A DIRECT TEST OF THE MIXTURE OF DISTRIBUTIONS HYPOTHESIS: MEASURING THE INFORMATION FLOW THROUGHOUT THE DAY

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A Direct Test of the Mixture of Distributions Hypothesis: Measuring the Information Flow Throughout the Day

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Abstract

This paper proposes and conducts direct tests of the mixture of distributions model for stock prices. By exploiting the model's bivariate conditional normality of price changes and trading volume, these restrictions can be tested under very weak assumptions regarding the daily flow of information to the market. As a technical byproduct, important parameters governing the distribution of this unobservable information flow are estimated.

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1 Introduction

There has been a recent surge in papers relating information flow to stock prices and volume. On the theory side, Tauchen and Pitts (1983), Admati and Pfleiderer (1988) and Foster and Viswanathan (1990a) directly relate moments of price changes and volume to the amount of information arriving to the market. With the availability of volume data and transactions data, several empirical papers investigate the statistical relationship between price changes and volume and find support for these models.[1] Alternatively, theories have been developed which predict volume and price changes in the absence of information arriving to the market, most notably Brock and Kleidon (1991). Gerety and Mulherin (1991) and Harris, McInish and Chakravarty (1991) provide some stylized evidence in favor of this model. In this paper, we further investigate the relationship between prices, volume and information flow in the context of a well-known financial theory, the mixture of distributions model (MODM).

MODM implies restrictions on security prices by assuming the flow of information about securities occurs at a random rate through time. This theory is appealing to financial economists because it explicitly models the impact information has on prices and volume. More recently, the popular ARCH/GARCH models have been represented in terms of mixture of distribution models. For example, Nelson (1990) shows that the discrete time version of continuous-time exponential ARCH models (see Nelson (1989)) can be written in terms of Clark's (1973) MODM. In addition to MODM's aesthetic appeal, Clark (1973), Harris (1986), and Tauchen and Pitts (1983), among others, show that empirical implications of the model are consistent with daily data on securities. Harris (1987) extends these implications to transactions data and presents additional evidence supporting MODM.

Although the existing evidence is heavily in favor of MODM, the results are for the most part anecdotal. That is, the type of distributional patterns generated from daily data appear consistent with those expected from a mixed distributional model. For example, Tauchen and Pitts (1983) perform maximum likelihood estimation of the model and find that the parameter estimates have the correct signs and are of reasonable magnitude. Harris (1986,1987) reports evidence confirming the model's

[1]See, for example, Harris (1986a), Jain and Joh (1988) Foster and Viswananthan (1990b) and Gallant, Rossi and Tauchen (1991), to name a few.

predictions regarding conditional and unconditional moments of price changes and trading volume. Nevertheless, little in the form of direct tests of MODM have taken place. Part of the reason for this is that the flow of information is unobservable, thus making direct tests of the theory nontrivial. In addition, the model's implied heteroskedasticity and autocorrelation properties of price changes further complicates matters.

This paper proposes a direct test of MODM. Because the model imposes restrictions on the joint moments of price changes and volume as a function of only a few parameters, it is possible to form overidentifying restrictions on the data. These restrictions can then be tested using the generalized method of moments (GMM) procedure of Hansen (1982). Perhaps even more important, characteristics of the distribution of the random rate of flow of information can be estimated. These estimates provide information concerning firm specific as well as more general market microstructure issues. This is especially interesting because the flow of information is completely unobservable to the econometrician and cannot therefore be studied directly.

As an application, we conduct tests of the MODM using transactions data on the Dow Jones 30 firms from 1982-1986. We document significant differences in the distribution of the information flow throughout the day. For example, the coefficient of variation (i.e. the ratio of variance to squared mean) tends to be at its lowest in the first hour and highest in the final hour of trading. In addition, many of the standard distributional assumptions concerning information flow (e.g. Poisson and Inverted Gamma) can be rejected at standard significance levels. The lognormal distribution (e.g. see Clark (1973)), however, appears to be the most consistent with the data.

Of particular importance, we also find that information flow by-itself cannot explain the differences in the distributional characteristics of stock prices and volume over the trading day. Within the framework of MODM, we view this as support for alternative theories of market microstructure which do not rely solely on information flow, such as Brock and Kleidon (1991). In particular, we find the greatest effects in the first and last trading hour, which is consistent with some implications from their model.

The paper is organized as follows. Section 2 presents the methodology for a direct test of MODM. In Section 3, we apply this test methodology to daily data on individual securities. Section 4 explores the intraday properties of MODM and in particular the information flow throughout different times in the day. Section 5 considers some extensions. Section 6 concludes.

2 MODM Test Methodology

In the Tauchen and Pitts (1983) framework, there are J traders with different expectations about the future and different risk profiles which cause them to have different reservation prices for security i. Traders take buy or sell positions in the security depending on whether their reservation prices exceed the market price. Under the equilibrium condition that markets clear, the market price for security i, P_i, is simply the average of all the reservation prices. As new information arrives to the market and traders adjust their reservation prices, the change in the market price is the average change in reservation prices.

By assuming that these changes are normally distributed, Tauchen and Pitts (1983) show that price changes and volume of trade approximately follow joint stochastically independent normals. By fixing the number of traders J and allowing the number of information events I to vary from day to day, the total price change and trading volume over the day is then just the sum over I within-day price changes and volumes of trade. The daily price change, ΔP, and daily trading volume, V, are, therefore, mixtures of independent normals with mixing variable I:[2]

$$\begin{aligned} \Delta P &\sim N(\mu_p I, \sigma_p^2 I | I) \qquad (1) \\ V &\sim N(\mu_v I, \sigma_v^2 I | I) \\ \text{with} \quad & cov(\Delta P, V | I) = 0. \end{aligned}$$

One of the attractive features of MODM is that it directly models price changes and trading volume as a function of the rate at which information arrives to the market. From an empirical point of view, however, this poses several problems for the researcher. First, price changes and volume are not conditionally normally distributed to the extent that weight is placed on the left tail of the distribution. There are two

[2]If the number of information events is large, Harris (1986) shows that (1) holds under the weaker assumption that reservation price changes follow any i.i.d. distribution.

ways to address this issue. One is to truncate the distribution; for example, one might consider an empirical version of the model which treats changes in log prices and log volume as joint conditional normals.[3] Another way is to treat the data as observationally equivalent to model (1) with the type of sample sizes found in the literature. This approach is followed in the extreme by Tauchen and Pitts (1983) (i.e. they perform a maximum likelihood analysis) and in a lesser manner by Harris (1986,1987) (i.e. he looks at higher order moments). To coincide with existing stylized facts and the form of MODM looked at in the literature, we choose the latter approach and test the model using higher order moments.

Second, while the bivariate conditional normality of ΔP and V has some justification on theoretical grounds, the model places no distributional restrictions on I, the information flow. Thus, techniques which rely heavily on complete parameterization of the model (like, for example, maximum likelihood) seem inappropriate. Third, because I enters the model nonlinearly there is no apparent way of eliminating I from functional equations for ΔP and V. Since I is unobservable, these functional forms, therefore, cannot be tested directly. Nevertheless, the model does place restrictions on the unconditional moments of ΔP and V and on their cross-moments. Because I enters their conditional normal distributions in a similar way, all higher order moments will be a function of only $(\mu_p, \sigma_p, \mu_v, \sigma_v)$ and the central moments of I, denote $m_1, m_2, m_3, \ldots\ldots$ Therefore, the unconditional moments and cross-moments of the *observable* variables, ΔP and V, will place overidentifying restrictions on the data. Under weak assumptions (namely, stationarity and ergodicity in the variables), we can then test MODM directly.

Previous empirical literature on MODM has focused on particular moments of the distribution of price changes and trading volume. For example, perhaps the most studied implication of MODM is that price changes will be both kurtotic and skewed relative to the normal distribution. A number of authors have tested this proposition and found that daily price changes are indeed both kurtotic and skewed (see, among others, Clark (1973), Epps and Epps (1976), and Harris (1987)). In addition to these pricing effects, a number of authors studied the implied restrictions on volume from the model discussed in equation (1). For example, daily trading volume is skewed for

[3]The test methodology we discuss in this paper carries through to this setting.

a wide cross-section of securities (see Harris (1986)). Of recent interest empirically for its implications for heteroskedasticity, squared price changes and trading volume are correlated (see Tauchen and Pitts (1983), Harris (1987), Lamoureux and Lastrapes (1990) and Gallant, Rossi and Tauchen (1991)).

While these stylized facts are consistent with MODM, there are also a number of reasons why they may be inconsistent with the model. First, all of these statistics have been interpreted separately; so, for example, any correlation that may exist between the kurtosis and skewness measures of ΔP and V have been ignored. Since MODM imposes all of these restrictions at any given time, a more accurate test, therefore, should test the model's implications jointly. Second, and a related point, MODM implies that the magnitudes of the estimated moments should be consistent across the moment conditions. This observation is difficult to interpret in a single restriction setting, but is straightforward to test in a joint restriction framework. Third, the literature has focused on "anomalous" behavior in the moments of ΔP and V and found they are consistent with MODM. MODM, however, also implies similar "anomalous" behavior in other moments. For example, the model implies some dependence between V^2 and ΔP; absence of dependence in this setting, therefore, can provide evidence against the MODM null hypothesis.

2.1 The GMM Test

This subsection develops a procedure for testing whether time series observations on price changes, ΔP, and trading volume, V, conform to the mixture of distributions model. This procedure is based on Hansen's (1982) GMM procedure. Let $X_t = (\Delta P_t, V_t)$, the vector of observables. If the multivariate series X_t conforms to MODM given in equation (1), then its unconditional moments should also conform to those of MODM:

$$E[h(X_t, \theta)] = 0 \tag{2}$$

where θ equals an M-vector of parameters governing MODM, namely the means and variances of X_t ($(\mu_p, \sigma_p, \mu_v, \sigma_v)$) and the central moments of I_t, $(m_1, m_2, m_3, \ldots\ldots)$, and where $h(\cdot)$ is an R-vector of functional forms implied by MODM.

In large samples, under the null hypothesis that X_t is distributed as MODM in equation (1), the sample moments of (2) should be close to zero:

$$g_T(\theta) \equiv \frac{1}{T}\sum_{t=1}^{T} h(X_t, \theta) \stackrel{T\to\infty}{\longrightarrow} 0.$$

The idea behind the GMM procedure is to find the values of the unknown parameters θ that set the sample vector $g_T(\theta)$ equal to zero. This will not be possible if the system is overidentified, i.e. if $M < R$. We can, however, set $M \times R$ linear combinations (denote A) of the R-vector $g_T(\theta)$ equal to zero:

$$Ag_T(\theta) = 0. \tag{3}$$

Hansen (1982) shows that the optimal choice of A in terms of minimizing the variance-covariance matrix of the parameter estimates $\hat{\theta}$ from (3) is $A = D_0{}' S_0^{-1}$ where $D_0 = E[\frac{\partial h(X_t,\theta)}{\partial \theta}]$ and $S_0 = \sum_{l=-\infty}^{+\infty} E[h(X_t,\theta)h(X_{t-l},\theta)']$. Hansen also provides the necessary distributional results for the parameter estimators $\hat{\theta}$ and for the overidentifying test statistic $J_T(\hat{\theta})$:[4]

- $\sqrt{T}(\hat{\theta} - \theta) \stackrel{asy}{\sim} N(0, [D_0{}' S_0^{-1} D_0]^{-1})$.
- $J_T \equiv T g_T(\hat{\theta}) S_0^{-1} g_T(\hat{\theta}) \stackrel{asy}{\sim} \chi^2_{R-M}$.

In practice, D_0 and S_0 are usually unknown; however, all that the theory requires for the distributional results to hold are consistent estimators; denote D_T and S_T, respectively.[5]

[4]Assumptions sufficient for these asymptotic results to hold are stationarity and ergodicity in price changes and volume, and the existence of twice the highest moment estimated. This latter assumption depends very much on the mixing variable I. For example, if I is distributed as an asymmetric stable distribution with characteristic exponent lesss than one, then price changes will follow a symmetric stable distribution and higher order moments do not exist. A number of distributional assumptions concerning the mixing variable I do not necessarily pose problems (see Section 2.2.

[5]One popular choice of consistent estimators are the sample estimates of D_0 and S_0. In practice, the sample estimates of S_0 are limited by sample size and are not constrained to be positive definite. There are, however, alternative autocorrelation and heteroskedastisticity consistent positve definite estimates for S_0; see, for example, Newey and West (1988). Note that under the null hypothesis of MODM, very few restrictions are placed on the autocovariances of the moments of I_t. Since autocorrelation in the moments of I_t will induce autocorrelation in $h(X_t, \theta)$, it is necessary to employ consistent estimates of S_0 in order, for example, to take account of any serial correlation or heteroskedasticity present in daily stock returns or trading volume.

To coincide with the previous literature, we focus on skewness, kurtosis and their corresponding cross-moments in the GMM estimation. As mentioned above, given their interpretation in the literature, these are natural choices of moment conditions to analyze.

2.1.1 Skewness

The most studied empirical characteristic of MODM has been the apparent skewness of price changes and volume. In addition, squared price changes and volume are significantly positively correlated. However, if squared price changes and volume are related, then under MODM given in (1) there will be a similar relationship between squared volume and price changes. Using the GMM approach outlined above, it is possible to test these properties jointly.

Specifically, under MODM in equation (1), it is possible to calculate the implied unconditional means, variances, skewness and corresponding cross-moments of the observable variables, ΔP_t and V_t. (See Harris (1987), Appendix A, for detailed calculations). As Tauchen and Pitts (1983) point out, however, MODM is invariant with respect to scalar transformations of I. Thus, if c is any positive constant and $I^* \equiv \frac{I}{c}$, then the model

$$\begin{aligned} \Delta P &\sim N\left([\mu_p c]I^*, [\sigma_p^2 c]I^* | I\right) \\ V &\sim N\left([\mu_v c]I^*, [\sigma_v^2 c]I^* | I\right) \\ \text{with} \quad & cov(\Delta P, V | I) = 0. \end{aligned}$$

cannot be differentiated empirically from MODM in equation (1). By normalizing $E[I^*] = 1$, however, it is possible to identify the transformed parameters $\mu_p^* = \mu_p m_1$, $\mu_v^* = \mu_v m_1$, $\sigma_p^{*2} = \sigma_p^2 m_1$, $\sigma_v^{*2} = \sigma_v^2 m_1$, $m_2^* = \frac{m_2}{m_1^2}$, and $m_3^* = \frac{m_3}{m_1^3}$. Under this transformation, the corresponding sample moment vector $g_T(\theta)$ of the moment conditions relating to skewness is given by

$$g_T(\theta) = \frac{1}{T}\sum_{t=1}^{T}\begin{pmatrix} \Delta P_t - \mu_p^* \\ V_t - \mu_v^* \\ (\Delta P_t - \mu_p^*)^2 - \sigma_p^{*2} - \mu_p^{*2} m_2^* \\ (V_t - \mu_v^*)^2 - \sigma_v^{*2} - \mu_v^{*2} m_2^* \\ (\Delta P_t - \mu_p^*)(V_t - \mu_v^*) - \mu_p^* \mu_v^* m_2^* \\ (\Delta P_t - \mu_p^*)^3 - 3\mu_p^* \sigma_p^{*2} m_2^* - \mu_p^{*3} m_3^* \\ (V_t - \mu_v^*)^3 - 3\mu_v^* \sigma_v^{*2} m_2^* - \mu_v^{*3} m_3^* \\ (\Delta P_t - \mu_p^*)^2 (V_t - \mu_v^*) - \sigma_p^{*2} \mu_v^* m_2^* - \mu_p^{*2} \mu_v^* (m_3^* + 2m_2^*) \\ (\Delta P_t - \mu_p^*)(V_t - \mu_v^*)^2 - \sigma_v^{*2} \mu_p^* m_2^* - \mu_v^{*2} \mu_p^* (m_3^* + 2m_2^*) \end{pmatrix}, \qquad (4)$$

where $\theta = (\mu_p^*, \mu_v^*, \sigma_p^{*2}, \sigma_v^{*2}, m_2^*, m_3^*)$.

The above restrictions in (4) represent a system of nine equations (i.e. the first three moments of each observable and their cross-moments) and six parameters governing the distribution of the observables, ΔP and V, and the unobservable, I. This leaves the econometrician with three overidentifying restrictions to test. By applying the GMM procedure in equation (3) to the sample moment vector in (4), the econometrician can estimate $\hat{\theta}$ and test MODM jointly. Specifically, the resulting $J_T(\hat{\theta})$ overidentifying statistic will have an asymptotic χ_3^2 distribution. If, for example, $J(\hat{\theta})$ exceeds exceeds 7.815, then we can reject MODM at the table 5% level of significance. Tests of these restrictions are conducted in Sections 3 and 4.

2.1.2 Kurtosis

Under MODM in equation (1), it is also possible to calculate the implied unconditional fourth moments and corresponding cross-moments of the observable variables, ΔP_t and V_t. In particular, along with the means and variances, it is possible to show that the sample moment vector $g_T(\theta)$ of these moment conditions is given by:

$$g_T(\theta) = \frac{1}{T}\sum_{t=1}^{T} \begin{pmatrix} \Delta P_t - \mu_p^* \\ V_t - \mu_v^* \\ (\Delta P_t - \mu_p^*)^2 - \sigma_p^{*2} - \mu_p^{*2} m_2^* \\ (V_t - \mu_v^*)^2 - \sigma_v^{*2} - \mu_v^{*2} m_2^* \\ (\Delta P_t - \mu_p^*)(V_t - \mu_v^*) - \mu_p^* \mu_v^* m_2^* \\ (\Delta P_t - \mu_p^*)^4 - 3\sigma_p^{*4}(1 + m_2^*) - 6\mu_p^{*2}\sigma_p^{*2}(m_3^* + m_2^*) - \mu_p^{*4} m_4^* \\ (V_t - \mu_v^*)^4 - 3\sigma_v^{*4}(1 + m_2^*) - 6\mu_v^{*2}\sigma_v^{*2}(m_3^* + m_2^*) - \mu_v^{*4} m_4^* \\ (\Delta P_t - \mu_p^*)^2 (V_t - \mu_v^*)^2 - \sigma_p^{*2}\sigma_v^{*2}(1 + m_2^*) - (\sigma_v^{*2}\mu_p^{*2} + \sigma_p^{*2}\mu_v^{*2})(m_3^* + m_2^*) - \mu_p^{*2}\mu_v^{*2} m_4^* \\ (\Delta P_t - \mu_p^*)^3 (V_t - \mu_v^*) - 3\mu_p^* \mu_v^* \sigma_p^{*2}(m_3^* + m_2^*) - \mu_v^* \mu_p^{*3} m_4^* \\ (\Delta P_t - \mu_p^*)(V_t - \mu_v^*)^3 - 3\mu_v^* \mu_p^* \sigma_v^{*2}(m_3^* + m_2^*) - \mu_p^* \mu_v^{*3} m_4^* \end{pmatrix}, \quad (5)$$

where $m_4^* = \frac{m_4}{m_1^4}$ and $\theta = (\mu_p^*, \mu_v^*, \sigma_p^{*2}, \sigma_v^{*2}, m_2^*, m_3^*, m_4^*)$.

The above restrictions in (5) represent a system of ten equations (the first two moments of each observable and their corresponding central and cross-moments of fourth order) and seven parameters governing the distributions of ΔP, V and I. As with the skewness conditions above, substituting the sample moment vector (5) into equation (3) and applying the GMM procedure will lead to an overidentifying restrictions test statistic which asymptotically follows a χ_3^2. MODM can then be evaluated at standard levels of significance.

2.2 The Distribution of Information Flow

In estimating the system of equations in (4) and (5), we are able to estimate the transformed information flow parameters m_2^*, m_3^* and m_4^*. A number of popular distributional models for the information mixing variable in turn imply restrictions on these parameters. Therefore, these restrictions can be tested using the asymptotic distributional results in Section 2.1.

In particular, let $\theta_I = (m_1, m_2, m_3, \ldots)$ and $\theta_I^* = (m_2^*, m_3^*, m_4^*, \ldots)$. Suppose the distribution of information flow $F(I, \theta_I)$ implies $Q(\theta_I^*) = 0$. Since the parameter estimators $\hat{\theta}_I^*$ are asymptotically normally distributed, we know that

$$\sqrt{T}[Q(\hat{\theta}_I^*) - Q(\theta_I^*)] \overset{asy}{\sim} N\left(0, \frac{\partial Q}{\partial \theta_I^*} V(\hat{\theta}_I^*) \frac{\partial Q}{\partial \theta_I^*}'\right),$$

where $V(\hat{\theta}_I^*)$ is the asymptotic variance-covariance matrix of the estimators $\hat{\theta}_I^*$. Using the fact that under the null the flow of information is distributed $F(I, \theta_I)$, we can derive the asymptotic distribution for a test statistic of this assumption in terms of the transformed parameters:

$$\gamma_{I^*} \equiv \frac{\sqrt{T}Q(\hat{\theta}_I^*)}{\sqrt{\left[\frac{\partial Q}{\partial \theta_I^*} V(\hat{\theta}_I^*) \frac{\partial Q}{\partial \theta_I^*}'\right]}} \overset{asy}{\sim} N(0,1).$$

For example, at the 5% two-sided significance level, if γ_{I^*} lies outside 1.96 we would reject the assumption that the information flow is distributed $F(I, \theta_I)$.

Below, we consider a number of possible distributional assumptions and their corresponding $Q(\theta_I^*) = 0$ restriction.

2.2.1 Symmetric Distribution

Since much of our intuition for distributions derives from the normal distribution, it seems appropriate to compare the implied moments from the data to that of the normal distribution. Of course, even though information flow cannot be distributed normally, implications of the normal distribution such as symmetry may be tested. An example of a symmetric distribution which could describe the information flow is the uniform distribution. This would suggest that information is distributed evenly over a given range. In particular, the testable restriction can be written as

$$\begin{aligned} Q(\theta_I^*) \equiv m_3^* &= 0 \\ m_4^* - \frac{9}{5} m_2^{*2} &= 0. \end{aligned}$$

2.2.2 Inverted Gamma Distribution

One choice of a distribution for the mixing variable (i.e. the information flow) which has both theoretical and empirical appeal is the inverted gamma distribution. Blattberg and Gonedes (1974) show, for example, that if the variance of returns are conditionally normal with the conditioning variable following an inverted gamma then stock returns will follow a student-t distribution. The student-t distribution seems to explain a number of features of realized returns. In addition, student-t has theoretical appeal and is consistent with a number of asset pricing models (see, for example, Ingersoll (1987). It is worthwhile, therefore, to test whether the mixing variable has transformed moments which are consistent with the inverted gamma distribution.

In particular, it is possible to show that the testable restriction for the inverted gamma distribution can be written as

$$Q(\theta_I^*) \equiv (m_4^* + 4m_3^* + 6m_2^* + 1)(1 - m_3^* - m_2^*) - (1 + m_2^*)^2(1 + m^*3 + 3m_2^*) = 0.$$

2.2.3 Poisson Distribution

The poisson distribution has popular roots in the social science literature. It explicitly provides a distribution for the random number of events which occur in a particular time period. In addition, it implies that the waiting time between events follows an exponential distribution. Because of its natural interpretation, it has been suggested as a convenient distribution for describing the amount of information flow arriving in the stock market each period (see, for example, Tauchen and Pitts (1983)).

In terms of the transformed parameters, it is possible to show that the relevant testable restrictions are

$$\begin{aligned} Q(\theta_I^*) \equiv m_3^* - m_2^{*2} &= 0 \\ m_4^* - m_2^{*3} - 3m_2^{*2} &= 0. \end{aligned}$$

2.2.4 Lognormal Distribution

The final distribution to be studied is the lognormal distribution. This distribution has the appealing property that the information flow is skewed. That is, most of the time a small amount of information flow arrives to the market. However, on occasion, large information flow will occur (e.g. around firm specific announcements). Moreover, the log of information flow can be interpreted as a normal random variable which is convenient in terms of our understanding of the information flow's distributional characteristics. As a result, this asumption has been suggested by a number of authors (see, for example, Clark (1973) and Tauchen and Pitts (1983)).

In terms of the transformed parameters, it is possible to show that the relevant testable restrictions for the lognormal case are

$$\begin{aligned} Q(\theta_I^*) \equiv m_3^* - m_2^{*3} - 3m_2^{*2} &= 0 \\ m_4^* + 4(1 + m_2^*)^3 + 3 - (1 + m_2^*)^6 - 6(1 + m_2^*) &= 0. \end{aligned}$$

3 Empirical Tests: Daily Data

In this section, we perform the two GMM tests discussed above (i.e. using the skewness and kurtosis based moment conditions in (4) and (5)) on daily data for individual securities. Specifically, data are collected on daily prices and volume for the Dow Jones 30 firms over the sample period 1982-1986.

3.1 Skewness Test Results

For each Dow Jones firm, Table 1 provides tests of the skewness moment restrictions given in (4). The second through seventh columns give the parameter estimates and their corresponding standard errors. The final column presents the overidentifying test statistic. Some observations regarding these results are in order.

First, a quick glance at the skewness-based parameter m_3^* shows that most of the firms exhibit a positively skewed information flow. The flow of information, therefore, is nonnormal. Positive skewness suggests that the weight of the distribution is in the left tails. Therefore, only a few information events occur throughout the day; nevertheless, on occasion, a considerable information flow will occur on any given day.

Second, the coefficient of variation parameter, m_2^*, is "similar" in magnitude across firms. So, for example, if the Dow Jones 30 firms have essentially the same average amount of information flow throughout the day, then the variance of this flow is the same. Moreover, the estimate for most firms is significantly less than one, which suggests that the standard deviation of the information flow is small relative to the average information flow.

Third, and perhaps most interesting, there is not much empirical support for MODM in the data. Only one-third of the firms have test statistics which do not reject MODM at the 5% significance level. Further, one of these firms exhibits a significantly negative σ_v^{*2}, which can also be considered a violation of the model. This is somewhat surprising given the amount of empirical support for the particular moment restrictions (skewness and cross-moments) utilized in equation (4). Apparently, the joint properties of these moments (i.e. magnitude and signs) do not conform to those predicted by MODM.

3.2 Kurtosis Test Results

Somewhat similar results are attained for tests of the kurtosis restrictions given in equation (5). These results are provided in Table 2. With respect to the parameter estimates, the flow of information is again positively skewed. The coefficient of variation, although slightly larger, is still less than one in most cases and is of similar magnitude across the firms. The point estimates of the measure m_4^* signify a high degree of kurtosis relative to say a normal distribution. This suggests that the distribution of flow of information has long thick tails.

Note though that there is again not much empirical support for MODM. Even though only one-third of the overidentifying statistics reject the model at the 5% significance level, an additional 9 firms have a significantly negative σ_v^{*2} which violates parameter restrictions of the model. While most previous research has documented that price changes are kurtotic relative to the normal distribution, it is interesting to note that the evidence here suggests this result does not come from MODM. An open question is whether alternative models (which can explain a high degree of kurtosis in price changes) can explain magnitudes and signs of related moments in a joint setting.

4 Empirical Tests: Intraday Data

The results from the previous section were generally not supportive of MODM. Restrictions from the model could be rejected for most of the Dow Jones 30 firms. There is some reason to believe, however, that the information flow has different properties throughout the day. It may be the case, therefore, that MODM provides a good description of transactions data (i.e. at different times in the day) but cannot explain aggregate data such as daily or longer intervals.

From a theoretical point of view, a number of papers suggest the information flow may differ over the day. Adamati and Pfleiderer (1988) and Foster and Viswanathan (1990a,1991) present models of intraday trading with strategic behavior on the part of agents. Their models, for example, imply volume and variance patterns throughout the day due to agents strategically releasing information at different times. In terms of the tests in this paper, their models suggest that the daily time period for investigating the distribution of information flow should be broken up into subperiods.

There is also a large empirical literature which documents various patterns in means, variance and volume throughout the day. For example, Wood, McInish and Ord (1985) document a U-shaped pattern in stock returns during the day. Harris (1986) provides additional evidence on intraday returns (both in means and variances of returns) and also investigates intraday cross-sectional properties of returns. French and Roll (1986) provide evidence to suggest that the variability of returns is related to the information flow to the market. Evidence on intraday patterns in volume is provided in both Jain and Joh (1988) and Foster and Viswananthan (1990b).

As an alternative to the information-based models, Brock and Kleidon (1991) explain intraday patterns in volume based on a model with investors facing periodic transactions demand. For example, they show that, at the open and close of trading, transactions demand and volume of trade will be higher due to stock market closure. There is some evidence in intraday volume and bid-ask patterns to support this model (see, for example, Harris, McInish and Chakravarty (1991)). In addition, Madhavan and Smidt (1991) develop and test a model where price changes may occur because of noninformation factors.[6]

Within the framework of the mixture of distributions model, we explore some of these issues more closely. As mentioned in the introduction, there has been almost no research on MODM's intraday properties. An exception is Harris (1987) who reports a number of properties in the transactions data which appear consistent with MODM. To the extent that MODM provides a good description of transactions data (as demonstrated in Harris (1987)), our test procedure can therefore be applied to investigate recent market microstructure issues. In particular, we can estimate properties of I over different times of the day. Conditional on the model working better at the transactions data level, this analysis can provide evidence either for or against recent explanations of "anomalous" intra-day behavior in stock returns.

In this section, we perform the two GMM tests discussed above (i.e. in (4) and (5)) on intraday data for individual securities. Specifically, data are collected on hourly prices and volume for the Dow Jones 30 firms over the sample period 1982-1986. For example, consider the last hour of trading. We collect hourly observations over 3:00 PM to 4:00 PM for the five-year period 1982-1986. The tests are then performed over

[6]Spiegel and Subrahmanyam (1991) also posit a model in which price changes can occur in the absence of information, such as variations in market makers' holdings. This can lead, for example, to time-varying expected returns within the day.

this five-year time series. We perform the GMM procedure for each of the one-hour time periods during the day.

In performing these tests, we need to be wary of possible dependence between the statistics across the day. Below, we provide some summary statistics on the autocorrelation and covariance patterns in volume and price changes. Although there is some degree of dependence in volume across the hours and across the Dow Jones firms, the correlation is in general not large.

4.1 Summary Statistics

Table 3A presents the mean correlation matrix across the hours for both price changes and volume for the Dow Jones 30 firms. With respect to price changes, the correlation is small in magnitude. Most of the correlations lie in the -.01 to .02 range. Apparently, price changes are uncorrelated across the hours. Volume is somewhat more correlated across the hours. One might expect the volume to be high throughout the day if related news gets released during the day. Nevertheless, the correlation is not large (on average around 25%). In particular, the correlation between the first and last hour of volume is no larger (and in fact sometimes smaller) than correlations between other hours. The general conclusion we reach from Table 3A is that although there is some degree of dependence across the hours it is generally small in magnitude.

Table 3B reports the first-order daily autocorrelation of price changes and volume for each hour. For example, column one presents $corr(\Delta P_{3pm-4pm:t}, \Delta P_{3pm-4pm:t-1})$ and $corr(V_{3pm-4pm:t}, V_{3pm-4pm:t-1})$. Price changes tend to have only minor correlation (between -3% — 3%) which is statistically insignificant from zero. On the other hand, volume's autocorrelation is approximately 16-18%. The exception is the opening hour which has a 30% autocorrelation on average. Given the overlap between the daily autocorrelations for each hour, this suggests a fundamental difference between the open and other hours. Possible explanations are either the different market mechanism at the opening or the aforementioned Brock and Kleidon (1991) paper dealing with transactions demand and market closure.

Finally, Table 3C provides the average correlation of price changes and volume for the Dow Jones 30 firms across the different hours. The cross-correlations for price changes are approximately 25%, which represents the common impact of market-wide information. Similarly, volume's cross-correlation, although somewhat smaller in magnitude (approximately 10%), also probably reflects market-wide news releases. Of some interest, the open and close have higher correlations (17% and 25% respectively), suggesting again different behavior throughout the day.

The conclusion we reach is that, although some dependence exists across the hours and across firms for price changes and volume, the level of dependence is not large and will therefore not have a significant impact on the joint properties of the individual test statistics and parameter estimators. Nevertheless, the individual results are subject to the caveat that some dependence is present in the statistics (see Section 5 for a discussion of some extensions to the individual GMM tests).

4.2 MODM Test Results

4.2.1 Skewness

The intraday results for the test of the skewness restrictions are presented in Table 4. In particular, we provide the average overidentifying test statistic, corresponding average p-value and the % of rejections over the 30 firms. In addition, similar results are provided for a test of equality of each of the parameters across the six different hours during the day. Because of the number of firms in our sample and the number of subperiods, however, we provide only summary statistics on the individual parameter estimates.

Because the standard errors of the estimators differ from firm to firm, the mean of the estimates across the firms will place too much weight on those estimators which are imprecise. Therefore, we provide both the mean and "weighted" mean across the 30 firms (i.e. weighted by its standard error where the weights add up to one) and the median across the 30 firms for each parameter estimate. These summary statistics are not intended to have a statistical interpretation. Instead, we chose these summary values because they are indicative of the types of patterns that develop across the day for each individual firm. For example, the summary results in Table 4 suggest a U-shaped pattern in the variance σ_p^{*2} but not in the mean μ_p^*. This suggests that changes in the distribution of the information flow throughout the day cannot be the sole explanation for empirical facts related to price changes and volume. This pattern is consistent with the majority of the firms in the sample.

In contrast to the daily tests, approximately only one-fifth of the firms exhibit significant rejection of the restrictions given in equation (4) across the first five hours.

The exception is during the final hour which yields over a 50% rejection rate. This is also evident from the average p-values which drop in value for the final hour (i.e. 14% p-value versus 30%). All in all, the restrictions at the transactions data level fit somewhat better than at the daily level. This coincides with results in Harris (1987) and theoretical models predicting distributional changes in the flow of information.

The tests for equality of the parameters across the day show very strong rejection rates across the Dow 30. In addition, given the low dependence in the variables across the hours, the similar patterns and significant rejections across the hour suggest common behavior within this sample. Of some interest, note that the U-shaped pattern in μ_v^* is consistent with a constant μ_v and a varying flow of expected information flow m_1 throughout the day. At face value, this suggests that the information flow on average starts off high, subsides during the middle of the day and then picks up again at the close. However, if μ_p is constant, this cannot explain the humped shape pattern in μ_p^*. Within the framework of MODM, some noninformation factor also plays a role in describing intraday patterns in volume and price changes, e.g. μ_p may vary throughout the day (as in Spiegel and Subrahmanyam (1991)). Further, the U-shaped pattern in σ_p^* and σ_v^* is consistent with U-shaped patterns in either the mean or variance of information flow.

4.2.2 Kurtosis

The kurtosis test results are provided in Table 5. With respect to the overidentifying test statistic, the evidence is even more in favor of MODM (at least relative to the skewness based statistic). In particular, the largest rejection rate during any particular hour is 24% (i.e. at the open) while for a number of hours the rate is below 10%. In fact, during the third hour of the day, none of the firms had significant overidentifying test statistics. The average p-values are generally in the low to mid range (23% to 42%), suggesting much less evidence against the MODM null at the transactions data level.

With respect to the distributional parameters of price changes and volume, tests for equality of the parameters throughout the day can be rejected at standard levels of significance. Note that the parameter estimates suggest patterns very similar to those obtained from the skewness restrictions. For example, μ_v^* is U-shaped while μ_p^* is for the most part humped shape throughout the trading day. Similar to the interpretation for the skewness restrictions, these patterns cannot be explained solely by time-dependent movements in the information flow throughout the day. This again suggests a role for noninformation factors in describing comovements of price changes and volume throughout the day. Furthermore, the magnitude of the parameter estimates is similar between the two sets of restrictions. For example, consider the median parameter estimates of ${\sigma_p^*}^2$. The values over the day are [.073,.045,.035,.029,.036,.042] and [.070,.045,.036,.041,.036,.063] for the skewness and kurtosis restrictions, respectively.

4.2.3 Distribution of Information Flow

Tables 4 and 5 also report estimates and equality tests for the information flow parameters m_2^*, m_3^* and m_4^* across the trading day. As a byproduct of these estimates, formal tests of particular distributional assumptions (such as the inverted gamma) are also calculated. From the tests for equality, there is strong evidence that m_2^* differs systematically throughout the day. The evidence, albeit weaker, suggests similar differences for m_3^*. On the other hand, m_4^* is not estimated precisely enough for us to form strong conclusions.

With respect to the parameter estimates, the values of $\hat{m}_3^*$ imply a positively skewed information flow. While no one discernible pattern develops across the firms, a common element in the results for both the skewness and kurtosis restrictions is that the opening hour appears to consistently have the lowest m_3^*. While this could suggest less skewness at the open, a more likely explanation is that the expected information flow (i.e. m_1) is at its highest during this period. A check of the other parameter estimates, $\hat{m}_2^*$ and $\hat{m}_4^*$, during the opening hour suggests that this is in fact the case. That is, almost all of the point estimates are at their lowest during the first trading hour, which is consistent with high expected values of the information flow.

The coefficient of variation m_2^* also displays patterns across the day. While the magnitude of the estimates differs for the skewness (i.e. in the .5 range) and kurtosis (i.e. in the 1.0 range) restrictions, the pattern is the same irrespective of the testable restrictions. In particular, firms' estimates of m_2^* display a humped shaped pattern with a spike at the close. Given the other estimates, this is consistent with a U-shaped pattern in the expected information flow throughout the day. To the extent that the last hour's spike is significant, this may be due to an increase in the variance of the information flow during the final hour. This coincides with the belief that firm specific news on occasion gets released towards the end of the trading day. Finally, the kurtosis parameter m_*^4 is not estimated as precisely and therefore does not imply any particular significant pattern. The one consistent result across all the trading hours, however, is that information flow tends to be more kurtotic than a normal distribution; that is, the tails are are thick relative to a normal.

Tables 4 and 5 also provide formal tests of commonly assumed distributions for the information flow such as the poisson, inverted gamma, lognormal and symmetric distributions. While the evidence using the kurtosis restrictions is somewhat weaker than that using the skewness restrictions, the majority of the sample rejects the null that the information flow is distributed either poisson, symmetric or inverted gamma. As an illustration, for the majority of the hours within the day, over two-thirds of the Dow Jones sample imply parameter estimates which are not consistent with the poisson distributional assumption (e.g. see Table 4, heading titled "averages for poisson information").

Of the distributions we investigated, the lognormal is rejected less frequently. A main implication of the lognormal model is that information flow is positively skewed. To the extent that the lognormal assumption is empirically reasonable, this should be useful for future research given the distribution's convenient statistical properties. Of interest to the prior literature, using treasury data, Tauchen and Pitts (1983) provide some empirical evidence that lognormal information flow provides a better fit than the poisson distribution. It is interesting to note that we find similar evidence in a completely different set-up.

5 Extensions

The tests were performed for each of the Dow Jones 30 firms separately. Clearly, we would like to calculate joint statistics across the firms. The problems with doing these joint tests, however, are twofold. First, the number of moment restrictions and parameters become unmanageable. For example, the kurtosis restrictions require 300 moments and 210 parameters. Second, the small sample properties of the variance-covariance matrix estimator will probably be quite poor since estimation would require 300 cross-moments from only 5 years of data (i.e. 1220 observations).

There are, however, a few ways to address these statistical difficulties. First, the limited summary statistics provided in Table 3 suggest the degree of dependence is not large. Hence, the similar patterns which emerge across the firms and the substantial rejection rate for a number of the tests can be given more weight than one might normally give under these circumstances (i.e. in the univariate rather than multivariate framework). Second, because the parameters on firm i only occur in firm i's equations, the derivative matrix given in the optimal choice of A (see equation (3)) will have many zeroes. This may make the estimation problem less complicated and hence more manageable. Third, it is possible to reduce the number of overidentifying restrictions without necessarily losing efficiency of the estimators. However, this would require some a priori knowledge regarding the importance of the various moment restrictions. In this paper, we chose to rely on the suggestive evidence given in Table 3. However, the reader should be aware of the inference problem in this multivariate framework. We hope in future research to study this problem in a more complete way.

Another area of potential research is to address some standard market microstructure induced biases such as the bid-ask spread and nonsynchronous trading. Given the Dow Jones sample, the latter bias will probably not play an important role; that is, all the stocks in the sample trade frequently throughout the day. Note though that the bid-ask spread will effect the estimation of the sample moments. However, the magnitude of the bias will probably be small given the small spreads for many of the firms. In order to address this question more directly, we could model the bid-ask spread empirically and jointly estimate its moments with equations (4) and (5). We leave this joint estimation problem as a topic for future research.

One final issue, which we do address below, is the fact that the skewness and kurtosis statistics have been reported separately. We did this to coincide with the previous literature's discussion of the MODM's testable restrictions. It seems worthwhile, however, to investigate the joint skewness and kurtosis properties of price changes and volume.

5.1 Combined Skewness and Kurtosis Tests

Most of the empirical literature on MODM has focused on moment restrictions in an individual setting. Joint tests of the more popular moment restrictions have been presented in Sections 2.1.1 and 2.1.2. Since these restrictions cover all those tested in the literature, it may be of interest to test all the restrictions (i.e. both skewness and kurtosis) jointly. Specifically, consider the sample moment vector:

$$
g_T(\theta) = \frac{1}{T}\sum_{t=1}^{T}\begin{pmatrix}
\Delta P_t - \mu_p^* \\
V_t - \mu_v^* \\
(\Delta P_t - \mu_p^*)^2 - {\sigma_p^*}^2 - {\mu_p^*}^2 m_2^* \\
(V_t - \mu_v^*)^2 - {\sigma_v^*}^2 - {\mu_v^*}^2 m_2^* \\
(\Delta P_t - \mu_p^*)(V_t - \mu_v^*) - \mu_p^* \mu_v^* m_2^* \\
(\Delta P_t - \mu_p^*)^3 - 3\mu_p^* {\sigma_p^*}^2 m_2^* - {\mu_p^*}^3 m_3^* \\
(\Delta V_t - \mu_v^*)^3 - 3\mu_v^* {\sigma_v^*}^2 m_2^* - {\mu_v^*}^3 m_3^* \\
(\Delta P_t - \mu_p^*)^2 (V_t - \mu_v^*) - {\sigma_p^*}^2 \mu_v^* m_2^* - {\mu_p^*}^2 \mu_v^* (m_3^* + 2m_2^*) \\
(\Delta P_t - \mu_p^*)(V_t - \mu_v^*)^2 - {\sigma_v^*}^2 \mu_p^* m_2^* - {\mu_v^*}^2 \mu_p^* (m_3^* + 2m_2^*) \\
(\Delta P_t - \mu_p^*)^4 - 3{\sigma_p^*}^4 (1 + m_2^*) - 6{\mu_p^*}^2 {\sigma_p^*}^2 (m_3^* + m_2^*) - {\mu_p^*}^4 m_4^* \\
(\Delta V_t - \mu_v^*)^4 - 3{\sigma_v^*}^4 (1 + m_2^*) - 6{\mu_v^*}^2 {\sigma_v^*}^2 (m_3^* + m_2^*) - {\mu_v^*}^4 m_4^* \\
(\Delta P_t - \mu_p^*)^2 (V_t - \mu_v^*)^2 - {\sigma_p^*}^2 {\sigma_v^*}^2 (1 + m_2^*) - ({\sigma_v^*}^2 {\mu_p^*}^2 + {\sigma_p^*}^2 {\mu_v^*}^2)(m_3^* + m_2^*) - {\mu_p^*}^2 {\mu_v^*}^2 m_4^* \\
(\Delta P_t - \mu_p^*)^3 (V_t - \mu_v^*) - 3\mu_p^* \mu_v^* {\sigma_p^*}^2 (m_3^* + m_2^*) - \mu_v^* {\mu_p^*}^3 m_4^* \\
(\Delta P_t - \mu_p^*)(V_t - \mu_v^*)^3 - 3\mu_v^* \mu_p^* {\sigma_v^*}^2 (m_3^* + m_2^*) - \mu_p^* {\mu_v^*}^3 m_4^*
\end{pmatrix}, \tag{6}
$$

where $m_4^* = \frac{m_4}{m_1^4}$ and $\theta = (\mu_p^*, \mu_v^*, {\sigma_p^*}^2, {\sigma_v^*}^2, m_2^*, m_3^*, m_4^*)$.

The above system represents a system of fourteen equations (the first four moments of each variable and corresponding cross-moments) and seven parameters governing the distribution of the relevant variables ΔP, V and I. Following the GMM procedure outlined in Section 2 leads to an overidentifying restrictions test statistic which follows an asymptotic χ^2_7 distribution. The test results are given in Table 6.

The results in Table 6 are in contrast to those provided in Tables 4 and 5. At both the daily and transactions data level, the overidentifying restrictions test provides strong evidence against MODM. For example, the rejection rate is at least 50% for the sample of firms, and (in the extreme case) the final hour of trading produces almost unanimous rejection across the firms (i.e. 88%). In addition, the average p-values are small across the hours of the day. Apparently, there is less evidence in favor of MODM using these joint restrictions. Note, however, that we have increased the estimation problem by four additional moments. It is an open question whether this increase causes significant changes in the small sample properties of the statistics to warrant the differences between Table 6 and Tables 4 and 5.

6 Conclusion

While the mixture of distributions hypothesis has at present both substantial theoretical and empirical appeal, no direct tests of the joint implications of the model have been tested. In this paper, we develop a general procedure for testing whether the data conforms to a mixture of distributions model. Of particular interest, the estimation procedure is robust to distributional parameterizations of the mixing variable I, the rate of information flow. As such, I can be nonnormal, autocorrelated, heteroskedastistic, and, of course, unobservable. Using the implied unconditional moments of the observable variables, the econometrician can deduce, however, distributional properties of I and test explicit distributional models of the information flow.

In applying this procedure to transactions data, the results are interesting for a number of reasons. First, although MODM is rejected for a number of firms, these rejections are by no means uniform across the sample. Second, we document significant differences in the information flow throughout the day. These results coincide with theoretical implications from models in which agents strategically release information throughout various times of the day. While many of the standard distribution assumptions for this flow can be rejected, however, we find that the estimates are close to parameter restrictions implied by lognormally distributed information flow. Third, the evidence suggests, however, that non-information factors also play a role in describing various intraday patterns in volume and price changes. We view these results as also providing support for non-information based models such as Brock and Kleidon (1991), Spiegel and Subrahmanyam (1991), among others.

Table 1: Skewness Tests–Daily

Table 1 presents tests of skewness restrictions implied by the mixture of distributions model. Colums 2-7 provide estimates and standard errors of the transformed parameters of price changes, volume and unobservable information flow, respectively. The final column provides the test of the overidentifying restrictions of the model.

Co.	μ_p^*	σ_p^{*2}	μ_v^*	σ_v^{*2}	m_2^*	m_3^*	χ_3^2
AA	0.0013	0.2110	0.3891	0.0096	0.3004	0.9245	13.6593
	(0.0178)	(0.0043)	(0.0191)	(0.0032)	(0.0524)	(0.4823)	(0.0034)
ALD	0.0057	0.2176	0.4047	0.0276	0.3822	2.1431	10.4392
	(0.0169)	(0.0062)	(0.0269)	(0.0069)	(0.1575)	(0.6237)	(0.0152)
AXP	0.0670	0.5166	0.9129	0.0440	0.1732	-2.1902	9.8650
	(0.0245)	(0.0110)	(0.0471)	(0.0342)	(0.0540)	(1.4385)	(0.0197)
BA	0.0452	0.3349	0.6432	-0.0117	0.4582	-0.2760	11.4522
	(0.0223)	(0.0062)	(0.0302)	(0.0059)	(0.0426)	(0.3812)	(0.0095)
BS	-0.0092	0.1583	0.1829	0.0228	0.7473	15.9540	1.7711
	(0.0099)	(0.0057)	(0.0103)	(0.0131)	(0.1086)	(12.7704)	(0.6212)
CAT	0.0014	0.2257	0.6141	-0.0040	0.5342	0.4034	7.9006
	(0.0217)	(0.0044)	(0.0357)	(0.0038)	(0.0778)	(0.2015)	(0.0481)
CHV	-0.0059	0.3408	0.3902	-0.0002	0.3917	0.5569	17.5604
	(0.0166)	(0.0061)	(0.0211)	(0.0052)	(0.0530)	(0.1335)	(0.0005)
DD	0.0016	0.2710	0.5911	-0.0007	0.3682	0.4158	14.5333
	(0.0137)	(0.0046)	(0.0342)	(0.0039)	(0.0517)	(0.0777)	(0.0023)
DIS	0.0616	0.2283	1.3690	0.0385	0.5773	3.3703	4.2550
	(0.0278)	(0.0075)	(0.0860)	(0.0095)	(0.1659)	(0.7066)	(0.2352)
EK	-0.0036	0.4695	0.9074	0.0234	0.3908	0.8299	8.0392
	(0.0267)	(0.0094)	(0.0481)	(0.0120)	(0.0657)	(0.2112)	(0.0452)
GE	0.0265	0.4449	0.9240	0.0270	0.2578	0.6786	13.9295
	(0.0229)	(0.0079)	(0.0537)	(0.0144)	(0.0577)	(0.2657)	(0.0030)
GM	0.0121	0.5644	0.9519	-0.0230	0.3220	0.2876	13.2016
	(0.0210)	(0.0082)	(0.0433)	(0.0098)	(0.0327)	(0.0751)	(0.0042)
GT	0.0070	0.3079	0.2772	0.0728	0.6030	8.6941	4.8755
	(0.0133)	(0.0105)	(0.0180)	(0.0338)	(0.1614)	(4.8054)	(0.1811)
IBM	0.0784	0.9657	2.0867	-0.0612	0.3765	0.7291	1.0878
	(0.0400)	(0.0144)	(0.1061)	(0.0449)	(0.0516)	(0.1990)	(0.7800)
IP	-0.0091	0.2259	0.7223	0.0385	0.6897	5.3167	6.0882
	(0.0166)	(0.0077)	(0.0404)	(0.0083)	(0.0990)	(0.9963)	(0.1074)
JPM	0.0622	0.1799	-2.4317	-0.0111	1.5717	3.9041	13.2280
	(0.0253)	(0.0071)	(1.2121)	(0.0190)	(0.5763)	(4.4774)	(0.0042)
KO	-0.0193	0.2673	0.7444	0.0208	0.3571	1.6709	19.5012
	(0.0135)	(0.0059)	(0.0499)	(0.0070)	(0.0729)	(0.3592)	(0.0002)
MCD	0.0350	0.2204	0.9399	0.0222	0.2575	1.7202	5.0641
	(0.0158)	(0.0052)	(0.0516)	(0.0046)	(0.0533)	(0.3711)	(0.1672)
MMM	-0.0103	0.1997	0.6279	0.0845	-1.7402	11.5241	9.4802
	(0.0136)	(0.0055)	(0.4365)	(0.0711)	(1.7767)	(20.6595)	(0.0235)
MO	0.0541	0.3143	1.2283	-0.0199	0.7708	1.6300	2.9595
	(0.0249)	(0.0069)	(0.0900)	(0.0117)	(0.1234)	(0.4911)	(0.3979)
MRK	0.0931	0.1802	1.6247	0.0097	0.3721	0.0018	6.0665
	(0.0344)	(0.0045)	(0.1045)	(0.0029)	(0.0667)	(0.4847)	(0.1084)
PG	0.0516	0.1956	1.1042	-0.0008	0.5174	1.0112	1.9057
	(0.0279)	(0.0039)	(0.0767)	(0.0030)	(0.0967)	(0.3811)	(0.5922)
S	-0.0036	0.4594	0.3982	0.0411	0.2807	0.8286	24.1035
	(0.0127)	(0.0090)	(0.0230)	(0.0145)	(0.0461)	(0.2388)	(0.0000)
T	0.0033	1.1061	0.2889	0.6163	0.0726	1.3581	7.6000
	(0.0137)	(0.0233)	(0.0178)	(0.0929)	(0.0372)	(0.3183)	(0.0550)
TX	-0.0053	0.5073	0.2250	0.2655	0.8025	8.7968	9.6096
	(0.0119)	(0.0200)	(0.0154)	(0.0892)	(0.2888)	(3.1704)	(0.0222)
UK	-0.0181	0.2962	2.7109	0.2388	-0.3074	22.1765	15.8228
	(0.0135)	(0.0128)	(1.6138)	(0.0411)	(0.2311)	(4.9901)	(0.0012)
UTX	0.0142	0.2421	0.6323	0.0522	0.1080	4.9112	3.2054
	(0.0164)	(0.0068)	(0.0306)	(0.0116)	(0.0500)	(1.9143)	(0.3610)
WX	0.0359	0.2863	0.5133	0.0228	0.3392	1.0115	28.7847
	(0.0204)	(0.0069)	(0.0268)	(0.0098)	(0.0731)	(0.4414)	(0.0000)
XON	0.0375	0.6328	0.2905	0.0322	0.1783	0.2039	5.7619
	(0.0145)	(0.0093)	(0.0173)	(0.0180)	(0.0431)	(0.0603)	(0.1238)
Z	0.0223	0.1644	0.4520	0.0398	1.0055	10.0994	16.2424
	(0.0139)	(0.0071)	(0.0288)	(0.0066)	(0.2034)	(2.0052)	(0.0010)

Table 2: Kurtosis Tests–Daily

Table 2 presents tests of kurtosis restrictions implied by the mixture of distributions model. Colums 2-8 provide estimates and standard errors of the transformed parameters of price changes, volume and unobservable information flow, respectively. The final column provides the test of the overidentifying restrictions of the model.

Co.	μ_p^*	σ_p^{*2}	μ_v^*	σ_v^{*2}	m_2^*	m_3^*	m_4^*	χ_3^2
AA	0.0069	0.2113	0.4013	0.0082	0.3261	0.2640	5.5859	8.0230
	(0.0182)	(0.0043)	(0.0196)	(0.0039)	(0.0691)	(0.1137)	(3.5110)	(0.0455)
ALD	0.0391	0.2186	0.4158	-0.0155	1.2886	1.7821	22.1463	2.6545
	(0.0143)	(0.0062)	(0.0266)	(0.0124)	(0.2631)	(1.1303)	(8.0110)	(0.4480)
AXP	0.0695	0.5279	0.9232	-0.0030	0.5010	0.0952	-11.1219	6.3828
	(0.0241)	(0.0112)	(0.0477)	(0.0301)	(0.0942)	(0.2166)	(14.3534)	(0.0944)
BA	0.0295	0.3295	0.6321	0.0001	0.3532	0.3343	0.3145	20.3889
	(0.0220)	(0.0056)	(0.0301)	(0.0080)	(0.0743)	(0.1017)	(0.3425)	(0.0001)
BS	-0.0066	0.1562	0.1673	0.0241	0.7038	0.7040	351.5406	3.7635
	(0.0104)	(0.0055)	(0.0095)	(0.0115)	(0.1460)	(0.6753)	(362.3806)	(0.2882)
CAT	0.0132	0.2212	0.5816	-0.0094	0.6251	0.3999	1.8778	15.3853
	(0.0218)	(0.0042)	(0.0334)	(0.0074)	(0.1448)	(0.2397)	(1.4057)	(0.0015)
CHV	0.0239	0.3504	0.4012	-0.0223	0.5681	0.6228	2.7493	14.1038
	(0.0177)	(0.0062)	(0.0211)	(0.0130)	(0.1057)	(0.1561)	(0.8818)	(0.0028)
DD	0.0634	0.2687	0.6193	-0.0375	0.8663	0.8273	5.4430	3.9449
	(0.0184)	(0.0045)	(0.0347)	(0.0104)	(0.1474)	(0.2507)	(2.1062)	(0.2675)
DIS	0.0954	0.2295	1.3305	0.0138	1.0873	0.6717	26.6692	1.6417
	(0.0321)	(0.0092)	(0.0807)	(0.0135)	(0.2696)	(0.6370)	(8.6599)	(0.6500)
EK	0.0458	0.4849	1.0212	-0.1822	1.2979	1.6074	13.3415	2.4256
	(0.0180)	(0.0098)	(0.0619)	(0.0653)	(0.2782)	(0.6163)	(5.5972)	(0.4889)
GE	0.0838	0.4428	0.9959	-0.0949	0.8430	0.7525	5.3728	1.8246
	(0.0240)	(0.0075)	(0.0568)	(0.0264)	(0.1344)	(0.2196)	(1.6236)	(0.6096)
GM	0.0251	0.5675	0.9388	0.0039	0.2436	0.2172	0.5100	7.8556
	(0.0265)	(0.0082)	(0.0428)	(0.0237)	(0.0716)	(0.0518)	(0.2726)	(0.0491)
GT	0.0173	0.3313	0.2798	0.0290	1.5947	4.2458	122.4714	3.3729
	(0.0151)	(0.0141)	(0.0206)	(0.0556)	(0.4278)	(2.7286)	(93.2073)	(0.3376)
IBM	0.0745	0.9600	2.1233	-0.1464	0.3742	0.4046	0.9254	3.6081
	(0.0405)	(0.0126)	(0.1048)	(0.0961)	(0.1077)	(0.1500)	(0.5916)	(0.3070)
IP	0.0574	0.1870	0.6787	-0.0052	0.7288	0.6039	6.7961	10.7188
	(0.0191)	(0.0041)	(0.0367)	(0.0044)	(0.1201)	(0.2150)	(3.9580)	(0.0133)
JPM	-9.9							
KO	0.0579	0.2881	0.7634	-0.0646	1.4279	1.4360	15.4820	2.9865
	(0.0175)	(0.0065)	(0.0532)	(0.0202)	(0.2429)	(0.5593)	(4.8095)	(0.3937)
MCD	0.0821	0.1985	1.0026	-0.0140	0.7427	0.5342	3.1990	5.5570
	(0.0248)	(0.0035)	(0.0555)	(0.0058)	(0.1470)	(0.2069)	(1.2499)	(0.1353)
MMM	-9.9							
MO	0.0634	0.3361	1.1269	-0.0579	1.1030	1.9445	13.0009	2.6333
	(0.0284)	(0.0072)	(0.0745)	(0.0343)	(0.3047)	(0.6476)	(6.2826)	(0.4517)
MRK	0.1233	0.1632	1.7693	-0.0141	1.0075	1.1176	12.0782	3.0953
	(0.0349)	(0.0032)	(0.1107)	(0.0045)	(0.1574)	(0.3435)	(5.1140)	(0.3772)
PG	0.0299	0.1865	0.8786	-0.0195	0.9665	0.7349	5.0476	6.2915
	(0.0199)	(0.0034)	(0.0578)	(0.0126)	(0.3603)	(0.5331)	(3.9130)	(0.0983)
S	0.0355	0.4830	0.3607	-0.0529	0.6312	0.5362	3.3727	26.4448
	(0.0166)	(0.0091)	(0.0193)	(0.0271)	(0.1157)	(0.1573)	(1.5041)	(0.0000)
T	0.0176	1.1865	0.2629	-0.6027	0.9844	0.5776	8.8948	1.0538
	(0.0092)	(0.0244)	(0.0200)	(0.2682)	(0.1895)	(0.3228)	(2.5718)	(0.7882)
TX	0.0091	0.4130	0.2345	0.1508	0.3736	5.2145	-6.2191	9.7072
	(0.0132)	(0.0142)	(0.0153)	(0.1142)	(0.7392)	(4.2459)	(116.6594)	(0.0212)
UK	0.0547	0.2907	0.6520	-0.0651	3.0031	5.8972	153.0128	3.0272
	(0.0226)	(0.0147)	(0.0461)	(0.0762)	(0.9199)	(5.2340)	(74.0144)	(0.3874)
UTX	0.0272	0.2269	0.6691	0.0166	0.2470	0.1160	2.9236	5.6803
	(0.0229)	(0.0049)	(0.0309)	(0.0035)	(0.0571)	(0.1468)	(2.8872)	(0.1282)
WX	0.0678	0.2948	0.5140	-0.0146	0.8064	1.0491	6.1871	14.1237
	(0.0193)	(0.0071)	(0.0273)	(0.0128)	(0.1307)	(0.4570)	(2.7685)	(0.0027)
XON	0.0439	0.6736	0.3040	-0.1438	0.5702	0.4458	2.1513	2.7873
	(0.0149)	(0.0096)	(0.0171)	(0.0724)	(0.1607)	(0.2035)	(1.1375)	(0.4256)
Z	0.0831	0.1014	0.4213	-0.0011	1.0851	0.3598	11.6295	13.9311
	(0.0168)	(0.0030)	(0.0243)	(0.0014)	(0.1406)	(0.2359)	(4.8621)	(0.0030)

Table 3: Summary Statistics – Intraday Data

Table 3 presents summary statistics for intraday data on price changes and volume across different hours during the day. Table 3A presents the mean correlation matrix across the hours for both price changes (i.e. the upper triangular portion) and volume (i.e. the lower triangular portion) for the Dow Jones 30 firms. Table 3B reports the first-order daily autocorrelation of price changes and volume for each hour across the 30 firms. Table 3C provides the average correlation of price changes and volume for the Dow Jones firms across the different hours. The time period covered is between 1982-1986 (i.e. 1225 observations).

Table 3A: Correlation Matrix

	Hour 1	Hour 2	Hour 3	Hour4	Hour 5	Hour 6
Hour 1		-.035	.082	.055	.012	.015
Hour 2	.382		.027	.023	-.007	.021
Hour 3	.309	.277		.002	.006	.043
Hour 4	.285	.239	.274		-.017	.036
Hour 5	.279	.243	.244	.265		.025
Hour 6	.259	.224	.213	.221	.261	

Table 3B: 1st Order Autocorrelation

Variable	Hour 1	Hour 2	Hour 3	Hour4	Hour 5	Hour 6
Price Change	.036	.019	.026	-.011	-.037	-.022
Volume	.305	.183	.167	.177	.168	.171

Table 3C: Cross-Correlation across Dow Jones 30

Variable	Hour 1	Hour 2	Hour 3	Hour4	Hour 5	Hour 6
Price Change	.212	.238	.239	.219	.323	.327
Volume	.179	.100	.108	.124	.113	.256

Table 4: Summary Results for Skewness Tests

Table 4 provides summary information on tests for the mixture of distributions model based on skewness restrictions implied by the model. The mean, weighted mean and median of the parameter estimates across the Dow Jones 30 firms are given in columns 2-8. Summary information on tests for equality of the parameters across the six trading hours are provided in the last three columns of the Table (columns 9-11). Below the estimates, Table 4 provides the average overidentifying test statistic, corresponding average p-value and % of rejections across the Dow Jones sample for each trading hour. Also provided are the tests for whether the information flow follows a given distribution — in particular, the table presents the average value of the restriction, the average p-value and % of rejections.

	Hour 1	Hour 2	Hour 3	Hour 4	Hour 5	Hour 6	Daily	Av. χ^2_5	Av. p-value	% Reject
	Mean, Weighted Mean and Median of Parameters							Equality Tests of Parameters		
μ_p^*	-.0069	-.0029	.0117	-.0040	-.0044	.0240	.0211			
	-.0056	-.0029	.0091	-.0029	-.0038	.0198		27.278	.0168	95.83
	-0.0080	-0.0035	0.0120	-0.0016	-0.0034	0.0278	0.0070			
σ_p^{*2}	.0938	.0573	.0456	.0411	.0462	.0637	.3578			
	.0739	.0467	.0368	.0358	.0399	.0514		555.09	.0000	100.00
	0.0728	0.0446	0.0350	0.0293	0.0364	0.0424	0.2710			
μ_v^*	.1201	.0760	.0617	.0542	.0786	.1531	.6905			
	.0945	.0546	.0471	.0403	.0512	.0879		126.687	.0000	100.00
	0.1209	0.0685	0.0540	0.0408	0.0689	0.1098	0.6279			
σ_v^{*2}	.0055	.0038	.0021	.0017	.0021	.0013	.0539			
	.0019	.0013	.0006	.0007	.0009	.0006		12.373	.1577	52.17
	0.0027	0.0015	0.0006	0.0007	0.0010	0.0006	0.0228			
m_2^*	.4312	.4719	.6640	.6369	.f4392	.7696	.3719			
	.3740	.4301	.5845	.5193	.3962	.6908		12.986	.0883	62.50
	0.3962	0.4450	0.6178	0.5024	0.3987	0.7040	0.3721			
m_3^*	7.8575	11.2618	13.0754	24.7827	14.3542	5.3712	3.6229			
	1.8287	2.1008	2.3304	1.7632	1.9912	2.0237		11.572	.1683	50.00
	2.7525	4.4967	3.2811	2.2533	3.3771	1.9188	1.0115			
Test of MODM										
χ_3^2	5.2129	4.7957	4.7548	6.5401	5.2097	8.9589	10.2664			
Av. p-value	0.2985	0.2829	0.3297	0.2336	0.3010	0.1444	0.1311			
% Reject	26.6667	13.3333	20.0000	34.6154	23.3333	51.8519	60.000			
	Averages for Poisson information									
$m_3^* - m_2^{*2}$	7.6339	10.9904	12.5561	24.2763	14.0938	4.6956	3.2252			
P. Value	0.0730	0.1524	0.1829	0.1208	0.0930	0.2515	0.1111			
% Reject	76.6667	60.0000	53.3333	61.5385	70.0000	37.0370	66.6667			
	Averages for Symmetric information									
m_3^*	7.8575	11.2618	13.0755	24.7827	14.3542	5.3712	3.6229			
P. Value	0.0674	0.1360	0.1598	0.0930	0.0899	0.2275	0.1014			
% Reject	76.6667	60.0000	60.0000	69.2308	70.0000	37.0370	73.3333			
	Averages for Lognormal information									
$m_3^* - m_2^{*3} - 3m_2^{*2}$	7.0470	10.2701	11.0388	22.7869	13.4079	2.6817	2.3410			
P. Value	0.1672	0.1615	0.3285	0.3379	0.1699	0.3109	0.1887			
% Reject	36.6667	50.0000	20.0000	15.3846	40.0000	22.2222	43.3333			

Table 5: Summary Results for Kurtosis Tests

Table 5 provides summary information on tests for the mixture of distributions model based on kurtosis restrictions implied by the model. The mean, weighted mean and median of the parameter estimates across the Dow Jones 30 firms are given in columns 2-8. Summary information on tests for equality of the parameters across the six trading hours are provided in the last three columns of the table (columns 9-11). Below the estimates, Table 4 provides the average overidentifying test statistic, corresponding average p-value and % of rejections across the Dow Jones sample for each trading hour. Also provided are the tests for whether the information flow follows a given distribution — in particular, the table presents the average value of the restriction, the average p-value and % of rejections.

	Hour 1	Hour 2	Hour 3	Hour 4	Hour 5	Hour 6	Daily	Av. χ^2_5	Av. p-value	% Reject
	Mean, Weighted Mean and Median of Parameters							Equality Tests of Parameters		
μ_p^*	-.0005	.0022	.0139	-.0012	.0027	.0290	.0476			
	.0019	.0024	.0476	.0369	.0339	.0427		30.32	.0168	94.12
	-0.0013	0.0042	0.0105	0.0044	0.0017	0.0345	0.0458			
σ_p^{*2}	0.0928	0.0526	0.0456	0.0413	0.0471	0.0664	0.3689			
	.0743	.0476	.0369	.0339	.0427	.0521		688.33	.0000	100.00
	0.0699	0.0451	0.0364	0.0410	0.0358	0.0632	0.2948			
μ_v^*	0.1223	0.0775	0.0604	0.0564	0.0816	0.1624	0.7310			
	.0986	.0645	.0553	.0471	.0684	.1098		135.47	.0000	100.00
	0.1176	0.0780	0.0545	0.0632	0.0704	0.1782	0.6520			
σ_v^{*2}	0.0012	0.0001	0.0001	0.0015	0.0002	-0.0006	-0.0471			
	.0004	.0000	.0001	.0002	-.0002	-.0002		15.09	.0668	70.59
	0.0011	0.0006	0.0003	0.0014	0.0001	-0.0001	-0.0141			
m_2^*	0.8707	1.1275	1.2608	1.0275	1.0955	1.3574	0.8697			
	.7053	.9065	.9874	.7916	.9399	1.1548		18.47	.0743	82.35
	0.7995	1.0046	1.2057	1.1571	1.0411	1.4671	0.8064			
m_3^*	1.0959	0.7808	1.2182	1.1023	0.9034	2.1626	1.1963			
	.7334	.6461	1.0478	.7495	.7240	1.4777		7.59	.2681	11.76
	0.6486	0.7514	1.1595	1.0237	0.8937	2.0834	0.6717			
m_4^*	111.0497	123.8348	210.7582	150.3827	255.5646	93.8035	28.0494			
	9.8446	13.0188	13.5704	7.1327	12.4004	15.9974		6.81	.3734	5.88
	12.9880	46.5823	31.5589	19.6952	40.0682	29.4073	5.5859			
Test of MODM										
χ_3^2	4.9606	4.8466	3.1985	6.1394	4.9530	5.0006	7.2647			
P. Value	0.3264	0.2821	0.4191	0.2296	0.3126	0.3763	0.2434			
% Reject	24.1379	11.1111	0.0000	21.7391	20.6897	8.3333	35.7143			
	Averages for Poisson information									
$Q(I^*)$	105.1846	16.6427	201.3003	142.8530	765.6338	82.5186	23.1340			
P. Value	0.2259	0.1703	0.1951	0.2082	0.1919	0.3143	0.1802			
% Reject	37.9310	66.6667	39.2857	43.4783	51.7241	25.0000	47.8571			
	Averages for Symmetric information									
$Q(I^*)$	108.8197	21.4689	207.2710	147.5469	772.0443	89.6797	26.1562			
P. Value	0.2163	0.1591	0.1801	0.1885	0.1776	0.3006	0.1592			
% Reject	41.3793	62.9630	50.0000	47.8261	55.1724	33.3333	46.4286			
	Averages for Lognormal information									
$Q(I^*)$	-67.8419	-228.8004	-185.7955	-85.8307	223.1166	-326.9036	-148.1564			
P. Value	0.3950	0.4371	0.4571	0.4297	0.3911	0.4486	0.3438			
% Reject	10.3448	18.5185	0.0000	17.3913	10.3448	0.0000	0.0000			
	Averages for Inverted Gamma information									
$Q(I^*)$	118.7652	55.0092	252.0893	156.4119	804.6394	128.0283	85.3581			
P. Value	0.2958	0.2361	0.1653	0.2561	0.1863	0.2463	0.2559			
% Reject	48.2759	51.8519	50.0000	34.7826	65.5172	54.1667	50.0000			

Table 6: Summary Results for Combined Skewness and Kurtosis Tests

Table 6 provides summary information on tests for the mixture of distributions model based on both skewness and kurtosis restrictions implied by the model. The mean, weighted mean and median of the parameter estimates across the Dow Jones 30 firms are given in columns 2-8. Summary information on tests for equality of the parameters across the six trading hours are provided in the last three columns of the Table (columns 9-11). Below the estimates, Table 4 provides the average overidentifying test statistic, corresponding average p-value and % of rejections across the Dow Jones sample for each trading hour.

	Hour 1	Hour 2	Hour 3	Hour 4	Hour 5	Hour 6	Daily	Av. χ^2_5	Av. p-value	% Reject
	Mean, Weighted Mean and Median of Parameters							Equality Tests of Parameters		
μ_p^*	-0.0062	-0.0037	0.0057	-0.0045	-0.0041	0.0162	-0.0001			
	-.0068	.0001	.0035	-.0044	-.0029	.0138		26.72	.0231	83.33
	-0.0063	-0.0039	0.0042	-0.0002	-0.0036	0.0227	0.0010			
σ_p^{*2}	0.0925	0.0560	0.0457	0.0426	0.0469	0.0630	0.3846			
	.0716	.0481	.0441	.0445	.0435	0538		771.46	.0000	100.00
	0.0694	0.0479	0.0363	0.0496	0.0355	0.0540	0.3493			
μ_v^*	0.1158	0.0763	0.0577	0.0578	0.0757	0.1429	0.7356			
	.0920	.0603	.0444	.0419	.0509	.1012		159.92	.0000	100.00
	0.1125	0.0756	0.0519	0.0656	0.0667	0.1462	0.8373			
σ_v^{*2}	0.0032	0.0014	0.0013	0.0008	0.0009	0.0008	0.0185			
	.0013	.0007	.0006	.0005	.0006	.0006		22.19	.0448	83.33
	0.0018	0.0007	0.0005	0.0005	0.0005	0.0006	0.0140			
m_2^*	0.3888	0.5010	0.6717	0.5931	0.4910	0.6400	0.3618			
	.3523	.4507	.5622	.5156	.4603	.5875		19.71	.0327	72.22
	0.4080	0.5058	0.6324	0.6746	0.5199	0.6830	0.3765			
m_3^*	0.8582	1.3436	2.1511	1.3465	1.1109	1.4579	0.6418			
	.5677	.7645	1.1230	.9133	.6861	1.0781		17.48	.0866	66.67
	0.5910	0.9295	1.5413	1.6798	0.8124	1.2693	0.5115			
m_4^*	-11.0496	200.3871	-51.2905	-6.5571	204.1630	14.5110	1.4293			
	-.5360	-.1418	.3440	1.7655	-.1202	3.3733		10.44	.1644	27.78
	-0.8110	-0.5602	1.7126	4.4923	0.2341	5.1768	1.0490			
Test of MODM										
χ^2_7	15.8491	16.5577	16.9708	15.6159	21.3753	24.2661	24.6613			
P. Value	0.0698	0.0631	0.0665	0.1118	0.0453	0.0509	0.0231			
% Reject	57.1429	57.6923	64.2857	47.6190	74.0741	88.0000	86.9565			

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