LIMITED MARKET PARTICIPATION AND VOLATILITY OF ASSET PRICES

by

Franklin Allen
Douglas Gale

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RODNEY L. WHITE CENTER FOR FINANCIAL RESEARCH
The Wharton School
University of Pennsylvania
Philadelphia, PA 19104-6367

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Franklin Allen

University of Pennsylvania

and

Douglas Gale*

Boston University

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ABSTRACT

Traditional theories of asset pricing assume there is complete market participation so all investors participate in all markets. In this case changes in preferences typically have only a small effect on asset prices and are not an important determinant of asset price volatility. However, there is considerable empirical evidence that most investors participate in a limited number of markets. We show that limited market participation can amplify the effect of changes in preferences so that an arbitrarily small degree of aggregate uncertainty in preferences can cause a large degree of price volatility. We also show that in addition to this equilibrium with limited participation and volatile asset prices, there may exist a Pareto-preferred equilibrium with complete participation and less volatility.

1. INTRODUCTION

The prices of financial assets such as stocks are volatile in comparison to most other prices. What causes this volatility? There are at least two common explanations. One is that it is caused by the arrival of new information about payoff streams and discount rates. Another is that changes in preferences lead to liquidity trading and this causes changes in asset prices. Traditional analyses of efficient markets have focused on the arrival of new information as the cause of asset price volatility and have placed hardly any emphasis on changes in preferences (see, e.g., Fama (1970) and Merton (1987a)).

Why is this? In standard models, such as the capital asset pricing model, it is assumed there is complete participation so all traders invest in all markets and everyone helps to absorb the shocks caused by liquidity trades. There is continuity in the sense that small changes in preferences cause small changes in asset prices. Since short run changes in preferences are usually assumed to be small, the implication is that these are of minor importance in determining asset price volatility. It is more plausible that information about payoff streams and discount rates changes significantly in the short run and that this causes the large changes in asset prices that are observed.

Is the assumption of complete market participation justified? There is extensive empirical evidence that it is not. Most investors do not diversify across different classes of asset or across different stocks so that there is limited market participation. For example, King and Leape (1984) analyze data from a 1978 survey of 6,010 US households with average wealth of almost $250,000. They categorize assets into 36 classes and find that the median number owned is 8. In a more recent study, Mankiw and Zeldes (1990) find that only a small proportion of consumers hold stocks; more surprisingly, perhaps, even among those with large liquid wealth only a fairly small proportion own stocks; of those with other liquid assets in excess of $100,000, only 47.7 percent hold stocks.

Other studies have found that investors' diversification within equity portfolios is also limited. Blume, Crockett and Friend (1974) develop a measure of portfolio diversification which takes into account the proportion of stocks held in individuals' portfolios. Using this measure, they find the average amount of diversification is equivalent to having an equally weighted portfolio with two stocks. Blume and Friend (1978) provide more detailed evidence of this lack of diversification. They find that a large proportion of investors have only one or two stocks in their portfolios and very few have more than ten. This observation cannot be explained by the argument that investors are mainly holding mutual funds. In King and Leape's (1984) study, only 1 percent of investors' wealth was in mutual funds compared to 22.3 percent held directly in equities. The Blume, Crockett and Friend (1974) and Blume and Friend (1978) studies are based on earlier data where the ownership of mutual funds is likely to be an even smaller proportion of wealth, given the growth in mutual fund holdings that took place during that period.

How can this limited market participation be explained? One possibility is that there is a fixed setup cost of participating in a market. In order to be active in a market, it is necessary to devote resources to initially learn about its basic features such as the distribution of asset returns and so forth. Once an asset has been purchased it is necessary to monitor its performance. Brennan (1975) has shown that with fixed setup costs of this kind it is only worth investing in a limited number of assets. King and Leape (1984) find evidence that is consistent with this type of theory.

The purpose of this paper is to investigate the effect of limited market participation and changes in preferences on the volatility of asset prices. We derive two main results. The first is that limited market participation, liquidity preference and risk aversion combine to amplify the effects of changes in preferences on asset prices. In particular, we show that an arbitrarily small degree of aggregate uncertainty in preferences can cause a large degree of volatility. The second result is that there can exist multiple Pareto-ranked equilibria. In one equilibrium asset prices are volatile. In another equilibrium, which is Pareto-preferred, asset prices are not very volatile.

We derive these results in a simple model with three dates (i.e. two periods). Investors have the same type of preferences as in Diamond and Dybvig (1983). They wish to concentrate all their consumption either at the end of the first period or at the end of the second period and they do not find out when they are to consume until the end of the first period. There are two types of investors. The first has a high probability of being an early consumer and this probability is known with certainty at the initial date; they have a high preference for liquidity. The second has a low probability of being an early consumer and this probability is a random variable which is not realized until the end of the first period; they have a low preference for liquidity. It is the aggregate uncertainty about the probability of the second group being early consumers that generates the price volatility in the model.

There are two financial markets. A short-lived asset where the payoff occurs after one period is traded in the first market. It can be thought of as short term debt. A long-lived asset which lasts two periods and only pays off at the end of the second period is traded in the second market. It can be thought of as equity. The return on the long-lived asset is higher than the two period return on the short-lived asset.

Limited market participation is captured by assuming that investors are initially free to choose the market in which they want to invest. Once this choice has been made, however, they cannot participate in the other market. Thus at the first date, agents invest their wealth in one of these two assets. If they invest in the short-lived asset they can consume at the end of the first period or reinvest their funds and consume at the end of the second period. If they invest in the long-lived asset, they can sell it to consume at the end of the first period or they can hold it and consume the payoff at the end of the second period.

The first result (that a small amount of aggregate uncertainty in preferences can lead to a large amount of price volatility) arises because investors with a high preference for liquidity who are risk averse are reluctant to participate in the volatile market for the long-lived asset. They fear that if they have to liquidate their holdings unexpectedly they may receive a low price. As a result, the market may be dominated by investors with a low preference for liquidity who are not very risk averse. It is optimal for these investors to hold small reserves of cash. Since the amount of money available in the market determines asset prices, small variations in the proportion of investors that are early consumers causes a significant amount of price volatility. Although there are investors in the economy who hold large reserves of cash which could potentially dampen this volatility, they have chosen not to participate in this market and cannot be called to do so at short notice.[1]

In the equilibrium described, low participation and illiquidity in the market are self-sustaining phenomena. The second result that there can be multiple Pareto-ranked equilibria occurs because the converse is also true; a high level of participation and a liquid market can also be self sustaining. This equilibrium is Pareto preferred to the other because all investors benefit from the higher returns to the long-lived asset and still enjoy reasonable liquidity.

LeRoy and Porter (1981) and Shiller (1981), among others, have argued that asset prices are characterized by excess volatility, that is they are more volatile than changes in payoff streams and discount rates would suggest. A number of authors have suggested that the degree of excess volatility found by these studies is due to the use of inappropriate econometric techniques (see Merton (1987a) and West (1988) for surveys of the empirical literature on excess volatility). It is also possible that there may be a number of other determinants of asset price volatility.[2] The first result above suggests that when there is limited market participation, even very small changes in preferences can lead to a significant degree of price volatility. The second result suggests that in addition to "statistical" excess volatility, there should also be concern about excess volatility in a welfare sense. The fact that the prices of financial assets are more volatile than other prices is not per se undesirable. However, if there exists a Pareto-preferred equilibrium with high participation and lower volatility, we can say that high volatility represents a market failure.

Our analysis is related to a number of other papers in the literature. The assumption of a fixed cost of participating in a market has been used in a number of other contexts. The closest are Merton (1987b), Hirshleifer (1988) and Cuny (1990). In Merton (1987b), there is a fixed cost of acquiring information about the returns to an asset which causes traders to invest in a limited number of assets. Among other things, Merton shows that this can lead to an empirically significant effect on asset returns. Hirshleifer (1988) assumes there is a fixed cost of participating in a futures market. These costs result in thin markets, which in turn causes the residual risk to be priced. In this case, risk premia are quite different from those when there are perfect frictionless markets. Cuny (1990) considers the optimal design of futures contracts assuming investors can only participate in one market. This assumption causes liquidity to vary across markets, as in our model, and makes the design problem a nontrivial one.[3]

Pagano (1989) develops a related model which captures the relationship between market thinness and volatility. He does not consider the role of limited market participation in amplifying stock price volatility. He does demonstrate the existence of multiple equilibria, but the reason they arise is rather different. In his model it is the small number of investors that leads to asset price volatility and multiple equilibria. If investors believe that the market will be small and volatile, only a few traders enter and the beliefs are self-fulfilling. If investors believe that the market will be large and asset prices stable, many will enter and the beliefs are again self-fulfilling. The theory is thus able to explain the well-documented empirical finding that thin markets are more volatile than thick ones (see, e.g., Cohen et al. (1976), Telser and Higimbotham (1977), Pagano (1986) and Tauchen and Pitts (1983)). Studies of excess volatility are based on data from the NYSE and other major exchanges which are usually considered to be thick so Pagano's theory may not be relevant. In contrast, in our model there is a large number of investors. It is the relative proportions of different types that is important. Prices are volatile when only investors with a low preference for liquidity participate in the market for the long-lived asset.

The rest of the paper is organized as follows. The model is described in Section 2. In Section 3, the benchmark case where there is no aggregate uncertainty is analyzed and in Section 4 aggregate uncertainty is introduced. Section 5 presents an example. Finally, Section 6 contains concluding remarks.

2. THE MODEL

To highlight the nature of the results, a simple stylized model is used.

(A.1) There are three points in time indexed by $t = 0,1,2$.

We assume investors' preferences are subject to random shocks as in Diamond and Dybvig (1983).

(A.2) There are two types of investor, $i = A,B$. The probability type i needs to consume at date 1 is λ_i and at date 2 is $1-\lambda_i$. At date 0, expected utility is

$$V_i(C_{i1},C_{i2}) = \lambda_i U_i(C_{i1}) + (1-\lambda_i)U_i(C_{i2}). \tag{1}$$

where C_{it} is consumption at date t and $U_i(.)$ is a continuous, concave and strictly increasing von Neumann-Morgenstern utility function.

For the purposes of discussion, it is helpful to think of type A's as being risk averse and having a high value of λ_A so that they have a high preference for liquidity. Type B's, on the other hand, are not as risk averse and have a lower value of λ_B. The assumption that liquidity preference and risk aversion are related in this way is not necessary and other possible cases can be analyzed using our framework.

There is a continuum of each type of investor and preference shocks are assumed to be independent across investors. This means that we can assume the law of large numbers applies. That is, we assume that the proportion of investors of type i who prefer early consumption is equal to the probability

λ_i. In the analysis of Section 3 it is assumed λ_i is a constant. In the analysis of Section 4, however, it is assumed that λ_i is itself a random variable which is realized at date 1.

(A.3) There are three assets:

(i) *money* with return 1;

(ii) a *short-lived productive asset* with return $R_S > 1$ per period;

(iii) a *long-lived productive asset* with return $R_L > R_S^2 > 1$ over two periods.

Money serves as the numeraire. Initially, investors are endowed with money W_i. One unit of money can be transformed into one unit of the other assets and these productive assets yield returns in the form of money. Thus the stocks of the productive assets are not fixed at the initial date. A unit of money held at the end of period t yields a unit of money in period t+1, for t = 0,1. The short-lived asset yields a return after one period. It is possible to invest in the short-lived asset at either date 0 or date 1. A unit of money invested in the short-lived asset at date t yields $R_S > 1$ units of money at date t+1, for t = 0,1. The long-lived asset yields a return after two periods. A unit of money invested in the long-lived asset at date 0 yields $R_L > R_S^2 > 1$ units of money at date 2.

(A.4) Financial claims to the assets are traded in financial markets, with the long term claim trading at price P at date 1. There are no short sales of financial claims.

The timing of returns is illustrated in Figure 1. The short term financial asset corresponding to the short-lived productive asset can be thought of as a short term bond. The long term financial claim corresponding to the long-lived productive asset can be thought of as equity since it pays off R_L at date 2 and there are no intermediate payoffs. In order to focus on the uncertainty due to changes in preferences, the payoffs on both assets are assumed to occur with certainty.

The model has a special market structure. It is intended to capture the fact that there are fixed costs of investing in an asset which leads to limited market participation. The fixed costs make it prohibitively expensive to invest in both assets. To keep things simple these costs are not modeled explicitly. Instead, we assume that the market structure allows the investor to invest in one or other of the assets but not both.

(A.5) At $t = 0$ investors must choose whether to invest in the market for the short term asset S or the market for the long term asset L. They cannot invest in both and it is not possible to switch markets once a choice has been made.

Investors' choice of markets is illustrated in Figure 2. Once they choose which market to enter at date 0, the two markets are segregated. First consider market S which is shown in the upper branch of the diagram. At $t = 0$ investors can either invest in S or money. Since $R_S > 1$, it is optimal for them to invest entirely in S. At $t = 1$ investors find out whether they are early consumers or late consumers. If an investor is an early consumer, he consumes the entire liquidation value of his portfolio $C_{i1} = R_S W_i$. If an investor is a late consumer he reinvests his entire portfolio in S and at date 2 he consumes the total return to his portfolio which is $C_{i2} = R_S{}^2 W_i$. Hence equilibrium in the short term market is relatively straightforward.

In market L, which is illustrated in the lower branch of Figure 2, investors can either invest in L or money. At $t = 0$ they choose a portfolio which is denoted (ℓ_i, m_i) where ℓ_i is type i's holding of L and m_i is their holding of money. At $t = 1$ they find out whether they are early consumers or late consumers. If an investor is an early consumer, he liquidates his holdings of L at price P and this together with his holding of money gives consumption $C_{i1} = m_i + P\ell_i$. If an investor is a late consumer, then at $t = 1$ he can use his money to buy m_i/P units of L so that at time 2 he can consume $C_{i2} = (m_i/P + \ell_i)R_2$. The reason it may be worth holding money in this market is that when $P < 1$ it is possible to buy L at a lower price at date 1 than at date 0. The lower is P, the more attractive holding money is.

Equilibrium in the long term market is not quite as straightforward as equilibrium in the short term market. At $t = 1$, a proportion λ_i of type i investors that are participating in the market sell their holdings of L and consume while the remainder use their money to buy L. Thus the equilibrium level of P depends on the relative number of people of each type in the market, their probabilities of selling λ_i and their initial portfolios (ℓ_i, m_i). It is important to note that $P \leq R_L$ since otherwise everybody would prefer money to L at $t = 1$ and P would not be an equilibrium price. When expressions for P are given below which could have values above R_L, it is implicit that the price is the minimum of the expression and R_L.

So far we have considered equilibrium in the short term market and equilibrium in the long term market. In order to ensure overall equilibrium, it is necessary to require that investors choose the market where their expected utility will be maximized.

Given that equilibrium in the short term market is assured since S dominates money, the three conditions that are important for overall equilibrium are the following.

(E.1) P clears the long term market.

(E.2) The portfolios of investors in the long term market maximize their expected utility.

(E.3) Investors choose the market which maximizes their expected utility.

Having outlined the basic model and defined equilibrium, we next consider the case where the parameters λ_i are non-stochastic. This helps to fix ideas and illustrate how the model works.

3. EQUILIBRIUM WITH NO AGGREGATE UNCERTAINTY

When λ_A and λ_B are non-stochastic, there is uncertainty for individuals but no aggregate uncertainty. The determinants of price at date 1 are known at $t = 0$ so it is possible to deduce what the equilibrium value of P must be.

At date 1, early consumers will want to sell their holdings of L. In order for an equilibrium to exist, there must be investors holding money which they can use to buy L. To see this suppose that nobody held money, then at date 1 there would be an excess supply of L. This cannot be an equilibrium unless $P = 0$. However, at this price it would be worth holding money between dates 0 and 1 in order to buy a large amount of L at date 1. Hence, in equilibrium investors must be holding money with which to buy L.

Investors will hold money between dates 0 and 1 if they are indifferent between money and L so in equilibrium it is necessary that

$$P = 1. \tag{2}$$

In fact, this is the only equilibrium price. If $P > 1$, L dominates money between dates 0 and 1 and nobody would hold money. This cannot be consistent with equilibrium since at date 1 there would an excess supply of L. If $P < 1$ then money dominates L between dates 0 and 1 and everybody would hold money. However, at date 1 everybody would try to switch to L since this dominates between dates 1 and 2. This cannot be consistent with equilibrium at a finite price since we know the supply of L is zero at date 1.

When $P = 1$, investors are indifferent between money and L between dates 0 and 1 but strictly prefer L between 1 and 2 since it pays off $R_L > 1$ whereas money only pays off 1. In equilibrium, investors therefore hold just enough money so that when the early consumers come to sell their holdings of L at date 1, the late consumers have exactly enough money to purchase them at $P = 1$. If investors held any more or less money than this the market would not clear at date 1.

Given $P = 1$, it can straightforwardly be seen that the utility for type i in market L, is given by

$$V_{iL} = \lambda_i U_i(W_i) + (1 - \lambda_i)U_i(W_i R_L). \tag{3}$$

For type i in market S,

$$V_{iS} = \lambda_i U_i(W_i R_S) + (1 - \lambda_i)U_i(W_i R_S^{2}). \tag{4}$$

Equilibrium can have a number of different forms depending on the ranking of V_{iL} and V_{iS} as illustrated in Table 1. A *separating* equilibrium occurs when the different types choose different markets. The fact that type A's have a higher value of λ than type B's means that the separating equilibrium will have type A's enter market S and type B's enter market L as shown in Table 1. A *pooling* equilibrium occurs when both types enter market S or market L. A *partial pooling* equilibrium occurs when both types enter one of the markets and one type enters the other market. In the example shown in Table 1, type B's are indifferent between the two markets and some may participate in S and some in L. Type A's on the other hand strictly prefer S and will only participate in that market.

It can be seen from (3) and (4) that the parameters R_S, R_L and W_i uniquely determine V_{iS} and V_{iL}. Hence the form of equilibrium is uniquely determined except in the special case where the parameters are such that $V_{AS} = V_{AL}$ or $V_{BS} = V_{BL}$ and all three types of equilibria exist. Thus when there is no aggregate uncertainty equilibria are generically either separating or pooling. We mainly focus on the pure separating and pooling cases in the sequel.

The preceding discussion can be summarized in a proposition.

Proposition 1: Suppose that $0 < \lambda_i < 1$ is non-stochastic for at least one i = A,B. Then equilibrium exists and equilibrium expected utilities are uniquely determined. Generically, equilibrium is unique and must be either separating or pooling.

4. EQUILIBRIUM WITH AGGREGATE UNCERTAINTY

In this section we study the effects of introducing aggregate uncertainty. In particular, it is assumed that λ_B is a random variable. Two main results are derived. The first is that the introduction of an arbitrarily small degree of aggregate uncertainty can lead to a separating equilibrium with significant price volatility even though with no aggregate uncertainty there is a pooling equilibrium. The second is that, under certain circumstances, a pooling equilibrium can exist as well as the separating equilibrium and this pooling equilibrium is Pareto preferred to the separating equilibrium. In order to prove these results we need the following conditions on the parameters.

(C.1) $0 < \lambda_A < 1$ is a constant.

(C.2) $$\lambda_B = \begin{cases} \epsilon \text{ with probability } \pi; \\ 0 \text{ with probability } (1-\pi). \end{cases}$$

(C.3) When $\epsilon = 0$, there is pooling in market L in equilibrium.

The first condition implies that type A's face some uncertainty at the individual level. The second condition implies that type B's face uncertainty at both the individual and the aggregate level. It is assumed that λ_B has a two-point support for ease of exposition. It is shown in the Appendix that the results of this section hold for continuous distributions. Note that when λ_B is random it is necessary to take expectations over λ_B in the definition of expected utility in (1). The third condition is necessary as a starting point for the two main results that are to be proved. It is satisfied provided R_L is large enough relative to R_S^2 for the long term market to be attractive to both types of investor.

Separating equilibrium with volatile asset prices

The first result we demonstrate is that introducing an arbitrarily small amount of aggregate uncertainty (i.e. $\epsilon > 0$) can lead to a separating equilibrium. This is true even though (C.3) is satisfied so that with no aggregate uncertainty (i.e. $\epsilon = 0$) there is a pooling equilibrium. There is thus a sharp discontinuity in the model when uncertainty is introduced.

Since λ_B has a two-point distribution, there are two possible values of P. We use the following notation.

$$P = \begin{cases} P_\epsilon \text{ when } \lambda_B = \epsilon; \\ P_0 \text{ when } \lambda_B = 0. \end{cases} \tag{5}$$

We consider the sequence of equilibria as $\epsilon \to 0$ and show that there can exist a separating equilibrium with type A's in market S and type B's in market L. We start by stating an existence result. This can be proved using standard techniques.

Lemma 1: For any distribution of investors across markets, there exist prices and portfolios that satisfy equilibrium conditions (E.1) and (E.2).

Proof: See Appendix.

It remains to show that as $\epsilon \to 0$, prices can be such that it is optimal for type A's to enter market S and type B's to enter market L so that there is a separating equilibrium and (E.3) is satisfied.

First consider market L and suppose that only type B's enter. When $\lambda_B = \epsilon$, early consumers sell their holdings of L. In order for an equilibrium value of P_ϵ to exist, it is necessary for investors to hold money between dates 0 and 1. If they did not there would be an excess supply of L at time 1. Similarly to the argument in the previous section, it is not possible to have $P_\epsilon = 0$ in equilibrium since investors would then find it worthwhile to hold money and purchase large amounts of L. Money must be held between dates 0 and 1, in equilibrium.

When $\lambda_B = 0$ there are no early consumers at $t = 1$ and hence there are no sellers of L. This means that the money investors hold between dates 0 and 1 is not used and the price P_0 must be such that the investors are willing to continue holding the money as well as their holdings of L between dates 1 and 2. This implies that in equilibrium $P_0 = R_L$ since otherwise investors would not be willing to hold money and L between dates 1 and 2. At $P_0 = R_L$ they are indifferent between the two assets, but if $P_0 < R_L$ they will want to hold only

L and if $P_0 > R_L$ they will want to hold only money.

Given $P_0 = R_L$, it follows that when $\lambda_B = \epsilon$, $P_\epsilon < 1$. If this were not so L would dominate money between dates 0 and 1 for both realizations of λ_B and nobody would hold money. However, as argued above, it is necessary for equilibrium that money be held.

What is it that determines the equilibrium value of P_ϵ? The market clearing condition when $\lambda_B = \epsilon$ is

$$\epsilon \ell_B P_\epsilon = (1-\epsilon) m_B. \tag{6}$$

The left hand side represents the sale of L by early consumers and the right hand side is the amount of money late consumers have to purchase L with. In contrast to standard theories where asset prices are determined by discounted present values, here the price is determined by the amount of money held by participants in the market.

It is possible to use this market clearing condition together with the investors' first order condition to demonstrate the following result.

Lemma 2:

$$P_{\epsilon 0} \equiv \lim_{\epsilon \to 0} P_\epsilon = \frac{\pi R_L}{R_L - (1 - \pi)} . \tag{7}$$

Proof: See Appendix.

It can be seen from Lemma 2 that when $\pi = 1$, $P_{\epsilon 0} = 1$. Intuitively, for large values of π, the probability that the state $\lambda_B = \epsilon$ occurs is relatively large and investors have an incentive to hold money in case they are early consumers. Hence P_ϵ is near 1. As π falls, the probability of being an early consumer falls so the need for money falls. To induce investors to hold money, it must become relati when $\pi = 0$, it can be seen from Lemma 2 that $P_{\epsilon 0} = 0$.

Thus even when ϵ is very small, prices can be very variable. In the limit,

$$P = \begin{cases} P_{\epsilon 0} < 1 \text{ with probability } \pi; \\ R_L > 1 \text{ with probability } 1 - \pi. \end{cases} \tag{8}$$

Note that these are not equilibrium prices. When $\epsilon = 0$, the equilibrium is as described in Section 3 with $P = 1$ in all states. However, by choosing ϵ small enough it is possible to get an equilibrium with prices arbitrarily close to these.

Since prices are volatile it follows that if type A's are sufficiently risk averse they will prefer market S. Hence we can have a separating equilibrium.

Proposition 2: If (C.1)-(C.3) are satisfied, and type A's are sufficiently risk averse, then the introduction of an arbitrarily small amount of aggregate uncertainty leads to a separating equilibrium.

Proof: See Appendix.

This result has been demonstrated for the case where λ_B has two-point support but it is shown in the proof in the Appendix that the result holds when λ_B has continuous support. This is true even when $\lambda_B = 0$ lies outside the support. What is important for the result, is that the distribution of λ_B converges to a limit of zero.

In our model, price changes are driven by preference shocks. Investors who learn that they need to consume early will liquidate their stock. With limited market participati much money the other traders in the market have with which to buy the stock. If there is a lot of money in the hands of traders, the market is liquid and prices may not change much. If there is little money, a small shock may cause a lot of volatility. Thus volatility can be amplified by portfolio decisions.

What determines the amount of money traders decide to include in their portfolios? Type A traders have high liquidity preference because they think there is a high probability they will be early consumers so they hold a lot of money. Type B traders have a low liquidity preference because they think the probability will be low so they hold little money.

This is where limited market participation, liquidity preference and risk aversion combine to generate high volatility. In the case considered in Proposition 2, the type A's with high liquidity preference have high risk aversion and the type B's with low liquidity preference have relatively low risk aversion. The equilibrium with high volatility arises because the expectation of high volatility will keep the type A traders who are highly risk averse out of the market. Since the only traders in the market are then type B's, who have low liquidity preference, the market will be illiquid and high volatility will become a self-fulfilling expectation.

Another possibility is that investors expect a low degree of price volatility and as a result both type A's and type B's enter market L. Since type A's have a high preference for liquidity, they will hold large reserves of money and as a result price volatility will be low. Expectations will again be self-fulfilling. Moreover, since price volatility is higher in the separating equilibrium, it also natural to suppose that the pooling equilibrium will be Pareto preferred. We consider these possibilities next.

Multiple Pareto-ranked equilibria

We start by demonstrating that when ϵ is small there exists a pooling equilibrium as well as a separating equilibrium. As before, we are interested in the sequence of equilibria as $\epsilon \to 0$. Suppose that both type A's and type B's enter market L. It follows from Lemma 1 that a price function and a pair of portfolios that satisfy equilibrium conditions (E.1) and (E.2) exist. It remains to show that (E.3) is satisfied and it is optimal for both A's and B's to enter market L.

Using N_i to denote the measure of investors of type i, the market clearing conditions (which are similar to (6)) give the following expressions for P_ϵ and P_0:[4]

$$P_\epsilon = \frac{N_A(1-\lambda_A)m_A + N_B(1-\epsilon)m_B}{N_A\lambda_A\ell_A + N_B\epsilon\ell_B} \; ; \tag{9}$$

$$P_0 = \frac{N_A(1-\lambda_A)m_A + N_Bm_B}{N_A\lambda_A\ell_A} \; . \tag{10}$$

When $\lambda_B = \epsilon$ there is less money in the market for L than when $\lambda_B = 0$. As a result $P_\epsilon < P_0$ as shown by (9) and (10). By the usual argument both money and L must be held in equilibrium. Given this it is not possible for both P_ϵ and P_0 to be above or below 1 since in that case either L or money would dominate and investors would not be willing to hold both. This implies $P_\epsilon \le 1 \le P_0$. From (9) and (10) it follows that,

$$\lim_{\epsilon \to 0} P_\epsilon = \lim_{\epsilon \to 0} P_0 = 1. \tag{11}$$

Hence as $\epsilon \to 0$, the equilibrium tends to the equilibrium with no aggregate uncertainty discussed in Section 3. Condition (C.3) implies that it is optimal for both types to choose market L in this case. Thus for sufficiently small ϵ, a pooling equilibrium exists in addition to the separating equilibrium.

The next step is to demonstrate that the pooling equilibrium is Pareto preferred to the separating equilibrium. It is clear that type A's are better off in the pooling equilibrium than the separating equilibrium since they could have chosen market S if it yielded them higher utility. It remains to show that type B's are better off.

In the separating equilibrium, type B's have expected consumption $EC_B^S = m_i + \ell_i R_L$ where (ℓ_i, m_i) is their optimal portfolio at time 0. When the probability that an investor of type B is an early consumer converges to zero, he will wish to take advantage of the high returns to holding asset L by putting all his wealth in asset L and none in money. Hence it can straightforwardly be shown using the investors' first order condition for the choice of portfolio that $\lim_{\epsilon \to 0} \ell_B = W_B$ so that

$$\lim_{\epsilon \to 0} EC_B^S = W_B R_L. \tag{12}$$

In the pooling equilibrium $P_\epsilon \approx P_0 \approx 1$ for small ϵ. If an investor survives to be a late consumer his consumption is therefore $W_B R_L$. Hence, as their probability of being an early consumer goes to zero, their expected consumption in the pooling equilibrium is given by

$$\lim_{\epsilon \to 0} EC_B^P = W_B R_L. \tag{13}$$

For small ϵ the expected consumption of the type B's is approximately the same in both the separating and the pooling equilibrium. However, in the separating equilibrium there is greater price volatility than in the pooling equilibrium and as a result consumption is more volatile. Hence the type B's are better off in the pooling equilibrium. This gives the following result.

Proposition 3: If (C.1)-(C.3) and the type A's are sufficiently risk averse then for sufficiently small ϵ, a pooling and a separating equilibrium exist. The pooling equilibrium is Pareto preferred to the separating equilibrium.

Proof: See Appendix.

Equilibria usually come in odd numbers and that is the case here, too. For the same parameter values there will also be a partial pooling equilibrium in which some type A investors are found in each market. The number of investors of type A who enter market L will be just sufficient to reduce price volatility to the point where the type A investors are indifferent between the two markets.

Proposition 2 showed that limited market participation could amplify the effect of changes in preferences on stock price volatility and this has implications for the debate on statistical excess volatility. In contrast, Proposition 3 suggests that there may be excess volatility in a *welfare* sense. With limited market participation there can be multiple Pareto-ranked equilibria. The high volatility equilibrium can be Pareto preferred to the low volatility equilibrium. This is the sense in which there is a market failure.

5. AN EXAMPLE

The results above have been concerned with the case where ϵ is arbitrarily small. This is an extreme case but one that is convenient for deriving analytic results. We next consider a numerical example to illustrate that the qualitative results do not just hold for small ϵ. The parameter values for the example are shown in Table 2.

In order to understand the factors determining whether a pooling or separating equilibrium occurs, it is helpful to start by considering the case where $\epsilon = 0$, so there is no aggregate uncertainty, and varying R_L while holding the other parameters constant. It follows from Proposition 1 that equilibrium is either separating or pooling, except for a negligible set of parameters. The effect of varying R_L on the type of equilibrium is illustrated in Figure 3.

If R_L is sufficiently small then both types will pool in market S. For $R_L < (R_S)^2 = 1.0609$, the short term technology dominates the long term one and both types will clearly prefer market S. As R_L increases and the long term technology becomes relatively more efficient, type B's, who have a high chance of being late consumers, will eventually find it worthwhile to invest in market L. When $R_L = (R_S)^2 = 1.0609$ the short term and long term technologies are equivalent for type B's, since $\lambda_B = 0$ and there is a partial pooling equilibrium. For $R_L > (R_S)^2 = 1.0609$ there is a separating equilibrium. As R_L increases further the long term technology becomes relatively more advantageous for type A's and when $R_L = 1.103$ they are indifferent between the two so there is a partial pooling equilibrium again. For $R_L > 1.103$ there is pooling in market L.

The main case of interest is where (C.3) is satisfied, i.e., where $R_L >$ 1.103. Figure 4 shows the types of equilibrium obtained for various values of ϵ and π when $R_L = 1.12$. For large values of ϵ, there is considerable volatility and as a result there is pooling in market S. For low ϵ and large values of π, it follows from Lemma 2 that there is relatively little price volatility. In this case there is a unique equilibrium which involves pooling in L. For low ϵ and π, there exist both separating and pooling equilibria. In most cases the pooling equilibrium is Pareto preferred to the separating. However, this is not always the case. To obtain a Pareto ranking of equilibria (Proposition 3), it was necessary to compare the expected consumption of type B's in the separating and pooling equilibria. For small ϵ, these are approximately the same. However, for larger ϵ they are not and in the area marked "Not Ranked" the type B's expected consumption (and their expected utility) are higher with separation than with pooling. For larger ϵ and moderate π, there is just a separating equilibrium, because even with both types in market L the price volatility is too large for type A's to want to stay. When $\pi = 0$ there is no aggregate uncertainty and (C.3) requires pooling in L. There is pooling for small π also. In the small shaded region near the origin the pooling is in market L; otherwise it is in market S.

The example illustrates that the phenomena highlighted by Propositions 2 and 3 do not rely on ϵ being small. For a wide range of parameter values separating equilibria with significant price volatility can exist. For many of these parameter values there also exists a pooling equilibrium which is Pareto preferred.

6. CONCLUDING REMARKS

There is considerable empirical evidence suggesting that investors only participate in a limited number of markets because of transaction costs. This paper has investigated the effect of limited market participation on asset price volatility. The first result is to demonstrate that even a small amount of aggregate uncertainty can lead to a large amount of price variation. In the particular model analyzed, it was shown that a small degree of uncertainty about liquidity preference can cause sufficient volatility in the market for the long-lived asset to drive out investors with a high liquidity preference and high risk aversion. Only investors with low risk aversion and low liquidity preference who hold little money are left in the market. As a result, small shocks in liquidity preference lead to a large fluctuation in prices. Investors from other markets could potentially dampen the fluctuations in prices, as standard theories suggest. They do not do so because they have already committed to transact elsewhere and cannot participate in the market for the long-lived asset. The second result is that there can exist a Pareto-preferred equilibrium with high participation and low volatility. In this case, investors with a high preference for liquidity holding large reserves of cash enter the market and this dampens price fluctuations.

The analysis in the paper is based on a simple stylized model and many important factors are omitted. For example, an important assumption is that short sales are prohibited. This assumption rules out borrowing. Also, the short-term provision of liquidity by market makers is excluded from the analysis. The effect of incorporating borrowing restrictions, margin requirements, market makers and a number of other features of actual markets into models with limited market participation is an interesting topic for future research.

APPENDIX

Notation

In proving the results in this Appendix, some notation which was not introduced in the text is useful. Let $0 \leq n_i \leq N_i$ denote the measure of investors of type i who enter market L at date 0, for $i = A,B$.

The expected utility of an investor of type i who enters market S is denoted by V^*_{iS} and defined by

$$V^*_{iS} = E[\lambda_i U_i(R_S W_i) + (1-\lambda_i)U_i(R_S{}^2 W_i)] \quad \text{for } i = A,B. \tag{A1}$$

An investor's expected utility from entering market L at date 0 will be:

$$V_{iL}(\ell_i, m_i; P) = E[\lambda_i U_i(m_i + P\ell_i) + (1-\lambda_i)U_i((m_i/P + \ell_i)R_L)]. \tag{A2}$$

An investor of type i will choose a portfolio (ℓ_i, m_i) to maximize $V_{iL}(\ell_i, m_i; P)$ subject to the budget constraint $\ell_i + m_i = W_i$. Let $V^*_{iL}(P)$ denote the maximized value of $V_{iL}(\ell_i, m_i; P)$ for any price function P.

Proof of Lemma 1

Let n_A and n_B be fixed and satisfy $n_A + n_B > 0$. Let $S_\xi = \{(\ell_A, \ell_B) \mid \xi \leq \ell_i \leq W_i,\ i = A,B\}$ for any $\xi > 0$. For any $\ell \in S_\xi$ there exists a unique price function defined by $\Pi(\ell) = \min\ \{R_L, \Sigma(1-\lambda_i)n_i(W_i - \ell_i)/\Sigma\lambda_i n_i \ell_i\}$. Then we can define a correspondence from S_ξ to itself by putting $G_i(\ell^0) = \arg\max V^*_{iL}(\ell_i, W_i - \ell_i; \Pi(\ell^0))$ and $G(\ell^0) = G_1(\ell^0) \times G_2(\ell^0)$ for any $\ell^0 \in S_\xi$. It is easily checked that G satisfies the properties of Kakutani's fixed point theorem. Let $\ell^\xi \in G(\ell^\xi)$ denote a fixed point for each value of $\xi > 0$. Letting ξ converge to zero we get a sequence $\{\ell^\xi\}$ of fixed points which contains a convergent subsequence. In the same notation, suppose that ℓ^ξ converges to ℓ^0. If $\ell^0_A + \ell^0_B = 0$ then $\Pi(\ell^\xi)$ converges to R_L, which implies that $\ell^\xi_i = W_i$ for ξ

sufficiently small, a contradiction. Similarly, if $\ell_A^0 + \ell_B^0 = W_A + W_B$ then $\Pi(\ell^\xi)$ converges to 0, which implies that $\ell_i^0 = 0$ for ξ sufficiently small, a contradiction. Thus, $0 < \ell_A^0 + \ell_B^0 < W_A + W_B$ and it is easily checked that $\Pi(\ell^0)$ and $\{(\ell_i^0, W_i - \ell_i^0)\}$ are the required price function and portfolios, respectively.∎

Proof of Lemma 2

Consider the decision problem facing investors at date 0. If they enter market S their optimal strategy is the same as in Section 3. Consider market L. The portfolio problem for a person of type i is

$$\max_{\ell_i, m_i} \quad \pi[\lambda_i U_i(m_i + \ell_i P_\epsilon) + (1-\lambda_i)U_i((m_i/P_\epsilon + \ell_i)R_L)]$$
$$+ (1-\pi)[\lambda_i U_i(m_i + \ell_i P_0) + (1-\lambda_i)U_i((m_i/P_0 + \ell_i)R_L)] \qquad \text{(A3)}$$

subject to
$$W_i = \ell_i + m_i \; . \qquad \text{(A4)}$$

Substituting for m_i using the budget constraint (A4), the first order condition is

$$\pi[\lambda_i U_i'(W_i+\ell_i(P_\epsilon-1))(P_\epsilon-1) + (1-\lambda_i)U_i'((W_i/P_\epsilon+\ell_i(1-1/P_\epsilon))R_L)(1-1/P_\epsilon)R_L]$$
$$+ (1-\pi)[\lambda_i U_i'(W_i+\ell_i(P_0-1))(P_0-1) + (1-\lambda_i)U_i'((W_i/P_0+\ell_i(1-1/P_0))R_L)(1-1/P_0)R_L]$$
$$= 0 \; . \qquad \text{(A5)}$$

Using (C.2), the market clearing condition (6) and the budget constraint (A4) in the first order condition (A5) and rearranging gives

$$\pi\left\{\epsilon U_B'\left(\frac{W_B P_\epsilon}{1-\epsilon+\epsilon P_\epsilon}\right)(P_\epsilon - 1) + (1-\epsilon)U_B'\left(\frac{W_B R_L}{1-\epsilon+\epsilon P_\epsilon}\right)\left(1-\frac{1}{P_\epsilon}\right)R_L\right\}$$

$$+ (1-\pi)U'\left(\frac{W_B(\epsilon P_\epsilon + (1-\epsilon)R_L)}{1-\epsilon+\epsilon P_\epsilon}\right)(R_L - 1) = 0 \ . \qquad \text{(A6)}$$

It can be seen from this that,

$$P_{\epsilon 0} = \lim_{\epsilon \to 0} P_\epsilon = \frac{\pi R_L}{R_L - (1 - \pi)} \ . \qquad (7)$$

■

Proof of Proposition 2

The result demonstrated informally in the text holds for continuous distribution functions. In particular, the result can be proved using the following version of (C.2):

(C.2) $\lambda_B = \theta/(Z+\theta)$, where $Z > 0$ is a constant and θ is a (non-degenerate) random variable with c.d.f. F. The support of F is contained in the compact interval $[a,b]$ where $0 \leq a < b$.

This rather odd way of parameterizing λ_B ensures that $(1-\lambda_B)/\lambda_B = Z/\theta$, which makes subsequent calculations easier.

Our aim is to show that when the liquidity preference of type B's, λ_B, is sufficiently small, a separating equilibrium always exists. More precisely, when Z is sufficiently large and type A investors are sufficiently risk averse, there exists an equilibrium in which $n_A = 0$ and $n_B = N_B$.

Let $\{Z^k\}$ be a sequence of positive numbers diverging to infinity. For every k, set $\lambda_B^k = \theta/(Z^k+\theta)$. Let P^k and $\{(\ell_i^k, m_i^k)\}$ be the price function and portfolios that support equilibrium in market L, relative to the distribution of investors $n = (0, N_B)$, for every k. The existence of this partial equilibrium is guaranteed by Lemma 1.

The next step is to show that for k sufficiently large, the chosen distribution of investors is indeed optimal. For each value of k, $P^k = \min\{R_B, Y^k/\theta\}$ where $Y^k = Z^k m_B^k/\ell_B^k$. Y^k is bounded. To see this note that if Y^k approaches infinity, P^k converges almost surely to R_L, in which case money is eventually dominated. But that implies that $m^k = 0$, a contradiction. Thus, $\{Y^k\}$ is bounded and contains a convergent subsequence. Without loss of generality we can take this to be the original sequence. Then P^k converges almost uniformly to $P^\infty = \min\{R_L, Y^\infty/\theta\}$, where Y^∞ is the limit of Y^k. Similarly, $\{(\ell_i^k, m_i^k)\}$ is bounded so we can choose a subsequence along which (ℓ_i^k, m_i^k) converges to $(\ell_i^\infty, m_i^\infty)$, for $i = A,B$. Without loss of generality we can take this to be the original sequence. The next lemma characterizes the limiting behavior of P^k and (ℓ_i^k, m_i^k).

Lemma 3: If (C.1)-(C.3) are satisfied then $\ell_B^\infty = W_i$, $\ell_A^\infty > 0$ and $(\ell_i^\infty, m_i^\infty) \in \arg\max V_{iL}(\ell_i, m_i; P^\infty)$ for $i = A,B$.

Proof: The proof proceeds in a number of steps. The first step is to show that $\ell_B^\infty = W_B$. It is clear that in the limit the investors of type B can attain an expected utility of $U_B(W_B R_L)$. If $\ell_B^\infty < W_B$ then $V_{BL}^*(P^k)$ cannot converge to $U_B(W_B R_L)$ since in the limit consumption at date 2 must be less than or equal to $\ell_B^\infty R_L + m_B < W_B R_L$, a contradiction.

The next step is to show that $E[1/P^\infty] = 1$. For each value of k, $\ell_B^k > 0$ and $m_B^k > 0$. If $\ell_B^k = 0$ (resp. $m_B^k = 0$) then $P^k = R_B$ (resp. 0) but that implies that money (resp. asset L) is dominated, a contradiction. At an interior optimum, the following first order condition must be satisfied:

$$E[\lambda_B^k U_B'(C_1^k)P + (1-\lambda_B^k)U_B'(C_2^k)R_L] = E[\lambda_B^k U_B'(C_1^k) + (1-\lambda_B^k)U_B'(C_2^k)R_L/P^k]. \tag{A7}$$

Taking limits, we get $E[U_B'(C_2^\infty)R_L] = E[U_B'(C_2^\infty)R_L/P^\infty]$, where $C_2^\infty = W_B R_L$, so $E[1/P^\infty] = 1$ as claimed.

The final step is to show that $\ell_A^\infty > 0$. From standard continuity arguments it can be shown that, for i = A,B, $(\ell_i^\infty, m_i^\infty) \in \arg\max V_{iL}(\ell_i, m_i; P^\infty)$. It is enough to set $\ell_A = 0$ and check the first-order condition:

$$\begin{aligned} &E[\lambda_A U_A'(W_A) + (1-\lambda_A)U_A'(W_A R_L/P^\infty)R_L/P^\infty] \\ &= E[(\lambda_A U_A'(W_A)P + (1-\lambda_A)U_A'(W_A R_L/P^\infty)R_L)/P^\infty] \\ &< E[\lambda_A U_A'(W_A)P + (1-\lambda_A)U_A'(W_A R_L/P^\infty)R_L]E[1/P^\infty] \\ &= E[\lambda_A U_A'(W_A)P + (1-\lambda_A)U_A'(W_A R_L/P^\infty)R_L], \end{aligned} \tag{A8}$$

a contradiction.■

An immediate corollary of Lemma 3 is that $V_{BL}^*(P^k)$ converges to $U_B(W_B R_L)$. Since $R_L > (R_S)^2$, it is clear that for k sufficiently large $V_{BL}^*(P^k) > V_{BS}^*$. Similarly, if we assume that type A's are sufficiently risk averse that $V_{AL}(P^\infty) < V_{AS}^*$, then for k sufficiently large $V_{AL}^*(P^k) < V_{AS}^*$. Thus for k sufficiently large the chosen distribution of investors is optimal and we have a separating equilibrium $(P^k, (\ell_i^k, m_i^k, n_i^k))$ where $n_A^k = 0$ and $n_B^k = N_B$ so that Proposition 2 is demonstrated.■

Proof of Proposition 3

Let (ℓ_s, m_s) denote type B's optimal portfolio choice and $(P_{s\epsilon}, R_L)$ denote prices in the separating equilibrium. Then type B's expected utility in this equilibrium will be

$$\pi[\epsilon U_B(C_{s1}) + (1-\epsilon)U_B(C_{s2})] + (1-\pi)U_B(C_{s3}) \tag{A9}$$

where

$$C_{s1} = m_s + \ell_s P_{s\epsilon}; \quad C_{s2} = (m_s/P_{s\epsilon} + \ell_s)R_L; \quad C_{s3} = m_s + \ell_s R_L. \tag{A10}$$

Suppose that the same portfolio had been chosen in the pooling equilibrium with prices $(P_{p\epsilon}, P_{p0})$. Then type B's expected utility would be

$$\pi[\epsilon U_B(C_{p1}) + (1-\epsilon)U_B(C_{p2})] + (1-\pi)U_B(C_{p3}) \tag{A11}$$

where

$$C_{p1} = m_s + \ell_s P_{p\epsilon}; \quad C_{p2} = (m_s/P_{p\epsilon} + \ell_s)R_L; \quad C_{p3} = (m_s/P_{p0} + \ell_s)R_L. \tag{A12}$$

For small ϵ, $P_{s\epsilon}$ is approximated by the expression in Lemma 2. By choosing π appropriately it is possible to obtain any value for $P_{s\epsilon}$ between 0 and 1. Choose π so that

$$P_{s\epsilon} = P_{p\epsilon} < 1 < P_{p0}. \tag{A13}$$

Using this in (A9) and (A11), it can be shown that the expected utility in the separating equilibrium must be less than the expected utility in the pooling equilibrium if the portfolio (ℓ_s, m_s) were chosen. Since this, in turn, is no better than the expected utility in the pooling equilibrium it follows that type B's prefer the pooling equilibrium. Hence for small ϵ and π satisfying (A13), the pooling equilibrium is Pareto preferred to the separating equilibrium.

What happens for other values of π? Provided ϵ is sufficiently small, it can also be shown that the pooling equilibrium is Pareto preferred. To see this note that from (11) $\lim_{\epsilon \to 0} P_{p\epsilon} = 1$. Hence for small ϵ, the case of interest is where

$$P_{s\epsilon} < P_{p\epsilon}. \tag{A14}$$

Now the expected consumption of a type B in the separating equilibrium is

$$EC_s = \pi[\epsilon C_{s1} + (1-\epsilon)C_{s2}] + (1-\pi)C_{s3} \tag{A15}$$

If a type B were to choose the portfolio (ℓ_s, m_s) in the pooling equilibrium, expected consumption would be

$$EC_p = \pi[\epsilon(C_{p1}) + (1-\epsilon)C_{p2}] + (1-\pi)C_{p3} . \tag{A16}$$

Using the market clearing condition (6) and (11) and rearranging, it can be shown that

$$EC_p - EC_s \approx m_s(R_L-1)(1-\pi)/R_L > 0. \tag{A17}$$

Since $P_{s\epsilon} < P_{p\epsilon} < 1 < P_{p0} < R_2$ it follows that

$$C_{s2} > C_{p2} > C_{p3} > C_{s3} > C_{p1} > C_{s1}. \tag{A18}$$

Now transfer expected consumption from C_{s2} to C_{s3} until $C_{s2}^{new} = C_{p2}$ or until $C_{s3}^{new} = C_{p3}$. Suppose $C_{s2}^{new} = C_{p2}$. The expected utility corresponding to the new allocation must be better than the expected utility in the separating equilibrium since there has been a transfer from a high consumption state to a low consumption state and expected consumption is the same. However, this new expected utility is still clearly worse than the expected utility in the pooling equilibrium. If $C_{s3}^{new} = C_{p3}$, then transfer expected consumption from C_{s2} to C_{s1} until $C_{s2}^{new} = C_{p2}$. It follows from (A17) that $C_{p1} > C_{s1}^{new}$. Hence we can use an argument similar to the previous one to show the pooling equilibrium is again preferred to the separating equilibrium.■

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Table 1

Types of Equilibria

Separating	-	Type A's enter market S: $V_{AS} > V_{AL}$ Type B's enter market L: $V_{BL} > V_{BS}$
Pooling	-	Both types enter market S or market L $V_{iL} > V_{iS} \quad i = A,B$
Partial Pooling	-	Both types enter one of the markets One type enters the other market e.g. $V_{BS} = V_{BL}$; $V_{AS} > V_{AL}$.

Table 2

A Numerical Example

Investors:

	Type A's	Type B's
Utility function:	$U = -\exp(-5C)$	$U = -\exp(-C)$
Initial Wealth:	$W_A = 1$	$W_B = 1$
Group Size:	$N_A = 9$	$N_B = 1$
Probability of being an early consumer:	$\lambda_A = 0.5$	$\lambda_B = \epsilon$ with pr. π $= 0$ with pr. $1-\pi$

Assets:

$$R_S = 1.03$$

Date:	0		1		2
Money:	1	$\longrightarrow$	1	$\longrightarrow$	1
Short term asset:	1	$\longrightarrow$	R_S (> 1)	$\longrightarrow$	R_S^2
Long term asset:	1	$\longrightarrow$	Trades at P	$\longrightarrow$	$R_L (> R_S^2)$

Figure 1

The Timing of Payoffs on Assets

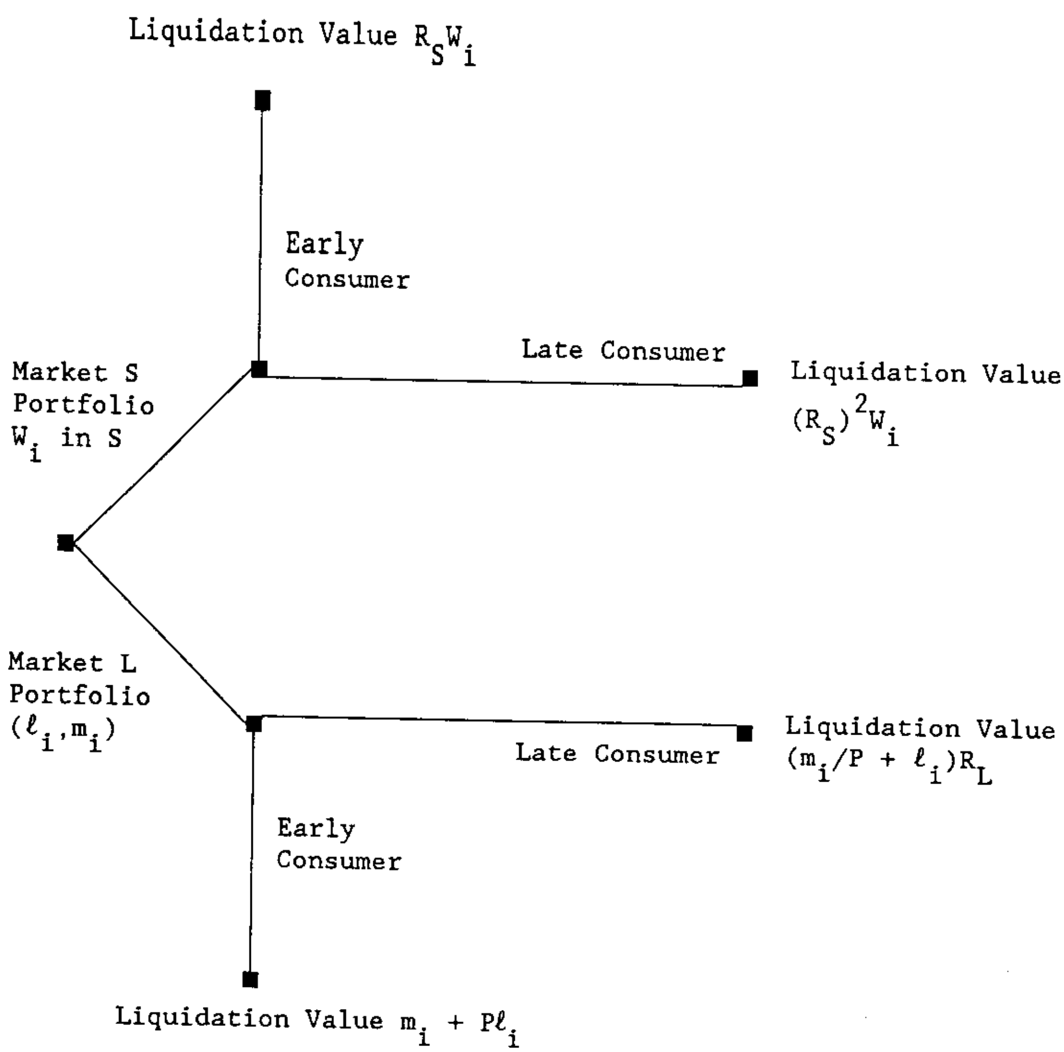

Figure 2

Investors' Choice of Market

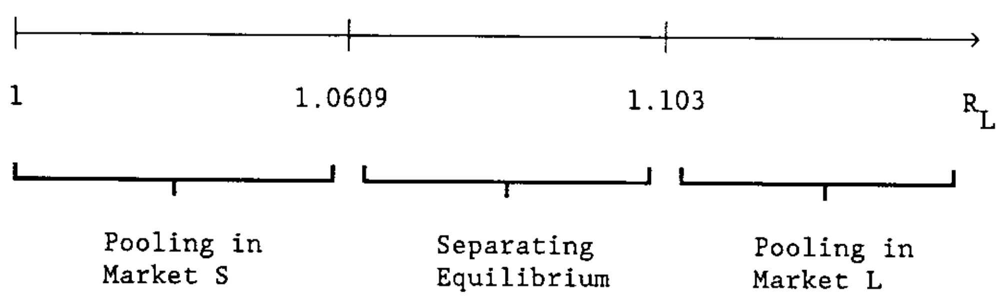

Figure 3

The Relationship between R_L and the Type of Equilibrium with No Aggregate Uncertainty

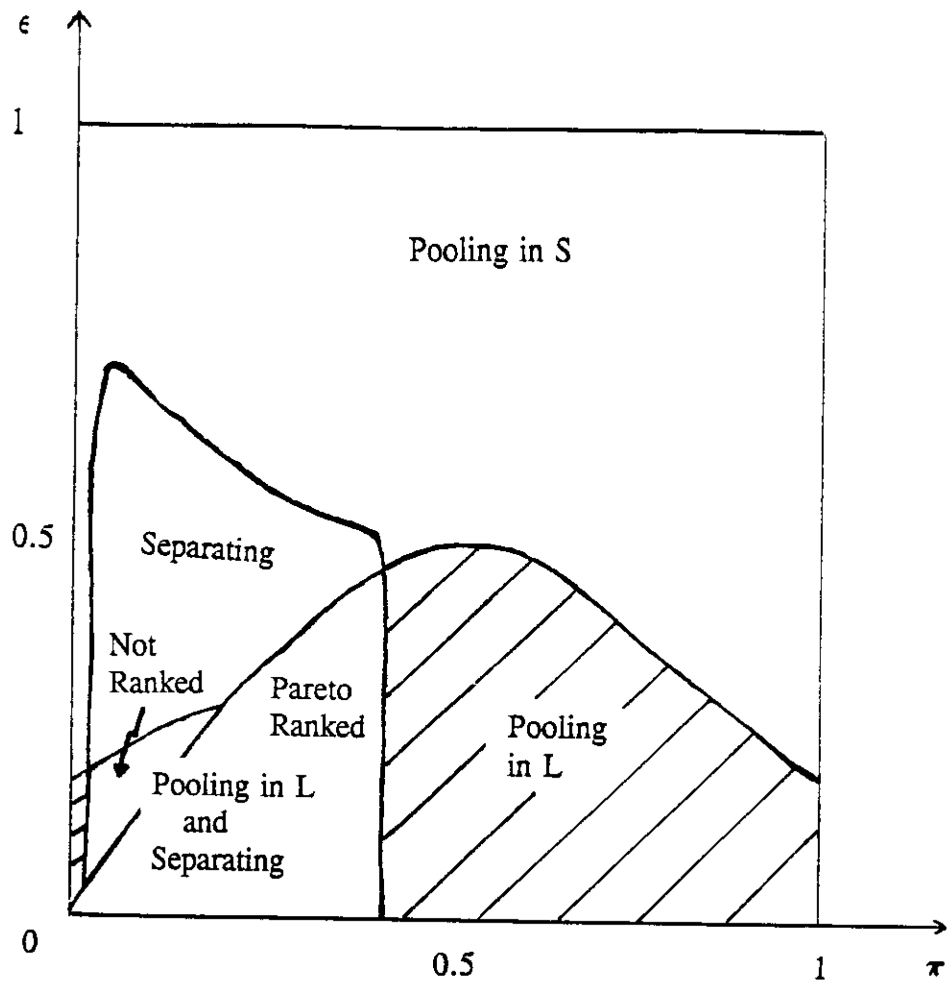

Figure 4

An Example with $R_L = 1.12$

Notes

[1]The costs that prevent instantaneous movement from one asset market to another have been stressed in a somewhat related context by Grossman (1988). His concern is that the use of synthetic securities to replace trading in options and futures markets reduces the amount of information flowing to the market. In particular, it may mean that arbitrageurs are not forewarned of impending demands for liquidity in the market. As a result, their capital may be committed elsewhere, so that they cannot react in time to prevent sharp falls in price. See also Grossman and Miller's (1988) explanation of the crash of October 1987 as being the result of a lack of funds to absorb a liquidity shock.

[2]A number of other theories of excess volatility have been suggested. These include those based on asymmetric information (see e.g., Allen and Gorton (1988) and Gennotte and Leland (1990)) and those based on noise traders (see, e.g., Shiller (1984) and DeLong et al. (1990). Recent empirical contributions attempting to categorize the causes of stock price volatility include Campbell and Shiller (1988a; 1988b) and Campbell and Kyle (1988).

[3]See also Chatterjee and Corbae (1990). They develop a model where there is a fixed cost of participating in the bond market and use it to address a number of issues in monetary theory.

[4]In the case where $\lambda_B = \epsilon$, the market clearing condition is

$$N_A\lambda_A\ell_A P_\epsilon + N_B\epsilon\ell_B P_\epsilon = N_A(1-\lambda_A)m_A + N_B(1-\epsilon)m_B,$$

where the left hand side is the sales of L by early consumers and the right hand side is the amount of money consumers have to purchase L with. Rearranging this gives (9). Similarly, when $\lambda_B = 0$, the market clearing condition is

$$N_A\lambda_A\ell_A P_0 = N_A(1-\lambda_A)m_A + N_B m_B,$$

and (10) follows immediately from this.