ADJUSTMENT OF CONSUMERS' DURABLES STOCKS: EVIDENCE FROM AUTOMOBILE PURCHASES

by

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ABSTRACT

Disturbances to aggregate expenditure on consumer durables are much more persistent than predicted by an optimizing representative consumer model. Recent work in macroeconomics has explored this "slowness to adjust" in the context of consumers facing transactions costs when purchasing durable goods. This approach produces infrequent household purchases and requires explicit attention to aggregation of households.

This paper considers household behavior directly, using panel data on automobile purchases and finds that about half of the households purchase automobiles subject only to transactions costs. Explicit aggregation shows that the cross-section distribution of households according to their durables stocks is quite similar to that theoretically predicted. Simulated aggregate expenditures using observed household slowness to adjust show persistence of aggregate shocks and response to income growth consistent with the aggregate data.

1. INTRODUCTION

Recent work in macroeconomics has explored the aggregate dynamics of expenditure on consumer durables as a result of consumers facing transactions costs.[1] Little evidence has been offered, however, to suggest that microeconomic agents actually behave in this manner or that the degree of households' slowness to adjust is sufficient to explain that observed in aggregate data. Further, aggregate data are insufficient to ascertain how such household slowness might respond to changes in economic conditions.

This paper takes up the issue of microeconomic behavior by considering panel data on household automobile purchases. I consider the transactions cost model versus a liquidity constraint, and find that about half of the households purchase automobiles subject only to transactions costs.

I then consider in detail the behavior of those facing the transactions cost. Theory predicts that these households should adjust their durables stock to a target share of their total wealth, and then allow it to depreciate until it reaches a critical share of wealth. They should then purchase a new durable good so that it again equals the target share of wealth. I calculate the parameters of this decision rule for the households and find that autos fall to about one half of their optimal value before households adjust their stocks. For the S-s households this value is unaffected by the level of income and wealth, but rises when income variability increases. Household data further allows estimation of the response of household waiting, and therefore aggregate slowness, to changes in economic conditions. I estimate that waiting between purchases by the S-s households is

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[1] Such models tend to produce a large swings in the number of buyers, as well as persistence of shocks to aggregate expenditure.

also unaffected by the level of income and wealth, but increases when their growth rate is low.

Finally, I consider explicitly the cross-section distribution of households according to their durables stocks. This distribution determines how many households' durables are near the point requiring adjustment, and thus provides the link between individual purchase decisions and the behavior of aggregate expenditures. I find this distribution quite similar to that theoretically derived and its evolution over time consistent with aggregate changes in the observed growth rate of income. Simulated aggregate expenditures exhibit the same rapid acceleration and subsequent slowdown observed in the actual data in the 1980s.

The paper is divided into six sections. The first section continues with the issues and literature. Part 2 presents the theoretical foundations for the transaction costs explanation of durables purchases. Part 3 uses the implications of the model to separate the households into those facing transactions costs and those facing a liquidity constraint. This is done both by exogenously splitting the sample, and also by endogenously estimating a switching model. The characteristics of the group following the transactions cost model are the subject of Part 4. I explicitly calculate the parameters of the households' decision rule and measure the effects of wealth, its growth rate and variance, and other factors on these decision rules. Part 5 presents the theoretical ergodic and empirical distributions of households' durables stocks relative to their wealth. I then simulate the response of the distribution to aggregate income growth during the 1980s and compare the resulting estimates of aggregate expenditure to the actual data. Section 6 concludes.

1.1 Previous Evidence

Most empirical studies of durable goods consumption have been at the aggregate level. Mankiw [1981] showed that in a representative agent framework, the stock of durables should follow a random walk, and thus durables purchases should follow an IMA(1,1) process of the form,

$C_t = C_{t-1} - (1-\delta)\epsilon_{t-1} + \epsilon_t$, where

C_t = Consumption at time t,

δ = depreciation rate of durable goods, and

ϵ_t = a shock to consumption at time t.

Estimation of this equation yields a series much like a random walk - the MA(1) term is typically much too small and often insignificant.

This persistence of shocks evident in Mankiw's finding was often modeled in the earlier literature as partial adjustment (see for example, Chow [1957]). Bernanke estimated durables purchases with convex costs of adjustment motivating the partial adjustment assumption. In panel data (Bernanke [1984]) he found that consumers' auto stocks did not over-respond to transitory income. In later work on aggregate data (Bernanke [1985]), however, he concluded that reasonable values of convex adjustment costs are not sufficient to explain the degree of persistence and sensitivity to transitory income in the aggregate time series.[2]

Shocks to aggregate expenditure are much more persistent than predicted by a frictionless representative consumer model, yet convex costs of adjustment do not adequately explain the observed persistence. Moreover, convex costs of adjustment suggest that

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2 Bernanke estimates an adjustment cost parameter of 35 percent and still finds that more than 70 percent of adjustment occurs in the first year.

consumers should optimally adjust frequently and in small increments.

Transactions costs, on the other hand, imply infrequent adjustment, but in larger increments. This strategy has been applied to durables by Grossman and Laroque [1990] and Bertola and Caballero [1990] and produces a decision rule in which durables are allowed to deviate from their optimal stock until some "threshold" amount is reached, and then the stock is adjusted to the optimal amount. Such models have been previously employed in other familiar applications, notably money holdings and inventories. Transactions costs can explain long delays in individual durable purchases, and may produce persistence or "slowness" in the aggregate time series.

This paper provides direct evidence of S-s behavior in household durables purchases and describes explicitly the characteristics of these decisions. In addition, I examine the empirical cross-section distribution of durables stocks relative to wealth and show that the observed microeconomic slowness to adjust explains much of the short run response of aggregate expenditure to changes in income growth, as well as the persistence of such changes in the 1980s.

2. THEORETICAL FOUNDATION

I consider durables in the context of the lifetime portfolio selection models of Samuelson [1969] and Merton [1969]. The models are adjusted to allow consumption of a durable good, and then lifetime utility is optimized to choose a consumer portfolio among the durable, a risky asset, and a riskless asset.[3] I first develop the solution in the frictionless case and then extend the

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[3] The extension to numerous risky assets is straightforward, but will be suppressed here for simplicity.

results to include a transactions cost. The results show that at the time of purchase, the transaction cost model has a proportional stock rule as in the frictionless case[4], so purchases track the level of wealth.

2.1 The Frictionless Case with Durable Goods

The problem of the consumer is to maximize the present discounted value of expected utility from consumption of a durable good. The consumption flow is assumed proportional to the stock of the good. The consumer derives income as a return on lifetime wealth, which can be invested in a portfolio of risky and riskless assets. The maximization problem is:

$$(1) \quad \text{Max } E \left\{ \int_0^{\infty} U(K_t) e^{-\gamma t} \, dt \right\}$$

subject to the dynamic budget constraint,

$$(2) \quad dW_t/dt = (\mu - r + \sigma dZ)A_t + rW_t - (r+\delta)K_t,$$

where K is the stock of durables, W is total wealth (including durables), and A is wealth held in the risky asset. The parameter γ is the subjective discount rate, δ is the depreciation rate of the durable good, μ and σ are the expected rate of return and standard deviation of the risky asset, r is the riskless rate of return, and dZ is the increment of a standard Brownian Motion.

If we further assume that U(K) takes the CRRA form, K^{α}/α, ($\alpha<1$, $\alpha\neq0$) the solution to this problem (shown in Appendix A) is a linear portfolio rule:

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[4] The proportion is not the same, however, so the level of purchases is different in the two models.

(3) $A_t/W_t = (\mu - r)/\sigma^2(1-\alpha)$, and

(4) $$K_t/W_t = \frac{1}{(r+\delta)(1-\alpha)} \left[\gamma - \alpha r - \frac{(\mu - r)^2 \alpha}{2\sigma^2(1-\alpha)} \right]$$

A constant proportion of wealth is held in the risky asset and in durable goods. In the case of the durable good, we see that when $\delta = 1$ (a nondurable good), the result is the same as the nondurable consumption case. Since $\alpha<1$, as depreciation decreases, more wealth is held in the durable good.

2.2 Individuals Facing a Transactions Cost

Suppose now that in addition to equations (1) and (2), the consumer optimizes subject to the payment of a transactions cost when purchasing the durable good. Following Grossman and Laroque (1990), assume this cost is proportional to the amount of durable good sold by the consumer:

(5) $W_{\tau+} = W_{\tau-} - \lambda K_{\tau-}$,

where $\tau+$ is the moment immediately following the durable purchase, $\tau-$ is the moment before, and λ is the proportional transaction cost. This cost may be thought of as a direct sales commission, sales taxes, a search cost, or the result of the "lemons principle" that undervalues used cars (see Akerlof [1970]).[5]

Grossman and Laroque [1990] show that optimal consumer behavior under these conditions is an "S-s" rule governing the state variable, $W/K \equiv y$, and the control variable, K. Behavior is characterized by three critical values of the state variable: a

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[5] The Akerlof equilibrium has no trade because of an informational failure in the secondary market that prevents the differentiation of "lemons". However, if there are liquidity traders, who must sell for liquidity reasons, then a market exists and the equilibrium price is lower than the value of "good" cars.

lower bound, an upper bound, and an internal return point. The upper and lower bounds trigger adjustment when they are reached, while the internal return point is the target value of y chosen at adjustment. Typically, W grows and K depreciates, so y increases over time. When y reaches the upper bound, the consumer readjusts his portfolio to increase K, returning y down to its target value. W would then continue to grow and K to depreciate until the upper bound is again reached, and the process repeats. Occasionally, however, W may fall, since its growth is random. If W falls more than K depreciates, then y decreases, and the consumer may hit the lower band of y, where he has too much of the durable good. The consumer would then adjust his stock of K downward, returning the state variable up to its target value. A sample path of this behavior is shown in Figure 1.

This solution precisely characterizes the durables choice at adjustment. Since the target point of the state variable (y) is a constant, at adjustment W/K always equals a constant (y^*), so K is a fixed fraction of W when purchases occur. If ι and τ are times when adjustment occurs ($\iota<\tau$), we have equation (6).

$$(6)\quad \ln\left[\frac{K_\tau}{K_\iota}\right] = \ln\left[\frac{W_\tau}{W_\iota}\right] = \xi_{\iota,\tau} \quad ,$$

where $\xi_{\iota,\tau}$ is the realized rate of return on wealth between ι and τ. This realization, of course, depends only on the stochastic process, the riskless and mean risky rates of return, and the portfolio rule.[6] None of these is history dependent.

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6 This assumes that the relative price of durables is fixed. If changes in the relative price of durables are the same for all households, then the growth rate of durables will equal that of wealth plus a constant term. Since the period $[\iota,\tau]$ varies over households, though, this term may not be constant across households. However, so long as it is not correlated with liquidity, the Eular equation implications will be unaffected.

Thus *from purchase to purchase* consumption growth is as in the frictionless case, even in the presence of a transactions cost. In between purchases, the stock deteriorates deterministically. The decision to purchase is governed by a "threshold" rule, since only when durables become much too large or too small relative to wealth, will the consumer adjust his stock.

3. SEPARATING LIQUIDITY CONSTRAINED HOUSEHOLDS

Household purchase decisions may deviate from their optimal path when consumers face binding liquidity constraints. Work on nondurables has shown these deviations to be empirically relevant for a substantial number of households[7]. Before examining the behavior of households facing transactions costs, therefore, it is necessary to separate those households facing a binding liquidity constraint. This is accomplished using an Euler equation test.

3.1 Empirically Differentiating Binding Liquidity Constrained Households from S-s Households: Exogenous Sample Splitting

Section 2.2 showed that conditional on having purchased a durable, we can perform a test equivalent to the random walk tests of consumption. The martingale property should be true for transactions cost theories, as seen in equation (6), since no information known at the time of the previous purchase (ι) should affect the current slope of the purchase path. This generalizes Hall's [1978] random walk result to purchases of

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[7] Hall and Mishkin [1982] and Zeldes [1989] are recent examples using data on food consumption.

durable goods.[8] However, in the case of a binding liquidity constraint, the martingale property fails and previously known variables that affect liquidity will also affect the slope of the purchase path.[9] I therefore examine whether past liquidity variables, such as income, affect the current slope of the purchase path.

I use data available in the *Survey of Consumer Finances* (1983 and 1986), compiled by the Federal Reserve. The data are 2422 (representative sample) households questioned in both 1983 and 1986. They provide information on their asset holdings, as well as liabilities, income, and major purchases.

I begin by identifying a priori those individuals I expect to be liquidity constrained and those I do not, and then test for differences in their behavior.[10] The *Survey* asks households whether they have been denied credit or received less credit than they requested any time in the past two years. The *Survey* further asks respondents whether or not they have been discouraged from applying for credit. I divide the sample into those with affirmative responses to either question and those with negative responses to both, then estimate the Euler equation for each:

$$(7) \quad \Delta \ln(K_i(\tau,\iota)) = \beta_0 + \beta_1 Y_i(\iota) + \beta_2 \, (\Delta DEMAND_i).$$

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8 One of the advantages of Hall's characterization for nondurables is that it does not rely on a specification for income dynamics. This result for durables was derived with perfectly diversifiable income in the form of W, and thus is less general than Hall's result.

9 Zeldes [1989] provides an example of this result. Zeldes uses food consumption, so purchases occur every period, and the purchase path can be calculated every period. Here, however, the Euler equation is tested from purchase to purchase, as in Equation (6).

10 This strategy is used frequently in the liquidity constraint literature. Zeldes [1989] is a recent example.

$\Delta \ln(K(\tau,\iota))$ is log of the ratio of automobile purchases between 1983 and 1986 (deflated to 1982 by the CPI for new cars) to last reported purchase of an automobile, $Y(\iota)$ is total income prior to the first purchase,[11] and ΔDEMAND is a vector of demand shift variables, specifically changes in household size and changes in number of driving-age members. For the S-s households, β_1 should be zero. For liquidity constrained households, the expected sign of β_1 is not unambiguous. For these households, given future income, higher current income tends to decrease the slope of consumption. However, current income may be correlated with income growth as well. This may actually increase the slope of the purchase path, if the correlation is large enough. Therefore, a significant coefficient of either sign on past income is evidence of a liquidity constraint, while the sign suggests the correlation between income levels and growth rates. The observed correlation in the sample is -0.3, so β_1 is expected to be negative if there are liquidity constraints.

The results of estimating this equation are reported in the first column of Table 1. I exclude the coefficients on the ΔDEMAND vector, but they have the expected signs and their inclusion or exclusion has little effect on the other results. For the liquid group, past income has a small negative effect on the slope of the consumption path, but this effect is not statistically significant. For the constrained group, past income has a substantial and statistically significant negative effect on the consumption profile. The constant term for the constrained group is larger than for the unconstrained group. This is consistent with the hypothesis that on average, given income and changes in demand, the constrained group should have a steeper consumption profile than the unconstrained group.

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[11] Total income includes both labor and capital income.

3.2 Endogenous Switching Estimation

There are several reasons to question the exogenous sample splitting method. First, the 1983 survey followed a period of credit upheaval in the United States, so individuals denied credit during this period may not be representative.[12] In addition some households may believe they are ineligible for credit and therefore not apply, while not identifying themselves as "denied" or "discouraged". On the other hand, a household might apply for a large amount of credit and receive less than requested, while remaining sufficiently liquid to purchase an automobile. Finally, this exogenous splitting method *a priori* identifies only 15 percent of the sample as constrained. This is lower than most other estimates. Campbell and Mankiw [1989], for example, estimate that "rule of thumb" consumers, who equate consumption and income, represent between 20 and 50 percent of the aggregate.

To address these issues, I use an endogenous method of sample splitting. Equation (6) for the constrained and unconstrained households describes two regimes; I estimate the regimes simultaneously along with the probability of being in each. This method is described in Goldfeld and Quandt [1976]. The switching model is estimated using a likelihood function including the two regimes, with observations assigned to each regime based on the value of a switching variable.[13]

I split the sample using several criteria, with similar

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[12] The 1980 credit controls induced not only constriction of credit supply but also voluntary restriction of demand, leading to a collapse of consumer credit. Further, the subsequent recession restricted access to credit for many households. In aggregate, consumer credit grew only 0.5 percent in 1980, 5 percent in 1981, and 4 percent in 1982, before jumping to 13 percent in 1983 (Table C-75, *Economic Report of the President, February 1990*).

[13] The reported results used weights calculated using the cumulative distribution function of a normal distribution, but were not sensitive to this choice.

results. The first variable used is total income. Income measures the liquidity flow available to meet consumption needs. The second measure accounts for the fact that income *relative* to demand determines whether liquidity constraints bind. For example, a household with relatively high income but even higher expected income might indeed face a binding constraint. The second measure therefore scales current income by a measure of desired consumption. I thus estimate a value for lifetime income, following Hall and Mishkin [1978], as the predicted value of a regression of total income on observable characteristics, such as age, education, occupation, and gender of head of household.[14] This is not permanent income (since it excludes the annuity value of transitory income and idiosyncratic permanent income), but is an important component.[15] The ratio of total income to lifetime income is a measure of the adequacy of current liquidity to meet desired consumption.

Equations (8) and (9) compose the estimated system, where the switching variable is S. Observations are assigned to the first regime if S is greater than the critical value (S^*), and to the second if less than S^*. The vectors β and α are then estimated along with S^*.

(8) $$\Delta \ln(K_i(\tau,\iota)) = \beta_0 + \beta_1 Y_i(\iota) + \beta_2 (\Delta DEMAND_i) \text{ if } S \geq S^*,$$

(9) $$\Delta \ln(K_i(\tau,\iota)) = \alpha_0 + \alpha_1 Y_i(\iota) + \alpha_2 (\Delta DEMAND_i) \text{ if } S < S^*.$$

The results of this estimation are reported in columns 2 and 3 of Table 1. Column 2 gives the results using income as the

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[14] The equation estimated was total income as a function of age, gender, race, marital status, education, and occupation of head of household, plus number of wage-earners, and dummies for retirement and unemployment of head of household.

[15] I also estimate lifetime income using the out-of-sample coefficients from Carroll's [1990] estimates on the *Consumer Expenditure Survey*. The results are virtually identical.

switching variable and the level of past income as an independent variable. Column 3 shows the results of using the ratio of income to lifetime income as the switching variable. Those households sorted into the high income group, I will call S-s households and those with low income "constrained".

The switching results strongly reinforce those of the exogenous sample splitting exercise. Using the level of income as the trigger, the endogenous switch occurs at a household income of $30228, which sorts 58 percent of the sample into the constrained group. Within this group, the coefficient on past income is negative and significant -- and fifty percent larger than in column 1. Evaluated at the switching point, this translates into a slope elasticity of -0.31 with respect to past income. For the households with high income, the coefficient on past income is indistinguishable from zero.

Column 3 of Table 1 reports the results of the switching estimation using the ratio of income as the trigger variable. The switching value is estimated at 1.06. The parameter estimates are qualitatively the same as those for the level of income switching, and quantitatively bounded by the earlier estimates.

These results suggest that there is a substantial group of households who purchase automobiles facing a transaction cost rather than a liquidity constraint. Throughout the remainder of the paper I use the switching results from the level of income (Table 1, column 2) to split the sample into S-s and liquidity constrained households. The data in Table 2 describe the characteristics of households below and above mean income. The constrained (low income) group has financial flows and stocks, such as income and assets, on average only 25 percent that of the liquid group. In addition among those who reported being denied credit, 75 percent are sorted into the low income group. In the automobile category, constrained households on average spend about one-third less than the unconstrained. They are 15 percent less likely than the average to obtain a loan, but when successful they borrow about the same amount (80 percent of purchase price) as their liquid counterparts.[16] The constrained households are more likely to be headed by non-whites, women, and single individuals. Not surprisingly, higher wage and skill occupations are overrepresented in the high income group.

4. BEHAVIOR OF CONSUMERS FACING A TRANSACTIONS COST FOR AUTOMOBILES

Having identified those households facing only a transactions cost, I now focus explicitly on describing their behavior. First, I calculate directly the S-s bands of these households. Then I consider how households choose the waiting time between their automobile purchases. I first estimate the determinants of the width of the bands. Then I consider the characteristics of the speed of movement through the bands. The results indicate that increasing the level of wealth does not reduce the waiting time between purchases. However, higher growth reduces the waiting time by moving households more quickly through the S-s bands. On the other hand, higher variability of movement through the bands widens the bands since households want to avoid a premature purchase that is costly to reverse.

4.1 Location of the S-s Bands

The primary characteristic of the the S-s model is the bands around which the consumer moves his durables stock. A consumer's S-s bands can be calculated by observing the value of (W/K) before and after the consumer adjusts his durables stock. The

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[16] In theory it is indeterminate whether constrained households should borrow more or less than their unconstrained counterparts -- they might demand more credit but appear less "credit worthy" and therefore face restricted supply.

observation before adjustment provides the trigger point, and the observation afterwards provides the target point. A complication, however, is the difficulty of measuring total lifetime wealth, W, which includes both financial and (unobservable) human wealth. Net worth is clearly the first component of total wealth, but human wealth is more problematic. In order to calculate human wealth, I assume a household's horizon is the difference between the age of the head and age 65; for household heads over 65 with labor income, I take the horizon to be one year. Human wealth is then the present discounted value of current income over this horizon, discounted at 5 percent annually. Define π_i as the ratio of human wealth to current income. The state variable, y, should then be the following sum:

$$(10) \quad y_i = \frac{\text{Net Worth}}{\text{Durables}} + \pi_i \frac{\text{Income}}{\text{Durables}} .$$

In the theoretical model, there is only one durable good. So if the denominator includes only automobiles, it is clear that we will underestimate durables and overestimate y. The simplest solution is to divide automobile stocks by their share in total durables, which in the sample is 10 percent. This produces an estimate of durables based only on automobiles and forms the denominator of y.[17]

Table 3 reports the means of these ratios and the resulting values of y. The first row reports the means of y calculated as above, taken at the trigger and target points.[18] The second row

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[17] This forms an estimate of automobiles as if they were the only durable consumption good available.

[18] To calculate the empirical trigger and target values of y, I take the values of durables, net worth and income in the year in which the household purchases a new automobile and use these to calculate y*. These values of income and net worth and the value of their automobile the previous year are used to calculate y^H or y^L, depending on whether the household adjusted their automobile

reports the theoretical values for these points. Theory indicates that at adjustment about 13 percent of total wealth should be held in durables. This would be allowed to drift down to about 6 percent before adjusting again, or up to 30 percent if wealth falls. The empirical values are very close to those theoretically predicted, and while the standard errors are large, the model seems to do well on average. The means of the components of y are reported in rows (3a) and (3b), and they follow the same general pattern as the theoretical prediction.[19]

The second half of Table 3 indicates observed band width. Theory predicts that y doubles from target to trigger (halving the ratio of durables to wealth) before adjusting, corresponding to values of two in the second column. Column 2 of the table shows that this is true for both the calculated value of y and its components. The last column in the second half of Table 3 suggests that if wealth is falling, consumers wait until the stock is 25 percent too large before adjusting it downward. This is smaller than the theoretical value, which is closer to 50 percent. The number of consumers who adjust downward in the sample is very small, however, -- only 4 percent.

These band widths are substantially wider than those estimated by Bertola and Caballero in the aggregate data.[20] They

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up or down. Since y is measured at a fixed interval (of years) and not at the moment of adjustment, band width is underestimated. The magnitude of the bias will vary with the speed of movement through the bands; this will be discussed in Sections 4.2 and 4.3.

[19] The last two rows in the top and bottom of the Table 3 give the band calculations for the components of y separately. These values follow the same pattern as those for the calculated y, suggesting that the band widths found for y do not depend on the particular formulation chosen for human wealth and autos.

[20] Bertola and Caballero construct aggregate durables stocks and wealth from the series for durables purchases and income. Setting the theoretically optimal durables stock to a share of wealth, they then estimate the S-s bands necessary to generate the

find that consumers adjust their durables stocks by 26 percent when adjusting. This may result from their wider classification of durable goods -- all durables rather than just automobiles. However, in subsequent work Caballero [1990] estimates that automobile stocks should increase fivefold at adjustment. My estimate is far less than this. The presence of liquidity constrained consumers in the aggregate data may account for this difference, but this effect is not clear. Their more rapid response to changes in income may lead to faster observed responses to aggregate shocks, and therefore lower estimated band width. However, the liquidity constraint may prevent them from responding to all but very large increases in income, and therefore slow their response and increase estimated band width.

4.2 Determinants of Band Width

The length of time that consumers wait between durables purchases is determined by the speed at which consumers move from target to trigger and the width between the S-s bands. This distance is chosen by consumers when they choose the bands.

Band width is determined by three characteristics. First, factors which systematically increase the speed of movement through the bands will increase band width, since the consumer wants to avoid frequent and costly adjustment. Intuitively, the state variable, W/K, can be translated into terms of the desired level of durables holdings, $K^*/K = \alpha W/K$. The triggers and targets can therefore be thought of as critical values of the preferred level of the durables stock relative to its actual value. As a result movement through the bands results from depreciation in K, as well as changes K^*. In the formulation of the model here, the only source of such changes in K^* is wealth. However, in a more general formulation, there would be several sources of such systematic changes in demand. For example, predictable increases

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observed deviations from this optimal stock.

in household size, number of drivers, and desire for quality or reliability, or negative drift in the relative price of durables over time would all tend to induce positive drift in K*. Therefore, in addition to the mean growth rate of wealth, these variables would increase band width.[21]

The second determinant of band width, risk, has two effects. The risk aversion effect tends to widen the bands since the consumer wants to avoid adjusting often in response to "jumpy" realizations of y. The portfolio effect, however, reduces the speed of movement by reducing investment in the riskier high return asset, and therefore tends to narrow the bands. Thus, increases in the variance of returns may increase band width, but this effect will be damped if the consumer substitutes away from this asset. Finally, an increase in the adjustment cost also broadens the bands.

To quantify these effects, I estimate the following equation:

$$11)\quad WIDTH = \beta_0 + \beta_1 LDUM + \beta_2 \ln(Y) + \beta_3 \ln(NW) + \beta_4(growth,Ylife) + \beta_5(\sigma^2 Y) + \beta_6(sales\ tax),$$

where LDUM is a dummy for downward adjustment of the durables stock, ln(Y) is the natural logarithm of 1985 income, ln(NW) is the natural logarithm of 1986 net worth, (growth,Ylife) is the growth rate of lifetime income (as estimated in Section 3), $\sigma^2 Y$ is the variance of total income from 1982 to 1985, and (sales tax) is the state sales tax rate. The dependent variable is the percent that the durables stock is adjusted relative to wealth at the time of the most recent purchase. I include both upward and downward adjustment in the sample since in the data the distance to the

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21 Increased depreciation also increases band width, but cross-section variation is difficult to identify.

lower bound is typically a constant fraction of the distance to the upper band; a dummy variable for adjustment from the lower band is included in the regression.

The first column of Table 4 gives the estimation results for the S-s households. The first two rows give the coefficients on the constant term and the dummy variable for downward adjustment. The second two rows give the results for income and net worth, neither of which should affect band width. The last three variables should all increase band width. The first is growth in lifetime income. To the extent that this measures only expected lifetime income, such growth should be included in the measure of W, and therefore be irrelevant for band width. However, this measure is calculated from demographic characteristics, so I infer that it also includes information about the household's changing demand for automobile services. Changing household size and age, for instance, may affect the demand for services and quality, and therefore systematically increase K*. Such changes should be largely predictable and should be factored into the choice of band width. Growth in K* should increase band width, so this drift term should enter with a positive sign.[22] The next variable is income variance, which should increase the variability of growth in the state variable, and therefore also increase band width. The final variable is the state sales tax rate, which increases the cost of adjusting automobile stocks and increases band width.

The first column of Table 4 is generally supportive of the above hypotheses. Income and net worth do not affect the choice of band width; both are insignificant and have very small coefficients. Lifetime income growth is significant and but has a small effect; evaluated at the means, an increase of five

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[22] I also used an out-of-sample estimate of lifetime income growth using the coefficients of Carroll [1990] calculated from the *Survey of Consumer Expenditure*. The results are unaffected.

percentage points increases band width by a percentage point.[23] Income variance has a significant and very large effect. Taken at the means, increasing income variability by one standard deviation (0.39) increases band width by 10 percent. This implies that the durables stock would be allowed to deviate from optimal by 55 percent rather than 50 percent before adjusting. The state sales tax rate is insignificant in the regression.

The second column of Table 4 reports the results of the same regression for the liquidity constrained households. These results show much different behavior than for the S-s group. Income and net worth both enter negatively and significantly, implying that households with higher income and assets keep their durables stocks closer to the optimal level. Lifetime income growth does not have a significant effect for this group. Income variance again enters, significantly increasing the width of the bands. Sales taxes are again insignificant.

These results suggest that income and net worth levels do not affect how closely the S-s households maintain their auto stocks relative to their optimal stocks. As income rises, therefore, this implies that households will not increase the frequency of adjustment. The S-s consumers' durables stocks will deviate from their optimal level to a greater extent when income variability are high, and also when lifetime income growth is high, though by a smaller amount. The former is consistent with Romer's [1990] evidence on consumer uncertainty and postponement of durables purchases during the Great Depression.[24] For the liquidity

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[23] The downward bias in measured band width increases with the speed of movement through the bands. This biases downward the coefficient on lifetime income growth, since it is correlated with the growth rate of y. The result reported above is therefore a lower bound.

[24] If income uncertainty is permanently increased, households widen their S-s bands, but will adjust more frequently due to higher variability of income. However, this also increases the

constrained consumers, however, income and net worth are important determinants of how closely they are able to track their optimal auto stocks.

4.3 Speed and Waiting Time

The time dimension of purchases is crucial for aggregation since most of the variation in the total durables expenditures is from the number of buyers rather than the amount purchased.[25] In household data, this dimension is the choice of when to buy, and given the band width, is determined by the speed of movement through the bands.

For a given set of preference and rate of return parameters, the bands are a fixed distance apart and independent of wealth. Given this distance, the waiting time evolves from the "speed" at which the consumer moves from "target" to "trigger", i.e. on the realized growth of the state variable y. This depends on the portfolio chosen by the consumer and does not depend on the level of wealth, but only on its growth.

To estimate the determinants of the speed of movement, I specify the following equation:

$$12)\quad \text{SPEED} = \beta_0 + \beta_1\text{LDUM} + \beta_2\ln(Y) + \beta_3\ln(NW) + \beta_4(\text{growth},Y\&NW) + \beta_5(\sigma^2 Y) + \beta_6(\text{sales tax}),$$

where SPEED is band width divided by the waiting time between the two most recent purchases, (growth,Y&NW) is the sum of the growth rates of income and net worth over 1982 to 1985, and the other variables are as in the previous section. The dependent variable

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probability that households adjust their durables stock downward, depressing aggregate expenditures. In the short run, the increase in band width may depress expenditures, as well.

25 See Bar-Ilan and Blinder [1988].

is the household's average speed of movement from the target to the trigger.[26] It should be unaffected by income and net worth. The realized growth rates of income and net worth, however, should increase the speed of the state variable. The income variance and sales tax measures should be accounted for by band width, and therefore should not affect speed.[27]

Table 5 reports the results of estimating equation (16). The first column reports the results for the S-s households. The first two rows report the coefficients on the constant term and the dummy for downward adjustment of the durables stock. The second two rows report the effects of income and net worth on the speed of movement. The final three rows contain the coefficients on actual income and wealth growth, income variance, and sales taxes.

The first column of Table 5 supports of the transactions cost view. Income and net worth do not affect the average speed of movement through the bands; their coefficients are small and insignificant. Income variance and sales taxes are also insignificant; their effect on waiting time is captured by band width. The only significant variable is the rate of growth of income and net worth. This enters very significantly and the magnitude implies that a doubling of the growth rate produces a 10 percent decline in the waiting time.[28]

For the liquidity constrained households in the second

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[26] This is the average growth rate of y.

[27] If asset returns negatively correlated with income uncertainty also had lower returns, then income variance would have a negative effect on speed as households diversify their risk.

[28] To the extent that band width is underestimated, speed will be underestimated as well, especially at high rates of movement. This would bias downward the coefficient on (growth,Y&NW), so the reported result is a lower bound.

column of Table 5, the results of equation (16) look quite different. Income and net worth both enter significantly and negatively. This may be largely the negative effect of these variables on band width (the numerator of SPEED) found in Table 4 for the liquidity constrained group. Income and wealth growth again significantly increase speed, while income variance and sales taxes are insignificant.

These results show that for the S-s households, the rate of growth of income and wealth is the primary determinant of how quickly they move through the S-s bands. This implies that for given band width, faster income growth will cause more households to "hit" the triggers in a given period, resulting in increased aggregate expenditures. This is also true in the case of a temporary increase in the level of income or when the increase in income growth is transitory, since the state variable increases faster than expected. In the short run, more households hitting the trigger will cause aggregate expenditure to rise since the number of buyers increases. Aggregate expenditure will therefore respond to changes in income by more than the amount of the change in individual expenditures, since the number of buyers will also be temporarily high.

In order to be more precise about the number of households "hitting" the triggers and making purchases, I now consider the cross-section distribution of households between the trigger points.

5. DISTRIBUTION OF DURABLES STOCKS RELATIVE TO WEALTH

The critical step in moving from the micro behavior described in Section 4 to the macroeconomic behavior of durables expenditures involves the cross-section distribution of households. This distribution determines how many households are near the trigger points, and therefore how many will adjust their durables stocks during a given period. I first present the theoretical limiting distribution of households and find it closely matches the observed empirical distribution. I then examine how the distribution responds to aggregate shocks and show that it can explain the magnitude and persistence of the response of durables expenditure to income growth in the 1980s.

5.1 The Ergodic Distribution of Durables Relative to Wealth

The ergodic distribution of y for a single individual gives the probability density function for his value of y at any point in time. In this case, under very general conditions, the ergodic distribution can be precisely characterized. With some simplifying assumptions about the portfolio chosen by the household[29], the ergodic distribution can be solved analytically. This distribution is shown in Figure 2. The shape, piecewise exponential and discontinuous at the return point, is similar to that found by Tsiddon [1988] and Bertola and Caballero [1990] in other models.

Under simple regularity conditions, when there is no aggregate uncertainty, this distribution is also the cross-section stationary distribution. This may be interpreted as the distribution to which the cross-section converges in the long run, so long as shocks to wealth are not correlated across individuals.

Taken as the cross-section stationary distribution, the shape is very intuitive. Households in the tails tend to hit the triggers and adjust to the target in the interior of the distribution. Around this point, the durable is a large part of the portfolio, so total wealth grows slowly, and households do not move quickly away. However, once they have begun to drift to the right, they hold more of their assets in the risky asset, y grows quickly, and the density falls.

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[29] This derivation and assumptions are shown in Appendix B.

This distribution is the stable ergodic distribution if there are no aggregate shocks. However, if there are also aggregate shocks, there is no stationary distribution and we would not expect to actually observe this shape at any point in time. The density would tend toward the ergodic distribution between[30] aggregate shocks, but it would not be stable.

5.2 The Empirical Distribution of Durables Stocks

To compare the ergodic density in Figure 2 to the empirical density, I first construct a Kernel Density Estimate of the distribution of y in 1983. Intuitively, this constructs a distribution around each of the data points and uses them as weights in a moving average. This systematically smooths the discrete histogram using a weighted moving average.[31] The resulting estimate is shown in Figure 3 as the empirical density, plotted together with the theoretical ergodic distribution. The empirical peak is very near the optimal adjustment point, with similar asymmetries. A Kolmogorov-Smirnov Goodness-of-Fit test confirms that this similarity cannot be statistically rejected (the p-value for rejection is more than 0.2).[32]

If this density is stable, however, the same number of households will hit the triggers each period; this would not provide much basis for aggregate dynamics. In fact, however, the distribution does shift over time. Figure 4 shows that over the period 1983 to 1986, the peak becomes more pronounced, implying that large numbers of households adjusted their automobile stocks

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30 Since all shocks are continuous in this model, the density tends toward the ergodic when idiosyncratic uncertainty is large relative to the aggregate.

31 This was done using the optimal bin widths from Silverman [1990].

32 This test statistic is the maximum of the absolute difference between the two curves.

relative to wealth. This is consistent with the fact that during this period real Personal Consumption Expenditures for automobiles averaged annual growth of 15 percent per year, while income and net worth growth averaged 3.2 and 4.1 percent, respectively.[33] We can reject that the 1983 and 1986 distributions are statistically identical with a p-value of 0.05 and further reject that the 1986 distribution is the same as the ergodic distribution with a p-value of 0.01.

The shape of the distribution and consequently, the dynamics of aggregate expenditure, are determined by the same factors that affect band width and speed of movement, noted in Section 4. Band width determines the state space of the distribution, while the speed of movement determines the density over the state space.

The results of Section 4 showed that the variance and growth rate of income and wealth are determinants of the S-s bands and the speed of movement. Increases in income variability have several aggregate effects. First, as shown in Section 4, increases in income variability tend to broaden the bands. This broadens the state space of the distribution of y. Second, higher variability increases the frequency of adjustment and the probability of adjusting the durables stock downward; this corresponds to shifting the density of y toward the lower bound. Third, if the increase in variability is primarily aggregate, (e.g. it increases aggregate relative to idiosyncratic uncertainty) the cross-section distribution of y will deviate more from the ergodic distribution and the resulting aggregate expenditure series will exhibit larger deviations from its frictionless counterpart.

Section 4 also showed that higher income and wealth growth increase the speed of movement through the bands. This increases

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[33] *Economic Report of the President, 1990*, and *Flow of Funds* Data.

the number of households hitting the trigger in any period, boosting aggregate expenditure.[34] The empirical distribution also changes, with more density concentrated near the peak, corresponding to the increased number of households adjusting.

These effects are quantified by a series of simulations starting from the 1983 observed cross-section. I assume aggregate shocks corresponding to actual growth in real per capita disposable income, with no idiosyncratic uncertainty.[35] I then simulate the response of the households to these shocks and calculate the kernel density estimate corresponding to each year. The simulated density estimate for 1986 is plotted in Figure 5 along with its empirical counterpart. The densities correspond closely, suggesting that much of the shift in the empirical distribution from 1983 to 1986 can be explained by the temporary increase in income growth in the early 1980s. Over this period rapid income growth caused households in the upper tail of the distribution to adjust to the target point. Households near the peak in 1983 showed rapid growth of y and move to the right in the distribution, and are close to adjusting again by 1986.

Figure 6 shows the evolution of the density over the entire period. Year zero is 1983, and the series continues through 1988. Year two is the second contour on the surface and shows little change in the distribution. Year three shows that the peak moves slightly toward the origin and becomes more pronounced in response to doubling the growth rate of income. This is the contour we observe in cross-section in Figure 5. When the growth rate returns to two percent in the subsequent 3 years, the peak becomes

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[34] An increase in the mean rate of income or wealth growth broadens the bands, but here I take mean growth as fixed.

[35] I assume that every household receives a 2.0 percent increase in real income in the first year. This increases to 4.0 percent in the second year, and then returns to 2.0 percent for the last two years.

higher and stabilizes near the target point. This implies that more households are adjusting each year, so aggregate expenditure has increased in response to higher income growth.

The implied increase in aggregate expenditures is made precise in Table 6. The upper half of Table 6 reports the growth in automobile expenditures corresponding to the above simulation. The first row gives the assumed income growth. The second row gives the growth in aggregate expenditures, while the third and fourth rows break this into the growth in number of buyers versus growth in average purchase. These simulations imply that the elasticity of expenditure with respect to income exceeds 10 in the short run, but the effect is depleted within three years. Almost all of the variation is in the number of buyers, while the average purchase corresponds more closely to the increase in wealth.

The middle of Table 6 reports the actual income, wealth, and expenditure aggregates over the period of the simulation. The first row gives the growth rates in real disposable income which motivate the assumptions for the simulations. The second row gives the growth in real net worth. The third and fourth rows give the break down of net worth for corporate equities and owner-occupied housing. The fifth row reports the growth rate of real expenditures on motor vehicles. The first row shows that real income surged in 1984, and then tapered off to less than one percent by 1987. Net worth grew faster than income in 1983, but slowed in 1984 before increasing in 1985 and tapering off thereafter. Expenditures on motor vehicles increased over 16 percent in 1983 and 1984; this corresponds to an average short run income elasticity of 5.8. The growth rate of expenditures then fell slightly in 1985, again in 1986, and became negative in 1987.

The simulations closely match the actual data. Simulated expenditure rises sharply in the first two years as the higher growth of income causes more households to hit the trigger point in a given period. The growth rate of expenditure declines when the growth rate of income stabilizes, since households are no longer hitting the trigger at an increasing rate. Expenditure declines in year four, when virtually all households have adjusted to the surge in the growth rate. After this, expenditure stabilizes with low frequency cycles. Overall, the temporary increase in the growth of income boosts expenditure for four years -- most strongly after the initial shock, and then tapers off.

This is in contrast to the predictions of a model with zero transaction cost. The last row of Table 6 shows that the frictionless model predicts the immediate response to higher income growth in 1983 and 1984, but then implies a rapid reduction in expenditure when growth slows in 1985 and 1987. This is because the model implies that expenditure should rise immediately in response to higher wealth, but then fall the next period since consumers will only need to replace the previous period's depreciation.

Simulated expenditures differ from the aggregate data in two ways. First, the initial growth in simulated expenditure is larger than the observed, and second, growth tapers off one year sooner in the simulations than it does in the actual data. There are two possible explanations for this result. First, growth in net worth is negatively correlated with that of income over this period. Total wealth growth is therefore smoother than that used in the experiment, smoothing the response of expenditure. Second, the aggregate data also include the liquidity constrained households. For these households, expenditure is determined largely by the timing of increases in income. If these increases are distributed throughout the sample period, then the measured growth rate is less volatile than the simulated rate, reducing the initial response and prolonging the adjustment period.

6. CONCLUSIONS

This paper considers the automobile purchases of households facing a transactions cost. I find that about half of the households in the sample behave in this manner, while the others are liquidity constrained. The decision rules of the households facing the transactions cost are not affected by the level of income or wealth, but are determined largely by their growth rates and variability. Higher variability broadens the S-s bands so durables deviate more from their optimal levels before households adjust. Higher growth of income or wealth, on the other hand, speeds households through the bands, increasing the number of buyers per period and stimulating aggregate expenditure. The empirical distribution of consumers' auto stocks relative to wealth fits that predicted by theory, but is not stationary. Simulations show that the changes in the distribution are consistent with observed growth in income over the period. Further, the model explains the actual acceleration and subsequent slowing of aggregate durables expenditure in the 1980s.

These results may provide an explanation for the persistence of shocks in aggregate durables expenditures. With transactions costs, disturbances to aggregate expenditures persist until all households have adjusted their durables stocks. Those close to the trigger point will react immediately to an aggregate disturbance, incorporating it into their purchases. Households further away from the trigger point will take longer to respond, only incorporating the aggregate shock when they purchase. Thus, the aggregate effect is only complete when all households have adjusted. While theoretically these effects can be negated if the economy is at steady state, as in Caplin and Spulber (1987), the cross-section evidence does not show a stationary distribution.[36]

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[36] Furthermore, the steady state result would not obtain if the distribution of wealth is not stationary.

Simulations of the microeconomic adjustment show that complete adjustment should take three to four years, consistent with the aggregate finding of Caballero [1990]. The simulations further show that the adjustment process is not smooth, but instead exhibits the rapid initial response and slow tapering off observed in the aggregate data.

Further, aggregate expenditure will respond vigorously to changes in income relative to a Life Cycle-Permanent Income model, even without liquidity constrained households. An increase in the level or growth of income moves households through the S-s bands more rapidly, increasing the number of purchasers per period. This temporarily increases aggregate expenditure, producing a large income elasticity of aggregate expenditure, although each individual purchase is still of the optimal amount.

Finally, these aggregate effects will arise even when individual purchases are consistent with Life Cycle-Permanent Income behavior. Individual purchases are selected as an optimal share of lifetime wealth, but the number of buyers also responds to innovations in permanent income. The product of these two effects produces a dichotomy between observations of a household and the aggregate, with the difference explained by the failure of the representative agent assumption and the dynamics of the cross-section distribution of households.

Future work will address in more detail the implications of the liquidity constrained households for aggregate expenditure, as well as the multivariate characteristics of models with transactions costs.

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APPENDIX A

DERIVATION OF OPTIMAL CONSUMPTION

As in equations (1) and (2) in the text, the consumer maximizes

$$\text{A.1)} \qquad E\left\{ \int_0^\infty U(K_t)e^{-\gamma t}\, dt \right\},$$

subject to the dynamic budget constraint,

$$\text{A.2)} \qquad dW_t/dt = (\mu - r + \sigma dZ)A_t + rW_t - (r+\delta)K_t \;,$$

where K is the stock of durables, W is total wealth, and F is wealth held in the risky asset. The parameter γ is the subjective discount rate, δ is the depreciation rate of the durable good, μ and σ are the expected rate of return and standard deviation of the risky asset, r is the riskless rate of return, and dZ is the increment of a standard Brownian Motion.

Define the functions I(.) and J(.) as follows:

$$\text{A.3)} \qquad I(W_t, t) \equiv \underset{K_s, W_s}{\text{Max}} \int_0^\infty e^{-\gamma s}\, U(K_s)\, ds$$

$$\text{A.4)} \qquad J(W_t, t) \equiv e^{\gamma t} I(W_t, t)$$

$J(W_t, t)$ is independent of time and can be written simply as J(W). The maximization can then be written as equation (A.5), with first order conditions (A.6) through (A.8).

A.5) $$0 = \max_{K_t, W_t} \left\{ U(K_t) - \gamma J(W_t) + J'(W_t)\left[rW_t + (\mu - r)A_t - (r+\delta)K_t \right] + (1/2)\sigma^2 A_t^{\ 2} J''(W_t) \right\}$$

A.6) $$U'(K_t) - (r+\delta)J'(W_t) = 0$$

A.7) $$(\mu - r)J'(W_t) + \sigma^2 A J''(W_t) = 0$$

A.8) $$(r-\gamma)J'(W_t) + J''(W_t)\left[rW_t + (\mu - r)A_t - (r+\delta)K_t\right] + (1/2)\sigma^2 A_t^{\ 2} J'''(W_t) = 0$$

As in the text, let $U(K) = K^\alpha/\alpha$, $I = (b/\alpha)e^{-\gamma t}W^\alpha$, and therefore $J = (b/\alpha)W^\alpha$, where b is a constant to be determined. Equation (A.7) can then be solved for A_t/W_t, giving the linear portfolio rule.

A.9) $$A_t/W_t = \frac{\mu - r}{\sigma^2(1-\alpha)}$$

Using the above functional forms, Equation (A.9) can then be substituted into Equation (A.6) to solve for K_t/W_t.

A.10) $$K_t/W_t = \frac{1}{(r+\delta)(1-\alpha)} \left[\gamma - \alpha r - \frac{(\mu - r)^2 \alpha}{2\sigma^2(1-\alpha)} \right]$$

This is the equation seen in text equation (4).

APPENDIX B
DERIVATION OF THE ERGODIC DISTRIBUTION

The derivation and implications of the ergodic distribution of y (defined there as $\equiv W/K - \lambda$) are found in Eberly [1991]. The following provides an overview.

Nondurable assets are held in a risky asset (A) and a riskless asset (B). The risky asset has expected rate of return μ and standard deviation σ. δ is the rate of depreciation of the durable good. Define x as A/K, one component of y. I first derive the distribution of x and then use it to characterize that of y. The equation of motion governing x is

B.1) $$dx = x(\mu+\delta)dt + x\sigma dW,$$

where dW is the increment of a Standard Brownian Motion. It is convenient in what follows to use z, defined as the natural log of x, which follows a Standard Brownian Motion with drift g and standard deviation σ when adjustment does not occur.

B.2) $$z \equiv \ln(x), \text{ where}$$
B.2a) $$dz = gdt + \sigma dW.$$

The movement of z is regulated by the S-s bounds in the state space of y. When g is large relative to σ, the relationship between x and y is monotonic, and therefore the bounds in the state space of y (yL, y*, and yH) can be transformed into the state space of x, and then to z. Call these bounds zL, z*, and zH. Z then follows a regulated Brownian Motion described by equation (B.2a) and the bounds.

The limiting distribution of this process is $\varphi(z)$:

$$\text{B.3)} \qquad \varphi(z) = \begin{cases} \dfrac{\Pr(T_l<T_h)\left\{\exp\left[(2g/\sigma^2)(z-zL)\right] -1 \right\}}{\Pr(T_h<T_l)(zH-zL) - (z^* - zL)} & \text{, if } z < z^* \\[2ex] \dfrac{\Pr(T_h<T_l)\left\{1 - \exp\left[(2g/\sigma^2)(zH-z)\right] \right\}}{\Pr(T_h<T_l)(zH-zL) - (z^* - zL)} & \text{, if } z > z^*, \end{cases}$$

where T_l and T_h are the hitting times for the lower and upper bounds, respectively. $\Pr(T_l < T_h)$ is then the probability of hitting the lower bound before the upper. This distribution is piecewise exponential with a discontinuity at the internal return point z^*.

x and y are related by the portfolio rule of the consumer, which determines how much of the risky asset is held.

$$\text{B.4)} \qquad x(y) = - \frac{(\mu-r)h'(y)}{\sigma^2 \; h''(y)} ,$$

where r is the riskless rate of return and h(y) is a transformation of the consumer's value function. Without the transaction cost, this would be the linear portfolio rule of Merton [1969]:

$$\text{B.5)} \qquad x(y) = - \frac{(\mu-r)}{\sigma^2 \; (1-\alpha)} y ,$$

where $(1-\alpha)$ is the coefficient of relative risk aversion.

Using (B.2) through (B.4) I can use a change of variable to characterize the limiting distribution of y:

B.6) $\Xi(y) = \varphi(z)/\ y'(z)$.

This requires that $y'(z)$ be a single-valued function. This is satisfied over the region [y*,yH] for all values of the parameters. Over the region [yL,y*), this is most likely to be satisfied when the drift in y is large relative to its variance. I assume that this condition holds for automobiles, which have high depreciation and therefore high drift in y.

TABLE 1. SUB-SAMPLE & SWITCHING ESTIMATION OF THE EULER EQUATION

Dependent Variable: %Δ(K(τ,τ-1)), percent change in purchases

	(1)	(2)	(3)
METHOD:	Sample Splitting	Switching	Switching
SORTING VARIABLE	Credit denied or Discouraged	Income	Total Income/ Lifetime
GROUP 1: LIQUID	Neither denied nor discouraged	High Income	High Income/ Lifetime
constant	2.2 (0.17)	1.81 (0.29)	2.04 (0.14)
past income ($10000)	-0.03 (0.03)	-0.00 (0.04)	-0.02 (0.02)
number of group observations	470	233	221
GROUP 2: CONSTRAINED	Either denied or discouraged	Low Income	Low Income/ Lifetime
constant	3.9 (0.51)	3.52 (0.50)	3.37 (0.42)
past income ($10000)	-0.30 (0.15)	-0.47 (0.25)	-0.41 (0.16)
number of group observations	80	317	329
switch variable critical value	N/A	$30228 (4703)	1.06 (0.04)

Notes: Column 1 reports the results of estimating equation (7) in the text. Columns 2 and 3 report the results of estimating the system in equations (8) and (9), using switching variables as noted. Lifetime income is the predicted value of total income based on demographic variables. Standard errors are in parentheses.

TABLE 2. CHARACTERISTICS OF THE SUB-SAMPLES, 1983

	Group 1	Group 2	Total
Income	>28,000	<28,000	
Number of Households	888	1,534	2,422
Financial Characteristics			
Mean Income	51,321	14,758	28,205
Mean Net Worth	202,666	50,331	106,399
Mean Check/Savings	6,518	2,582	4,026
Mean Real Assets	178,839	46,303	95,143
Reported Credit Denied	85	249	334
Automobile Purchases			
Mean Waiting Time Given Income	29	32	30
Mean Amount of Car Purchase	7185	4782	
% of New Car Buyers Obtaining Loans	40.9	29.7	36.4
Mean Percent Financed if Loan Obtained	76.0	79.8	77.9
Demographics			
Mean Age of Head	45.2	47.0	46.3
% Nonwhite Head	8.1	16.2	13.2
% Female Head	5.1	34.6	23.7
% Married	88.1	52.7	65.8
% Prof & Technical	22.7	12.3	16.2
% Managers & Admin	21.4	7.0	12.3
% Sales & Clerical	10.5	15.1	13.4

Note: The data are computed from the 1983 *Survey of Consumer Finance*. The statistics represent the means (unless otherwise noted) of each series for the high income sub-sample, the low income sub-sample and the entire representative sample.

TABLE 3. OBSERVED VALUES OF THE S-s BANDS

State Variable $y + \lambda \equiv W/K$	Observed Means: target at adjustment	trigger to adjust: downward	upward
1. Calculated	7.69 (6.09)	16.9 (18.5)	7.6 (5.4)
2. Theoretical	7.67	17.1	3.4
3a. Net Worth / Automobile Stock	19.36 (26.49)	37.80 (85.88)	14.73 (13.85)
3b. Income / Automobile Stock	4.87 (3.50)	11.34 (11.26)	4.69 (3.16)
	Normalized by the target value[1]		
1. Calculated	1.00	2.20	0.98
2. Theoretical	1.00	2.23	0.45
3a) Net Worth / Automobile Stock	1.00	1.95	0.76
3b) Income / Automobile Stock	1.00	2.33	0.96

Note: Table 3 gives the observed values for the target point, y^*, and the trigger points, yL and yH, as well as the band width. The bands are calculated by taking the ratios of wealth to durables times preceding adjustment of the automobile stock, and then again after adjustment. Note that downward adjustment of y represents an increase in the durables stock. Standard errors are in parentheses. The theoretical values correspond to parameter values of a 5 percent transaction cost, depreciation of 10 percent, mean risky rate of return of .059 and standard deviation 0.22, and riskless rate of .01. The asset returns and variance are from Ibbotsen and Sinquefeld [1989]. The depreciation rate corresponds to a scrappage time of 10 years (Moody's [1988]).

1. In each case the variables in the upper part of the table are normalized by dividing each row by its value in the first column, the value chosen at adjustment. The second two columns can now be interpreted as measures of how far the consumer allows the state variable to deviate from its target value before adjusting.

TABLE 4. DETERMINANTS OF BAND WIDTH

DEPENDENT VARIABLE = Fraction the State Variable (y) is adjusted when adjustment occurs

INDEPENDENT VARIABLES	S-s HOUSEHOLDS[1]	LIQUIDITY CONSTRAINED
Constant	0.85 (0.80)	2.27 (0.71)
Dummy for downward adjustment	0.12 (.06)	.55 (.08)
Income ($10,000)	-0.04 (.08)	-.14 (.07)
Net Worth ($10,000)	0.01 (.02)	-.04 (.016)
Real Lifetime Income Growth	0.15 (.08)	-.02 (.06)
Year-to-year Real Income Variance	0.09 (.01)	0.21 (.02)
State Sales Tax Rate (% points)	0.02 (0.03)	-.01 (.03)
R-Squared	.27	.33
Number of Observations	550	602

Note: Standard errors are in parentheses. The Table reports the results of estimating the following equation:

$$\text{WIDTH} = \alpha + \beta_1(\text{LDUM}) + \beta_2\text{LN}(85\ \text{income}) + \beta_3\text{LN}(86\ \text{Net Worth}) + \beta_4\text{LN}(\text{lifetime income growth},82\text{-}85) + \beta_5(\text{income variance},82\text{-}85) + \beta_6(\text{state sales tax rate}),$$

where width is calculated as abs[(target/trigger)-1].

1. The S-s households are those endogenously sorted into the S-s group by the switching regression of Section 3.2. The switching variable used is the level of income.

TABLE 5. DETERMINANTS OF SPEED THROUGH BANDS

DEPENDENT VARIABLE = Band Width/Waiting Time = Speed of Movement Through the Bands

INDEPENDENT VARIABLES	Ss HOUSEHOLDS[1]	LIQUIDITY CONSTRAINED
Constant	0.02 (0.03)	0.27 (0.06)
Dummy for downward adjustment	0.012 (.003)	.026 (.006)
Income	0.0005 (.0033)	-.018 (.006)
Net Worth	-.001 (.001)	-.006 (.0018)
Actual (Income+ Wealth) Growth	0.004 (.001)	.009 (.002)
Year-to-year Real Income Variance	0.000 (.000)	0.000 (.000)
State Sales Tax Rate (% points)	0.001 (.001)	-.002 (.002)
R-Squared	.08	.33
Number of Observations	550	602

Note: Standard errors are in parentheses. The Table reports the results of estimating the following equation:

$$SPEED = \alpha + \beta_1(LDUM) + \beta_2 LN(85\ income) + \beta_3 LN(86\ Net\ Worth) + \beta_4 LN(wealth + income\ growth, 82\text{-}85) + \beta_5(income\ variance, 82\text{-}85) + \beta_6(state\ sales\ tax\ rate),$$

where speed = band width/waiting time. Band width is calculated as in Table 4, and waiting time is the number of months between the two most recent purchases.

1. The S-s households are those endogenously sorted into the S-s group by the switching regression of Section 3.2. The switching variable used is the level of income.

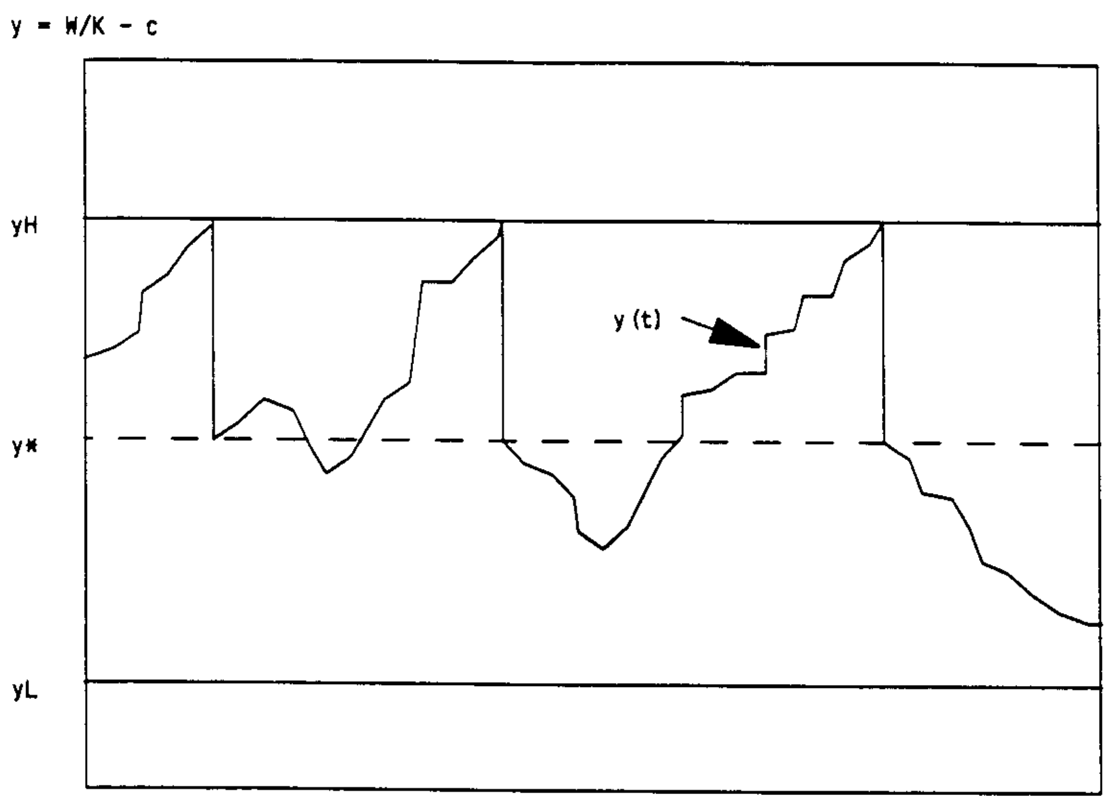
TABLE 6. SIMULATED AND ACTUAL AGGREGATE AUTO EXPENDITURES

Simulations: (% changes)	YEAR 1	2	3	4
Income	2.0	4.0	2.0	2.0
Expenditure	27.6	23.5	9.8	-5.5
Number of Buyers	24.8	21.5	9.1	-5.8
Average Purchase	2.2	1.6	1.0	1.0

Aggregate Income, Wealth, Consumption Expenditure (% changes)	YEAR 83	84	85	86	87
Real Disposable Income	2.0	4.9	2.0	2.6	0.6
Real Net Worth	6.5	2.4	5.8	4.1	2.5
Corporate Equities	12.0	-1.5	27.9	12.7	-7.3
Owner-Occupied Housing	7.8	3.2	2.8	4.2	6.0
Real Auto Expenditure	16.4	16.7	11.1	7.2	-4.2
Frictionless Prediction of Auto Expenditure (% changes)	17.0	22.3	-19.8	6.9	-16.4

Notes: The simulations were run beginning from the observed empirical distribution of y for 1983. The optimal return and trigger points used are those from Table 3. There are no idiosyncratic shocks, and the aggregate shocks are those to income noted above. The aggregates for real per capita disposable income and real Personal Consumption Expenditure for motor vehicles are annual NIPA data. The wealth data are nominal from the Flow of Funds, deflated by the CPI for new cars. The Frictionless Prediction of Auto Expenditure is the prediction of the optimal consumption model with a representative consumer without a transaction cost.

SAMPLE PATH OF OPTIMAL S-s BEHAVIOR

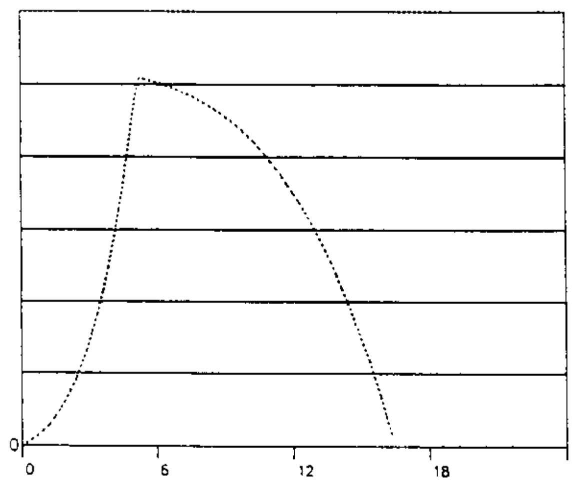


When y (t) reaches yL or yH, the consumer pays the transaction cost and adjusts K so that y=y*. K then depreciates and W grows stochastically until yL or yH is reached again, and the process continues.

Figure 2

THEORETICAL ERGODIC

DISTRIBUTION OF THE STATE VARIABLE

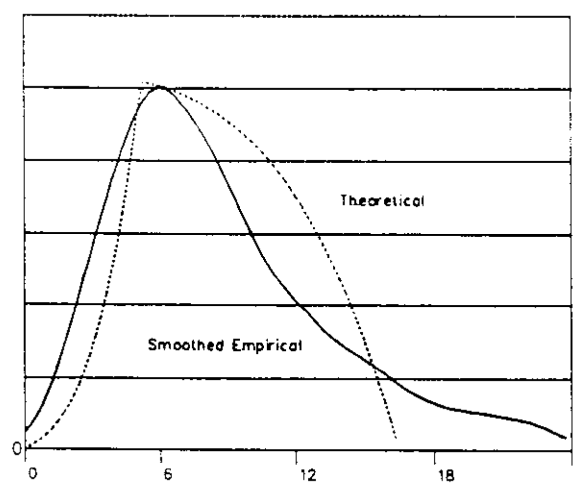


Notes: This was derived assuming depreciation of 10%, riskless rate of 1%, mean risky rate of 5.9%, and CRRA of 1.1

Figure 3

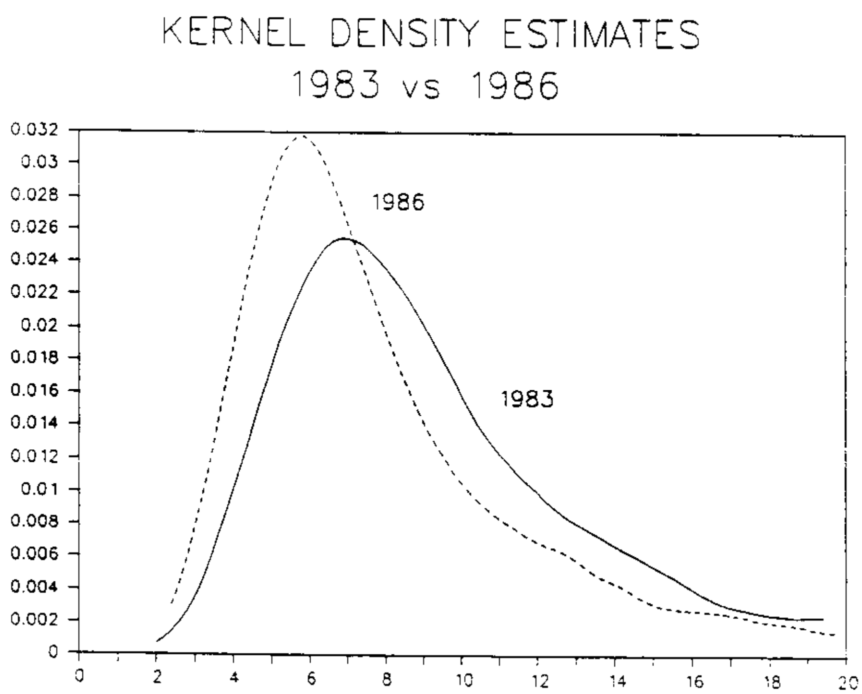
THEORETICAL VERSUS 1983 EMPIRICAL

DISTRIBUTION OF THE STATE VARIABLE



Kolmogorov-Smirnov Goodness of Fit Statistic = 03, p-value < 0.2

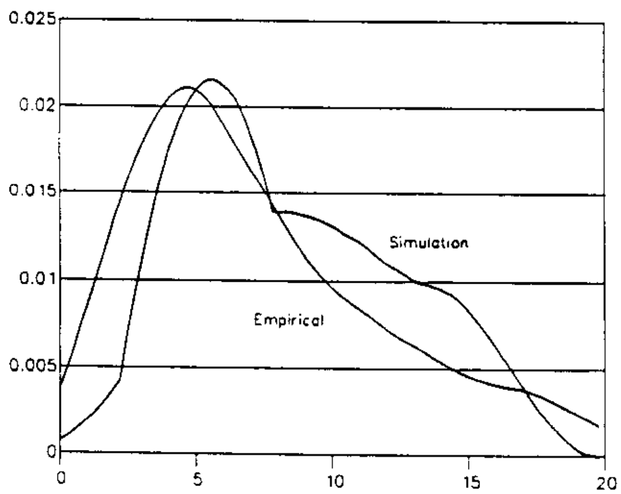
Figure 4



Bin Width = 0.2
Number of Observations: 1983=771, 1986=815
Window Width = 2.45*(nobs)**(-1/5)

Figure 5

SIMULATED AND EMPIRICAL
1986 DENSITY ESTIMATES



Notes: The simulation was run beginning from the 1983 empirical distribution. Aggregate shocks of 2 percent in the first year, 4 percent in the second year, and 2 percent in the last 2 years are assumed. There is no idiosyncratic uncertainty. Optimal bands are as in Table 5.

Figure 6

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SIMULATED DISTRIBUTION OF Y