RATIONAL EXPECTATIONS AND STOCK MARKET BUBBLES

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1. INTRODUCTION

What determines stock prices? Are they determined by expectations about future dividends so they trade at their "fundamental value", or are they "bubbles" which are determined by crowd psychology, fads or some other arbitrary factor? These questions are central to the issue of whether stock markets allocate resources efficiently or not.

There is no wide agreement on how the empirical evidence on stock prices should be interpreted. There have been a number of extreme historical episodes, including the Dutch tulipmania (1634-37), the Mississippi Bubble (1719-20) and the South Sea Bubble (1720) where asset prices rose very quickly and then dramatically collapsed. Some authors have argued that these examples provide evidence of bubbles (see, e.g., Kindleberger [1978]) while others have argued that assets traded at their fundamental values during these episodes (see, e.g., Garber [1990]). More recent evidence has been presented which suggests that the volatility of stock prices is too large to be explained by variations in underlying dividend streams (see, e.g., Leroy and Porter [1981] and Shiller [1981]). However, the interpretation of these studies has also been challenged by a number of authors who claim the econometric techniques used are flawed (see, e.g., Flavin [1983], Kleidon [1986a,b], Marsh and Merton [1986] and West [1988]).

In addition to the empirical debate on the existence of bubbles, there has been an extensive theoretical debate on whether bubbles are consistent with rational behavior. An important strand of the literature has developed models of bubbles where at least some traders are irrational (see, e.g., De Long et al. [1990] and Shleifer and Summers [1990]).

Another strand has adopted the more traditional approach of assuming all traders are rational. An argument that is often used in this framework to support the position that stock prices reflect fundamental values relies on backward induction. Suppose that at time T an asset is known to have a final payoff P_T. Then at time T-1 it must be worth the discounted present value of P_T, otherwise there would be an arbitrage opportunity. By extending this argument backwards appropriately, it can be seen that at any point in time the value of a stock must be equal to the present discounted value of its future dividends.

One of the situations in which this argument fails to work is when there is an infinite horizon so that there is no final payoff. There is a large literature on the possibility of assets trading above their fundamental value in this case, starting with Samuelson's [1958] overlapping generations model explaining the existence of fiat money (Camerer [1989] contains a survey of these and other theories of bubbles). It has also been shown that bubbles can still exist when there is a finite horizon provided there are an infinite number of trading opportunities (Allen and Gorton [1991] and Bhattacharya and Lipman [1990]).

The purpose of this note is to show that even if there is a finite number of trading opportunities the backward induction argument may fail. We present an example where the market price of a security is above the present value of its future dividends even though every trader is rational and knows the dividends with certainty. The reason is that traders do not know other traders' beliefs. Ex ante, all traders have a common knowledge common prior, but because of private information it is not common knowledge that everybody believes the stock price will fall. Everybody realizes the stock is overpriced but each person thinks he may be able to sell it at a higher price to somebody else before the true value becomes publicly known.

A number of recent papers have emphasized the importance of the common knowledge assumption in related contexts. Abel and Mailath [1990] show that there exist cases where a project can obtain financing even though everybody believes it has a negative net present value. Kraus and Smith [1990] demonstrate that there can be a change in asset prices, even though there is no new information about security payoffs, because some traders' beliefs about other traders' beliefs change. Jackson and Peck [1990] present an infinite horizon overlapping generations model where rational traders attempt to deduce "market psychology" by examining the past movements of prices and show that bubbles can exist.

We proceed as follows. Section 2 contains an example of a fulfilled expectations equilibrium with a bubble where investors do not deduce information from their own trades. Section 3 presents an example of a rational expectations equilibrium with a bubble where investors do deduce information from their own trades. Finally, Section 4 concludes with a discussion of these results.

2. FULFILLED EXPECTATIONS EQUILIBRIA WITH BUBBLES

In this section, we will define a fulfilled expectations equilibrium and present an example of such an equilibrium in which there is a bubble. Before proceeding, we will present some notation and definitions.

We consider a market with I risk averse or risk neutral traders: $i = 1,\ldots,I$. There are a finite number of states of the world represented by Ω. The traders have a common prior over Ω; the prior assigns positive probability to each state in Ω. There are two assets, a riskless, divisible asset (money) and a risky asset. There exist a finite number of shares of the risky asset each of which will pay a dividend which may depend on the state of the world. We represent by $d(\omega)$ the dividend per share to be paid in state $\omega \in \Omega$. Prior to the payment of the dividend, there are a finite number of periods in which the traders can exchange claims on the asset at a price p which depends upon the true state and the period. We will denote by $x_{it}(\omega)$ trader i's net trade in this asset in period t when ω is the realized state. Short sales of the asset are prohibited. Trader i's information about the state of the world at the beginning of period $t = 1,\ldots T$ is represented by a partition of the space Ω, Π_{it}. We will denote by $s_{it}(\omega)$ the event in Π_{it} containing the state ω. This represents the information trader i has at time t when the state $\omega \in \Omega$ has occurred. Trader i has a concave utility function over final consumption. We will assume that the discount rate is zero, although it will be clear that this will not alter any conclusions.

DEFINITION 1: A *fulfilled expectations equilibrium* is a price function P which associates with each state $\omega \in \Omega$ and each period $t = 1,\ldots,T$ a price $p = P(\omega,t)$, and a set of net trades $x_{it}(\omega)$ such that for each i and t:

(a) $x_{it}(\omega)$ maximizes i's expected utility conditional on i's private information s_{it}, and the information conveyed by the price $S(p) \equiv P^{-1}(p)$,

(b) The market clears: $\Sigma_i x_{it}(\omega) = 0$,

(c) $P(\cdot,t)$ is measurable with respect to the partition generated by the individuals' partitions at time t, $[\Pi_{it}]_{i=1,\ldots I}$.

(d) x_{it} is measurable with respect to the partition generated by the intersection of the individuals' partitions.

It is worthwhile to say a few things about this definition before going on. First, the definition is quite standard. Part (a) simply says that for each trader and for each period, the trades are utility maximizing using the individual's private information and any information that may be contained in the price. Parts (c) and (d) are simply restrictions restricting the price and trades to be feasible given the totality of information available.

More importantly we note that an individual's trades may not be measurable with respect to the partition generated by the price function P and his private information. If this is the case, the trader could have obtained additional information from the trade he was to carry out. In the next section we will consider a stricter equilibrium notion in which traders are assumed not to be able to obtain such information.

We will say that for a given equilibrium there is a price bubble in an asset if there is a state of the world such that when that state of the world is realized, all traders know that the price of the asset is higher than its "fundamental value". A precise notion of the fundamental value of an asset in a world of risk averse traders is difficult. We will take a very conservative view of what constitutes a bubble by demanding that each trader know that with probability 1 the dividends the asset pays will be less than the current price. More formally, we say that the price function P exhibits a *bubble* in state s at time t if $\forall$ i, $\forall$ $s \in S^{-1}P(\omega,t) \cap s_{it}$, $d(\omega) < P(\omega,t)$. Clearly weaker notions of bubbles would be interesting; e.g., taking expected dividends over the indicated information sets, etc. Since our purpose in this note is to show that bubbles can exist in equilibrium, the examples are more striking given the strong notion of a bubble we are using.

Before giving the example formally, we will describe it in words. There are three periods and two traders holding quantities of an asset that yields a state dependent dividend in the third period. The dividend will be either 10 or 22 depending upon the state. Each trader will have private information regarding the state of the world. In the event that the dividend is low (that is, that the dividend will be 10), with positive probability a trader's private information will reveal this. The information partitions are such that in some states of the world, a trader will know the value of the asset to be 10, but not know whether the other trader knows this. If the other trader does not know that the value is 10 in the first period, the information structure, and dividends, are such that this trader's estimate of the value of the asset will go up in the second period. Given this information structure, there will be an equilibrium in which for one state of the world each trader is informed that the asset has value 10 (and thus be priced at 10 two periods hence) and yet be willing to buy or hold the asset at a higher price (13 to be precise) in the hopes that he can sell it at a still higher price in the next period (before the other trader knows the value to be 10).

It is important to emphasize that there is no irrationality on the traders' parts. A trader who buys or holds the asset at the price of 13 which he knows ultimately will have value 10 does so knowing that the other trader also may have been informed that the value will be 10. In this case, each will have bought or held the asset in the hope of selling it to the other trader. In this case, the price will go to 10 and each trader will lose money. Precisely offsetting this will be the (expected) profits a trader would make in the case the other trader did not know the asset had value 10. The formal example follows.

There are two traders and two assets, one riskless and one risky. Each trader has a von Neumann-Morgenstern utility function $u(m) = m$. There are ten equally likely states of the world, ω_j, $j = 1,\ldots,10$. We list the state dependent dividends below:

$$d(\omega_j) = \begin{cases} 10 & j = 1,2,3 \\ 22 & j = 4,5 \\ 10 & j = 6,\ldots,10 \end{cases}$$

There are three periods; all consumption takes place at the end of the third period. The asset pays dividends in the last period. We will denote by Π_{it} $i = A,B$ $t = 1,2,3$ the partition of trader i in period t. The partitions are listed below.

$$\Pi_{A1} = \{\omega_1,\omega_2\}\{\omega_3,\omega_4,\omega_5,\omega_6,\omega_7,\omega_8,\omega_9,\omega_{10}\}$$

$$\Pi_{B1} = \{\omega_1,\omega_3\}\{\omega_2,\omega_4,\omega_5,\omega_6,\omega_7,\omega_8,\omega_9,\omega_{10}\}$$

$$\Pi_{A2} = \{\omega_1,\omega_2\}\{\omega_3,\omega_4\}\{\omega_5,\omega_6\}\{\omega_7,\omega_8\}\{\omega_9,\omega_{10}\}$$

$$\Pi_{B2} = \{\omega_1,\omega_3\}\{\omega_2,\omega_5\}\{\omega_4,\omega_6\}\{\omega_7,\omega_9\}\{\omega_8,\omega_{10}\}$$

$$\Pi_{A3} = \Pi_{B3} = \{\omega_1\}\{\omega_2\}\{\omega_3\}\{\omega_4\}\{\omega_5\}\{\omega_6\}\{\omega_7\}\{\omega_8\}\{\omega_9\}\{\omega_{10}\}$$

We claim that there is a fulfilled expectations equilibrium such that in period 1 both traders know that the dividend will be 10 yet each buys the asset at a higher price in the hope that the other trader does not know the asset has value 10. The traders are completely rational in that they realize that there is some chance that the other trader does know the true value and is behaving in the same way.

The prices in this equilibrium are given in Table 1 below.

TABLE 1

state	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	ω_9	ω_{10}
period										
1	13	13	13	13	13	13	13	13	13	13
2	10	16	16	16	16	16	10	10	10	10
3	10	10	10	22	22	10	10	10	10	10

To see that these are equilibrium prices, we begin by observing that clearly in the third period the prices are the only prices consistent with the state contingent dividends of the asset and the fact that each trader has perfect information in this period. Next, note that in the second period, if the trader A observed an event other than $\{\omega_1,\omega_2\}$ and trader B observed an event other than $\{\omega_1,\omega_3\}$, the price conveys no information beyond that the traders already possessed. Also note that for every event other than these two events, the expected dividend in the third period is equal to the price in the second period (the expectation being taken over each of the possible events that either trader could have observed). For example, if person A observes $\{\omega_5,\omega_6\}$ in the second period and person B observes $\{\omega_4,\omega_6\}$, the price is 16 since the expected dividend for both information sets is 16.

If either trader observes in the second period the event that contains ω_1, the price will be different for the two states contained in this event.

Thus, his private information plus that revealed by the price will give him perfect information. If the state is ω_1, the price of the asset is 10, making each trader indifferent to buying, selling or holding the asset. If he sees this event but the state is not ω_1, the price will be above the value (and hence, the price of the asset next period) and he will want to sell any of the asset he owns. The restriction on short sales means that he can sell no more than his holdings of the asset in this period. Thus, the prices in the second period are consistent with equilibrium and the prices in the third period.

It is clear that the price reveals no information in the first period since it is constant across the states. It is straightforward to verify that for every state of the world and for each individual the expected price in the second period is equal to 13, the price of the asset in the first period (the expectation being taken over an individual's observed event). Hence, the prices given in Table 1 constitute a fulfilled expectations equilibrium.

There are several points to observe about the example.

1. *It is possible that a bubble exists so that all traders know that the price of an asset will fall in the future but are still willing to hold the asset.*

To see this, suppose that state ω_1 arises. In this state both traders observe an event which guarantees that the asset has value 10 yet each is willing to hold (or buy) the asset at price 13.

2. *All traders may know that the asset will drop in price and yet it is not common knowledge.*

When trader A sees the event $\{\omega_1,\omega_2\}$, he knows that the true state of the world is either ω_1 or ω_2. If the state is ω_1, trader B has observed the event $\{\omega_1,\omega_3\}$ and thus knows the value to be 10. If the state is ω_2, however, trader B has observed the event $\{\omega_2,\omega_4,\omega_5,\omega_6,\omega_7,\omega_8,\omega_9,\omega_{10}\}$; the expected value of the asset given this information is 13, the average of its dividends over the eight equally likely states in the event that he has observed. When the state is ω_2, trader B will observe the event $\{\omega_2,\omega_4\}$ in period 2 and the expected value for the asset will go up to 16. Thus if the state is ω_1, both traders know that the value of the asset is 10, yet neither knows whether the other knows this.

3. *There is a second equilibrium with no bubble.*

Consider the prices given in Table 2 below.

TABLE 2

state	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	ω_9	ω_{10}
period										
1	10	10	10	94/7	94/7	94/7	94/7	94/7	94/7	94/7
2	10	10	10	22	22	10	10	10	10	10
3	10	10	10	22	22	10	10	10	10	10

It can easily be checked that the prices given in this table constitute a second equilibrium in which there is no bubble. Here, if either of the two traders receives a signal that the asset has value 10, the price immediately reflects this fact.

3. RATIONAL EXPECTATIONS EQUILIBRIA WITH BUBBLES

In the example above, we have shown that there exists a fulfilled expectations equilibrium in which there is a bubble. There is a shortcoming of the example, however, to which we alluded following the definition of a fulfilled expectations equilibrium. In the example, we did not specify what net trades were to take place in each state; we only showed that for any state other than ω_1 or ω_2, each trader will be indifferent to all net trades and that in the states ω_1 and ω_2 exactly one of the traders wants to sell his holdings. If we had specified the net trades to take place in each state, the trade that an individual trader made in some states would have revealed information in addition to the information that could be extracted from the price and the trader's private information. (See Kreps [1977] or Tirole [1982] for a discussion of the possibility of traders learning from quantities). To illustrate this, Table 3 below contains a set of net trades for trader A consistent with the equilibrium prices in Table 1. (The changes in trader B's holdings are, of course, just the negative.)

TABLE 3

state	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	ω_9	ω_{10}
period										
1	0	0	0	0	0	0	0	0	0	0
2	0	-1	+1	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0

These net trades are the minimal trades consistent with equilibrium; in those states that a trader knows that the asset will fall in price the next period, he sells his holdings. It is clear that these net trades contain information in addition to that conveyed by a trader's private signal and the information contained in the price. For example, if trader A has observed the event $\{\omega_3,\omega_4\}$, the price provides no additional information since it is constant over this event. As indicated in the previous section, trader A is indifferent to all net trades with only the information provided by observing the event $\{\omega_3,\omega_4\}$. However, under the proposed net trades shown in Table 3, trader A buys 1 unit of the asset in ω_3 and doesn't trade in ω_4. Thus his expected gain for the proposed net trades is negative.

To summarize, if the traders were to condition their beliefs on the net trade in addition to the information they could extract from their private signals and the price, they would not be indifferent to all net trades. There is no consensus as to whether or not it should be assumed that traders will extract information from their net trades. The example in the previous section should cause one to be skeptical about equilibria in which traders could learn something from their trades in addition to the information they glean from prices and their private information. In that example, either of the two traders could follow a simple trading strategy that would yield higher expected utility than they receive in the equilibrium: sell out in period 1 and never buy. That trading strategy yields expected utility 13 for the share of the asset initially owned. If a trader carried out the net trades in table 1 as part of the equilibrium in the previous section, his expected utility from the asset and/or the proceeds from selling the asset would be only 12.4. In other words, a trader doesn't have to be incredibly sophisticated to be able to do better than he could within that equilibrium.

It isn't our aim to provide a definitive market equilibrium concept for a world in which traders have differential information. We would like to show, however, that the phenomenon exhibited in the example doesn't rely upon the traders not extracting information available from their personal net trades. Toward this end, we will follow Tirole [1982] in defining a rational expectations equilibrium in a way that prevents traders from extracting information from their net trades in addition to that extracted from the price and their private signals.

DEFINITION 3: A *rational expectations equilibrium* is a fulfilled expectations equilibrium in which for each trader i and each period t, the net trade function $x_{it}(\omega)$ is measurable with respect to the partition $\Pi_{jt} \cap S^{-1}(P(\omega,t))$.

If a trader's net trade function is measurable with respect to the partition $\Pi_{jt} \cap S^{-1}(P(\omega,t))$ clearly no information can be gained in addition to that obtainable from the trader's private information and the price. As we have explained, while the example is an example of a fulfilled expectations equilibrium in which, for some states of the world, all traders know that an asset is overpriced, the price is not a rational expectations equilibrium price function.[1] We will next modify the example in a way that makes it a rational expectations equilibrium. The modification will be done in two steps. First, we will introduce a third trader and second, we will modify the dividends in one state. This third trader will be risk neutral and trade so that the other two traders to have trades that are functions only of their private signals and the information conveyed by the price function. Trader C's partitions for the three periods are

$$\Pi_{C1} = \{\omega_1,\omega_2,\omega_3,\omega_4,\omega_5,\omega_6,\omega_7,\omega_8,\omega_9,\omega_{10}\}$$

$$\Pi_{C2} = \{\omega_1\}\{\omega_2,\omega_3,\omega_4,\omega_5\}\{\omega_6\}\{\omega_7,\omega_8,\omega_9,\omega_{10}\}$$

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[1] We note that we only demonstrated one set of net trades that were consistent with the price function and showed that these trades were not consistent with a rational expectations equilibrium. This leaves open the possibility that there might be other trades that are consistent with a rational expectations equilibrium. It is straightforward to verify that, in fact, there are no such trades possible.

$\Pi_{C3} = \{\omega_1\}\{\omega_3\}\{\omega_2\}\{\omega_4\}\{\omega_5\}\{\omega_6\}\{\omega_7\}\{\omega_8\}\{\omega_9\}\{\omega_{10}\}$.

His valuations of the asset are the same as those given in Section 2 for traders A and B for each state except for ω_6; for this state, trader C's value is 16. The prices for this economy are given in Table 4.

TABLE 4

state	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	ω_9	ω_{10}
period										
1	13	13	13	13	13	13	13	13	13	13
2	10	16	16	16	16	16	10	10	10	10
3	10	10	10	22	22	16	10	10	10	10

The only non-zero trades for this economy are in period 2. They are shown in Table 5 below.

TABLE 5

state	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	ω_9	ω_{10}
trader										
A	-1	-1	+1	+1	-1	-1	0	0	0	0
B	-1	+1	-1	-1	+1	-1	0	0	0	0
C	+2	0	0	0	0	+2	0	0	0	0

It is straightforward to verify the following:

1. For every trader and for every state of the world, the trades are functions of the private signals and the information conveyed by the price function; no additional information can be extracted from the trades.

2. For each trader i and for every state of the world, the trades maximize expected utility conditional on i's private information and the information conveyed by the price, except for state ω_6. When the state is ω_6, both traders A and B have seen an event for which the price of the asset is 16 and they are to sell the asset. The difficulty is that given their private information (the price and their net trades yield no further information) they are selling an asset which will have value of 16 or 22 with equal probability for 16. Clearly, this is not optimal. We will alter the dividends and the risk aversion of traders A and B to make the trades optimal.

The alteration consists of letting the asset have a random return in ω_6.[2] In particular, the asset will return either 22 or 0 in this state. The probabilities of the two amounts are 16/22 and 6/22 respectively. Thus, the expected value of the asset is 16. Since trader C is risk neutral, he is willing to pay the price 16 in period 2 for the asset. We will make both traders A and B risk averse in a way that will make them prefer selling the asset for 16 to holding the asset and bearing the risk of a return of 0.

Assume that each of the traders A and B hold one unit of the asset to begin with and have initial money holdings of 16. Both trader A and B have a piece-wise linear utility function with a kink at 26:

$$u(m) = km - 26(k - 1) \text{ if } m < 26$$

$$= m \text{ if } m \geq 26$$

where $k > 1$. It is illustrated in Figure 1. If trader A holds the asset, he will have a final wealth of either 38 or 16. The expected wealth is 35; from this it can be shown that the expected utility from holding the asset is

[2] Formally we could divide the state ω_6 into two distinct states $\tilde{\omega}_6$ and $\hat{\omega}_6$ with the dividends and probabilities associated with these states chosen as below.

b in the diagram. Selling the asset for 16 yields a non-random final wealth of 32, which yields a final expected utility of a in the diagram. For sufficiently large k, it can easily be seen that b > a. Thus, even though the price is below the expected return on the asset, both traders A and B have higher expected utility if they sell their single unit of the asset rather than hold. Note that they have only enough money to buy a single unit at most. We also note that the utility functions are such that in any state other than ω_6, the traders act the same as risk neutral traders would.

This completes the example. The traders are now maximizing in state ω_6, which was the last state that had to be checked.

4. DISCUSSION

Clearly the example is "razor-edged", that is, there are a number of perturbations of the economic environment which would destroy the qualitative characteristics of the equilibrium. To the extent that the example is not robust, it has few implications about real asset markets. We will discuss the various ways that the example is not robust and the importance of that non-robustness.

Dividends and Probabilities

(a) Values of the dividends. In the example, the dividends take on only two values, 10 and 22. If the dividends were different in different states, this might reveal some of the information. If the asymmetry of information is eliminated, there will be no possibility of bubbles. Thus, it might be argued that since small deviations of the dividends across states could eliminate the bubbles, they are not important. This is simply a variation of the fact that rational expectations are generically revealing in many circumstances.

One has to be careful of such arguments, however. A trader in the example faces two kinds of uncertainty. There is uncertainty about the fundamentals (whether the dividend will be 10 or 22) and uncertainty about what other traders know about the fundamentals. It may be reasonable that the actual payout in two different states differs at least a <u>little</u> bit if those two states are meant to capture some physical difference. But in the example there are really only <u>two</u> states necessary to describe the physical differences that may arise (that is, whether the dividend is 10 or 22). The other states are artificial in a sense; they are introduced to model the second type of uncertainty, namely what other traders know. For example, states ω_1, ω_2, and ω_3 should be thought of as a single physical state that might be known to either of the first two traders but not the other or to both of the traders. While it might be reasonable to believe that the dividend should vary across physical states, it should definitely not be different in two states whose sole difference is whether or not someone knows when that state has arisen.

(b) Probabilities. We argued above that it would not be reasonable to alter the value of the asset so that it differed across states in which the fundamentals were the same. This suggests that the non-genericity associated with the fact that the values of the asset are constant across these states is irrelevant. There is a second feature of the example that is more problematic, however. The example had equally likely states. Small changes in these probabilities would have changed the values of the asset so as to reveal more information than in the example. Unlike the case of the value of the asset, there is no reason for the probabilities not to be altered across the states.

They reflect the probabilities attached to various sets of people knowing some fundamental variable; there is no reason that there should be any particular pattern to these probabilities.

While it is true that the example is highly non-generic with respect to these probabilities, we would argue that this feature stems from the finite state space that we have used for expository purposes. In models with an infinite signal space, generic non-fully revealing equilibria may exist.[3] While we have not constructed an analog of the example in such models, we have no reason to expect that this cannot be done.

No-trade theorem

Milgrom and Stokey [1982] proved a result that is sometimes called the no-trade theorem. It says that if two traders agree on an ex-ante efficient allocation of goods, then after they get new information, there is no possibility of a transaction with the property that it is common knowledge that both traders are willing to carry out the transaction and at least one trader strictly prefers the transaction to no (further) trade. Our example is not in conflict with this result for several reasons. First, that result doesn't rule out the possibility of it's being common knowledge that a set of risk neutral traders are engaging in fair gambles. Second, the initial distribution of the asset is not efficient. In state ω_6, the asset must be transferred to the third trader for efficiency, since he is the only trader who is risk neutral over the relevant gambles in this state. The assumptions of the no-trade theorem are violated, hence it is not surprising that the conclusions may be as well.

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[3] See, e.g., Ausubel [1990].

This last point is worth elaborating upon. To the extent that there is a bubble of the sort exhibited in the example, we are compounding socially useful trade (efficient distribution of risk among the traders) with superfluous gambles. If traders are strictly risk averse, there is a social loss to that part of trade that is essentially a gamble. There would, in a sense, be a social gain to the elimination of the gambling aspect. That doesn't prevent speculation from occurring in equilibrium, however. Traders will still find it rational to trade in a world with bubbles so long as the gain from the beneficial redistribution of risk outweighs the loss from the costly gambles associated with the bubbles.

If equilibria without bubbles have efficiency advantages over those exhibiting bubbles, perhaps traders have a way to assure that only those equilibria without bubbles arise. For example, traders might simply trade to an efficient distribution of the assets immediately. Once the allocation is efficient, the no-trade theorem applies, and no further trade would occur (at least no trade that is socially detrimental). There are two responses to this. First, our models typically don't allow analysis of which of several equilibria in a model might arise. Given any equilibrium and its path of prices over time, each trader by definition is doing as well as he can.

There is a second and more important reason to be unconvinced by the suggestion that there might be trade to an efficient distribution of assets with no further trade necessary. This would be possible in an Arrow-Debreu world with complete state-contingent markets (as assumed in Milgrom-Stokey), but in a world of incomplete markets there may be no distribution of assets that is not only efficient at the present time, but will remain efficient over time with probability one. As the uncertainty in the environment unfolds, it may be necessary for there to be redistributions of the assets to regain an efficient distribution of risk. Even if markets were essentially complete through dynamic trading strategies, there would be no reason that the sort of bubbles exhibited in the example would be ruled out.

In general, the only thing that can be ruled out is that the expected loss from the riskiness of the bubble be no larger than the expected gains from efficient distribution of risk.

Other observable information

At the beginning of Section 3, we pointed out that the example in Section 2 of a fulfilled expectations equilibrium with a bubble might be considered flawed by the fact that traders were not extracting information available from their personal net trades. That, in fact, was the motivation for modifying the example in Section 3 to generate a rational expectations equilibrium with a bubble. As mentioned there, it is never absolutely compelling what information we should assume that traders condition on when forming their beliefs. There is (usually) general agreement that the traders should condition on their private information and that information conveyed by the price. It is not agreed whether traders should condition on their personal net trades; it is also not agreed whether traders should condition on some aggregate market variables, such as volume of trade. One possibility is that the aggregate volume of trade cannot be observed by individual traders. If aggregate volume could be observed and was conditioned on, it would provide additional information which would upset the example. It seems likely that the example might be modified further so that volume was constant, and hence conveyed no information. We have not carried out this exercise, however. Clearly, though, if traders conditioned on the identities of the traders with whom they transact, the example would be upset and could probably not be "fixed up". That traders might have this information and condition on it seems unreasonable for the sorts of markets we have in mind.

Indivisibilities

We assumed that the asset was available in indivisible shares. This wasn't necessary for the first example (the fulfilled expectations example in Section 2). It is necessary in the modification in Section 3. In that modification trader C was to buy the shares of the asset in state ω_6. Both traders A and B are selling the asset for a price of 16 when the expected value of the asset, given their beliefs, is higher than 16. If the asset were divisible, they would not sell all the asset, but would retain some portion. The indivisibility makes them choose between holding at minimum one unit or holding none. The form of their risk aversion leads them to choose none. A similar result could be obtained with divisible assets if there are transactions costs that are non-linear so that it is cheaper to trade in round lots rather than odd lots. Such transactions costs could also lead to traders either selling all or none of their holding of stock.

Concluding remarks

Examples have been presented where the backward induction argument that stock prices reflect fundamental values fails. Bubbles can exist because traders are unaware of other traders' beliefs; beliefs are not common knowledge. Even though they all know an asset is priced above its fundamental value, they can all rationally believe that they may be able to sell it to somebody else at a higher price before its true value is publicly revealed. Although the particular examples presented were rather special, it was argued that the phenomenon they illustrate can occur in a wide range of situations.

BIBLIOGRAPHY

Abel, A. and G. Mailath [1990], "Financing Losers in Competitive Markets," Rodney White Working Paper 2-90, University of Pennsylvania.

Allen, F. and G. Gorton [1991], "Rational Finite Bubbles," Mimeo, University of Pennsylvania.

Ausubel, L. [1990], "Partially-Revealing Rational Expectations Equilibrium in a Competitive Economy," *Journal of Economic Theory*, 50, 93-126.

Bhattacharya, S. and B. Lipman [1990], "Ex Ante versus Interim Rationality and the Existence of Bubbles," Mimeo, Carnegie Mellon University.

Camerer, C. [1989], "Bubbles and Fads in Asset Prices: A Review of Theory and Evidence," *Journal of Economic Surveys* 3, 3-41.

De Long, J. B., A. Shleifer, L. Summers and R. Waldmann [1990], "Noise Trader Risk in Financial Markets," *Journal of Political Economy*, 98, 703-738.

Flavin, M. [1983], "Excess Volatility in Financial Markets: A Reassessment of of the Empirical Evidence," *Journal of Political Economy*, 91, 929-956.

Garber, P. [1990], "Famous First Bubbles," *Journal of Economic Perspectives*, 4, 35-54.

Harrison, M. and D. Kreps [1978], "Speculative Investor Behavior in a Stock Market with Heterogeneous Expectations," *Quarterly Journal of Economics*, 89, 519-542.

Jackson, M. and J. Peck [1990], Speculation and Price Fluctuations with Private, Extrinsic Signals," Mimeo, Northwestern University.

Kindleberger, C. [1978], *Manias, Panics and Crashes*, New York: Basic Books.

Kleidon, A. [1986a], "Variance Bounds Tests and Stock Price Valuation Models," *Journal of Political Economy*, 94, 953-1001.

Kleidon, A. [1986b], "Bias in Small Sample Tests of Stock Price Rationality," *Journal of Business*, 59, 237-61.

Kraus, A. and M. Smith, [1990], "Beliefs about Beliefs: The Effect on Equilibrium of Non-Payoff-Relevant Uncertainty," Mimeo, University of British Columbia.

Kreps, D. [1977], "A Note on 'Fulfilled Expectations' Equilibria," *Journal of Economic Theory*, 14, 32-44.

Leroy, S. and R. Porter [1981], "The Present Value Relation: Tests Based on Implied Variance Bounds," *Econometrica*, 49, 555-574.

Marsh, T. and R. Merton (1986), "Dividend Variability and Variance Bounds Tests for Rationality of Stock Prices," *American Economic Review*, 76, 483-98.

Milgrom, P. and N. Stokey [1982], "Information, Trade and Common Knowledge," *Journal of Economic Theory*, 26, 17-27.

Samuelson, P. [1958], "An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money," *Journal of Political Economy*, 66, 467-482.

Shiller, R. [1981], "Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?" *American Economic Review*, 71, 421-436.

Shleifer, A. and L. Summers [1990], "The Noise Trader Approach to Finance," *Journal of Economic Perspectives*, 4, 19-33.

Tirole, J. [1982], "On the Possibility of Speculation under Rational Expectations," *Econometrica* 50, 1163-81.

West, K. [1988], "Bubbles, Fads and Stock Price Volatility Tests: A Partial Evaluation," *Journal of Finance*, 43, 639-56.

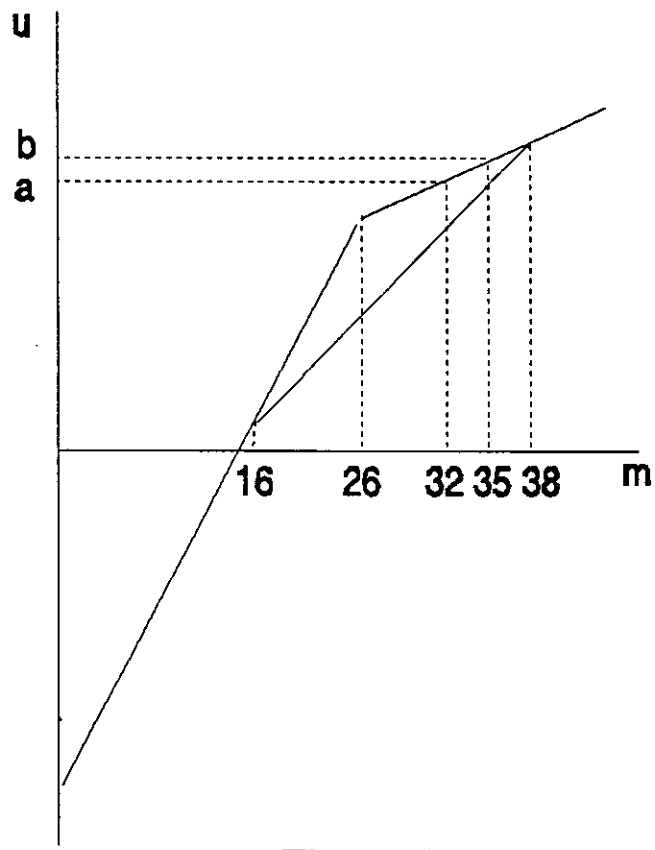

Figure 1