DYNAMIC WEALTH REDISTRIBUTION, TRADE AND ASSET PRICING

by

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ABSTRACT

We relate wealth redistribution, asset pricing, and trade in financial assets by introducing heterogeneous agents into a Lucas tree-model. Heterogeneity of agents causes trade in financial assets and dynamic wealth redistribution. When consumers have time-separable, constant elasticity utilities with constant time-discount factors, the price-representative consumer has declining temporal relative risk aversion and intertemporal discount factors. Resulting asset prices "over-react": Adverse aggregate consumption shocks cause wealth redistribution towards more risk averse consumers, reinforcing the adverse market value effect. Interest rates, risk premia, return volatility, and trade volume exhibit time-variance.

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I. Introduction

Much research over the past decade has been concerned with the empirical implications of Lucas's (1978) "tree" model. In the Lucas model asset prices are derived from the marginal rates of substitution of a single representative consumer-investor who derives her entire consumption from a tree with a known stochastic pattern of harvests over time. Using aggregate data, researchers have uncovered major problems in reconciling the model with the historical record. Hansen and Singleton (1982), for example, reject the Euler equation tests arising out of a stock market pricing model with constant relative risk aversion. Mehra and Prescott (1985) conclude that given the smoothness of consumption in the past century and given "reasonable" preferences, the premium for equity over bonds is too high. Hansen and Jagannathan (1991) find that tests of the model result in unrealistic volatility bounds.

In this paper we introduce agents with heterogeneous tastes into the Lucas model. Although the appropriateness of postulating a single "representative" agent has come under repeated attack (see, for example, Kirman 1992), such models continue to be used because they present such a simple and convenient paradigm for modelling economic aggregates. On the other hand, it is not altogether clear whether consumer heterogeneity *per se* can in fact explain the empirical puzzles cited above. We suggest that it can.

In an attempt to explain the Mehra-Prescott "equity premium puzzle," multiple agents have recently been introduced to the Lucas model by Mankiw (1986), Aiyagari and Gertler

(1991), Marcet and Singleton (1990), Telmer (1991), Weil (1991), and Constantinides and Duffie (1992). In these models agents who are ex-ante identical end up heterogeneous ex-post, due to the existence of idiosyncratic risk in (labor) income and due to market imperfections that restrict the availability of full insurance markets. In contrast, in our model the capital market is Arrow-Debreu complete, and agents are ex-ante different in their tastes.

One basic observation that motivates our approach is the well-known disparity between the distributions of consumption and wealth. The Gini coefficients for measuring inequality of household wealth and consumption in the United States in 1986-88 were 0.77 and 0.29 respectively (Cutler and Katz 1992, Wolff 1992). A recent study finds that in 1989 the top one percent of wealth holders held 36.2% of the total non-human worth of United States households and 62.5% of the business assets and corporate stock held by households (Kennickell and Woodburn 1992). While the average consumer is at about the 70th percentile of households, the average wealth holder, when human capital is included, is apparently closer to the middle of the top quintile of households. Wealthier individuals, therefore, do not consume in proportion to their wealth, and hence the "representative" consumer is not the same person as the "representative" investor-wealthholder.[1] One would expect a model of the key dynamic regularities of the capital market to shed light on the determinants of the distribution of wealth. Yet it is clear that a representative agent model cannot fulfill this task.

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[1] Mankiw and Zeldes (1991) report that only 11.9% of a representative sample of families hold equity in excess of $10,000. Families which do not own stock have 62% of disposable income and account for 68% of food expenditures.

In introducing heterogeneous agents to the Lucas model, we follow most of the related literature by assuming that each consumer's utility belongs to the class of time-separable, constant elasticity utility functions with a constant time discount factor.[2] Our agents, however, differ in their tastes. This heterogeneity of agents accords with the Knightian vision that the accumulation of personal wealth is a byproduct of a combination of luck and entrepreneurial spirit (see Khilstrom and Laffont 1979). Recent studies suggest that there are significant differences among agents in their subjective rates of time discount (Lawrence 1991) and in their attitude towards risk.[3] It should be emphasized, though, that the essence of our argument concerns the joint assumption of identical tastes and of a particular preference relation common in the literature, satisfying separability, constant elasticity, and constant discounting. As section III below demonstrates, asset pricing in the economy can still be explained by postulating a fictitious "pricing-representative" agent, albeit one whose preferences are considerably more complicated than those of the agents he represents.

In our model we assume that agents' endowments are perfectly correlated and that there are no differences in their beliefs. However, due to the heterogeneity in preferences,

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[2] For the implications of alternative preferences for the representative agent see Constantinides (1990) and Kandel and Stambaugh (1991).

[3] Disparities in the subjective rate of time preference are analyzed by Lawrence (1991). Marshall (1920 ,pp.187-91) and Fisher (1991, chapters 4.9-10, 18.2) have emphasized the cultural influences that cause people to differ from one another in their subjective rates of intertemporal substitution. MacCrimmon and Wehrung (1990), analyzing the attitude towards risk of top-level business executives, find that the most successful were the biggest risk takers. Mankiw and Zeldes (1991) estimate that the risk aversion of non-stockholders is substantially higher than that of stockholders.

agents will differently adjust their consumption and the composition of their portfolios, in response to the realization of each harvest. In particular, the consumption stream of a less risk averse individual will be significantly more variable than aggregate consumption.

As has been shown by Constantinides (1982), the asset prices that arise in the economy with heterogeneous agents could in fact be rationalized as if originating (locally) from the preferences of a fictitious representative agent. What we demonstrate, however, is that even though all the consumers are endowed with "reasonable" constant elasticity, constant time-discount utility functions, the induced preferences for the representative agent are not of that type. Thus we cast considerable doubt on the reasonableness of the simple preference structure with which the representative agent is typically endowed in the literature.

In particular, we show that in our model the representative agent's preferences exhibit a temporarily diminishing time discount factor and temporarily declining relative risk aversion. Even though the distribution of aggregate consumption is stationary, these "odd" properties of the "pricing-representative" agent lead to time-variation in the risk-free rate of return, in the market risk premium, and in the variances of the riskless rate and the market risky rate of return. The variability over time in these financial variables should be understood to result both from the properties of the "pricing-representative" agent as well as from endogenous changes in the distribution of wealth.

Indeed the full richness of the Lucas tree-model with heterogeneous agents can only be appreciated by noting the allocational dimensions that underlie asset pricing. Taste differences imply that when financial markets are introduced into the model, zero-supply riskless assets will exist and will serve a major role in securing a more stable consumption pattern for more risk averse agents. Moreover, we show that dynamic trade in assets will take place, as agents adjust their portfolio holdings because of changes in the wealth distribution.

In our model trade is conducted for fundamental consumption reasons. When the harvest is bountiful and wealth is redistributed towards less risk averse agents, who own a relatively higher proportion of the risky asset, these agents will expand their stock ownership by purchasing stock from the more risk averse agents. In a downturn, when wealth is redistributed the other way, the less risk-averse agents will seek to supplement consumption by selling off some shares in a downward market.

As a result of these trade patterns, our simulation produces a positive relation between volume of trade in shares and price changes, as well as an asymmetry in that relation between upturns and downturns. These predictions provide a simple rationale for the empirical finding surveyed by Karpoff (1987). These findings, on the whole, indicate that price increases are correlated with larger volumes, and that the ratio of volume to the absolute value of price change is higher for upturns.

We argue that this pattern of wealth redistribution and trade may also provide a structural explanation for some asset pricing anomalies, such as the non-stationarity of returns, excessive risk premia, and the phenomenon of apparent market "over-reaction." When the Lucas tree happens to be particularly bountiful, the wealth distribution tilts towards less risk-averse agents, who have heavier stock investment, and the valuation of the tree tilts towards representing their tastes; the reverse happens when the fruit of the tree is particularly meager.[4] Thus, endogenous changes in the wealth distribution reinforce the variability (and riskiness) of share prices. Personal characteristics and "luck" result in endogenous wealth redistribution that feeds back on asset prices.

In an important recent paper Dumas (1989) explores a production model with two heterogeneous agents and also finds time variation in the distribution of wealth and consumption. In his model however, the market risky return is exogenously determined by the stochastic yield on constant-return real investment, and only the riskless rate of return is endogenous. Dumas does not explore the characteristics of the representative agent.

Surprisingly, there are few models that shed any light on the existence of dynamic trade in assets. Most of the actual trade in assets is clearly conducted by those who reorganize their portfolios, a feature that can be ill-understood in a framework with identical beliefs (and, according to Milgrom and Stokey 1982, can also be ill-understood in a model of asymmetric information; see also Huffman 1987). In our model, by contrast, dynamic trade in financial assets serves a fundamental allocational role.

The structure of the paper is as follows: In the following section we set out the model. In Section III we characterize the preferences of the "pricing-representative" agent in a model with multiple, heterogeneous, consumers. In section IV we simulate an economy whose aggregate characteristics are identical to Mehra and Prescott's (1985) example. We show that the Mehra-Prescott targets for the market risk premium and the risk-free rate can be achieved when there are two very different consumers in the economy. Our example also

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[4] See Bewley (1982-90?) and Dumas (1989) for similar results.

gives rise to dynamic wealth allocations and trade in financial assets. In Section V we summarize our results.

II. The Lucas economy with heterogeneous agents

Following Lucas (1978), Mehra and Prescott (1985) and most of the related literature, we assume an exchange economy with one good whose aggregate endowment across time and states of nature is given. In order to concentrate on the heterogeneity in tastes and to stay as close as possible to the framework where only one agent exists, we assume that the initial endowment of each agent i is a fraction w_i of the economy's endowment.

Let S_t be the list of the alternative histories (events) up to period t. For event $s \varepsilon S_t$ let C_{ts} be the strictly positive total endowment of consumption, and let c^i_{ts} be the amount consumed by agent i in the date-event pair ts. We normalize the scale of consumption so that C_0, the total consumption in the initial period, equals to unity.[5] Each one of the H agents has time-separable, expected utility preferences that take the form:

$$U^i(\{c^i_{ts}\}) = u^i(c^i_0) + \sum_{t=1}^{\infty} \beta^t_i \sum_{s \in S_t} \pi_{ts} u^i(c^i_{ts}), \quad where\ u^i(c) = \frac{c^{1-\gamma_i}}{1-\gamma_i}. \tag{1}$$

All agents are assumed to share the strictly positive probability assessment π_{ts} of each event. Agent i is thus characterized by her subjective rate of time discount β_i, her constant degree of

[5] Since all consumer's utility functions will be assumed to exhibit constant proportional risk aversion, this assumption is harmless.

relative risk aversion γ_i, and her initial fraction of ownership in the aggregate endowment, w_i. We assume $\beta_i > 0$, $\gamma_i > 0$, and $w_i > 0$.

The pattern of trade and dynamic wealth distribution that emerges in this economy depends on the structure of markets that is assumed to exist. We do not intend that the structure of markets will leave potential beneficial trade unexploited. Thus, we assume that even if there is trade over time in stocks and bonds, this trade can be interpreted to mimic the once-and-for-all trade that would have taken with a full initial set of Arrow-Debreu markets.

Let p_{ts} denote the Arrow-Debreu equilibrium price of contingent consumption in state s in period t. Each agent selects a consumption program which maximizes $U^i(\{c^i_{ts}\})$ in (1) given her budget constraint,

$$c_0^i + \sum_{t=1}^{\infty} \sum_{s \in S_t} p_{ts} c_{ts}^i = w_i \left[1 + \sum_{t=1}^{\infty} \sum_{s \in S_t} p_{ts} C_{ts} \right] \tag{2}$$

Let $\{c^i_{ts}\}$ be the equilibrium allocation in the economy. As is well-known, given the particular preferences assumed here, all c^i_{ts} will be strictly positive; that is, there will be no corner solutions. In equilibrium, all consumers' marginal rates of substitution equal the state prices:

$$p_{ts} = (\beta_i)^t \pi_{ts} \frac{u^{i\prime}(c_{ts}^i)}{u^{i\prime}(c_0^i)} = (\beta_i)^t \pi_{ts} \left[\frac{c_0^i}{c_{ts}^i} \right]^{\gamma_i} \qquad all\ i,t,s \in S_t \tag{3}$$

The prices p_{ts} are determined in a Walrasian equilibrium, so as to equate the demand and supply for all the state-contingent goods:

$$\sum_{i=1}^{H} c_{ts}^{i} = C_{ts} \qquad all\ t,s \in S_t \quad . \tag{4}$$

The equilibrium prices are, of course, a function of the agents' characteristics $\{\beta_i, \gamma_i, w_i\}$ and of market parameters $\{C_{ts}, \pi_{ts}\}$.

Denote the probability-normalized price by:

$$q_{ts} \equiv \frac{p_{ts}}{\pi_{ts}} \quad . \tag{5}$$

By combining (3) and (4), we can view q_{ts} as determined by:

$$\sum_{i=1}^{H} c_0^{i} \left[\frac{\beta_i^{t}}{q_{ts}} \right]^{\frac{1}{\gamma_i}} = C_{ts}, \quad all\ t,s \in S_t \tag{6}$$

Here, the equilibrium normalized prices are determined by the aggregate consumption quantities, the agents' taste parameters, and their shares of initial period consumption (instead of the initial distribution of wealth). In fact the equilibrium conditions (3)-(4) and agents' budget constraints (2) establish a one-to-one relation between agents' initial distribution of wealth and the distribution of initial consumption. In particular, given the latter, and given q_{ts} as determined by (6), we can consider the wealth as determined by:

$$w_i = c_0^{i} \frac{1 + \sum_{t=1}^{\infty} \sum_{s \in S_t} q_{ts} \pi_{ts} \left[\frac{\beta_i^{t}}{q_{ts}} \right]^{\frac{1}{\gamma_i}}}{1 + \sum_{t=1}^{\infty} \sum_{s \in S_t} q_{ts} \pi_{ts} C_{ts}} \quad . \tag{7}$$

III. Identifying preferences for a representative individual

Our aim in this section is to identify the characteristics of a "pricing-representative" agent in the economy. We define such an agent as a consumer whose tastes are such that if all the H agents in the economy had tastes identical to her then all the equilibrium state prices in the economy would remain unchanged. It is important to note that we are looking for someone who can mimic the given economy, rather than mimic the economy with the given $\{\beta_i, \gamma_i, w_i,\}$ for, say, any possible set of aggregate endowments $\{C_{ts}\}$.[6]

We assume that a the utility function for the representative agent takes the separable form:

$$U^*(\{C_{ts}\}) = u_0^*(C_0) + \sum_{t=1}^{\infty} (\beta_t^*)^t \sum_{s \in S_t} \pi_{ts} u_t^*(C_{st}) \tag{8}$$

We require from this function that its implied marginal rates of substitution be equal to the equilibrium state prices p_{ts}:

$$(\beta_t^*)^t \left[\frac{u_t^{*\prime}(C_{ts})}{u_0^{*\prime}(C_0)} \right] = \frac{p_{ts}}{\pi_{ts}} = q_{ts} \qquad \textit{for all } t,\ s \in S_t \tag{9}$$

The form of the representative utility function in (8) assumes time-separability and expected utility maximization, but does not impose the additional assumptions assumed in (1) above for each individual, namely the time invariance of the utility function and of the time discount factor and the constant elasticity temporal utility function.

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[6] What is important is that the representative consumer is well-defined when the time 0 consumptions allotments, c_0^i, are taken as given and when the future aggregate consumption can take on a continuum of values.

The number of restrictions that are placed on the choice of β_t^* and $u_t^*(.)$ by condition (9) is equal to the number of different states in time t. We plan to identify these unknowns by widening that set of restrictions.

By analogy to equation (6), we now define the function $q_t(C)$ by the implicit condition:

$$\sum_{i=1}^{H} \frac{c^i_0}{C} \left[\frac{(\beta_i)^t}{q_t(C)} \right]^{\frac{1}{\gamma_i}} = 1 \tag{10}$$

It follows from the definition of q_{ts} above and the equilibrium condition (6) that for any t and $C = C_{ts}$,

$$q_t(C_{ts}) = q_{ts} \tag{11}$$

That is, for aggregate consumption C in period t, the function $q_t(C)$ coincides with the probability-weighted equilibrium state prices q_{ts} posited above. Clearly, the functions $q_t(C)$ depend on all investors' taste parameters, $\{\beta_i, \gamma_i\}$ and on the equilibrium shares of initial consumption $\{c^i_0\}$.

Condition (9) for the representative preferences can now be restated as requiring that for every t and for $C = C_{ts}$:

$$(\beta_t^*)^t \left[\frac{u_t^{*\prime}(C)}{u_0^{*\prime}(1)} \right] = q_t(C) \tag{12}$$

We plan to identify u_t^* by requiring now that for every t, (11) will hold not **only** for level of C such that $C = C_{ts}$, but as an identity for **all** C. We further impose the normalization: $u^{*\prime}_t(1) = 1$ for every t.

Under these additional restrictions, by setting C = 1, condition (9) identifies the time-discount factor β^*_t of the representative consumer:

$$(\beta_t^*)^t = q_t(1) \tag{13}$$

Conditions (12) and (13) then further identify the temporal utility function $u_t^*(.)$ by the differential equation:

$$u_t^{*\prime}(C) = \frac{q_t(C)}{q_t(1)} \tag{14}$$

Since equations (13) and (14) are assumed to hold as an identity for all (positive) values of C, by differentiating (14) we can then define the temporal degree of relative risk aversion of the representative investor:

$$\gamma_t^*(C) = -\frac{C\, q_t'(C)}{q_t(C)} \tag{15}$$

<u>Proposition 1</u>: β_t^* is bounded from above by $\text{Max}_i\{\beta_i\}$ and from below by $\text{Min}_i\{\beta_i\}$.

<u>Proof</u>:

By combining (10) and (13), β_t^* is defined as the implicit solution for the condition:

$$\sum_{i=1}^{H} c_{i0} \left[\frac{\beta_i}{\beta_t^*}\right]^{\frac{t}{\gamma_i}} = 1 \quad . \tag{16}$$

Suppose, to the contrary that $\beta_t^* > \text{Max}_i\{\beta_i\}$. In this case $[\beta_i/\beta_t^*]^{t/\gamma_i} < 1$ for all i. Since the c_{i0} sum to one, the left hand side of (16) would then be lower than the right hand side. In a similar manner one can rule out the possibility that $\beta_t^* < \text{Min}_i\{\beta_i\}$. | |

Proposition 2: β_t^* increases overtime towards $\text{Max}_i\{\beta_i\}$; that is, the representative rate of time discount approaches the subjective time discount of the most patient investor.

Proof:

From (16), for any t and k, $1 = \sum c_{i0}\, [\beta_i/\beta_t^*]^{t/\gamma_i} \geq c_{k0}[\beta_k/\beta_t^*]^{t/\gamma_k} > 0$. Thus for any t and k, $\beta_t^* \geq (c_{k0})^{\gamma_k/t}\beta_k$. Since $c_{k0} > 0$, as $t \to \infty$ $(c_{c0})^{\gamma_k/t} \to \infty$, thus establishing that as $t \to \infty$, β_t^* is bounded from below by β_k for every k. Thus, as $t \to \infty$, β_t^* is bounded from below by $\text{Max}_i\{\beta_i\}$12. The proof is completed by recognizing from the preceding proposition that for every t, β_t^* is also bounded from above by $\text{Max}_i\{\beta_i\}$. | |

Proposition 3: For all t and C, $\gamma_t^*(C)$ can be considered as a harmonic weighted average of individuals' γ_i's. Thus, for all t and C, $\gamma_t^*(C)$ is bounded from above by $\text{Max}_i\{\gamma_i\}$ and from below by $\text{Min}_i\{\gamma_i\}$.

Proof:

By equation (6) $q_t(C)$ is determined as the solution of the implicit condition:

$$F(C,q) \equiv \sum_i^H \frac{c_{i0}}{C}\left[\frac{(\beta_i)^t}{q}\right]^{\frac{1}{\gamma_i}} = 1 \quad . \tag{17}$$

It follows that:

$$\frac{dF}{dC} = -\sum_i \frac{c_{io}}{C^2}\left[\frac{(\beta_i)^t}{q}\right]^{\frac{1}{\gamma_i}} = -\frac{1}{C} \quad . \tag{18}$$

and

$$\frac{dF}{dq} = \frac{-1}{Cq}\sum_i \frac{c_{i0}}{\gamma_i}\left[\frac{(\beta_i)^t}{q}\right]^{\frac{1}{\gamma_i}} \tag{19}$$

$$= -\frac{1}{q}\sum_i \frac{\alpha_i}{\gamma_i} \quad .$$

where

$$\alpha_i \equiv \frac{c_{i0}}{C}\left[\frac{(\beta_i)^t}{q}\right]^{\frac{1}{\gamma_i}} \quad . \tag{20}$$

This means that:

$$q_t^{\prime}(C) = \frac{-dF/dC}{dF/dq} = \frac{-q_t(C)/C}{\sum_i \alpha_i/\gamma_i} \quad . \tag{21}$$

The α_i's, which, like q, depend on t, C, and the initial consumption shares c_{i0}, can be interpreted as weights, since they are positive, and, by (6) sum to one. By its definition in (15), therefore,

$$\gamma_t^*(C) = \frac{1}{\sum_i \alpha_i/\gamma_i} \quad . \tag{22}$$

This completes the proof of the proposition.| |

Proposition 4: The representative agent displays decreasing relative risk aversion.

Proof:

By differentiation of (20):

$$\frac{\partial \alpha_i}{\partial C} = \frac{\alpha_i}{C}\left[\frac{\gamma^*}{\gamma_i} - 1\right] . \tag{23}$$

Thus as C increases, the weight α_i of those investors with relatively low degree of risk aversion increases. From (22) and (23),

$$\frac{\partial \gamma^*}{\partial C} = \frac{-\sum_i \frac{1}{\gamma_i}\frac{\partial \alpha_i}{\partial C}}{\left[\sum_i \alpha_i/\gamma_i\right]^2}$$

$$= \frac{(\gamma^*)^2}{C}\left\{\sum_i \left[\frac{\alpha_i}{\gamma_i} - \gamma^*\left(\frac{\alpha_i}{\gamma_i^2}\right)\right]\right\} \tag{24}$$

$$= \frac{(\gamma^*)^3}{C}\left\{\sum_i \left[\frac{\alpha_i}{\gamma_i}\right]^2 - \sum_i \frac{\alpha_i}{\gamma_i^2}\right\} .$$

It follows that $\partial\gamma/\partial C < 0$ if and only if:

$$\frac{1}{\gamma^{*2}} = \left[\sum_i \frac{\alpha_i}{\gamma_i}\right]^2 > \sum_i \frac{\alpha_i}{\gamma_i^2} . \tag{25}$$

Looking at the random variable X that obtains the value $1/\gamma_i$ with the probability α_i, we see that the left-hand side above is $E(X^2)$ and the right-hand side is $[E(X)]^2$, so that by the definition of the variance, we have established our claim.||

Proposition 5: (i) If $C \rightarrow \infty$ then $\gamma_t^*(C) \rightarrow Min_i\{\gamma_i\}$; (ii) if $C \rightarrow 0$, then $\gamma_t^*(C) \rightarrow Max_i\{\gamma_i\}$.

Proof:

Let $\gamma_k = Min_i\{\gamma_i\}$. By (22) it is sufficient to prove that for all j, if $\gamma_j > Min_i\{\gamma_i\}$, then as $C \rightarrow \infty$, $\alpha_j \rightarrow 0$. Suppose to the contrary that there is a consumer j where $\gamma_j > Min_i\{\gamma_i\}$ and where as $C \rightarrow \infty$, $\alpha_j > \epsilon_0 > 0$. By (22) then, there has to be $\epsilon_1 > 0$ such that $[\gamma_t^*(C)/\gamma_k - 1] > \epsilon_1$. This proves (i); the proof of (ii) is similar.||

Proposition 6: (i) If $\beta_i = \beta$ (that is, if all consumers have identical subjective rates of discount), then the representative relative risk aversion function $\gamma_t^*(C)$ does not change over time.

(ii) If consumers' rates of time discount vary, then as $t \rightarrow \infty$, $\gamma_t^*(C)$ approaches a weighted average of only those consumers with the highest β_i (the most patient). In particular if $\beta_k > \beta_i$ for all i other than k, then as $t \rightarrow \infty$, $\gamma_t^*(C) \rightarrow \gamma_k$. That is, if there is a single most patient investor, then as time approaches infinity the representative relative risk aversion function approaches his constant rate of relative risk aversion.

Proof:

(i) Even though we consider here time as discrete, the functions $q_t(C)$ defined in (10), $\gamma_t^*(C)$ defined in (15), and α_i defined in (20) are defined, and differentiable for any positive t. From the definition of α_i,

$$\frac{\partial\alpha_i}{\partial t} = \frac{\alpha_i}{\gamma_i}\left[Ln\beta_i - \frac{\partial q/\partial t}{q}\right] \qquad (26)$$

Because $\Sigma\, \alpha_i \equiv 1$, it has to be the case that $\Sigma\, [\partial\alpha_i/\partial t] = 0$. Thus, from (26),

$$\sum_j \frac{\alpha_j}{\gamma_j} Ln\,\beta_j = \frac{\partial q/\partial t}{q} \sum_j \frac{\alpha_j}{\gamma_j} \quad . \tag{27}$$

By combining (26) and (27),

$$\frac{\partial \alpha_i}{\partial t} = \frac{\alpha_i}{\gamma_i}\left[Ln\beta_i - \sum_j \eta_j Ln\beta_j\right] \tag{28}$$

where,

$$\eta_j = \frac{\alpha_j/\gamma_j}{\sum_k \alpha_k/\gamma_k} \quad , \tag{29}$$

so that the η_j's are positive weights that sum to one. From (29) it is evident that when all the $ß_i$'s are the same, the weights α_i do not vary over time. Thus, from (22), $\gamma_t^*(C)$ will also be independent of time.

(ii) Given (22), it is sufficient to prove that if $ß_i < Max_j\{ß_j\}$, then $\alpha_i \to 0$ as $t \to \infty$. Suppose to the contrary that $ß_i < Max_j\{ß_j\}$ and that for some level C, $\alpha_i > \epsilon_0 > 0$ for all t sufficiently large. In that case, also η_i, in (29) has to be bound away from zero. If $ß_k = Max_j\{ß_j\}$, then from (28) there exists $\epsilon_1 > 0$ such that $[Lnß_k - \Sigma\eta_j\, Lnß_j] > \epsilon_1$. From (28), this means that $\partial Ln\alpha_k/\partial t > \epsilon_1/\gamma_k > 0$. This last differential inequality contradicts the condition that α_k cannot be greater than one.||

IV. A Calibration Example

In this section we report on simulations of a two-consumer version of the Mehra-Prescott model, a special case of the tree economy described above.[7] The simulation assumes that the stochastic process which generates the aggregate harvest is a two-state mean-reverting Markov chain in the growth rate of consumption. In our model the distribution of wealth between the two agents provides an additional state variable. This gives rise to phenomena which cannot exist in a Lucas economy with a single representative consumer, such as time-varying stock returns and interest rates (and hence time-varying market risk premia), and a positive correlation between the volume of stock traded on the stock market and the value of the market.

We denote one-plus-the-growth rate of aggregate consumption by G_d in a "down" state and by G_u in an "up" state of the world. Thus, given aggregate consumption C at date t, consumption at date t+1 will be CG_u or CG_d. For dates $t \geq 1$, the four Markov transition probabilities: φ_{dd}, φ_{du}, φ_{ud} and φ_{uu} are constant. To start off the model, we assume that at date 0 initial aggregate consumption is 1 and that the probabilities of having aggregate consumption G_d or G_u at date 1 are equal. We replicate Mehra and Prescott's specification of a mean-reverting process by setting $\varphi_{ud} = \varphi_{du} = 0.57 > \varphi_{dd} = \varphi_{uu} = 0.43$, $G_u = 1.05$, and $G_d = 0.98$. This gives an average aggregate consumption growth of 1.5% and a standard deviation of this growth of 3.5%, close to the historic United States averages as reported by Mehra-Prescott (1985).

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[7] This section outlines the framework of the simulation and surveys its results. A detailed explanation of the computational method used is given in the Appendix to the paper.

In selecting the specification of the two agents' preferences we have several considerations in mind: (i) We want the first agent to be similar to an individual in the upper tail of the total wealth distribution. As noted in the introduction to the paper, such individuals typically hold most of the shares of corporate stock and have a relatively low consumption-to-wealth ratio. We conceive of these individuals as having low relative risk aversion and as being relatively patient. (ii) We want our second agent to be representative of individuals in the lower deciles of the total wealth distribution. The evidence cited in the Introduction suggests that these individuals typically hold very little stock; instead their wealth consists mostly of human capital, some real estate, and some safe financial assets. These individuals are portrayed here as more risk averse and more impatient, than the first type of agent. (iii) Finally, we wish to choose our agent parameters (as well as a market leverage ratio) so that we can approximate the Mehra-Prescott targets for the equity premium, the risk-free rate, and the standard deviation of market returns. We shall thus illustrate the power of incorporating heterogeneity among consumers into our equilibrium model, by showing that the economy can be replicated by two very different consumers, neither of whom could, singly, replicate the parameters of the economy.

To capture these features we assume that the first consumer-investor is both more patient and less risk averse than the second. In particular, we set: $(\beta_1,\gamma_1) = (1.12,3)$ and $(\beta_2,\gamma_2) = (0.93,36)$. In heuristic support of this specification, we argue that the evidence that researchers typically use concerning the tastes of "the" representative agent can, in a sense, be dichotomized. We interpret the time discount factor that researchers obtain from micro-based information, as characterizing the majority of individuals, that is our second agent. The attitude towards risk that is indirectly estimated from corporate stock pricing (e.g., as in Friend and Blume 1975) we interpret to be that of the first agent. We thus view our specification of $\beta_2 = 0.93$ and $\gamma_1 = 3$ as well within the range that other researchers consider as "reasonable."

This leaves us with the need to specify the two complementary parameters. Our choice of a higher-than-one subjective discount factor for the first agent is motivated by the idea that this agent represents the upper tail of wealthholders, whose desire to accumulate may be qualitatively different from that of the "average" individual.[8] Benninga and Protopapadakis (1991) and Kocherlakota (1991) have argued for a similar specification of the representative agent; they show that, due to the existence of uncertainty, it is not unreasonable to assume that agents have a larger-than-one time discount factor. Finally, the risk aversion of the second agent has to be high enough to explain her holding relatively little stock. We do not consider our choice of $\gamma_2 = 36$ to be unreasonable. By comparison, Kandel and Stambaugh (1991) argue for the possibility of a value of 29 for the representative agent, while Mankiw and Zeldes (1991) estimate a value of 35 for the average stockholder (and a value of 100 for their entire sample of consumers, including both stockholders and non-stockholders).

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[8] As Marshall (1920, pp. 189-190) suggested, these are people "who find an intense pleasure in seeing their hoards of wealth grow up ... with scarcely any thought for the happiness that may be got from it ... prompted partly by the instincts of the chase, by the desire to outstrip their rivals ...." They find "pleasure in amassing wealth for its own sake" or prefer "to leave their stored-up wealth intact for their families .... The greatest savings are made by those who ... desire to be found at their death richer than they had been thought to be."

For the parameters chosen, we can closely approximate the Mehra-Prescott historical figures. Table 1 gives the simulated model values when the market leverage equals 28.27% and when consumer 1 has holds 33.08% of the aggregate wealth. For comparison we also give the Mehra-Prescott targets and the simulation results when there is only one agent (either of type 1 or of type 2) in the economy.

There are several interesting aspects to the results reported in Table 1. First, we note that while neither of the consumers individually can approximate the Mehra-Prescott targets, the two consumers in combination can approximate these targets. Second, as suggested by the propositions of Section III, it is apparent that the simulated results for the two-consumer economy do not at all look like an "average" of the individual consumers' simulations. To better understand this phenomenon, we plot (Figure 1) the graph of the risk-free rate, the market return and the standard deviation of the market returns against the proportion of the total wealth in the economy initially held by the first agent (holding leverage unchanged).

Figure 1 demonstrates that consumer heterogeneity may introduce significant non-linearities in all of the market parameters. When the first agent ($\beta_1 = 1.12$, $\gamma_1 = 3$) holds a high proportion of the economy's wealth, she effectively prices the assets in the economy. Since she has a very high β, both the risk-free rate of interest and the expected market return become negative. On the other hand, since consumer 1 is relatively less risk-averse than consumer 2, the standard deviation of the market return is relatively low.

When the second agent ($\beta_2 = 0.93$, $\gamma_2 = 36$) holds a high fraction of total wealth, the expected return and standard deviation of the market are high. Since consumer 2's high risk aversion causes her to place a great premium on the risk-free asset, this rate again becomes negative.

What is not evident from Figure 1 is the very great difference between "up" and "down" states. In Table 2 we calculate these state prices for each consumer separately. If consumer i prices an the equilibrium by herself, then her state prices are given by:

$$\begin{aligned} q^i_{uu} &= \beta_i \phi_{uu} G_u^{-\gamma_i}, \quad q^i_{ud} = \beta_i \phi_{ud} G_d^{-\gamma_i} \\ q^i_{du} &= \beta_i \phi_{du} G_u^{-\gamma_i}, \quad q^i_{dd} = \beta_i \phi_{dd} G_d^{-\gamma_i} \end{aligned} \tag{30}$$

Note that both consumers demand higher risk-free rates, market returns, and market risk-premia in the "down" states as opposed to the "up" states. The risk-free rate of interest in the one-agent economies is given by

$$r^i_{fu} = 1/(q^i_{uu} + q^i_{ud}) \ , \quad r^i_{fd} = 1/(q^i_{du} + q^i_{dd}) \tag{31}$$

In Table 3 we give market data in the one-consumer economies in the "up" and the "down" states.[9] Note that consumer 2's desire to have very little variability in her consumption causes her to have a very large state price for units of consumption in the "down" state, whether or not this state's predecessor is a good or a bad outcome.

The consumption patterns of the two agents are very different. The initial consumption to wealth ratios of the two individuals are 2.11% for consumer 1 and 3.73% for consumer 2. For comparison, if both individuals had identical tastes similar to that of the first or second agents, the consumption to wealth ratio would be 0.73% and 7.14%

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[9] The rate reported in Table 1 for the two one-agent economies are simple averages of those in Table 3.

respectively. Furthermore, the consumption pattern of the second agent is much more stable. The correlation of consumer 2's period 30 consumption with aggregate period 30 consumption is 0.2267, while consumer 1's consumption has a 0.9990 correlation with aggregate consumption (see Figure 2). In the extreme case of an "up" predecessor, consumer 2's desire for balanced consumption (a result of her high risk aversion) causes a negative interest rate. Consumer 1, on the other hand has a negative implied interest rate in both cases because of her very high time discount factor.

Another feature of a two-consumer world is that, as one might expect from Propositions 2 and 6, the wealth distribution over time becomes skewed towards the more patient consumer. This is illustrated in Figure 3, which shows 6 simulated random runs of the time paths of the wealth distribution between the two consumers in the first 20 periods. Because consumer 1 is more patient than consumer 2, she is more willing to postpone her consumption. Because she is less risk averse, consumer 1 is willing to take bigger gambles, and, on average, benefit from these gambles.[10]

The last feature which heterogeneity introduces into the Lucas model is that trade in financial assets is endogenized. At Date 0, consumer 1 (with 33.08% of the aggregate wealth) shorts 4.27% of the corporate debt in order to finance her holding of 48.36% of the

[10] The tendency for the most patient, least risk-averse consumer to become the wealthiest consumer in the economy is clear from Propositions 2 and 5. Invoking casual empiricism, we conjecture that this phenomenon mirrors real-world phenomena. We speculate that these real-world phenomena are, in the long-run, mitigated by the several factors: First, there is a tendency for societies to impose wealth transfers on the rich in order to decrease the skewness of income distributions. Second, the children and heirs of miserly, patient individuals are perhaps themselves spendthrift and risk-averse (and vice versa). Incorporating these factors into a general equilibrium framework presents an interesting modelling challenge which we have left for future research.

market portfolio. As the wealth distribution between the two consumers changes, they trade financial assets between them to support their optimal consumption choices. To present the relation between the volume of trade and price changes, we must first calculate the portfolio composition--stock and bond holdings--of each of our two consumers. The level of trade in assets results from the implicit dynamics of portfolio consumption. Note that since the Modigliani-Miller Theorem holds in our model, the consumption of consumers is not a function of market leverage. The composition of consumer portfolios is, however, dependent on the amount of leverage. In Figure 4 we plot changes in the shareholdings of consumer 1 (as a percentage of the market held) against percentage changes in the market price (on the y-axis). We present data for the first six dates of our model. There is a clear positive correlation between the amount of stock bought or sold by consumer 1 and the change in the market price.[11] In particular, then, our model thus makes a definite prediction concerning the identity of the agents who sell or purchase stock when the market price increases: It predicts that increases in market prices are accompanied by purchases of less risk-averse, more patient consumers. We do not know of any formal evidence on this issue, although the anecdotal evidence about the massive distress sale by speculators in 1929 does fit our predictions. It also accords with the evidence cited in Karpoff (1987), who cites 12 papers which find a positive correlation between market price changes and trading volume.

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[11] Had we instead plotted pairs consisting of changes in market prices and changes in consumer 1's consumption, we would have also found a strong positive correlation. This confirms the findings reported by Mankiw and Zeldes (1991).

V. Conclusion

In this paper we have studied the effect of introducing consumer heterogeneity into the standard Lucas tree model of consumption. On a theoretical level we have shown that although in an economy with disparate consumers a "representative" consumer exists, this consumer does not inherit the properties (constant proportional risk aversion, constant time discount factor) of the consumers who form the constituents of the economy. The representative consumer has decreasing relative risk aversion and a pure-time discount factor which is a function of her wealth.

These "odd" properties of the aggregative agent can provide a simple explanation for some apparent anomalies of asset pricing. In particular the characterization of countercyclical "representative" relative risk aversion implies that procyclicality of market asset prices will be amplified. Thus, in the simple case when aggregate consumption growth follows a stationary stochastic process, the ratio of market wealth to consumption will be positively related to the level of consumption, rather than be constant as would be the case if the representative consumer displayed constant relative risk aversion. A downturn in aggregate consumption will lead to an increase in risk aversion in the economy which reinforces the fall in the value of the market.

To demonstrate the properties of asset pricing in the Lucas-tree economy with heterogeneous agents, we presented a two-consumer economy which could replicate the historical averages prescribed by Mehra and Prescott. The characterization of the two agents in that example seems to us as more defensible than that of other recent attempts to resolve the Mehra-Prescott puzzle in a representative-agent model. Our first agent, representing the very rich, who hold the bulk of stock, has a very reasonable degree of risk aversion but has a less-than-conventional (higher than one) discount factor. Our second agent, representing the not-very-rich, has a very conventional discount factor, but has a very high degree of risk aversion, as would be necessary to justify her very limited stock holding.

Other than our ability to provide an answer to the Mehra-Prescott asset-pricing puzzle, this simulation of the two-agent economy has interesting structural phenomena. In line with the theoretical propositions we obtained, the "odd" properties of the representative agent provide a simple rationale for time-variation in the risk-free rate of return, in the market risk premium, and in the variances of the riskless rate and the market risky rate of return. This non-stationarity reflects a fundamental dynamic interaction between the distribution of wealth, the heterogeneous composition of portfolios and asset prices.

As our simulations display, the heterogeneity of agents' tastes implies that the composition of their portfolios between shares in the tree and one-period bonds will differ. This means that each harvest realization will affect the agents differently, causing a redistribution of wealth, and a need to reshuffle their portfolios, that is to trade. The magnitude of trade is here endogenously determined, and in the example here the volume of trade in stock is positively related to the magnitude of changes in the market value. We suggest that a framework of dynamic equilibrium with heterogeneous agents may be suited not only for analysis of asset trade in closed economies, but may also aid in understanding trade patterns across nations.

Appendix: Details on simulation procedures

In simulating the extended Mehra-Prescott model with two agents, we followed the following steps. We first arbitrarily select a distribution of initial consumption. This choice, implies by (3) a unique efficient distribution of consumption in every subsequent date and event, and also determines the Arrow-Debreu contingent prices. In turn, the contingent prices determine the safe interest rate and the value of the tree (aggregate wealth) at each date-event. The latter is computed at each date-event as the present value of the subsequent harvests from that node onwards,including the current harvest. A similar procedure for the present value of each agent's consumption, defines each agent's wealth in every node.

More specifically, the binomial structure of the model means that at any date t, there are t+1 possible aggregate consumption levels: $G_d^{t-j}G_u^j$, $j=0,\ldots,t$. Thus, the date ,t, and the number of "up" moves in aggregate consumption since date 0, j, is sufficient to determine the aggregate consumption level. The mean-reversion in the transition probabilities implies that market prices will depend also on whether the current state is an "up" or a "down" one. Thus, for our purposes, an adequate description of the state of the world is given by the triple (t,j,g), where $j = 0,\ldots,t$ denotes the number of "up" moves in the history, and where $g \in \{d,u\}$ indicates whether the current state is a "down" or an "up" state. In the reported simulations, $t = 0, \ldots, 29$.[12]

Given the initial distribution of consumption, c^1_0 and c^2_0, and the condition for Pareto efficiency, (3), the pair (t,j) is sufficient to determine the division of current consumption,

[12] Computational considerations dictate this finite horizon; none of the results are affected qualitatively by this assumption.

$c^1_{t,j,u} = c^1_{t,j,d}$ and $c^2_{t,j,u} = c^2_{t,j,d}$. To calculate market prices and interest rates, we need to define state contingent prices. We use the common marginal rates of substitution (below, those of the first agent) to define the one-period state prices.

Two states can follow from a state (t,j,g), namely (t+1,j,d) and (t+1,j+1,u), with probabilities φ_{gd} and φ_{du}, respectively. Using the efficiency condition, (3), the one-period two contingent prices in state (t,j,g), for g =u,d, are:

$$p^g_{t+1,j,d} = \beta_1 \phi_{gd} \left[\frac{c^1_{t+1,j,d}}{c^1_{t,j,g}} \right]^{-\gamma_1} , \quad p^g_{t+1,j+1,u} = \beta_1 \phi_{gu} \left[\frac{c^1_{t+1,j+1,u}}{c^1_{t,j,g}} \right]^{-\gamma_1} . \tag{32}$$

The market value of the entire tree at state (t,j,g) is then given by the recursive relation:

$$M_{t,j,g} = G_d^{t-j} G_u^j + p^g_{t+1,j,d} M_{t+1,j,d} + p^g_{t+1,j+1,u} M_{t+1,j+1,u} . \tag{33}$$

The one-period risk-free interest rate at (t,j,g) is given by:

$$r_{t,j,g} = \frac{1}{p^g_{t+1,j,d} + p^g_{t+1,j+1,u}} - 1 . \tag{34}$$

Correspondingly, following (t,j,g) two possible rates of return may be realized on owning the tree, namely:

$$\begin{aligned} R^g_{t+1,j,d} &= \frac{M_{t+1,j,d}}{M_{t,j,g} - G_d^{t-j} G_u^j} - 1 \\ R^g_{t+1,j+1,u} &= \frac{M_{t+1,j+1,u}}{M_{t,j,g} - G_d^{t-j} G_u^j} - 1 . \end{aligned} \tag{35}$$

Trade in shares and changes in the market price

To identify the extent of trade, we must first calculate the share and bond holdings of each of our two consumers. For this, we solve for the wealth level of agent 1, by working our way backwards recursively, as we did for the market value. If $W^i_{t,j,g}$ is agent i's wealth in the beginning (before consumption) of date t, state j,g, then,

$$W^i_{t,j,g} = c^i_{t,j,g} + p^g{}_{t+1,j,d} W^i_{t+1,j,d} + p^g{}_{t+1,j+1,u} W^i_{t+1,j+1,u} \quad . \tag{36}$$

Let now $s^i_{t,j,g}$ denote the proportion of the market (tree) held by agent i and $b_{t,j,g}$ denote the amount of one period bonds purchased in the triplet (t,j,g). The agent's portfolio has to satisfy[13]

$$\begin{aligned} W^i_{t+1,j,d} &= s^i_{t,j,g} M_{t+1,j,d} + b^i_{t,j,g}(1+r_{t,j,g}) \\ W^i_{t+1,j+1,u} &= s^i_{t,j,g} M_{t+1,j+1,u} + b^i_{t,j,g}(1+r_{t,j,g}) \end{aligned} \tag{37}$$

The composition of the portfolio problem may now be easily found:

[13] Note that we have subtracted aggregate consumption at the coming date [(t+1,j,d) or (t+1,j+1,u)] from the market value at that date, since our definition of the market value includes contemporaneous consumption.

$$s^i_{t,j,g} = \frac{W^i_{t+1,j+1,u} - W^i_{t+1,j,d}}{M_{t+1,j+1,u} - M_{t+1,j,d}}$$

$$b^i_{t,j,g} = \frac{M_{t+1,j+1,u}\, W^i_{t+1,j,d} - M_{t+1,j,d}\, W^i_{t+1,j+1,u}}{[M_{t+1,j+1,u} - M_{t+1,j,d}]\,(1+r_{t,j,g})} \tag{38}$$

We see immediately that the share holdings are unaffected by whether a state is up or down; that is: $s_{t,j,u} = s_{t,j,d}$. On the other hand the bond holdings are affected by the state of the world.[14]

In turn, once the dynamic composition of portfolio is computed, one can identify the extent and direction of the trade in shares and in bonds required by the rebalancing of portfolios. Following a triple, (t,j,g), were the first agent holds a fraction $s^1_{t,j,g}$ of the stock (tree), in the next period a fraction $s^1_{t+1,j,d} - s^1_{t,j,g}$ or a fraction $s^1_{t+1,j+1,u} - s^1_{t,j,g}$ will be traded. These trade possibilities correspond respectively to a changes in the stock value in the magnitudes: $M_{t+1,j,d} - M_{t,j,g}$ and $M_{t+1,j+1,u} - M_{t,j,g}$.

Putting leverage into the model

Leverage does not affect the total value of the tree (firm, market), neither does it affect the allocation of consumption between the two consumers or the interest rate. Its only debt obligations).

[14] Note that the way we have written the share holdings, the above statement means that the percentage of the market the consumer wishes to hold is not affected by whether he is in an up or a down state. Of course, the value of his shares is affected (the bond holding is given in "dollars").

For simplicity, we assume that at each triplet (t,j,g) the firm issues one-period bonds whose magnitude is independent of (j,g), obliging the firm to pay D_t units of consumption in period t+1. If L is the constant average ratio of the firm's debt obligation to its total (post-consumption) market value, the firm's debt obligation D_t is thus:

$$D_t = \frac{L}{2t} * \sum_{j,g} [M_{t,j,g} - G_d^{t-j} G_u^j] \quad . \tag{39}$$

We divide the summation by 2t, since for each t there are 2t possible (j,g) events.

The introduction of leverage does not affect the recursive definitions of the total market value of the tree (firm) $M_{t,j,g,}$ as in (31) above, or of agents' wealth, as in (34) above. However, in accordance with the Modigliani-Miller proposition, it does effect the rate of return on equity. Given the assumed leverage policy, the rate of return on equity, following a triple, (t,j,g), will be given by:

$$\begin{aligned} R^g_{t+1,j,d} &= \frac{M_{t+1,j,d} - D_t}{M_{t,j,g} - G_d^{t-j} G_u^j - D_t /(1+r_{t,j,g})} - 1 \\ R^g_{t+1,j+1,u} &= \frac{M_{t+1,j+1,u} - D_t}{M_{t,j,g} - G_d^{t-j} G_u^j - D_t /(1+r_{t,j,g})} - 1 \quad . \end{aligned} \tag{40}$$

To obtain the impact of leverage on portfolio composition all we have to do is to substitute the terms $[M_{t+1,j,d}$ - Dt] and $[M_{t+1,j+1,u} - D_t]$ for the terms $M_{t+1,j,d}$ and $M_{t+1,j+1,u}$ respectively, in (35) and (36). In this case, the proportion of share holdings will be affected by whether it is an up or a down state.

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TABLE 1
SUMMARY OF SIMULATION RESULTS

	Mehra-Prescott Target	simulation result, both consumers	consumer 1 only	consumer 2 only
risk-free rate	0.80%	0.80%	-7.28%	-2.72%
expected market return	6.98%	6.98%	-6.60%	13.63%
standard deviation of market return	16.54%	18.75%	5.65%	18.99%

Note: The Table 1 column for "both consumers" gives the simulated model values when the market leverage equals 28.27% and when consumer 1 has holds 33.08% of the aggregate wealth. Consumer 1 has $\gamma_1 = 3$ and $\beta_1 = 1.12$; consumer 2 has $\gamma_2 = 36$ and $\beta_2 = 0.93$.

TABLE 2
STATE PRICES IN "UP" AND "DOWN" STATES

	q_{uu}	q_{ud}	q_{du}	q_{dd}
consumer 1 $\gamma = 3$, $\beta = 1.12$	0.4160	0.6783	0.5515	0.5117
consumer 2 $\gamma = 36$, $\beta = 0.93$	0.0690	1.0970	0.0915	0.8376

TABLE 3
MARKET RETURNS IN THE "DOWN" AND "UP" STATES
for each consumer separately

	consumer 1 only		consumer 2 only	
	"down" state	"up" state	"down" state	"up" state
r_f	-5.94%	-8.62%	8.80%	-14.24%
ER_m	-5.25%	-7.95%	27.75%	- 0.50%
$sigma_m$	5.77%	5.53%	21.10%	16.89%
market risk premium	0.69%	0.66%	18.95%	13.75%

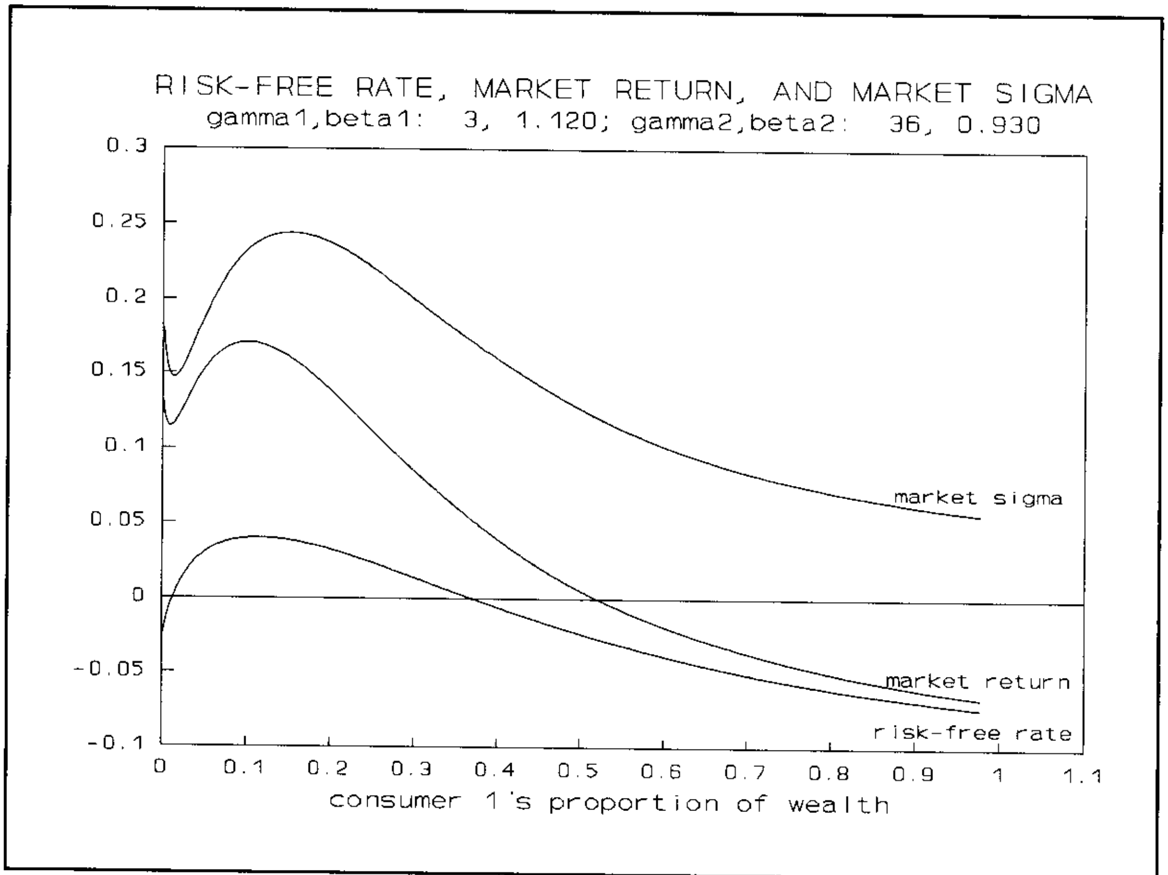

RISK-FREE RATE, MARKET RETURN, AND MARKET SIGMA
gamma1,beta1: 3, 1.120; gamma2,beta2: 36, 0.930
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
consumer 1's proportion of wealth
market sigma
market return
risk-free rate

Figure 1

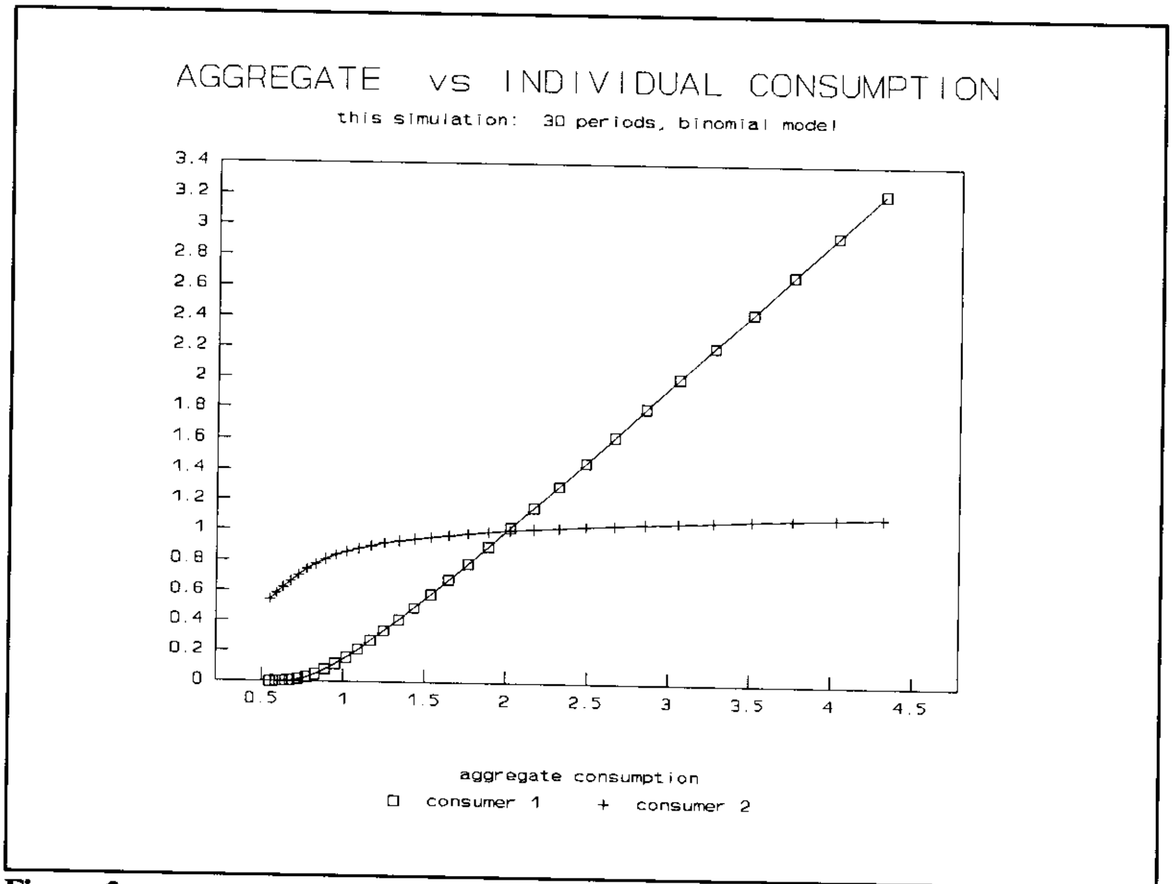
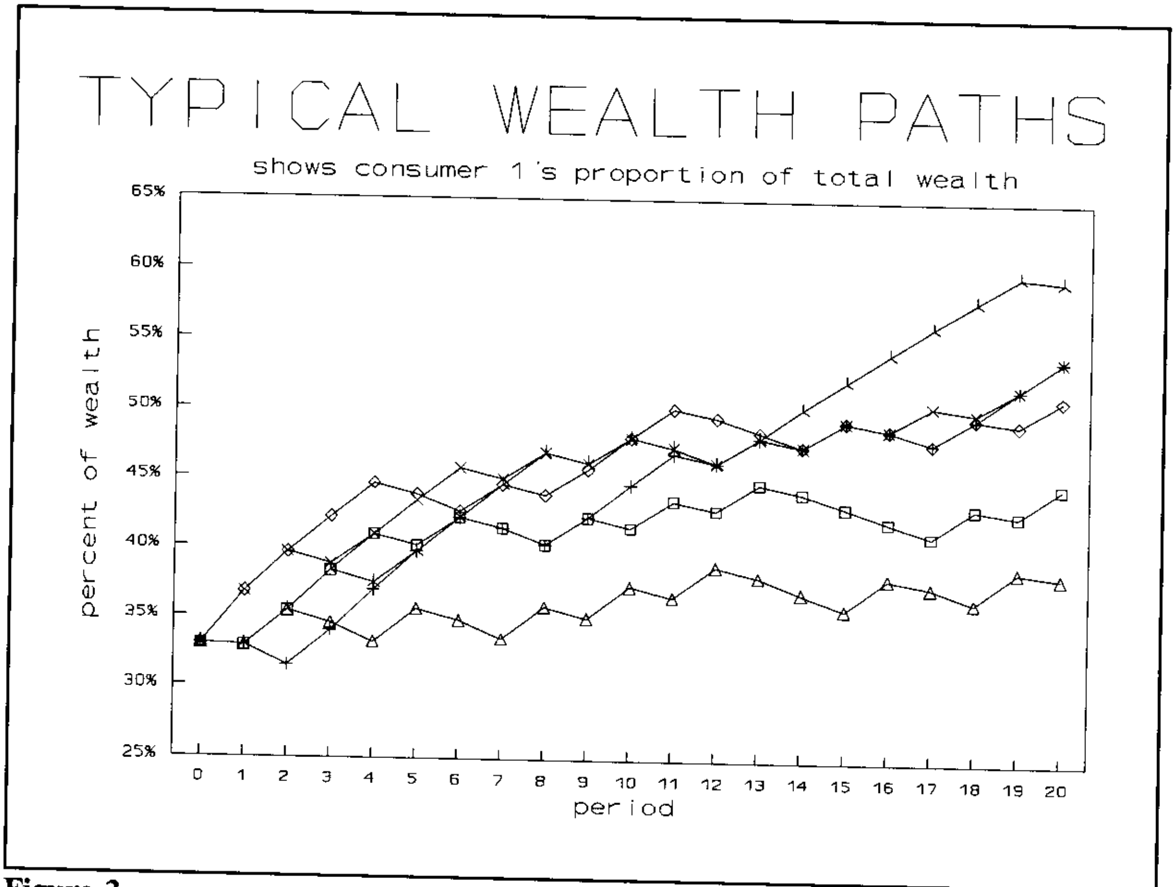

AGGREGATE vs INDIVIDUAL CONSUMPTION
this simulation: 30 periods, binomial model
3.4
3.2
3
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
aggregate consumption
□ consumer 1
+ consumer 2

Figure 2


TYPICAL WEALTH PATHS
shows consumer 1's proportion of total wealth
percent of wealth
65%
60%
55%
50%
45%
40%
35%
30%
25%
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
period
Figure 3

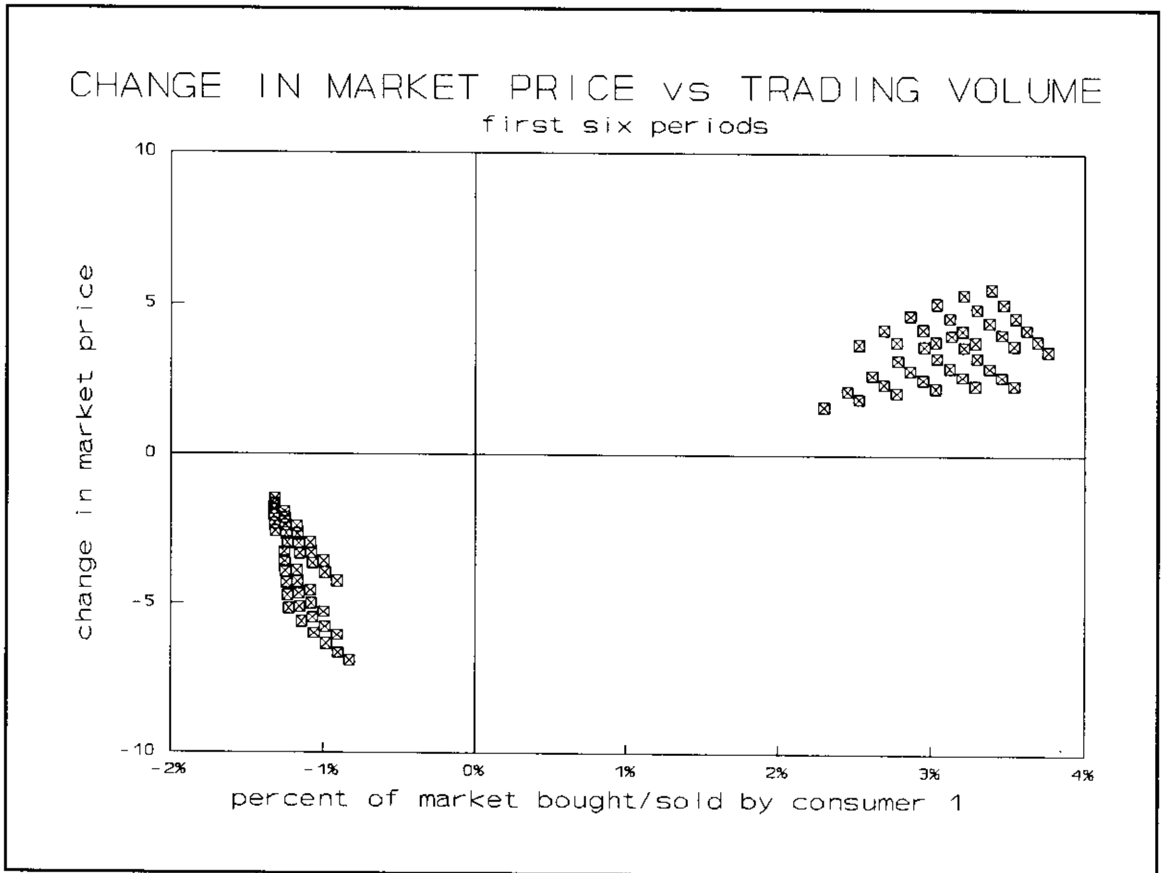

CHANGE IN MARKET PRICE vs TRADING VOLUME
first six periods
change in market price
10
5
0
-5
-10
-2%
-1%
0%
1%
2%
3%
4%
percent of market bought/sold by consumer 1

Figure 4