

**REAL INTEREST RATES AND INFLATION:  
AN *EX-ANTE* EMPIRICAL ANALYSIS**

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## **Real Interest Rates and Inflation: An *Ex-Ante* Empirical Analysis**

We develop a method of measuring *ex-ante* real interest rates using prices of index and nominal bonds. Employing this method and newly available data, we directly test the Fisher hypothesis that the real rate of interest is independent of inflation expectations. We find a negative correlation between *ex-ante* real interest rates and expected inflation. This contradicts the Fisher hypothesis but is consistent with the theories of Mundell and Tobin, Darby and Feldstein, and Stulz. We also find that nominal interest rates include an inflation risk premium that is positively related to a proxy for inflation uncertainty.

## Real Interest Rates and Inflation: An *Ex-Ante* Empirical Analysis

### Introduction

In this paper we develop a method of measuring *ex-ante* real interest rates. Using this method and newly available data, we study empirically the relations among real and nominal interest rates, inflation rates, and inflation uncertainty. Many studies of these relations use the term *real interest rate* when referring to the *ex-post* real return on a nominal bond, constructed as the difference between the *ex-ante* nominal interest rate and the *ex-post* inflation rate. We differentiate between the *ex-ante* real rate of interest, which is measured using only prices observed at the beginning of each month, and the *ex-post* real rate of return on nominal bonds.

*Ex-ante* measurement of real interest rates is possible when there exists a market for bonds that are linked to the Consumer Price Index (CPI), henceforth, index bonds. Observing prices of index bonds, however, is not sufficient for the calculation of *ex-ante* real interest rates. First, because the CPI is announced with a lag, the price level is not directly observed on the day an index bond is bought. Second, because the level of the CPI on the bond's expiration day is announced after the bond holders are paid, index bonds are usually not indexed during the month preceding their maturity. Thus, the price of an index bond reflects not only the *ex-ante* real rate of interest but also market expectations of the yet unannounced past inflation and the nominal forward interest rates for the period in which the bond is not indexed.

We develop a method to measure a single *ex-ante* real interest rate by using simultaneously observed prices of five index and nominal bonds. Two equal maturity bonds, indexed and nominal, are used to infer market expectations of the yet unannounced past inflation. Two additional nominal bonds are used to calculate the forward nominal interest rate for the unindexed period. The prices of index bonds, inferred expectations, and forward rates are used to calculate *ex-ante* real interest rates.

To implement our method we need prices of index and nominal bonds with a particular maturity structure. Such prices are observable in Israel, where the government has simultaneously

issued index and nominal bonds since 1984. While index bonds have been issued by other governments (typically in periods of high inflation), to the best of our knowledge this is the first time index and nominal bonds of equal maturity are *simultaneously* traded.

We use our method and the Israeli bond prices to study three related questions. First, we examine whether *ex post* real returns on nominal bonds are a good proxy for *ex ante* real rates of interest. Second, we test whether the real rate of interest is uncorrelated with inflation expectations. Third, we analyze nominal interest rates to see whether they include a premium for inflation uncertainty.

Prior studies of these issues can be grouped into three sets based on the way inflation expectations and real interest rates are measured or estimated. The first set (e.g., Friedman (1980) and Figlewsky and Wachtel (1981)) uses survey data on investor expectations. The results of these studies are hard to interpret since it is not clear how prices aggregate the individual expectations which are the subject of analysis. In the second set (e.g., Mishkin (1981) and Hamilton (1985)), inflation expectations and real interest rates are estimated from time series of realized (*ex-post*) values and other economic data under the Rational Expectations (RE) hypothesis. The main problem with this method is that the RE hypothesis does not provide any guidance in choosing among available data and statistical models. Studies in the third set (e.g., Cukierman (1974), Fama (1975), Paunio and Suvanto (1977), Boschen and Newman (1987), Huberman and Schwert (1985), Yariv (1990), and Woodward (1990)) examine prices that directly depend on either real interest rates or inflation expectations. Our study belongs to the third set.<sup>1</sup> Because we have complete data of equal maturity nominal and index bond prices, we use observed prices to replace assumptions made by prior studies.

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<sup>1</sup> Our method extends a method developed by Kandel, Ofer, and Sarig (1991, 1993) and by Yariv (1991). Kandel, Ofer, and Sarig (1991) use the method to differentiate between expected and unexpected inflation. Kandel, Ofer, and Sarig (1993) and Yariv (1991) study expectations about inflation that has already occurred but has not yet been announced.

For example, when testing the Fisher (1930) hypothesis, we do not have to assume that the real rate is constant as Fama (1975) does. Nor do we have to assume, as in Paunio and Suvanto (1977), Boschen and Newman (1987) and Yariv (1990), that the term structure of interest rates is flat and that nominal interest rates include no inflation risk premium.

Our method and data allow us to *directly* test the Fisher hypothesis. Fisher argues that deterministic changes in inflation expectations cause equal changes in nominal interest rates, leaving the real interest rate unaffected. In a stochastic setting, we interpret the Fisher hypothesis to mean that the real interest rate process and the inflation process are independent of each other. This independence holds even when inflation regimes change over time. In particular, the real interest rate process neither changes *across* regimes nor depends on inflation *within* each regime.

We find that inflation expectations and *ex-ante* real rates of interest are negatively correlated. This finding contradicts the Fisher hypothesis. It is in line, however, with the arguments of several theoretical models: Mundell (1963), Tobin (1965), and Fischer (1979) show that an increase in expected inflation increases the capital stock and reduces real return to capital; Darby (1975) and Feldstein (1976) suggest that because inflation gains are taxed as if they were real gains, the after-tax real rate of interest declines when inflation expectations increase; Stulz (1986) argues that because investors are uncertain about the monetary policy, real interest rates are negatively correlated with inflation when inflation and output are negatively correlated.

The paper is organized as follows. In Section I we explain the method of extracting *ex-ante* real interest rates from five-tuples of nominal and index bonds. Section II describes the data. Section III compares *ex-ante* real interest rates to *ex-post* real rates of return. In Section IV we examine the relation between inflation expectations and interest rates. Section V analyzes the inflation risk premium included in nominal bond prices. Section VI concludes the paper.



through April 25. Using equation (A13) in the Appendix to value the indexed part, this bond's price-- $P_2$ --can be written as:

$$P_2 = (1 - \alpha_2) \cdot \frac{(F_2 + C_2)}{1 + {}_{3/1}N_{4/25}} + \alpha_2 \cdot \frac{(F_2 + C_2) \cdot \frac{\hat{i}_{February}}{I_2^{base}}}{(1 + R_{March})(1 + {}_{4/1}N_{4/25})} \quad (1)$$

where

- $F_2$  = face value of the index bond,
- $C_2$  = the after-tax coupon rate of the bond,<sup>2</sup>
- $I_2^{base}$  = the index of the month during which bond 2 was issued (base index),
- $R_{March}$  = the real interest rate for March,
- ${}_{3/1}N_{4/25}$  = the nominal interest rate between March 1 and April 25,
- ${}_{4/1}N_{4/25}$  = the forward nominal interest rate (as of March 1) for the period April 1 through April 25, and
- $\hat{i}_{February}$  = the expectations of investors (as of March 1) of the February CPI.<sup>3</sup>

The values of  $F_2$ ,  $C_2$ ,  $\alpha_2$ , and  $I_2^{base}$  are known characteristics of index bond 2. The price  $P_2$  and the yield  ${}_{3/1}N_{4/25}$  are observable. Using the prices of two nominal bonds, one maturing on April 1 and the other on April 25, we calculate  ${}_{4/1}N_{4/25}$ . Finally, as detailed below, we extract  $\hat{i}_{February}$  from

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<sup>2</sup> In Israel, *all* buyers of index bonds are subject to a tax of 35% on the inflation-adjusted interest payments prorated to the number of months between the last coupon payment and their point of purchase. Hence, the marginal investor's tax rate applies at most to only one sixth of the annual coupon payments of the bonds in our sample and has only a small effect on our results.

<sup>3</sup> In general,  $\hat{i}_{February}$  may include a premium for past-inflation risk. In the theoretical Appendix we assume that the current price level is known. Relaxing this assumption is analytically difficult since investor opportunity sets are not well defined when current prices are not known. Kandel, Ofer, and Sarig (1993) show empirically that differences between expectations of *past* inflation embedded in bond prices and actual inflation rates are small in magnitude and uncorrelated with subsequent risks. For our calculations it is sufficient that on March 1 the same  $\hat{i}_{February}$  is embedded in all index bond prices.



prices of an index bond that is not indexed beyond February and an equal-maturity nominal bond.

Consider, for example, an index bond maturing on March 21. We denote this bond by the subscript 1 and indicate its maturity date on the time-line by an arrow marked by (1). The last index to be announced before this bond matures is the February index, which will be announced on March 15. Using the notation introduced in equation (1), the price of this index bond on March 1, denoted by  $P_1$ , is given by

$$P_1 = \frac{(1 - \alpha_1)(F_1 + C_1) + \alpha_2(F_1 + C_1) \frac{i_{February}}{i_1^{base}}}{1 + {}_{3/1}N_{3/21}} \quad (2)$$

where  ${}_{3/1}N_{3/21}$  is the nominal interest from March 1 to March 21 calculated from the price of the equal-maturity nominal bond. The values of  $F_1$ ,  $C_1$ ,  $\alpha_1$ , and  $i_1^{base}$  are known characteristics of index bond 1. Since the price  $P_1$  and the yield  ${}_{3/1}N_{3/21}$  are observable,  $i_{February}$  can be calculated by equation (2) and used in equation (1) to calculate  $R_{March}$ .

The above procedure yields an *ex-ante* real interest rate for each month for which a suitable five-tuple set of bonds exist.<sup>4</sup> For each month we also obtain a one-month default-free nominal rate of interest. In line with previous research, we define the *ex-post* real rate of return as the difference between the nominal rate of interest and the realized rate of inflation.

## II. Data

The data consist of Israeli inflation rates and prices of index and nominal bonds for the period of September 1984 through March 1992. Data are available for all months in the sample period

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<sup>4</sup> In Israel, every Thursday is the maturity day of at least one nominal bond. Some index bonds mature on other days of the week. When a nominal bond with exactly the same maturity day does not exist, we interpolate the yields of two nominal bonds with maturities surrounding the maturity of the index bond.

except for four months in 1987 (July, August, November, and December) for which no suitable bond data are available.

Realized inflation rates are calculated and published by the Central Bureau of Statistics (CBS) using ten components with pre-determined weights. The CPI and its ten components are announced on the last business day before the 16th of the following month.<sup>5</sup>

Bond prices are taken from the official publications of the Tel-Aviv Stock Exchange. Index bonds are partially or fully indexed with indexation rates ranging from 80% to 100%. Coupon rates of index bonds range from 2% to 7% and are paid annually. Upon issuance, index-bond maturities range from six to twenty years. Nominal bonds promise a single payment of par at maturity. The characteristics of Israeli nominal bonds are very similar to those of US treasury bills. Upon issuance, nominal bonds have maturities ranging from one month to one year.

Trading in bonds on the Tel Aviv Stock Exchange is done once a day as an auction. Since there are no market makers, there are no separate bid and ask prices. Transaction costs are levied as a percentage of the total value of trades and run from 0.1% to 1% depending on trade size.

The sample period includes two subperiods with different inflation levels: September 1984 through July 1985 and August 1985 through March 1992. The first subperiod includes 11 monthly observations from a period of high and volatile inflation. The second subperiod follows an austerity program adopted by the Israeli government in July 1985, and is a period of relatively low and stable inflation.

Table I presents summary statistics for the monthly rates of inflation, *ex-ante* real interest, *ex-post* real return on nominal bonds, and nominal interest for the period September 1984 through March 1992 and for the two subperiods. All monthly rates are normalized to a 30-day month.

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<sup>5</sup> While we would like to use the *true* price level on the last day of each month, the CBS actually *samples* prices throughout the month. This sampling adds noise to some of our dependent variables.

In the high inflation period, the average monthly inflation rate was 15.2%, which is an annual inflation rate of 443%. At its peak, the monthly inflation rate was 26.5% or 1578% per annum! On the other hand, immediately following the adoption of the austerity measures, the monthly inflation rate dropped to a level of about 1.5% per month, or about 19% per annum.

The real interest rate, measured either *ex-ante* or *ex-post*, reaches a low of about -10% per month in the high inflation period. While this rate may seem very low in moderately inflationary regimes, it is plausible in a period where the monthly inflation rate is as high as 26%.<sup>6</sup> Note that the difference in the average *ex ante* real interest rates across the two inflation sub-periods is not consistent with the Fisher hypothesis, which we formally test in Section IV.

In Table I we also report the first-order autocorrelations of all series. Figure 1 presents the series' first 15 autocorrelations. While the first-order autocorrelations of the inflation and nominal interest series are high, the higher-order autocorrelations vanish after 12 lags. Moreover, as reported in Table II, we reject the hypothesis that any series includes a unit root using Dickey-Fuller (1979, 1981) and Phillips-Perron (1988) tests with significance levels of 5% or less. Schwert (1987, 1989) and Mishkin (1992) show that these tests reject the null hypothesis of a unit root too often in small samples and when a moving-average component is present. They conclude that US inflation and nominal rate series are non-stationary but cointegrated. Similarly, Huberman and Schwert (1985) conclude that the Israeli inflation series was not stationary in 1958-1981. Recently, however, Evans and Lewis (1994) suggest that the non-stationarity found in the US data is actually an artifact of switching inflation regimes. In our sample period there are clearly two inflation regimes and switching inflation regimes may also explain Huberman and Schwert's findings. Note that unlike studies that use *ex-post* returns on nominal bonds to study the properties of the real rate of interest, we measure the real interest rate directly. This measurement does not depend on whether the inflation

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<sup>6</sup> Throughout the high inflation period the access of Israeli investors to foreign markets was curtailed.

regime changes or not. Nonetheless, in our tests we consider the potential effect of a regime switch and show that our analysis is consistent both with a single inflation regime and switching inflation regimes.

### III. *Ex-Ante* Real Interest Rates and *Ex-Post* Real Rates of Return

Many studies of real interest rates use an *ex-post* measure of the real rate of interest, constructed as the difference between the *ex-ante* nominal interest rate and the *ex-post* inflation rate. In this section we compare real interest rates calculated by our method, on the basis of *ex-ante* information only, to *ex-post* real rates of return.

Figure 2 presents monthly *ex-ante* real interest rates and *ex-post* rates of return. Both the *ex-post* and the *ex-ante* rates are more volatile in the high inflation period than in the low inflation period. The figure also suggests that *ex-post* real rates of return and *ex-ante* real interest rates are less than perfectly correlated with each other. To formally examine the relation between these rates, we define the following terms:

- $\pi$  = log of one plus the actual one-month inflation rate,
- $e\pi$  = expected value of  $\pi$ ,
- $\sigma^2$  = variance of  $\pi$ ,
- $n$  = log of one plus the one-month nominal interest rate,
- $r^{xa}$  = log of one plus the *ex-ante* one-month real interest rate.

Using this notation, the *ex-post* (continuously compounded) real return to the holder of a nominal bond is

$$r^p \equiv n - \pi. \quad (3)$$

We estimate the regression equation:

$$r_t^{\mathcal{P}} = \alpha + \beta r_t^{\mathcal{M}} + v_t. \quad (4)$$

To interpret the regression coefficients, we decompose the nominal rate into three components:

$$n = r^{\mathcal{M}} + e\pi + \lambda, \quad (5)$$

where  $\lambda$  is an inflation premium. This decomposition reflects the results of Fischer (1975), Barro (1976), Benninga and Protopapadakis (1983), and Cox, Ingersoll, and Ross (1985). They show that under uncertain inflation, nominal interest rates may include an inflation risk premium. This premium may be either positive or negative depending on how inflation risk covaries with the marginal utility of consumption. Note that, because of the log transformation,  $\lambda$  is not just an inflation risk premium. Specifically, assuming that  $\pi$  is normally distributed, we can decompose  $\lambda$  as

$$\lambda = \kappa - \frac{1}{2}\sigma^2, \quad (6)$$

where  $\kappa$  is a (pure) risk premium reflecting investors' risk aversion and the correlation between inflation and investors' wealth. Thus, even if inflation is independent of the marginal utility of consumption,  $\lambda = -\frac{1}{2}\sigma^2$ . Also note that under the Fisher hypothesis,  $r^{\mathcal{M}}$  is uncorrelated with  $e\pi$  and with  $\lambda$ .

The (transformed) actual inflation rate can be written as

$$\pi = e\pi + \delta, \quad (7)$$

where  $\delta$  is a zero-mean expectation error. Using equations (5) and (7) we can write expression (3) for  $r^{\mathcal{P}}$  as:

$$r^{\mathcal{P}} = r^{\mathcal{M}} + \lambda - \delta. \quad (8)$$

We assume rational expectations, which implies that  $\delta$  is uncorrelated with  $r^{\mathcal{M}}$  and  $\lambda$ . Hence, from

equation (8), the slope coefficient of the estimated regression equation (4) is

$$\beta = 1 + \frac{\text{cov}(\lambda, r^{aa})}{\text{var}(r^{aa})}. \quad (9)$$

Thus, it is meaningful to test whether  $\beta$  equals one only if the inflation premium is uncorrelated with  $r^{aa}$ . This occurs when the real interest rate is constant or under the Fisher hypothesis. This is true regardless of whether there is a unique inflation regime or whether inflation regimes change over time (under the assumed rationality of expectations). To account for the potential changes in the variance of the unexpected inflation across regimes, our tests are heteroskedasticity adjusted.

In Table III we report the estimation of regression equation (4). The estimated  $\beta$  is significantly less than one at the 5% level. It appears that there is an inflation premium that is negatively correlated with the real interest rate.<sup>7</sup> This contradicts the Fisher hypothesis. Therefore, we proceed to examine the relations between inflation, inflation uncertainty, and real and nominal interest rates.

#### IV. Inflation and Interest Rates

Numerous empirical studies attempt to estimate the relation between inflation and interest rates using series of nominal interest rates and realized inflation rates. Interpretation of the results of these studies requires making assumptions regarding the determinants of interest rates. For example, Fama (1975) and others test the Fisher hypothesis by testing whether the slope coefficient in the regression equation:

$$\pi = \alpha_0 + \beta_0 \cdot r + \epsilon \quad (10)$$

equals one. Using equations (5) and (7), this can be written as:

Under rational expectations,  $\delta$  is uncorrelated with  $r^{aa}$ ,  $\epsilon\pi$ , and  $\lambda$ . Hence,

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<sup>7</sup> This finding is consistent with Wakeman and Bhagat (1984) who show that US Treasury Bill returns include an inflation premium.

$$(e\pi + \delta) = \alpha_0 + \beta_0 \cdot (r^{xa} + e\pi + \lambda) + \epsilon. \quad (11)$$

$$\beta_0 = \frac{\text{cov}(\pi, n)}{\text{var}(n)} \quad (12)$$

$$= \frac{\text{var}(e\pi) + \text{cov}(e\pi, r^{xa}) + \text{cov}(e\pi, \lambda)}{\text{var}(e\pi) + \text{var}(r^{xa}) + \text{var}(\lambda) + 2\text{cov}(e\pi, r^{xa}) + 2\text{cov}(e\pi, \lambda) + 2\text{cov}(r^{xa}, \lambda)},$$

which implies that the Fisher hypothesis (i.e., that  $r^{xa}$  is independent of  $e\pi$  and  $\lambda$ ), is not sufficient for  $\beta_0$  to equal one. Ostensibly,  $\beta_0 = 1$  if both the real rate of interest,  $r^{xa}$ , and the inflation premium,  $\lambda$ , are non-stochastic. (For similar conclusions see Nelson and Schwert (1977) and Mishkin (1992).)

Using the series of *ex-ante* real interest rates we can directly examine theories about the relations between inflation and interest rates. First, we estimate the regression:

$$\pi = \alpha_1 + \beta_1 r^{xa} + \eta, \quad (13)$$

which, under rational expectations, directly estimates the relation between the *ex-ante* real rate of interest and expected inflation. Since we do not use nominal interest rates to estimate this relation, its interpretation is not predicated upon any assumed property of either the nominal interest rate ( $n$ ) or of the inflation premium ( $\lambda$ ). We then estimate the second component of equation (10):

$$\pi = \alpha_2 + \beta_2 \cdot (n - r^{xa}) + \zeta. \quad (14)$$

Table IV reports estimates of the combined regression equation (10) and of the component regression equations (13) and (14). For the complete sample, the estimated slope coefficient of equation (10) is close to one, while for the low inflation period it is 0.319. Comparable estimates using US data differ across sample periods. For example, Fama (1975) estimates a slope coefficient of 0.97 for the period January 1953 through August 1971, while Mishkin (1992) estimates a slope coefficient of 0.718 for the period of January 1953 through December 1990.

Carlson (1977), Barsky (1987), and Mishkin (1992) argue that the estimated slope coefficient of equation (10) varies over time since there are changes in the way inflation expectations and real interest rates vary over time. From equation (12) it can be seen that when changes in inflation expectations dominate the movement of the nominal rate, the estimated coefficient is close to one. On the other hand, when variation in the nominal rate is dominated by variation in the real rate of interest, the estimated coefficient is close to zero. This prediction is born out in our sample. In the high and volatile inflation period, movement in the nominal interest rate is largely driven by changes in inflation expectations. Accordingly, the estimated coefficient for the complete sample is close to one. In the low and stable inflation period, movement in the nominal interest rate is largely driven by changes in the real rate of interest. Therefore, the estimated coefficient for the low inflation subperiod is less than one.

Equation (13) allows us to directly test the Fisher hypothesis by examining the relation between real interest rates and expected inflation. If the real rate of interest is independent of inflation,  $\beta_1 = 0$ . The coefficient equals zero even when there are multiple inflation regimes since the Fisher hypothesis implies independence of inflation and real interest rates both *within* and *across* regimes. The alternative theories of Mundell (1963), Tobin (1965), Fischer (1979), Darby (1975), Feldstein (1976), and Stulz (1986) imply that real interest rates are negatively related to inflation expectations, i.e.,  $\beta_1 < 0$ . Note that when there are multiple inflation regimes  $\eta$  may be heteroskedastic. Therefore, we use heteroskedasticity-consistent tests.

For the entire sample we obtain a negative estimate for  $\beta_1$ , which is significantly different from zero at the 0.1% level. The negative correlation between *ex-ante* real interest rates and expected inflation contradicts the Fisher hypothesis. However, these findings are consistent with the results of Fama (1990), Fama and Gibbons (1982), Huizinga and Mishkin (1984), Pennacchi (1991), and Summers (1983), all of whom use alternative methods to estimate the relation between real interest



rates and inflation in the United States.

We do not find a negative relation between expected inflation and *ex-ante* real interest rates in the low and stable inflation subperiod, possibly because the dispersion of the explanatory variable is too small to allow us to empirically detect the relation. Alternatively, taxation of inflation gains may affect the real rate of interest in a period of low inflation less than it affects the real interest rate in a high inflation period.

To interpret the coefficient of regression equation (14), we use equations (5) and (7) to rewrite it as:

$$(e\pi + \delta) = \alpha_2 + \beta_2 \cdot (e\pi + \lambda) + \zeta. \quad (15)$$

From this equation it follows that, under rational expectations,

$$\beta_2 = \frac{\text{var}(e\pi) + \text{cov}(e\pi, \lambda)}{\text{var}(e\pi) + \text{var}(\lambda) + 2\text{cov}(e\pi, \lambda)}. \quad (16)$$

Thus, a test of the equality of  $\beta_2$  to one is a joint test of the rationality of inflation expectations and of a constant inflation premium. This test is independent of the time series properties of the real rate of interest ( $r^m$ ). If the inflation premium covaries positively with the level of expected inflation,  $\beta_2$  will be less than one. If the variation in expected inflation is much larger than the variation of the risk premium,  $\beta_2$  will be close to one.

For the complete sample we estimate a slope coefficient of 0.78, which is significantly different from one at the 1% significance level. This could be interpreted as evidence against the rationality of inflation expectation. Within the rational expectations paradigm, however, this suggests that an inflation premium is included in the nominal interest rate and that this premium covaries positively with the level of inflation. In the low inflation period we find a much smaller  $\beta_2$  coefficient. From equation (16), this may reflect a lower ratio of the variance of expected inflation to the

variance of inflation premium in the low inflation period than in the high inflation period. This explanation is consistent with the lower coefficient of variation of inflation in the high inflation period than in the low inflation period. In the next section we explicitly examine the hypothesis that an inflation risk premium is included in the nominal rate and that it covaries positively with inflation uncertainty.

## V. Inflation Premium and Inflation Uncertainty

In this section we examine the properties of the inflation premium included in nominal interest rates. Subtracting the calculated *ex-ante* real rate of interest and the *realized* rate of inflation from the nominal interest rate we obtain:

$$PR \equiv n - r^{aa} - \pi = \lambda - \delta, \quad (17)$$

where the second equality follows from equations (5) and (7). Hence,  $PR$  is the difference between the inflation premium and inflation expectation errors. Using equation (6) we get

$$E(PR) = \lambda - E(\delta) = \kappa - \frac{1}{2}\sigma^2. \quad (18)$$

Thus, even if inflation is uncorrelated with investors' marginal rates of substitution (which implies that  $\kappa = 0$ ),  $E(PR) = -\frac{1}{2}\sigma^2 < 0$ .

Table V presents summary statistics for  $PR$  for the complete sample as well as for the two inflationary regimes. As can be seen, the mean value of  $PR$  is positive in both subperiods, albeit in an insignificant way, and is higher in the high and volatile inflation subperiod than in the low and stable inflation subperiod. The difference between the average  $PR$ 's of the different periods is significantly different from zero (p-value 0.1%). Note that the significant difference is *not* due to the presence of half the inflation variance in  $E(PR)$ . This is because inflation volatility in the high inflation period is *larger* than the inflation volatility in the low inflation period. Thus, the difference between the pure risk premia (i.e., the difference between the  $\kappa$ 's of the two subperiods) is *larg-*

er than the difference between the *PR*'s of the two periods.<sup>8</sup>

To provide additional evidence for the existence of an inflation risk premium, we show that *PR* is related to a measure of inflation uncertainty. The monthly inflation rate is calculated as a weighted average of changes in prices of goods included in the CPI basket according to predetermined weights. In any given month, each investor does not transact in all goods included in the CPI basket. Since investors observe only a subset of the prices in the economy, their information about inflation is less accurate in months with large changes in relative prices than in months with small changes in relative prices. Therefore, we use the dispersion of price changes across the ten components of the CPI as a measure of inflation uncertainty. The dispersion in month  $t$ , denoted by  $D_t$ , is calculated as the weighted average of the squared deviations of the ten CPI components relative to the monthly inflation rate, using the weights of the components in the CPI<sup>9</sup>:

$$D_t = \sqrt{\sum_{i=1}^{10} w_{i,t} (\pi_{i,t} - \pi_t)^2} \quad (19)$$

where  $w_{i,t}$  is the weight of the  $i$ -th component of the CPI,  $\pi_{i,t}$  is the rate of change of the  $i$ -th component of the CPI, and  $\pi_t$  is the overall inflation rate in month  $t$ .

The last dispersion information investors have on the first day of month  $t$  is the CPI's dispersion of month  $t-2$ , which was announced in month  $t-1$ . Therefore, we regress the *PR*'s on the  $D_{t-2}$ 's. If the nominal interest rate includes an inflation premium and if  $D_{t-2}$  is a good proxy for month  $t$ 's inflation uncertainty, the estimated slope coefficient will be positive. Under rational expectations,  $D_{t-2}$  is uncorrelated with  $\delta_t$ .

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<sup>8</sup> The mean untransformed *PR*'s of the two subperiods are also positive and insignificantly different from zero. The difference between the means of the *untransformed PR*'s of the two subperiods is different from zero with p-value of 0.5%.

<sup>9</sup> This measure is discussed by Fischer (1981) and is used, for example, by Blejer and Leiderman (1980) and Parks (1978).

The estimated relation between  $PR$  and inflation uncertainty, as proxied by the dispersion of relative price changes, is reported in Table VI. For the complete sample the estimated slope coefficient is positive and significantly different from zero (p-value 0.033).<sup>10</sup> For the low and stable inflation period, the estimated coefficient is also positive but in an insignificant way. The lack of significance in the stable inflation subperiod can be attributed to too little variation in inflation uncertainty within this subperiod as evident by this period's low variation in the dispersion of relative price changes. The documented positive relation between  $PR$  and our proxy for inflation uncertainty suggests that nominal interest rates contain an inflation risk premium that is large when inflation uncertainty is high. This is consistent with the findings of Barnea, Dotan, and Lakonishok (1979) who show that US nominal interest rates are higher when expert predictions of inflation are dispersed.

## VI. Conclusions

In this paper we develop a method to extract *ex-ante* real rates of interest from simultaneously observed prices of index and nominal bonds. Using newly available data and this method, we directly test the Fisher hypothesis that the real rate of interest is independent of inflation expectations.

We find a negative correlation between *ex-ante* real interest rates and expected inflation. This contradicts the Fisher hypothesis. It is consistent, under diminishing returns to capital, with the Mundell-Tobin argument that high inflation expectations cause higher capital accumulation. It is also consistent with the Darby-Feldstein argument that taxation of inflation gains causes the real rate of interest to be negatively correlated with expected inflation and with Stulz's argument that uncertainty about monetary policy leads to such a negative correlation.

We also find that nominal interest rates include an inflation premium. The premium is high

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<sup>10</sup> Note that the positive coefficient is *not* due to the dependence of  $PR$  on  $\sigma^2$ . If inflation volatility is positively serially correlated, the regression coefficient of  $PR_t (= \kappa_t - \frac{1}{2}\sigma_t^2)$  on  $D_{t-2}$  would be *smaller* than the regression coefficient of the (unobserved) risk premium,  $\kappa_t$ , on  $D_{t-2}$ .

when inflation uncertainty is high and is positively related to a proxy for inflation uncertainty. The existence of a risk premium is consistent with theoretical analysis of nominal contracts under uncertain inflation.

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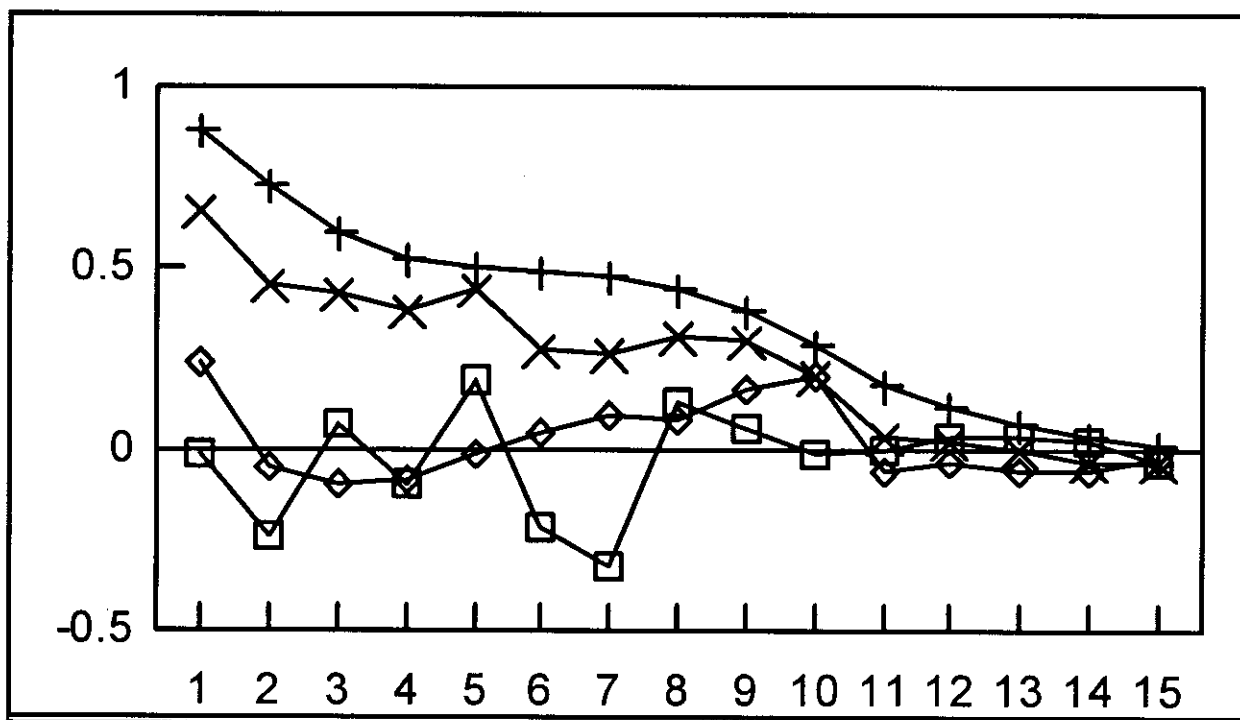
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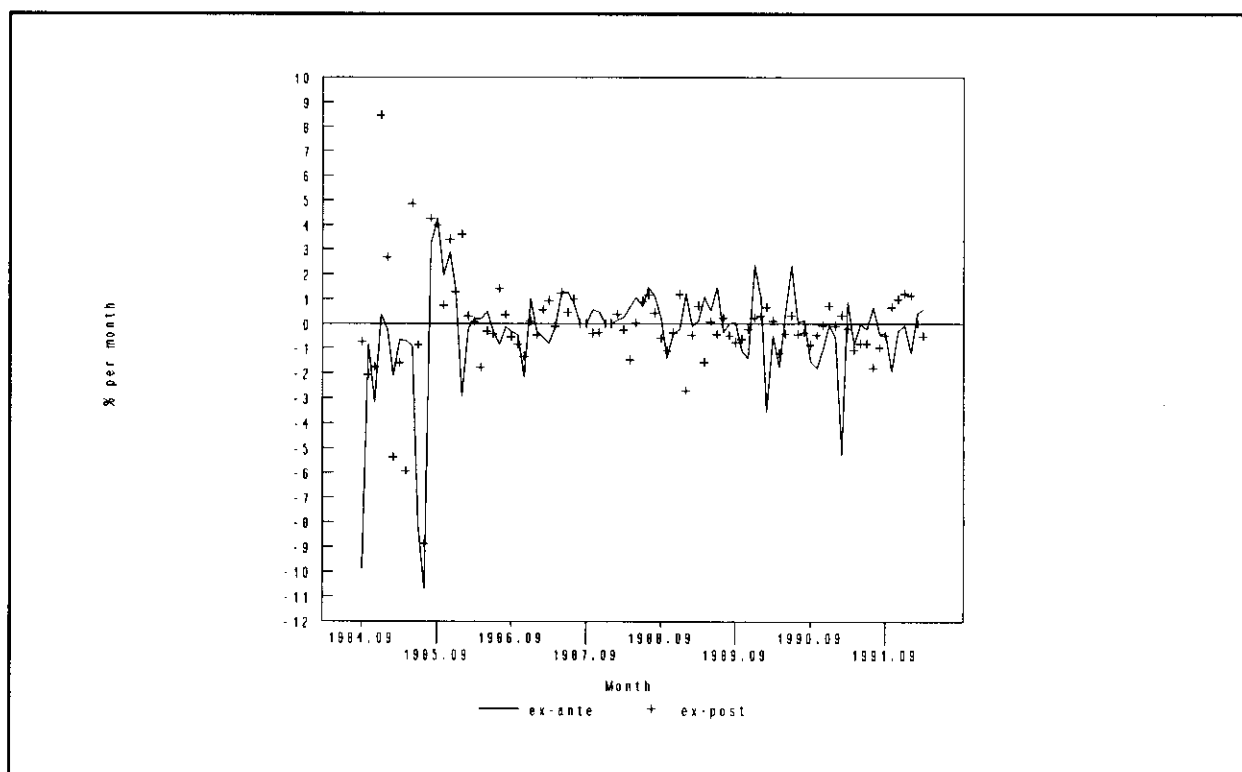


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**FIGURE 1. Autocorrelations of Data Series** The first 15 autocorrelations of the monthly *ex-ante* real interest rates ( $\diamond$ ), nominal interest rates (X), *ex-post* real rate of returns ( $\square$ ), and *actual* inflation rates (+) series in Israel between September 1984 and March 1992. The *ex ante* real interest rates are calculated from prices of index and nominal bonds simultaneously observed at the beginning of each month. The nominal interest rates are the yields to maturity of one-month nominal bonds. The *ex-post* real rates of return are calculated using nominal interest and realized inflation rates. All rates are normalized to 30 day months.



**FIGURE 2. *Ex-ante* Real Rates of Interest and *Ex-post* Real Returns of Nominal Bonds** Monthly *ex ante* real interest rates and *ex-post* real rates of return on nominal bonds in Israel between September 1984 and March 1992. The *ex ante* real interest rates are calculated from prices of index and nominal bonds simultaneously observed at the beginning of each month and are depicted by a solid line. The *ex-post* real rates of return are calculated using yields to maturity of one-month nominal bonds and realized inflation rates and are depicted by '+'. All rates are normalized to 30 day months.



**TABLE I**  
**Summary Statistics**

Summary statistics for monthly rates of *ex-ante* real interest ('Ex-Ante'), *ex-post* real return on nominal bonds ('Ex-Post'), nominal interest ('Nominal'), and inflation ('Inflation') in Israel. All rates are expressed in terms of percent per 30-day month. The *ex ante* real interest rates are calculated from prices of index and nominal bonds simultaneously observed at the beginning of each month. The nominal interest rates are the yields to maturity of one-month nominal bonds. The *ex-post* real rates of return are calculated using the nominal interest and realized inflation rates. S.D. denotes the standard deviations of the variables and  $\rho$  denotes their first-order autocorrelations.

The sample period includes two subperiods with different inflation levels: September 1984 through July 1985 and August 1985 through March 1992. The first subperiod is a period of high and volatile inflation while the second subperiod, which follows an austerity program adopted by the Israeli government in July 1985, is a period of relatively low and stable inflation.

**September 1984 - March 1992**

Variable	Mean	S.D.	$\rho^a$	Min.	Max.
Ex-Ante	-0.392	2.245	0.234	-10.677	4.281
Ex-Post	-0.052	2.061	-0.005	-8.879	8.445
Nominal	3.067	4.480	0.868	0.669	20.913
Inflation	3.179	5.362	0.640	-1.294	26.493

**September 1984 - July 1985**

Variable	Mean	S.D.	$\rho$	Min.	Max.
Ex-Ante	-3.362	4.126	0.240	-10.677	0.387
Ex-Post	-1.030	4.943	-0.046	-8.879	8.445
Nominal	13.681	4.455	0.693	7.881	20.913
Inflation	15.150	7.709	0.186	3.614	26.493

**August 1985 - March 1992**

Variable	Mean	S.D.	$\rho$	Min.	Max.
Ex-Ante	0.038	1.424	0.261	-5.265	4.281
Ex-Post	0.089	1.205	0.407	-2.270	4.261
Nominal	1.531	1.205	0.743	0.670	8.197
Inflation	1.446	0.993	0.200	-1.294	4.516

**TABLE II**  
**Tests for Unit Roots**

Tests for the existence of unit roots in Israeli monthly rates of *ex-ante* real interest ('Ex-Ante'), *ex-post* real return on nominal bonds ('Ex-Post'), nominal interest ('Nominal'), and inflation ('Inflation'). All rates are expressed in terms of percent per 30-day month. The *ex ante* real interest rates are calculated from prices of index and nominal bonds simultaneously observed at the beginning of each month. The nominal interest rates are the yields to maturity of one-month nominal bonds. The *ex-post* real rates of return are calculated using the nominal interest and realized inflation rates. D-F(n) refers to the Dickey-Fuller (1981) test for unit roots with n additional lags. P-P(n) refers to the Phillips-Perron (1988) test for unit roots with n additional lags, which allows non-i.i.d. disturbance terms in the autoregressive process. Both tests are based on estimated OLS t-statistics. The 10% (5%) critical values of the D-F statistic is -1.60 (-1.95) for both samples. The critical values for the P-P statistic are the same as those of the D-F statistic.

The sample period includes two subperiods with different inflation levels: September 1984 through July 1985 and August 1985 through March 1992. The first subperiod is a period of high and volatile inflation while the second subperiod, which follows an austerity program adopted by the Israeli government in July 1985, is a period of relatively low and stable inflation.

**September 1984 - March 1992**

Variable	D-F(0)	D-F(11)	P-P(0)	P-P(11)
Ex-Ante	-8.048	-3.970	-8.146	-8.503
Ex-Post	-9.091	-2.364	-9.202	-10.286
Nominal	-4.637	-3.745	-4.693	-6.415
Inflation	-4.792	-6.562	-4.850	-4.773

**August 1985 - March 1992**

Variable	D-F(0)	D-F(11)	P-P(0)	P-P(11)
Ex-Ante	-6.672	-1.815	-6.765	-6.704
Ex-Post	-6.018	-4.150	-6.102	-6.042
Nominal	-12.470	-1.614	-12.645	-13.676
Inflation	-7.068	-2.046	-7.167	-7.901

**TABLE III**  
**The Relation between *Ex-Ante* Real Interest and *Ex-Post* Real Returns**

Estimates of the regression equation  $r^{sp} = \alpha + \beta \cdot r^{xa} + v$  where  $r^{sp}$  denotes *ex-post* real returns on nominal bonds and  $r^{xa}$  denotes *ex-ante* real interest rates. The *ex ante* real interest rates are calculated from prices of index and nominal bonds simultaneously observed at the beginning of each month. All rates are continuously compounded and are expressed in terms of percent per 30-day month. Standard errors of the coefficient estimates appear in parentheses under the coefficients. The first standard error is estimated using the White (1980) correction for heteroskedasticity. The second standard error is also corrected for autocorrelation of order 12 using Newey and West (1987) weighing matrix.  $R_{adj}^2$  is the adjusted  $R^2$ .

The sample period includes two subperiods with different inflation levels: September 1984 through July 1985 and August 1985 through March 1992. The first subperiod is a period of high and volatile inflation while the second subperiod, which follows an austerity program adopted by the Israeli government in July 1985, is a period of relatively low and stable inflation.

Period	$\alpha$	$\beta$	$R_{adj}^2$
September 1984 - March 1992	0.080 (0.188) (0.181)	0.377 (0.155) (0.147)	0.171
August 1985 - March 1992	0.070 (0.129) (0.182)	0.222 (0.158) (0.168)	0.060

**TABLE IV**  
**The Relation between Interest Rates and Inflation**

Regression estimates of the relations between monthly rates of inflation ( $\pi$ ), nominal interest ( $n$ ), and *ex-ante* real interest ( $r^{xa}$ ). The inflation rate is the continuously compounded rate of change in the Consumer Price Index. The nominal interest rates are the yields to maturity of one-month nominal bonds. The *ex ante* real interest rates are calculated from prices of index and nominal bonds simultaneously observed at the beginning of each month. All rates are expressed in terms of percent per 30-day month. Standard errors of the coefficient estimates appear in parentheses under the coefficients. The standard error is corrected for heteroskedasticity and autocorrelation of order 12 using Newey and West (1987) weighing matrix.  $R_{adj.}^2$  is the adjusted  $R^2$ .

The sample period includes two subperiods with different inflation levels: September 1984 through July 1985 and August 1985 through March 1992. The first subperiod is a period of high and volatile inflation while the second subperiod, which follows an austerity program adopted by the Israeli government in July 1985, is a period of relatively low and stable inflation.

**September 1984 - March 1992**

Equation	Intercept	Slope	$R_{adj.}^2$
$\pi = \alpha_0 + \beta_0 n + \varepsilon$	-0.108 (0.214)	1.064 (0.044)	0.820
$\pi = \alpha_1 + \beta_1 r^{xa} + \eta$	2.489 (0.854)	-1.249 (0.289)	0.350
$\pi = \alpha_2 + \beta_2 (n - r^{xa}) + \zeta$	0.393 (0.135)	0.781 (0.037)	0.826

**August 1985 - March 1992**

Equation	Intercept	Slope	$R_{adj.}^2$
$\pi = \alpha_0 + \beta_0 n + \varepsilon$	0.944 (0.089)	0.326 (0.032)	0.143
$\pi = \alpha_1 + \beta_1 r^{xa} + \eta$	1.432 (0.081)	0.161 (0.117)	0.044
$\pi = \alpha_2 + \beta_2 (n - r^{xa}) + \zeta$	1.347 (0.121)	0.060 (0.091)	-0.006

**TABLE V**  
**Summary Statistics for *PR***

Summary statistics for a measure of the monthly inflation premium (*PR*) included in monthly nominal interest rates (*n*). *PR* is calculated as the (logarithmic) difference between the nominal interest rate and the sum of (1) the *ex-ante* real interest rate ( $r^{aa}$ ), and (2) the realized inflation rate ( $\pi$ ). The *ex ante* real interest rates are calculated from prices of index and nominal bonds simultaneously observed at the beginning of each month. The nominal interest rates are the yields to maturity of one-month nominal bonds. All rates are continuously compounded and are expressed in terms of percent per 30-day month. S.D. denotes the standard deviation.  $\rho$  denotes the first order serial correlation.

The sample period includes two subperiods with different inflation levels: September 1984 through July 1985 and August 1985 through March 1992. The first subperiod is a period of high and volatile inflation while the second subperiod, which follows an austerity program adopted by the Israeli government in July 1985, is a period of relatively low and stable inflation.

Period	Mean	S.D.	$\rho$	Min.	Max.
September 1984 - March 1992	0.340	2.359	0.117	-5.415	9.640
September 1984 - July 1985	2.358	4.847	0.028	-5.415	9.640
August 1985 - March 1992	0.048	1.589	0.087	-3.948	6.111



**TABLE VI**  
**The Relation between Inflation Uncertainty and Inflation Premium**

Estimates of the regression equation  $PR = \gamma_0 + \gamma_1 D + \varepsilon$ , where  $PR$  is a measure of the inflation premium included in monthly nominal interest rates and  $D$  denotes the monthly dispersion of relative price changes.  $PR$  is calculated as the (logarithmic) difference between the nominal interest rate ( $n$ ) and the sum of the *ex-ante* real interest rate ( $r^a$ ) and the realized inflation rate ( $\pi$ ). All rates are continuously compounded and are expressed in terms of percent per 30-day month. The *ex ante* real interest rates are calculated from prices of index and nominal bonds simultaneously observed at the beginning of each month. The nominal interest rates are the yields to maturity of one-month nominal bonds. The dispersion of relative price changes within the Consumer Price Index (CPI) proxies for inflation uncertainty. The dispersion in month  $t$  is computed as the weighted average of the squared deviations of the ten components of the Israeli CPI relative to the monthly inflation rate, using the weights of the components in the CPI. Standard errors of the coefficient estimates appear in parentheses under the coefficients. The standard error is corrected for heteroskedasticity and autocorrelation of order 12 using Newey and West (1987) weighing matrix.  $R^2_{adj.}$  is the adjusted  $R^2$ .

The sample period includes two subperiods with different inflation levels: September 1984 through July 1985 and August 1985 through March 1992. The first subperiod is a period of high and volatile inflation while the second subperiod, which follows an austerity program adopted by the Israeli government in July 1985, is a period of relatively low and stable inflation.

Period	Intercept	Slope	$R^2_{adj.}$
September 1984 - March 1992	-0.593 (0.418)	0.336 (0.158)	0.056
August 1985 - March 1992	-0.116 (0.333)	0.075 (0.107)	-0.009

### Appendix

In this appendix we examine the relations among the real interest rate, the prices of default-free zero-coupon indexed and nominal bonds, investor marginal rates of substitution, and inflation. We do so in the setting of Lucas (1978) with a representative investor with power utility function. For this investor the marginal rate of substitution between current consumption and one-month and two-month ahead consumption equal, respectively:

$$mrs_1 = \beta \cdot g_1^{-\gamma}, \quad (A1)$$

$$mrs_2 = \beta^2 \cdot g_1^{-\gamma} \cdot g_2^{-\gamma}, \quad (A2)$$

where  $\beta$  is the time-preference coefficient,  $g_t$  is one plus the consumption growth rate of month  $t$ , and  $\gamma > 0$  is the coefficient of relative risk aversion.

From the first order conditions of an investor holding a one-month nominal bond we get that the price of the bond, denoted by  $P_1^N$ , satisfies:

$$P_1^N = E_0 \left( \beta \cdot \frac{g_1^{-\gamma}}{\Pi_1} \right), \quad (A3)$$

where  $E_0$  denotes time-0 expectations and  $\Pi_1$  denotes one plus the first month's inflation.

Similarly, for a two-month nominal bond we get:

$$P_2^N = E_0 \left( \beta^2 \cdot \frac{g_1^{-\gamma} \cdot g_2^{-\gamma}}{\Pi_1 \cdot \Pi_2} \right), \quad (A4)$$

where  $\Pi_2$  denotes one plus the second month's inflation.

For the indexed bonds, the investor expects cash flows that are proportional to the current level of the index, denoted by  $I_0$ , and to the rate of future inflation. As explained in the introduction, an indexed bond compensates its holder for inflation occurring up to one month before its maturity.

Hence, the first order condition for a *two month* indexed bond is:

$$P_2^I = E_0\left(\beta^2 \cdot \frac{g_1^{-\gamma} \cdot g_2^{-\gamma}}{\Pi_2} \cdot I_0\right) . \quad (\text{A5})$$

Similarly, for a one-month indexed bond, *which is no longer indexed*, the first order condition is:

$$P_1^I = E_0\left(\beta \cdot \frac{g_1^{-\gamma}}{\Pi_1} \cdot I_0\right) . \quad (\text{A6})$$

The price of a unit of the consumption good to be delivered for sure in one month is:

$$P_1^R = E_0(\beta \cdot g_1^{-\gamma}) . \quad (\text{A7})$$

Using this price, we define the real interest rate as:

$$R_1 \equiv \frac{1}{P_1^R} - 1 . \quad (\text{A8})$$

Assuming investors know  $I_0$  and that all variables are serially independent we get the real interest rate by the following sequence of computations:

· From equations (A3) and (A6)

$$I_0 = \frac{P_1^I}{P_1^N} ; \quad (\text{A9})$$

· From equations (A3) and (A4)

$$E_0\left(\beta \frac{g_2^{-\gamma}}{\Pi_2}\right) = \frac{P_2^N}{P_1^N} ; \quad (\text{A10})$$

· Substituting (A7), (A9), and (A10) into (A5)

$$P_2^I = P_1^R \cdot \left( \frac{P_2^N}{P_1^N} \right) \cdot \left( \frac{P_1^I}{P_1^N} \right). \quad (\text{A11})$$

Substituting from equations (A11) to equation (A8) the real interest rate is given by:

$$R_1 = \left( \frac{1}{P_2^I} \right) \cdot \left( \frac{P_2^N}{P_1^N} \right) \cdot \left( \frac{P_1^I}{P_1^N} \right) - 1. \quad (\text{A12})$$

Equation (A12) shows how to calculate the real rate from pure-discount bond prices. Actual bond contracts provide for coupon payment, partial indexation, non-standard maturity dates etc. The operationalization of equation (A12) for the empirical analysis of this paper is given in Section III where we incorporate these added factors.

Note that nominal forward rate between the end of the first month and the end of the second one, denoted by  ${}_1N_2$ , is given by

$$1 + {}_1N_2 = \frac{P_1^N}{P_2^N}. \quad (\text{A13})$$

Thus, using equations (A8) and (A9) we can rewrite equation (A11) as

$$P_2^I = \frac{1}{(1 + R_1)} \cdot \frac{I_0}{(1 + {}_1N_2)}.$$