RATIONAL EXPECTATIONS, INFLATION AND THE NOMINAL INTEREST RATE

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Abstract

There is a substantial empirical literature, beginning with Fama [1975], that utilizes regressions of the inflation rate in a given period on initial interest rates (or inflation differentials on the slope of the initial yield curve) to test the Fisher hypothesis and/or to provide forecasts of inflation. Both uses depend critically on the maintained hypothesis that asset market prices fully incorporate all relevant current information about future yields. This paper will investigate the plausibility of the rational expectations hypothesis for real returns in markets for one-period default-free bonds, will show that under normal macroeconomic assumptions it cannot be expected to hold, and will consider the consequences of its failure for the interpretation of empirical results.

In the first section, we show that deviations of the market forecast of inflation from the rational expectations forecast will cause the slope coefficient in regressions of next period's inflation on current nominal yields to be asymptotically biased away from its expected value of 1, invalidating tests of the Fisher hypothesis based on these regressions.

In Section II we examine the deviation of the market forecast from the rational expectations forecast and show that it cannot under reasonable assumptions be identically equal to zero. In Section III we investigate the effect on this deviation of rational investors who correct their own inflation forecasts to take advantage of the information on others' private signals that is contained in current asset price.

In Section IV we attempt to assess the implications of the failure of the rational expectations hypothesis for the interpretation of a variety of empirical results. Section V concludes.

I

The *ex post* real return on a one-period bond, r'_{t+1}, is definitionally related to the initial price per dollar of face value, P_t , and the realized inflation rate, π_{t+1}:

$$\begin{aligned} r'_{t+1} &= -\ln P_t - \pi_{t+1} \\ &= i_t - \pi_{t+1} \, . \end{aligned}$$

The Fisher hypothesis relates the *ex ante* real return to the nominal rate and the market's expectation of inflation in a similar way:

$$i_t = \hat{r}_t + \hat{\pi}_t \, . \tag{1}$$

Here $\hat{r}_t$ is the market-clearing real rate at the end of period t and $\hat{\pi}_t$ is the market forecast of next period's inflation at that time, defined as the rate which, if held by all market participants, would lead to the price and nominal yield actually observed.

Obviously $\hat{\pi}_t$ is a function of individual investors' expectations, $\hat{\pi}_t^j$, where j runs from 1 to N across market participants. Some individuals' expectations may be based on naive models—e.g., that next period's inflation rate will be zero, or equal to the current rate, or equal to the average rate over the last 3 or 5 or 10 years. Other investors may utilize more plausible, but fairly low-cost techniques, such as autoregressive time series models or small-scale econometric models. Some undoubtedly utilize sophisticated econometric models which, while they are likely to be more accurate on average, are much more expensive than simplistic approaches. It is important to note that there are several large-scale models that are used routinely for macroeconomic forecasting and that they do not generate identical forecasts of inflation. At most one of them can be correct, but they all survive because existing data are insufficient to distinguish conclusively among them.

The rational expectations hypothesis holds that the bond market will so aggregate these diverse individual forecasts as to produce a market forecast, $\hat{\pi}_t = i_t - \hat{r}_t$, with very special characteristics. Let the rational expectations forecast of π_{t+1} be π_t^* and let I stand for all information available to market participants at t. Then

$$\pi_{t+1} = \pi_t^* + \epsilon_{t+1} , \quad (2)$$

where

$$\pi_t^* = E(\pi_{t+1} \mid I)$$

and ϵ_{t+1} is a stochastic variable with zero mean and is independent of any element of the information set I. If in a given time series sample $\hat{\pi}_t \equiv \pi_t^*$ and $\hat{r}_t$ is constant at $\bar{r}$, then from (1)

$$i_t = \bar{r} + \pi_t^* ,$$

and it is obvious that a regression of π_{t+1} on the nominal rate, i_t, will yield a constant term with probability limit $-\bar{r}$ and a slope coefficient with probability limit 1. The finding of a slope coefficient significantly different from 1 rejects either the Fisher hypothesis (1) or the assumption $\hat{\pi}_t \equiv \pi_t^*$ or the assumption $\hat{r}_t = \bar{r}$.

If, however,

$$\hat{\pi}_t = \pi_t^* + v_t , \quad (3)$$

where v_t is a random variable, then by substituting (3) and (1) in (2), we have

$$\pi_{t+1} = i_t - \bar{r} - v_t + \epsilon_{t+1}$$

and the probability limit of the slope coefficient, $\hat{b}$, is

$$\text{plim}(\hat{b}) = \frac{\sigma_i^2 + \text{cov}(i_t, -v_t + \epsilon_{t+1})}{\sigma_i^2} \ .$$

Since in this case

$$i_t = \bar{r} + \pi_t^* + v_t \ ,$$

we have

$$\text{plim}(\hat{b}) = 1 - \frac{\sigma_v^2 + \text{cov}(\pi_t^*, v_t)}{\sigma_i^2} \ . \tag{4}$$

It follows that, even in the special case of a constant *ex ante* real rate, the evidence from regressions of inflation on the lagged nominal rate are worthless as a test of the Fisher hypothesis unless it can be shown that the market forecast of inflation is identically equal to the rational expectations forecast. A similar argument holds with respect to changes in the inflation rate when regressed against the slope of the initial yield curve.

Depending on the size of the bias term, such regressions may still provide useful forecasts of the time path of future inflation, but that must be tested through comparison with alternative methods of prediction.

In the next section we take a closer look at the rational expectations forecast of inflation, the market forecast implicit in the current price of a one-period bond, and the difference between them, v_t. We will show that v_t, as a function of random variables, cannot be identically equal to zero.

II

In specifying the rational expectations forecast, we make the standard assumption that inflation is one of several macroeconomic variables jointly determined each period by a system of stochastic structural equations. End-of-period asset prices are also determined by a stochastic equation system, assumed separable from the first in that outcomes are affected by macroeconomic outcomes in t, but affect only those in t+1 or later.

We further assume that inflation can be expressed in reduced form as a function of past realizations of system variables and contemporaneous values of exogenous variables plus an uncorrelated disturbance term. Let

$$\pi_{t+1} = \varphi(X_t, Z_{t+1}) + \epsilon_{t+1} ,$$

where X_t is a vector of system variables realized in period t or earlier and Z_{t+1} is the vector of exogenous variables for period t+1. If the elements of Z_{t+1} are precisely known at t, then the rational expectations forecast is simply

$$\pi_t^* = \varphi(X_t, Z_{t+1}) . \tag{5}$$

It is likely, however, that some elements of Z_{t+1} are not known at t, but may be predicted with some accuracy from, say, anticipatory data. To the extent that plans or attitudes of economic agents are independent of the elements of X_t, they may well proxy for exogenous factors operating in t+1 and so must be taken into account in the rational expectations forecast. For example, the exogenous component of next period's government expenditure (presumably reflecting legal and political considerations) may be estimated from budget data, while the exogenous components of investment (reflecting technological change or entrepreneurs' "animal spirits") and of consumption (reflecting tastes and consumer attitudes) may be predicted to some extent from published surveys of spending plans.

In this case we may define

$$\begin{aligned} \pi_t^* &= \varphi\big[X_t, E(Z_{t+1} \mid S_t)\big] \qquad (5a) \\ &= \psi(X_t, S_t) , \end{aligned}$$

where S_t is a vector of indicator variables that do not enter into the "true" macroeconomic model, but are correlated with elements of Z_{t+1}. The nominal yield on one-period bonds may itself serve as an indicator variable to the extent that it incorporates private signals as to next period's realization of exogenous variables.

We next specify the market forecast of inflation, $\hat{\pi}_t$, in terms of the market for a one-period default-free bond. Let the demand of investor j at the end of period t be a function of the *ex ante* real return for the following period, a vector of prices (P_t') of alternative assets, the investor's tastes (T^j) and wealth (W_t^j), and vectors X_t^j and S_t^j of macroeconomic realizations and private signals that affect expectations as to returns on alternative assets in $t+1$. Then the market-clearing condition is

$$\sum_j \left[D^j(i_t - \hat{\pi}_t^j, P_t', X_t^j, S_t^j, T^j, W_t^j)\right] + u_t^j = Q_t \ , \tag{6}$$

where u_t^j is a random variable reflecting current liquidity needs (or simply a noise component) and Q_t is supply of the asset, assumed to be known with certainty.

Let (6) be one of a system of stochastic equations which clear asset markets at the end of period t. It is convenient to think of demand and perhaps supply in asset markets as affected by certain subsets, X_t^A and S_t^A, of the components of X_t and S_t, while the prices generated affect only realizations in $t+1$ or later. If i_t can be expressed in reduced form as a function of the $\hat{\pi}_t^j$ $(j = 1, 2, \ldots, N)$, the vectors X_t^A and S_t^A and vectors of investor characteristics, T and W_t, then we may write

$$i_t = F(\hat{\pi}_t^{(1)}, \hat{\pi}_t^{(2)}, \ldots, \hat{\pi}_t^{(N)}, X_t^A, S_t^A, T, W_t) + \eta_t \ , \tag{7}$$

where η_t is a random disturbance related to the u_t^j (and to disturbances affecting the demand or supply of other assets).

The market forecast of inflation, $\hat{\pi}_t$, may now be defined as that value which, if expected by all market participants, would have produced the value of i_t actually observed, holding everything else constant. Conceptually this would be determined by substituting $\hat{\pi}_t$ for each of the $\hat{\pi}_t^j$ in (7) and solving for this variable.

We note that $\hat{\pi}_t$, thus defined, is a function of i_t, which is in turn a function of the actual $\hat{\pi}_t^j$. These are themselves functions of (some subset of) the components of X_t and the signalling variables in S_t:

$$\hat{\pi}_t^j = \psi^j(X_t, S_t) \ . \tag{8}$$

At most one of the diverse investor forecasts can be identical with π_t^*. Thus

$$\hat{\pi}_t \quad = \quad G(\psi^{(1)}, \psi^{(2)}, \ldots, \psi^{(N)}, X_t^A, S_t^A, T, W_t, \eta_t)$$

and

$$v_t \;=\; G - \psi(X_t, S_t) = v(X_t, S_t, T, W_t, \eta_t)\ . \tag{9}$$

Since the components of X_t and S_t, as well as η_t, are random variables, v_t cannot be identically equal to zero except in the trivial case when all the coefficients in (9) are zero.

III

It is nevertheless of some interest to investigate the impact on v_t (and more specifically on its variance) if rational market participants use the observed nominal rate, i_t, as one of the signals affecting their inflationary expectations.

Let us suppose an initial situation in which investors do not use i_t in formulating their expectations and in which $v_t \neq 0$. Note that neither the market forecast, $\hat{\pi}_t$, nor the market-clearing *ex ante* real rate, $\hat{r}_t$, is observed, but only their sum, i_t. Thus investors cannot directly compare $\hat{\pi}_t$ with the subsequently observed value of π_{t+1} in an attempt to improve their forecasts. The situation is thus somewhat different from the standard asset-market analysis in which investors can use a historical relationship relationship between asset price and its one-period lag to extract information about the private signals of other market participants.

However a rational investor in the market for one-period bonds will attempt to eliminate systematic errors in his personal prediction, $\hat{\pi}_t^j$, through comparison with π_{t+1} and will further recognize that i_t contains possibly useful information about the inflationary expectations of others. Let us assume that he the reformulates his expectation on the basis of a linear regression of π_{t+1} on i_t and his private signal $\hat{\pi}_t^j$:

$$\hat{\pi}_{It}^j = \hat{b}_I^j\, i_t + \hat{c}_I^j\, \hat{\pi}_t^j$$

(with all variables measured from their means). Let all other investors reformulate their inflation predictions in a similar way. This behavior should reduce the errors of individual investors, especially those using naive models, and presumably it will bring the market forecast closer to the rational expectations forecast.

Note that i_t may be thought of as measuring π_t^* subject to two sources of error:

$$i_t = \pi_t^* + v_t + \hat{r}_t \ . \tag{10}$$

How useful the inclusion of i_t will be in improving individual forecasts depends in large part on the variance of the measurement error.

Under the new regime, the asset demand function is changed and so is the market-clearing value of i_t for given values of non-expectational variables. Substituting the new forecasts, $\hat{\pi}_{It}^j$, which are functions of i_t, for the private signals, $\hat{\pi}_t^j$ in (7) and solving the resulting implicit function for i_t, we have (with the time subscripts now suppressed):

$$i = F_I(\hat{\pi}^{(1)}, \hat{\pi}^{(2)}, \ldots, \hat{\pi}^{(N)}, X^A, S^A, T, W) + \eta_I \ . \tag{7a}$$

This changes the market forecast of inflation to $\hat{\pi}_I$—obtained by substituting (7a) in (1)—and the market's error to v_I $(= \hat{\pi}_I - \pi^*)$ and thus changes the relationship of i to the rational expectations forecast and to next period's observed inflation. Now

$$i - \pi^* = v_I + \hat{r}$$

is the measurement error in i as a proxy for π^* and if the variance of v_I is less than that of v, investors may further improve their forecasts by recomputing their regressions with data generated under the new regime. Once this is done, the inflation forecast of investor j will be

$$\hat{\pi}_{II}^j = \hat{b}_{II}^j \, i + \hat{c}_{II}^j \, \hat{\pi}^j \ .$$

Again this refinement of individual forecasts changes the market forecast, requiring still further recomputation of the regressions used by individual investors. Let us suppose that this process leads eventually to an equilibrium in which

$$\hat{\pi}_E^j = \hat{b}_E^j \, i + \hat{c}_E^j \, \hat{\pi}^j \ . \tag{11}$$

We should also consider the possibility that forecasts may be improved by taking account of the impact on the *ex ante* real rate of (components of) X^A and S^A and of serial correlation in the random disturbances affecting asset markets. Rational investors who are aware of this possibility will expand their forecasting equations to include additional variables related to the conditional expectation of $\hat{r}$ in the current period. In this case the forecasting equation of investor j will converge to:

$$\hat{\pi}^j_E = -\hat{r}^j + \hat{b}^j_E\, i + \hat{c}^j_E\, \tilde{\pi}^j\ , \tag{11a}$$

where $\hat{r}^j$ is a function of the added variables, with coefficients as estimated by regression analysis, and is positively correlated with $\hat{r}$. The effect is to introduce negative covariance between v_E, which is a function of the $\hat{\pi}^j_E$, and $\hat{r}$, thus reducing the variance of $i - \pi^*$ $(= v_E + \hat{r})$.

We conclude that the variance in $i - \pi^*$ present in the initial regime can be reduced but not eliminated through the replacement of $\hat{\pi}^j$ with $\hat{\pi}^j_E$. In particular, some part of the variance due to $\hat{r}$ will remain unless all of the $\hat{r}^j$ are equal to $\hat{r}$. Even apart from this, so long as several alternative models remain in use (yielding distinct inflation estimates), the market's error, v_E, must remain a function of random variables, including components of X and S. Thus its variance will not vanish and informational efficiency will not be achieved.

This result is entirely consistent with a body of literature that dicusses the impact of random disturbances in asset markets on their informational efficiency and outlines the characteristics of a noisy rational expectations equilibrium. See, for example, Diamond and Verrecchia [1981], Grossman [1976] and [1981], Grossman and Stiglitz [1980] and Hellwig [1980]. For a recent investigation of the possible divergence of the inflation expectations of bond market participants from that implied by the conventional definition of rationality, see Evans and Lewis [1995, forthcoming].

Variance in $i - \pi^*$ leads to asymptotic bias in individuals' forecasting regressions (11a). In practice, sampling error may also be important unless the equilibrium represented by (11) or (11a) has been long in place. To make some judgment as to the probable magnitude of the bias, we must consider whether or not individual mistakes are likely to reinforce each other.

If, in the initial regime, the market error, v, is uncorrelated with the rational expectations forecast, π^*, we would expect values of $\hat{b}^j$ and $\hat{c}$ to lie between zero and 1. Furthermore, we expect the $\hat{b}^j$ to move toward 1 and the $\hat{c}^j$ to move toward zero as individual forecasts become more accurate and i therefore becomes a better proxy for π^*. For naive investors, $\hat{c}^j$ may well vanish entirely. The general downward bias in the $\hat{b}^j_E$, due to the remaining variance in $i - \pi^*$, produces a general tendency toward underestimates in periods when π^* is above its mean and a general tendency toward overestimation when π^* is below its mean. Thus individual forecast errors are likely to move up or down together depending on the value of π^*, serving to reinforce each other.

Some reinforcement may also arise from the term in $\tilde{\pi}^j$ in periods when a rarely occurring event enters the picture—for example, the oil shocks of the 1970s. Prior to that time most analysts probably had not built oil prices into their inflation models and so most models consistently underestimated in that period. Similarly, most analysts may well have been slow to take full account of the impact on inflation of the changes in Federal Reserve policy initiated in 1979. Expectations data from sources such as the Livingstone Survey suggest that inflation was persistently underestimated in 1973–74 and 1977–80 and persistently overestimated in 1981–86. (In such periods the assumption that v is uncorrelated with π^* cannot be expected to hold and the $\hat{b}^j$ may fall outside the range from zero to 1.)

In summary, there is no safety in numbers. While certain noise elements in individuals' private signals may average out in large markets, systematic factors that affect individual errors similarly will remain. Important among these are (1) variance in π^*, which (if biases in the $\hat{b}^j$ are similar among investors) leads to similar forecasting errors, and (2) the common delay in adjusting individual forecasting relationships when changes in regime occur.

IV

What does it mean for empirical studies relating inflation to lagged interest rates if market forecasts of inflation deviate persistently from the rational expectations forecast?

There is a very long history of such empirical work. The earliest studies failed to establish any power of interest rates to predict subsequent inflation. To the extent that investors' expectations are adaptive, this is not surprising, since a regression based on nominal yields could not be expected to improve on univariate times series analysis in this case. But if investors are rational and do indeed have relevant information that goes beyond the past history of inflation, then bond yields should have something interesting to tell us about future inflation over various time horizons.

Fama's 1975 paper explores this possibility for one-month Treasury bills over the period from January 1953 to July 1971 and for maturities up to 6 months over the period from March 1959 to July 1971. His tests, although slightly different in form, are equivalent to regressing the inflation rate in period $t+1$ against the initial yield on a one-period security:

$$\pi_{t+1} = \hat{a} + \hat{b}\, i_t + e_{t+1} \ ,$$

(where e_{t+1} is the regression residual) and determining whether the slope coefficient is significantly different from 1. Substituting from (10), the above equation may be written

$$\pi_{t+1} = \hat{a} + \hat{b}(\pi_t^* + v_t + \hat{r}_t) + e_{t+1} \ ,$$

where v_t and $\hat{r}_t$ may be treated as error components in i_t as a measure of π_t^*, due respectively to imperfections in the market forecast of inflation and to the presence of a component in i_t that is noise in this context. It follows that the estimated regression slope will take on its "true" value of 1 only if the sum of the error components has zero variance. Otherwise estimates of $\hat{b}$ are asymptotically biased, with the magnitude of the bias depending on the variance of the measurement error relative to that of π_t^* and on any covariance it may have with π_t^*.

Thus Fama's test is actually a joint test of three separate hypotheses: the Fisher equation ($i_t = \hat{r}_t + \hat{\pi}_t$), the rational expectations hypothesis ($v_t = 0$), and the constancy of the *ex ante* real rate ($\hat{r}_t = \bar{r}$). A fourth hypothesis is also required in a world where taxes exist: the marginal tax rate T must be zero for participants in the asset market.

Allowing for taxes, the Fisher equation becomes

$$i_t(1 - T) = \pi_t^* + v_t + \hat{r}_t \ , \qquad (12)$$

implying a "true" slope coefficient of $1-T$ when i_t is used as a proxy for π_t^*. The assumption that T is zero is not necessarily a bad one in the period that Fama examined (though suspect in an era of money market mutual funds). At that time Treasury bills were predominantly held by investors whose demand was not likely to have been highly tax sensitive. A large volume was held by banks as secondary reserves, for which purpose their extremely high liquidity was important. Secondly, savings and loan associations were holders, in part because they were severely restricted in the assets, other than home mortgages, that they were permitted to hold. Financial intermediaries represented in effect a captive market.

Also Treasury bills were held by nonfinancial businesses as transactions balances to bridge short gaps between receipts and expenditures and by some wealthy individuals who may have had unusual liquidity needs or unusually high risk aversion. A final group of holders, pension funds and state and local retirement funds, were growing in importance during this period, and these nontaxable institutions may well have been the marginal investors in Treasury bills.

Fama's findings were in fact quite favorable to his joint hypothesis. For his full sample and for his final subperiod (August 1964 to July 1971), slope coefficients close to 1 were found for one-month bills, with modest standard errors and reasonable correlation coefficients. For the two remaining subperiods, slope estimates were very different, standard errors were large and correlations tiny. In the case of 6-month bills, the slope coefficient was found to be significantly less than 1.

How we interpret these tests of the Fisher equation depends on the credibility we attach to the two maintained hypotheses. In a time of structural change, it is hard to believe that even the most sophisticated investors can accurately adapt their inflation-forecasting models to the new situation based on a handful of observations. It is also hard to believe in the long-term constancy of the real rate under structural change.

I would argue that significant changes did occur in the economy over Fama's sample. The Eisenhower years were characterized by low inflation and a slowing of output growth. By 1960 output had fallen well below the long-term trend then projected. In the Kennedy/Johnson years, real growth was almost twice as high, with the combined impact of the Vietnam War and the Great Society programs pushing the country into inflation and the first sizeable non-recession government deficits since the end of World War II.

In the final subperiod of Fama's sample, which begins in 1964 and heavily influences the overall results, CPI inflation rose from an average of 1.1% over the previous 5 years to 4.2% by 1968; the GNP deflator rose from an average of 1.8% in the 5 years preceding 1964 to 5% by 1968. Is it really plausible that investors in the Treasury bill market were able to predict these large departures from the norm with perfect accuracy? Just as important, is it plausible that the *ex ante* real rate remained constant in the 1960s while output growth exploded, the personal savings rate remained essentially unchanged, and government saving clearly deteriorated?

I would like to propose an alternative explanation for Fama's results. especially in the final subperiod. For simplicity assume that the real rate remained constant, but that investors persistently underestimated inflation in this period according to the relationship

$$\hat{\pi}_t = d\pi_t^* + \delta_t \ , \quad 0 < d < 1 \ ,$$

where δ_t is a random variable uncorrelated with π_t^*. Then $\mathrm{cov}(\pi_t^*, v_t) = (d-1)\sigma_{\pi^*}^2$, and the estimated slope coefficient is

$$\hat{b} = \frac{d\sigma_{\pi^*}^2}{d^2\sigma_{\pi^*}^2 + \sigma_\delta^2} \ . \tag{13}$$

The effects of persistent understatement and those of random error in the market forecast of inflation (relative to the rational expectations forecast) have offsetting effects on bias:

$$\hat{b} - 1 = \frac{d(1-d)\sigma_{\pi^*}^2 - \sigma_\delta^2}{d^2\sigma_{\pi^*}^2 + \sigma_\delta^2} \ .$$

We note that

$$\hat{b} - 1 \gtrless 0 \ , \quad \text{according as} \quad d(1-d) \gtrless \frac{\sigma_\delta^2}{\sigma_{\pi^*}^2} \ .$$

A value of $\hat{b}$ close to 1 is thus quite consistent with a scenario that seems relatively plausible for this particular period as compared with Fama's two maintained hypotheses. The events of the 1960s were quite unlike anything that had been experienced in the previous three decades.

In response to Fama's paper, questions were raised about the constancy of the *ex ante* real rate and attempts were made, notably by Fama and Gibbons [1982], to control for the impact of variations in this rate upon the estimated relationship of inflation to the nominal rate. In the absence of indexed bonds, such as have existed in the United Kingdom since 1984, it is not easy to determine whether or how much *ex ante* rates change over time. All that can be observed are changes in *ex post* rates and these confound the changes that are sought with unanticipated changes in inflation.

Let the *ex post* rate observed in period $t+1$ be r'_{t+1}. Then

$$r'_{t+1} = \hat{r}_t - (\pi_{t+1} - \hat{\pi}_t)$$

and the change in the *ex post* rate is

$$r'_{t+2} - r'_{t+1} = (\hat{r}_{t+1} - \hat{r}_t) - (\pi_{t+2} - \hat{\pi}_{t+1}) + (\pi_{t+1} - \hat{\pi}_t) \ . \tag{14}$$

If errors in inflation forecasts are not simply white noise, but also reflect discrepancies between the market forecast and the rational expectations forecast, then changes in *ex post* real rates may not be a useful proxy for the changes in *ex ante* real rates that must be controlled.

Fama and Gibbons found slope coefficients of around 1.3 for the period 1953–1977 for monthly (or quarterly) inflation rates when these are regressed against lagged yields on one-month (or 90-day) Treasury bills, without controlling for variation in real rates. This is consistent with the hypothesis of persistent underestimation of inflation which was discussed above and is even more plausible for the longer sample now covered. However, it is *not* possible to explain the finding of a coefficient greater than 1 by invoking variations in the real rate under rational expectations. If $\hat{\pi}_t \equiv \pi^*_t$, but the *ex ante* rate varies, we have

$$\hat{b} = \frac{\sigma^2_{\pi^*} + \operatorname{cov}(\pi^*_t, \hat{r}_t)}{\sigma^2_{\pi^*} + \sigma^2_{\hat{r}} + 2\operatorname{cov}(\pi^*_t, \hat{r}_t)} \ . \tag{15}$$

If $\operatorname{cov}(\pi^*_t, \hat{r}_t) \leq 0$, then $\hat{b}$ is unambiguously biased downward from the theoretically expected value of 1.

While it is not evident *a priori* what to expect regarding the sign of $\operatorname{cov}(\pi^*_t, \hat{r}_t)$, some interesting insights are provided in a forthcoming paper by Evans and Lewis [1995]. They find evidence of cointegration between inflation and the *ex post* real rate, with permanent shocks to inflation associated with permanent shocks of opposite sign to the *ex post* real rate. But under rational expectations

$$r'_{t+1} = \hat{r}_t - \epsilon_{t+1}$$

where ϵ_{t+1} is the white noise component of π_{t+1}. Thus whatever permanent shocks are found in the *ex post* real rate must carry over to similar shocks to the *ex ante* real rate; and the expectation must be for negative covariance of π_{t+1} (and hence π_t^*) with $\hat{r}_t$. It is therefore not possible, under conventional rational expectations assumptions, to attribute the Fama and Gibbons finding of a slope coefficient well above 1 to noise introduced by variations in the *ex ante* real rate.

The situation is less clear, however, if we relax the rational expectations assumption. Evans and Lewis go on to show that, under reasonable assumptions about investor behavior in a world where shifts occur in the inflation-generating process, it will no longer be the case that unanticipated inflation is white noise. In our notation, $\hat{\pi}_t$ may differ from π_t^*; and shocks to the *ex post* real rate may carry over into shocks to $\hat{\pi}_t - \pi_t^*$, leaving the real *ex ante* rate stationary. This would remove the presumption of a negative covariance between π_t^* and $\hat{r}_t$ that exists under rational expectations and leave indeterminate the sign of the bias in $\hat{b}$ due to variation in the *ex ante* real rate.

Still mysterious is the decline in the slope coefficients found by Fama and Gibbons when they permit the *ex ante* real rate to behave as a random walk. This result must depend on $\text{cov}(\pi_t^*, \hat{r}_t)$. Only if this covariance is positive and sufficiently large—a result strongly rejected under rational expectations by the empirical evidence of a negative covariance between π_{t+1} and the *ex post* real rate—can variation in the *ex ante* real rate lead to upward bias in the slope coefficient. Only in this case can controlling for variation in $\hat{r}_t$, which must move the slope coefficient toward 1, lead to a *reduction* in this coefficient.

A series of papers by Mishkin [1981, 1988, 1990, 1993] have used more recent data to investigate the relationship between future differentials in inflation rates and current yield-curve spreads. Again it is possible to interpret a slope coefficient insignificantly different from 1 as supporting the Fisher hypothesis, but now another maintained hypothesis must be added: the constancy of the relevant term premia.

Mishkin [1990] uses data from February 1964 to December 1986 and three subperiods to relate inflation changes over time spans up to one year to the initial yield spreads on one- to twelve-month Treasury bills. For example, Mishkin relates the differential between inflation over the next month and over the next quarter to the yield spread between one-month and three-month Treasury bills, or the differential between inflation over the next 6 months and over the next year to the yield spread between 6-month and 12-month bills.

For the one-to-three-month differentials, slope coefficients are negative throughout, and except for the final subperiod they are significantly less than 1, soundly rejecting at least one of the four joint hypotheses involved. Rejection occurs also for the 6-to-9-month differential in the first subperiod, ending in October 1979. Otherwise the data fail to reject the joint hypothesis, but this is small comfort in view of the large standard errors (.260 to .415 for the full sample).

A study by Blough [1994] uses a similar approach to relate the inflation changes between next year and the following year to the current yield spread between one- and two-year Treasury or other low-risk bonds. Using four data sets, he is able to cover four subperiods beginning as early as 1922 and extending through 1990. The results are similar to Mishkin's. In no case is it possible to reject the four-fold joint hypothesis by finding a slope coefficient significantly different from 1, but standard errors are so large (.62 and up) that this hardly constitutes convincing support. The estimated slopes in the latest of four non-overlapping subperiods (1971–1990) and also in longer periods (1950–1990 and 1923–1990) incorporating this subperiod are significantly greater than zero, supporting the usefulness of lagged interest rates in predicting inflation.

V

Because the maintained hypotheses are so unrealistic, the empirical studies described above provide no convincing evidence either for or against the Fisher relationship. That rejections are occasionally found is not surprising in such a situation. That failures to reject also occur gives us little information, in view of the size of the standard errors found in in most regressions and the fact that failure in one of the maintained hypotheses may simply compensate for failure in another. This appears to be the case when it is possible to judge the likely direction of the impact of individual failures on the estimated slope coefficient.

In the best of all possible worlds, we might find a time series sample in which *ex ante* real rates and term premia were both constant. But if we believe that inflation is determined jointly with other macroeconomic variables by a system of stochastic equations whose parameters are not precisely known, then the forecasts of individual analysts—and the market forecast itself—will deviate randomly from a rational expectations forecast based on the reduced form of the structural model. We have shown that such random error will bias estimates of the slope coefficient in the types of regressions reported, invalidating hypothesis testing based on the magnitude of these estimates.

Individual errors are likely to be particularly large and predominantly in the same direction when structural changes occur. There is good reason to believe that the oil shocks of the 1970s, to say nothing of the change in Federal Reserve policy initiated in 1979 and its eventual success by 1982, caused significant shifts in the inflation-generating process during the sample periods analyzed. (For statistical evidence, see Evans and Lewis [1995].)

What is highly likely is that current bond yields (and yield spreads) do contain some useful information on future inflation levels and patterns. To the extent that bond market participants possess some forward-looking information on inflation and use it in their portfolio decisions, we may expect yield curve data to make a net contribution to whatever we have learned from univariate time series analysis (see Fama and Gibbons [1984]) and very possibly to forecasts based on full-blown econometric models. For example, investors may have knowledge that sheds some light on the future value of exogenous components of investment or consumption. Like other types of anticipatory data, bond yields should be used wherever possible to enhance forecasts based exclusively on *ex post* data.

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