Germany and the US to suggest that different financial systems deal with nondiversifiable risk in different ways. The model shows that it is theoretically possible for an intermediated financial system to achieve a higher level of welfare than a market-based system. It is tempting, then, to compare the US and German financial systems in the light of this example. It is often suggested that German banks hold high levels of hidden reserves, which they rely on when asset returns are low. Even if this form of intertemporal smoothing is limited by comparison with the theoretical schemes considered above, it may nonetheless be an improvement over competitive financial markets in terms of reducing nondiversifiable risk. Thus, the German financial system, with its reliance on financial intermediaries, may have some advantages over the US, which relies more on financial markets.

Given this interpretation, Proposition 5 has important policy implications. It suggests that opening the German financial system to foreign competition, for example by creating a single European market in financial services, could threaten intertemporal smoothing and make Germans worse off in the long run. Of course, risk sharing is not the only consideration in the choice of optimal financial systems. Other important issues are discussed in Allen and Gale (1995).

8 Appendix

8.1 Proof of Proposition 1

Suppose that $(s_t, x_t) = (0, 1)$ for every t. Then the necessary and sufficient conditions for the optimality of this portfolio are:

$$
\begin{aligned}
u'(e - p_t)p_t &= E_t[v'(p_t + y_t)(p_t + y_t)] \\
u'(e - p_t) &\geq E_t[v'(p_t + y_t)].
\end{aligned}
$$

If these conditions are satisfied with $p_t = p_{t+1} = p$, then $\{(s_t, x_t, p_t)\}$ is a stationary, Markov equilibrium. Substituting $p_t = p_{t+1} = p$ in the first-order conditions, it is clear that the first condition implies the second. Hence, we only need to find a solution to the equation

$$
u'(e - p) = E[v'(p + y_t)(p + y_t)/p] \leq E[v'(p + y_t)]E[(p + y_t)/p].
$$

Since $u'(\cdot), v'(\cdot) > 0$, the left hand side is clearly less than the right when p is sufficiently small. On the other hand, for p sufficiently large the right hand side must exceed the left; otherwise, taking the limit as $p \to \infty$, and noting that $E[(p + y_t)/p] \to 1$, we have

$$
\sup u'(\cdot) \leq \inf v'(\cdot),
$$

a contradiction. Thus, for some intermediate value of p the first-order condition must be satisfied and this value of p is the equilibrium asset price. ∎

8.2 Proof of Lemma 1

Recall that $s_{t+1} = \max\{0, s_t + \omega_t - \bar{\omega}\}$, so that $E[s_{t+1}|s_t] \geq s_t$. Define $f(s) = \frac{1}{s+1} \in [0, 1]$ for any $s \geq 0$. Then

$$
\begin{aligned}
E[f(s_{t+1})|s_t] &\geq f(E[s_{t+1}|s_t]) \\
&\geq f(s_t)
\end{aligned}
$$

since f is convex and decreasing. Putting $F_t \equiv f(s_t)$, $\{F_t\}$ is a bounded supermartingale, so by the Martingale Convergence Theorem $F_t \to F_\infty$ almost

surely as $t \to \infty$. Since ω_t has positive variance, it is clear that $F_\infty = 0$, almost surely. Convergence almost surely implies convergence in measure so, for any $\epsilon > 0$, there is a finite T such that

$$\Pr[F_t < \epsilon] > 1 - \epsilon, \forall t > T.$$

Suppose that we want to start the intertemporal smoothing plan at date T when the reserves have grown to $s_T = S$. We have shown that for any $\epsilon > 0$ there is a $T' > T$ such that $\Pr[s_t = 0] < \epsilon$ for all $t > T'$. Keeping ϵ and T fixed, it is clear that by making S sufficiently large, the probability that $s_t = 0$ for any $T \leq t \leq T'$ can be made less than ϵ. Then we have shown that for any $\epsilon > 0$ there is a level of initial reserves S sufficiently large that, starting the intertemporal smoothing plan at date T the probability of $s_t = 0$ at any $t \geq T$ is less than ϵ as required. ∎

8.3 Proof of Proposition 5

Index the values of y_t by $s = 1, ..., S$ and let $c_1 \in \mathbf{R}$ and $c_2 \in \mathbf{R}^S$. Then we can write the expected utility of an agent who consumes c_1 in the first period and c_{2s} in the second period if state s occurs as $u(c_1) + V(c_2)$, where $V(c_2) \equiv \sum_{s=1}^{S} \pi_s v(c_{2s})$. Let $C \subset \mathbf{R} \times \mathbf{R}^S$ be a compact set such that $c \in C$ implies that

$$u'(c_1) \geq \sum_{s=1}^{S} \pi_s v'(c_{2s})$$

and, for any $c \in C$, let

$$\Delta(c) = \{\delta \in \mathbf{R} \times \mathbf{R}^S | u(c_1 + \delta_1) + V(c_2 + \delta_2) \geq u(c_1) + V(c_2)\}.$$

From the concavity of $u(\cdot)$ and $v(\cdot)$ and the gradient inequality, it follows that

$$u'(c_1)\delta_1 + \sum_{s=1}^{S} \pi_s v'(c_{2s})\delta_{2s} \geq 0.$$

Then $c \in C$ implies that

$$\max \delta_{2s} \geq -\delta_1.$$

We now prove a slightly stronger result.

Lemma 2 For any $\epsilon > 0, c \in C$ and $\delta \in \Delta(c)$, there exists $\lambda > 1$ such that $\delta_1 \leq -\epsilon$ implies that $\max \delta_{1s} \geq \lambda \delta_1$.

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Proof. The proof is by contradiction. Suppose that, contrary to what we want to prove, for some $\epsilon > 0$ and any $\lambda > 1$ we can find $c \in C$ and $\delta \in \Delta(c)$ such that $\delta_1 \leq -\epsilon$ and $\max \delta_{2s} < \lambda \delta_1$. Then we can find a sequence (c^k, δ^k) such that, for each k, $c^k \in C$, $\delta^k \in \Delta(c^k)$, $\delta_1^k \leq -\epsilon$ and

$$\lim_{k \to \infty} \max_s \delta_{2s}^k / |\delta_1^k| = 1.$$

C is compact, so there exists a convergent subsequence of $\{c^k\}$. Since u and v are concave, there is no loss of generality in assuming that $\delta_1^k = -\epsilon$. Then $\{\delta^k\}$ is bounded above and $\delta^k \in \Delta(c^k)$ implies it is bounded below as well, so $\{\delta^k\}$ has a convergent subsequence as well. There is no loss of generality, then, in taking $\{(c^k, \delta^k)\}$ to be a convergent sequence with a limit (c^0, δ^0) say. By continuity,

$$u'(c_1^0) \geq \sum_s \pi_s v'(c_{2s}^0),$$

$$u(c_1^0 + \delta_1^0) + V(c_2^0 + \delta_2^0) \geq u(c_1^0) + V(c_2^0)$$

and $\max \delta_{2s}^0 = -\delta_1^0$. However, the second inequality and the strict concavity of u and v imply that

$$u'(c_1^0)\delta_1^0 + \sum_{s=1}^{S} \pi_s v'(c_{2s}^0)\delta_{2s}^0 > 0$$

which contradicts the other two relations. This completes the proof of the lemma ∎.

Now, turning to the proof of Proposition 5, let $\{(c_{1t}, c_{2t}, s_t)\}$ denote the equilibrium allocation and let $\{(c'_{1t}, c'_{2t}, s'_t)\}$ denote another feasible allocation which satisfies the disintermediation constraint. Suppose to begin with that $s_t = s'_t = 0$ for every date t. Let $\delta_t \equiv (c'_{1t}, c'_{2t+1}) - (c_{1t}, c_{2t+1})$ denote the difference in generation t's consumption in the two allocations. The equilibrium allocation satisfies

$$u'(c_{1t}) \geq E[v'(c_{2t+1})|y^t], (c_{1t}, c_{2t+1}) \geq 0, c_{1t} \leq e, \text{ and } c_{2t} \leq e + y_t.$$

The first inequality is the first-order condition, the second holds by assumption and the last two follow from the budget constraints and the fact that $p_t = p_{t+1}$. If we define C by putting

$$C = \{(c_1, c_2) \in \mathbf{R}_+ \times \mathbf{R}_+^S | u'(c_1) \geq E[v'(c_2)], c_1 \leq e, c_2 \leq e + \max y_s$$

then it is clear that $(c_{1t}, c_{2t+1}) \in C$ for every t. Furthermore, $\delta_t \in \Delta(c_{1t}, c_{2t+1})$ for each t. Hence, the conditions of the lemma are satisfied.

Suppose that, contrary to what we want to prove, some generation is *ex post* better off under the alternative allocation than it would be in equilibrium. Without loss of generality, we can assume that generation 1 is better off. Since the initial generation is no worse off, and there is no possible gain from using the storage technology, the improvement in generation 1's welfare must come from a transfer from generation 2, which implies that in some state(s), $\delta_{12} < 0$. Since generation 2 is *ex post* no worse off, there must be some state in which $\delta_{22} \geq -\lambda\delta_{12}$ for some $\lambda > 1$. The increase in generation 2's second-period consumption can only come from a reduction in generation 3's first-period consumption, and since $\delta_{13} = -\delta_{22} \geq \delta_{12}$ our lemma implies that $\delta_{23} \geq -\lambda\delta_{13} \geq -\lambda^2\delta_{12}$ in some state(s). Continuing in this way, we can find a sequence of states $(y_1, y_2, ...)$ such that at each date t, generation t reduces its first-period consumption by $-\lambda^{t-1}\delta_{12}$ and increases its second-period consumption by at least $\lambda^t\delta_{12}$. Since $\lambda > 1$, this will become infeasible in finite time.

Now suppose that there may be changes in the holding of the safe asset. Other things being equal, an increase in storage will have the effect of reducing the first-period consumption and increasing the second-period consumption of a given generation by the same amount, but will not reduce the ratio λ in the inequalities above. The preceding argument will continue to hold, with δ_{2t+1} interpreted as the transfer of consumption between generations $t+1$ and t. Again, there is no feasible sequence of transfers that will make some generation better off *ex post* without make some other generation(s) worse off. ■

References

Allen, F. and D. Gale (1994). "Limited Market Participation and Volatility of Asset Prices," *American Economic Review* 84, 933-955.

Allen, F. and D. Gale (1995). "A Welfare Comparison of Intermediaries and Financial Markets in Germany and the U.S.," *European Economic Review* 39, 179-209.

Altonji, J., F. Hayashi and L. Kotlikoff (1992). "Is the Extended Family Altruistically Linked? Direct Tests Using Microdata," *American Economic Review* 82, 1177-1198.

Altonji, J., F. Hayashi and L. Kotlikoff (1996). "Risk Sharing Between and Within Families," *Econometrica* 64, 261-294.

Bhattacharya, S. and A. J. Padilla (1996). "Dynamic Banking: A Reconsideration," *Review of Financial Studies* 9, 1003-1032.

Fulghieri, P. and R. Rovelli (1994). "Capital Markets, Financial Intermediaries, and the Supply of Liquidity in a Dynamic Economy," working paper, IGIER, Milan.

Gale, D. (1994). "The Efficient Design of Public Debt," in F. Allen and D. Gale, eds., *Financial Innovation and Risk Sharing*, MIT Press, Cambridge, MA.

Gordon, R. A. and H. R. Varian (1988). "Intergenerational Risk Sharing," *Journal of Public Economics* 37, 185-202.

McCallum, B. T. (1987). "The Optimal Inflation Rate in an Overlapping-Generations Economy with Land," in W. A. Barnett and K. J. Singleton, eds., *New Approaches to Monetary Economics*, Cambridge University Press, Cambridge, U.K.

Qi, J. (1994). "Bank Liquidity and Stability in an Overlapping Generations Model," *Review of Financial Studies* 7, 389-417..

Schectman, J. (1976). "An Income Fluctuation Problem," *Journal of Economic Theory* 12, 218-241.

Scheinkman, J. (1980) "Notes on Asset Trading in an Overlapping Generations Economy," working paper, University of Chicago.
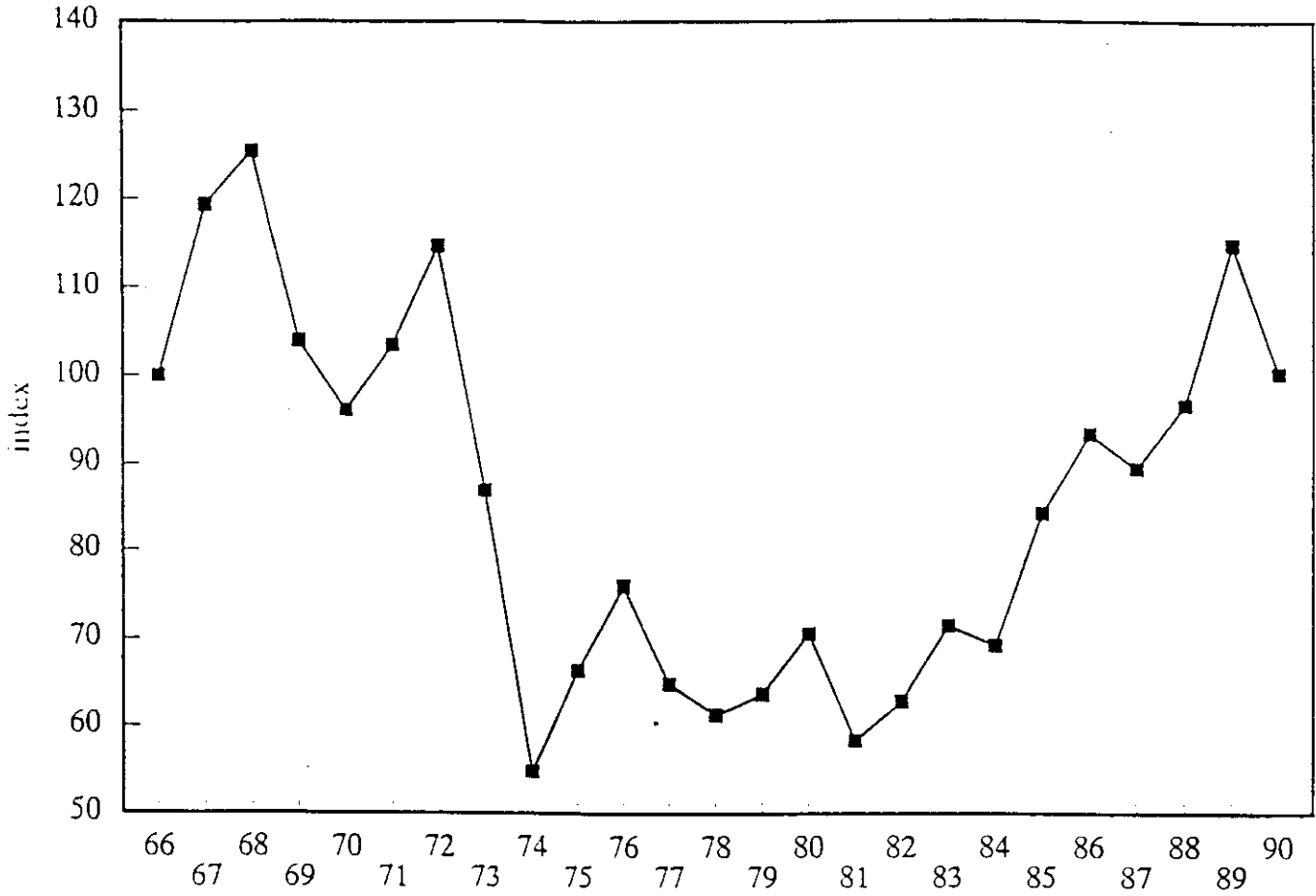
NYSE Index

constant dollars, 1966=100

Figure 1

The Variation of Real U.S. Stock Prices 1966-1990