

**THE DYNAMICS OF INFORMATION
INCORPORATION INTO ASSET PRICES:
AN EMPIRICAL ANALYSIS**

by

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The Dynamics of Information Incorporation into Asset Prices: An Empirical Analysis

ABSTRACT

The incorporation of diverse information into asset prices is empirically examined in an actual securities market with multiple rounds of trade. Using prices of Israeli index and nominal bonds of equal maturity, we calculate implied expectations of inflation that has already occurred but for which the official statistic has not yet been announced. 'Learning' is defined as the convergence of these expectations to the actual level of inflation in the period after the end of the month but before the announcement of the official statistic. We find that the variance of the inflation expectation errors decreases with trading days in this period. The decline in the variance suggests that investors learn, by repeatedly observing prices, about the distribution of other investors' information. We also find a positive relation between the dispersion of relative price changes and the size of inflation-expectation errors on the first round of trade. The correlation diminishes as investors learn about the distribution of inflation information in the economy.

The Dynamics of Information Incorporation into Asset Prices: An Empirical Analysis

I. INTRODUCTION

This paper analyzes the dynamics of the incorporation of diverse information into asset prices. Specifically, we examine whether repeated rounds of trade are required for bond prices to reflect investors' inflation information or whether prices already reflect this information after one round of trading. This investigation is motivated by recent theoretical analyses of Noisy Rational Expectation Equilibria (NREE), where investors can repeatedly trade on the basis of the same fundamental information. In NREE models, investors are endowed with diverse private information about the value of an asset and (rationally) condition their valuation of the asset on its equilibrium price. Recent models show that second-round prices may be closer to the true value of the asset than first-round prices even though no new fundamental information is generated between the rounds.

In line with these models, we assume that, by the end of each month, investors have observed diverse signals about the inflation rate of that month. After the end of the month, but before the official inflation statistic of the previous month is announced, investors can repeatedly trade in various assets. In the first trading round, investors' information about last month's inflation is aggregated into asset prices, possibly in a noisy manner. In later rounds, investors' information is augmented by the latest prices. Although there is no specific theory for information aggregation in more than two trading rounds, we consider two hypotheses motivated by NREE models with two rounds of trade. According to the first hypothesis, prices in later rounds reflect no more information about the last month's inflation than do first-round prices. Consequently, investors do not learn from a sequence of prices about the distribution of inflation information in the economy more than they learn from first-round prices. We call this the 'no learning' hypothesis. According to the 'learning' hypothesis, which is also contained in the NREE framework, investors can use a sequence of prices to learn more about the distribution of inflation information than they learn from single-round prices. Hence, while no new fundamental information is generated between rounds of trade, prices in later rounds reflect more information about the last month's

inflation than do first-round prices. These hypotheses correspond to the 'no-change-in-expectation' and to the 'change-in-expectation' equilibria of Grundy and McNichols (1989) and Kraus and Smith (1989).¹

We use newly available data on prices of Israeli nominal and index bonds with equal maturity. The official monthly inflation statistic, to which index bond payments are linked, is announced about two weeks after the end of each month. Hence, the maturity payment of index bonds that mature shortly after announcement days are not adjusted for inflation that occurs in this two-week period. The prices of these index bonds do, however, reflect investors' expectations about last month's inflation rate, which has already occurred. We develop a method to extract implied aggregate expectations about last month's inflation from pairs of nominal and index bonds. The evolution of the implied inflation expectations over the trading days following the end of the month but before the announcement of the official inflation statistic allows us to examine the dynamics of information aggregation into asset prices.²

Within the NREE framework, with or without learning, the average expectation error is zero. Hence, the first moment of the inflation expectation errors cannot be used to distinguish between the two hypotheses. However, the two hypotheses have different implications for the evolution of the second moments of inflation expectation errors over trading days. Under the 'learning' hypothesis, prices in later rounds of trade reflect more information than prices in earlier rounds. Hence, under the 'learning' hypothesis, the variance of inflation expectation errors declines as more trading takes place. On the other hand, under the 'no learning' hypothesis the variances of inflation expectation errors on all trading days are the same.

¹ Other models of information aggregation into asset prices in repeated rounds of trade include Milgrom and Stokey (1982), Brown and Jennings (1989), Dow and Gorton (1990), and Blume, Easley, and O'Hara (1991).

² Most previous examinations of information aggregation into asset prices use data from simulated markets. (E.g., Forsythe and Lundholm 1990, Forsythe, Palfrey, and Plott 1984, and Copeland and Friedman 1987.) A notable exception is Huberman and Schwert (1985) who also use Israeli bond data. Their focus, however, is on the incorporation of inflation shocks into prices as they occur. Yariv (1989) also uses Israeli index bond data to examine the rationality of inflation expectations.

The two hypotheses also differ in their implications for the relation between inflation expectation errors and the dispersion of relative price changes. Since each investor observes only a subset of all prices in the economy, investors' signals about the last month's inflation are less accurate in months with high dispersion of relative price changes than in months with low dispersion of relative price changes. Therefore, over months, the dispersion of relative price changes and inflation expectation errors in the first round of trading are positively correlated. Under the 'learning' hypothesis, the covariance of relative price dispersion and inflation expectation errors declines with trading rounds. Under the 'no learning' hypothesis, the covariance of relative price dispersion and inflation expectation errors is the same in all trading rounds.

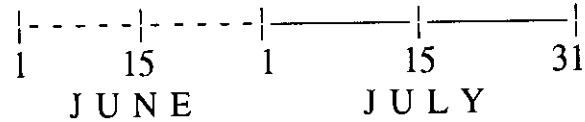
Note that, in our setting, a decline in the variance of inflation expectation errors and in their covariance with the dispersion of relative price changes can be due to learning only from prices observed in the period examined. This is because the price of each good in the preceding month has been observed by at least one agent in the economy. Hence, all information about the last month's inflation rate is contained in the union of the separate information sets of all investors. Assuming that investors are rational in processing information, the only learning that can take place in the two-week period examined is from newly observed prices, that is, learning which results from trading in this period. The prices used in this learning process, however, are not limited to prices of index bonds: investors can learn from prices observed while trading in both real and financial assets.

The paper is organized as follows. In Section II, we explain the estimation and the statistical testing of the two learning hypotheses. In Section III, we describe the data. The dynamics of inflation expectations is examined in Section IV. Section V concludes the paper.

II. EMPIRICAL IMPLICATIONS OF THE TWO LEARNING HYPOTHESES

On the last business day before the 16th day of each month, the Israeli Central Bureau of

Statistics (CBS) announces the Consumer Price Index (CPI) of the previous month. Consider the time-line illustrated below and an index bond that matures on July 31st.



The inflation adjustment of index bonds is based on inflation rates officially announced prior to their maturity. The last inflation rate announced before this bond matures is the June inflation rate announced on July 15th. Hence, while the bond is still traded in July, its maturity payment is adjusted only for inflation occurring up to June 30th. We analyze this bond's prices between July 1st and July 15th, the period preceding the announcement of the June index. In this period, the bond is no longer indexed and its price reflects the market's expectations of the June inflation rate, which has already occurred but has not yet been announced.

Usually, there are ten trading days in the period between the end of a month and the announcement of the official inflation rate. The Appendix describes how simultaneously observed nominal and index bond prices allow us to calculate implied inflation expectations for the preceding month. Let $\pi_{j,t}^p$, $j = 1, \dots, 10$ denote month t 's inflation expectations calculated from bond prices observed on trading day j (of month $t+1$), and π_t denote month t 's actual inflation rate. On trading day j , the inflation expectation error, $\delta_{j,t}$, is:

$$\delta_{j,t} = \pi_{j,t}^p - \pi_t. \quad (1)$$

NREE implies that the unconditional mean of δ_j is zero for all j under both learning hypotheses. Therefore, the mean error cannot be used to distinguish between these hypotheses. The second moments of the δ_j 's, however, can be used to differentiate between the hypotheses.

Under the 'learning' hypothesis, since prices in later rounds of trade reflect more information than prices in earlier rounds, the variance of the δ_j 's declines as more trading takes place. Moreover, because

investors do not forget information known in preceding days, we model the within-month dynamic change of the δ_j 's as an autoregressive process:³

$$\delta_{j,t} = \beta_j \cdot \delta_{j-1,t} + \eta_{j,t} \quad j = 2, \dots, 10, \quad (2)$$

where the η_j 's are the daily innovations in the expectation error process. NREE implies that the δ_j 's are independent across months and that

$$\text{VAR}(\delta_j) \equiv \sigma_{\delta_j}^2 \geq \sigma_{\delta_{j+1}}^2 \quad j = 1, \dots, 9. \quad (3)$$

A necessary condition for non-increasing variances is that the absolute values of the β_j 's are less than 1.

Under the 'no learning' hypothesis, implied inflation expectations do not converge to actual inflation rates. Yet, the δ_j 's may be affected by disturbances such as liquidity shocks, non-synchronous trading, recording errors, and price rounding. We assume that these disturbances are i.i.d. Therefore, the 'no learning' hypothesis, which is nested in the NREE hypothesis, further implies that:

$$\begin{aligned} \sigma_{\delta_j}^2 &= \sigma_{\delta_i}^2 & i, j &= 1, \dots, 10, \\ \beta_j &= 0 & j &= 2, \dots, 10. \end{aligned} \quad (4)$$

The 'learning' hypothesis is the complement of the 'no learning' hypothesis in NREE.

The two hypotheses also differ in their implications for the relation between inflation expectation errors and the dispersion of relative price changes. The monthly inflation rate is a weighted average of the changes in prices of goods included in the CPI basket according to predetermined weights. In any given month, each investor does not transact in all goods included in the CPI basket. Since investors observe only a subset of the prices in the economy, their signals about the last month's inflation are less

³ For the dependency of the second-round price on the first-round price, see, for example, Grundy and McNichols (1989) equation (30) and Kraus and Smith (1989) equation (14).

accurate in months with high dispersion of relative price changes than in months with low dispersion of relative price changes. NREE implies that less accurate individual signals about the last month's inflation, under both learning hypotheses, entail large inflation expectation errors.⁴ Therefore, under both learning hypotheses, across months

$$\text{COV}(\delta_i, D) > 0 \quad i = 1, \dots, 10, \quad (5)$$

where D_t is a measure of the dispersion of relative price changes in month t .

Under the 'learning' hypothesis, since inflation expectations converge to actual inflation rates, the covariance of the updated inflation expectation errors and D declines over trading days:

$$\text{COV}(\delta_i, D) \geq \text{COV}(\delta_j, D) \quad i < j, \quad (6)$$

with a strict inequality for some i and j . Under the 'no learning' hypothesis, the covariance is the same on all trading days.

The learning hypotheses are specified relative to the CBS measure of inflation, π , which is the average of all price changes in Israel according to known weights. The CBS estimates π by sampling prices at randomly selected locations. Therefore, the official inflation statistic, π^a , contains a sampling error, ζ , common to all observations in the same month:

$$\pi_t^a = \pi_t + \zeta_t. \quad (7)$$

We assume that ζ is a 'white noise'. Moreover, since the CBS's sampling error is not predictable, forecasting π^a is equivalent to forecasting π . We estimate the moments of the inflation expectation errors (δ_j 's) using two methods that account for the sampling error of the CBS.

⁴ See, for example, Grossman and Stiglitz (1980) and Diamond and Verrecchia (1981), for the effect of noise traders on prices as a function of the accuracy of investors' signal about the true value of the asset.

A. Errors Measured Relative to Official Inflation Rates

Let $\epsilon_{j,t}$ denote month t 's inflation expectation error on trading day j (in month $t+1$) relative to the announced inflation rate of month t , π_t^a (announced in month $t+1$):

$$\epsilon_{j,t} \equiv \pi_{j,t}^p - \pi_t^a = \delta_{j,t} - \zeta_t. \quad (8)$$

Let ϵ denote the vector of $\epsilon_{j,t}$'s ordered first by month (t) and then by day (j). Given our assumptions about the ζ 's and the δ 's, the variance-covariance matrix of ϵ is given by:

$$\mathbf{E}(\epsilon\epsilon') = \Sigma \otimes \mathbf{I}, \quad (9)$$

where Σ is the 'within month' covariance matrix of the ϵ 's (defined below according to the two learning hypotheses), \mathbf{I} is a $T \times T$ identity matrix, and T is the number of months in our sample.

Under the 'learning' hypothesis, the δ_j 's form an autoregressive process, the variance of which decreases over trading days. Therefore, under the 'learning' hypothesis, the elements of Σ are given by:

$$\begin{aligned} \Sigma_{j,j} &= \sigma_{\delta_j}^2 + \sigma_{\zeta}^2 \\ \Sigma_{j,k} &= \left(\prod_{i=j+1}^k \beta_i \right) \sigma_{\delta_j}^2 + \sigma_{\zeta}^2 \quad j < k, \end{aligned} \quad (10)$$

where β_j , $j = 2, \dots, 10$, is the autoregressive coefficient of day j and σ_{ζ}^2 denotes the variance of ζ .

Under the 'no learning' hypothesis, the δ 's also form an autoregressive process, albeit a degenerate one ($\beta_j = 0$, $j = 2, \dots, 10$), and the variances of the δ 's are constant:

$$\begin{aligned} \Sigma_{j,j} &= \sigma_{\delta}^2 + \sigma_{\zeta}^2 \\ \Sigma_{j,k} &= \sigma_{\zeta}^2, \quad j \neq k. \end{aligned} \quad (11)$$

We employ a maximum likelihood (ML) procedure that explicitly accounts for the above covariance structure and assumes that the δ 's and ζ are normally distributed. We simultaneously estimate the autoregressive coefficients and the standard deviations of the δ_j 's and ζ . Likelihood ratio tests are

used to test the predictions of the 'no learning' hypothesis versus the 'learning' hypothesis.

The implications of the two learning hypotheses regarding the relation between the dispersion of relative price changes, the D's, and inflation expectation errors, the δ_j 's, carry over to the relation between the D's and the ϵ_j 's. Specifically, under the 'learning' hypothesis:

$$\begin{aligned} \text{COV}(\epsilon_j, D) &= \text{COV}(\delta_j, D) - \text{COV}(\zeta, D) \\ &> \text{COV}(\delta_{j+1}, D) - \text{COV}(\zeta, D) = \text{COV}(\epsilon_{j+1}, D). \end{aligned} \quad (12)$$

Under the 'no learning' hypothesis, since $\text{COV}(\delta_j, D)$ is the same for all j :

$$\text{COV}(\epsilon_j, D) = \text{COV}(\epsilon_i, D) \quad i, j = 1, \dots, 10. \quad (13)$$

B. Errors Measured Relative to the Tenth Day's Implied Inflation Expectations

If learning takes place, implied inflation expectations will eventually converge to actual inflation rates. In our setting, inflation expectations imputed from bond prices observed on the tenth trading day are the most accurate inflation expectations that investors can form before the official inflation rate is announced. Subtracting the tenth day's imputed inflation expectation from the imputed expectations of the first through the ninth days in this month, we get:

$$\omega_{j,t} \equiv \pi_{j,t}^P - \pi_{10,t}^P = \delta_{j,t} - \delta_{10,t}, \quad j = 1, \dots, 9. \quad (14)$$

Ostensibly, in this specification of the model, the CBS sampling error (ζ) is replaced by the error remaining after ten days of trading (δ_{10}).

Let ω denote the vector of $\omega_{j,t}$'s ordered first by month (t) and then by day (j). For the reasons given in Section II.A, the variance-covariance matrix of ω is given by:

$$E(\omega \omega') = \Omega \otimes I, \quad (15)$$

where Ω is now the 'within month' covariance matrix of the ω 's. Under the 'learning' hypothesis, since within each month the δ_j 's form an autoregressive process, the elements of Ω are given by:

$$\Omega_{j,j} = \text{VAR}(\delta_j - \delta_{10}) = \sigma_{\delta_j}^2 - 2\left(\prod_{k=j+1}^{10} \beta_k\right)\sigma_{\delta_j}^2 + \sigma_{\delta_{10}}^2, \quad (16)$$

$$\Omega_{j,k} = \left(\prod_{i=j+1}^k \beta_i\right)\sigma_{\delta_j}^2 - \left(\prod_{i=j+1}^{10} \beta_i\right)\sigma_{\delta_j}^2 - \left(\prod_{i=k+1}^{10} \beta_i\right)\sigma_{\delta_k}^2 + \sigma_{\delta_{10}}^2 \quad j < k.$$

Under 'no learning', Ω can be obtained by using zero β 's and constant σ 's in equation (16):

$$\begin{aligned} \Omega_{j,j} &= 2 \cdot \sigma_{\delta}^2, & j &= 1, \dots, 9. \\ \Omega_{j,k} &= \sigma_{\delta}^2, & j &\neq k. \end{aligned} \quad (17)$$

We estimate the model's parameters of the δ 's with a ML procedure similar to that described in subsection II.A, and use similar likelihood ratio tests to test the 'learning' hypotheses with this specification of the model.

Measuring inflation expectation errors relative to the tenth day's errors provides us with sharper predictions regarding the evolution of the covariance between inflation expectation errors, δ_j 's, and the dispersion of relative price changes, D . Specifically, under the 'no learning' hypothesis:

$$\text{COV}(\omega_j, D) = \text{COV}(\delta_j, D) - \text{COV}(\delta_{10}, D) = 0. \quad (18)$$

On the other hand, under the 'learning' hypothesis:

$$\begin{aligned} \text{COV}(\omega_j, D) &= \text{COV}(\delta_j, D) - \text{COV}(\delta_{10}, D) \\ &> \text{COV}(\delta_{j+1}, D) - \text{COV}(\delta_{10}, D) = \text{COV}(\omega_{j+1}, D). \end{aligned} \quad (19)$$

IV. DATA

We use prices of index and nominal bonds which the Israeli government has been issuing simultaneously since 1984. These bonds are actively traded by both individual and institutional investors on the Tel-Aviv Stock Exchange and represent about two thirds of the market value of all securities listed on the exchange and about half of the volume of trade. In December 1989, the aggregate market value

of the index bonds was \$20.8 billion and the market value of the nominal bonds was \$0.9 billion.

Upon issuance, the index bonds have maturities ranging from 6 to 20 years. The indexation rate of these bonds ranges from 80% to 100%. Coupon rates range from 2% to 7%. In our sample, six bonds have an indexation rate of 100%, six bonds have an indexation rate of 90%, and the rest have an indexation rate of 80%; coupon rates were 4% for seventeen bonds, 6% for two bonds, and 7% for the rest of the bonds. The period during which the bonds in our sample were effectively nominal ranges from 28 days to 42 days.

Nominal bonds have, upon issuance, maturities ranging from one month to one year. These bonds promise a single payment of par at maturity and, therefore, prior to maturity are traded at discount. In their characteristics and trading mechanism, Israeli nominal bonds are very similar to US Treasury bills.

The prices of the index and the nominal bonds are taken from the official publications of the Tel-Aviv Stock Exchange. The CBS estimates of the inflation rates and their components are taken from the CBS publications. We consider the 39 months in the period October 1984 through May 1990 in which there were ten trading days prior to the announcement of the CPI. Our measure of the relative price change dispersion, D , is the weighted average of the squared deviations of the ten CPI components relative to the monthly inflation rate, using the weights of the components in the CPI.

V. RESULTS

To help assess the difference between the two specifications of the estimation process, we plot, in Figure 1, the CBS's statistics and the inflation expectations imputed from the tenth trading day's prices for the 39 months used in this study. The first five months of the sample are from a period of high inflation, before the Israeli government adopted an anti-inflationary program in July 1985. In this period, the monthly inflation rate averaged 15.4% with a standard deviation of 7.2%. In the low inflation period

(i.e., the remaining 34 months), the monthly inflation rate averaged 1.5% with a standard deviation of 1.1%. As can be seen in the figure, the two variables are highly correlated even in the low-inflation sub-period (the correlation coefficient is 0.49).

The constraints on the parameters of the model imposed by either the hypothesis of rational inflation expectations or the 'learning' hypothesis include inequality constraints. This means that the customary χ^2 distribution of the Likelihood Ratio Test (LRT) statistic cannot be applied, since the number of degrees of freedom is not well defined. Tests for inequality constraints have been developed for some special cases, but the distributions of the test statistics are difficult to compute.⁵ Therefore, we base our inference on an empirical distribution derived from simulated data. For each test, we generate 300 sets of 39x10 random errors, the distribution of which is based on the ML estimates of the parameters under the null hypothesis. For each simulated data set we calculate the maximum values of the log-likelihood function both under the null and the alternative hypotheses as well as the usual LRT statistic. The empirical distribution of the resulting 300 statistics is the simulated small-sample distribution of the statistic under the null hypothesis.

We first test the NREE hypothesis, which asserts that, with or without learning, $E(\delta_j) = 0$ and σ_j^2 's are non-increasing, implying that, for all j 's, $|\beta_j| < 1$. The alternative hypothesis imposes no restrictions on the model's parameters. Tables 1 and 2 report the unrestricted and restricted ML estimates of the parameters for the two estimation specifications and the respective values of the log-likelihood function.⁶ The test statistics for the NREE hypothesis, which are twice the difference between the log-likelihood values in Tables 1 and 2, are 11.362 when the model is estimated relative to π^a , and 7.074 when the π_{10}^p specification is used. The corresponding p-values, which are based on the simulated

⁵ See, for example, Kudo (1963), Kodde and Palm (1986), and Wolak (1989).

⁶ In the second specification of the model, only the differences between the $E(\delta_j)$'s and $E(\delta_{10})$ can be estimated. These differences are not exhibited in Table 1 as they can be easily calculated from the daily mean prediction errors reported in column 1 of Table 1.

distributions of the statistic under the null hypothesis, are 0.81 and 0.89, respectively.

Based on these results, we conclude that inflation expectations on all trading days in the period examined comply with restrictions imposed by the NREE hypothesis. Specifically, estimated mean expectation errors are insignificantly different from zero and the variances are non-increasing over trading days. The estimates reported in Tables 1 and 2 further indicate that the autoregressive coefficients vary across trading days and that there is a marked decrease in the standard deviations of the inflation errors as more trading takes place. To facilitate comparison of the σ_δ 's estimated under the NREE hypothesis using the two specifications of the model, Figure 2 plots the two series of estimates together. The observed decline in the estimated variances appears to be inconsistent with 'no learning.' We formally test this hypothesis under the assumption that investor expectations are rational.

Recall that, under the 'no learning' hypothesis, the variances of the δ_j 's are constant over trading days and that the β_j 's equal zero. The estimated constant standard deviation of the δ_j 's under this hypothesis is 0.651% in both specifications of the estimation process. The values of the log-likelihood function under these restrictions are -95.277 and -69.876, respectively. The resulting LRT statistics relative to the alternative of an NREE, which is the union of the two learning hypotheses, are 256.386 and 263.986, respectively. Based on the simulated distribution of the statistic under the null of 'no learning,' the null can be rejected at significance levels of less than 1/3%⁷.

Next we test the learning hypotheses by examining the evolution over trading days of the relation between inflation expectation errors and the dispersion of relative price changes. In Table 3 we report the covariances and the correlation coefficients of the ϵ 's and of the ω 's with D. We also report the probability of observing correlations of at least this magnitude under the hypothesis of zero correlation.⁸

⁷ Since we use a simulated distribution based on 300 simulations, this corresponds to an estimated p-value of no more than 1/300.

⁸ Under the null hypothesis that the two variables are uncorrelated, $(n-2)\hat{\rho}/(1-\hat{\rho}^2)$ has a t distribution with n-2 degrees of freedom, where $\hat{\rho}$ is the estimated correlation coefficient.

Consistent with an NREE, ϵ_t is positively correlated with D . The 'no learning' hypothesis, under which the covariance of the δ 's does not change as more trading takes place, further implies that the ω 's are uncorrelated with D (equation 18). The hypothesis that the ω 's and D are uncorrelated can be rejected for trading days one through five at commonly used significance levels, indicating that the 'learning' hypothesis fits the data better than the 'no learning' hypothesis.⁹

In Figure 3 we plot the estimated covariances of the ϵ 's and of the ω 's with D . The decline in the covariances is apparent. We use a non-parametric sign test to verify that the covariances of different trading days are decreasing. For errors measured relative to π^a , 39 of the 45 possible comparisons of covariances are in the direction predicted by the 'learning' hypothesis. The p-value derived from a Binomial distribution $B(45, 1/2)$ is smaller than 10^{-4} . When measured relative to π_{10}^a , 34 of the 36 possible comparisons fit the 'learning' hypothesis. The associated p-value is again smaller than 10^{-4} .

In sum, the evolution of the variances and the covariances of inflation expectation errors over trading days supports the 'learning' hypothesis, namely that investors learn from a sequence of prices about the distribution of inflation expectations in the economy more than they learn from single-round prices.

The figures of the evolution of the variances and the covariances of inflation expectation errors over trading days suggest that learning is most pronounced in the first two days. To test whether learning is limited to the first two days we estimate the Σ matrix specified in equation (10) under the restriction that no 'learning' occurs after day two. The value of the log-likelihood function under this restriction is -59.939, yielding an LRT statistic relative to the hypothesis that learning occurs over the entire ten-day period of 185.71 with a p-value of less than $1/3\%$. Similarly, testing whether the covariances of the δ 's with D stop declining after day two yields a p-value, from a $B(36, 1/2)$ distribution, of less than 10^{-4} .

⁹ The correlations of the ϵ 's and the ω 's with D increase to more than 0.8 when both variables are deflated by π . Therefore, this rejection is not due to differences in the levels of inflation over time.

Thus, we can reject the conjecture that learning ceases after trading day two.

The preceding analysis is carried out under the assumption that there is a zero risk premium associated with uncertainty about inflation that has already occurred. We formally check this supposition and find that it cannot be rejected and that even with risk premia included, the hypothesis of 'no learning' is not supported by the data. Further diagnostic checks also suggest that our results do not depend on the assumptions of no tax on interest income, or of no transaction costs, or that under 'no learning' the δ_j 's are uncorrelated within a month, or that the learning process is the same in periods of high and of low inflation.

It should be noted that our results can also be explained by phenomena outside the NREE framework such as lags in information collection and processing at the investor level. In Israel, however, large organizations (e.g., banks, brokerage houses) routinely estimate the inflation rate and actively trade bonds. These organizations, for various institutional reasons, need accurate inflation estimates before the end of each month. Consequently, it is unlikely that lags in information processing drive these results. Another possible explanation outside the NREE framework is that agents start with expectations that are not necessarily rational, but equilibrium converges to an NREE as the agents learn. See, for example, Bray (1982) and Marcet and Sargent (1988).

VII. CONCLUSIONS

In this study we empirically examine the incorporation of diverse information into asset prices in an actual securities market where investors are allowed to repeatedly trade on the basis of the same fundamental information. Specifically, we use prices of Israeli index and nominal bonds of equal maturity to calculate implied expectations of inflation that has already occurred but for which the official statistic has not yet been announced.

We analyze prices of index bonds observed after the end of each month but before the official

announcement of that month's inflation rate. In this period, index bond prices reflect investors' private information about the last month's inflation, updated by prices observed in the current month. If investors learn from these prices about the distribution of inflation information in the economy more than they know in the first round of trading, inflation expectations impounded in index bond prices will converge to the actual inflation rate. This prediction, which we call the 'learning' hypothesis, implies that the variance of inflation expectation errors and the covariance between these errors and a measure of investor uncertainty regarding last month's inflation decrease with trading days.

We find that both the variance and the covariance decline with trading days. Thus, even when all inflation information is contained in the union of all investors' information sets before the first round, investors learn from a sequence of prices. Furthermore, while most of the learning seems to occur within the first two trading days, our results indicate that the learning process continues throughout the ten trading days preceding the announcement of the official inflation statistics.

TABLE 1

ESTIMATED MEANS, STANDARD DEVIATIONS, AND PERSISTENCE COEFFICIENTS
OF INFLATION EXPECTATION ERRORS;

Unrestricted Model^a

Process Estimated Relative to:	Official Inflation Statistic			Tenth Day's Implied Inflation	
	Mean Error (%)	Standard Deviation (%)	Auto- regressive Coefficient	Standard Deviation (%)	Auto- regressive Coefficient
Day					
1	0.034	1.490		1.607	
2	0.115	0.874	0.528	1.080	0.602
3	0.213	0.936	0.889	1.073	0.872
4	0.225	0.872	0.850	1.084	0.946
5	0.220	0.888	0.8567	1.026	0.843
6	0.274	0.817	0.737	0.711	0.601
7	0.211	0.852	0.857	0.845	0.960
8	0.292	0.991	0.948	1.066	1.062
9	0.385	0.732	0.635	0.926	0.758
10	0.356	0.731	0.815	0.714	0.682
Value of Log-Likelihood function:			38.597		65.571

^a The model estimated is:

$$(\delta_{t,j} - \mu_j) = \beta_j \cdot (\delta_{t,j-1} - \mu_j) + \eta_{t,j} \quad j = 2, \dots, 10$$

where $\delta_{t,j}$ is the inflation expectation error for the j 'th trading day in month t . The model is estimated either relative to the announced inflation rate or relative to the tenth day's implied inflation rate. The model's parameters are the autoregressive coefficients (β_j 's) for each trading day $j=2, \dots, 10$, the means (μ_j 's) for each trading day $j=1, \dots, 10$, the standard deviations of the δ 's (σ_j 's), and the standard deviation of the CBS error (σ_ϵ). The model is estimated using a maximum likelihood procedure with 39 months and ten trading days in each month.

TABLE 2

ESTIMATED STANDARD DEVIATIONS AND PERSISTENCE COEFFICIENTS
OF INFLATION EXPECTATION ERRORS;

Noisy Rational Expectations Equilibrium^a

Process Estimated Relative to:	Official Inflation Statistic		Tenth Day's Implied Inflation	
	Standard Deviation (%)	Auto- regressive Coefficient	Standard Deviation (%)	Auto- regressive Coefficient
Day				
1	1.537		1.532	
2	0.936	0.546	1.047	0.595
3	0.936	0.833	1.047	0.867
4	0.897	0.874	1.042	0.927
5	0.897	0.846	0.977	0.827
6	0.819	0.733	0.743	0.636
7	0.819	0.809	0.743	0.770
8	0.819	0.747	0.743	0.694
9	0.628	0.598	0.605	0.584
10	0.628	0.752	0.167	0.276

Value of Log-Likelihood function:	32.916	62.034
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^a The model estimated is:

$$\delta_{t,j} = \beta_j \cdot \delta_{t,j-1} + \eta_{t,j} \quad j = 2, \dots, 10$$

where $\delta_{t,j}$ is the inflation expectation error for the j 'th trading day in month t . The model is estimated either relative to the announced inflation rate or relative to the tenth day's implied inflation rate. The model's parameters are the autoregressive coefficients (β_j 's) for each trading day $j=2, \dots, 10$, the standard deviations of the δ 's (σ_j 's), and the standard deviation of the CBS error (σ_τ). The model is estimated using a maximum likelihood procedure with 39 months and ten trading days in each month under the restriction that $E(\delta_j) = 0$ and that σ_j^2 decreases in j .

TABLE 3

THE RELATION BETWEEN INFLATION EXPECTATION ERRORS AND
RELATIVE PRICE CHANGE DISPERSION ON DIFFERENT TRADING DAYS¹

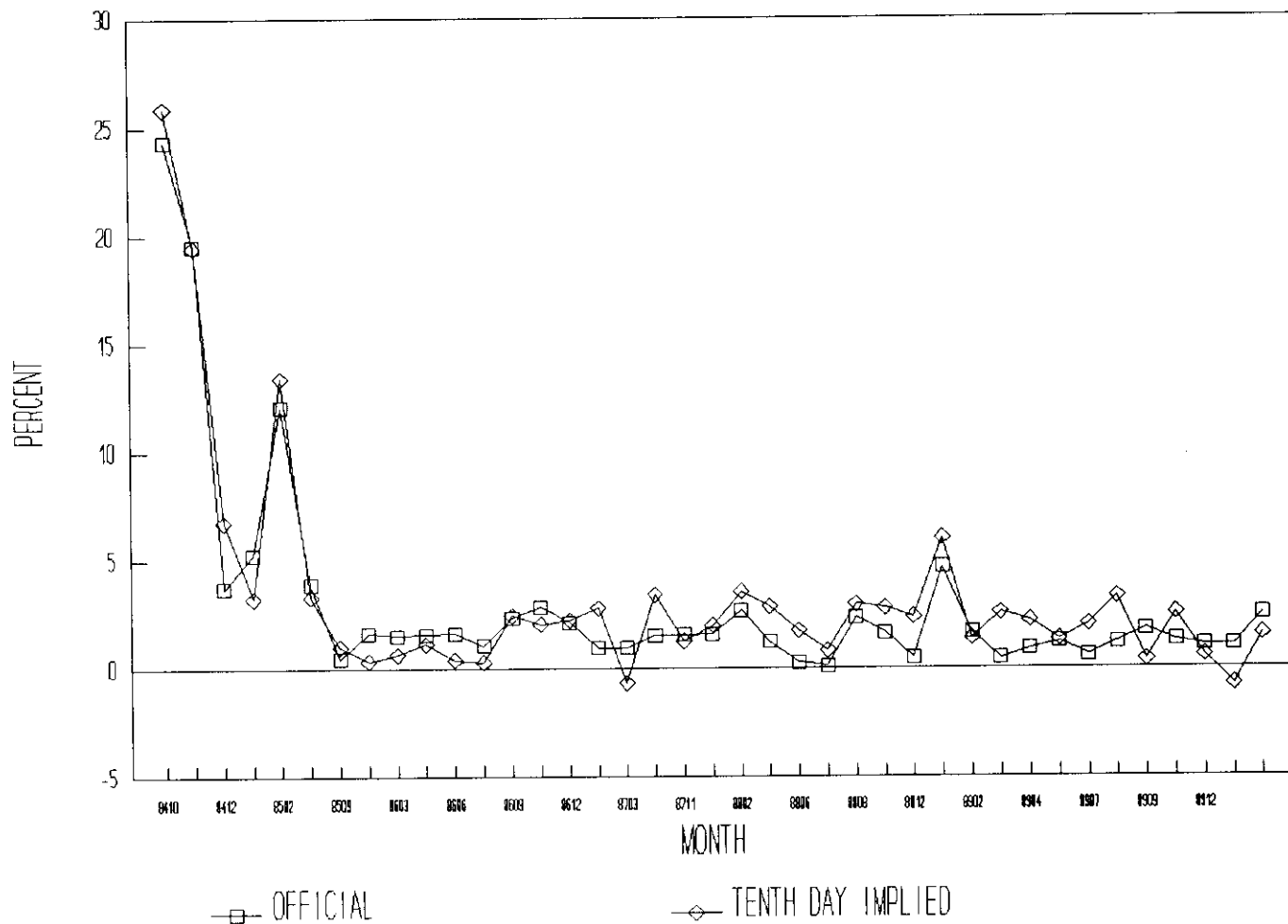
<u>Day</u>	Process Estimated Relative to:	Official Inflation Statistic		Tenth Day's Implied Inflation	
		Corr ²	Cov ³	Corr ²	Cov ³
1		0.381 (0.017)	0.547	0.591 (0.001)	0.996
2		0.250 (0.124)	0.212	0.599 (0.001)	0.548
3		0.243 (0.137)	0.256	0.506 (0.001)	0.480
4		0.198 (0.227)	0.208	0.470 (0.003)	0.446
5		0.111 (0.503)	0.105	0.328 (0.041)	0.303
6		-0.016 (0.925)	-0.018	0.102 (0.535)	0.070
7		0.036 (0.828)	0.033	0.095 (0.565)	0.061
8		-0.106 (0.520)	-0.105	0.172 (0.296)	0.112
9		0.026 (0.874)	0.023	0.017 (0.916)	0.009
10		-0.068 (0.683)	-0.061		

¹ Relative price change dispersion is measured by the weighted average of the squared deviations of price changes of the ten components of the CPI relative to that month's inflation. The weights are the weights of the components in the CPI.

² 'Corr' denotes the correlation coefficient between inflation expectation errors and relative price change dispersion. Numbers in parentheses are p-values for the test that the correlation coefficients are zero. P-values are calculated using the T_{n-2} distribution for the statistic $(n-2)\rho/(1-\rho^2)$, where ρ is the estimated correlation coefficient.

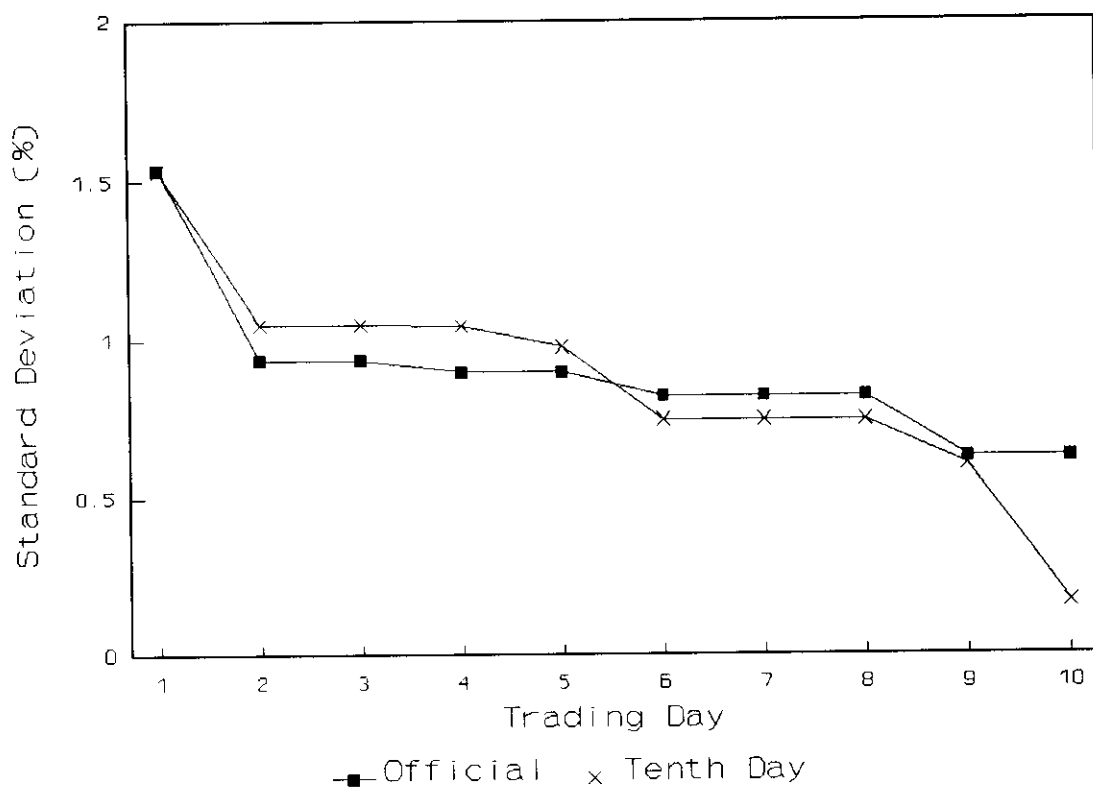
³ 'Cov' denotes the covariance between inflation expectation errors and relative price change dispersion.

Figure 1: Monthly inflation rates as estimated by the Central Bureau of Statistics and as implied from index and nominal bond prices.



The "official" rate is announced by the Central Bureau of Statistics (CBS) about two weeks after each month-end. The "tenth-day-implied" rate is implied by the relative pricing of index and nominal bonds on the tenth day of trading after the end of the month but before the announcement of the CBS.

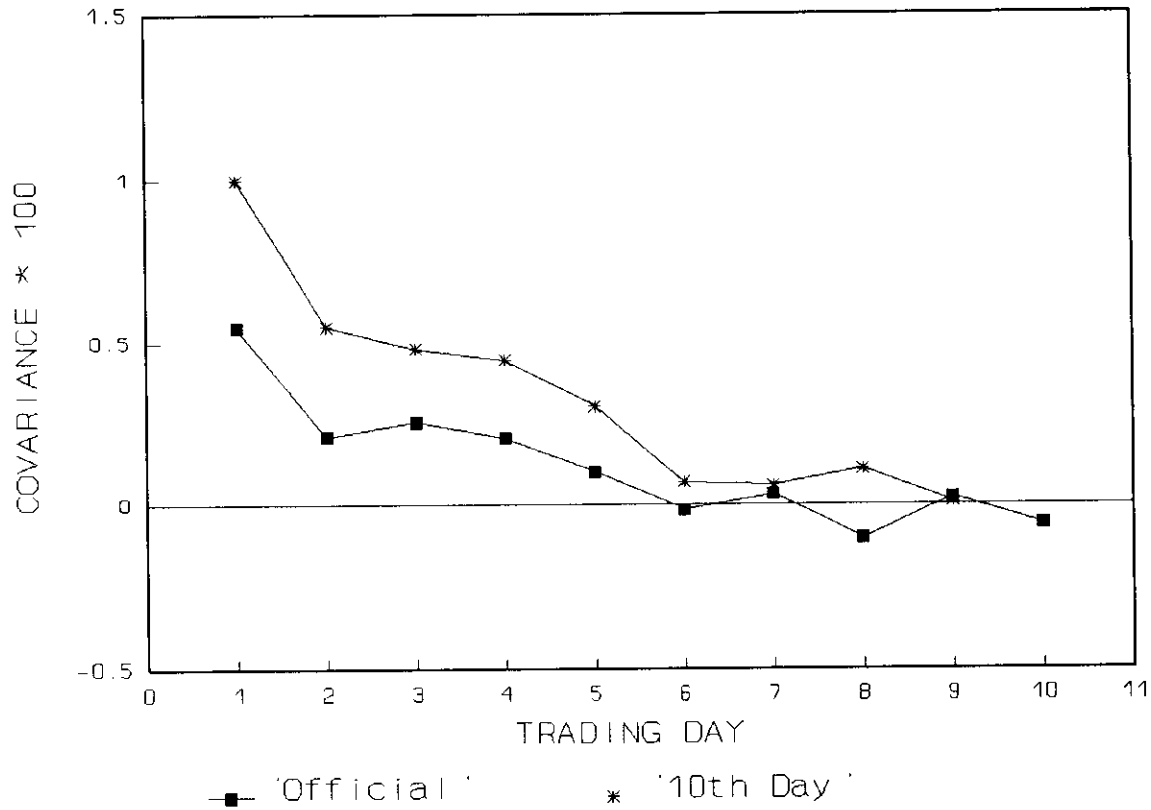
Figure 2: Estimated standard deviations of inflation expectation errors on trading days following the end of a month but preceding the announcement of the official inflation statistic. All estimates use 39 months of observations between October 84 and May 90.



The "Official" line depicts the maximum likelihood estimates of the standard deviations of inflation expectation errors relative to the CBS's announced inflation rate under the NREE hypothesis.

The "Tenth day" line depicts the maximum likelihood estimates of the standard deviations of inflation expectation errors relative to the inflation expectations imputed from bond prices on the tenth trading day after the end of the month but before the announcement of the CBS under the NREE hypothesis.

Figure 3: Estimated covariances between the dispersion of relative price changes and inflation expectation errors on trading days following the end of a month but preceding the announcement of the official inflation statistic. All estimates use 39 months of observations between October 84 and May 90.



The "Official" line depicts the estimates of the covariances of inflation expectation errors relative to the CBS's announced inflation rate with the dispersion of relative price changes.

The "10th Day" line depicts the estimates of the covariances of inflation expectation errors relative to the inflation expectations imputed from bond prices on the tenth trading day after the end of the month but before the announcement of the CBS with the dispersion of relative price changes.

APPENDIX: CALCULATION OF IMPLIED INFLATION EXPECTATIONS

We refer throughout this exposition to the time line illustrated in Section II. The maturity payoff of an index bond consists of a par value, normalized at 100, plus the last annual coupon. All buyers of index bonds are taxed at the rate of 35% on the interest payments prorated to the number of months between the last coupon payment and their point of purchase.¹⁰ Hence, the marginal investor's tax rate applies only to 1/12 of the coupon payments of the bonds in our sample, and, therefore, is expected to have only a small effect on our results. Indeed, we find that our analysis is insensitive to the assumed marginal tax rate and, hence, assume that it is zero.

Let α denote the indexation rate of the bond. The after tax nominal and indexed components of the payment are respectively:

$$CF_{\text{ind}} = 100 \cdot \alpha + \text{coupon} \cdot 100 \cdot \alpha \cdot (1 - 0.35 \cdot 11/12)$$

The inflation adjustment is calculated as the ratio of the last index known on payment day to the base index, I_{Base} , which is the index of the issue month. We examine bond prices on trading days in the period of July 1st through July 15th, i.e., before the June index is announced. In this period, this bond prices reflect investor expectations about June's inflation rate, which has already occurred. Let π_j^p denote day j 's expectations of the CBS statistic of the June inflation measured relative to the already announced May index, I_{May} . Then:

$$CF_j = CF_{\text{nom}} + CF_{\text{ind}} \cdot \left(\frac{I_{\text{May}}}{I_{\text{Base}}} \right) \cdot (1 + \pi_j^p).$$

where CF_j is day j 's expectation of the maturity payment of the index bond.

The index bond is not linked to inflation after June 30th. Therefore, in the period examined, the index bond's nominal expected return equals the yield-to-maturity of a nominal bond with an identical maturity day, possibly adjusted for a risk premium. Note that any risk premium is for uncertainty which pertains to events that have already

¹⁰ This provision of the tax code is intended to prevent interest-income shifting to low tax investors just before coupon payments.

taken place. We test for the existence of such risk premia and find that they are not significantly different from zero. Moreover, our main results remain virtually the same when non-zero risk premia are included in the estimation. Hence, our analysis is presented under the assumption that there is no premium for this type of uncertainty.

Let n_j denote day j 's yield-to-maturity of an equal-maturity nominal bond.¹¹ The price of the index bond, denoted by P_j , on that day is:

$$P_j = \frac{CF_{\text{nom}} + CF_{\text{ind}} \cdot \left(\frac{I_{\text{May}}}{I_{\text{Base}}} \right) \cdot (1 + \pi_j^P)}{1 + n_j}$$

Observing the price of the index bond, P_j , and the nominal interest rate n_j , this equation allows us to calculate the expected inflation rate of the preceding month, π_j^P , for each trading day j in the period examined.

¹¹ In our empirical work, when an equal-maturity nominal bond does not exist, we interpolate the yields of two nominal bonds with maturities surrounding the maturity of the index bond. The difference in maturities between the closest-in-maturity nominal bond and the index bond is always less than 3 days.

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