SECURITY PRICES AND MARKET TRANSPARENCY

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Abstract

Many recommendations for reforming securities markets are predicated on the belief that providing information on order flow and other market variables to traders (i.e., increasing *market transparency*) will increase liquidity and improve price efficiency. This paper demonstrates that market transparency can actually increase price volatility and lower market liquidity. This occurs even though transparency increases the precision of traders' predictions about the asset's value. In a sufficiently large market, transparency always reduces volatility and improves market quality. We use these results to assess policy proposals concerning the disclosure of trading information.

The relation between trading arrangements and security prices has been the subject of great interest following the events of October 1987. A major concern is the ability of market mechanisms to accommodate substantial variation in order flow without increased price volatility. After the crash, several investigative commissions proposed trading halts following large price movements ('circuit breakers') to reduce instability. A trading halt, it is argued, would allow information on transitory order imbalances to be publicized, providing opportunities for speculative traders to place stabilizing orders.[1]

Underlying these policy recommendations is the belief that price variability would be reduced by making the market more *transparent* to investors by providing them with more information regarding current market conditions. Similar logic applies in many other contexts in speculative markets. For example, it has been argued that 'sunshine trading,' where traders can pre-disclose their trading intentions, will reduce the price impact of a large order by allowing the demanders of liquidity to coordinate their trades with the suppliers of liquidity. Related arguments concern the timing and quality of trade reporting, the desirability of explicitly identifying demands generated by automated trading strategies such as portfolio insurance, and the extent to which off-floor traders should be permitted access to public limit orders.

This paper investigates the impact of disclosing information about order flow imbalances on security prices and market liquidity. The imbalances in question can arise from several potential sources, for example, from a trading system that accumulates small market orders for execution at the opening or from orders generated by the dynamic portfolio hedging strategies (such as portfolio insurance) of large institutional firms.[2] Since unanticipated order flow shocks generate transitory price movements, the argument goes, speculative traders and market makers will be better able to absorb these imbalances in a system that provides transparency (by disclosing the size of these imbalances), thereby reducing price volatility and enhancing market liquidity.

While the argument above appears intuitive at first glance, there are reasons to believe that greater transparency may produce just the opposite of the desired results. Order flow shocks are a source of price volatility. However, the uncertainty associated with the direction and magnitude of these trades is an important source of noise in speculative markets.[3] Disclosing information about the magnitude of order flow shocks changes the information structure, and therefore alters the trading strategies of liquidity providers. As suggested above, speculative traders/market makers coordinate their strategies to absorb the order flow shock. But the reduction in the perceived level of noise trading reduces market liquidity as speculative traders and market makers demand a greater risk premium to accommodate the announced order imbalance and undisclosed liquidity-motivated trades. This effect tends to widen the deviation between the market clearing price and the conditional expected value of the security, possibly increasing price volatility.

A model of market transparency that captures this intuition should incorporate the following two elements: (i) Some portion of liquidity-motivated trade cannot be distinguished from speculative trades based on private information, so preannouncement is possible only for some trades. This condition is necessary to prevent market failure altogether in the event of disclosure; it also allows us to assess the impact of disclosure on endogenous liquidity trade. In the familiar framework of Kyle (1985), there is one source of exogenous noise, so that increased transparency cannot induce less efficient pricing. (ii) The providers of liquidity (speculative traders and market makers) act strategically; in particular, they recognize the influence of their orders on market prices. This condition is needed for transparency to affect the trading strategies of the market's liquidity providers. For strategic behavior to be relevant, the number of market participants must be finite. While this assumption complicates our analysis, it allows us to perform comparative statics as market size changes, an important consideration since disclosure issues are especially relevant for thinly traded securities.

Our model has these two features; it is based on the model of Kyle (1989), where strategic agents with rational expectations trade a risky security in an auction market. Order flow is also subject to transitory imbalances that represent informationless trade. Unlike Kyle's model, the speculative traders and market makers in our model trade to hedge their portfolios as well as on the basis of their information, providing a source of endogenous liquidity-motivated trade. We show that the portfolio hedging demands cannot be separated from the speculative demands of these traders, as discussed in condition (i) above. Since there are a finite number of traders, as discussed in condition (ii), traders recognize their influence on prices in their trading strategies. The strategic behavior of traders is complicated because prices convey noisy information signals about fundamental values and rational traders condition their beliefs on these signals.

To focus attention on transparency, we consider two trading protocols for this market at opposite ends of the spectrum in terms of the provision of market information to traders. Under the first protocol, the market operates as an auction where all orders are executed simultaneously at a single price. This system is not transparent because traders have no information concerning order imbalances. The second protocol we consider for this model is transparent in that information on order imbalances is publicized before order submission. If the order imbalance originates from the accumulation of small orders through an automated trading system, the transparent system can be interpreted as a open electronic book system. If the imbalance arises from informationless hedging strategies (e.g., portfolio insurance) the transparent system corresponds to a system where the initiators of these strategies preannounce their trades.

There are three principal results. First, we demonstrate that transparency can exacerbate the price volatility generated by transitory order imbalances, even though we show that these imbalances are in themselves a source of price volatility. Market quality can also suffer with lower liquidity and higher implicit transaction costs. Transparency also reduces the volume of endogenous (undisclosed) liquidity trading. However, the informativeness of prices is strictly greater as a result of transparency. Second, we prove that transparency can induce market failure even in a large auction market. This can occur because the reduction in the perceived level of noise trading lowers market liquidity, making prices more sensitive to undisclosed liquidity-based trades. Third, we show that transparency reduces price volatility and increases market liquidity if the market is sufficiently large, provided that transparency does not cause market failure as noted above. In a thick market, disclosure leads to more stable prices because traders' strategies are not significantly altered by disclosure.

We use these results to evaluate various policy proposals relating to information disclosure. For example, our results show that sunshine trading may worsen the price for a liquidity-motivated trade. The results also have implications for circuit breaker mechanisms. Without adequate provision for disseminating information about trading halts beyond the trading floor, a circuit breaker may exacerbate price volatility. Paradoxically, a circuit breaker is least likely to function as desired for thinly traded assets.

The issues discussed here are related to several recent studies. Forster and George (1991) and Lindsey (1991) analyze models based on the Kyle (1985) model where some market participants have information on market conditions while others have information about the mean or the variance of the distribution of noise trading. Since there is only one source of non-information trading in the Kyle (1985) model, increased transparency cannot increase price variability as is the case here.

Admati and Pfleiderer (1991) provide a model of sunshine trading where some liquidity traders can preannounce the size of their orders while others cannot.[4] They show that those investors who are able to preannounce their trades enjoy lower trading costs, but the costs for liquidity traders who are unable to preannounce their trades rises. Intuitively, preannouncement identifies the trade as informationless, but increases the adverse selection costs for other traders. Similarly, Röell (1990) provides a model of dual-capacity trading where brokers are also permitted to act as dealers. Disclosure is an issue in this model because brokers can identify certain customer trades as liquidity-motivated, much as in the Admati-Pfleiderer (1991) model. In our model, the strategic response of market participants implies that liquidity-motivated traders who can preannounce the size of their trades may choose not to do so.

Transparency may have adverse consequences by other means. Diamond and Verrecchia (1991) model the effect on market liquidity of corporate disclosure of information on fundamental asset values (as opposed to disclosure regarding market conditions) that reduces information asymmetry. In their model, a reduction in information asymmetry reduces the cost of capital, increasing the security's price and increasing market liquidity. Beyond some point, however, disclosure has adverse consequences since it induces market makers to exit the market, lowering liquidity and price. Similarly, Fishman and Hagerty (1991) investigate the effect of rules that force insiders to disclose their trades. Since only insiders know whether their trades are liquidity- or information-motivated, their disclosures can mislead other traders providing a rationale for profitable price manipulation.

The arguments for circuit breakers are related to discussions about transparency. Greenwald and Stein (1991) provide a model of a circuit breaker system where outsiders submit market orders to market makers. Uncertainty over the number of these traders (and hence the magnitude of order imbalances) creates price volatility independently of the volatility in fundamentals. In the Greenwald-Stein model, a trading halt benefits traders because it reduces the risk created by price volatility.

Spiegel and Subrahmanyam (1991) develop an intertemporal model of a circuit breaker mechanism. They show that there are benefits to circuit breakers since traders who are excluded from trading during the halt are batched with other traders, leading to better execution. In our model, a circuit breaker can exacerbate price volatility. However, when combined with communications systems to encourage potential traders to enter the market in the event of a halt, a circuit breaker will reduce volatility.

Gennotte and Leland (1990) model crashes whose source is uncertainty regarding the magnitude of non-information based demands. The lack of information on the part of liquidity providers concerning these demands can lead to discontinuities in pricing, producing crashes. In this context, disclosure takes the form of announcement of the magnitude of these hedging demands. Our analysis would indicate that predisclosure may not always be effective in preventing crashes of this type.

The paper is organized as follows: In Section I we describe the theoretical framework and in Section II we analyze a non-transparent trading mechanism. Section III examines a transparent market mechanism and Section IV compares the two equilibria using a variety of criteria. We demonstrate conditions under which transparency affects market performance and discuss a number of applications of our results. Finally, Section V concludes the paper. All proofs are contained in the appendix.

I. The Analytical Framework

Consider an economy with two financial assets: a risky asset and a riskless bond. The risky asset is traded at time 0 in an auction market. We discuss two alternative trading protocols for this market that differ in the amount of market information provided to traders at the time of order submission. The exact details of the protocols are explained after we establish the basic framework and notation.

The value of the risky asset in period 1 in the future is represented by a random variable, $\tilde{v}$. The stochastic payoff, $\tilde{v}$, can be thought of either as a liquidating dividend or the full information price following a public announcement. In this market, there are N traders (indexed by $i = 1, \ldots, N$), each of whom enters the market with an initial endowment of x_i of the risky asset and initial wealth W_{0i}. Endowments of the risky asset are independently normally distributed across traders with mean 0 and precision (The precision is the inverse of the variance.) denoted by $\psi > 0$.[5] The private information of trader i is the pair (x_i, y_i). Traders can be thought of as individual investors (referred to as speculative traders) or as market makers, and in the latter case, x_i is interpreted as their inventory position. Another interpretation for x_i is that off-exchange investors place orders with market makers, so that $-x_i$ is the net demand of these investors, i.e., the amount of securities that market maker i has promised to deliver.

Each trader receives a private information signal concerning the payoff of the risky asset. Trader i 's information signal, $\tilde{y}_i$, is a random variable $\tilde{y}_i = v + \tilde{\epsilon}_i$ where v is the realized value of the risky asset and $\tilde{\epsilon}_i$ is normally distributed with mean 0 and precision $\tau > 0$. The realization of the information signal $\tilde{y}_i$ is denoted y_i. Assuming all traders have diffuse prior beliefs, trader i's prior distribution of $\tilde{v}$ is normal with mean y_i and precision τ.[6]

We assume traders have negative exponential expected utility functions over final period wealth, W_{1i}, of the form $U(W) = -e^{-\rho W}$, where $\rho > 0$ is the coefficient of risk aversion. Let q_i represent the order quantity of trader i, with the convention that security purchases are represented by positive numbers and trader sales by negative numbers. Information is not the sole motivation for trade in this model. Since traders are risk averse and enter the market with non-zero endowments, a portion of transaction volume arises from portfolio hedging.

As a result, liquidity trading arises endogenously.

In addition to the demands of the speculative traders/market makers, there are demands from other traders who are not present on the exchange floor. Let $\tilde{Z}$ represent the stochastic order imbalance generated by these accumulation of these orders. We assume that $\tilde{Z}$ is normally distributed with zero mean and variance σ_z^2. The realized order imbalance is denoted Z, with the convention that $Z > 0$ represents excess demand and $Z < 0$ represents excess supply of the risky asset.

The noise shock is a convenient summary statistic for extrinsic uncertainty regarding the aggregate supply of the risky asset. Different interpretations of the order flow shock correspond to different views of transparency. First, if Z is interpreted instead as the order imbalance arising from consolidating small trades (that are not information-motivated) through an automated system, transparency corresponds to reporting the net effect of automated trading to market participants. Second, if we interpret Z as the order of a large uninformed trader, transparency takes the form of sunshine trading, where a trader can pre-announce his trading intentions. This can be formally modeled within the current framework.[7] Third, if the shock represents the net portfolio hedging demands of institutional traders, transparency corresponds to predisclosure of these trades. This predisclosure may be mandated by exchange rules concerning 'program trading' or may occur because brokers can infer the nature of their customers' trades. Fourth, we can extend our analysis to allow the mean of $\tilde{Z}$ to be a linear function of price. In this case, $\tilde{Z}$ can be thought of as the consolidation of limit orders, and transparency corresponds to an open display of the limit book to floor traders prior to the market's opening. We turn now to the specifics of market organization.

II. Trading without Transparency

The first protocol we consider for the auction market at time 0 is one where traders submit orders (either price-contingent limit orders or market orders) that are accumulated for simultaneous execution at a single market clearing price.[8] This protocol is non-transparent because traders cannot observe the equilibrium price and volume until after the market has cleared. Trading is modeled as a game characterized by the number of players, their reward functions, information signals, endowments, and beliefs.

Let W_{0i} denote the cash holdings (initial wealth) of trader i. The private information of trader i is represented by $I_i = (W_{0i}, x_i, y_i) \in \Re^3$ for $i = 1, \ldots, N$.[9] The trader's (pure) strategy is a mapping $q_i : \Re \times \Re^3 \rightarrow \Re$ is the demand function for trader i, mapping price and the initial state into desired order quantity. The trading mechanism is described as a game $M^{nt} = (N, u(\cdot), \{I_i\}, \{q_i(\cdot)\})$. We define an equilibrium for this trading mechanism using a Bayes-Nash solution concept.

Definition 1 *An equilibrium for the mechanism M^{nt} is a price p^*, and a set of strategy functions, $\{q_i(p; I_i)\}$, such that:*
(i) Excess demand is zero at the equilibrium price:

$$\sum_{i=1}^{N} q_i(p^*; I_i) + Z = 0$$

(ii) Trader i maximizes the expected utility of final period wealth, W_{1i}, given the strategies of other traders:

$$q_i(p^*; I_i) \in \underset{\{q_i\}}{\operatorname{argmax}} \{E[u(\tilde{W}_{1i}) \mid I_i \wedge p^*]\}$$

where:

$$\tilde{W}_{1i} = (\tilde{v} - p^*)q_i + \tilde{v}x_i + W_{0i}$$

for $i = 1, \ldots, N$.

Condition (i) requires that the equilibrium price clears the market. Condition (ii) requires that the strategy function selected by trader i be a best response to the conjectured strategy functions of other traders. In forming their best-response, traders use Bayes' rule to form their beliefs using the statistical information generated by the mechanism, including the price. The price not only determines the equilibrium allocation but also conveys information to traders with rational expectations. Thus, traders' probability assessments are determined endogenously in equilibrium. Implicit in condition (ii) is the idea that traders choose strategies knowing that they not only influence prices directly through their order size, but also indirectly through the effect their actions have on the beliefs of other players. Proposition 1 demonstrates that there exists a linear equilibrium for the non-transparent trading protocol, i.e., M^{nt} has a solution with linear price and strategy functions.

Proposition 1 *There exists an equilibrium for the exchange mechanism M^{nt} described above where the optimal strategy functions, $\{q_i(p; I_i)\}$, and market clearing price, p^*, are given by:*

$$\begin{aligned} q_i(p; I_i) &= \gamma[(1-\delta)(y_i - p) - \alpha x_i] \\ p^* &= \frac{1}{N}\left(\sum_{i=1}^{N}\left[y_i - \left(\frac{\rho}{\tau}\right)x_i\right] + (N-1)\lambda Z\right) \end{aligned}$$

for $i = 1, \ldots, N$, where α, δ, γ, and λ are positive constants described in the appendix.

Proposition 1 provides a closed-form solution for the non-transparent system M^{nt}.[10] A trader's optimal strategy has two components: the first component is $\gamma(1-\delta)(y_i - p)$, i.e., a constant proportion of the difference between the information signal and price. This represents the *speculative* or information-motivated part of trade. The second component, $-\alpha\gamma x_i$, is a negative fraction of the endowment, and represents *portfolio hedging.* This component is not based on private information signals.[11]

From the discussion above, liquidity trading has two dimensions: (i) Order imbalances, Z, that are subject to disclosure, and (ii) Portfolio hedging by traders, $-\alpha\gamma x$, that cannot be distinguished from their speculative trades. The expected volume of portfolio trading is proportional to the standard deviation of risky asset endowments, $1/\sqrt{\psi}$, while the expected size of the order imbalances is proportional to σ_z.[12] An increase in the volume of noise trading leads to larger expected order imbalances and hence larger absolute deviations from the full information price, i.e., larger expected risk premia.

Proposition 2 (Noise Trading and Price Variability): *Price variability increases with the volume of liquidity (portfolio hedging and noise) trading.*

Proposition 2 confirms our intuition that greater noise is associated with greater price volatility. This result forms the basis for the arguments in favor of greater transparency and disclosure. We show, however, that providing information about imbalances need not always reduce price volatility. The proposition implies a positive relation between price variability and volume is positive since total transaction volume increases with the amount of noise trading. The result is also consistent with empirical evidence of a positive relation between transitory order imbalances and price movements in auction markets. For example, Haller and Stoll (1989), using data from the Frankfurt Stock Exchange, conclude that "even in auction markets, prices are driven away from their true underlying value by temporary imbalances of orders."

III. Trading in a Transparent Market

We turn now to an analysis of a *transparent* market where information on current market conditions is disseminated to traders prior to trading.[13] Consider a trading protocol or mechanism, denoted M^t, where Z is displayed to traders before they submit their demands. The only difference between this system and the non-transparent system M^{nt} is that trader i ($i = 1, \ldots, N$) observes Z before choosing $q_i(p)$. We retain the Bayes-Nash equilibrium concept of Definition 1. The next result establishes the existence of a well-defined solution to the game M^t.

Proposition 3 *If $\tau < \tau^* \equiv \frac{\rho^2(N-2)}{\psi N}$, there exists a linear Bayes-Nash equilibrium for the transparent mechanism M^t where the optimal strategy functions, $\{q_i(\,\cdot\,; I_i)\}$, and market clearing price, p^*, are given by:*

$$\begin{aligned} q_i(p; I_i) &= \gamma_1(1-\delta_1)(y_i - p^*) - \gamma_1\alpha_1 x_i - \gamma_1\delta_1\lambda_2 Z, \\ p^* &= \frac{1}{N}\left\{\sum_{i=1}^{N}\left[y_i - \left(\frac{\rho}{\tau}\right)x_i\right] + \left[\frac{1-N\zeta}{\gamma_1(1-\delta_1)}\right]Z\right\}. \end{aligned}$$

for $i = 1, \ldots, N$, where $\gamma_1, \delta_1, \alpha_1, \zeta$, and λ_2 are positive constants defined in the appendix.

There are two major differences between this equilibrium and the solution to M^{nt}. First, the conditions necessary for the existence of a linear equilibrium are more stringent. The precision of private information, τ, must be bounded above for existence. Even with large numbers of traders, existence is not assured since $\lim_{N\to\infty}\tau^* = \rho^2/\psi > 0$. Interestingly, this result obtains even though agents are symmetric as far as the *quality* of their information signals is concerned. Intuitively, if traders obtain high quality information signals (i.e., τ is high) and there is very little endogenous liquidity trading (i.e., ψ is high and ρ is low), traders are unwilling to reveal their information to others and less willing to share risk by trading. Thus, transparency may induce market failure.

Second, unlike Proposition 1, a trader's strategy, q_i, depends not only on price but also on the disclosed volume, Z. Since the coefficient of Z is negative, traders' speculative actions partly accommodate the order flow shock. This effect is offset by changes in the strategies of traders that increases the price impact of noise shocks and reduces the volume of endogenous liquidity trading, as shown below:

Proposition 4 *Traders' strategy functions are less price sensitive in a transparent mechanism:*

$$\gamma_1(1-\delta_1) < \gamma(1-\delta).$$

Further, transparency lowers the volume of portfolio hedging trade.

The strategy functions in a transparent market are less price sensitive than the strategies in a non-transparent system. Traders scale back the size of their order for any given discrepancy between their signal and the price. Equivalently, traders demand a larger deviation between the transaction price and their signals in order to absorb a given number of securities. Consequently, as traders' demand schedules become less responsive to price, the sensitivity of prices to portfolio hedging demands that are not disclosed increases.

Proposition 4 also shows that transparency has another effect on trader's demand, i.e., a reduction in endogenous liquidity trading. This result is additional source of concern regarding disclosure because it implies less risk sharing; it is analogous to the Admati-Pfleiderer result that liquidity traders who cannot preannounce are hurt by sunshine trading.

To summarize this discussion, transparency has important effects on traders' strategies: First, by allowing traders to place orders to offset transitory shocks, transparency tends to stabilize prices. Second, by decreasing the price-responsiveness of trader's demands, transparency tends to increase the price movements in response to liquidity demand. Third, the reduction in the volume of endogenous liquidity trading tends to create more informative prices and less variability. Consequently, the net effect on price volatility and market liquidity is unclear. It is to this issue we now turn.

IV. Trading Arrangements and Performance

A. Price Volatility, Informativeness, and Transparency

Our discussion above suggests the possibility that price variability may be higher in a transparent system. Before we consider this question, we consider the effect of transparency on the informativeness of prices. Our measure of informativeness is the precision of the forecast of the final period asset value, $\tilde{v}$. Formally, a trading protocol M^a is said to be *more informative* than another protocol M^b if both market participants and outsiders with market information can form more accurate assessments of the fundamental value under M^a.

Proposition 5 *Prices are more informative under a transparent market protocol.*

Intuitively, prices in a non-transparent system are noisy signals of fundamental value because information signals are imprecise and because prices reflect risk premia induced by order flow shocks and undisclosed liquidity trading. In a transparent protocol, traders and outsiders can eliminate the influence of order flow shocks on prices to form more accurate forecasts of the asset's fundamental value. Greater informativeness, however, does not imply less price variability, since price variability also reflects variation in the risk premium induced by non-information based demands. The following proposition provides conditions under which price variability is higher in a transparent mechanism.

Proposition 6 *There exists a constant $k_N \in (0,1)$ such that price variability is lower in a transparent mechanism if:*

$$\frac{\tau\psi}{\rho^2} < k_N.$$

Price variability is higher in a transparent mechanism if:

$$k_N < \frac{\tau\psi}{\rho^2} < 1$$

and σ_z^2 is sufficiently high.

Corollary 1 (Large Markets): *Price variability is always lower under transparency if the market is sufficiently large, provided that an equilibrium exists with a transparent system.*

With disclosure, traders can condition their trades on the known imbalance, tending to reduce the price impact of the shock. However, in a thin market, disclosure reduces the perceived level of noise trade, as shown by Proposition 4, lowering liquidity. This can occur even though (from Proposition 2) noise trading itself is destabilizing.

This argument suggests that information disclosure can reduce price variability if the market is sufficiently competitive, given that transparency does not induce market failure. This is easy to verify. From the proof of Proposition 6, $\lim_{N\to\infty} k_N = 1$ so that price variability is always lower in a transparent mechanism if the market is sufficiently competitive. Transparency increases stability in markets that are already large and liquid. Given a finite number of traders, however, there is a *range* of values for τ, ρ, σ_z^2, and ψ under which transparency is destabilizing. This implies that there is no critical size above which transparency leads to greater stability. Finally, the result shows that the equilibria for M^{nt} and M^t are generically different unless $\sigma_z^2 = 0$. The key point is that a non-transparent system can operate in economies where a transparent system may not be viable.

From Proposition 6, it follows that the expected absolute risk premium (measured by the deviation of the clearing price from the full information price of the security, $E[|\tilde{p} - v|]$) can be larger if traders are permitted to submit their demands after information on order imbalances is disclosed. If we view the mechanism M^t as describing the rules for sunshine trading, it follows that the expected losses of the uninformed trader can, depending on the parameters, be larger than the losses from non-disclosure. It follows that uninformed traders with large liquidity-based trades need not always be better off under sunshine trading.

B. Liquidity, Transaction Costs, and Welfare

Price variability is only one aspect of market performance. In this section, we consider the impact of transparency on liquidity, implicit transaction costs, and social welfare. A measure of market liquidity is *market depth*, which Kyle (1985) defines as the order flow necessary to induce a unit price change.[14] Following Kyle, let Υ represent market depth, where:

$$\Upsilon = \left(\frac{dp^*}{dZ}\right)^{-1}.$$

A related issue concerns traders' perceptions of their impact on the equilibrium price. Since prices move in the direction of the trade there is a bid-ask spread implicit in a single-price auction.

In the appendix we show that the equilibrium price can be expressed as an increasing function of trader i 's demand, which we denote $p(q_i)$. Then we can define the implicit bid-ask spread $s(\cdot)$ as the difference between the market clearing price if an order of size q_i was to buy and the price if the order was to sell:

$$s(q_i) = p(|q_i|) - p(-|q_i|).$$

Proposition 7 shows that there is a trade-off between price variability and depth, but the implicit spread is higher in a transparent market.

Proposition 7 *Price variability and market depth are inversely related; the mechanism with greater price volatility also provides less liquidity. However, the implicit bid-ask spread for an order of given size is strictly greater in a transparent market.*

A more liquid market offers greater price stability. The uninformed investors who given rise to the imbalance Z suffer lower expected losses in deeper markets. This fact is consistent with the concern about the effect of price variability on the 'small investor' in public policy debates. The implicit spread, however, is strictly greater in a transparent market. In a transparent market the actions of an informed trader convey more information than under a non-transparent system. A trader's action has an indirect effect through the actions of others (represented by the conjectural variation) in addition to the direct effect on prices, leading to wider implicit spreads. This proposition is testable since implicit spreads can be estimated from the serial covariance of transaction prices using a formula due to Roll (1984). Haller and Stoll (1989) compute implied spreads for stocks on the Frankfurt Stock Exchange (which is organized as an auction market) in this manner and find that implicit spreads are significantly positive.

Although the measures of market quality discussed so far figure prominently in policy discussions, they are not necessarily relevant from a welfare perspective. As the impact of transparency depends on the parameters of the model we would expect the same to be true for welfare. Under our assumptions, traders care about only the mean and the variance of future wealth. However, improved information need not increase utility because it can eliminate opportunities for risk-sharing. In particular, we can show that the volume of endogenous liquidity trading is strictly lower in a transparent market. As a result, we cannot make unambiguous welfare statements about market organization. Cost considerations would favor the non-transparent system, since there is no need to create systems to provide disclosure.

C. Price Movements and Circuit Breakers

We are now in a position to evaluate the effectiveness of circuit breaker mechanisms that involve trading halts when price movements exceed pre-set limits. Suppose that $\tilde{v} = v_0 + \tilde{d}$, where v_0 is a constant (which we interpret as the asset's price at the end of the last trading period) and $\tilde{d}$ is a stochastic increment. The price movement over the trading day is then $\tilde{\eta} \equiv (\tilde{p} - v_0)$. Consider the following trading protocol. Trading begins under the protocol described in the game M^{nt}, but prices are restricted to the bands imposed by the limits. If $|\eta| \geq L$, where $0 \leq L < v_0$ is the price limit, trading is suspended and existing orders are canceled. If this occurs, the order imbalance is disclosed, and the game M^{nt} is replayed.[15] Denote this new game by $M^c = \{M^{nt}, L\}$. The relevant question then is whether the eventual price movement under the game M^c is greater than under the game M^{nt} alone, i.e., whether disclosing the order imbalance reduces the absolute size of the price movement.

To simplify matters, we assume that the order imbalance arising from noise traders does not change during the halt. This assumption may be justified if the small traders that are the source of the imbalance do not continuously monitor trading or if news of the halt is not widely disseminated. Speculative traders and market makers do revise their orders, including the amount of trading related to portfolio hedging.

Proposition 8 *If $\tau < \tau^*$, the game M^c has a well-defined solution. The eventual price movement under the circuit breaker mechanism M^c can be larger in absolute size than the price movement under an auction mechanism M^{nt} with no price limits.*

The proposition shows that circuit breakers can exacerbate the price movement due to a temporary order imbalance. The problem is greatest for thinly traded assets. However, Proposition 6 implies that if the trading halt is sufficiently long that new investors can enter the market, the circuit breaker will have the desired effect. From a policy viewpoint, our analysis highlights the need to develop communications systems to disseminate widely market information during trading halts. Unless such communication links exist to bring new traders into the market, circuit breakers may have unintended consequences.

V. Conclusions

Underlying many policy recommendations for improving the operation of financial markets is the presumption that price volatility would be reduced by providing investors with information regarding order flow, thereby increasing market transparency. This argument arises in many different contexts, including discussions regarding market anonymity (including sunshine trading, dual-capacity trading, and access to public limit orders), the effectiveness of circuit breaker mechanisms, and the timing and quality of reporting on automated trading. This paper investigates the impact of disclosing information on market imbalances during the process of price formation on security prices and market liquidity. The imbalances in question can arise from several sources, for example, the automated batching of small orders or from portfolio hedging strategies.

There are three principal results: First, transparency can exacerbate the price volatility generated by order imbalances, even though these imbalances a source of price volatility. Market quality can also suffer with lower liquidity and higher implicit transaction costs. Second, transparency can induce market failure even in a large auction market. Third, transparency reduces price volatility and increases market liquidity if the market is sufficiently large and there is sufficient noise trading.

We use these results to assess various policy proposals related to market transparency. The results suggest that a liquidity-motivated trader can worsen execution by predisclosing the trade. Similarly, a circuit breaker mechanism that halts trading to publicize order imbalances originating from automated trading systems can exacerbate price volatility. Our analysis points to the need to have in place communications links to induce new traders to enter the market in the event of a trading halt. Paradoxically, the potentially adverse effects of transparency are likely to be greatest in thin markets.

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Notes

[1] See, e.g., Brady Commission (1988) and Greenwald and Stein (1988) who discuss these arguments. Blume, MacKinlay, and Terker (1989) examine empirically the effect of order imbalances on price movements on October 19 and 20, 1987.

[2] For example, the Small Order Execution System (SOES) on NASDAQ batches small market and limit orders to market makers. NASDAQ explicitly prohibits 'professional traders' from using SOES. Similarly, the New York Stock Exchange (NYSE) has systems, such as the DOT system, to automatically accumulate and rout small orders to the trading floor.

[3] It is well known that some source of non-information based trading is necessary for the operation of a speculative market. See, for example, Milgrom and Stokey (1982) and Black (1986).

[4] This means that there must be some method from preventing informed traders from predisclosing their trades to mimic liquidity traders. Further, there must exist some mechanism to ensure that the preannouncer does not manipulate the market by placing offsetting trades.

[5] Negative endowments are interpreted as short positions. The assumption of a zero mean is simply a convenient normalization. We also assume the supply of the risky asset is "widely" distributed, so that the initial endowment gives no information about the endowments of other traders.

[6] The assumption of independent signals is mathematically equivalent to specifying a diffuse prior distribution for traders. This assumption can be relaxed to allow correlation among conditional expectations, but at the cost of considerable complexity.

[7] If prices are unbiased predictors of true value (as we show), a risk-averse uninformed trader will hedge a constant fraction of the endowment using a market order. Thus, Z represents the aggregate amount of hedging demand. Informed traders will not choose to disclose their trades if given the opportunity to do so, since this means revealing valuable information to competitors.

[8] Examples include batch markets for inactive stocks in some European stock exchanges and closed-bid auctions of various types.

[9] We assume throughout the paper that $N > 2$, to eliminate trivial cases where, say, a monopolist insider sets arbitrarily high prices. This would not be a problem if the mean of $\tilde{Z}$ were price dependent.

[10] The proposition does not rule out the existence of equilibria with non-linear strategies. We do not examine these possibilities. The linear equilibrium is a natural object of attention since it imposes the fewest computational burdens on agents, exhibits stability, and yields a closed-form solution.

[11] In our framework, if x_i is not observable to traders other than i, there is no credible way for trader i to separate the speculative component of his or her order from the portfolio hedging component.

[12] This follows from the normality of $\tilde{x}_i$ and $\tilde{Z}$.

[13] Examples include the opening procedures for continuous trading systems (e.g., Toronto CATS) where traders receive indicated prices based on the current excess demand prior to the opening. The NYSE opening auction provides some transparency by displaying information on the overnight accumulation of market orders through the Opening Automated Report Service (OARS).

[14] The definition applies only to anonymous market orders since price contingent orders effectively determine the price. This qualification is unnecessary in Kyle (1985) where all traders submit market orders and the price is determined by competitive market makers who observe only the net order flow.

[15] There is no halt if in the second iteration prices again cross the limit.

Appendix

Proof of Proposition 1:
The proof constructs the Bayes-Nash equilibrium by solving for a trader's best response to the conjectured strategies adopted by other traders and then shows the conjectures are consistent.
Step 1: (Traders' Conjectures and the Clearing Price) For notational convenience index traders other than i by $h = 1, \ldots, i-1, i+1, \ldots, N$. Suppose trader i conjectures that all other traders adopt linear strategy functions:

$$q_h(p; I_h) = A_h(I_h) - Bp \tag{1}$$

where $I_h = (W_{0h}, x_h, y_h)$ and B is a constant. Note that I_h is private information and is known only to trader h, so that A_h is also private information. Suppose trader i places an order for q_i. Using (1) and the market clearing condition we obtain:

$$\sum_{h \neq i}^{N} A_h - (N-1)Bp + q_i + Z = 0. \tag{2}$$

From equation (2) we can express price as a function of q_i, denoted $p(q_i)$:

$$p(q_i) = p_{-i} + \lambda q_i \tag{3}$$

where p_{-i} and λ are defined as follows:

$$p_{-i} = \frac{\sum_{h \neq i}^{N} A_h + Z}{(N-1)B} \tag{4}$$

$$\lambda = \frac{1}{(N-1)B}. \tag{5}$$

For trader i, p_{-i} is a random variable (denoted $\tilde{p}_{-i}$) because neither A_h nor Z is observable at the time of order submission. To construct the trader's strategy, we first consider the case where trader i actually observes the realization of $\tilde{p}_{-i}$. We relax this in Step 2 below. Trader i then chooses her optimal order quantity q_i^* to maximize her expected utility of wealth. With $p^* = p(q_i^*)$, final period wealth is:

$$W_{1i} = (v - p^*)q_i^* + vx_i + W_{0i} \tag{6}$$

where W_{0i} is the initial cash holdings of trader i. It is well known that maximizing the expected utility of wealth (assuming $\tilde{W}_{1i}$ is normally distributed) is equivalent to maximizing:

$$E[\tilde{W}_{1i} \mid I_i \wedge p^*] - \frac{\rho}{2}\sigma^2(\tilde{W}_{1i} \mid I_i \wedge p^*) \tag{7}$$

where $E[\cdot \mid \cdot]$ and $\sigma^2(\cdot \mid \cdot)$ are the conditional expectation and variance operators. Substituting (6) into equation (7), we obtain:

$$E[\tilde{v}|I_i \wedge p^*](q_i + x_i) - p(q_i)q_i + W_{0i} - (\rho/2)\sigma^2(\tilde{v}|I_i \wedge p^*)(q_i + x_i)^2.$$

Trader i's optimal order quantity, q_i^* is found by differentiating this expression with respect to q_i^*:

$$E[\tilde{v} \mid I_i \wedge p^*] = p(q_i^*) + p'(q_i^*)q_i^* + \alpha_i(q_i^* + x_i) \tag{8}$$

where $\alpha_i \equiv \rho\sigma^2(v|I_i \wedge p^*)$. The second order condition for a maximum is:

$$2p'(q_i^*) + p''(q_i^*)q_i^* + \alpha_i < 0. \tag{9}$$

Step 2: (Construction of the Best Response Function) Suppose trader i conjectures that $\tilde{p}_{-i}$ is normally distributed with mean v and variance $1/\pi$ and that $Cov(p_{-i}, y_i) = Cov(p_{-i}, x_i) = 0$. We will show that in equilibrium trader i's conjectures concerning $q_h(\cdot)$ and p_{-i} are correct. Trader i cannot invert the market clearing price p^* to infer the realization of $\tilde{p}_{-i}$ because p^* is not observed at the time of order submission. We claim that trader i can effectively condition on p^*, and thus p_{-i}, by submitting a demand schedule that specifies her demand at every price. To show this is feasible, suppose the realization of $\tilde{p}_{-i}$ is p^*_{-i}. Then, trader i's optimal order quantity q_i^* is determined from (8) with trader i acting as the 'marginal' trader who determines the price. This yields a point $(p^*_{-i}, q_i^*) \in \Re^2$. From (3), the corresponding price is $p(q_i^*)$ so that uniquely associated with the point (p^*_{-i}, q_i^*) is the pair $(p(q_i^*), q_i^*)$, a point on trader i's demand schedule. Repeating this exercise for $p^*_{-i} \in \Re$ leads to a curve in $\Re^2$, the graph of $(p(q_i^*), q_i^*)$, trader i 's demand function in (p, q)-space.

The next step is to construct the trader's demand function, denoted $q_i(p; I_i)$, as outlined above, given one point on it. Suppose trader i believes p_{-i} is the realization of $\tilde{p}_{-i}$. Under our assumptions concerning the distribution of y_i and p_{-i}, trader i's conditional expectation of $\tilde{v}$ is:

$$E[\tilde{v}\,|I_i \wedge p^*] = (1-\delta)y_i + \delta p_{-i}$$

where $\delta = \pi/(\pi + \tau)$ is a constant. Further, it can be shown that $\sigma^2(\tilde{v}|I_i \wedge p^*) = (\pi + \tau)^{-1}$. From (8), the optimal order quantity $q_i^* = q_i(p^*)$ satisfies:

$$q_i^* = \frac{E[\tilde{v}|I_i \wedge p^*] - p^* - \alpha_i x_i}{\alpha + \lambda} \tag{10}$$

where $\alpha = \rho/(\pi + \tau)$ is a constant for all i. In (10), q_i^* depends upon $E[\tilde{v}|\cdot]$ which depends the conjectured value of p_{-i} through p^*. We must ensure that each price-quantity pair on the demand schedule is consistent with the posterior beliefs generated by observing the realization of that particular trade. To do this, substitute the value of $E[\tilde{v}|\cdot]$ given by (15) into equation (10). Since $p_{-i} = p^* - \lambda q_i^*$, we obtain:

$$q_i(p^*; I_i) = \gamma(1-\delta)(y_i - p^*) - \gamma\alpha x_i \tag{11}$$

where:

$$\gamma \equiv \frac{1}{(\alpha + \lambda + \delta\lambda)}. \tag{12}$$

Equation (11) is the optimal demand given $p = p^*$; the demand schedule, $q_i(p; I_i)$, is the graph of $(q_i(p^*), p^*)$ for $p^* \in \Re_+$. Inspection of (11) shows that the optimal strategy of trader i is of the form conjectured in (1) with:

$$A_i(I_i) = \gamma[(1-\delta)y_i - \alpha x_i] \tag{13}$$

$$B = \gamma(1-\delta). \tag{14}$$

Step 4: (Existence) Traders have the same price sensitivity although their reservation prices A_i vary with the initial state I_i. To show the strategy functions are well- defined, we must express the unknown values of the strategy functions, $\alpha, \lambda, \gamma, \delta$, and π in terms of the parameters N, ρ, τ, and ψ, and verify that the initial conjectures are satisfied. From the definition of B and λ write:

$$\lambda = \frac{1}{(N-1)\gamma(1-\delta)}. \tag{15}$$

Since $\alpha = \rho/(\pi + \tau)$ and $\delta = \pi/(\pi + \tau)$, $\alpha/(1-\delta) = (\rho/\tau)$. Then, from (11), and the market clearing condition, the equilibrium price is:

$$p^* = \frac{1}{N}\left(\sum_{i=1}^{N}\left[y_i - \left(\frac{\rho}{\tau}\right)x_i\right] + \frac{Z}{\gamma(1-\delta)}\right). \tag{16}$$

The formula for p_{-i} is identical to (16), except that the summation excludes trader i:

$$p_{-i} = \frac{1}{N-1}\left(\sum_{h \neq i}^{N}\left[y_h - \left(\frac{\rho}{\tau}\right)x_h\right] + \frac{Z}{\gamma(1-\delta)}\right). \tag{17}$$

The variance of p_{-i} is given by:

$$\frac{1}{\pi} = \omega_0 + \frac{\omega_1}{\gamma^2(1-\delta)^2} \tag{18}$$

where:

$$\omega_0 = \frac{\tau^{-1} + \psi^{-1}(\rho/\tau)^2}{(N-1)} \tag{19}$$

$$\omega_1 = \frac{\sigma_z^2}{(N-1)^2} \tag{20}$$

are constants. When $\sigma_z^2 > 0, \omega_1 > 0$ and (18) can be inverted to express γ as a function of π:

$$\gamma = \left(\frac{\pi + \tau}{\tau}\right)\sqrt{\frac{\omega_1}{(\pi^{-1} - \omega_0)}}. \tag{21}$$

Write (21) as $\gamma = g(\pi)$. Next, we derive a second function relating γ to π. The equilibrium values of γ and π are the solutions to these two non-linear equations. Substituting (15) into (12), we obtain:

$$\frac{1}{\gamma} = \alpha + \frac{(1+\delta)}{(N-1)\gamma(1-\delta)}.$$

Write this equation as:

$$\gamma = \frac{(\pi + \tau)[(N-2)\tau - 2\pi]}{(N-1)\rho\tau}. \tag{22}$$

Equation (22) expresses γ as a function of π. Denote this function by $\gamma = f(\pi)$. A solution to the game M^{nt} exists if there exists π^*, where $\pi^* > 0$, solving $f(\pi^*) = g(\pi^*) > 0$.

Step 5: (Equilibrium) From (21), $g(\pi)$ is a continuous function of π on the interval $(0, \pi_0)$, where π_0 is:

$$\pi_0 = \frac{1}{\omega_0} \tag{23}$$

where ω_0 is defined in (19). Then, $g(\pi) \geq 0$ and $g'(\pi) > 0$. It is easy to show that $g(0) = 0$ and that $g(\pi) \to \infty$ as $\pi \to \pi_0$. Examining (22), $f(\pi)$ is continuous in π, for $\pi \in [0, \pi_0]$. Putting $\pi = 0$ in (22) yields $f(0) = (\tau/\rho)(N-2)/(N-1)$, which is strictly positive for $N > 2$.

Note that $|f(\pi)| < \infty$, for $\pi \in [0, \pi_0]$. Define $R(\pi) \equiv f(\pi) - g(\pi)$. Clearly, $R(\pi)$ is continuous in π on the interval $[0, \pi_0)$. Note that $R(0) > 0$ and that the limit of $R(\pi)$, as $\pi \uparrow \pi_0$ is $-\infty$. Applying the Intermediate Value Theorem, there exists $\pi^* \in (0, \pi_0)$ such that $R(\pi^*) = 0$. Since $g(\pi)$ is positive for $\pi \in (0, \pi_0)$, $\gamma^* > 0$. Finally, from (17) it is easy to verify the initial conjectures that p_{-i} is normally distributed with $E[p_{-i}] = v$ and $Cov(p_{-i}, y_i) = Cov(p_{-i}, x_i) = 0$. This argument establishes the proposition. ∎

Proof of Proposition 2:
First, we establish a useful Lemma.

Lemma 1 *The unconditional variance of the equilibrium price is a linear function of the variance of* $\tilde{p}_{-i}$*:*

$$\sigma^2(p^*) = \frac{N-1}{N}\left(\omega_0 + \frac{N-1}{N}(\pi^{-1} - \omega_0)\right)$$

where ω_0 *is a constant defined in equation (19).*

Proof of Lemma 1:
Taking the variance of p^* in equation (16) and substituting the definition of ω_0 in (19) and the definition of π in (18) into this expression yields the Lemma. ∎

Now consider two possible values for σ_z^2 say σ_1^2 and σ_2^2, where $\sigma_2^2 > \sigma_1^2 > 0$. From (22), it follows that $f(\cdot)$ does not depend on σ_z^2. However, the function $g(\cdot)$ depends on σ_z^2 through the constant ω_1. Let $g_1(\pi)$ represent equation (21) when $\sigma_z^2 = \sigma_1^2$ and similarly define $g_2(\pi)$. From (21):

$$g_2(\pi) > g_1(\pi)$$

for $\pi \in (0, \pi_0)$. Suppose π_1 is the solution to $f(\pi) = g_1(\pi)$, where $f(\cdot)$ is defined by (22). Consider the function $R_2(\pi) \equiv f(\pi) - g_2(\pi)$. From (21), $R_2(\pi_1) < 0$. As $R_2(0) > 0$, the Intermediate Value Theorem implies there exists $\pi_2 \in (0, \pi_1)$ such that $R_2(\pi_2) = 0$. Since $\pi_2 < \pi_1$, it follows from Lemma 1 that $\sigma^2(p^*)$ is higher when $\sigma_z^2 = \sigma_2^2$. Next, observe that an increase in ψ reduces ω_0, but does not affect $f(\cdot)$ in (22). A fall in ω_0 decreases the value of $g(\pi)$ for all $\pi \in (0, \pi_0)$. Since $f(\cdot)$ is unaffected by a change in ψ, it follows from the arguments above that π^* increases as ψ increases. Applying Lemma 1, the variance of the market clearing price, $\sigma^2(p^*)$, falls as π^* rises and ω_0 decreases. ∎

Proof of Proposition 3:
Suppose that trader i believes that $q_h(p)$ is a linear policy:

$$q_h(p; q_{-h}) = A_h(I_h) - Bp - CZ \tag{24}$$

for $h, j = 1, \ldots, N; j \neq h$. The market clearing condition is:

$$\sum_{h \neq i}^{N} A_h - (N-1)Bp^* - C(N-1)Z + q_i + Z = 0. \tag{25}$$

In this mechanism p^* conveys statistical information concerning I_h to other traders. To make this explicit, suppose trader i regards (A_h/B) as a draw from an independent normal distribution with mean v and precision $\pi_1/(N-1)$. This will be shown to be correct in equilibrium. Define:

$$\mu_{-i} = \frac{\sum_{h\neq i}^{N} A_h}{B(N-1)}. \tag{26}$$

Under our assumptions, trader i conjectures that μ_{-i} is the realization of a normally distributed random variable $\tilde{\mu}_{-i}$ with mean v and precision π_1 that is uncorrelated with y_i and x_i. Substituting equation (26) into (25), we can express the price facing trader i as:

$$p^* = \mu_{-i} + \lambda_1 q_i + \lambda_2 Z \tag{27}$$

where:

$$\begin{aligned} \lambda_1 &= \frac{1}{B(N-1)} \\ \lambda_2 &= \frac{1-C(N-1)}{B(N-1)}. \end{aligned}$$

The trader's conditional expectation, given the realization of $\tilde{\mu}_{-i}$, is:

$$E[\tilde{v}|I_i \wedge \mu_{-i}] = (1-\delta_1)y_i + \delta_1 \mu_{-i} \tag{28}$$

where $\delta_1 = \pi_1/(\pi_1 + \tau)$ is a constant. From (8) trader i acts as the marginal trader who actually determines the price:

$$E[\tilde{v}|I_i \wedge p^*] = p^* + \lambda_1 q_i + \alpha_1(q_i + x_i).$$

where $\alpha_1 = \rho/(\tau + \pi_1)$. Substitute $\mu_{-i} = p^* - \lambda_1 q_i - \lambda_2 Z$ and (28) into this equation. The demand at the equilibrium price p^* is given by $q_i(p^*)$, where:

$$q_i(p^*) = \gamma_1(1-\delta_1)(y_i - p^*) - \gamma_1\alpha_1 x_i - \gamma_1\delta_1\lambda_2 Z \tag{29}$$

where

$$\gamma_1 \equiv \frac{1}{\alpha_1 + \lambda_1 + \delta_1\lambda_1}. \tag{30}$$

From equation (29), the best-response strategy of a trader i has the same form as conjectured with:

$$A_i = \gamma_1[(1-\delta_1)y_i - \alpha_1 x_i] \tag{31}$$

$$B = \gamma_1(1-\delta_1) \tag{32}$$

$$C = \gamma_1\delta_1\lambda_2. \tag{33}$$

The precision of μ_{-i} can be calculated directly:

$$\pi_1 = \frac{1}{\omega_0}$$

where ω_0 was defined above in equation (19). Using the definition of λ_1, we obtain:

$$\lambda_1 = \frac{1}{(N-1)(1-\delta_1)\gamma_1}. \tag{34}$$

Since the value of π_1 is identical the value of π_0 in equation (23). Substituting equation (34) into equation (30), we obtain an expression for γ_1 in terms of π. It is readily verified that this expression is given by equation (22). Then, since $\omega_1 = 0$ by definition, we can solve for γ_1 by substituting $\pi = \pi_0$ in (22):

$$\gamma_1 = \frac{(\pi_0 + \tau)[(N-2)\tau - 2\pi_0]}{(N-1)\rho\tau}. \tag{35}$$

Then, we can easily show that $\delta_1 = \pi_0/(\tau + \pi_0)$, and $\alpha_1 = \rho/(\tau + \pi_0)$. Next, we verify that λ_2 is well-defined. Substituting (33) and the definition of λ_1 into equation (34) we obtain:

$$\lambda_2 = \frac{1}{(N-1)\gamma_1},$$

which is positive if $\gamma_1 > 0$. To insure the proposed solution is economically meaningful, γ_1 must be positive. From (35), $\gamma_1 > 0$ implies:

$$\frac{(N-2)\tau}{2} > \frac{1}{\omega_0}.$$

We can rearrange this inequality to obtain:

$$\tau < \frac{(N-2)\rho^2}{N\psi}. \tag{36}$$

Let $\tau^* \equiv \frac{(N-2)\rho^2}{N\psi}$, so that the second order condition (9) is satisfied if $\tau < \tau^*$. From (29), the equilibrium price is:

$$p^* = \frac{1}{N}\left\{\sum_{i=1}^{N}\left[y_i - \left(\frac{\rho}{\tau}\right)x_i\right] + \left[\frac{1 - N\zeta}{\gamma_1(1-\delta_1)}\right]Z\right\}, \tag{37}$$

where $\zeta \equiv \delta_1/(N-1) \in (0, \frac{1}{2})$. This establishes the existence of a linear equilibrium. ∎

Proof of Proposition 4:

From equation (22) and the definition of δ, we can write $\gamma(1-\delta)$ as:

$$\gamma(1-\delta) = \frac{(N-2)\tau - 2\pi}{\rho(N-1)}. \tag{38}$$

Now, γ_1 is given by equation (35), and using the definition of δ_1, we can write:

$$\gamma_1(1-\delta_1) = \frac{(N-2)\tau - 2\pi_0}{\rho(N-1)}. \tag{39}$$

Then, upon comparing (38) and (39) and noting that since $\pi < \pi_0$, it follows that $\gamma_1(1-\delta_1) < \gamma(1-\delta)$.

For the second part, note that from Proposition 1, the portfolio hedging component of $q_i(p; I_i)$ is $-\gamma\alpha x_i$ in M^{nt}. The expected volume of portfolio hedging is given by $N\alpha\gamma E[|\tilde{x}_i|]$, where $\gamma\alpha = \gamma(1-\delta)(\rho/\tau)$. Since $\tilde{x}_i$ is distributed normally with mean zero, the expected volume of endogenous liquidity trading is proportional to $\frac{\gamma(1-\delta)\rho}{\tau\sqrt{\psi}}$. Similarly, under M^t the expected volume of endogenous liquidity trading is proportional to $\frac{\gamma_1(1-\delta_1)\rho}{\tau\sqrt{\psi}}$, with the same factor of proportionality. The second part of the proposition follows immediately from the slope conditions derived above. ∎

Proof of Proposition 5:
For an outside observer, the price in the non-transparent protocol is an unbiased signal of fundamental value. Using Lemma 1, this forecast has precision:

$$\frac{N^2}{(N-1)^2\pi^{-1}+(N-1)\omega_0}$$

In a transparent system, an outsider observing $\{p^*, Z\}$ can construct the statistic $p^* - \lambda_2 Z$, which is an unbiased estimator of $\tilde{v}$. This estimate can be shown to have precision:

$$\frac{N^2}{(N-1)^2\pi_0^{-1}+(N-1)\omega_0}.$$

Since $\pi < \pi_0$ for all N, the transparent system is more informative for outside observers. Now consider a trader. In a non-transparent system, it can be shown (see, e.g., DeGroot (1970), page 169) that the precision of a trader's forecast of $\tilde{v}$ is $\tau + \pi$ which is strictly less than $\tau + \pi_0$, the corresponding forecast precision in a transparent system. ∎

Proof of Proposition 6:
The unconditional variance of p^* under the mechanism M^t can be determined directly from (37):

$$\sigma_1^2(p^*) = \frac{N-1}{N}\omega_0 + \left[\frac{\sigma_z(1-\zeta N)}{N\gamma_1(1-\delta_1)}\right]^2$$

where $\zeta = \delta_1/(N-1)$.

The variability of prices under the mechanism M^{nt} is given by Lemma 1. The difference in price variability under M^{nt} and M^t is:

$$\sigma^2(p^*) - \sigma_1^2(p^*) = \left[\frac{\sigma_z}{N\gamma(1-\delta)}\right]^2 - \left[\frac{\sigma_z(1-\zeta N)}{N\gamma_1(1-\delta_1)}\right]^2. \tag{40}$$

From (40), $\sigma^2(p^*) < \sigma_1^2(p^*)$ if and only if:

$$\gamma_1(1-\delta_1) < (1-\zeta N)\gamma(1-\delta). \tag{41}$$

Using equations (38) and (39), equation (41) can be written as:

$$(N-2)\tau - 2\pi_0 < (1-\zeta N)[(N-2)\tau - 2\pi].$$

Rearrange this to obtain:

$$2\pi < (N-2)\tau - \frac{(N-2)\tau - 2\pi_0}{1-\zeta N}. \tag{42}$$

Observe that $\delta_1 = 1/(1+\tau/\pi_0)$. Then $\zeta = \delta_1/(N-1)$ is:

$$\zeta = \frac{1}{N + (\rho^2/\tau\psi)} \tag{43}$$

using the definition of π_0. Note that $(1-\zeta N)$ is independent of σ_z^2. Let

$$\Xi \equiv (N-2)\tau - \frac{(N-2)\tau - 2\pi_0}{1-\zeta N}.$$

Ξ is the right hand side of (42). The following lemma shows that $\Xi > 0$ for a range of parameter values.

Lemma 2 *There exists a constant, $k_N > 0$, which is independent of ρ, τ, and ψ such that $\Xi > 0$ in equilibrium if $\frac{\tau\psi}{\rho^2} > k_N$.*

Proof of Lemma 2:
Observe that Ξ is independent of the level of noise σ_z^2. Now $\Xi > 0$ is equivalent to:

$$(N-2)\tau > \frac{(N-2)\tau - 2\pi_0}{1-\zeta N}.$$

Using the definition of ζ in (43) and the definition of π_0, this reduces to:

$$\frac{N^2 - 4N + 2}{N^2} < \frac{\tau\psi}{\rho^2}. \tag{44}$$

From Proposition 1, when $\sigma_z^2 = 0$, existence requires that equation (36) hold. Comparing (36) with (44), we see that $\Xi > 0$ in equilibrium if the parameter values satisfy:

$$\frac{(N^2 - 4N + 2)}{N^2} < \frac{\tau\psi}{\rho^2} < \frac{(N-2)}{N}. \tag{45}$$

Define k_N by $k_N \equiv (N^2 - 4N + 2)/N^2$. Note that for all $N \geq 2$, $k_N < (N-2)/N$. Only if the precision of endowments or private information is sufficiently low, i.e., $\frac{\tau\psi}{\rho^2} < k_N$, is $\Xi < 0$. ∎

To complete the proof of the proposition note that by Lemma 2, if ψ (or τ) is sufficiently high, i.e., if $\tau\psi > (\rho^2 k_N)$, then $\Xi > 0$. From Lemma 1, we see that π is strictly decreasing in σ_z^2. Therefore, there exists a critical value for σ_z^2 such that if σ_z^2 is sufficiently high, the inequality (42) is satisfied and $\sigma_1^2(p^*) > \sigma^2(p^*)$. This proves the proposition. ∎

Proof of Proposition 7:
In the non-transparent system, depth is the inverse of the derivative of the price functional (16) with respect to Z:

$$\Upsilon = N\gamma(1-\delta). \tag{46}$$

Under transparency, we use equation (37) to obtain:

$$\Upsilon_1 = \frac{N\gamma_1(1-\delta_1)}{1-\zeta N}. \tag{47}$$

From equations (40) and (41), $\sigma^2(p^*) < \sigma_1^2(p^*)$ if and only if

$$\gamma_1(1-\delta_1) < \gamma(1-\delta)(1-\zeta N).$$

Dividing both sides of this expression by $(1-\zeta N)$, we see that $\sigma^2(p^*) < \sigma_1^2(p^*)$ if and only if $\Upsilon_1 < \Upsilon$. Therefore, price variability and market depth are inversely related. Turning now to the implicit costs of trading, the implicit spread function follows from equation (3):

$$s(q_i) = 2\lambda|q_i|.$$

Similarly, from (27) the implicit spread in a transparent market is:

$$s_1(q_i) = 2\lambda_1|q_i|.$$

From Proposition 4, it follows that $\lambda_1 > \lambda$ so that for all $q_i \in \Re$, $s_1(q_i) > s(q_i)$. ∎

Proof of Proposition 8:

First suppose that the strategic traders do not deliberately trigger a limit move. We establish an equilibrium for this game, and then consider the possibility of market manipulation. The relevant price space in the first stage of the game is $\mathcal{P} = (v_0 - L, v_0 + L)$. The equilibrium for the game M^c takes one of two forms: First, if there exists an the equilibrium price $p^* \in \mathcal{P}$ for the game M^{nt} where the strategies $\{q_i(\cdot)\}$ are restricted to $\mathcal{P}$, then the solution to M^c is identical to the solution to M^{nt}. Second, if the equilibrium price for the game M^{nt} is not in $\mathcal{P}$, trading is suspended, and the orders at the limit price are disclosed. Traders resubmit their orders and the final equilibrium allocation is determined. To prove that equilibrium exists for the game M^c, we need to demonstrate that the second stage of the game has a well-defined solution.

We begin by analyzing the case where trading has been suspended following a limit move. Assume without loss of generality a negative price movement, and denote the limit price by $p_L \equiv (v_0 - L)$. The information disseminated to traders is represented by $\{Q_L, Z\}$, where $Q_L = \sum_i q_i(p_L) + Z$ is the excess demand at the limit price, p_L. Then, trader i forms the statistic:

$$\hat{p} \equiv \frac{Q_L - Z}{\gamma(1-\delta)(N-1)} + \frac{p_L}{N-1}.$$

This statistic has mean v and precision π_0. Suppose that trader i conjectures the demand functions of other traders in the second stage of the game is given by equation (29). The market clearing condition (25) then implies that p^* is a linear function of q_i, as given by equation (27). Observing p^* is equivalent to observing p_{-i}, which in turn is equivalent to observing $\hat{p}$. Therefore, the conditional expectation is given by (28), and following the derivation of the solution to the game M^t, we see that the conjectures are satisfied in equilibrium, and the demands and price are as given by Proposition 3. In this case, M^c is equivalent to M^t. Now consider the actual movements in price under the two systems. Now, the price movement in the game M^c, denoted by η^c, is either η (the actual price movement in the game M^{nt}) if $|\eta| < L$, or η_1 if $|\eta| > L$. Then, using Propositions 1 and 3, we can write:

$$\tilde{\eta^c} = \tilde{\eta} + \theta \tilde{Z}$$

where θ is a constant given by:

$$\theta \equiv \left[\frac{1-\zeta N}{\gamma_1(1-\delta_1)} - \frac{1}{\gamma(1-\delta)}\right] \tag{48}$$

From (40), if prices are more variable under a transparent system, i.e., if $\sigma_1^2(\tilde{p}) > \sigma^2(\tilde{p})$, then $\theta > 0$. When $\eta < 0$ and $Z < 0$, equation (48) implies that $\eta_1 < \eta$ if $\theta > 0$, so the eventual price movement under M^c can be greater than under M^{nt} alone. There may be an alternative equilibrium where traders deliberately trigger limit price movements. In this event, the game M^c reduces directly to M^t and the proposition is proved. ∎