AN INTERTEMPORAL MODEL
OF SEGMENTATION

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Abstract

This paper develops an intertemporal model of international capital market segmentation. Within the model, under various forms of segmentation/integration, the equilibrium asset prices and allocations, the risk-free interest rate, and the intertemporal consumption behavior and welfares of two countries are derived and compared. It is shown that the equilibrium interest rate is increased on integration, and that integrating markets may be significantly welfare decreasing for one of the countries. Conditions that may lead to a decrease in welfare are investigated. The conclusions as to the effects of segmentation on asset prices in the mean-variance model of the existing finance segmentation literature are also shown to break down in an intertemporal model.

I. Introduction

Beginning with the seminal work of Black (1974) and Stulz (1981), the strand of literature theoretically studying the effects of international capital market segmentation/integration has been developed in a two-period static mean-variance framework (Subrahmanyam (1975), Errunza and Losq (1985, 1989), Eun and Janakiramanan (1986), Merton (1987) (in a domestic context), Hietala (1989), Bergstrom, Rydqvist and Sellin (1993)). The primary focus of these papers has been on asset prices, portfolio choice, and on welfare implications, in particular the diversification benefits/losses due to enhanced/reduced risk-sharing opportunities.

A shortcoming of these existing models is that the international bond market is used only to facilitate risk-sharing between countries, and not for intertemporal borrowing or lending against a better or worse future income. This omission is unfortunate because many of the countries (such as less-developed countries) facing segmentation issues (such as restrictions on capital flow) have economies which are critically dependent on foreign debt predominantly for intertemporal consumption smoothing purposes. For less-developed countries, intertemporal consumption smoothing objectives may be just as important, if not more so, than diversification objectives, and hence should not be overlooked.[1] An international segmentation model should address whether such economies are affected differently by integration than are economies less dependent on foreign debt. A related drawback of the current static models is that their lack of intertemporal consumption leads to indeterminacy in the international borrowing rate (representing the cost of intertemporal consumption smoothing). Despite the fact that countries' participation in the international bond market is found to be affected by segmentation, these models specify the international interest rate exogenously to be unaffected by capital market segmentation.

Our objective in this paper is to extend the previous work to incorporate intertemporal consumption behavior and an endogenously determined international borrowing rate in order to re-examine the price and welfare implications of segmentation in a richer, more realistic framework. Our focus is to evaluate the robustness of the previous conclusions, without deviating from the standard mean-variance framework in any dimension other than the intertemporal aspect. In this way meaningful comparisons can be made with the existing literature. Our results show significant, non-trivial differences from the previous conclusions, both in the asset price and the welfare implications.

[1] An additional important consideration for less-developed countries is default-risk, which we do not attempt to model in our analysis.

A major conclusion of this theoretical investigation is that, in our model, the international interest rate is unambiguously increased on market integration. The intuition is that the new diversification opportunities effectively "improve" the risky future consumption (by decreasing its variance) relative to the certain initial consumption, and so countries would like to borrow more against that future wealth, hence pushing up the interest rate. As a result of this increasing interest rate, the asset price comparisons of the existing segmentation literature are not robust to the incorporation of intertemporal consumption into the model. Although the covariance structure of the assets' risk premia within an economy are unchanged, the price effects of market integration are all modified. The existing literature found restricting trade in one asset to unambiguously reduce its price while leaving all other asset prices unchanged. We on the contrary find the restricted asset's price to either be increased or decreased on segmentation, while the other (unrestricted) asset's price to unambiguously be increased. More generally, our analysis reveals two additional factors driving the effect of segmentation on the asset prices, the extent of the lost diversification opportunities, and the attractiveness of the asset in question. We also study how the increasing interest rate interacts with countries' intertemporal consumption smoothing behavior. We identify four factors influencing countries' consumption smoothing and international borrowing behavior, and relate this behavior to welfare implications of integration. In the special case of both countries having identical attitudes to risk, we find that as markets are integrated, the extent of countries' intertemporal consumption smoothing decreases, and hence the amount of international bond trading for that consumption smoothing decreases.

While it has long been recognized that the opening of new markets is not always welfare improving (e.g., Pomery (1984)) because of possibly negative effects on the terms-of-trade, the welfare results of the finance segmentation literature have not so far revealed any such effects. The common conclusion of the previous literature (Subrahmanyam (1975, 1975a), Errunza and Losq (1989)) is that the welfare of all countries is improved on market integration, thus making a strong case for the diversification benefits of international securities markets integration. These papers, then, provide no rationalization for the existence of market segmentation, despite its prevalence in the world. Indeed Stulz (1994) points out that there has been little theoretical explanation of why international investment barriers exist.[2] In our model, however, we show that

[2]Gordon and Varian (1989) do demonstrate that if governments can behave as non-price-takers they may affect asset terms-of-trade through taxation so as to improve their country's welfare. Their framework is very different from ours, since they consider complicated forms of taxation rather than simply opening and closing markets. They briefly mention (but do not fully analyze) the case of taxation affecting the interest rate, but they state that this would be achieved through a taxation on riskless income, effectively segmenting the bond market. An important distinction in our model is that we find segmentation in the risky asset markets to affect the riskless interest rate. All these differences make it very hard to compare their results with those of the existing international finance

the welfare of one country may decrease on market integration and identify the circumstances under which this can arise. For example, a country which initially has less consumption available (has a consumption supply shortage) relative to the other country or which is expected to have a relatively more productive economy in the future, and hence is currently smoothing consumption forward (such as a less-developed country) will be the type of country whose welfare may decrease on integration. This is a result of the increasing interest rate on integration, which may make such a country worse off, as the cost of its borrowing for intertemporal consumption smoothing is increasing. Our results point to international borrowing as a significant factor in determining whether a country will be better or worse off on integration. However, there are circumstances when a country is a lender in the international bond market yet is still made worse off on integration despite the higher interest rate. To estimate the magnitude of the welfare decrease on integration, we appeal to a measure commonly used to quantify welfare changes (Cole and Obstfeld (1991)). We find the welfare decrease to be pronounced for a set of parameters with reasonable interpretation. We also introduce a measure to disentangle the impact on welfare of the diversification and the consumption smoothing effects of segmentation.

Sellin and Werner (1993) also develop an international model of segmentation with an endogenous interest rate but do not address the intertemporal consumption behavior and welfare implications. They show the interest rate to increase under a different form of integration, in a continuous-time Cox, Ingersoll, Ross (1985) model with two log-utility countries whose production technologies have the same growth rates and volatilities. Our paper is complementary to theirs in that we study a two-period mean-variance type framework, where we additionally allow heterogeneity of the countries' preferences, endowments and productivities. Of the current mean-variance literature, Subrahmanyam (1975a) is the exception who allows an interest rate in the integrated market to differ from those in the segmented markets, but he must still specify the interest rates exogenously and he still finds welfare to always be improved on integration. Uppal (1992, 1993) has a model with an endogenous interest rate but in the context of segmentation in the good market. In the context of studying the determinants of asset trade, Svensson (1988) develops an intertemporal model like ours, but does not address the issue of market segmentation. Another related paper which extends the mean-variance framework to include more realistic (entrepreneurial) investors and to break down one of the major price conclusions of the existing literature is Stulz and Wasserfallen (1992).

The remainder of the paper is organized as follows. Section II outlines the framework of the

segmentation literature.

mean-variance segmentation models. In Section III, we briefly review the static mean-variance model of the existing literature, and its relevant results. In Section IV, we present an intertemporal extension of previous work, and study asset prices, allocations, interest rates, consumption behavior, and welfares of each country. Section V concludes. The Appendix provides the proofs of all propositions and corollaries.

II. The Set-Up

We consider a world with two countries, X and Y, characterized by the following:

- There are two periods, $t = 0$ and $t = 1$.
- There is a single perishable consumption good; all quantities are expressed in units of this good.
- Investors in each country i ($i = X, Y$) have negative exponential utility, $u_i(c_t) = -\exp\{-a_i c_t\}/a_i$, $a_i > 0$, where c_t is consumption at date t. The number of investors in each country is normalized to one.
- There is a riskless bond and two risky securities. The bond is in zero net supply with an initial price of 1, and gross (risk-free) rate of return of R. The two risky securities are in positive net supply of one, security 1 initially owned by country X, security 2 by country Y. Each share of security j ($j = 1, 2$) at time 0 has price P_j paying off $\tilde{P}_j$ at time 1. We assume $\tilde{P}_j$ is normally distributed with mean μ_j, and variance σ_j^2, and has a correlation of ρ with the other risky security.

Following Errunza and Losq (1985), we discuss the following market structures, with increasing levels of integration, dictating which securities each country is allowed to trade in:

Autarky (A): Each country trades in its own risky security; countries X and Y cannot risklessly lend/borrow from each other.

Segmentation (S): Each country trades in its own risky security; countries X and Y may risklessly lend/borrow from each other.

Mild Segmentation (MS): Country X trades in its own risky security; country Y trades in securities 1 and 2; countries X and Y may risklessly lend/borrow from each other.

Integration (I): Countries X and Y can both trade in all three securities.

Moving from autarky to segmentation, the international bond market is opened; moving from segmentation to mild segmentation, country X opens its security market 1 to Y while Y keeps its security market closed to X[3]; moving to integration Y opens its market 2 to X. These forms of segmentation can be viewed as extreme types of legal investment barriers. In many countries the trading restrictions imposed are often milder (Black (1974), Stulz (1981), Eun and Janakiramanan (1986)). Our discussion will focus on the last three market structures, with autarky seen as a benchmark case. Autarky is less interesting because both agents must be better off under some type of integration, since they can always choose their autarkic strategies in any of the other market structures.[4]

III. The Static Two Period Mean-Variance Model of Segmentation

We now briefly review the static mean-variance model of the existing finance segmentation literature, and its relevant results. In this model, each country $i(i = X, Y)$ solves

$$\max_{\alpha_{i1},\alpha_{i2}} E\left[u_i(\tilde{W}_i)\right]$$

$$\text{subject to} \quad \tilde{W}_i = W_i R + \alpha_{i1}(\tilde{P}_1 - RP_1) + \alpha_{i2}(\tilde{P}_2 - RP_2)$$

$$\text{and appropriate restrictions on} \quad (\alpha_{i1}, \alpha_{i2}) ,$$

where W_i,$\tilde{W}_i$ denote respectively, the time-0 and time-1 wealths of agent i, and α_{ij} denotes the number of shares of security j demanded by country i. There is no consumption at time 0, and at time 1 each country converts all its wealth into consumption from which it derives utility. Analysis and intuition of a mean-variance model are aided by the introduction of the concept of each country's certainty equivalent time-1 wealth (CEQ_i) defined by $E[u_i(\tilde{W}_i)] \equiv u_i(CEQ_i)$.

[3]We consider this asymmetric form of segmentation following Errunza and Losq (1985) to capture capital flow restrictions involving less developed countries, which may be able to enforce restrictions on inflow better than on outflow.

[4]Other types of segmentation that could be considered would be to have a segmented bond market throughout. This case is less interesting because it is less realistic and because market structures segmentation and mild segmentation are both effectively autarky meaning that all countries must be at least as well off in the integrated market.

Result:[5] *In the two period mean-variance model, the equilibrium asset prices and security demands under alternative market structures are as presented in Table I.*

Table I.

Asset Prices and Demands under Alternative Market Structures

	Segmentation (S)	*Mild Segmentation (MS)*	*Integration (I)*
Asset Prices			
P_1	$\frac{1}{R}\left[\mu_1 - a_X\sigma_1^2\right]$	$\frac{1}{R}\left[\mu_1 - A(\sigma_1^2 + \rho\sigma_1\sigma_2)\right]$	$\frac{1}{R}\left[\mu_1 - A(\sigma_1^2 + \rho\sigma_1\sigma_2)\right]$
P_2	$\frac{1}{R}\left[\mu_2 - a_Y\sigma_2^2\right]$	$\frac{1}{R}\left[\mu_2 - A(\sigma_2^2 + \rho\sigma_1\sigma_2) - \frac{a_Y}{a_X}A(1-\rho^2)\sigma_2^2\right]$	$\frac{1}{R}\left[\mu_2 - A(\sigma_2^2 + \rho\sigma_1\sigma_2)\right]$
Security Demands of Country X			
α_{X1}	1	$\frac{A}{a_X}\left(1 + \rho\frac{\sigma_2}{\sigma_1}\right)$	$\frac{A}{a_X}$
α_{X2}	0	0	$\frac{A}{a_X}$
α_{X0}	0	$\frac{AP_1}{a_Y}\left(1 - \rho\frac{\sigma_2}{\sigma_1}\frac{a_Y}{a_X}\right)$	$A\left(\frac{P_1}{a_Y} - \frac{P_2}{a_X}\right)$

where $A \equiv \frac{a_X a_Y}{a_X + a_Y}$, *the aggregate world risk aversion coefficient. Country Y holds the remainder of all assets. Furthermore, we have*

$$P_1^{MS} = P_1^{I}, \quad P_2^{MS} < P_2^{I},$$

$$V_i^{S} < V_i^{MS} < V_i^{I}, \quad i = X, Y, \quad \rho \neq \pm 1, \quad \sigma_1 a_X \neq \rho\sigma_2 a_Y,$$

where V_i^k *denotes the welfare (expected total utility at the equilibrium allocations) of country i under the market structure k. When* $\sigma_1 a_X = \rho\sigma_2 a_Y$, $V_i^S = V_i^{MS}$; *when* $\rho = \pm 1$, $V_i^{MS} = V_i^{I}$.

The lower price for asset 2 in the mildly segmented market relative to the integrated can be explained by the fact that, since asset 2 has now become closed to country X, there is less demand for it. Hence, to clear its market, asset 2's price must decrease to encourage country Y to demand more; asset 2 now sells at a "super discount". Clearly asset demands are changing as we move from the integrated to the mildly segmented market and change X's opportunity set. The result that the price of asset 1 remains unchanged is, then, somewhat unintuitive. In this model, any change in demands due to country X's changing opportunity set are exactly compensated by asset 2's price change only.

[5] Errunza and Losq (1985) and Eun and Janakiramanan (1986) derive the risky asset prices and demands presented in Table I and the asset price comparisons stated below. They do not explicitly compute the bond demands, but this is straightforward. Errunza and Losq (1989) derive the welfare comparison between the mildly segmented and the integrated markets, and Subrahmanyam (1975) between the segmented and integrated markets; the comparison of segmented and mildly segmented is then a straightforward adaptation.

The welfare of both countries increases as we integrate markets because the countries seek to share the risk associated with the risky assets. As the markets integrate, the opportunities for such diversification increase, enabling each country to reduce the variance of its time-1 wealth, hence increasing its certainty equivalent time-1 wealth, and thereby increasing its welfare. This idea is often used to conclude that market integration is often better for all parties involved.

From the risky asset and bond demands we see that, in the integrated market, the countries share the risk in proportion to their risk tolerance by holding A/a_i of each risky asset, using the riskless bond to be able to afford this. In the mildly segmented markets they attempt to share the risk as much as possible given the restriction that X cannot trade in asset 2, again lending and borrowing to and from each other to afford this. Here, country X in addition to its integrated market asset 1 holding, holds a "hedge portfolio", $\rho\sigma_2/\sigma_1$ of asset 1, the portfolio that is the best substitute for asset 2's payoff (e.g., Errunza and Losq (1985)). In the segmented market the two countries are unable to share any of the risk and so do not trade in the riskless bond. In summary, in this model the bond is traded between countries only to allow them to afford the risk-sharing between them.

We also note that in this static model, since there is consumption only at one point in time, the risk-free rate R is indeterminate and hence specified exogenously. Given that the demands for the bond are changing across market structures this suggests to us that it would be appropriate to develop a model with an endogenously determined interest rate. As we have just discussed, the role of the international bond market in the current model is purely to facilitate risk-sharing. However, in international capital markets certainly not all borrowing and lending is due to risk-sharing. There is considerable evidence of countries, such as less-developed countries, borrowing against their future income in order to afford consumption today (i.e., smoothing their consumption intertemporally). There is also clear evidence that many less-developed countries have legal restrictions or barriers concerning international investment (e.g., Eun and Janakiramanan (1986, Table I), Errunza, Losq and Padmanabhan (1992, Table 3)). It would then be useful to develop a model of international segmentation in which the effects of intertemporal consumption smoothing and an endogenous interest rate are taken into account, and to re-examine the results concerning the quantities of interest in Table I, and in particular the welfare results.

IV. An Intertemporal Model of Segmentation

In this section we introduce an international model of segmentation with intertemporal consumption in order to address some of the aforementioned shortcomings of the existing models. It is of course of independent interest to generalize to a multi-period model of international segmentation, and this model provides a step in that direction, by addressing the intertemporal nature of consumption.

The new features we add to the mean-variance model of the previous section are the following:

- In addition to a unit of its security, country X is endowed with $\gamma\epsilon$ units of consumption good and country Y with $(1-\gamma)\epsilon$ units at time 0, where $0 < \gamma < 1$.[6]
- In addition to deriving utility from time-1 wealth, the agents now also derive satisfaction from time-0 consumption.

In this model, each country i solves

$$\max_{c_i, \alpha_{i1}, \alpha_{i2}} u_i(c_i) + \beta_i E\left[u_i(\tilde{W}_i)\right]$$

$$\text{subject to} \quad c_i + W_i = e_i$$

$$\text{and} \quad \tilde{W}_i = W_i R + \alpha_{i1}(\tilde{P}_1 - RP_1) + \alpha_{i2}(\tilde{P}_2 - RP_2)$$

$$\text{and appropriate restrictions on} \quad (\alpha_{i1}, \alpha_{i2}) \, ,$$

where $e_X \equiv \gamma\epsilon + P_1$, $e_Y \equiv (1-\gamma)\epsilon + P_2$ are the values of each country's endowments at time 0. To be compatible with the existing models, we assume $u_i(c_i)$ is the negative exponential utility function. The parameter β_i is country i's impatience parameter, with $0 < \beta_i < 1$, $i = X, Y$.

One major difference in this model is that the new choice variable of c_i in the country's optimization problem leads to the additional intertemporal first order condition,

$$R = \frac{u_i'(c_i)}{\beta_i E\left[u_i'(\tilde{W}_i)\right]}, \quad i = X, Y.$$

This condition is the familiar statement that the interest rate must equal each country's marginal rate of substitution between time-0 and time-1 (certainty equivalent) consumption, at the optimum. The first order conditions for the optimal asset demands are identical to those in the previous model (see the appendix).

[6]This could alternatively be interpreted as the payoff from some securities traded before time 0 where each country holds a given number of these securities.

We analyze quantities of interest by appealing to general equilibrium restrictions.[7] We define *equilibrium* to be a triplet (R, P_1, P_2) such that, given all agents follow their optimal consumption/portfolio strategies, all markets clear, i.e.,

$$\sum_{i=X,Y} c_i = \epsilon; \quad \sum_{i=X,Y} \alpha_{ij} = 1, \quad j = 1,2; \quad \sum_{i=X,Y} W_i = \sum_{j=1,2} P_j.$$

A. Interest Rate and Asset Prices

Proposition 1 compares the endogenous interest rate under the various market structures.

Proposition 1:[8] *The equilibrium interest rate is monotonically increased by market integration when $\rho \neq \pm 1$, $\sigma_1 a_X \neq \rho\sigma_2 a_Y$, i.e.,*

$$R^S < R^{MS} < R^I.$$

$R^S = R^{MS}$ *if and only if* $\sigma_1 a_X = \rho\sigma_2 a_Y$; $R^{MS} = R^I$ *if and only if the two assets are perfectly positively or negatively correlated,* $\rho = \pm 1$.

Recall that the equilibrium risk-free rate must equate to each country's marginal rate of substitution between time-0 consumption and time-1 (certainty-equivalent) wealth. If we imagine a world with a "representative agent" who consumes all the time-0 consumption in the economy and all the certainty equivalent time-1 wealth, R must also equal his marginal rate of substitution. As discussed in Section III, constant absolute risk aversion (CARA) utility agents attempt to decrease the variance of their time-1 wealth and as markets integrate are less restricted from doing this. As a result, on integration the aggregate variance of their wealths decreases and the aggregate certainty equivalent time-1 wealth increases. Since the aggregate time-0 consumption is unchanged, the marginal rate of substitution and hence the interest rate must increase. In other words, since countries are made effectively "better off" at time 1 due to diversification effects they would like to consume some of this extra "benefit" at time 0 by borrowing against the time-1 wealth at time 0. But in aggregate there is a fixed supply of the consumption good at time 0, so the risk-free rate must go up to counteract this borrowing tendency. The extent of the interest rate increase on market integration is driven by the extent of the diversification benefits (in the sense of increased aggregate certainty equivalent time-1 wealth) due to integration. The proof of Proposition 1 reveals that the ratios R^I/R^{MS} and R^{MS}/R^S depend only on the risk aversion and the variance-covariance matrix of the asset payoffs, and not on the countries' mean growths,

[7] This is as in the mean-variance model of Section III, with the addition of clearing in the time-0 consumption.
[8] The second inequality is consistent with a result in Sellin and Werner (1993).

impatience parameters or endowments. Hence, within our model, the benefits of integration depend only on countries' risk aversion and the asset payoff parameters.

Under the CARA framework, countries' risk sharing objectives are the same as in the static model, leading to identical expressions for equilibrium demands for the two risky assets. This implies a separation of countries' diversification and intertemporal smoothing objectives; countries first choose the risky asset demands in the unique way to share their risk and then choose the bond demands to smooth their consumption intertemporally. This separation feature is convenient since it allows us to highlight and isolate the effects of the intertemporal element we have introduced. Given the identical diversification objectives, not surprisingly the equilibrium risky asset prices and covariance structure of asset risk premia in the intertemporal model are given by the identical expressions to those in Table I. However, since the interest rate changes across market structures, all asset price changes are affected.

In light of this, in Proposition 2 we re-examine the price results for the risky assets in Table I.

Proposition 2: *The equilibrium asset prices in the three market structures are given by the identical price expressions in Table I, and obey the following inequalities:*

$$
\begin{aligned}
P_1^S &> P_1^{MS} \quad \textit{if and only if} \quad \frac{a_X}{a_Y} < \frac{\rho\sigma_2}{\sigma_1} + \frac{1}{A\sigma_1^2}\left(\frac{R^{MS}}{R^S} - 1\right)(\mu_1 - a_X\sigma_1^2) \\
P_2^S &> P_2^{MS} \quad \textit{if and only if} \quad \frac{a_Y}{a_X} < \frac{\rho\sigma_1}{\sigma_2} + \frac{1}{A\rho^2\sigma_2^2}\left(\frac{R^{MS}}{R^S} - 1\right)(\mu_2 - a_Y\sigma_2^2) \\
P_1^{MS} &\geq P_1^I \\
P_2^{MS} &> P_2^I \quad \textit{if and only if} \quad \frac{a_Y}{a_X} < \frac{1}{A(1-\rho^2)\sigma_2^2}\left(1 - \frac{R^{MS}}{R^I}\right)(\mu_2 - A(\sigma_2^2 + \rho\sigma_1\sigma_1)) \\
P_1^S &> P_1^I \quad \textit{if and only if} \quad \frac{a_X}{a_Y} < \frac{\rho\sigma_2}{\sigma_1} + \frac{1}{A\sigma_1^2}\left(\frac{R^I}{R^S} - 1\right)(\mu_1 - a_X\sigma_1^2) \\
P_2^S &> P_2^I \quad \textit{if and only if} \quad \frac{a_Y}{a_X} < \frac{\rho\sigma_1}{\sigma_2} + \frac{1}{A\sigma_2^2}\left(\frac{R^I}{R^S} - 1\right)(\mu_2 - a_Y\sigma_2^2)\,.
\end{aligned}
$$

Consequently, the static model underestimates an asset price decrease due to market integration.

This proposition reveals that the risky asset price results of the previous literature (e.g., Errunza and Losq (1985), Eun and Janakiramanan (1986), and Merton (1987)) are not robust to this extension to an intertemporal model. Whereas the previous models concluded that the price of the "unrestricted" asset (asset 1) was unaffected on moving from the integrated to the mildly segmented market, in the intertemporal model the price unambiguously increases. The price of the restricted asset (asset 2) may increase or decrease, whereas in the previous models it unambiguously decreased on moving from integrated to mildly segmented. As discussed earlier, when we move from integration to mild segmentation, the demand for asset 2 decreases because country X no longer has access to it, but now it also increases because agents substitute away from the lower interest rate bond. So the net effect on asset 2 demand and hence its price is ambiguous. Agents similarly substitute away from the bond into asset 1 and hence its price must increase. Recall that there was no direct effect on asset 1's price of the changing opportunity sets, so the only effect now is via the bond.

The remaining asset price comparisons across market structures are also altered in this intertemporal model. The conditions for the price comparisons in the static model can be obtained from Proposition 2, by setting all the interest rates equal, so that the last term on the right hand side of each inequality becomes zero. The change in an asset's price on moving from segmentation to mild segmentation or integration, in the static model depends only on how risk averse one country is relative to the other (a_X/a_Y) and on how much of a good "hedge" that asset is for the other ($\rho\sigma_2/\sigma_1$ or $\rho\sigma_1/\sigma_2$). In the intertemporal model, however, all asset price changes are also dependent on two extra driving factors: the extent to which both countries benefit from diversification (the interest rate ratio), and the attractiveness (certainty equivalent) of the security's payoff in the original market. While one would expect the attractiveness of an asset's payoff to be a driving factor in its price behavior, the appearance in our model of the diversification factor is particularly appealing since risky assets are specifically used for diversification purposes. The implication of these additional factors is that the bias of the static model compared with the intertemporal model is to underestimate a price decrease on integration, overestimate a price increase, or even predict an increase when the intertemporal model predicts a decrease. This is because the increase in interest rate on integration tends to "suppress" all asset prices, as agents would like to substitute into the higher interest rate bond. The greater the diversification benefits from integration, the more pronounced this effect will be.

B. Intertemporal Consumption Smoothing and International Debt

Proposition 3 uncovers the causes and direction of a country's intertemporal consumption smoothing, and borrowing/lending in the bond market. We will see later a close connection between a country's direction of smoothing and the welfare implications of integration. The notation $CEQ_i^{\bar{k}}$ or $\alpha_{i0}^{\bar{k}}$ refers to the corresponding expression in the model of Section III (with no intertemporal consumption smoothing).

Proposition 3: *Equilibrium time-0 consumption of country X under market structure k is given by*

$$c_X^k = \gamma\epsilon - \frac{1}{1+R^k}\left[A_X\gamma\epsilon - A_Y(1-\gamma)\epsilon\right] + \frac{1}{1+R^k}\left[A_X CEQ_X^{\overline{k}} - A_Y CEQ_Y^{\overline{k}}\right] + \frac{1}{(1+R^k)a_X}\ln\left(\frac{\beta}{\beta_X}\right),$$

and the equilibrium bond demand of country X under market structure k by

$$\alpha_{X0}^k = \alpha_{X0}^{\overline{k}} + \frac{1}{1+R^k}\left[A_X\gamma\epsilon - A_Y(1-\gamma)\epsilon\right] - \frac{1}{1+R^k}\left[A_X CEQ_X^{\overline{k}} - A_Y CEQ_Y^{\overline{k}}\right] - \frac{1}{(1+R^k)a_X}\ln\left(\frac{\beta}{\beta_X}\right),$$

where

$$A_X \equiv a_X/(a_X+a_Y), \qquad A_Y \equiv a_Y/(a_X+a_Y), \quad \beta \equiv \left(\beta_X^{1/a_X}\beta_Y^{1/a_Y}\right)^A,$$

$$\begin{aligned}
\left[A_X CEQ_X^{\overline{S}} - A_Y CEQ_Y^{\overline{S}}\right] &\equiv A_X(\mu_1 - a_X\sigma_1^2/2) - A_Y(\mu_2 - a_Y\sigma_2^2/2), \\
\left[A_X CEQ_X^{\overline{MS}} - A_Y CEQ_Y^{\overline{MS}}\right] &\equiv \left[A_X CEQ_X^{\overline{S}} - A_Y CEQ_Y^{\overline{S}}\right] + \frac{A^3}{2a_X^3 a_Y}(\sigma_1 a_X/a_Y - \rho\sigma_2)^2(a_X^2 - a_Y^2), \\
\left[A_X CEQ_X^{\overline{I}} - A_Y CEQ_Y^{\overline{I}}\right] &\equiv \left[A_X CEQ_X^{\overline{MS}} - A_Y CEQ_Y^{\overline{MS}}\right] + \frac{A^3}{2a_X^3 a_Y}(1-\rho^2)\sigma_2^2(a_X^2 - a_Y^2),
\end{aligned}$$

and $\alpha_{X0}^{\overline{k}}$ are the bond demands as given in Table I. The expressions for Y are analogous.

Intuitively, one might expect a country to smooth consumption to time 0 under three cases: either (1) it has relatively less endowment at time 0; or (2) it is relatively more productive at time 1; or (3) it is more impatient, than the other country. Cases (1) and (2) are particularly pertinent to less-developed countries who have a consumption supply shortage now and would like to borrow against future productivity. Proposition 3 states this intuition more rigorously also taking into account the extra factor of (4) countries' differing risk aversions. Country X's time-0 consumption is equal to its time-0 endowment plus three consumption smoothing terms driven by: (1) its "excess" time-0 endowment over the other country, normalized by their risk aversion parameters, $[A_X\gamma\epsilon - A_Y(1-\gamma)\epsilon]$; (2) its "excess" time-1 certainty equivalent wealth $[A_X CEQ_X^{\overline{k}} - A_Y CEQ_Y^{\overline{k}}]$; and (3) its relative impatience $\ln(\beta/\beta_X)$. (Note that $CEQ_i^{\overline{k}}$ can be thought of as the time-1 certainty equivalent wealth of country i "before smoothing", i.e., the certainty equivalent wealth country i would have had if it had consumed exactly its own endowment at time 0, held its optimal risky asset holdings and used the bond only to afford these risky asset holdings.)

We now ask: how is this intertemporal consumption affected by market integration or segmentation? For example, will the integration of a less-developed country's capital markets tend to make it smooth more or less consumption from the future? In general, there is both a direct effect of integration changing the asymmetry between countries, and an indirect effect via the change in the interest rate R. The second smoothing term in country X's consumption (the time-1 relative advantage term) captures the direct effect of integration. If countries have the same attitudes to risk ($a_X = a_Y$), this time-1 relative advantage term is unchanged on integration because both countries benefit "equally" from the new diversification opportunities. However, if for example, country X is more risk averse than country Y ($a_X > a_Y$), country X benefits more from new diversification opportunities and hence its time-1 relative advantage increases while country Y's relative advantage decreases. The indirect effect of integration via the change in R is captured by the fraction $1/(1+R)$ multiplying each smoothing term in Proposition 3.[9]

Let us consider the case of countries having the same attitudes to risk ($a_X = a_Y = a$), where there is only the indirect effect of integration via the interest rate. Corollary 1 summarizes how a country's intertemporal smoothing, and hence borrowing/lending is affected by integration.

Corollary 1: *Let $a_X = a_Y = a$. Assume $\rho \neq \pm 1$ and $\sigma_1 \neq \rho\sigma_2$. The condition*

$$[(1/2 - \gamma)2\epsilon] + [(\mu_1 - a\sigma_1^2/2) - (\mu_2 - a\sigma_2^2/2)] + \ln(\beta/\beta_X)/a > 0$$

is necessary and sufficient for $c_X^S > c_X^{MS} > c_X^I > \gamma\epsilon$ and also necessary and sufficient for $\alpha_{X0}^S < 0$, i.e., country X is a net buyer of consumption at time 0, becomes less of a net consumer as markets integrate, and is a borrower in the segmented market.[10] *The condition for Y is analogous.*

The condition in Corollary 1 reveals that if country X has less than half the endowment at time 0 ($\gamma < 1/2$), or its asset pays more certainty equivalent wealth $(\mu_1 - a\sigma_1^2/2) > (\mu_2 - a\sigma_2^2/2)$, or it is more impatient than country Y ($\beta_X < \beta_Y$), country X tends to smooth extra consumption from time 1, but less and less as markets integrate and the smoothing fraction $1/(1+R)$ decreases. That is, if a country is smoothing consumption to time 0 it will do less smoothing as markets integrate and the cost of this smoothing (R) increases.

Given this discussion, it is also of interest to consider the net foreign investment in each country. The net foreign investment in a country is defined as the total wealth invested in that country's asset less that country's total wealth (Stulz(1983)). In our model, the net foreign investment in country X must equal country X's net time-0 consumption ($c_X - \gamma\epsilon$). Hence we conclude from Corollary 1 that for countries having the same attitudes to risk ($a_X = a_Y$) when a country has less than half the endowment, a relative advantage at time 1, and is more impatient,

[9] If X, for example, has positive excess time-0 endowment, it smooths a fraction $1/(1+R)$ of the excess to time-1 (whose value at time-1 is $R/(1+R)$ fraction of the excess), and keeps the remaining $R/(1+R)$ for time 0; X is smoothing this excess time 0 endowment "equally" between time 0 and time 1, so as to equate the marginal rates of substitution of the two countries.

[10] When $\sigma_1 = \rho\sigma_2$ we have the same conclusion, but $c_X^{MS} = c_X^S$; when $\rho = \pm 1$ we have $c_X^I = c_X^{MS}$.

there is positive net foreign investment in that country, but that level decreases as we integrate markets.

If countries have differing attitudes to risk ($a_X \neq a_Y$), we also need to take into account that countries benefit asymmetrically from new diversification opportunities. We can focus on the effects of differing risk aversions alone by taking the case where the two countries are effectively identical in other respects as summarized in Corollary 2.

Corollary 2: *Let* $\gamma = 1/2$, $\beta_X = \beta_Y$, $A_X(\mu_1 - a_X\sigma_1^2/2) = A_Y(\mu_2 - a_Y\sigma_2^2/2)$. *Then*

(i) $c_X^S = c_Y^S = \epsilon/2$ *and* $\alpha_{X0}^S = \alpha_{Y0}^S = 0$, *i.e., there is no intertemporal consumption smoothing nor riskless lending and borrowing, in the segmented market.*

(ii) If $a_X > a_Y$ *then* $c_X^S < c_X^{MS}, c_X^I$ *and* $c_Y^S > c_Y^{MS}, c_Y^I$, *i.e., country* X *is a net buyer of consumption at time 0 and country* Y *a net seller, in the mildly segmented or integrated markets.*

(iii) If $a_X < a_Y$, *then* $c_X^S > c_X^{MS}, c_X^I$ *and* $c_Y^S < c_Y^{MS}, c_Y^I$.

If $a_X > a_Y$, in the segmented market, country X has no relative advantage or disadvantage over Y at any time and so consumes only its own endowment $\epsilon/2$. In the mildly segmented market, country X gains a relative advantage at time 1 over Y because it is more risk averse and so benefits more from the diversification opportunities. Country X smooths some of this benefit out to time 0 and so is a net buyer of time-0 consumption. In the integrated market, country X gains a further relative advantage due to the further diversification opportunities and so would smooth more to time 0, but at the same time R increases so the fraction it smooths decreases, so the net effect on his time 0 consumption is ambiguous. Hence differing risk aversions across countries complicate our conclusions about the effects of integration.

We now discuss countries' participation in the international bond market. In our intertemporal model, the bond market is used to afford risk-sharing as well as to smooth consumption intertemporally. Hence, there is a close relationship between a country's international debt and the intertemporal consumption smoothing behavior just discussed. Indeed, in the bond demands in Proposition 3, the first term ($\alpha_{X0}^{\bar{k}}$) is the amount country X chooses for risk-sharing and the remaining terms are the amount of additional borrowing (or lending) for intertemporal consumption smoothing driven by the same four factors. For example, if country X is identical to country Y in all respects, except that its asset is more productive, X always smooths consumption to time 0, and always borrows risklessly for consumption smoothing; and that amount of borrowing decreases as markets integrate. Since there is no risk-sharing in the segmented market, ($\alpha^{S}_{X0} = 0$), country X borrows from country Y under segmentation, in this case. The net sign of the borrowing and lending is, however, ambiguous under mild segmentation or integration because countries also use the bond to afford risk-sharing. Our analysis here of the net lending and borrowing behavior resembles that of Svensson (1988). He only considers the case of an integrated market since his focus is not on segmentation but on the determinants of trade in the bond. His results are related to our Proposition 3, but he restricts himself to only letting countries differ in one respect at a time (e.g., impatience) rather than deriving general joint conditions as we do.

C. Welfare Implications

The results of Subsections A and B, on interest rates, asset prices, intertemporal consumption smoothing behavior, together turn out to have important implications on each country's welfare. Proposition 4 presents necessary conditions for a country to be worse off under integration. Conditions (i)–(iv) were stated to be equivalent in Corollary 1; we repeat here for completeness.

Proposition 4: *Let $a_X = a_Y = a$. Assume $\rho \neq \pm 1$ and $\sigma_1 \neq \rho\sigma_2$. Then equivalent necessary conditions for country X to be worse off on integration, i.e., $V^{S}_{X} > V^{MS}_{X} > V^{I}_{X}$, are*

(i) $c^{S}_{X} > c^{MS}_{X} > c^{I}_{X}$,

(ii) $c_X > \gamma\epsilon$ *under all market structures,*

(iii) $\{[(1/2 - \gamma)2\epsilon] + [(\mu_1 - a\sigma_1^2/2) - (\mu_2 - a\sigma_2^2/2)] + \ln(\beta/\beta_X)/a\} > 0$,

(iv) $\alpha^{S}_{X0} < 0$.

When $\rho = \pm 1$, we have $c^{MS}_{X} = c^{I}_{X}$ and $V^{MS}_{X} = V^{I}_{X}$; when $\sigma_1 = \rho\sigma_2$, we have $c^{S}_{X} = c^{MS}_{X}$ and $V^{S}_{X} = V^{MS}_{X}$, but the remaining inequality conditions are as above. The conditions are exactly analogous for country Y.

For the case of countries not differing in their risk aversions, if country X is worse off on integration, country X must be relatively more productive at time 1 than country Y, have relatively less time 0 endowment or be more impatient (as stated in part (iii) of Proposition 4). Then, country X smooths consumption from time 1 to time 0, as stated in part (ii) (for example, a less-developed country). Condition (ii) is intuitive since, as markets integrate, the cost, R, of this smoothing increases, which is disadvantageous to country X and may cause X's welfare to decrease. Analytic sufficient conditions for the welfare to decrease on integration may also be derived, but are economically less informative. The remainder of our discussion hereafter is numerical, with the objective of highlighting economic interpretation of the welfare implications.

Figures 1 and 2 plot country X and Y's welfares and bond demands across market structures for a given set of exogenous parameters with reasonable interpretation. Figure 1 is for countries identical except for the productivity of their assets, plotted against increasing levels of mean per annum growth in country X between 4 and 10%. Country X's asset has both higher mean growth and is more risky than country Y's, but country X's certainty equivalent payoff remains higher than country Y's. Figure 2 is for countries identical except for their initial good endowments, plotted against increasing levels of country X's fraction of the initial endowment between 0.1 and 0.9. We do not attempt a full calibration of our model in selecting the parameters of Figures 1 and 2, since our model is in a 2-period CARA utility framework, under which calibration is non-standard. However, for a given horizon of $T = 25$ years, the exogenous parameters of Figures 1 and 2 are chosen to correspond to reasonable mean growths, volatility of growths, interest rates and impatience parameters when standardized to per annum terms. The mean growths standardized to per annum are computed according to the formulae $(\mu_1/\gamma\epsilon)^{1/T} - 1$ and $(\mu_2/(1-\gamma)\epsilon)^{1/T} - 1$, i.e., they are the per annum growths that would have to exist if each year's growth were identically and independently distributed. The per annum interest rates as presented later in the Tables are calculated from $R^{1/T} - 1$, and the per annum impatience parameters from $\beta_i^{1/T}$. To compute the volatility of growth standardized to per annum would involve an approximation, so we do not emphasize those values here.[11]

The graphs demonstrate that in contrast to the welfare implications of the static model (Subrahmanyam (1975, 1975a), Errunza and Losq (1989)), within the intertemporal model, there exist circumstances under which a country can be worse off as a result of integration. Consistent with Proposition 4, when a country is worse off on integration, it is a borrower in the segmented market; its risky asset has a high (before-trade) certainty-equivalent payoff relative to the other asset (Figure 1) or it has a relatively low initial good endowment (Figure 2). Figure 1 also demonstrates that being a borrower in the international bond market is not necessary for a country to become worse off on integration. Indeed, even though country X is a lender under mild segmentation, country X is still made worse off on integration despite an associated increase in the interest rate.

[11] A back-of-the-envelope calculation of the volatility of the (arithmetic) rate of growth per annum ($\sigma_1/\gamma\epsilon T$ and $\sigma_2/(1-\gamma)\epsilon T$) yields 5% for country X and 4% for country Y for the given parameter values of Figure 1, and 1.7–15% for both countries in Figure 2.

In order to evaluate the economic significance of the welfare changes illustrated in the Figures, we calculate welfare measures quantifying the effects of segmentation/integration. We define a welfare function for country i by

$$V_i(c, CEQ) = -\frac{1}{a_i} \exp\{-a_i c\} - \frac{\beta_i}{a_i} \exp\{-a_i CEQ\} .$$

Then under market structure k, country i's equilibrium welfare is given by

$$V_i^k = V_i(c_i^k, CEQ_i^k) ,$$

where the arguments are evaluated at the equilibrium. To get an estimate of the magnitude of our particular welfare results, following Cole and Obstfeld (1991), we may compute the fractional permanent reduction/enhancement in consumption (at both points in time) required to provide the same welfare reduction/increase. Accordingly, we define the total measure $\delta^{k,\ell}$ of the welfare change on moving from market structure k to market structure ℓ by

$$V_i\Big((1+\delta)c_i^k, (1+\delta)CEQ_i^k\Big) = V_i\Big(c_i^\ell, CEQ_i^\ell\Big) .$$

In order to disentangle the effects on welfare of diversification and intertemporal consumption smoothing, we decompose this total measure into two components. The measure $\overline{\delta}^{k,\ell}$ due to diversification opportunities alone is defined by

$$V_i\Big((1+\overline{\delta})c_i^k, (1+\overline{\delta})CEQ_i^k\Big) = V_i\Big(c_i^k, CEQ_i^k + CEQ_i^{\overline{\ell}} - CEQ_i^{\overline{k}}\Big) ,$$

where the right hand side is the fictitious welfare in market structure k, but with the enhanced/reduced diversification opportunities $(CEQ_i^{\overline{\ell}} - CEQ_i^{\overline{k}})$ of market structure ℓ. Finally a measure $\hat{\delta}^{k,\ell}$ due to intertemporal consumption smoothing effects may be defined by

$$(1+\hat{\delta}) = (1+\delta)/(1+\overline{\delta}) .$$

Tables II and III report these welfare measures (δ's in percentages) and the equilibrium interest rates (per annum) on integrating markets. Our results are derived under a very special model (2-period and CARA preferences) and so their detailed implications about reality should be treated cautiously. The results are merely suggestive of the approximate size of possible welfare changes.

Table II corresponds to the parameter values in Figure 1. There is a region, for fairly high growth in country X (6–8% p.a.) relative to country Y (2% p.a.) in which country X's welfare decrease on any type of integration is equivalent to a permanent consumption reduction of 0.6–3.0%. Most of these values would be considered as significant by literature making use of such welfare measures (for example, van Wincoop (1994)). The measure of welfare change due to diversification alone is always positive since both countries always benefit from extra diversification opportunities on integration, and the magnitude of this welfare improvement can be quite high (up to 16.3%). The measure of the welfare change due to intertemporal consumption smoothing may be negative or positive depending on whether the country is smoothing consumption forward or backward. For a country to be made worse off on integration the intertemporal consumption smoothing effect must be sufficiently negative to outweigh the diversification effect.

The diversification measure decreases monotonically for both countries as we move from left to right of Table II. This is because we are increasing the mean world output, but keeping the volatility of output fixed, so output is becoming relatively less risky and hence diversification relatively less important. Since a country's extent of consumption smoothing is related to its bond holding in the segmented market (as seen in Proposition 3), not surprisingly, the consumption smoothing measure of welfare change behaves similarly to the country's bond holding under segmentation. For Country Y this measure is hump-shaped in country X's mean output; for country X it is an inverted hump. For the particular parameter values chosen, for country X the trend in the consumption-smoothing measure dominates the trend in the diversification measure, leaving its total measure also inverted humped. For country Y the diversification trends dominate leaving the total measure monotonically decreasing.

Table III corresponds to the parameter values of Figure 2. Either country may become worse off on integration when it is endowed with sufficiently little of the initial endowment. For less than 0.2 of the initial endowment the welfare reduction on any form of integration is equivalent to a permanent reduction in consumption of 1.2–11.9%. As we move from left to right of Table III the diversification measure of welfare improvement decreases monotonically for country X and increases monotonically for country Y. This is simply because country X's endowment is increasing, so fractionally any welfare improvement becomes relatively less important, and vice versa for country Y. Unlike in Table II, the consumption smoothing measure for both countries is also monotonic in their fraction of the initial endowment, becoming more pronounced as the asymmetry between the countries increases, and as before behaving similarly to their bond holdings under segmentation. As a result, the total welfare measure is also monotonic.

In summary, the analysis reveals that there are two effects of integration on countries' welfare. The first is the improved diversification opportunities, increasing a country's welfare, and the second is due to intertemporal consumption smoothing effects, which is beneficial to a country smoothing consumption backward and detrimental to a country smoothing consumption forward (such as less-developed countries). This second effect is primarily due to the increase in interest rate on integration which is also indirectly due to increased diversification opportunities. Tables II and III report circumstances for a range of parameter values with reasonable interpretation, where the detrimental consumption-smoothing effects in one country outweigh the benefits of diversification leading to a pronounced net welfare decrease on integration. A more general conclusion, though, is that there is a clear breakdown of the traditional view in the international finance literature that all countries' welfares must be increased by market integration.

Since both countries benefit from diversification, but only one suffers from the intertemporal smoothing effects, an immediate question arises as to whether the other country would be willing to subsidize the country who suffers in order to encourage markets to open. Indeed, in Tables II and III we observe that the country benefitting from diversification and intertemporal smoothing invariably benefits more than the other country suffers. This suggests that one country might be willing to subsidize the other, but this point should be treated with caution since these are fractional measures of welfare changes.

D. Extensions

The extension of the analysis and results in this paper to the case of more than two assets is straightforward. The prices of many assets in the mean-variance model of Section II have been derived by, for example, Errunza and Losq (1985), Eun and Janakiramanan (1986), who showed unconstrained assets' prices to remain unchanged across economies (analogous to $P_1^{MS} = P_1^I$) and constrained assets' prices to increase when they become unconstrained (analogous to $P_2^{MS} < P_2^I$). In our intertemporal model, the increase in interest rate on integration (Proposition 1) carries through for the case of many assets, so we would again conclude that unconstrained assets' prices decrease on integration, whereas the constrained assets' prices behave ambiguously. Propositions 3 and 4 of our paper would be analogous in the many asset case.

Errunza and Losq (1989) extend the static mean-variance framework to the case of more than two countries. Unlike the case of many assets, there is a significant difference when many countries are introduced. When there are only two countries present, an inflow restriction in an asset on one country is tantamount to an outflow restriction in that same asset on the other country. This is why our seemingly asymmetric mildly segmented market structure actually affects both countries symmetrically. With more than two countries one can capture the idea of closing an asset to one country, while the other countries may still trade amongst themselves in that asset, so real asymmetric cases can be modeled. In a many-country model, our interest rate result would still carry through; R would still increase as markets were opened. Our results on consumption smoothing and welfare would be complicated by the possibility of asymmetric diversification benefits across the various countries. It is clear, though, that the welfare of any one country may still decrease on some type of integration.

A further feature that could be captured in a multi-country model is the effect of a country's size on the welfare results. At first blush, one might conjecture a smaller country to have less impact on the international interest rate, and so to suffer less by opening its markets if it is smoothing consumption forward. However, in our model of only two countries, an opening of a market by a small country is tantamount to the opening of that market to the whole world, and so we cannot capture this intuition. A more realistic analysis of a country's size would require a multi-country model.

V. Conclusion

We have extended the current mean-variance international segmentation literature to include intertemporal consumption and an endogenous interest rate. We retain the mean-variance framework (using exponential utility) in order to compare our conclusions with the previous work and maintain tractability.[12] We revisit the asset price, demand and welfare results of the previous literature and gain several new insights. The interest rate is found to be monotonically increasing as markets are integrated. We also find a breakdown of the effects of market segmentation on asset prices derived in the earlier literature. In analyzing countries' intertemporal consumption behavior we identify four factors which determine the direction and the extent of a country's consumption smoothing. We also find an interrelationship between a country's consumption smoothing, its riskless lending or borrowing, and the influence of market integration on its welfare. We show that a country's welfare may decrease significantly on market integration, specifically for countries which are smoothing consumption forward, or are borrowing against future wealth, in the international bond market.

[12] If, for example, constant relative risk aversion preferences were used, complicated wealth effects would arise from which it would be hard to disentangle the effects of segmentation and of adding an intertemporal component to the model.

APPENDIX

Proof of Proposition 1: Negative exponential utility and normally distributed time-1 wealth imply that the certainty-equivalent time-1 wealth of country i is given by

$$\begin{aligned} CEQ_i &= \mathrm{E}[\tilde{W}_i] - a_i \mathrm{var}(\tilde{W}_i)/2 \\ &= W_i R + \alpha_{i1}(\mu_1 - RP_1) + \alpha_{i2}(\mu_2 - RP_2) - a_i\alpha_{i1}^2\sigma_1^2/2 - a_i\alpha_{i2}^2\sigma_2^2/2 - a_i\alpha_{i1}\alpha_{i2}\rho\sigma_1\sigma_2. \end{aligned}$$

So, in the integrated market each country i solves

$$\max_{W_i, \alpha_{i1}, \alpha_{i2}} u_i(e_i - W_i) + \beta_i u_i(CEQ_i).$$

The first order conditions for this problem lead to

$$\mu_1 - RP_1 - a_i\alpha_{i1}\sigma_1^2 - a_i\alpha_{i2}\rho\sigma_1\sigma_2 = 0, \tag{1}$$

$$\mu_2 - RP_2 - a_i\alpha_{i2}\sigma_2^2 - a_i\alpha_{i1}\rho\sigma_1\sigma_2 = 0, \tag{2}$$

and

$$R = \frac{u_i'(e_i - W_i)}{\beta_i u_i'(CEQ_i)}. \tag{3}$$

Applying the market clearing condition $\alpha_{Xj} + \alpha_{Yj} = 1$ in each risky security j to (1) and (2), we get the equilibrium price of each risky security as a function of the interest rate R as given in Table I. Then substituting these equilibrium price expressions into the asset demand functions given by (1) and (2), yields the equilibrium risky asset demands for each country.

In determining asset prices and demands under other market structures the above procedure is repeated with the explicit trading restrictions imposed on asset demands in the optimization problem (e.g., in MS set $\alpha_{X2} = 0$) yielding again the expressions as in Table I.

Now we show that R increases on integration. From (3), we have under all market structures

$$R = \frac{u_X'(c_X)}{\beta_X u_X'(CEQ_X)} = \frac{u_Y'(c_Y)}{\beta_Y u_Y'(CEQ_Y)},$$

implying

$$R = \left(R^{1/a_X} R^{1/a_Y}\right)^A = \frac{\exp\{-A(c_X + c_Y)\}}{\beta \exp\{-A(CEQ_X + CEQ_Y)\}},$$

where $\beta \equiv \left(\beta_X^{1/a_X}\beta_Y^{1/a_Y}\right)^A$. Now, by the definition of CEQ_i and the market clearing condition in the bond market, we have

$$CEQ_X + CEQ_Y = \mu_1 + \mu_2 - (a_X \mathrm{var}(\tilde{W}_X) + a_Y \mathrm{var}(\tilde{W}_Y))/2.$$

Hence, we obtain

$$(4)\quad R = \frac{\exp\{-A\epsilon\}}{\beta\exp\{-A(\mu_1+\mu_2)+A(a_X\mathrm{var}(\tilde{W}_X)+a_Y\mathrm{var}(\tilde{W}_Y))/2\}},\quad \text{in all market structures.}$$

Now, substituting the equilibrium asset demands from Table I into $\mathrm{var}(\tilde{W}_i)=a_i\alpha_{i1}^2\sigma_1^2/2+a_i\alpha_{i2}^2\sigma_2^2/2+a_i\alpha_{i1}\alpha_{i2}\rho\sigma_1\sigma_2$, after some straightforward manipulations we get

$$\begin{aligned}
a_X\mathrm{var}(\tilde{W}_X^I)+a_Y\mathrm{var}(\tilde{W}_Y^I) &= A(\sigma_1^2+\sigma_2^2+2\rho\sigma_1\sigma_2)\\
a_X\mathrm{var}(\tilde{W}_X^{MS})+a_Y\mathrm{var}(\tilde{W}_Y^{MS}) &= A(\sigma_1^2+\sigma_2^2+2\rho\sigma_1\sigma_2)+a_YA(1-\rho^2)\sigma_2^2/a_X\\
a_X\mathrm{var}(\tilde{W}_X^S)+a_Y\mathrm{var}(\tilde{W}_Y^S) &= A(\sigma_1^2+\sigma_2^2+2\rho\sigma_1\sigma_2)+a_YA(1-\rho^2)\sigma_2^2/a_X+a_YA(a_X\sigma_1/a_Y-\rho\sigma_2)^2/a_X
\end{aligned}$$

which imply

$$a_X\mathrm{var}(\tilde{W}_X^S)+a_Y\mathrm{var}(\tilde{W}_Y^S)\geq a_X\mathrm{var}(\tilde{W}_X^{MS})+a_Y\mathrm{var}(\tilde{W}_Y^{MS})\geq a_X\mathrm{var}(\tilde{W}_X^I)+a_Y\mathrm{var}(\tilde{W}_Y^I)$$

with the first inequality holding with equality if and only if $\sigma_1 a_X=\rho\sigma_Y a_Y$ and the second if $\rho^2=1$. Finally the above inequality and equation (4) yield the desired results. *Q.E.D.*

Proof of Proposition 2: The asset prices in Table I are derived again as outlined in the proof of Proposition 1. The inequalities are derived by simple algebra from the price expressions in Table I. We obtain the comparisons of the static model by setting all interest rates equal and so conclude the final statement, using Proposition 1. *Q.E.D.*

Proof of Proposition 3: In the proof of Proposition 1, we showed the derivation of countries' equilibrium risky asset demands. Now, suppose that, instead of following their optimal consumption strategies, each country consumed its own endowment ($\gamma\epsilon$ or $(1-\gamma)\epsilon$) at time 0, held its equilibrium risky asset holdings and held enough bond to afford to do this. The associated bond demands and certainty equivalent wealths for each country would be the same as those in the model of Section II, which we denote by $\alpha_{i0}^{\overline{k}}$ and $CEQ_i^{\overline{k}}$. Substitution of the risky asset demands from Table I yields the $\alpha_{i0}^{\overline{k}}$ of Table I, and the $CEQ_i^{\overline{k}}$ as follows:

$$\begin{aligned}
CEQ_X^{\overline{S}} &= \mu_1-a_X\sigma_1^2/2\\
CEQ_X^{\overline{MS}} &= \mu_1-A(\sigma_1^2+\rho\sigma_1\sigma_2)+A^2(\sigma_1^2+\sigma_2^2+\rho\sigma_1\sigma_2)/(2a_X)-A^2(1-\rho^2)\sigma_2^2/(2a_X)\\
CEQ_X^{\overline{I}} &= \mu_1-A(\sigma_1^2+\rho\sigma_1\sigma_2)+A^2(\sigma_1^2+\sigma_2^2+\rho\sigma_1\sigma_2)/(2a_X)\\
CEQ_Y^{\overline{S}} &= \mu_2-a_Y\sigma_2^2/2\\
CEQ_Y^{\overline{MS}} &= \mu_2-A(\sigma_2^2+\rho\sigma_1\sigma_2)+A^2(\sigma_1^2+\sigma_2^2+\rho\sigma_1\sigma_2)/(2a_Y)-A^2a_Y(1-\rho^2)\sigma_2^2/(2a_X^2)\\
CEQ_Y^{\overline{I}} &= \mu_2-A(\sigma_2^2+\rho\sigma_1\sigma_2)+A^2(\sigma_1^2+\sigma_2^2+\rho\sigma_1\sigma_2)/(2a_Y)
\end{aligned}$$

These expressions directly yield the quoted expressions for $[A_X CEQ_X^{\bar{k}} - A_Y CEQ_Y^{\bar{k}}]$.

Now define Δc_X as the difference between country X's equilibrium consumption and its time 0 endowment, i.e., $c_X \equiv \gamma\epsilon + \Delta c_X$. Then we have

$$\begin{aligned} \alpha_{X0}^{k} &= e_X - (\gamma\epsilon + \Delta c_X) - \alpha_{X1}P_1 - \alpha_{X2}P_2 \\ &= \alpha_{X0}^{\bar{k}} - \Delta c_X \end{aligned} \tag{5}$$

and hence

$$\begin{aligned} CEQ_X^{k} &= (\alpha_{X0}^{\bar{k}} - \Delta c_X)R + \alpha_{X1}\mu_1 + \alpha_{X2}\mu_2 - a_X\alpha_{X1}^2\sigma_1^2/2 - a_X\alpha_{X2}^2\sigma_2^2/2 - a_X\alpha_{X1}\alpha_{X2}\rho\sigma_1\sigma_2 \\ &= CEQ_X^{\bar{k}} - \Delta c_X R. \end{aligned} \tag{6}$$

Analogous expression may be derived for country Y. Then since $\Delta c_X = -\Delta c_Y$ by clearing in time-0 consumption, we conclude $CEQ_X + CEQ_Y = CEQ_X^{\bar{k}} + CEQ_Y^{\bar{k}}$. To derive Δc_X we use (4) and (3) to obtain, with the use of (6),

$$\begin{aligned} R &= \frac{\exp\{-A\epsilon\}}{\beta\exp\{-A(CEQ_X + CEQ_Y)\}} = \frac{\exp\{-A\epsilon\}}{\beta\exp\{-A(CEQ_X^{\bar{k}} + CEQ_Y^{\bar{k}})\}} \\ &= \frac{\exp\{-a_X c_X\}}{\beta_X\exp\{-a_X CEQ_X\}} = \frac{\exp\{-a_X(\gamma\epsilon + \Delta c_X\}}{\beta_X\exp\{-a_X(CEQ_X^{\bar{k}} - \Delta c_X R)\}} \end{aligned}$$

implying

$$A(\epsilon - CEQ_X^{\bar{k}} - CEQ_Y^{\bar{k}}) + \ln\beta = a_X(\gamma\epsilon + \Delta c_X - CEQ_X^{\bar{k}} + \Delta c_X R) + \ln\beta_X.$$

Rearranging and manipulating yields

$$\Delta c_X = -\frac{1}{1+R}[A_X\gamma\epsilon - A_Y(1-\gamma)\epsilon] + \frac{1}{1+R}[A_X CEQ_X^{\bar{k}} - A_X CEQ_Y^{\bar{k}}] + \frac{1}{(1+R)a_X}\ln(\beta/\beta_X).$$

Then $c_X = \gamma\epsilon + \Delta c_X$ and (5) yield the expressions for X's time-0 consumption and bond demands. c_Y may be obtained from $c_Y = \epsilon - c_X$. Lastly, when $a_X = a_Y = a$, X's consumption in all three markets is given by

$$c_X^k = \gamma\epsilon + \frac{1}{1+R^k}\{[1/2 - \gamma)2\epsilon] + [(\mu_1 - a\sigma_1^2/2) - (\mu_2 - a\sigma_2^2/2)] + \ln(\beta/\beta_X)\}$$

and X's bond demand in the segmented market is given by

$$\alpha_{X0}^{s} = -\frac{1}{1+R^s}\{[1/2 - \gamma)2\epsilon] + [(\mu_1 - a\sigma_1^2/2) - (\mu_2 - a\sigma_2^2/2)] + \ln(\beta/\beta_X)\}.$$

Q.E.D.

Proof of Corollary 1: Since Proposition 1 implies that, when $\rho \neq \pm 1$, $\sigma_1 \neq \rho\sigma_2$, we have

$$0 < \frac{1}{1+R^I} < \frac{1}{1+R^{MS}} < \frac{1}{1+R^S}$$

the stated necessary and sufficient condition for $\gamma\epsilon < c_X^I < c_X^{MS} < c_X^S$ or for $\alpha_{X0}^S < 0$ is immediate from the expressions in Proposition 3. *Q.E.D.*

Proof of Corollary 2: Substitution into the expressions of Proposition 3 yields the desired results. *Q.E.D.*

Proof of Proposition 4: Each country's welfare is defined by

$$V_i \equiv u_i(c_i) + \beta_i E[u_i(\tilde{W}_i)] = u_i(c_i) + \beta_i u_i(CEQ_i). \tag{7}$$

Again we use our extra first order condition (3)

$$R = \frac{u_i'(c_i)}{\beta_i u_i'(CEQ_i)} = \frac{u_i(c_i)}{\beta_i u_i(CEQ_i)},$$

where the second equality arises because of the negative exponential utility, since $u_i'(c) = \exp(-a_i c_i) = -a_i u_i(c_i)$. We substitute for $u_i(CEQ_i)$ in (7) to obtain

$$V_i = \frac{1+R}{R} u_i(c_i) = -\frac{1+R}{a_i R} \exp\{-a_i c_i\}. \tag{8}$$

Since from Proposition 1 we have $(1+R^I)/R^I < (1+R^{MS})/R^{MS} < (1+R^S)/R^S$, from (8) we have that

$$c_i^I \geq c_i^{MS} \text{ implies } V_i^I \geq V_i^{MS} \text{ and } c_i^{MS} \geq c_i^I \text{ implies } V_i^{MS} \geq V_i^I.$$

Hence $V_X^I < V_X^{MS} < V_X^S$ implies condition (i) $c_X^I < c_X^{MS} < c_X^S$, by the contrapositive. By inspection of X's consumption in the proof of Proposition 3, we see that (i) is necessary and sufficient for (ii). Then Corollary 1 states that (iii) and (iv) are equivalently necessary and sufficient for (i) and (ii). *Q.E.D.*

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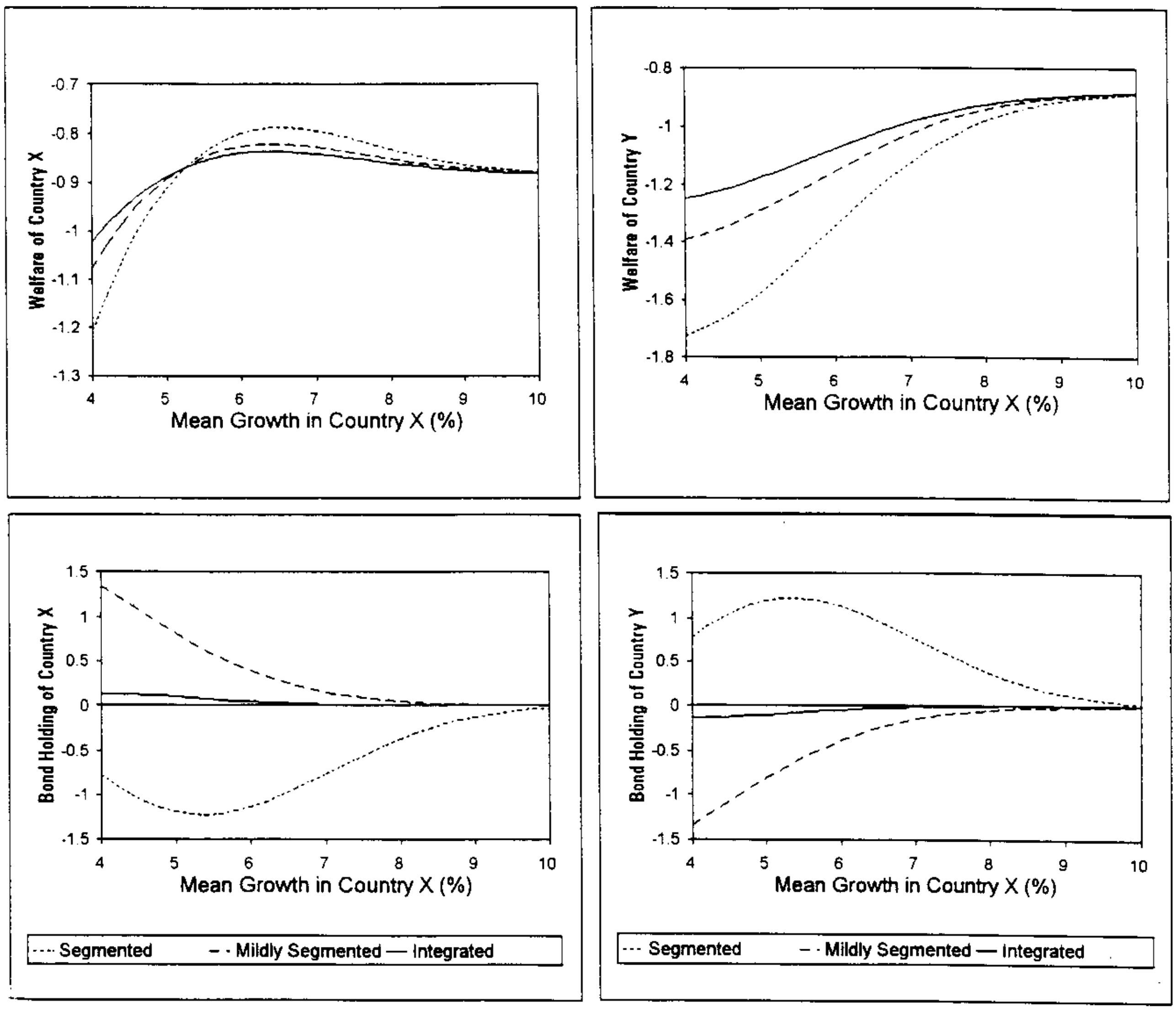

Figure 1: Welfares and bond holdings of the two countries plotted against mean annual growth of Country X, under the various market structures. Plots are against the expected payoff on asset 1, $\mu_1 = 18.66 - 75.84$, for fixed values of $\mu_2 = 11.48$, $\sigma_1 = 8.75$, $\sigma_2 = 7.00$, $\rho = 0$, $\epsilon = 14$, $\gamma = 0.5$, $\beta_X = \beta_Y = 0.78$, and $a_X = a_Y = 0.229$. For a horizon of 25 years, these parameters correspond to a mean per annum growth of 4–10% in country X and 2% in country Y, and an impatience parameter of 0.99 per annum in both countries. Countries are identical in all respects, except that $\mu_1 - a_X\sigma_1^2/2 > \mu_2 - a_Y\sigma_2^2/2$.

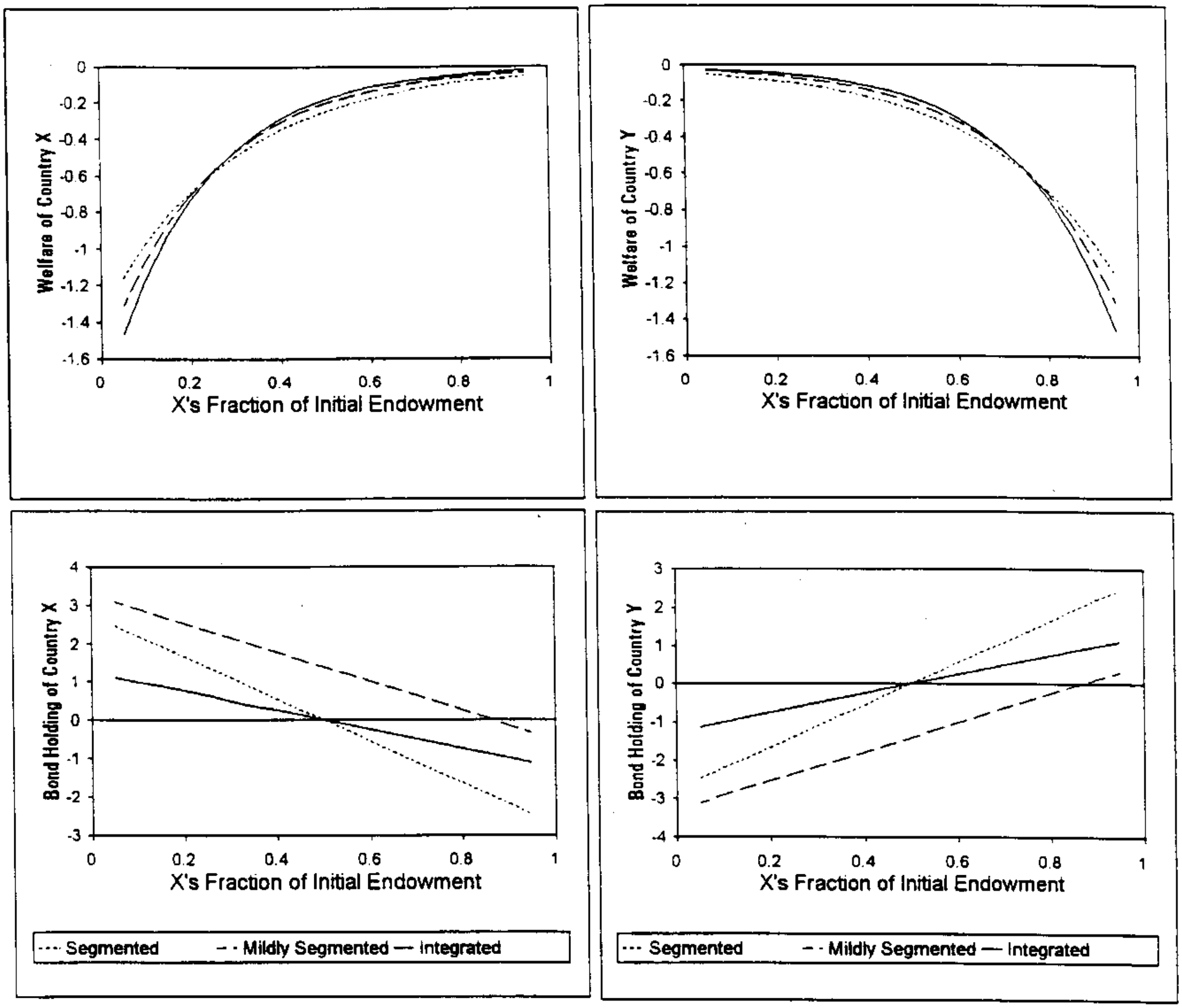

Figure 2: Welfares and bond holdings of the two countries plotted against the fraction of time-0 good endowed to country X, under the various market structures. Plots are against $\gamma = 0.0 - 1.0$, for fixed values at $\mu_1 = \mu_2 = 12.98$, $\sigma_1 = \sigma_2 = 5.25$, $\rho = 0$, $\epsilon = 14$, $\beta_X = \beta_Y = 0.78$, and $a_X = a_Y = 0.4$. For a horizon of 25 years, these parameters correspond to mean per annum growths ranging from 0.1–9.3% in each country (depending on γ), and an impatience parameter of 0.99 per annum in both countries. Countries are identical in all respects (including $\mu_1 - a_X\sigma_1^2/2 = \mu_2 - a_Y\sigma_2^2/2$), except their initial good endowment.

Mean Growth in Country X (% p.a.)			4	5	6	7	8	9	10
Interest Rates (% p.a.)		S	1.8	4.2	7.3	11.2	16.4	23.2	32.3
		MS	3.9	6.3	9.4	13.5	18.8	25.7	34.9
		I	5.2	7.7	10.8	14.9	20.3	27.3	36.7
Measure of Country X's Welfare Change on Integration (%)	Total	S->MS	6.2	1.0	-1.6	-2.1	-1.3	-0.5	-0.1
		MS->I	2.7	0.3	-0.8	-1.0	-0.6	-0.2	-0.1
		S->I	9.0	1.3	-2.3	-3.0	-1.9	-0.8	-0.2
	Diversification	S->MS	8.9	5.3	2.9	1.3	0.5	0.1	0.02
		MS->I	4.1	2.5	1.4	0.6	0.2	0.1	0.01
		S->I	13.2	7.7	4.1	1.9	0.7	0.2	0.03
	Consumption Smoothing	S->MS	-2.5	-4.1	-4.4	-3.4	-1.8	-0.6	-0.1
		MS->I	-1.3	-2.1	-2.2	-1.6	-0.8	-0.3	-0.1
		S->I	-3.7	-5.9	-6.1	-4.8	-2.6	-1.0	-0.2
Measure of Country Y's Welfare Change on Integration (%)	Total	S->MS	14.4	12.9	9.9	5.8	2.6	0.8	0.2
		MS->I	6.4	5.7	4.3	2.5	1.1	0.3	0.1
		S->I	21.6	19.1	14.3	8.4	3.7	1.2	0.3
	Diversification	S->MS	11.1	7.1	3.8	1.6	0.5	0.1	0.02
		MS->I	4.8	3.0	1.6	0.7	0.2	0.1	0.01
		S->I	16.3	10.4	5.5	2.4	0.8	0.2	0.03
	Consumption Smoothing	S->MS	3.0	5.4	5.9	4.1	2.1	0.7	0.2
		MS->I	1.5	2.6	2.7	1.8	0.9	0.2	0.1
		S->I	4.6	7.9	8.3	5.9	2.9	1.0	0.3

Table II: Measures of welfare changes as markets are integrated, for varying levels of mean per annum growth in country X. Interest rates and welfare changes (δ's expressed as percentages, see text) are reported for the parameter values of Figure 1.

X's Fraction of Initial Endowment			0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Interest Rates (% p.a.)		S	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8
		MS	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0	4.0
		I	6.3	6.3	6.3	6.3	6.3	6.3	6.3	6.3	6.3
Measure of Country X's Welfare Change on Integration (%)	Total	S->MS	-5.9	-1.2	2.1	4.5	6.3	7.8	9.0	9.9	10.7
		MS->I	-6.6	-2.1	0.7	2.7	4.1	5.2	6.0	6.7	7.2
		S->I	-11.9	-3.2	2.9	7.2	10.6	13.2	15.4	17.1	18.6
	Diversification	S->MS	12.0	9.8	8.3	7.2	6.3	5.7	5.2	4.7	4.3
		MS->I	9.0	6.9	5.6	4.7	4.1	3.6	3.2	2.9	2.6
		S->I	20.1	16.4	13.9	12.0	10.6	9.5	8.6	7.8	7.2
	Consumption Smoothing	S->MS	-16.0	-10.0	-5.7	-2.5	0.0	2.0	3.6	5.0	6.1
		MS->I	-14.3	-8.4	-4.6	-1.9	0.0	1.5	2.7	3.7	4.5
		S->I	-26.6	-16.8	-9.7	-4.3	0.0	3.4	6.3	8.6	10.6
Measure of Country Y's Welfare Change on Integration (%)	Total	S->MS	10.7	9.9	9.0	7.8	6.3	4.5	2.1	-1.2	-5.9
		MS->I	7.2	6.7	6.0	5.2	4.1	2.7	0.7	-2.1	-6.6
		S->I	18.6	17.1	15.4	13.2	10.6	7.2	2.9	-3.2	-11.9
	Diversification	S->MS	4.3	4.7	5.2	5.7	6.3	7.2	8.3	9.8	12.0
		MS->I	2.6	2.9	3.2	3.6	4.1	4.7	5.6	6.9	9.0
		S->I	7.2	7.8	8.6	9.5	10.6	12.0	13.9	16.4	20.1
	Consumption Smoothing	S->MS	6.1	5.0	3.6	2.0	0.0	-2.5	-5.7	-10.0	-16.0
		MS->I	4.5	3.7	2.7	1.5	0.0	-1.9	-4.6	-8.4	-14.3
		S->I	10.6	8.6	6.3	3.4	0.0	-4.3	-9.7	-16.8	-26.6

Table III: Measures of welfare changes as markets are integrated, for varying fraction of time-0 good endowed to country X. Interest rates and welfare changes (δ's expressed as percentages, see text) are reported for the parameter values of Figure 2.