LIMITED MARKET PARTICIPATION AND VOLATILITY OF ASSET PRICES

by

Franklin Allen
Douglas Gale

2-92
(revision of 14-91)

RODNEY L. WHITE CENTER FOR FINANCIAL RESEARCH
The Wharton School
University of Pennsylvania
Philadelphia, PA 19104-6367

LIMITED MARKET PARTICIPATION

AND

VOLATILITY OF ASSET PRICES*

by

Franklin Allen

University of Pennsylvania

and

Douglas Gale

Boston University

Latest Revision: November 1, 1991

*We would like to thank Nancy Stokey, David Hirshleifer and participants of seminars at Baruch College, Duke, the Minneapolis Federal Reserve, Princeton, Temple, the University of Pennsylvania and UCLA for helpful comments. Financial support from the National Science Foundation is gratefully acknowledged by both authors.

ABSTRACT

Traditional theories of asset pricing assume there is complete market participation, in the sense that all investors participate in all markets. In that case, preference shocks typically have only a small effect on asset prices and are not an important determinant of asset price volatility. However, there is considerable empirical evidence that most investors participate in a limited number of markets. We show that limited market participation can amplify the effect of preference shocks, so that an arbitrarily small degree of aggregate uncertainty about preferences causes a large degree of price volatility. We also show there may exist Pareto-ranked equilibria, where the Pareto-preferred equilibrium is characterized by a different pattern of participation and lower volatility.

1. INTRODUCTION

The prices of financial assets such as stocks are volatile in comparison to most other prices. There are at least two possible explanations of this volatility. One is that it is caused by the arrival of new information about payoff streams and discount rates. Another is that changes in preferences lead to liquidity trading and this causes changes in asset prices. Traditional analyses of efficient markets have focused on the arrival of new information as the cause of asset price volatility and have placed hardly any emphasis on changes in preferences (see, e.g., Fama (1970) and Merton (1987a)).

Why is this? Standard models, such as the capital asset pricing model, assume there is complete participation: all traders invest in all markets and everyone helps to absorb the shocks caused by liquidity trades. There is continuity in the sense that small changes in preferences cause small changes in asset prices. Since short-run changes in preferences are usually assumed to be small, the implication is that these are of minor importance in determining asset-price volatility. In contrast, it is plausible to assume that information about payoff streams and discount rates changes significantly in the short run and that this causes the large changes in asset prices that we observe.

However, there is extensive empirical evidence that the assumption of complete market participation is not justified. Most investors do not diversify across different classes of assets or across different stocks. For example, King and Leape (1984) analyze data from a 1978 survey of 6,010 US households with average wealth of almost $250,000. They categorize assets into 36 classes and find that the median number owned is 8. In a more recent study, Mankiw and Zeldes (1991) find that only a small proportion of consumers hold stocks; more surprisingly, perhaps, even among those with large liquid wealth, only a fairly small proportion own stocks; of those with liquid assets in excess of $100,000, only 47.7 percent hold stocks.

Other studies have found that investors' diversification within equity portfolios is also limited. Blume, Crockett and Friend (1974) develop a measure of portfolio diversification which takes into account the proportion of stocks held in individuals' portfolios. Using this measure, they find the average amount of diversification is equivalent to having an equally weighted portfolio with two stocks. Blume and Friend (1978) provide more detailed evidence of this lack of diversification. They find that a large proportion of investors have only one or two stocks in their portfolios and very few have more than ten. This observation cannot be explained by the argument that investors are mainly holding mutual funds. In King and Leape's (1984) study, only 1 percent of investors' wealth was in mutual funds compared to 22.3 percent held directly in equities. The Blume, Crockett and Friend (1974) and Blume and Friend (1978) studies are from an earlier period when it is likely that the ownership of mutual funds was an even smaller proportion of wealth, given the growth in mutual fund holdings that has occurred.

How can this limited market participation be explained? One possibility is that there is a fixed setup cost of participating in a market. In order to be active in a market, it is necessary to devote resources initially to learning about the basic features of the market, such as the distribution of asset returns and so forth. Once an asset has been purchased, it is necessary to monitor its performance. Brennan (1975) has shown that with fixed setup costs of this kind, it is only worth investing in a limited number of assets. King and Leape (1984) find evidence that is consistent with this type of theory.

The purpose of the present paper is to investigate the effect of limited market participation and preference shocks on the volatility of asset prices.

We derive two main results. The first is that limited market participation, liquidity preference and risk aversion combine to amplify the effects of changes in preferences on asset prices. In particular, we show that an arbitrarily small degree of aggregate uncertainty in preferences can cause a large degree of volatility. The second result is that there can exist multiple Pareto-ranked equilibria. In one equilibrium asset prices are volatile. In another equilibrium, which is Pareto-preferred, asset prices are not very volatile.

We derive these results in a simple model with three dates. Investors have random preferences of the type introduced by Diamond and Dybvig (1983). They wish to concentrate all their consumption either at the second date or at the third date but they do not learn which until the second date. In addition, the proportion who will be early consumers is a random variable which is not realized until the second date. It is this aggregate uncertainty about the proportion of early consumers that generates the price volatility in the model. There are two types of investors. The first type have a high probability of being an early consumer; as a result, they have a high preference for liquidity. The second type have a low probability of being an early consumer so they have a low preference for liquidity.

There are two financial markets. In the first market, a short-term asset which produces a payoff the date after purchase is traded. In the second market, a long-term asset which only pays off the second date after purchase is traded. The return on the long-term asset is higher than the return on a sequence of short-term assets.

Limited market participation is captured by assuming that investors are initially free to choose the market in which they want to invest but once this choice has been made, they cannot participate in the other market. Thus, at the first date, agents invest their wealth in one of these two assets. If they invest in the short-term asset they can consume at the second date or reinvest their funds and consume at the third date. If they invest in the long-term asset, they can sell it to pay for consumption at the second date, or they can hold it and consume the payoff at the third date.

The first result—that a small amount of aggregate uncertainty in preferences can lead to a large amount of price volatility—arises because investors with a high preference for liquidity are risk averse and reluctant to participate in the volatile market for the long-term asset. They fear that if they have to liquidate their holdings unexpectedly they may receive a low price. As a result, the market may be dominated by investors who have a low preference for liquidity and are much less risk averse. It is optimal for these investors to hold small reserves of cash. Since the amount of money available in the market determines asset prices, small variations in the proportion of investors who are early consumers causes a significant amount of price volatility. Although there are investors in the economy holding large reserves of cash, which could potentially be used to dampen this volatility, they cannot be called upon to do so at short notice, because they have chosen not to participate in this market.[1]

In the equilibrium described, low participation and illiquidity in the long-term market are self-sustaining phenomena. The second result—the existence of multiple, Pareto-ranked equilibria—occurs because the converse is also true: a high level of participation and liquidity in the long-term market can also be self sustaining. The second equilibrium is Pareto preferred to the first because all investors benefit from the higher returns to the long-lived asset and still enjoy reasonable liquidity.

LeRoy and Porter (1981) and Shiller (1981), among others, have argued that asset prices are characterized by *excess volatility*: they are more volatile than changes in payoff streams and discount rates would predict. A number of authors have suggested that the degree of excess volatility found by these studies can be attributed to the use of inappropriate econometric techniques (see Merton (1987a) and West (1988) for surveys of the empirical literature on excess volatility). It is also possible that there may be a number of other determinants of asset-price volatility.[2] The first result above suggests that when there is limited market participation, even very small changes in preferences can lead to a significant degree of price volatility. The second result suggests that in addition to "statistical" excess volatility, there should also be concern about excess volatility in a welfare sense. The fact that the prices of financial assets are more volatile than other prices is not per se undesirable. However, if there exists a Pareto-preferred equilibrium with lower volatility, we can say that high volatility represents a market failure.

Our analysis is related to a number of other papers in the literature. The assumption of a fixed cost of participating in a market has been used in various settings. The closest are Merton (1987b), Hirshleifer (1988) and Cuny (1990). In Merton (1987b), there is a fixed cost of acquiring information about the returns to an asset, which causes traders to invest in a limited number of assets. Among other things, Merton shows that this can lead to an empirically significant effect on asset returns. Hirshleifer (1988) assumes there is a fixed cost of participating in a futures market. These costs result in thin markets, which in turn cause the residual risk to be priced. In this case, equilibrium risk premia are quite different from those in perfect, frictionless markets. Cuny (1990) considers the optimal design of futures contracts assuming investors can only participate in one market. This assumption causes liquidity to vary across markets, as in our model, and makes the design problem a nontrivial one.[3]

Pagano (1989) develops a related model which captures the relationship between market thinness and volatility. In his model, a finite number of investors receive idiosyncratic demand shocks. Thus, individual demands are more volatile than aggregate demands. If investors believe that the market will have few traders and will be volatile, only a few traders enter. If investors believe that a market will have many traders and stable asset prices, many traders enter. In each case, beliefs will be self-fulfilling because the volatility of demand and hence prices is inversely related to the number of traders. The theory is thus able to explain the well-documented empirical finding that thin markets are more volatile than thick ones (see, e.g., Cohen et al. (1976), Telser and Higimbotham (1977), Pagano (1986) and Tauchen and Pitts (1983)). The empirical studies of excess volatility are based on data from the NYSE and other major exchanges. Pagano's explanation of high volatility may not be relevant to these markets, which are usually considered to be thick. In contrast, what matters in our model is the kind of trader who enters a market. Markets are always thick because there is a large number of traders of each type. Prices are volatile when the only investors in the market have a low preference for liquidity.

The rest of the paper is organized as follows. The model is described in Section 2. In Section 3, the benchmark case where there is no aggregate uncertainty is analyzed and in Section 4 aggregate uncertainty is introduced. Section 5 presents an example. Finally, Section 6 contains concluding remarks.

2. THE MODEL

To highlight the nature of the results, a very stylized model is used.

(A.1) There are three dates, indexed by $t = 0,1,2$.

We assume investors' preferences are subject to random shocks as in Diamond and Dybvig (1983).

(A.2) There are two types of investor, $i = A,B$. The proportion of type i who want to consume at date 1 is λ_i; the proportion who want to consume at date 2 is $1-\lambda_i$. The proportions λ_i may be random variables, which are not realized until date 1. Their means are $\bar{\lambda}_i$. Expected utility at date 0 is

$$EV_i(C_{i1},C_{i2}) = E(\lambda_i U_i(C_{i1}) + (1-\lambda_i)U_i(C_{i2})), \qquad (1)$$

where C_{it} is consumption at date t and $U_i(.)$ is a continuous, concave and strictly increasing von Neumann-Morgenstern utility function.

There is assumed to be a continuum of investors of each type. An investor's probability of being an early consumer is equal to the proportion of his type who are early consumers. Thus, at date 1, when the proportion of type i investors who are early consumers is realized, the probability of being an early consumer is equal to λ_i. At date 0, before λ_i is realized, the probability of a type i investor being an early consumer is equal to the expected proportion of early consumers, i.e. $\bar{\lambda}_i$.

For the purposes of discussion, it is helpful to think of type A's as being risk averse and $\bar{\lambda}_A$ being large so they have a high liquidity preference. Type B's, on the other hand, are less risk averse and have a small value of $\bar{\lambda}_B$ so their liquidity preference is low. These assumptions are not necessary; we can obtain similar results under a variety of assumptions.

(A.3) There are three financial assets:

(i) *money* with return 1;

(ii) a *short-term asset* with return $R_S > 1$ one date after purchase;

(iii) a *long-term asset* which trades at price P one date after purchase and has return $R_L > R_S^2 > 1$ two dates after purchase.

Money serves as the numeraire. One unit of money held at date t yields one unit of money at date t+1, for t = 0,1. Initially, investors are endowed with an amount of money W_i.

The short and long-term assets are financial claims on real investments. The supply of the short-term asset is therefore endogenous at dates 0 and 1 and the supply of the long-term asset is endogenous at date 0 but fixed at date 1. The returns to the assets are determined by the technologies underlying the real investments and are measured in terms of money.[4] One unit of money invested in the short-term asset at date t yields $R_S > 1$ units of money at date t+1, for t = 0,1. One unit of money invested in the long-term asset at date 0 yields $R_L > R_S^2 > 1$ units of money at date 2. The long-term asset does not yield a return at date 1 but can be traded at price P in a financial market.

The timing of returns is illustrated in Figure 1. In order to focus on the uncertainty due to changes in preferences, the payoffs on both assets are assumed to occur with certainty.

(A.4) There is no borrowing of money and there are no short sales of the short and long-term assets.

To capture the fact that there are fixed costs of investing in financial assets, we assume the market has a special structure. We assume the fixed costs make it prohibitively expensive to invest in both assets, which leads to limited market participation. To keep things simple, the setup costs are not modeled explicitly. Instead, we assume that the market structure allows the investor to invest in one or other of the assets but not both.

(A.5) At date 0, investors must choose whether to invest in the market for the short-term asset S or the market for the long-term asset L. They cannot invest in both and it is not possible to switch markets once a choice has been made.

Investors' choice of markets is illustrated in Figure 2. Once they choose which market to enter at date 0, the two markets are segregated. First consider market S, which is shown in the upper branch of the diagram. At date 0 investors can invest either in S or in money. Since $R_S > 1$, it is optimal for them to invest entirely in S. At date 1, investors find out whether they are early consumers or late consumers. If an investor is an early consumer, he consumes the entire liquidation value of his portfolio $C_{i1} = R_S W_i$. If the investor is a late consumer, he reinvests his entire portfolio in S and at date 2 he consumes the total return to his portfolio, $C_{i2} = R_S^2 W_i$.

In market L, which is illustrated in the lower branch of Figure 2, investors can invest either in L or in money. At date 0, they choose a portfolio which is denoted (ℓ_i, m_i), where ℓ_i and m_i are type i's holding of L and money, respectively. At date 1, they find out whether they are early consumers or late consumers. If an investor is an early consumer, he liquidates his holdings of L at price P and this together with his holding of money gives him consumption $C_{i1} = m_i + P\ell_i$. If the investor is a late consumer, he can use his money to buy m_i/P units of L at date 1 and consume $C_{i2} = (m_i/P + \ell_i)R_L$ at date 2. It is worth holding money in this market because when $P < 1$, L can be bought at a lower price at date 1 than at date 0. The lower P is, the more attractive holding money is.

Equilibrium in the long-term market is not quite as straightforward as equilibrium in the short-term market. At date 1, a proportion λ_i of type i investors who are participating in the market sell their holdings of L and consume. The remainder use their money to buy this supply of L. The equilibrium level of P depends on the relative number of people of each type in the market, the proportion of each type λ_i who sell the long-term asset, and their initial portfolios (ℓ_i, m_i). It is important to note that $P \leq R_L$, since otherwise everybody would prefer money to L at date 1 and the market would not clear.

So far, we have considered equilibrium in the short and long-term markets in isolation. In order to ensure overall equilibrium, it is necessary that investors choose the market where their expected utility will be maximized.

Given that equilibrium in the short-term market is assured since S dominates money, the three conditions that characterize overall equilibrium are:

(E.1) P clears the long-term market at date 1.

(E.2) The portfolios of investors in the long-term market maximize their expected utility at date 0.

(E.3) Investors choose which market to enter at date 0 in order to maximize their expected utility.

Having outlined the basic model and defined equilibrium, we next consider the case where the parameters λ_i are non-stochastic. This helps to fix ideas and illustrate how the model works.

3. EQUILIBRIUM WITH NO AGGREGATE UNCERTAINTY

When λ_A and λ_B are non-stochastic, and $0 < \lambda_i < 1$ for i = A, B, there is uncertainty for individuals but no aggregate uncertainty. The determinants of P, the price of the long-term asset at date 1, are known at date 0. So at date 0, investors can deduce what the equilibrium value of P must be.

At date 1, early consumers will want to sell their holdings of L. For the market to clear, there must be investors holding money which they can use to buy L. To see this, suppose that nobody held money; then at date 1 there would be an excess supply of L. This cannot be an equilibrium unless P = 0. However, at this price it would be worth holding money between dates 0 and 1 in order to buy a large amount of L at date 1. Hence, in equilibrium some investors must be holding money with which to buy L.

Investors will hold money between dates 0 and 1 if they are indifferent between money and L, that is, if

$$P = 1. \tag{2}$$

In fact, this is the only equilibrium price. If $P > 1$, L dominates money between dates 0 and 1 and nobody will hold money. This is inconsistent with equilibrium, since at date 1 there would be an excess supply of L. If $P < 1$ then money dominates L between dates 0 and 1, everybody will hold money and the stock of L will be zero. However, at date 1 everybody will try to switch to L, since this dominates money between dates 1 and 2. This too is inconsistent with equilibrium, since we know the supply of L will still be zero at date 1.

When $P = 1$, investors are indifferent between money and L between dates 0 and 1. However, they strictly prefer L between 1 and 2, since it pays $R_L > 1$ whereas money pays only 1. In equilibrium, investors therefore hold just enough money so that when the early consumers come to sell their holdings of L at date 1, the late consumers have exactly enough money to purchase them at $P = 1$. If investors held any more or less money than this the market would not clear at date 1.

Given $P = 1$, it can be seen that the utility of type i in market L, is given by

$$V_{iL} = \lambda_i U_i(W_i) + (1 - \lambda_i) U_i(W_i R_L). \qquad (3)$$

For type i in market S,

$$V_{iS} = \lambda_i U_i(W_i R_S) + (1 - \lambda_i) U_i(W_i R_S^2). \qquad (4)$$

Equilibrium can have a number of different forms, depending on the ranking of V_{iL} and V_{iS} as illustrated in Table 1. A *separating* equilibrium occurs when the different types choose different markets. The fact that type A's have a higher value of λ than type B's means that the separating equilibrium will have type A's enter market S and type B's enter market L, as shown in Table 1. A *pooling* equilibrium occurs when both types enter market S or market L. A *partial pooling* equilibrium occurs when both types enter one of the markets and one type enters the other market. In the example shown in Table 1, type B's are indifferent between the two markets and some may participate in S and some in L. Type A's on the other hand strictly prefer S and will only participate in that market.

It can be seen from (3) and (4) that the parameters R_S, R_L and W_i uniquely determine V_{iS} and V_{iL}. Hence the form of equilibrium is uniquely determined except in the special case where the parameters are such that $V_{AS} = V_{AL}$ or $V_{BS} = V_{BL}$ and all three types of equilibria exist. Thus, when there is no aggregate uncertainty, equilibria are generically either separating or pooling. We mainly focus on the pure separating and pooling cases in the sequel.

The preceding discussion is summarized in a proposition.

Proposition 1: Suppose that $0 < \lambda_i < 1$ is non-stochastic for i = A,B. Then equilibrium exists and equilibrium expected utilities are uniquely determined. Generically, equilibrium is unique and must be either separating or pooling.

When there is no aggregate uncertainty in preferences the price of the long-term asset is constant at $P = 1$. It is shown in the next section that even a small amount of aggregate uncertainty can result in a significant degree of price volatility.

4. EQUILIBRIUM WITH AGGREGATE UNCERTAINTY

In this section we assume that λ_B, the proportion of type B investors that consume at date 1, is a random variable. This means that in addition to uncertainty at the individual level there is also uncertainty at the aggregate level, since at date 0 nobody knows the proportion of type B's who will be early consumers.

(A.6) There is no aggregate uncertainty concerning the preferences of type A's. Formally, $\lambda_A = \bar{\lambda}_A$ with probability one, for some constant $0 < \bar{\lambda}_A < 1$.

(A.7) There is aggregate uncertainty about the preferences of type B's: λ_B is a random variable with mean $0 < \bar{\lambda}_B < 1$. $\lambda_B = \theta/(Z+\theta)$, where $Z > 0$ is a constant and θ is a (non-degenerate) random variable with c.d.f. F. The support of F is contained in the compact interval $[a,b]$ where $0 \le a < b$.

Separating equilibrium with volatile asset prices

Our first result is to show that an arbitrarily small amount of aggregate uncertainty about preference shocks can support a large amount of price volatility in market L. This is proved for the case where $Z \to \infty$ so the distribution of λ_B collapses toward 0.[5]

Proposition 2: Consider an economy where (A.1) - (A.7) are satisfied. If type A investors are sufficiently risk averse, and Z is large enough so λ_B is sufficiently close to zero, there will exist a separating equilibrium in which asset prices are volatile and type A's choose market S but type B's choose market L.

Proof: See Appendix.

We can illustrate Proposition 2 using the special case in which the distribution of λ_B has a two-point support:

(A.7')
$$\lambda_B = \begin{cases} \epsilon \text{ with probability } \pi > 0; \\ 0 \text{ with probability } (1-\pi). \end{cases}$$

Suppose that (A.7') is satisfied and consider the sequence of parameter values as $\epsilon \to 0$. We shall show that for ϵ sufficiently small, there exists a separating equilibrium in which type A's choose market S and type B's choose market L. We begin by stating an existence result.

Lemma 1: For any distribution of investors across markets, there exist prices and portfolios that satisfy equilibrium conditions (E.1) and (E.2).

Proof: See Appendix.

Now suppose there is a given distribution in which all investors of type A are assigned to market S and all investors of type B to are assigned to market L. Lemma 1 guarantees the existence of market-clearing prices for this distribution of investors between markets. It remains to show that as $\epsilon \to 0$, this distribution of investors is optimal, i.e., satisfies (E.3).

Since λ_B has a two-point distribution, there are two possible values of P. We use the following notation.

$$P = \begin{cases} P_\epsilon \text{ when } \lambda_B = \epsilon; \\ P_0 \text{ when } \lambda_B = 0. \end{cases} \tag{5}$$

First consider market L and suppose that only type B's have entered. When $\lambda_B = \epsilon$, early consumers sell their holdings of L at date 1. In order for the market to clear, these investors must hold money between dates 0 and 1. If not, there would be an excess supply of L at date 1 and the price of L would fall to zero. By the argument in the previous section, it is impossible to have $P_\epsilon = 0$ in equilibrium, since investors would then find it worthwhile to hold money and purchase large amounts of L. So equilibrium requires $P_\epsilon > 0$ and money must be held between dates 0 and 1.

When $\lambda_B = 0$ there are no early consumers at $t = 1$ and hence no sellers of L. Then the money investors hold between dates 0 and 1 is carried over to date 2. The equilibrium price P_0 must be such that the investors are indifferent between holding money and L between dates 1 and 2. This requires $P_0 = R_L$. At any other price, one of the assets would be dominated.

Given $P_0 = R_L$, it follows that when $\lambda_B = \epsilon$, $P_\epsilon < 1$. Otherwise, L would dominate money between dates 0 and 1 for both realizations of λ_B and nobody would hold money. However, as argued above, it is necessary for equilibrium that money be held.

What is it that determines the equilibrium value of P_ϵ? The market-clearing condition when $\lambda_B = \epsilon$ is

$$\epsilon \ell_B P_\epsilon = (1-\epsilon) m_B. \tag{6}$$

The left-hand side represents the value of L sold by early consumers: a measure ϵ of investors each sells his holding ℓ_B of L at a price of P_ϵ. The right-hand side is the amount of money late consumers have to purchase L with: a measure $(1 - \epsilon)$ of investors each supplies his holding m_B of money. In contrast to standard theories, where asset prices are determined by discounted present values, here the price is determined by the amount of money held by participants in the market.

We use the market-clearing condition, together with the investors' first-order condition, to demonstrate the following result.

Lemma 2:

$$P_{\epsilon 0} = \lim_{\epsilon \to 0} P_\epsilon = \frac{\pi R_L}{R_L - (1 - \pi)} . \tag{7}$$

Proof: See Appendix.

It **can be seen from** Lemma 2 that when π is near 1, $P_{\epsilon 0}$ is also near 1. Intuitively, for large values of π, the probability that the state $\lambda_B = \epsilon$ occurs is relatively large and investors have an incentive to hold money in case they are early consumers. Hence P_ϵ is near 1. As π falls, the probability of being an early consumer falls, so the need for money falls. To induce investors to hold money, it must become relatively more attractive so P_ϵ falls.

Thus, even when ϵ is very small, prices can be highly volatile. In the limit, as $\epsilon \to 0$,

$$P = \begin{cases} P_{\epsilon 0} < 1 \text{ with probability } \pi; \\ R_L > 1 \text{ with probability } 1 - \pi. \end{cases} \tag{8}$$

By choosing ϵ small enough, it is possible to find an equilibrium with prices arbitrarily close to these.[6] Since prices are volatile it follows that if type A's are sufficiently risk averse they will prefer market S. Also, given $R_L > R_S^2$ from (A.3), it can be seen that if ϵ is small enough type B's will prefer market L. Hence, there will exist a separating equilibrium.

This result has been demonstrated for the case covered by (A.7′), where λ_B has a two-point support, but it is shown in the Appendix that the result holds for (A.7) when λ_B is continuously distributed. This is true even when the support does not contain $\lambda_B = 0$. What is important is that the distribution of λ_B converge to zero in the limit.

In our model, price changes are driven by preference shocks. Investors who learn that they need to consume early will liquidate their stock of the illiquid asset. With limited market participation, the amount by which the price changes depends on how much money the other traders in the market have available to buy the stock with. If there is a lot of money in the hands of traders, the market is liquid and prices may not change much. If there is little money, a small shock may cause a lot of volatility. Thus, volatility can be amplified by portfolio decisions.

What determines the amount of money traders decide to include in their portfolios? Type A traders have high liquidity preference because they think there is a high probability they will be early consumers. So they hold a lot of money. Type B traders have a low liquidity preference, because they think the probability will be low. So they hold little money.

This is where limited market participation, liquidity preference and risk aversion combine to generate high volatility. In the case considered in Proposition 2, the type A's with high liquidity preference have high risk aversion and the type B's with low liquidity preference have relatively low risk aversion. The equilibrium with high volatility arises because the expectation of high volatility will keep the type A traders who are highly risk averse out of the market. Since the only traders in the market are then type B's, who have low liquidity preference, the market will be illiquid and high volatility will become a self-fulfilling expectation.

Multiple, Pareto-ranked equilibria

Proposition 2 shows that under certain conditions there will exist a separating equilibrium in which the expectation of price volatility will keep type A's out of market L. It is interesting to contrast this equilibrium with that described in Proposition 1 where there is no aggregate uncertainty and the probability of being an early consumer is $\bar{\lambda}_A$ for type A and $\bar{\lambda}_B$ for type B. It follows from Proposition 1 that equilibrium is generically either pooling or separating in this case. Since $R_L > R_S^2$, type B's will prefer market L provided $\bar{\lambda}_B$ is sufficiently small. Whether the equilibrium involves pooling or separating then depends on whether type A's prefer market L, i.e. on the condition

$$\text{(A.8)} \qquad \bar{\lambda}_A U_A(W_A) + (1 - \bar{\lambda}_A)U_A(W_A R_L) > \bar{\lambda}_A U_A(W_A R_S) + (1 - \bar{\lambda}_A)U_A(W_A R_S^2).$$

If R_L is sufficiently large relative to R_S^2, (A.8) will be satisfied and the unique equilibrium will be pooling when there is no aggregate uncertainty.

Suppose that everything remains the same except λ_B is random with mean $\bar{\lambda}_B$. Proposition 2 tells us that under certain conditions there exists a different kind of equilibrium, a separating equilibrium in which type A's choose market S. Introducing aggregate uncertainty changes the nature of the equilibrium set: it adds a separating equilibrium that cannot exist in the absence of aggregate uncertainty.

In fact, if (A.8) is satisfied and there is a very small amount of aggregate uncertainty, the separating equilibrium described in Proposition 2 will not be the only equilibrium. There will also exist a pooling equilibrium. To see this, consider the unique pooling equilibrium for the economy with no aggregate uncertainty and parameters $\bar{\lambda}_A$ and $\bar{\lambda}_B$. Holding the distribution of investors across markets constant and introducing a small amount of uncertainty will only change asset price volatility by a small amount. If both types preferred market L initially, they will still prefer it and pooling in market L remains an equilibrium. Moreover, since price volatility is higher in the separating equilibrium, it is also natural to suppose that the pooling equilibrium will be Pareto preferred. We consider these possibilities next assuming (A.7′) is satisfied.

We start by demonstrating that when ϵ is small there exists a pooling equilibrium as well as a separating equilibrium. As before, we are interested in what happens as $\epsilon \to 0$. Suppose that both type A's and type B's enter market L. It follows from Lemma 1 that a price function and a pair of portfolios that satisfy equilibrium conditions (E.1) and (E.2) exist. It remains to show that (E.3) is satisfied and it is optimal for both A's and B's to enter market L.

Using N_i to denote the measure of investors of type i, the market-clearing condition when $\lambda_B = \epsilon$ (which is similar to (6)) gives the following expression for P_ϵ:[7]

$$P_\epsilon = \frac{N_A(1-\lambda_A)m_A + N_B(1-\epsilon)m_B}{N_A\lambda_A\ell_A + N_B\epsilon\ell_B} \quad (9)$$

A similar expression can be derived when $\lambda_B = 0$. However, in this case the constraint that P_0 is bounded above by R_L may bind (recall that if P_0 were above R_L nobody would hold asset L between dates 1 and 2). Hence:[8]

$$P_0 = \text{Min}\left\{R_L, \frac{N_A(1-\lambda_A)m_A + N_B m_B}{N_A \lambda_A \ell_A}\right\}. \tag{10}$$

When $\lambda_B = \epsilon$ there is less money available to purchase L than when $\lambda_B = 0$, since some of investors are spending their cash on consumption. As a result $P_\epsilon < P_0$ as shown by (9) and (10). By the usual arguments, both money and L must be held in equilibrium. Given this, it is not possible for both P_ϵ and P_0 to be above 1 or below 1, for this would mean that one of the assets was dominated and investors would not be willing to hold both. This implies $P_\epsilon \leq 1 \leq P_0$. From (9) and (10) it follows that,

$$\lim_{\epsilon \to 0} P_\epsilon = \lim_{\epsilon \to 0} P_0 = 1. \tag{11}$$

Hence, as $\epsilon \to 0$, the equilibrium prices tend to the equilibrium prices of the economy with no aggregate uncertainty discussed in Section 3. Condition (A.6) implies that it is optimal for both types to choose market L when λ_B is nonstochastic and $P = 1$. Thus, for sufficiently small ϵ, a pooling equilibrium exists in addition to the separating equilibrium when λ_B is stochastic.

The next step is to demonstrate that the pooling equilibrium is Pareto preferred to the separating equilibrium. It is clear that type A's are better off in the pooling equilibrium than the separating equilibrium since they could have chosen market S if it yielded them higher utility. It remains to show that type B's are better off.

In the separating equilibrium, a type B has expected consumption $EC_B^S = m_i + \ell_i R_L$ where (ℓ_i, m_i) is his optimal portfolio at time 0. When the probability that an investor of type B is an early consumer converges to zero, he will wish to take advantage of the high returns to holding asset L by putting more of his wealth in asset L and less in money. It can straightforwardly be shown using the investor's first-order condition for the choice of portfolio that $\lim_{\epsilon \to 0} \ell_B = W_B$ so

$$\lim_{\epsilon \to 0} EC_B^s = W_B R_L. \qquad (12)$$

In the pooling equilibrium $P_\epsilon \approx P_0 \approx 1$ for small ϵ. If a type B investor survives to be a late consumer his consumption therefore tends to $W_B R_L$. Hence, as his probability of being an early consumer goes to zero, his expected consumption also tends to $W_B R_L$:

$$\lim_{\epsilon \to 0} EC_B^p = W_B R_L. \qquad (13)$$

It follows from (12) and (13) that for small ϵ the expected consumption of the type B's is approximately the same in both the separating and the pooling equilibrium. However, in the separating equilibrium there is greater price volatility than in the pooling equilibrium and as a result consumption is more volatile. Hence, the type B's are better off in the pooling equilibrium. This gives the following result.

Proposition 3: If (A.1)-(A.6), (A.7′) and (A.8) are satisfied and the type A's are sufficiently risk averse then for sufficiently small ϵ, a pooling and a separating equilibrium exist. The pooling equilibrium is Pareto-preferred to the separating equilibrium.

Proof: See Appendix.

Equilibria usually come in odd numbers and that is the case here, too. For the same parameter values there will also be a partial pooling equilibrium in which some type A investors are found in each market. The number of investors of type A who enter market L will be just sufficient to reduce price volatility to the point where the type A investors are indifferent between the two markets.

Proposition 2 showed that limited market participation could amplify the effect of changes in preferences on stock price volatility and this has implications for the debate on statistical excess volatility. In contrast, Proposition 3 suggests that there may be excess volatility in a *welfare* sense. With limited market participation there can be multiple, Pareto-ranked equilibria. The high volatility equilibrium can be Pareto preferred to the low volatility equilibrium. This is the sense in which there may be a market failure.

5. AN EXAMPLE

The results above have been concerned with the case where λ_B is arbitrarily small. This is an extreme case but one that is convenient for deriving analytic results. We next consider a numerical example to illustrate that the qualitative results do not just hold for small λ_B. The parameter values for the example are shown in Table 2.

In order to understand the factors determining whether a pooling or separating equilibrium occurs when λ_B is nonstochastic, it is helpful to start by considering the case where there is no aggregate uncertainty. Suppose $\bar{\lambda}_B$ = 0.1. It follows from Proposition 1 that equilibrium is either separating or pooling, except for a negligible set of parameters. The effect of varying R_L on the type of equilibrium, while holding the other parameters constant, is illustrated in Figure 3.

If R_L is sufficiently small then both types will pool in market S. For $R_L < (R_S)^2 = 1.0609$, the short-term asset dominates the long-term one and both types will clearly prefer market S. As R_L increases and the long-term asset becomes relatively more attractive, type B's, who have a high chance of being late consumers, will at some point find it worthwhile to invest in market L. When $R_L = 1.0644$ the short-term and long-term assets are equivalent for type B's (i. e. $\bar{\lambda}_B U_B(W_B) + (1 - \bar{\lambda}_B)U_B(W_B R_L) = \bar{\lambda}_B U_B(W_B R_S) + (1 - \bar{\lambda}_B)U_B(W_B R_S^2)$) and there is a partial pooling equilibrium. For $R_L > 1.0644$ there is a separating equilibrium. As R_L increases further the long term asset becomes relatively more advantageous for type A's and when $R_L = 1.1027$ they are indifferent between the two (i.e., $\bar{\lambda}_A U_A(W_A) + (1 - \bar{\lambda}_A)U_A(W_A R_L) = \bar{\lambda}_A U_A(W_A R_S) + (1 - \bar{\lambda}_A)U_A(W_A R_S^2)$) so there is a partial pooling equilibrium again. For $R_L >$ 1.1027 there is pooling in market L.

The main case of interest is where (A.8) is satisfied (i.e., where $R_L >$ 1.1027 in the example for Figure 3). Figure 4 shows the types of equilibrium obtained for various values of ϵ and π when $R_L = 1.12$. For large values of ϵ, there is considerable volatility and as a result there is pooling in market S. For low ϵ and large values of π, it follows from Lemma 2 that there is relatively little price volatility. In this case there is a unique equilibrium which involves pooling in L. For low ϵ and π, there exist both separating and pooling equilibria. In most cases the pooling equilibrium is Pareto preferred to the separating. However, this is not always the case. To obtain a Pareto ranking of equilibria (Proposition 3), it was necessary to compare the expected consumption of type B's in the separating and pooling equilibria. For small ϵ, these are approximately the same. However, for larger ϵ they are not and in the area marked "Not Ranked" the type B's expected consumption (and their expected utility) are higher with separation than with pooling. For larger ϵ and moderate π, there is just a separating equilibrium, because even with both types in market L the price volatility is too large for type A's to want to stay. There is pooling for small π. In the shaded region near the origin the pooling is in market L; otherwise it is in market S.

The example illustrates that the phenomena highlighted by Propositions 2 and 3 do not rely on λ_B being small. For a wide range of parameter values separating equilibria with significant price volatility can exist. For many of these parameter values there also exists a pooling equilibrium which is Pareto preferred.

6. CONCLUDING REMARKS

There is considerable empirical evidence suggesting that investors only participate in a limited number of markets because of transaction costs. This paper has investigated the effect of limited market participation on asset price volatility. The first result is to demonstrate that even a small amount of aggregate uncertainty can lead to a large amount of price variation. In the particular model analyzed, it was shown that a small degree of uncertainty about liquidity preference can cause sufficient volatility in the market for the long-term asset to drive out investors with a high liquidity preference and high risk aversion. Only investors with low risk aversion and low liquidity preference who hold little money are left in the market. As a result, small shocks in liquidity preference lead to a large fluctuation in prices. Investors from other markets could potentially dampen the fluctuations in prices, as standard theories suggest. They do not do so because they have already committed to transact elsewhere and cannot participate in the market for the long-term asset. The second result is that there can exist a Pareto-preferred equilibrium with high participation and low volatility. In this case, investors with a high preference for liquidity holding large reserves of cash enter the market and this dampens price fluctuations.

The analysis in the paper is based on a simple stylized model and many important factors are omitted. For example, an important assumption is that short sales are prohibited. This assumption rules out borrowing. Also, the short-term provision of liquidity by market makers is excluded from the analysis. The effect of incorporating borrowing restrictions, margin requirements, market makers and a number of other features of actual markets into models with limited market participation is an interesting topic for future research.

APPENDIX

Notation

In proving the results in this Appendix, some notation which was not introduced in the text is useful. Let $0 \le n_i \le N_i$ denote the measure of investors of type i who enter market L at date 0, for i = A,B.

The expected utility of an investor of type i who enters market S is denoted by V^*_{iS} and defined by

$$V^*_{iS} = E[\lambda_i U_i(R_S W_i) + (1-\lambda_i)U_i(R_S{}^2 W_i)] \quad \text{for } i = A,B. \tag{A1}$$

An investor's expected utility from entering market L at date 0 will be:

$$V_{iL}(\ell_i,m_i;P) = E[\lambda_i U_i(m_i + P\ell_i) + (1-\lambda_i)U_i((m_i/P + \ell_i)R_L)]. \tag{A2}$$

An investor of type i will choose a portfolio (ℓ_i,m_i) to maximize $V_{iL}(\ell_i,m_i;P)$ subject to the budget constraint $\ell_i + m_i = W_i$. Let $V^*_{iL}(P)$ denote the maximized value of $V_{iL}(\ell_i,m_i;P)$ for any price function P.

Proof of Lemma 1

Let n_A and n_B be fixed and satisfy $n_A + n_B > 0$. Let $S_\xi = \{(\ell_A,\ell_B) \mid \xi \le \ell_i \le W_i,\ i = A,B\}$ for any $\xi > 0$. For any $\ell \in S_\xi$ there exists a unique price function defined by $\Pi(\ell) = \min\{R_L, \Sigma(1-\lambda_i)n_i(W_i-\ell_i)/\Sigma\lambda_i n_i \ell_i\}$. Then we can define a correspondence from S_ξ to itself by putting $G_i(\ell^0) = \arg\max V^*_{iL}(\ell_i,W_i-\ell_i;\Pi(\ell^0))$ and $G(\ell^0) = G_1(\ell^0)\times G_2(\ell^0)$ for any $\ell^0 \in S_\xi$. It is easily checked that G satisfies the properties of Kakutani's fixed point theorem. Let $\ell^\xi \in G(\ell^\xi)$ denote a fixed point for each value of $\xi > 0$. Letting ξ converge to zero we get a sequence $\{\ell^\xi\}$ of fixed points which contains a convergent subsequence. In the same notation, suppose that ℓ^ξ converges to ℓ^0. If $\ell^0_A + \ell^0_B = 0$ then $\Pi(\ell^\xi)$ converges to R_L, which implies that $\ell^\xi_i = W_i$ for ξ

sufficiently small, a contradiction. Similarly, if $\ell_A^0 + \ell_B^0 = W_A + W_B$ then $\Pi(\ell^\xi)$ converges to 0, which implies that $\ell_i^0 = 0$ for ξ sufficiently small, a contradiction. Thus, $0 < \ell_A^0 + \ell_B^0 < W_A + W_B$ and it is easily checked that $\Pi(\ell^0)$ and $\{(\ell_i^0, W_i - \ell_i^0)\}$ are the required price function and portfolios, respectively.■

Proof of Lemma 2

Consider the decision problem facing investors at date 0. If they enter market S their optimal strategy is the same as in Section 3. Consider market L. The portfolio problem for a person of type i is

$$\max_{\ell_i, m_i} \pi[\lambda_i U_i(m_i + \ell_i P_\epsilon) + (1-\lambda_i)U_i((m_i/P_\epsilon + \ell_i)R_L)]$$

$$+ (1-\pi)[\lambda_i U_i(m_i + \ell_i P_0) + (1-\lambda_i)U_i((m_i/P_0 + \ell_i)R_L)] \qquad \text{(A3)}$$

subject to $$W_i = \ell_i + m_i\ . \qquad \text{(A4)}$$

Substituting for m_i using the budget constraint (A4), the first order condition is

$$\pi[\lambda_i U_i'(W_i+\ell_i(P_\epsilon-1))(P_\epsilon-1) + (1-\lambda_i)U_i'((W_i/P_\epsilon+\ell_i(1-1/P_\epsilon))R_L)(1-1/P_\epsilon)R_L]$$

$$+ (1-\pi)[\lambda_i U_i'(W_i+\ell_i(P_0-1))(P_0-1) + (1-\lambda_i)U_i'((W_i/P_0+\ell_i(1-1/P_0))R_L)(1-1/P_0)R_L]$$

$$= 0\ . \qquad \text{(A5)}$$

Using (A.7'), the market clearing condition (6) and the budget constraint (A4) in the first order condition (A5) and rearranging gives

$$\pi\left\{\epsilon U_B'\left(\frac{W_B P_\epsilon}{1-\epsilon+\epsilon P_\epsilon}\right)(P_\epsilon - 1) + (1-\epsilon)U_B'\left(\frac{W_B R_L}{1-\epsilon+\epsilon P_\epsilon}\right)\left(1-\frac{1}{P_\epsilon}\right)R_L\right\}$$

$$+ (1-\pi)U'\left(\frac{W_B(\epsilon P_\epsilon + (1-\epsilon)R_L)}{1-\epsilon+\epsilon P_\epsilon}\right)(R_L - 1) = 0 \ . \qquad \text{(A6)}$$

It can be seen from this that,

$$P_{\epsilon 0} \equiv \lim_{\epsilon \to 0} P_\epsilon = \frac{\pi R_L}{R_L - (1 - \pi)} \ . \qquad (7)$$

■

Proof of Proposition 2

The result demonstrated informally in the text for two-point distributions satisfying (A.7') holds for continuous distribution functions satisfying (A.7). Our aim is to show that when the liquidity preference of type B's, λ_B, is sufficiently small, a separating equilibrium always exists. More precisely, when Z is sufficiently large and type A investors are sufficiently risk averse, there exists an equilibrium in which $n_A = 0$ and $n_B = N_B$.

Let $\{Z^k\}$ be a sequence of positive numbers diverging to infinity. For every k, set $\lambda_B^k = \theta/(Z^k+\theta)$. Let P^k and $\{(\ell_i^k, m_i^k)\}$ be the price function and portfolios that support equilibrium in market L, relative to the distribution of investors $n = (0, N_B)$, for every k. The existence of this partial equilibrium is guaranteed by Lemma 1.

The next step is to show that for k sufficiently large, the chosen distribution of investors is indeed optimal. For each value of k, $P^k = \min\{R_L, Y^k/\theta\}$ where $Y^k = Z^k m_B^k/\ell_B^k$. Y^k is bounded. To see this note that if Y^k approaches infinity, P^k converges almost surely to R_L, in which case money is eventually dominated. But that implies that $m^k = 0$, a contradiction. Thus, $\{Y^k\}$ is bounded and contains a convergent subsequence. Without loss of generality we can take this to be the original sequence. Then P^k converges almost uniformly to $P^\infty = \min\{R_L, Y^\infty/\theta\}$, where Y^∞ is the limit of Y^k. Similarly, $\{(\ell_i^k, m_i^k)\}$ is bounded so we can choose a subsequence along which (ℓ_i^k, m_i^k) converges to $(\ell_i^\infty, m_i^\infty)$, for $i = A,B$. Without loss of generality we can take this to be the original sequence. The next lemma characterizes the limiting behavior of P^k and (ℓ_i^k, m_i^k).

Lemma 3: If (A.1)-(A.7) are satisfied then $\ell_B^\infty = W_i$, $\ell_A^\infty > 0$ and $(\ell_i^\infty, m_i^\infty) \in \arg\max V_{iL}(\ell_i, m_i; P^\infty)$ for $i = A,B$.

Proof: The proof proceeds in a number of steps. The first step is to show that $\ell_B^\infty = W_B$. It is clear that in the limit the investors of type B can attain an expected utility of $U_B(W_B R_L)$. If $\ell_B^\infty < W_B$ then $V_{BL}^*(P^k)$ cannot converge to $U_B(W_B R_L)$ since in the limit consumption at date 2 must be less than or equal to $\ell_B^\infty R_L + m_B < W_B R_L$, a contradiction.

The next step is to show that $E[1/P^\infty] = 1$. For each value of k, $\ell_B^k > 0$ and $m_B^k > 0$. If $\ell_B^k = 0$ (resp. $m_B^k = 0$) then $P^k = R_B$ (resp. 0) but that implies that money (resp. asset L) is dominated, a contradiction. At an interior optimum, the following first order condition must be satisfied:

$$E[\lambda_B^k U_B'(c_1^k)P + (1-\lambda_B^k)U_B'(c_2^k)R_L] = E[\lambda_B^k U_B'(c_1^k) + (1-\lambda_B^k)U_B'(c_2^k)R_L/P^k]. \qquad \text{(A7)}$$

Taking limits, we get $E[U_B'(c_2^\infty)R_L] = E[U_B'(c_2^\infty)R_L/P^\infty]$, where $c_2^\infty = W_B R_L$, so $E[1/P^\infty] = 1$ as claimed.

The final step is to show that $\ell_A^\infty > 0$. From standard continuity arguments it can be shown that, for $i = A,B$, $(\ell_i^\infty, m_i^\infty) \in \arg\max V_{iL}(\ell_i, m_i; P^\infty)$. It is enough to set $\ell_A = 0$ and check the first-order condition:

$$E[\lambda_A U'_A(W_A) + (1-\lambda_A)U'_A(W_A R_L/P^\infty)R_L/P^\infty]$$

$$= E[(\lambda_A U'_A(W_A)P + (1-\lambda_A)U'_A(W_A R_L/P^\infty)R_L)/P^\infty]$$

$$< E[\lambda_A U'_A(W_A)P + (1-\lambda_A)U'_A(W_A R_L/P^\infty)R_L]E[1/P^\infty]$$

$$= E[\lambda_A U'_A(W_A)P + (1-\lambda_A)U'_A(W_A R_L/P^\infty)R_L], \tag{A8}$$

a contradiction.■

An immediate corollary of Lemma 3 is that $V^*_{BL}(P^k)$ converges to $U_B(W_B R_L)$. Since $R_L > (R_S)^2$, it is clear that for k sufficiently large $V^*_{BL}(P^k) > V^*_{BS}$. Similarly, if we assume that type A's are sufficiently risk averse that $V_{AL}(P^\infty) < V^*_{AS}$, then for k sufficiently large $V^*_{AL}(P^k) < V^*_{AS}$. Thus for k sufficiently large the chosen distribution of investors is optimal and we have a separating equilibrium $(P^k,(\ell_i^k,m_i^k,n_i^k))$ where $n_A^k = 0$ and $n_B^k = N_B$ so that Proposition 2 is demonstrated.■

Proof of Proposition 3

Let (ℓ_s,m_s) denote type B's optimal portfolio choice and $(P_{s\epsilon},R_L)$ denote prices in the separating equilibrium. Then type B's expected utility in this equilibrium will be

$$\pi[\epsilon U_B(C_{s1}) + (1-\epsilon)U_B(C_{s2})] + (1-\pi)U_B(C_{s3}) \tag{A9}$$

where

$$C_{s1} = m_s+\ell_s P_{s\epsilon}; \quad C_{s2} = (m_s/P_{s\epsilon}+\ell_s)R_L; \quad C_{s3} = m_s+\ell_s R_L. \tag{A10}$$

Suppose that the same portfolio had been chosen in the pooling equilibrium with prices $(P_{p\epsilon},P_{p0})$. Then type B's expected utility would be

$$\pi[\epsilon U_B(C_{p1}) + (1-\epsilon)U_B(C_{p2})] + (1-\pi)U_B(C_{p3}) \tag{A11}$$

where

$$C_{p1} = m_s+\ell_s P_{p\epsilon}; \quad C_{p2} = (m_s/P_{p\epsilon}+\ell_s)R_L; \quad C_{p3} = (m_s/P_{p0}+ \ell_s)R_L. \tag{A12}$$

For small ϵ, $P_{s\epsilon}$ is approximated by the expression in Lemma 2. By choosing π appropriately it is possible to obtain any value for $P_{s\epsilon}$ between 0 and 1. Choose π so that

$$P_{s\epsilon} = P_{p\epsilon} < 1 < P_{p0}. \tag{A13}$$

Using this in (A9) and (A11), it can be shown that the expected utility in the separating equilibrium must be less than the expected utility in the pooling equilibrium if the portfolio (ℓ_s, m_s) were chosen. Since this, in turn, is no better than the expected utility in the pooling equilibrium it follows that type B's prefer the pooling equilibrium. Hence for small ϵ and π satisfying (A13), the pooling equilibrium is Pareto preferred to the separating equilibrium.

What happens for other values of π? Provided ϵ is sufficiently small, it can also be shown that the pooling equilibrium is Pareto preferred. To see this note that from (11) $\lim_{\epsilon \to 0} P_{p\epsilon} = 1$. Hence for small ϵ, the case of interest is where

$$P_{s\epsilon} < P_{p\epsilon}. \tag{A14}$$

Now the expected consumption of a type B in the separating equilibrium is

$$EC_s = \pi[\epsilon C_{s1} + (1-\epsilon)C_{s2}] + (1-\pi)C_{s3} . \tag{A15}$$

If a type B were to choose the portfolio (ℓ_s, m_s) in the pooling equilibrium, expected consumption would be

$$EC_p = \pi[\epsilon(C_{p1}) + (1-\epsilon)C_{p2}] + (1-\pi)C_{p3} . \tag{A16}$$

Using the market clearing condition (6) and (11) and rearranging, it can be shown that for small ϵ

$$EC_p - EC_s \approx m_s(R_L - 1)(1-\pi)/R_L > 0. \tag{A17}$$

Since $P_{s\epsilon} < P_{p\epsilon} < 1 < P_{p0} < R_L$ it follows that

$$C_{s2} > C_{p2} > C_{p3} > C_{s3} > C_{p1} > C_{s1}. \tag{A18}$$

Now transfer expected consumption from C_{s2} to C_{s3} until $C_{s2}^{new} = C_{p2}$ or until $C_{s3}^{new} = C_{p3}$. Suppose $C_{s2}^{new} = C_{p2}$. The expected utility corresponding to the new allocation must be better than the expected utility in the separating equilibrium since there has been a transfer from a high consumption state to a low consumption state and expected consumption is the same. However, this new expected utility is still clearly worse than the expected utility in the pooling equilibrium. Hence the pooling equilibrium is preferred to the separating equilibrium. If $C_{s3}^{new} = C_{p3}$, then transfer expected consumption from C_{s2} to C_{s1} until $C_{s2}^{new} = C_{p2}$. It follows from (A17) that $C_{p1} > C_{s1}^{new}$. Hence we can use an argument similar to the previous one to show the pooling equilibrium is again preferred to the separating equilibrium.■

REFERENCES

Allen, F. and G. Gorton (1988), "Rational Finite Bubbles," Working Paper 41-88, Rodney White Center, University of Pennsylvania.

Brennan, M. (1975), "The Optimal Number of Securities in a Risky Asset Portfolio when there are Fixed Costs of Transacting: Theory and Some Empirical Results," *Journal of Financial and Quantitative Analysis* 10, 483-496.

Blume, M., J. Crockett and I. Friend (1974), "Stock Ownership in the United States: Characteristics and Trends," *Survey of Current Business*, 16-40.

Blume, M. and I. Friend (1978), The Changing Role of the Individual Investor: A Twentieth Century Fund Report, John Wiley and Sons: New York.

Campbell, J. and A. Kyle (1988), "Smart Money, Noise Trading and Stock Price Behavior," NBER Technical Working Paper 71.

Campbell, J. and R. Shiller (1988a), "Stock Prices, Earnings and Expected Dividends," *Journal of Finance* 43, 661-676.

Campbell, J. and R. Shiller (1988b), "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors," *Review of Financial Studies* 1, 195-228.

Chatterjee, C. and D. Corbae (1990), "Endogenous Market Participation and the General Equilibrium Value of Money," University of Iowa (mimeo).

Cohen, K., W. Ness Jr., H. Okuda, R. Schwartz and D. Whitcomb (1976), "The Determinants of Common Stock Returns Volatility: An International Comparison," *Journal of Finance* 31, 733-740.

Cuny, C. (1990), "The Role of Liquidity in Futures Market Innovations," University of California, Irvine (mimeo).

Delong, B., A. Shleifer, L. Summers and R. Waldmann (1990), "Noise Trader Risk in Financial Markets," *Journal of Political Economy* 98, 703-738.

Diamond, D. and P. Dybvig (1983), "Bank Runs, Deposit Insurance and Liquidity," *Journal of Political Economy* 91, 401-419.

Fama, E. (1970), "Efficient Capital Markets: A Review of Theory and Empirical Work," *Journal of Finance* 35, 383-417.

Gennotte, G. and H. Leland (1990), "Market Liquidity, Hedging, and Crashes," *American Economic Review* 80, 999-1021.

Grossman, S. (1988), "An Analysis of the Implications for Stock and Futures Price Volatility of Program Trading and Dynamic Hedging Strategies," *Journal of Business* 61, 275-298.

Grossman, S. and M. Miller (1988), "Liquidity and Market Structure," *Journal of Finance* 43, 617-633.

Hirshleifer, D. (1988), "Residual Risk, Trading Costs, and Commodity Futures Risk Premia," *Review of Financial Studies* 1, 173-193.

King, M. and J. Leape (1984), "Wealth and Portfolio Composition: Theory and Evidence," Working Paper 1468, NBER: Cambridge, MA.

LeRoy, S. and R. Porter (1981), "The Present Value Relation: Tests Based on Implied Variance Bounds," *Econometrica*, 49, 555-574.

Mankiw, G. and S. Zeldes (1991), "The Consumption of Stockholders and Non-Stockholders," *Journal of Financial Economics* 29, 97-112.

Merton, R. (1987a), On the Current State of the Stock Market Rationality Hypothesis," in *Macroeconomics and Finance: Essays in Honor of Franco Modigliani*, R. Dornbusch, S. Fischer and J. Bossons (eds.) MIT Press: Cambridge, MA; 99-123.

Merton, R. (1987b), "A Simple Model of Capital Market Equilibrium with Incomplete Information," *Journal of Finance* 42, 483-510.

Pagano, M. (1986), "Market Size, The Informational Content of Stock Prices and Risk: A Multiasset Model and Some Evidence," Discussion Paper 144, CEPR: London, U.K.

Pagano, M. (1989), "Endogenous Market Thinness and Stock Price Volatility," *Review of Economic Studies* 56, 269-288.

Shiller, R. (1981), "Do Stock Prices Move Too Much to Be Justified by Subsequent Changes in Dividends?" *American Economic Review* 71, 421-436.

Shiller, R. (1984) "Stock Prices and Social Dynamics," *Brookings Papers on Economic Activity* 2, 457-498.

Tauchen, G. and M. Pitts (1983), "The Price Variability-Volume Relationship on Speculative Markets," *Econometrica* 51, 485-505.

Telser, L. and H. Higimbotham (1977), "Organized Futures Markets: Costs and Benefits," *Journal of Political Economy* 85, 969-1000.

West, K. (1988), "Dividend Innovations and Stock Price Volatility," *Econometrica* 56, 37-61.

Table 1

Types of Equilibria

Separating	-	Type A's enter market S: $V_{AS} > V_{AL}$ Type B's enter market L: $V_{BL} > V_{BS}$
Pooling	-	Both types enter market S or market L $V_{iL} > V_{iS} \quad i = A,B$
Partial Pooling	-	Both types enter one of the markets One type enters the other market e.g. $V_{BS} = V_{BL}$; $V_{AS} > V_{AL}$.

Table 2

A Numerical Example

Investors:

	Type A's	Type B's
Utility function:	$U = -\exp(-5C)$	$U = -\exp(-C)$
Initial Wealth:	$W_A = 1$	$W_B = 1$
Group Size:	$N_A = 9$	$N_B = 1$
Probability of being an early consumer:	$\bar{\lambda}_A = 0.5$	$\lambda_B = \epsilon$ with pr. π $= 0$ with pr. $1-\pi$

Assets:

$$R_S = 1.03$$

Date:	0		1		2
Money:	1	→	1	→	1
Short-term asset:	1	→	R_S (> 1)	→	R_S^2
Long-term asset:	1	→	Trades at P	→	$R_L (> R_S^2)$

Figure 1

The Timing of Payoffs on Assets

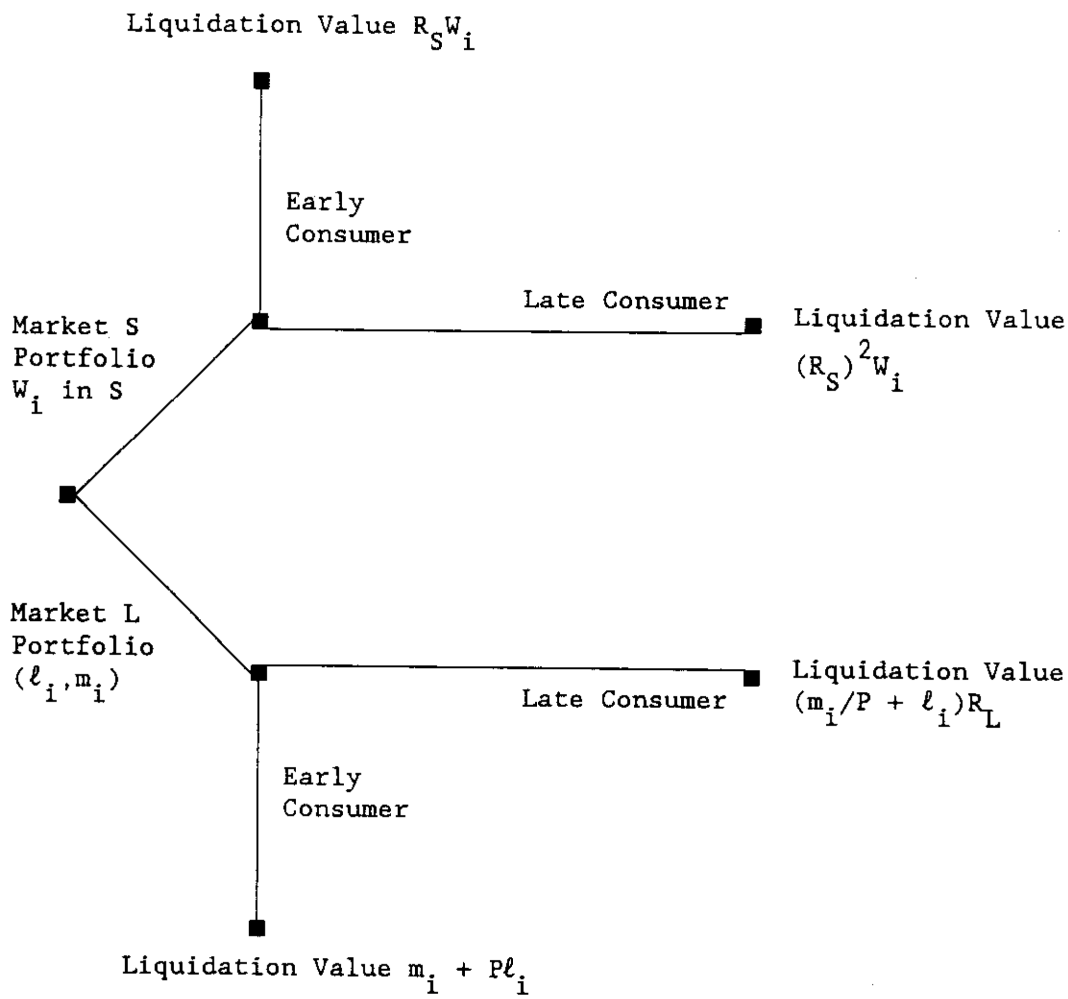

Figure 2

Investors' Choice of Market

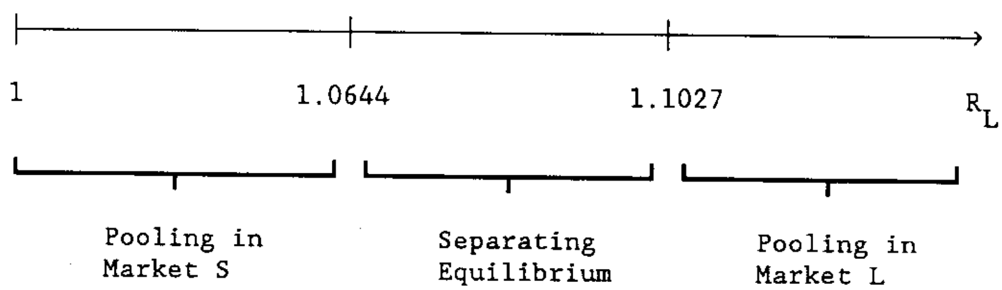

Figure 3

The Relationship between R_L and the Type of Equilibrium

with No Aggregate Uncertainty when $\bar{\lambda}_B = 0.1$

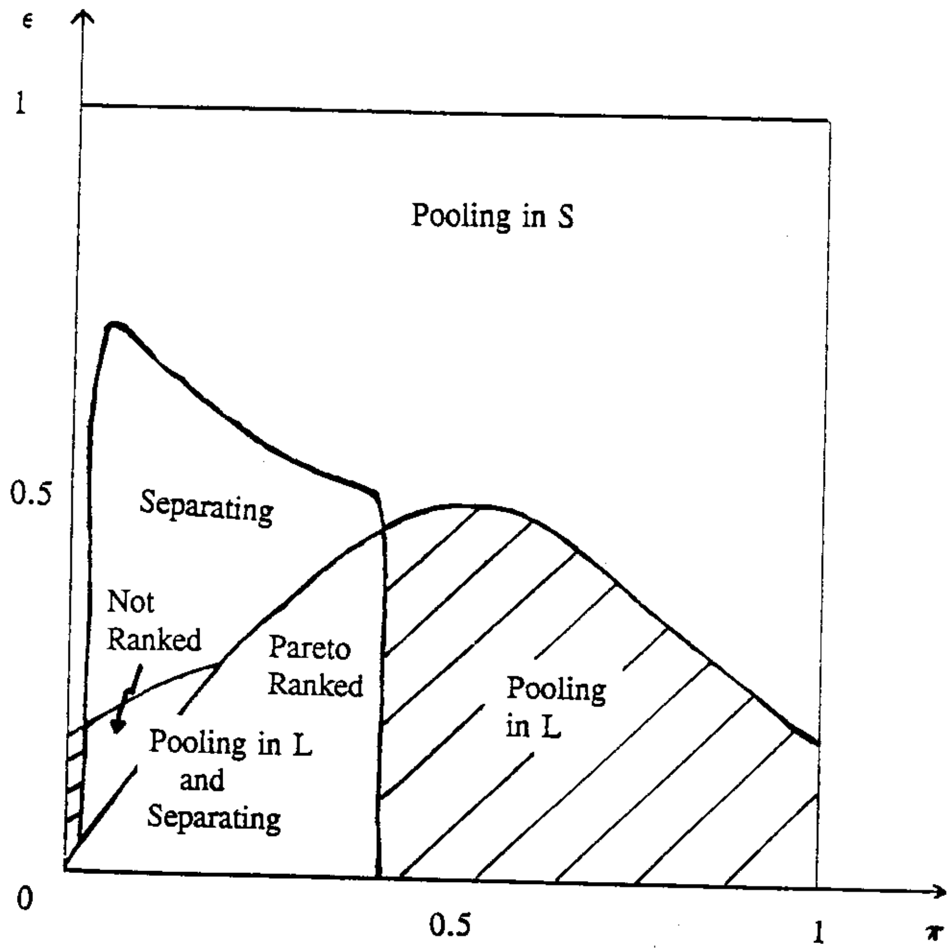

Figure 4

An Example with $R_L = 1.12$

Notes

[1]The costs that prevent instantaneous movement from one asset market to another have been stressed in a somewhat related context by Grossman (1988). His concern is that the use of synthetic securities to replace trading in options and futures markets reduces the amount of information flowing to the market. In particular, it may mean that arbitrageurs are not forewarned of impending demands for liquidity in the market. As a result, their capital may be committed elsewhere, so that they cannot react in time to prevent sharp falls in price. See also Grossman and Miller's (1988) explanation of the crash of October 1987 as being the result of a lack of funds to absorb a liquidity shock.

[2]A number of other theories of excess volatility have been suggested. These include those based on asymmetric information (see e.g., Allen and Gorton (1988) and Gennotte and Leland (1990)) and those based on noise traders (see, e.g., Shiller (1984) and DeLong et al. (1990). Recent empirical contributions attempting to categorize the causes of stock price volatility include Campbell and Shiller (1988a; 1988b) and Campbell and Kyle (1988).

[3]See also Chatterjee and Corbae (1990). They develop a model where there is a fixed cost of participating in the bond market and use it to address a number of issues in monetary theory.

[4]The financial assets can be thought of as claims on firms. These firms are set up by entrepreneurs who invest funds in constant returns to scale technologies. There is a short-term technology and a long-term technology. One unit of money used to buy inputs for the short-term technology at date t leads to output at date t+1 which is worth R_S, for t = 0,1. This money is paid out to the investors holding the financial claims on the firm. Since the technology is constant returns to scale, the number of firms adjusts to the amount of money available in market S at dates 0 and 1 and the supply of short-term financial assets is endogenous at these dates. Similarly, one unit of money used for inputs in the long-term technology at date 0 leads to output at date 2 worth R_L which is paid out to investors. The supply of long-term financial assets adjusts to the amount of money available in market L at date 0 but is fixed at date 1.

[5]The rather odd way of parameterizing λ_B in (A.7) ensures that $(1-\lambda_B)/\lambda_B = Z/\theta$, which makes calculations in the Appendix easier.

[6]A natural question here is what happens in the limit when $\epsilon \to 0$ and $\lambda_B = 0$ is non-stochastic? Note that Proposition 1 does not apply since it requires $0 < \lambda_i < 1$. When $\lambda_B = 0$ with probability one, the type B investors are certain that they will be late consumers. They do not hold money and they do not trade at date 1. As long as money and L have the same *expected* return between dates 0 and 1, type B investors will be happy to invest only in L. The price of L at date 1 is then largely indeterminate. As long as the volatility is high enough to keep the type A's out of the market, there will be a separating equilibrium. Of course, there is nothing in the model driving this uncertainty. It is extrinsic or "sunspot" uncertainty. One example of such an equilibrium is provided by the limiting distribution of prices as $\epsilon \to 0$.

These equilibria are not robust, in the sense that as soon as $\lambda_B > 0$ with positive probability, the sunspot equilibria disappear. For the market to clear at date 1 in the states where $\lambda_B > 0$, money will always be held between dates 0 and 1. This ensures $P = R_L$ in the states where $\lambda_B = 0$. Thus, sunspot equilibria only exist when no trade ever takes place at date 1 in the long-term market. In addition to the sunspot equilibria, an equilibrium with no price volatility where $P = 1$ also exists as in Proposition 1.

[7]When $\lambda_B = \epsilon$, the market clearing condition is

$$N_A\lambda_A\ell_A P_\epsilon + N_B\epsilon\ell_B P_\epsilon = N_A(1-\lambda_A)m_A + N_B(1-\epsilon)m_B,$$

where the left hand side is the sales of L by early consumers and the right hand side is the amount of money consumers have to purchase L with. Rearranging this gives (9).

[8]If $\lambda_B = 0$, the market clearing condition when $P_0 < R_L$ is

$$N_A\lambda_A\ell_A P_0 = N_A(1-\lambda_A)m_A + N_B m_B,$$

and (10) follows from this.