CALL POLICIES WITH FLOTATION COSTS: A DOG CHASING ITS TAIL

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12-95

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Current Version: February 1995.

ABSTRACT

This paper presents a characterization of callable bond pricing and call decision when there are transactions costs. When capital structure is kept constant a firm that has outstanding callable bonds refinances them with similarly structured callable bonds. Since refinancing is costly, firms will delay the call decision. Given that the firm's cash flows differ from investors' cash flows, the valuation of the callable bond will be different for the firm and for the investors. We find that the investors' valuation function exhibits three important empirical regularities for low interest rates: Inverse convexity, negative duration and market prices higher than call prices. In the region between the next optimal refinancing rate and the first time that the price of the bonds equals the call price, the market valuation of the bonds has a hump.

To simplify the problem we have assumed that the firm will replace the outstanding bond with an identically structured bond. Because the firm will be replacing a seasoned bond with a new one, it will be pasting a function with itself at two different maturities. A head for the new issue and a tail for the seasoned bond. By following this procedure we collapse into a single step the problem of figuring out when to replace a callable bond with another callable bond that needs to be priced before pricing the former. This exchange of bonds will occur at a lower rate than the normal call rate when cash in hand is used. Small transaction costs may justify waiting past the call price if the firm wants to keep a callable bond in its capital structure.

We conclude that transaction costs alone may be enough to explain the overvaluation of callable bonds with respect to the call price. We use a general one-factor interest rate process in continuous time that nests most of the popular one-factor interest rate models used by researchers and practitioners.

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[1] We would like to thank Süleyman Başak, Domenico Cuoco, Bruce Grundy, Haluk Unal, and Zvi Wiener for helpful conversations. All remaining errors are ours. Comments welcome.

I. Introduction

This paper prices callable bonds in a stochastic world and in the presence of flotation costs incurred when issuing debt to obtain the funds to call the outstanding bond. This paper also obtains the optimal call decision under these conditions for a general interest rate process. This is the first paper to do both things for a general interest rate process.

Brennan and Schwartz (1977) solved the pricing of callable bonds without transaction costs or call premium for a simple interest rate process given by: $dr = \sigma dW$. Weingarten (1967) solved the problem of pricing callable bonds in a perfect foresight model. Recently, Mauer (1993) solved part of this problem for infinitely lived bonds and a particular interest rate process given by $dr = \alpha r^2 dt + \sigma r^{3/2} dW$ which allowed him to obtain closed-form solutions.

In this paper we assume a general interest rate process that subsumes most single-factor models. We will also present a characterization of the critical interest rate at which bonds should be called. Pricing callable bonds is done in two steps, first the bond is priced from the point of view of the borrower who not only has to pay a call premium to the investor but also has to pay underwriting costs to the investment bank helping in the flotation of new bonds. Once the pricing for the borrower is done, the critical interest rate is used to price the bond from the point of view of the investors who only receive the call price.

These two valuations are very different and the market valuation will have four basic properties for low interest rates: (I) the market bond price will exceed the call price by an amount similar in magnitude to the flotation costs. Since these costs are non-trivial (3%-5%) the market price can be significantly larger than the call price. (ii) the market price presents a hump before it is called; (iii) the bond has negative duration and (iv) the bond exhibits inverse convexity.

Dunn and Spatt (1986) proposed to solve a similar model in which not only the next refinancing had to be considered, but all future refinancings as well.

Weingarten (1967) solved the problem of refinancing callable bonds with flotation costs in a perfect foresight model. He assumed that to decide on when to refinance the firm needed to know all future refinancings. He analyzed the recursive problem of having to consider the optimal refinancing of the subsequent bond when solving for the value of the current bond. Since the firm is assumed infinitely lived, the problem becomes one of infinite recursion. For example, for a 17% coupon bond there would be a need to analyze all bonds with lower coupons. Without costs, an infinite number of bonds will have to be priced. When studying the decision to call a bond, one would have to keep track of when all subsequent bonds will be called. This is possible in a perfect foresight model, but it cannot be used with stochastic interest rates.

In this paper a callable bond is replaced by another callable bond collapsing into one step the problem of looking at all future refinancings as in Weingarten (1967). Because the bond used to provide the funds for the refinancing is also callable, it correctly incorporates all future decisions to call.

We obtain sizeable over valuations with reasonable parameter values. In general, the overvaluation is slightly smaller than the flotation costs. Vu (1986) finds no evidence of large valuation over call price, in fact he reports three basic regularities regarding callable bond behavior: (I) Only one of the bonds in his sample was called when the call price exceeded the call price by more than 2% (ii) 75% of all bonds were called below call price, actually the average under-valuation was 4.7%, and (iii) when called some bonds actually sold below par.

Given the results in Vu (1986) we propose that transaction costs alone could explain the instances in which market value exceeds call price. Therefore, there would be no need to rely on alternative explanations for the over valuation. The empirical question to be resolved at this point is whether callable non-convertible bonds trade at prices in excess of call price by an amount significantly higher than transactions costs. Crabbe and Helwege (1993) find that information theories do not explain call behavior.

The economic explanation of the result that transaction costs alone could be enough to explain overvaluation is fairly simple. Without refinancing costs[2] the firm would float new bonds to replace bonds issued at par as soon as interest rates drop by a small amount. This is because the firm does not have to worry about giving up the option to refinance later at even lower rates as would be the case if the refinancing was done with non-callable debt[3]. However, the presence of an initial blackout period in the newly issued bond may reduce the refinancing opportunities.

The remainder of the paper is organized as follows, section II presents the model and the modeling choices; section III solves the problem; section IV provides numerical results; finally section V provides a summary and concluding remarks

II. Economic environment and other modeling assumptions

The following assumptions and modeling choices were made in order for us to solve the callable bond problem.

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[2]Refinancing costs are both the call premium and proper flotation costs. We are trying to explain why the use of non-callable debt to finance a call would by itself delay the call decision. Without call premium or refinancing transactions costs using non-callable bonds to call current callable bonds would delay the call decision because the firm would give up completely the option to refinance later at an even lower rate.

[3]Mauer (1993) proposed this framework to price the call option. To do that, a non-callable is the right choice because the option is the difference between the callable and non-callable bond.

A0) It will be assumed that there is one firm which has as its objective to minimize the value of outstanding coupon bonds given by $G^B(r,t;r_n)$. The superscript B stands for borrower; r_n is the nominal rate on the bond. The bond has current maturity of t (it has t years until expiration) and it was originally a T-year bond. The coupon is paid continuously at a rate $pd\tau$ per unit of time. In general it might be the case that $p \neq r_n$ but we assume them to be the same.

A1) The interest rate process is a one factor Ito-process given by the following stochastic differential equation at current time τ

(1) $$dr = k(L - r)d\tau + \sigma r^{\gamma/2} dW.$$

It is necessary to impose restrictions on the parameters L, k and γ so that interest rates never become negative, L, k and $\gamma \geq 0$. We also assume $\gamma \leq 2$ to satisfy the growth condition so that bond prices do not explode. $\sigma r^{\gamma/2}$ is the volatility of the process and dW is the increment of a standard Wiener process. By changing the different parameters in this process, many single-factor models can be obtained. In this sense it has the same spirit as the process assumed by Chan, Karolyi, Longstaff and Sanders (1992)[4].

A2) The bond being priced is callable after an initial blackout period $[0,t_B]$ at a call price $cp(G^B,t)$, which is in general any function of time and/or bond price.

A3) Other than market movements, there are no additional stochastic variables that affect bond prices in this economy. In particular, default risk is zero. With respect to market risk see assumption (A7).

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[4] In fact, it is the same process, we have decided to present it in its more natural form with a long term interest rate and a speed of reversion parameter. In their notation $\alpha = Lk$, and $\beta = -k$.

A4) The firm prefers to replace a callable bond with another callable bond, as it is often done in practice.

A5) In addition to (A4), it will be assumed for computational ease that the bond used to refinance only differs in maturity from the outstanding bond. Its maturity is equal to the original maturity T. This is only used to find the critical rate at which bonds should be switched. The actual bond used to call may be structured with coupon rates that reflect current market conditions.

A6) When calling a bond, the firm incurs flotation costs given by $f(G^B(.))$ where $f(.)$ is a cost function and $G^B(.)$ is the market price of the bond being replaced.

A7) All financial instruments that depend on interest rates as given in (A1) will be priced using a market risk adjustment of λr, where the constant λ is the price of interest rate risk.

A8) In addition to flotation costs, there are no corporate taxes or other cashflows related to these bonds.

A9) Investors have no taxes or transaction costs and possess the same information as the firm. In particular, it will be assumed that investors know when the firm will refinance. This implies that they know the cost structure of the refunding decision $f(G^B(.))$.

One assumption is non-conventional and therefore needs further explanation: "The replacement of a callable bond with an identically structured callable bond." This assumption was made to avoid the infinite regress problem of Weingarten (1967) in which one needs to solve for all future refinancings before one can solve for the current one.

As stated before, the only purpose of this assumption is to obtain the critical interest rate at which bonds should be exchanged. This assumption should not be taken literally because the structure of new bonds may be such that it matches market conditions at the time of issuance. In particular, the coupon it pays may be different from the previous one.

This assumption should also be contrasted with a parallel one made by Mauer (1993) where a callable bond was replaced by a non-callable bond. If the objective is to price the call options, this is the right assumption because it is the difference between the callable and non-callable bond. But, it distorts the call decision we are after in this paper. To highlight this difference further, assume a perfect world where there are no taxes, or costs, further assume that there is no call premium. In this world, if a callable bond is going to be replaced with another callable bond it will be done as soon as its price increases by any small amount above par. If, in contrast, the callable bond is going to be replaced by a non-callable bond, then rates will be allowed to drop further before the bond is called because the firm will be giving up any option to refinance at lower rates. Assumption (A4) guarantees that the firm maintains this option to refinance later at lower rates.

This analysis also helps to understand why results regarding overvaluation are so significant. If the firm were to maintain its policy of refinancing as soon as rates dropped by a small amount, there would be a significant accumulation of flotation costs that would grow very quickly. To avoid most of those costs, the firm waits until the benefits obtained from one refinancing compensate for the current and future flotation costs properly discounted.

Given the assumption for this economy, the evolution of ***any*** coupon bond will be given by the following partial differential equation (PDE).

(2) $$0.5\,\sigma^2 r^{\gamma} G_{rr}(\cdot) + \left(kL - (L+\lambda)r\right) G_r(\cdot) - G_t(\cdot) + p = r\,G(\cdot).$$

This relationship is obtained by equating the risk adjusted return on a bond with the risk free rate. All the equation is expressing is the fact that the expected return on bonds is equal to the risk free rate once the risk adjustment factor has been included (λr). Including the risk adjustment factor allows this specification to incorporate a pure expectations hypothesis as in Brennan and Schwartz (1977) by making λ=0, as well as a more general risk premium model. Equation (2) must be satisfied by every coupon bond (remember that p is the continuously paid coupon rate) in this partial equilibrium one-factor model of the economy.

The only way to differentiate one bond from another in this economy is to look at the bond covenants and imbedded options[5]. These represent boundary behavior that the bond must satisfy. It is these boundary conditions what make callable bonds different from any other bond. Since our interest is the study of callable bonds, we will state the boundary conditions that the bond must satisfy.

Before doing that we would like to draw attention to the difference between the valuation of the bond by the issuing corporation (borrower) and the valuation of the same bond by the market (investors). The only difference between these valuations is the fact that flotation costs are paid by the corporation and go to third parties as opposed to investors. The crucial difference between these two valuations has been generally absent in the literature until now, with the exception of Timmis (1985) and Dunn and Spatt (1986). Stanton (1993) and Mauer (1993) also have a very similar setup[6].

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[5]By making $p = 0$ a pure discount bond could be studied.

[6]Timmis (1985) is the earliest reference that we have found to this difference although he and Stanton (1993) analyze the behavior of individual mortgages as opposed to callable bonds.

Investors will be assumed to know the costs incurred by the firm when refinancing (A9). They would solve the problem from the point of view of the firm and use the critical interest rate that triggers refinancing as a boundary condition to solve their own problem; r^* will be that critical rate. With that process in mind the boundary conditions for the firm will be stated first.

(3) a). $$\lim_{r \to \infty} G^B(r, t; r_n) = 0$$

the economic value of a bond when rates increase without bounds is zero.

b. $$G^B(r, 0; r_n) = 1$$

at maturity, the borrower returns the face value of the bond, if the bond has not already been called. We normalize the face value to one. Finally, a differential boundary condition is obtained at the reflecting barrier when $r = 0$:

c. $$kL\, G_r(\cdot) - G_t(\cdot) + p = 0 \quad \text{if} \quad r = 0$$

the natural boundary of the interest rate process assumed in (1) provides this additional boundary condition for the bond. In addition, we have the blackout period condition that prevents the bond from being called. At the end of the initial blackout period the bond is called if $r = r^*$. We approximate the value $G^B(r=0, \tau \le t_B; r_n)$ during the blackout period as the present value of $G^B(r=0, \tau=t_B; r_n)$ plus the coupon stream setting to zero the probability of the short-term rate entering the continuation region[7] at $\tau = t_B$, conditional on $r_t = 0, \tau < t_B$. This approximation works quite well but causes small numerical distortions for $r < 0.004$ (forty basis points).

So far, there are four conditions and, in principle, for this bond, that should be enough to solve

[7]The continuation region for this problem is the region in the interest rate domain where the bond is not called, that is, the process is allowed to "continue".

the PDE given in (2). Indeed, they are the boundary conditions for a straight, non-callable, non-convertible bond. Finally, there are the call boundary conditions if the firm decides to replace a callable bond with another callable bond. In this case the following condition must be satisfied:

(4) $G^B(r^*, t; r_n) = G^B(r^*, T; r_n)$

where $G^B(r^*, T; r_n)$ includes $f(G^B(\cdot))$, the transaction costs function, which is the flotation costs incurred when calling the bond.

What the firm owes after refinancing should be equal to what it used to owe minus the refinancing costs represented by $f(G^B(\cdot))$. All this equation conveys is the fact that at the point of switching bonds the marginal benefits from refinancing should be the same as the marginal costs. In other words that under the switching strategy there should be no net cashflow. Since equation (2) was obtained using the expected capital appreciation of the bond, the firm is actually taking into account not only the present refinancing decision, but all future ones as well.

The implicit assumption is that the objective of the firm is to minimize the value of the bonds to favor shareholders (A0). It is beyond the scope of the current paper to study the optimality of the capital structure or the optimality of the bond minimization strategy. Brennan and Schwartz (1977) and most researchers since then have used identical specifications.

What is new in this paper is the comparison of the usual call policy with the assumption (A5) that not only will a callable bond be replaced by another callable bond but that the stated coupon rate is the same. Although the usual call policy is preferred in the absence of transactions costs, switching directly from a callable bond into another one may save one transaction if the firm wants a callable bond in its capital structure. The implication of our assumption is that the firm will owe no more than what it owes when it refinances. This allows for a solution of the infinite regress problem of Weingarten (1967)[8]. In principle, it means that the firm will be replacing the outstanding bonds with identical bonds, except that they will have the initial maturity and will be selling at a premium because of the lower interest rate in the current market conditions. But our replacement strategy need not be taken literally, because all we are after is the critical interest rate at which the firm will call the bond. Indeed, the new bond may be issued at rates which are consistent with current market condition when the refinancing decision is made. In terms of the numerical implementations it means that condition (4) will be used to locate the free boundary where it is optimal to call the bond.

In the name of clarity and simplicity a flat call premium schedule will be assumed after the initial blackout period. Adding a deterministic general call schedule would be simple to implement and add nothing to the understanding of the problem. Also, only proportional transaction costs will be used[9].

These two assumptions imply that

(5) $$f(G^B(\cdot)) = G^B(\cdot)\, f(t)$$

where $f(t)$ represents the transaction (flotation) costs which are assumed proportional to the value of the bond being issued. We chose the transaction costs of the refinancing decision to depend on time because at maturity the bond should be refinanced in any event. Therefore, the incremental transaction costs due to the call, which affects the switching decision, depends on the

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[8]Dunn and Spatt (1986) also propose a solution technique similar to Weingarten (1967).

[9]For this particular problem it makes no difference whether the costs are fixed or proportional.

time left to maturity for the bond being replaced. For simplicity we assume that $f(t)$ is proportional to the time until expiration.

So far we have discussed the firm's problem. From the investors' perspective, the only difference is in the boundary condition (4) which they will replace with

6. a) $G^M(r^*, t; r_n) = (1 + \pi)$ [the superscript M indicates market values.]

Investors are only going to receive the call premium (remember that the face was normalized to one), they do not receive the transactions cost for the issuance of the new bonds.

II. b. The frictionless case:

In an economy without transactions costs (or call premium), taxes, or other market imperfections it would not make any difference whether the firm maintains its capital structure constant or not. The solution to the model will therefore be obtained by making $f = \pi = 0$ (no flotation costs or call premium) and using equation (6 b) as the firm's boundary condition

6. b) $G^M(r^*, t) = G^B(r^*, t) = 1.$

This would imply that the firm is using cash to call the bonds and that investors only receive the face amount. How the firm obtains the funds would be irrelevant in this frictionless economy. The Modigliani Miller conditions would be satisfied and capital structure would be irrelevant. This is the simplest refinancing of callable bonds as solved by Brennan and Schwartz (1977). In the current paper it is assumed that the firm refinances the callable bond with another callable bond of identical specification, maintaining also constant the amount of money owed[10].

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[10]This is very important because the argument used to support the claim that it does not matter how a corporation finances a call says that if a holder of a callable bond needs to be paid \$1.09, then it does not matter how this money is obtained. There will be a non-callable bond that sells at par (\$1.00), but there would also be a callable bond that sells at \$0.95 (the difference is the price the firm pays for the call option). By selling 1.1474 callable bonds instead of 1.09 non-

The result from the frictionless model will be used as a benchmark in this paper.

The space and time spent analyzing and discussing the boundary conditions is essential because, as was said earlier, in this economy all coupon bonds satisfy the PDE given in (2). The only difference between one bond and another is just the set of boundary conditions that must be satisfied. Imposing the wrong boundary condition would give the wrong answer. Many researchers have used an incorrect boundary condition. Some people impose a boundary condition that says that the bond should be called when the *market* price (as opposed to the issuer's price) is equal to the premium plus transactions costs. This would give as a result a model where at the critical interest rate the value of the bond will be equal to one plus the value of the premium plus the transactions costs. Such a model does not satisfy the empirical regularities that the current model captures, but more importantly it is not correct.

The problem with the latter condition is that it does not incorporate the fact that market investors do not receive the transaction costs. The market valuation should be made with the right amount of funds that investors receive, and that is only face plus call premium. These funds are received when the firm finds it optimal, subject to the investment bank flotation costs.

The right boundary condition for the borrower is given by equation (4) and for the investors by equation (6). They allow the model to explain market valuations larger than call premium (by an amount similar in size to transactions costs).

The typical argument used to support the claim that the observed market value of callable bonds could not be explained by transactions costs alone could be summarize as follows. The

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callable bonds the firm can call the current bond using callable bonds. However this financing strategy minimizes expected costs over the interval $(0,t)$. Our switching strategy minimizes costs over the interval $(0,T_C+T)$. Were T_C is the point in time at which the switching takes place.

company's valuation G^B is larger than the market valuation ,G^M , but it cannot be too much bigger than the call price because costs have to be discounted. The difference between the two is the transactions cost f that the borrower pays a third party. This implies that G^B is an upper bound for G^M. Since G^B will never be higher than the call premium by more than $f\, G^M$, it will be barely larger than call price.

Under the bond switching policy the proper boundary condition for the borrower's valuation is not $1 + \pi + f$[11], but $G^B(r^*, T; r_n)$ to reflect the fact that the company is switching to this new bond and that it is the only source of funds. Since the nominal rate r_n is higher than the market rate r^* at which the bond is being refinanced, the bond is selling at a premium and $G^B(r^*, T; r_n)$ could be larger than the call boundary condition. This allows more room for G^M to be larger than the call price. In general, G^M will be larger than the call price by an amount very similar to the transactions costs involved when refinancing. For the numerical exercises performed for this paper[12], $G^B(r^*, T; r_n)$ is very similar in size to the call boundary condition, but the difference between $G^B(r^*, T; r_n)$ and $G^M(r^*, T; r_n)$ at the switching point is only somewhat smaller than f.

III Results

After having discussed the model we can present and analyze some of the numerical results. As a reference point we choose the benchmark parameter values given in the following table. The parameters are from CKLS and we use 25 years as bond maturity.

$\gamma = 1.5$

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[11]This is the standard call boundary condition.

[12]See Figure 6 which shows the difference between the market and the borrowers valuations.

$L = 0.02$ (the short-term interest rate has a long-term value of 2%)

$k = 0.2$ (the short term interest rate reverts in five years)

$\sigma = 0.045$

$\lambda = 0.02$

$p = 0.08$ [continuous coupon rate]

$t = 25$

$1+f+\pi = 1.09$

$f = 0.03$ (This is the maximum amount of transaction costs if the switching occurs at 25 years to maturity).

The PDE in equation (2) was solved by finite differences after making a change of variable from $r\in[0,\infty)$ to $s\in[0,1]$. The number of time points=200, number of interest rate points=200 [for a transformation r = (1-s/Xs)]. X was chosen equal to 10 so that most of the interest rate points fell bellow 15%.

Figure 1 shows the value of a callable bond as a function of time to maturity and instantaneous interest rates. We assume the call to occur at $\pi + f$[13]. The introduction of a three-year initial blackout period, in Figure 2, allows for the bond price to be initially higher than the refinancing costs at very low interest rates. Note that the partial derivative of the bond value with respect to time is discontinuous at ($t = t_B$[14], $r = 0$), where it jumps from $-p$ to zero.

To start the description of the results we will first look at the vector of critical rates (r^*) as

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[13]Note that the interest rate scale is ordinal. The graph with this scale is clearer than the one with the cardinal numbers.

[14]t_B is the end of the blackout period.

a function of time to expiration (t). If the firm wishes to maintain a callable bond in its capital structure it may decide to forgo the opportunity of calling the bond until it is optimal to exchange the existing callable bond for a new one. For computational ease we assume that the new bond has all the original characteristics of the old bond (assumption (A5)). Such a strategy is clearly dominated by an outright call in a world without transactions costs, however, we will show that it may be justifiable when there are transactions costs, even if they are small. This is because not going through an intermediate bond between the two callable ones saves one transaction. Figure 3 contrasts the critical call rate with the critical switching rate. It is apparent that bond switching becomes optimal at rates lower than the call rates. In the interval between the two critical rates, holding the original callable bond is suboptimal if transactions costs are not taken into account. To measure the cost of waiting until the lower critical rate is reached, Table 1 reports the values of the callable bond under the two call policies for a variety of initial interest rates and blackout periods. Panel A presents the results for the switching strategy, panel B for the call strategy and panel C the difference. By construction the difference is zero for r=0. After that, the difference decreases monotonically with interest rates. The cost of using the critical rate of the switching strategy rather than the higher call policy is very small unless the short term rate is approaching the call region. The difference in call value under the two policies is highlighted in Figure 4, which shows the difference between the bond values computed under the two policies. In the call region the difference between the two bonds is close to transaction costs[15]. As rates increase the price difference drops rather quickly to zero. Also, after the blackout period, as maturity

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[15]For extremely low values of the interest rate ($r < 0.004$) and at the time of first issue, there are numerical inaccuracies that cause an area of relative high differences. This only happens during the blackout period because of the approximate boundary condition that we use there.

approaches the difference decreases because the advantage of the switching strategy decreases as maturity nears. As we said before, at maturity the bond has to be refinanced in any event. Figure 5 also shows a series of cross sections at nine and a half years after the end of the initial blackout. This figure shows the pattern previously described when interest rates increase. Similarly, Table 2 presents the results for two maturities, 12.5 and 6.25 years to maturity. The difference is constant until rates reach 7.15% and then decrease monotonically to zero for higher rates.

It appears that moderate flotation costs may dominate the cost of following the switching policy rather than the usual call policy. Changing the blackout period has almost no effect. Finally, Figure 6 shows two cross section: one of $G^M(r)$ and the other of $G^B(r)$ at a given point in time evidencing the hump in $G^M(r)$ at low rates. $G^B(r)$ is indeed an upper bound for $G^M(r)$ but the latter is very close to the former. This is one of the most significant results of this paper because it shows that for interest rates in the neighborhood of the critical refinancing rate the ***market*** price of a callable non-convertible bond presents the empirical regularities that we anticipated in $G^M(r)$: (I) a "hump", (ii) inverse convexity, and (ii) negative duration for low rates.

Figures 7 and 8 show critical rates for 10 different values of the call or refinancing costs (the sum of the call premium and the flotation costs to switch). From the point of view of the corporation (the borrower), it does not make any difference whether the call costs include flotation costs as well as a call premium. The only difference between the two is the fact that the call premium is part of the debt contract and cannot be changed or altered, while transaction costs could be negotiated with the underwriters.

For the firm, call premium and transaction costs just make it costly to refinance. To understand better the effects of the call premium and transactions costs as deterrents of refinancing we have changed the call or refinancing costs ($f + \pi$) from 0.001 (10 basis points) to 0.1210 (1210 bps).

There are two striking and significant effects of this experiment. First of all, the amount that interest rates need to fall for the bonds to be called the day the bonds are first callable after the blackout period (when time to maturity is 22 years) is directly affected by the costs (this is the intersection of the critical rate curve with the vertical axis). This value changes from 7.99% needed at 10 bps to 7.01% at 1210 bps. The other result is the time until expiration for which NO decrease in interest rates will trigger refinancing (the intercept with the horizontal axis). For refinancing costs equal to 10 bps the firm will refinance even one week before expiration, while for costs equal to 1210 bps, once bonds have eighteen months until expiration, there is no positive rate low enough that will trigger refinancing because the firm will not recover the costs of refinancing.

This behavior explains clearly why we observe decreasing call premium schedules or even an initial blackout period when refinancing is not possible (this could be interpreted as infinite refinancing costs). As expiration nears, the importance of refinancing costs increases dramatically.

Table 3 presents the most salient values of the same experiment performed with a blackout period of three years as in our benchmark case. In addition to the blackout period there is an "optimal" no-switch region where replacing the outstanding bond will not pay. The initial blackout period causes a small increase in the delay.

We can now look again at Figures 7 and 8 and observe an important result regarding the costs of refinancing and the relationship for small values of refinancing costs (again, this is both call premium and transaction costs per se). As typical with transaction costs models in continuous time, the effect of costs per unit of costs decreases as refinancing costs increase. This means that the strongest effect is felt for the first epsilon of transaction costs (that is, in real markets, the first basis point is the most important). As we increase refinancing costs, the delay before refinancing increases at a deceasing rate. Given results in other papers (see for example Delgado and Dumas (1994)) it is tempting to conjecture that there will be initially a cubic relationship between the delay of refinancing and refinancing costs. Due to the existence of the contractual call premium, refinancing costs are larger than 500 basis points and such a relationship would be of limited practical use. This analysis, however, highlights the effect of pure transaction costs (as opposed to refinancing costs that include call premium) on the refinancing decision. Since most of the effect of refinancing costs is due to the built-in call premium, transaction costs are not as important as they could be without the call premium. This should not be taken to mean that they are irrelevant. Because transaction costs go to underwriters, as opposed to investors, they create the "hump" in the market price of callable bonds, and delay further the call decision (see Figure 6 and our previous discussion of that figure).

Our results regarding the behavior of the critical rate as a function of the time to expiration should be contrasted with results obtained when the interest rate process allows for negative interest rates as it would be the case with a simple Vasicek (1977) model. Figure 9 shows the critical rate for different values of volatility for the following particular interest rate process $dr = \sigma dW$, which is the one used by Brennan and Schwartz (1977). Two very important features of this process should be highlighted. The most striking one is the fact that the relationship between the critical rate and time to expiration is actually inverted, that is, the critical rate increases as we get closer to expiration (time to expiration goes to zero). With $k=L=\gamma=0$, the critical rate increases as the bond gets closer to expiration because the process $dr = \sigma dW$ allows for negative interest rates. Actually, rates can be infinitely negative (to perfectly replicate Brennan and Schwartz we also assumed zero refinancing costs, that is zero call premium and zero flotation costs, but we have kept our three year blackout period). This means that the critical rate will intercept the two axis just as before, but the one with the horizontal axis will give us the smallest maturity for which it is optimal to refinance. As figure 9 shows, all curves reach the coupon rate of 8% at maturity to indicate that rates do not have to be smaller than 8% to optimally refinance. The critical rate decreases from that point on as maturity increases. For $\sigma=0.049$ we see that the intercept at t=22 (when the bond is callable for the first time) is 1.54%. The economic rational of this behavior is based on the fact that, as we said before, interest rates are allowed to became negative. Not only is it that as maturity increases the critical rate decreases, as we increase volatility, but also there are values of volatility for which it does not matter how low interest rate became the day the bond is callable for the first time, it will never be optimal to refinance for positive interest rates. For these curves, there is an intercept with the horizontal axis which represents the minimum expiration for which it is optimal to refinance at positive interest rates. This is due to the fact that the longer the maturity the larger the probability that between now and expiration interest rates will become negative. As is the case in all option valuation models increases in time to expiration and volatility produce results in the same direction: the value of the option increases.

Table 4 presents these results in a compact manner, again, the "optimal" blackout region is represented by the zeroes that start for some values of σ after eighteen months to indicate that for most of its life the bond will not be called. This analysis implies that for these models the choice of interest rate process is crucial. In particular, whether the process allows for negative interest rates because under those conditions the results are rather counterintuitive. The critical refinancing rate increases as maturity approaches. This result is not only present for the Brennan and Schwartz's model, Vasicek's (1977) model also has the same properties because it also allows negative rates.

The second type of experiment that is important to perform is the change in volatility to see the effects that volatility has on the critical interest rate. Figures 10 and 11 present the results of these experiments. As expected, increases in volatility produce a decrease in the critical rate (or equivalently they delay the call decision). These curves were obtained by changing volatility (σ) from 0.001 to 0.151. An important feature of this behavior should be pointed out, the effect of increases in volatility decrease with time to maturity. They decrease so much that the differences in the critical rates for significantly different volatilities vanish at one point. In the case of Figures 10 and 11 this happens at about ten months to expiration. This is the point after which it does not matter how low interest rates get, it will not be optimal to refinance. Basically, what increases in volatility do is to rotate clockwise the critical rate schedule using the intercept with the horizontal axis as a fulcrum. As volatility is increased, the interest rate needed to trigger refinancing at the point at which the bond is first callable (22 years to maturity) increases almost proportionately to volatility while after ten months to maturity there will be no effect. Table 5 presents results numerically for the most relevant values. What is important to note is that for small times to maturity (two years or less) σ has very little effect on the critical rates (the values in each row are the same).

An additional experiment that sheds light on the understanding of refinancing with transaction costs is one where the mean of the interest rate process is changed. Note that the interest rate process assumed in equation (1) is in general mean reverting with long term mean L and speed of reversion k. As we increase the long term mean of the interest rate process the critical interest rate at which it is optimal to refinance also increases because the probability of lower rates decreases. Figure 12 and Table 6 give us these results. The most relevant fact is the existence of a non-monotonic critical rate (it is more clear in this figure but it can also be seen in figure 10). It is due to the fact that as maturity approaches the advantages of switching decrease because at expiration the bond will have to be refinanced. This is contrasted with the fact that as maturity approaches transaction costs are have more impact on the switching decision because there is less time to amortize the costs. The combination of these two forces causes the non-monotonic behavior. It is important to note that changes in the market price of risk (λ) have very similar results to changes in L. A look at equation (2) will confirm this statement.

The most interesting exercise to be performed is, of course, the market price of callable bonds at different maturities and the determination of whether they are significantly larger than call price. As Figure 6 shows, the market price of callable bonds not only can be greater than the call price, but the amount of the overpricing can be as large as the size of the flotation costs. For the parameter values we have chosen, the overpricing is of the order of 3%. One of the most important results of this paper is the fact that indeed transaction costs alone can explain callable bond overpricing given that flotation costs are non-trivial and that 3% to 5% is a rather conservative range for flotation costs. Also, as discussed before, we obtain negative duration and inverse convexity for the market price of the bond.

IV Conclusions

This paper has shown that with transactions costs alone and very reasonable parameter values (estimated by Chan, Karoly, Longstaff and Sanders (1992)) it is possible to obtain market prices of callable bonds that exceed call price by an amount similar in magnitude to the costs of raising funds to call the outstanding bonds.

The choice of interest rate process drastically affects the results obtained. In particular, if the assumed process allows for negative rates one can obtain counterintuitive results. With the possibility of negative rates the borrower has an incentive to delay refinancing in the expectation that future rates might be significantly lower (negative). As maturity approaches, the probability of negative rates decreases and the incentive to wait for lower rates also decreases. In the presence of transactions costs this tendency is tempered by the time needed to take advantage of the reduced interest rate payments.

Since a model that implies negative rates is not realistic our conclusions regarding the effect of flotation costs do not take them into consideration. Flotation costs appear to cause a "hump" in the market price of callable bonds. In addition, market prices can be larger than call prices by an amount very similar to the flotation costs. Finally, transaction costs reduce the value of the critical rate at which it is optimal to refinance bonds. This extends the period of time during which callable bonds will not be refinance regardless of how low interest rates became.

Unanswered questions left for future empirical research are the measure of the overpricing of callable bonds over call price and the determination of whether this overpricing can be significantly larger than reasonable flotation costs.

Table 1: Prices of bonds when first issued as a function of market interest rates and size of Blackout period. The first column shows the market short term rate in percentages and the first row the size of the initial blackout period in years. A Comparison of switching against calling.

A: BOND SWITCHING

r\TB	5.25	4.875	4.5	4.125	3.75	3.375	3	2.625	2.25	1.875
0.00 %	1.51865	1.488782	1.458908	1.429028	1.399143	1.369252	1.339355	1.309453	1.279545	1.249632
1.03	1.424492	1.402781	1.380864	1.358733	1.336379	1.313796	1.290973	1.267902	1.244572	1.220972
2.03	1.385722	1.365831	1.345824	1.325701	1.305464	1.285112	1.264648	1.244073	1.223389	1.202597
3.02	1.348162	1.329997	1.311804	1.293591	1.275368	1.257147	1.238939	1.220759	1.202622	1.184544
4.04	1.310871	1.294382	1.277952	1.261598	1.245341	1.229203	1.213207	1.197381	1.181756	1.166364
5.03	1.275377	1.260447	1.245659	1.231041	1.216621	1.202434	1.188516	1.174908	1.161656	1.148814
6.08	1.239282	1.225900	1.212746	1.199857	1.187272	1.175036	1.163201	1.151824	1.140968	1.130708
7.01	1.207995	1.195925	1.184156	1.172735	1.161710	1.151138	1.141084	1.131619	1.122825	1.114795
8.07	1.173423	1.162768	1.152496	1.142662	1.133329	1.124565	1.116449	1.109072	1.102533	1.096946
9.10	1.140779	1.131426	1.122533	1.114165	1.106394	1.099302	1.092982	1.087533	1.083059	1.079656
10.07	1.111154	1.102953	1.095281	1.088211	1.081824	1.076210	1.071461	1.067665	1.064882	1.063104
12.11	1.051356	1.045384	1.040069	1.035489	1.031715	1.028800	1.026749	1.025491	1.024860	1.024625
14.04	0.998017	0.993885	0.990472	0.987818	0.985920	0.984712	0.984058	0.983775	0.983684	0.983665
16.01	0.946508	0.943918	0.942002	0.940714	0.939955	0.939578	0.939429	0.939384	0.939375	0.939374
18.36	0.888451	0.887163	0.886357	0.885920	0.885722	0.885651	0.885631	0.885627	0.885627	0.885627
20.19	0.845323	0.844651	0.844290	0.844126	0.844065	0.844048	0.844045	0.844044	0.844044	0.844044

B: CALLING THE BOND

r\TB	5.25	4.875	4.5	4.125	3.75	3.375	3	2.625	2.25	1.875
0.00 %	1.518650	1.488782	1.458908	1.429028	1.399143	1.369252	1.339355	1.309453	1.279545	1.249632
1.03	1.402639	1.380376	1.357903	1.335211	1.312295	1.289145	1.265753	1.242111	1.218208	1.194033
2.03	1.364544	1.344093	1.323517	1.302819	1.281997	1.261054	1.239990	1.218807	1.197507	1.176090
3.02	1.327638	1.308904	1.290131	1.271327	1.252502	1.233666	1.214830	1.196007	1.177212	1.158461
4.04	1.290994	1.273929	1.256910	1.239952	1.223074	1.206298	1.189646	1.173144	1.156821	1.140707
5.03	1.256115	1.240603	1.225217	1.209983	1.194927	1.180081	1.165481	1.151165	1.137178	1.123568
6.08	1.220644	1.206676	1.192916	1.179398	1.166161	1.153247	1.140705	1.128589	1.116960	1.105888
7.01	1.189897	1.177237	1.164856	1.152797	1.141107	1.129840	1.119058	1.108828	1.099229	1.090348
8.07	1.155921	1.144672	1.133782	1.123301	1.113289	1.103813	1.094947	1.086778	1.079402	1.072930
9.10	1.123838	1.113889	1.104373	1.095350	1.086890	1.079072	1.071986	1.065731	1.060418	1.056160
10.07	1.094722	1.085924	1.077625	1.069895	1.062813	1.056468	1.050957	1.046381	1.042831	1.040361
12.11	1.035951	1.029385	1.023447	1.018219	1.013782	1.010212	1.007553	1.005787	1.004801	1.004377
14.04	0.983540	0.978838	0.974846	0.971625	0.969203	0.967556	0.966585	0.966117	0.965946	0.965903
16.01	0.932973	0.929882	0.927496	0.925803	0.924734	0.924155	0.923901	0.923816	0.923796	0.923793
18.36	0.876089	0.874439	0.873345	0.872709	0.872395	0.872269	0.872231	0.872223	0.872221	0.872221
20.19	0.833925	0.833012	0.832486	0.832228	0.832124	0.832091	0.832083	0.832082	0.832082	0.832082

C: DIFFERENCE

r\TB	5.25	4.875	4.5	4.125	3.75	3.375	3	2.625	2.25	1.875
0.00 %	0	0	0	0	0	0	0	0	0	0
1.03	0.021853	0.022405	0.022961	0.023522	0.024084	0.024651	0.025220	0.025791	0.026364	0.026939
2.03	0.021178	0.021738	0.022307	0.022882	0.023467	0.024058	0.024658	0.025266	0.025882	0.026507
3.02	0.020524	0.021093	0.021673	0.022264	0.022866	0.023481	0.024109	0.024752	0.025410	0.026083
4.04	0.019877	0.020453	0.021042	0.021646	0.022267	0.022905	0.023561	0.024237	0.024935	0.025657
5.03	0.019262	0.019844	0.020442	0.021058	0.021694	0.022353	0.023035	0.023743	0.024478	0.025246
6.08	0.018638	0.019224	0.019830	0.020459	0.021111	0.021789	0.022496	0.023235	0.024008	0.024820
7.01	0.018098	0.018688	0.019300	0.019938	0.020603	0.021298	0.022026	0.022791	0.023596	0.024447
8.07	0.017502	0.018096	0.018714	0.019361	0.020040	0.020752	0.021502	0.022294	0.023131	0.024016
9.10	0.016941	0.017537	0.018160	0.018815	0.019504	0.020230	0.020996	0.021802	0.022641	0.023496
10.07	0.016432	0.017029	0.017656	0.018316	0.019011	0.019742	0.020504	0.021284	0.022051	0.022743
12.11	0.015405	0.015999	0.016622	0.017270	0.017933	0.018588	0.019196	0.019704	0.020059	0.020248
14.04	0.014477	0.015047	0.015626	0.016193	0.016717	0.017156	0.017473	0.017658	0.017738	0.017762
16.01	0.013535	0.014036	0.014506	0.014911	0.015221	0.015423	0.015528	0.015568	0.015579	0.015581
18.36	0.012362	0.012724	0.013012	0.013211	0.013327	0.013382	0.013400	0.013404	0.013406	0.013406
20.19	0.011398	0.011639	0.011804	0.011898	0.011941	0.011957	0.011962	0.011962	0.011962	0.011962

Table 2. Price Differences Between a bond that is "called" and a bond that is "switched". The parameter values are those of the benchmark case. The first column shows the interest rate in percentages and the top row the time until expiration of the bonds. Until rates are about 7.15% the difference is almost constant. After that, the difference quickly drops to zero as rates increase.

r \ t	*12.5*	*6.25*
0.000000	0.026231	0.026231
0.071518	0.026231	0.026231
0.079094	0.025000	0.025047
0.080699	0.024770	0.024810
0.089225	0.023600	0.023529
0.091037	0.023362	0.023237
0.098685	0.022396	0.021808
0.100703	0.022150	0.021358
0.109243	0.021151	0.019015
0.111503	0.020897	0.018272
0.118614	0.020122	0.015658
0.121101	0.019860	0.014672
0.128944	0.019060	0.011505

Table 3: Critical Switching Rates as Refinancing Costs are Changed, and as Functions of Maturity. The first column represents the time to maturity of the bond in years and the first row the total refinancing costs $1+\pi+f$. The zeroes after 22 years represent the initial blackout period. The zeroes close to expiration are "optimal" non-switching because costs will not be recovered.

$t \backslash 1+\pi+f$	*1.001*	*1.008*	*1.016*	*1.023*	*1.031*	*1.038*	*1.046*	*1.053*	*1.061*	*1.068*
0.125	0	0	0	0	0	0	0	0	0	0
0.25	0.076	0.016	0	0	0	0	0	0	0	0
0.375	0.079	0.048	0.018	0	0	0	0	0	0	0
0.5	0.079	0.06	0.039	0.018	0	0	0	0	0	0
0.625	0.079	0.066	0.05	0.034	0.019	0	0	0	0	0
0.75	0.079	0.07	0.057	0.044	0.032	0.019	8E-04	0	0	0
0.875	0.079	0.073	0.062	0.051	0.04	0.029	0.019	0.008	0	0
1	0.079	0.074	0.066	0.056	0.047	0.038	0.028	0.019	0.01	0
1.25	0.079	0.076	0.07	0.063	0.056	0.048	0.041	0.033	0.026	0.019
1.5	0.079	0.078	0.074	0.069	0.062	0.056	0.05	0.043	0.038	0.031
1.75	0.079	0.079	0.076	0.072	0.067	0.062	0.056	0.051	0.045	0.04
2	0.079	0.079	0.078	0.074	0.07	0.066	0.062	0.057	0.051	0.047
2.25	0.079	0.079	0.078	0.076	0.073	0.07	0.066	0.061	0.057	0.053
2.5	0.079	0.079	0.078	0.078	0.074	0.072	0.069	0.065	0.061	0.057
3	0.079	0.079	0.078	0.078	0.076	0.074	0.073	0.07	0.067	0.065
3.5	0.079	0.079	0.078	0.078	0.078	0.076	0.076	0.074	0.072	0.07
4	0.079	0.079	0.078	0.078	0.078	0.076	0.076	0.074	0.074	0.073
22	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0

Table 4. Critical Switching Rates when volatility is changed for the case when $L = k = \gamma = 0$. Also, there are no transaction costs or Call Premium, but the initial Blackout Period was set to three years. The first column shows time to maturity in years while the first row the values of sigma. The "optimal" policy is not to call when time to maturity is large because the probability of negative rates in this case increases with time to maturity.

t/σ	*0.04*	*0.049*	*0.058*	*0.067*	*0.076*	*0.085*	*0.094*	*0.103*	*0.112*	*0.121*
0.125	0.0800	0.0800	0.0800	0.0800	0.0800	0.0800	0.0800	0.0800	0.0800	0.0800
0.25	0.0791	0.0791	0.0791	0.0791	0.0791	0.0791	0.0791	0.0791	0.0775	0.0775
0.375	0.0745	0.0760	0.0745	0.0730	0.0730	0.0715	0.0701	0.0687	0.0673	0.0659
0.5	0.0715	0.0715	0.0701	0.0687	0.0673	0.0659	0.0633	0.0620	0.0595	0.0571
0.625	0.0687	0.0701	0.0673	0.0646	0.0633	0.0608	0.0583	0.0548	0.0525	0.0493
0.75	0.0659	0.0673	0.0646	0.0620	0.0595	0.0559	0.0525	0.0493	0.0462	0.0413
0.875	0.0633	0.0659	0.0620	0.0595	0.0559	0.0525	0.0482	0.0442	0.0404	0.0351
1	0.0620	0.0633	0.0608	0.0571	0.0525	0.0493	0.0452	0.0404	0.0351	0.0287
1.125	0.0595	0.0620	0.0583	0.0548	0.0503	0.0462	0.0413	0.0359	0.0294	0.0223
1.25	0.0583	0.0608	0.0559	0.0525	0.0472	0.0432	0.0377	0.0318	0.0243	0.0159
1.375	0.0571	0.0595	0.0548	0.0503	0.0452	0.0404	0.0342	0.0272	0.0190	0.0093
1.5	0.0548	0.0583	0.0525	0.0482	0.0432	0.0377	0.0310	0.0229	0.0142	0.0026
1.75	0.0525	0.0559	0.0503	0.0442	0.0385	0.0318	0.0243	0.0148	0.0026	0
2.125	0.0493	0.0525	0.0462	0.0394	0.0326	0.0250	0.0148	0.0017	0	0
2.625	0.0452	0.0482	0.0413	0.0342	0.0257	0.0159	0.0013	0	0	0
3.375	0.0394	0.0442	0.0359	0.0272	0.0165	0.0008	0	0	0	0
4.5	0.0334	0.0385	0.0287	0.0178	0.0017	0	0	0	0	0
6.75	0.0243	0.0302	0.0184	0.0008	0	0	0	0	0	0
13.625	0.0103	0.0190	0.0004	0	0	0	0	0	0	0
22	0.0053	0.0154	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0

Table 5. Critical Rates as we change sigma (σ) for different times to maturity. The first column contains the times to maturity in years while the first row shows the values of sigma. Note that for times to maturity less than two years the critical rates does not depend on the level of volatility. Basically, the time left is so small that the probability of drastically lower rates is not affected by the level of sigma.

$t \setminus \sigma$	*0.04*	*0.049*	*0.058*	*0.067*	*0.076*	*0.085*	*0.094*	*0.103*	*0.112*	*0.121*
0.125	0	0	0	0	0	0	0	0	0	0
0.75	0	0	0	0	0	0	0	0	0	0
1	0.0093	0.0093	0.0093	0.0093	0.0093	0.0093	0.0093	0.0093	0.0093	0.0093
1.125	0.0184	0.0184	0.0184	0.0184	0.0184	0.0184	0.0184	0.0184	0.0184	0.0184
1.25	0.0257	0.0257	0.0257	0.0257	0.0257	0.0257	0.0257	0.0257	0.0257	0.0257
1.375	0.0318	0.0318	0.0318	0.0318	0.0318	0.0318	0.0318	0.0318	0.0318	0.0318
1.5	0.0368	0.0368	0.0368	0.0368	0.0368	0.0368	0.0368	0.0368	0.0368	0.0359
1.75	0.0442	0.0442	0.0442	0.0442	0.0442	0.0442	0.0442	0.0442	0.0442	0.0442
2	0.0503	0.0503	0.0503	0.0503	0.0503	0.0503	0.0503	0.0503	0.0503	0.0493
2.25	0.0559	0.0559	0.0559	0.0559	0.0559	0.0548	0.0548	0.0548	0.0548	0.0536
2.5	0.0595	0.0608	0.0595	0.0595	0.0595	0.0595	0.0583	0.0583	0.0571	0.0571
2.75	0.0633	0.0633	0.0633	0.0633	0.062	0.062	0.0608	0.0608	0.0595	0.0595
3	0.0659	0.0673	0.0659	0.0659	0.0646	0.0646	0.0633	0.0633	0.062	0.0608
3.5	0.0701	0.0715	0.0701	0.0687	0.0687	0.0673	0.0659	0.0659	0.0646	0.0633
4	0.073	0.073	0.0715	0.0715	0.0701	0.0687	0.0687	0.0673	0.0659	0.0659
4.5	0.073	0.073	0.073	0.0715	0.0715	0.0701	0.0687	0.0687	0.0673	0.0659
5	0.073	0.073	0.073	0.073	0.0715	0.0715	0.0701	0.0687	0.0673	0.0673
5.5	0.073	0.073	0.073	0.073	0.0715	0.0715	0.0701	0.0687	0.0687	0.0673
6	0.073	0.073	0.073	0.073	0.0715	0.0715	0.0701	0.0701	0.0687	0.0673
22	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0

Table 6: Critical switching rates as the long term value of the instantaneous rates (L) is changed from 1 basis point to 721 basis points. As L is decreased the critical rate also decreases because the interest rate will tend towards that value. Note the non-monotonic relationship for the critical rate. For L equal to one basis point the highest critical rate is 744 basis points at six years to maturity. For L equal to 321 the highest value is 730 at ten years.

t \ *L*	*0.0001*	*0.0081*	*0.0161*	*0.0241*	*0.0321*	*0.0401*	*0.0481*	*0.0561*	*0.0641*	*0.0721*
0	0	0	0	0	0	0	0	0	0	0
0.875	0.010878	0.010878	0.010346	0.009820	0.008787	0.008278	0.007278	0.006786	0.001708	0.001708
1	0.020287	0.020287	0.019644	0.018382	0.017763	0.017151	0.015949	0.015359	0.014775	0.013629
1.25	0.034231	0.034231	0.032589	0.031785	0.030992	0.030210	0.028678	0.027927	0.026455	0.025734
1.5	0.044163	0.044163	0.042234	0.041292	0.040363	0.038546	0.037658	0.036783	0.035069	0.034231
1.75	0.051407	0.051407	0.050322	0.048204	0.047170	0.045150	0.044163	0.043191	0.041292	0.039448
2	0.057107	0.057107	0.055929	0.054770	0.052510	0.051407	0.049255	0.048204	0.046152	0.044163
2.25	0.062026	0.062026	0.060764	0.059524	0.057107	0.055929	0.053631	0.051407	0.050322	0.048204
2.5	0.065948	0.065948	0.064617	0.063310	0.060764	0.059524	0.057107	0.054770	0.052510	0.051407
2.75	0.070087	0.070087	0.068682	0.065948	0.064617	0.062026	0.059524	0.058305	0.055929	0.053631
3	0.071518	0.071518	0.070087	0.068682	0.067302	0.064617	0.062026	0.060764	0.058305	0.055929
3.5	0.074462	0.074462	0.072976	0.071518	0.070087	0.068682	0.065948	0.064617	0.062026	0.058305
4	0.074462	0.074462	0.072976	0.072976	0.072976	0.071518	0.068682	0.067302	0.064617	0.060764
4.5	0.074462	0.074462	0.074462	0.072976	0.072976	0.071518	0.070087	0.068682	0.065948	0.063310
5	0.074462	0.074462	0.072976	0.072976	0.072976	0.072976	0.071518	0.070087	0.067302	0.064617
6	0.074462	0.074462	0.072976	0.072976	0.072976	0.072976	0.072976	0.071518	0.070087	0.067302
10	0.072976	0.072976	0.072976	0.072976	0.072976	0.072976	0.071518	0.071518	0.071518	0.070087
20	0.072976	0.072976	0.072976	0.071518	0.071518	0.071518	0.071518	0.070087	0.070087	0.068682
22	0	0	0	0	0	0	0	0	0	0
25	0	0	0	0	0	0	0	0	0	0

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Figure 1: Borrower's Bond Value
No Blackout Period

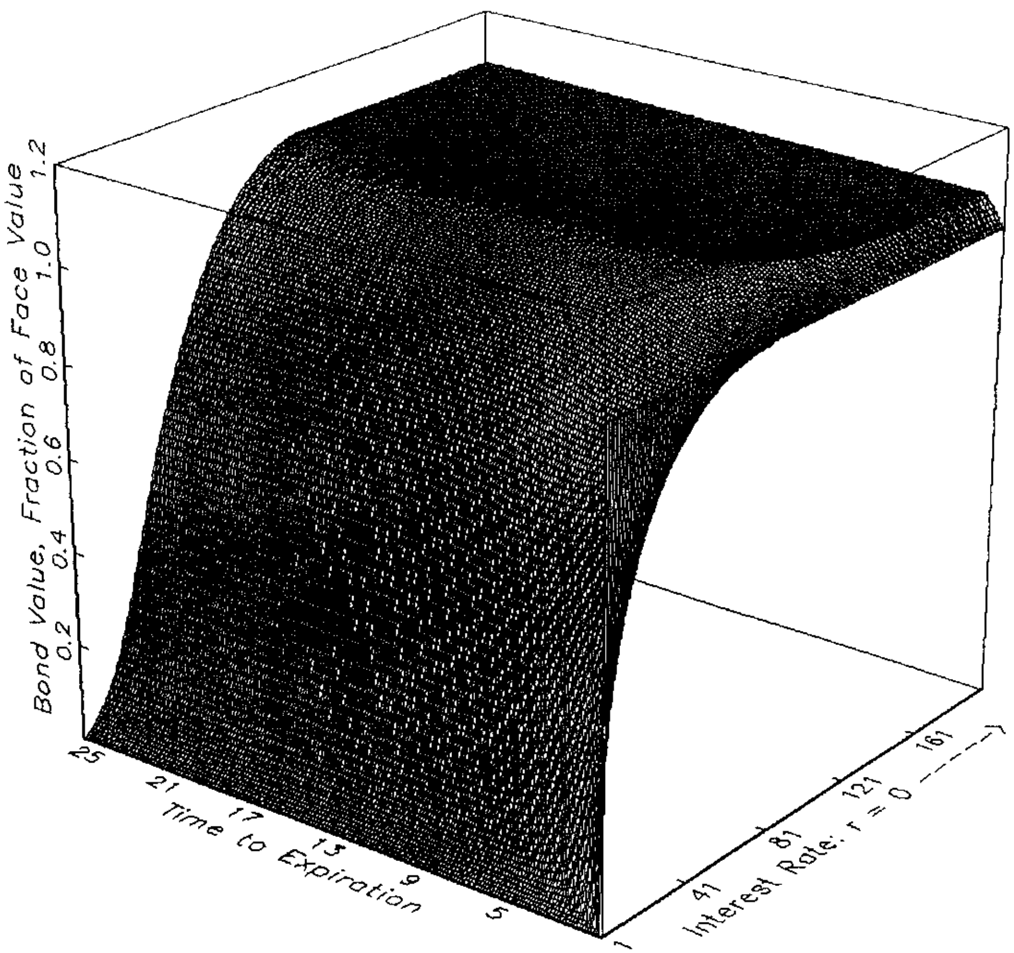

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Figure 2: Borrower's Bond Value
Three Year Blackout Period

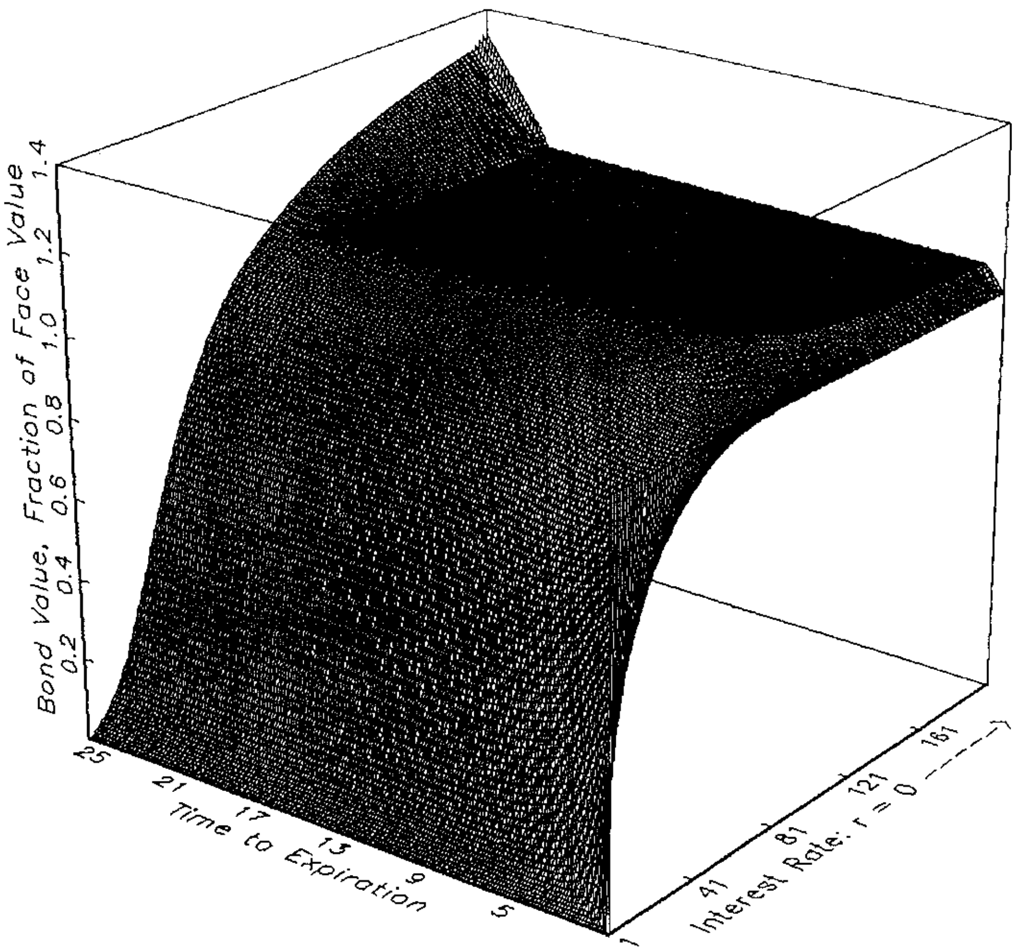

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Figure 3: Critical Rates Compared
Call and Switching Strategies

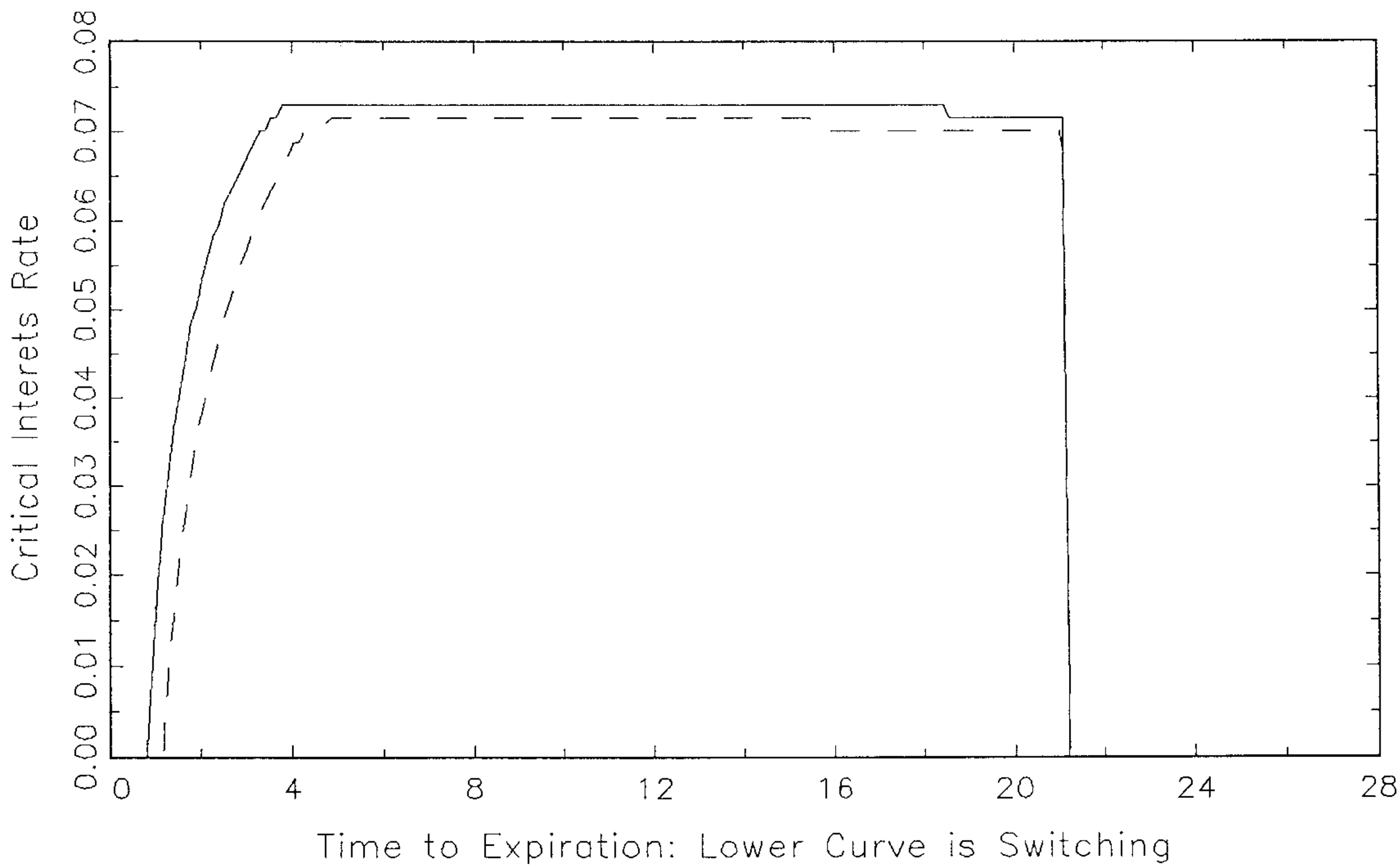

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Figure 4: Bond Price Differences: Call vs. Switching

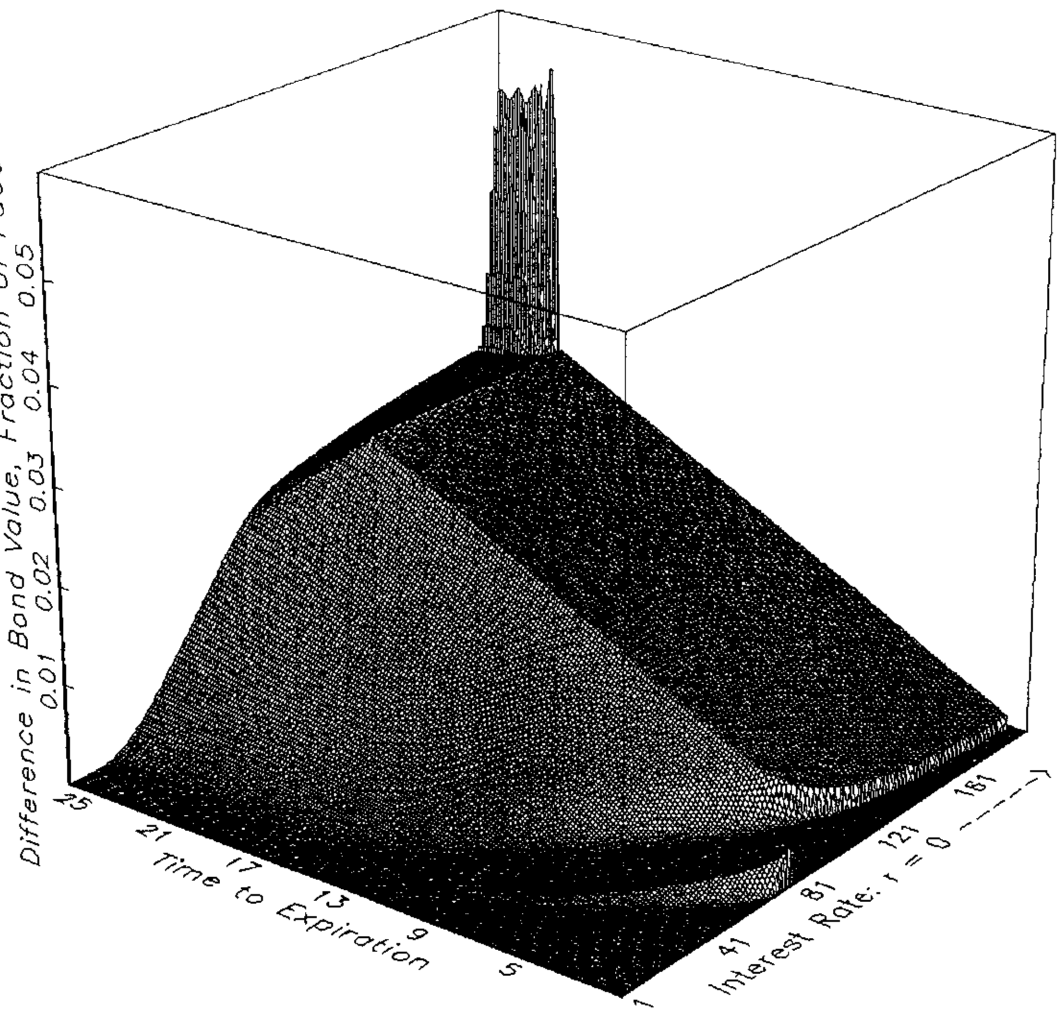

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Figure 5: Crossections (6) of Price Differences: Call vs. Switching
Time to Matirity Equal to 12.5 years

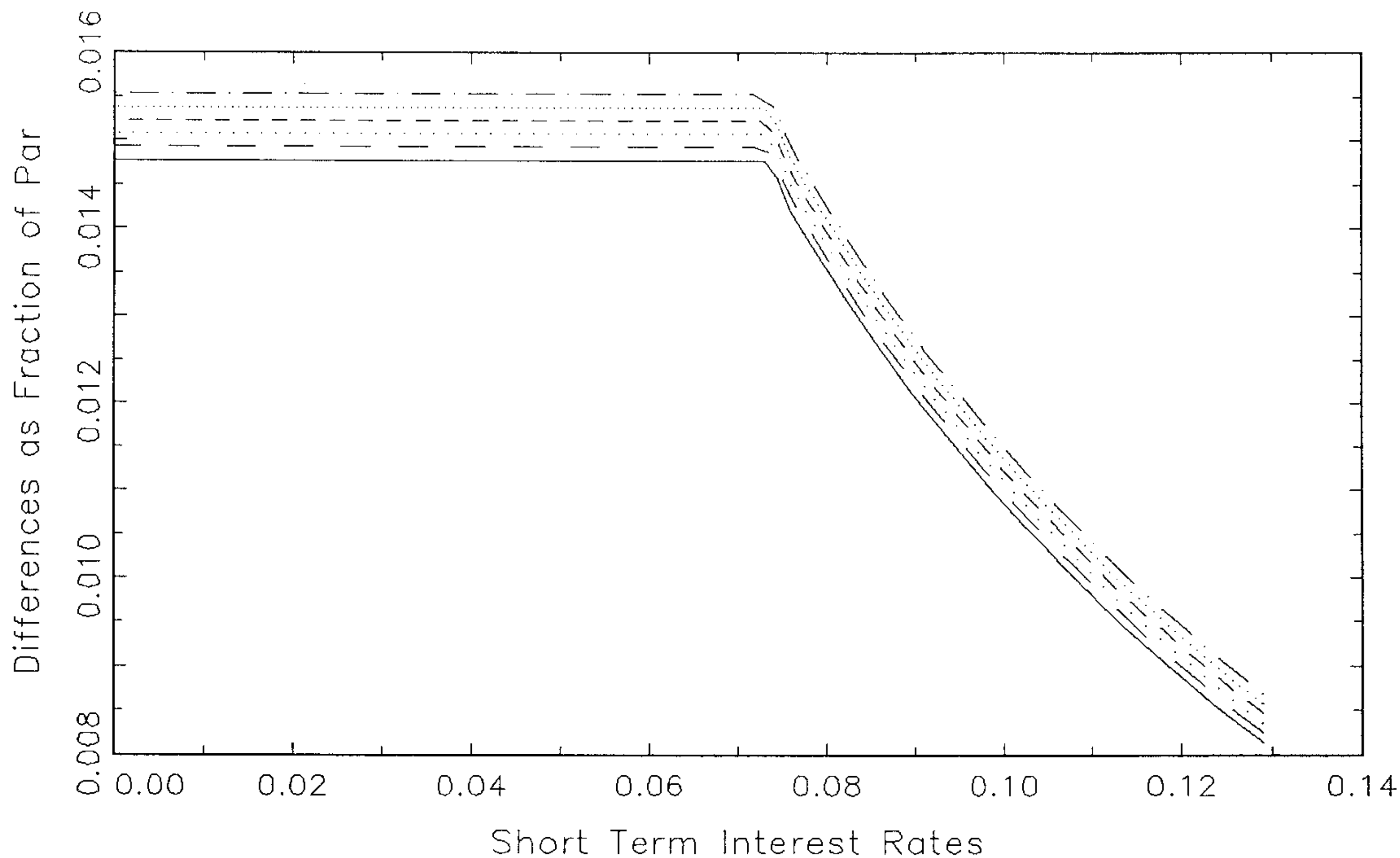

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Figure 6: Crossections (20) of Bond Prices

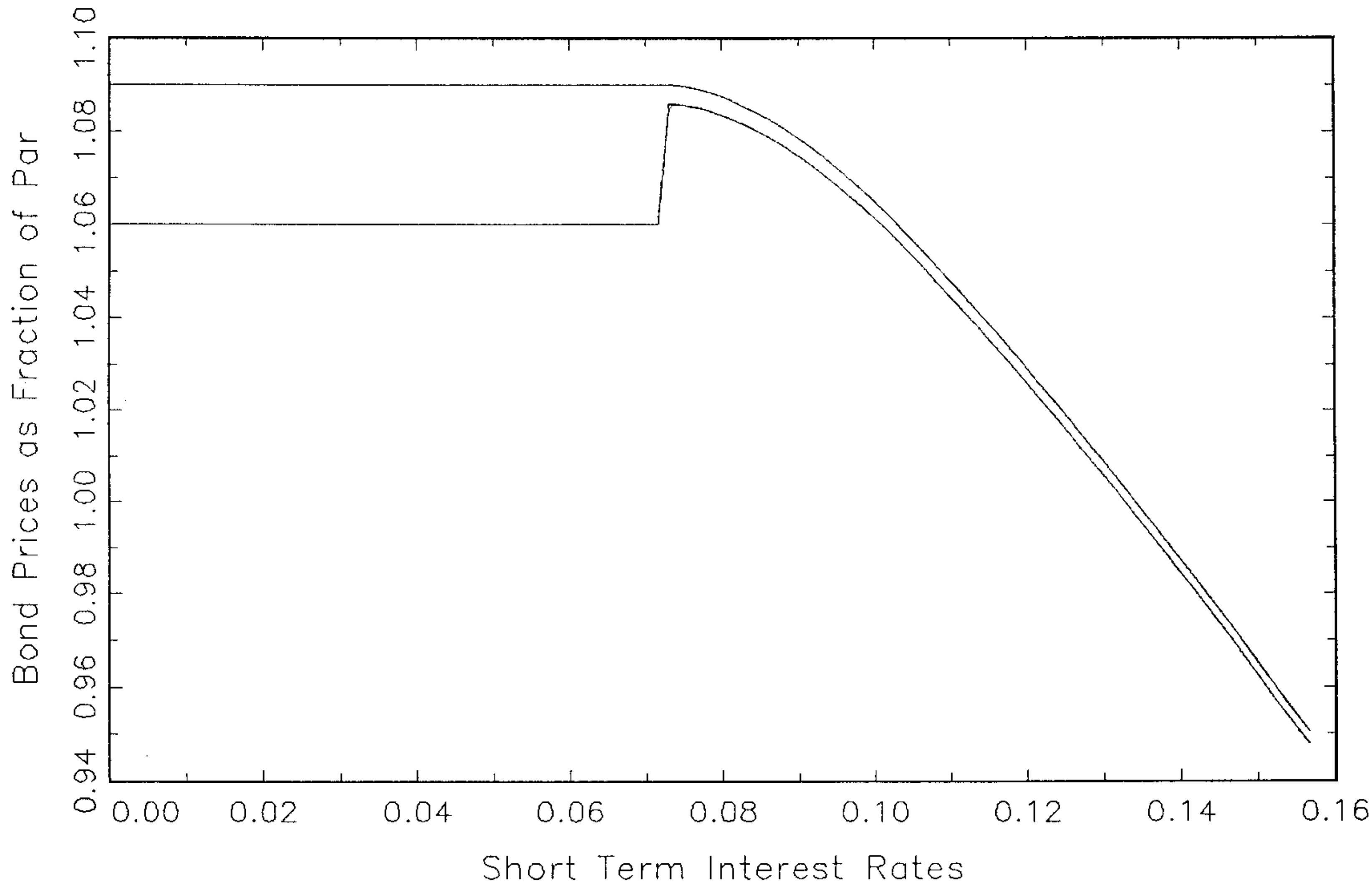

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Figure 7: Critical Rates Compared
Changes in Refinancing Cost

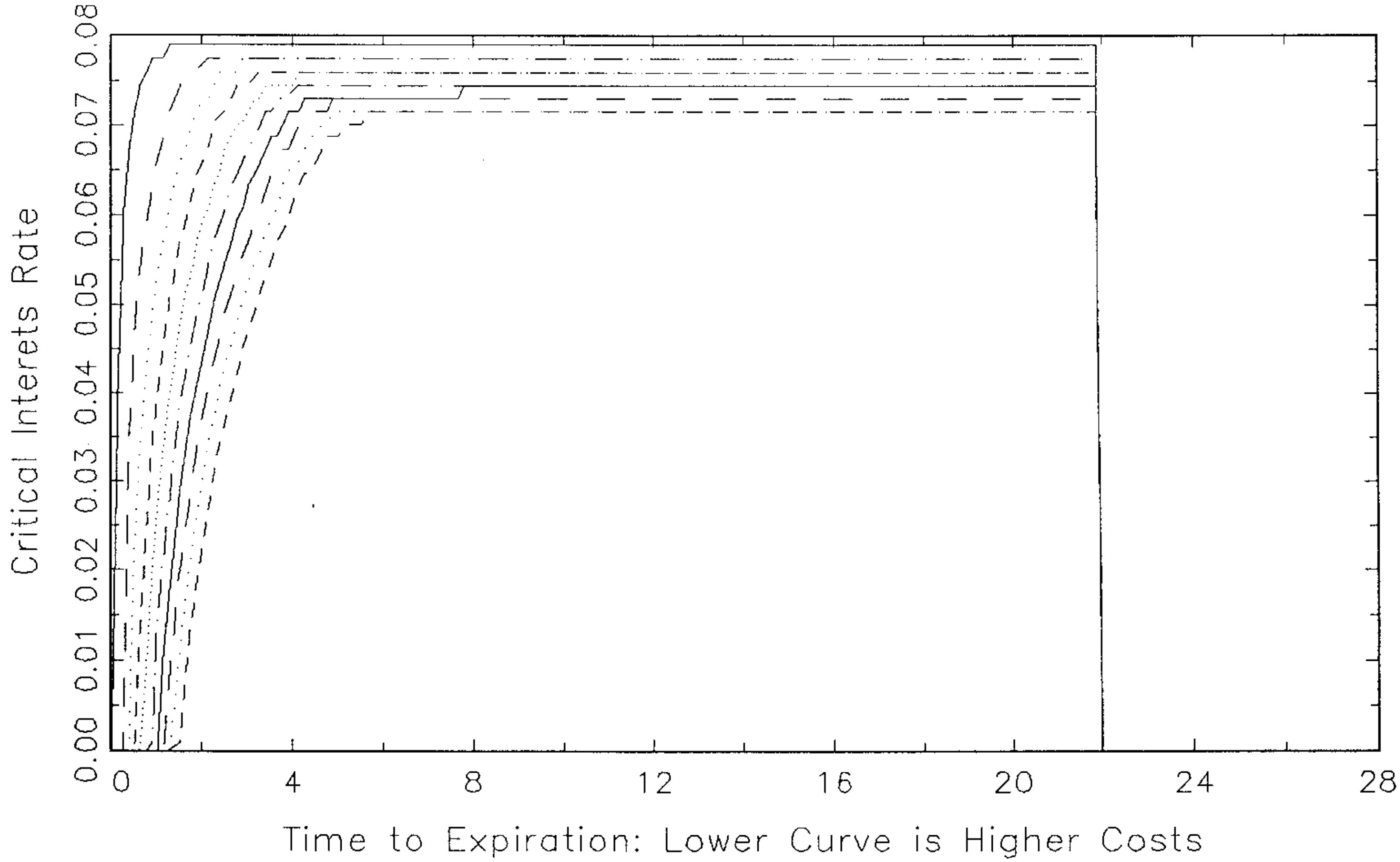

Figure 8: Critical Switching Rates
Varying Refinancing Costs:
Call Premium Plus Flotation Costs

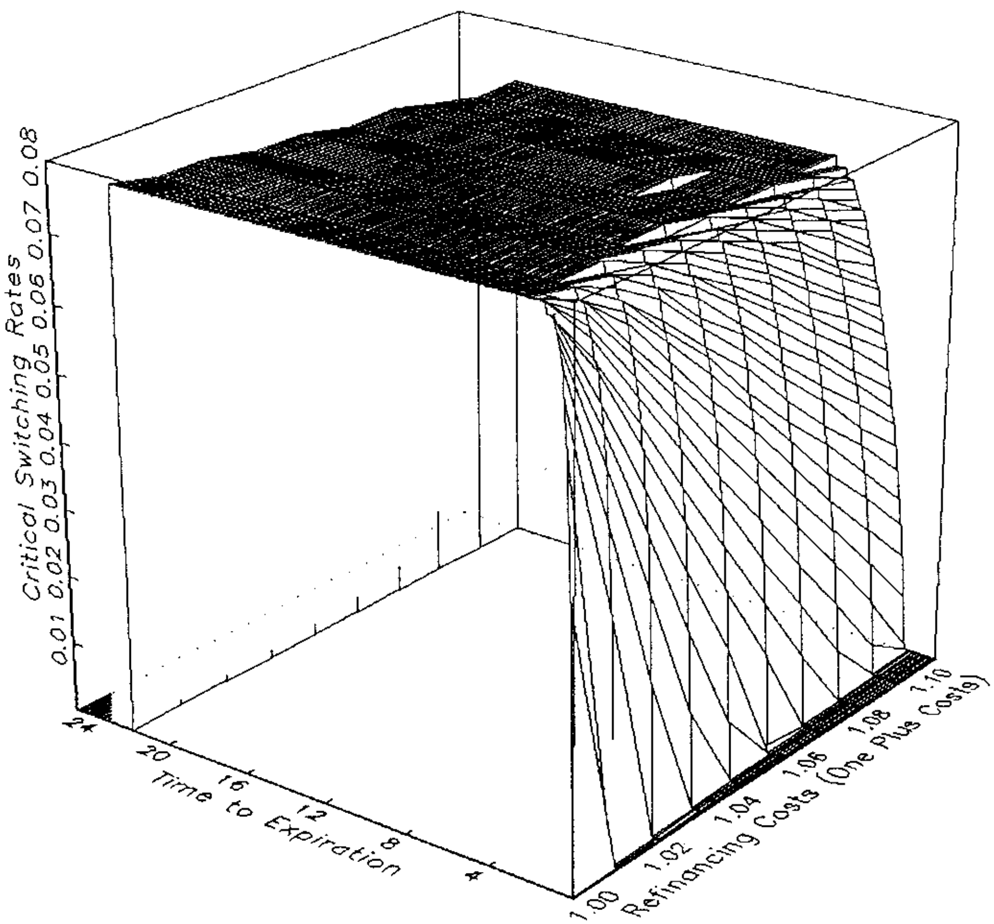

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Figure 9: Critical Rates Compared For Changes in Volatility (Sigma) k=L=gamma=0

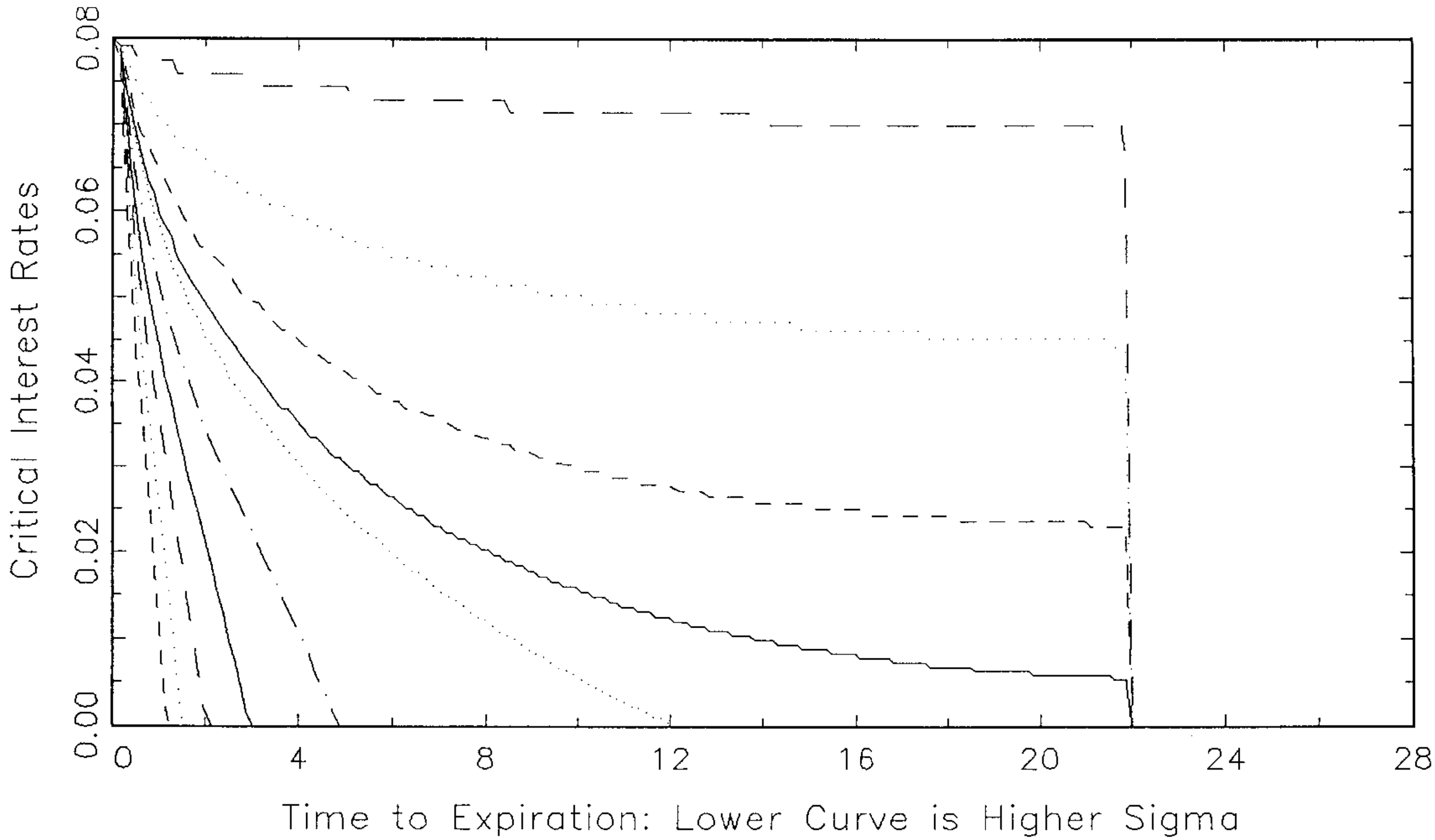

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Figure 10: Critical Rates Compared For Changes in Volatility (Sigma)

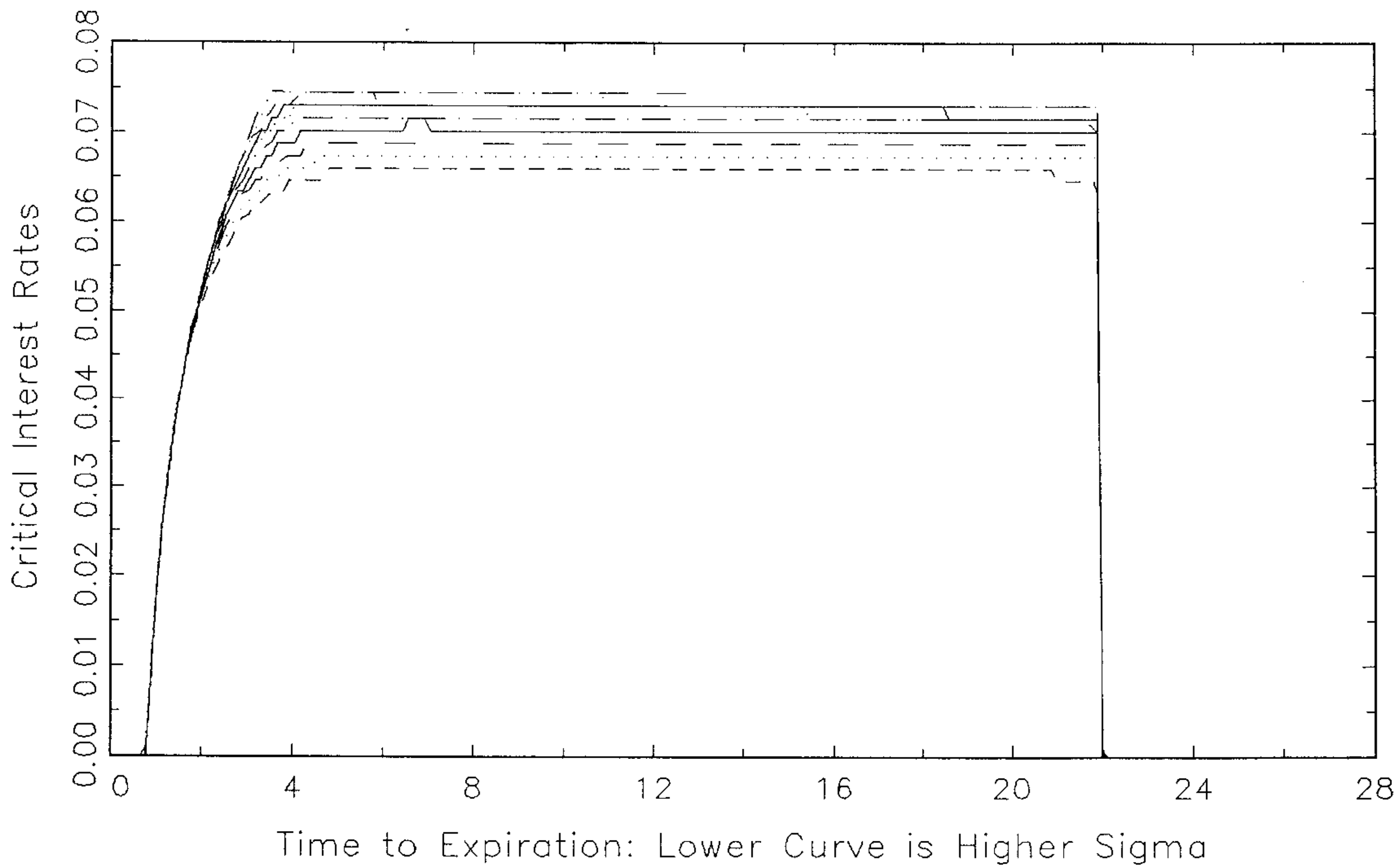

Figure 11: Critical Switching Rates for Varying Sigmas

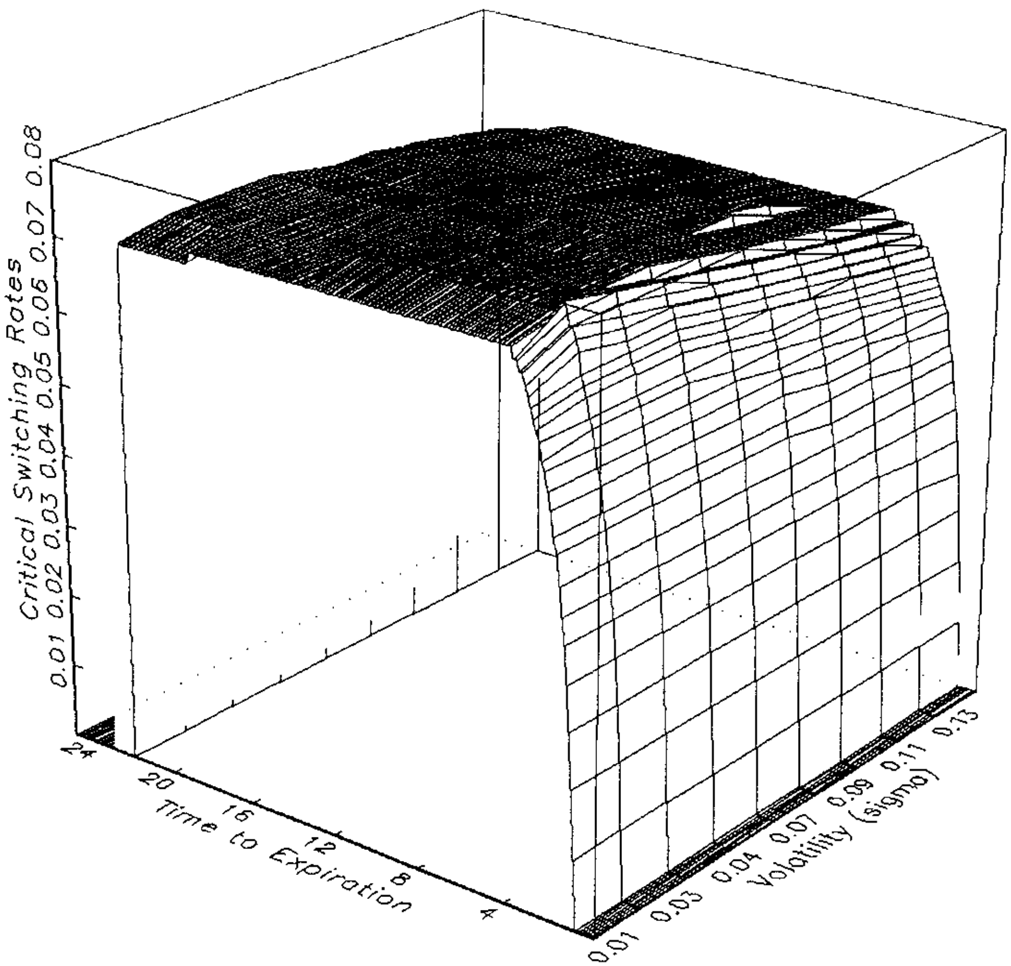

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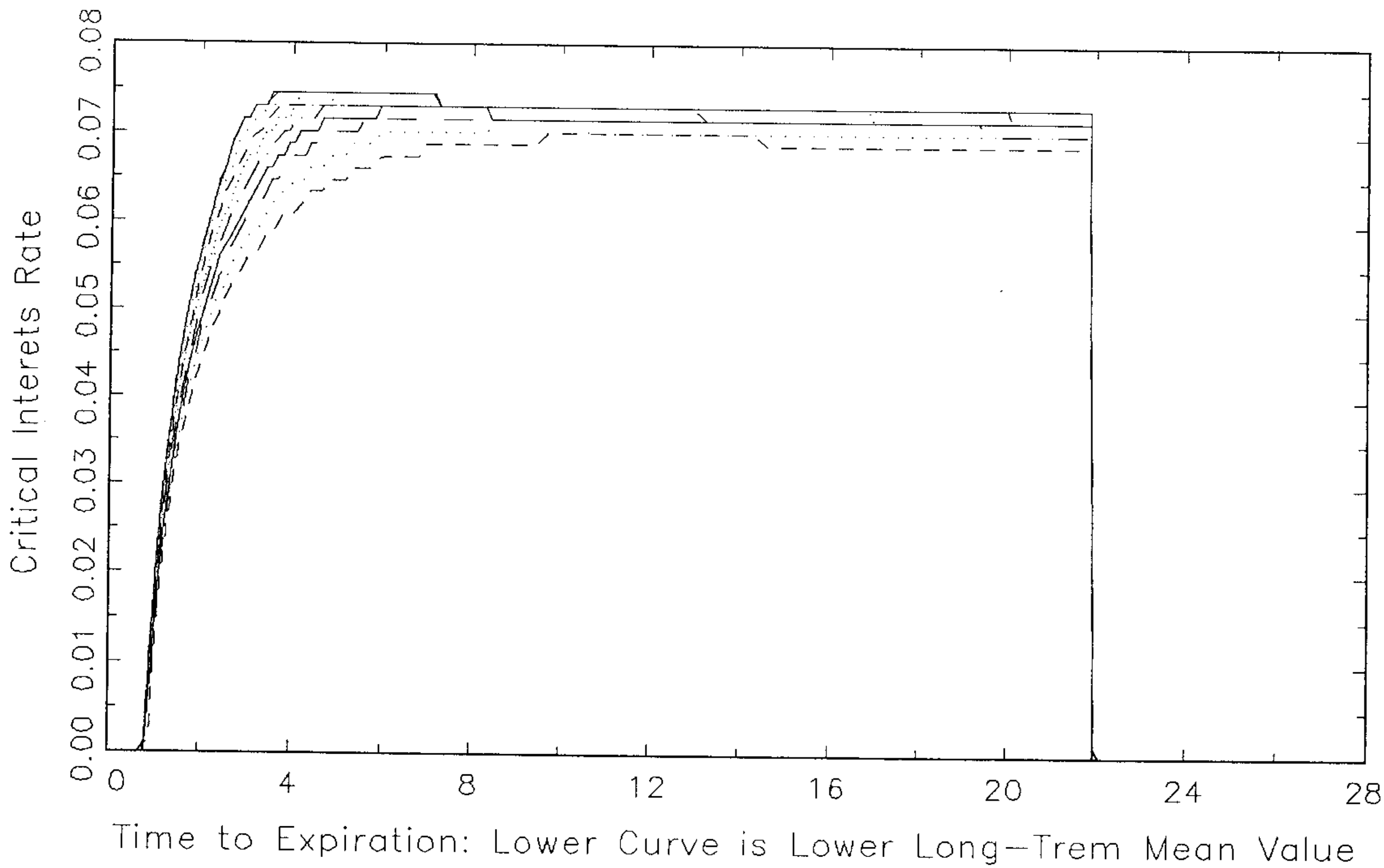
Figure 12: Critical Rates Compared For Changes in Long Term Mean (L)



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