AN INDEX-CONTINGENT TRADING MECHANISM: ECONOMIC IMPLICATIONS

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Abstract

In many stock exchanges around the world there is a "call" or "batch" transaction at the opening of the trading day. Currently, an essential problem in the application of this trading mechanism is that orders in one security cannot be conditioned on prices of other securities. As a result, precise portfolio considerations cannot be based simultaneously on the prices of all securities, and there is no way to trade an asset instantly on the basis of information derived from prices of other securities. Wohl (1994) suggests and analyzes the feasibility of a trading mechanism that facilitates conditioning on an index (a weighted average of stock prices) that is determined simultaneously with the prices of all assets. In this paper we analyze the economic implications of an index-contingent trading mechanism in the framework of rational expectations equilibrium models with liquidity traders. We show that an index-contingent trading mechanism may contribute to the efficiency of prices and to the reduction of losses and risks sustained by liquidity traders. In particular, we compare two trading systems, with and without index-conditioning, and find that in the system with index-conditioning (i) price fluctuations around "true" values are lower, (ii) expected trading costs of liquidity traders are lower, (iii) the variance of the payoffs to the liquidity liquidity traders is lower, (iv) the expected utility of informed traders is lower, and (v) the expected volume of trade is higher than in a model without cross-conditioning.

1 Introduction

In many stock exchanges around the world "call" or "batch" is used as a method of trading, either alone or in conjunction with continuous trading. Stock exchanges that use a computerized limit order book (for example, the stock exchanges of New York, Tokyo, Toronto, Paris, Frankfurt and Sydney) have a call transaction at the beginning of each trading day, and a continuous trading after that. The trading volume at the opening is significant. For example, Stoll and Whaley (1990) report that, on average, the volume at the opening of the New-York Stock Exchange (NYSE) is more than 10% of the daily volume of trade for the years 1982–86. Similar proportions are reported for the Tokyo Stock Exchange by Amihud and Mendelson (1991).

Currently, an essential problem in the application of this trading mechanism is that orders in one security cannot be conditioned on prices of other securities. As a result, precise portfolio considerations cannot be based simultaneously on the prices of all securities, and there is no way to trade an asset instantly on the basis of information derived from prices of other securities. The need for cross-conditioning has first been broached by Beja and Hakansson (1979) and is emphasized by Amihud and Mendelson (1985, 1988, 1990) and Miller (1991). It is also supported by several empirical studies that document, for various stock exchanges, high volatility of returns at the beginning of the trading day and that the variance of the open-to-open returns is higher than the close-to-close return variance. These findings suggest that, following the morning opening, investors "correct" the prices in accordance with the newly observed prices of other stocks.[1]

Wohl (1994) proposes a trading system with call transactions (hereafter, a call system) and index-contingent limit orders. In his call system, an investor's demand for a stock may be conditioned on an index (a weighted average of stock prices which is *simultaneously determined* with the clearing prices of all stocks), in addition to the stock's price itself . A typical buy order has the following format: "Buy Q shares of stock j if $P_j \leq C + LP_j^0 \left(\frac{I - I^0}{I^0}\right)$."

[1]See Wood, McInish and Ord (1985), Amihud and Mendelson (1987, 1989, 1991), Stoll and Whaley (1990), Amihud, Mendelson, and Murgia (1990), and Wohl and Kandel (1994)

The constants Q, C and L are provided by the investor, P_j is the clearing price of stock j, and I is the index based on all clearing prices. P_j^0 and I^0 are the previous day's (or previous round of trade's) values of P_j and I. When $L = 0$, this order is the usual limit order. When $L \neq 0$, the limit is adjusted to the market movements. For example, let $P_j^0 = 100$ and $I^0 = 1000$, and assume the investor chooses $Q = 200$,$C = 102$ and $L = \frac{1}{2}$. If the market doesn't move ($I = I^0 = 1000$), then the order will be: "Buy 200 shares of stock j if $P_j \leq 102$." If the market goes up by 1% ($I = 1010$), then the order will be: "Buy 200 shares of stock j if $P_j \leq 102 + \frac{1}{2}$." Wohl demonstrates that this call system is technically feasible: under the regularity condition $L < 1$, there exists a unique solution for any set of index-contingent limit orders in all stocks, where "solution" means a set of prices and a corresponding index under which all markets for individual stocks are cleared.[2]

In this paper we analyze the economic implications of an index-contingent call system in the framework of rational expectations equilibrium (hereafter REE) models with liquidity traders. Admati (1985) presents a REE model with liquidity traders and many risky assets. She assumes that each trader's net demand for every asset is a function of the equilibrium prices of all assets. Her assumption is equivalent to the assumption that, in a call system, each trader submits a set of contingent limit orders, where the quantity demanded depends on the clearing prices of *all* assets. Obviously, this trading system is impractical even in our era of powerful computers.[3] Wohl's proposal of index-contingent limit orders represents one step in the direction of a realistic call system with cross-conditioning.

We present a model that allows us to compare two call systems: the usual system, where the equilibrium demand for a risky asset may depend only on the equilibrium price of the asset itself, and the suggested system, where the order may also be conditioned on an index. We assume that every trader has private information about one risky asset and actively

[2]Miller (1991) proposes the introduction of similar contingent limit orders in order to increase the market's efficiency and mitigate the impact of intermarket arbitrage in a system with *continuous trading and a specialist*. These orders are automatically marked up or down by a prespecified movement in the future market or of another *observable* index. Brown and Holden (1993) examine the advantages of Miller's proposal for retail traders.

[3]See Admati (1989, 1991) for a discussion of the problems associated with the implementation of REE and how they are related to realistic trading mechanisms.

trades only this risky asset and the riskless asset. In the spirit of REE models we assume that informed investors place their limit orders in accordance with their private information and the information they gain during the trading from the equilibrium price of the asset they actively trade and the equilibrium value of the index. We derive investors' demand functions and the equilibrium prices. We compare the two call systems and find that, in the system with index-conditioning, (i) price fluctuations around "true" values are lower, (ii) expected trading costs of liquidity traders are lower, (iii) the variance of the payoffs to the liquidity traders is lower, (iv) the expected utility of informed traders is lower, and (v) the expected volume of trade is higher than in a system without index-conditioning.

Section 2 contains the assumptions of our model. Section 3 describes the equilibria under the two trading mechanisms. Section 4 derives the economic implications of the proposed system. Section 5 describes an extension of the model. We conclude the paper in Section 6. Proofs of lemmas and propositions are provided in an Appendix.

2 The Model

2.1 Assets

A riskless asset and a set of s risky assets (stocks) are traded. For the sake of simplicity, we assume that the elapsed time between the trading and the receipt of the assets' payoffs is negligible. The riskless asset is in perfectly elastic supply at the (normalized) price of one and will pay one unit immediately after the trading with certainty. The risky assets are in fixed supply. Risky asset j will pay immediately after the trading

$$\tilde{F}_j = \tilde{I} + \tilde{\delta}_j, \tag{2.1}$$

where $\tilde{I}$ is a common factor, $\tilde{\delta}_j$ is an idiosyncratic payoff (independent of $\tilde{I}$ and $\tilde{\delta}_k$, $k \neq j$, $E\tilde{\delta}_j = 0$), and

$$(1/s) \cdot \sum_{j=1}^{s} \tilde{\delta}_j \sim N(0, V_{\bar{\delta}}) \ , \quad V_{\bar{\delta}} = \text{constant}/s. \tag{2.2}$$

We assume that the number of risky assets, s, is "large".

2.2 Traders

There are two types of investors: informed investors and liquidity traders. Every informed investor, in addition to a proxy for the common factor, receives private information on one risky asset only and actively trades only this asset and the riskless asset. For each risky asset j, exactly $n \geq 2$ informed investors have private information. In the remainder of this paper, the term "traders" will refer to the group of informed traders (as opposed to "liquidity traders"). Trader ij is the i'th trader of asset j. Following are further assumptions on the information and behavior of the traders:

a. Before trading starts, trader ij observes two random variables, proxying for the common factor and the idiosyncratic payoff of asset j:

$$I_{ij} = \tilde{I} + \tilde{e}_{ij}, \tag{2.3}$$

and

$$\delta_{ij} = \tilde{\delta}_j + \tilde{\epsilon}_{ij}, \tag{2.4}$$

where $\tilde{e}_{ij}$ and $\tilde{\epsilon}_{ij}$ are normally distributed random variables with zero means and variances equal to V_e and V_ϵ, respectively. The variables are independent of each other, $\tilde{I}$, $\tilde{\delta}$, and all other δ's e's and ϵ's.

b. Based on his private information, trader ij forms a joint "*subjective distribution*" of $\tilde{I}$ and $\tilde{\delta}_j$:

$$\begin{bmatrix} \tilde{I} \\ \tilde{\delta}_j \end{bmatrix} \sim N\left(\begin{bmatrix} I_{ij} \\ \delta_{ij} \end{bmatrix}, \begin{bmatrix} V_e & 0 \\ 0 & V_\epsilon \end{bmatrix} \right). \tag{2.5}$$

This distribution is the posterior distribution of $\tilde{I}$ and $\tilde{\delta}_j$ when the variances V_e and V_ϵ are known and the investor has a diffuse (improper) prior distribution of $\tilde{I}$ and $\tilde{\delta}_j$.[4]

c. Every trader has an initial portfolio that contains K units of the risk-free asset and Q/s units of each of the risky assets. The trader's utility is based on the total wealth after trading, W:

$$U(W) = -e^{-aW} \tag{2.6}$$

where a, the risk aversion coefficient, is the same for all traders.

d. In seeking to maximize their expected utility, traders ignore any random variable with expected value and variance that are equal to constant/s (recall that s is a "large" number).

e. Traders ignore their own impact on prices.

[4]See Zelner (1971), p. 20.

f. Liquidity traders' orders are based on factors other than private information or share prices. Their total supply of asset j is $\widetilde{X}_j \sim N(0, V_x)$; $\widetilde{X}_j$ is independent of $\widetilde{X}_k$, $k \neq j$, and of $\tilde{I}$, $\tilde{\delta}_k$, $\tilde{e}_{ik}$, and $\tilde{\epsilon}_{ik}$ for all i, k.

2.3 Trading System

Denote the clearing price of asset j by $\widetilde{P}_j$. Define:

$$\tilde{I}_s = (1/s) \cdot \sum_{j=1}^{s} \widetilde{P}_j \, . \tag{2.7}$$

Every trader may choose l_{ij} and submit a demand function that depends on the variable $(\widetilde{P}_j - l_{ij}\tilde{I}_s)$. This variable will hereafter be termed "signal" and be denoted by S_{ij}. Denote trader ij's demand for asset j by d_{ij}. If the trader selects $l_{ij} = 0$, then d_{ij} depends only on the price of asset j but not on the index. Obviously, orders can indicate a fixed amount of shares which depends neither on the price nor on the index.

Equilibrium prices satisfy the condition that, for each asset j, total demand is equal to total supply. As the price of the riskless asset is normalized to be 1, buying (selling) a share at the price P_j decreases (increases) the number of riskless units held by trader ij by P_j.

3 Equilibrium

Lemma 3.1 *Assume the model presented in Section 2 and assume that, for a given l_{ij}, trader ij can condition his demand for asset j on the signal $\widetilde{S}_{ij} = \widetilde{P}_j - l_{ij}\tilde{I}_s$. Assume there exists an equilibrium with the following price for asset j:*

$$\widetilde{P}_j = \widetilde{F}_j + \tilde{\tau}_j + \tilde{\theta}_j \, , \tag{3.1}$$

where

$$\tilde{\tau}_j \sim N(0, V_\tau) \tag{3.1a}$$

independent of $\tilde{I}$ and $\tilde{\delta}_j$,

$$E(\tilde{\theta}_j) = 0 \, , \quad V(\tilde{\theta}_j) = \text{constant}/s \, , \tag{3.1b}$$

(3.2) $$V\left[(1/s)\cdot\sum_{k=1}^{s}(\tilde{P}_j-\tilde{I})\right]=\text{constant}/s\ ,$$

and $E(\cdot)$ and $V(\cdot)$ denote the random variable's expectation and variance, respectively. Then trader ij's demand for asset j will be:

(3.3) $$d_{ij}=\frac{(1-l_{ij})I_{ij}+\delta_{ij}-\tilde{S}_{ij}}{a[(1-l_{ij})^2V_e+V_\epsilon]}\ .$$

The result in Lemma 3.1 is for a fiven value l_{ij}. When $l_{ij}=0$, the it resembles the result in Grossman (1976). In the following lemma it is assumed that the trader can choose any value l_{ij} in the range $[0,L]$, $L\leq 1$, and condition his demand on $\tilde{S}_{ij}=(\tilde{P}_j-l_{ij}\tilde{I}_s)$.

Lemma 3.2 *Assume the equilibrium characteristics of Lemma 3.1. Assume that trader ij can choose any value l_{ij} in the range $[0,L]$, $L\leq 1$, and condition his demand for asset j on the signal $\tilde{S}_{ij}=(\tilde{P}_j-l_{ij}\tilde{I}_s)$. Then, the trader will choose a signal S_{ij} with $l_{ij}=L$ and his demand for asset j will be:*

(3.4) $$d_{ij}=\frac{(1-L)I_{ij}+\delta_{ij}-(\tilde{P}_j-L\tilde{I}_s)}{a[(1-L)^2V_e+V_\epsilon]}\ .$$

The intuition for the result in Lemma 3.2 is straightforward. Under the equilibrium characteristics of Lemma 3.1, $\tilde{I}_s$ (the trading index) is a much more precise assessment of $\tilde{I}$ (the common factor) than I_{ij} (the assessment of trader ij). Therefore, every trader prefers to use maximal weight of $\tilde{I}_s$ in the assessment of $\tilde{I}$ (and the payment $\tilde{F}_j$), and chooses $l_{ij}=L$. If $L=1$, the trader need not use his initial assessment I_{ij}, and the demand function of trader ij does not provide any private information about $\tilde{I}$. In order to force the traders to provide information about $\tilde{I}$, we limit their choice to $L<1$.

Proposition 3.3 *Assume the model in Section 2 where each trader ij can choose l_{ij} in the range $[0,L]$, $L<1$, and condition his demand for asset j on the signal $\tilde{S}_{ij}=(\tilde{P}_j-l_{ij}\tilde{I}_s)$. Then there exists an equilibrium with the following price for asset j:*

(3.5) $$\tilde{P}_j=\tilde{F}_j+\tilde{\tau}_j+\tilde{\theta}_j\ ,$$

where

$$\tilde{\tau}_j = -\frac{a[V_\epsilon + (1-L)^2 V_e]}{n} \cdot \widetilde{X}_j + \frac{(1-L)}{n} \sum_{i=1}^{n} \tilde{e}_{ij} + \frac{1}{n} \sum_{i=1}^{n} \tilde{\epsilon}_{ij}, \tag{3.6}$$

and

$$\tilde{\theta}_j = \frac{L}{s} \Big\{ -\frac{a[V_\epsilon + (1-L)^2 V_e]}{(1-L)n} \sum_{k=1}^{s} \widetilde{X}_k + \frac{1}{(1-L)} \sum_{k=1}^{s} \tilde{\delta}_k + \frac{1}{n} \sum_{k=1}^{s} \sum_{i=1}^{n} \tilde{e}_{ik} + \frac{1}{n(1-L)} \sum_{k=1}^{s} \sum_{i=1}^{n} \tilde{\epsilon}_{ik} \Big\}. \tag{3.7}$$

Proposition 3.3 implies that the price P_j is the sum of the true payoff $\widetilde{F}_j$ and two disturbances. The variance of the disturbance $\tilde{\tau}_j$ is decreasing in L. The second disturbance, $\tilde{\theta}_j$, reflects the "echo effect" of an index-conditioning: a noise in the demand for one asset affects the prices of all assets. As the variance of this disturbance is of order $\frac{1}{s}$, it is negligible as s increases to infinity.

4 Economic Implications

All the propositions in this section are expressed in terms of functions that are monotonic in L. To see the differences between the trading systems, recall that in the current system, without index-contingent limit orders, $L = 0$.

Proposition 4.1 *For every asset j, $\lim_{s\to\infty} V(\widetilde{P}_j - \widetilde{F}_j)$ is decreasing in L.*

Proposition 4.1 implies that price fluctuations around the "true" value are lower in the system with index-conditioning than in the current system. The possibility of using $\tilde{I}_s$ in limit orders leads to transfer of information about $\tilde{I}$, from and to the traders of different assets, via $\tilde{I}_s$. This leads to more efficient pricing. Because the model does not include liquidity traders' preferences and endowments, we cannot measure the improvement in the liquidity traders' utility. Instead, we analyze their total trading costs.[5] For every asset j, this total cost is

$$\widetilde{X}_j(\tilde{F}_j - \tilde{P}_j) = -\widetilde{X}_j(\tilde{\tau}_j + \tilde{\theta}_j). \tag{4.1}$$

[5]See Admati and Pfleiderer (1991) for a similar analysis of liquidity traders' total trading costs in a model with "sunshine trading", where some liquidity traders preannounce the size of their orders.

where the equality follows from (3.5).

Proposition 4.2 *For every asset j, the expected trading costs of the liquidity (uninformed) traders are decreasing in L as $s \to \infty$.*

Proposition 4.3 *For every asset j and for every realization X_j, $\lim_{s\to\infty} V(\tilde{P}_j \cdot X_j)$, the limit variance of the payoff to the liquidity traders of asset j conditional on X_j, is decreasing in L.*

Propositions 4.2 and 4.3 show that in a system with index- contingent trading the expected trading costs of the liquidity traders are lower and the variance of the payoffs to the liquidity traders is lower than in a system without it. The following proposition shows that the expected utility of the informed traders is also lower in a system with index contingent trading than in a system without it.

Proposition 4.4 *The expected utility of the informed traders is decreasing in L.*

It is interesting to note that the informed traders, as a group, will probably wish to prevent implementation of an index-contingent trading mechanism while, if the mechanism is established, all of them will use the conditioning by the index (Lemma 3.2). Finally we examine the trading volume under the suggested call systems. Intuitively, the index-contingent trading mechanism leads to more precise information about the common factor and, therefore, to more aggressive trading by the informed traders. This intuition is formally presented in the following proposition:

Proposition 4.5 *For every trader ij, $E \lim_{s\to\infty} |d_{ij}|$ is increasing in L.*

5 An Extension of the Model

In this section we briefly describe an extension of the model where the payoffs of the risky assets have different sensitivities to the common factor $\tilde{I}$:

$$\tilde{F}_j = \alpha_j + \beta_j \tilde{I} + \tilde{\delta}_j$$

where

$$\frac{1}{s}\sum_{j=1}^{s}\alpha_j = 0 ,$$

and

$$\frac{1}{s}\sum_{j=1}^{s}\beta_j = 1 .$$

In the extended model, if traders are limited to choose $l \leq L$, then trader ij will choose $l_{ij} = \beta_j$ if $\beta_j \leq L$, and $l_{ij} = L$ if $\beta_j > L$. When $l_{ij} = \beta_j \leq L$, the traders of asset j do not provide any information about $\tilde{I}$ to the mechanism. Hence, a sufficient condition for an equilibrium to exist is that the number of assets for which $\beta_j > L + \zeta$ (for some $\zeta > 0$) increases to infinity as $s \to \infty$.

The economic implications of the extended model are very similar to those in Section 4. Propositions 4.1 – 4.5 specify several economic values and moments and show that they are increasing or decreasing functions of L. It can be shown that, in the extended model, these functions are strictly increasing or decreasing in L when $L \leq \beta_j$ but do not change as a function of L for $L > \beta_j$. If allowed, the informed investor will limit his orders with $l_{ij} = \beta_j$. Increasing L beyond β_j will not change the trader's optimal behavior and the equilibrium characteristics.

6 Conclusions

A feasible call trading mechanism that enables conditioning on an index is proposed by Wohl (1994). The economic implications of this mechanism are examined in this paper in the framework of REE models with liquidity traders. We show that the index-contingent trading mechanism can contribute to the efficiency of prices and to the reduction of losses and risks sustained by liquidity traders. In particular, we compare two call systems, with and without an index-contingent trading, and find that in the system with index-conditioning (i) price fluctuations around "true" values are lower, (ii) expected trading costs of the liquidity traders are lower, (iii) the variance of the payoffs to the liquidity traders is lower, (iv) the expected utility of informed traders is lower, and (v) the expected volume of trade is higher than in a model witout index-conditioning. It is interesting to note that informed traders, as a group, will probably wish to prevent implementation of an index-contingent trading mechanism. However, if the mechanism is used, all of them will employ the conditioning by the index, rather than by the asset's price alone.

Appendix

Proof of Lemma 3.1

Consider trader ij. If he assumes the existence of the equilibrium (3.1)–(3.2), his wealth after trading will be given by

$$\begin{aligned}
\widetilde{W}_{ij} &= K + (Q/s)\cdot\sum_{k=1}^{s}\widetilde{F}_k + d_{ij}(\widetilde{F}_j - \widetilde{P}_j) \\
&= K + (Q/s)\cdot\sum_{k=1}^{s}(\tilde{I} + \tilde{\delta}_k) - d_{ij}(\widetilde{F}_j - \widetilde{P}_j) \\
&= K + Q\tilde{I} + (Q/s)\cdot\sum_{k=1}^{s}\tilde{\delta}_k - d_{ij}(\tilde{\tau}_j + \tilde{\theta}_j)\ ,
\end{aligned}$$

where the first equality is implied by equations (2.1)–(2.2) and the second equality is implied by equation (3.1). By assumption (d) on the behavior of the investors (see subsection 2.2) and assumption (3.1b) on $\tilde{\theta}_j$, the traders ignore $\tilde{\theta}_j$ in the maximization of their expected utility. Similarly, by (2.2) they ignore

$$Q/s\cdot\sum_{k=1}^{s}\tilde{\delta}_k\ .$$

We will denote by $\approx$ the approximation used by trader ij when ignoring the above variables. In his optimization problem, the trader's wealth will be (approximately)

$$\widetilde{W}_{ij} \approx K + Q\tilde{I} - d_{ij}\tilde{\tau}_j\ . \tag{A.1}$$

By the assumption of traders' utility function (2.6) and the fact that $\tilde{\tau}_j$ and $\tilde{I}$ are normally distributed we get the trader's expected utility:

$$EU = -e^{-a[K+QE(\tilde{I})-d_{ij}E(\tilde{\tau}_j)]+\frac{1}{2}a^2[Q^2V(\tilde{I})+d_{ij}^2V(\tilde{\tau}_j)]}\ , \tag{A.2}$$

where the moments in the exponent are conditional on the variables in the trader's information set: I_{ij}, δ_{ij}, and S_{ij}. The maximum of this expression is obtained when the exponent reaches its minimum value. The first-order condition for minimum is:

$$\frac{\partial\ \text{exponent}}{\partial d_{ij}} = E(\tilde{\tau}_j) + (1/2)a^2\cdot 2d_{ij}V(\tilde{\tau}_j) = 0\ .$$

The second-order condition is satisfied because $a^2V(\tilde{\tau}_j)$ is always positive. The first order condition implies that:

$$d_{ij} = -\frac{E(\tilde{\tau}_j)}{aV(\tilde{\tau}_j)} \tag{A.3}$$

where $E(\tilde{\tau}_j)$ and $V(\tilde{\tau}_j)$ are conditional on I_{ij}, δ_{ij}, and S_{ij}. Next, we derive these moments. From (2.1), (2.7), and (3.1):

$$\begin{aligned}\tilde{P}_j - l_{ij}\tilde{I}_s &= \tilde{I} + \tilde{\delta}_j + \tilde{\tau}_j + \tilde{\theta}_j - (l_{ij}/s) \cdot \sum_{k=1}^{s}(\tilde{I} + \tilde{\delta}_k + \tilde{\tau}_k + \tilde{\theta}_k) \\ &= (1 - l_{ij})\tilde{I} + \tilde{\delta}_j + \tilde{\tau}_j + \tilde{\theta}_j - (l_{ij}/s)\sum_{k=1}^{s}(\tilde{\delta}_k + \tilde{\tau}_k + \tilde{\theta}_k) \ .\end{aligned}$$

It can be verified that, conditional on I_{ij} and δ_{ij}, the expected value of the summation is of order $(1/s)$. Similarly, the conditional variance is:

$$V\left[(l_{ij}/s) \cdot \sum_{k=1}^{s}(\tilde{\delta}_k + \tilde{\tau}_k + \tilde{\theta}_k)\right] = V\left[(l_{ij}/s) \cdot \sum_{k=1}^{s}(\tilde{P}_k - \tilde{I})\right]$$

which, by assumption (3.2), is equal to constant$/s$. Based on assumption (d) about traders' behavior (subsection 2.2), we can ignore these components. By assumption (3.1b) we can ignore θ_j for the same reasons and get:

$$\tilde{P}_j - l_{ij}\tilde{I}_s \approx (1 - l_{ij})\tilde{I} + \tilde{\delta}_j + \tilde{\tau}_j \ . \tag{A.4}$$

The joint distribution of $\tilde{\tau}_j$ and S_{ij}, conditional on I_{ij} and δ_{ij}, is bivariate normal:

$$\begin{pmatrix} \tau_j \\ S_{ij} \end{pmatrix} \sim N\left\{\begin{pmatrix} 0 \\ (1 - l_{ij})I_{ij} + \delta_{ij} \end{pmatrix}, \begin{pmatrix} V_\tau & V_\tau \\ V_\tau & (1 - l_{ij})^2V_e + V_\epsilon + V_\tau \end{pmatrix}\right\} \ .$$

Therefore, as in Raiffa & Schlaifer (1961, page 250):

$$E(\tilde{\tau}_j \mid S_{ij}, I_{ij}, \delta_{ij}) = 0 + \frac{V_\tau}{(1 - l_{ij})^2V_e + V_\epsilon + V_\tau}\left[S_{ij} - (1 - l_{ij})I_{ij} - \delta_{ij}\right] , \tag{A.5}$$

and

$$V(\tilde{\tau}_j \mid S_{ij}, I_{ij}, \delta_{ij}) = V_\tau - \frac{V_\tau}{(1 - l_{ij})^2V_e + V_\epsilon + V_\tau} = \frac{[(1 - l_{ij})^2V_e + V_\epsilon]V_\tau}{(1 - l_{ij})^2V_e + V_\epsilon + V_\tau} \ . \tag{A.6}$$

Using (A.3) we get trader ij's demand for asset j

$$\begin{aligned}d_{ij} &= \frac{(1 - l_{ij})I_{ij} + \delta_{ij} - S_{i,j}}{(1 - l_{ij})^2V_e + V_\epsilon + V_\tau} \cdot V_\tau \cdot \frac{(1 - l_{ij})^2V_e + V_\epsilon + V_\tau}{[(1 - l_{ij})^2V_e + V_\epsilon] \cdot a} \\ &= \frac{(1 - l_{ij})I_{ij} + \delta_{ij} - S_{ij}}{a[(1 - l_{ij})^2V_e + V_\epsilon]} \ .\end{aligned} \tag{A.7}$$

Q.E.D.

Proof of Lemma 3.2

Consider trader ij (with the corresponding private information, I_{ij} and δ_{ij}). Assume the trader chooses $l_{ij} = l < L$ (l is strictly less than L) and conditions his demand on $\widetilde{S}_{ij}$. We show that this assumption contradicts the assumption that the trader is an expected utility maximizer. From Lemma 3.1 trader ij's optimal demand for asset j will be

$$\tilde{d}_{ij} = \frac{(1-l)I_{ij} + \delta_{ij} - \widetilde{S}_{ij}}{a[(1-l)^2 V_e + V_\epsilon]}$$

and from (A.1) his wealth will be (approximately)

$$\widetilde{W}_{ij} \approx K + Q\tilde{I} - \tilde{d}_{ij}\tilde{\tau}_j \,.$$

When the trader chooses l_{ij} he does not yet know the realization of $\widetilde{S}_{ij}$. Thus, he chooses l_{ij} that maximizes his expected utility conditional only on I_{ij} and δ_{ij}. Assume the investor chooses $l_{ij} = l^*$, $l < l^* \leq L$ and conditions his demand on $(\widetilde{P}_j - l^*\tilde{I})$. Assume the investor's demand is not the optimal demand for this case:

$$\tilde{d}^*_{ij} = \frac{(1-l^*)I_{ij} + \delta_{ij} - (\widetilde{P}_j - l^*\tilde{I}_S)}{a[(1-l)^2 V_e + V_\epsilon]} \,, \tag{A.8}$$

and his wealth will be (approximately)

$$\widetilde{W}^*_{ij} \approx K + Q\tilde{I} - \tilde{d}^*_{ij}\tilde{\tau}_j \,. \tag{A.9}$$

We show that the trader's expected utility is greater even in this case than in the case where $l_{ij} = l$. Denote

$$C = \frac{1}{a[(1-l)^2 V_e + V_\epsilon]}. \tag{A.10}$$

Substituting (A.10) into (A.8) and then (A.8) into (A.9) yields

$$\widetilde{W}_{ij} \approx K + Q\tilde{I} - C[(1-l)I_{ij} + \delta_{ij} - (\widetilde{P}_j - l\tilde{I}_S)]\tilde{\tau}_j \,.$$

From (A.4) and (2.5) we get

$$\begin{aligned} \widetilde{W}_{ij} &\approx K + Q\tilde{I} - C[(1-l)\tilde{e}_{ij} + \tilde{\epsilon}_{ij} - \tilde{\tau}_j]\tilde{\tau}_j \\ &= K + Q\tilde{I} - C(\tilde{\epsilon}_{ij}\tilde{\tau}_j - \tilde{\tau}_j^2) - C(1-l)\tilde{e}_{ij}\tilde{\tau}_j \end{aligned} \tag{A.11}$$

and similarly,

$$\text{(A.12)} \qquad \begin{aligned} \widetilde{W}^*_{ij} &\approx K + Q\tilde{I} - C[(1-l^*)\tilde{e}_{ij} + \tilde{\epsilon}_{ij} - \tilde{\tau}_j]\tilde{\tau}_j \\ &= K + Q\tilde{I} - C(\tilde{\epsilon}_{ij}\tilde{\tau}_j - \tilde{\tau}_j^2) - C(1-l^*)\tilde{e}_{ij}\tilde{\tau}_j \,. \end{aligned}$$

We shall show that every-risk averse trader (like ij) prefers $\widetilde{W}^*_{ij}$ to $\widetilde{W}_{ij}$. Define

$$\text{(A.13)} \qquad \tilde{\tau}_{ji} = \tilde{\tau}_j - \frac{1-L}{n}\tilde{e}_{ij}.$$

From (A.11), (A.12), (A.13), and (3.6) we get

$$\begin{aligned} \widetilde{W}^*_{ij} &\approx K + Q\tilde{I} - C\left(\tilde{\epsilon}_{ij}(\tilde{\tau}_{ji} + \frac{1-L}{n}\tilde{e}_{ij}) - (\tilde{\tau}_{ji} + \frac{1-L}{n}\tilde{e}_{ij})^2 + (1-l^*)\tilde{e}_{ij}(\tilde{\tau}_{ji} + \frac{1-L}{n}\tilde{e}_{ij})\right) \\ &= K + Q\tilde{I} - C\left(\tilde{\epsilon}_{ij}\tilde{\tau}_{ji} + \frac{1-L}{n}\tilde{e}_{ij}\tilde{\epsilon}_{ij} - \tilde{\tau}_{ji}^2 - \frac{1-L}{n}(\frac{1-L}{n} - (1-l^*))\tilde{e}_{ij}^2 - (2\frac{1-L}{n} - (1-l^*))\tilde{e}_{ij}\tilde{\tau}_{ji}\right) \end{aligned}$$

Since $\tilde{e}_{ij}^2 \geq 0$ and $l^* > l$, every risk-averse trader prefers the random wealth $\widetilde{W}^*_{ij}$ to the random wealth

$$\text{(A.14)} \quad K + Q\tilde{I} - C\left(\tilde{\epsilon}_{ij}\tilde{\tau}_{ji} + \frac{1-L}{n}\tilde{e}_{ij}\tilde{\epsilon}_{ij} - \tilde{\tau}_{ji}^2 - \frac{1-L}{n}(\frac{1-L}{n} - (1-l))\tilde{e}_{ij}^2 - (2\frac{1-L}{n} - (1-l^*))\tilde{e}_{ij}\tilde{\tau}_{ji}\right) .$$

Note also that

$$\text{(A.15)} \quad \widetilde{W}_{ij} \approx K + Q\tilde{I} - C\left(\tilde{\epsilon}_{ij}\tilde{\tau}_{ji} + \frac{1-L}{n}\tilde{e}_{ij}\tilde{\epsilon}_{ij} - \tilde{\tau}_{ji}^2 - \frac{1-L}{n}(\frac{1-L}{n} - (1-l))\tilde{e}_{ij}^2 - (2\frac{1-L}{n} - (1-l))\tilde{e}_{ij}\tilde{\tau}_{ji}\right) .$$

The difference between (A.14) and (A.15) is only the last term. We shall show that under the definition of Rothschild & Stiglitz (1970) the term $\tilde{e}_{ij}\tilde{\tau}_{ji}$ is "noise." Rothschild & Stiglitz (1970) showed that if:

1) $\widetilde{Y} = \widetilde{X} + \widetilde{Z}$, and

2) $E(\widetilde{Z} \mid X) = 0$ for every realization of $\widetilde{X}$,

then every risk averter prefers $\widetilde{X}$ to $\widetilde{Y}$ ("Y is equal to X plus noise"). It also can be shown that if $\widetilde{Z}$ has a symmetric distribution around 0 then every risk averter prefers $(\widetilde{X} + a\widetilde{Z})$ to $(\widetilde{X} + b\widetilde{Z})$ where $|b| > |a|$.

(a) Because $\tilde{e}_{ij}$ and $\tilde{\tau}_{ji}$ are independent of $\tilde{I}$, $E(\tilde{e}_{ij}\tilde{\tau}_{ji} \mid \tilde{I}\,) = E(\tilde{e}_{ij}\tilde{\tau}_{ji})$ for every realization of $\tilde{I}$. $E(\tilde{e}_{ij}\tilde{\tau}_{ji}) = E(\tilde{e}_{ij})E(\tilde{\tau}_{ji}) = 0$ because $\tilde{e}_{ij}$ and $\tilde{\tau}_{ji}$ are not correlated.

(b) For every realization, H, of $\tilde{\epsilon}_{ij}\tilde{\tau}_{ji}$, $E(\tilde{e}_{ij}\tilde{\tau}_{ji} \mid \tilde{\epsilon}_{ij}\tilde{\tau}_{ji} = H\,) = E\left(\tilde{e}_{ij}\frac{H}{\tilde{\epsilon}_{ij}}\right) = HE\left(\frac{\tilde{e}_{ij}}{\tilde{\epsilon}_{ij}}\right)$ and because of the independence between $\tilde{e}_{ij}$ and $\tilde{\epsilon}_{ij}$ we get $HE(\tilde{e}_{ij}) \cdot E\left(\frac{1}{\tilde{\epsilon}_{ij}}\right) = 0$.

(c) Because $\tilde{e}_{ij}$ and $\tilde{\tau}_{ji}$ are independent, $E(\tilde{e}_{ij}\tilde{\tau}_{ji} \mid \tilde{\tau}_{ji}^2 = H\,) = E(\tilde{e}_{ij}) \cdot E(\tilde{\tau}_{ji} \mid \tilde{\tau}_{ji}^2 = H\,) = 0$.

(d) Because $\tilde{e}_{ij}$ and $\tilde{\tau}_{ji}$ are independent, $E(\tilde{e}_{ij}\tilde{\tau}_{ji} \mid \tilde{e}_{ij}^2 = H\,) = E(\tilde{\tau}_{ji}) \cdot E(\tilde{e}_{ij} \mid \tilde{e}_{ij}^2 = H\,) = 0$.

Using (a)–(d) we have verified that $\tilde{e}_{ij}\cdot\tilde{\tau}_{ji}$ is "noise" with respect to the rest of the expressions (A.14) and (A.15). Because $n > 2$ and $0 \leq l < l^* \leq L$

$$\left|\frac{2(1-L)}{n} - (1-l^*)\right| < \left|\frac{2(1-L)}{n} - (1-l)\right| .$$

The above inequality implies that the absolute value of the multiplier of the symmetric noise $\tilde{e}_{ij} \cdot \tilde{\tau}_{ji}$ is greater in (A.15) than in (A.14). Therefore, every risk averse investor prefers $\widetilde{W}_{ij}^*$ to $\widetilde{W}_{ij}$, illustrating that the assumption of $l_{ij} = l$ is inconsistent with the assumption of the trader being risk averse. **Q.E.D.**

Proof of Proposition 3.3

Assume (3.5), (3.6) and (3.7) for asset j. We first show that these assumptions imply the equilibrium conditions assumed in Lemma 3.1. Because $\widetilde{X}_j$, $\tilde{e}_{ij}$, and $\tilde{\epsilon}_{ij}$ are normally distributed with zero expectations, $\tilde{\tau}_j$ is also of the same type, and, therefore, assumption (3.1a) is satisfied. $E(\tilde{\theta}_j) = 0$ because $\tilde{\theta}_j$ is the sum of random variables with zero expectations. By assumption (2.2) and by the independence among $\widetilde{X}_j$, $\tilde{e}_{ij}$, and $\tilde{\epsilon}_{ij}$, we get that $V(\tilde{\theta}_j) = \text{constant}/s$ and assumption (3.1b) is satisfied. Next, we verify assumption (3.2). By equations (2.1) and (3.5),

$$\widetilde{P}_j = \tilde{I} + \tilde{\delta}_j + \tilde{\tau}_j + \tilde{\theta}_j ,$$

and therefore,

$$(1/s)\cdot\sum_{k=1}^{s}(\widetilde{P}_k-\tilde{I})=(1/s)\cdot\sum_{k=1}^{s}(\tilde{\delta}_k+\tilde{\tau}_k+\tilde{\theta}_k)\,.$$

By substituting the expressions for $\tilde{\tau}_k$ and $\tilde{\theta}_k$ from (3.6) and (3.7), we get:

$$\begin{aligned}(1/s)\cdot\sum_{k=1}^{s}(\widetilde{P}_k-\tilde{I}) &= \frac{1}{s}\sum_{k=1}^{s}\Bigg\{\tilde{\delta}_k-\frac{a[V_\epsilon+(1-L)^2V_e]}{n}\widetilde{X}_k+\frac{1-L}{n}\sum_{i=1}^{n}\tilde{e}_{ij}+\\ &\quad+\frac{1}{n}\cdot\sum_{i=1}^{n}\tilde{\epsilon}_{ik}-L\,\frac{a[V_\epsilon+(1-L)^2V_e]}{(1-L)sn}\sum_{k=1}^{s}\widetilde{X}_k\\ &\quad+\frac{L}{1-L}\cdot\frac{1}{s}\cdot\sum_{k=1}^{s}\tilde{\delta}_k+\frac{L}{ns}\sum_{k=1}^{s}\sum_{i=1}^{n}\tilde{e}_{ij}+\frac{L}{ns(1-L)}\sum_{k=1}^{s}\sum_{i=1}^{n}\tilde{\epsilon}_{ik}\Bigg\}\\ &= \left(1+\frac{L}{1-L}\right)\frac{1}{s}\cdot\sum_{k=1}^{s}\tilde{\delta}_k-\left(1+\frac{L}{1-L}\right)\cdot\frac{a[V_\epsilon+(1-L)^2V_e]}{sn}\sum_{k=1}^{s}\widetilde{X}_k\\ &\quad+\frac{1}{sn}\sum_{i=1}^{n}\sum_{k=1}^{s}\tilde{e}_{ik}+\left(1+\frac{L}{1-L}\right)\frac{1}{sn}\sum_{i=1}^{n}\sum_{k=1}^{s}\tilde{\epsilon}_{ik}\,.\end{aligned}$$

This is a sum of four random variables, each of which is independent of the others, with variance equal to constant/s. Hence, assumption (3.2) is satisfied. Because the assumptions of Lemma 3.1 are satisfied and the signals are limited to L by Lemma 3.2, trader ij's demand for asset j is:

$$d_{ij}=\frac{(1-L)I_{ij}+\delta_{ij}-(\widetilde{P}_j-L\tilde{I}_s)}{a[(1-L)^2V_e+V_\epsilon]}\,.$$

In order to achieve equilibrium in every asset j, the total demand by informed traders $D_j=\sum_{j=1}^{n}d_{ij}$ must be equal to the amount offered by the liquidity traders $\widetilde{X}_j$. In other words:

$$\widetilde{X}_j=\sum_{i=1}^{n}\frac{\left[(1-L)I_{ij}+\delta_{ij}-\widetilde{P}_j+(L/s)\cdot\sum_{k=1}^{s}\widetilde{P}_k\right]}{a[(1-L)^2V_e+V_\epsilon]}\,.$$

By exchanging sides, we get:

$$\text{(A.16)}\qquad n\left[\widetilde{P}_j-(L/s)\sum_{k=1}^{s}\widetilde{P}_k\right]=-a[(1-L)^2V_e+V_\epsilon]\widetilde{X}_j+(1-L)\sum_{i=1}^{n}I_{ij}+\sum_{i=1}^{n}\tilde{\delta}_{ij}\,.$$

Summing up for all assets yields:

$$n(1-L)\sum_{k=1}^{s}\widetilde{P}_k=-a[(1-L)^2V_e+V_\epsilon]\cdot\sum_{k=1}^{s}\widetilde{X}_k+(1-L)\sum_{k=1}^{s}\sum_{i=1}^{n}\tilde{I}_{ik}+\sum_{k=1}^{s}\sum_{i=1}^{n}\tilde{\delta}_{ik}\,.$$

The sum of the prices is obtained by dividing by $n(1-L)$ and assigning to each ik $\tilde{\delta}_{ik} = \tilde{\delta}_k + \tilde{\epsilon}_{ik}$ and $\tilde{I}_{ik} = \tilde{I} + \tilde{e}_{ik}$:

$$\sum_{k=1}^{s} \widetilde{P}_k = -\frac{a[(1-L)^2 V_e + V_\epsilon]}{n} \cdot \frac{1}{1-L} \sum_{k=1}^{s} \widetilde{X}_k + (1/n) \sum_{i=1}^{n} \sum_{k=1}^{s} (\tilde{I} + \tilde{e}_{ik}) + \frac{1}{n(1-L)} \sum_{i=1}^{n} \sum_{k=1}^{s} (\tilde{\delta}_k + \tilde{\epsilon}_{ik}) \, .$$

Multiplying by L/s yields

(A.17)
$$(L/s) \sum_{k=1}^{s} \widetilde{P}_k = \frac{[(1-L)^2 V_e + V_\epsilon]}{ns} \cdot \frac{L}{1-L} \sum_{k=1}^{s} \widetilde{X}_k + L\tilde{I} + \frac{L}{ns} \sum_{i=1}^{n} \sum_{k=1}^{s} \tilde{e}_{ik} + \frac{Ln}{sn(1-L)} \cdot n \cdot \sum_{k=1}^{s} \tilde{\delta}_k + \frac{L}{1-L} \cdot \frac{1}{ns} \sum_{i=1}^{n} \sum_{k=1}^{s} \tilde{\epsilon}_{ik} \, .$$

In equation (A.16) assign to every ij the signals $\tilde{\delta}_{ij} = \tilde{\delta}_j + \tilde{\epsilon}_{ij}$ and $\tilde{I}_{ij} = \tilde{I} + \tilde{e}_{ij}$, and exchange sides. We get

(A.18)
$$\widetilde{P}_j = -\frac{a[(1-L)^2 V_e + V_\epsilon]}{n} \widetilde{X}_j + (1-L)\tilde{I} + \frac{1-L}{n} \sum_{i=1}^{n} \tilde{e}_{ij} + \tilde{\delta}_j + \frac{1}{n} \sum_{i=1}^{n} \tilde{\epsilon}_{ij} + \frac{L}{s} \sum_{k=1}^{s} \widetilde{P}_k \, .$$

Substitute (A.17) into (A.18) and get (3.5)-(3.7). **Q.E.D.**

Proof of Proposition 4.1

Directly from Proposition 3.3.

Proof of Proposition 4.2

By Proposition 3.3, for every j:

(A.19)
$$\widetilde{P}_j = \widetilde{F}_j + \tilde{\tau}_j + \tilde{\theta}_j \, .$$

The liquidity traders' trading costs are $-\widetilde{X}_j(\tilde{\tau}_j + \tilde{\theta}_j)$. From equations (3.6) and (3.7), and by the independence of $\widetilde{X}_j$ and $\widetilde{X}_k$, $k \neq j$, and of $\tilde{\delta}_k$, $\tilde{e}_{ik}$, and $\tilde{\epsilon}_{ik}$ for all i, k, we get that the expected trading costs of the liquidity traders of asset j is:

$$E\widetilde{X}_j^2 \cdot \frac{a[(1-L)^2V_e + V_\epsilon]}{n} \cdot \left(1 + \frac{1}{S(1-L)}\right)$$

$$= V_x \cdot \frac{a[(1-L)^2V_e + V_\epsilon]}{n} \cdot \left(1 + \frac{1}{S(1-L)}\right) .$$

As $s \to \infty$, the above expression is decreasing in L. **Q.E.D.**

Proof of Proposition 4.3

The total payoff to the liquidity traders of asset j is $X_j \cdot \widetilde{P}_j$. Conditional on the realization X_j the variance of this payoff is:

$$V(X_j \widetilde{P}_j) = X_j^2 V(\widetilde{P}_j) = X_j^2 V(\widetilde{F}_j + \tilde{\tau}_j + \tilde{\theta}_j) .$$

As $V(\tilde{\theta}_j) = \text{constant}/s$, we get

$$\begin{aligned} \lim_{s \longrightarrow \infty} V(\widetilde{P}_j \cdot X_j) &= X_j^2 \cdot \lim_{s \longrightarrow \infty} V(\widetilde{F}_j + \tilde{\tau}_j + \tilde{\theta}_j) \\ &= X_j^2 V(\widetilde{F}_j + \tilde{\tau}_j) = X_j^2 [V(\widetilde{F}_j) + V(\tilde{\tau}_j)] , \end{aligned}$$

where $V(\widetilde{F}_j)$ is independent of L and, from equation (3.6), $V(\tilde{\tau}_j)$ is decreasing in L ($L < 1$).

Q.E.D.

Proof of Proposition 4.4

Consider trader ij. His wealth after trading is:

$$\widetilde{W}_{ij} \approx K + Q\tilde{I} - \tilde{d}_{ij}\tilde{\tau}_j .$$

From Lemma 3.2

$$\tilde{d}_{ij} = \frac{(1-L)I_{ij} + \delta_{ij} - (\widetilde{P}_j - L\tilde{I}_S)}{a[(1-L)^2V_e + V_\epsilon]} .$$

Using (A.4) we get:

$$\tilde{d}_{ij} \approx \frac{(1-L)\tilde{e}_{ij} + \tilde{\epsilon}_{ij} - \tilde{\tau}_j}{a[(1-L)^2V_e + V_\epsilon]}$$

and

(A.20) $$\widetilde{W}_{ij} \approx K + Q\tilde{I} - \frac{(1-L)\tilde{e}_{ij} + \tilde{\epsilon}_{ij} - \tilde{\tau}_j}{a[(1-L)^2V_e + V_\epsilon]}\,\tilde{\tau}_j .$$

Denote

$$\tilde{\tau}_j^i = \tilde{\tau}_j - \tfrac{1}{n}[(1-L)\tilde{e}_{ij} + \tilde{\epsilon}_{ij}], \tag{A.21}$$

and

$$V_\tau^i = \mathrm{Var}(\tilde{\tau}_j^i). \tag{A.22}$$

The random wealth in (A.20) becomes

$$\widetilde{W}_{ij} \approx K + Q\tilde{I} - \frac{\left(1+\frac{1}{n}\right)[(1-L)\tilde{e}_{ij} + \tilde{\epsilon}_{ij}] - \tilde{\tau}_j^i}{a[(1-L)^2 V_e + V_\epsilon]} \left[\tilde{\tau}_j^i + \tfrac{1}{n}(1-L)\tilde{e}_{ij} + \tilde{\epsilon}_{ij}\right]. \tag{A.23}$$

Because

a) $\tilde{e}_{ij}$, $\tilde{\epsilon}_{ij}$, $\tilde{\tau}_j^i$ are normally distributed; and

b) $\tilde{e}_{ij}$, $\tilde{\epsilon}_{ij}$ and $\tilde{\tau}_j^i$ are independently distributed;

(A.23) can be written as

$$\widetilde{W}_{ij} \overset{d}{\approx} K + Q\tilde{I} - \left(1+\tfrac{1}{n}\right) \frac{\tilde{y}\sqrt{(1-L)^2 V_e + V_\epsilon} - \widetilde{Z}\sqrt{V_\tau^i}}{a[(1-L)^2 V_e + V_\epsilon]} \left(\widetilde{Z}\sqrt{V_\tau^i} + \tfrac{1}{n}\tilde{y}\sqrt{(1-L)^2 V_e + V_\epsilon}\right), \tag{A.24}$$

where $\overset{d}{\approx}$ means "has the same distribution by the trader's approximation," and $\widetilde{Z}$ and $\tilde{y}$ are independent standard normal variables. From (3.6), (A.21), (A.22)

$$\begin{aligned} V_\tau^i &= a^2[(1-L)^2 V_e + V_\epsilon]^2 V_x / n^2 + [(1-L)^2 V_e + V_\epsilon] \cdot \left(\tfrac{n-1}{n^2}\right) \\ &= \tfrac{n-1}{n^2}[(1-L)^2 V_e + V_\epsilon] \left[1 + \frac{a^2[(1-L)^2 V_e + V_\epsilon] \cdot V_x}{n-1}\right]. \end{aligned} \tag{A.25}$$

By substituting (A.25) in (A.24) we get

$$\widetilde{W}_{ij} \overset{d}{\approx} K + Q\tilde{I} - \frac{\tilde{y}\widetilde{Z}\sqrt{\frac{n-1}{n^2}}\sqrt{1 + \frac{a^2[(1-L)^2 V_e + V_\epsilon]V_x}{n-1}}}{a} + \frac{\widetilde{Z}^2 \frac{n-1}{n^2}\left[1 + \frac{a^2[(1-L)^2 V_e + V_\epsilon]}{n-1} V_x\right]}{a} - \frac{\frac{1}{n}\left(1+\frac{1}{n}\right)\tilde{y}^2}{a}. \tag{A.26}$$

Denote

$$b_L = \sqrt{\frac{n-1}{n^2}}\sqrt{1 + \frac{a^2[(1-L)^2 V_e + V_\epsilon]V_x}{n-1}}, \tag{A.27}$$

and substitute (A.27) into (A.26) to get

(A.28) $$\widetilde{W}_{ij} \overset{d}{\approx} K + Q\tilde{I} + \tfrac{1}{a}\left[\tilde{Z}^2 b_L^2 + \tilde{Z}\tilde{y}b_L - \tfrac{1}{n}\left(1+\tfrac{1}{n}\right)\tilde{y}^2\right] .$$

Consider a mechanism in which the bound is l, $0 \le l < L$ instead of L. We shall show that in the l-mechanism the trader can get greater expected utility than in the L-mechanism. Assume the investor's demand is

$$\tilde{d}_{ij}^* = \frac{(1-l)I_{ij} + \delta_{ij} - (\tilde{P}_j - l\tilde{I}_S)}{a[(1-l)^2 V_e + V_\epsilon]} \cdot \frac{b_L^2}{b_l^2}$$

then, the trader's wealth after trading will be

$$\begin{aligned} W_{ij}^l &\overset{d}{\approx} K + Q\tilde{I} + \frac{b_L^2}{b_l^2}\cdot\frac{1}{a}\left[\tilde{Z}^2 b_l^2 + \tilde{Z}\tilde{y}b_l - \frac{1}{n}\left(1+\frac{1}{n}\right)\tilde{y}^2\right] \\ &= K + Q\tilde{I} + \frac{1}{a}\left[\tilde{Z}^2 b_L^2 + \left(\frac{b_L}{b_l}\right) b_L\tilde{Z}\tilde{y} - \frac{b_L^2}{b_l^2}\left(1+\frac{1}{n}\right)\tilde{y}^2\right] \end{aligned}$$

where

$$b_l = \sqrt{\frac{n-1}{n^2}}\sqrt{1 + \frac{a^2[(1-l)^2 V_e + V_\epsilon]V_x}{n-1}} .$$

$\widetilde{W}_{ij}^l$ is preferred by any risk averter to $\widetilde{W}_{ij}$ because:

a) The term $-\left(1+\frac{1}{n}\right)\tilde{y}^2$ is multiplied by $0 < \frac{b_L^2}{b_l^2} < 1$;

b) The term $-\tilde{Z}\tilde{y}b_L$ is "noise" [see Rothschild & Stiglitz (1970) and the proof of Lemma 3.2] , symmetric around zero, and multiplied by $0 < \frac{b_L}{b_l} < 1$.

Therefore, *decreasing* L increases the expected utility of the traders. **Q.E.D.**

Proof of Proposition 4.5

Equation (3.4) presents trader ij's demand for asset j,

$$d_{ij} = \frac{(1-L)I_{ij} + \delta_{ij} - (\tilde{P}_j - L\tilde{I}_s)}{a[(1-l)^2 V_e + V_\epsilon]} ,$$

which, together with (3.5)-(3.7) implies that

$$\lim_{s\to\infty} d_{ij} = \frac{[(1-L)\tilde{e}_{ij} + \tilde{\epsilon}_{ij} - \tilde{\tau}_j]}{a[(1-L)^2 V_e + V_\epsilon]} .$$

Using equation (3.6) we get:

$$E \lim_{s \to \infty} |d_{ij}| = E|\tilde{Z}| \frac{\sqrt{[(1-L)^2 V_e + V_\epsilon] \cdot \left(\frac{n-1}{n}\right) + V_\tau}}{a[(1-L)^2 V_e + V_\epsilon]} ,$$

where $\tilde{Z}$ is a standardized normal variable. The derivative of the above expression with respect to L is positive. Q.E.D.

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